CHAPTER 21

PATTERNS OF FIELD IN SPACE

OVERVIEW

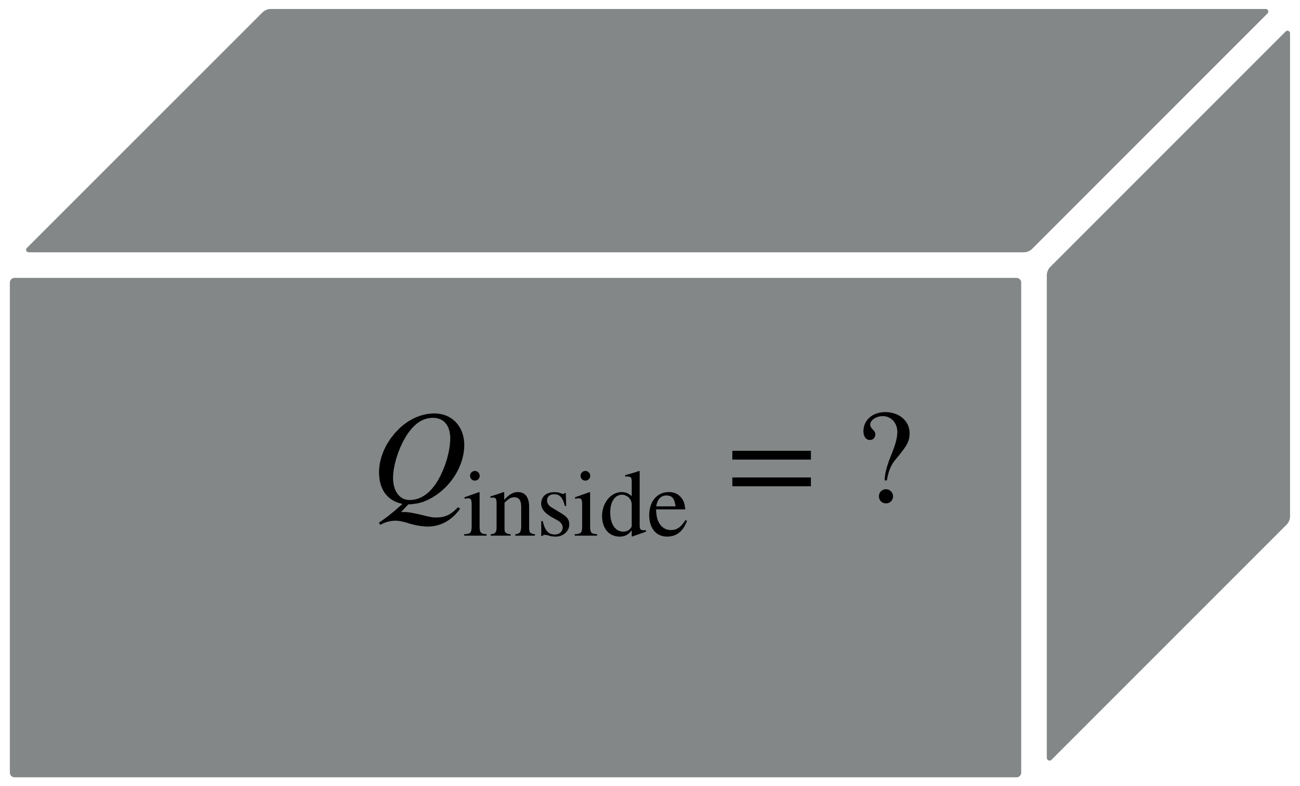
- Electric charges create electric and magnetic fields
- Usually, we start with an arrangement of charge or current and use it to calculate the resulting field

OVERVIEW

- Electric charges create electric and magnetic fields
- Usually, we start with an arrangement of charge or current and use it to calculate the resulting field
- It is sometimes useful to reason in the opposite direction: based on what this field looks like, what could be causing it?

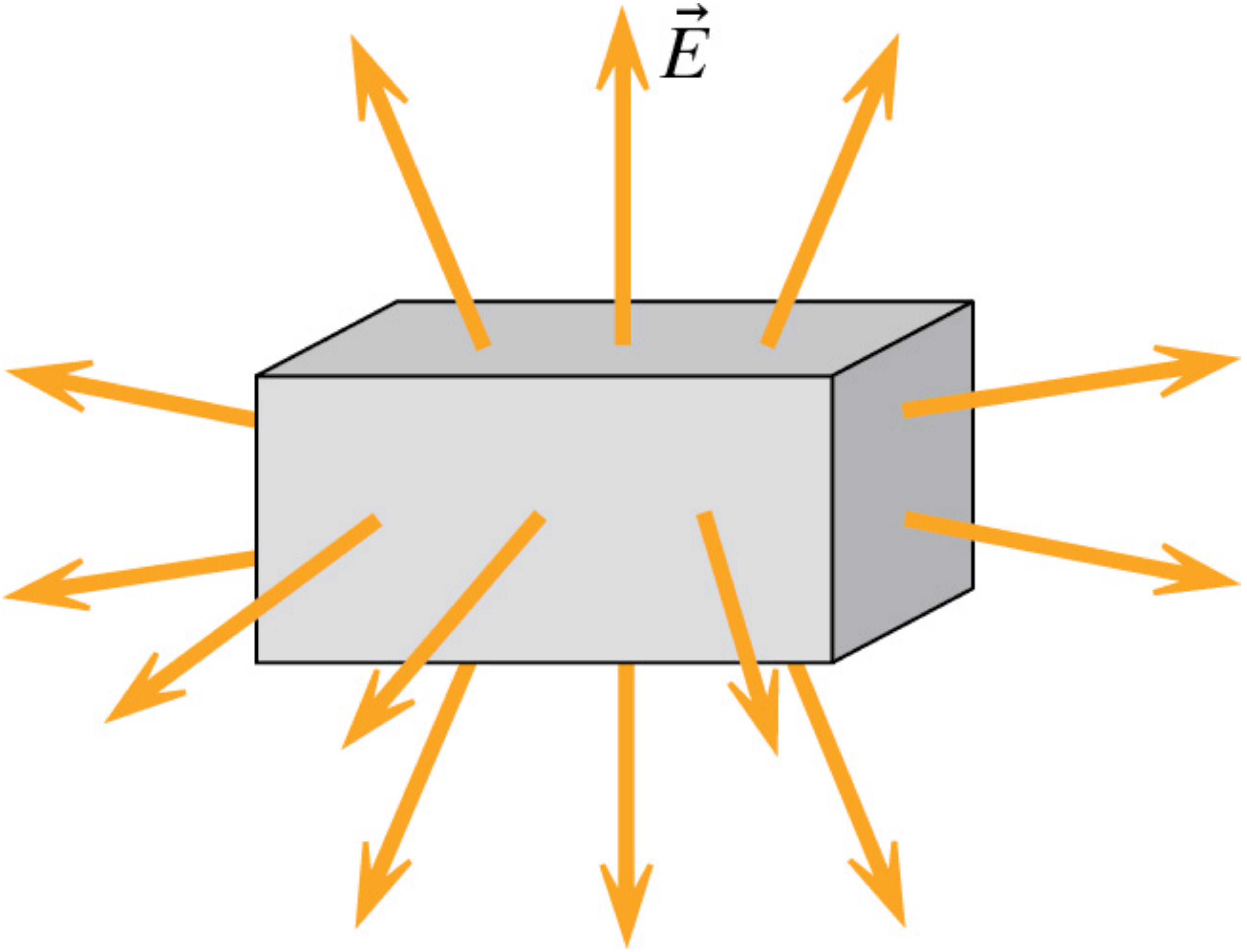
A THOUGHT EXPERIMENT

- Don't know what is in the box
- Can measure electric field all over the surface of the box
- What can we determine about the charge inside the box?




WHAT'S IN
THE BOX..?

- What charge (if any) is inside of this box?



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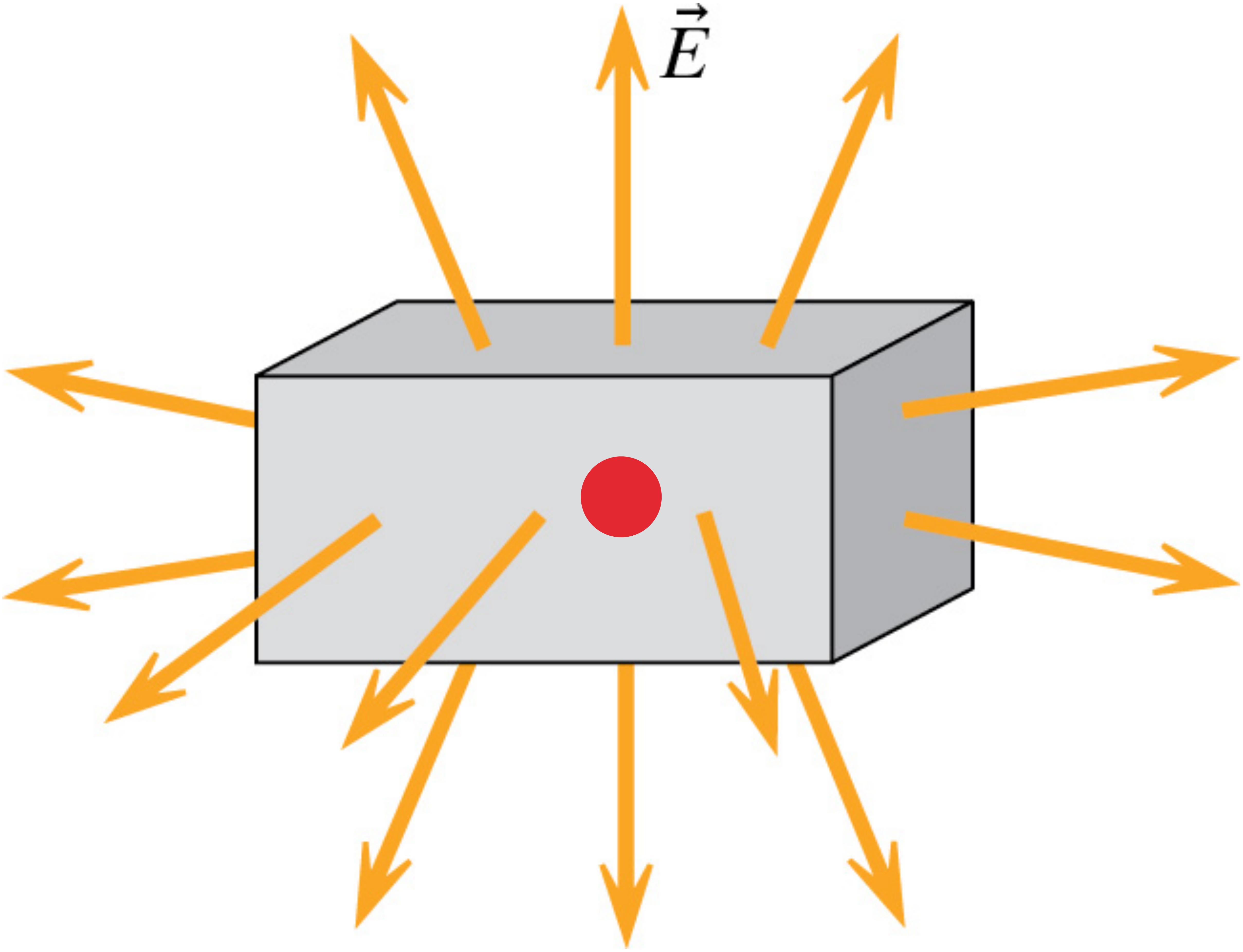

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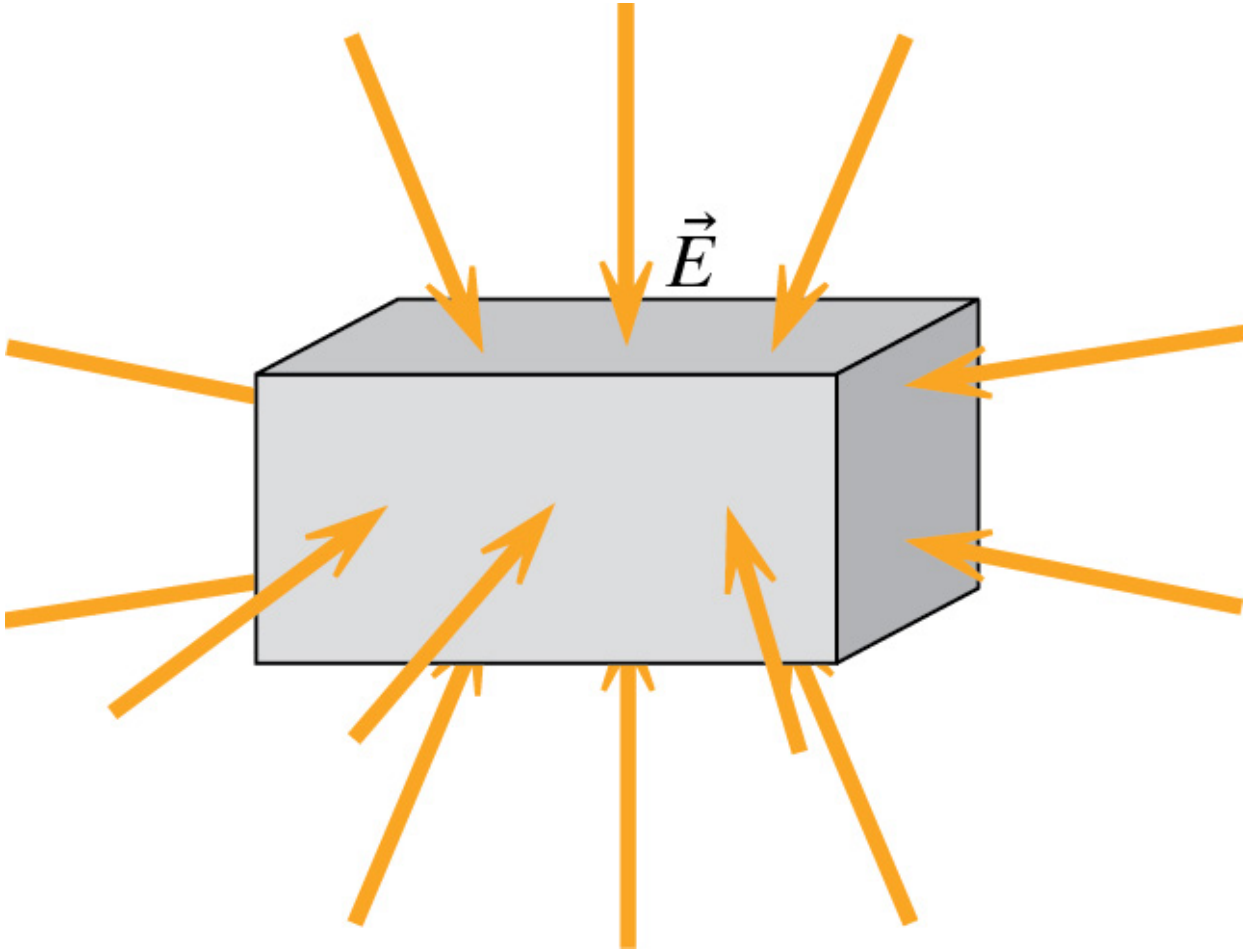

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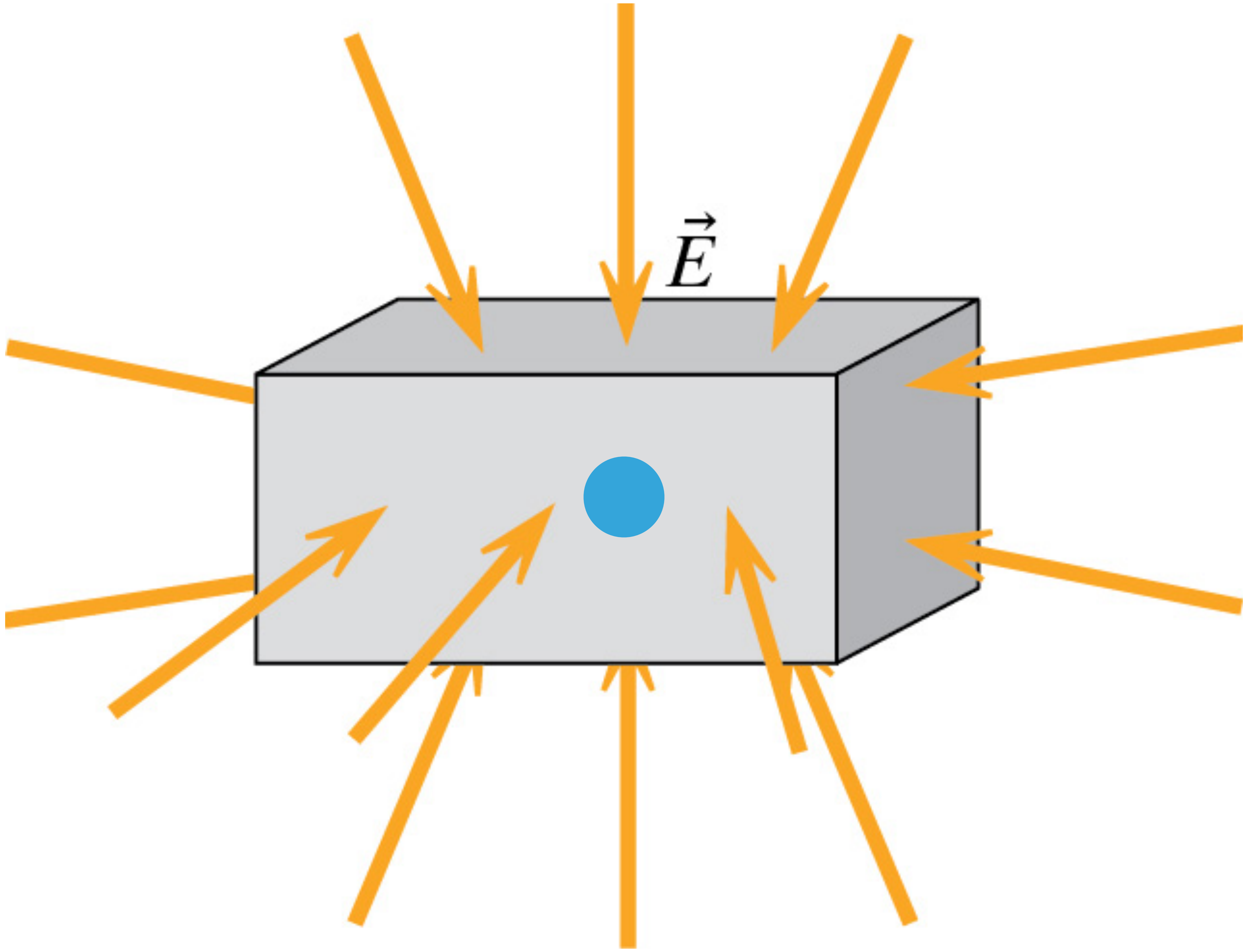

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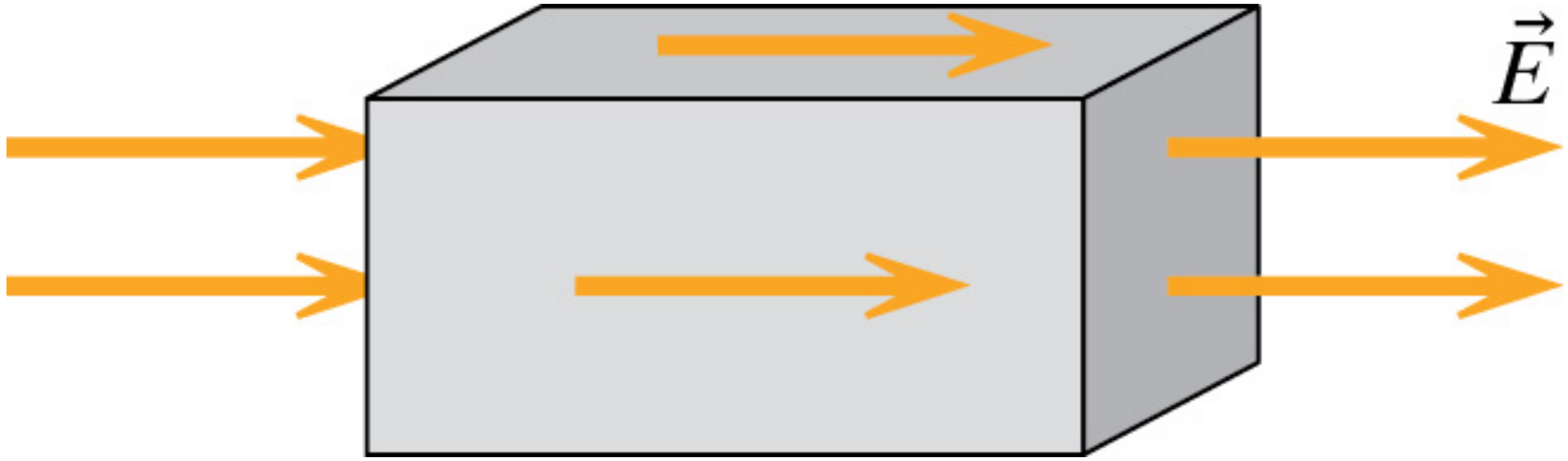

