Motivate the chapter

Core concept of the class

Fields

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r}$$

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \ldots$$

$$= \frac{1}{4\pi\epsilon_0} \int \frac{dq}{|\vec{r}|^2} \hat{r}$$

$$\vec{B} = \frac{\mu_0}{4\pi} \int \frac{I \, d\vec{l} \times \hat{r}}{|\vec{r}|^2}$$

$$\vec{B} = \vec{B}_1 + \vec{B}_2 + \vec{B}_3 + \ldots$$

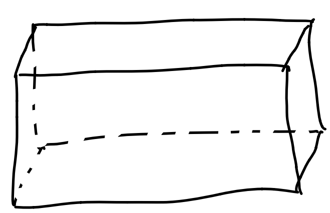
Given q or I, how to find E/B?

Gauss' Law

- Instead of using Q to find E, we use E to find Q

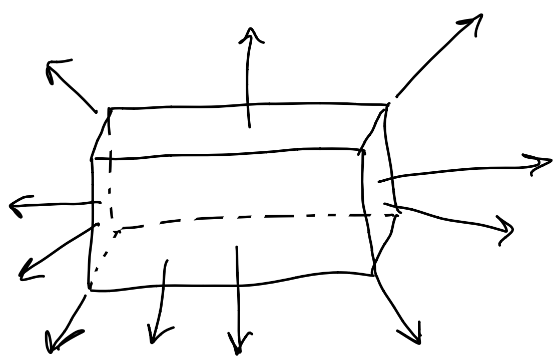
- Sounds backwards & unhelpful, but stay with me

A thought experiment

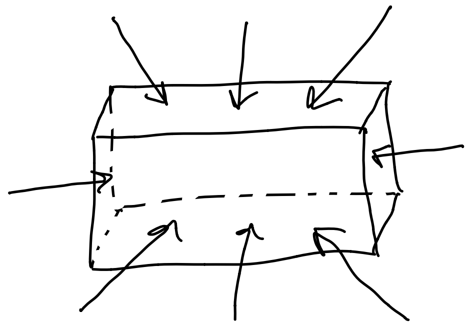


(Draw & copy!)

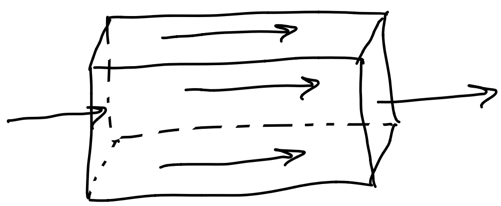
Measure $\vec{E}$ along the surface

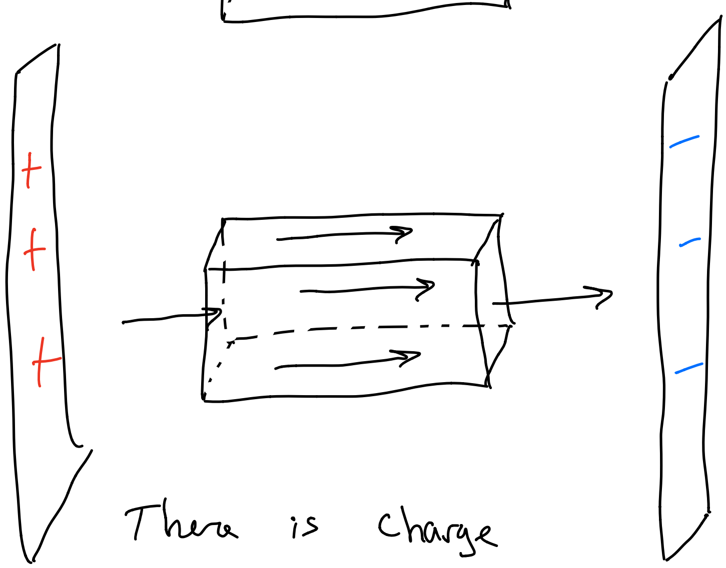


There must be some positive charge inside!



