CHAPTER 19

MACROSCOPIC CIRCUIT ANALYSIS

OVERVIEW

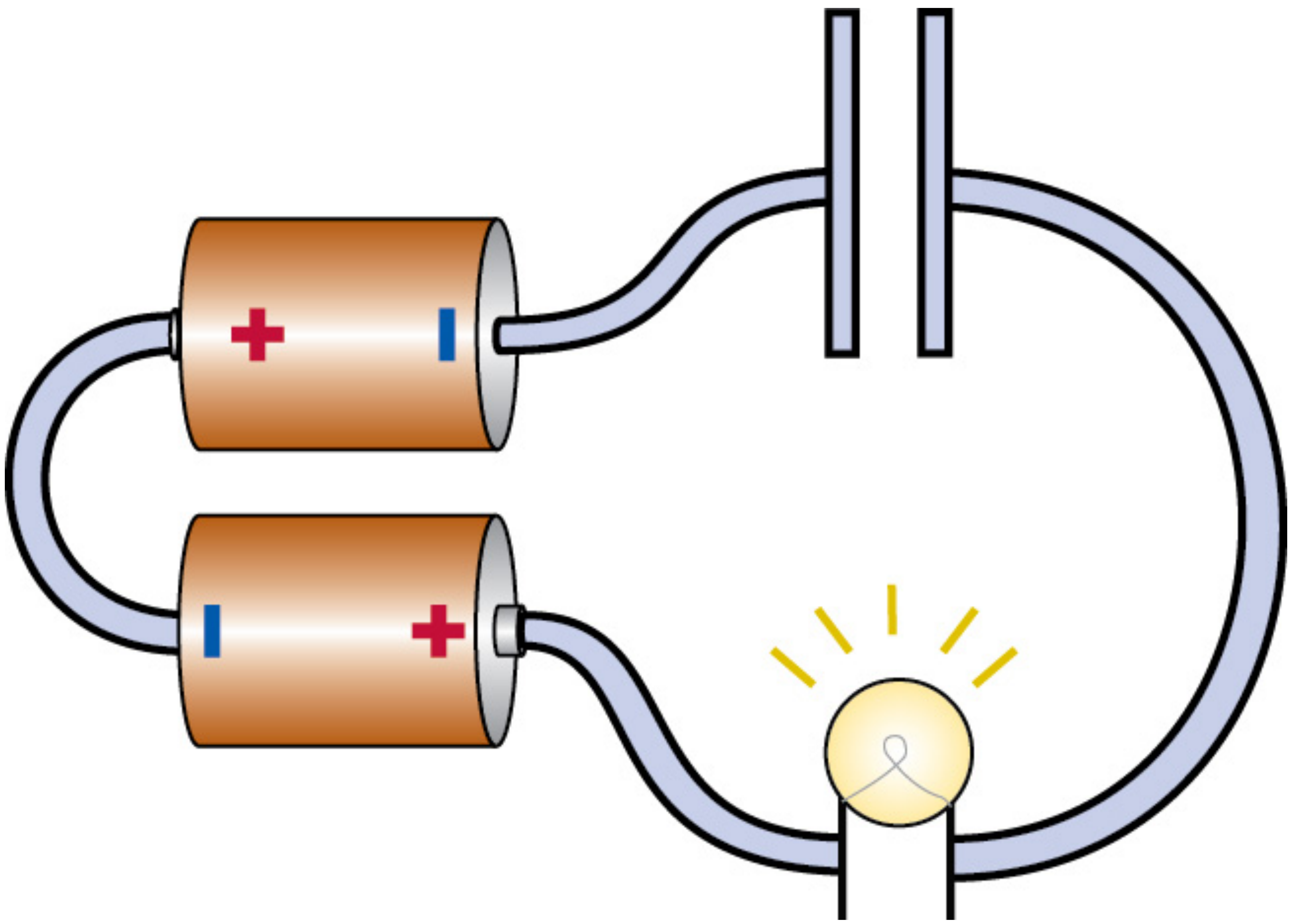
- Last chapter, we developed a qualitative sense of the microscopic behavior of a circuit in terms of fundamental principles
    - Where does the field come from?
    - What is the function of a battery?
    - What is a resistor?
    - Charge & energy conservation

OVERVIEW

- In this chapter, we will apply this understanding to understand circuits *macroscopically*
  - How do circuit elements behave within a circuit?
    - Capacitors & resistors
  - Calculate ΔV across different circuit elements
  - Calculate conventional current I in every part of a circuit

