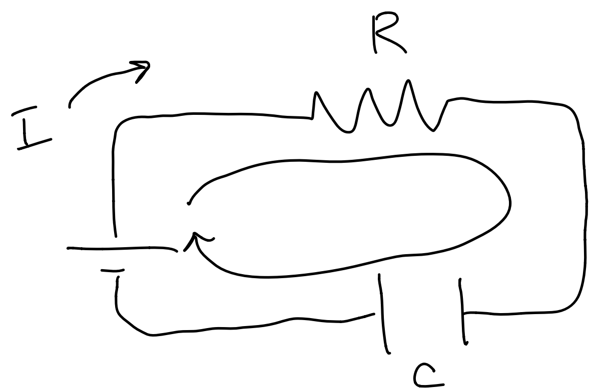
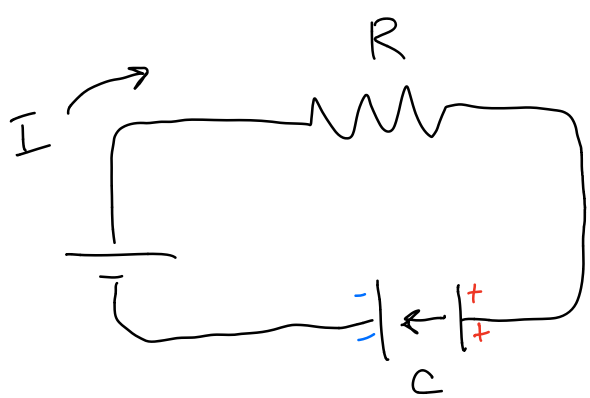
RC circuit: Charging



$$\mathcal{E} - IR + \Delta V_{cap} = 0$$

ΔV_{cap}? $$|\Delta V_{cap}| = \frac{Q}{C}$$



$$\Delta V = -\vec{E} \cdot \Delta\vec{r} < 0$$

$$\Delta V_{cap} = -\frac{Q}{C}$$

$$\mathcal{E} - IR - \frac{1}{C}Q = 0$$

$$I = \frac{\mathcal{E} - \frac{Q}{C}}{R}$$

As Q increases, I decreases

Q increases until $\frac{Q}{C} = \mathcal{E}$

$$Q = C\mathcal{E}$$

When $I = 0$

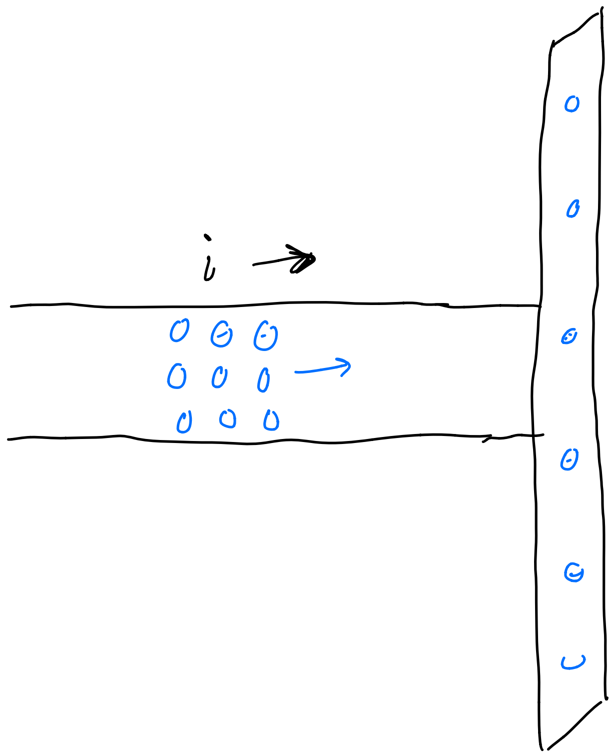
$$\mathcal{E} - IR - \frac{1}{C}Q = 0 \longrightarrow \mathcal{E} - \frac{1}{C}Q = 0$$

$$\mathcal{E} = \frac{Q}{C}$$

Q increases until $\Delta V_{cap} = \Delta V_{BAT}$

What dictates the time?

How are Q + I related?



How much charge ΔQ is added in a time Δt?

