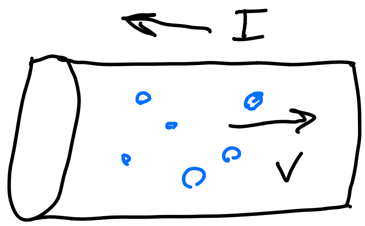
On Monday:

$I = |q| n A v = |q| i$ conventional current



$i = n A v = \frac{\#\ \text{elec}}{\Delta t}$ electron current

$i \longrightarrow$

We will use conventional current, I

Units of I?

$$I = |q| n A v = \frac{C}{m^3} m^2 \frac{m}{s} = \frac{C}{s}$$

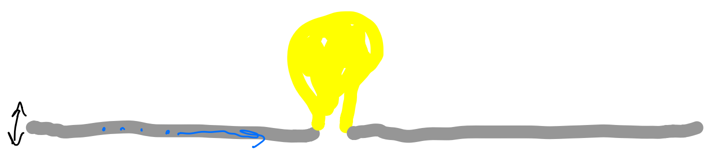
$$1 \frac{C}{s} := 1 \text{ Ampere} = 1 \text{ "amp"}$$

$$1 \frac{C}{s} = 1 A$$

Example: Drift velocity

$\swarrow$

Current in a 60 W bulb



$R_{wire} = 1\,mm$

$n \approx \frac{10^{28}}{m^3}$

$I = 0.5\,A$

$v = ?$

$$I = |q| n A v$$

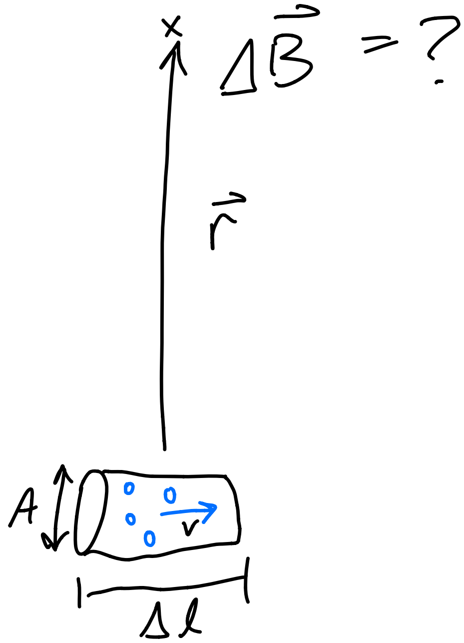
$$v = \frac{I}{q n A}, \quad A = \pi R^2, \quad q = e$$

$$= \frac{0.5}{(1.6\times10^{-19})(10^{28})(\pi(0.001)^2)}$$

$$v \approx 10^{-4}\,\frac{m}{s} = 0.1\,\frac{mm}{s}$$

SLOW

Magnetic field of a current



$\Delta \ell$, A are small compared to $\vec{r}$

Treat like a single point charge

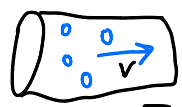
$$\Delta\vec{B} = \frac{\mu_0}{4\pi} q \frac{\vec{v} \times \hat{r}}{r^2}$$

$$q = {}^{-}n V = {}^{-}n A \Delta \ell$$

$$\Delta\vec{B} = \frac{{}^{-}\mu_0}{4\pi} \frac{n A \Delta \ell \, \vec{v} \times \hat{r}}{r^2} \qquad \vec{v} = v\hat{v}$$

$$\Delta\vec{B} = \frac{{}^{-}\mu_0}{4\pi} \frac{I \Delta \ell \, \hat{v} \times \hat{r}}{r^2}$$

$$\Delta \vec{B} = \frac{\mu_0}{4\pi} \frac{I \Delta l \, \hat{v} \times \hat{r}}{r^2}$$