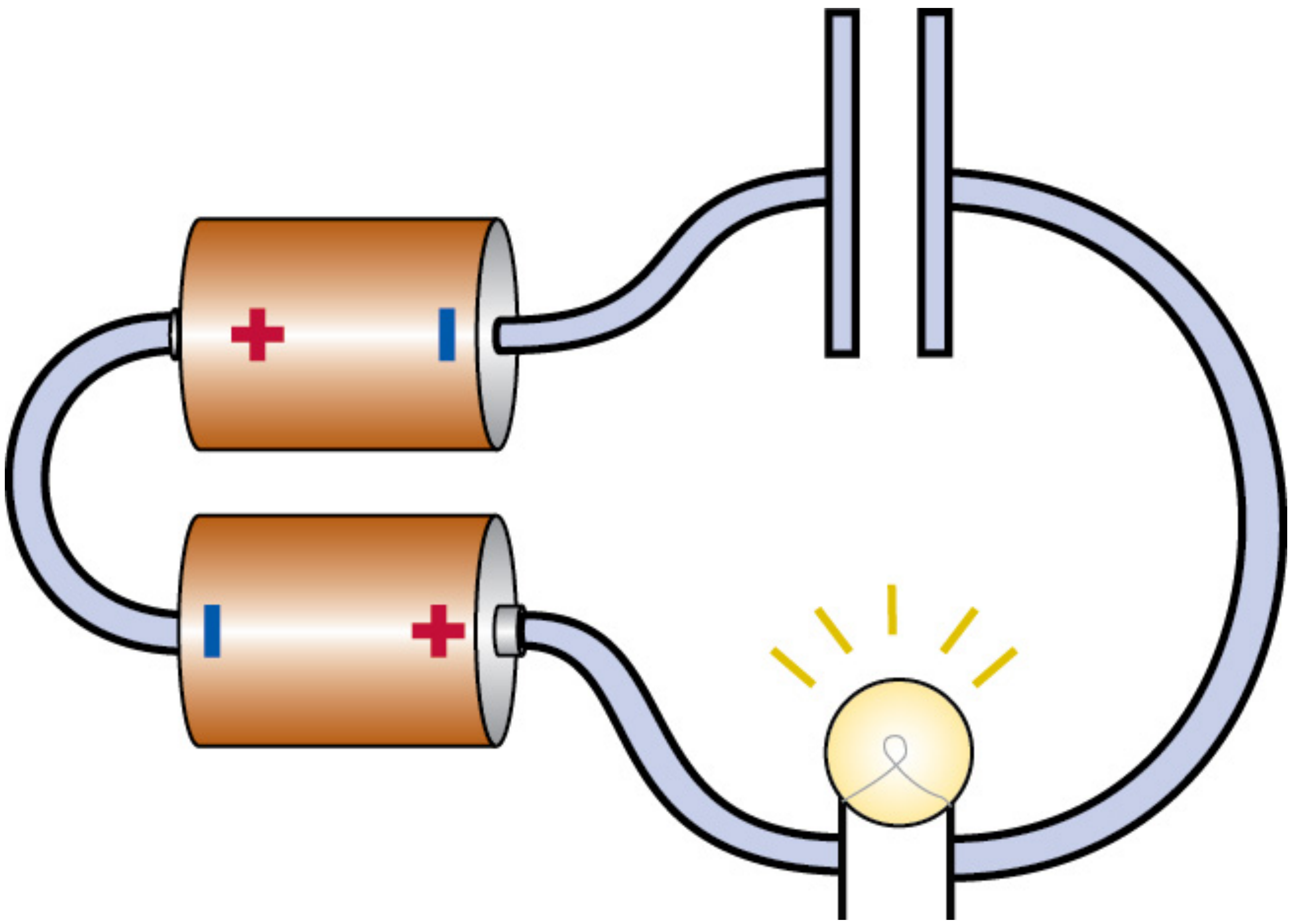
CONSIDER THIS CIRCUIT

- Assume it is initially disconnected



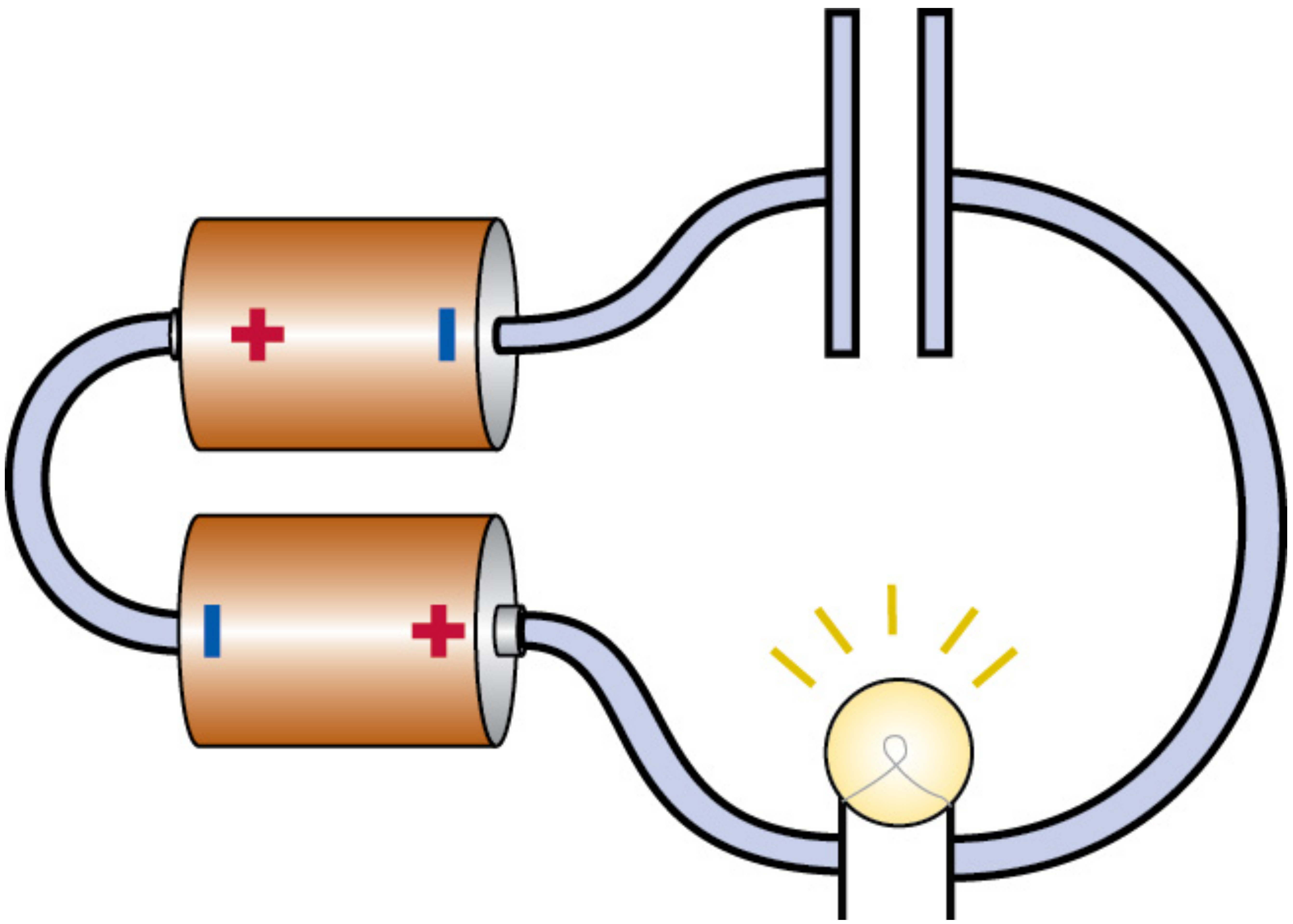
CONSIDER THIS CIRCUIT

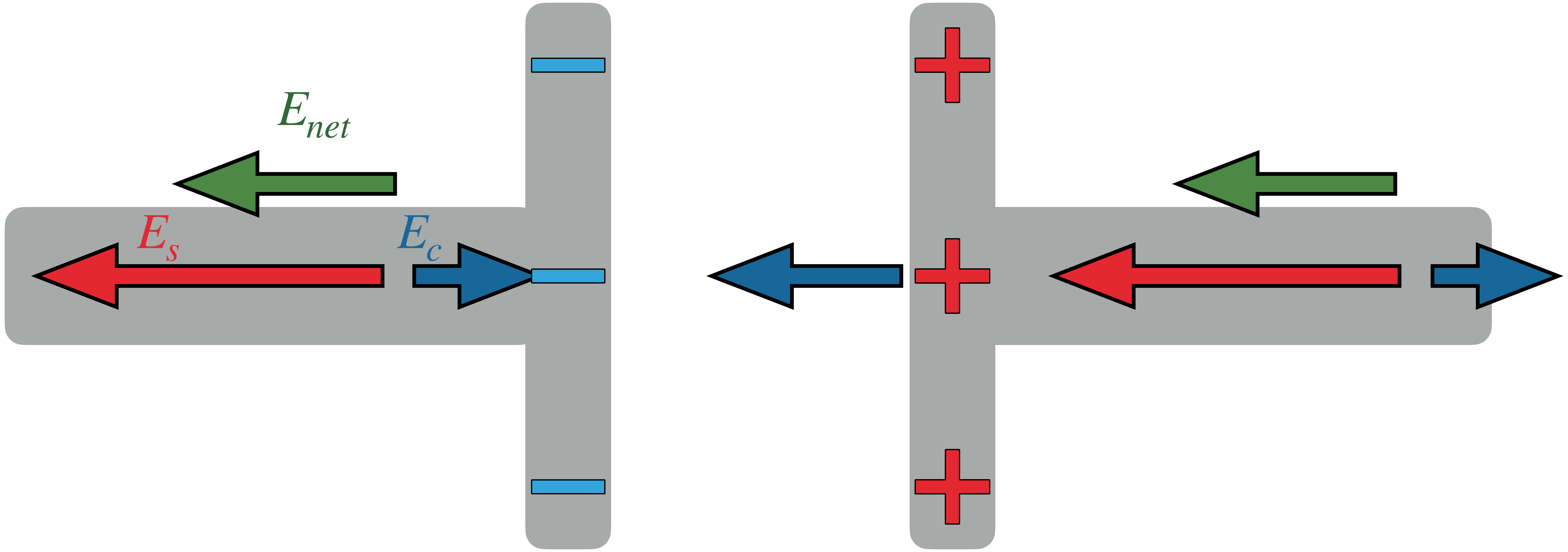
- Assume it is initially disconnected
- What happens when I connect the circuit?

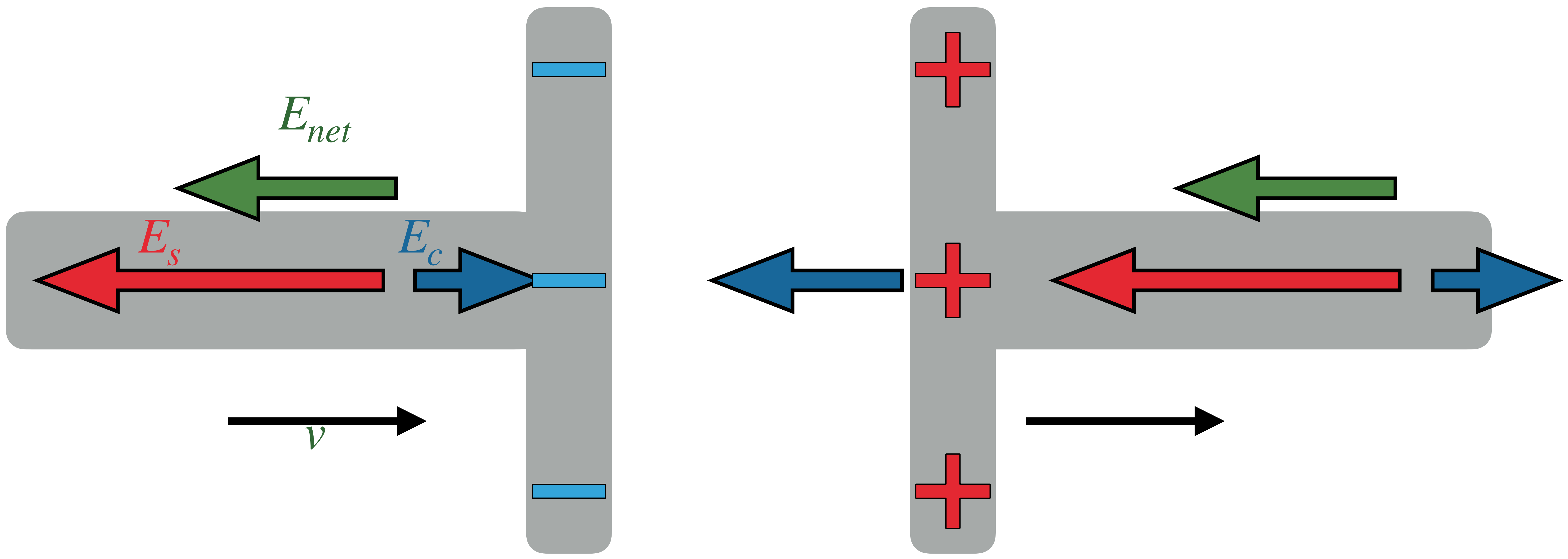


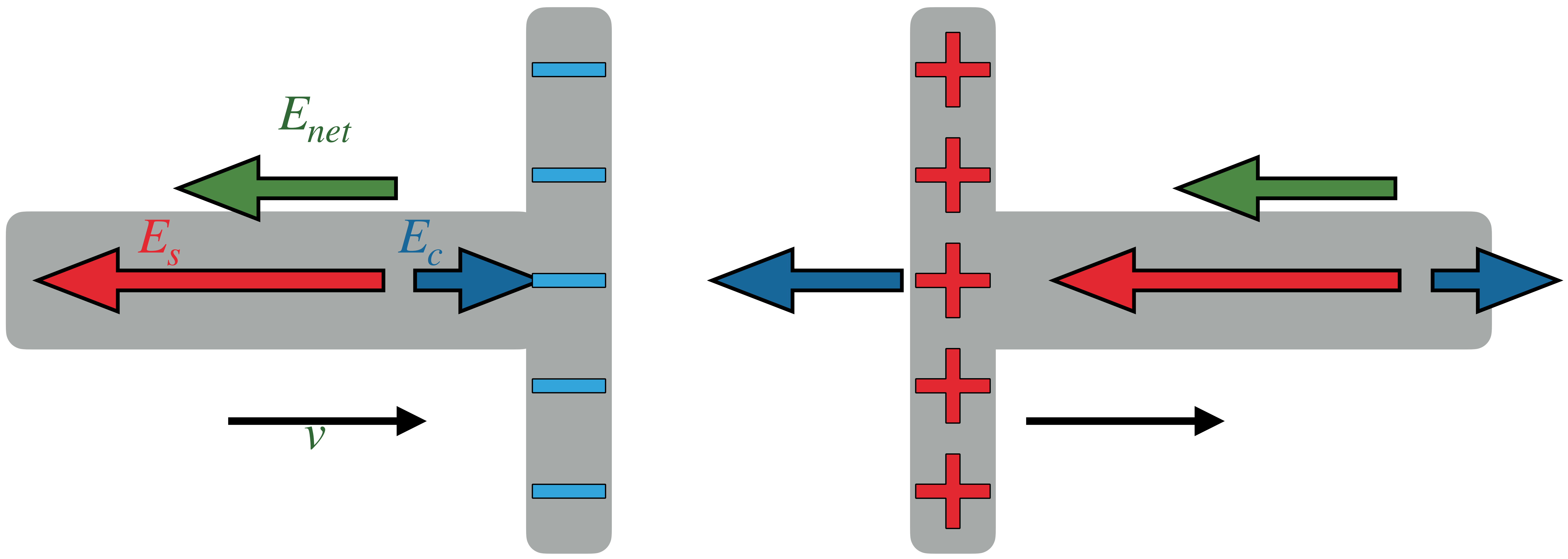
CONSIDER THIS CIRCUIT

- Assume it is initially disconnected
- What happens when I connect the circuit?
- Why?

