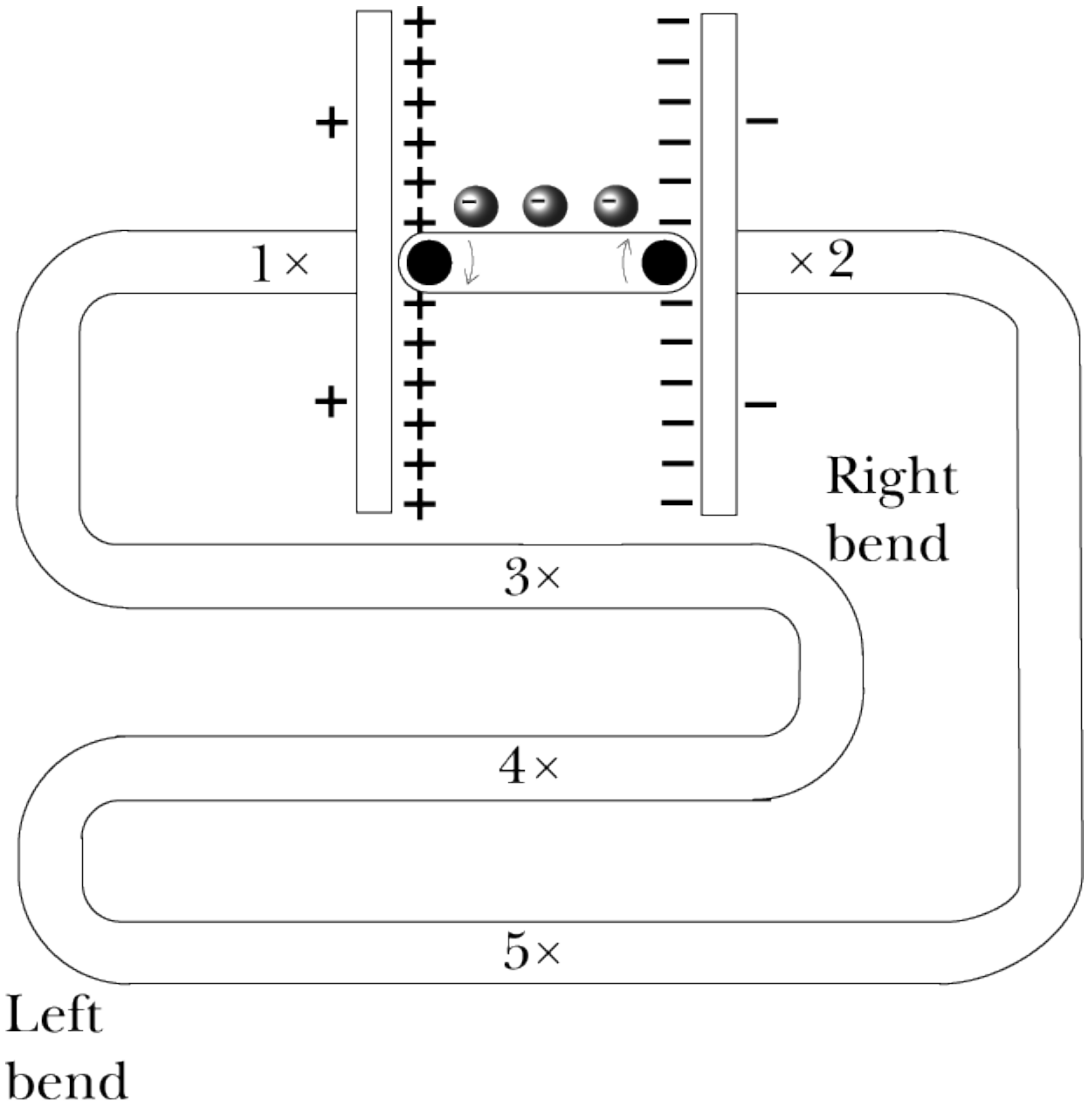
INSIDE A CIRCUIT

- Circuits are not in equilibrium
- Electric field within circuit causes electrons to move with average speed $\overline{v}$
    - $\bar{v} = uE$

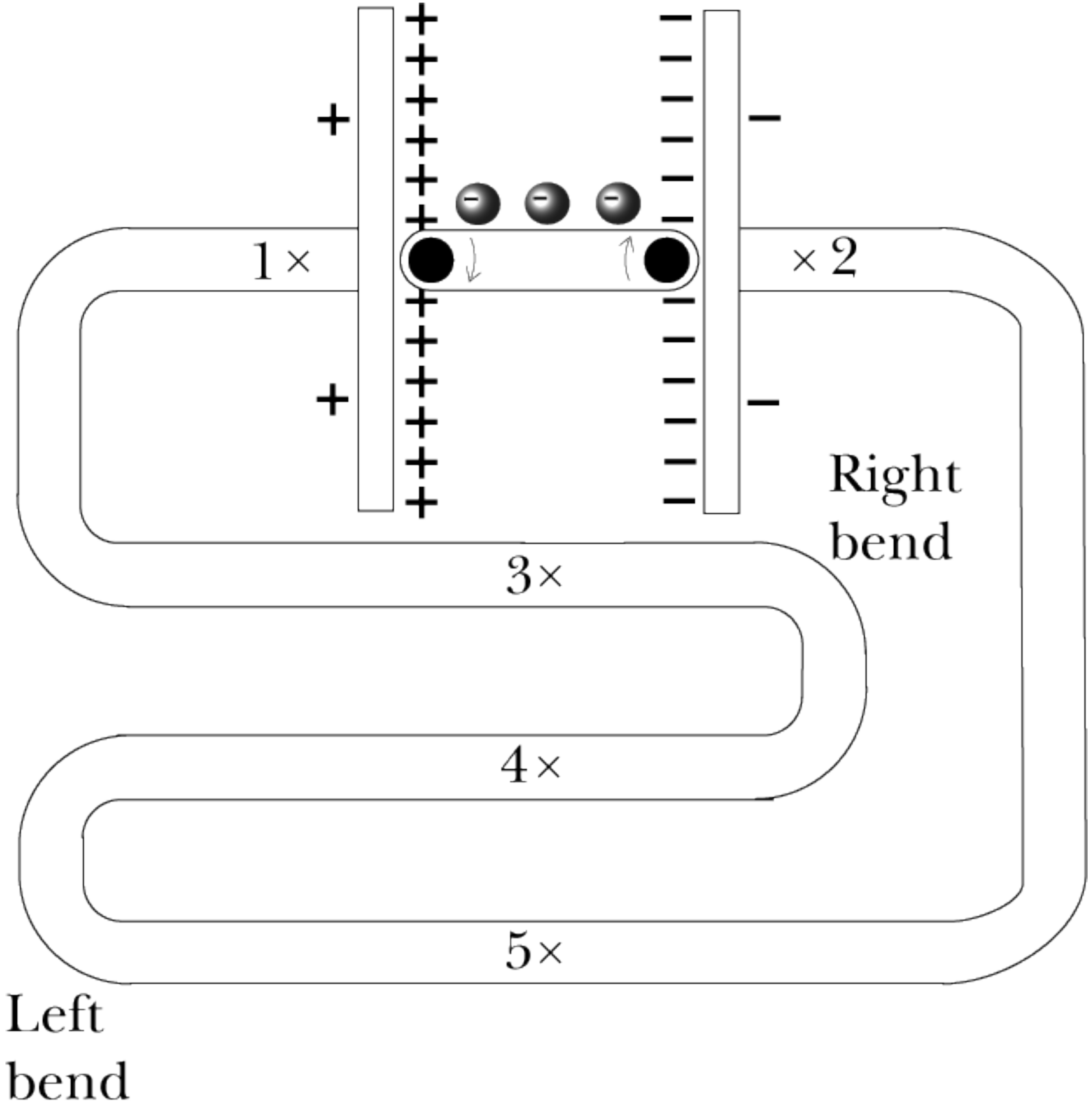
INSIDE A CIRCUIT

- Circuits are not in equilibrium
- Electric field within circuit causes electrons to move with average speed $\overline{v}$
    - $\bar{v} = uE$
- Conservation of charge + steady state –> node rule
    - Current in = current out

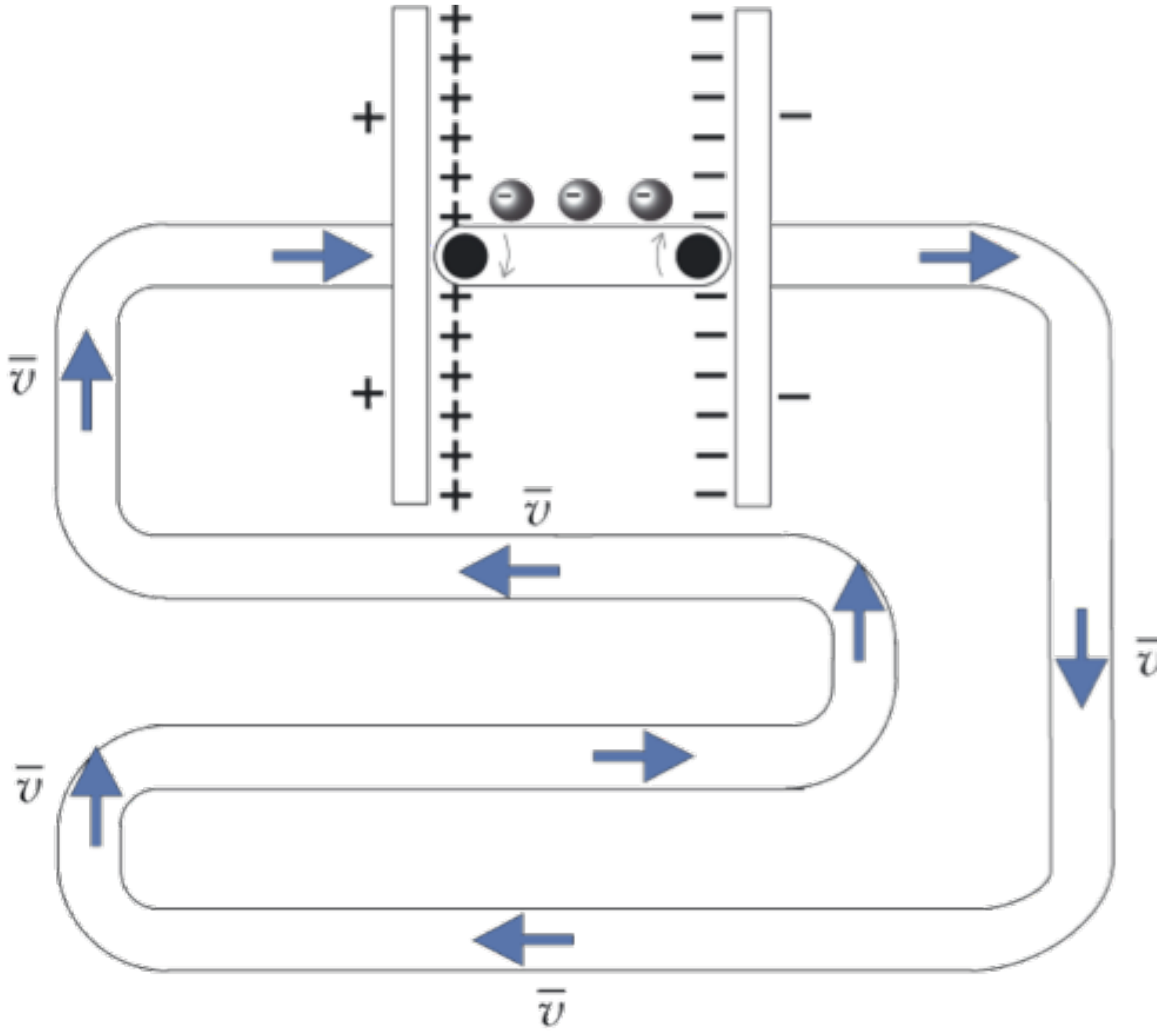


1×
×2
Right
bend
3×
4×
5×
Left
bend

What should be the drift velocity of electrons at different locations in this circuit, in the steady state?



Given this pattern of drift velocity, what should the electric field be inside the wire?



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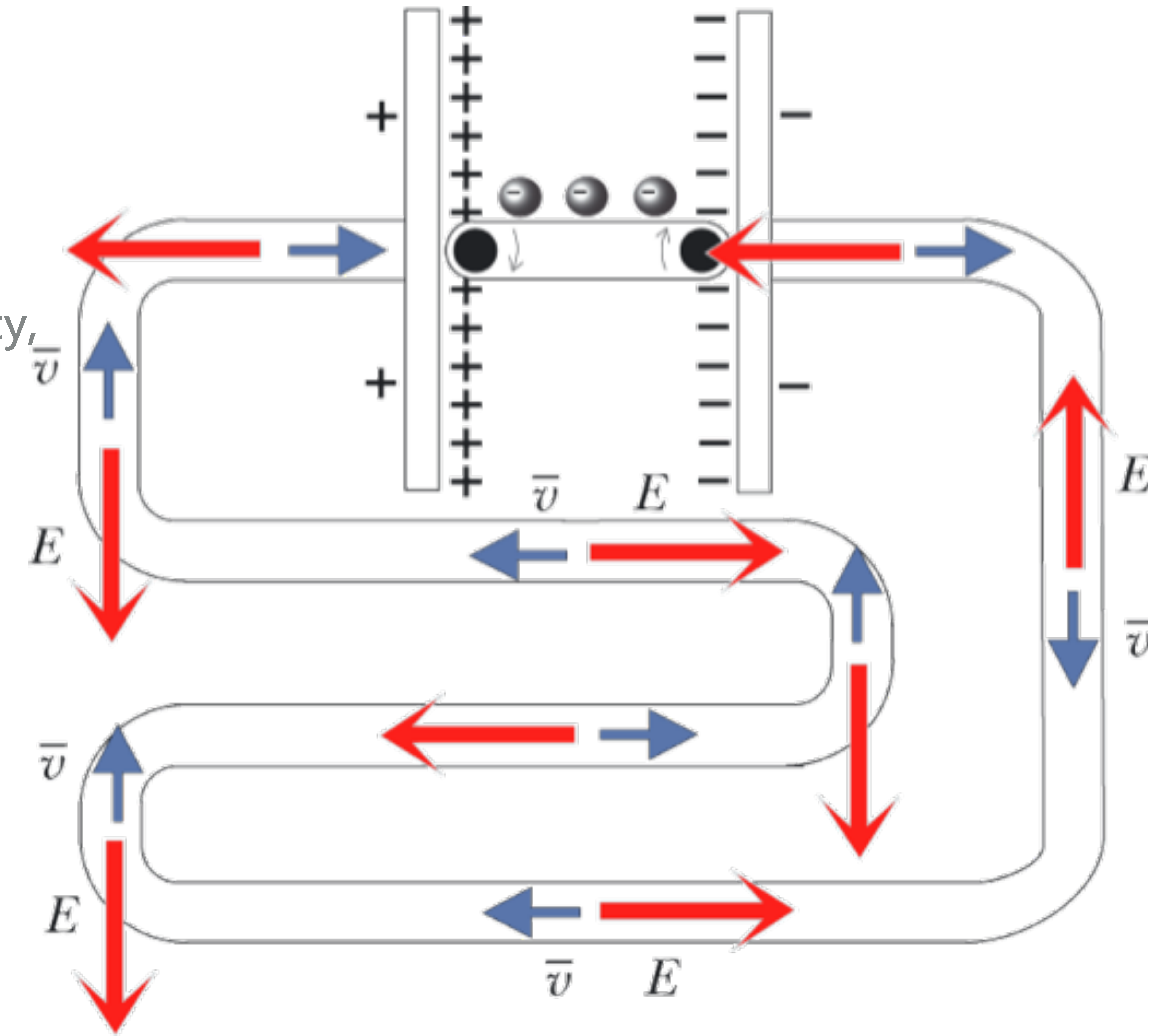
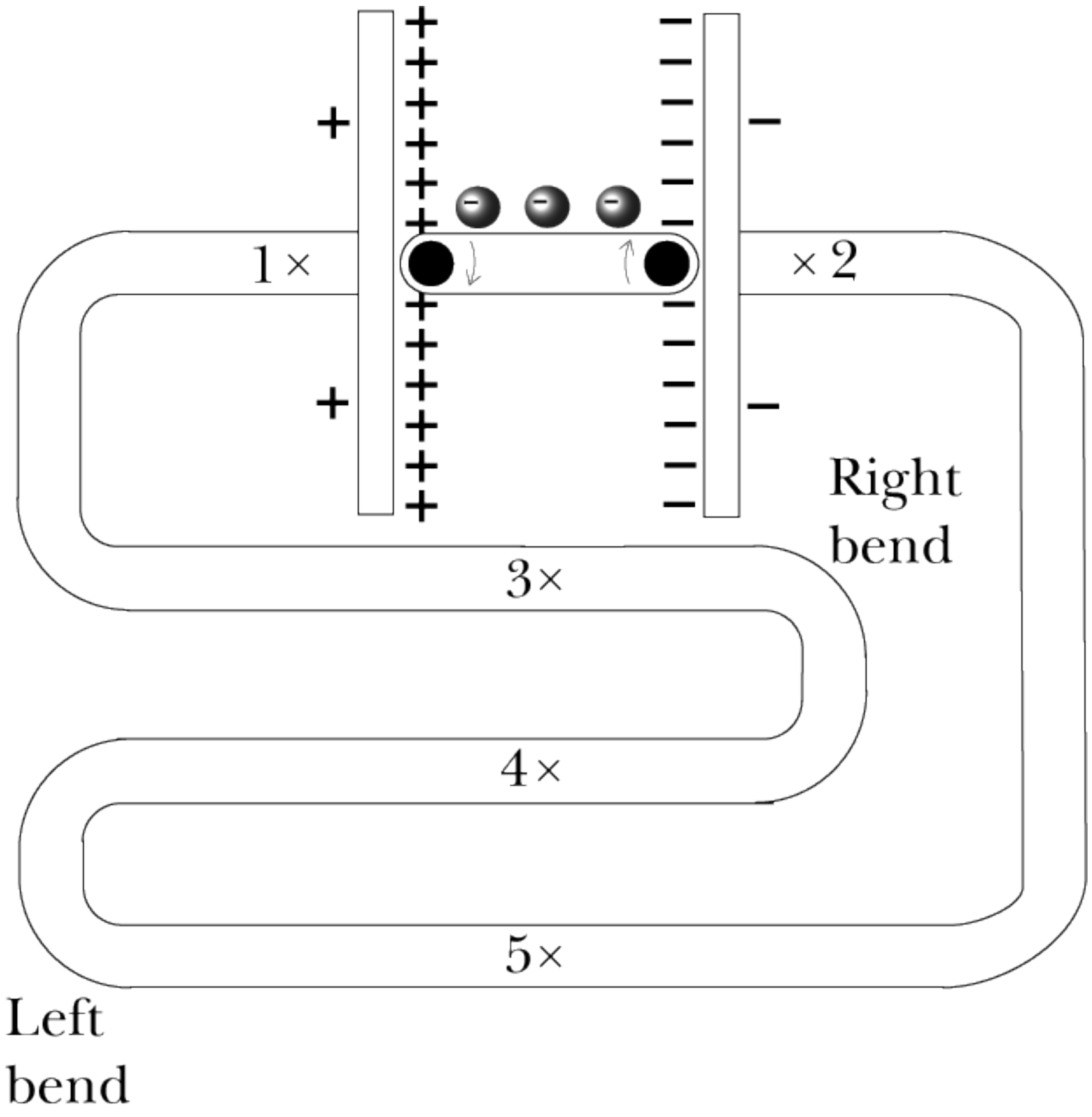


Assume that the only excess charges are the ones on the battery.

What does the electric field look like inside the wire, due ONLY to the charges on the battery?



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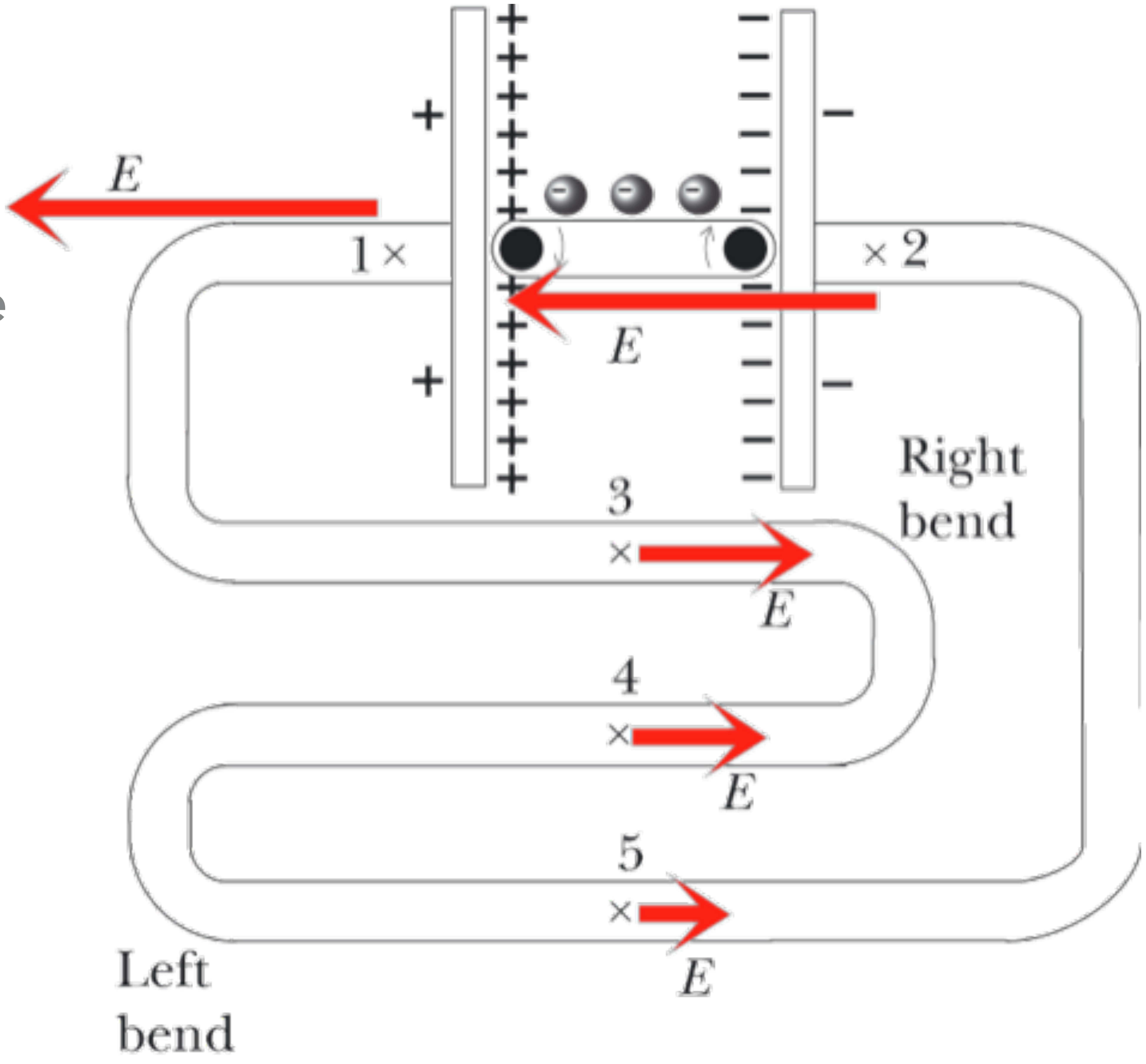

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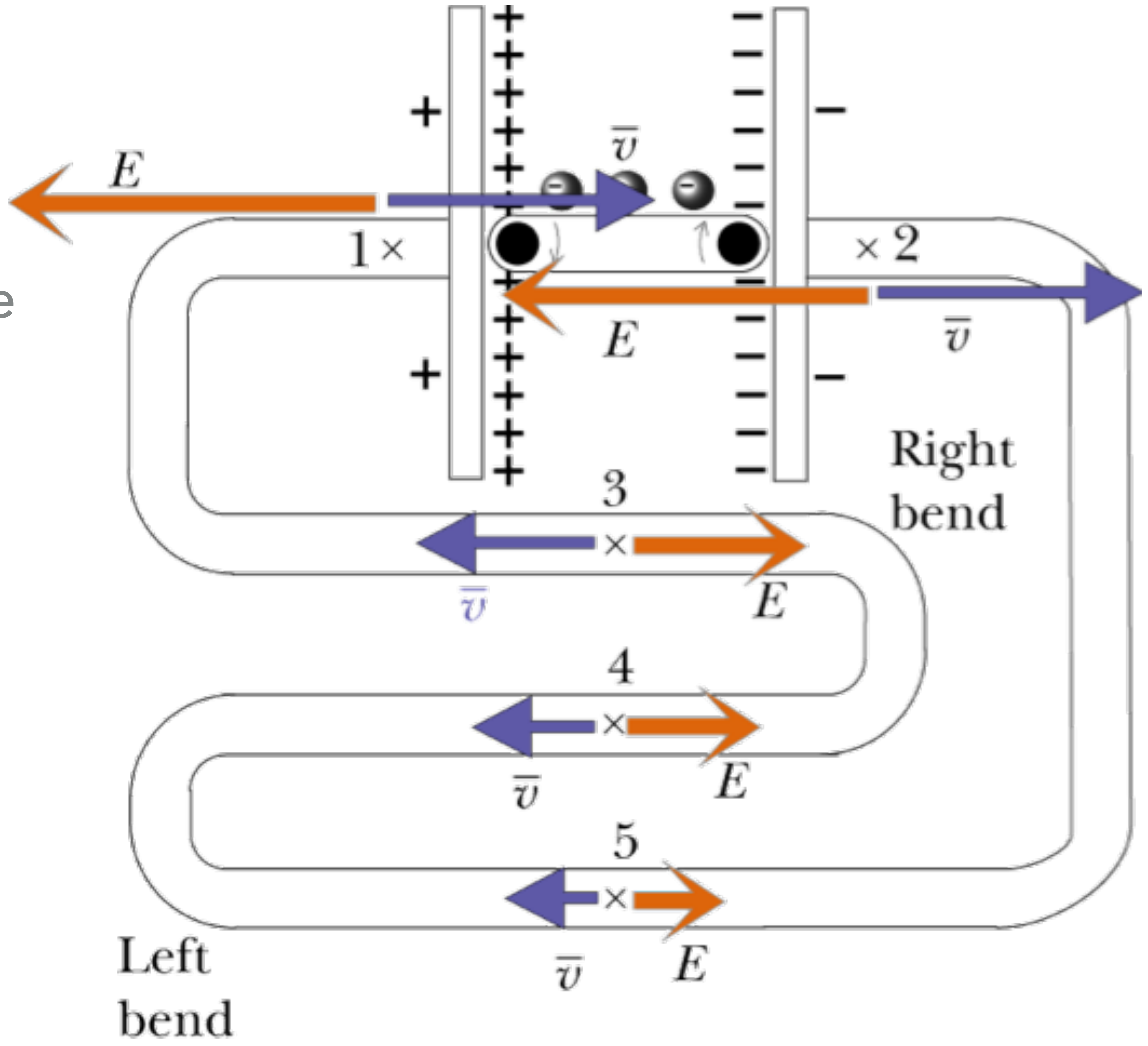
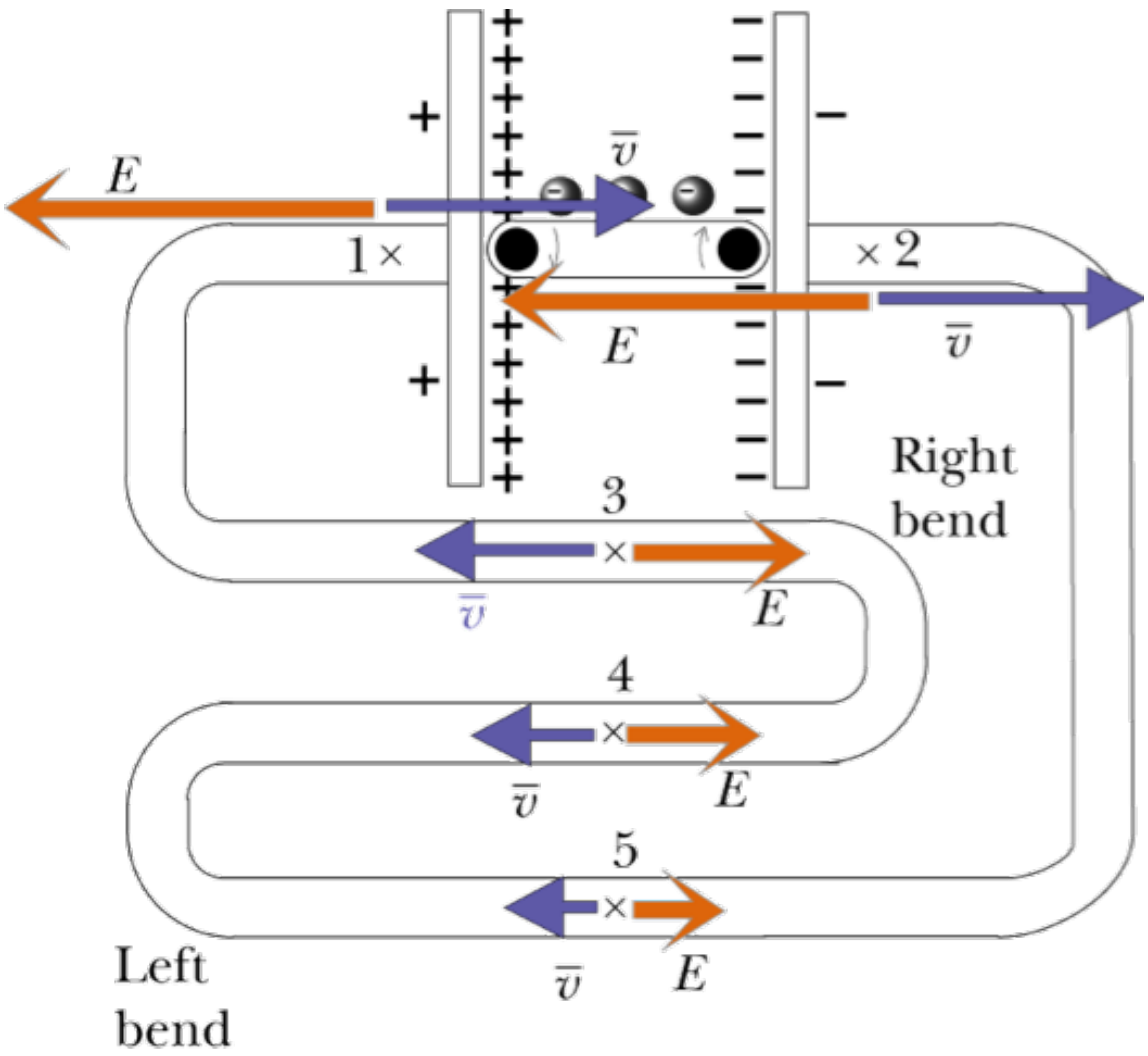


Is this a stable state? (Will the circuit stay like this indefinitely?)



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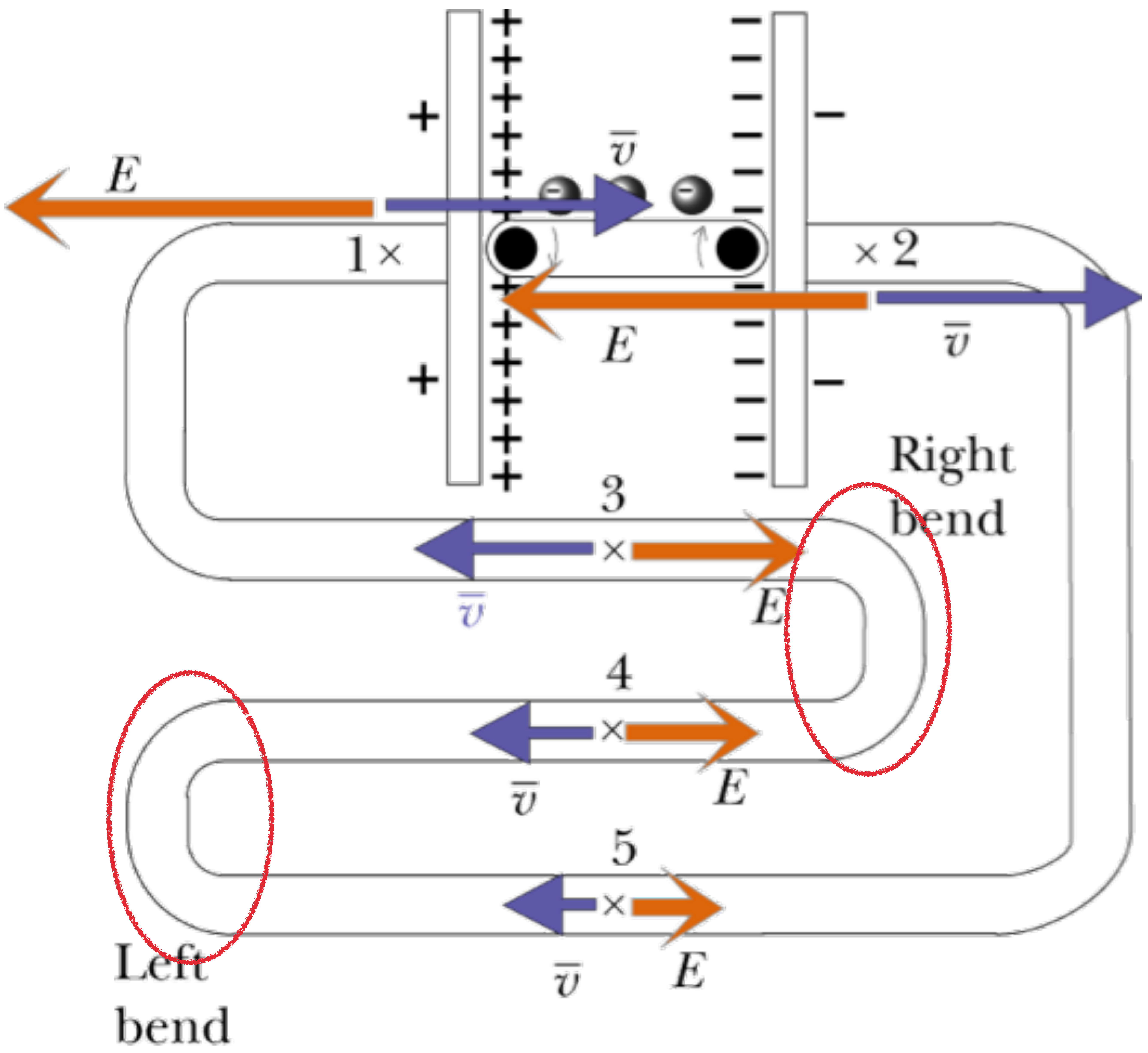

