Vectors:

What is a vector?

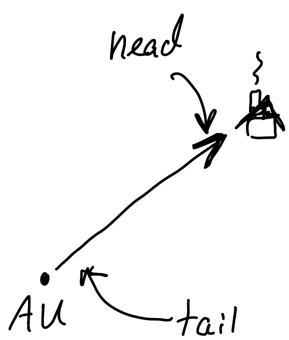
- Quantity with direction + magnitude

My house is 5 miles NE of AU

magnitude — 5 miles; direction — NE

The plane is moving with a speed of 300 m/s to the west.

We use arrows to represent vectors.



Position vector:

- the location of the object (head)
- the location of the reference point (tail)

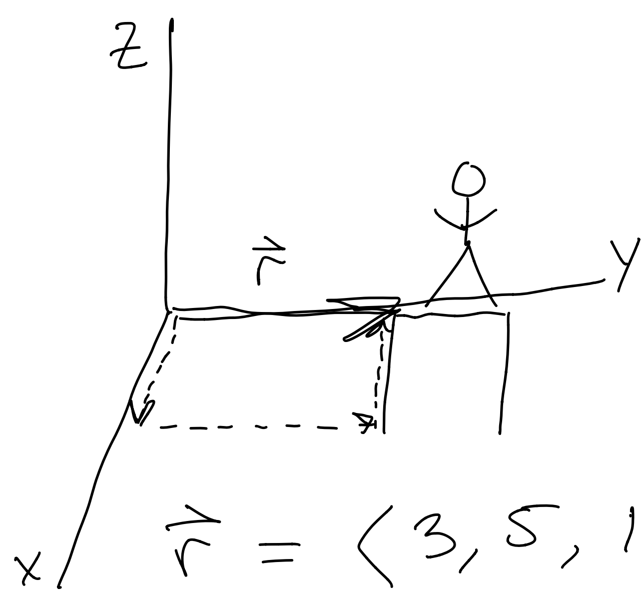
We live in a 3D Universe

Front - back

left - right

up - down

So we need three numbers to specify a direction/location



$$\vec{r} = \langle 3, 5, 1 \rangle \, m$$

$$\vec{r} = (3\hat{x} + 5\hat{y} + \hat{z})\, m$$

Adding vectors

At some point in time,
a hockey puck has momentum

$$\vec{p}_1 = \langle 3, 4 \rangle \; kg\frac{m}{s}$$

A second puck has momentum

$$\vec{p}_2 = \langle -1, 5 \rangle \; \frac{kg\,m}{s}$$

What is the total momentum of the system of two pucks?

$$\vec{p} = \vec{p}_1 + \vec{p}_2 = \langle 3, 4 \rangle \frac{kg\,m}{s} + \langle -1, 5 \rangle \frac{kg\,m}{s}$$

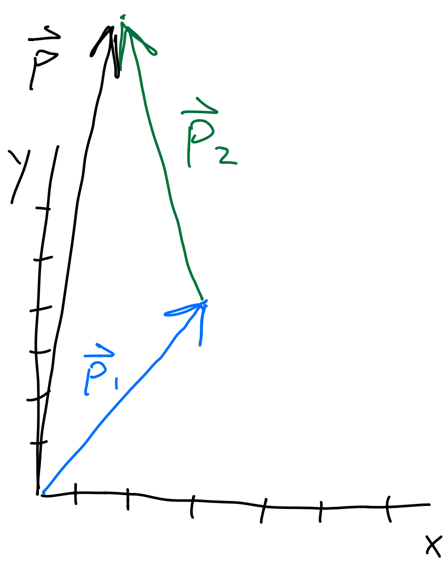
$$\vec{p} = \langle 2, 9 \rangle \frac{kg\,m}{s}$$

Graphically

1) Draw 1^{st} vector

2) Draw 2^{nd} vector w/ tail from head of first

3) Draw final vector with tail @ origin, head @ head of 2^{nd}



$$\vec{p} = \langle 2, 9 \rangle \frac{kg\, m}{s}$$

What is $|\vec{p}|$? (magnitude?)

$$\vec{A} = \langle A_x, A_y, A_z \rangle$$

$$|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

$$|\vec{p}| = \sqrt{2^2 + 9^2} \approx 9.22 \ \frac{kg\,m}{s}$$

What is the direction of $\vec{p}$?

$\hat{p}$?

Direction vectors (unit vectors)

- point in same direction
- mag of 1

$$\vec{p} = |\vec{p}|\,\hat{p} \longrightarrow \hat{p} = \frac{\vec{p}}{|\vec{p}|}$$