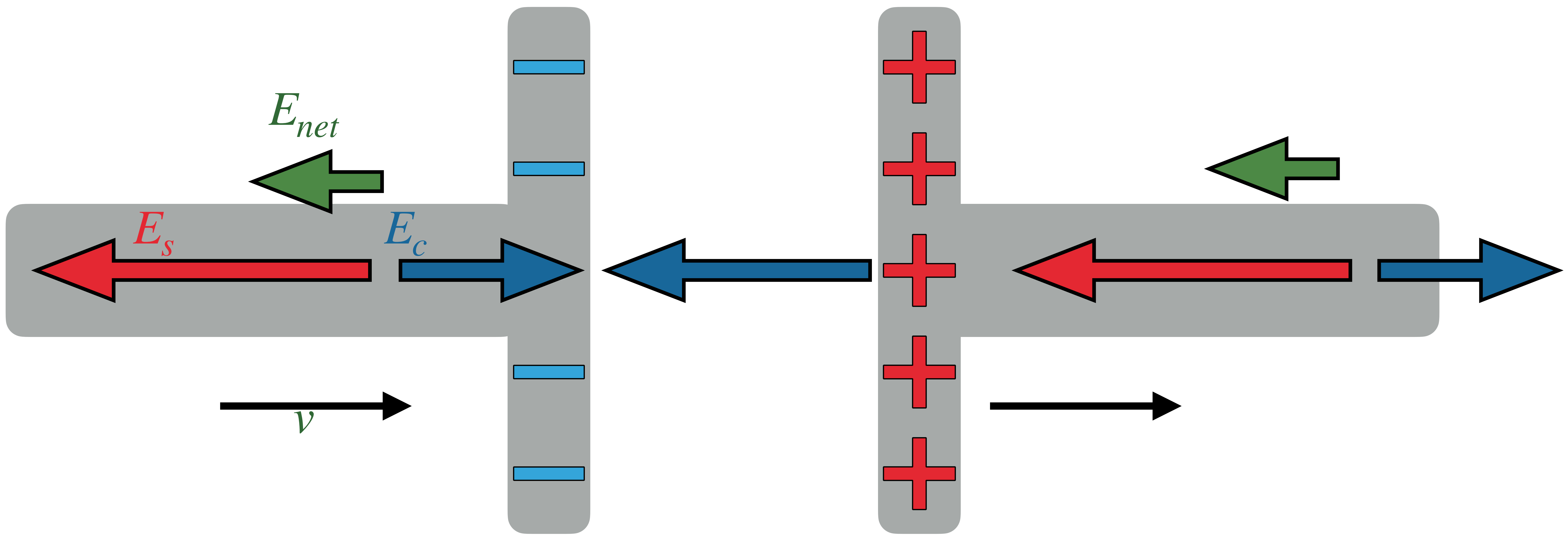



E_{net}
E_s
E_c

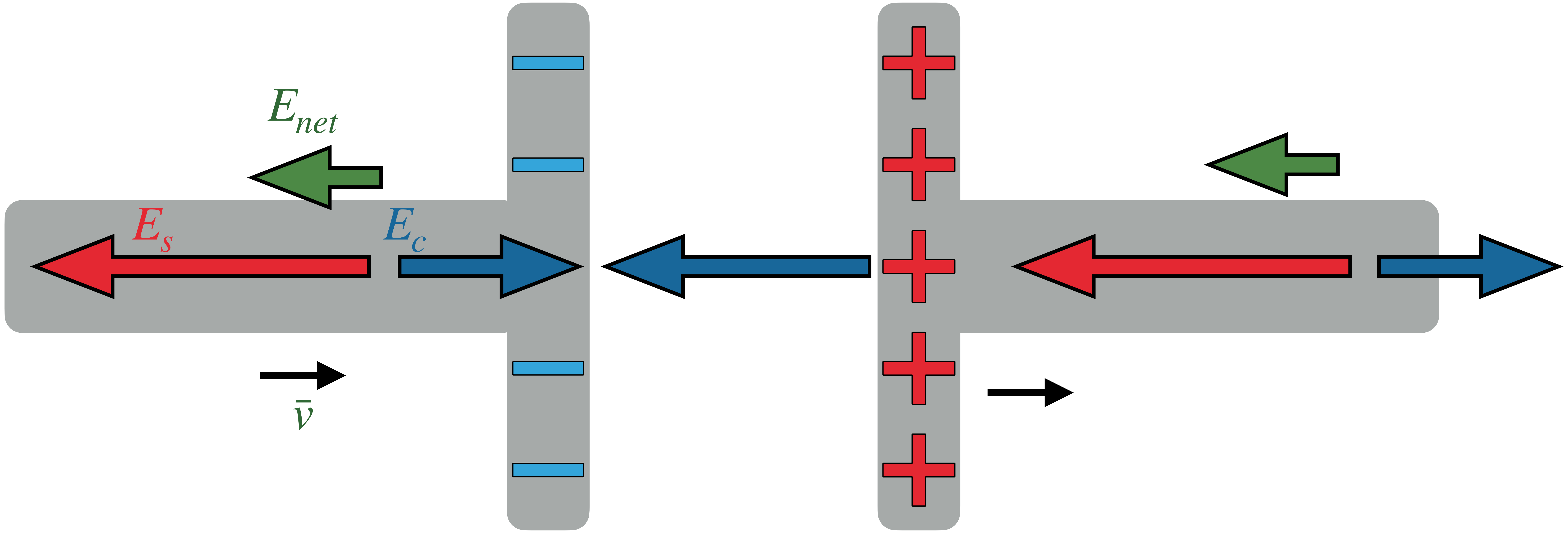

E_{net}
E_s
E_c
v



E_{net}
E_s
E_c
v

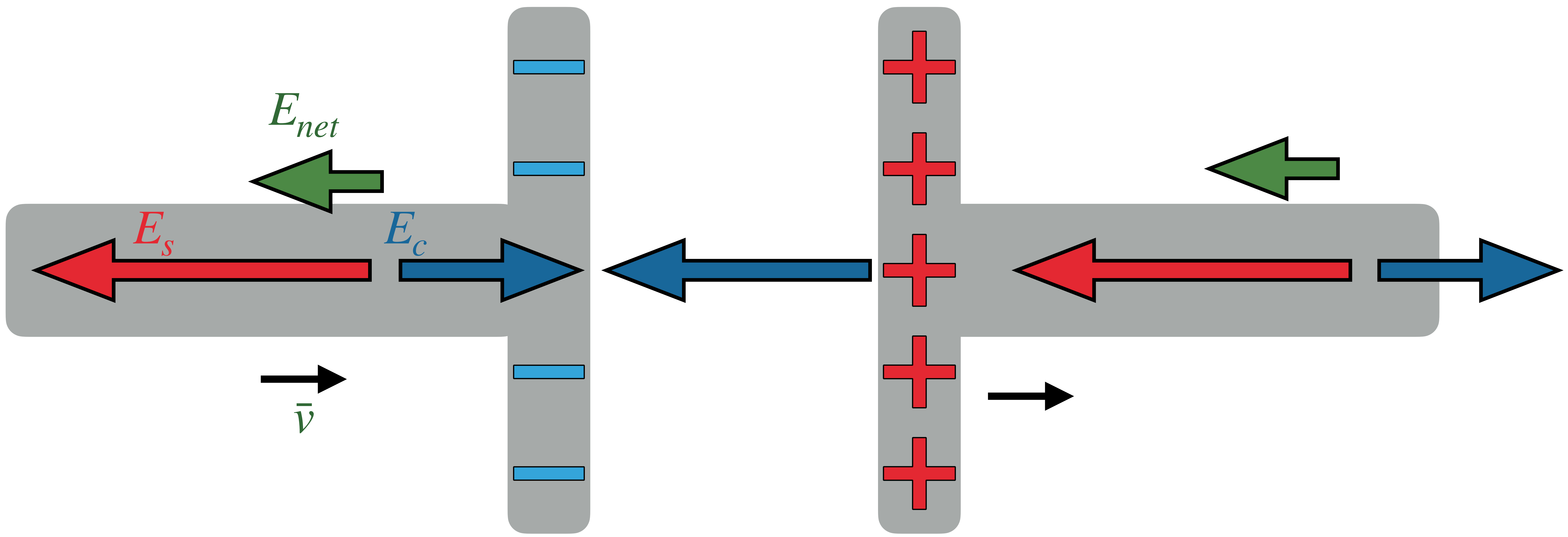


E_{net}
E_s
E_c
v

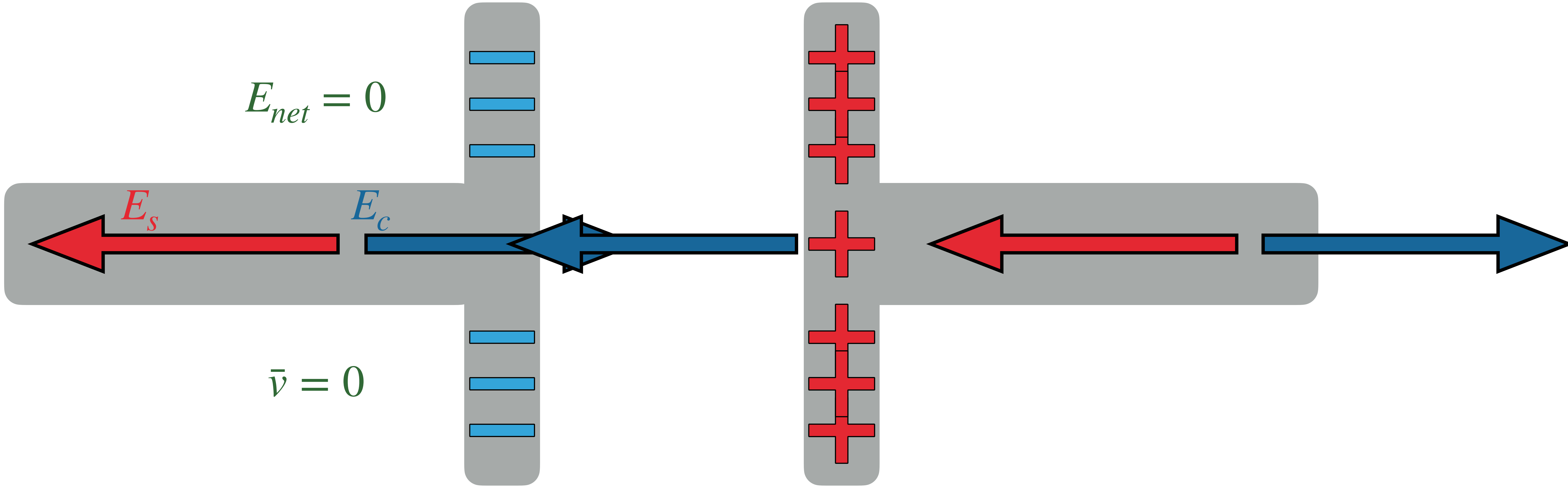


E_{net}
E_s
E_c
$\bar{v}$

- Capacitor continues charging until ______?

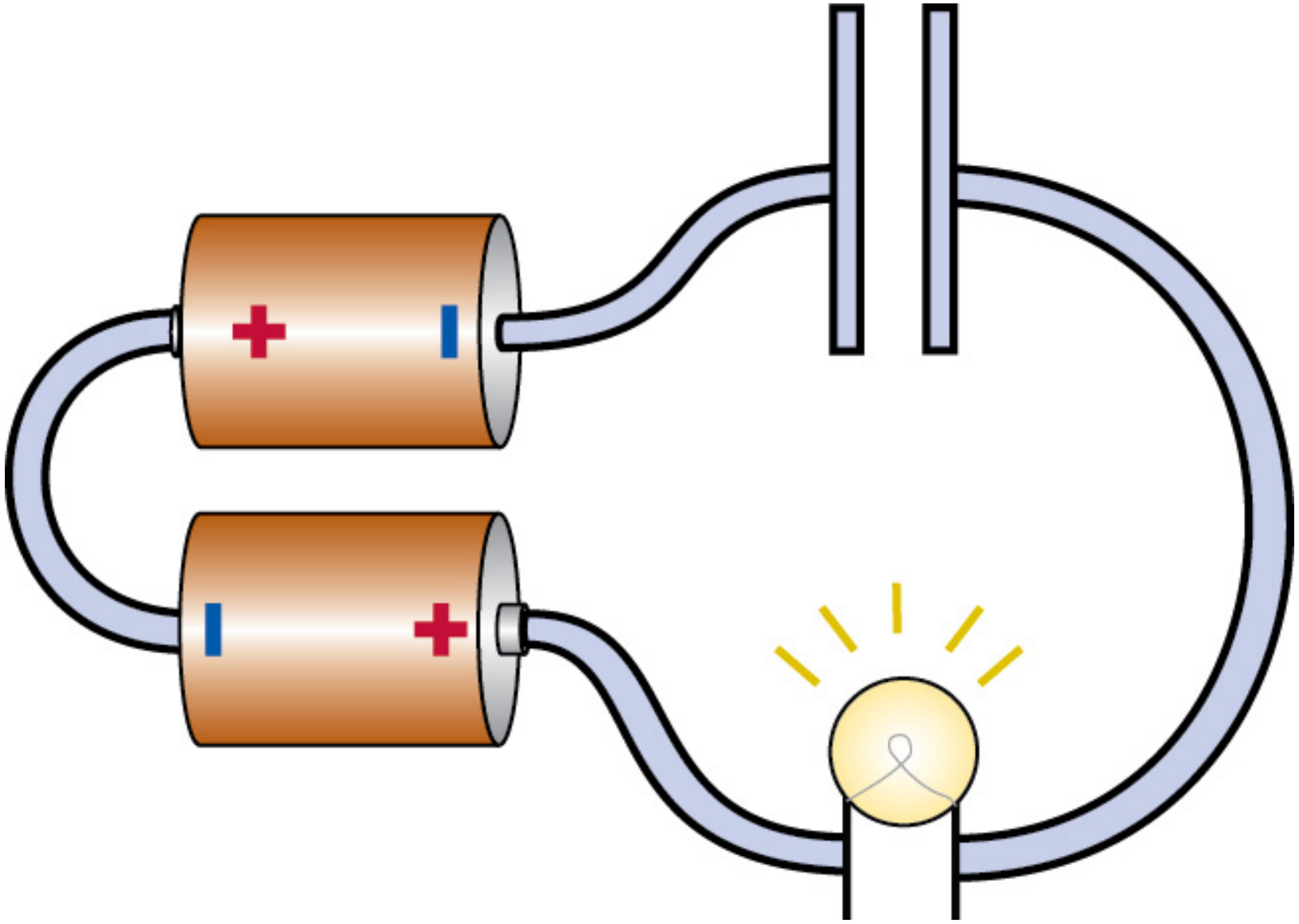


- Capacitor continues charging until $E_{net} = 0$



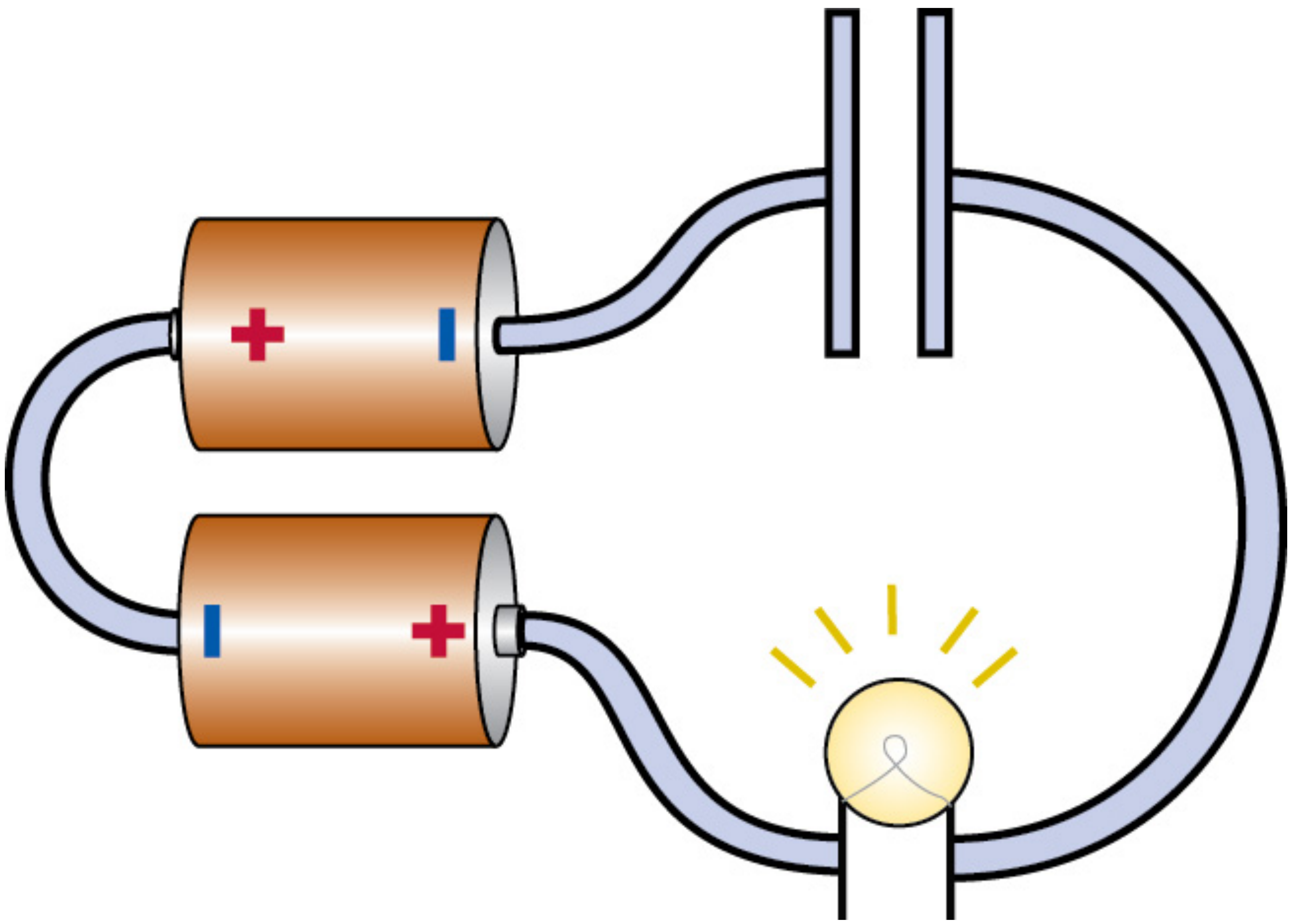
CAPACITOR CHARGING CIRCUIT

- Capacitor is initially "ignored"
- Charge accumulates on the capacitor, decreases current
    - Battery power diverted to capacitor, bulb dims



