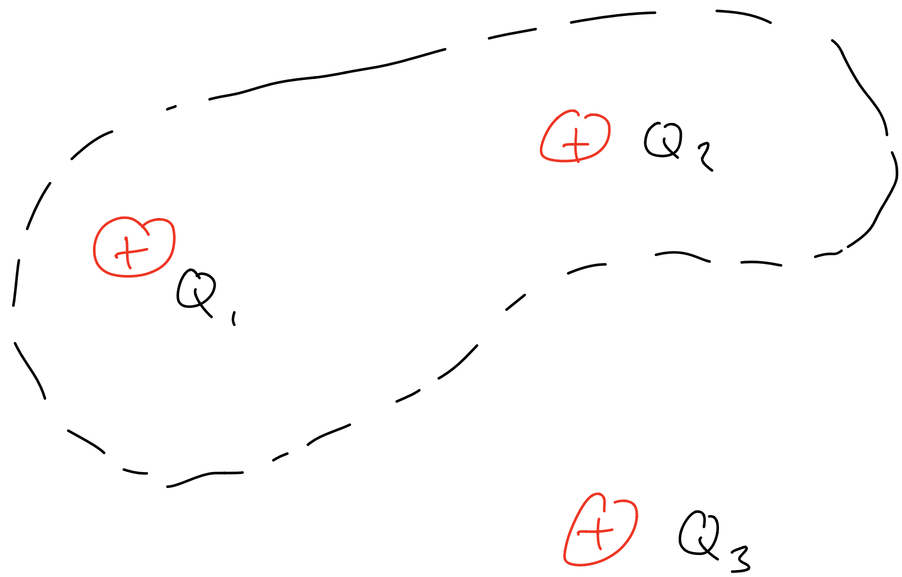



$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$$

$$Q_{inside} = Q_1 + Q_2$$

$$\oint \vec{E} \cdot \hat{n} \, dA = \frac{Q_{inside}}{\epsilon_0}$$

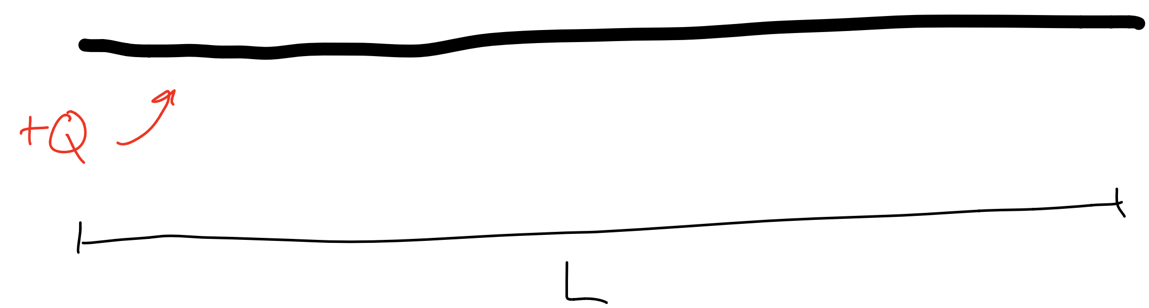
$$\oint \left(\vec{E}_1 + \vec{E}_2 + \vec{E}_3\right) \cdot \hat{n} \, dA = \frac{Q_1 + Q_2}{\epsilon_0}$$

$E$: net field from all charges, inside OR out

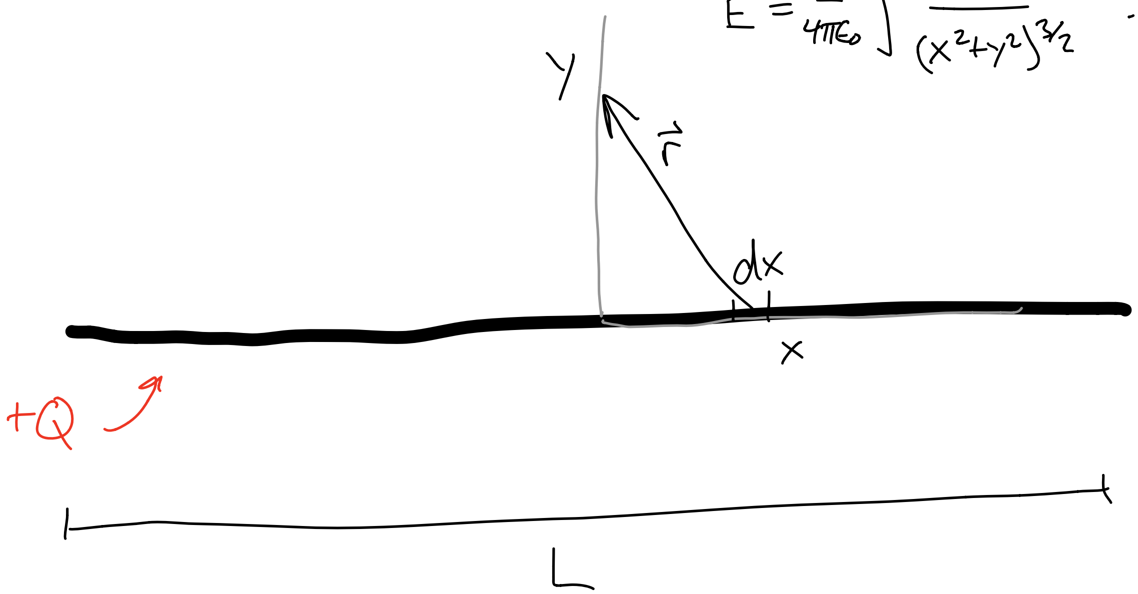
$Q_{inside}$: Just the charge inside the surface