Probably some negative charge



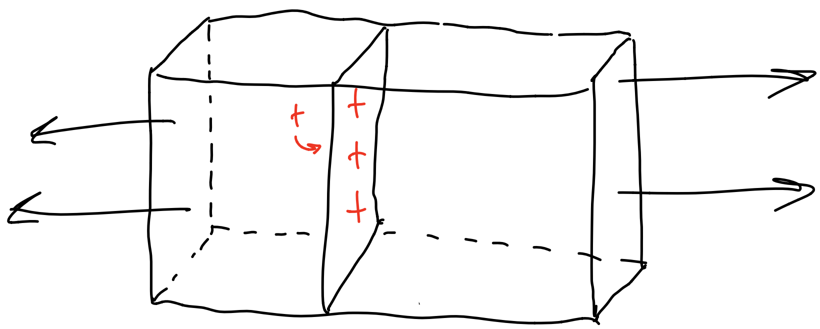


There is charge
somewhere,
but not in
the
box

Basic idea:

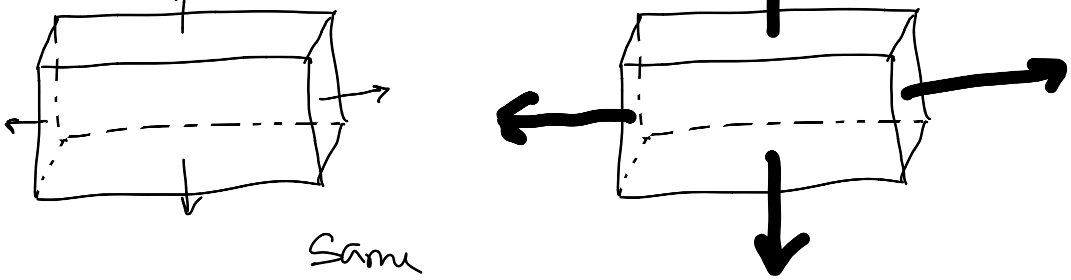
Pattern of $\vec{E}$ on a surface $\Rightarrow$ q inside surface

Example:



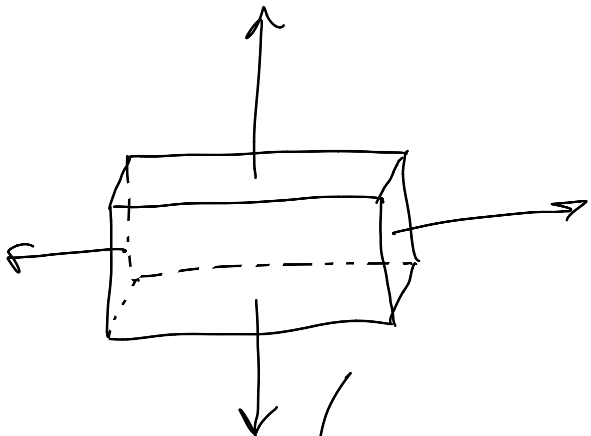
Let's try to quantify this

How does E on the box surface relatee to Q in the box?