CAPACITOR CHARGING CIRCUIT

- Charge continues to accumulate until fringe field of capacitor cancels field in wire
- Current stops flowing
  - Bulb is completely dark



CAPACITORS IN CIRCUITS

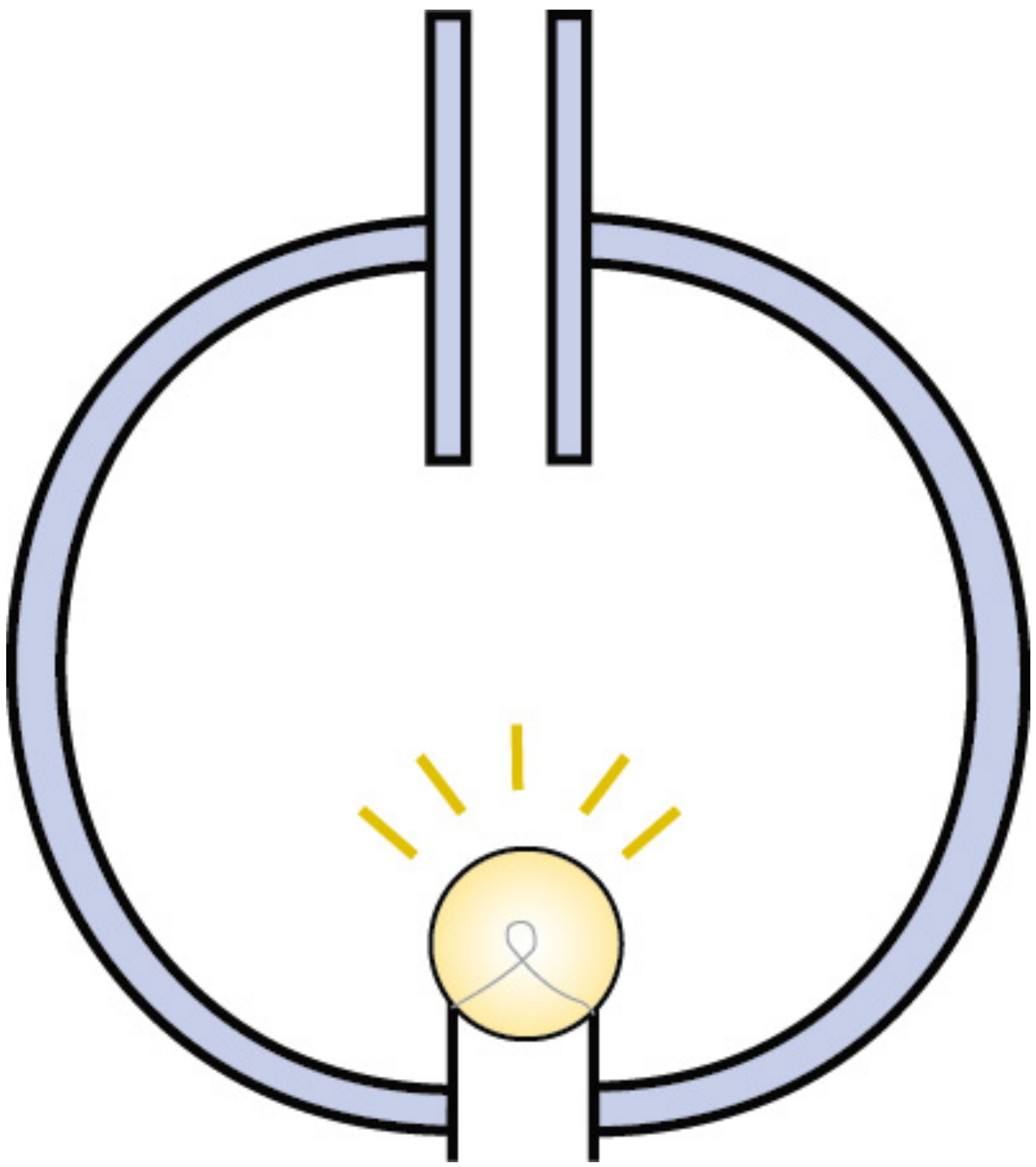
Charge on the capacitor: $Q = C\Delta V_c$

C is the **capacitance** of the capacitor (how much charge can it hold?)

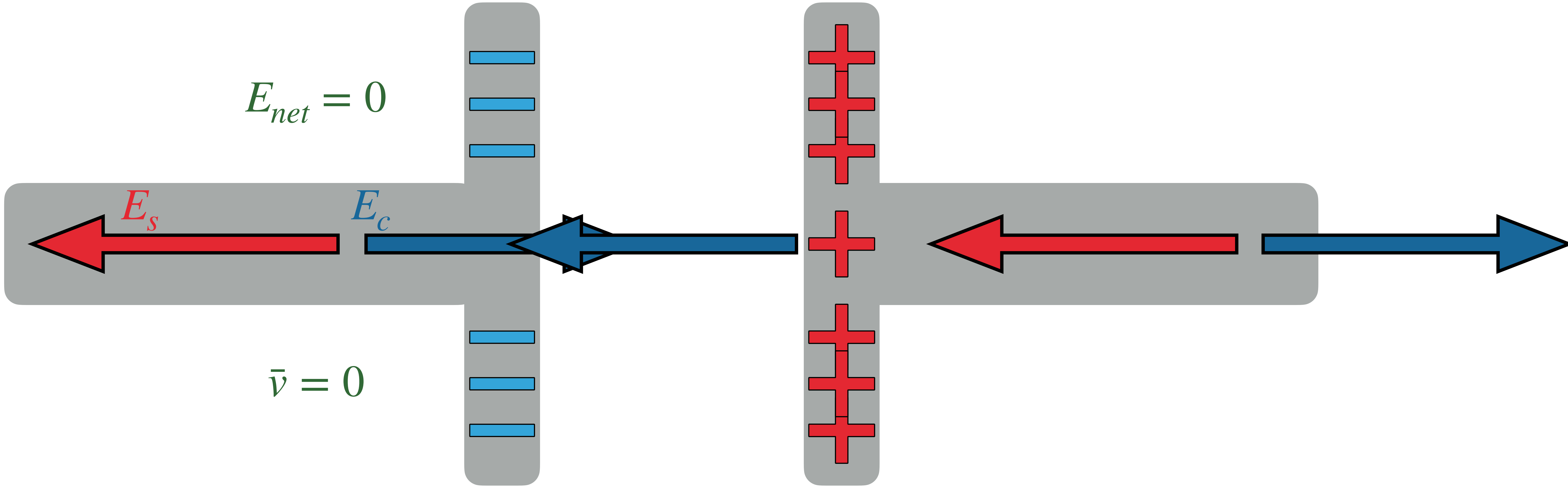
- For same ΔV_c , higher C means more charge stored on the capacitor

CAPACITORS IN CIRCUITS

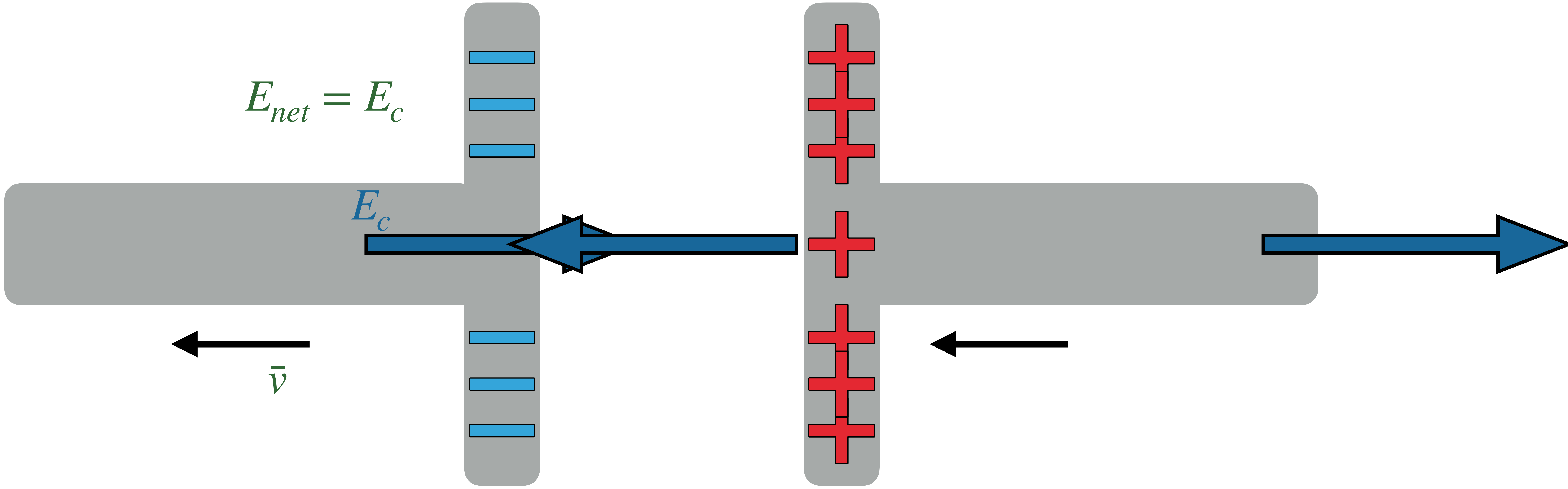
Full charged capacitor, remove the battery



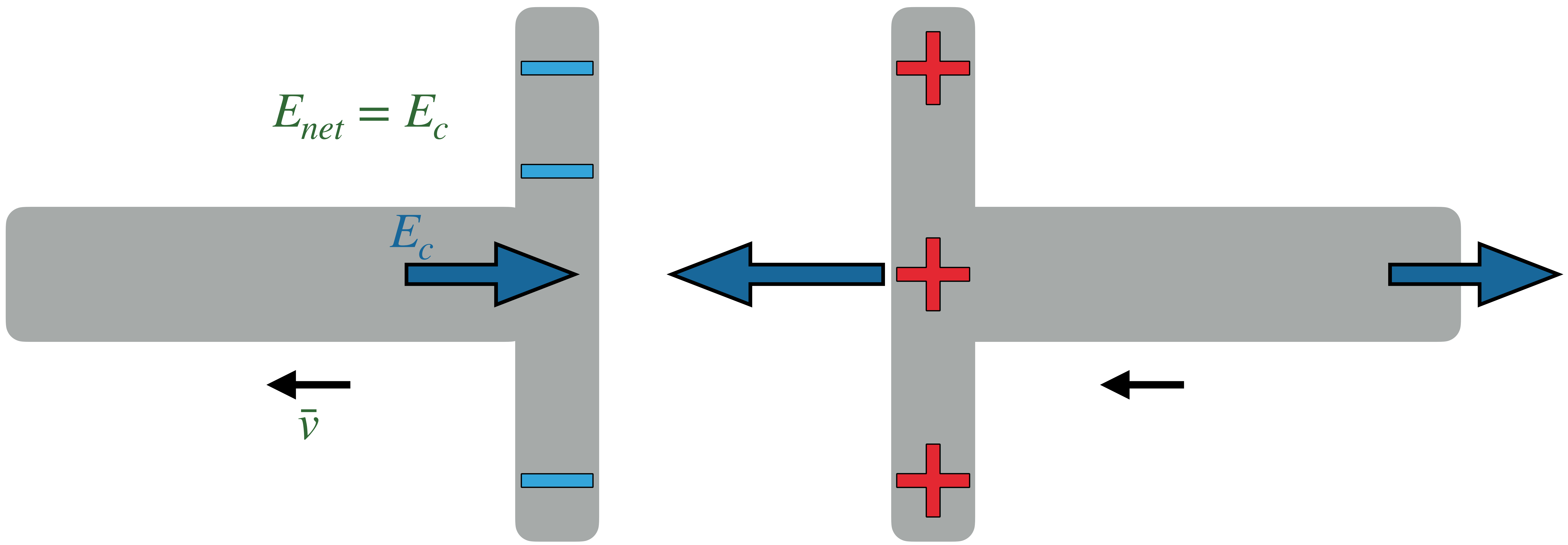
- Fully charged capacitor with battery connected



