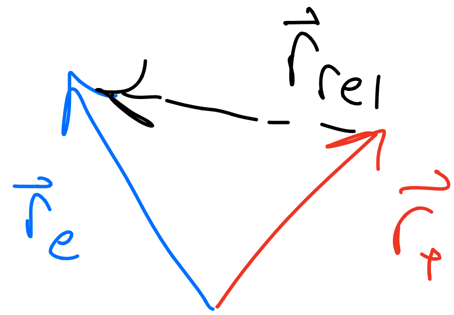
1. $\vec{A} = \langle 3, 5, 1 \rangle$

$|\vec{A}| = \sqrt{3^2 + 5^2 + 1^2} = 5.92$

2. $\vec{r}_e = \langle 4, 0, -2 \rangle$

$\vec{r}_p = \langle 0, 2, 3 \rangle$



FROM $\vec{r}_p$ TO $\vec{r}_e$

$$\vec{r}_{rel} = \vec{r}_e - \vec{r}_p$$
$$= \langle 4, 0, -2 \rangle - \langle 0, 2, 3 \rangle$$
$$= \langle 4, -2, -5 \rangle$$

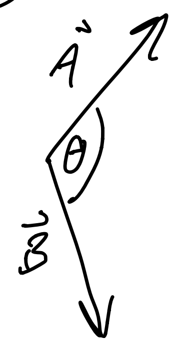
3. $\vec{A} = \langle 9, 5, 8 \rangle$

$\vec{B} = \langle -3, -5, 4 \rangle$

$\vec{A} \cdot \vec{B} = (9)(-3) + (5)(-5) + (8)(4)$

$= -27 - 25 + 32$

$= -20$ ①



$\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}| \cos\theta$

$|\vec{A}| = 13.04 \quad |\vec{B}| = 7.07$

$$\frac{\vec{A} \cdot \vec{B}}{(13.04)(7.07)} = \cos\theta$$

$$\frac{-20}{92.20} = \cos\theta$$

$\theta = \arccos(-0.22) = 1.79 \text{ rad}$

(102.5°)

4. $\vec{a} = \langle -3, 7, 1 \rangle$

$\hat{a} = \dfrac{\vec{a}}{|\vec{a}|}$; $|\vec{a}| = 7.68$

$\hat{a} = \left\langle \dfrac{-3}{7.68}, \dfrac{7}{7.68}, \dfrac{1}{7.68} \right\rangle$

$\hat{a} = \langle -0.39, 0.91, 0.13 \rangle$

direction of x-axis $= \hat{x} = \langle 1,0,0 \rangle$

$\hat{a} \cdot \hat{x} = \underbrace{|\hat{a}|}_{1}\underbrace{|\hat{x}|}_{1} \cos\theta$

$\hat{a} \cdot \hat{x} = \cos\theta \longrightarrow \theta = \arccos(-0.39 + 0 + 0)$

$\theta = 1.97$ rad

112.95°

5. $\vec{B} = \langle 3, -9, 7 \rangle$

$\vec{v} = \langle 8, 4, 6 \rangle$

$\vec{A} = \langle a_x, a_y, a_z \rangle = a_x \hat{x} + a_y \hat{y} + a_z \hat{z}$

$\vec{B} = \langle b_x, b_y, b_z \rangle = b_x \hat{x} + b_y \hat{y} + b_z \hat{z}$

$$\vec{A} \times \vec{B} = (a_x \hat{x} + a_y \hat{y} + a_z \hat{z}) \times (b_x \hat{x} + b_y \hat{y} + b_z \hat{z})$$

$$= a_x \hat{x} \times b_x \hat{x} + a_x \hat{x} \times b_y \hat{y} + a_x \hat{x} \times b_z \hat{z}$$
$$+ a_y \hat{y} \times b_x \hat{x} + a_y \hat{y} \times b_y \hat{y} + a_y \hat{y} \times b_z \hat{z}$$
$$+ a_z \hat{z} \times b_x \hat{x} + a_z \hat{z} \times b_y \hat{y} + a_z \hat{z} \times b_z \hat{z};$$

$$a_x \hat{x} \times b_y \hat{y} = a_x b_y (\hat{x} \times \hat{y})$$

$\hat{x} \times \hat{x} = 0$, $\hat{x} \times \hat{y} = \hat{z}$, $\hat{x} \times \hat{z} = -\hat{y}$

$\hat{y} \times \hat{x} = -\hat{z}$, $\hat{y} \times \hat{y} = 0$, $\hat{y} \times \hat{z} = \hat{x}$

