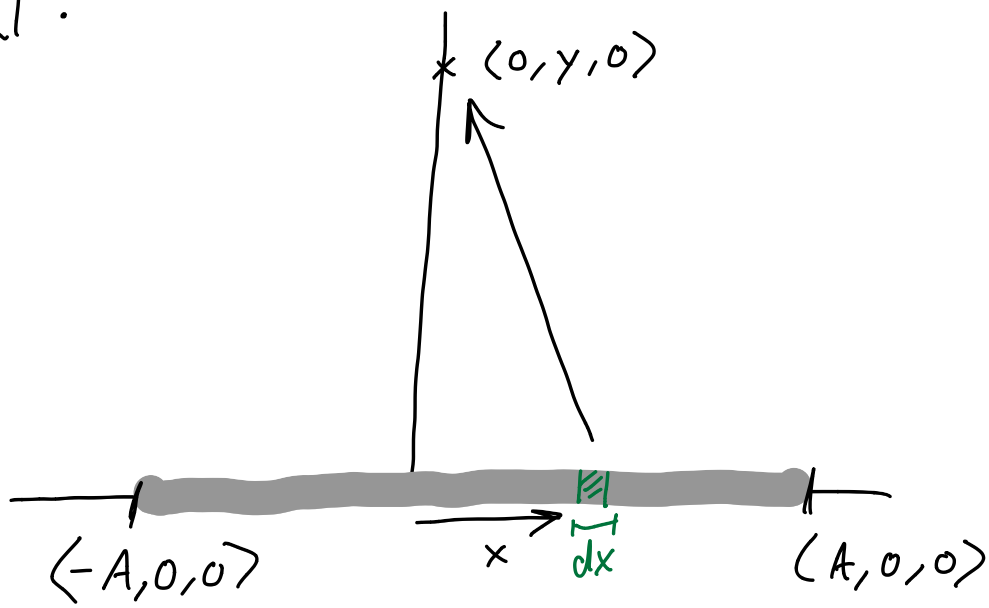
P21:



a) Total Charge: $-Q$

Length of rod: $2A$

Charge/length $= -Q/2A$

b) $dQ = \frac{\text{charge}}{\text{length}} dx = -\frac{Q}{2A} dx$

c) $\vec{r}_{src} = \langle x, 0, 0 \rangle$

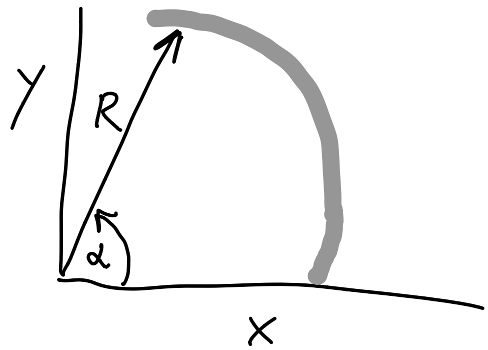
$\vec{r}_{obs} = \langle 0, y, 0 \rangle$

$\vec{r} = \langle -x, y, 0 \rangle$

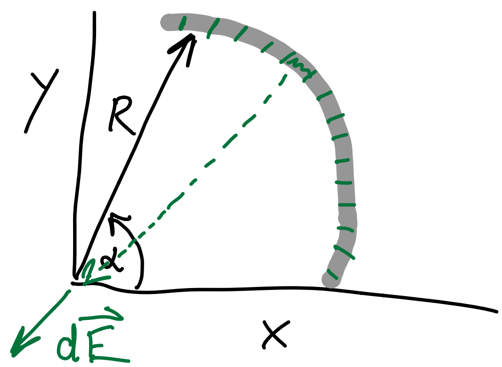
d) $|\vec{r}| = \sqrt{x^2 + y^2}$

e) Where is the charge? Distributed over x

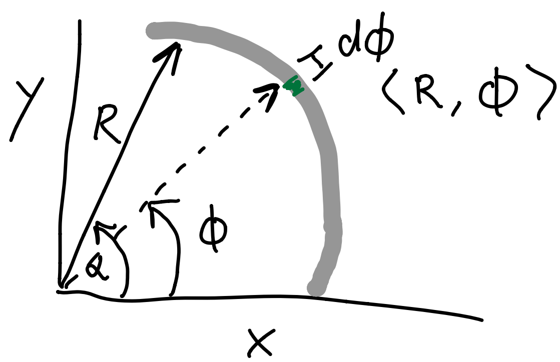
P27:



1) cut the charged object into tiny pieces



2) use polar coordinates ρ, ϕ



- How much charge dq on a small piece of size $d\phi$?