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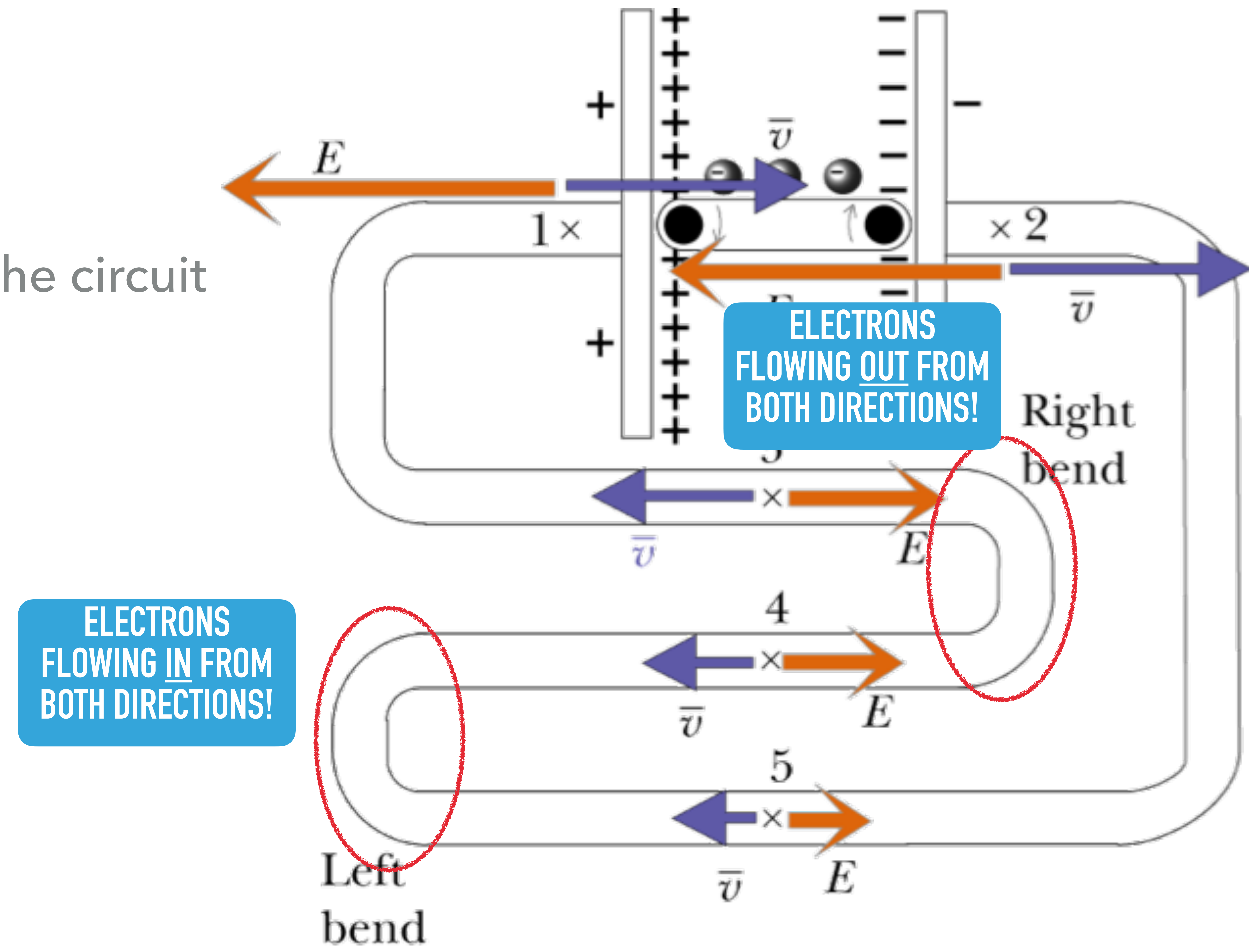
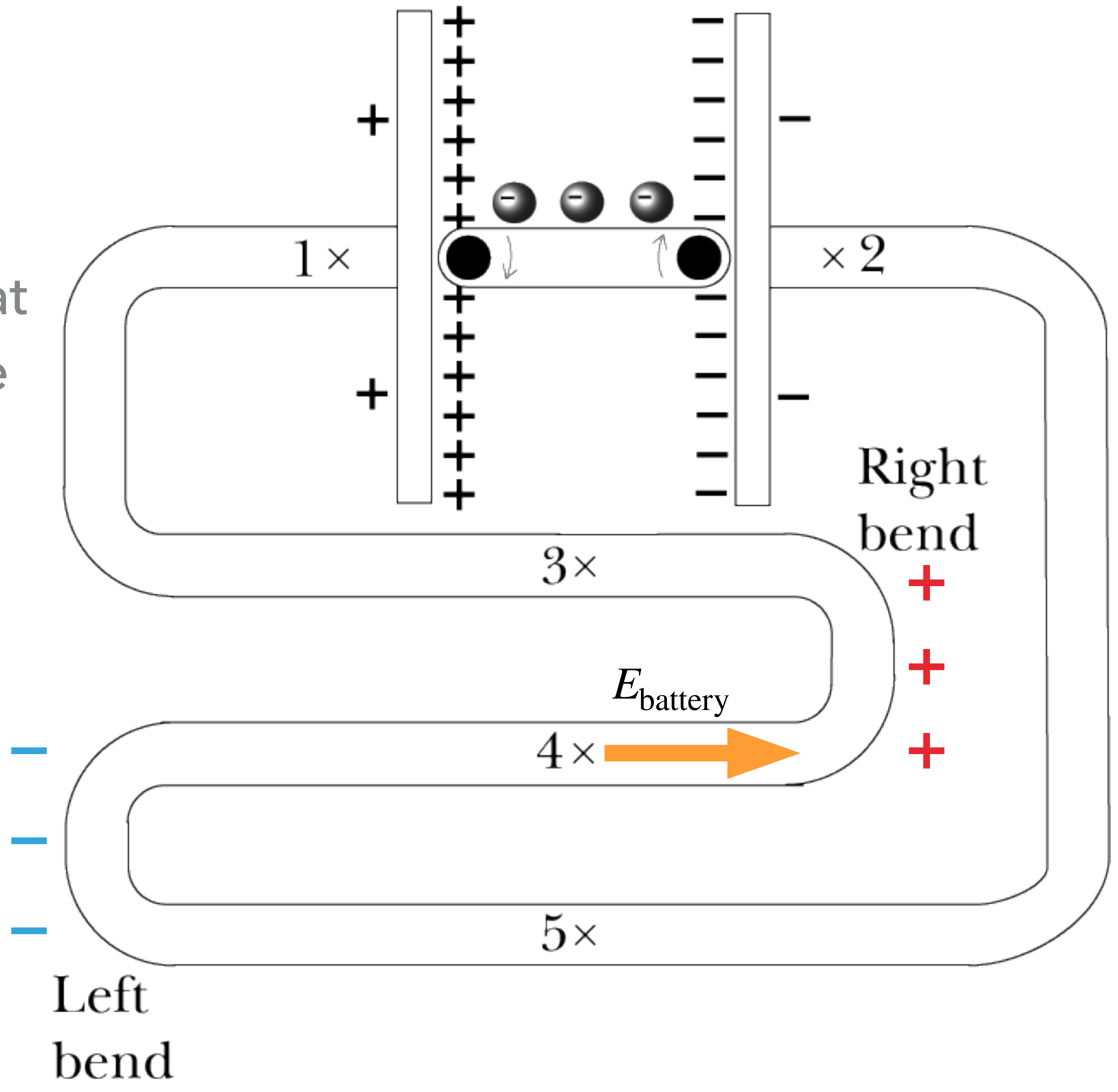
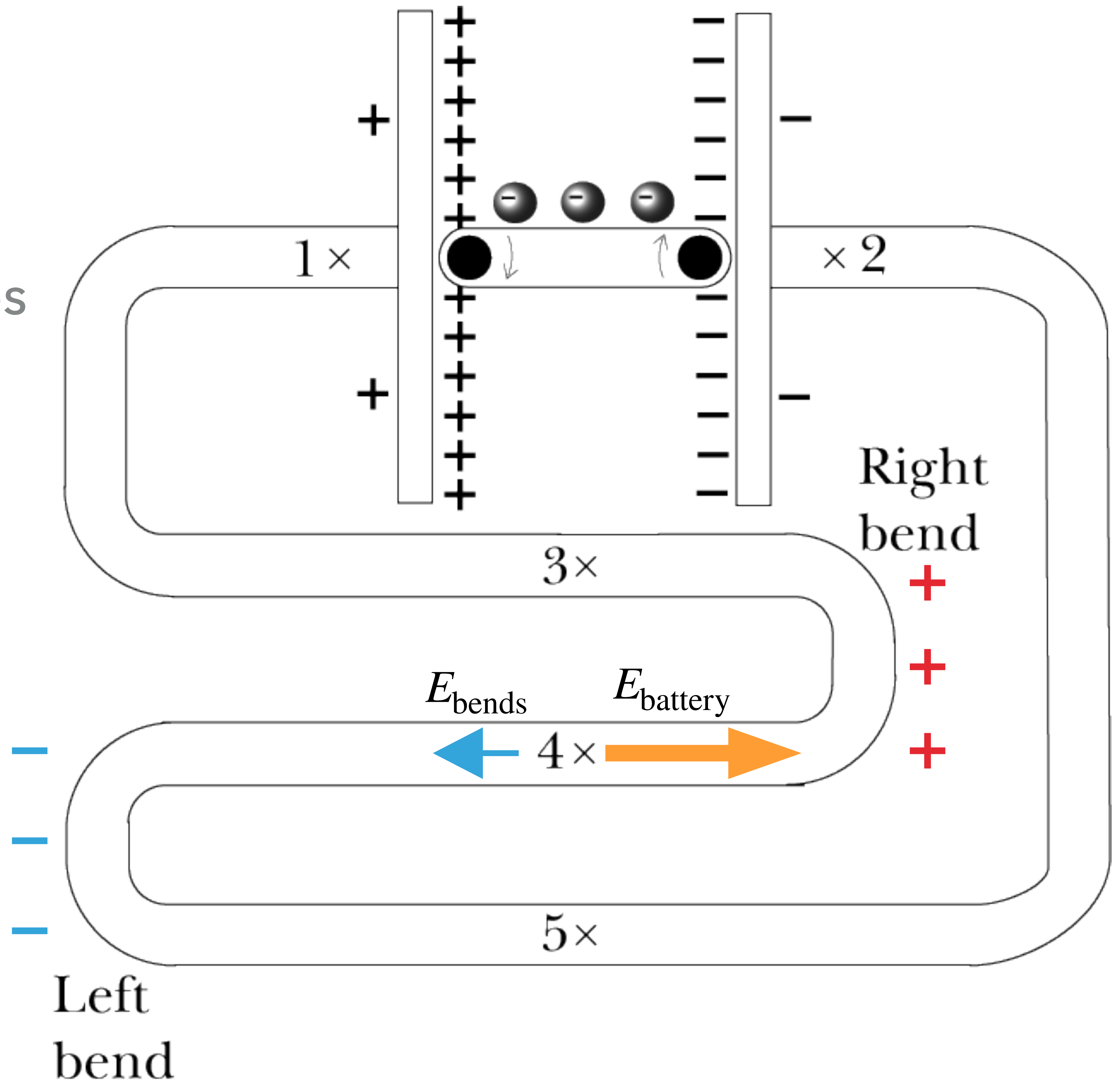


Dipole battery field creates fields that don't follow the wire, causing charge buildup



Charge buildup in wire bends creates a counteracting electric field!



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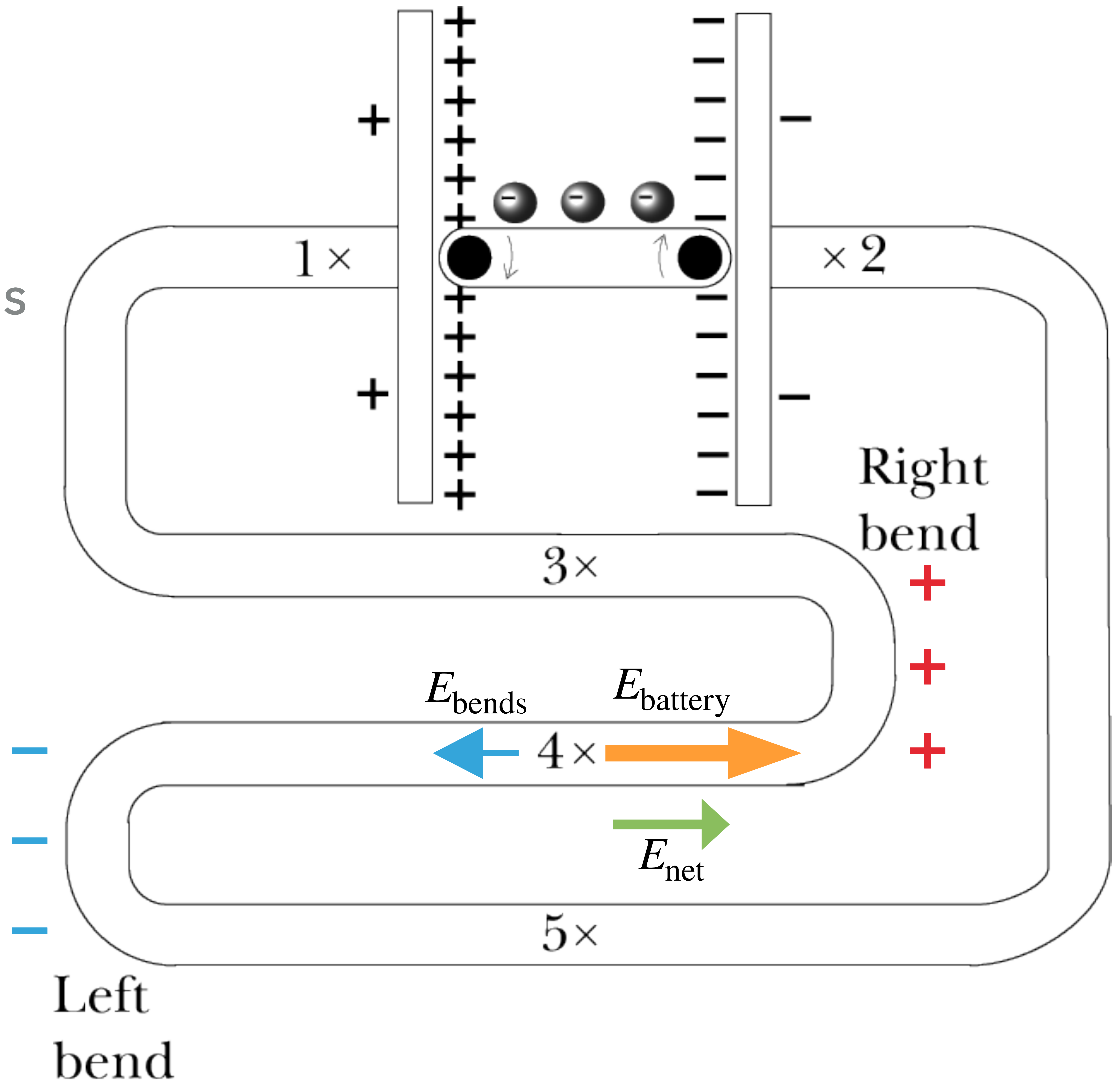
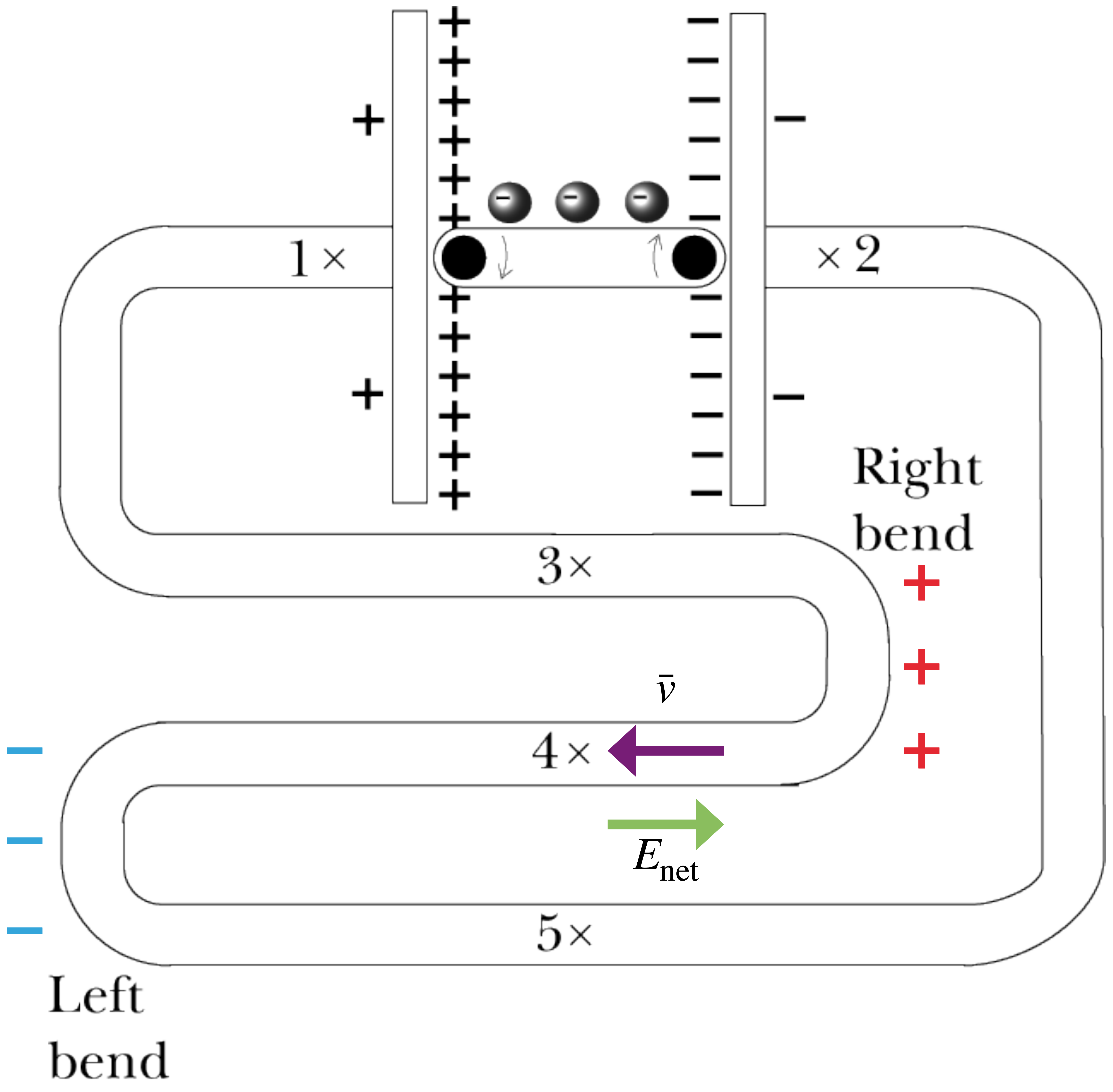


Net field still points in "wrong" direction, so charge buildup continues



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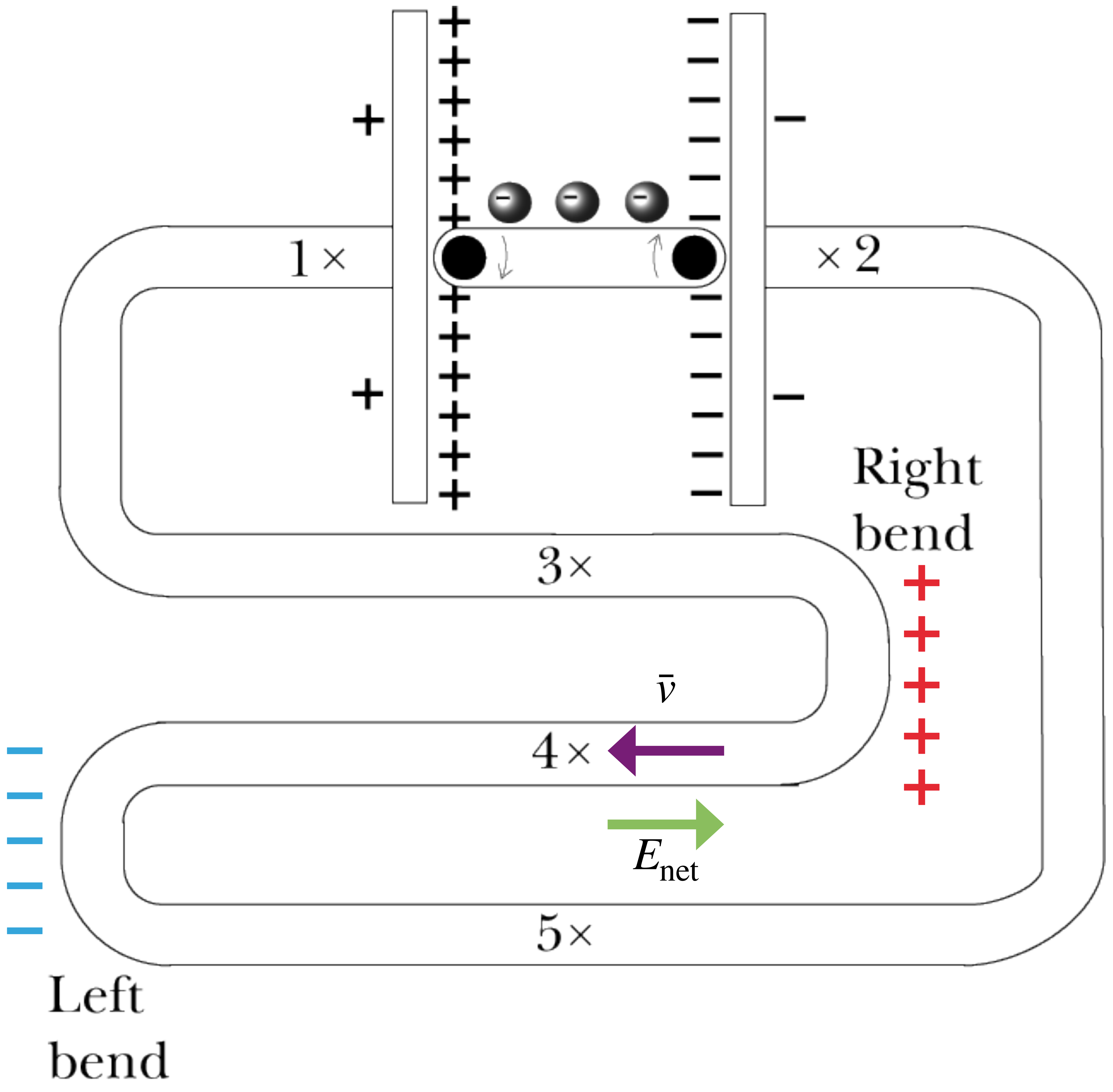

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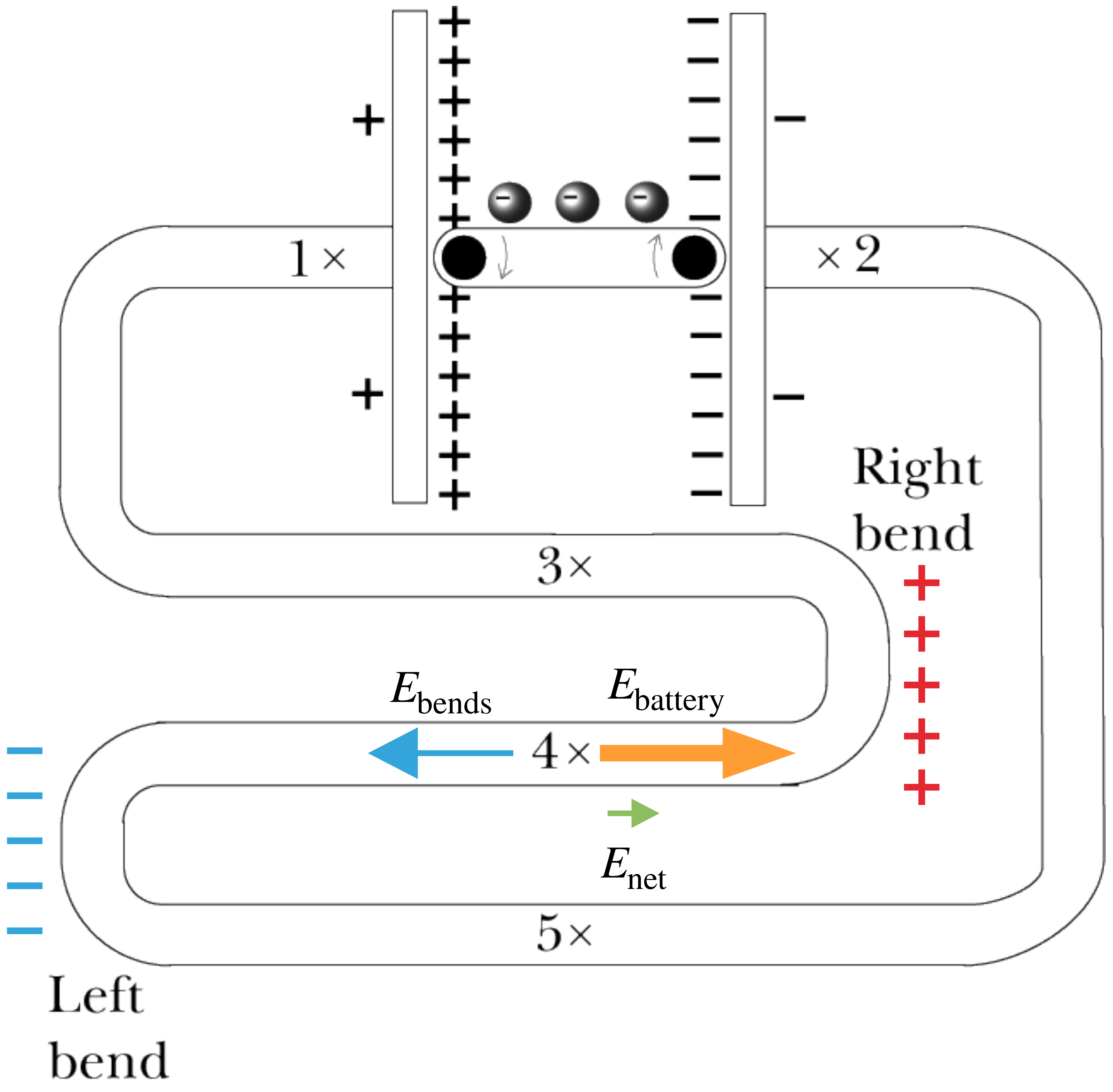
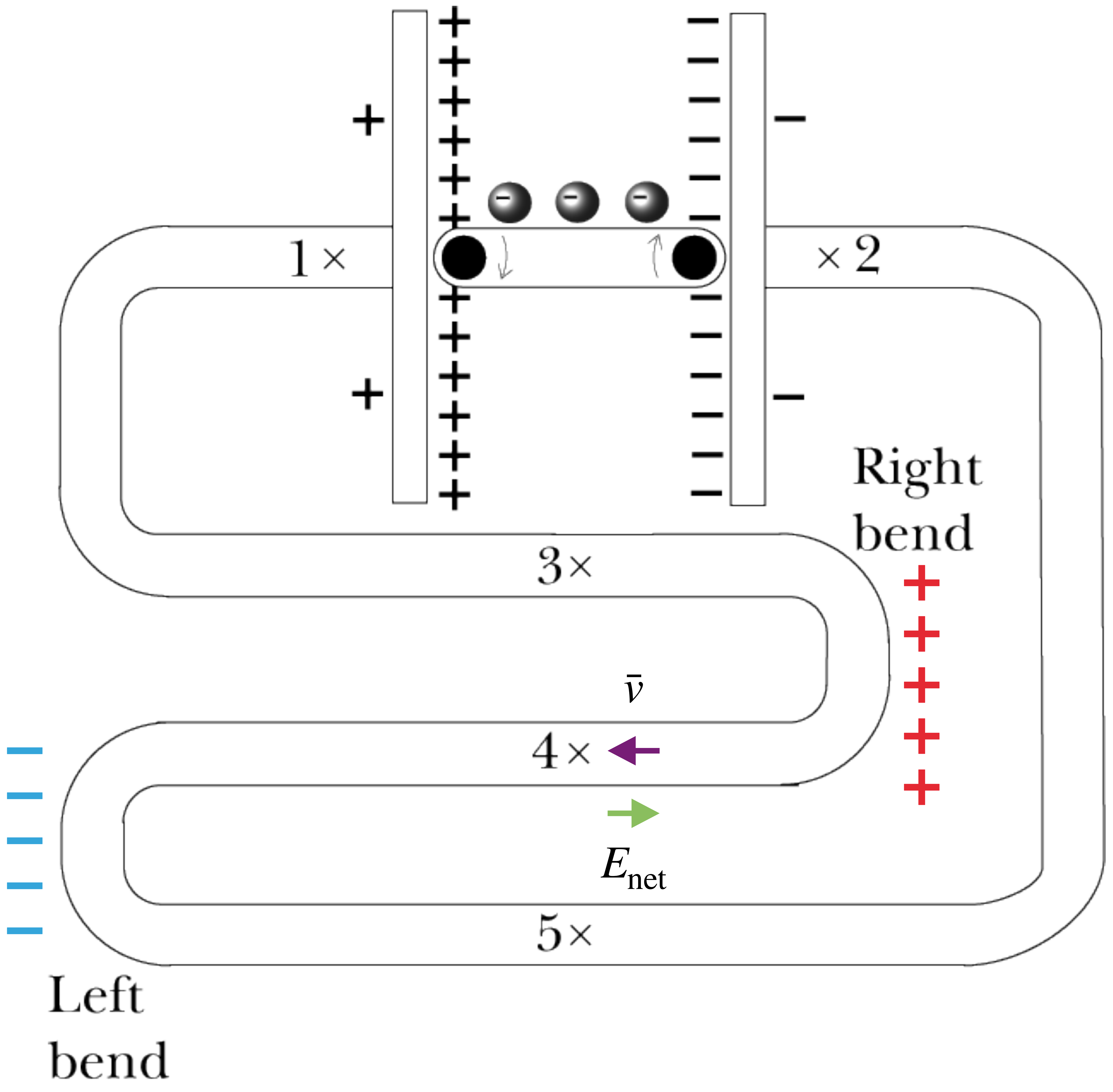
Continued charge buildup increases strength of E_{bends}



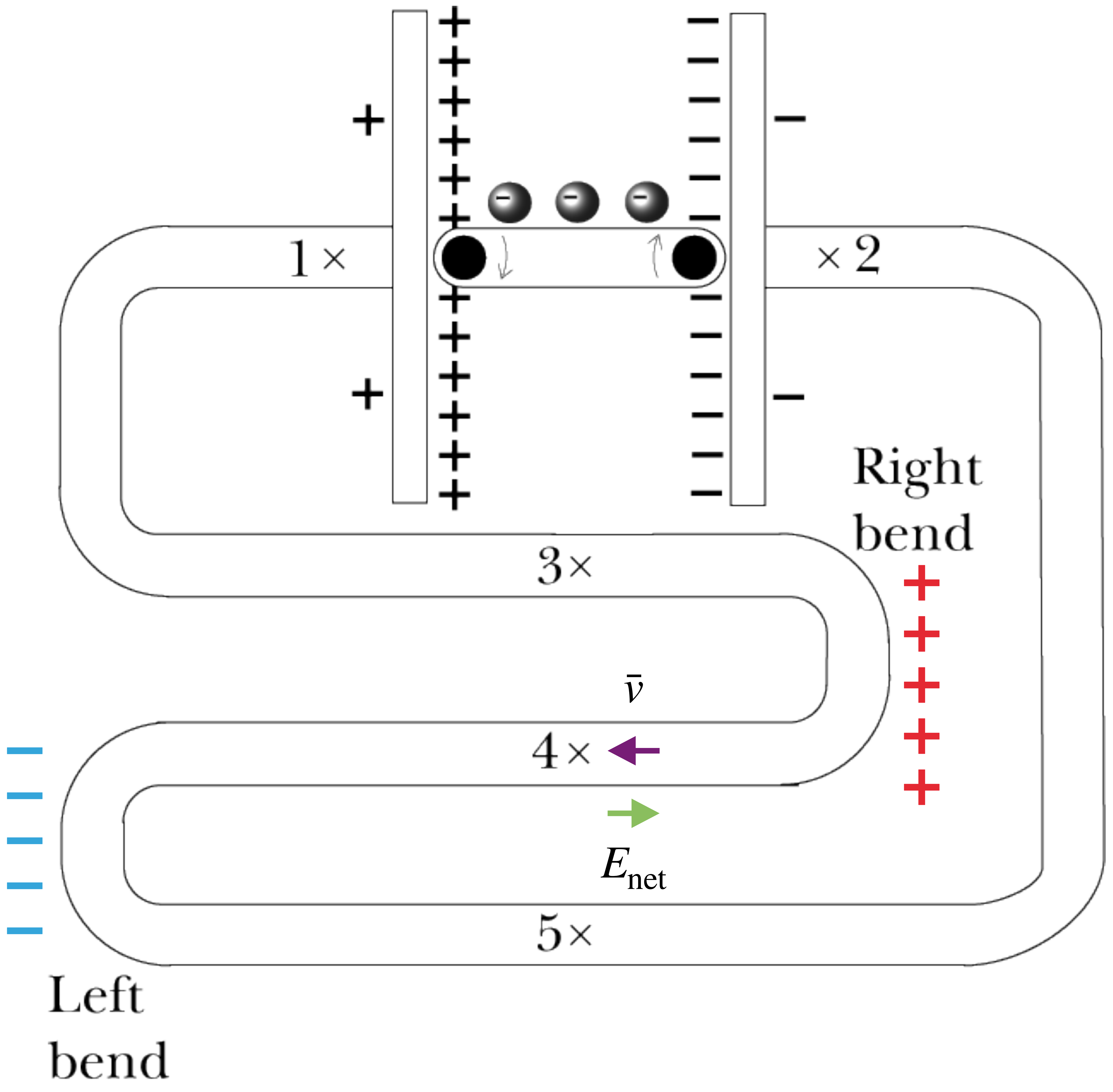
Net field still points in "wrong" direction, so charge buildup continues

Continued charge buildup increases strength of E_{bends}

Decreases $\bar{v}$ into the left bend

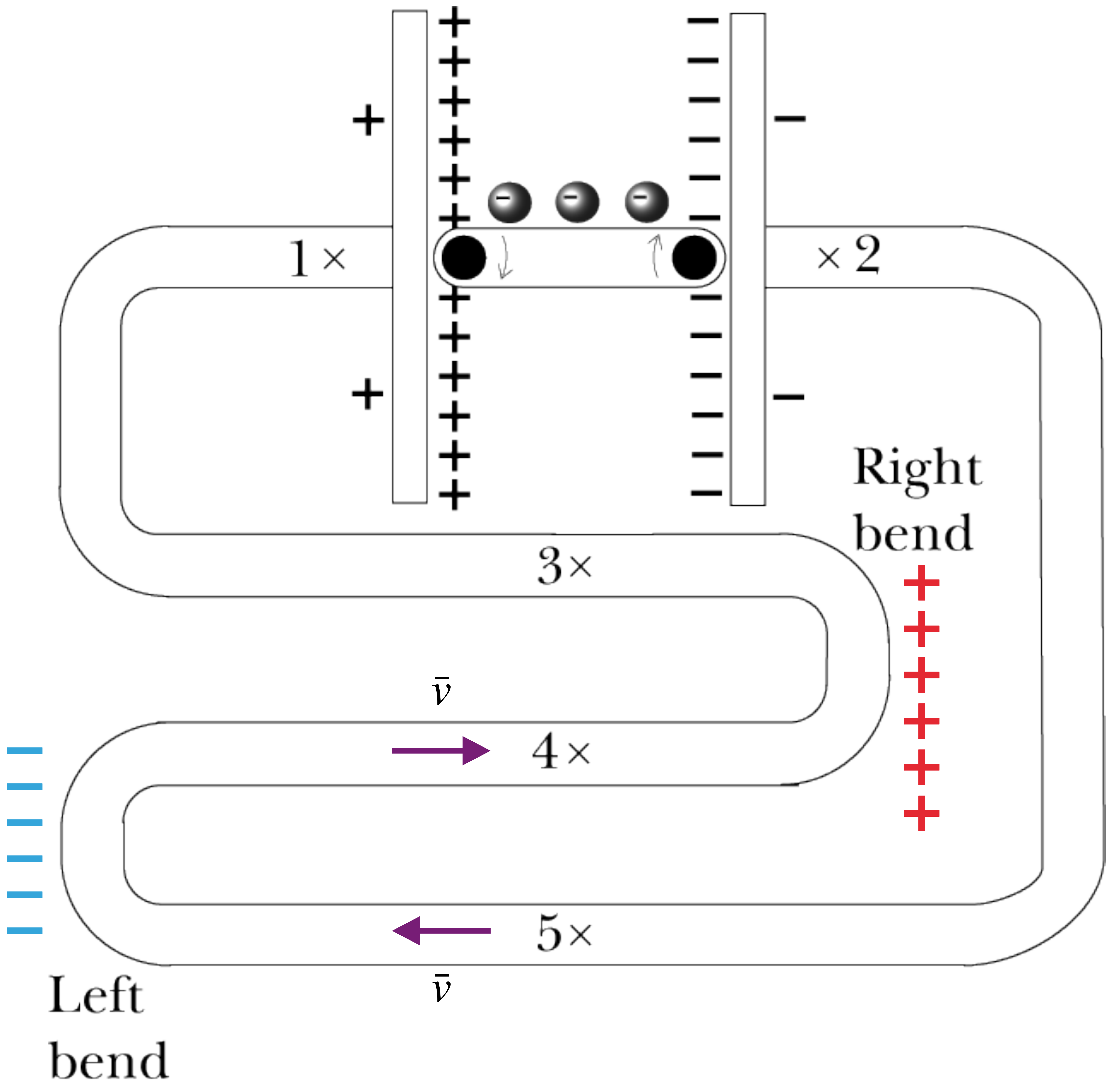


How long does charge buildup continue?



How long does charge buildup continue?

