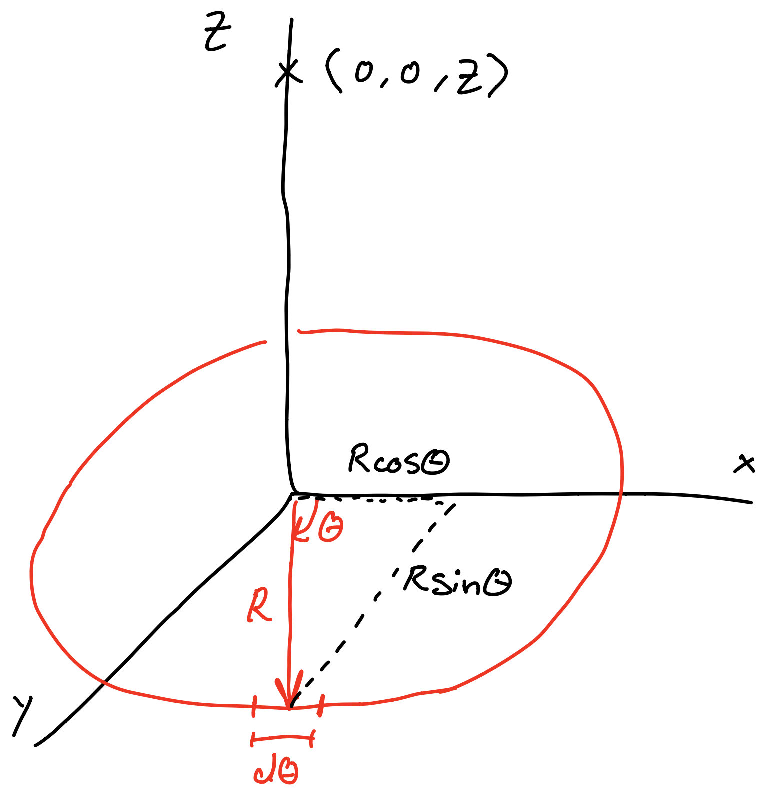
Field of charged ring



Chg density $\lambda = \frac{Q}{2\pi}$

$$dQ = \frac{Q}{2\pi} d\theta$$

$\vec{r}_{src} = \langle R\cos\theta, r\sin\theta, 0 \rangle$

$\vec{r}_{obs} = \langle 0, 0, z \rangle$

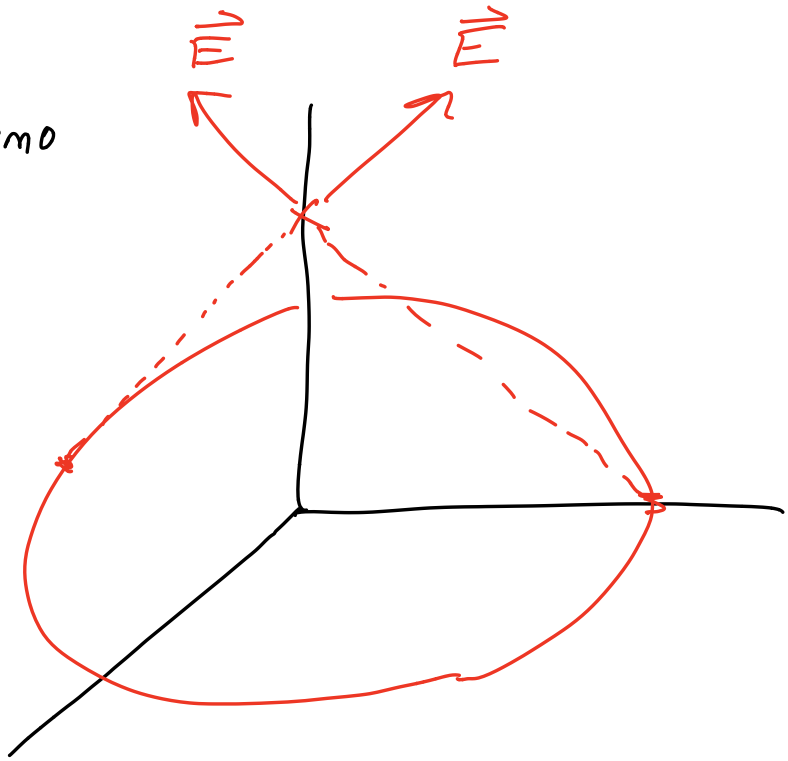
$\vec{r} = \langle -R\cos\theta, -R\sin\theta, z \rangle$

