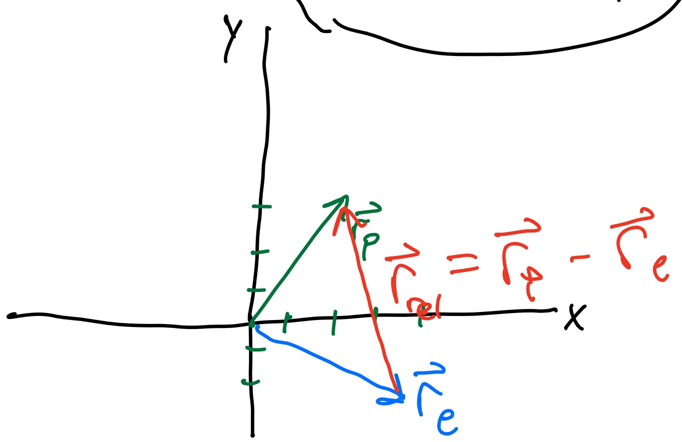
1. $\vec{A} = \langle 3, 5, 1 \rangle$

$|\vec{A}| = \sqrt{3^2 + 5^2 + 1^2} = 5.92$

2. $\vec{r}_p = \langle 2, 3, 0 \rangle$

$\vec{r}_e = \langle 4, -2, 0 \rangle$

$\vec{r}_{rel} = \vec{r}_p - \vec{r}_e = \langle -2, 5, 0 \rangle$



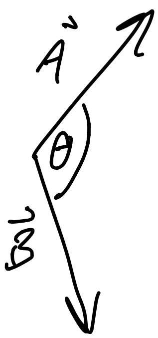
3. $\vec{A} = \langle 9, 5, 8 \rangle$

$\vec{B} = \langle -3, -5, 4 \rangle$

$\vec{A} \cdot \vec{B} = (9)(-3) + (5)(-5) + (8)(4)$

$= -27 - 25 + 32$

$= -20$ ①



$\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}| \cos\theta$

$|\vec{A}| = 13.04 \quad |\vec{B}| = 7.07$

$$\frac{\vec{A} \cdot \vec{B}}{(13.04)(7.07)} = \cos\theta$$

$$\frac{-20}{92.20} = \cos\theta$$

$\theta = \arccos(-0.22) = 1.79 \text{ rad}$

(102.5°)

4. $\vec{a} = \langle -3, 7, 1 \rangle$

$\hat{a} = \frac{\vec{a}}{|\vec{a}|}$; $|\vec{a}| = 7.68$

$\hat{a} = \left\langle \frac{-3}{7.68}, \frac{7}{7.68}, \frac{1}{7.68} \right\rangle$

$\hat{a} = \langle -0.39, 0.91, 0.13 \rangle$

direction of x-axis $= \hat{x} = \langle 1,0,0 \rangle$

$\hat{a} \cdot \hat{x} = |\hat{a}||\hat{x}| \cos\theta$

$|\hat{a}| = 1$, $|\hat{x}| = 1$

$\hat{a} \cdot \hat{x} = \cos\theta \longrightarrow \theta = \arccos(-0.39 + 0 + 0)$

$\theta = 1.97$ rad

$112.95°$

5. $\vec{B} = \langle 3, -9, 7 \rangle$

$\vec{v} = \langle 8, 4, 6 \rangle$

$\vec{A} = \langle a_x, a_y, a_z \rangle = a_x\hat{x} + a_y\hat{y} + a_z\hat{z}$

$\vec{B} = \langle b_x, b_y, b_z \rangle = b_x\hat{x} + b_y\hat{y} + b_z\hat{z}$

$$\vec{A} \times \vec{B} = (a_x\hat{x} + a_y\hat{y} + a_z\hat{z}) \times (b_x\hat{x} + b_y\hat{y} + b_z\hat{z})$$

$$= a_x\hat{x} \times b_x\hat{x} + a_x\hat{x} \times b_y\hat{y} + a_x\hat{x} \times b_z\hat{z}$$
$$+ a_y\hat{y} \times b_x\hat{x} + a_y\hat{y} \times b_y\hat{y} + a_y\hat{y} \times b_z\hat{z}$$
$$+ a_z\hat{z} \times b_x\hat{x} + a_z\hat{z} \times b_y\hat{y} + a_z\hat{z} \times b_z\hat{z};$$

$$a_x\hat{x} \times b_y\hat{y} = a_x b_y (\hat{x} \times \hat{y})$$

$\hat{x} \times \hat{x} = 0$, $\hat{x} \times \hat{y} = \hat{z}$, $\hat{x} \times \hat{z} = -\hat{y}$

$\hat{y} \times \hat{x} = -\hat{z}$, $\hat{y} \times \hat{y} = 0$, $\hat{y} \times \hat{z} = \hat{x}$