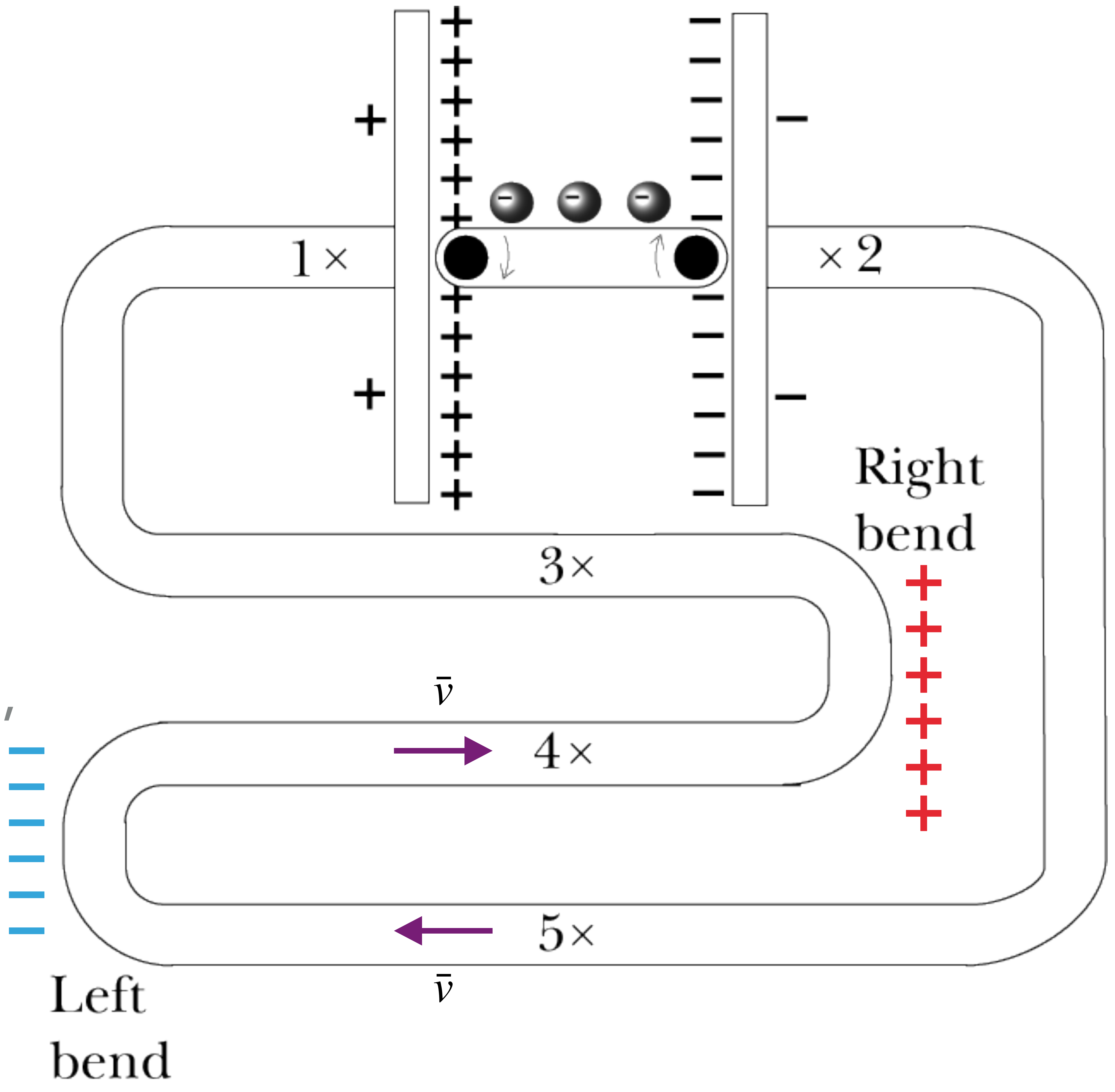
Until $\bar{v}$ into the bend equals $\bar{v}$ out of the bend



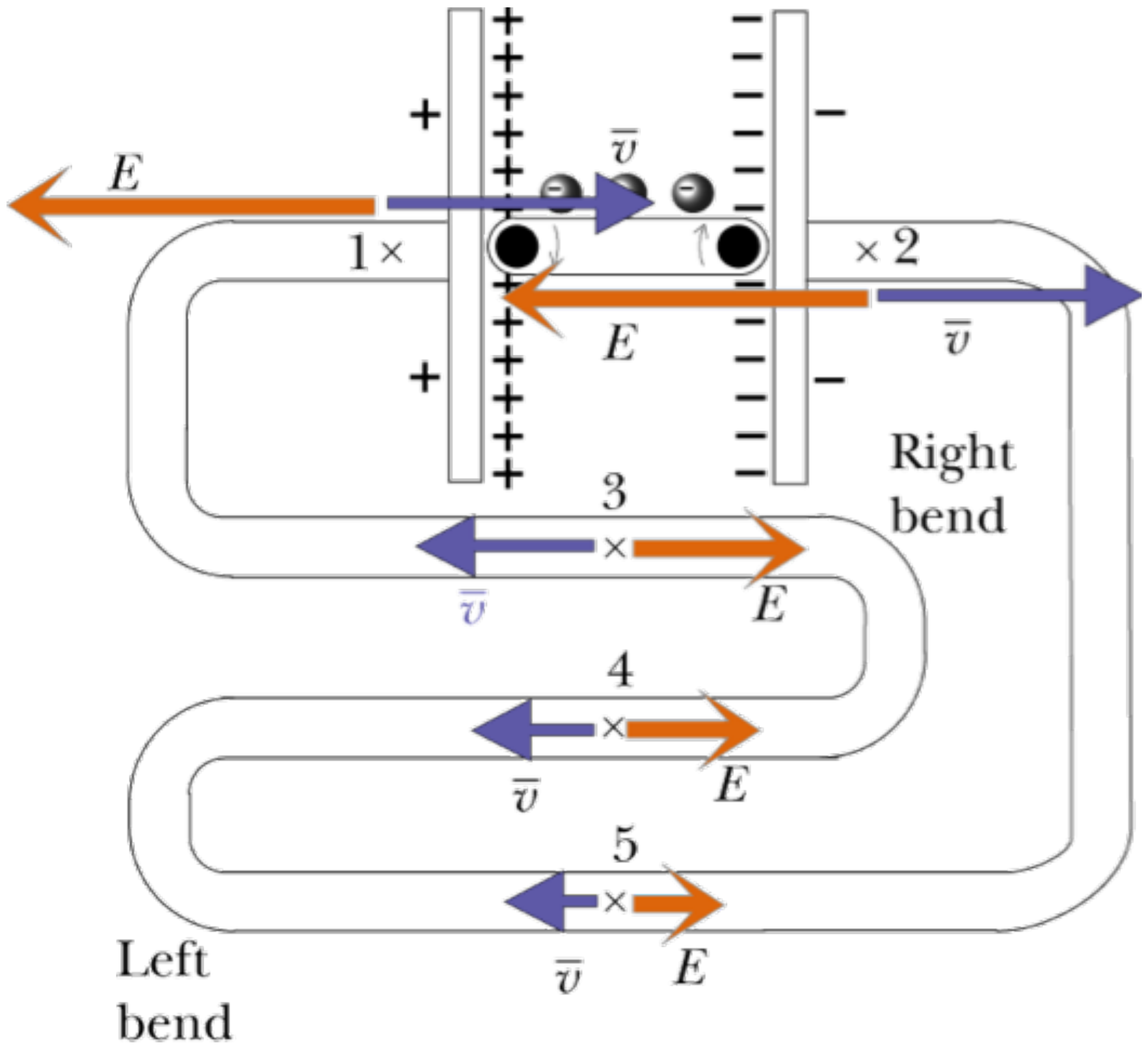
How long does charge buildup continue?

Until $\bar{v}$ into the bend equals $\bar{v}$ out of the bend

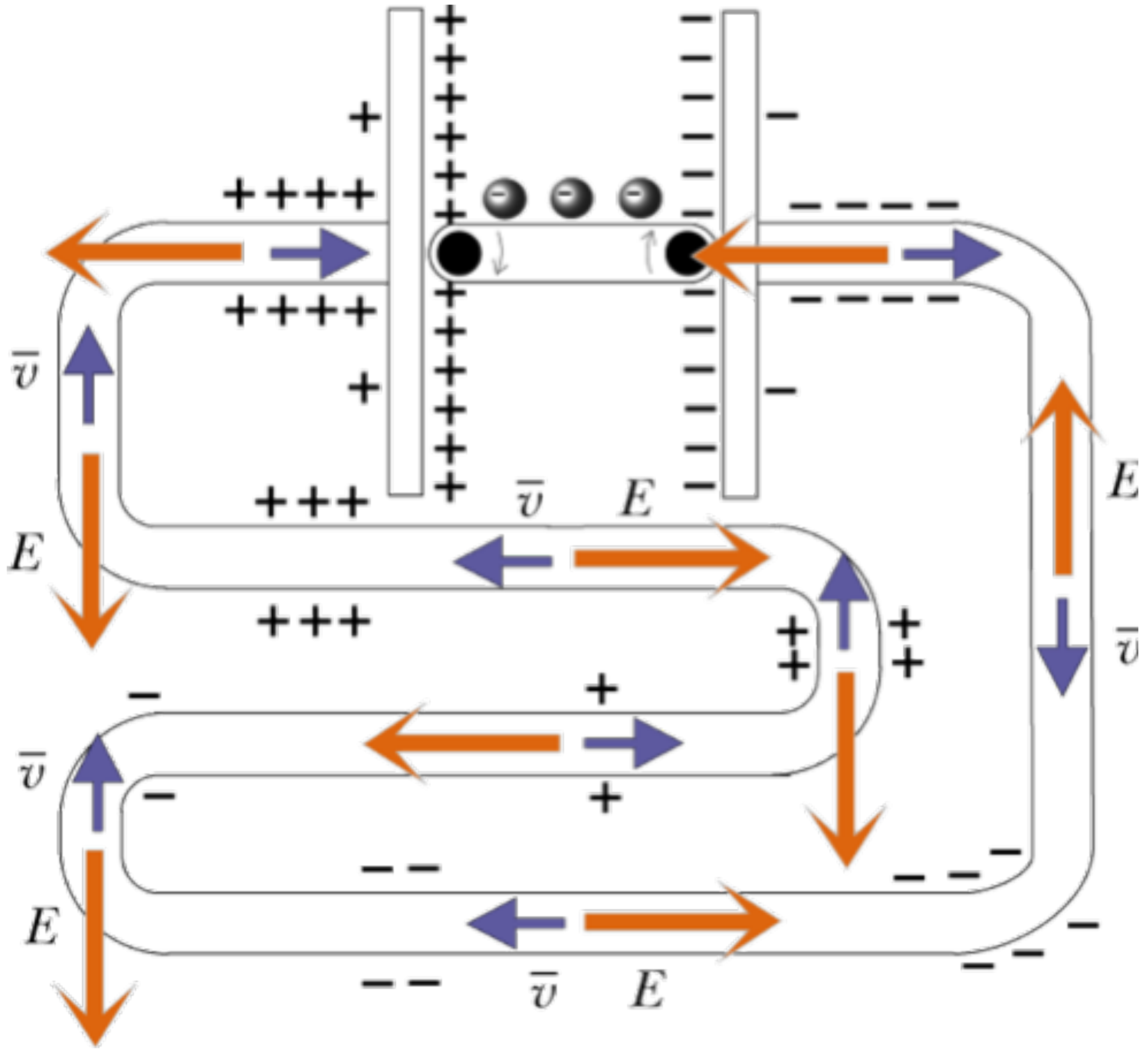
This happens all throughout the wire, not just at bendy points



BOTTOM LINE

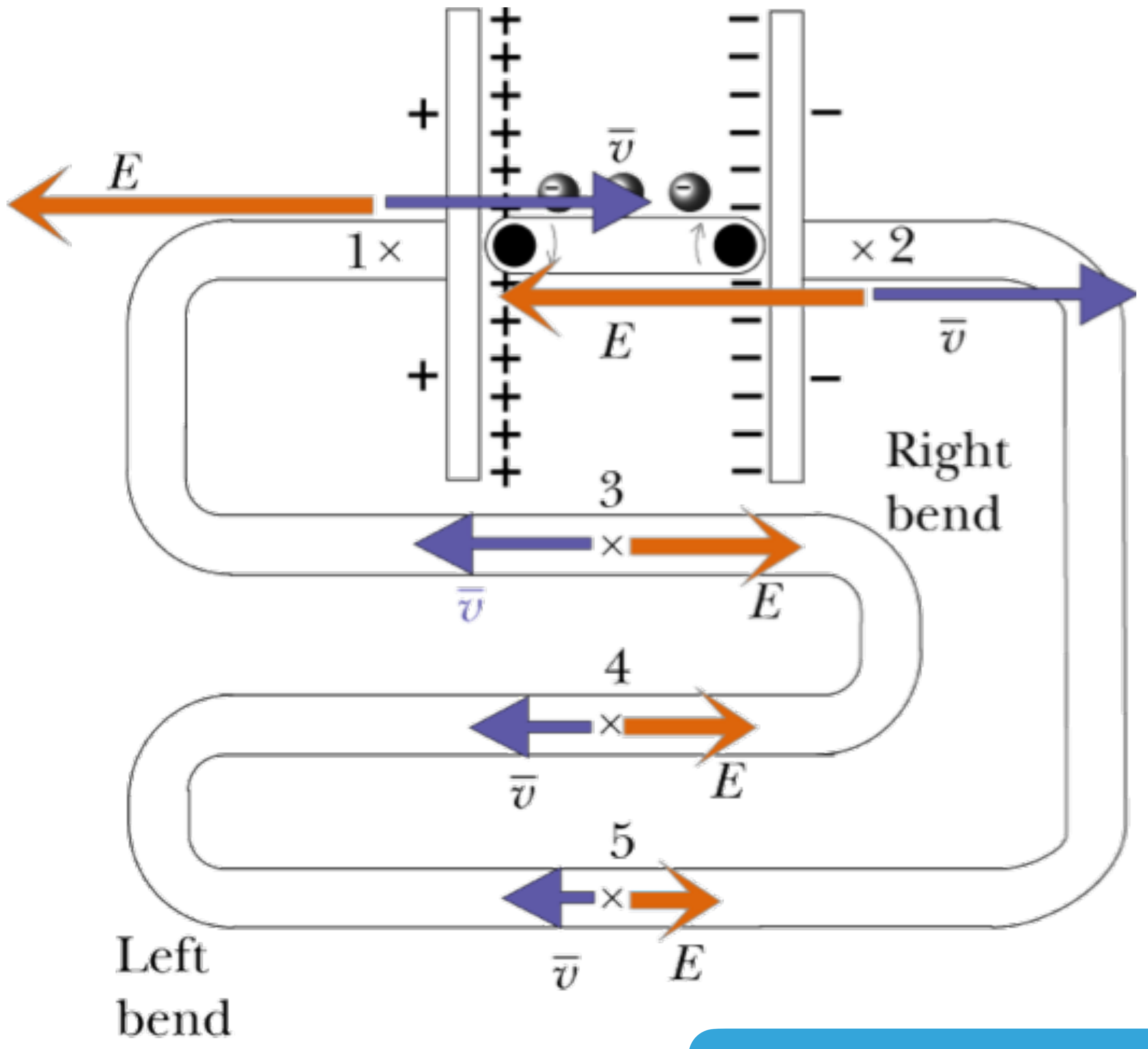


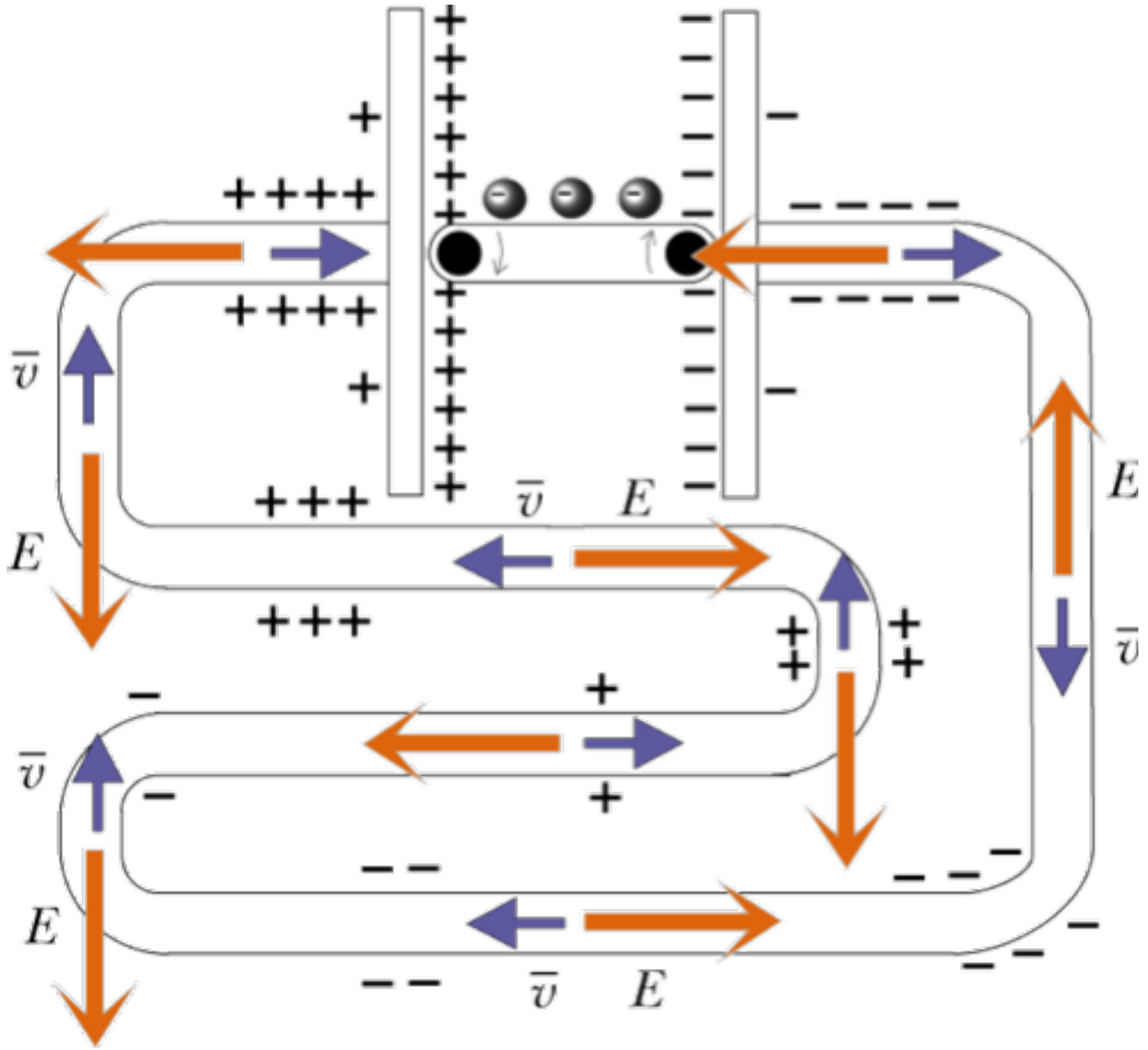
WHEN CIRCUIT IS FIRST CONNECTED, FIELD IS ONLY FROM THE BATTERY, CAUSING CHARGE BUILDUP THROUGHOUT THE CIRCUIT



CIRCUIT QUICKLY REACHES EQUILIBRIUM, WHERE $\bar{v}$ AND E_{net} ARE THE SAME EVERYWHERE

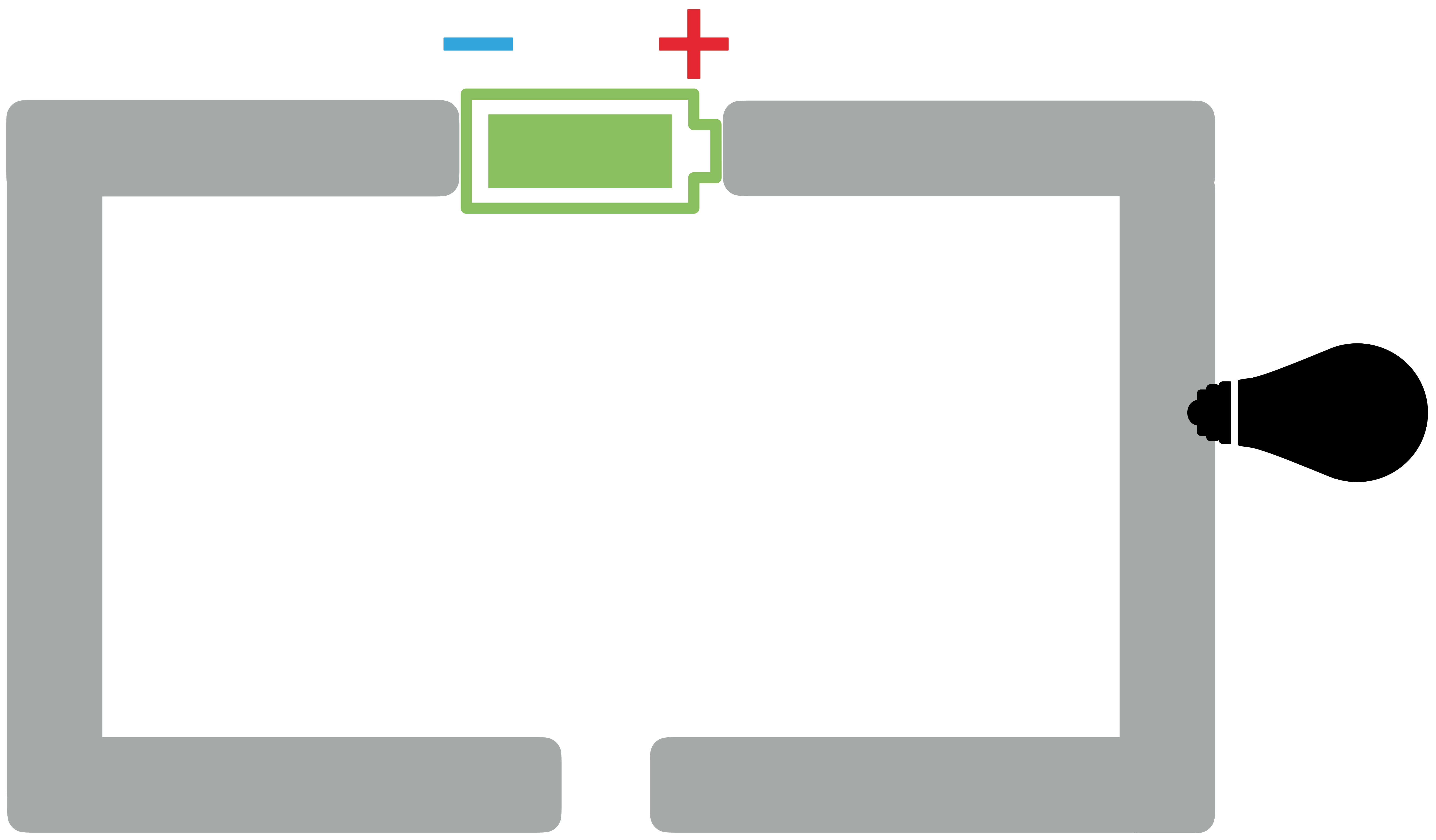
BOTTOM LINE





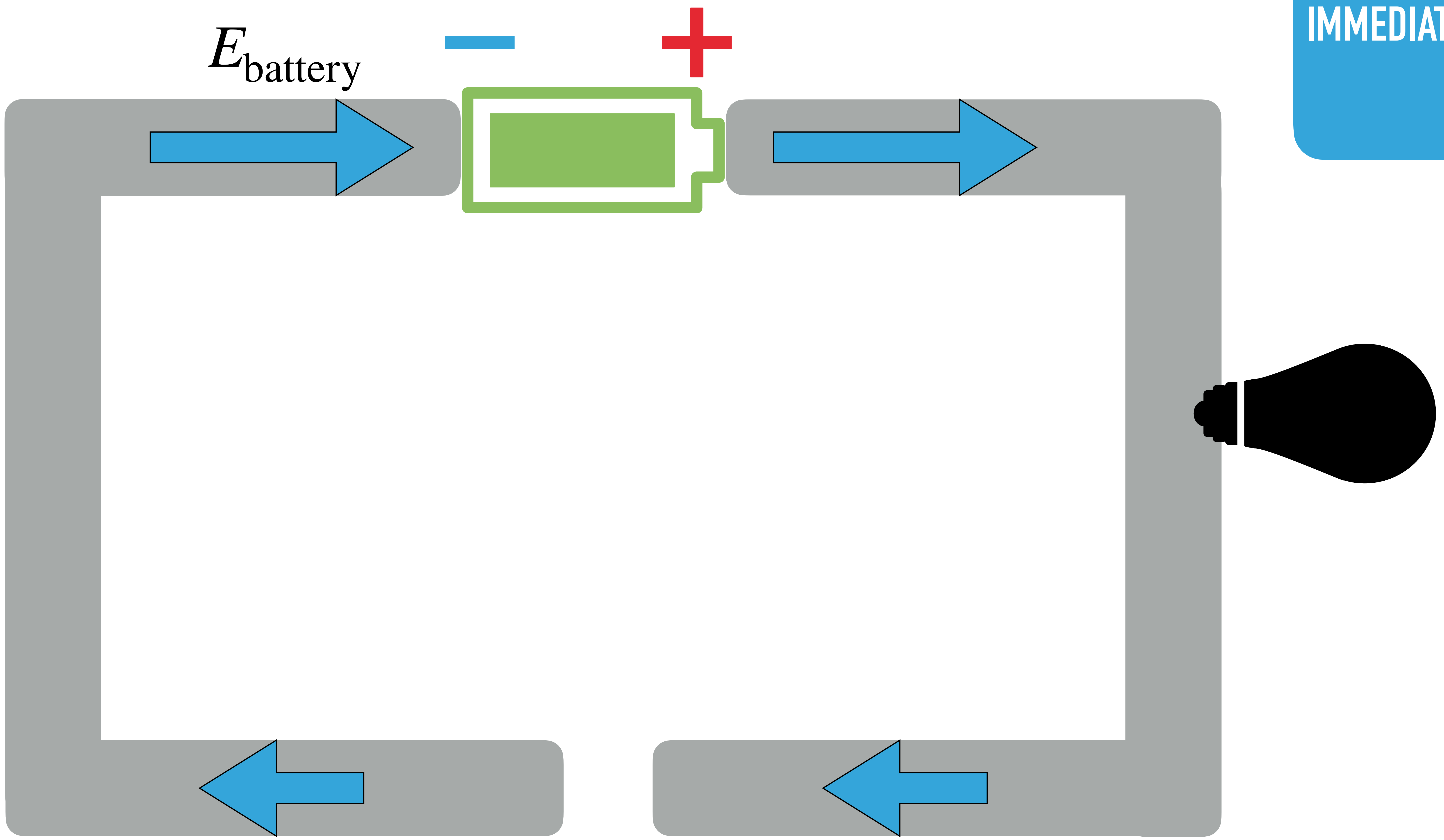
E FIELD INSIDE A CIRCUIT COMES FROM THE BATTERY <u>AND</u> THE BUILDUP OF CHARGES ON THE SURFACE OF THE WIRES

WHY DOESN'T CURRENT FLOW WHEN THE CIRCUIT IS DISCONNECTED?



WHY DOESN'T CURRENT FLOW WHEN THE CIRCUIT IS DISCONNECTED?

IMMEDIATELY AFTER BATTERY IS CONNECTED



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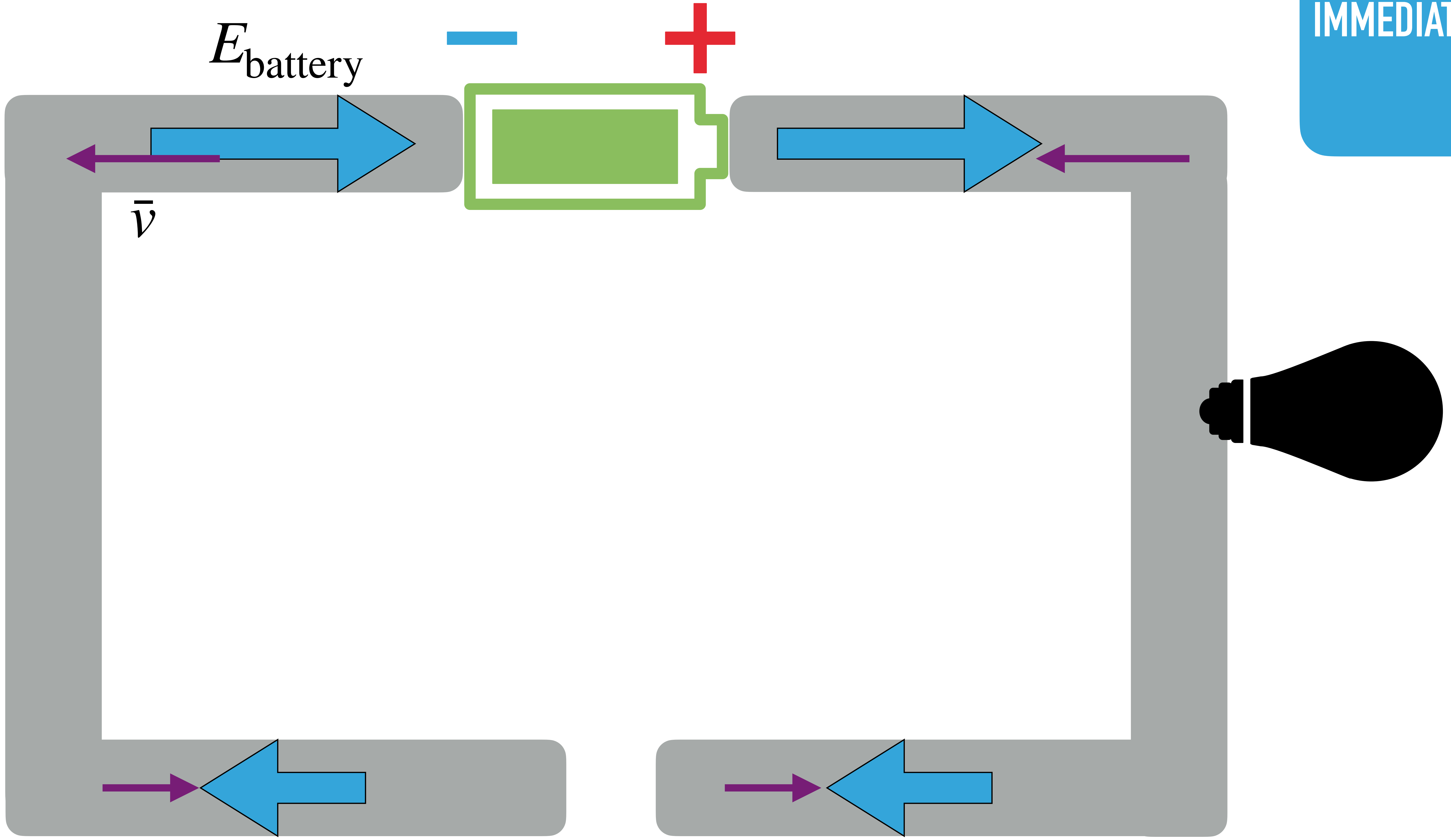

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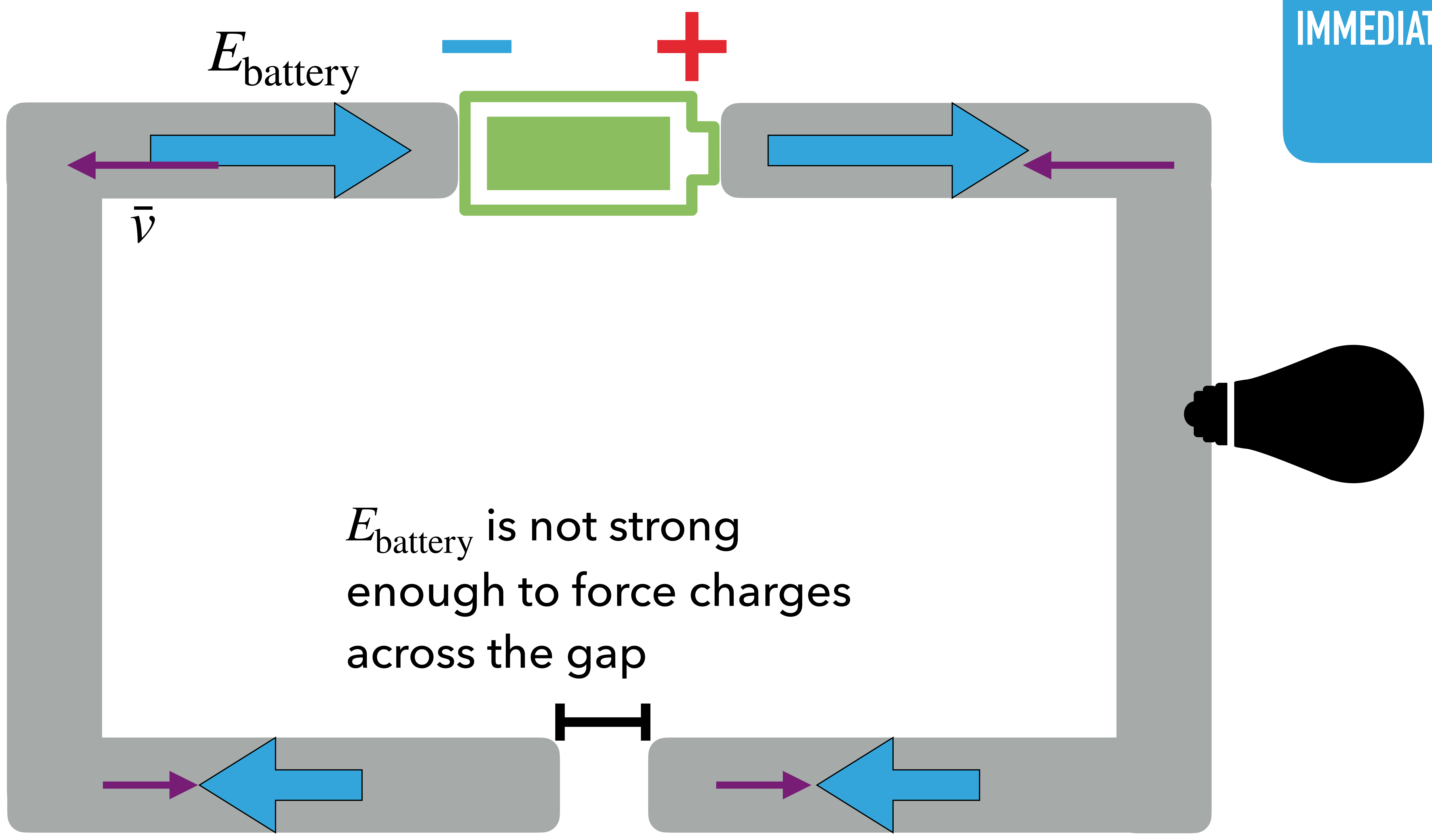
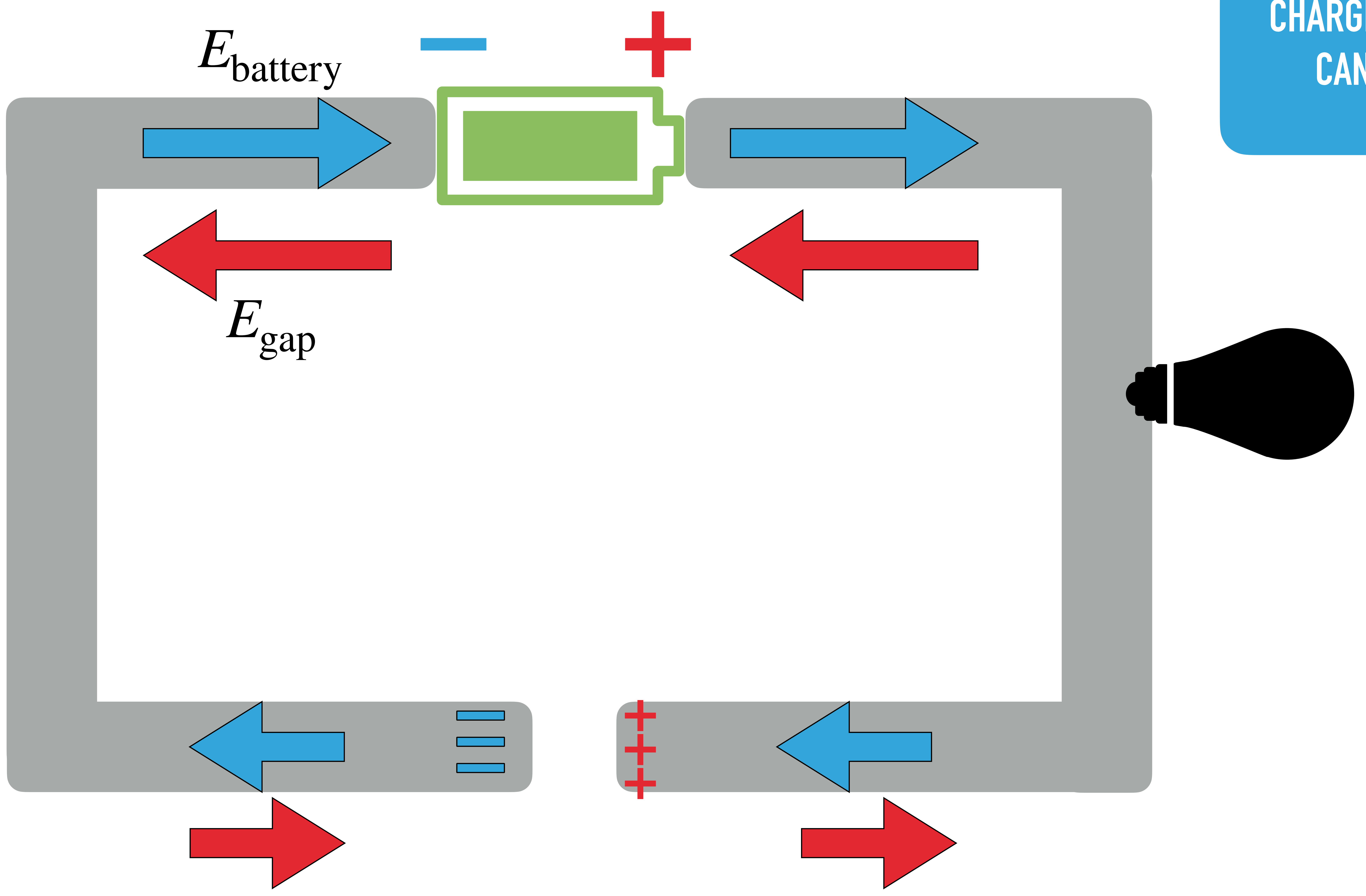


E_{battery} is not strong enough to force charges across the gap

WHY DOESN'T CURRENT FLOW WHEN THE CIRCUIT IS DISCONNECTED?