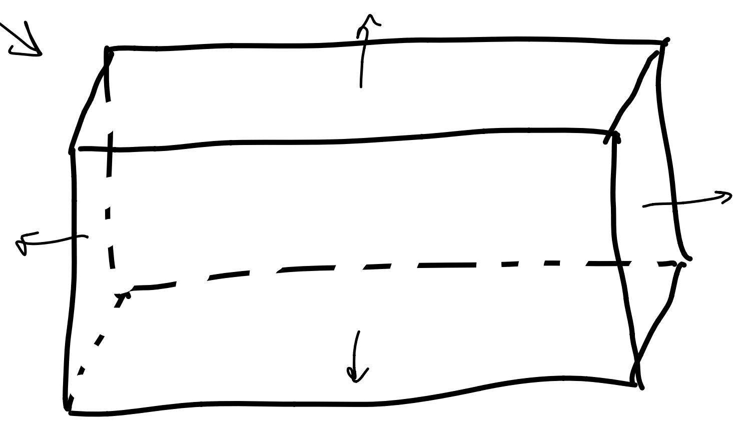


Same size box

larger E $\longrightarrow$ larger q



larger box, same q



E is smaller because we measure it farther away

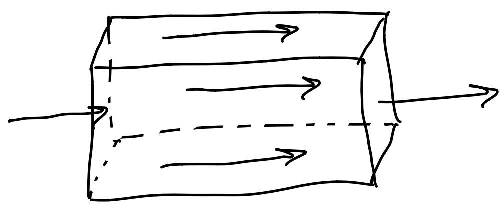
For fixed q, if A increases, E decreases

if A decreases, E increases

$q \propto |\vec{E}|A$

What else?

