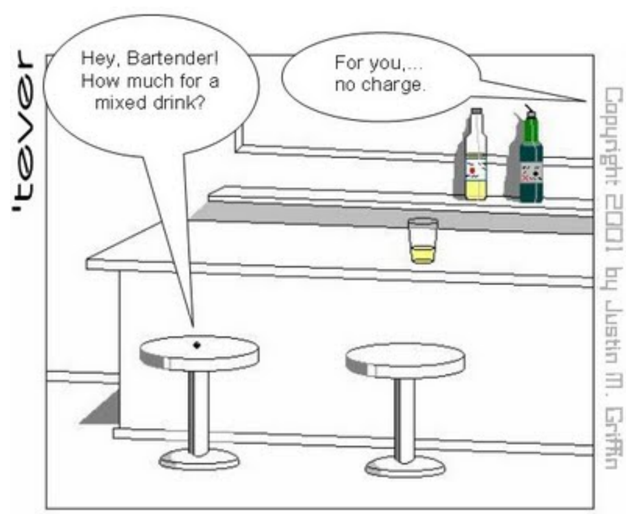
PHYS 2250 Exam I

Thursday, October 7, 2021

***Instructions**: You will have at least 2 hours to complete this exam. Take a deep breath and relax! Read each question carefully, and let me know if anything is unclear. Partial credit may be awarded, so you are encouraged to clearly and legibly show your work for each problem. Write your name on every extra sheet you use, and clearly label what problem you are working on. Staple this to the back of your exam when you turn it in. You may use any information contained within this exam, as well as a calculator.*
Good luck!

Name: ______________________________



A neutron walks into a bar...

Potentially useful information

Unit analysis

Power	Prefix	Name
10^{12}	T	tera
10^{9}	G	giga
10^{6}	M	mega
10^{3}	k	kilo
10^{0}	—	—
10^{-3}	m	milli
10^{-6}	μ	micro
10^{-9}	n	nano

Constants

$$\epsilon_0 = 8.85 \times 10^{-12}\ \mathrm{C}^2/\left(\mathrm{N} \cdot \mathrm{m}^2\right)$$

$$\frac{1}{4\pi\epsilon_0} = k = 9 \times 10^9\ \mathrm{N} \cdot \mathrm{m}^2/\mathrm{C}^2$$

$$e = 1.6 \times 10^{-19}\ \mathrm{C}$$

$$m_{proton} = 1.67 \times 10^{-27}\ \mathrm{kg}$$

$$m_{electron} = 9.11 \times 10^{-31}\ \mathrm{kg}$$

Electrostatics

Dipole, on-axis:

$$|\vec{E}| \approx \frac{1}{4\pi\epsilon_0}\frac{2p}{r^3}$$

Dipole, perpendicular:

$$|\vec{E}| \approx \frac{1}{4\pi\epsilon_0}\frac{p}{r^3}$$

Uniformly charged rod, middle axis:

$$|\vec{E}| = \frac{1}{4\pi\epsilon_0}\left[\frac{Q}{r\sqrt{r^2 + (L/2)^2}}\right]$$

Uniformly charged ring, central axis:

$$|\vec{E}| = \frac{1}{4\pi\epsilon_0}\frac{Qz}{\left(R^2 + z^2\right)^{3/2}}$$

Uniformly charged disk, central axis:

$$|\vec{E}| = \frac{Q/A}{2\epsilon_0}\left[1 - \frac{z}{\left(R^2 + z^2\right)^{1/2}}\right]$$

Capacitor:

$$|\vec{E}| = \frac{Q/A}{\epsilon_0}$$

Uniformly charged shell:

$$|\vec{E}|(r < R) = 0$$

$$|\vec{E}|(r > R)| = \frac{Q}{4\pi\epsilon_0}\frac{1}{r^2}$$

Uniformly charged solid sphere:

$$|\vec{E}|(r < R) = \frac{Q}{4\pi\epsilon_0}\frac{r}{R^3}$$

$$|\vec{E}|(r > R) = \frac{Q}{4\pi\epsilon_0}\frac{1}{r^2}$$

10 pts

1. Answer all of the questions

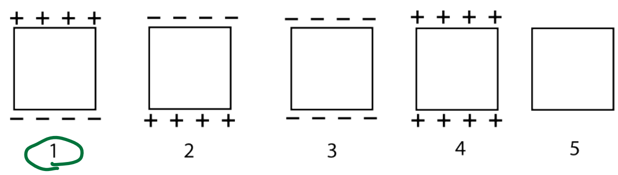
   (a) You hold a negatively charged rod (rod 1) near a second rod (rod 2), which is suspended by an insulating thread, and find that the second rod is attracted to the first one. Which of the following are possible? (Check all that apply)

   +3

   ☑ Rod 2 is positively charged
   ☐ Rod 2 is negatively charged
   ☑ Rod 2 is neutral

   (b) In the diagram below, there is an electric field in the upward direction due to charges not shown. Which diagram best describes the charge distribution on a neutral metal block?

