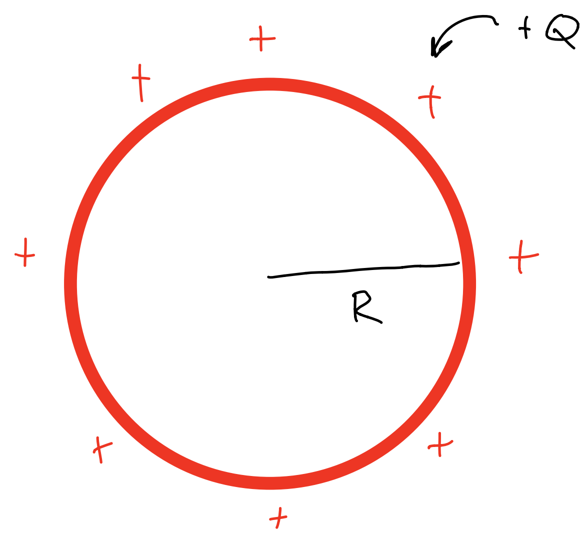
A spherical shell:

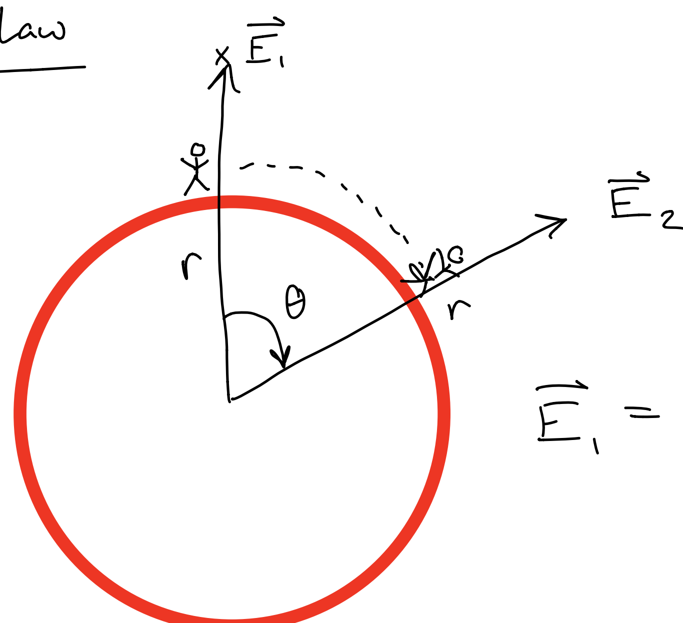


In Ch 15:

$$\vec{E} = \begin{cases} 0, & r < R \\ \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}, & r \geq R \end{cases}$$

Prove with Gauss' Law

What does E look like?