$\hat{z} \times \hat{x} = \hat{y}$, $\hat{z} \times \hat{y} = -\hat{x}$, $\hat{z} \times \hat{z} = 0$

$$\vec{A} \times \vec{B} = a_x \hat{x} \cancel{\times b_x \hat{x}} + a_x \hat{x} \times b_y \hat{y} + a_x \hat{x} \times b_z \hat{z}$$
$$+ a_y \hat{y} \times b_x \hat{x} + a_y \hat{y} \cancel{\times b_y} \hat{y} + a_y \hat{y} \times b_z \hat{z}$$
$$+ a_z \hat{z} \times b_x \hat{x} + a_z \hat{z} \times b_y \hat{y} + a_z \hat{z} \cancel{\times b_z} \hat{z};$$

$$= a_x b_y \hat{z} + a_x b_z (-\hat{y})$$
$$+ a_y b_x (-\hat{z}) + a_y b_z \hat{x} +$$
$$+ a_z b_x \hat{y} + a_z b_y (-\hat{x})$$

$$\vec{A} \times \vec{B} = (a_y b_z - a_z b_y)\hat{x} + (a_z b_x - a_x b_z)\hat{y} + (a_x b_y - a_y b_x)\hat{z}$$

$$\vec{B} = \langle 3, -9, 7 \rangle$$

$$\vec{v} = \langle 8, 4, 6 \rangle$$

$$\vec{B} \times \vec{v} = (-9 \cdot 6 - 7 \cdot 4)\hat{x} + (7 \cdot 8 - 3 \cdot 6)\hat{y}$$

$$(3 \cdot 4 - (-9) \cdot 8)\hat{z}$$

$$\boxed{\vec{B} \times \vec{v} = -82\hat{x} + 38\hat{y} + 84\hat{z}}$$

6. $\vec{A} = \langle 4, 5, -7 \rangle$

$\vec{B} = \langle 6, -2, 2 \rangle$

if $\vec{A} \perp \vec{B}$, then $\vec{A} \cdot \vec{B} = 0$

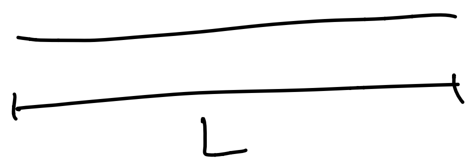
$\vec{A} \cdot \vec{B} = 4 \cdot 6 - 5 \cdot 2 - 7 \cdot 2$

$= 24 - 10 - 14 = 0$

$\vec{A} \cdot \vec{B} = 0$

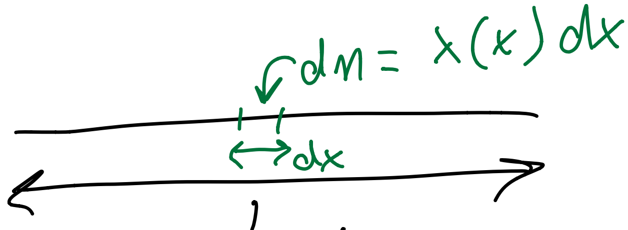
ORTHOGONAL ✓

7.



$$\frac{M}{L} = \lambda_0 \Rightarrow M = \lambda_0 L$$

8.

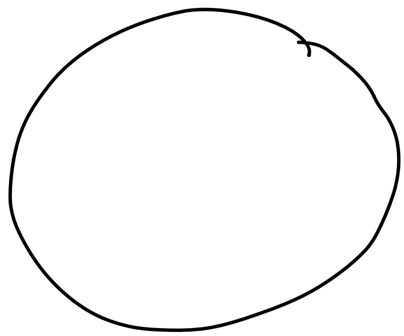


$$M = \int dm = \int_0^L \lambda(x)\,dx = \int_0^L \lambda_0 \left(\frac{x}{L}\right)^3 dx$$

$$= \frac{\lambda_0}{L^3} \int_0^L x^3 dx = \frac{\lambda_0}{L^3} \left(\frac{1}{4} x^4\right)\Bigg|_0^L = \frac{\lambda_0}{4} L$$

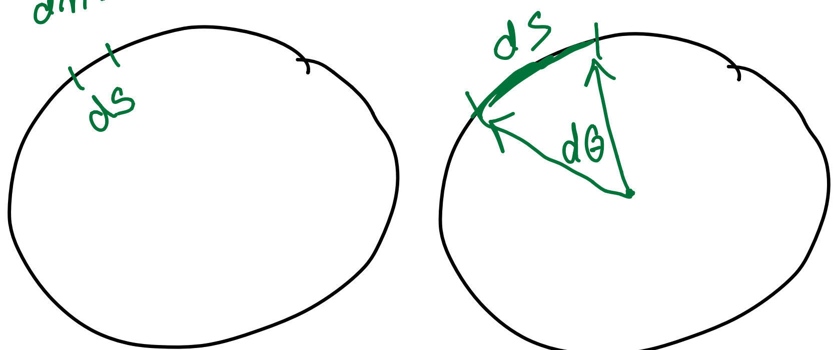
$$\boxed{M = \frac{1}{4} \lambda_0 L}$$

9.