## Usage:

What is $\vec{E}$ near the center of the wire?




$$\vec{E} = \frac{1}{4\pi\epsilon_0}\int \frac{Q/L \, dx}{(x^2+y^2)^{3/2}} \quad \dots$$

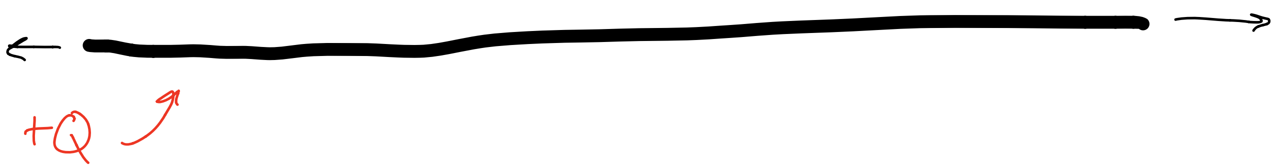


# An easier way:

1) Assume $L \longrightarrow \infty$

(near the center, close to surface)

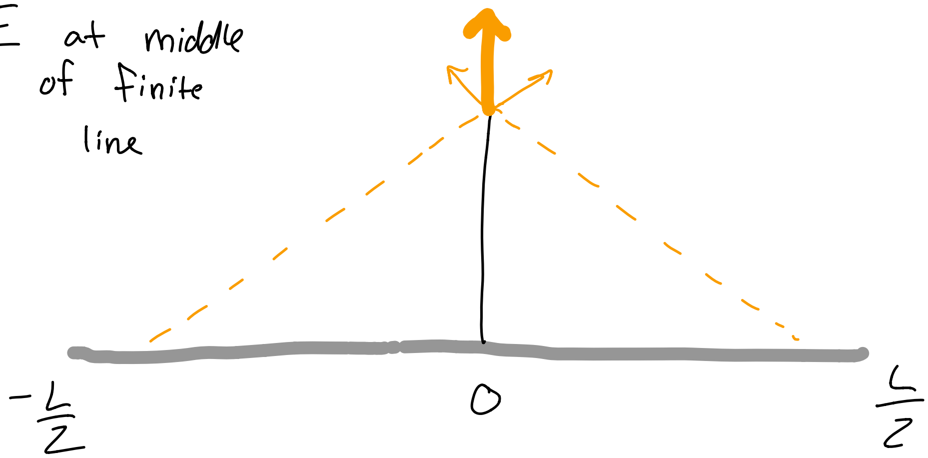
chg density $\lambda = Q/L$



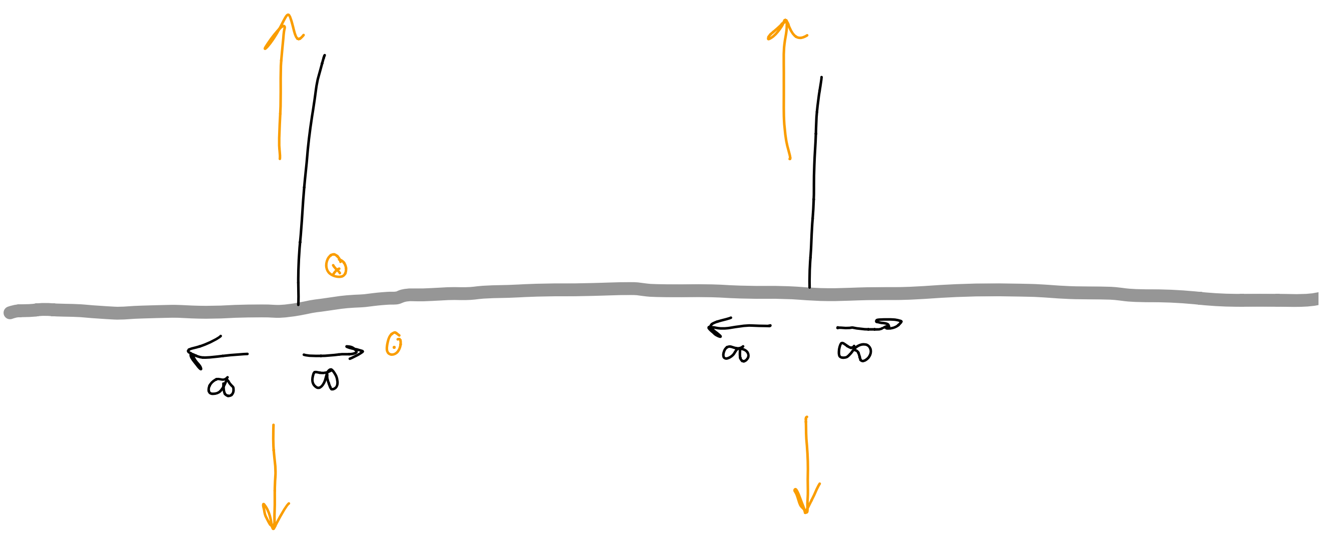

2) Utilize symmetry

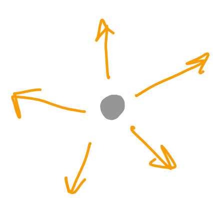
we dont fully know $\vec{E}$, but we know what it looks like

E at middle of finite line




if $L$ is $\infty$, we are <u>always</u> at the midpoint!



By symmetry, we know:

- Direction of $\vec{E}$ is $\perp$ to line of charge
- Magnitude of $\vec{E}$ only depends of distance away from line

Recall:

Gauss' Law:

$$\oint \vec{E} \cdot \hat{n} \, dA = \frac{1}{\epsilon_0} q_{inside}$$

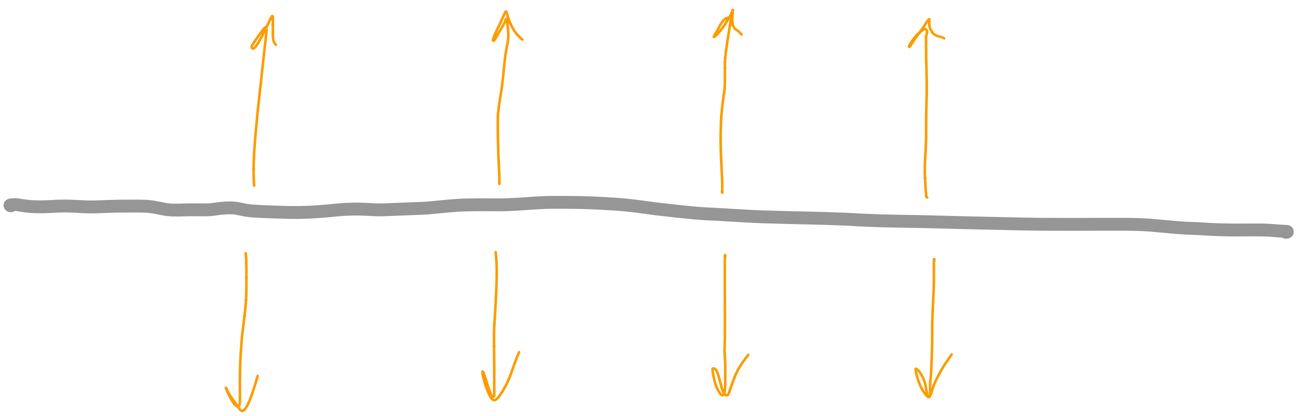
True for <u>any</u> surface

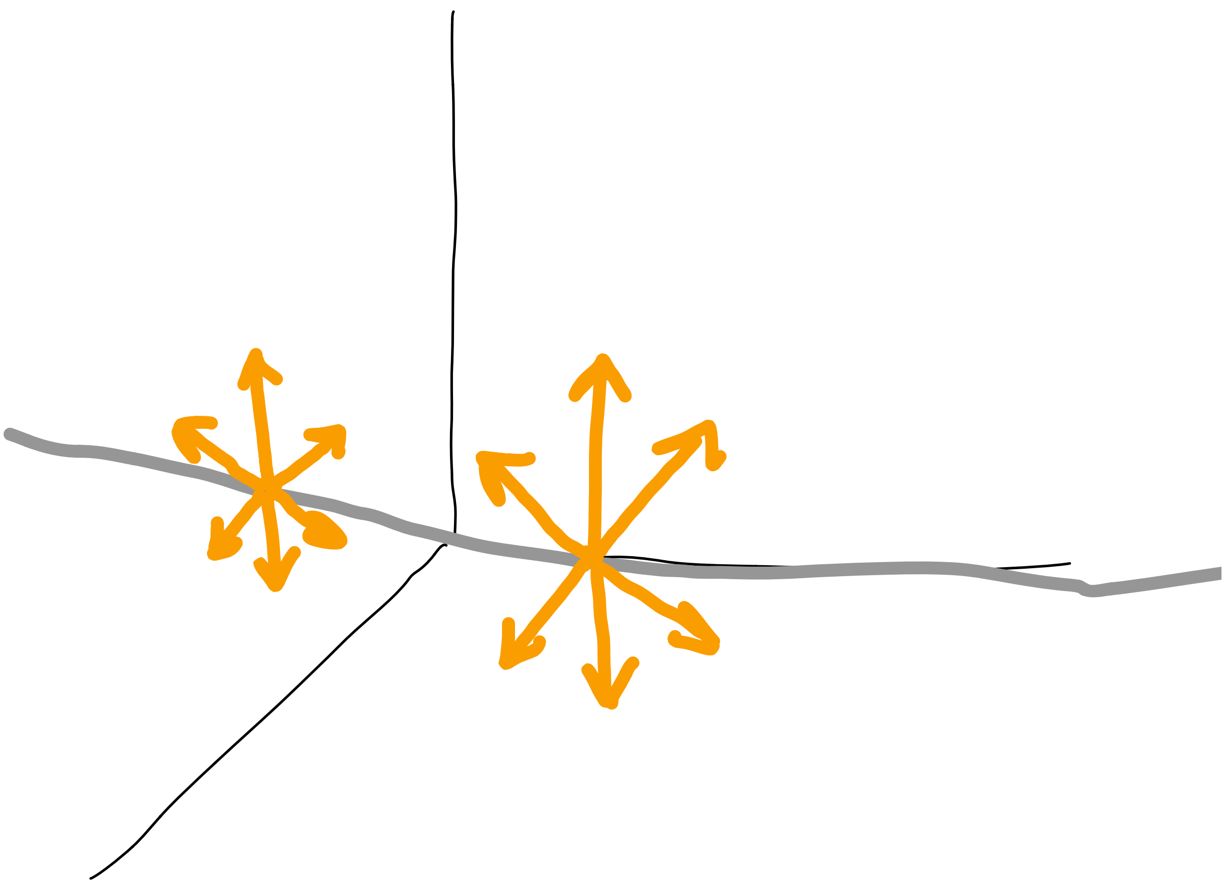
Can we pick a surface that will simplify