CHARGE BUILDUP QUICKLY CANCELS E_{battery}



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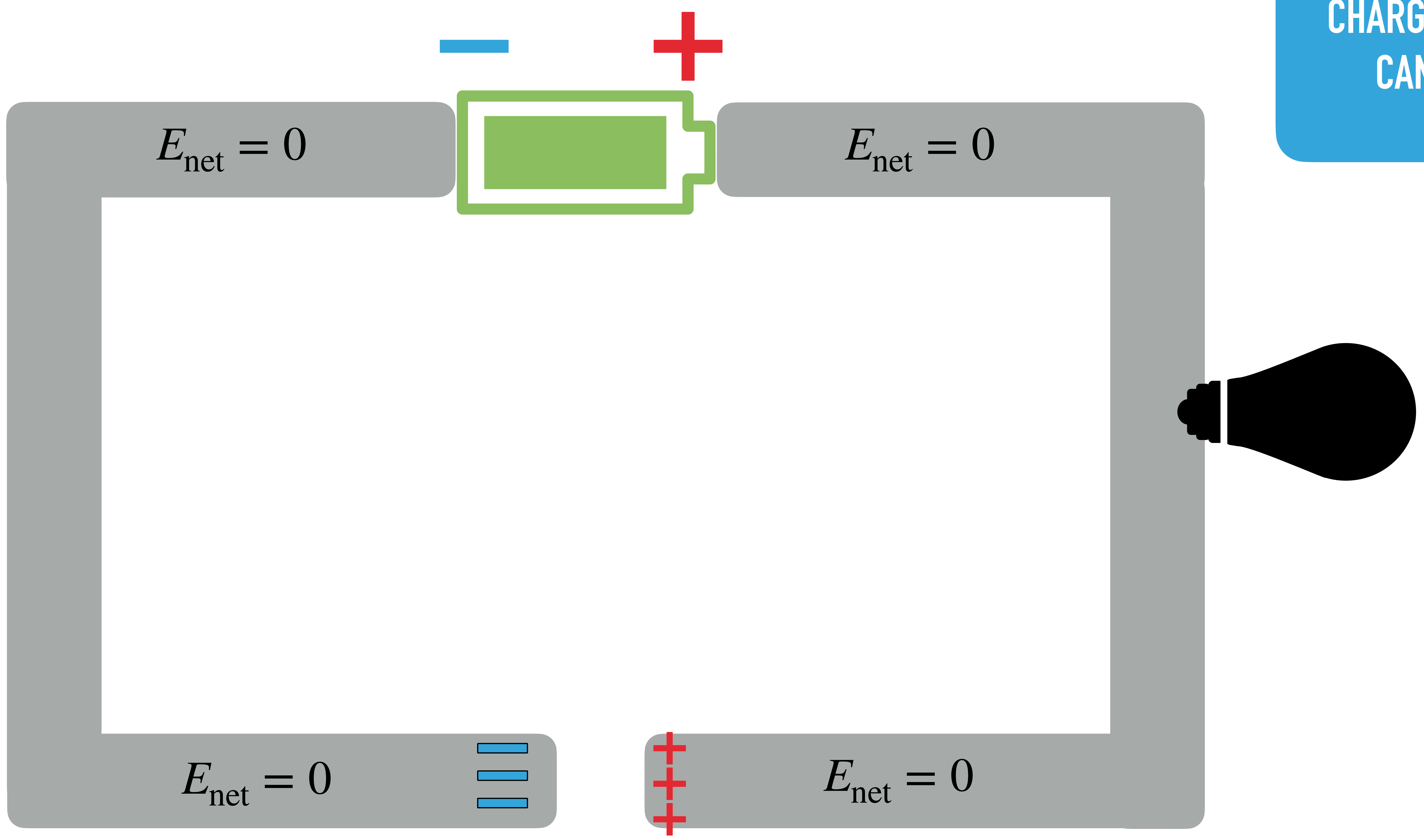

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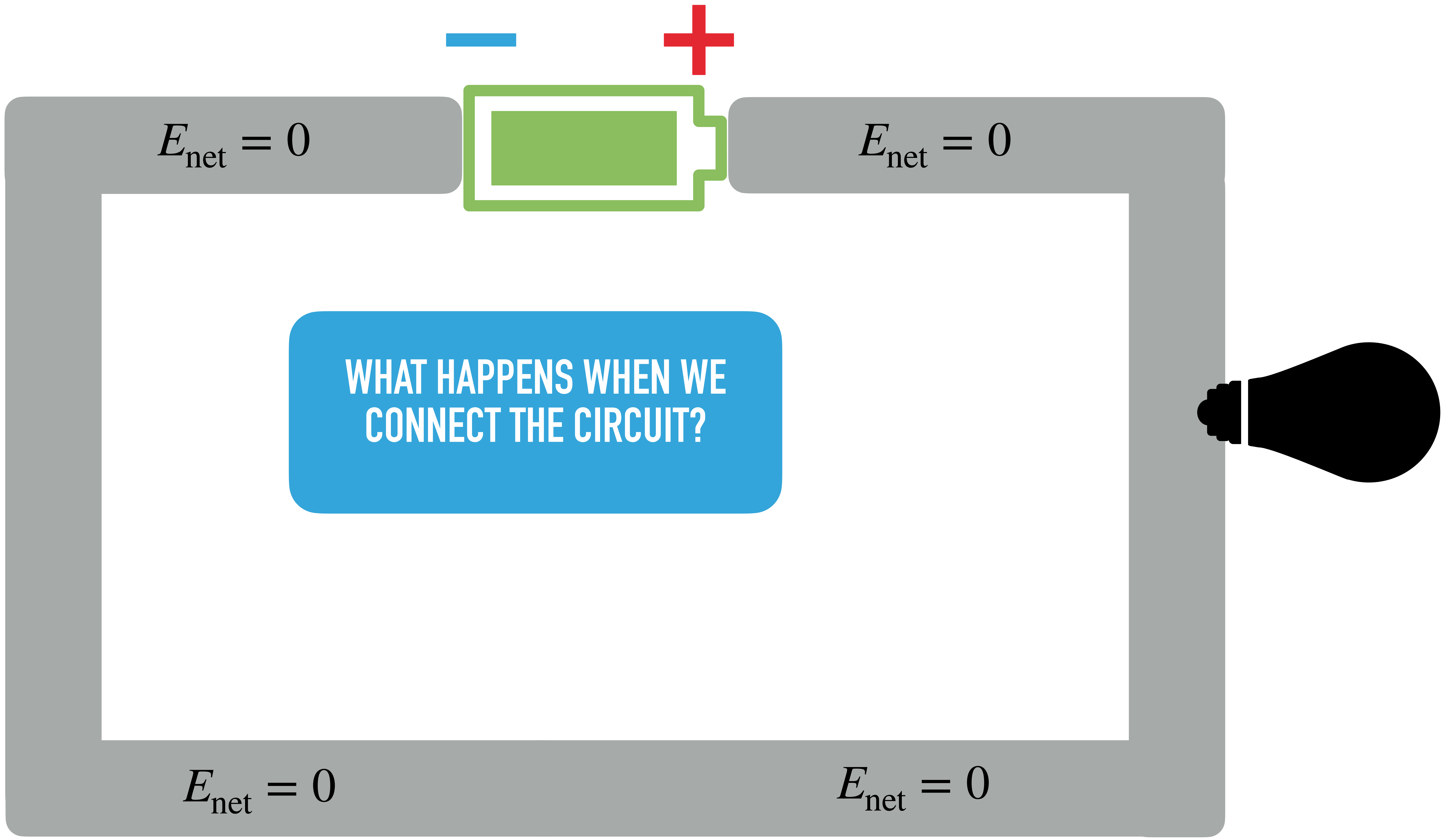

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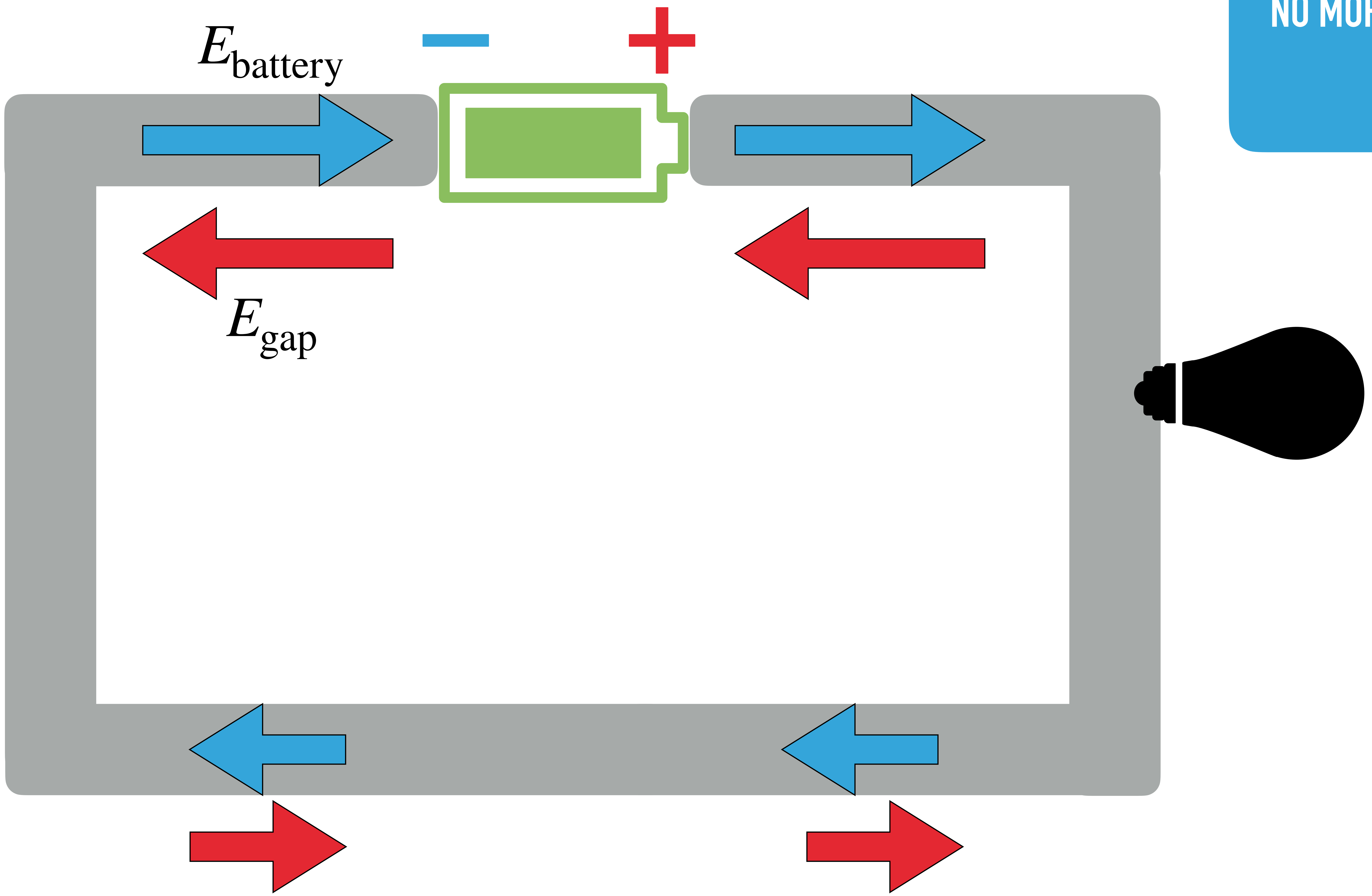

NO MORE E_{gap} TO CANCEL E_{battery}

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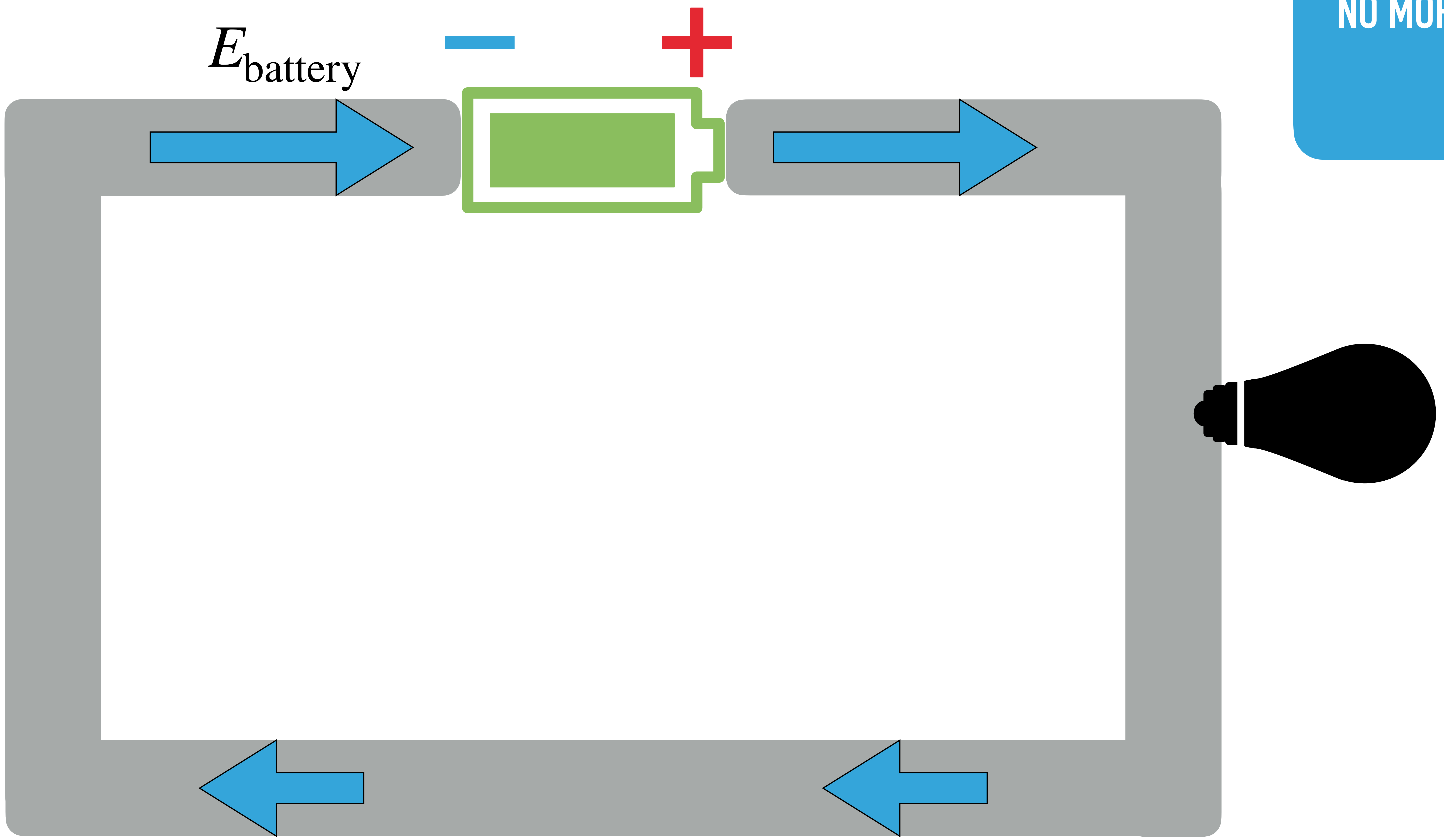

NO MORE E_{gap} TO CANCEL $E_{battery}$

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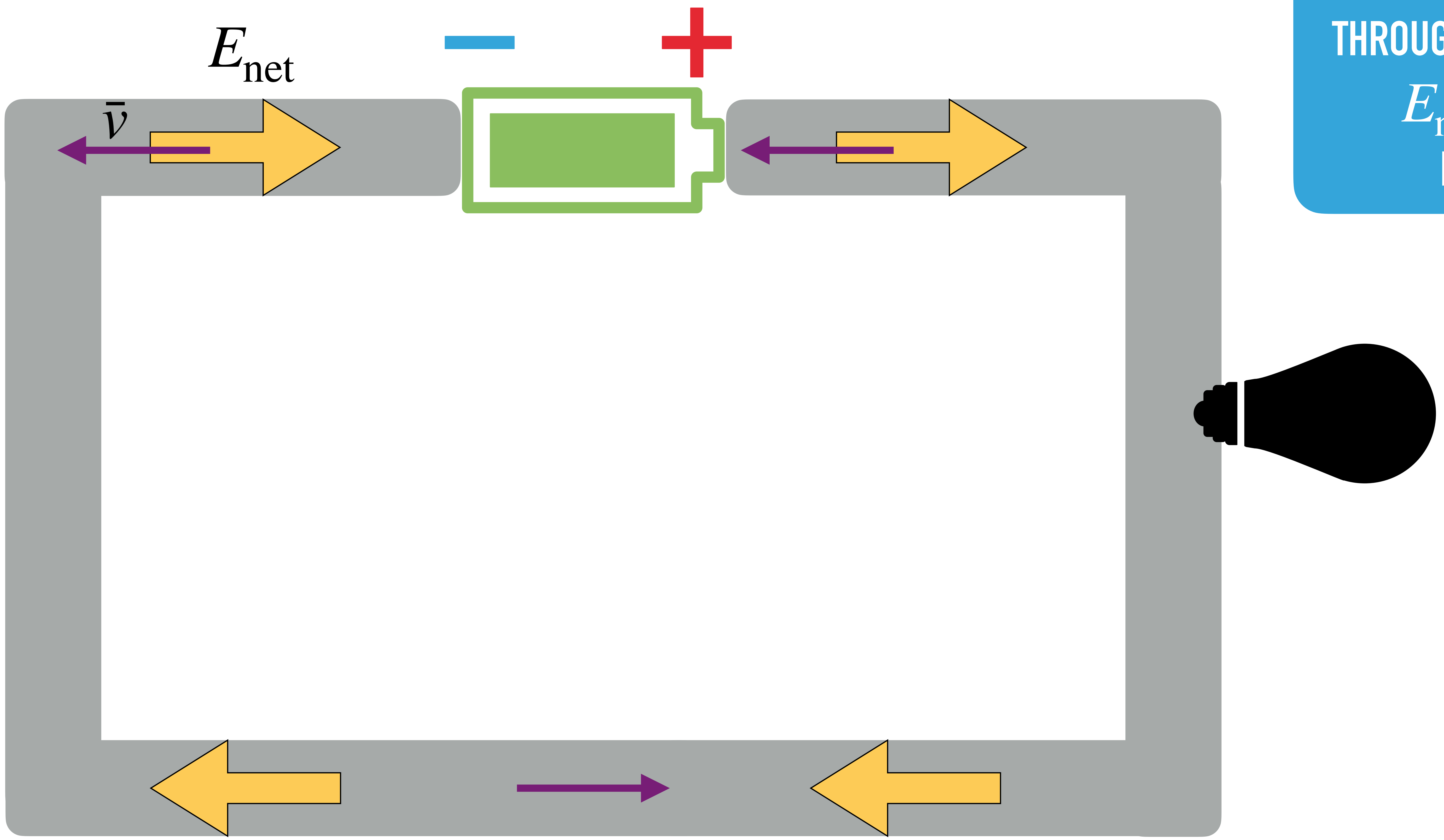
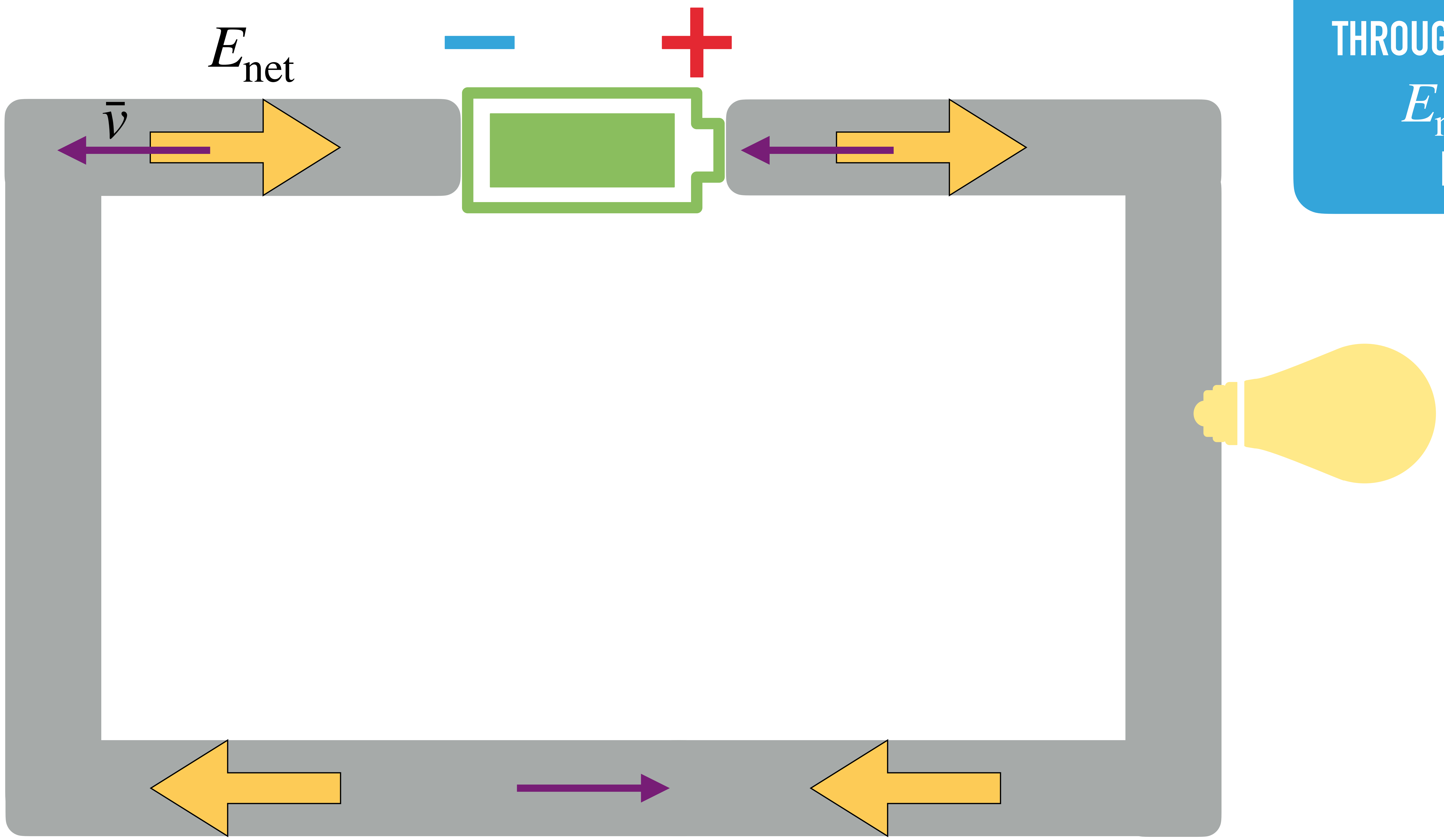


CHARGE BUILDUP THROUGHOUT CIRCUIT UNTIL E_{net} IS THE SAME EVERYWHERE

WHY DOESN'T CURRENT FLOW WHEN THE CIRCUIT IS DISCONNECTED?



CHARGE BUILDUP THROUGHOUT CIRCUIT UNTIL E_{net} IS THE SAME EVERYWHERE

WHY DOES THE LIGHT COME ON RIGHT AWAY?

- Drift speed of electrons in wire: $\bar{v} \sim 10^{-4}$ m/s

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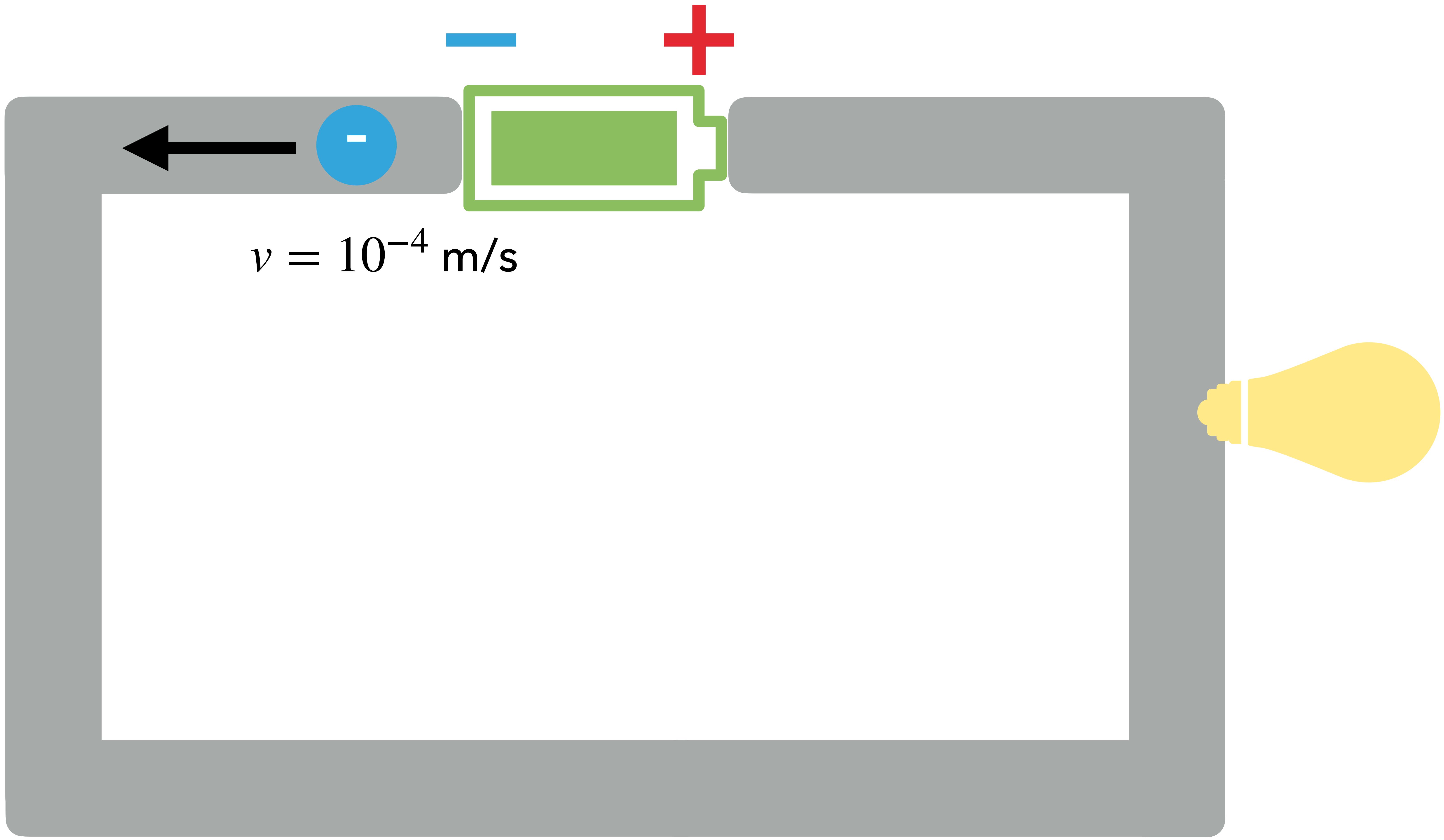
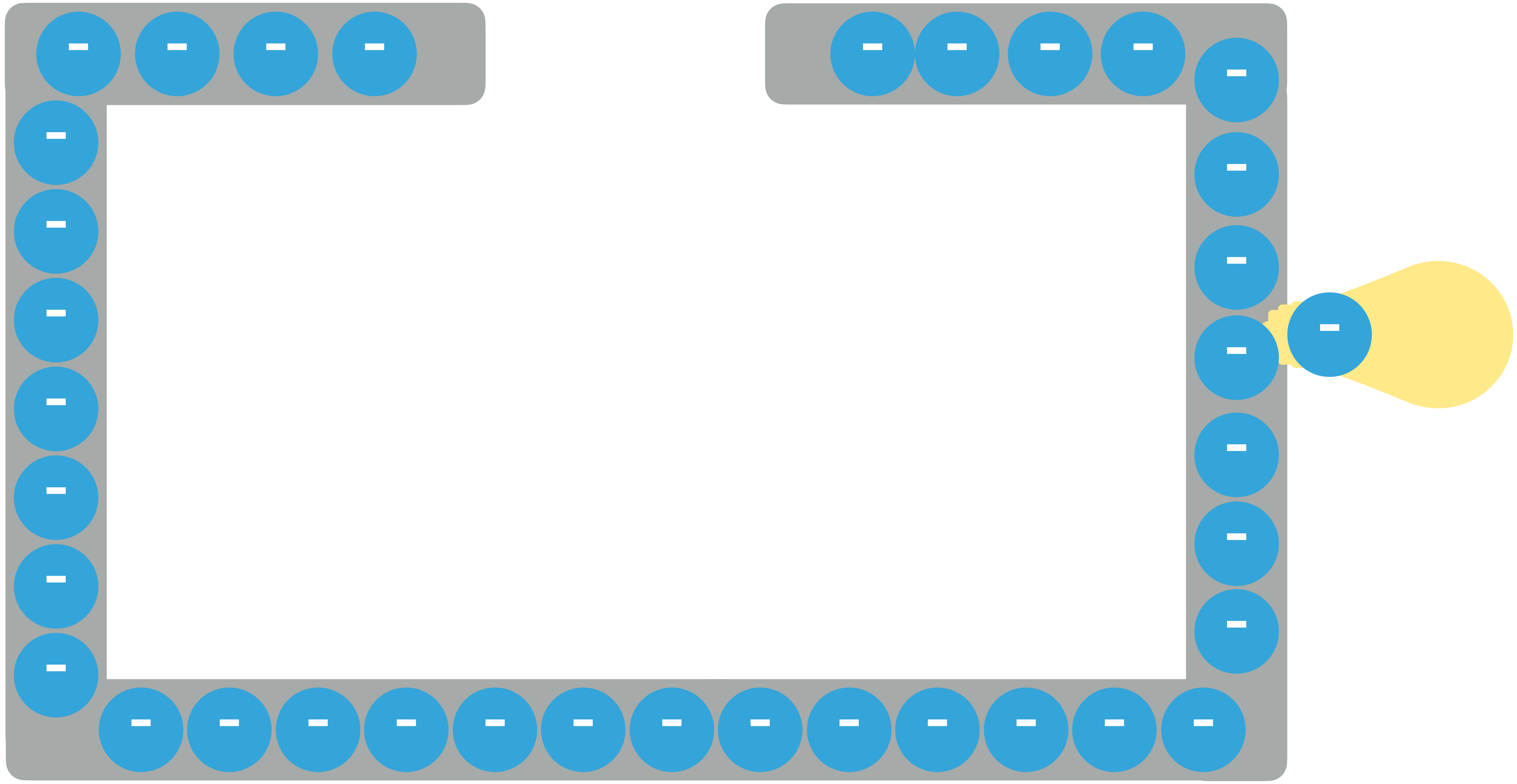


WHY DOES THE LIGHT COME ON RIGHT AWAY?

- Drift speed of electrons in wire: $\bar{v} \sim 10^{-4}$ m/s
- There are already free electrons *all throughout the wire!*
    - We don't need to wait for an electron to move from battery to bulb

WHY DOES THE LIGHT COME ON RIGHT AWAY?