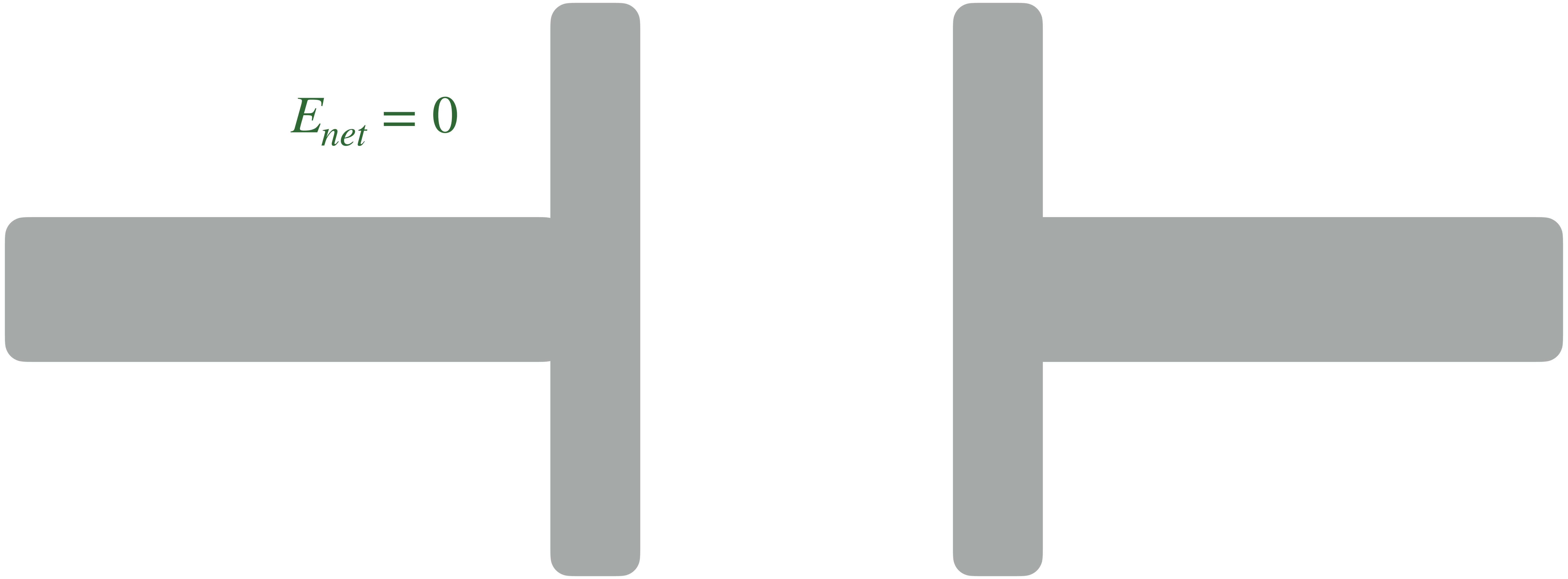
- Now remove the battery
- Charge leaves capacitor (current flows)
- Bulb lights up brightly



- Charge leaves capacitor –> E_c weakens
- Current weakens, bulb dims



- Eventually, capacitor is completely discharged
- $E_{net} = 0$, no more current
    - Bulb is off



CAPACITORS IN CIRCUITS

<u>Uncharged capacitor connected to a battery:</u>

- Initially acts like a piece of wire (no effect on the circuit)
- Builds up charge until it completely stops current flow (acts like an open wire)
- Fully charged capacitor: $Q = C\Delta V_c = C\varepsilon$

CAPACITORS IN CIRCUITS

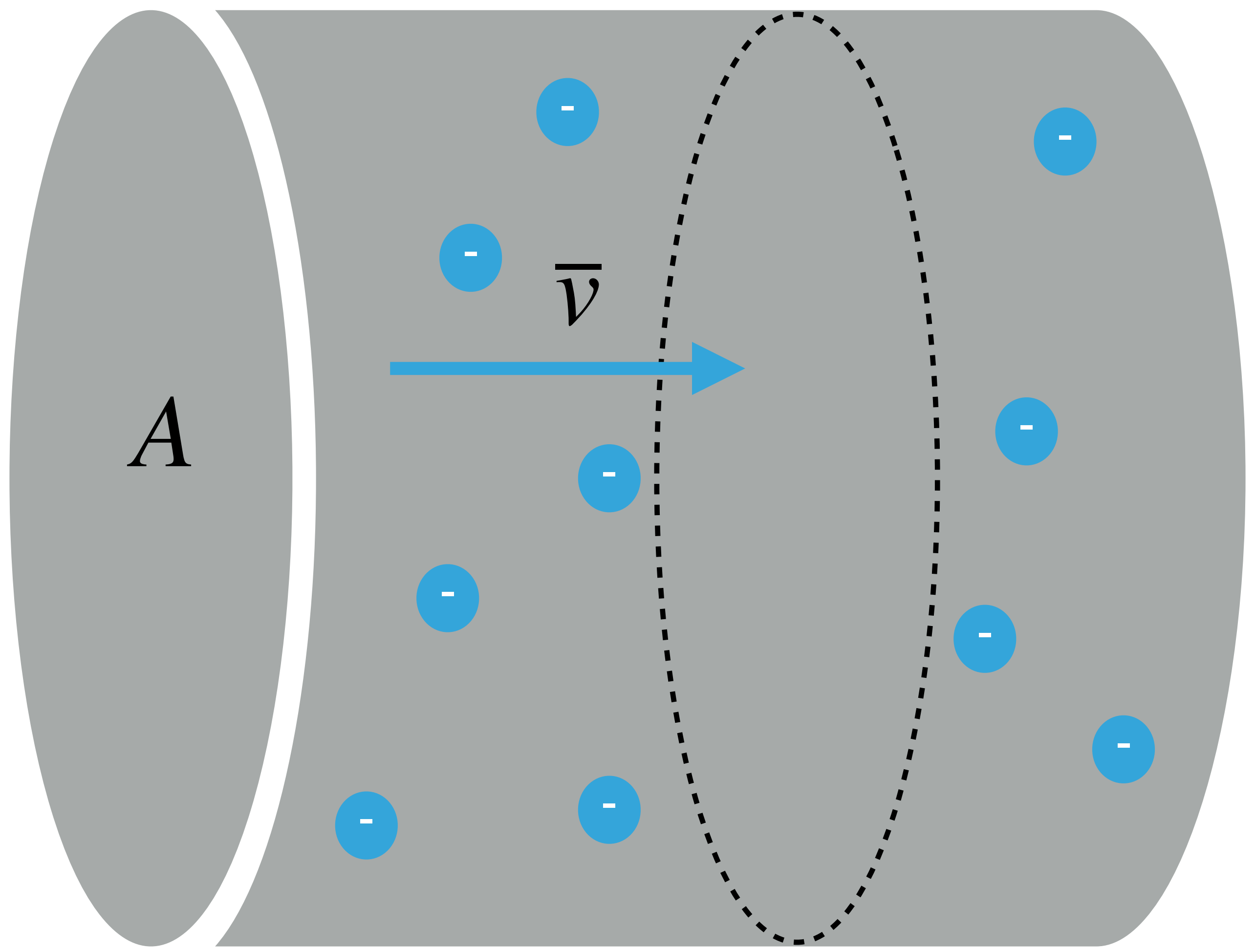
Fully capacitor disconnected from battery

- Initially acts like a battery with “emf” of $\Delta V = Q/C$
- ΔV drives current through circuit
- Capacitor slow discharges, decreasing ΔV, until current flow stops

MICRO VS MACRO

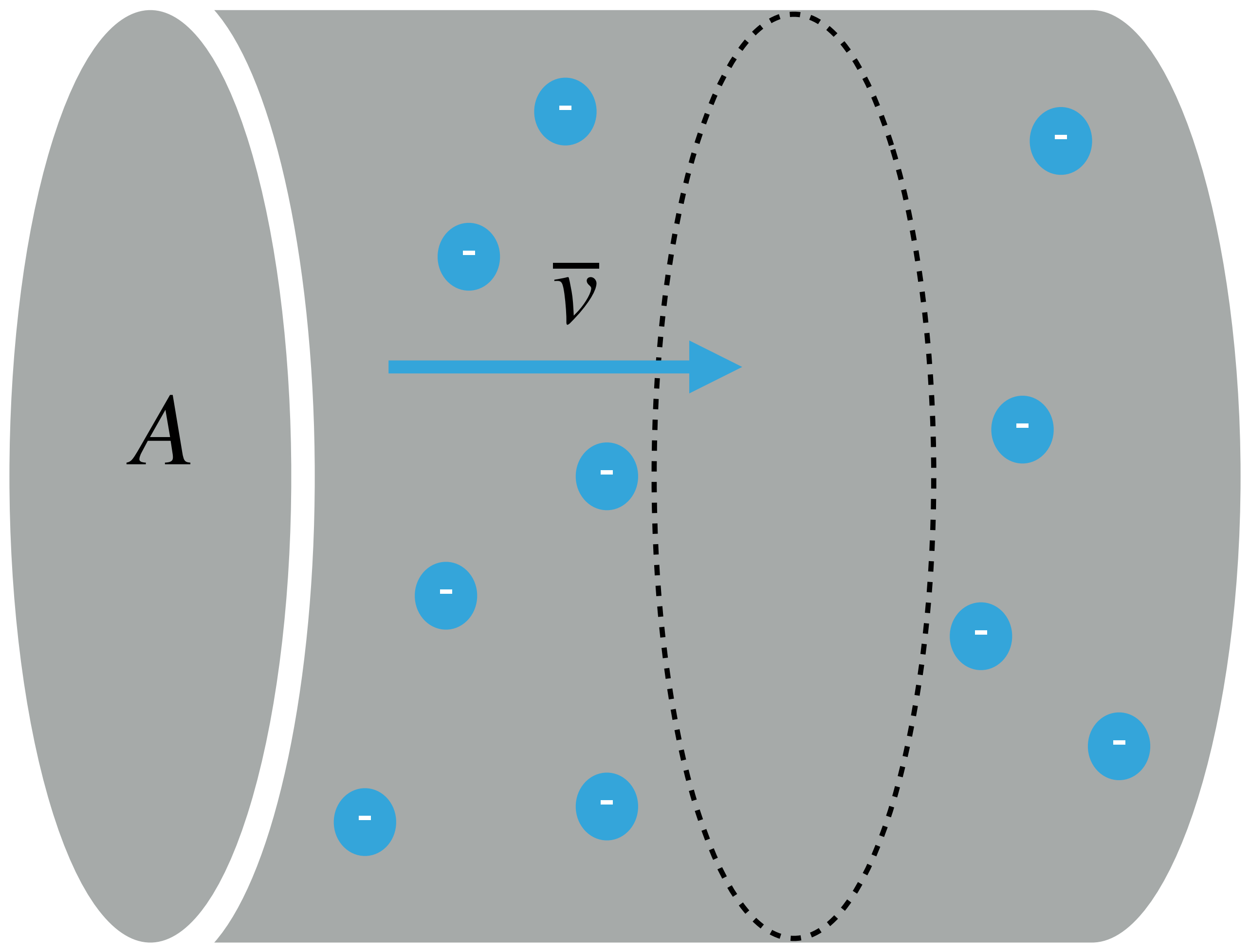
- In chapter 18, we learned how to describe circuits in terms of their atomic-level properties: i, n, u, E, etc
- These quantities are more "fundamental", but hard to work with in practice (hard to measure!)
- Easier to work with conventional current I, potential difference ΔV, etc