$\hat{z} \times \hat{x} = \hat{y}$, $\hat{z} \times \hat{y} = -\hat{x}$, $\hat{z} \times \hat{z} = 0$

$$\vec{A} \times \vec{B} = \cancel{a_x\hat{x} \times b_x\hat{x}} + a_x\hat{x} \times b_y\hat{y} + a_x\hat{x} \times b_z\hat{z}$$
$$+ a_y\hat{y} \times b_x\hat{x} + \cancel{a_y\hat{y} \times b_y\hat{y}} + a_y\hat{y} \times b_z\hat{z}$$
$$+ a_z\hat{z} \times b_x\hat{x} + a_z\hat{z} \times b_y\hat{y} + \cancel{a_z\hat{z} \times b_z\hat{z}};$$

$$= a_x b_y \hat{z} + a_x b_z (-\hat{y})$$
$$+ a_y b_x (-\hat{z}) + a_y b_z \hat{x} +$$
$$+ a_z b_x \hat{y} + a_z b_y (-\hat{x})$$

$$\vec{A} \times \vec{B} = (a_y b_z - a_z b_y)\hat{x} + (a_z b_x - a_x b_z)\hat{y} + (a_x b_y - a_y b_x)\hat{z}$$

$$\vec{B} = \langle 3, -9, 7 \rangle$$

$$\vec{v} = \langle 8, 4, 6 \rangle$$

$$\vec{B} \times \vec{v} = (-9 \cdot 6 - 7 \cdot 4)\hat{x} + (7 \cdot 8 - 3 \cdot 6)\hat{y}$$

$$(3 \cdot 4 - (-9) \cdot 8)\hat{z}$$

$$\boxed{\vec{B} \times \vec{v} = -82\hat{x} + 38\hat{y} + 84\hat{z}}$$

6. $\vec{A} = \langle 4, 5, -7 \rangle$

$\vec{B} = \langle 6, -2, 2 \rangle$

if $\vec{A} \perp \vec{B}$, then $\vec{A} \cdot \vec{B} = 0$

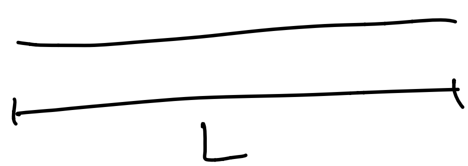
$\vec{A} \cdot \vec{B} = 4 \cdot 6 - 5 \cdot 2 - 7 \cdot 2$

$= 24 - 10 - 14 = 0$

$\vec{A} \cdot \vec{B} = 0$

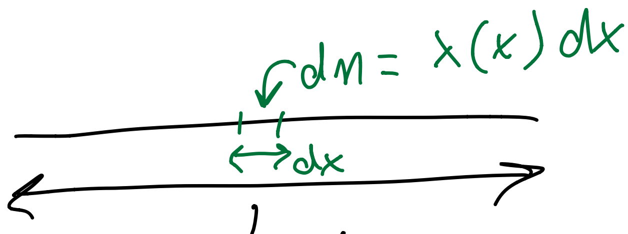
ORTHOGONAL ✓

7.



$$\frac{M}{L} = \lambda_0 \Rightarrow M = \lambda_0 L$$

8.

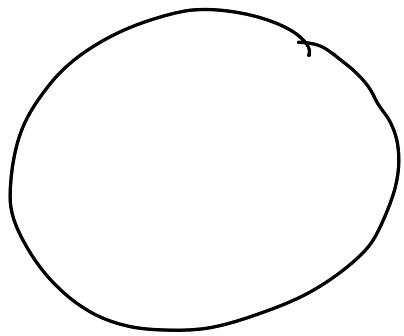


$$M = \int dm = \int_0^L \lambda(x)\,dx = \int_0^L \lambda_0 \left(\frac{x}{L}\right)^3 dx$$

$$= \frac{\lambda_0}{L^3} \int_0^L x^3 dx = \frac{\lambda_0}{L^3} \left(\frac{1}{4} x^4\right)\Bigg|_0^L = \frac{\lambda_0}{4} L$$

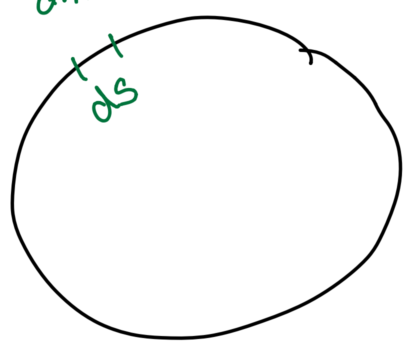
$$\boxed{M = \frac{1}{4} \lambda_0 L}$$

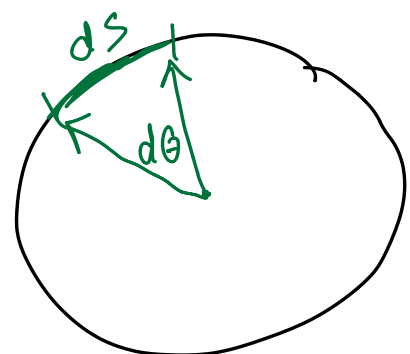
9.