length $L = 2\pi R \Rightarrow R = \dfrac{L}{2\pi}$

$M = \int dm$ $\quad dm = \lambda(\theta)ds$



$$M = \int \lambda(\theta) ds$$

$$ds = \frac{d\theta}{2\pi} \times 2\pi R$$

$$ds = d\theta R$$

$$= \int \lambda(\theta) R \, d\theta$$

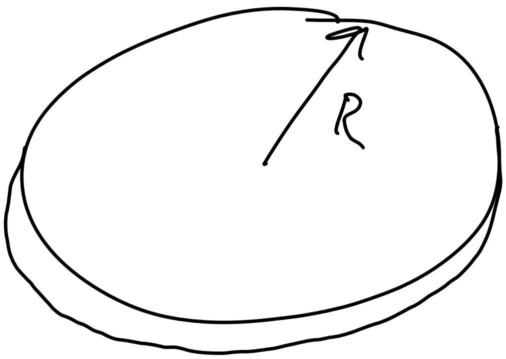
$$= \int \lambda_0 \theta R \, d\theta$$

$$= \lambda_0 R \left. \frac{\theta^2}{2} \right|_0^{2\pi} = \frac{\lambda_0 R}{2} 4\pi^2$$

$$= 2\pi^2 \lambda_0 R$$

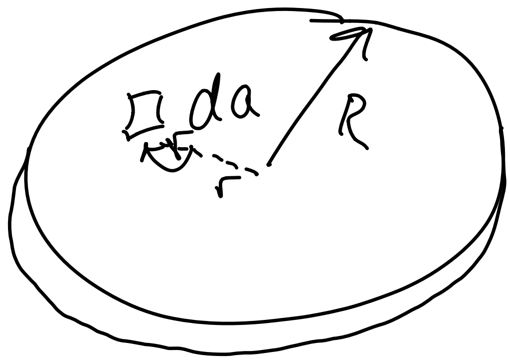
$$= 2\pi^2 \lambda_0 \frac{L}{2\pi} = \boxed{\pi \lambda_0 L = M}$$

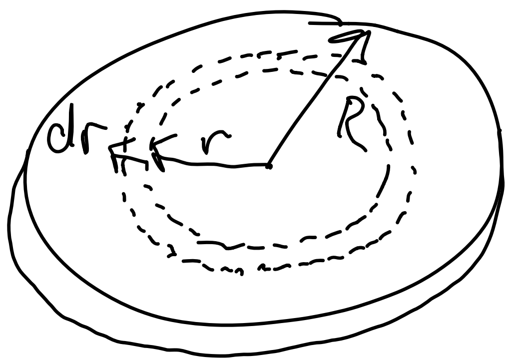
10.



$$M = \int dm$$

$$dm = \sigma(r)\,da$$





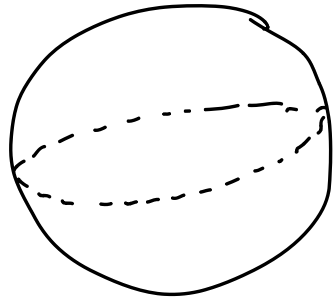
$$da = 2\pi r\,dr$$

$$dm = \sigma(r)\,2\pi r\,dr$$

$$M = \int_0^R \sigma(r)\,2\pi r\,dr = \int_0^R \sigma_0 \left(\frac{r}{R}\right)^2 2\pi r\,dr$$

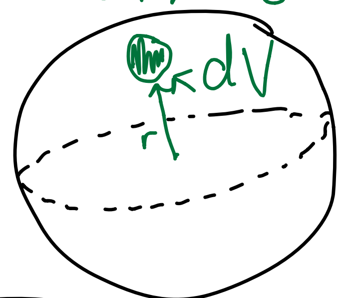
$$M = \frac{\sigma_0}{R^2}\,\frac{1}{4} R^4\, 2\pi = \frac{1}{2}\sigma_0 \pi R^2$$

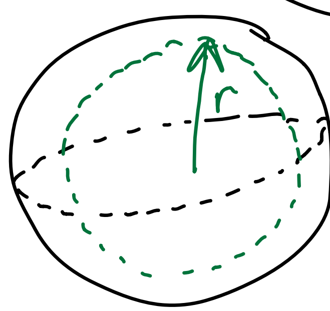
11.



$$M = \int dm$$

$$dm = \rho(r)\,dV$$





$$dV = 4\pi r^2 dr$$

$$M = \int dm$$

$$= \int_0^R \rho(r)\,4\pi r^2 dr$$

$$= \int_0^R \rho_0 \frac{r}{R}\, 4\pi r^2 dr = \frac{4\pi\rho_0}{R}\,\frac{1}{4}R^4$$

$$\boxed{M = \pi \rho_0 R^3}$$