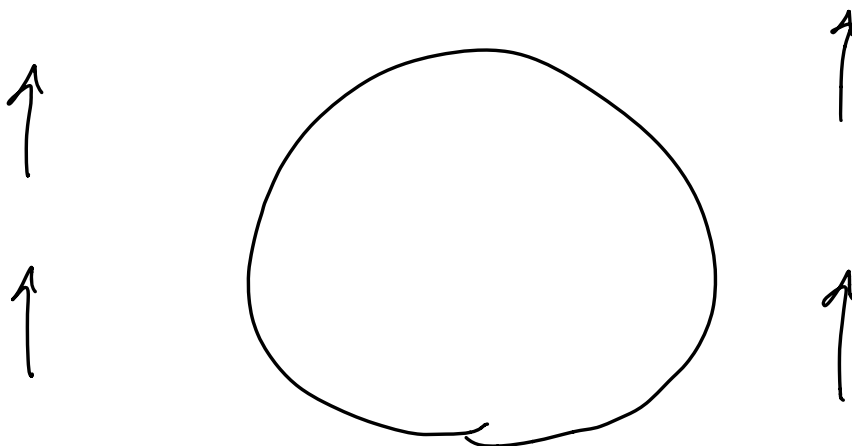


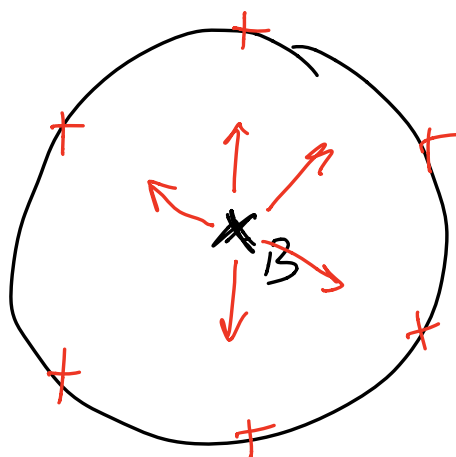
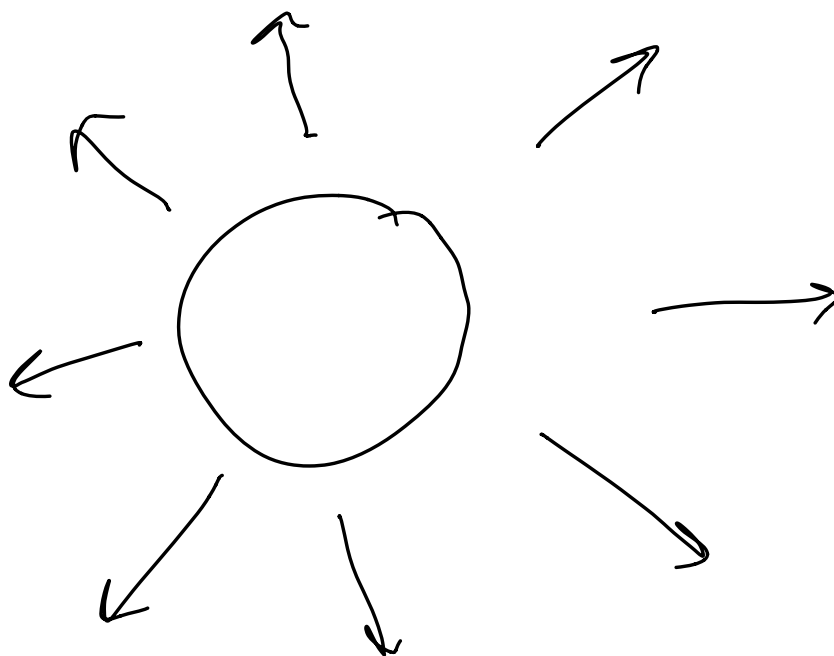
$\hat{E}$  at  $A$