MICRO VS MACRO



MICRO VS MACRO

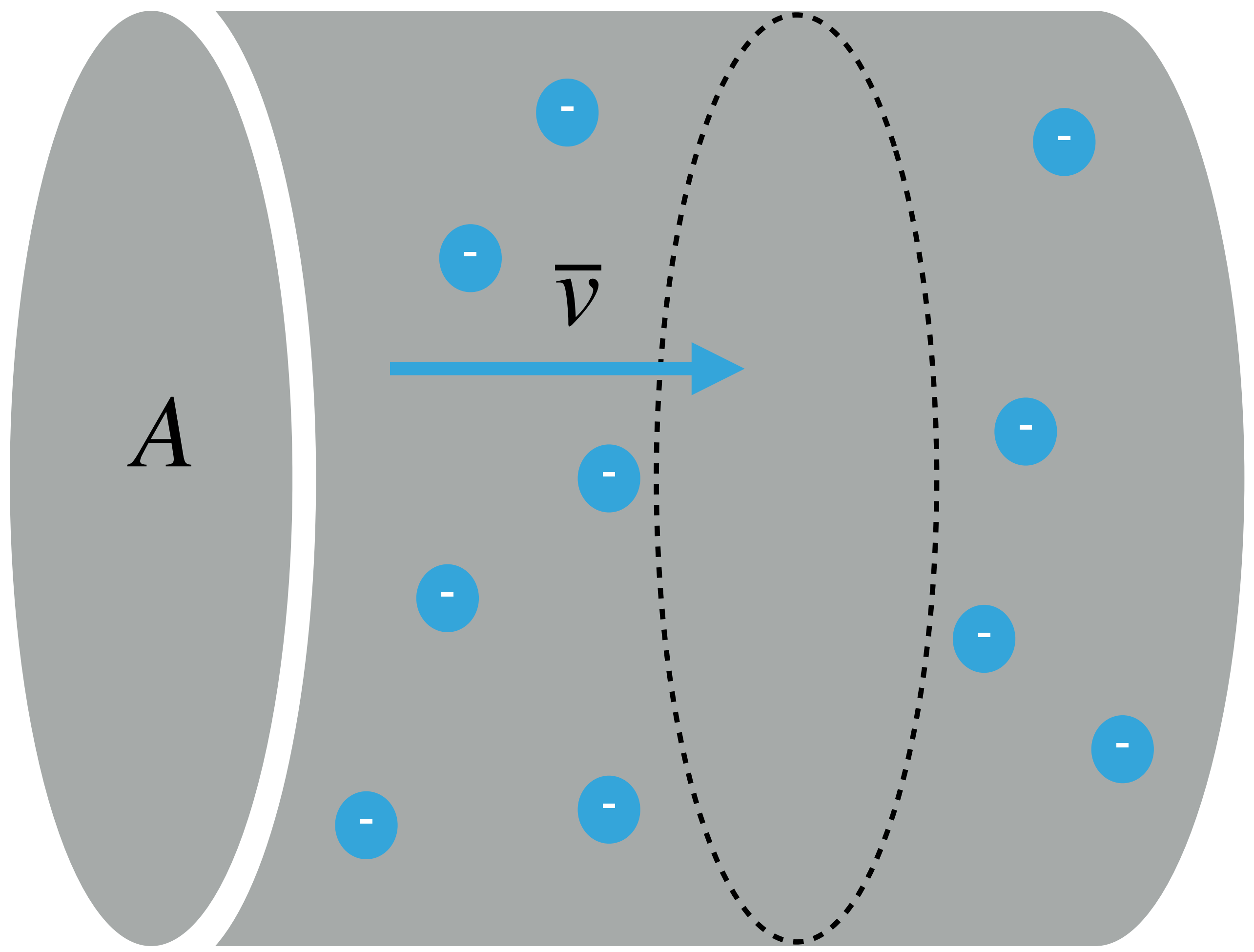
$$J = |q| nuE$$



MICRO VS MACRO

$$J = |q| nuE$$

	$\lvert q \rvert$	n	u	$\lvert q \rvert nu$
Copper	-1.6×10^{-19} C	8.5×10^{28} m^{-3}	4.5×10^{-3} $\frac{\text{m/s}}{\text{N/C}}$	6.1×10^{7} $\frac{\text{C}^2}{\text{m}^2\text{Ns}}$
Tungsten	-1.6×10^{-19} C	6×10^{28} m^{-3}	1.8×10^{-3} $\frac{\text{m/s}}{\text{N/C}}$	1.7×10^{7} $\frac{\text{C}^2}{\text{m}^2\text{Ns}}$
etc				



ELECTRICAL CONDUCTIVITY

$$\sigma = |q| \, nu$$

- σ is called the "electrical conductivity"
    - Lumps together all relevant properties of the material

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$$\sigma = |q|\, nu$$

- σ is called the "electrical conductivity"
    - Lumps together all relevant properties of the material
- Higher σ $\implies$ less field needed for same current

CONSIDER

Length L of wire with cross-sectional area A and conductivity σ

What is $\left|\Delta V\right|$ across the wire?

ELECTRICAL RESISTANCE

"Resistance" $R = \dfrac{L}{\sigma A}$

- Combines geometry and inherent conductivity
- Units: The "ohm" (Ω)

OHM'S LAW

$$I = \frac{\Delta V}{R}$$

- Electric potential difference causes charges to move (current)
- Higher $\Delta V \implies$ more charge motion $\implies$ higher current
- Higher R (low conductivity, fewer free electrons in wire, more collisions with atomic nuclei, etc) $\implies$ lower current

EXAMPLE

A resistor is made of nichrome wire (made of nickel, iron, and chromium) which has a cross-sectional area of $80\ \mu\mathrm{m}^2$.

- How long of a wire do you need to construct a $220\ \Omega$ resistor?
- What is the current through this resistor when connected to a $9\ \mathrm{V}$ battery?

$\sigma_{\mathrm{Nichrome}} = 0.10 \times 10^7\ \Omega^{-1}\mathrm{m}^{-1}$

EXAMPLE

The resistor is connected to the battery via a length of thick copper wire (cross-sectional area of $0.3\ \mathrm{mm}^2$)

- How much copper wire do you need to use to match the resistance of the Nichrome resistor?
- $\sigma_{\mathrm{Copper}} = 6.0 \times 10^7\ \Omega^{-1}\mathrm{m}^{-1}$

EXAMPLE

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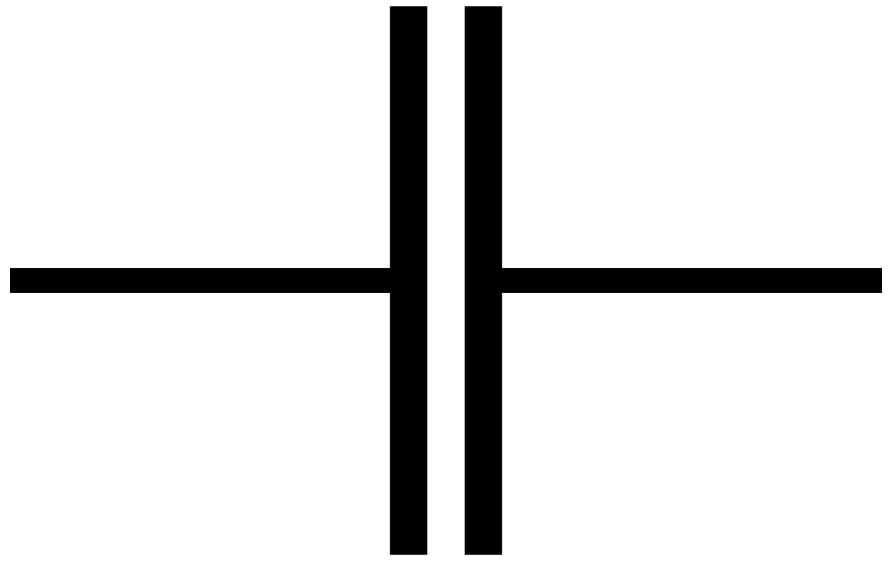
CONCLUSION? WE CAN SAFELY IGNORE CONNECTING WIRES IN CIRCUITS

CIRCUIT ELEMENTS

- For our purposes, a "circuit element" is structure placed in the circuit which uses (or supplies) significant energy (ΔV)
    - Capacitors
    - Resistors
    - Batteries