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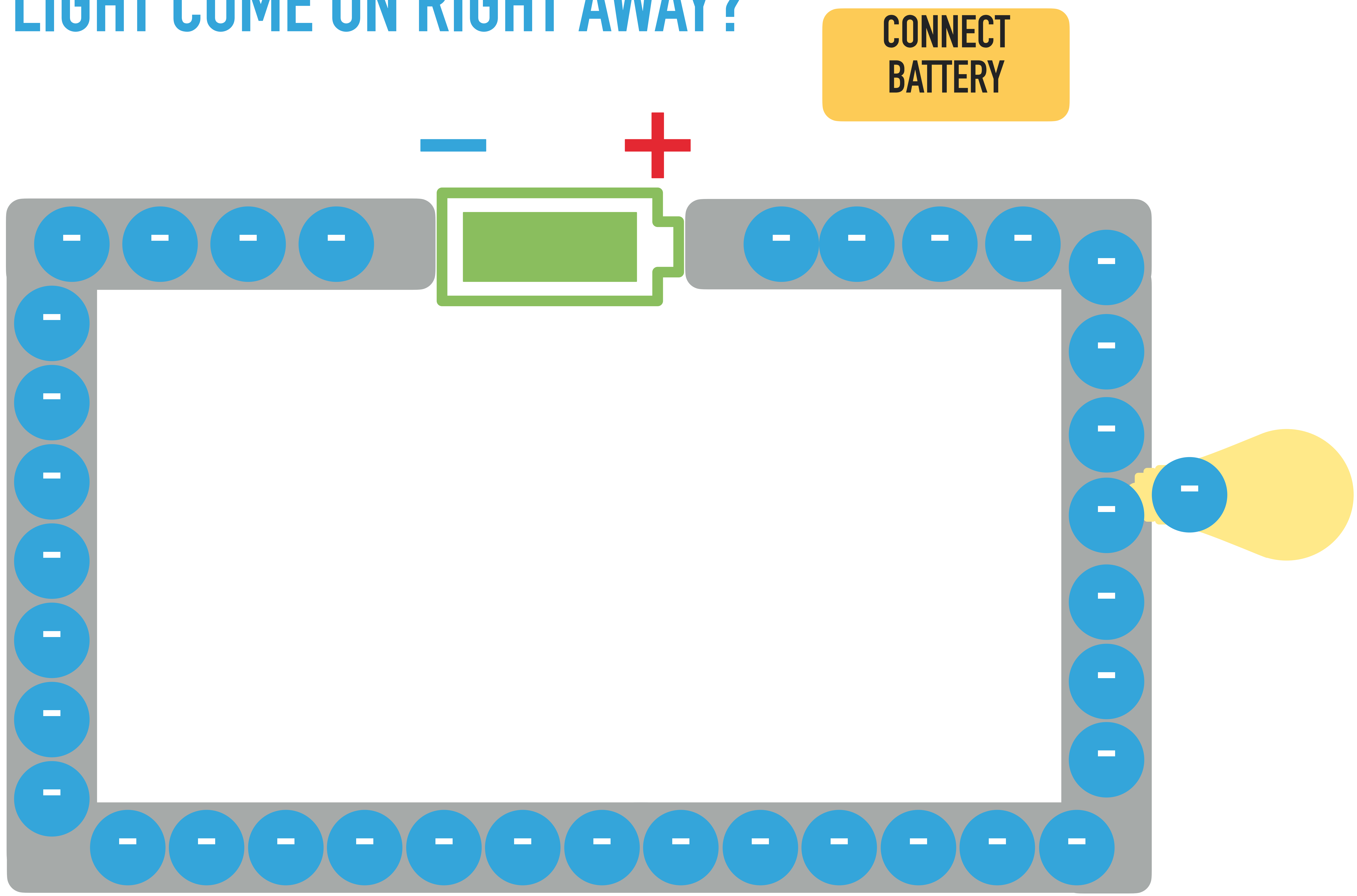

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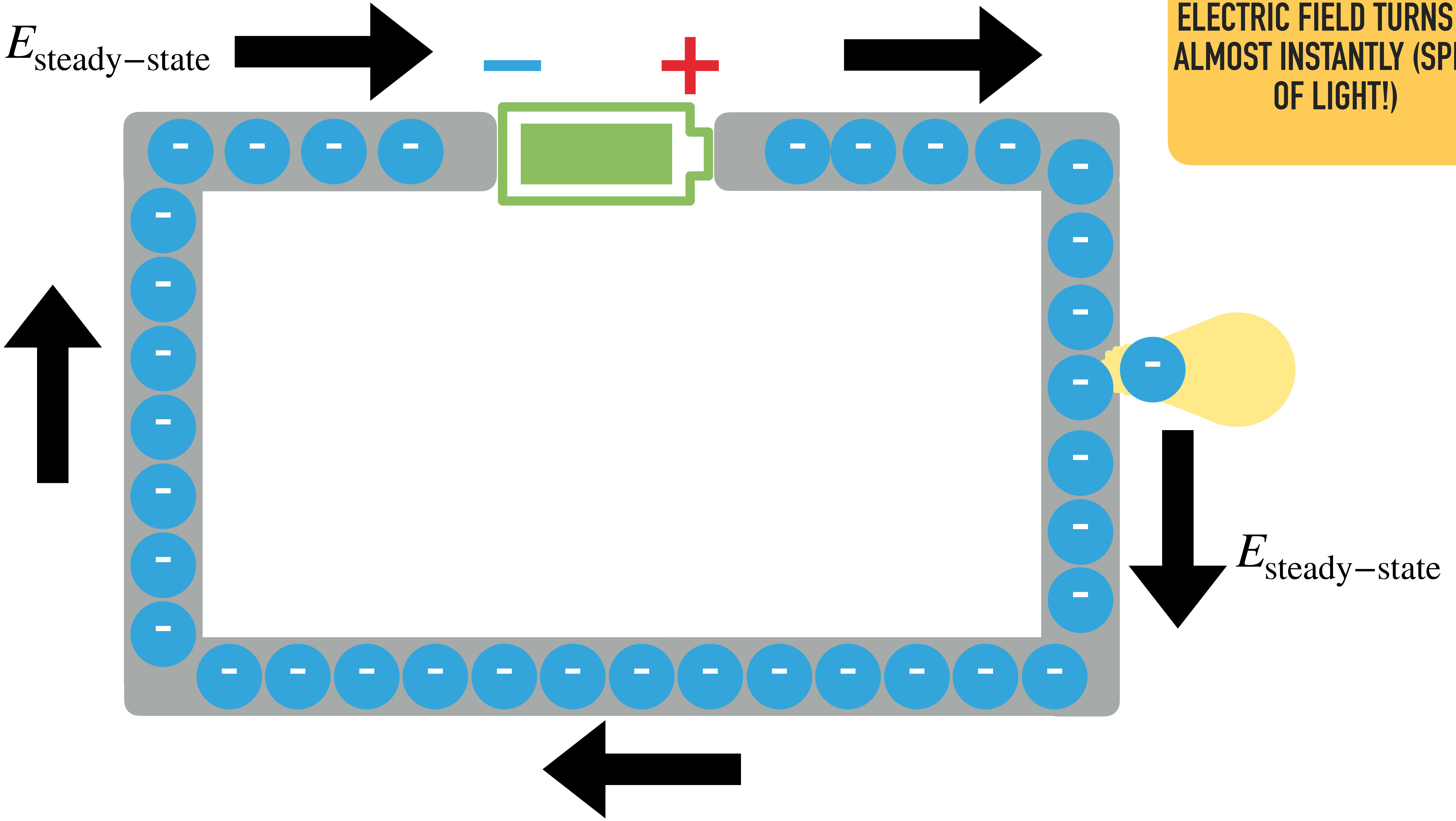
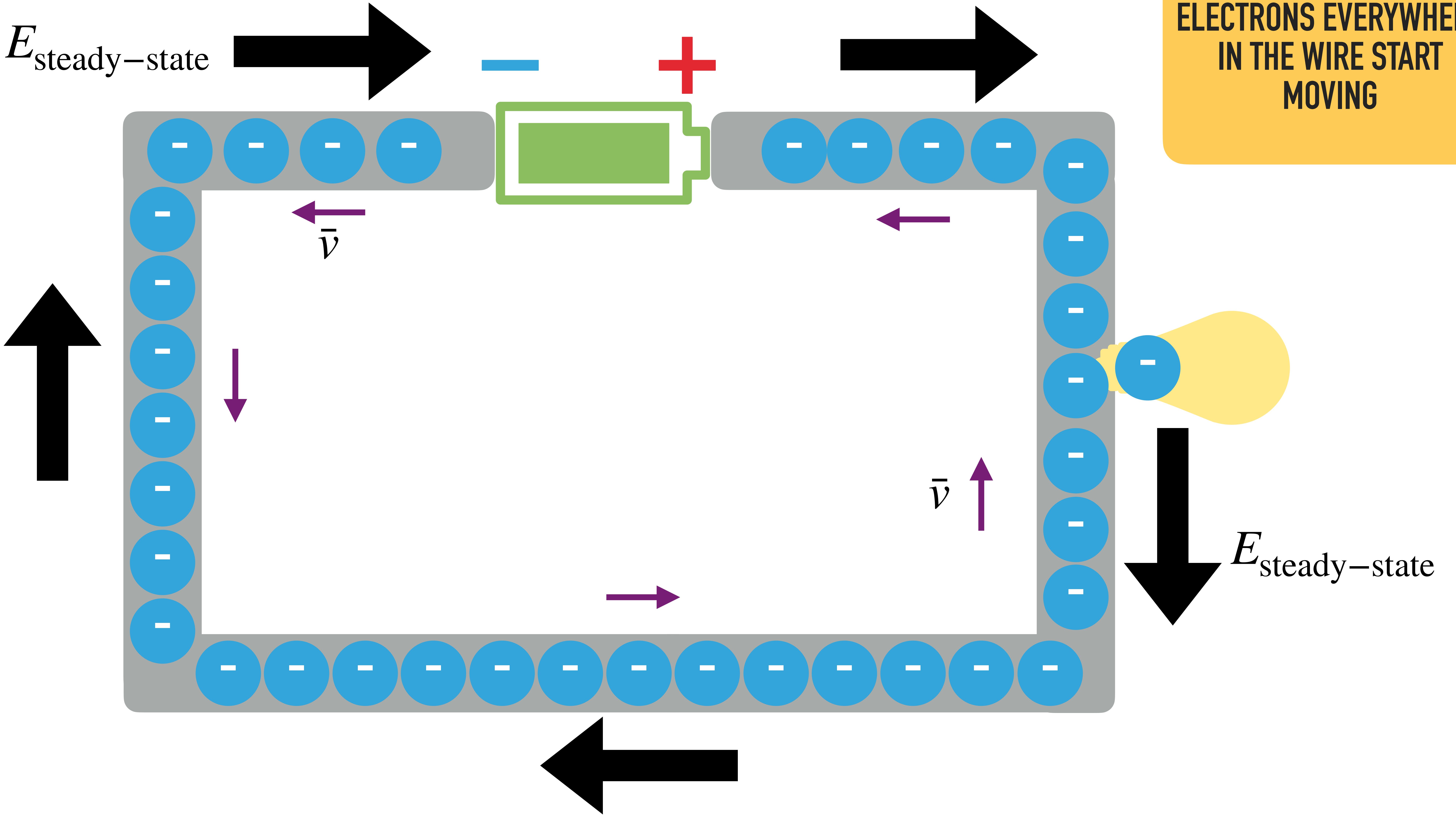


ELECTRIC FIELD TURNS ON ALMOST INSTANTLY (SPEED OF LIGHT!)

WHY DOES THE LIGHT COME ON RIGHT AWAY?



ELECTRONS EVERYWHERE IN THE WIRE START MOVING

WHAT GOES ON INSIDE A CIRCUIT?

WHAT GOES ON INSIDE A CIRCUIT?

- Current flow caused by electric field
- Immediately after connecting the circuit, takes a small amount of time to reach the steady state
    - In the steady state, current and field are not changing with time
    - Period of time before steady state: charge buildup along the wires until $E_{\text{battery}} + E_{\text{wires}}$ produces a current which satisfies the node rule
    - This short (~ a few ns) period of time is called the initial transient

THE NODE RULE

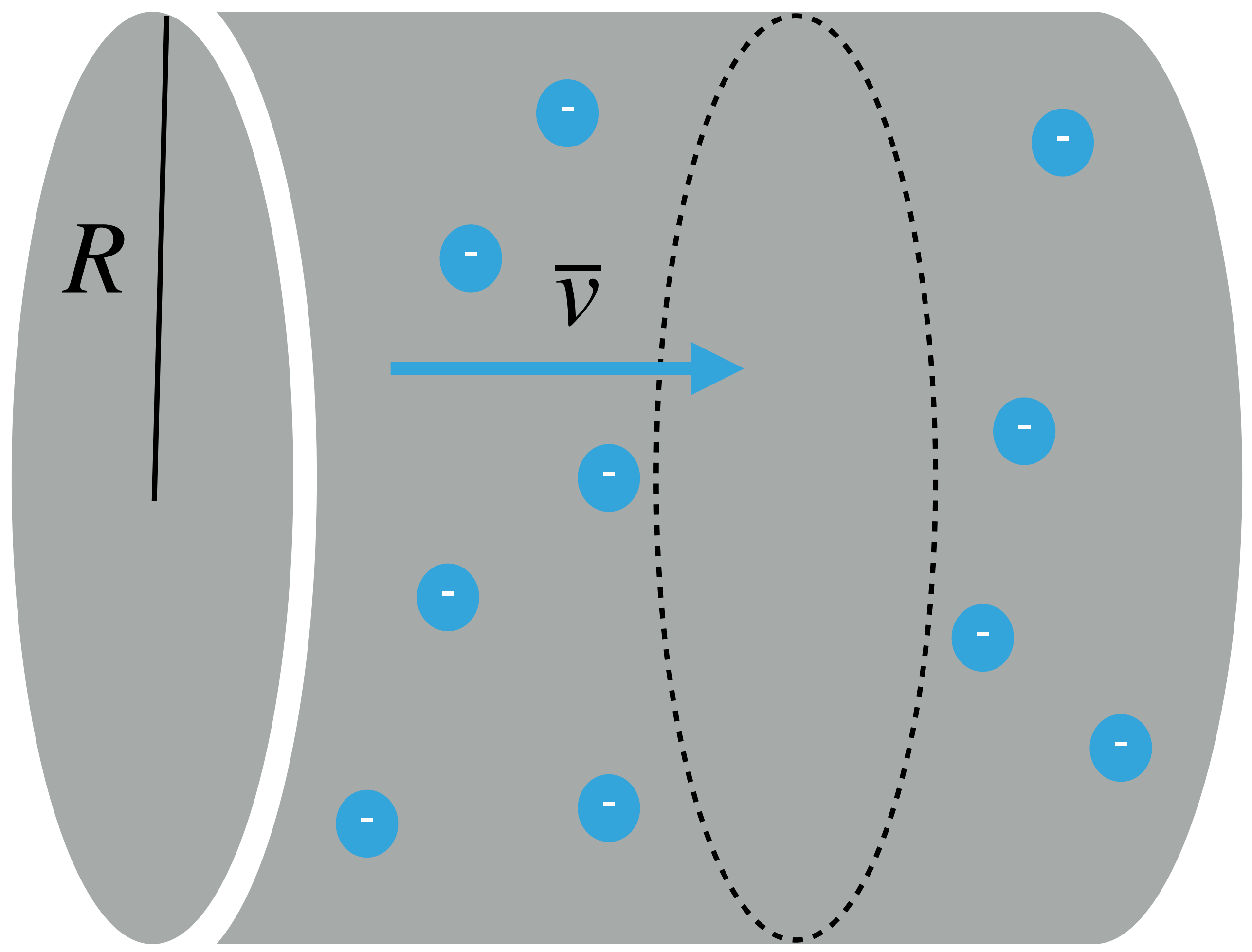
- In the steady state: at any point in the circuit, current flowing in must equal current flowing out
  - $\sum i_{\text{in}} = \sum i_{\text{out}}$
  - Consequence of charge conservation

WHAT DRIVES THE CURRENT?

WHAT DRIVES THE CURRENT?

$$i = nA\overline{v}$$

$$A = \pi R^2$$



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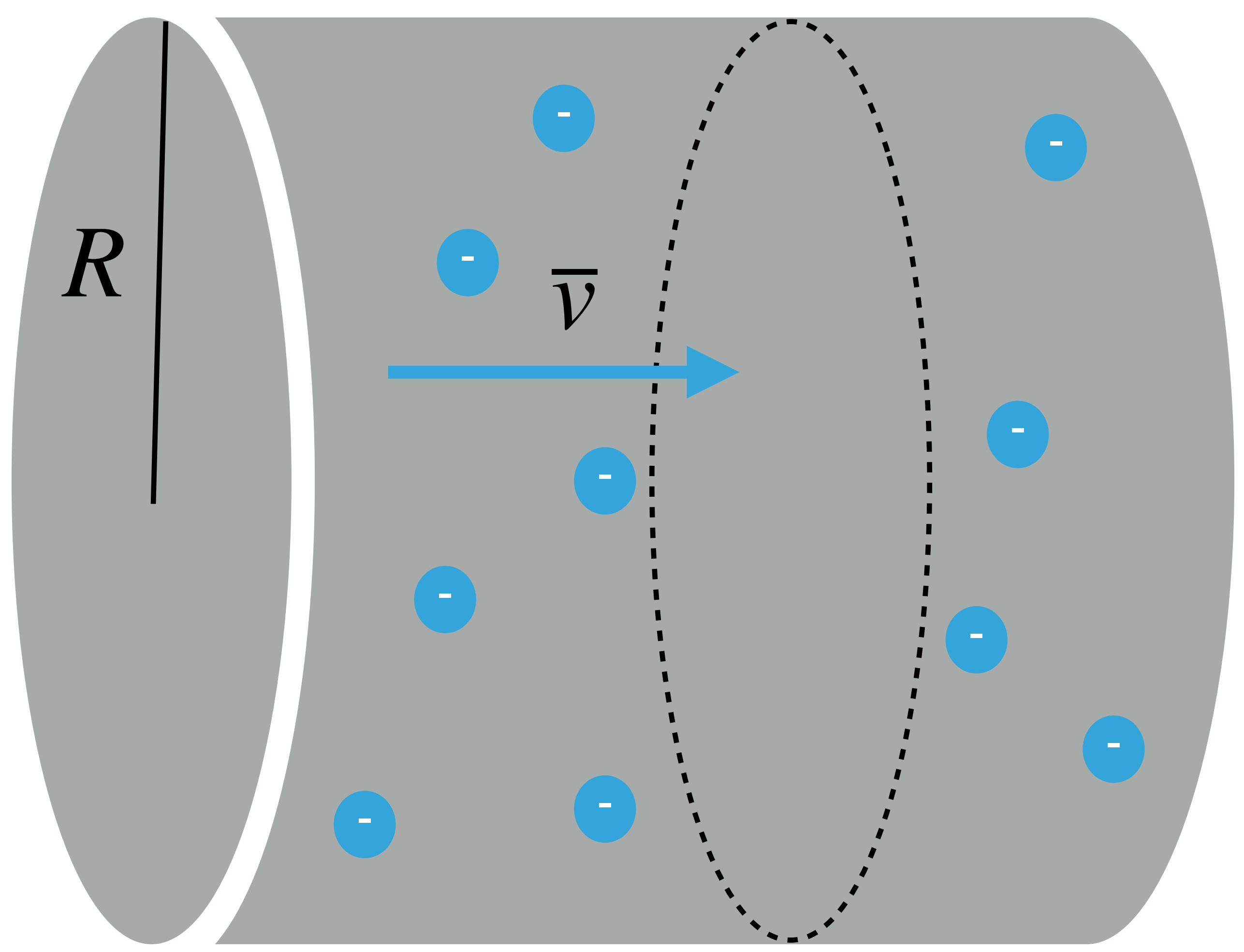


To increase current:

- Use a different wire (thicker and/or higher electron density)
  - $n_{Cu} \approx 8.5 \times 10^{28}\ \dfrac{e^-}{\text{m}^2}$
  - $n_{Al} \approx 17 \times 10^{28}\ \dfrac{e^-}{\text{m}^2}$

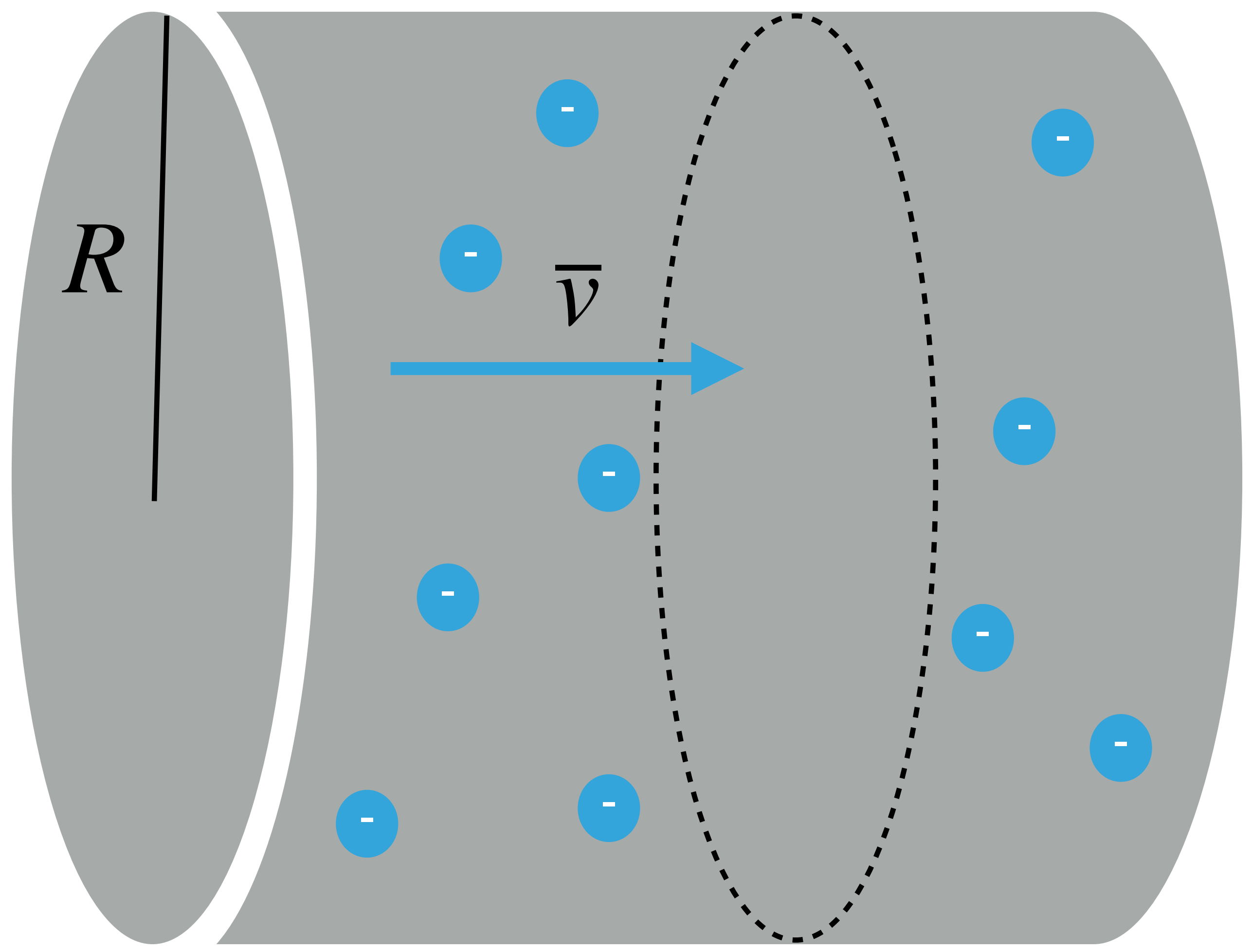
WHAT DRIVES THE CURRENT?

$i = nA\overline{v}$

$A = \pi R^2$

To increase current *through the same wire:*

- Have to increase $\overline{v}$



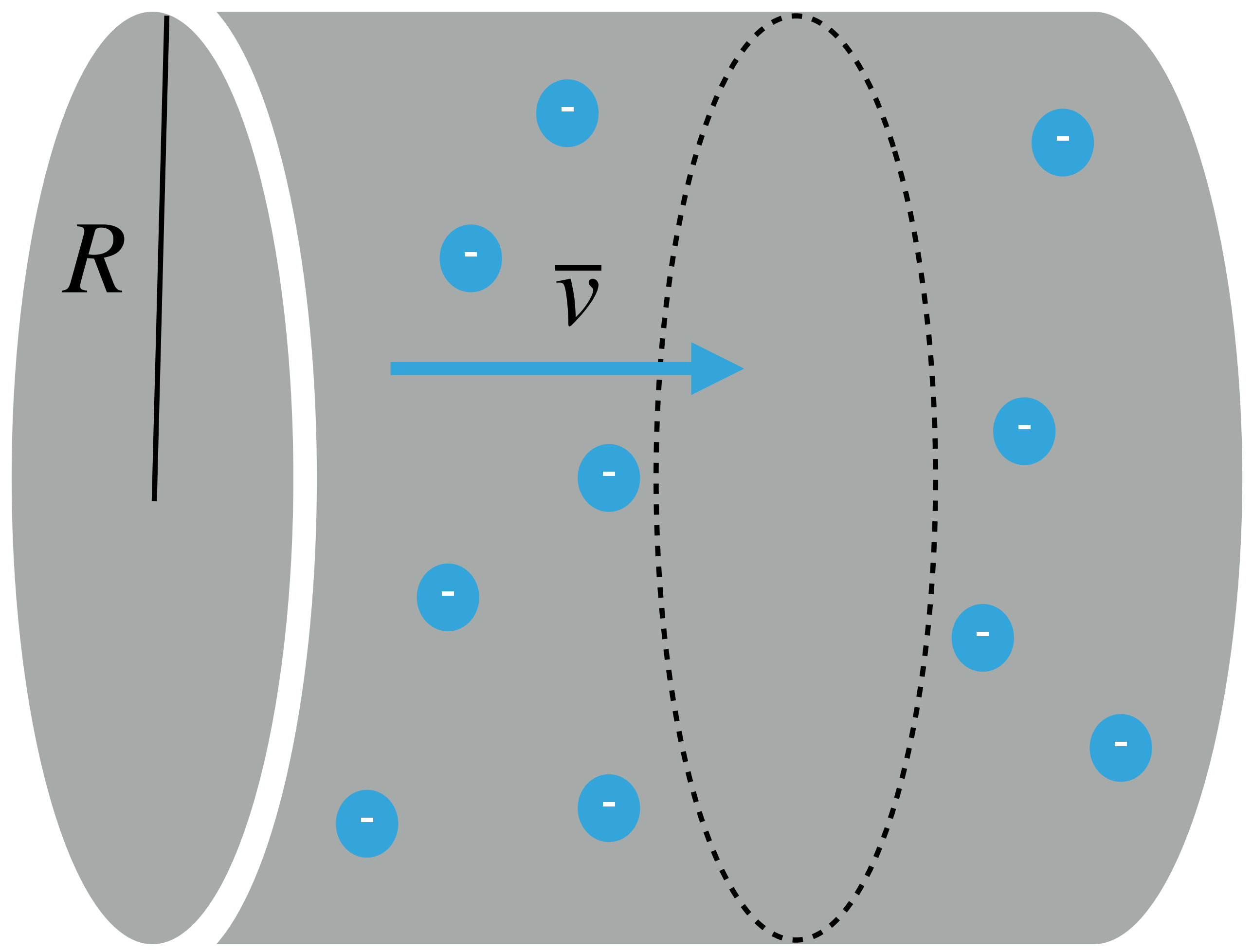
WHAT DRIVES THE CURRENT?

$i = nA\overline{v}$

$A = \pi R^2$

To increase current *through the same wire:*

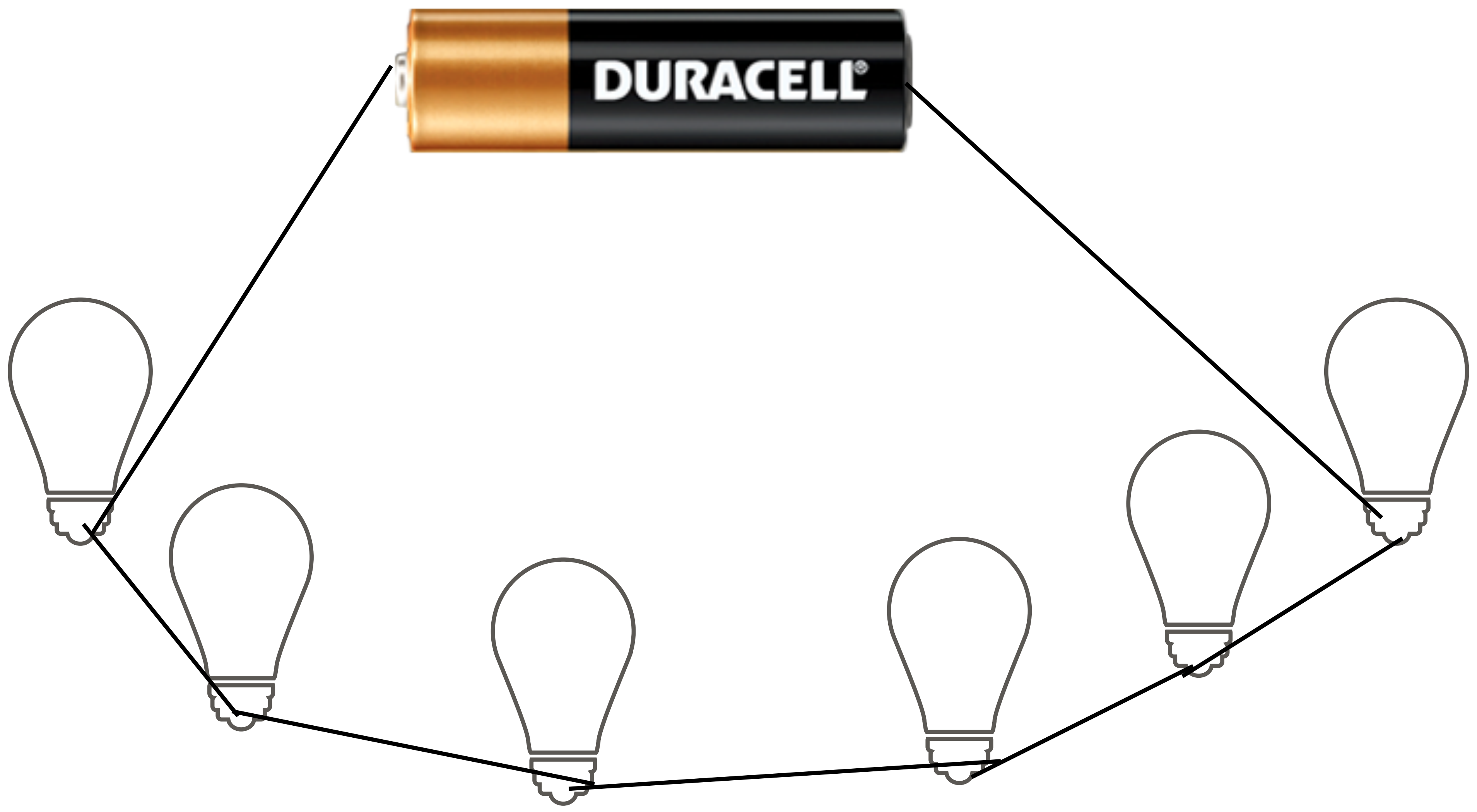
- Have to increase $\overline{v}$
- $\overline{v} = uE$
  - E drives the current



WHAT IS "USED UP" IN A LIGHTBULB

- Current is the same before and after a lightbulb
- Current determined by E
- It is the electron's energy that is used up in order to power the bulb

- If current only depends on E, and E comes from the battery (and surface charges), what is stopping us from powering infinitely many lightbulbs with a single battery?



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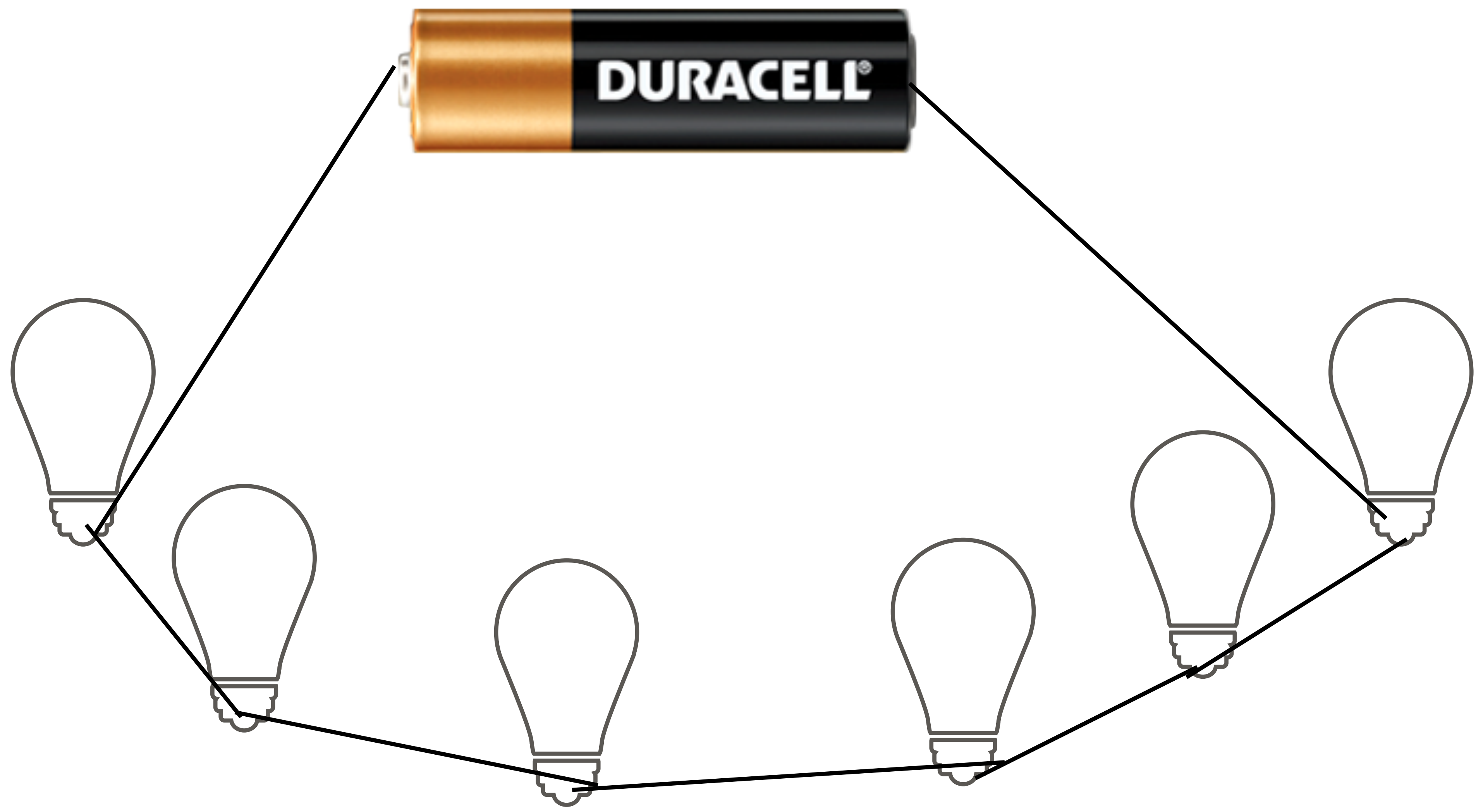
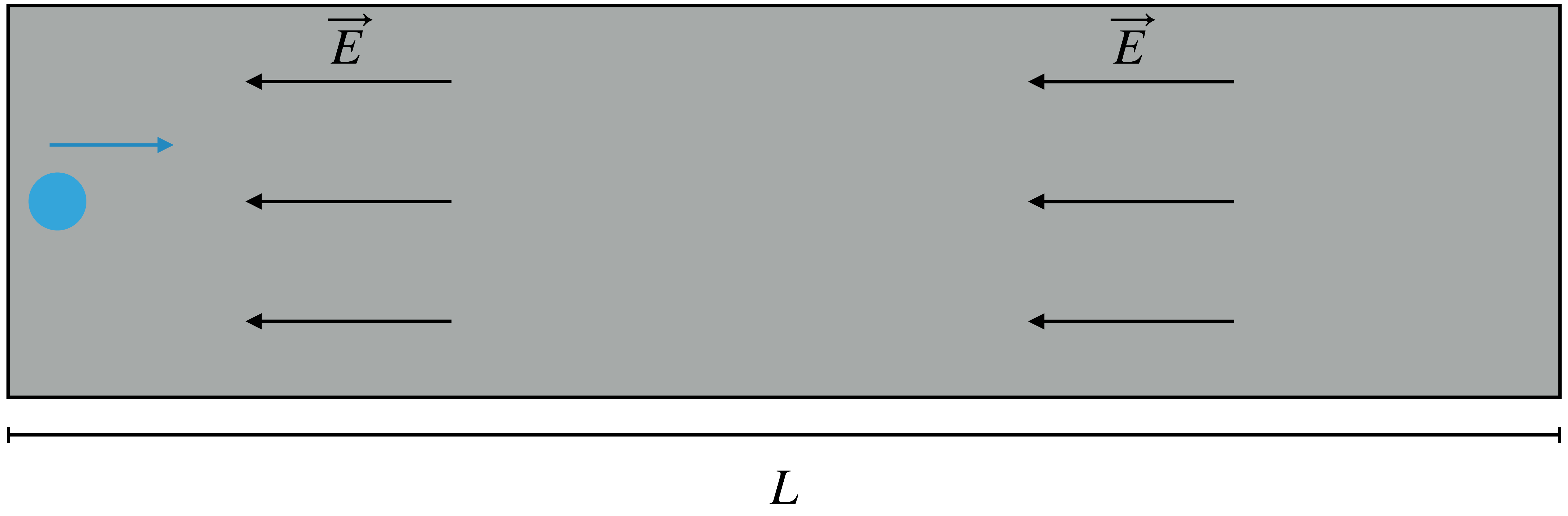


ENERGY CONSERVATION!

ENERGY LOST IN A PIECE OF WIRE

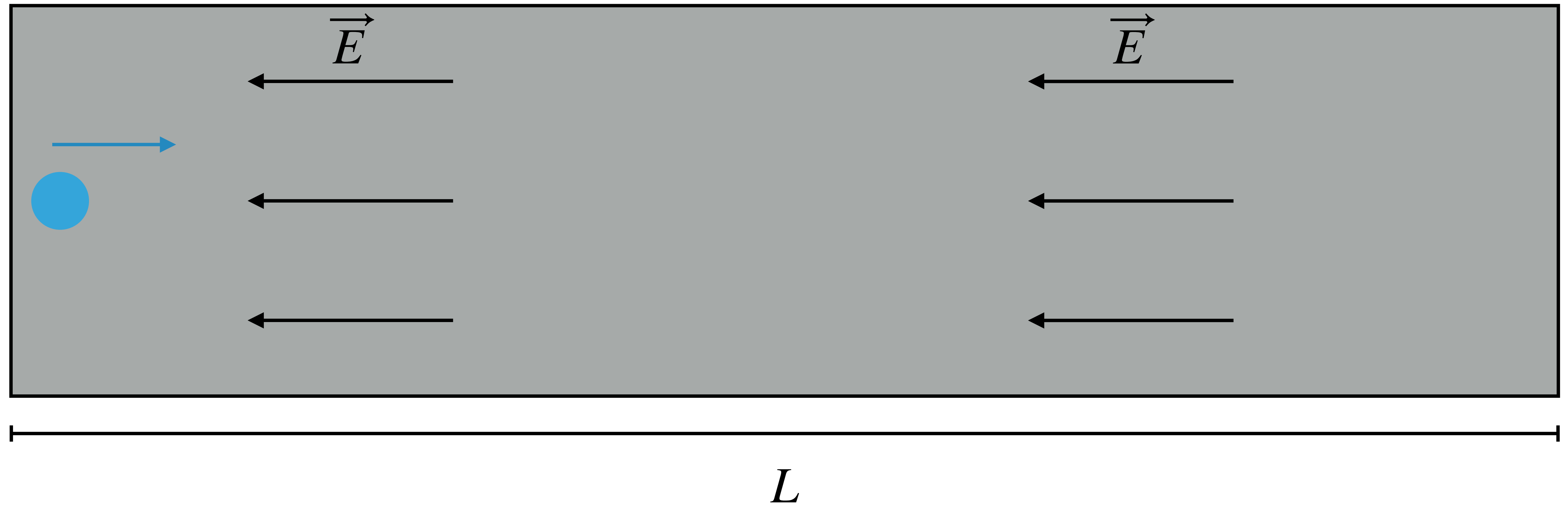
- Change in electron's energy along the wire?