Suppose

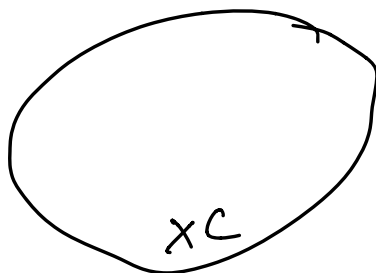
$A$  is not radial



Not Symmetric



$= 0$



$= 0$

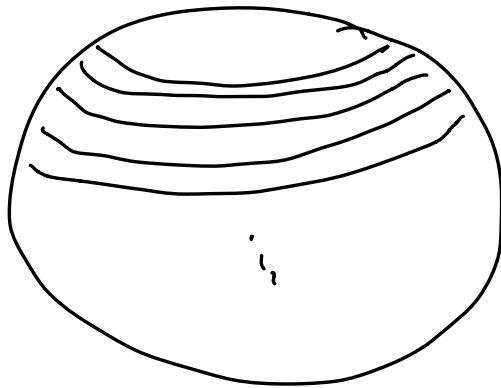
Jupyter

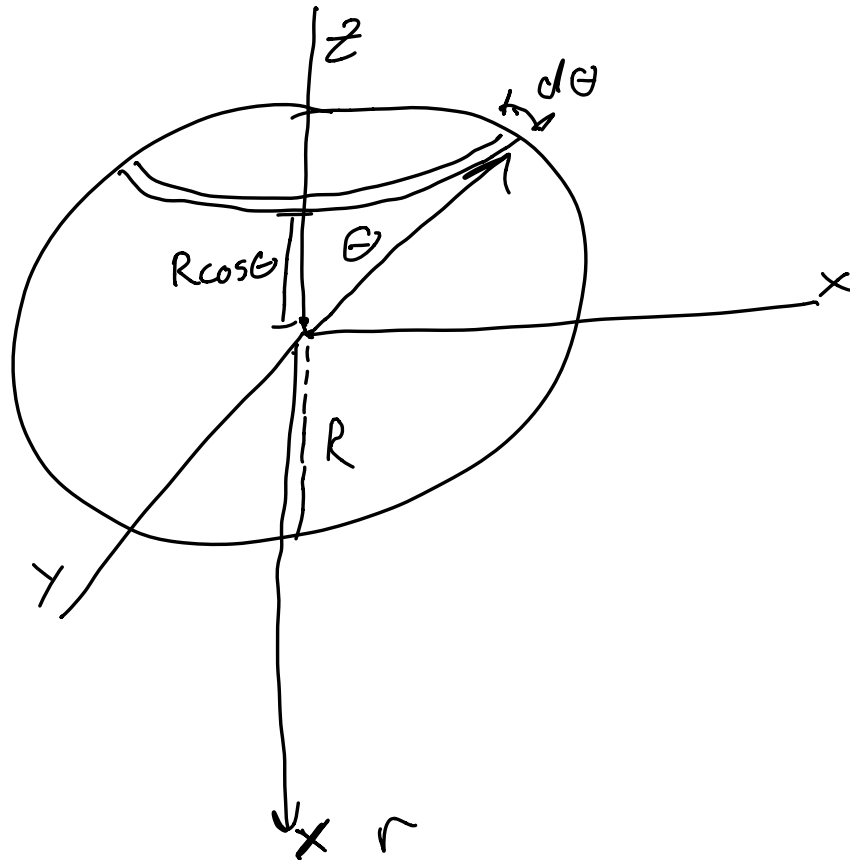
$$\vec{E}_{\text{inside}} = 0$$

$$\vec{E}_{\text{outside}} = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} \hat{r}$$

Point  
Charge!

Another way





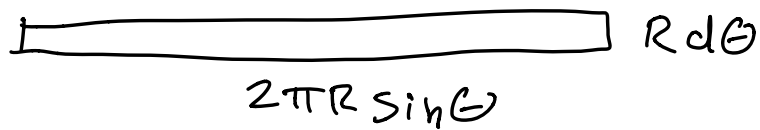
$$dE = \frac{k \Delta Q \ z}{(R^2 + z^2)^{\frac{3}{2}}}$$

$$z = r + R \cos \theta$$

$$\text{charge dens} = \frac{Q}{4\pi R^2}$$

$$\Delta Q = \text{dens} \times A_{\text{ring}}$$

↓



$$2\pi R \sin\theta$$

$A_{\text{ring}}$

$$\Delta Q = \frac{Q}{4\pi R^2} 2\pi R^2 \sin\theta = \frac{Q}{2} \sin\theta$$

$$dE = k \frac{Q}{z} \sin\theta \frac{z}{(R^2 + z^2)^{3/2}}$$

$$z = r + R \cos\theta$$

$$dE = \frac{kQ}{z} \frac{(r + R \cos\theta)}{[R^2 + (r + R \cos\theta)^2]^{3/2}} \sin\theta d\theta$$