   +3



   (c) Which of the following are true statements? (Check all that apply)

   +3

   ☑ In equilibrium, the average velocity of electrons $\bar{v}$ within a metal is 0
   ☐ The electric field inside of a metal is 0 under all circumstances
   ☐ Excess charge on an insulator cannot reside on the surface
   ☐ Conductors cannot be polarized
   ☑ In a metal any excess charge must be on the surface

   (d) Which is a better test of the existence and sign of the charge on an object, repulsion or attraction to a charged tape? Why? Explain briefly but clearly.

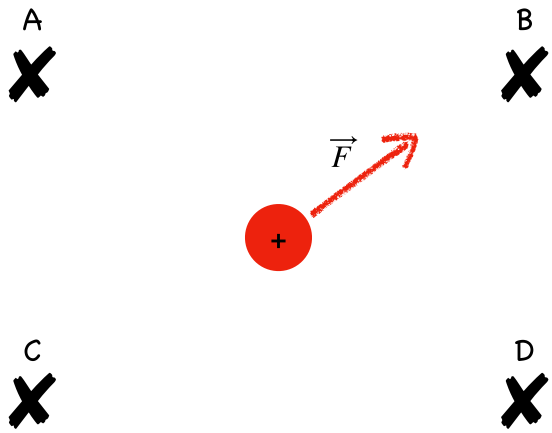
+1

Repulsion. Attraction is caused by either chg diff <u>or</u> polarization.

Repulsion can only be caused by charges.

15 pts

2. In the region of space shown in the figure, there is an electric field which is produced by charges that are not shown in the diagram.



+5 (a) The positively charged particle feels a force in the direction indicated. Which arrow (a-j) best describes the direction of the electric field vector at the location of the particle?

b

+5 (b) Now the positively charged particle is completely removed and replaced with a negatively charged particle. The electric field in the region is unchanged. Which arrow (a-j) best describes the direction of the force on the negatively charged particle?

f

+5 (c) Suppose you discover that the source of the electric field in this region is a single negatively charged particle. Which of the locations in the figure (A, B, C, or D) is a possible location for this negative charge?

B

3. At a certain point in space, a point charge with a charge of $+5\ \mu$C experiences a force $\vec{F} =< -1, 2, 0 >$ N due to the presence of an electric field.

9

(a) What is the electric field vector $\vec{E}$ responsible for this force? Express your result as a vector with appropriate units.

$$\vec{F} = q\vec{E} \quad (+2)$$

$$\vec{E} = \frac{\vec{F}}{q}$$

$$\vec{E} = \frac{\langle -1, 2, 0 \rangle \text{ N}}{5 \times 10^{-6} \text{ C}} = \boxed{\langle -2\times10^{5}, 4\times10^{5}, 0 \rangle \frac{\text{N}}{\text{C}}}$$

−1 for wrong units

−2 for not vector

−1 for not $\mu C \rightarrow 10^{-6}$ C

6

(b) The original $+5\ \mu$C charge is now removed and replaced with another point charge with a charge of $-2\ \mu$C. What is the force $\vec{F}$ experienced by this charge? Express your result as a vector with appropriate units.

$$\vec{F} = q\vec{E}$$

$$= (-2\times10^{-6} \text{ C}) \langle -2\times10^{5}, 4\times10^{5}, 0 \rangle \frac{\text{N}}{\text{C}}$$

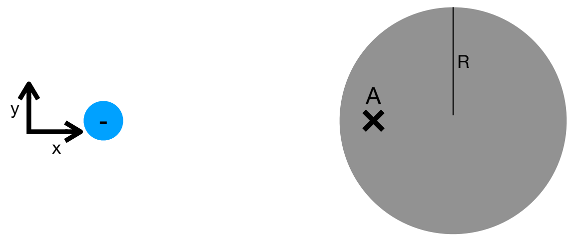
$$\boxed{\vec{F} = \langle 0.4, -0.8, 0 \rangle \text{ N}}$$

−1 for wrong units

−2 for not vector

−1 for not $\mu C \rightarrow 10^{-6}$ C

4. In the figure below, the gray sphere has a radius $R = 14$ cm and is positioned with its center at the origin. The gray sphere is a conductor and is in equilibrium. In this problem, the origin of the coordinate system is the center of the sphere and the x and y directions are depicted in the diagram.