$$\int \vec{E} \cdot \hat{n} \, dA \, ?$$

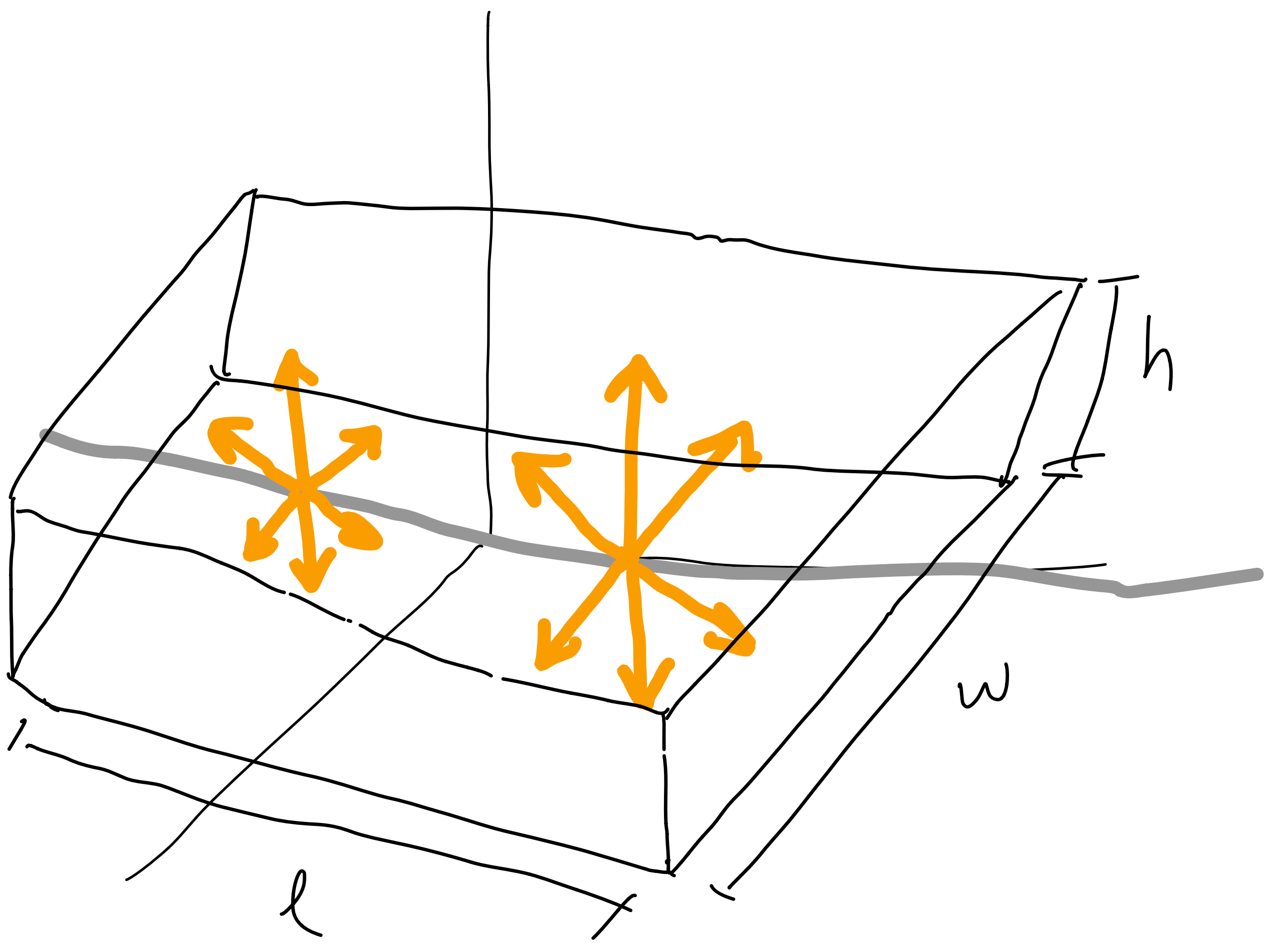
How? We don't know $\vec{E}$?

We know some things about E ...