ENERGY LOST IN A PIECE OF WIRE

- Change in electron's energy along the wire?

$$\Delta U = -eEL$$

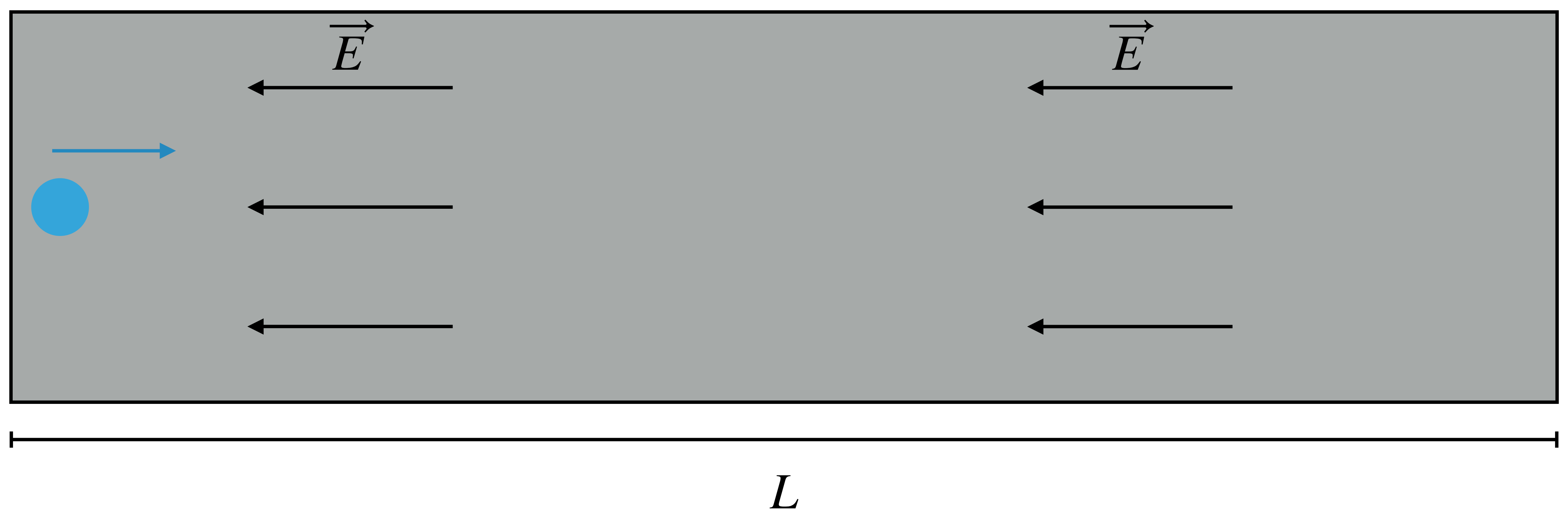


ENERGY LOST IN A PIECE OF WIRE

- Change in electron's energy along the wire?

$\Delta U = -eEL$

What happens to ΔK?



ENERGY LOST IN A PIECE OF WIRE

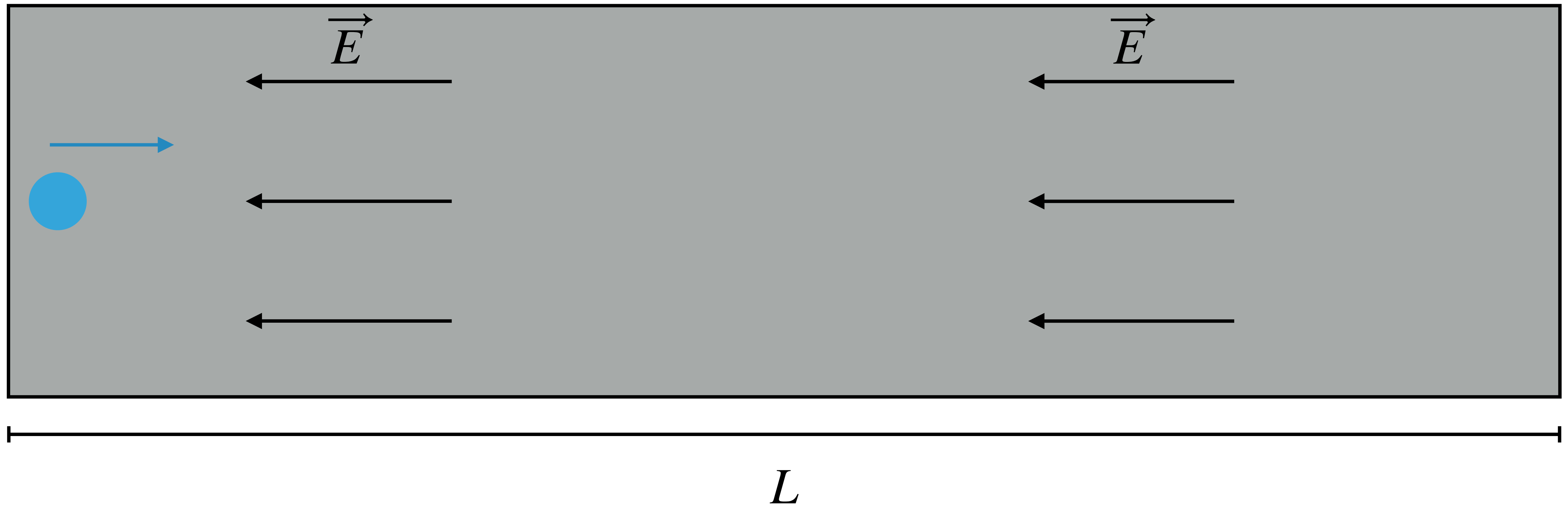
- Change in electron’s energy along the wire?

$\Delta U = -e\Delta V_{\text{wire}} = -eEL$

What happens to ΔK?

In steady state, $\overline{v}$ doesn’t change

Kinetic energy also doesn’t change!



ENERGY LOST IN A PIECE OF WIRE

- Change in electron's energy along the wire?

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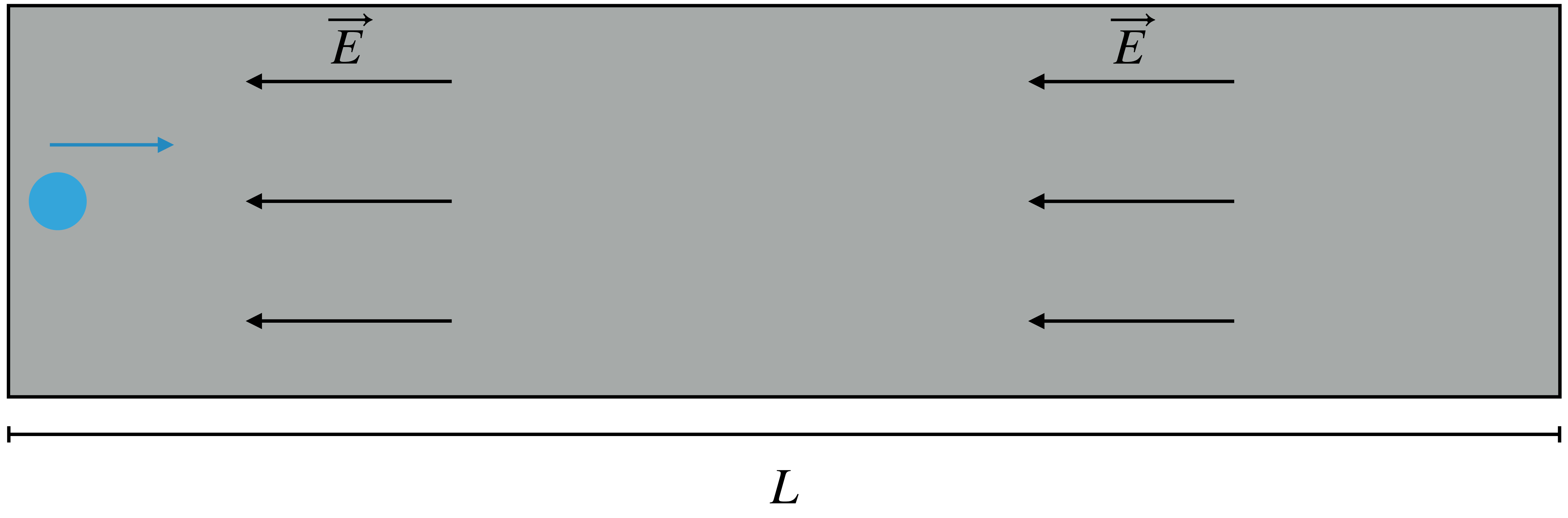
In steady state, $\overline{v}$ doesn't change

Kinetic energy also doesn't change!

ENERGY GAINED BY ELECTRIC FIELD IS LOST IN COLLISIONS WITHIN THE WIRE (RAISES WIRE TEMPERATURE)



THE “LOOP RULE”

- In steady-state:

$$\Delta V_1 + \Delta V_2 + \Delta V_3 + \ldots = 0$$

Consequence of energy conservation

THE EMF OF A BATTERY

- The energy input per charge of a battery is called the "electromotive force"
    - Not actually a force, name sticks around anyway
    - Will abbreviate by "emf" or using the variable ε
- For a perfect battery (no energy lost to collisions across the battery):
    - $\Delta V_{\text{battery}} = \varepsilon$

USING THE LOOP RULE

- How to apply the loop rule to a circuit

1. Pick a starting point

USING THE LOOP RULE

- How to apply the loop rule to a circuit

1. Pick a starting point
2. Pick a loop direction

USING THE LOOP RULE

- How to apply the loop rule to a circuit

1. Pick a starting point
2. Pick a loop direction
3. Calculate ΔV in the chosen direction and sum

ANALYZING CIRCUITS

Start with fundamental principles and write down set of equations

1. Node rule $i_{\mathrm{in}} = i_{\mathrm{out}}$
2. Loop rule $\sum_{\mathrm{loop}} \Delta V = 0$

ANALYZING CIRCUITS

Start with fundamental principles and write down set of equations

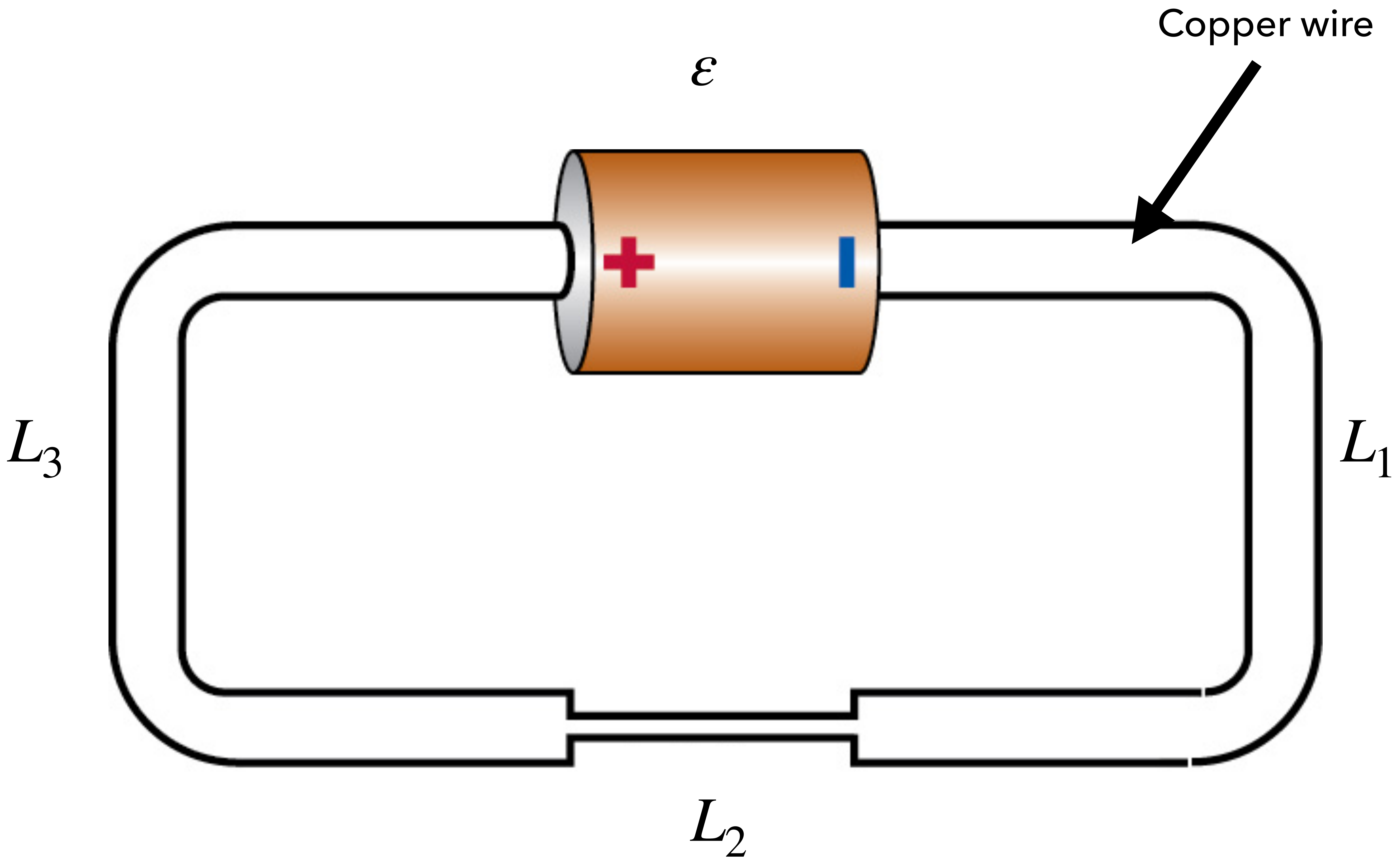
1. Node rule $i_{\text{in}} = i_{\text{out}}$
2. Loop rule $\sum_{\text{loop}} \Delta V = 0$

Now solve system of equations for quantities you want.

Remember:

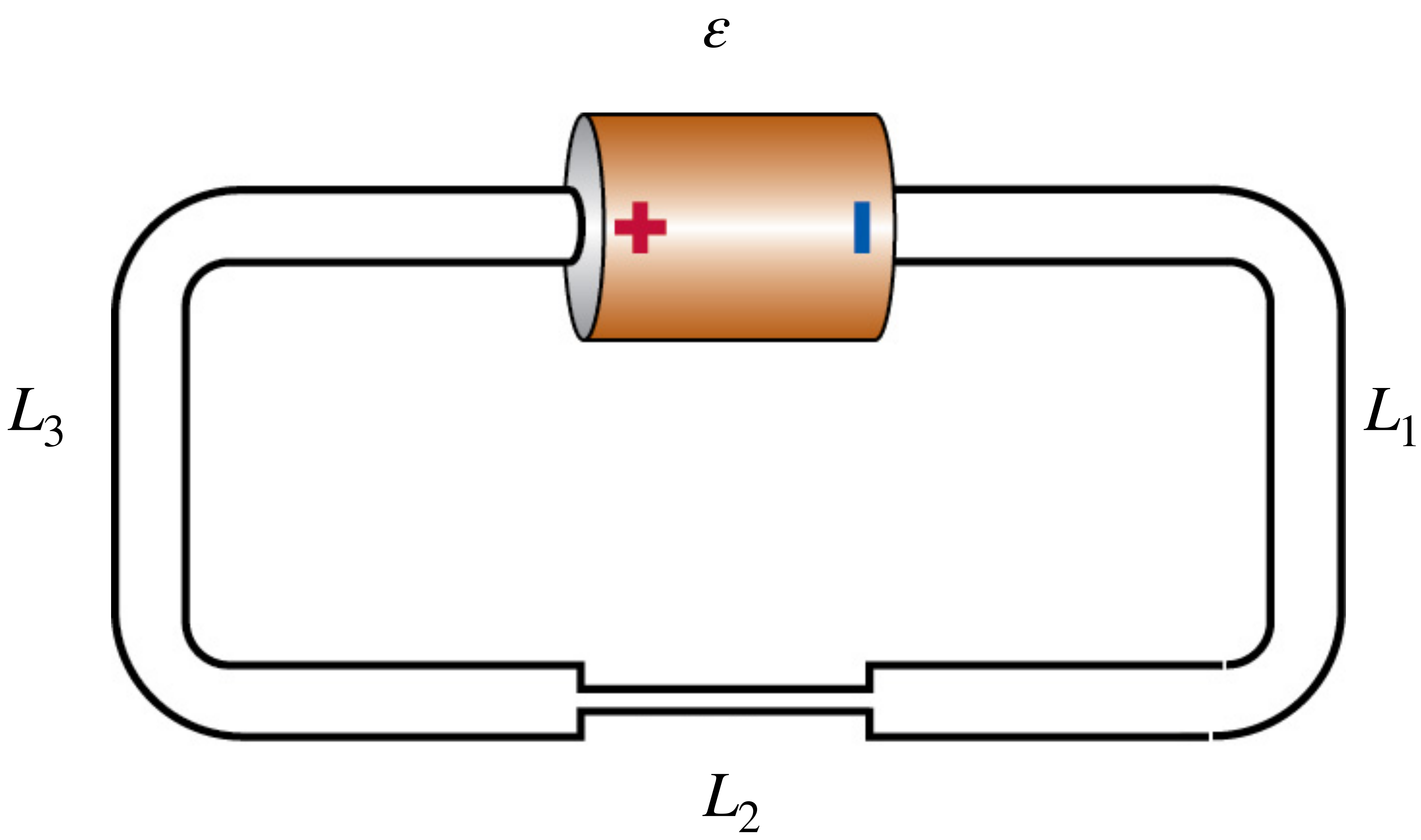
- $i = nA\bar{v}$
- $\bar{v} = uE$

EXAMPLE



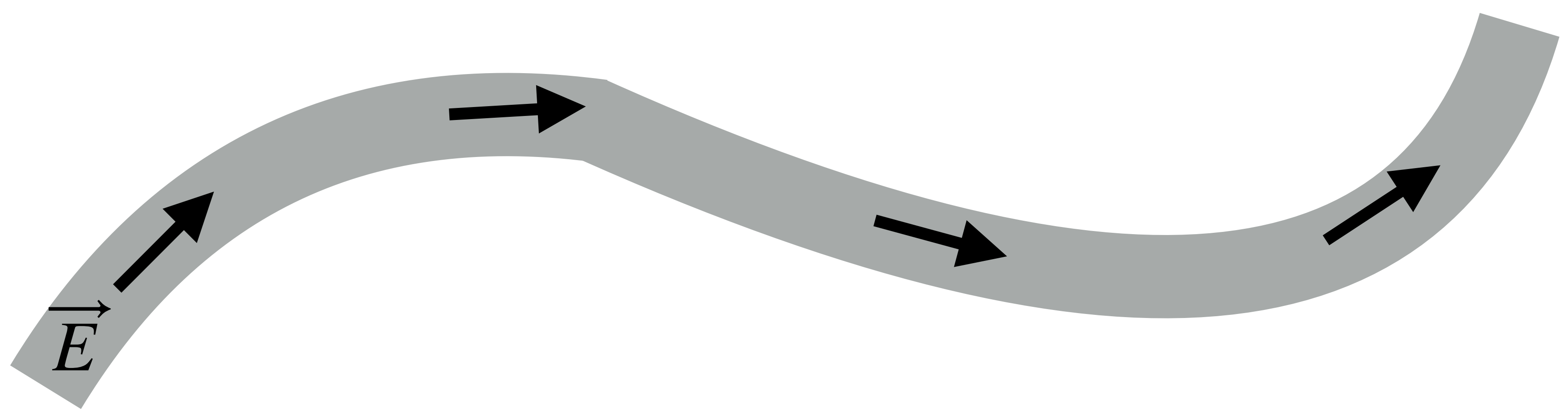
EXAMPLE

Find the field and current in every part of the circuit



USING THE LOOP RULE

ΔV along a "bendy" wire of length L



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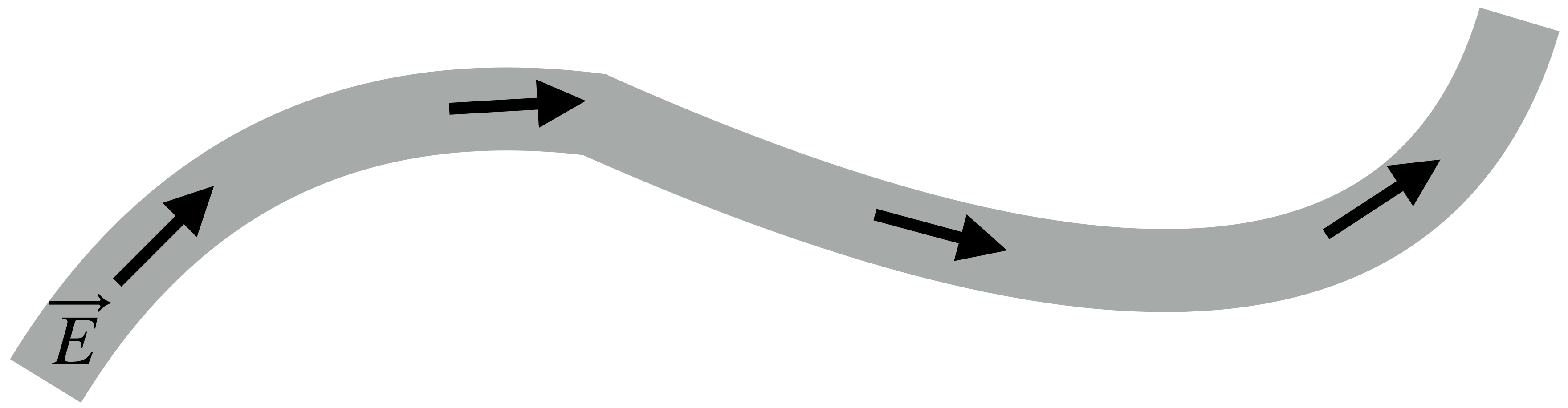
Note: $\vec{E}$ is **always** parallel to the wire



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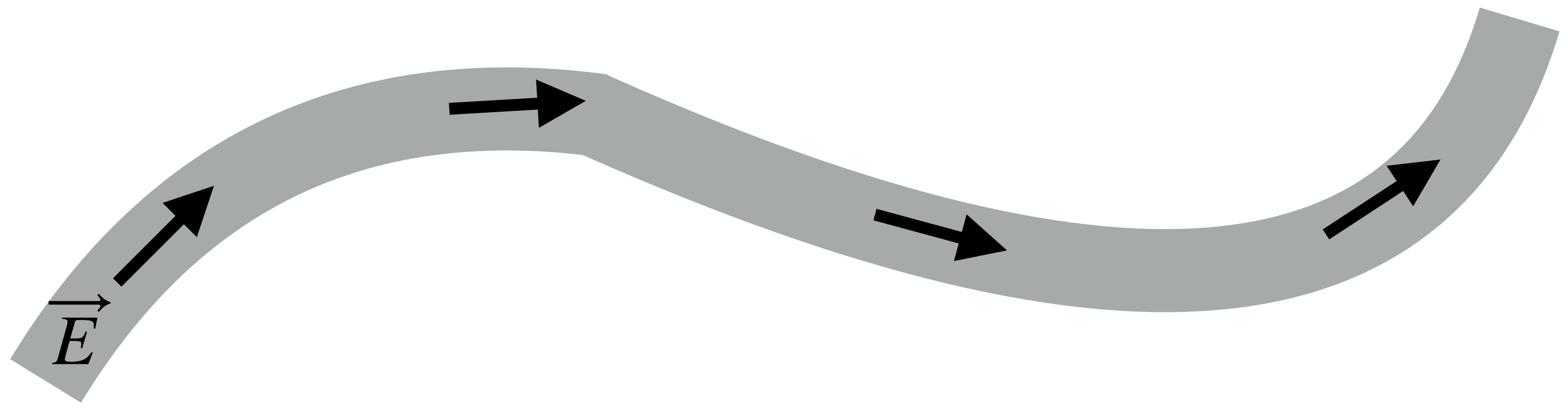

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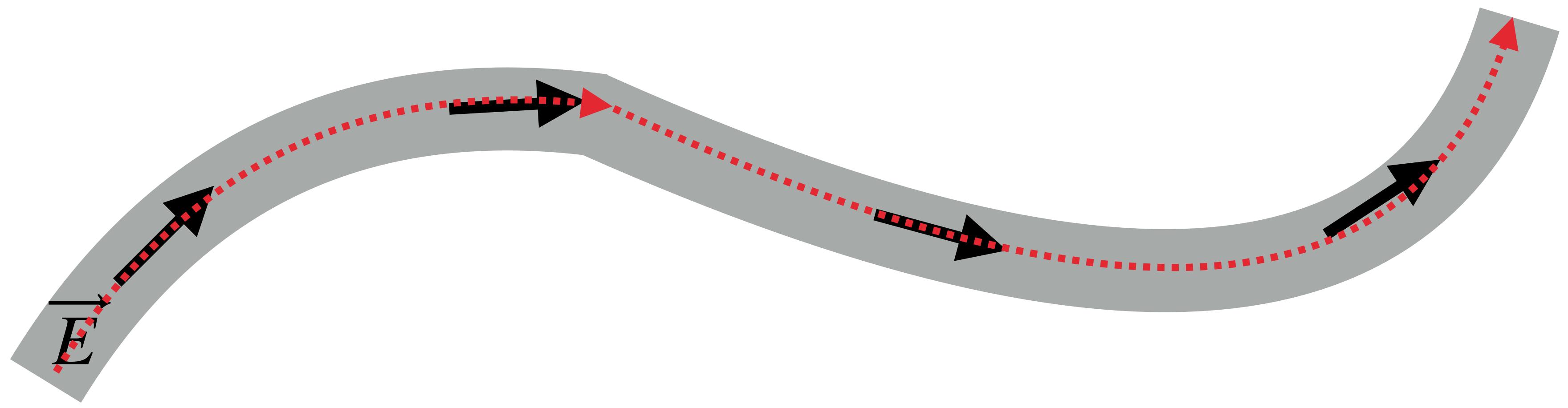

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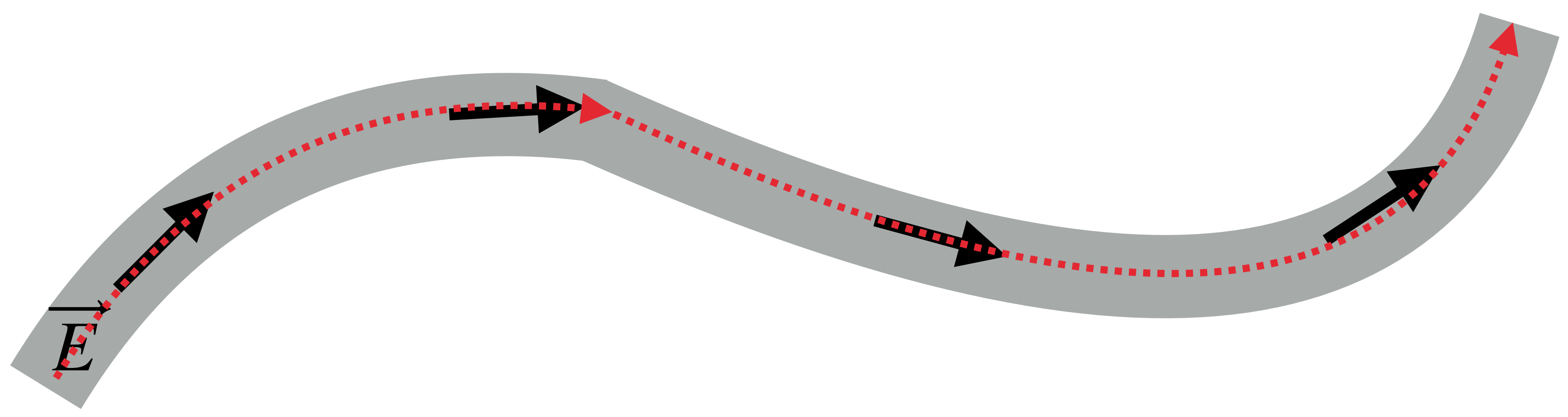
$$\left|\overrightarrow{E} \cdot \Delta\vec{r}\right| = EL$$

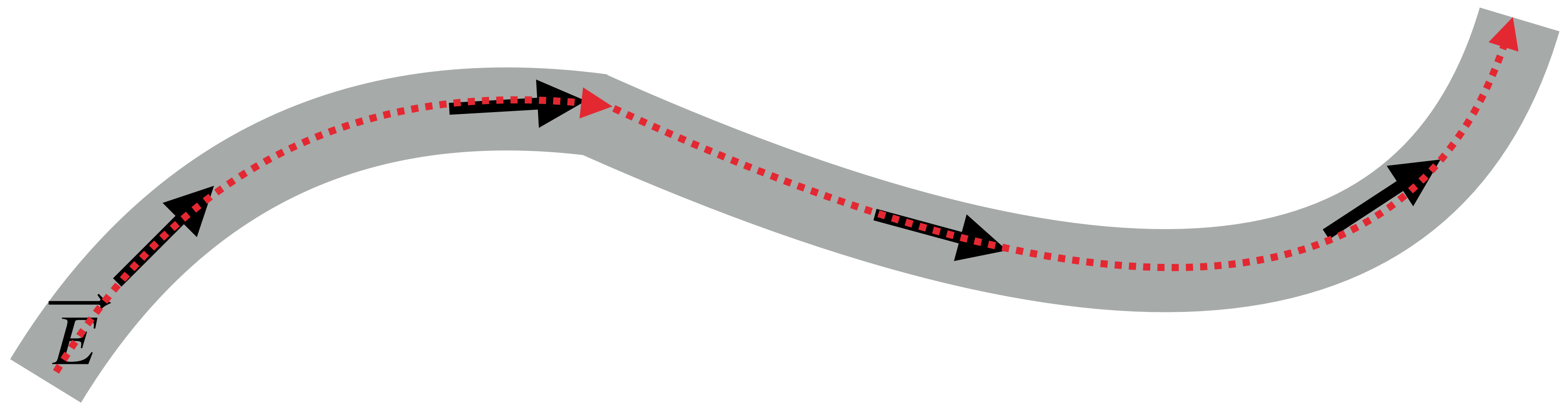


USING THE LOOP RULE

ΔV along a "bendy" wire of length L

$\Delta V = \pm EL$

Sign depends on path direction



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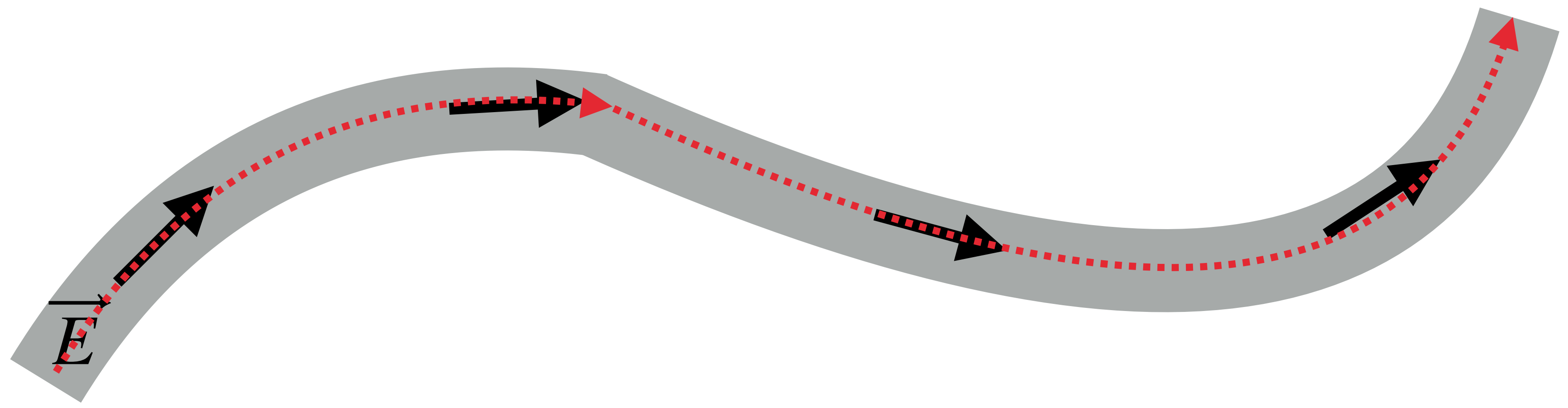
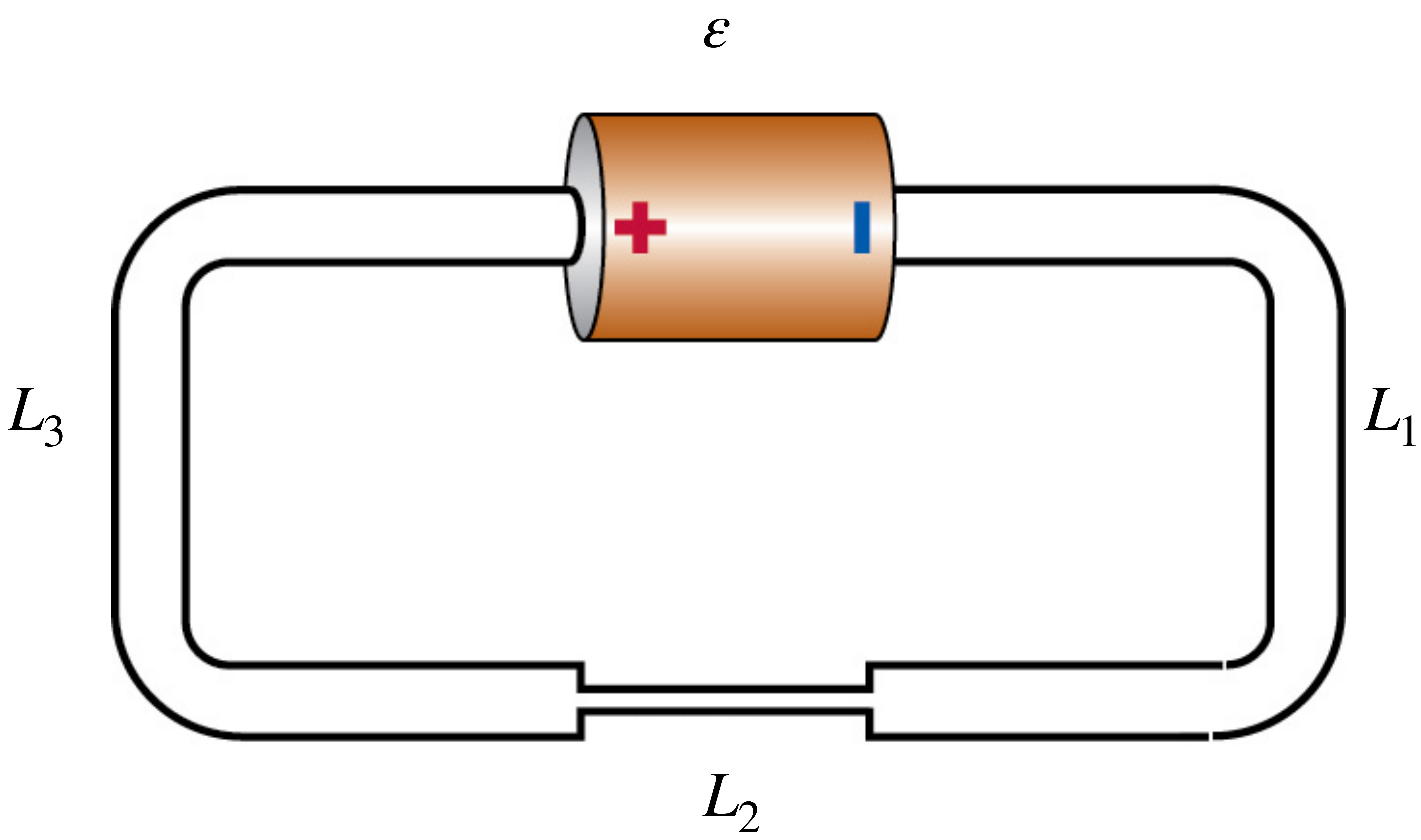