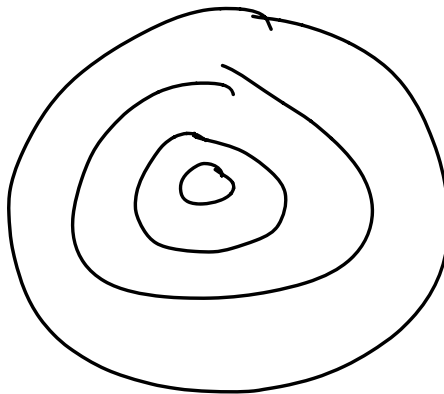
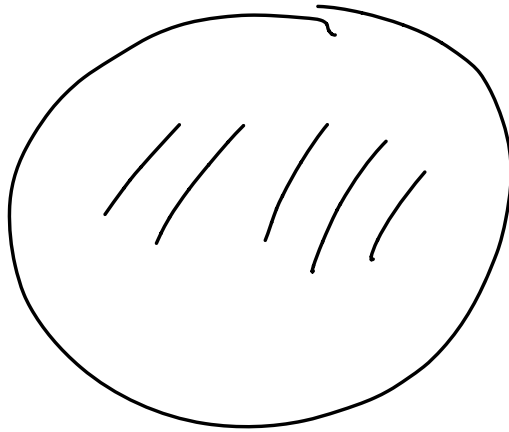
$$E = \frac{kQ}{z} \int_0^\pi \frac{(r + R \cos\theta)}{[R^2 + (r + R \cos\theta)^2]^{3/2}} \sin\theta d\theta$$

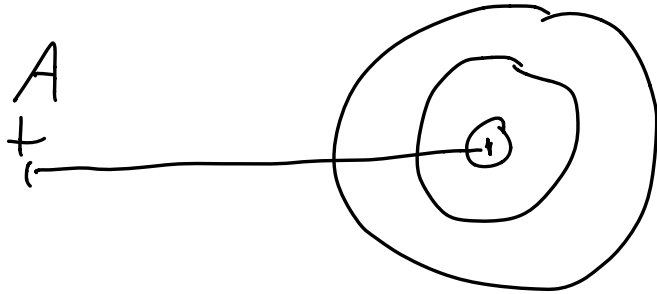
Complicated!

$$\vec{E} = \begin{cases} \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} \hat{r}, & r > R \\ 0, & r < R \end{cases}$$


---

Field of a solid sphere





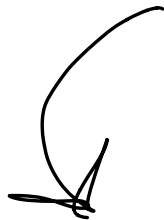
$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$$

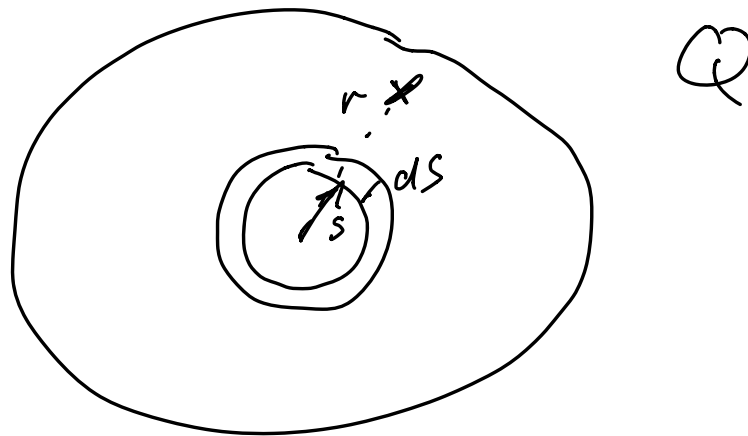
$$= \frac{k \Delta Q}{r^2} \hat{r} + \frac{k \Delta Q}{r^2} \hat{r} + \frac{k \Delta Q}{r^2} \hat{r}$$

$$\sum \Delta Q = Q$$

$$\vec{E}_{\text{outside}} = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} \hat{r}$$

inside





$$\text{dens} = \frac{Q}{V} = \frac{Q}{\frac{4}{3}\pi R^3}$$

$$V_{\text{shell}}$$

$$A = 4\pi s^2$$

A diagram of a rectangular shell with length  $l$ , width  $w$ , and height  $h$ . The volume is labeled  $V = A \cdot h$ .

$$V = 4\pi s^2 ds$$

$$dQ = Q \frac{4\pi s^2 ds}{\frac{4}{3}\pi R^3} = \frac{3Q s^2 ds}{R^3}$$



$$dE = \frac{1}{4\pi\epsilon_0} \frac{dq}{r^2}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{3Q s^2 ds}{r^2 R^3}$$

$$E = \frac{1}{4\pi\epsilon_0} \frac{3Q}{R^3 r^2} \int_0^r s^2 ds$$

$$= \frac{1}{4\pi\epsilon_0} \frac{3Q}{R^3 r^2} \frac{1}{3} r^3$$

$$E = \frac{Q}{4\pi\epsilon_0} \frac{r}{R^3}$$

$$E_{\text{solid sphere}} = \begin{cases} \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} & ; r > R \\ \frac{1}{4\pi\epsilon_0} \frac{Q r}{R^3} & ; r < R \end{cases}$$