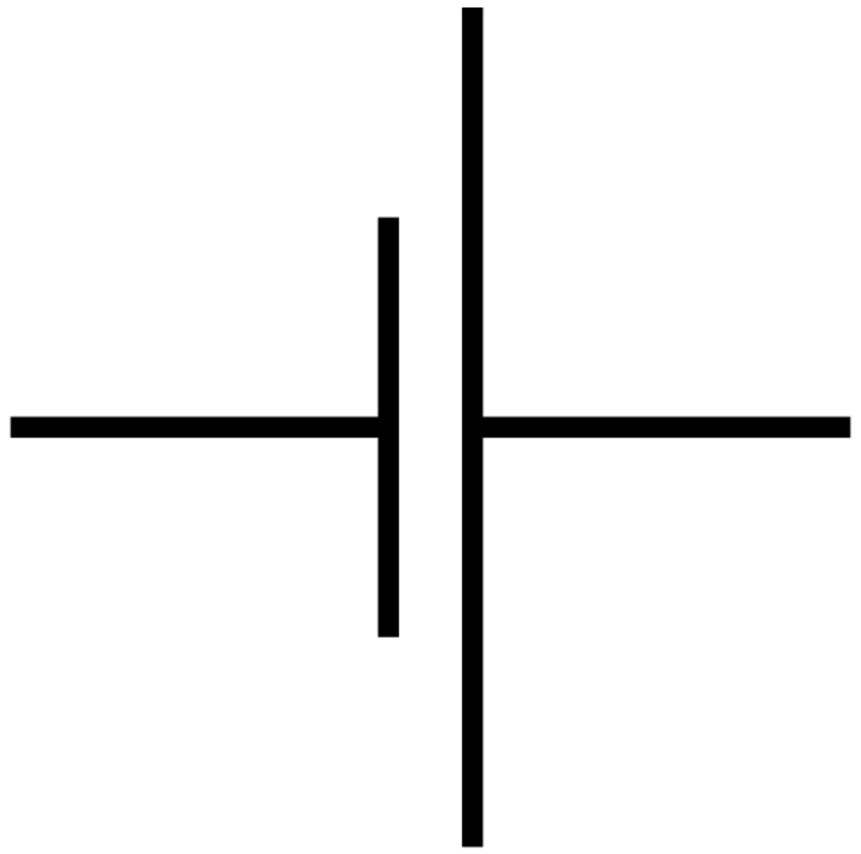
CIRCUIT DIAGRAMS

Capacitor



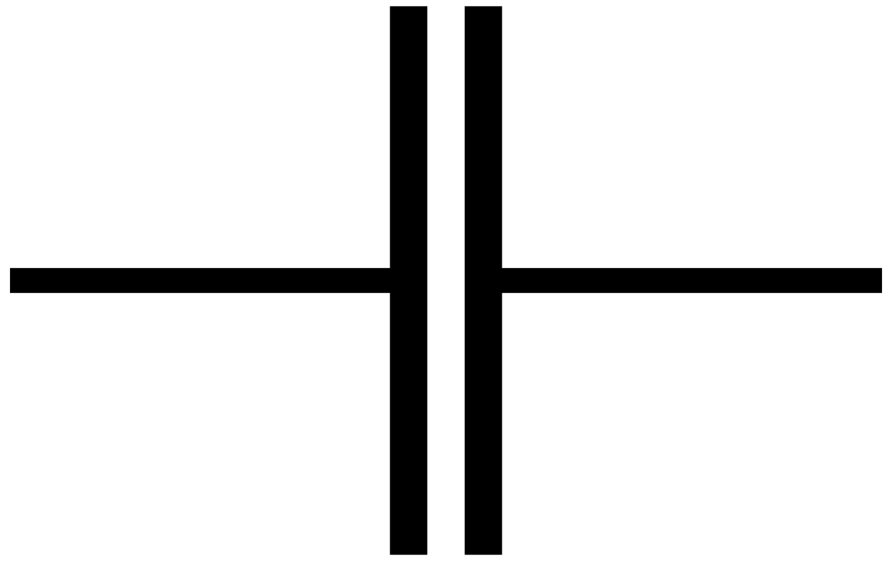
Resistor

Battery



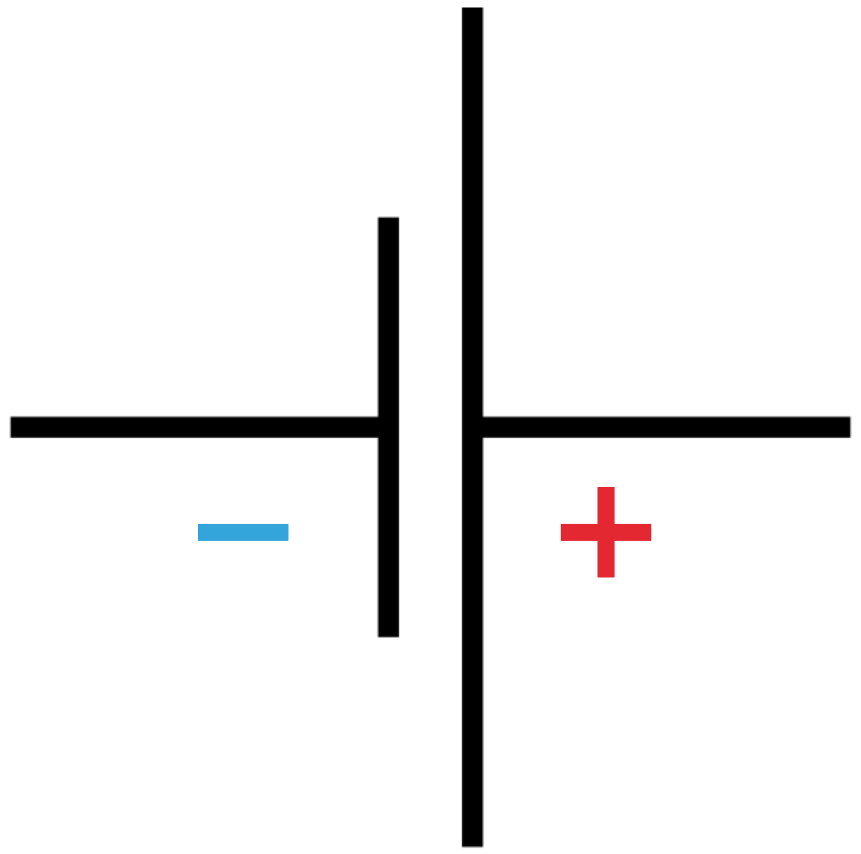
CIRCUIT DIAGRAMS

Capacitor



Resistor

Battery



THE MICRO-MACRO CONNECTION

Microscopic View

$\overline{v} = uE$

$i = nA\overline{v} = nAuE$

Macroscopic View

$J = \sigma E$

$I = \left| q \right| nA\overline{v} = \frac{1}{R}\Delta V$

THE MICRO-MACRO CONNECTION

Microscopic View

Node Rule:

$$\sum_{\text{node}} i_{\text{in}} = \sum_{\text{node}} i_{\text{out}}$$

Loop Rule:

$$\sum_{\text{loop}} \Delta V = 0$$

Macroscopic View

Node Rule:

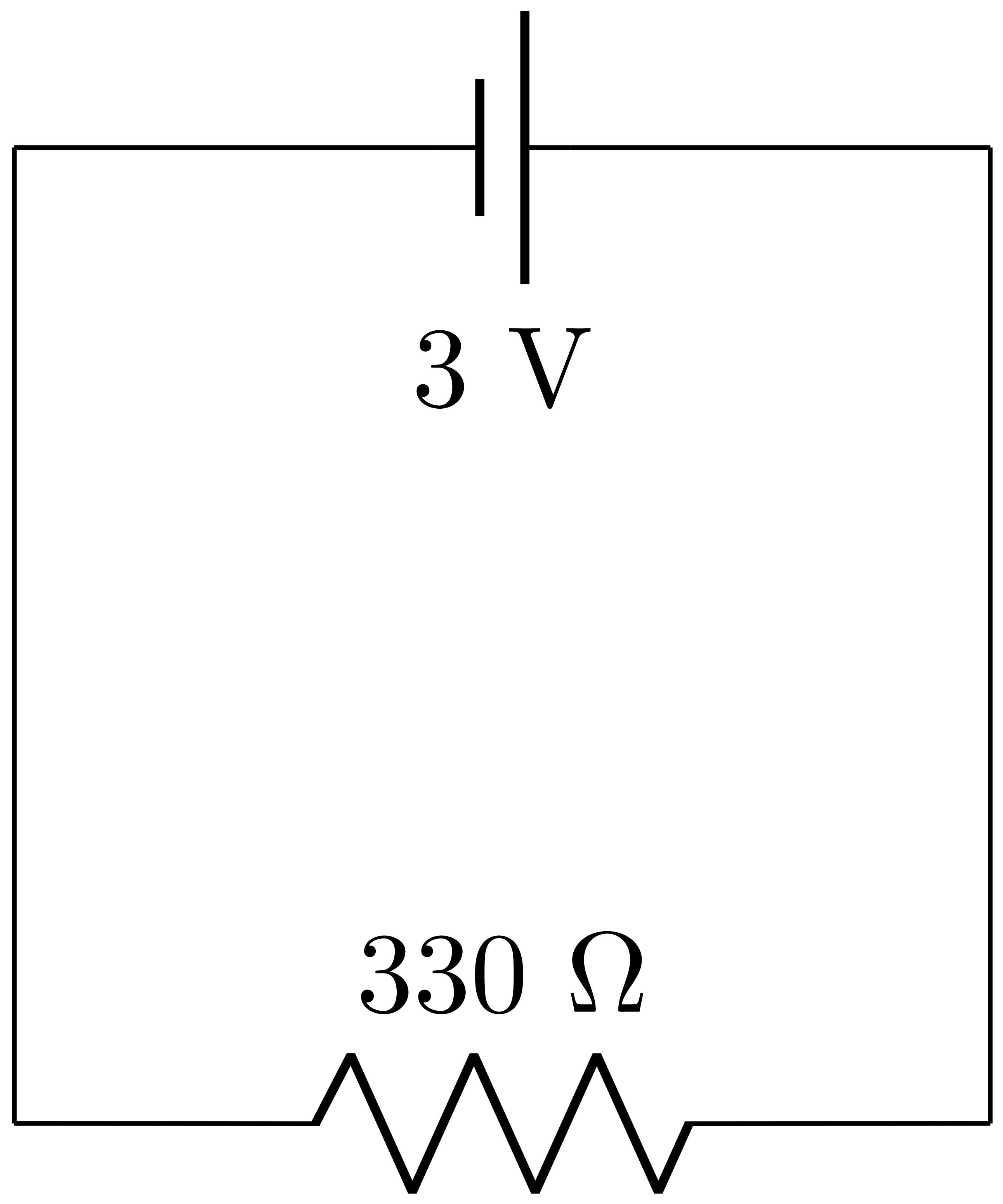
$$\sum_{\text{node}} I_{\text{in}} = \sum_{\text{node}} I_{\text{out}}$$

Loop Rule:

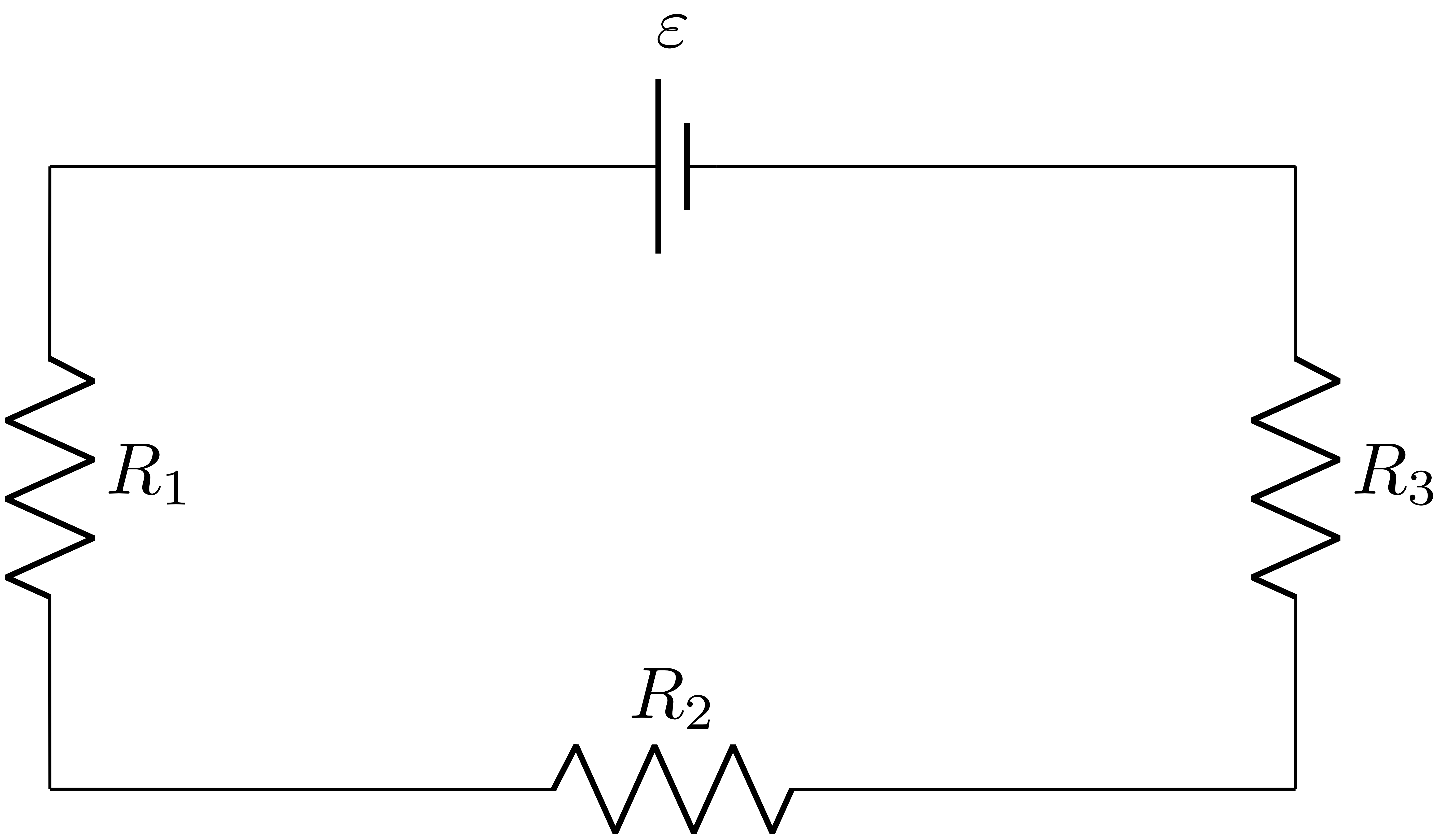
$$\sum_{\text{loop}} \Delta V = 0$$

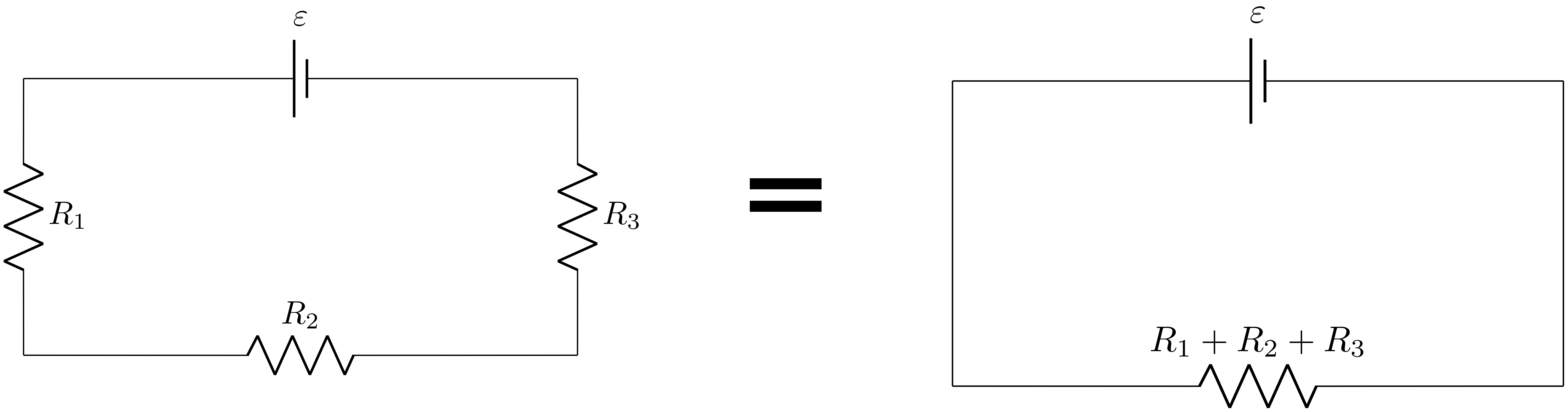
EXAMPLE

Current through and voltage across the resistor?