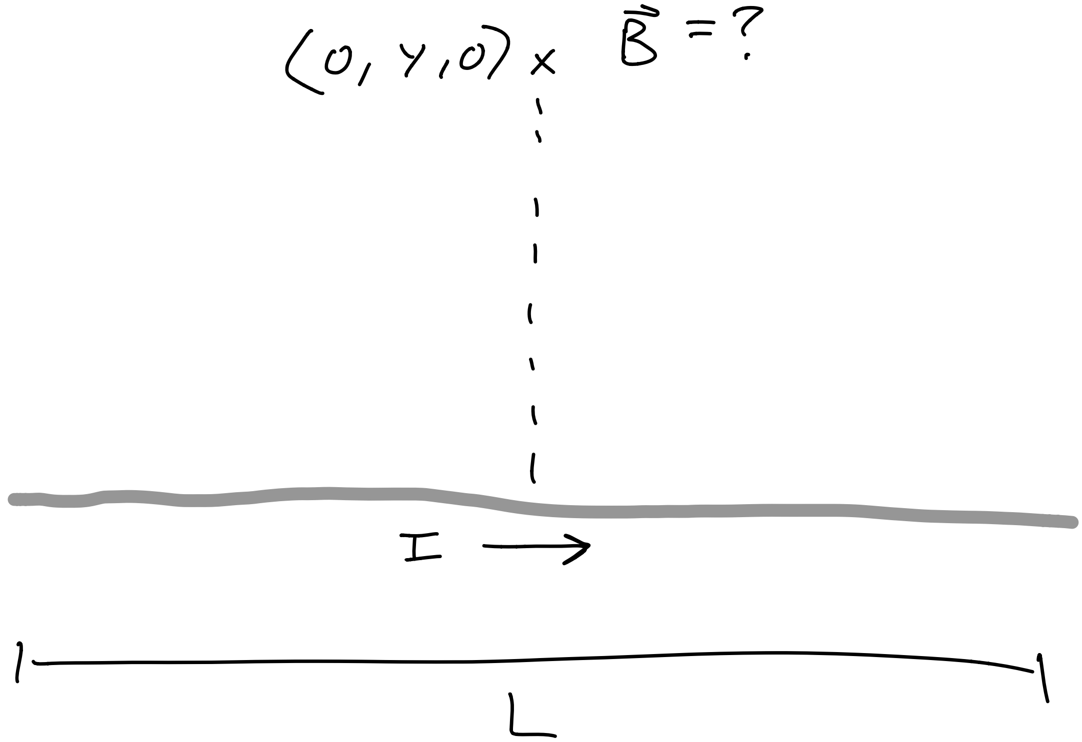


$\leftarrow I$

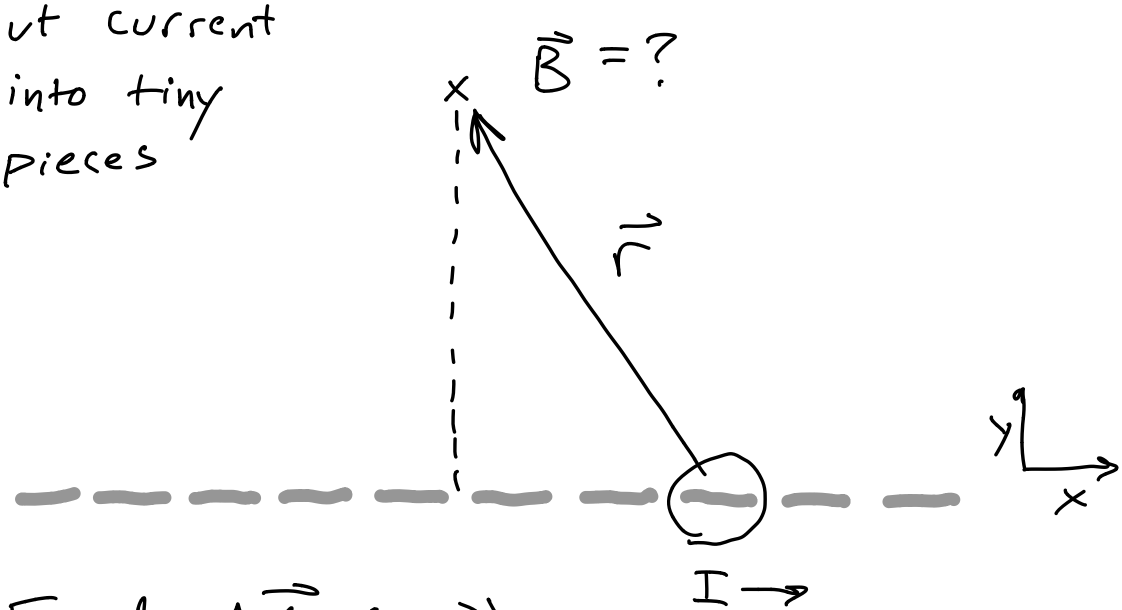
$\leftarrow \Delta l \qquad \Delta \vec{l} = \Delta l(-\hat{v})$