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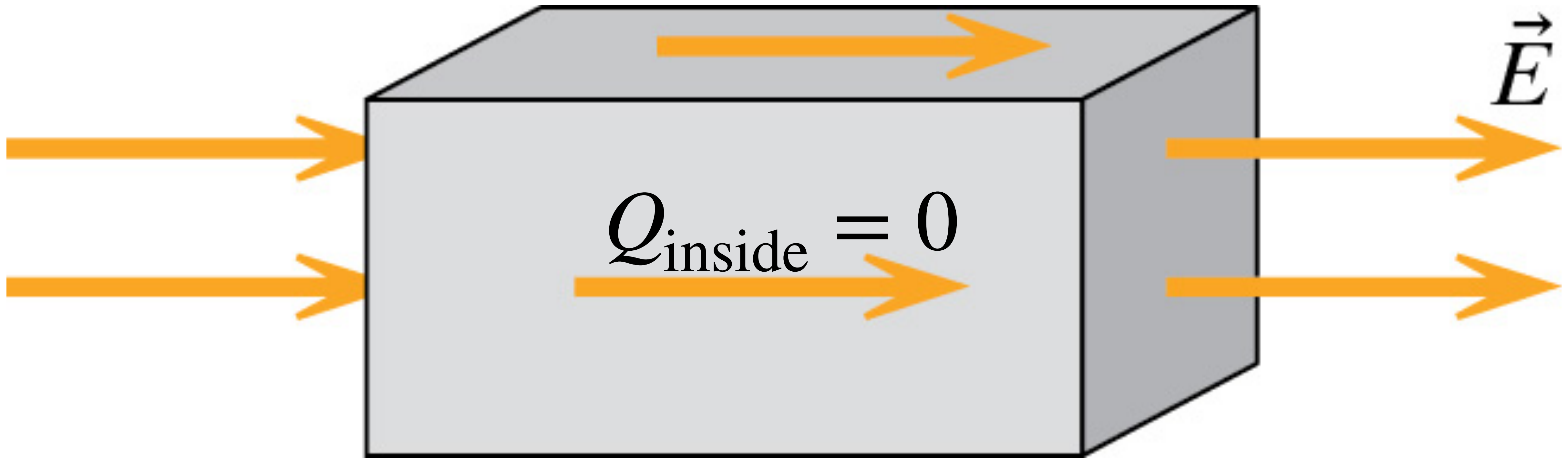

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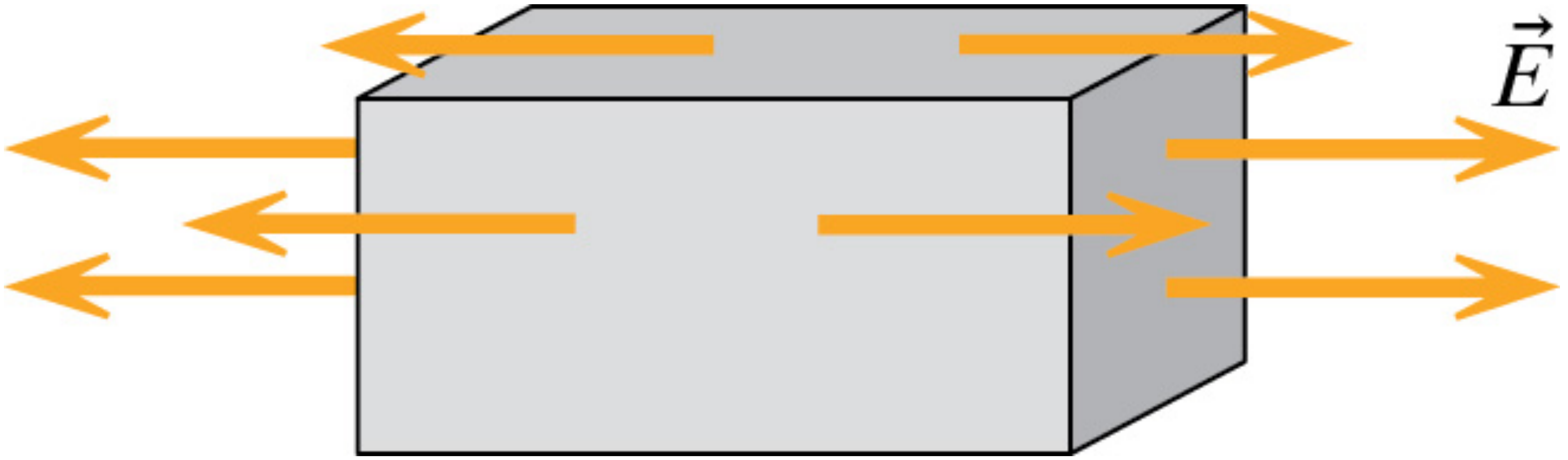

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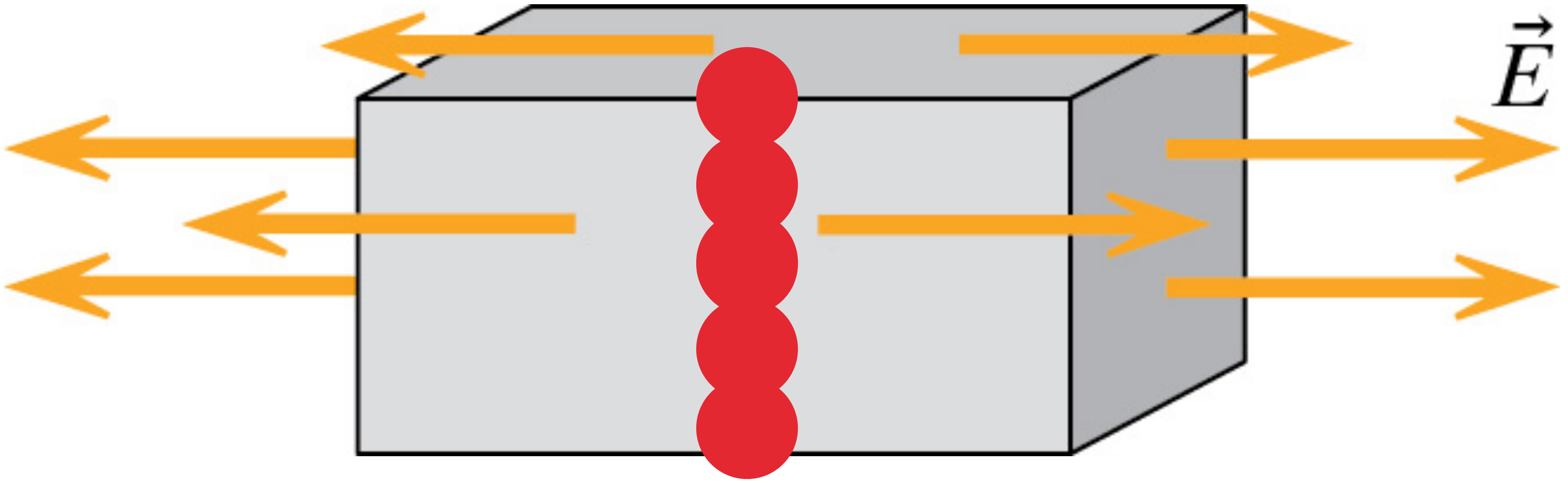
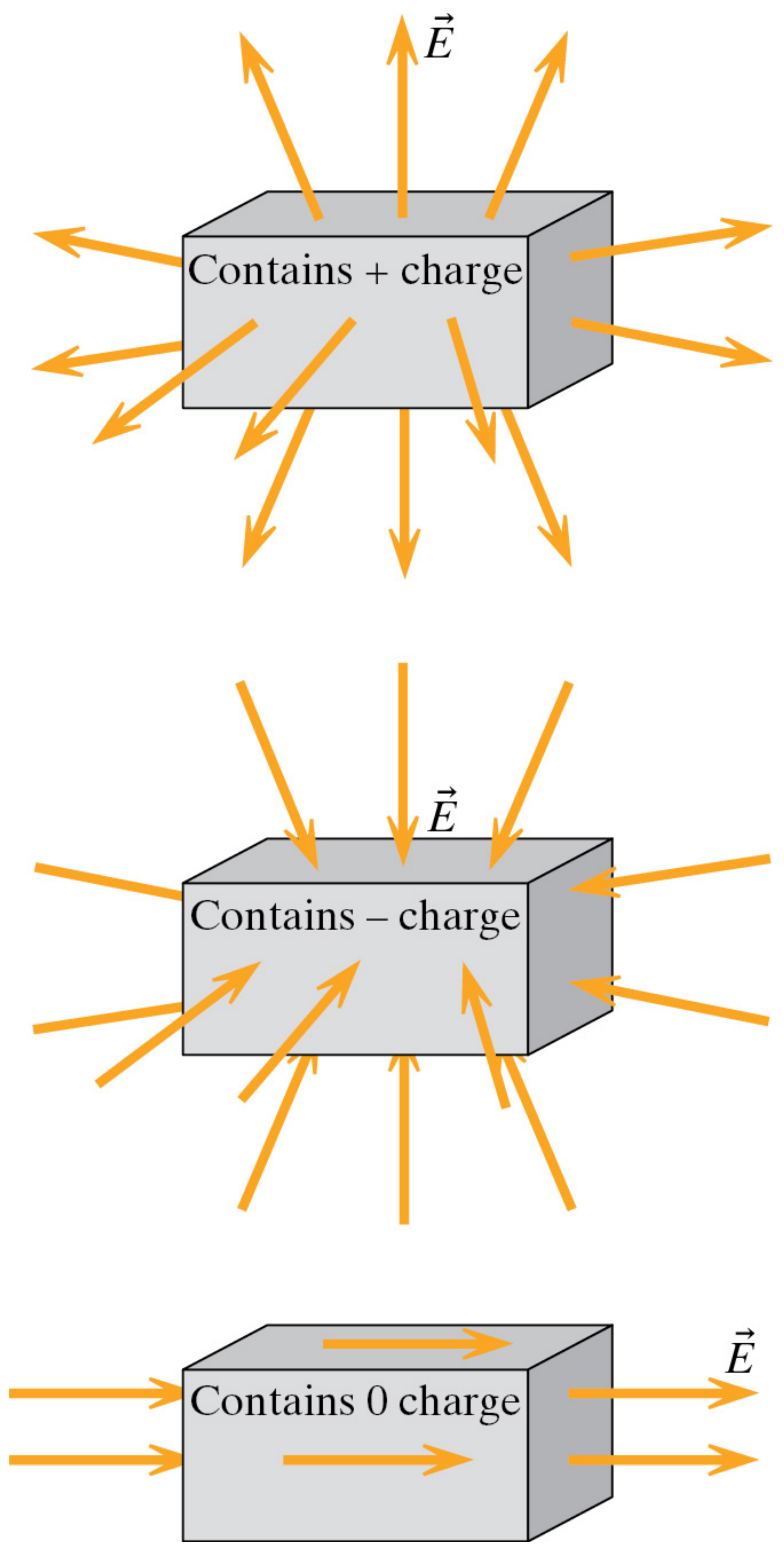
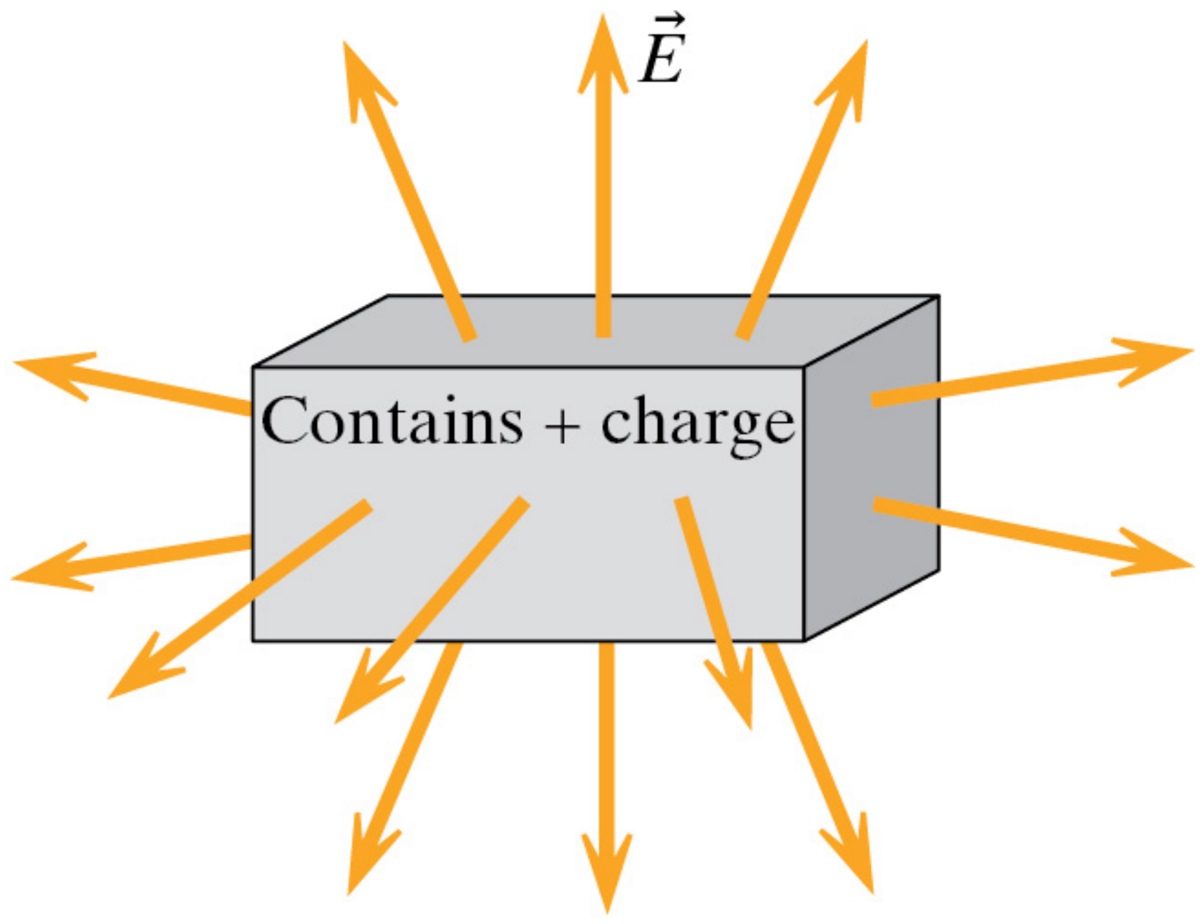


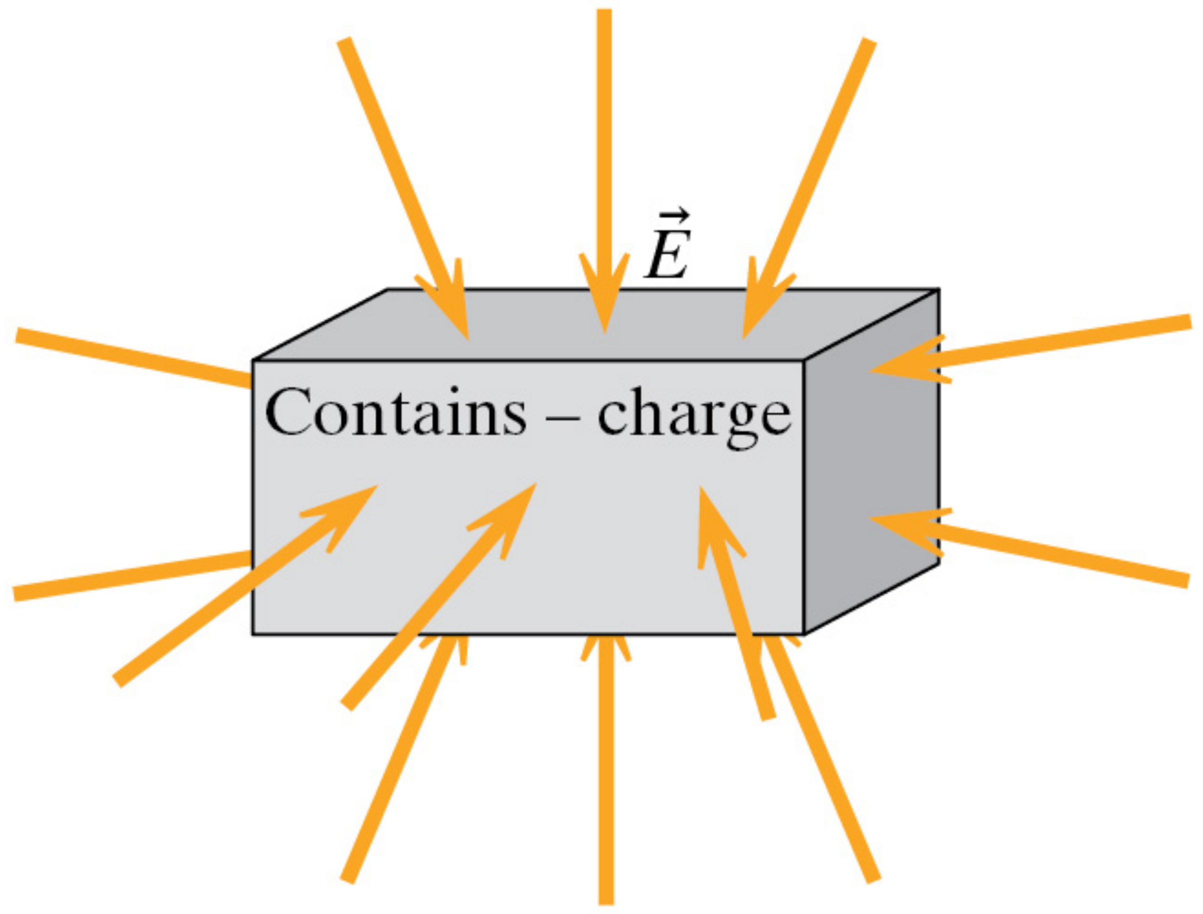
- Conclusion: pattern of $\vec{E}$ ***on*** a surface $\Longleftrightarrow$ charge ***inside*** the surface



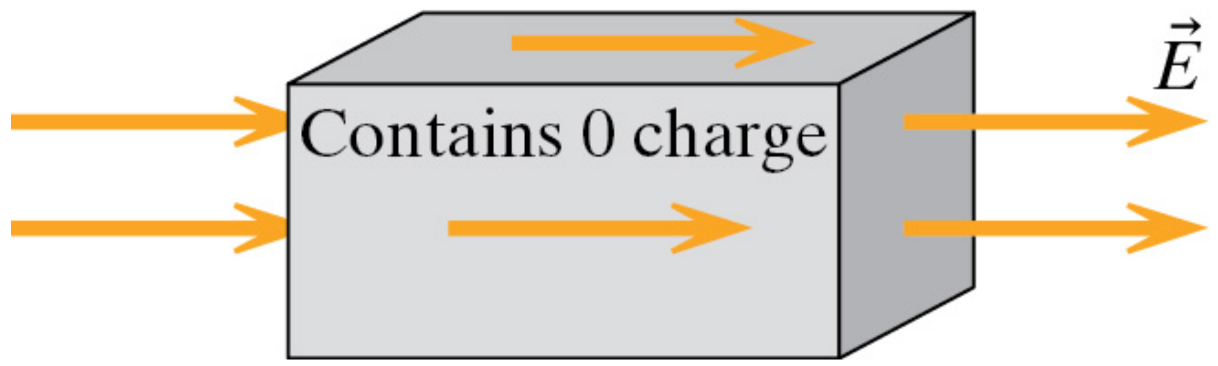
More field pointing out than in → positive charge inside surface



More field pointing in than out → negative charge inside surface



Same field in and out → no charge inside surface



QUANTIFYING THE RELATIONSHIP

- The net “amount” of field leaving or entering the surface is related to the sign and quantity of enclosed charge
- How do we quantify the “amount of field through a surface” ?