MULTIPLE RESISTORS IN SERIES




ε
R1
R3
R2
ε
R1 + R2 + R3

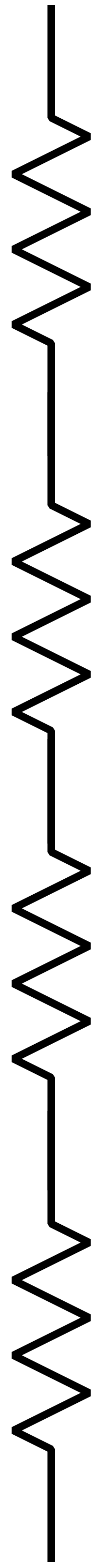
CIRCUIT ELEMENTS CONNECTED IN SERIES

$$I_1 = I_2 = I_3 = \ldots = I_n = I$$

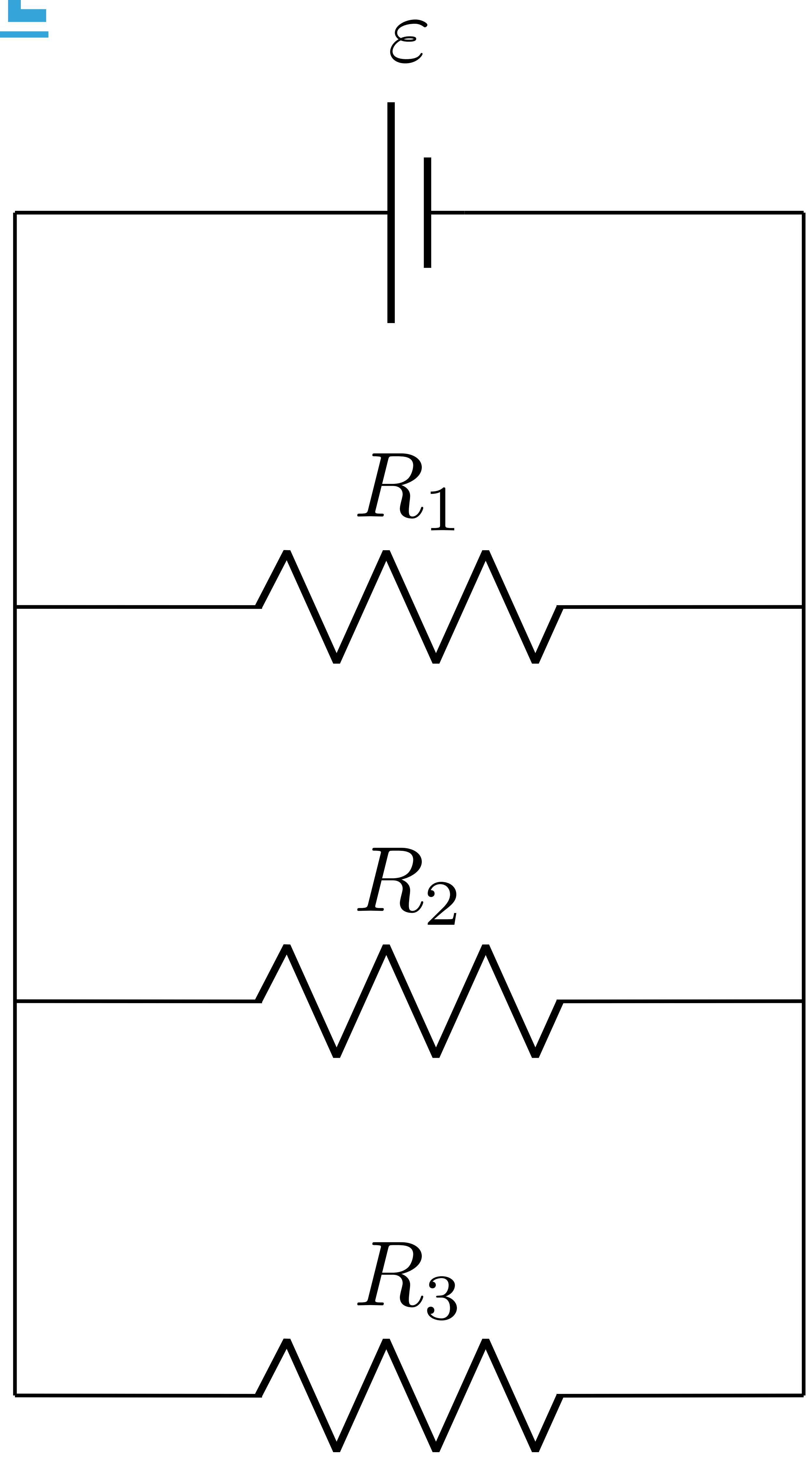
$$\Delta V = \Delta V_1 + \Delta V_2 + \Delta V_3 + \ldots + \Delta V_n$$

$$I = \frac{\Delta V}{R_1 + R_2 + R_3 + \ldots + R_n} = \frac{\Delta V}{R_{\text{eqiv}}}$$

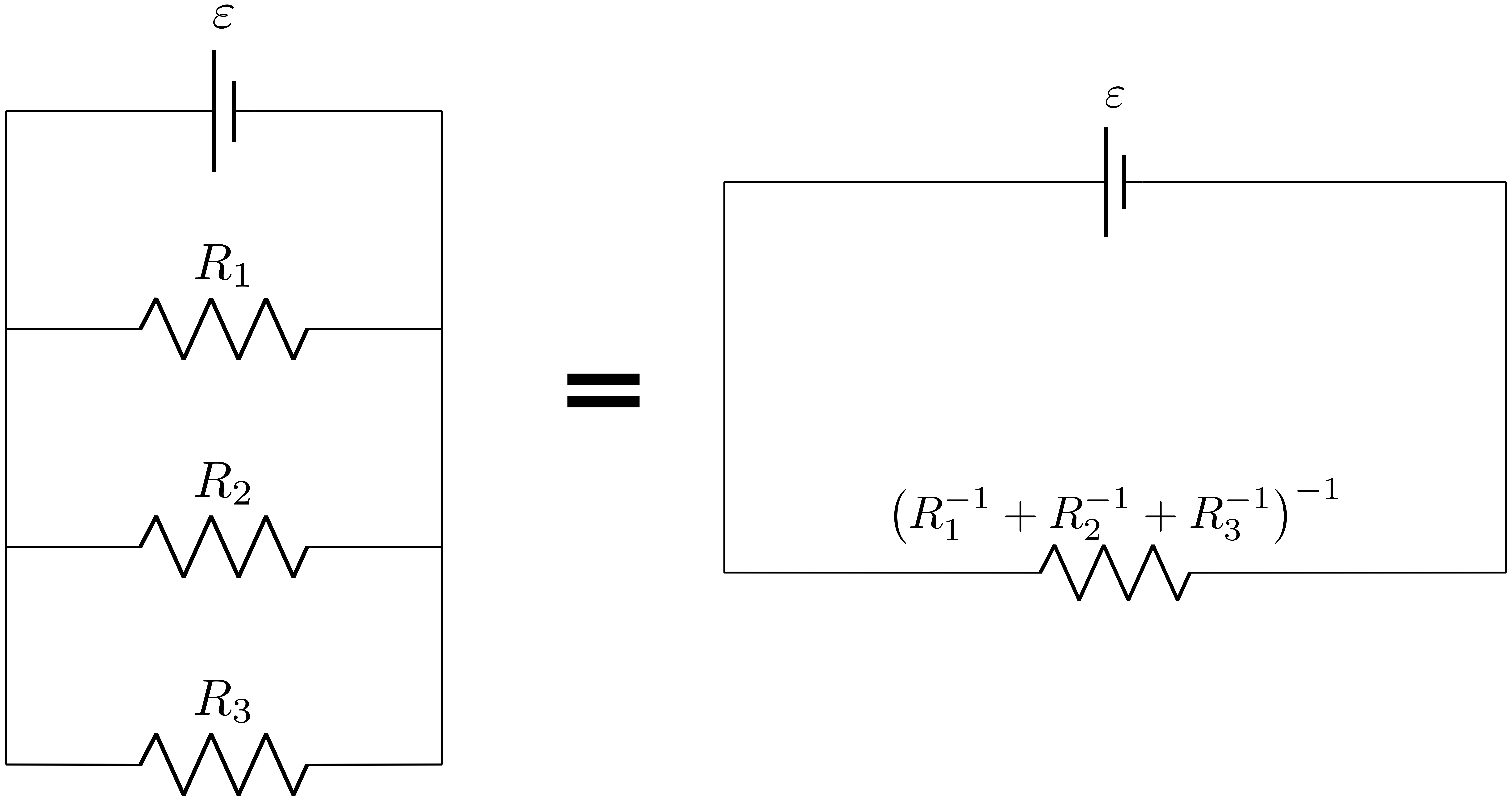
$$R_{\text{equiv}} = R_1 + R_2 + R_3 + \ldots + R_n$$



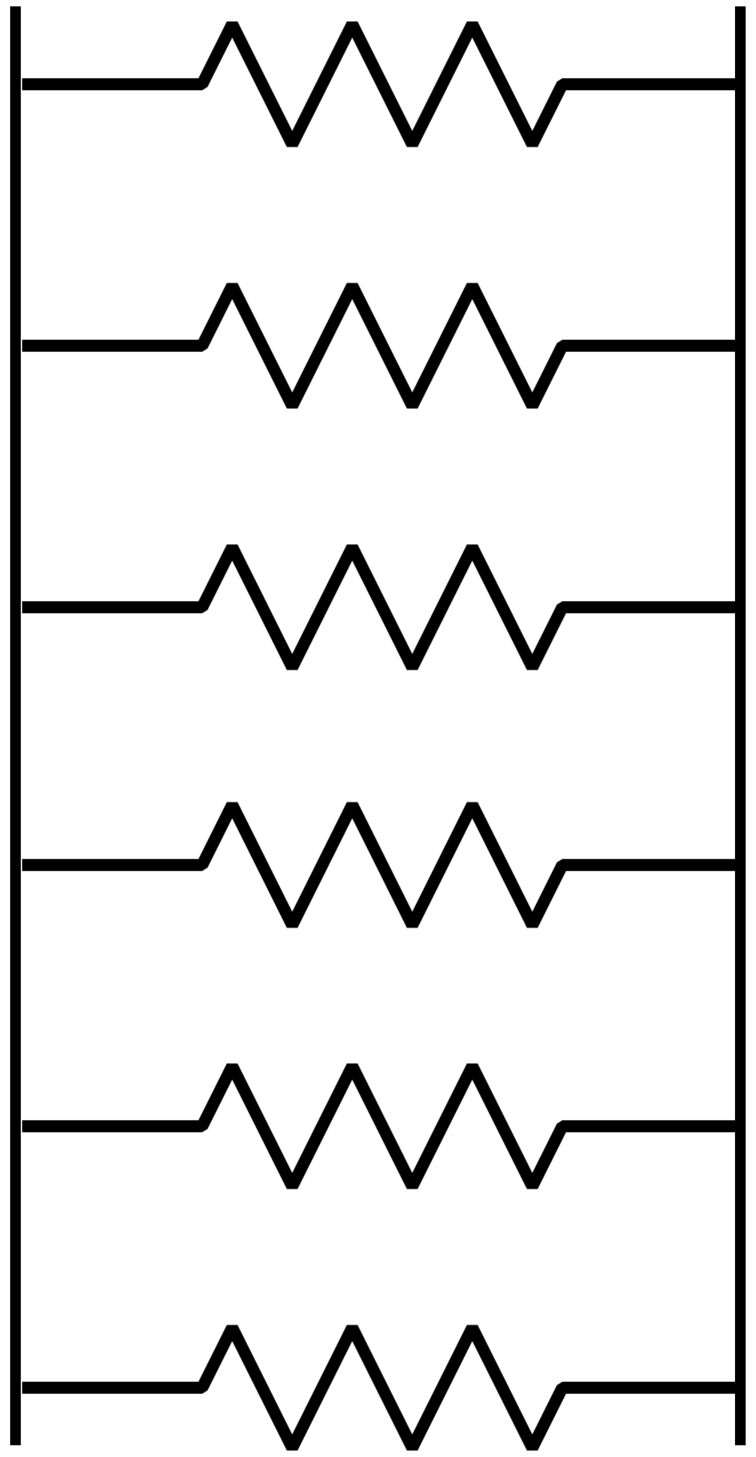
MULTIPLE RESISTORS IN PARALLEL



MULTIPLE RESISTORS IN PARALLEL



CIRCUIT ELEMENTS CONNECTED IN PARALLEL



$$I = I_1 + I_2 + I_3 + \ldots + I_n$$

$$\Delta V = \Delta V_1 = \Delta V_2 = \Delta V_3 = \ldots = \Delta V_n$$

$$I = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots + \frac{1}{R_n}\right)\varepsilon = \frac{\varepsilon}{R_{\text{equiv}}}$$

$$\frac{1}{R_{\text{equiv}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots + \frac{1}{R_n}$$