(a) A negative point charge $q = -12$ nC is brought near the conducting sphere and placed at the position $\vec{r}_{\text{chg}} =< -26, 0, 0 >$ cm. What is the net electric field $\vec{E}_{\text{net}}$ at the position A (the position of A is given by $\vec{r}_A =< -12, 0, 0 >$ cm)?

10

$\vec{E}_{net} = 0$ (conductor in equilibrium)

(b) What is the induced field $\vec{E}_{\text{ind}}$, the electric field produced by the polarized charges in the conductor, at point A?

10

$$\vec{E}_{net} = \vec{E}_{pt} + \vec{E}_{ind} = 0$$

$$\vec{E}_{ind} = -\vec{E}_{pt} \quad (+2)$$

$$\vec{E}_{pt} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r}$$

$$\vec{r}_{src} = \langle -.26, 0, 0 \rangle \text{ m}$$

$$\vec{r}_{obs} = \langle -.12, 0, 0 \rangle \text{ m}$$

$$\vec{r} = \langle 0.14, 0, 0 \rangle \text{ m}$$

$$\vec{E}_{pt} = \left(9\times10^{9}\,\frac{Nm^2}{C^2}\right)\frac{(-12\times10^{-9}\,C)}{(.14m)^2}\langle 1, 0, 0\rangle$$

$$\vec{E}_{pt} = \langle -5510, 0, 0\rangle \,\frac{N}{C} \qquad (+2)$$

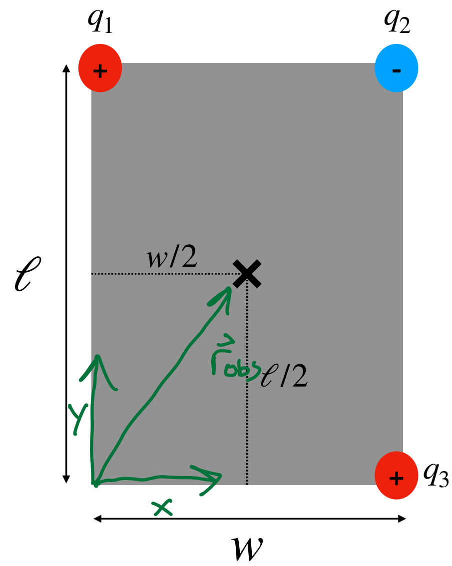
$$\vec{E}_{ind} = -\vec{E}_{pt} = \langle 5510, 0, 0\rangle \,\frac{N}{C}$$

−1 for wrong units

−2 for not vector

−1 for not $nC \rightarrow 10^{-9}\,C$

5. Three point charges are mounted onto the corners of a rectangle of length $\ell = 15$ cm and width $w = 10$ cm. If $q_1 = 6\ \mu$C, $q_2 = -4\ \mu$C, and $q_3 = 2\ \mu$C, what is the electric field at the center of the rectangle?



$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 \qquad (+3)$$

Choose origin. Can be anywhere. Must be same for each charge!

Origin = bottom left corner

$\underline{\vec{E}_1:}$ (+3)

$$\vec{r}_{src} = \langle 0, \ell \rangle$$

$$\vec{r}_{obs} = \langle \tfrac{w}{2}, \tfrac{\ell}{2} \rangle$$

$$\vec{r} = \langle \tfrac{w}{2}, -\tfrac{\ell}{2} \rangle \quad (+1) \qquad \vec{r} = \langle 0.05, -0.075 \rangle \text{ m}$$

$$|\vec{r}| = \left(\left(\tfrac{w}{2}\right)^2 + \left(\tfrac{\ell}{2}\right)^2 \right)^{\frac{1}{2}} = 0.090 \text{ m}$$

$$\hat{r} = \frac{\langle \frac{w}{\cancel{2}}, -\frac{\ell}{\cancel{2}} \rangle}{\frac{1}{\cancel{2}}(w^2 + \ell^2)^{\frac{1}{2}}} \qquad \hat{r} = \langle 0.55, -0.83 \rangle$$

$$\vec{E}_1 = \frac{1}{4\pi\epsilon_0} \frac{q_1}{\left(\frac{1}{4}\right)(\omega^2+\ell^2)} \frac{\langle \omega, -\ell \rangle}{(\omega^2+\ell^2)^{\frac{1}{2}}}$$

$$\vec{E}_1 = \frac{4}{4\pi\epsilon_0} \frac{q_1}{(\omega^2+\ell^2)^{\frac{3}{2}}} \langle \omega, -\ell \rangle$$