QUANTIFYING THE RELATIONSHIP

- The net “amount” of field leaving or entering the surface is related to the sign and quantity of enclosed charge
- How do we quantify the “amount of field through a surface” ?
- Need to quantify **amount** and **direction** of field over an **entire surface**
    - **Electric flux**

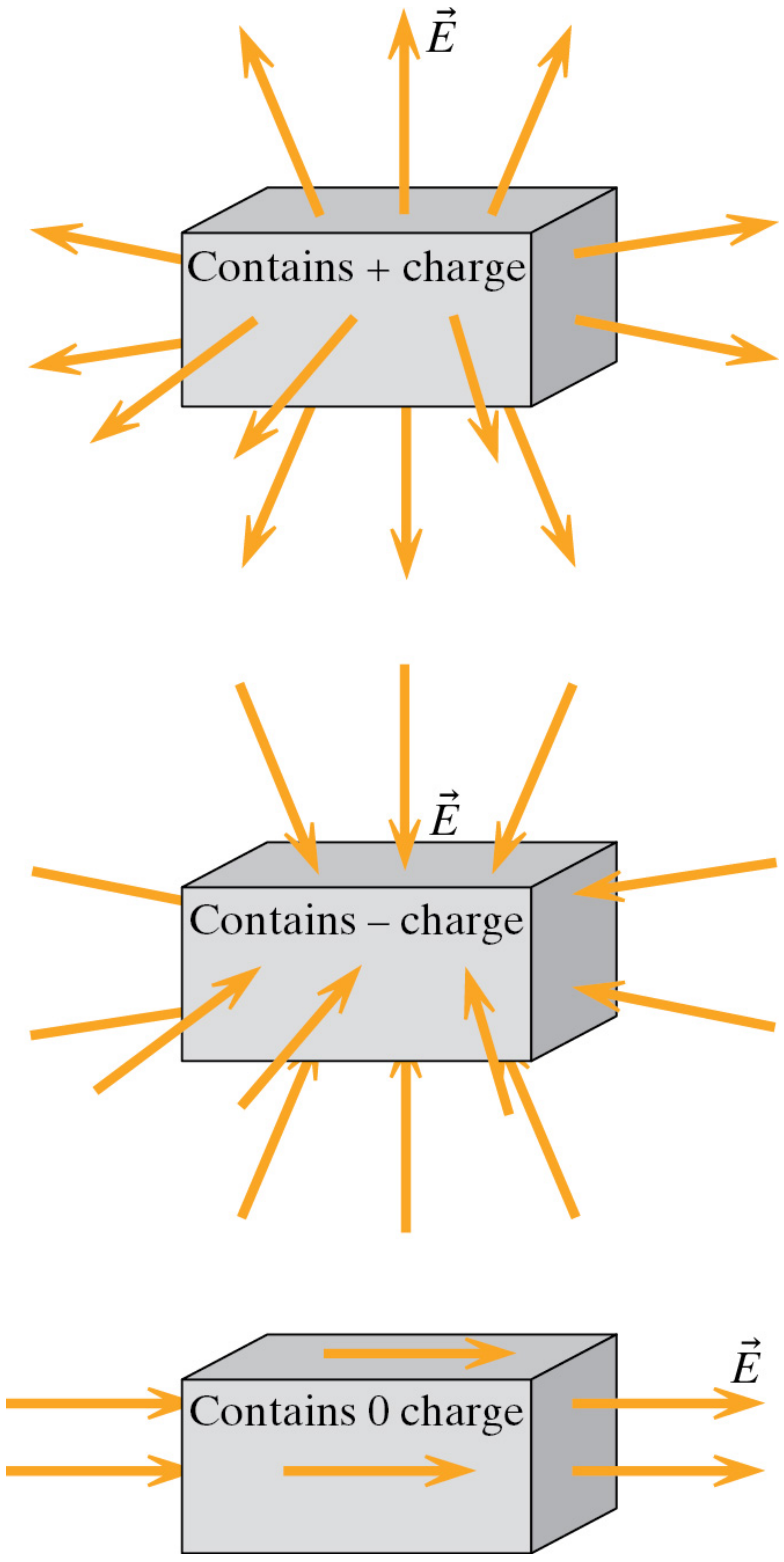
ELECTRIC FLUX

- Electric Flux: the total “amount” of field escaping a closed surface
- What properties should flux have?

flux > 0

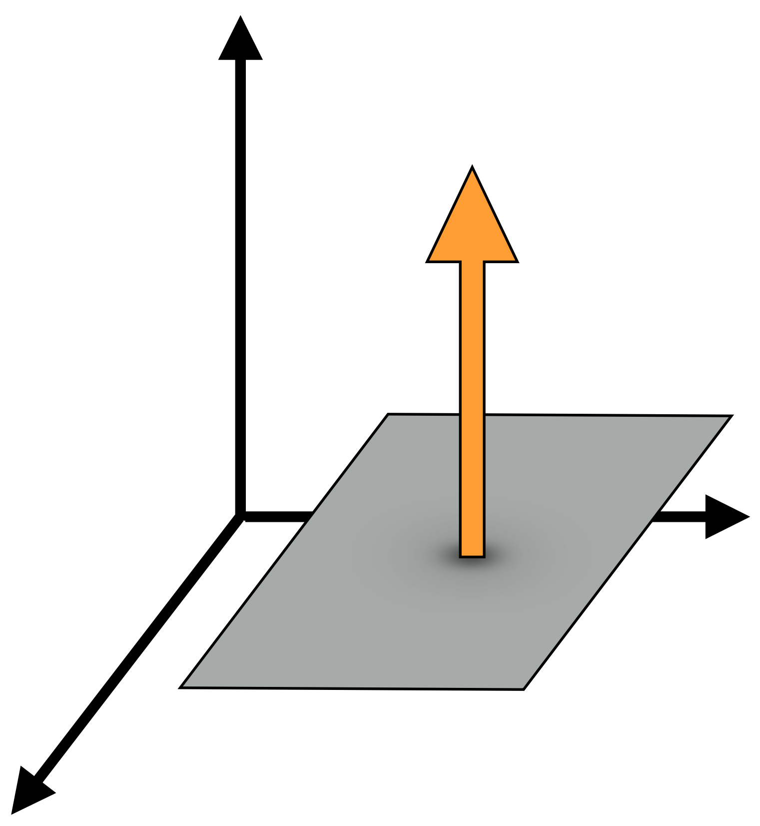
flux < 0

flux = 0

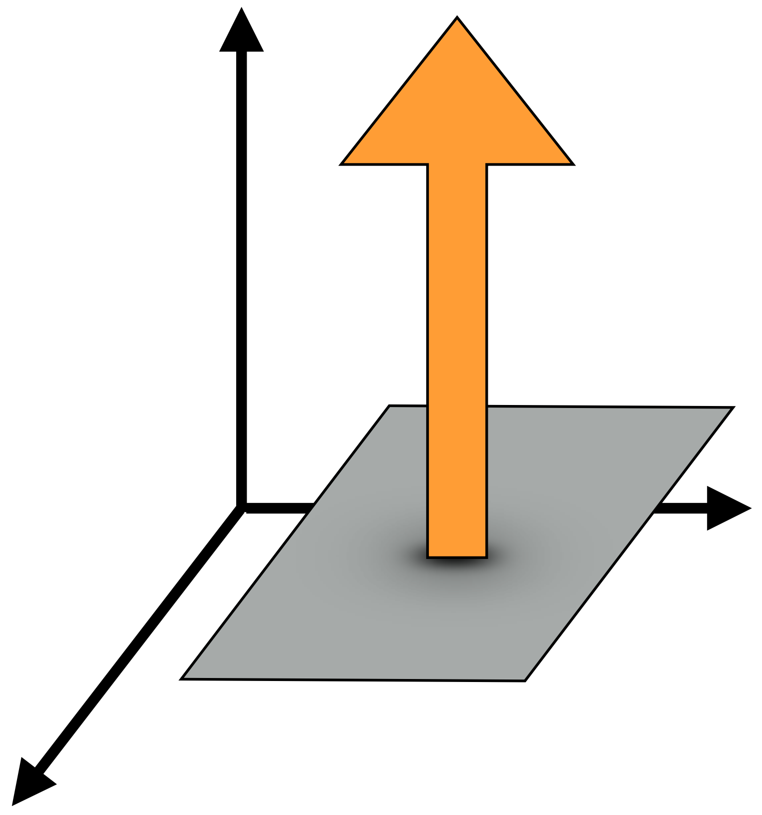


PROPERTIES OF ELECTRIC FLUX

- "Amount" of field through a surface depends on magnitude of the field at the surface



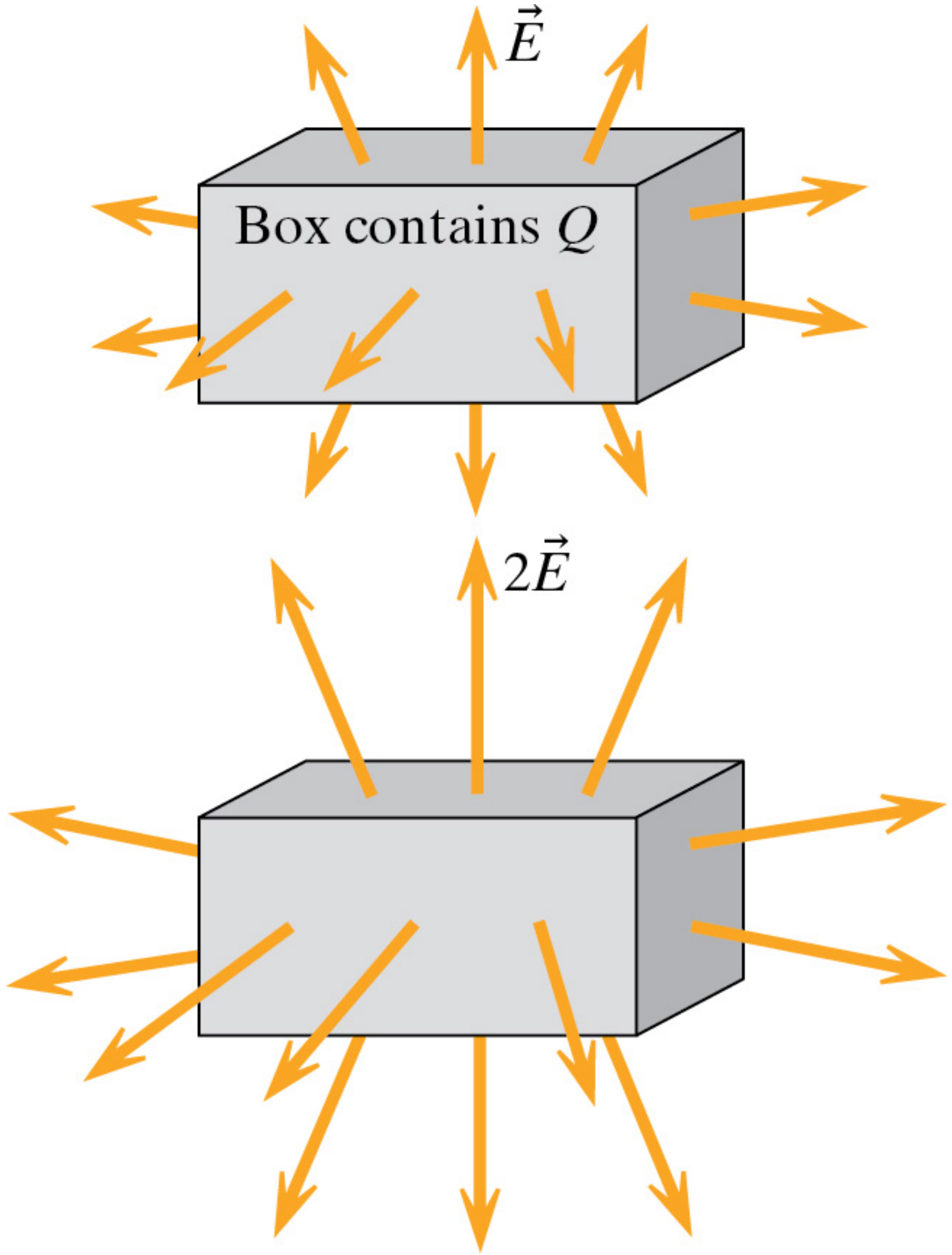
smaller flux



larger flux

PROPERTIES OF ELECTRIC FLUX

- Magnitude is important because it indicates the *amount* of charge

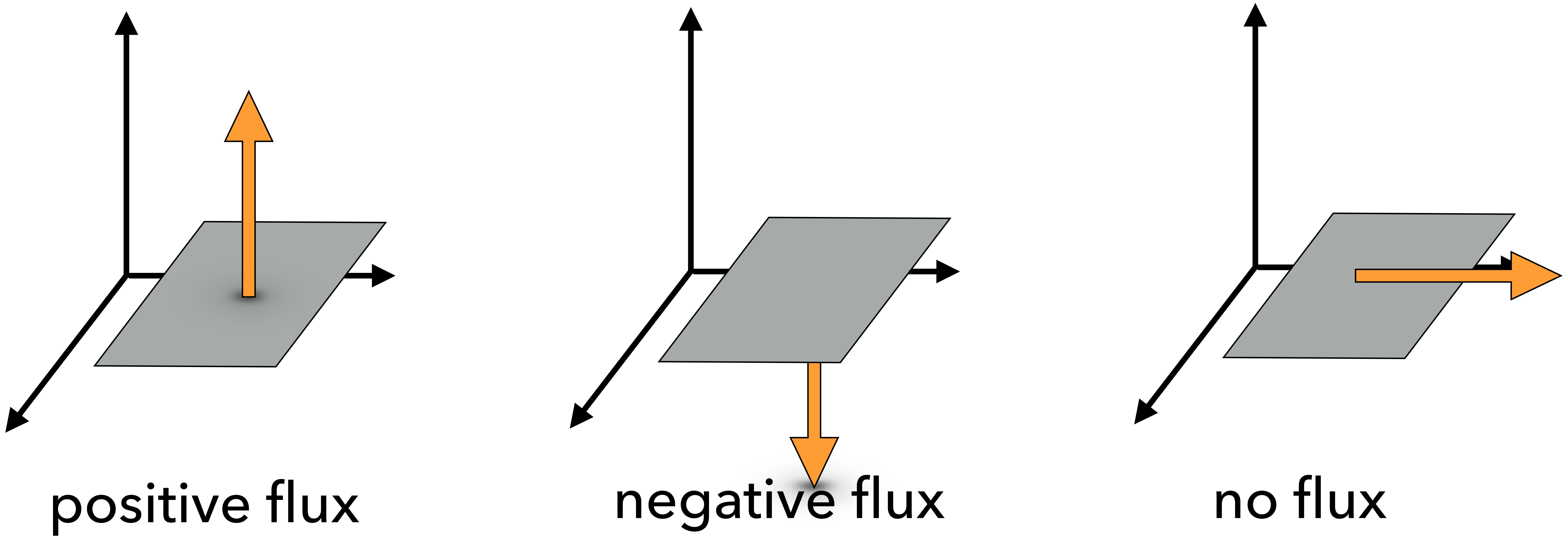


PROPERTIES OF ELECTRIC FLUX

1. Flux is proportional to field magnitude

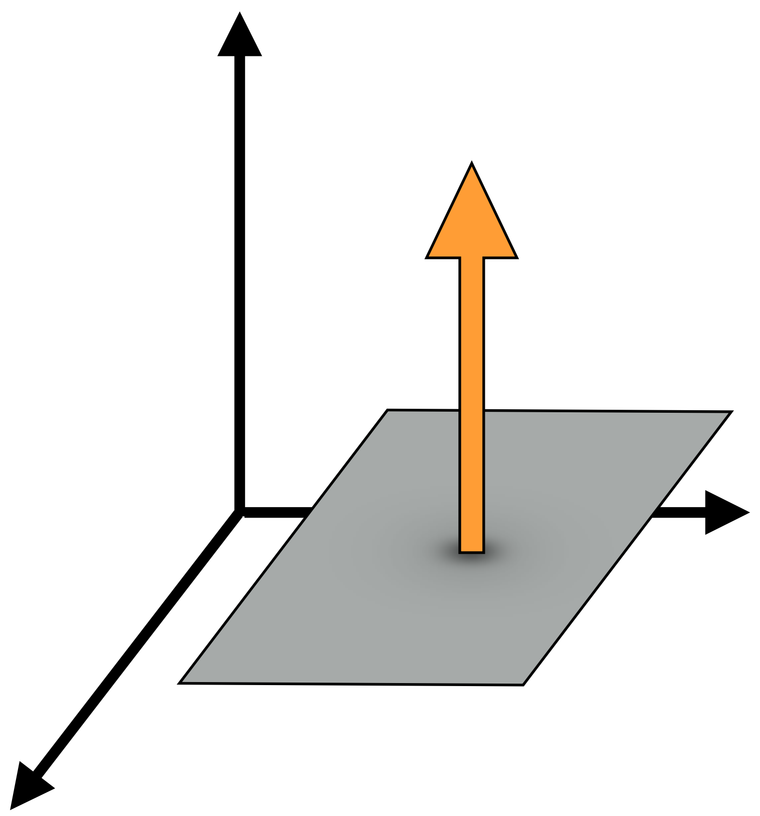
PROPERTIES OF ELECTRIC FLUX

- "Amount" of field through a surface depends on *direction of field through the surface*

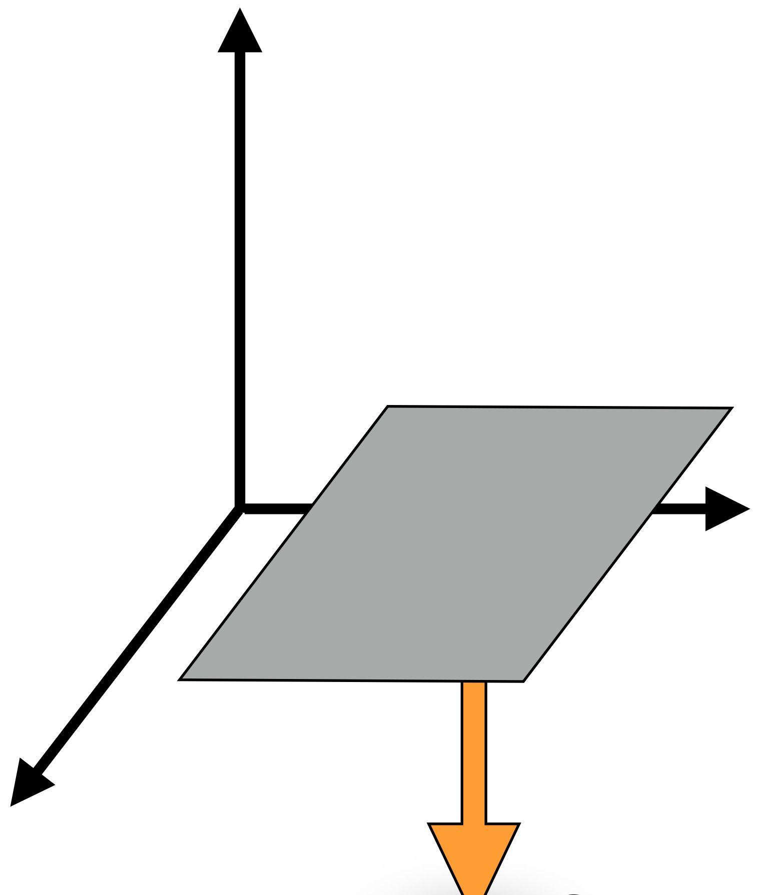


PROPERTIES OF ELECTRIC FLUX

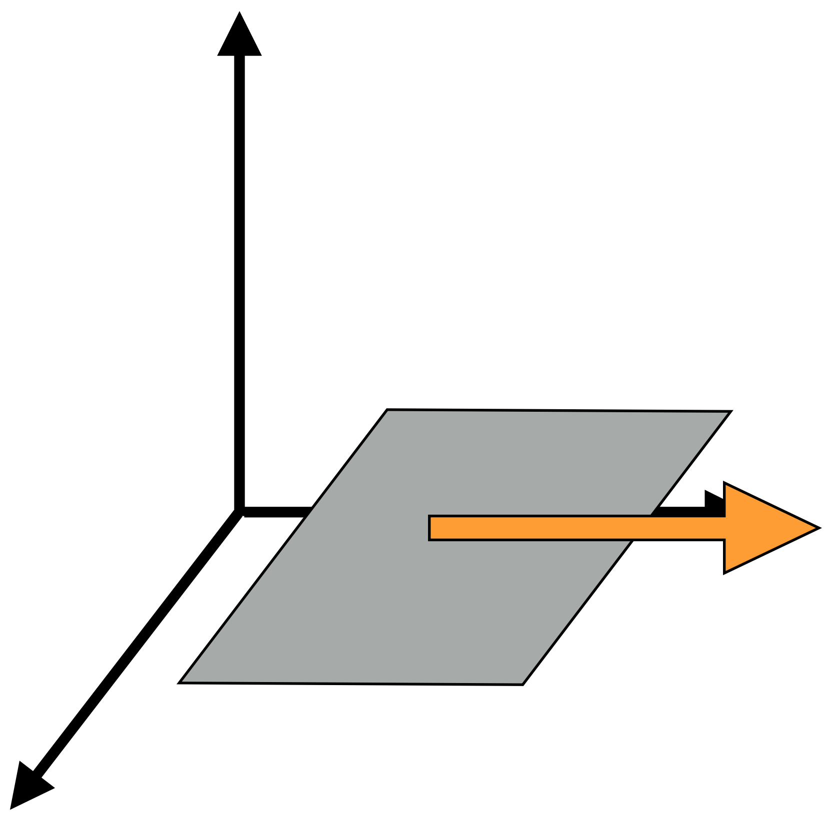
- Field pointing away from surface: positive flux
- Field pointing into surface: negative flux
- Field in same plane as surface: zero flux