Not just magnitude, but direction



$|\vec{E}|A > 0$, but $q_{inside} = 0$

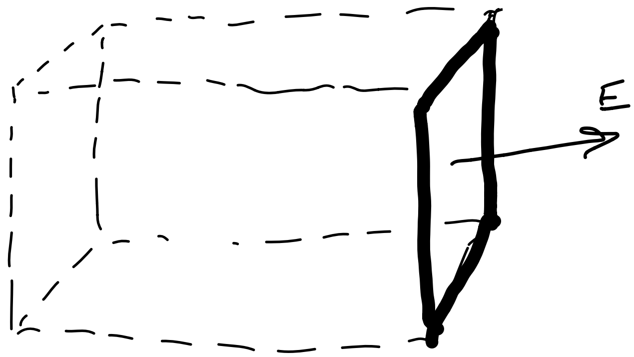
Count E_{in} different from E_{out}

If E is mostly out of the box,

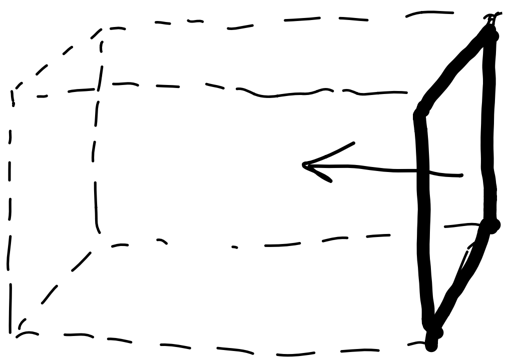
then q is $+$

If $E_{in} > E_{out}$, q is $-$

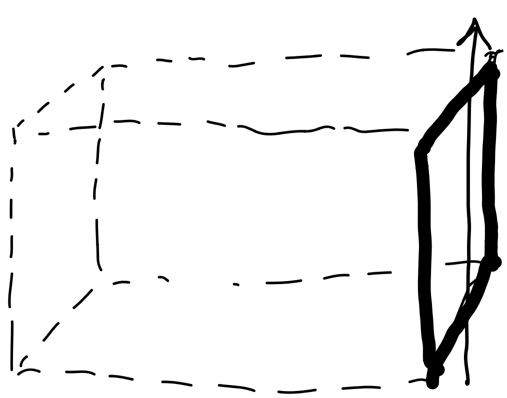
$E_{in} = E_{out}$ q is 0



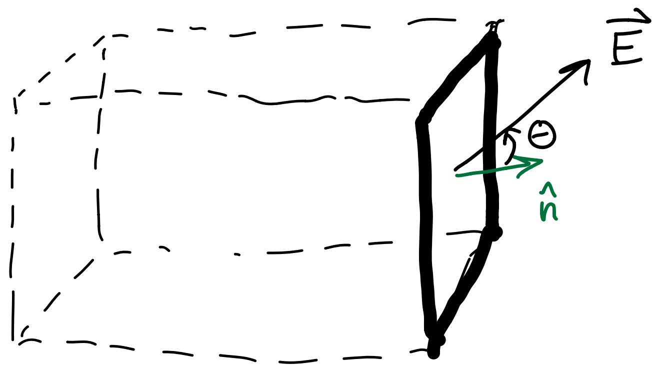
positive



negative

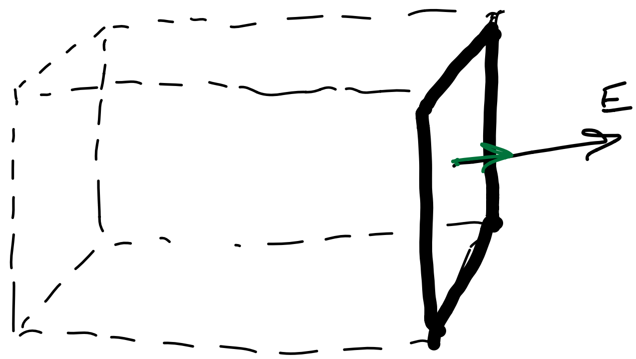


0



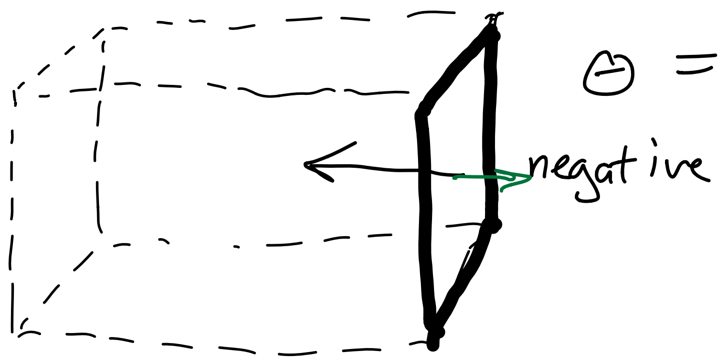
$\vec{E}$
θ
$\hat{n}$

$\Theta = 0°$

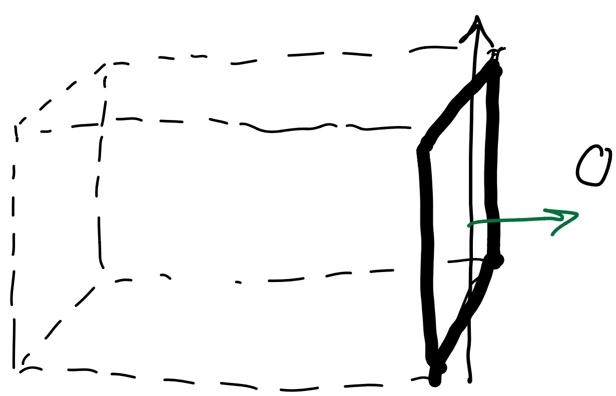


positive

$\Theta = 180°$



$\Theta = 90°$



What function is:

positive for $\theta = 0$

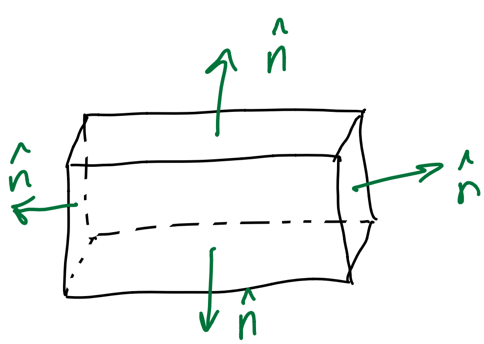
negative for $\theta = 180°$

0 for $\theta = 90°$

$\cos\theta$

$$q \propto |\vec{E}| A \cos\theta$$

$$q \propto \vec{E} \cdot \hat{n} A$$



Electric flux:

$$\Phi_E = \vec{E} \cdot \hat{n} \, \Delta A$$

(if A is thru whole box,
ΔA is a small piece)