Charge spread uniformly from 0 to α

Total Charge : $-Q$

"Length" : α

Density : $-Q/\alpha$

$$dq = -\frac{Q}{\alpha} d\phi$$

- What is $\vec{r}$?

$$\vec{r}_{src} = \langle R, \phi \rangle = R\cos\phi\,\hat{x} + R\sin\phi\,\hat{y}$$

$$\vec{r}_{obs} = \langle 0, 0 \rangle$$

$$\vec{r} = \langle -R\cos\phi, -R\sin\phi \rangle$$

$$|\vec{r}| = R, \quad \hat{r} = \langle -\cos\phi, -\sin\phi \rangle$$

$$d\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{dq}{|\vec{r}|^2} \hat{r}$$

$$d\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{-\frac{Q}{\alpha} d\phi}{R^2} \langle -\cos\phi, -\sin\phi \rangle$$

- Integration limits?

$$\phi_{min} = 0, \quad \phi_{max} = \alpha$$

$$\vec{E} = \int_0^\alpha \frac{-1}{4\pi\epsilon_0} \frac{Q}{\alpha} \frac{1}{R^2} \, d\phi \, \langle -\cos\phi, -\sin\phi \rangle$$

$$\vec{E} = \frac{-1}{4\pi\epsilon_0} \frac{Q}{\alpha} \frac{1}{R^2} \left\langle \int_0^\alpha -\cos\phi, \int_0^\alpha -\sin\phi \right\rangle$$

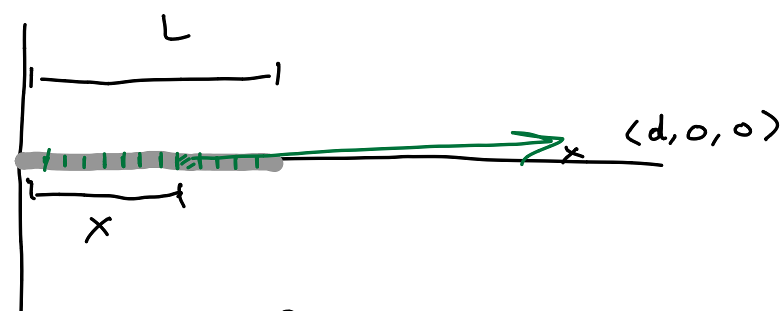
$$\vec{E} = \frac{-1}{4\pi\epsilon_0} \frac{Q}{\alpha} \frac{1}{R^2} \left\langle -\sin(\alpha), \cos(\alpha) - 1 \right\rangle$$

units: $\frac{Nm^2}{C^2} \cdot \frac{C}{m^2} = \frac{N}{C}$ ✓

(α has no units)

P29:

a)



b) $dq = \frac{Q}{L} dx$

$$\vec{r}_{obs} = \langle d, 0, 0 \rangle$$
$$\vec{r}_{src} = \langle x, 0, 0 \rangle$$
$$\vec{r} = \langle d-x, 0, 0 \rangle$$

$$d\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{1}{r^2} \hat{r}$$

$$|\vec{r}| = d - x$$
$$\hat{r} = \langle 1, 0, 0 \rangle$$

$$d\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{\frac{Q}{L} dx}{(d-x)^2} \langle 1, 0, 0 \rangle$$

c) $$\vec{E} = \int_0^L \frac{1}{4\pi\epsilon_0} \frac{\frac{Q}{L} dx}{(d-x)^2} \hat{x}$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{Q}{L} \int_0^L \frac{dx}{(d-x)^2} \hat{x}$$

$$u = d - x \qquad u(x=0) = d$$
$$du = -dx \qquad u(x=L) = d - L$$

$$\vec{E} = \frac{-1}{4\pi\epsilon_0} \frac{Q}{L} \int_d^{d-L} \frac{du}{u^2} \hat{x}$$
$$= \frac{1}{4\pi\epsilon_0} \frac{Q}{L} \left[\frac{1}{d-L} - \frac{1}{d} \right]$$

$$= \frac{1}{4\pi\epsilon_0} \frac{Q}{L} \left[\frac{d}{d(d-L)} - \frac{d-L}{d(d-L)} \right]$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{Q}{d(d-L)} \hat{x}$$