positive flux



negative flux

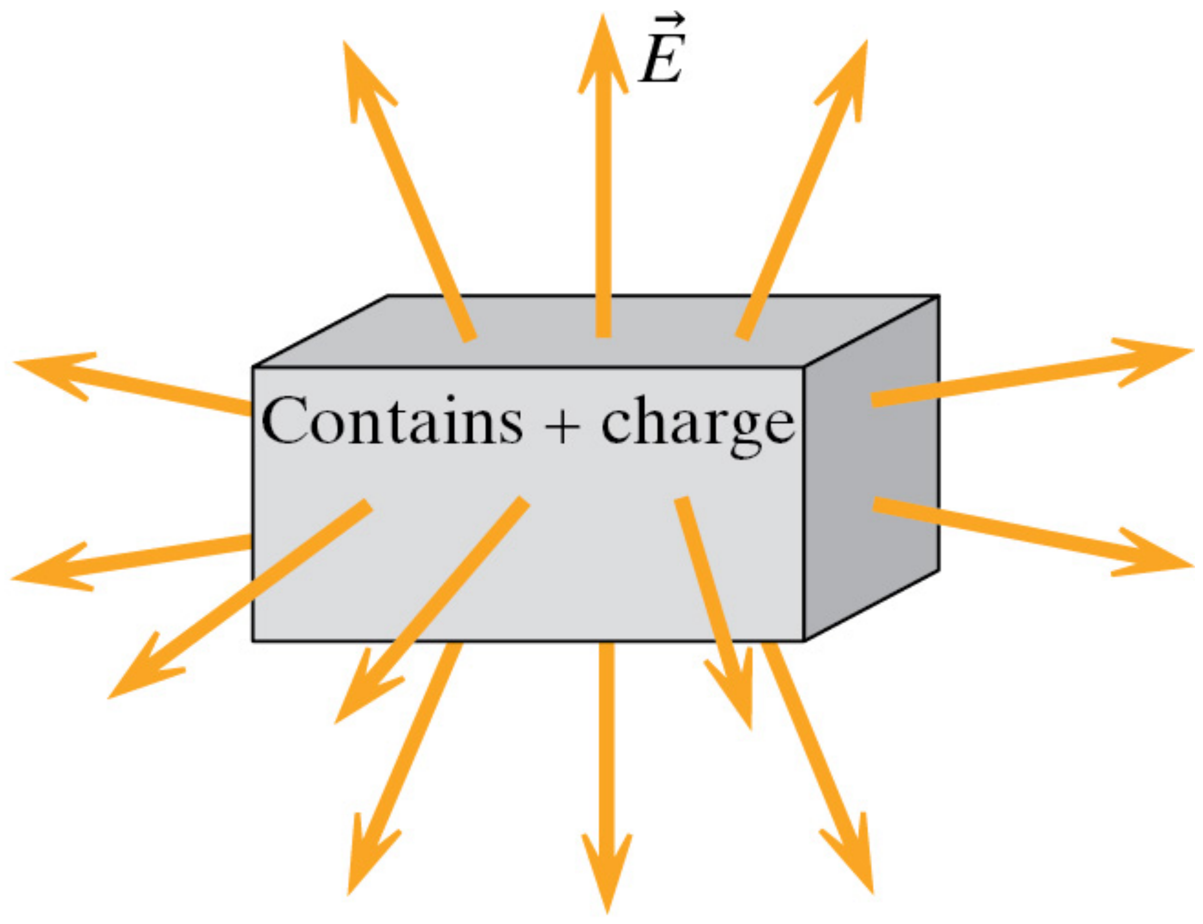


no flux

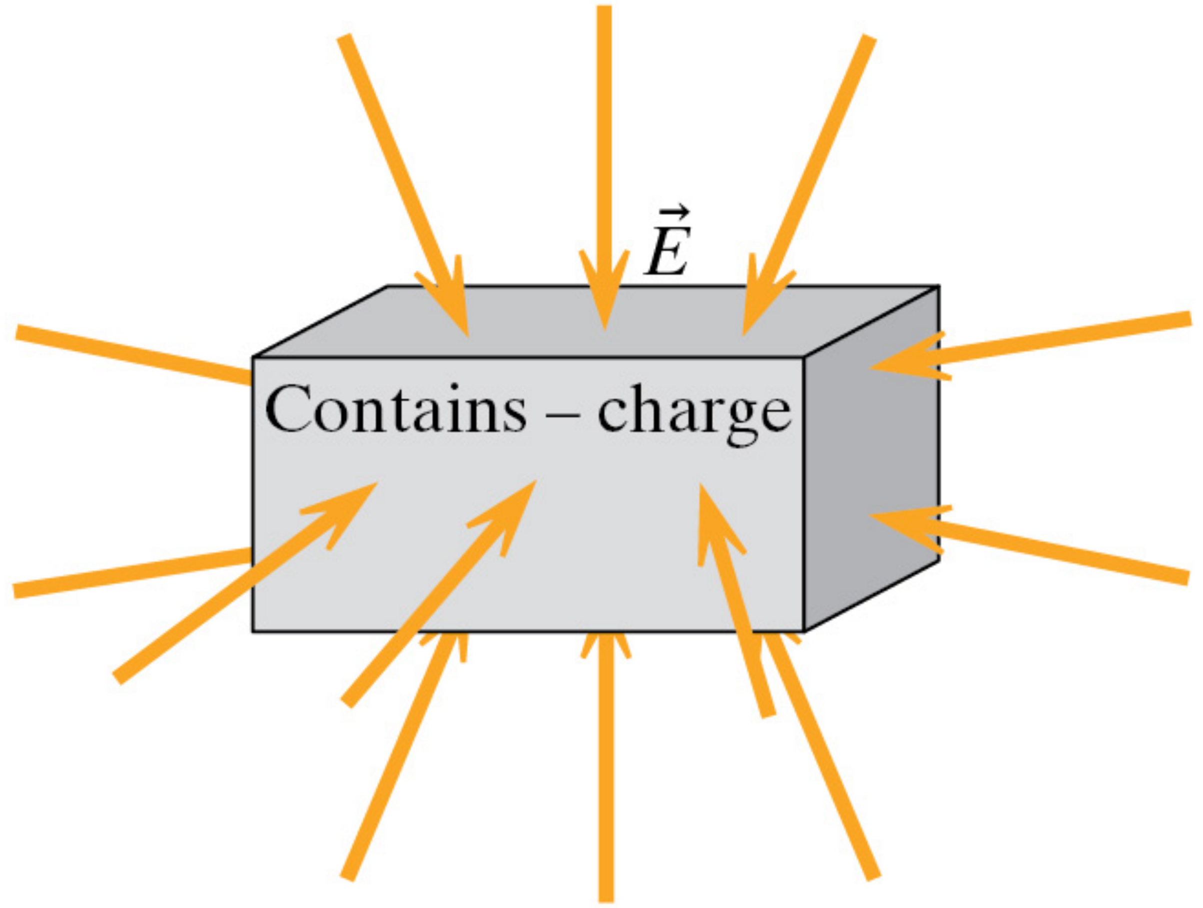
PROPERTIES OF ELECTRIC FLUX

- Direction is important because it indicates the sign of the charge

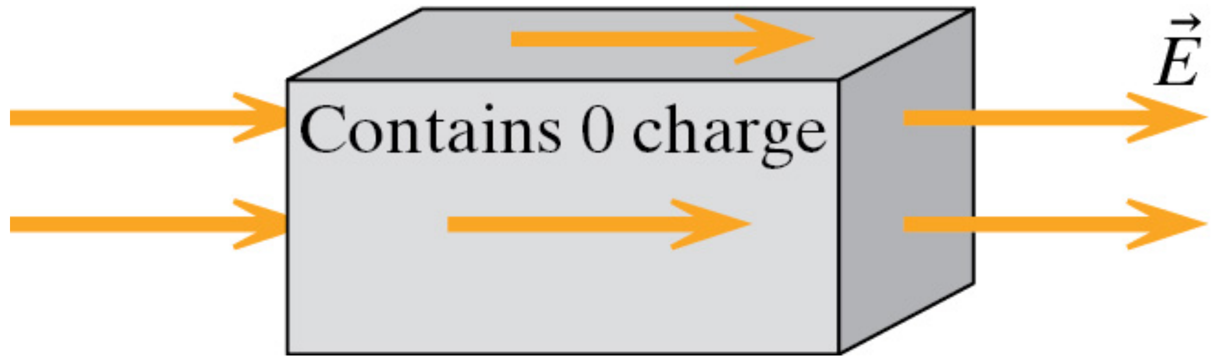
flux > 0



flux < 0

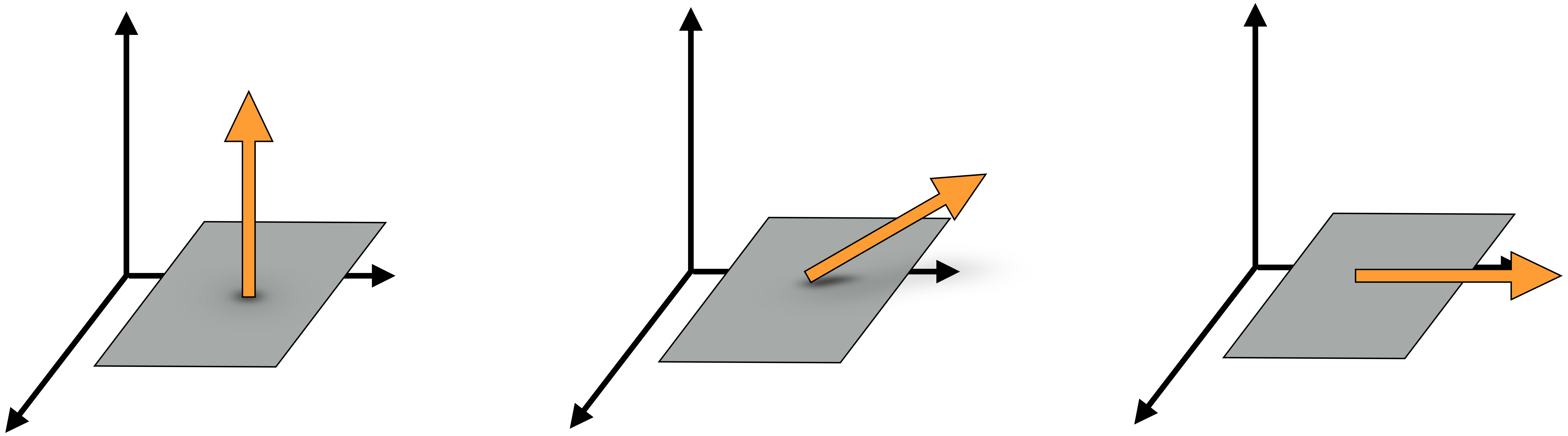


flux = 0



PROPERTIES OF ELECTRIC FLUX

- In general, flux varies with angle



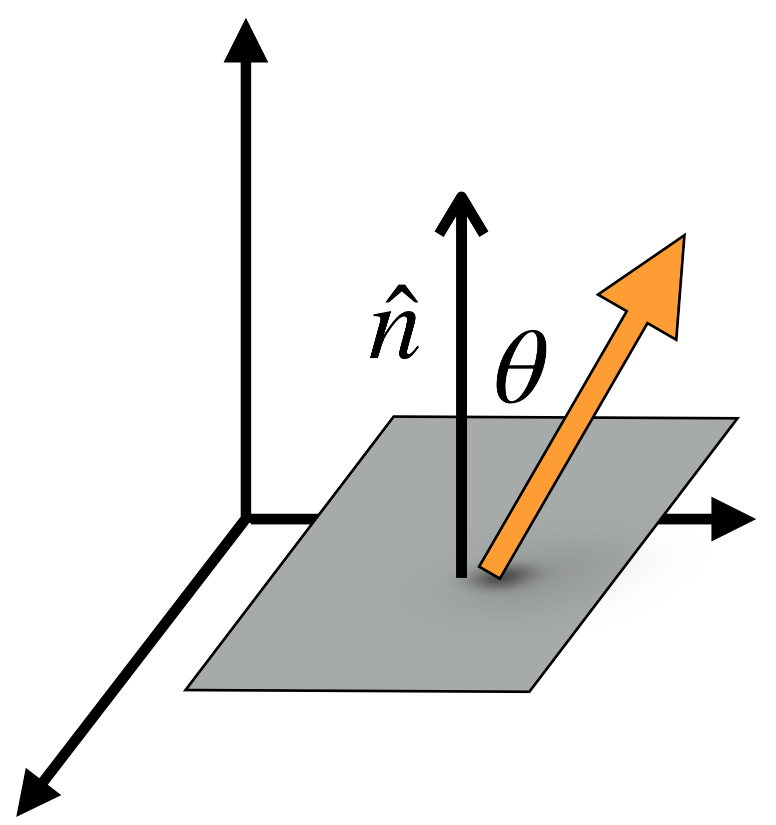
larger flux smaller flux no flux


LARGEST “RAIN FLUX”
SMALLER “RAIN FLUX”
ZERO “RAIN FLUX”

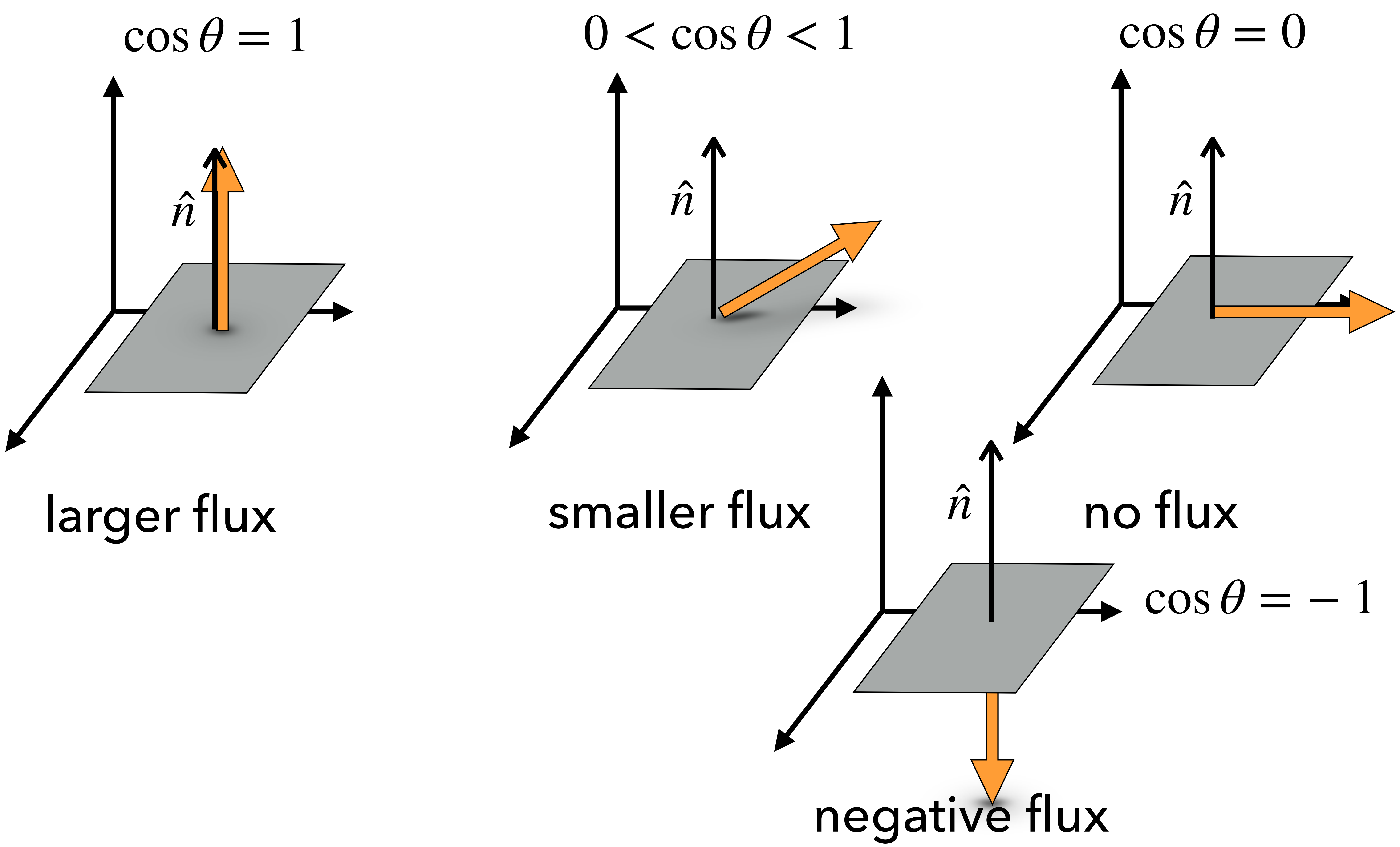
PROPERTIES OF ELECTRIC FLUX

$\text{Flux} \propto \cos\theta$

θ: angle between $\vec{E}$ and direction perpendicular to surface ($\hat{n}$)

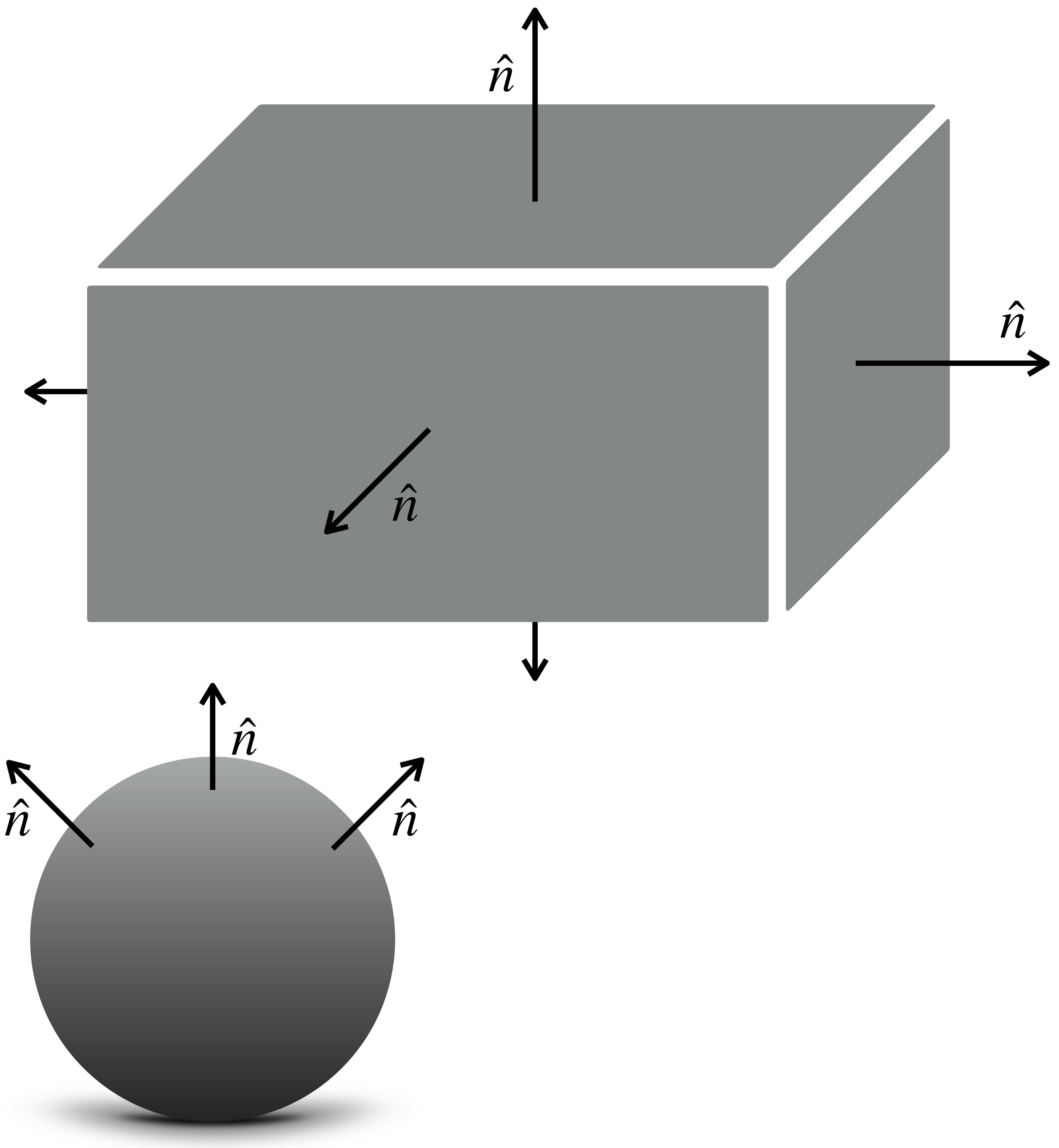


PROPERTIES OF ELECTRIC FLUX



PROPERTIES OF ELECTRIC FLUX

- For a closed surface, $\hat{n}$ is points perpendicularly *away* from the interior of the surface
- θ is the angle between $\vec{E}$ and $\hat{n}$