(+2)

$$\vec{E}_1 = \langle 3.7 \times 10^6, -5.5 \times 10^6 \rangle \ \frac{N}{C}$$

$\vec{E}_2$: (+3)

$$\vec{r}_{src} = \langle \omega, \ell \rangle$$

$$\vec{r}_{obs} = \left\langle \frac{\omega}{2}, \frac{\ell}{2} \right\rangle$$

$$\vec{r} = -\left\langle \frac{\omega}{2}, \frac{\ell}{2} \right\rangle = -\frac{1}{2} \langle \omega, \ell \rangle$$

(+1)

$$\vec{r} = \langle -0.05, -0.075 \rangle \ m$$

$$|\vec{r}| = \sqrt{\frac{1}{4}(\omega^2+\ell^2)} = \frac{1}{2}(\omega^2+\ell^2)^{\frac{1}{2}}$$

$$= 0.090 \ m$$

$$\hat{r} = \frac{-\frac{1}{2}\langle \omega, \ell \rangle}{\frac{1}{2}(\omega^2+\ell^2)^{\frac{1}{2}}} = \frac{-\langle \omega, \ell \rangle}{(\omega^2+\ell^2)^{\frac{1}{2}}}$$

$$\hat{r} = \langle -0.55, -0.83 \rangle$$

$$\vec{E}_2 = \frac{1}{4\pi\epsilon_0} \frac{q_2}{(\frac{1}{2})(w^2+l^2)} \frac{\langle w, l \rangle}{(w^2+l^2)^{\frac{1}{2}}}$$

$$\vec{E}_2 = \frac{2}{4\pi\epsilon_0} \frac{q_2}{(w^2+l^2)^{\frac{3}{2}}} \langle w, l \rangle \qquad (+2)$$

$$\vec{E}_2 = \langle 2.5 \times 10^6, 3.7 \times 10^6 \rangle \frac{N}{C}$$

$\vec{E}_3$:

$$\vec{r}_{src} = \langle w, 0 \rangle$$

$$\vec{r}_{obs} = \langle \frac{w}{2}, \frac{l}{2} \rangle$$

$$\vec{r} = \langle -\frac{w}{2}, \frac{l}{2} \rangle = \langle -.05, .075 \rangle m$$

$$\vec{r} = \frac{1}{2} \langle -w, l \rangle$$

$$|\vec{r}| = \frac{1}{2} (w^2+l^2)^{\frac{1}{2}}$$

$$\hat{r} = \frac{\cancel{\frac{1}{2}}\langle -w, \ell \rangle}{\cancel{\frac{1}{2}}(w^2+\ell^2)^{\frac{1}{2}}} = \langle -0.55, 0.83 \rangle$$

$$\vec{E}_3 = \frac{1}{4\pi\epsilon_0} \frac{q^3}{(\frac{1}{2})(w^2+\ell^2)^{\frac{3}{2}}} \langle -w, \ell \rangle$$

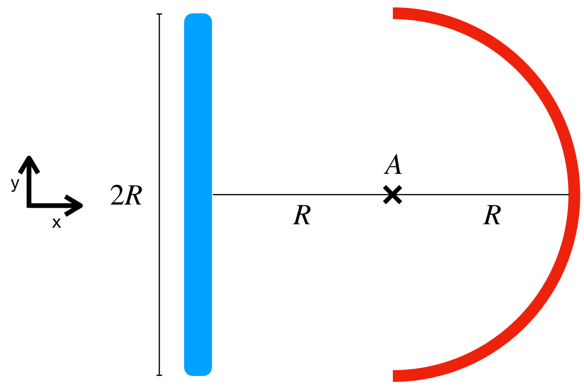
$$\vec{E}_3 = \frac{2}{4\pi\epsilon_0} \frac{q_3}{(w^2+\ell^2)^{\frac{3}{2}}} \langle -w, \ell \rangle \qquad (+z)$$

