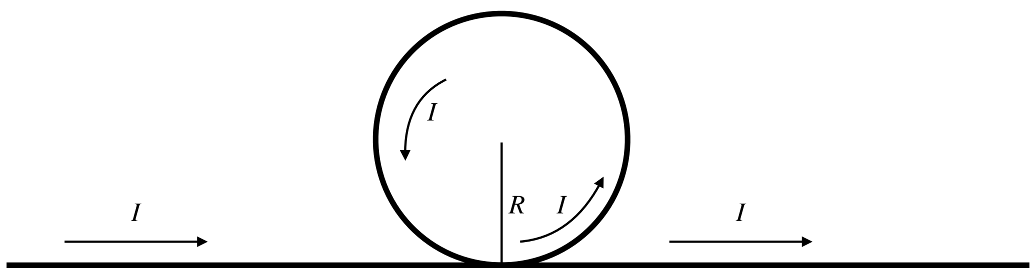
Quiz 5

The following information may or may not be of use:

$$\text{Electron current: } i = nA\bar{v}$$
$$\text{Conventional current: } I = |q|\, i$$
$$\text{Bio-Savart: } \Delta\vec{B} = \frac{\mu_0}{4\pi}\frac{I\Delta\vec{l}\times\hat{r}}{r^2}$$
$$\text{Straight Wire: } \left|\vec{B}\right| = \frac{\mu_0}{4\pi}\frac{LI}{r\sqrt{r^2+(L/2)^2}}$$
$$\text{Very long straight wire: } \left|\vec{B}\right| \approx \frac{\mu_0}{4\pi}\frac{2I}{r}$$
$$\text{Center of loop of current: } \left|\vec{B}\right| = \frac{\mu_0}{4\pi}\frac{2\pi R^2 I}{(z^2+R^2)^{3/2}}$$

A very long wire carrying a current I is kinked in the middle so that it forms a small loop of radius R. What is the magnetic field (both magnitude and direction) at the center of the loop? Express your answer in terms of μ_0, I, and R. Use a coordinate system where $+x$ points to the right ($\rightarrow$) and $+y$ points up ($\uparrow$).



Solution

$\vec{B} = \vec{B}_{\text{long wire}} + \vec{B}_{\text{loop}}$

$\left|\vec{B}_{\text{long wire}}\right| \approx \frac{\mu_0}{4\pi}\frac{2I}{r}$

$r = R \implies \left|\vec{B}_{\text{long wire}}\right| \approx \frac{\mu_0}{4\pi}\frac{2I}{R}$

Right hand rule: $\hat{B}_{\text{long wire}} = \hat{z}$

$\vec{B}_{\text{long wire}} \approx \frac{\mu_0}{4\pi}\frac{2I}{R}\hat{z}$

$\left|\vec{B}_{\text{loop}}\right| = \frac{\mu_0}{4\pi}\frac{2\pi R^2 I}{(z^2+R^2)^{3/2}}$

$z = 0 \implies \left|\vec{B}_{\text{loop}}\right| = \frac{\mu_0}{4\pi}\frac{2\pi R^2 I}{(R^2)^{3/2}} = \frac{\mu_0}{4\pi}\frac{2\pi I}{R}$

Right hand rule: $\hat{B}_{\text{loop}} = \hat{z}$

$\vec{B}_{\text{loop}} = \frac{\mu_0}{4\pi}\frac{2\pi I}{R}\hat{z}$

$\vec{B} = \vec{B}_{\text{long wire}} + \vec{B}_{\text{loop}} = \frac{\mu_0}{4\pi}\frac{2I}{R}\hat{z} + \frac{\mu_0}{4\pi}\frac{2\pi I}{R}\hat{z} = \frac{\mu_0}{4\pi}\frac{2I}{R}\left(1+\pi\right)\hat{z}$

$\vec{B} = \frac{\mu_0}{4\pi}\frac{2I}{R}\left(1+\pi\right)\hat{z}$