$$d\vec{E} = \frac{KQ}{2\pi} \frac{1}{(R^2+z^2)^{\frac{3}{2}}} d\theta \langle -R\cos\theta, R\sin\theta, z \rangle$$

$$E_x = -(\quad) \int_0^{2\pi} \cos\theta \, d\theta = 0$$

$$E_y = 0$$

Show Demo Program



$$E_x + E_y = 0$$

$$E_z = \frac{KQ}{2\pi} \frac{z}{(R^2+z^2)^{\frac{3}{2}}} \int_0^{2\pi} d\theta$$

$$E_z = \frac{KQz}{(R^2+z^2)^{3/2}}$$

$E_z (z >> R)$

$R \to 0$

$$E_z = \frac{KQz}{(z^2)^{3/2}} = \frac{KQ}{z^2} \quad (\text{pt chg}) \checkmark$$

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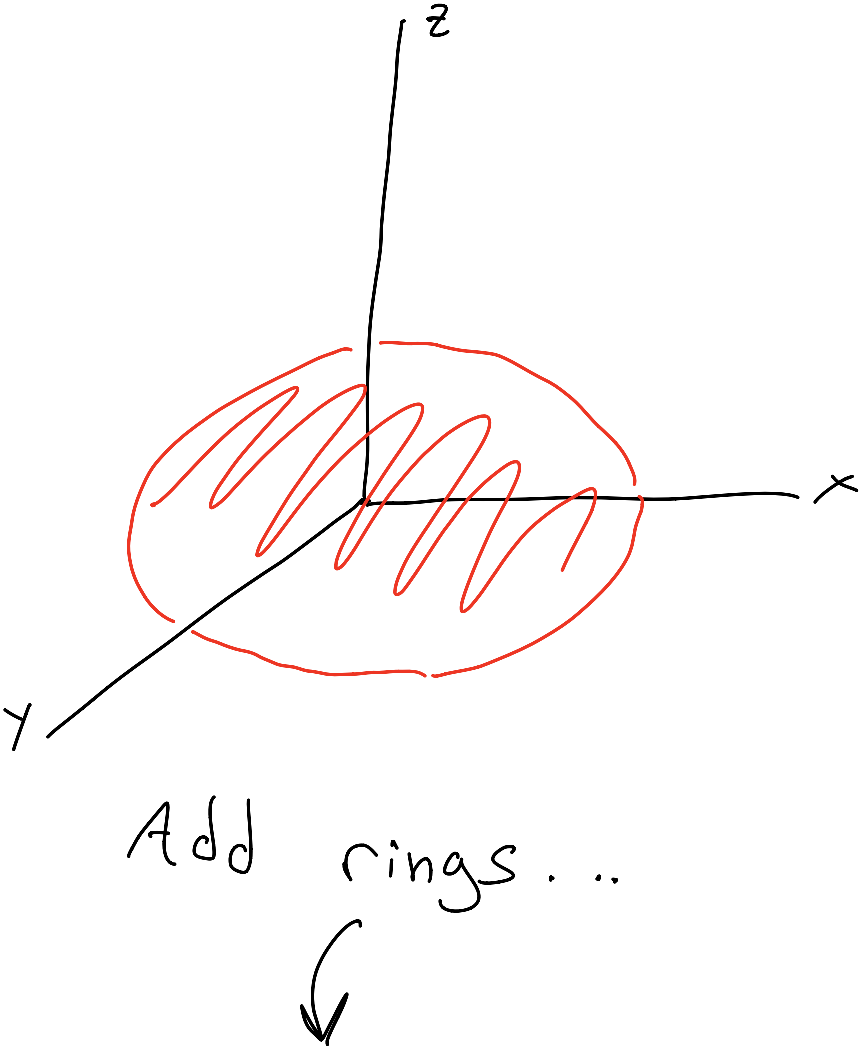
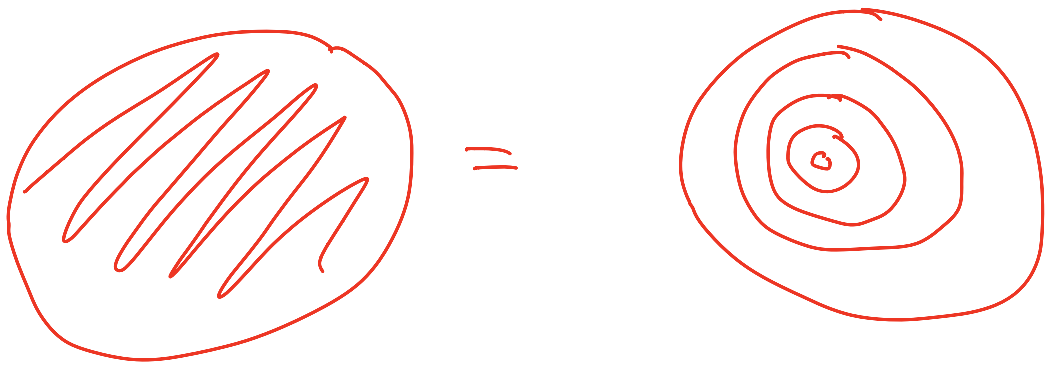
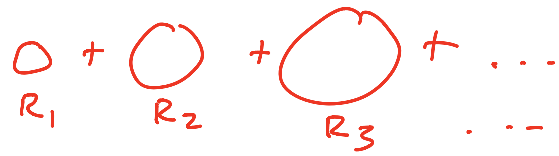
Uniformly Q disk