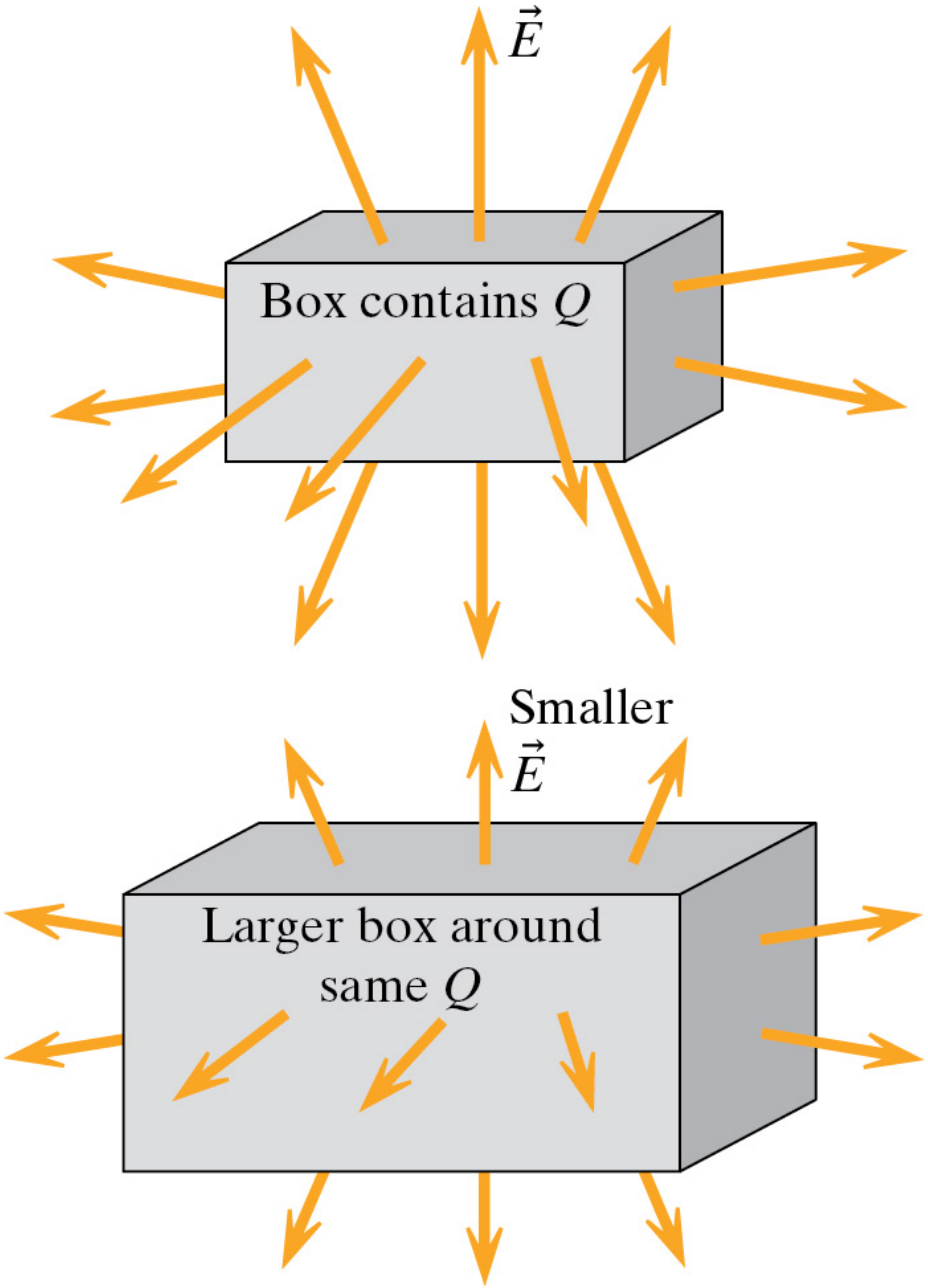
PROPERTIES OF ELECTRIC FLUX

1. Flux is proportional to field magnitude
2. Flux is proportional to $\cos\theta$

PROPERTIES OF ELECTRIC FLUX

- Flux should depend on the surface area of the box

$$\text{flux} \propto E \cdot A$$

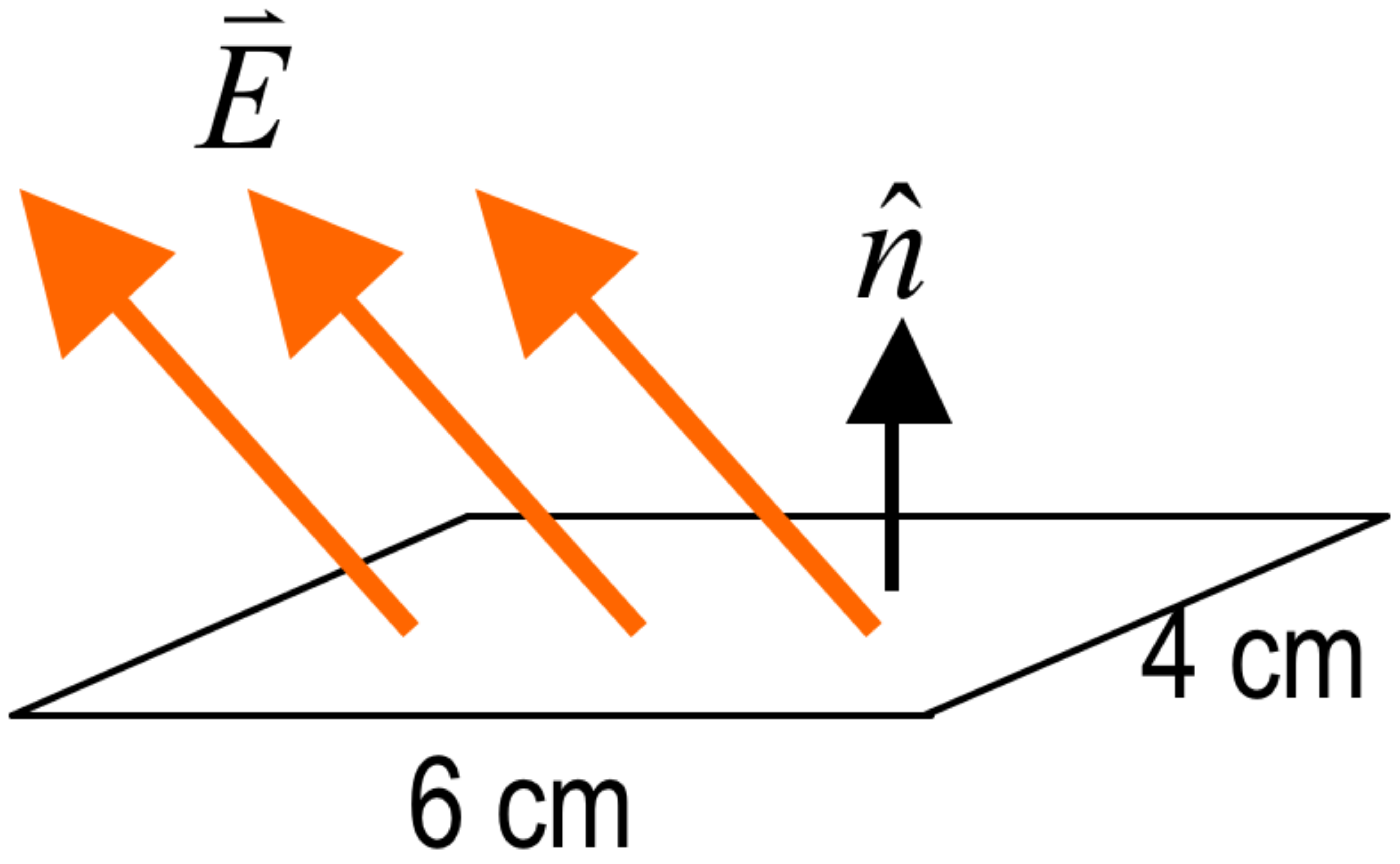


PROPERTIES OF ELECTRIC FLUX

1. Flux is proportional to field magnitude
2. Flux is proportional to $\cos\theta$
3. Flux is proportional to total surface area A

EXAMPLE

What is the electric flux through this surface?

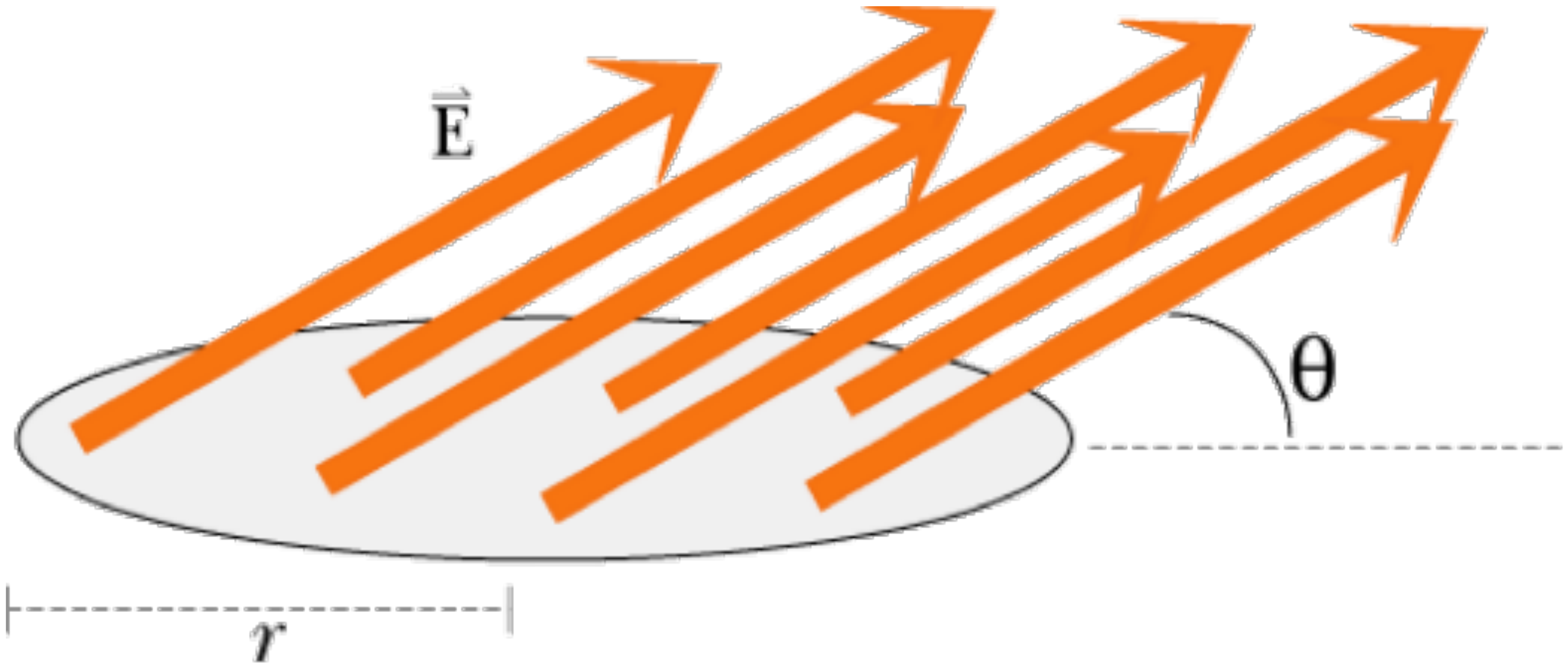


$$\vec{E} = \langle -230, 370, 0 \rangle \text{ V/m}$$

$$\hat{n} = \langle 0, 1, 0 \rangle$$

EXAMPLE

What is the electric flux through this surface?

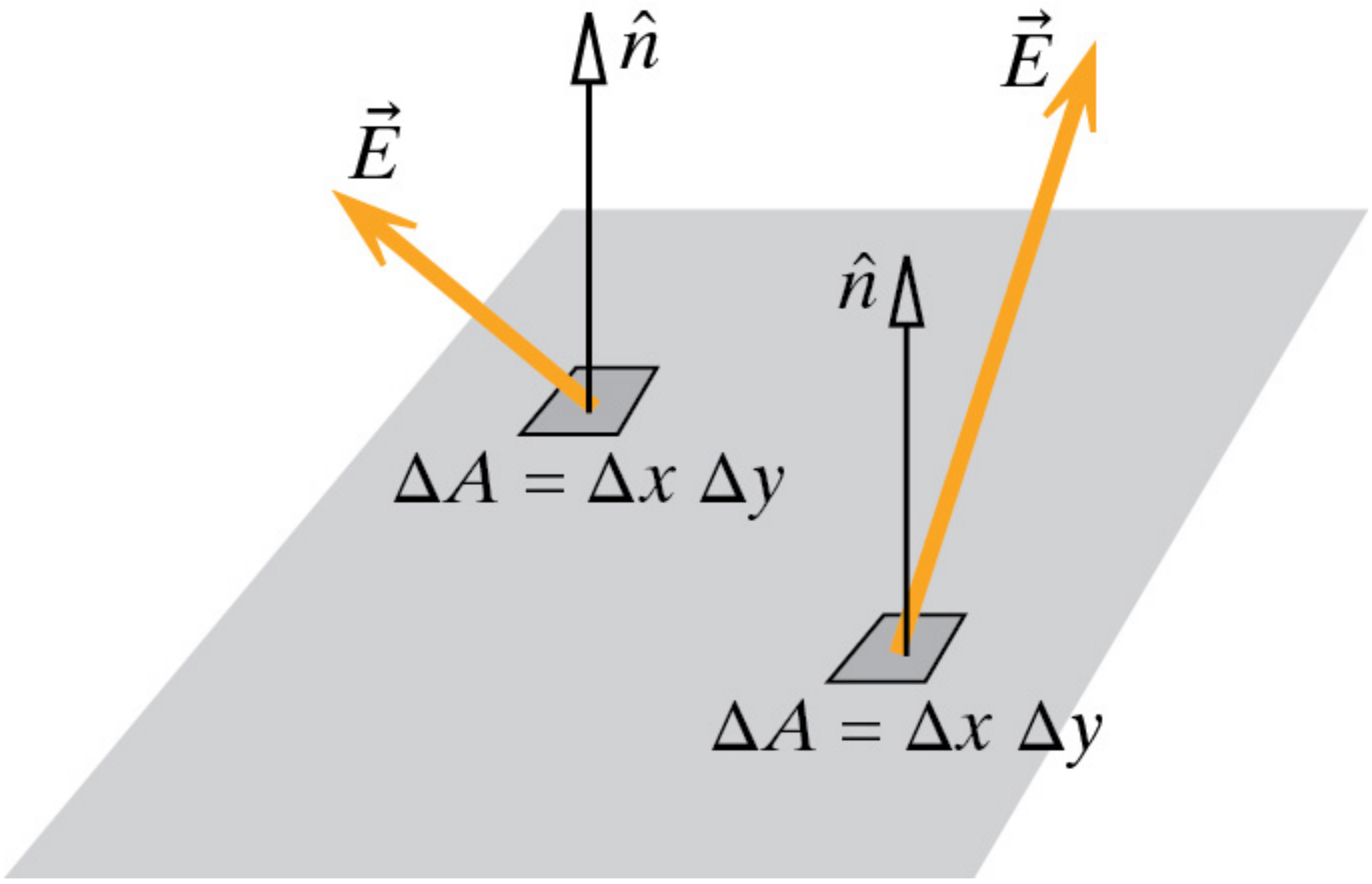


E = 600 V/m
r = 3 cm
θ = 25 degrees

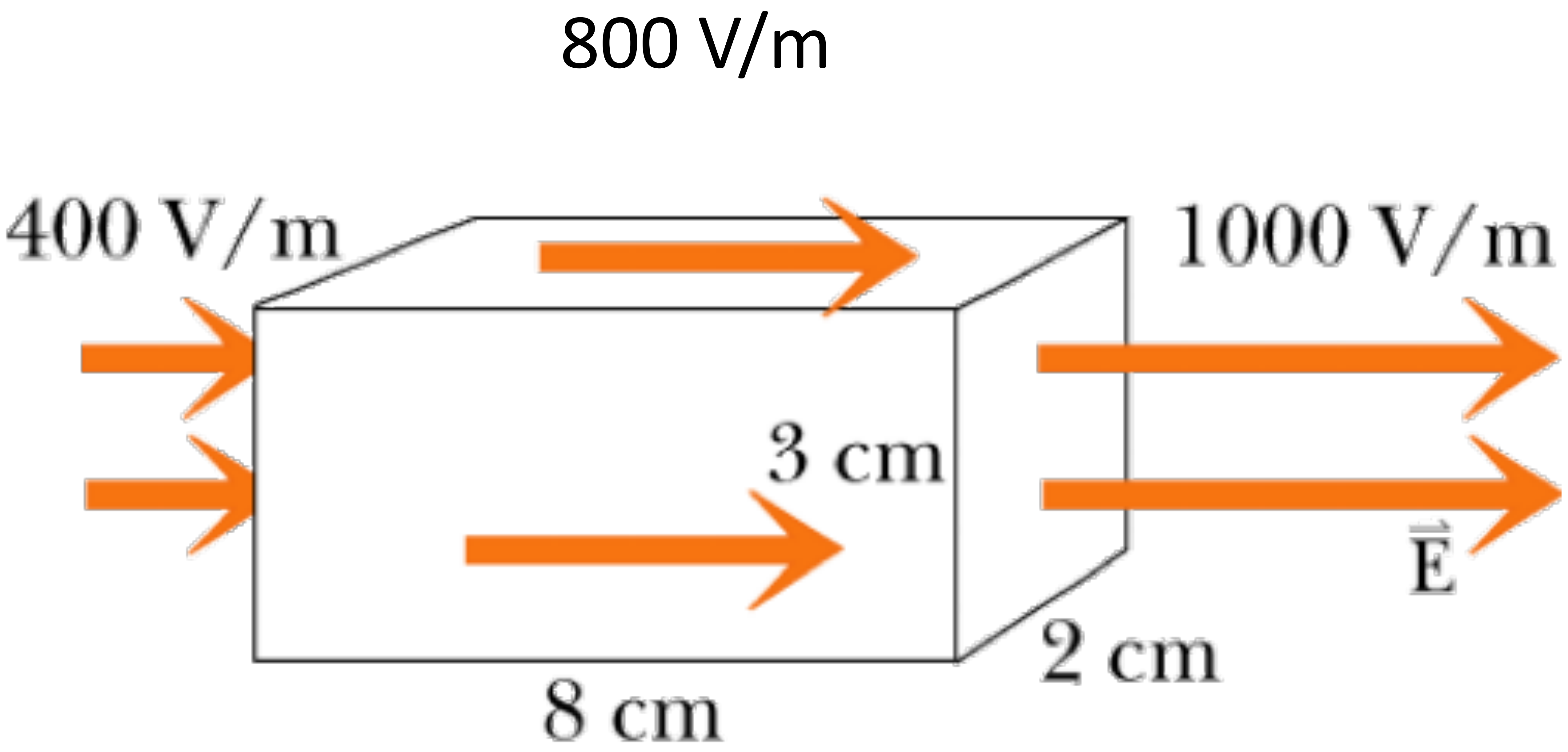
DEFINITION OF ELECTRIC FLUX

Definition of electric flux:

$$\boxed{\phi_{\text{el}} = \sum_{\text{surface}} \vec{E} \cdot \hat{n} \Delta A}$$



- What is the electric flux through the box?