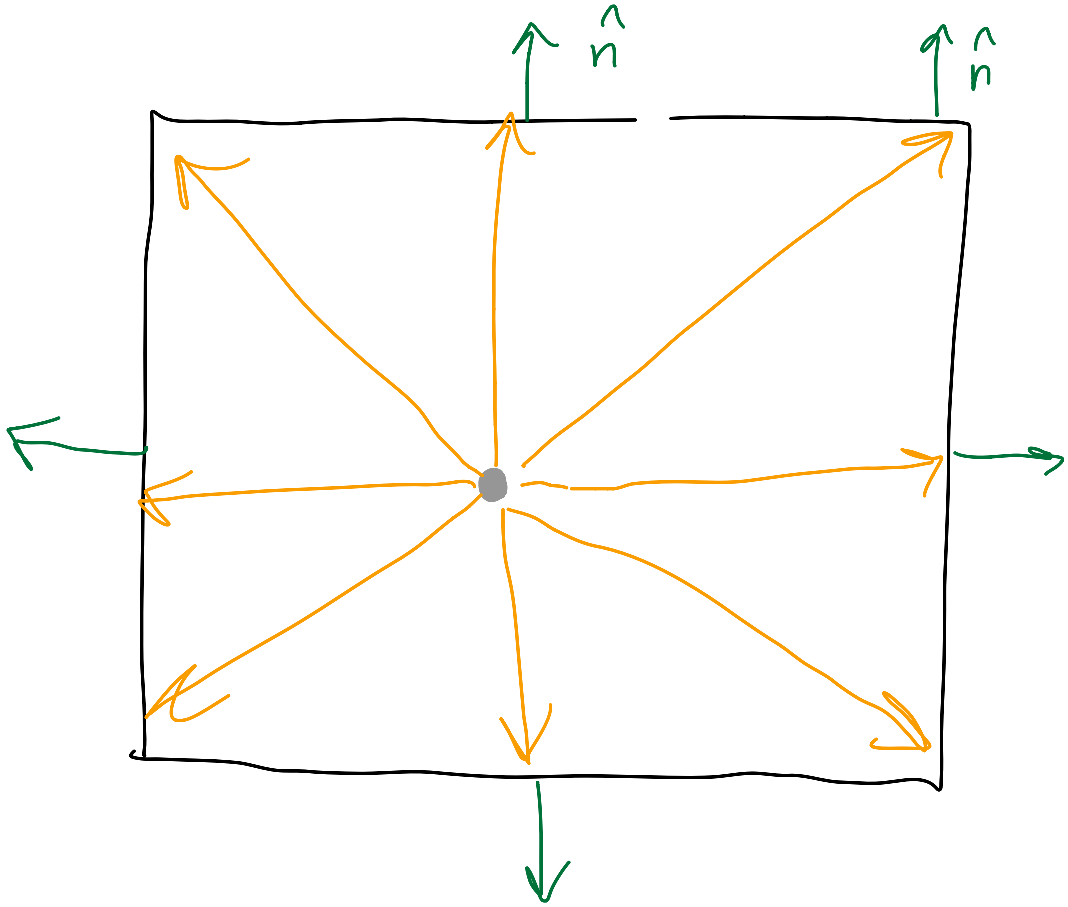




Pick a surface




$$q_{inside} = \frac{Q}{L}\, l$$



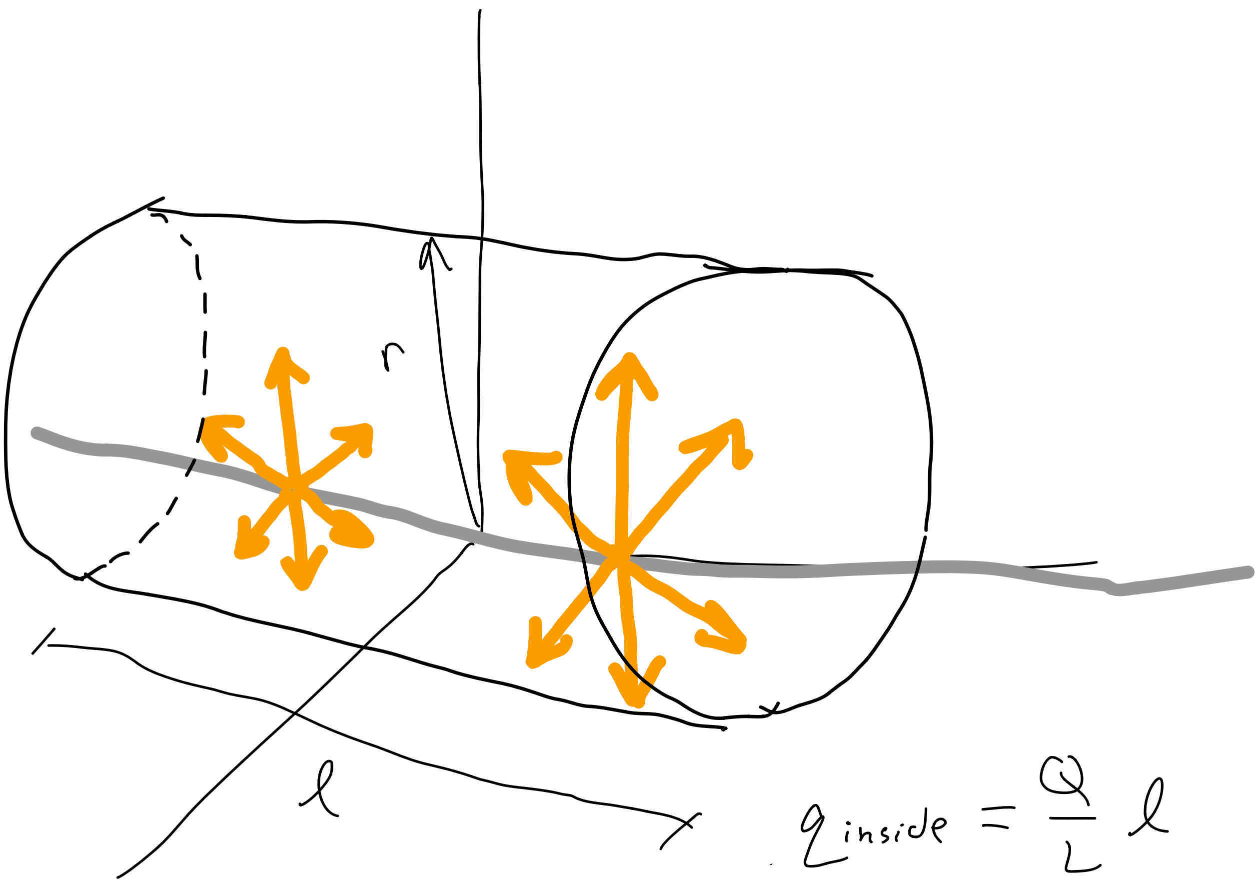

$\vec{E}$ is not uniform along the box

$$\oint \vec{E}\cdot\hat{n}\,dA = \frac{q_{inside}}{\epsilon_0}$$

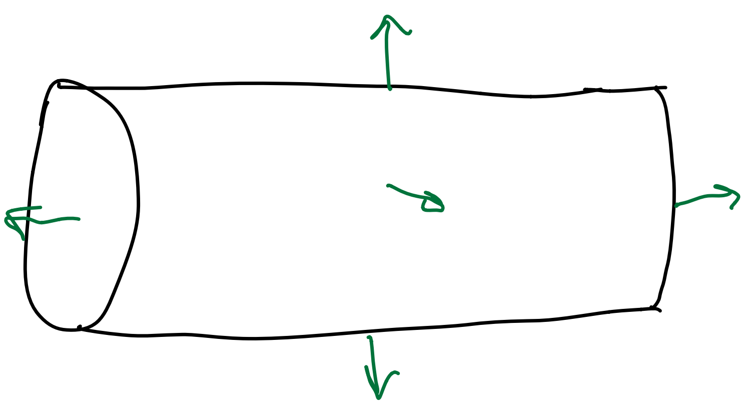
is still <u>true</u> but not <u>useful</u>

What shape do we want?

