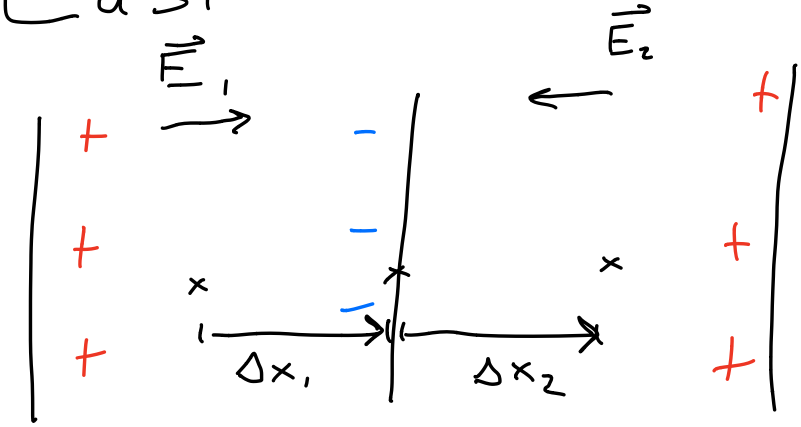
Lecture Outline

- Potential of a continuously varying field
  - Integral of E dot dr
  - Example: point charge
- Path integration
  - Path chosen does not matter
    - Example
  - Potential difference around closed path is 0
    - Example
    - Energy conservation
- Potential at a single point
  - Potential difference between point and infinity
  - Charged ring potential

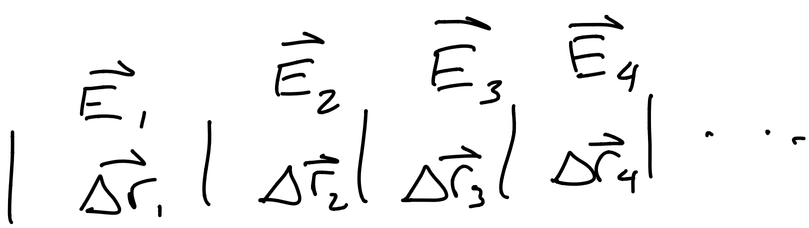
Last lecture



$$\Delta V = -\vec{E}_1 \cdot \Delta \vec{x}_1 - \vec{E}_2 \cdot \Delta x_2$$

$$= -\langle E_{1x}, 0, 0\rangle \cdot \langle \Delta x_1, 0, 0\rangle - \langle -E_{2x}, 0, 0\rangle \cdot \langle \Delta x_2, 0, 0\rangle$$

$$\Delta V = -E_{1x}\Delta x_1 + E_{2x}\Delta x_2$$



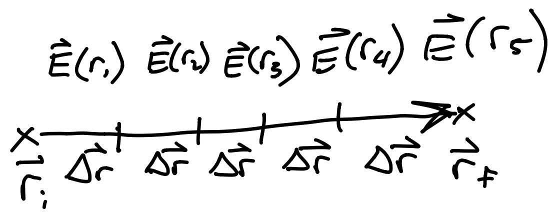
$$\Delta V = -\vec{E}_1 \cdot \Delta\vec{r}_1 - \vec{E}_2 \cdot \Delta\vec{r}_2 - \vec{E}_3 \cdot \Delta\vec{r}_3 \ \ldots$$

$$\Delta V = \sum_{i}^{N} -\vec{E}_i \cdot \Delta\vec{r}_i$$

What if $\vec{E}$ is continuously changing?