Example:

Q21.2a

$$\text{Flux} = \vec{E} \cdot \hat{n} \, \Delta A$$

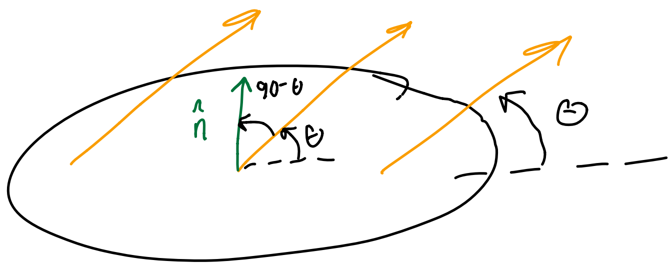
$$= \langle -230, 370, 0 \rangle \cdot \langle 0, 1, 0 \rangle \, (0.06 \times 0.04)$$

$$= (370)(0.0024)$$

$$= 0.889 \ \text{Vm}$$

Q 21.2b

$$\Phi_E = \vec{E} \cdot \hat{n} \, \Delta A$$

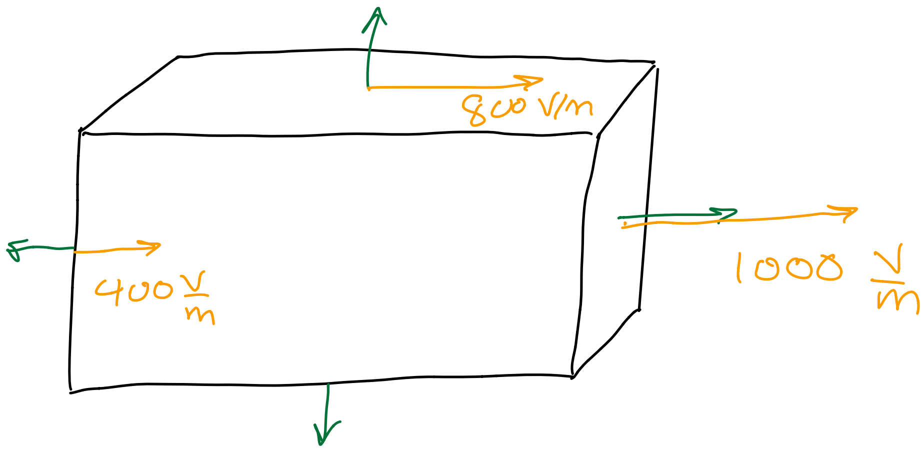


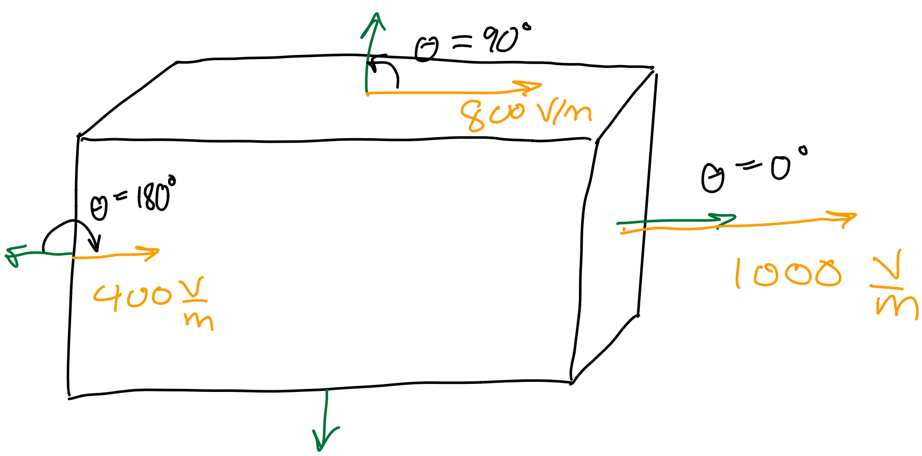
$$\vec{E} \cdot \hat{n} = E\cos\left(\frac{\pi}{2} - \theta\right)$$