Path in *same* direction as $\vec{E}$: $\Delta V = -\, EL$

Path in *opposite* direction: $\Delta V = EL$

EXAMPLE

$$\varepsilon - E_3 l_3 - E_2 L_2 - E_1 L_1 = 0$$

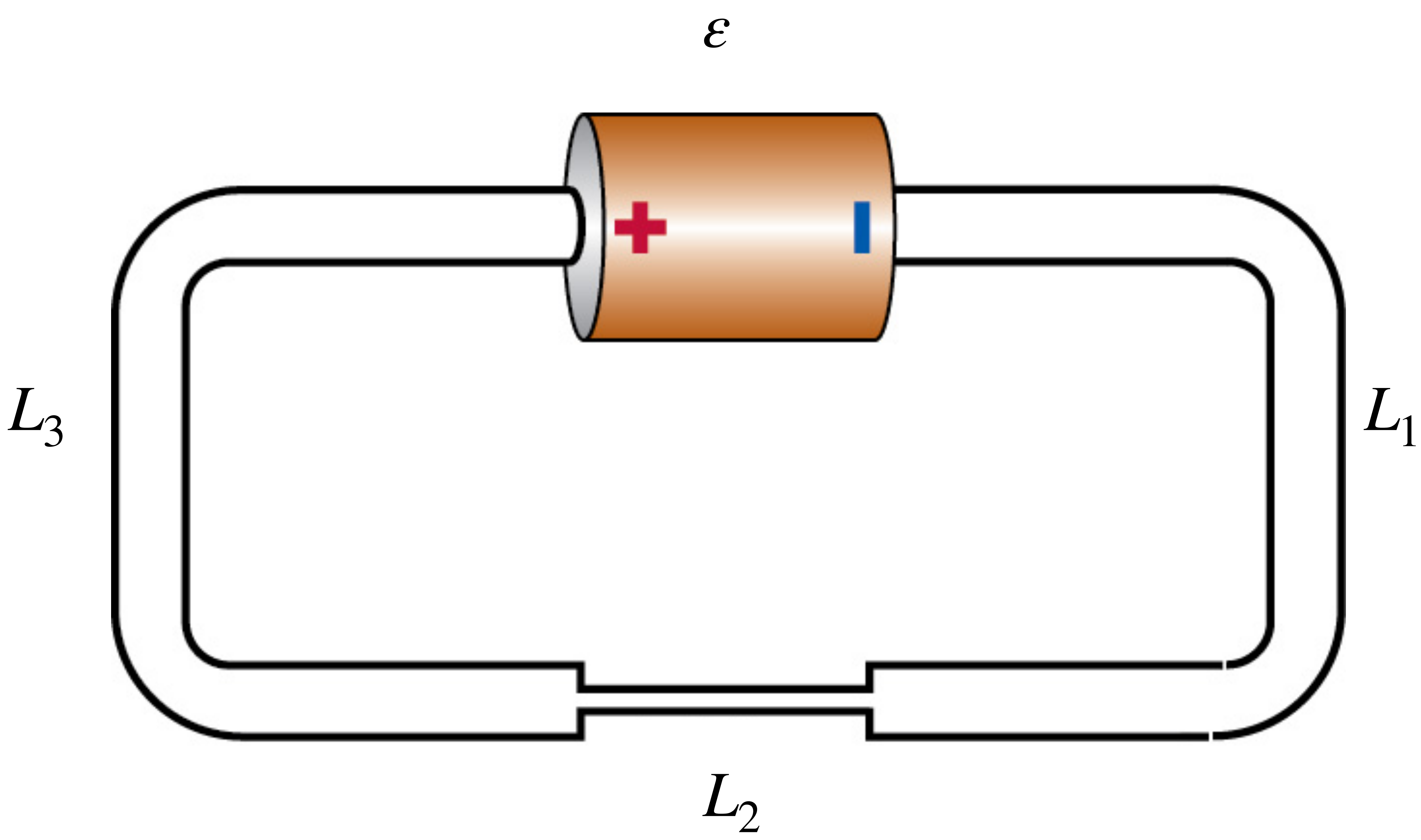


EXAMPLE

$$\varepsilon - E_3 l_3 - E_2 L_2 - E_1 L_1 = 0$$

$$i_1 = i_2$$

$$i_2 = i_3$$

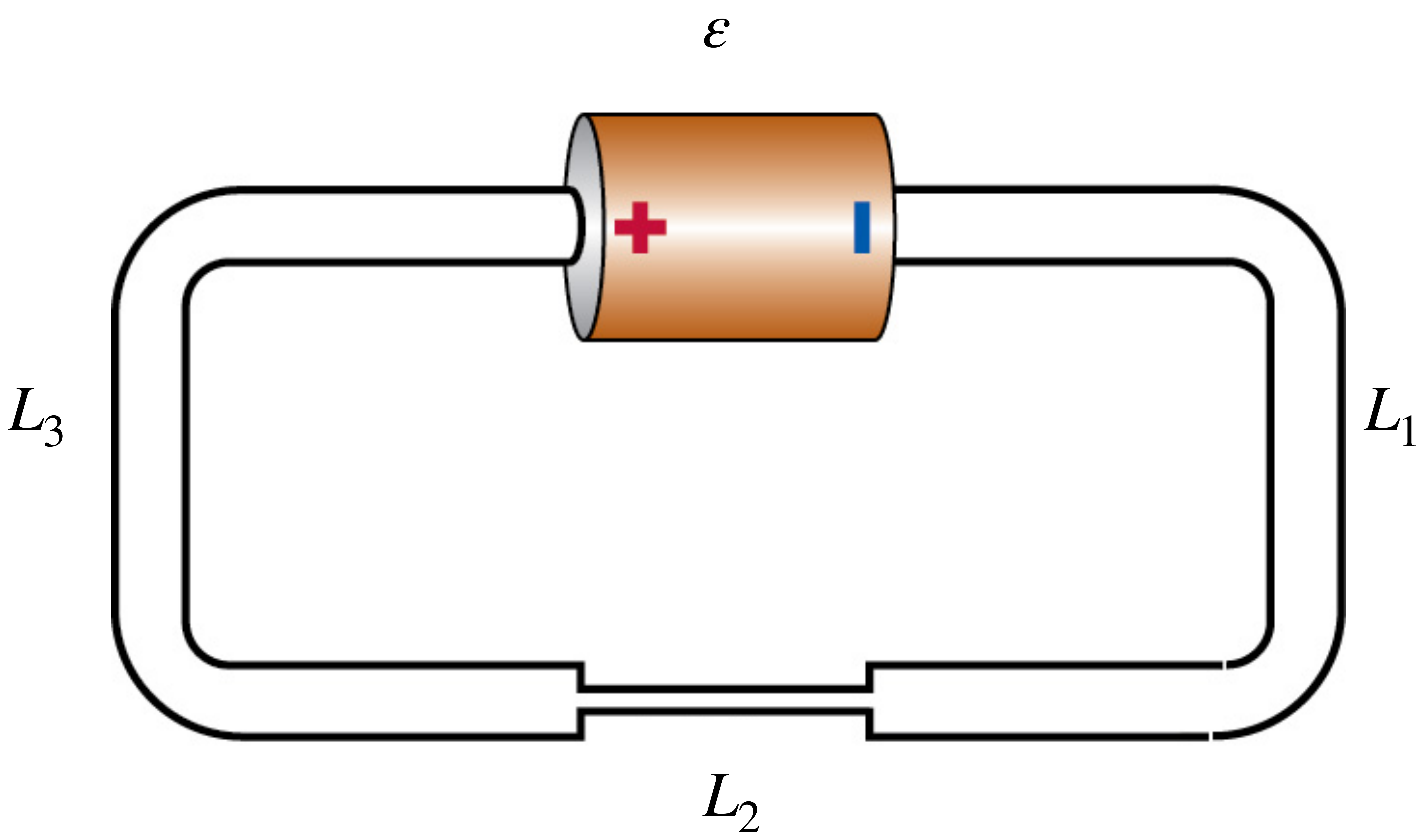


EXAMPLE

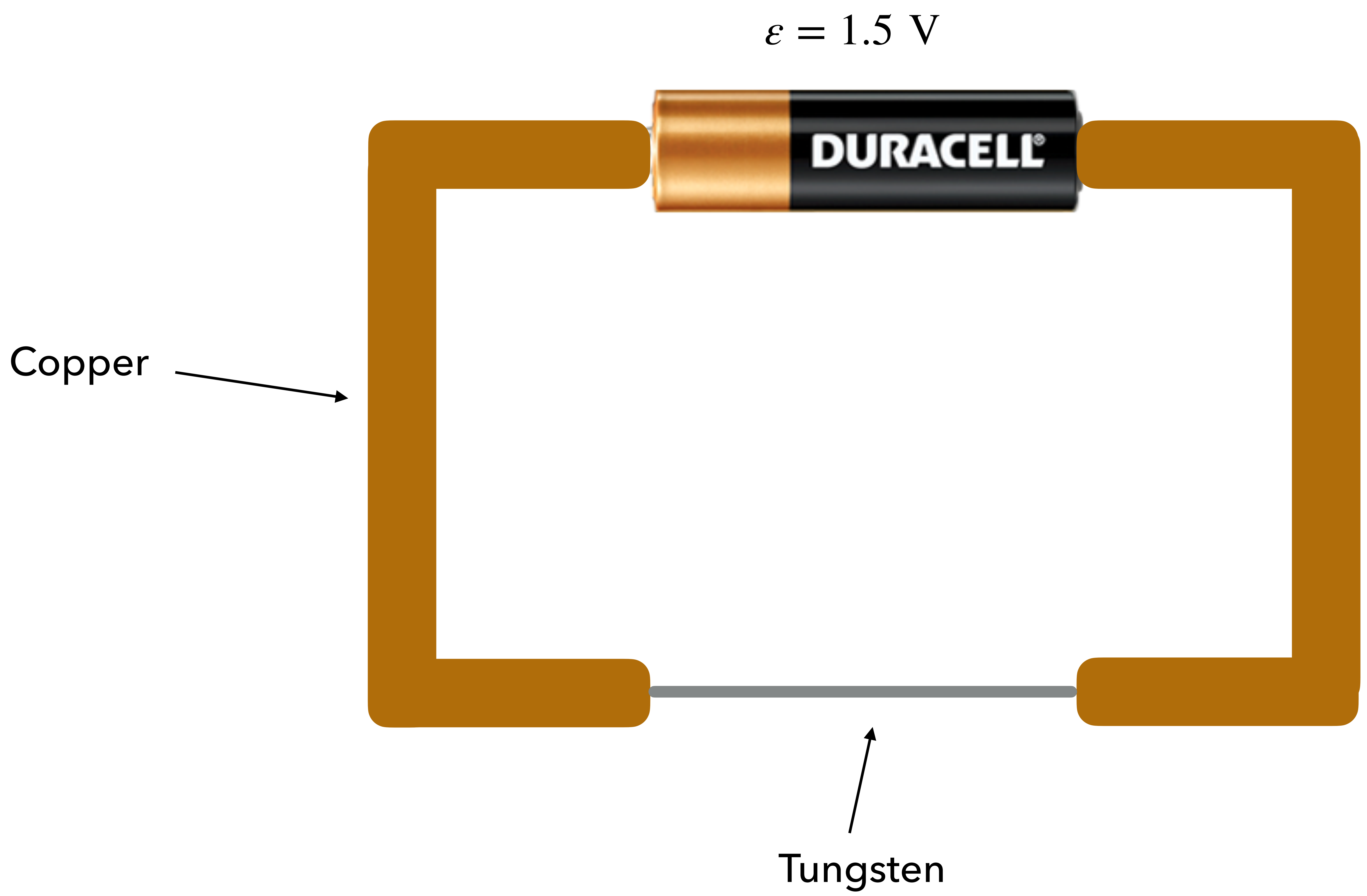
$$\varepsilon - E_3 l_3 - E_2 L_2 - E_1 L_1 = 0$$

$$n_1 A_1 u_1 E_1 = n_2 A_2 u_2 E_2$$

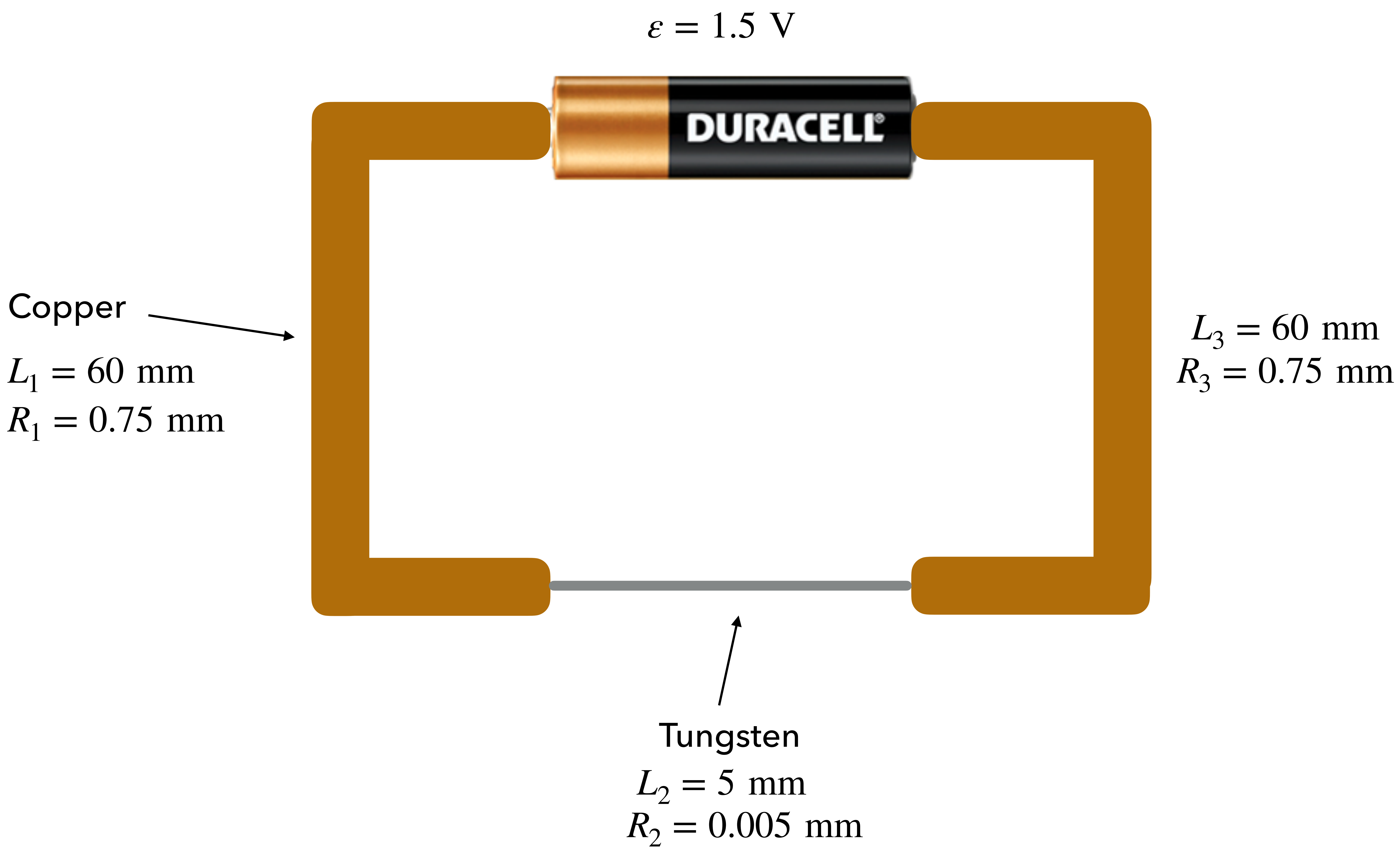
$$n_2 A_2 u_2 E_2 = n_3 A_3 u_3 E_3$$



EXAMPLE



EXAMPLE

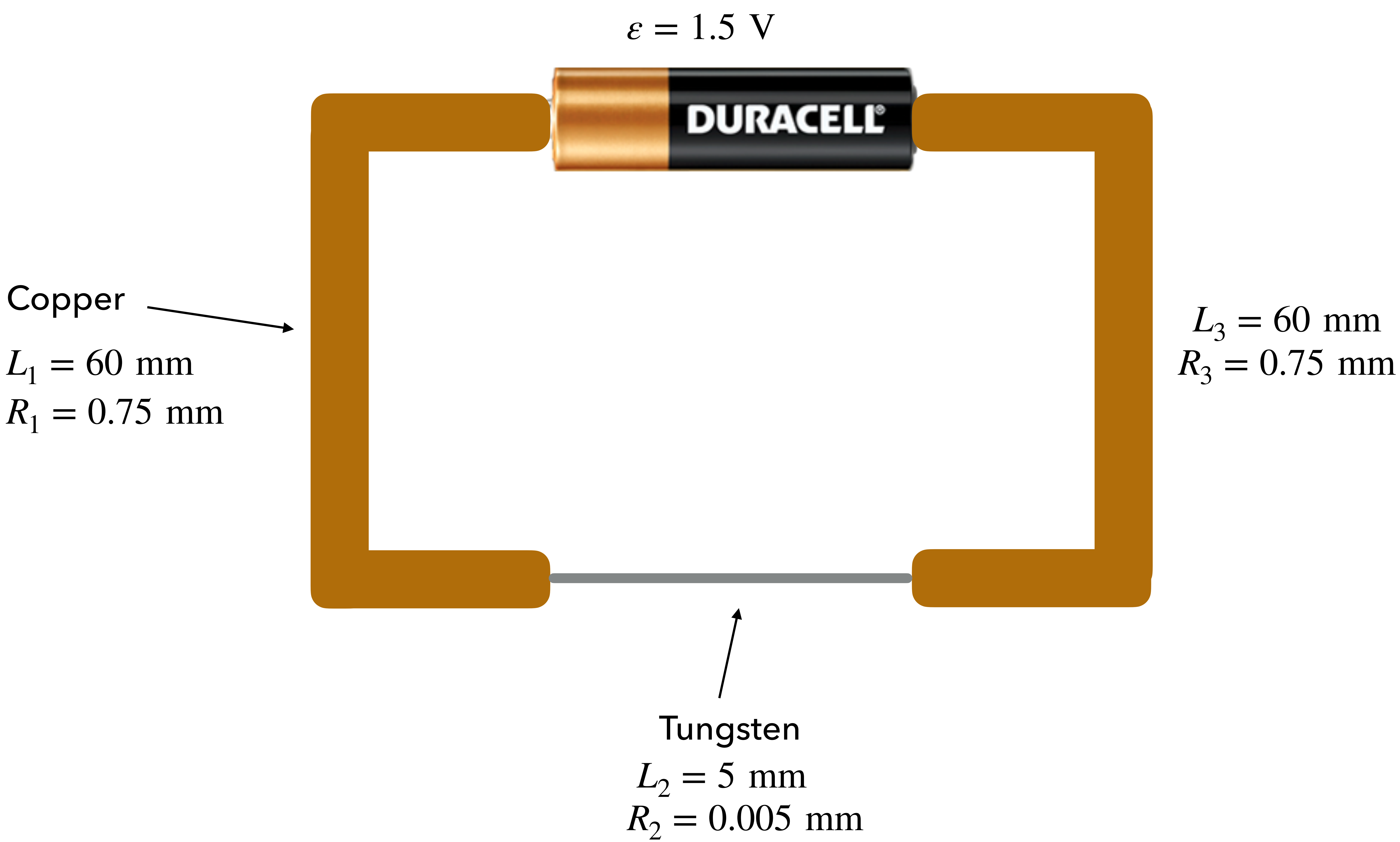


$\varepsilon = 1.5\ \text{V}$
DURACELL
Copper
$L_1 = 60\ \text{mm}$
$R_1 = 0.75\ \text{mm}$
$L_3 = 60\ \text{mm}$
$R_3 = 0.75\ \text{mm}$
Tungsten
$L_2 = 5\ \text{mm}$
$R_2 = 0.005\ \text{mm}$

EXAMPLE

Loop Rule:

$$\varepsilon - E_1L_1 - E_2L_2 - E_3L_3 = 0$$



EXAMPLE

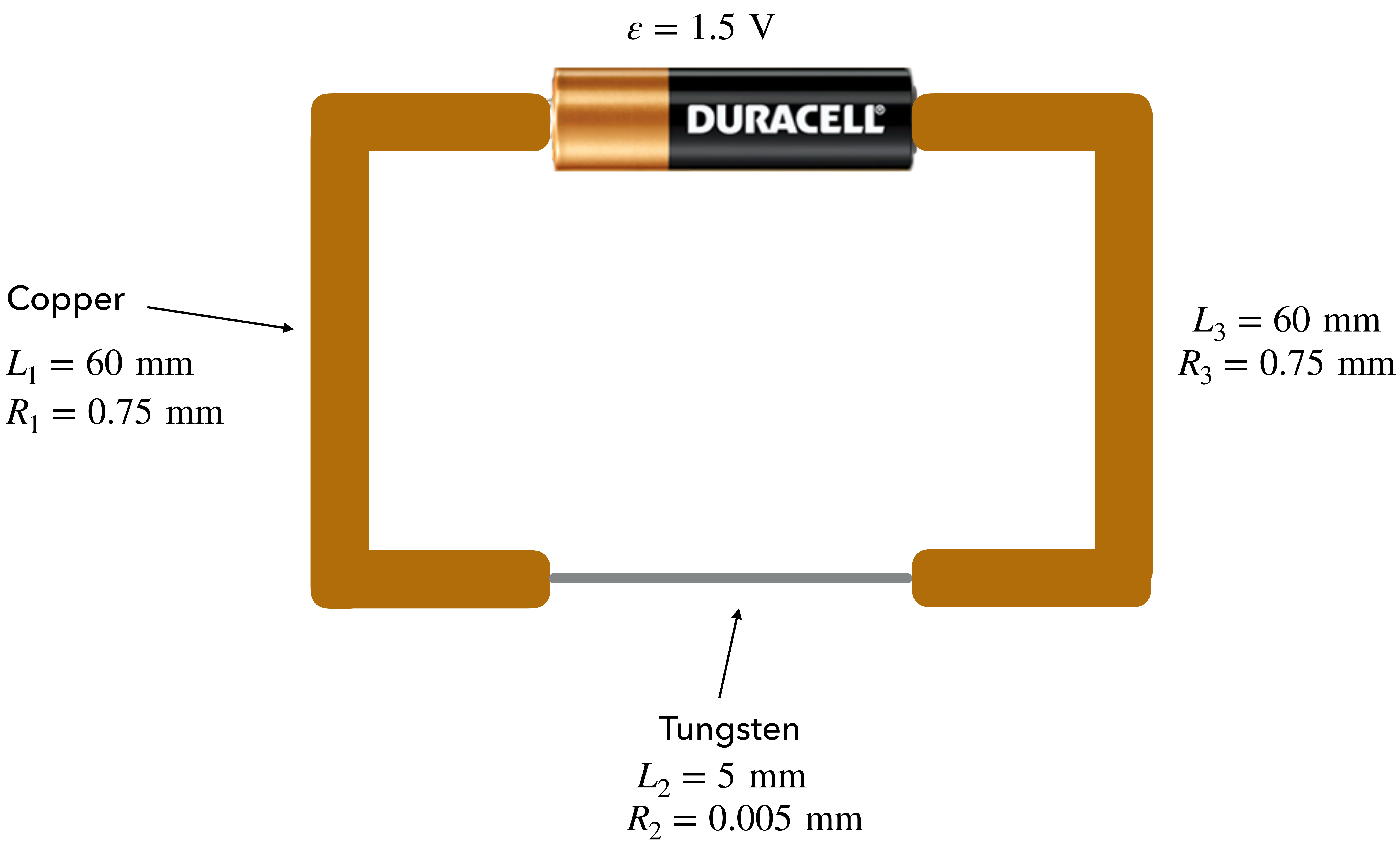
Loop Rule:

$$\varepsilon - E_1L_1 - E_2L_2 - E_3L_3 = 0$$

Node Rule:

$$n_1A_1u_1E_1 = n_2A_2u_2E_2$$

$$n_2A_2u_2E_2 = n_3A_3u_3E_3$$

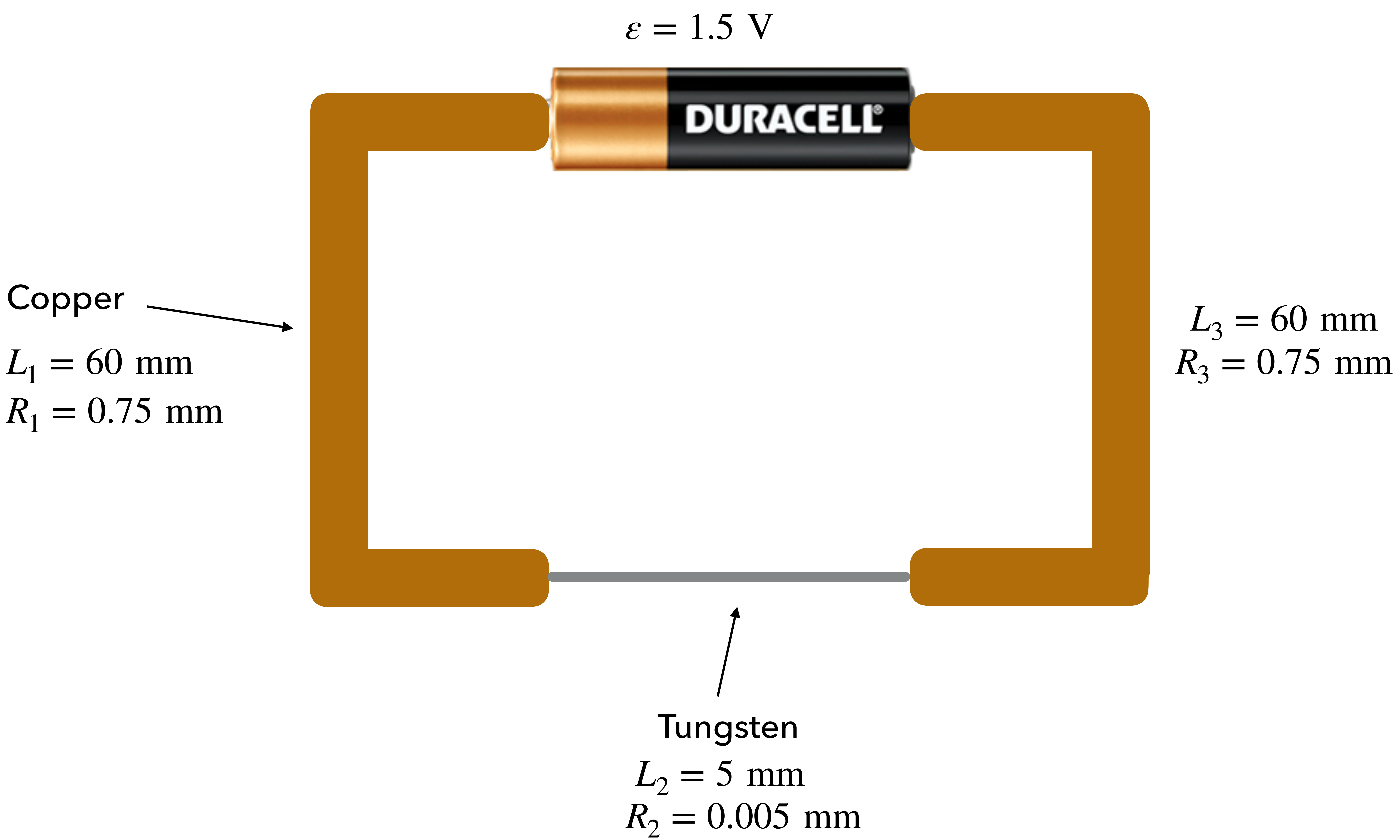


EXAMPLE

$$E_1 = \frac{\varepsilon}{L_1 + L_3 + \frac{n_1 A_1 u_1}{n_2 A_2 u_2} L_2}$$

$$E_3 = E_1$$

$$E_2 = \frac{n_1 A_1 u_1}{n_2 A_2 u_2} E_1$$



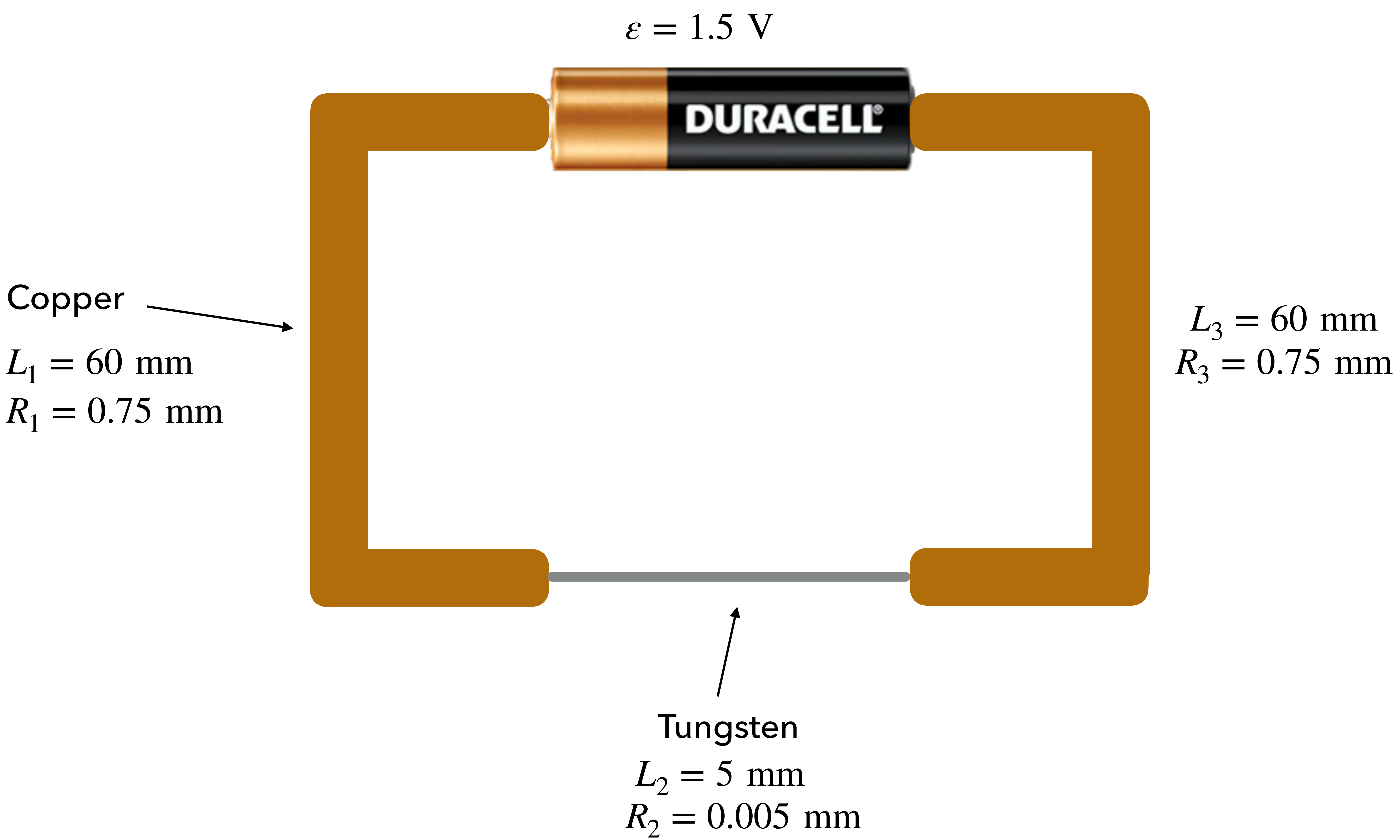
EXAMPLE

$$n_1 = n_{Cu} = 8.5 \times 10^{28}\ \mathrm{m^{-3}}$$

$$u_1 = u_{\mathrm{Cu}} = 4.5 \times 10^{-3}\ \frac{\mathrm{m/s}}{\mathrm{N/C}}$$

$$n_2 = n_W = 6 \times 10^{28}\ \mathrm{m^{-3}}$$

$$u_2 = u_W = 1.8 \times 10^{-3}\ \frac{\mathrm{m/s}}{\mathrm{N/C}}$$



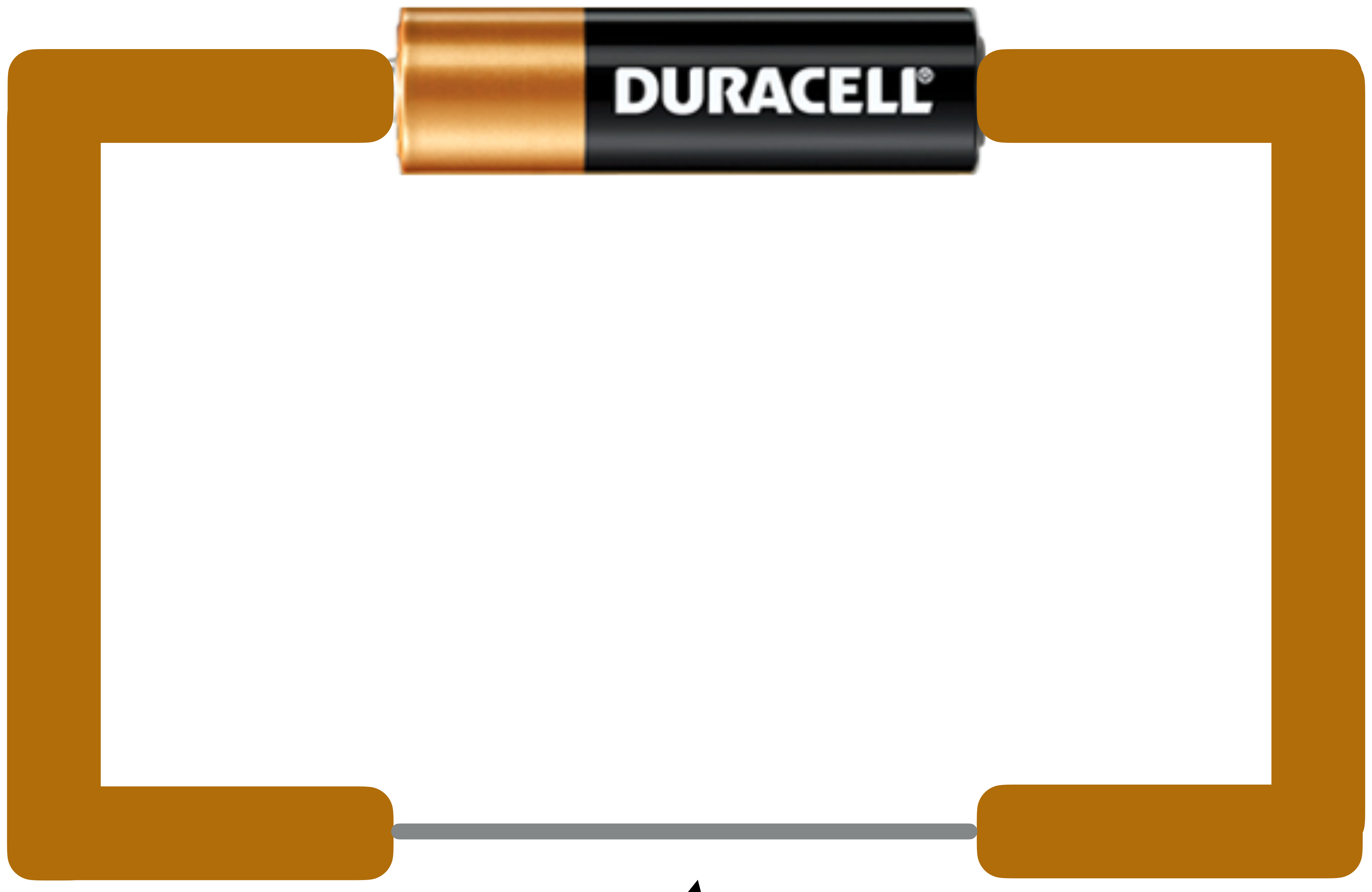
EXAMPLE

$$E_1 = 3.8 \times 10^{-3}\ \text{N/C}$$

$$E_3 = 3.8 \times 10^{-3}\ \text{N/C}$$

$$E_2 = 299.9\ \text{N/C}$$

ABOUT 80,000 TIMES STRONGER THAN E_1 OR E_2!