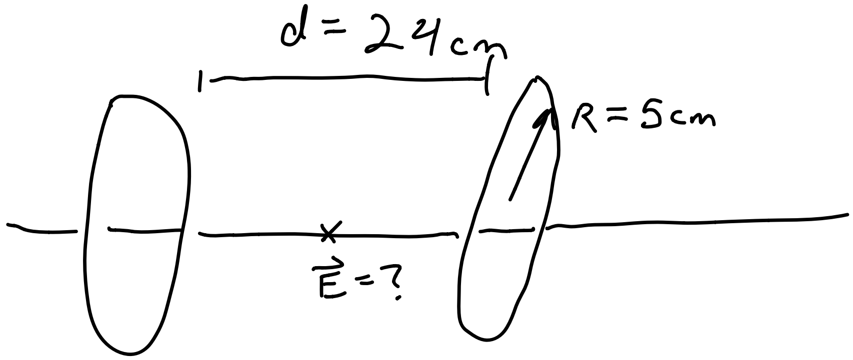
units $= \frac{N}{C}$ ✓

$d >> L$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{Q}{d^2\left(1-\frac{L}{d}\right)} \hat{x} \approx \frac{1}{4\pi\epsilon_0} \frac{Q}{d^2} \hat{x}$$

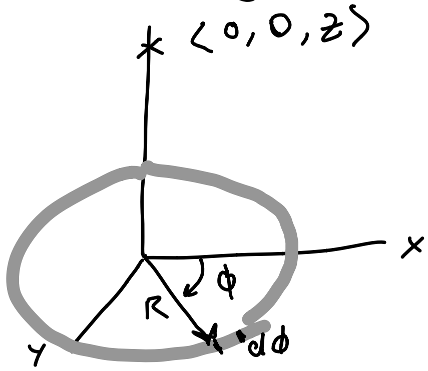
point chg ✓

P31:



$$\vec{E} = \vec{E}_{left} + \vec{E}_{right}$$

$\vec{E}_{ring}$:



$$dQ = \frac{Q}{2\pi} d\phi$$

$$\vec{r}_{obs} = \langle 0, 0, z \rangle$$

$$\vec{r}_{src} = \langle R\cos\phi, R\sin\phi, 0 \rangle$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \int_0^{2\pi} \frac{Q}{2\pi} \frac{d\phi}{(R^2+z^2)^{3/2}} z \hat{z}$$

$$\vec{E}_{\text{ring, on-axis}} = \frac{1}{4\pi\epsilon_0} \frac{Qz}{(R^2+z^2)^{3/2}} \hat{z}$$

$$\vec{E} = \vec{E}_{left} + \vec{E}_{right}$$

$$= \frac{1}{4\pi\epsilon_0} \left[\frac{Q_{left}\,\hat{x}}{[R^2+(d/2)^2]^{3/2}} - \frac{Q_{right}\,\hat{x}}{[R^2+(d/2)^2]^{3/2}} \right]$$

$$= \frac{1}{4\pi\epsilon_0} \frac{1}{[R^2+(d/2)^2]^{3/2}} \left(Q_{left} - Q_{right} \right) \hat{x}$$