$$\Phi_E = 600\cos(90-25)\,\pi(0.03)^2$$

$$= 0.717 \text{ V m}$$

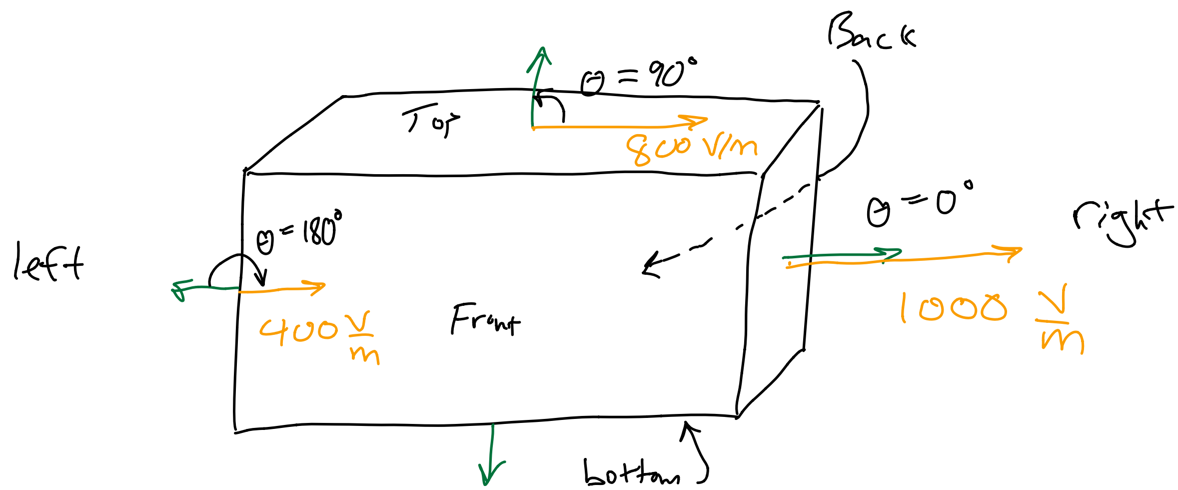
Q 21.2c





Add flux of each surface (6 of them)

left, right, top, bottom, front, back



Left:

$$\Phi_{left} = \vec{E} \cdot \hat{n}\,\Delta A$$

$$= E\Delta A \cos(180)$$

$$= -(400)(0.03)(0.02)$$

$$= -0.24 \text{ V·m}$$

Right

$$\Phi_{right} = \vec{E} \cdot \hat{n}\,\Delta A$$

$$= (1000)\cos(0)(0.03)(0.02)$$

$$= 0.6 \text{ V·m}$$

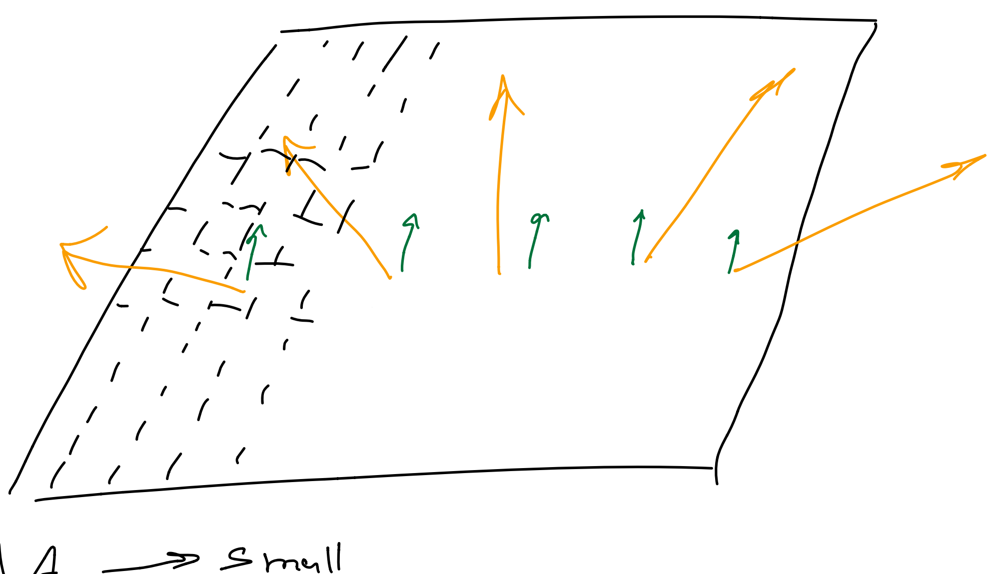
Top : $\vec{E} \cdot \hat{n} \, \Delta A$