try a cylinder




$$q_{inside} = \frac{Q}{L} \ell$$



$\hat{n}$ is everywhere $\perp$ to the surface

- On cylinder body, this is $\perp$ to axis of the cylinder, same as $\vec{E}$!

- On cylinder caps, this is $\parallel$ to axis, $\perp$ to $\vec{E}$

$$\oint \vec{E}\cdot\hat{n}\,dA = \int_{\text{body}} \vec{E}\cdot\hat{n}\,dA + \int_{\text{caps}} \vec{E}\cdot\hat{n}\,dA$$

$$= \int_{\text{body}} E\,dA$$

The body of the cylinder is a constant distance away so $E$ is constant.

$E$ comes out of the integral!

$$\int_{body} E\, dA = E \int_{body} dA = E\, 2\pi r \ell$$

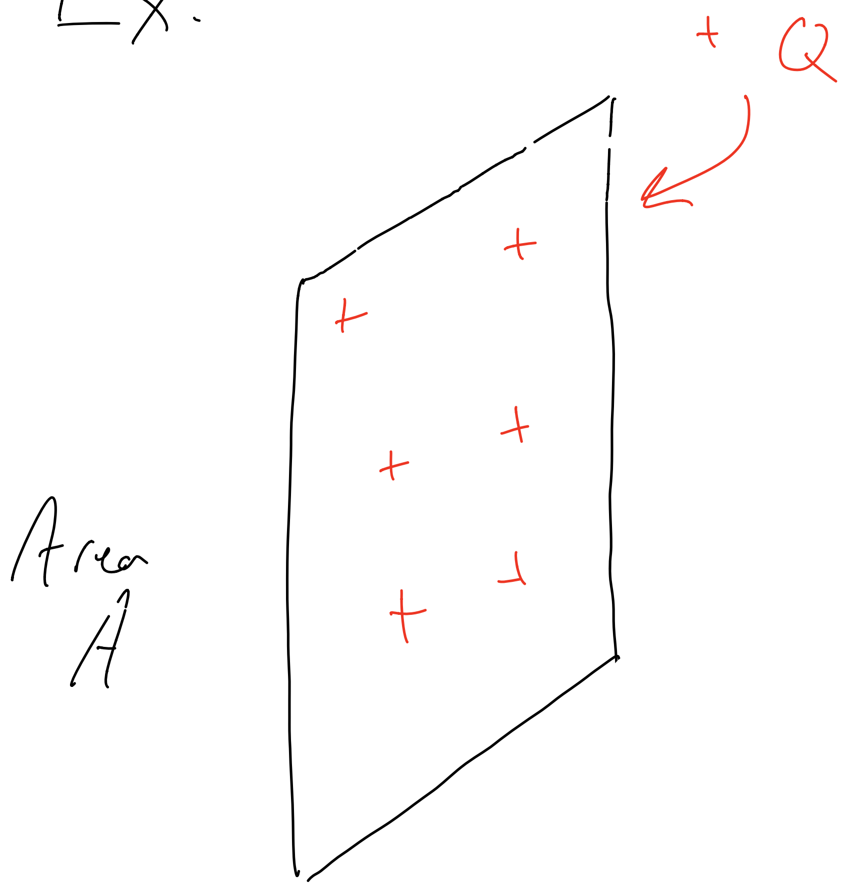
$$\oint \vec{E} \cdot \hat{n}\, dA = \frac{1}{\epsilon_0} q_{inside}$$