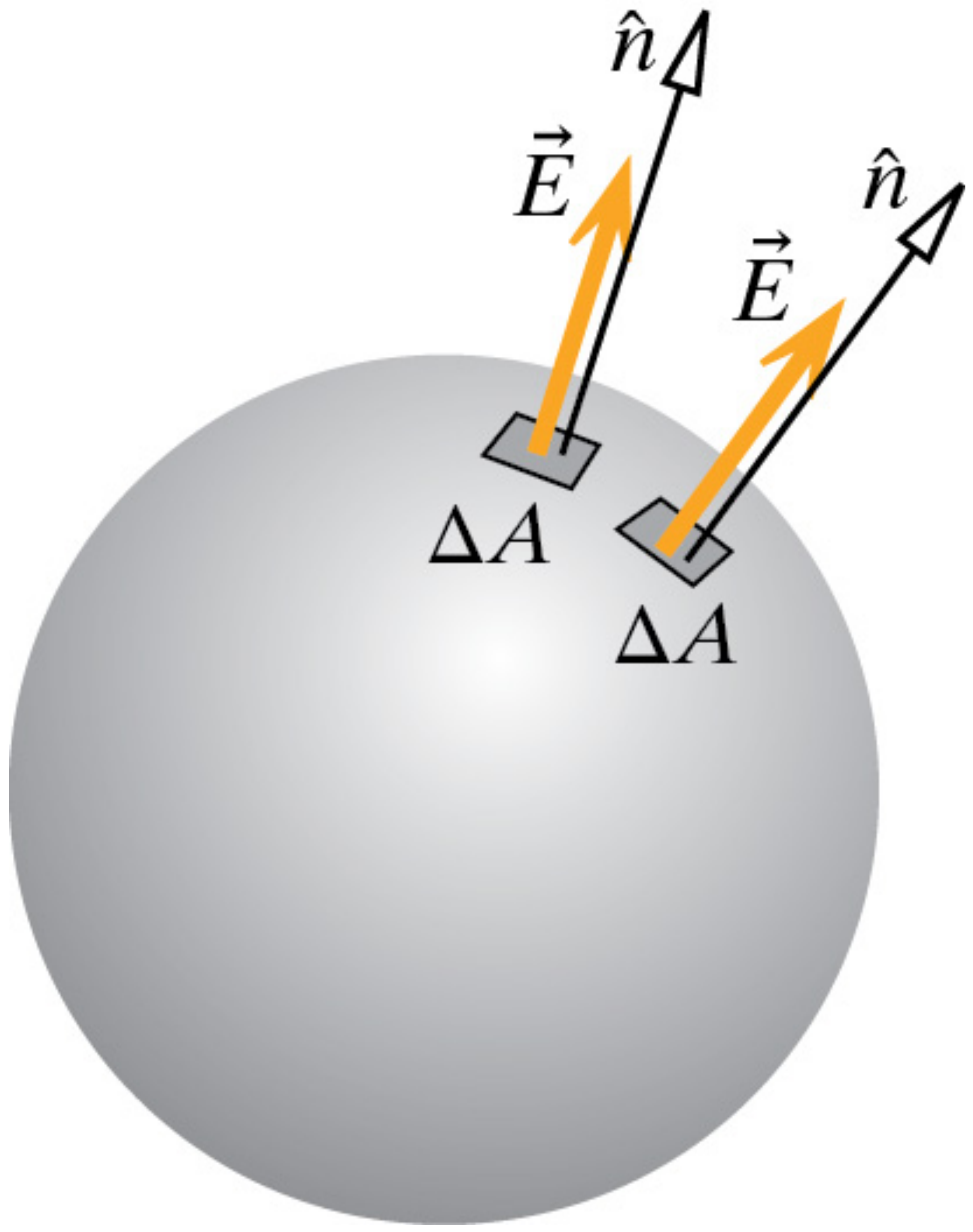


FLUX FROM CONTINUOUSLY CHANGING FIELD

$$\phi_{\text{el}} = \lim_{\Delta A \to 0} \sum_{\text{surface}} \vec{E} \cdot \hat{n} \Delta A = \oint \vec{E} \cdot \hat{n} dA$$

RELATING FLUX TO CHARGE

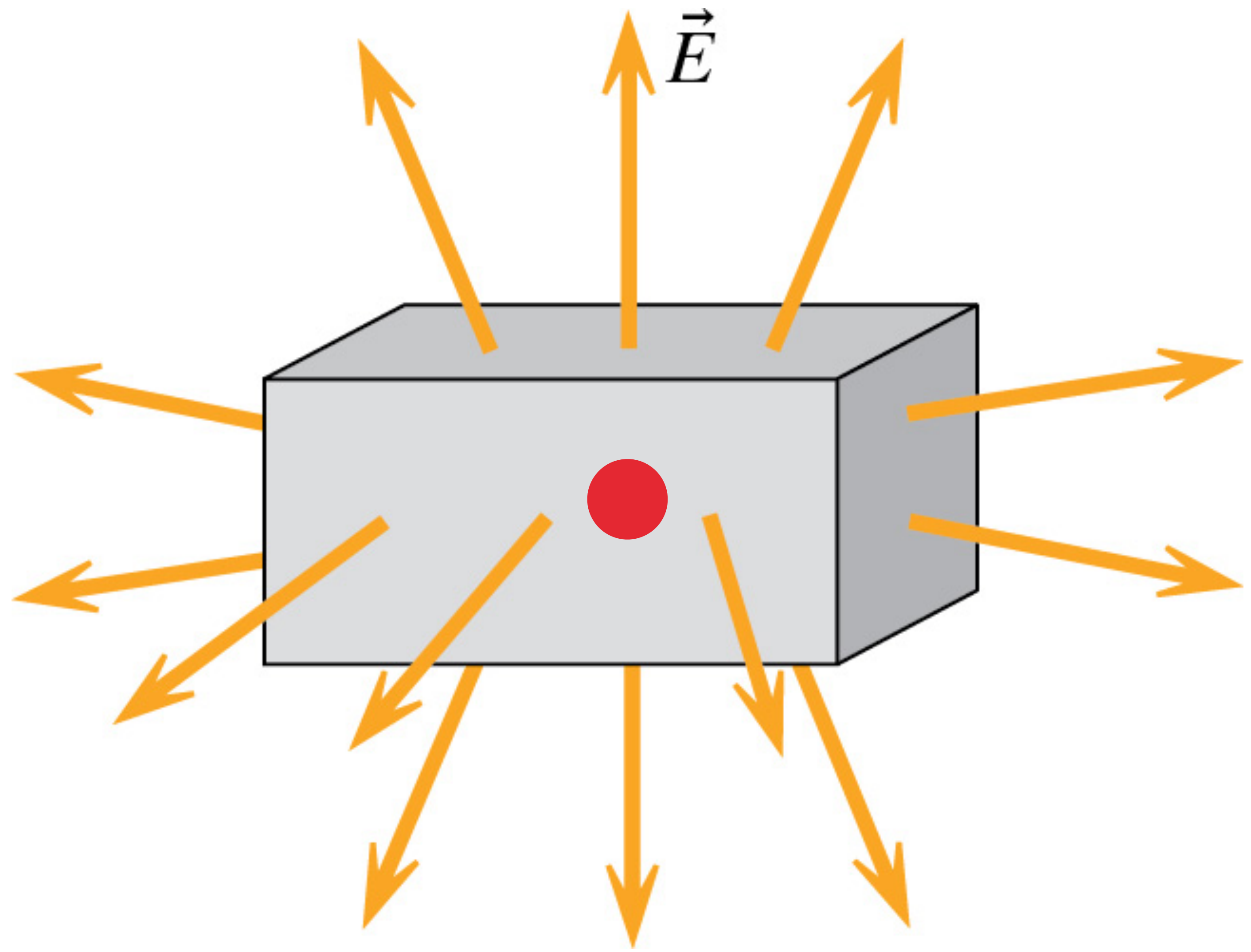
- We now have a quantitative definition of electric flux
- We suspect that net electric flux through a surface is **proportional** to the charge enclosed by the surface
- Let us quantify this relationship


$\hat{n}$
$\vec{E}$
$\hat{n}$
$\vec{E}$
ΔA
ΔA

GAUSS'S LAW

- Net electric flux through any closed surface is equal to the charge enclosed by the surface, divided by ϵ_0

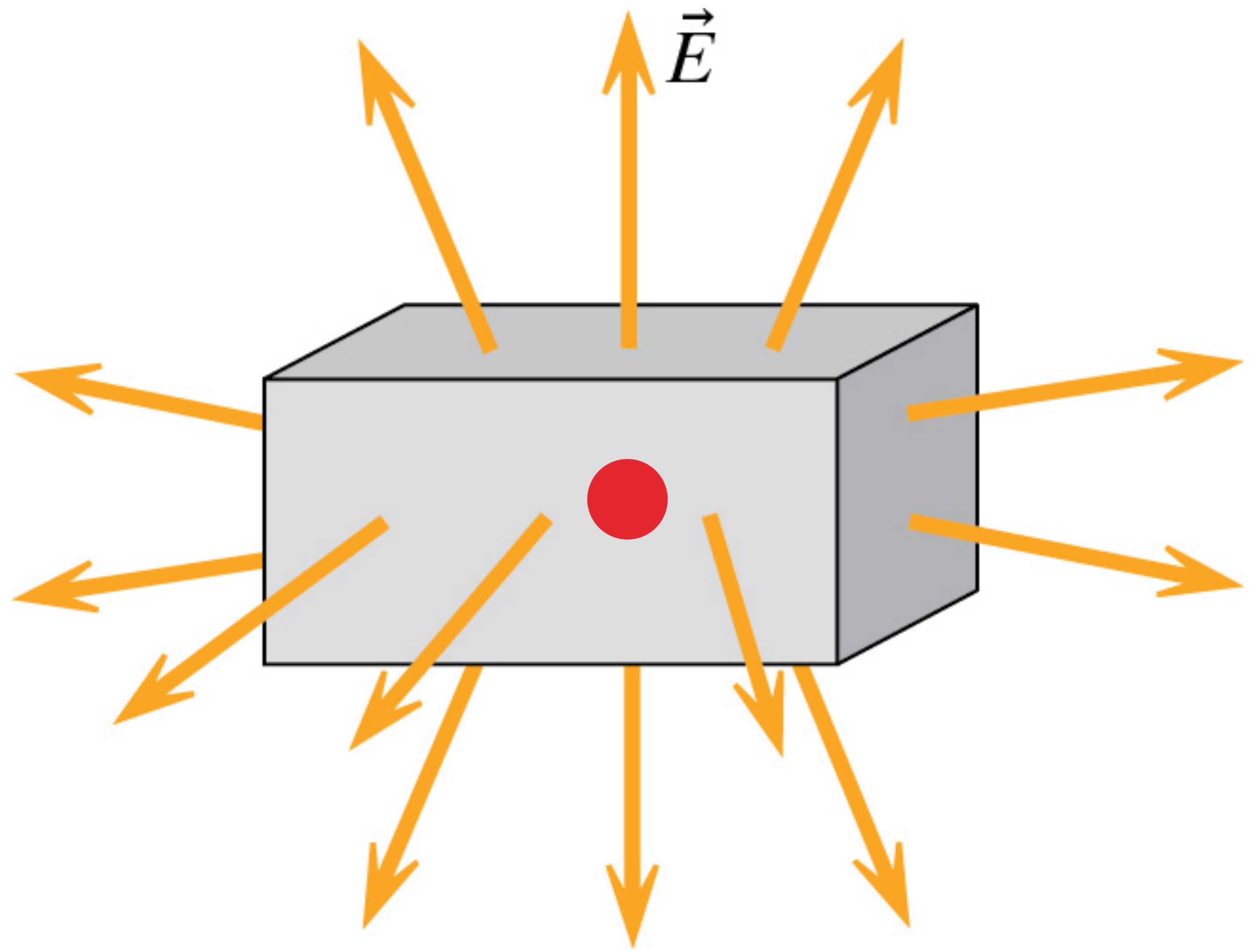
$$\boxed{\sum_{\text{closed surface}} \vec{E} \cdot \hat{n} \Delta A = \frac{q_{\text{in}}}{\epsilon_0}}$$



GAUSS'S LAW

- Net electric flux through any closed surface is equal to the charge enclosed by the surface, divided by ϵ_0

$$\boxed{\oint_S \vec{E} \cdot \hat{n} dA = \frac{q_{\text{in}}}{\epsilon_0}}$$



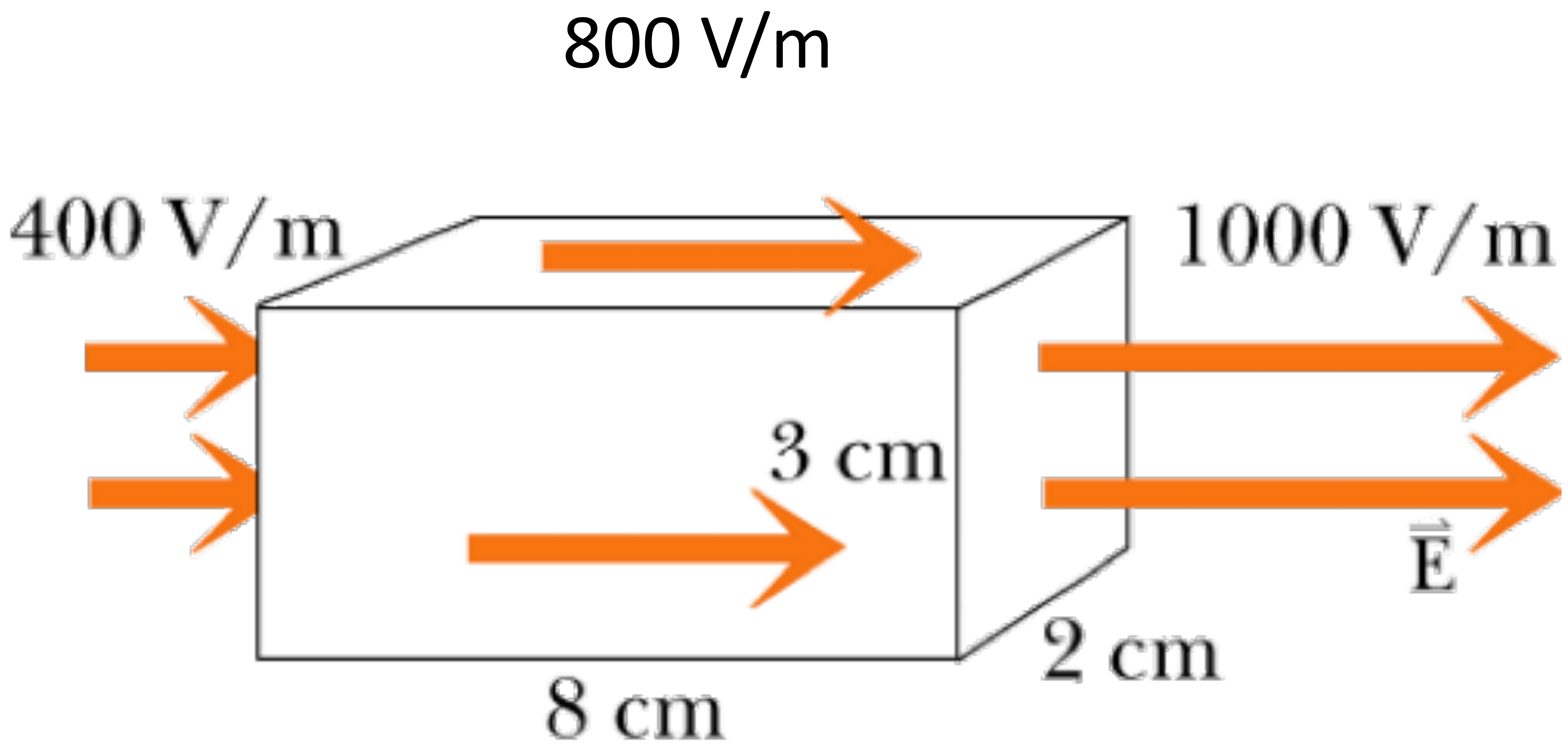
PROPERTIES OF GAUSS'S LAW

- The "Gaussian surface" which encloses the charge is purely conceptual
- The **size** of the surface enclosing the charge does not matter!
- The **shape** of the surface enclosing the charge does not matter!

PROPERTIES OF GAUSS'S LAW

- The "Gaussian surface" which encloses the charge is purely conceptual
- The **size** of the surface enclosing the charge does not matter!
- The **shape** of the surface enclosing the charge does not matter!
- Charge *outside of the surface* contributes **zero** net flux

- What is the electric charge inside the box?



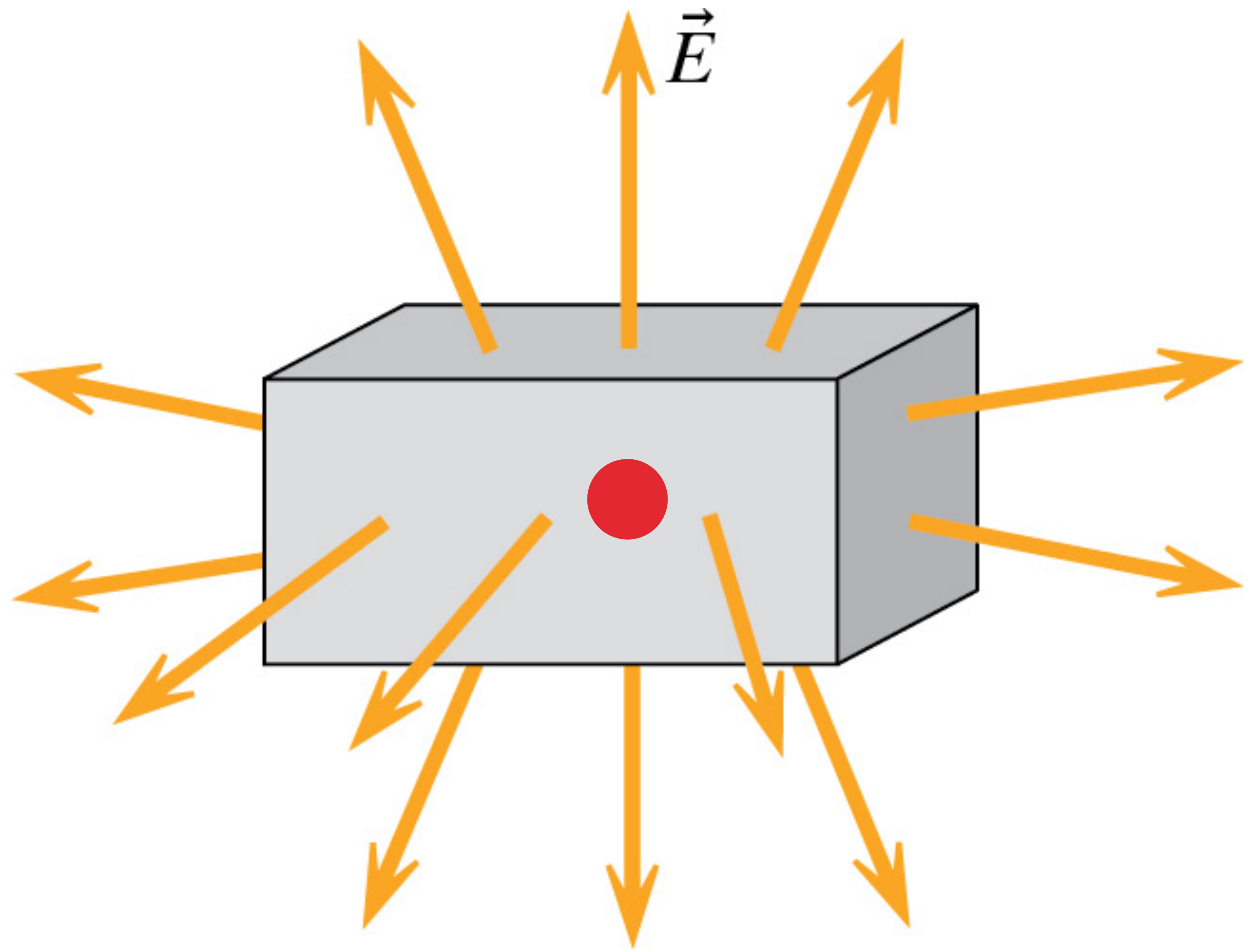
NEXT TWO WEEKS

- Chapter 20 Homework due Wednesday (Dec 1)
    - Quiz on Friday, Dec 3
- Finish Ch 21 **today**
- Ch 22: Wednesday and Friday (maybe some of Monday)
- Ch 23 next week
- Ch 21-22 Homework due by end of semester