$E \Delta A \cos(90)$

$\Phi_E = 0.36 \ V \cdot m$

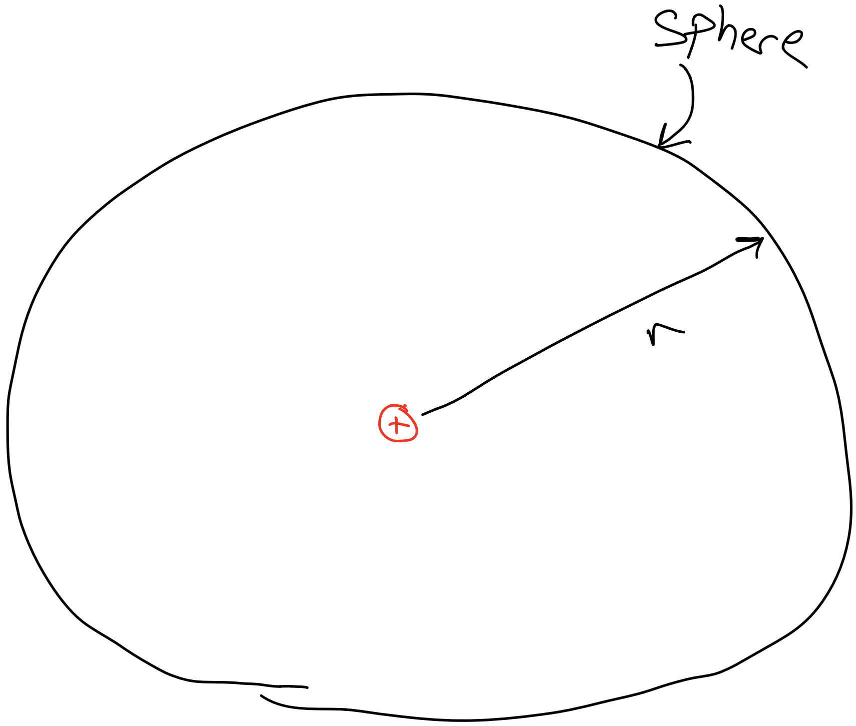
Electric Flux Through a surface

$$\Phi_E = \sum_{\text{surface}} \vec{E} \cdot \hat{n} \Delta A$$



$\Delta A \longrightarrow$ small

$$\boxed{\Phi_E = \oint \vec{E} \cdot \hat{n} \, dA}$$



$$\Phi_E = \oint \vec{E} \cdot \hat{n}\, dA$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r}$$

$$\hat{n} = \hat{r}$$

$$\Phi_E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \int dA$$

$$= \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} 4\pi r^2 = \frac{q}{\epsilon_0}$$

$$\Phi_E = \frac{q}{\epsilon_0}$$

Gauss' Law

For ANY closed surface

$$\boxed{\Phi_E = \oint \vec{E} \cdot \hat{n} \, da = \frac{q_{inside}}{\epsilon_0}}$$