$$\vec{E} = \vec{E}(r)$$

Ex: $$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$$



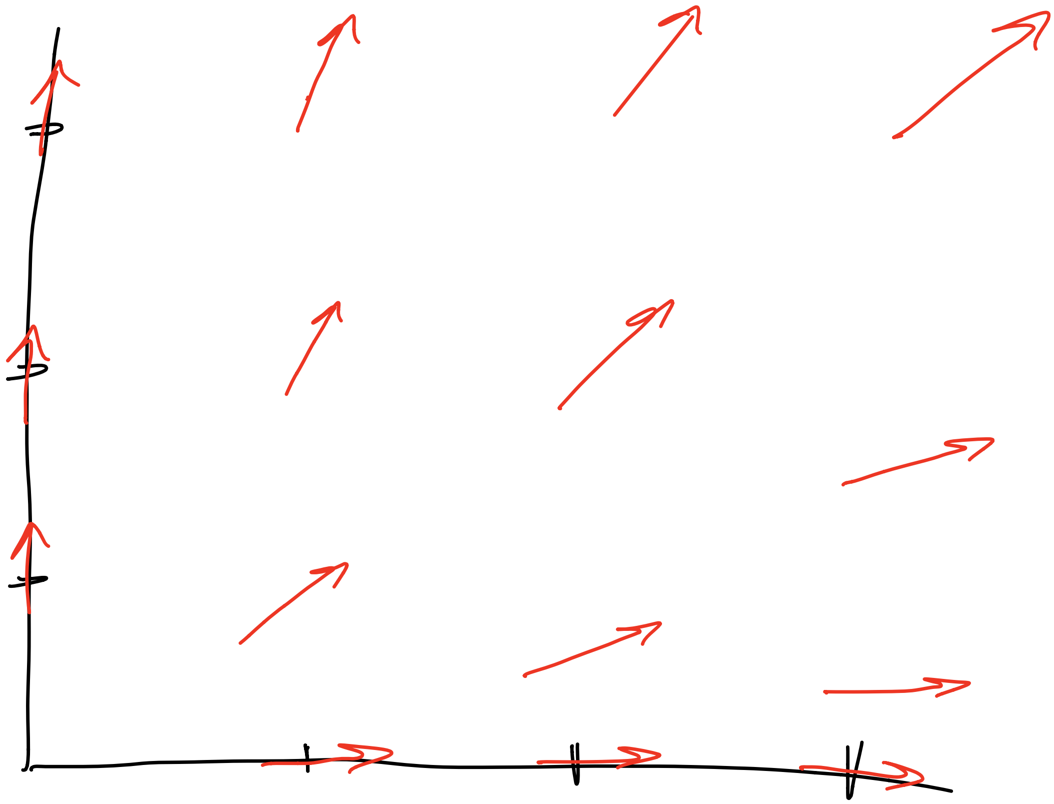
$$\Delta V \approx \sum_i^N -\vec{E}(r_i)\Delta\vec{r}$$

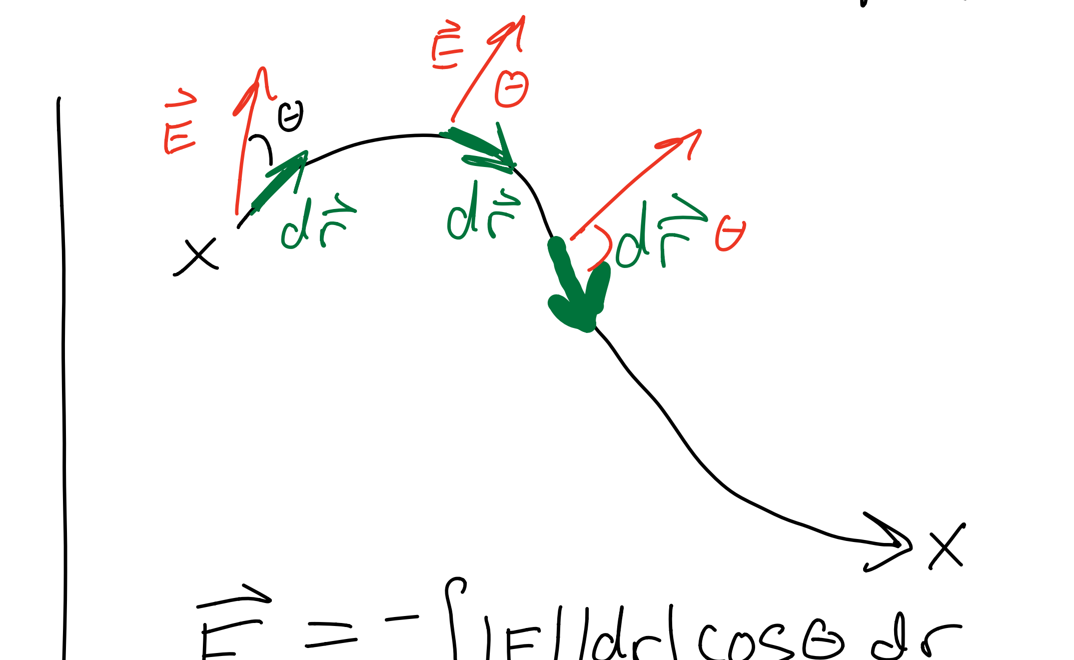
$$\Delta V = \lim_{\Delta r \to 0} \sum_i^N -\vec{E}(r_i)\Delta\vec{r} = -\int_{r_i}^{r_f} \vec{E}\cdot d\vec{r}$$