Answer $|q| i = I$

$$I = \frac{\Delta Q}{\Delta t}$$

$$I = \frac{dQ}{dt}$$

$$\mathcal{E} - IR - \frac{1}{C}Q = 0$$

$$\frac{d}{dt}$$

$$-R\frac{dI}{dt} - \frac{1}{C}\frac{dQ}{dt} = 0$$

$$\frac{dI}{dt} = -\frac{1}{RC} I$$

Separable

$$\frac{1}{I}dI = -\frac{1}{RC}dt$$

$$\int \frac{1}{I}dI = -\frac{1}{RC}\int dt$$

$$\ln(I) + A = -\frac{1}{RC}(t + B)$$

$$\ln(I) = -\frac{1}{RC}t - \frac{1}{RC}B - A$$

$$\ln(I) = -\frac{1}{RC}t + D$$

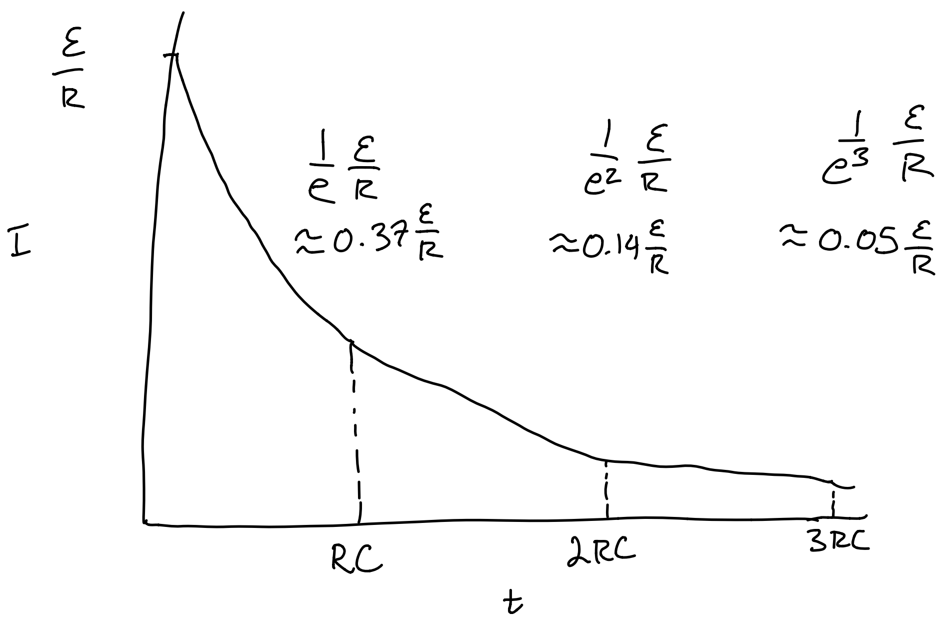
$$e^{\ln(I)} = I = e^{-\frac{1}{RC}t} e^{D}$$

$$I = e^{D} e^{-\frac{1}{RC}t}$$

Charging: $I(0) = \frac{\mathcal{E}}{R}$

$I(0) = \frac{\mathcal{E}}{R} = e^{D}$, $D = \ln\left(\frac{\mathcal{E}}{R}\right)$

$$\boxed{I(t) = \frac{\mathcal{E}}{R} e^{-\frac{t}{RC}}}$$



RC: Time constant

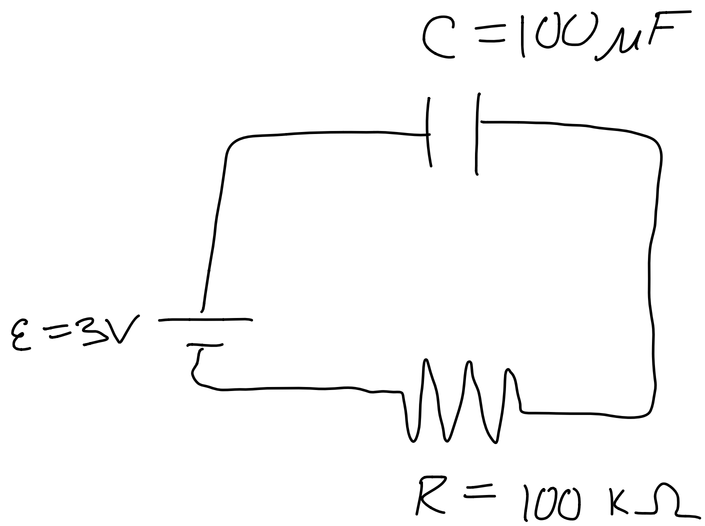
$1\,\Omega \cdot 1\,F = 1s$

LAB Example:

$100\,\Omega \cdot 100\,\mu F = (10^{2})(10^{-4})\,\Omega F = 10^{-2}\,s$

$100{,}000\,\Omega \cdot 470\,\mu F = (10^{5})(4.7\times10^{-4})\,\Omega F = 47\,s$

Example:



Charge on the capacitor after 5 s?

$$\mathcal{E} - \frac{1}{C}Q - IR = 0$$

$$Q = C(\mathcal{E} - IR)$$

$$I(t) = \frac{\mathcal{E}}{R} e^{-\frac{t}{RC}}, \quad Q(t) = C\left(\mathcal{E} - \frac{\mathcal{E}}{R} R e^{-\frac{t}{RC}}\right)$$

$$Q(t) = C\mathcal{E}\left(1 - e^{-t/RC}\right)$$

$$RC = 10^5 \times 10^{-4}\ \Omega F = 10\ s$$

$$I(t=5) = \frac{\mathcal{E}}{R} e^{-\frac{5}{10}} = \frac{\mathcal{E}}{R} e^{-\frac{1}{2}} = \frac{3}{10^5} \frac{1}{\sqrt{e}}$$

$$I = 18.2\,\mu A$$

$$Q = C(\mathcal{E} - IR)$$

$$= 10^{-4}(3 - 1.82\times10^{-5}\times10^{5})$$

$$= 10^{-4}(3 - 1.82)$$

$$Q = 1.18\times10^{-4}\ C = 118\,\mu C$$

How much power dissipated in the resistor?

$$P = IV, \quad V = IR$$

$$P = I^2 R = (1.82\times10^{-5})^2(10^5) = 33\,\mu W$$

Final charge on the capacitor?

Technically it's never "done" charging

After $t = 5RC$: $Q = 99\%$

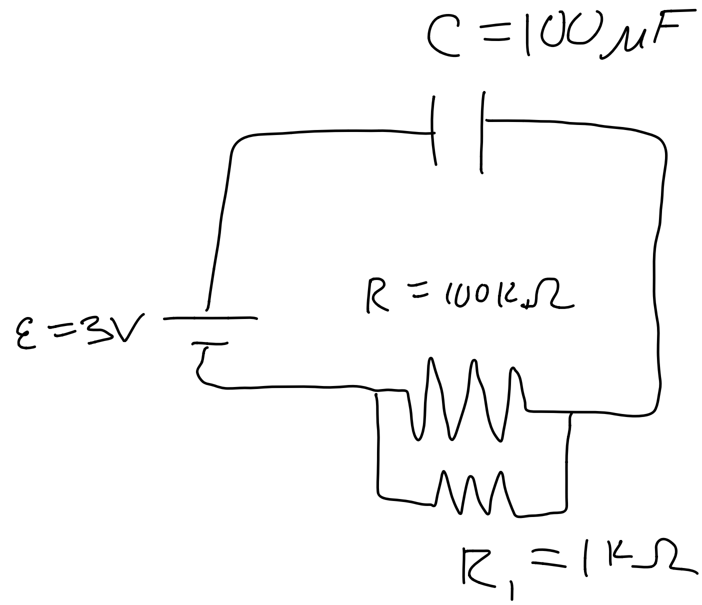
$t = 10RC$: $Q = 99.99\%$

$t \to \infty$ $\quad Q \to 100\%$

$$Q = C\varepsilon$$

$$Q = 3 \times 10^{-4}\ C$$

$$Q = 300\ \mu C$$



When will the capacitor be 50% charged?

$$\frac{1}{R_{eq}} = \frac{1}{10^5} + \frac{1}{10^4}$$

$$R_{eq} = 9091\ \Omega$$

$$Q(t) = C\mathcal{E}\left(1 - e^{-t/RC}\right)$$

$$C\mathcal{E} = Q_{final}$$

$$Q(t) = Q_{final}\left(1 - e^{-t/RC}\right)$$

$$\frac{Q}{Q_{final}} = 1 - e^{-t/RC} = \frac{1}{2}$$

$$e^{-t/RC} = \frac{1}{2}$$

$$\frac{-t}{RC} = \ln\left(\frac{1}{2}\right)$$

$$t = -RC\ln\left(\frac{1}{2}\right)$$

$$t = RC\ln(2)$$

$$RC = (9091)(10^{-4}) = 0.91\ s$$

$$t = (0.91)(0.69)$$

$$t = 0.63\ s$$

Final Q?

Still $Q = C\mathcal{E} = 300\,\mu C$

R does not affect final charge