$$\boxed{\Delta \vec{B} = \frac{\mu_0}{4\pi} \frac{I \Delta \vec{l} \times \hat{r}}{r^2}}$$

$\vec{B} = ?$



1) cut current into tiny pieces



2) Find $\Delta\vec{l}$ $(d\vec{l})$

Magnitude: size of the piece

dx

Direction: Direction of current: $\hat{x}$

$$d\vec{l} = \langle dx, 0, 0 \rangle$$

I is the same everywhere

3) Find $\vec{r}$

$$\vec{r} = \vec{r}_{obs} - \vec{r}_{src}$$

$$\vec{r}_{obs} = \langle 0, y, 0 \rangle$$

$$\vec{r}_{src} = \langle x, 0, 0 \rangle$$

$$\vec{r} = \langle -x, y, 0 \rangle$$

4) Write $d\vec{B}$

$$d\vec{B} = \frac{\mu_0}{4\pi} I \frac{d\vec{\ell} \times \hat{r}}{r^2}$$

$$d\vec{B} = \frac{\mu_0}{4\pi} I \frac{dx\,\hat{x} \times \hat{r}}{r^2}$$

$$|\vec{r}| = \sqrt{x^2+y^2}$$

$$\hat{r} = \left\langle \frac{-x}{\sqrt{x^2+y^2}}, \frac{y}{\sqrt{x^2+y^2}}, 0 \right\rangle$$

$$= \frac{1}{\sqrt{x^2+y^2}} \langle -x, y \rangle$$

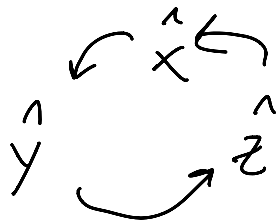
$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\,dx\,\hat{x} \times \langle -x, y \rangle}{(x^2+y^2)^{3/2}}$$

$$d\vec{B} = \frac{\mu_0 I\,dx}{4\pi(x^2+y^2)^{3/2}} \; \hat{x} \times (-x\hat{x} + y\hat{y})$$

$$\hat{x} \times (-\hat{x} + y\hat{y})$$

$$= -\hat{x} \times \hat{x} + y\,\hat{x} \times \hat{y}$$

$$= 0 + y\hat{z}$$



$$d\vec{B} = \frac{\mu_0 I y\,dx}{4\pi(x^2+y^2)^{3/2}}\hat{z}$$

5) Bounds of integral

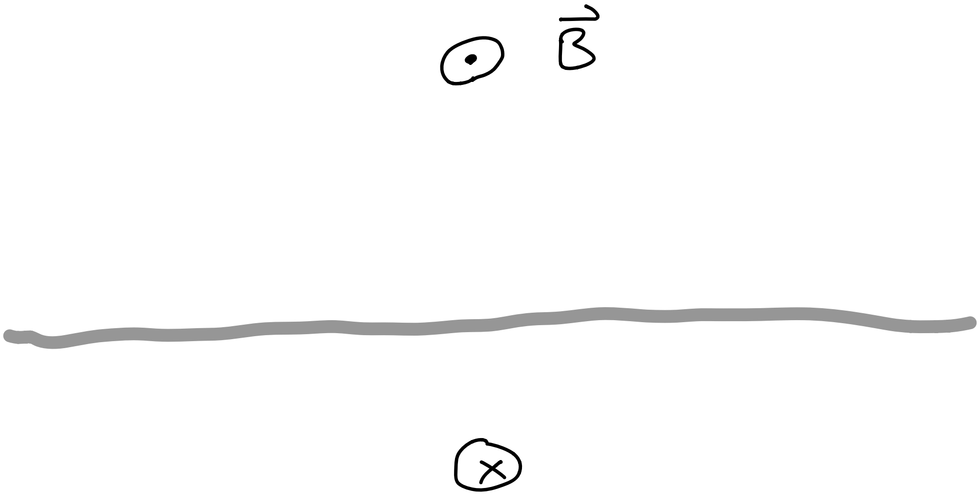
$$\vec{r}_{src} = \langle x, 0, 0 \rangle$$