General definition for ΔV

$$\boxed{\Delta V = -\int_{r_i}^{r_f} \vec{E}\cdot d\vec{r}}$$

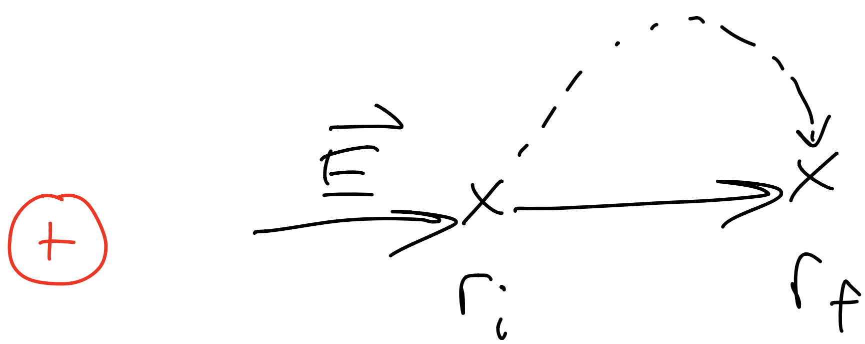
Path Integral





$$\vec{E} = -\int |E||dr| \cos\theta \, dr$$

Point charge



choose a path $\parallel$ to $\vec{E}$

$$\Delta V = -\int_{r_i}^{r_f} \vec{E} \cdot d\vec{r} = -\int_{r_i}^{r_f} E\,dr\cos\theta$$

$$\theta = 0$$

$$\Delta V = -\int_{r_i}^{r_f} E(r)\,dr$$

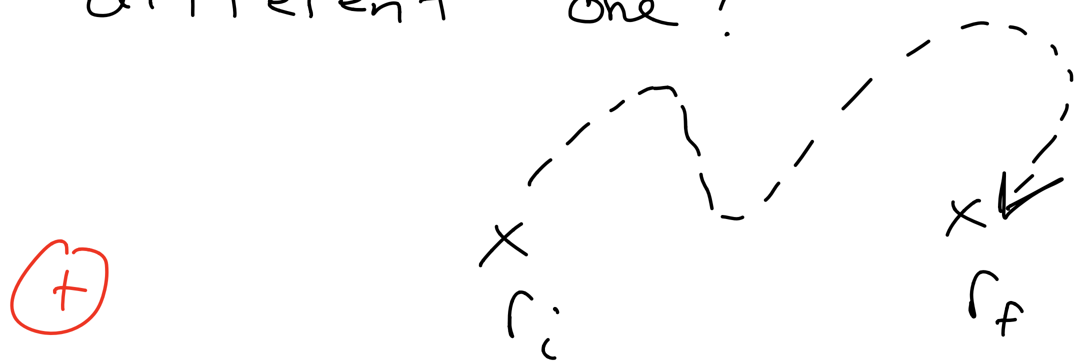
$$\Delta V = -\int_{r_i}^{r_f} \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}\,dr$$

$$\Delta V = \frac{1}{4\pi\epsilon_0} q\left[\frac{1}{r_f} - \frac{1}{r_i}\right]$$

if $r_i < r_f$, $\Delta V < 0$ ✓

We chose the path $\parallel$ to $\vec{E}$

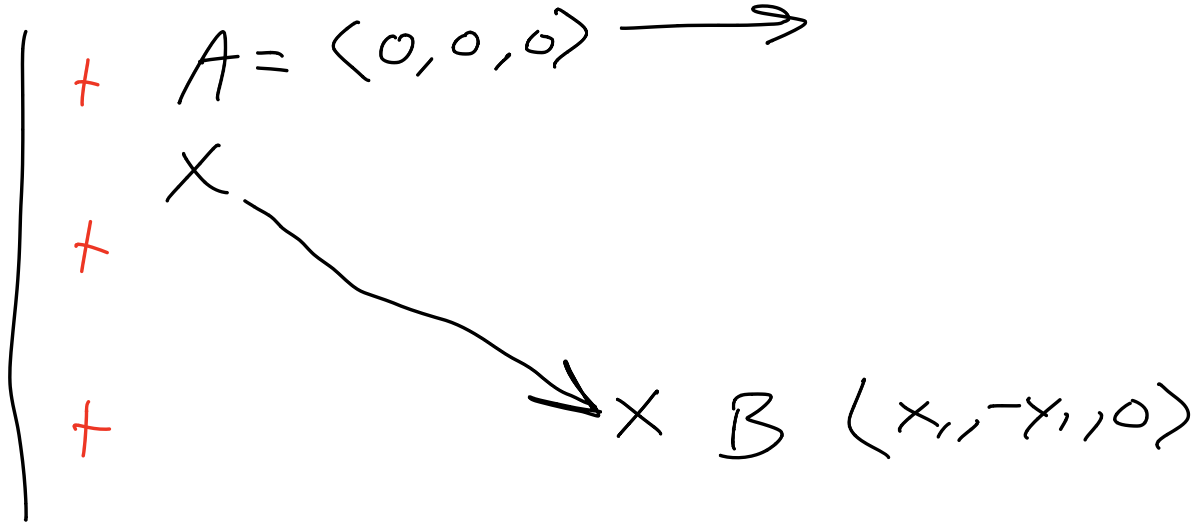
What if we chose a different one?