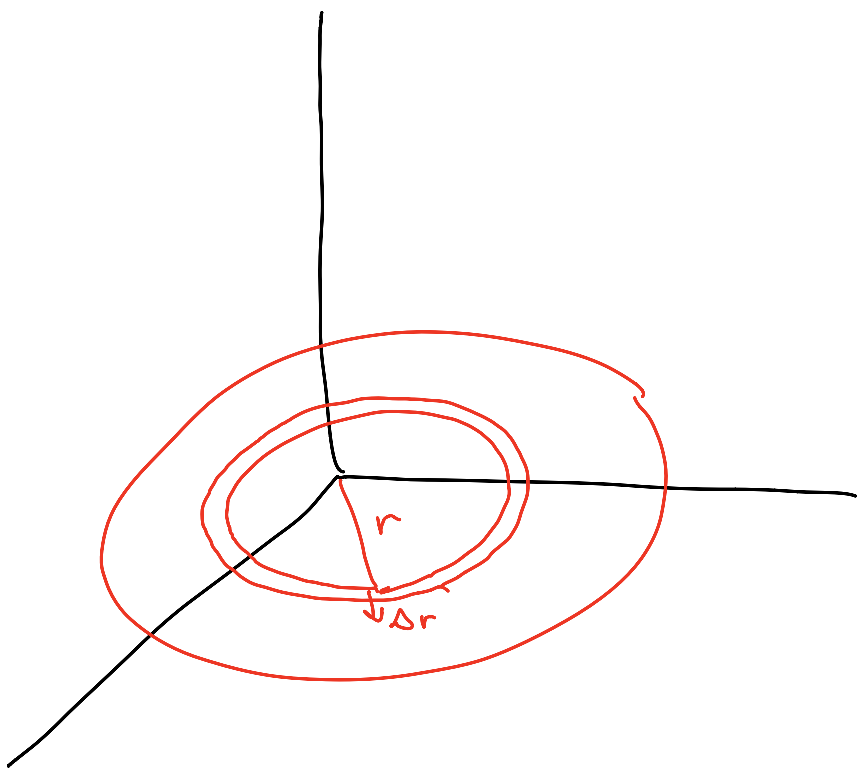


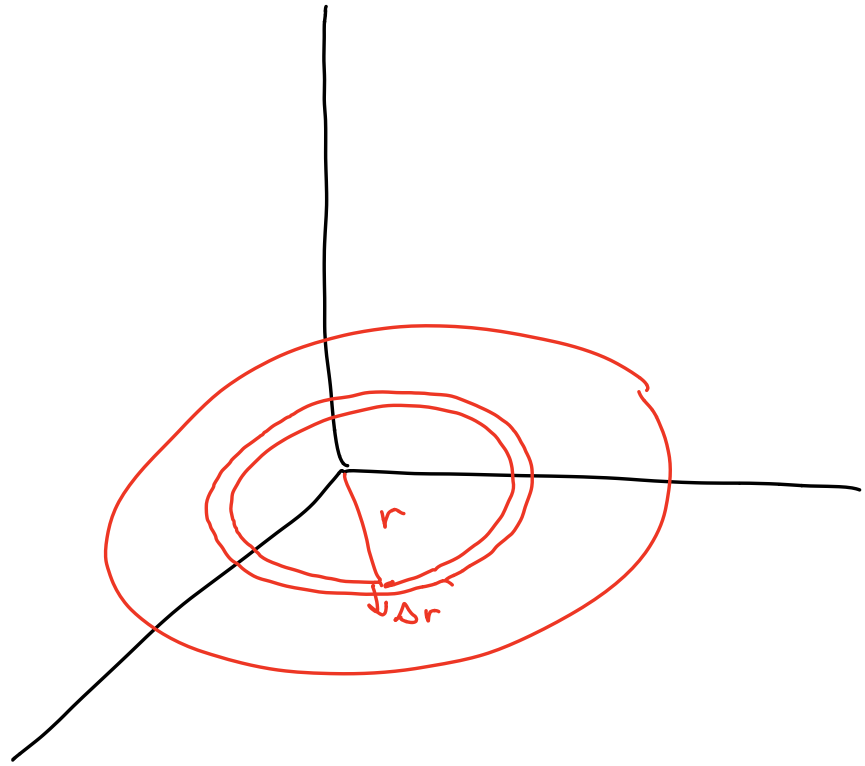
Add rings...



Disk = 

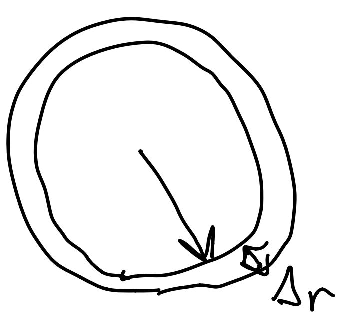
$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \dots$$





$$d\vec{E} = \frac{kz\,\Delta Q}{(r^2+z^2)^{3/2}}$$

ΔQ ?   chg density is $\dfrac{Q}{\pi R^2}$



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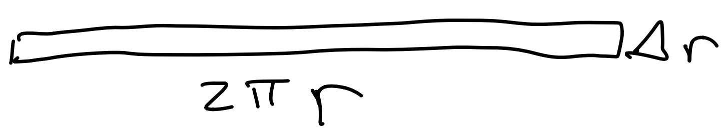


$A = 2\pi r \Delta r$

$$\Delta Q = \frac{Q}{\pi R^2} 2\pi r \Delta r$$

$$= \frac{2 Q r \Delta r}{R^2}$$

$$d\vec{E} = \frac{K z \Delta Q}{(r^2+z^2)^{3/2}} = \frac{K z 2 Q r \Delta r}{R^2 (r^2+z^2)^{3/2}} \hat{z}$$

$$\vec{E} = \frac{2QKz}{R^2} \int_0^R \frac{r\,dr}{(r^2+z^2)^{3/2}} \hat{z}$$

$$u = r^2+z^2, \quad du = 2r\,dr$$

$$\vec{E} = \frac{1}{2\epsilon_0} \left(\frac{Q}{\pi R^2}\right) \left[1 - \frac{z}{(R^2+z^2)^{1/2}}\right] \hat{z}$$

$$\vec{E} = \frac{Q/A}{2\epsilon_0} \left[1 - \frac{z}{(R^2+z^2)^{\frac{1}{2}}}\right] \hat{z}$$

What if z is small (but not 0)