$$\hat{p} = \frac{\langle 2, 9 \rangle \frac{kg\,m}{s}}{9.22 \frac{kg\,m}{s}}$$

$$\hat{p} = \langle 0.22, 0.98 \rangle$$

$$|\hat{p}| = 1$$

Q: in a certain coordinate system, an electron is at position

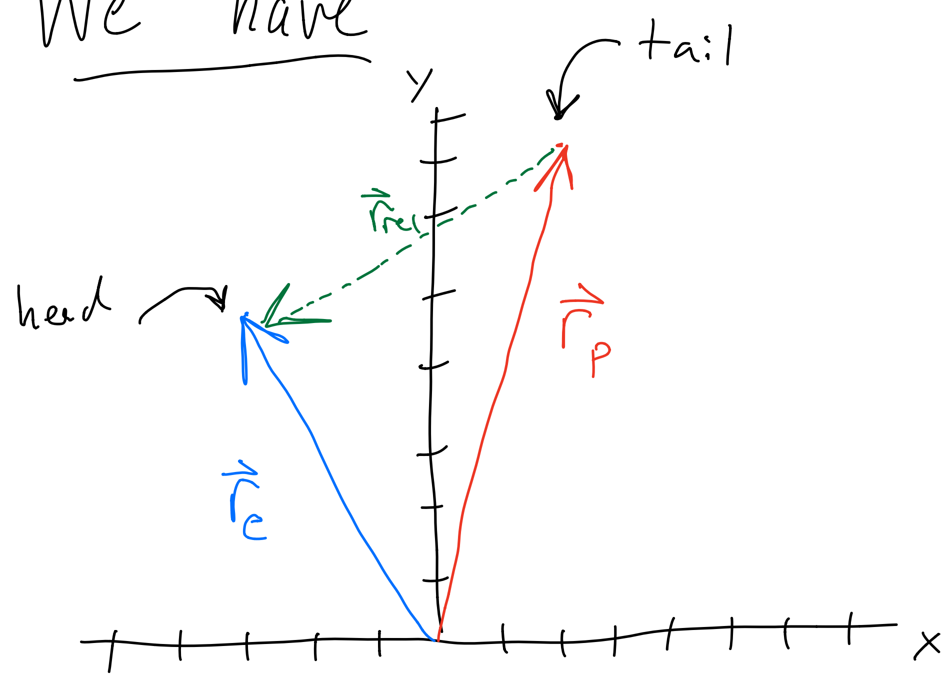
$$\vec{r}_e = \langle -3, 5 \rangle \text{ m}$$

A proton is at

$$\vec{r}_p = \langle 2, 7 \rangle$$

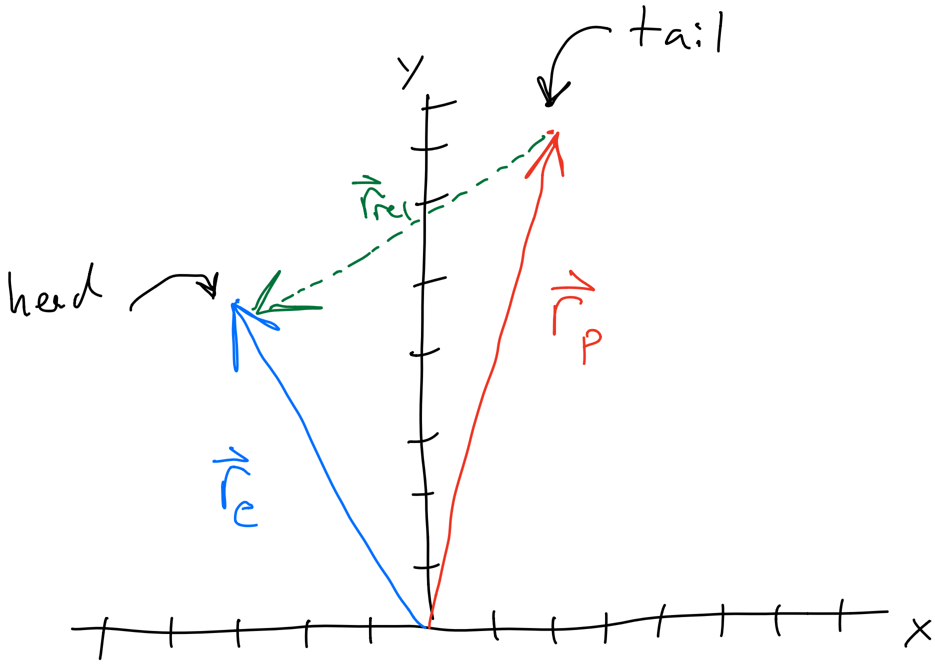
What is the position of the electron relative to the proton?

We have



position of e, relative to position of p.

- New vector, $\vec{r}_{rel}$, originates from the proton
- Ends at the electron



how do we find components of $\vec{r}_{rel}$?

$$\vec{r}_e = \vec{r}_p + \vec{r}_{rel}$$

$$\vec{r}_{rel} = \vec{r}_e - \vec{r}_p$$

Position of $\vec{B}$ rel to $\vec{A}$

is $\vec{B} - \vec{A}$

$$\vec{r}_{rel} = \langle -3, 5 \rangle m - \langle 2, 7 \rangle m$$

$$\vec{r}_{rel} = \langle -5, -2 \rangle m$$

Can we do multiplication w/ vectors?

Yes!

1) Scalar multiplication

$$K\vec{r} = K(r_x\hat{x} + r_y\hat{y} + r_z\hat{z})$$

$$= Kr_x\hat{x} + Kr_y\hat{y} + Kr_z\hat{z}$$

Does direction change?

No, only magnitude

$\vec{r} = \langle 4, 2, 0 \rangle$, $\frac{1}{2}\vec{r} = \langle 2, 1, 0 \rangle$

Can we multiply 2 vectors together?

A box is displaced by $\Delta\vec{r} = \langle 2, 4 \rangle$ m

While const $\vec{F} = \langle 3, -1 \rangle$ N is applied

$$W = \vec{F} \cdot \Delta\vec{r}$$

$$= (3)(2) + (-1)(4)$$

$$W = 6 - 4 = 2 \text{ J}$$

$$\vec{A} = \langle A_x, A_y, A_z \rangle$$

$$\vec{B} = \langle B_x, B_y, B_z \rangle$$

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$