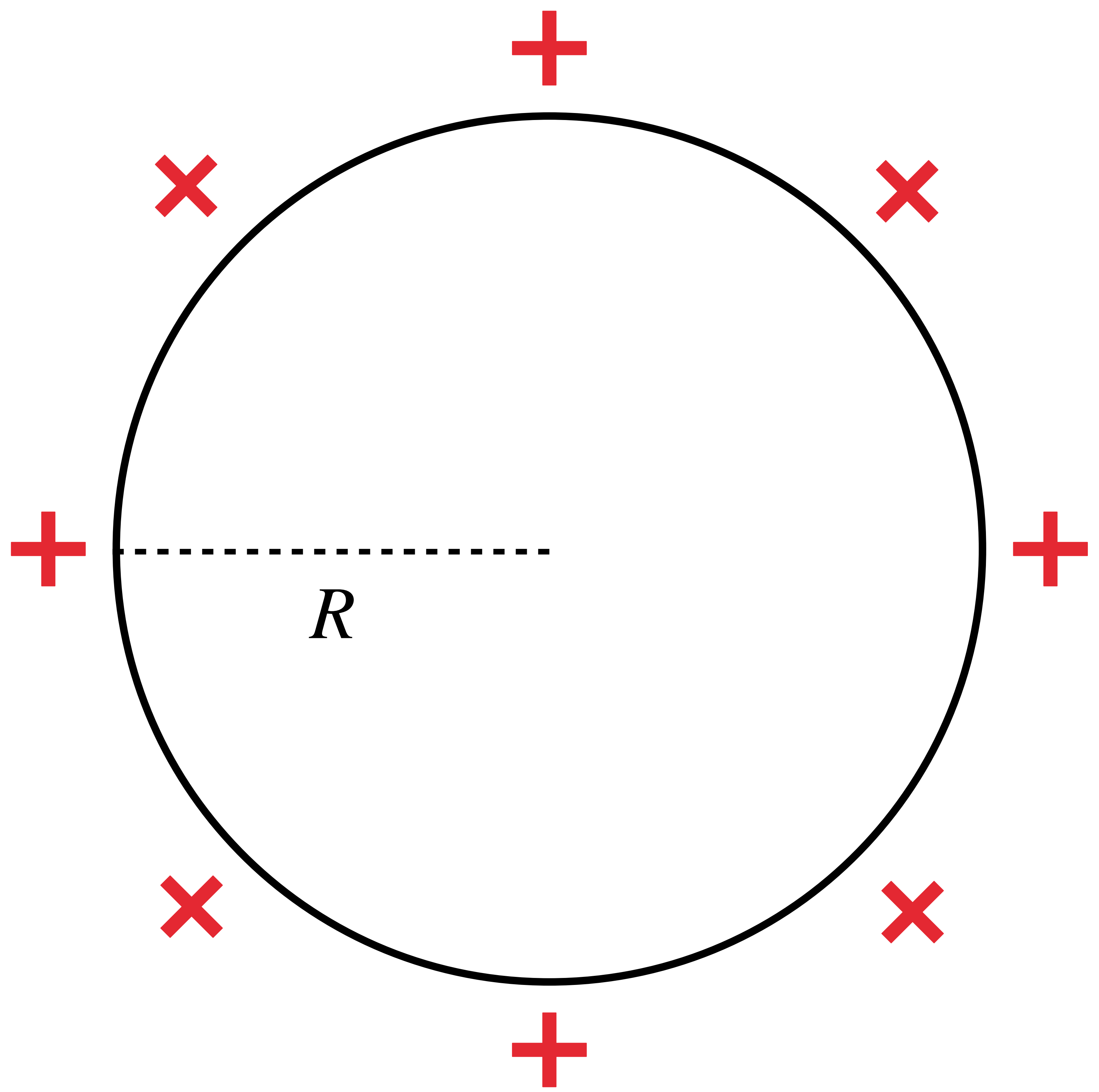
GAUSS'S LAW

- Net electric flux through any closed surface is equal to the charge enclosed by the surface, divided by ϵ_0

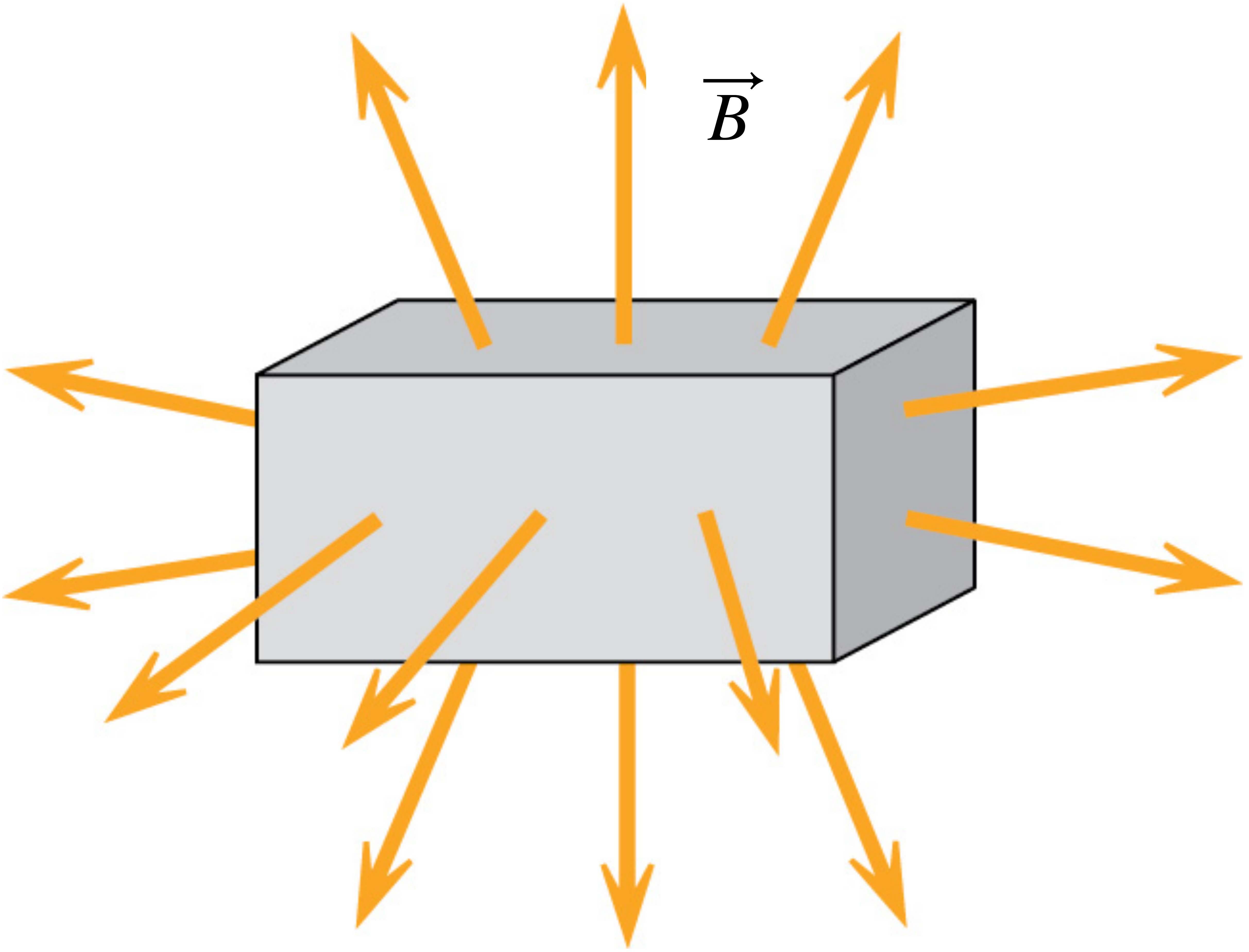
$$\boxed{\oint_S \vec{E} \cdot \hat{n} dA = \frac{q_{\text{in}}}{\epsilon_0}}$$



EXAMPLE: CHARGED SPHERICAL SHELL

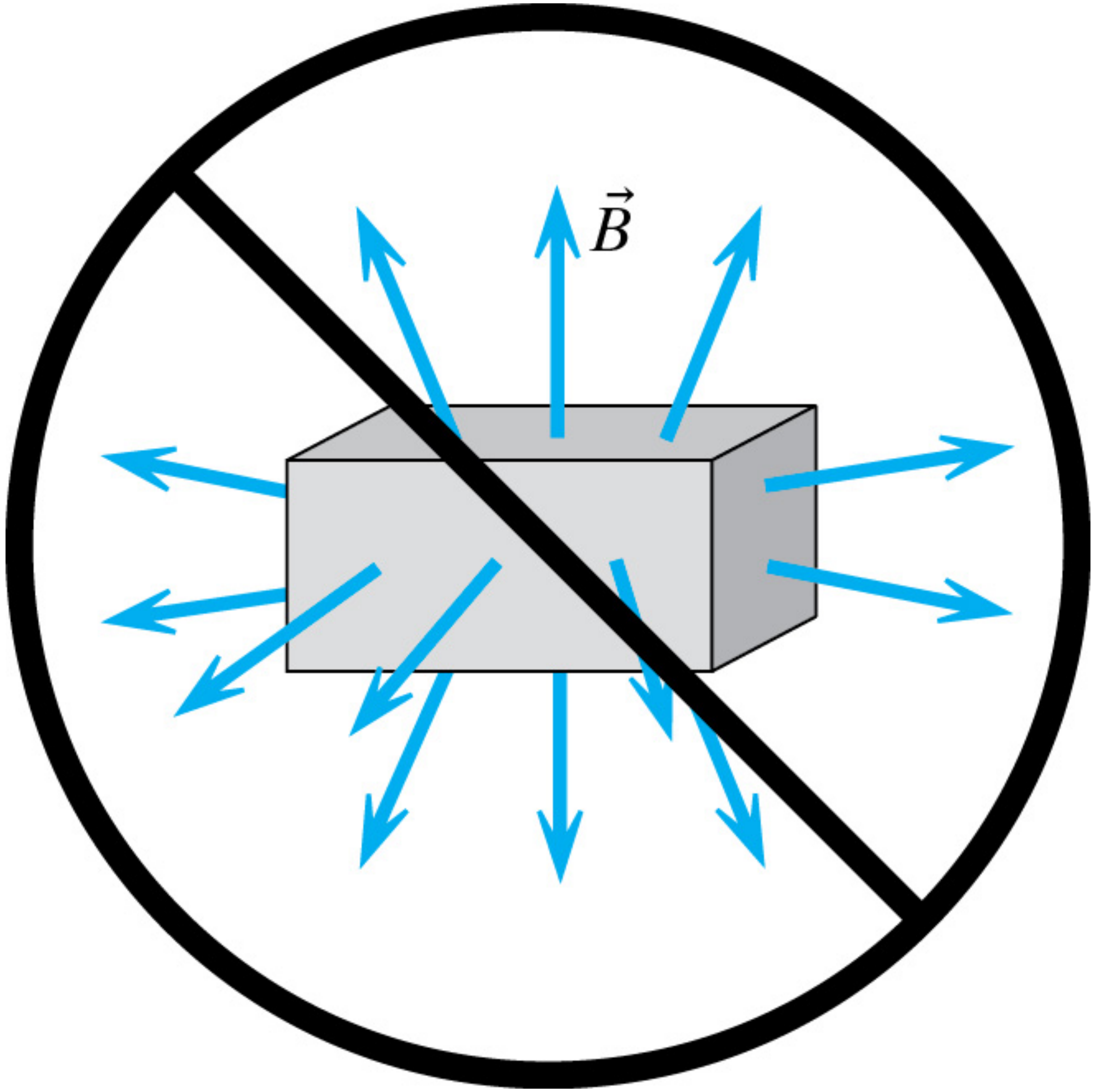


GAUSS'S LAW FOR MAGNETIC FIELDS?



GAUSS'S LAW FOR MAGNETIC FIELDS?

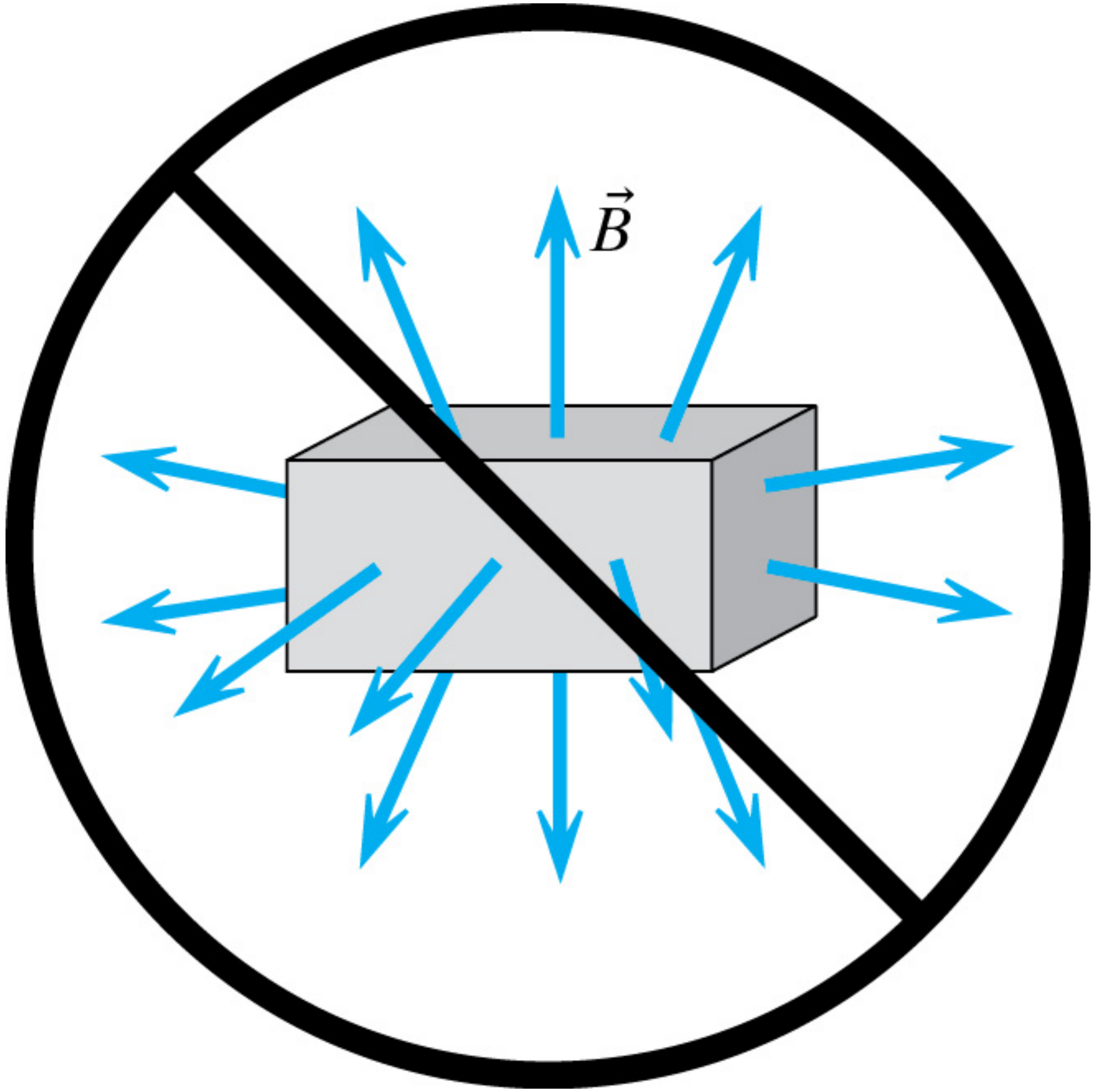
- Magnetic flux through a <u>*closed*</u> surface is *always* **zero**
- No magnetic monopoles!



GAUSS'S LAW FOR MAGNETIC FIELDS?

- Magnetic flux through a <u>*closed*</u> surface is *always* **zero**
- No magnetic monopoles!

$$\phi_{\text{mag}} = \oint \overrightarrow{B} \cdot \hat{n} dA = 0$$

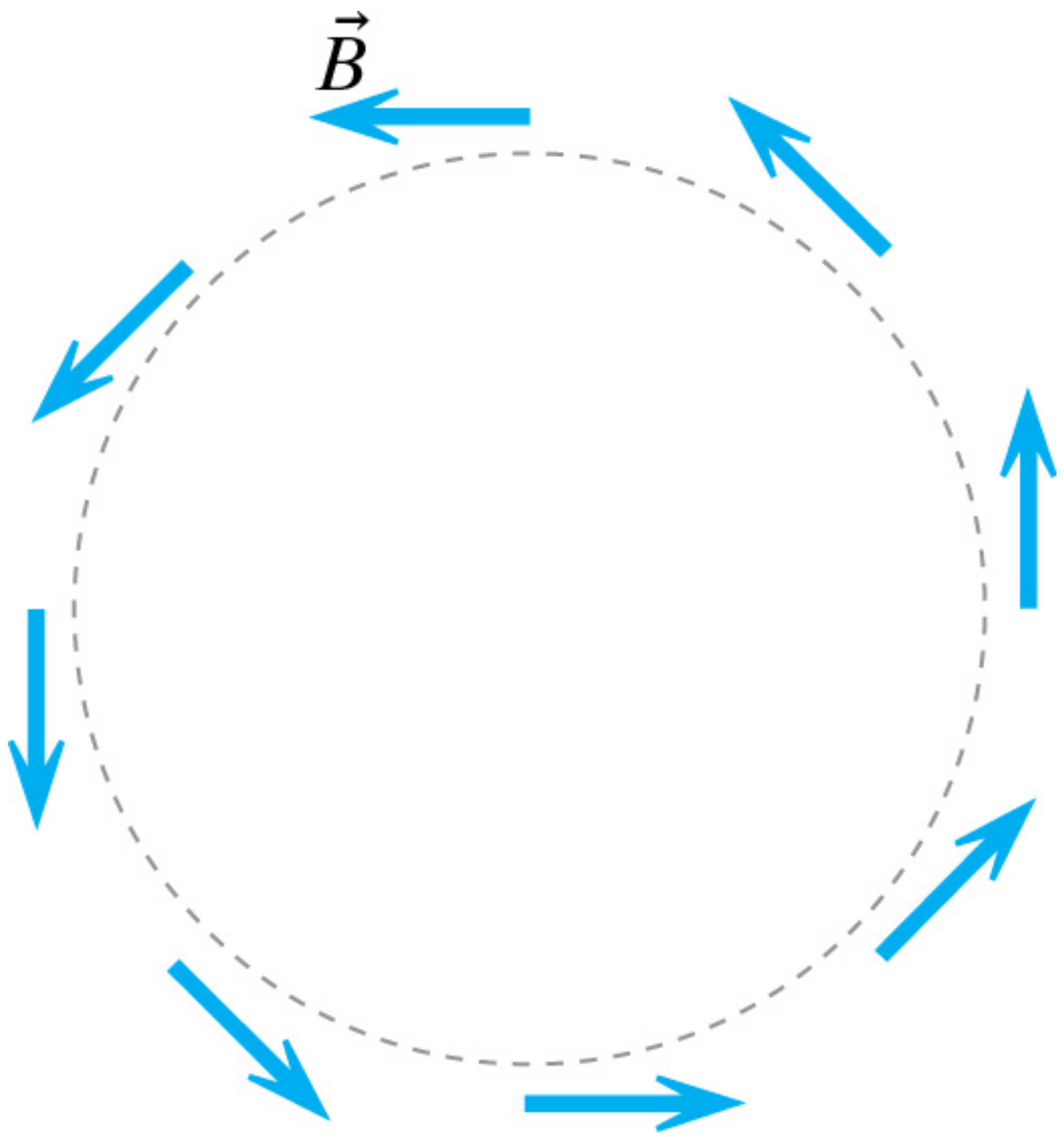


PATTERN OF MAGNETIC FIELD

- Magnetic flux tells us nothing about current
- Can we find any relationship that does?