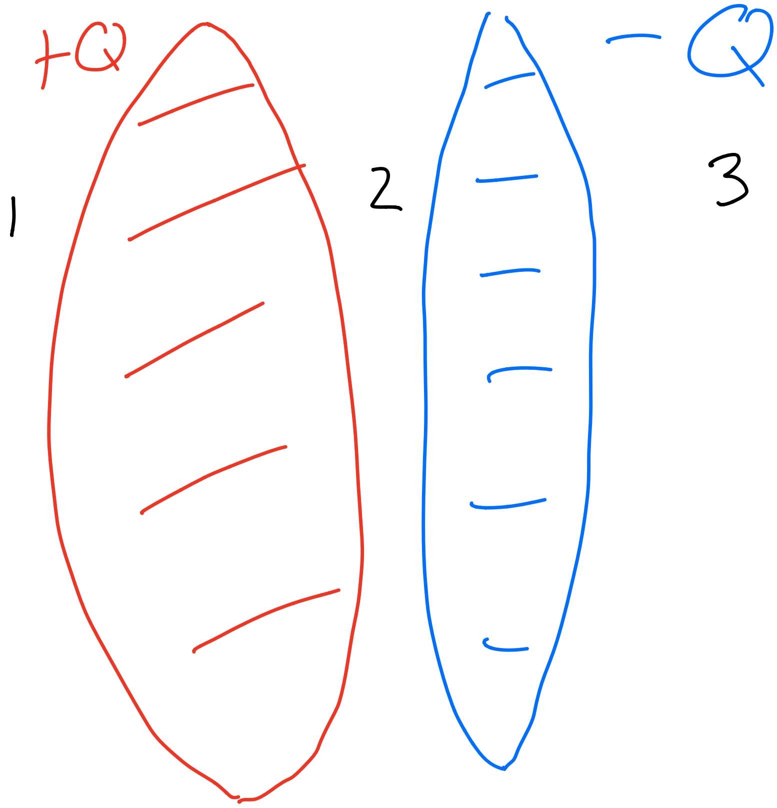
$R^2 + z^2 \approx R^2$

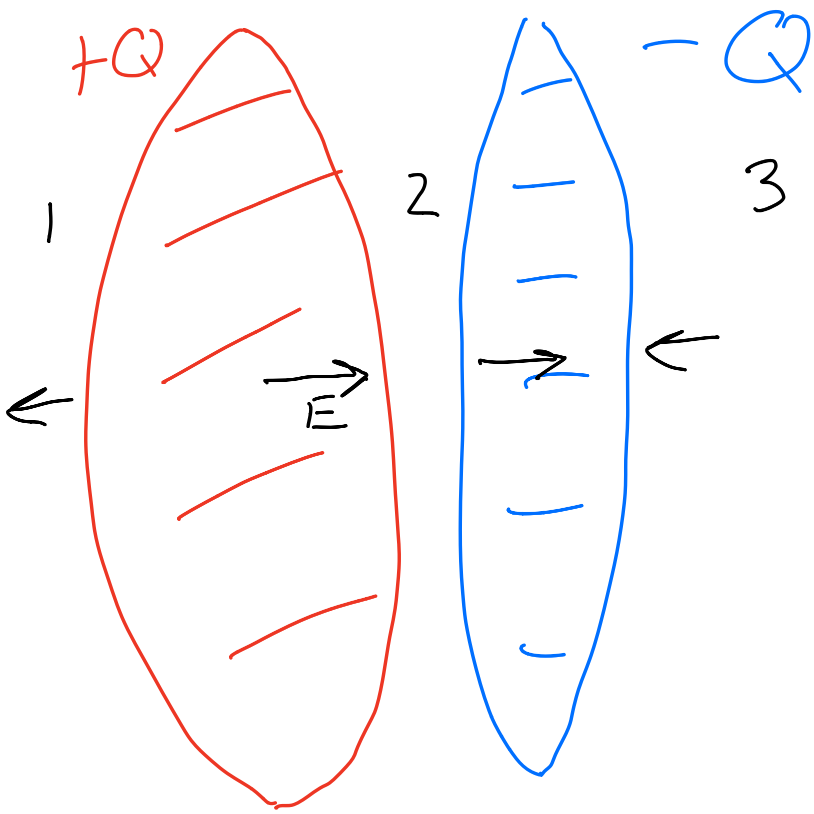
$$\vec{E} \approx \frac{Q/A}{2\epsilon_0}\left[1 - \frac{z}{R}\right]$$