$$\vec{E}_3 = \langle -1.2 \times 10^6, \; 1.8 \times 10^6 \rangle \frac{N}{C}$$

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$$

$$= \boxed{\langle 4.9 \times 10^6, \quad 0 \quad \rangle \frac{N}{C}}$$

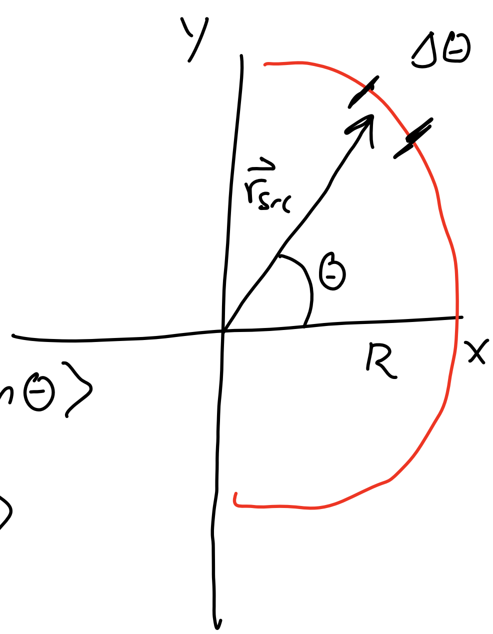
6. A thin, semi-circular ring of radius R carries a total charge $+Q$. A distance R to the left of the center of the ring is a thin negatively charged rod carrying charge $-Q$. The length of the rod is the same as the diameter of the ring: $2R$. Using the coordinate directions defined in the figure, what is the electric field vector at point A, which is at the center of the semi-ring and lies along the middle axis of the rod? Express your answer in terms of the variables defined in the problem: Q and R, as well as the constant ϵ_0 or k. You do not need to evaluate any integrals that arise (you may express your answer in the form of one or more integrals, but make sure to specify the bounds of integration).



$\vec{E} = \vec{E}_{ring} + \vec{E}_{rod}$ (+ 2)

$\underline{\vec{E}_{ring}}$

$\Delta Q = Q \dfrac{\Delta\theta}{\pi}$



$\vec{r}_{src} = R\langle \cos\theta, \sin\theta \rangle$

$\vec{r}_{obs} = \langle 0, 0 \rangle$

$\vec{r} = -R\langle \cos\theta, \sin\theta \rangle$

$|\vec{r}| = R$

$$\hat{r} = -\langle \cos\theta, \sin\theta \rangle$$

(+4)

$$\Delta\vec{E} = \frac{-1}{4\pi\epsilon_0} \frac{Q\,\Delta\theta}{\pi R^2} \langle \cos\theta, \sin\theta \rangle$$

$$\vec{E} = -\frac{1}{4\pi\epsilon_0} \frac{Q}{\pi R^2} \left\langle \underbrace{\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos\theta \, d\theta}_{\substack{\sin(\frac{\pi}{2}) - \sin(-\frac{\pi}{2}) \\ 1 - -1 \\ = 2}}, \underbrace{\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin\theta \, d\theta}_{\substack{-\cos(\frac{\pi}{2}) \\ + \cos(-\frac{\pi}{2}) \\ = 0}} \right\rangle$$

$$\vec{E}_{ring} = \frac{-2}{4\pi\epsilon_0} \frac{Q}{\pi R^2} \langle 1, 0 \rangle$$

$$|\vec{E}_{rod}| = \frac{1}{4\pi\epsilon_0} \left[\frac{Q}{r\left[r^2 + \left(\frac{L}{2}\right)^2\right]^{\frac{1}{2}}} \right]$$

$r = R$

$L = 2R$

(+2)

$$|\vec{E}_{rod}| = \frac{1}{4\pi\epsilon_0} \frac{Q}{R\left(R^2 + R^2\right)^{\frac{1}{2}}} = \frac{1}{4\pi\epsilon_0} \frac{Q}{\sqrt{2}\, R^2}$$

$$\vec{E}_{rod} = \left\langle -\frac{1}{4\pi\epsilon_0} \frac{\sqrt{2}}{2} \frac{Q}{R^2} \,,\, 0 \right\rangle$$

$$\vec{E} = \vec{E}_{rod} + \vec{E}_{ring}$$

$$= \left\langle \frac{-2}{4\pi\epsilon_0} \frac{Q}{\pi R^2} - \frac{\frac{\sqrt{2}}{2}}{4\pi\epsilon_0} \frac{Q}{R^2} \,,\, 0 \right\rangle$$

$$\vec{E} = \frac{-Q}{4\pi\epsilon_0 R^2} \left\langle \frac{2}{\pi} + \frac{\sqrt{2}}{2} \,,\, 0 \right\rangle$$