$R = 0.05\,m$

$d = 0.24\,m$

$Q_{left} = 31 \times 10^{-9}\,C$

$Q_{right} = -31 \times 10^{-9}\,C$

$$\boxed{\vec{E} = 3.048 \times 10^{4}\,\hat{x}\,\frac{N}{C}}$$

b) $\vec{F} = q\vec{E} = (-9 \times 10^{-9}\,C)(3.048 \times 10^{4}\,\hat{x}\,\frac{N}{C})$

$$\boxed{\vec{F} = -2.743 \times 10^{-4}\,\hat{x}\,N}$$

P 41: $$|\vec{E}| = \frac{Q/A}{\epsilon_0}$$

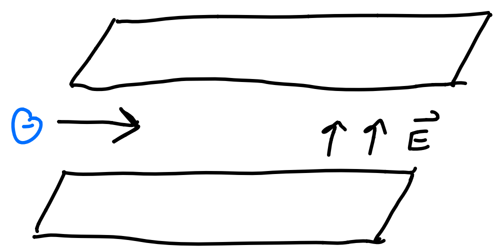
$$Q_{max} = A\epsilon_0 |\vec{E}_{max}|$$

$$= \pi R^2 \epsilon_0 \left(3\times10^6 \tfrac{N}{C}\right)$$

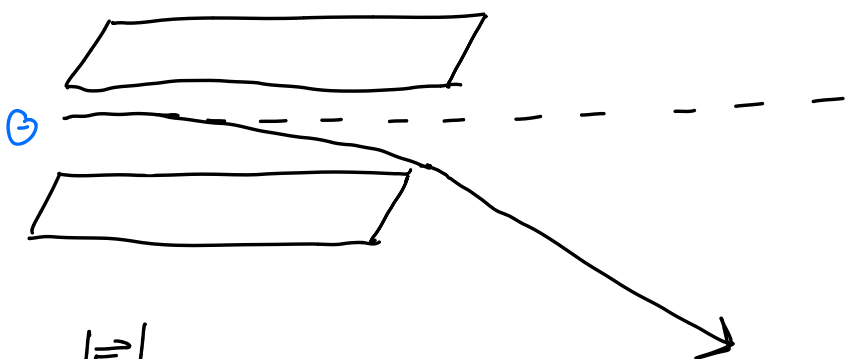
$$R = 0.47\ m$$

$$\boxed{Q_{max} = 1.84\times10^{-5}\ C = 10.84\ \mu C}$$

P45:



a)



b) $|\vec{F}| = q|\vec{E}|$

$$|\vec{a}| = \frac{|\vec{F}|}{m} = \frac{q|\vec{E}|}{m} = \frac{e}{m_e} 10^5, \quad e = 1.6\times10^{-19}\ C$$

$$m_e = 9.11\times10^{-31}\ kg$$

$$\boxed{|\vec{a}| = 1.8 \times 10^{16} \ m/s^2}$$

c) $|\vec{E}| = \left| \frac{Q/A}{\epsilon_0} \right|$

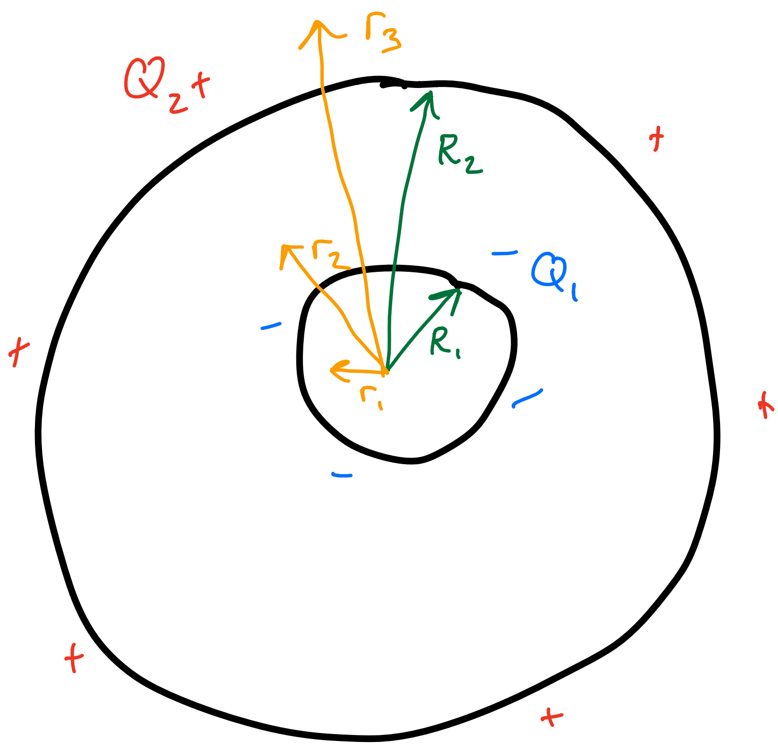
$|Q| = A \epsilon_0 E$

$A = (0.03)(0.12)$

$|Q| = 3.18 \times 10^{-9} \ C$

$\vec{E}$ is toward upper plate,
so $Q_{upper} = -3.18 \times 10^{-9} C$

P51:



Superposition of charged shells:

$$\vec{E} = \begin{cases} \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} \hat{r}, & r > R \\ 0, & r < R \end{cases}$$

$$\vec{E} = \underset{\text{inner}}{\vec{E}_1} + \underset{\text{outer}}{\vec{E}_2}$$

Region 1: $r < R_1 < R_2$

$$\vec{E} = 0 + 0 = 0$$

Region 2: $R_1 < r < R_2$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{Q_1}{r^2} \hat{r} + 0$$

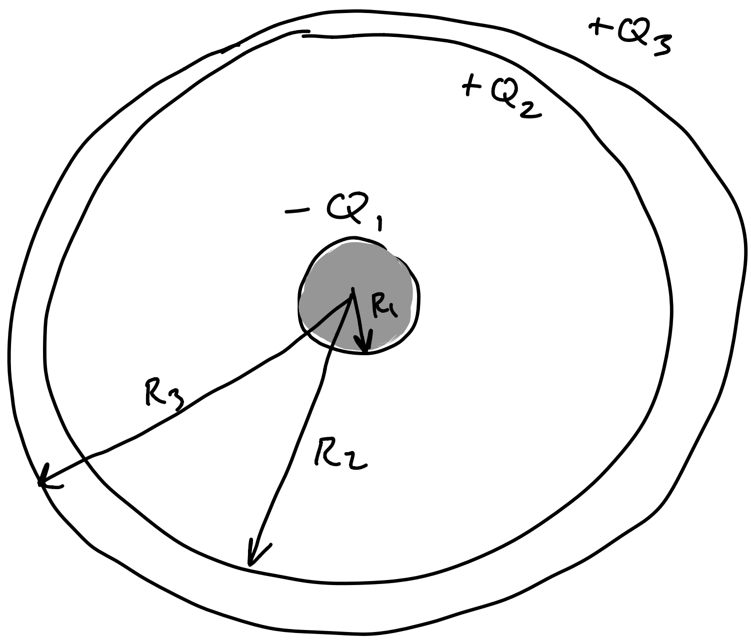
$$= \frac{1}{4\pi\epsilon_0} \frac{(-25\times10^{-9})}{(0.07)^2} = \boxed{-4.59\times10^4 \hat{r}}$$

Region 3: $r > R_2 > R_1$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{1}{r^2} (Q_1 + Q_2), \quad r = 0.1m$$

$$\boxed{\vec{E} = 3.51\times10^4 \frac{N}{C} \hat{r}}$$

P58:



a) $r < R_1$

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$$

solid (R_1) inner (R_2) outer (R_3)

$$\vec{E} = 0$$

$$E_1 = E_2 = E_3 = 0$$

b) $\vec{E}_1 = \frac{-1}{4\pi\epsilon_0} \frac{Q_1}{r^2} \hat{r}$

c) $\vec{E} = 0$ (inside a metal)

d) $\vec{E}_1 = \frac{-1}{4\pi\epsilon_0} \frac{Q_1}{r^2} \hat{r}$ $\quad \vec{E}_2 = \frac{1}{4\pi\epsilon_0} \frac{Q_2}{r^2} \hat{r}$

$$\vec{E}_3 = \frac{1}{4\pi\epsilon_0} \frac{Q_3}{r^2} \hat{r}$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{1}{r^2}\left(Q_2 + Q_3 - Q_1\right)$$

e) $\vec{E}_{metal} = \frac{-1}{4\pi\epsilon_0}\frac{Q_1}{r^2}\hat{r} + \frac{1}{4\pi\epsilon_0}\frac{Q_2}{r^2}\hat{r} = 0$

so $Q_1 = Q_2$

$Q_2 = 5\,nC$

f) $\vec{E} = 0$ inside, no polarization