length $L = 2\pi R \Rightarrow R = \frac{L}{2\pi}$

$M = \int dm$ $\quad dm = \lambda(\theta) ds$



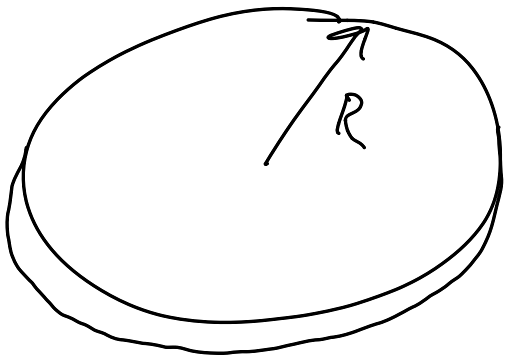


$$
\begin{aligned}
M &= \int \lambda(\theta) ds \\
&= \int \lambda(\theta) R d\theta \\
&= \int \lambda_0 \theta R d\theta \\
&= \lambda_0 R \frac{\theta^2}{2}\Big|_0^{2\pi} = \frac{\lambda_0 R}{2} 4\pi^2 \\
&= 2\pi^2 \lambda_0 R \\
&= 2\pi^2 \lambda_0 \frac{L}{2\pi} = \boxed{\pi \lambda_0 L = M}
\end{aligned}
$$

$ds = \frac{d\theta}{2\pi} \times 2\pi R$

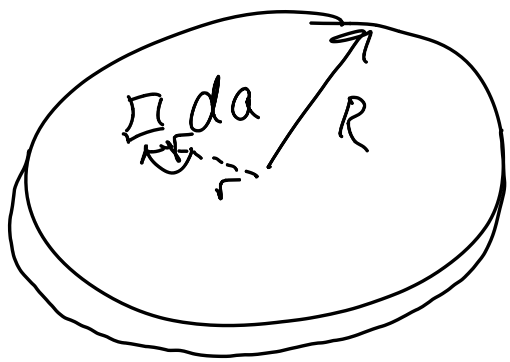
$ds = d\theta R$

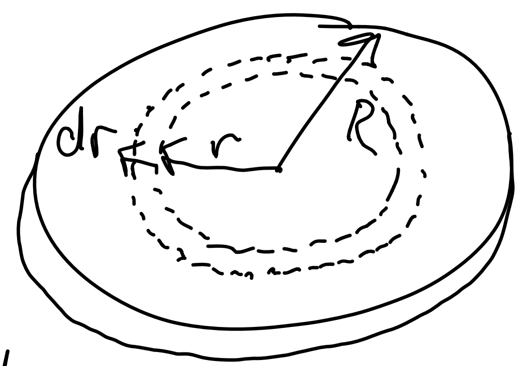
10.



$$M = \int dm$$

$$dm = \sigma(r)\,da$$





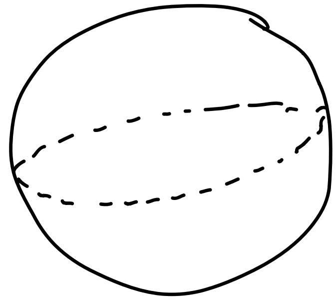
$$da = 2\pi r\,dr$$

$$dm = \sigma(r)\,2\pi r\,dr$$

$$M = \int_0^R \sigma(r)\,2\pi r\,dr = \int_0^R \sigma_0 \left(\frac{r}{R}\right)^2 2\pi r\,dr$$

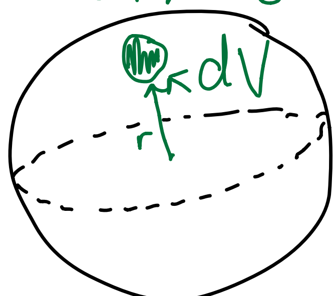
$$M = \frac{\sigma_0}{R^2}\,\frac{1}{4}R^4\,2\pi = \frac{1}{2}\sigma_0 \pi R^2$$

11.

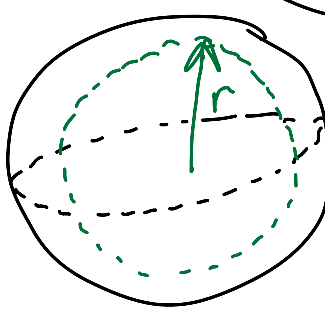


$$M = \int dm$$

$$dm = \rho(r)\,dV$$



$$M = \int dm = \int_0^R \rho(r)\, 4\pi r^2\, dr$$



$$dV = 4\pi r^2\, dr$$

$$= \int_0^R \rho_0 \frac{r}{R}\, 4\pi r^2\, dr = \frac{4\pi\rho_0}{R}\,\frac{1}{4}\,R^4$$

$$\boxed{M = \pi \rho_0 R^3}$$