$$E\,(2\pi r \ell) = \frac{1}{\epsilon_0} \frac{Q}{L} \ell$$

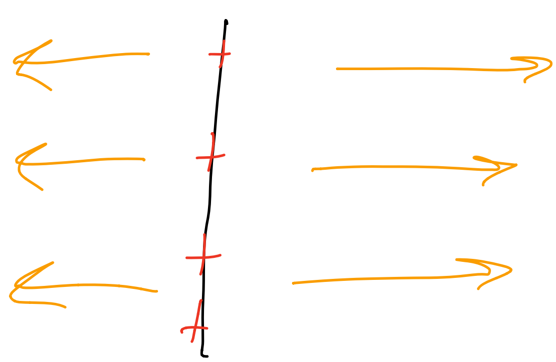
$$E = \frac{Q/L}{\epsilon_0} \frac{1}{2\pi r}$$

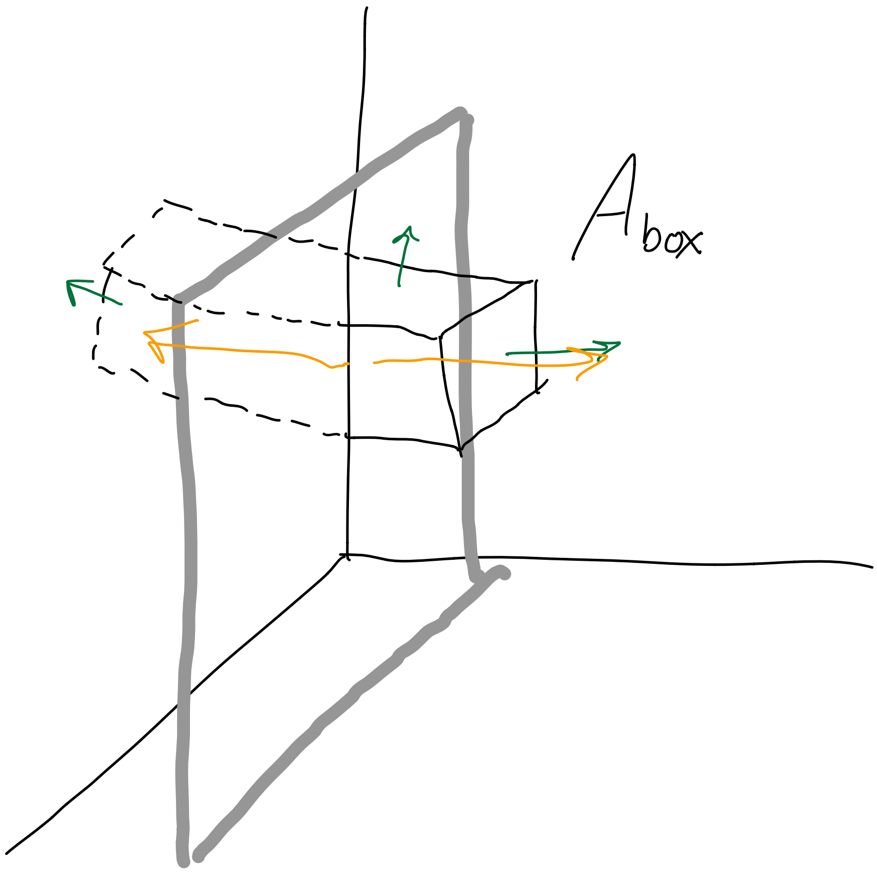
Compare to Ch 15 pg 595

Ex:




What is $\vec{E}$ near the center + close to surface?






$$\oint \vec{E}\cdot\hat{n}\,dA = \int \vec{E}_{left}\cdot\hat{n}\,dA + \oint \vec{E}_{right}\cdot\hat{n}\,dA$$

$$E_{left} = E_{right} = E$$

$$\vec{E}\cdot\hat{n}\,dA = E\,dA$$

$$E\int_{left} dA + E\int_{right} dA = 2EA_{box}$$

$$q_{inside} = \frac{Q}{A} A_{box}$$

$$2 E A_{box} = \frac{1}{\epsilon_0} \frac{Q}{A} A_{box}$$

$$\underline{E} = \frac{Q/A}{2\epsilon_0}$$

Compare to pg 601

ch 15