Same answer

E_x

$$\vec{E} = \langle E_x, 0, 0 \rangle$$



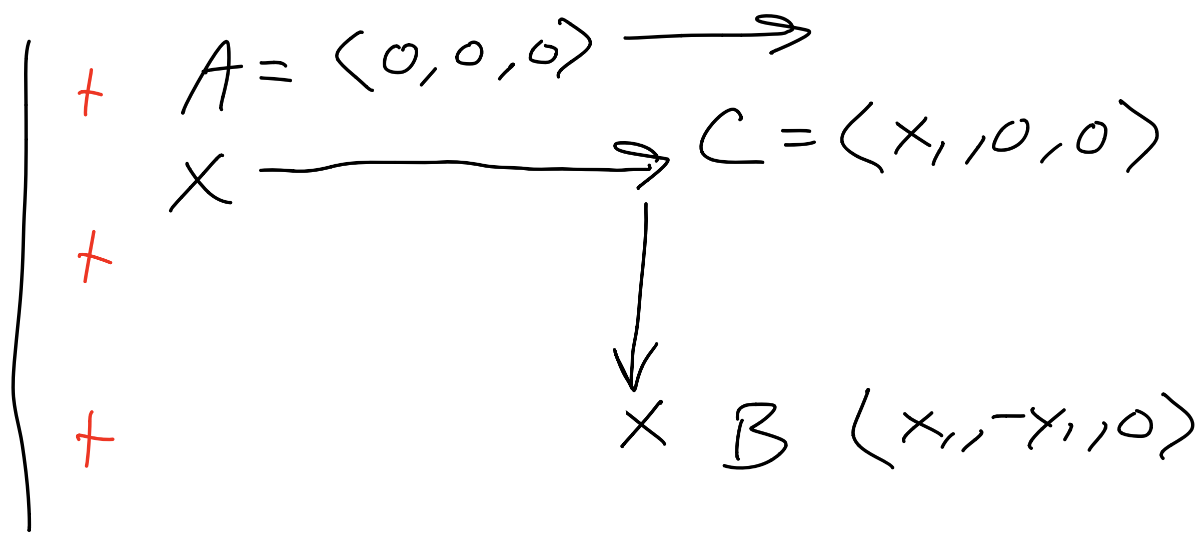
$$\Delta \vec{r} = \vec{B} - \vec{A} = \langle x_1, -y_1, 0 \rangle$$

$$\Delta V = -\vec{E} \cdot \Delta \vec{r}$$

$$= -E_x x_1 - 0(-y_1) + 0$$

$$\Delta V = -E_x x_1$$

$$\vec{E} = \langle E_x, 0, 0 \rangle$$



$$\Delta V = \Delta V_{AC} + \Delta V_{CB}$$

$$= -\langle E_x, 0, 0 \rangle \cdot \langle x_1, 0, 0 \rangle$$

$$- \langle E_x, 0, 0 \rangle \cdot \langle 0, -y_1, 0 \rangle$$

$\Delta V = -E_x x_1$ Same

Round trip $\Delta V = 0$

Suppose $\Delta V > 0$

then, for an electron,

$\Delta U = -e\Delta V$

$\Delta K > 0$

Potential @ one location

- Question:
  - What is the potential energy of two charges who are infinitely far apart?
  - It should be zero (zero electric field)