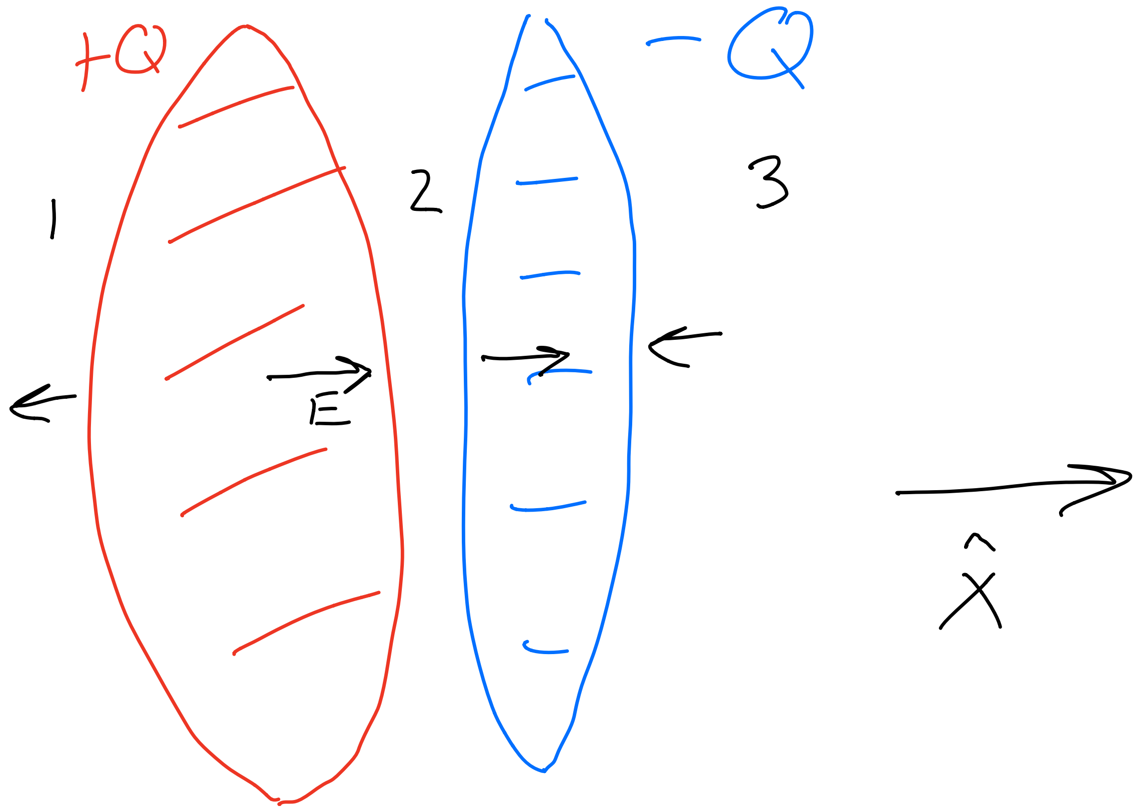
$$\vec{E} \approx \frac{Q/A}{2\epsilon_0}\,\hat{z}$$

No distance dependence!

Capacitors


+Q
−Q
1
2
3


+Q
−Q
1
2
3
E



2) $\vec{E}_2 = \vec{E}_+ + \vec{E}_-$

$$= \frac{Q/A}{2\epsilon_0}\hat{x} + \frac{Q/A}{2\epsilon_0}\hat{x}$$

$$\vec{E}_2 = \frac{Q/A}{\epsilon_0}\hat{x}$$

$$\vec{E}_1 = \vec{E}_+ + \vec{E}_-$$
$$= -\frac{Q/A}{2\epsilon_0}\hat{x} + \frac{Q/A}{2\epsilon_0}\hat{x}$$
$$= 0$$

$$\vec{E}_2 = \frac{Q/A}{2\epsilon_0}\hat{x} - \frac{Q/A}{2\epsilon_0}\hat{x} = 0$$

$$\vec{E}_{cap} = \begin{cases} \frac{Q/A}{\epsilon_0}\hat{x}, & \text{in between plates} \\ 0, & \text{else} \end{cases}$$

Fringe Field

$$|\vec{E}_{Fringe}| \approx \frac{Q/A}{2\epsilon_0}\frac{s}{R}, \quad \frac{s}{R} \ll 1 \longrightarrow 0$$