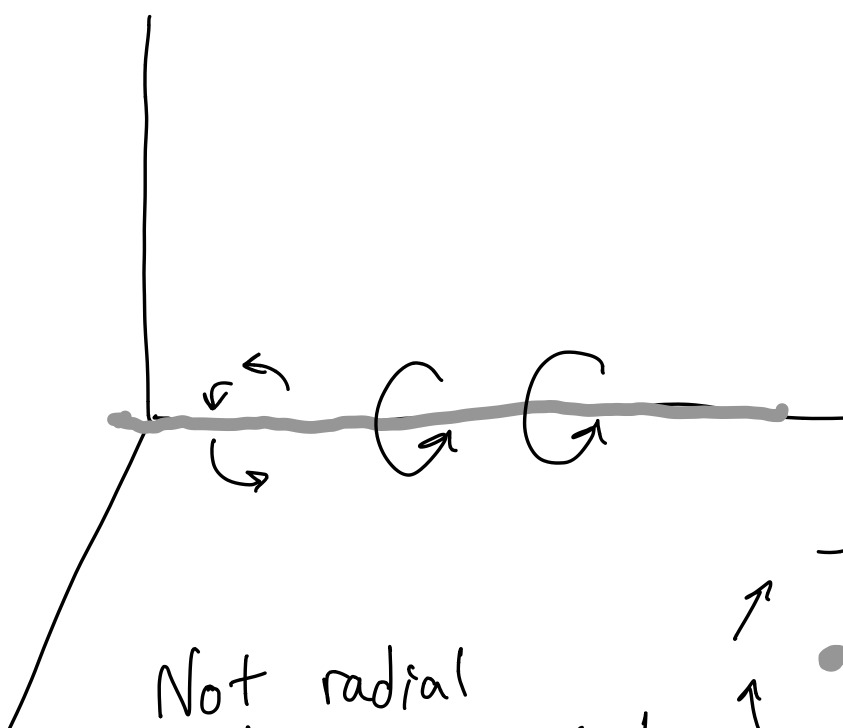
$$x_{min} = \frac{-L}{2} \qquad x_{max} = \frac{L}{2}$$

$$\vec{B} = \int_{-L/2}^{L/2} \frac{\mu_0 I}{4\pi} \frac{y}{(x^2+y^2)^{3/2}} dx\,\hat{z}$$

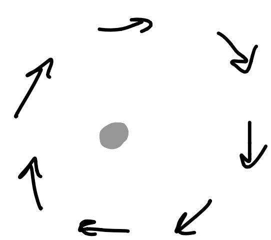
$$\vec{B} = \frac{\mu_0 I y}{4\pi} \int_{-L/2}^{L/2} \frac{dx}{(x^2+y^2)^{3/2}} \hat{z}$$

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{L I}{y\sqrt{y^2 + (L/2)^2}} \hat{z}$$





Not radial
but azimuthal



$$\vec{B} = \frac{\mu_0}{4\pi} \frac{LI}{y\sqrt{y^2 + (L/2)^2}} \hat{z}$$

if $L >> x$

$$|\vec{B}| = \frac{\mu_0}{4\pi} \frac{2I}{y}$$

$\vec{B}$ of long straight wire:

$$\boxed{\vec{B} \approx \frac{\mu_0}{4\pi} \frac{2I}{r}}$$

Direction: circling the wire (RHR)