if $B = \infty$

$$\Delta V = V_A - V_B$$
$$= V_A - V_\infty$$
$$\Delta V = V_A - 0$$

"Potential at a single point" just means

$$\Delta V = V - V_\infty, \quad V_\infty = 0$$

In this case, just call it V

$$\Delta U \rightarrow U = qV$$

Ex

$$\Delta V = V_{r_f} - V_{r_i}$$

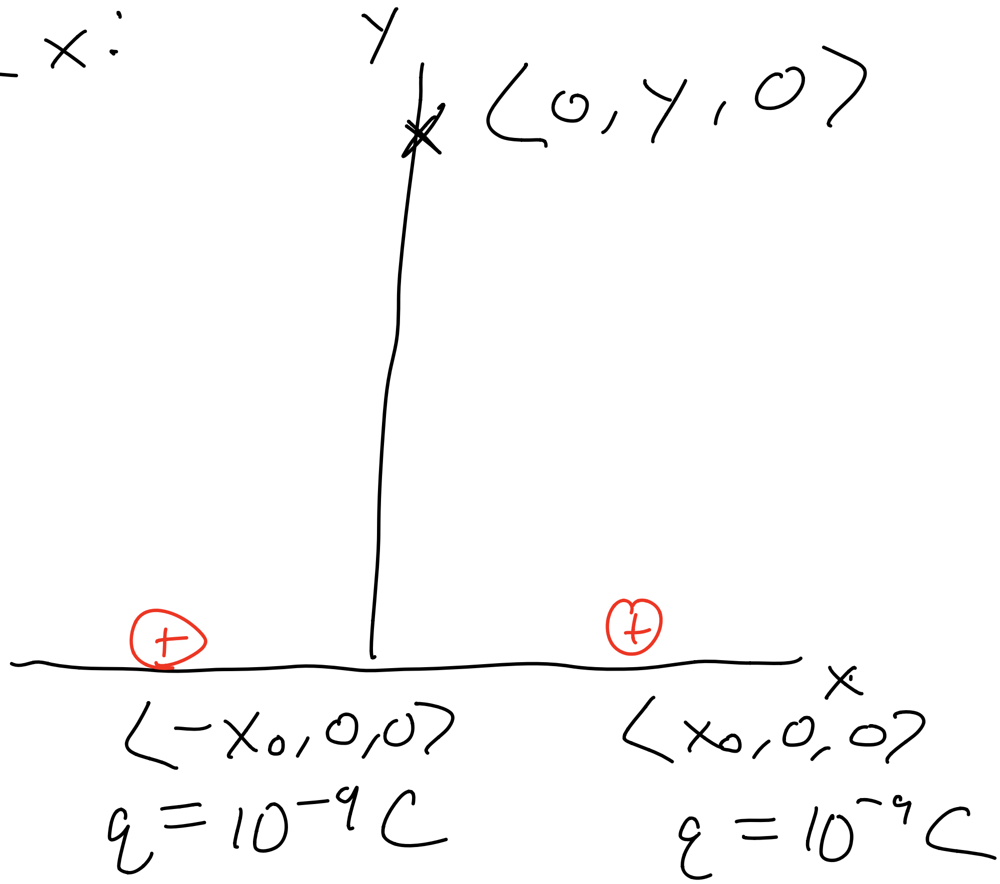
$$\Delta V = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{r_f} - \frac{1}{r_i}\right]$$

$$r_i \longrightarrow \infty$$

$$\Delta V = V = \frac{q}{4\pi\epsilon_0}\frac{1}{r_f}$$

$$V(r) = \frac{q}{4\pi\epsilon_0}\frac{1}{r}$$

Ex:



$$V = V_1 + V_2$$

$$V = \frac{q}{4\pi\varepsilon_0}\frac{1}{r_1} + \frac{q}{4\pi\varepsilon_0}\frac{1}{r_2}$$

$$r_1 = \sqrt{x_0^2 + y^2}$$

$$r_2 = \sqrt{x_0^2 + y^2}$$

$$V = \frac{2q}{4\pi\epsilon_0} \frac{1}{\sqrt{x_0^2 + y^2}}$$

$$E_y = -\frac{d}{dy} V$$

$$\frac{d}{dy}\left(x_0^2 + y^2\right)^{-\frac{1}{2}}$$

$$= -\frac{1}{2}(2y)\left(x_0^2 + y^2\right)^{-3/2}$$

$$E_y = \frac{2q}{4\pi\epsilon_0} \frac{y}{\left(x_0^2 + y^2\right)^{3/2}}$$