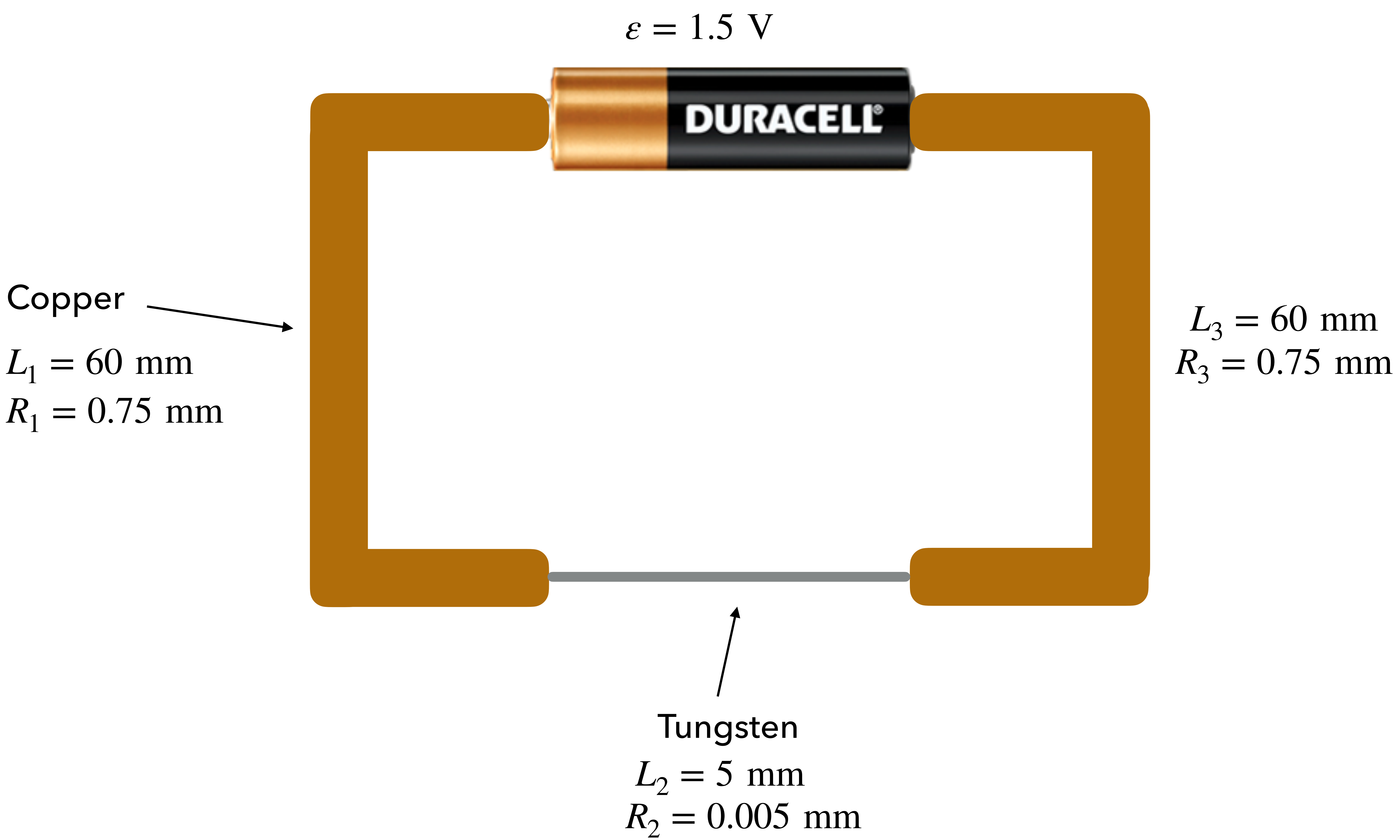


EXAMPLE

$$\Delta V_1 = E_1 L_1 = 2.2 \times 10^{-4} \text{ V}$$

$$\Delta V_3 = 2.2 \times 10^{-4} \text{ V}$$

$$\Delta V_3 = 1.49 \text{ V}$$



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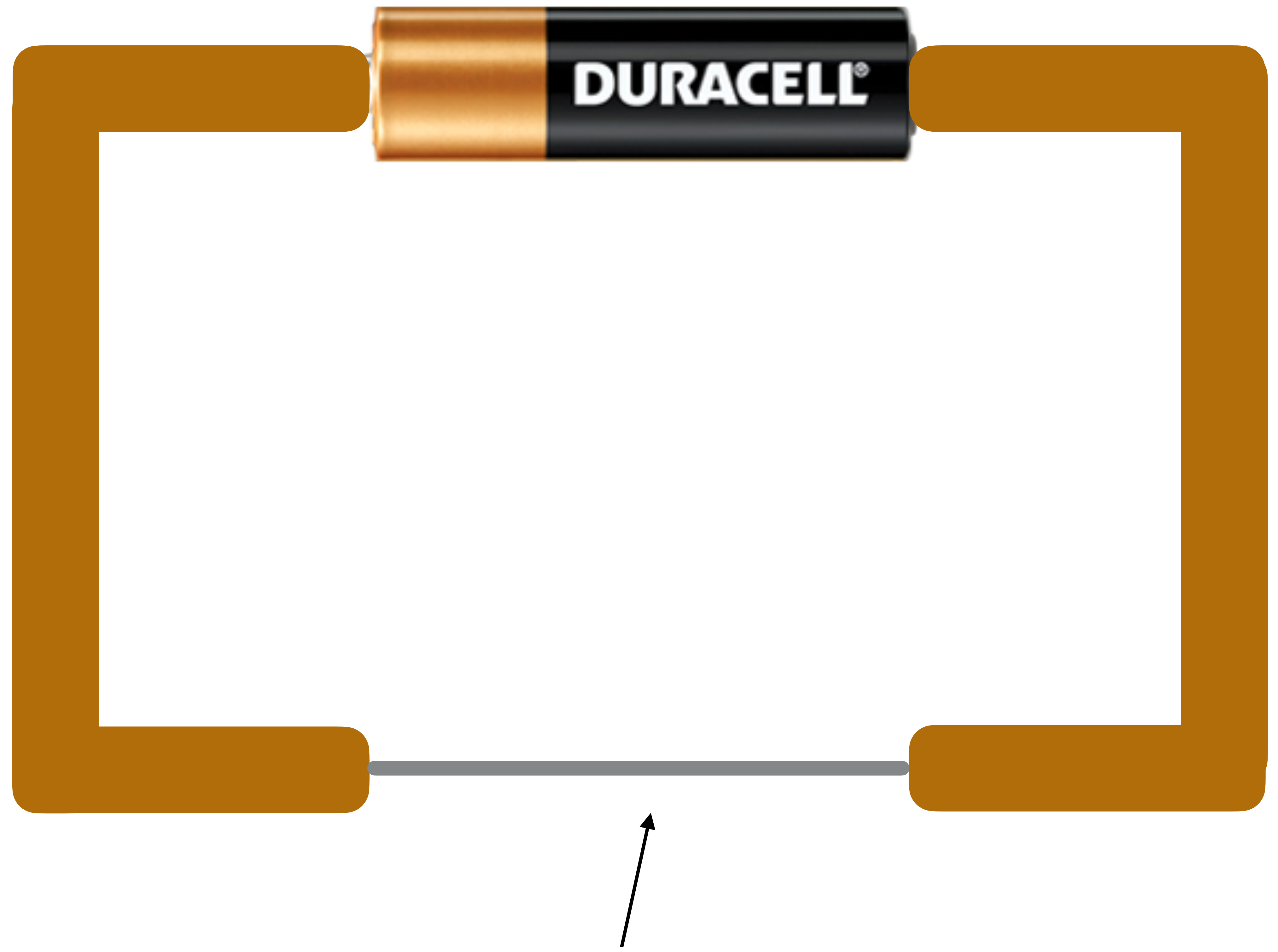

>99% OF THE BATTERY'S ENERGY IS USED TO DRIVE CURRENT THROUGH THE TUNGSTEN WIRE

EXAMPLE

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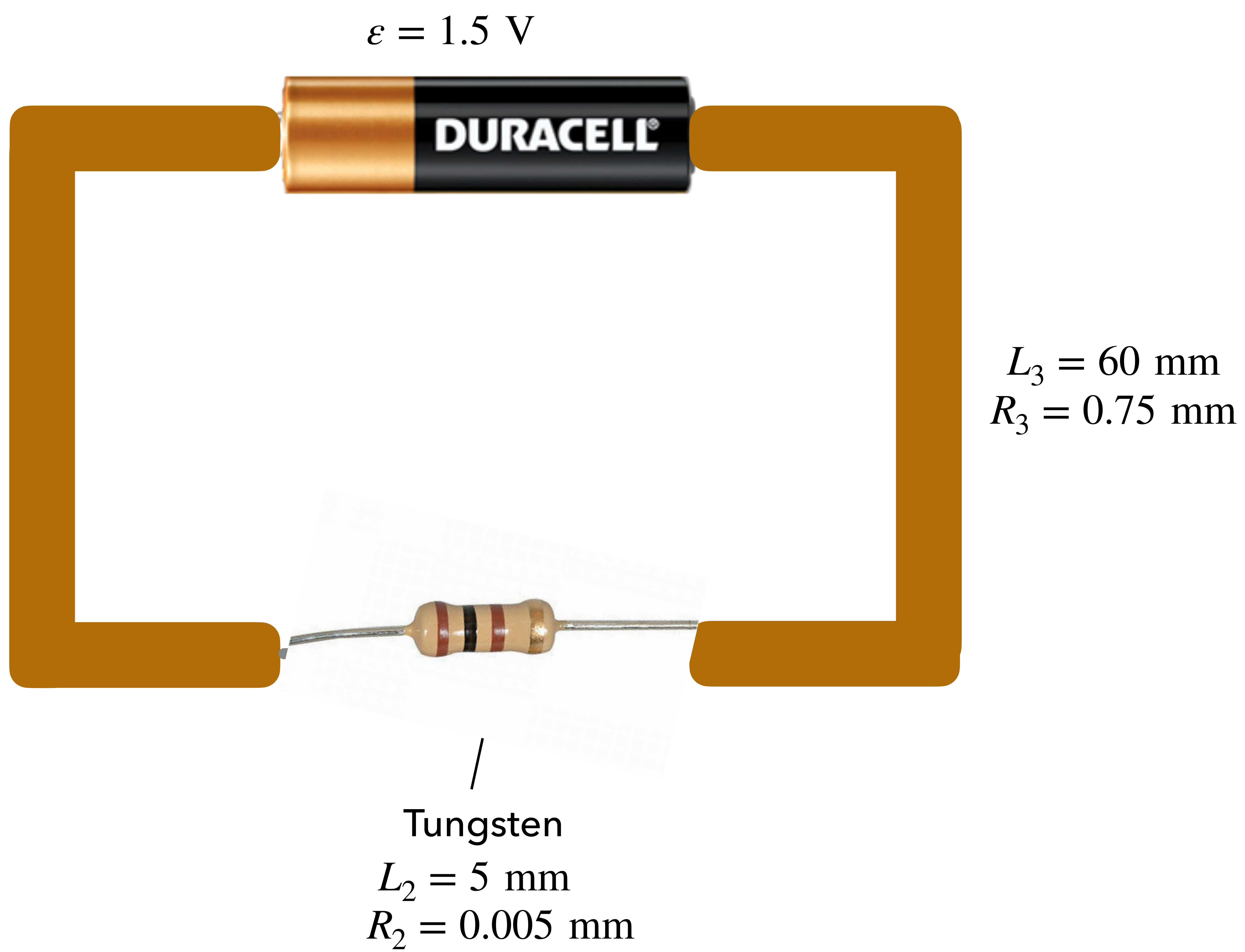


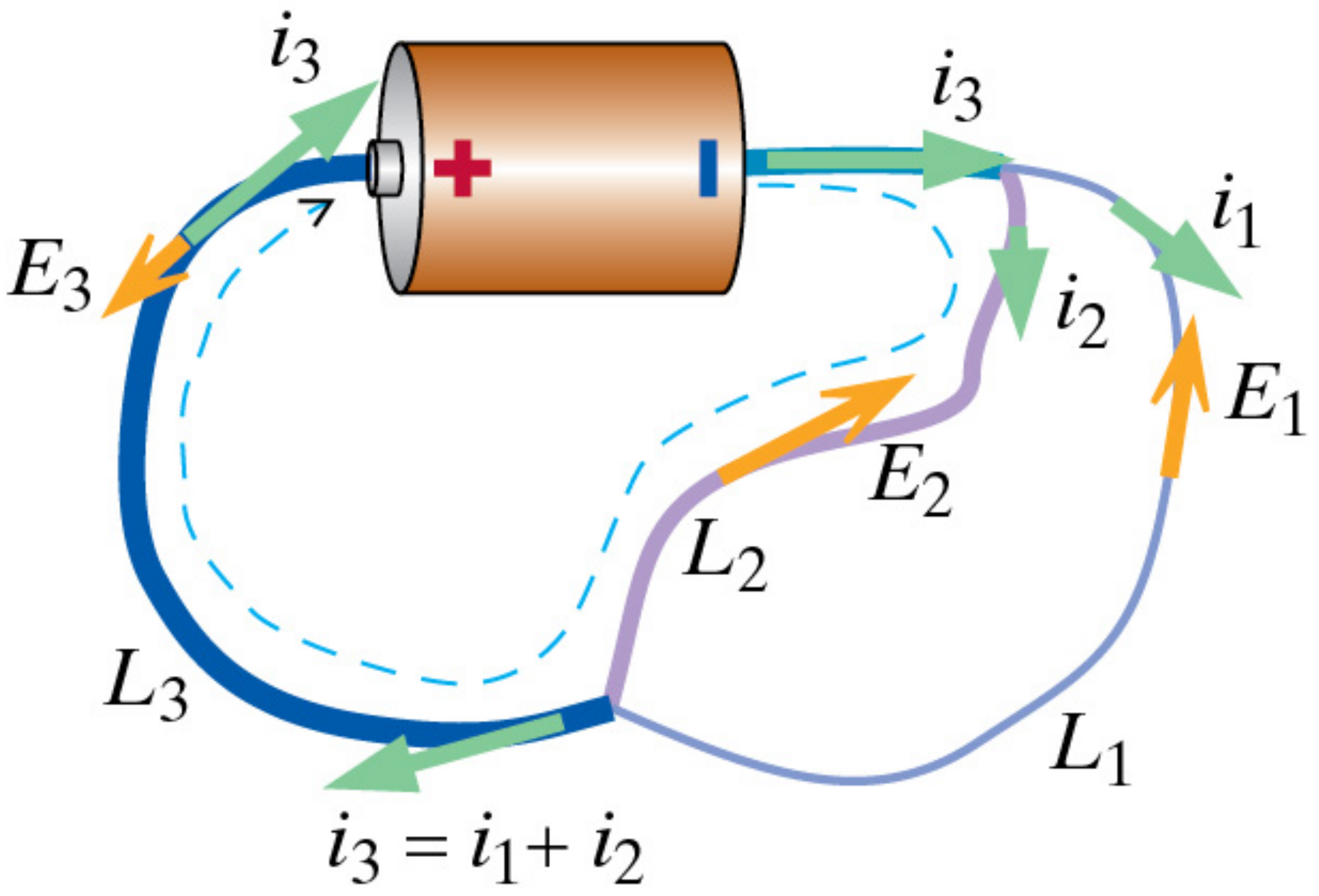
>99% OF THE BATTERY'S ENERGY IS USED TO DRIVE CURRENT THROUGH THE TUNGSTEN WIRE

RESISTORS

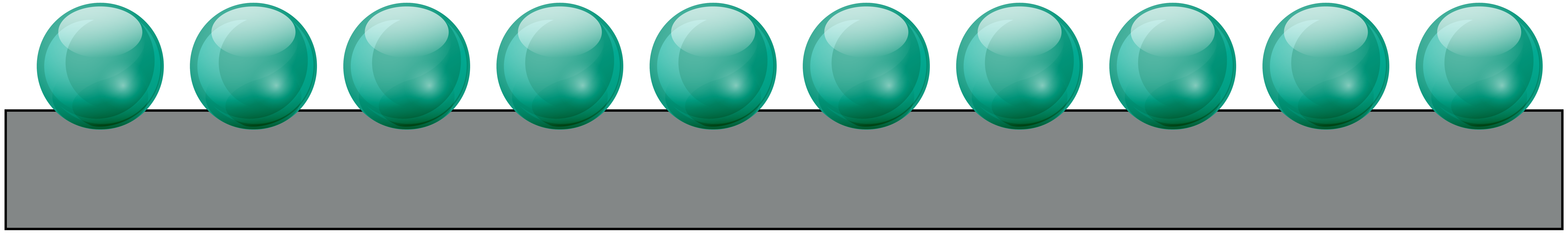
To make a resistor:

- Use material with relatively low electron density and electron mobility
- Use a relatively thin wire

CIRCUITS WITH MULTIPLE PATHS



A GRAVITATIONAL CIRCUIT



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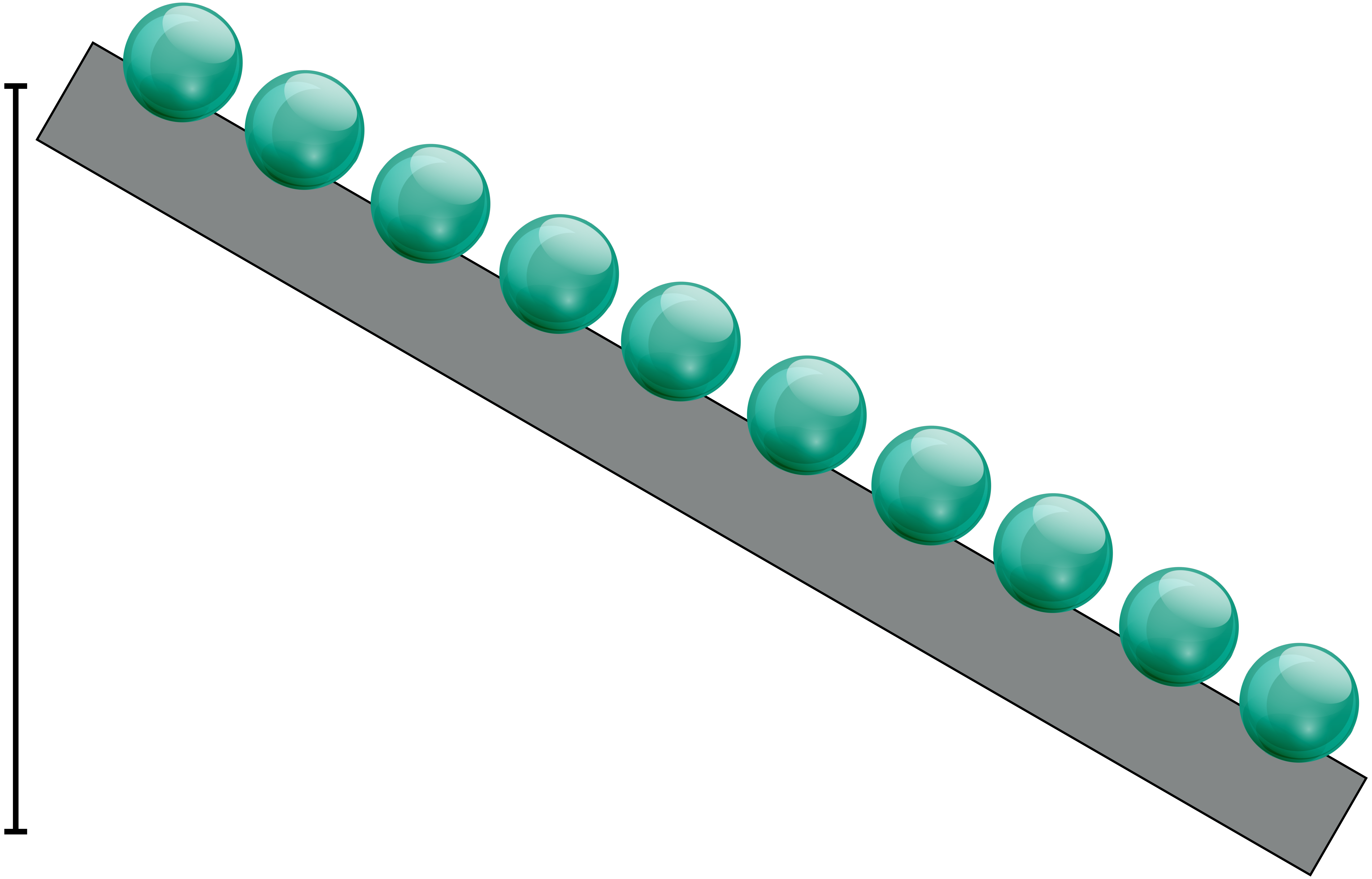

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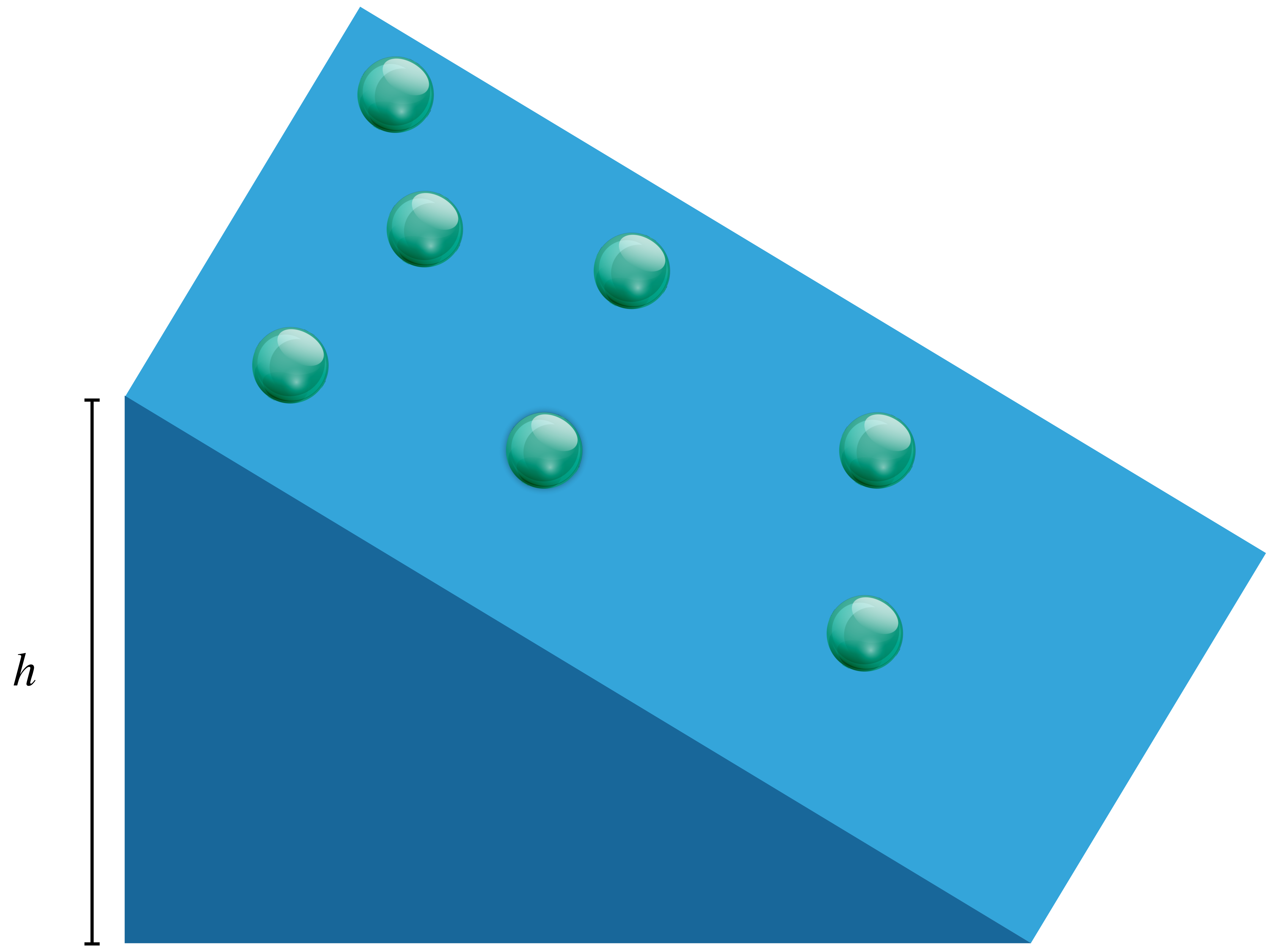

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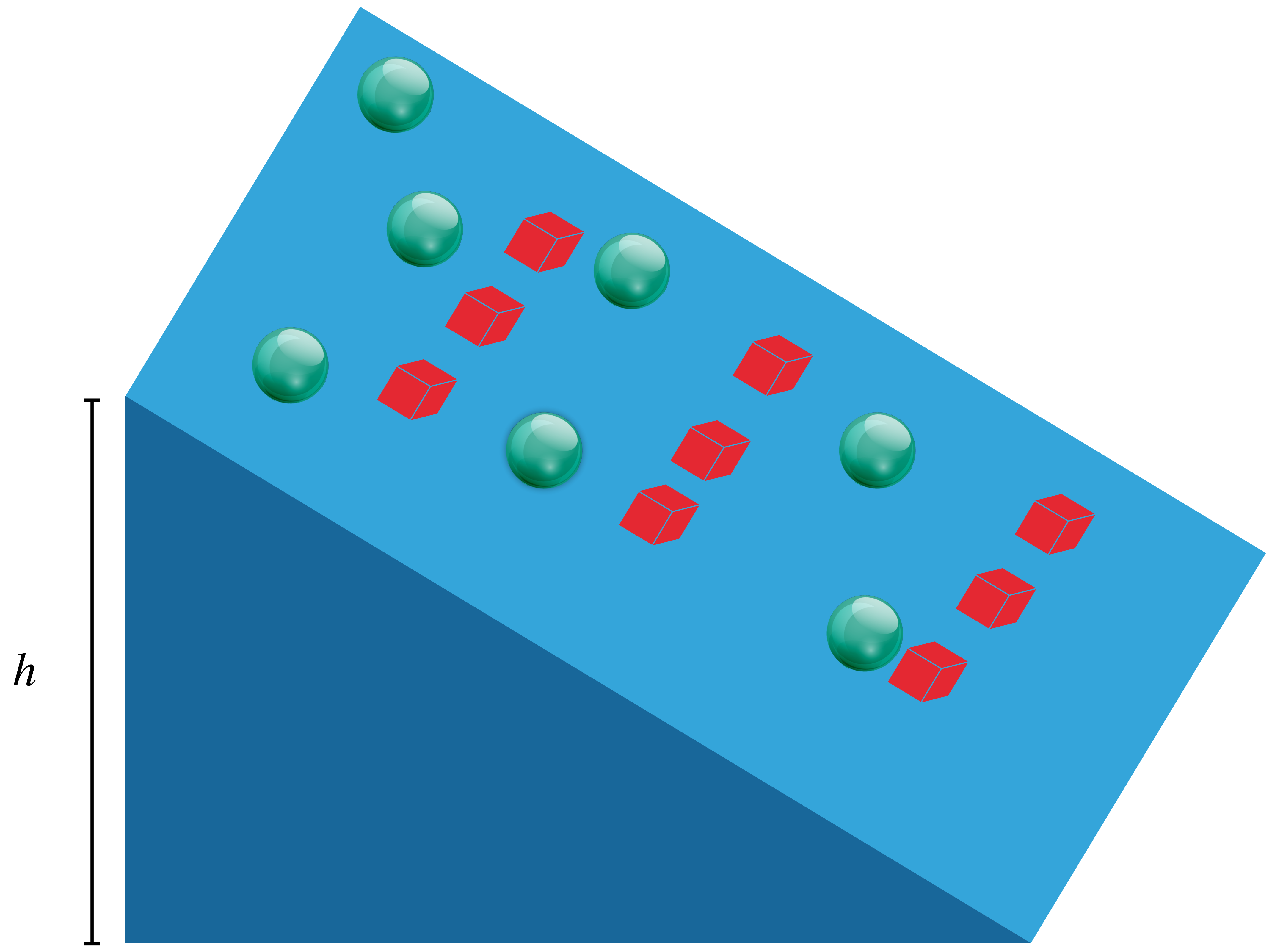
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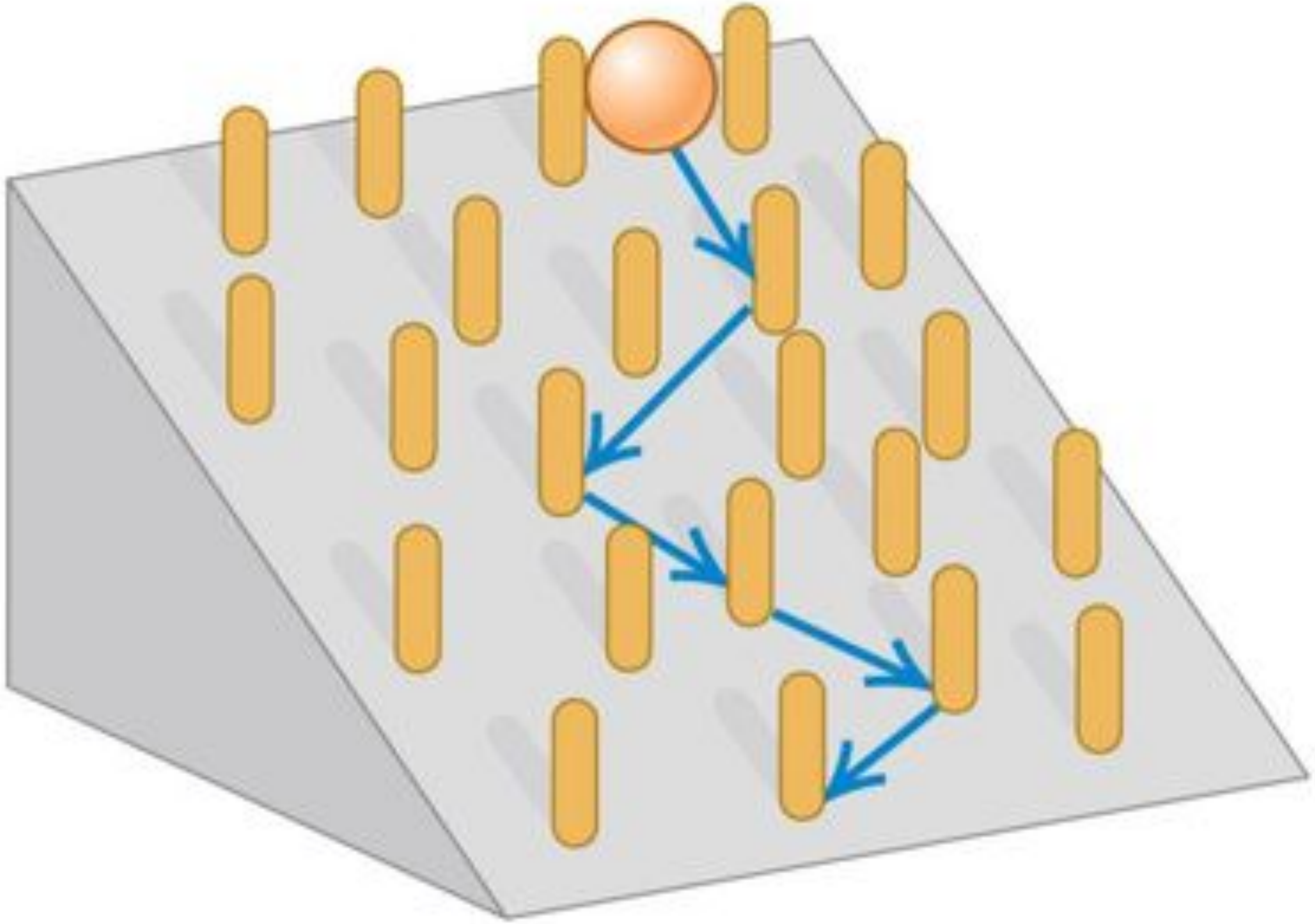

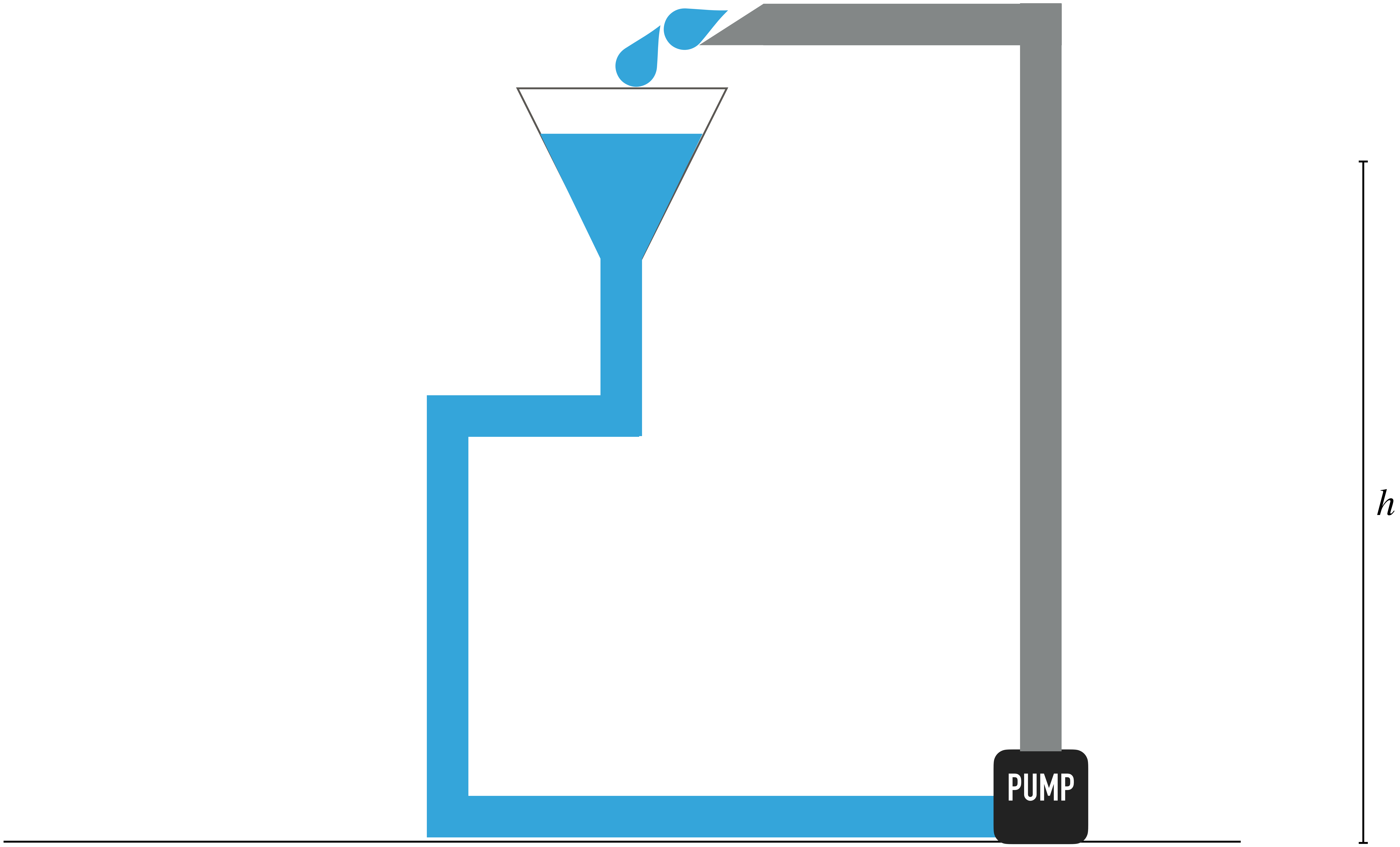
PUMP
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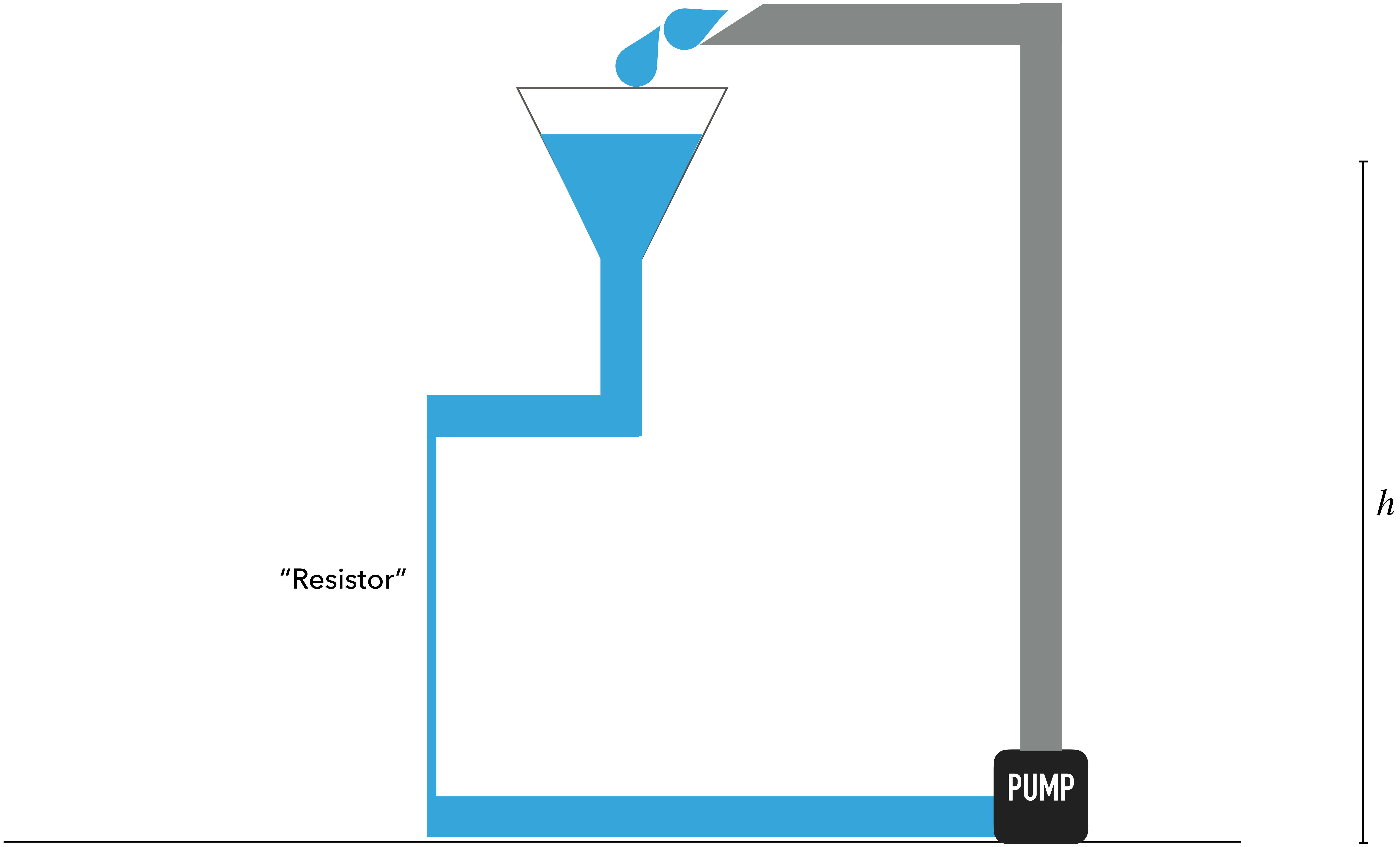
"Resistor"
PUMP
h

CHAPTER 18 SUMMARY

- The node rule: what is it and where does it come from?

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  - In steady state: $\sum i_{\text{in}} = \sum i_{\text{out}}$

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  - Charge conservation

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  - $$\sum_{\text{loop}} \Delta V = 0$$
  - Energy conservation

CHAPTER 18 SUMMARY

- What causes current to flow in a circuit?

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- How does the charge “know” which direction to go?

CHAPTER 18 SUMMARY

- Electron drift speed is very slow. Why do lights come on almost instantly?