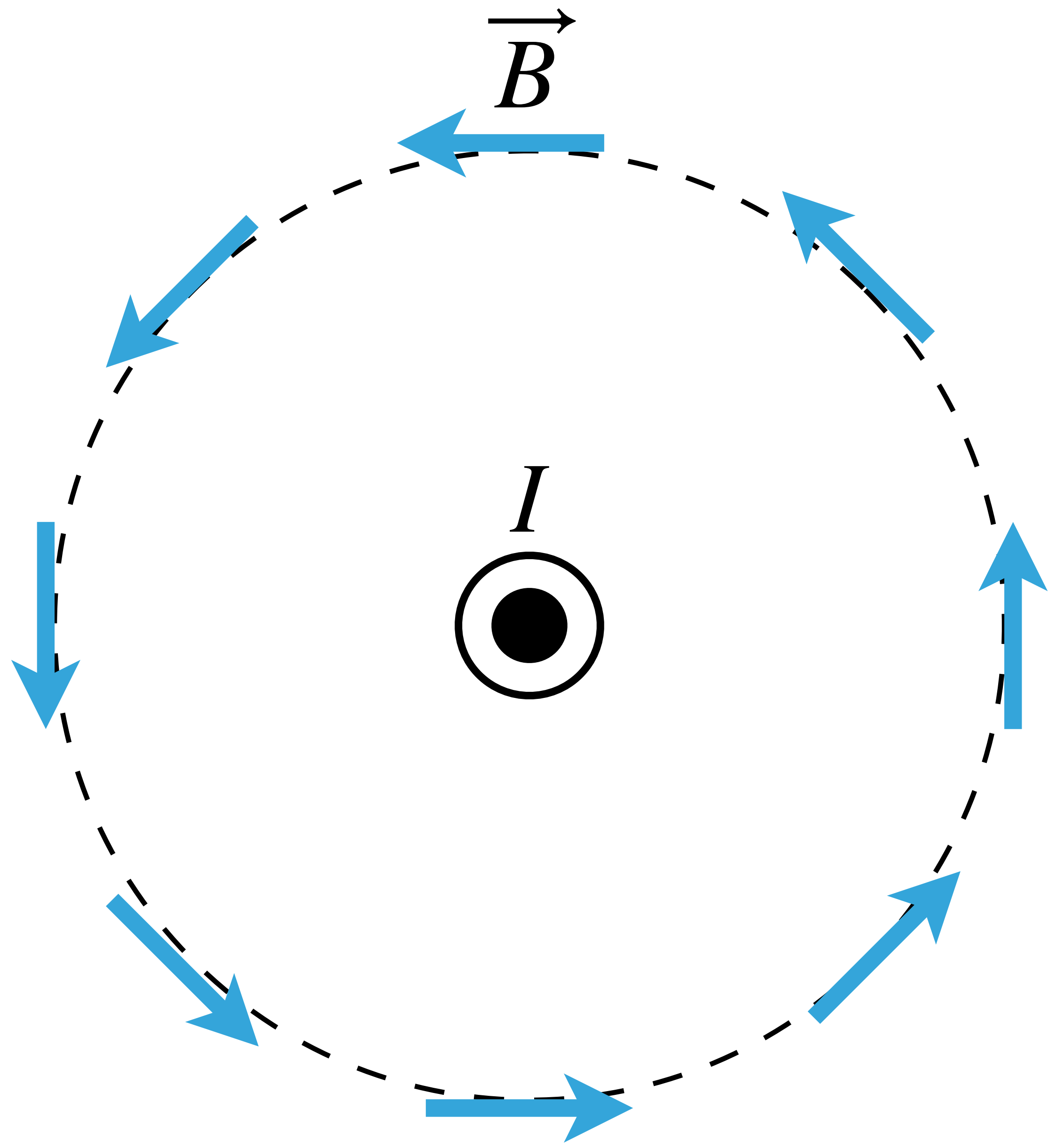
PATTERN OF MAGNETIC FIELD

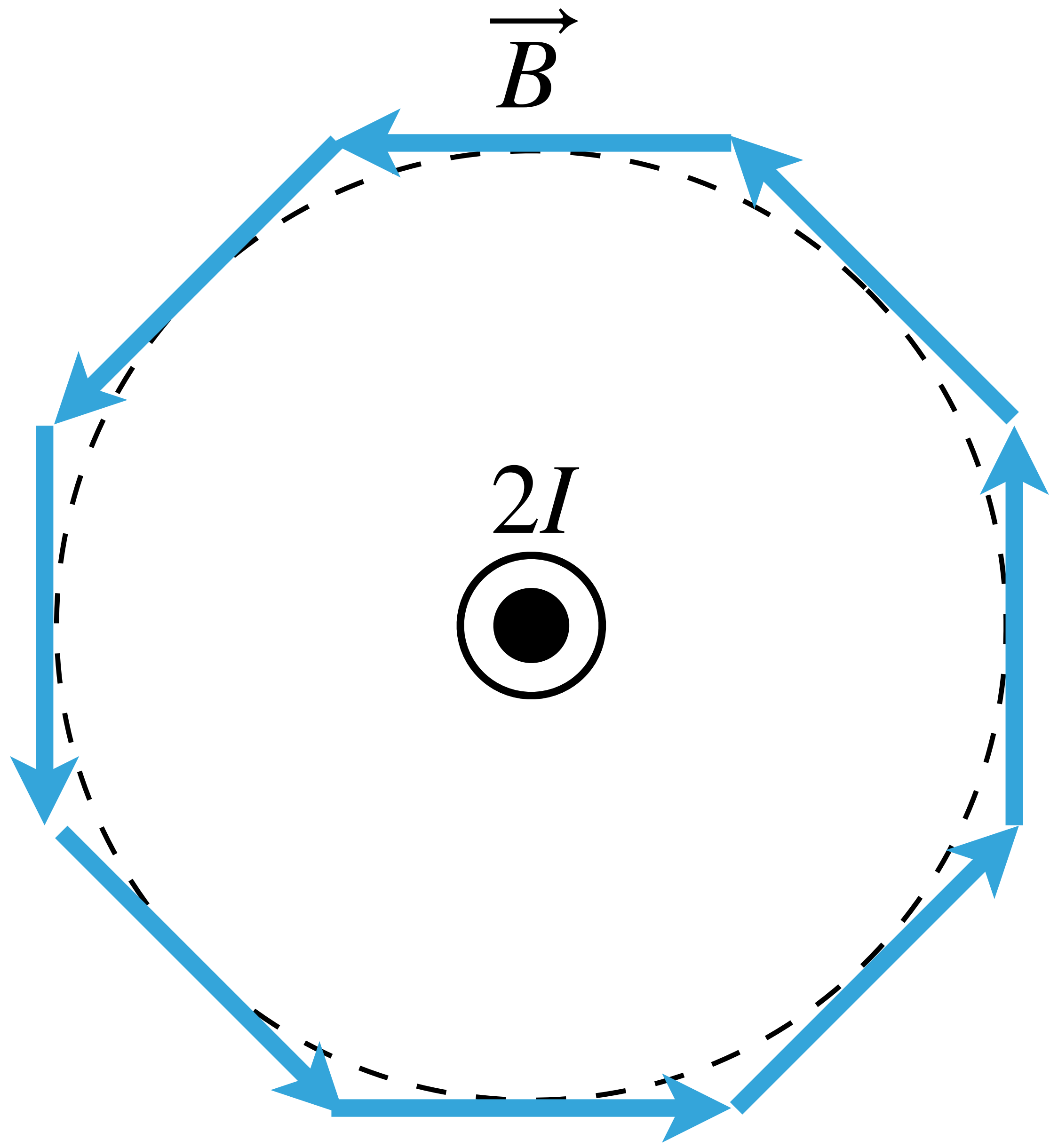
- Can we relate the "amount" of field around a closed loop to the current circled by the loop?



PROPERTIES

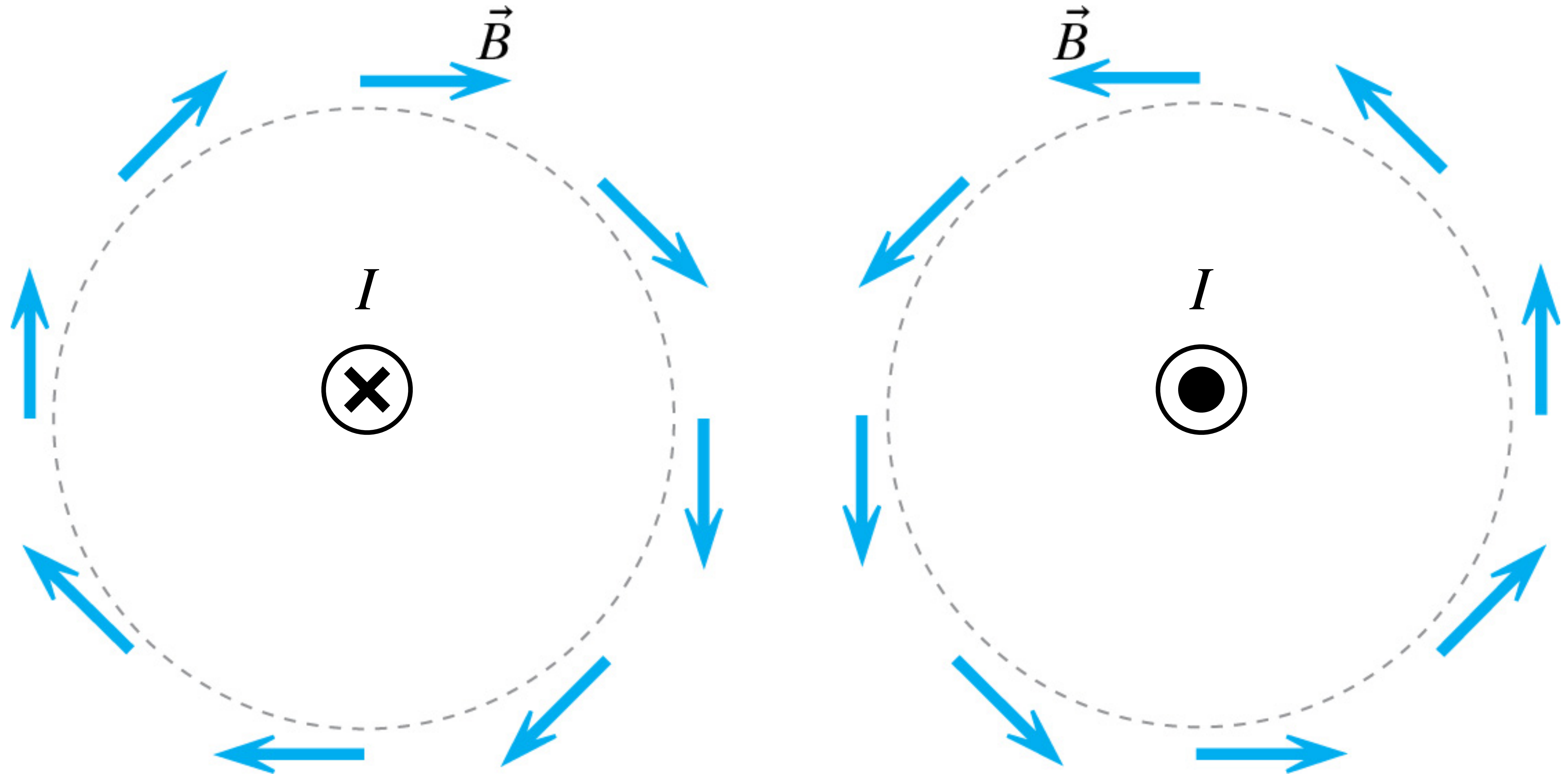
- Greater current –> greater $\overrightarrow{B}$ along the loop surrounding the current





PATTERN OF MAGNETIC FIELD

- Direction of $\overrightarrow{B}$ along the loop indicates direction of current through the loop



AMPERE'S LAW

Around a loop surrounding a distribution of current:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{inside}}$$

FUNDAMENTAL EQUATIONS

FUNDAMENTAL EQUATIONS

$$\oint \vec{E} \cdot \hat{n} dA = \frac{\sum q_{\text{inside}}}{\epsilon_0}$$

(Gauss's Law for electricity)

FUNDAMENTAL EQUATIONS

$$\oint \overrightarrow{E} \cdot \hat{n} dA = \frac{\sum q_{\text{inside}}}{\epsilon_0}$$

(Gauss's Law for electricity)

USED TO DERIVE COULOMB'S LAW

FUNDAMENTAL EQUATIONS

$$\oint \overrightarrow{E} \cdot \hat{n} dA = \frac{\sum q_{\text{inside}}}{\epsilon_0}$$

(Gauss's Law for electricity)

$$\oint \overrightarrow{B} \cdot \hat{n} dA = 0$$

(Gauss's Law for magnetism)

FUNDAMENTAL EQUATIONS

$$\oint \overrightarrow{E} \cdot \hat{n} dA = \frac{\sum q_{\text{inside}}}{\epsilon_0}$$

(Gauss's Law for electricity)

$$\oint \overrightarrow{B} \cdot \hat{n} dA = 0$$

(Gauss's Law for magnetism)

NO MAGNETIC MONOPOLES

FUNDAMENTAL EQUATIONS

$$\oint \overrightarrow{E} \cdot \hat{n} dA = \frac{\sum q_{\text{inside}}}{\epsilon_0}$$ (Gauss's Law for electricity)

$$\oint \overrightarrow{B} \cdot \hat{n} dA = 0$$ (Gauss's Law for magnetism)

$$\oint \overrightarrow{B} \cdot d\vec{l} = \mu_0 \sum I_{\text{inside loop}}$$ (Ampere's Law)

FUNDAMENTAL EQUATIONS

$$\oint \overrightarrow{E} \cdot \hat{n} dA = \frac{\sum q_{\text{inside}}}{\epsilon_0}$$

(Gauss's Law for electricity)

$$\oint \overrightarrow{B} \cdot \hat{n} dA = 0$$

(Gauss's Law for magnetism)

$$\oint \overrightarrow{B} \cdot d\vec{l} = \mu_0 \sum I_{\text{inside loop}}$$

(Ampere's Law)

USED TO DERIVE BIOT-SAVART LAW

FUNDAMENTAL EQUATIONS

$$\oint \vec{E} \cdot \hat{n} dA = \frac{\sum q_{\text{inside}}}{\epsilon_0}$$ (Gauss's Law for electricity)

$$\oint \vec{B} \cdot \hat{n} dA = 0$$ (Gauss's Law for magnetism)

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \sum I_{\text{inside loop}}$$ (Ampere's Law, **incomplete**)

FUNDAMENTAL EQUATIONS

$$\oint \vec{E} \cdot \hat{n} dA = \frac{\sum q_{\text{inside}}}{\epsilon_0}$$

(Gauss's Law for electricity)

$$\oint \vec{B} \cdot \hat{n} dA = 0$$

(Gauss's Law for magnetism)

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \sum I_{\text{inside loop}}$$

(Ampere's Law, **incomplete**)

$$\oint \vec{E} \cdot d\vec{l} = 0$$

(Faraday's law)

FUNDAMENTAL EQUATIONS

$$\oint \vec{E} \cdot \hat{n} dA = \frac{\sum q_{\text{inside}}}{\epsilon_0}$$ (Gauss's Law for electricity)

$$\oint \vec{B} \cdot \hat{n} dA = 0$$ (Gauss's Law for magnetism)

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \sum I_{\text{inside loop}}$$ (Ampere's Law, **incomplete**)

$$\oint \vec{E} \cdot d\vec{l} = 0$$ (Faraday's law, **incomplete**)

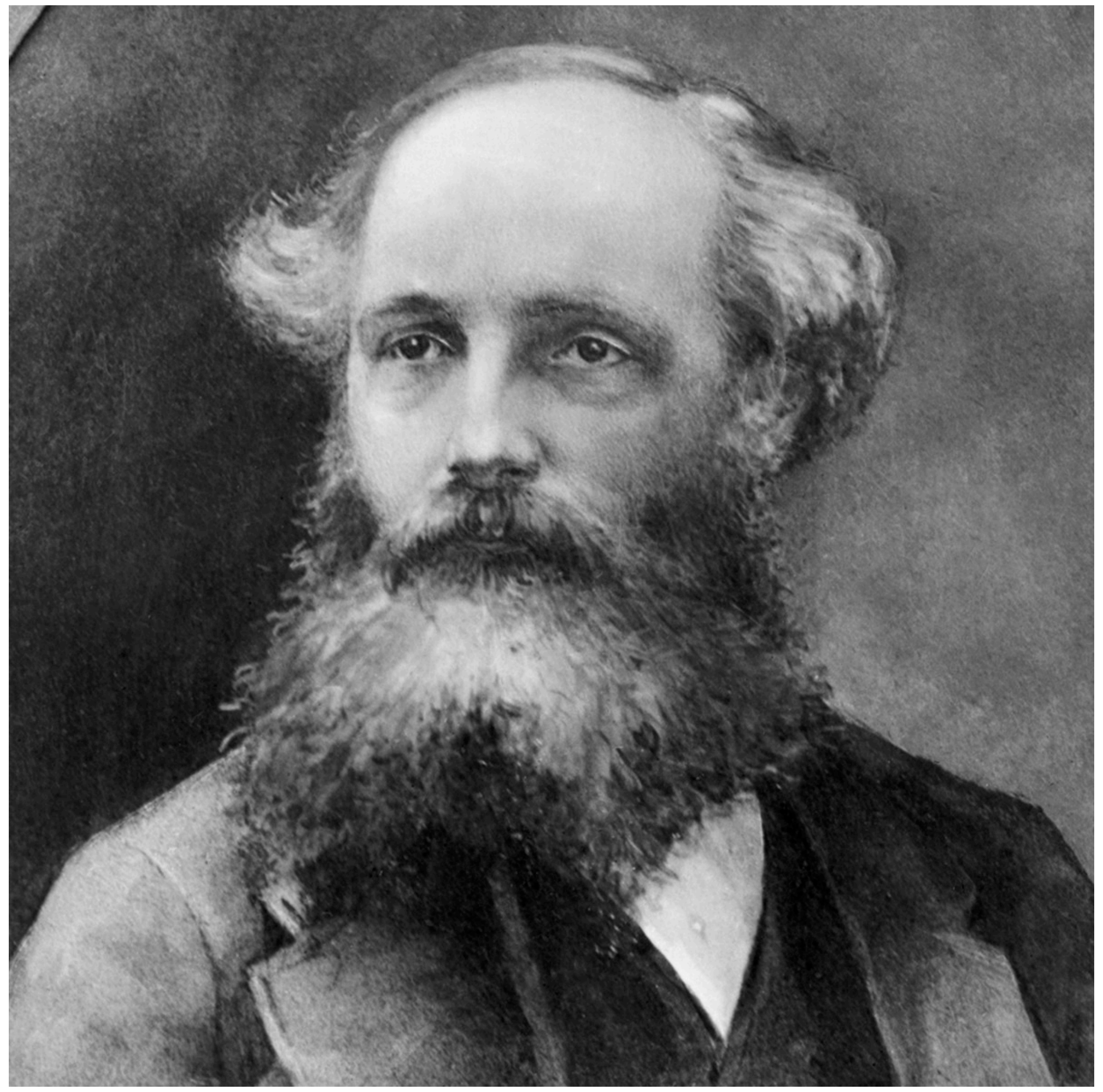
MAXWELL'S EQUATIONS

$$\oint \vec{E} \cdot \hat{n} dA = \frac{\sum q_{\text{inside}}}{\epsilon_0}$$ (Gauss's Law for electricity)

$$\oint \vec{B} \cdot \hat{n} dA = 0$$ (Gauss's Law for magnetism)

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \sum I_{\text{inside loop}}$$ (Ampere's Law, **incomplete**)

$$\oint \vec{E} \cdot d\vec{l} = 0$$ (Faraday's law, **incomplete**)



MAXWELL'S EQUATIONS

$$\oint \vec{E} \cdot \hat{n} dA = \frac{\sum q_{\text{inside}}}{\epsilon_0}$$ (Gauss's Law for electricity)

$$\oint \vec{B} \cdot \hat{n} dA = 0$$ (Gauss's Law for magnetism)

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \sum I_{\text{inside loop}}$$ (Ampere's Law, **incomplete**) CHAPTER 23

$$\oint \vec{E} \cdot d\vec{l} = 0$$ (Faraday's law, **incomplete**) CHAPTER 22