$\vec{E}_1 = \vec{E}_2$

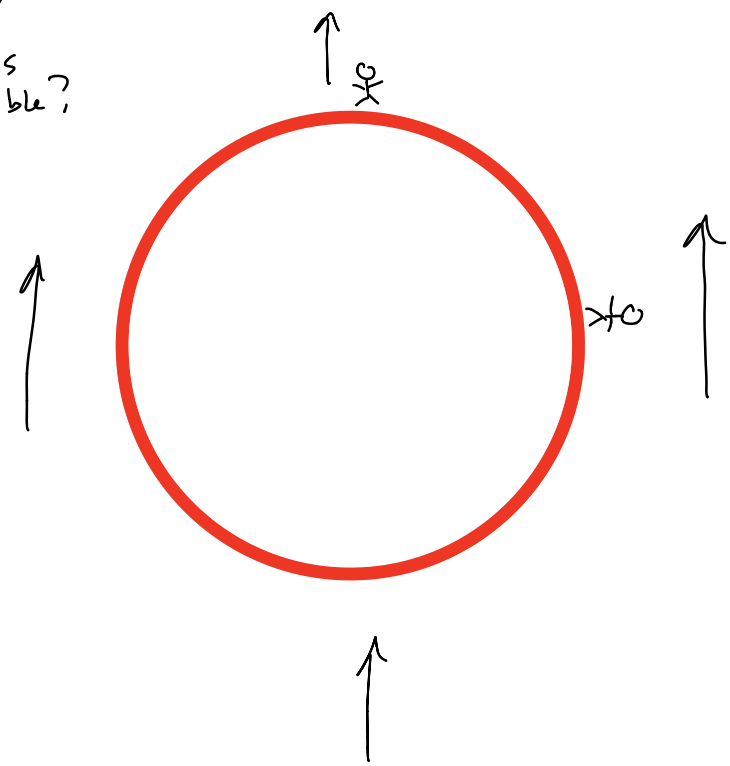
Rotational Symmetry

No way to determine where I am on the surface

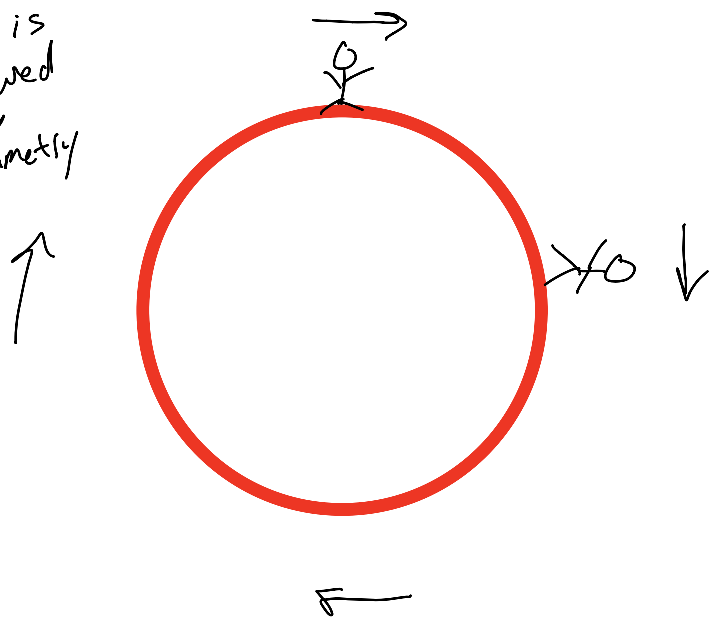
$\vec{E}$ should <u>only</u> depend on r

Direction?

is this possible?



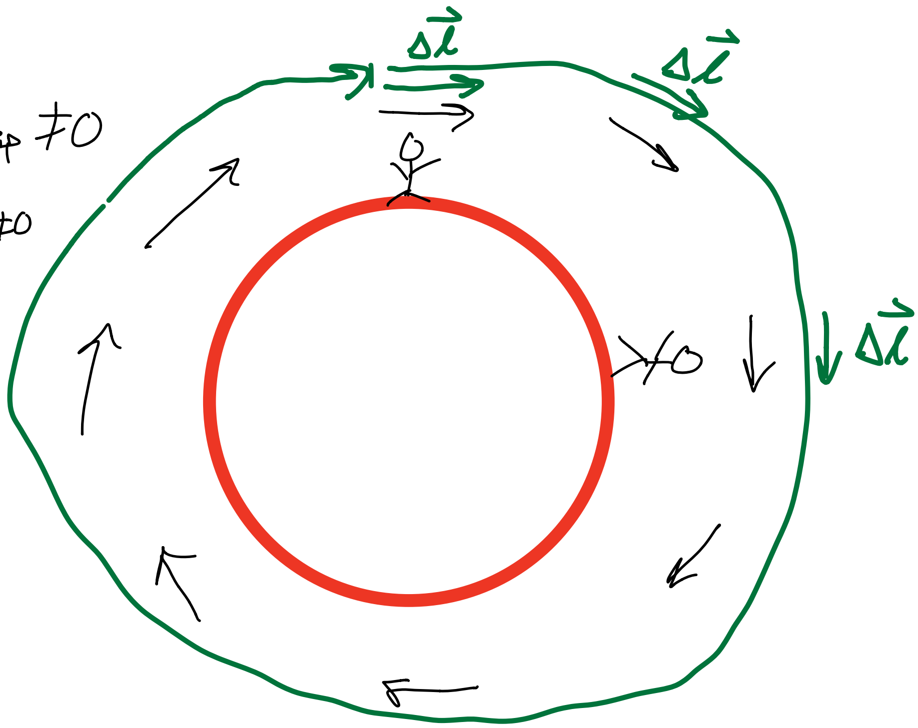
This is allowed by symmetry



But
$\Delta V_{\text{round trip}} \neq 0$

$-\sum \vec{E} \cdot \Delta \vec{l} \neq 0$

impossible!

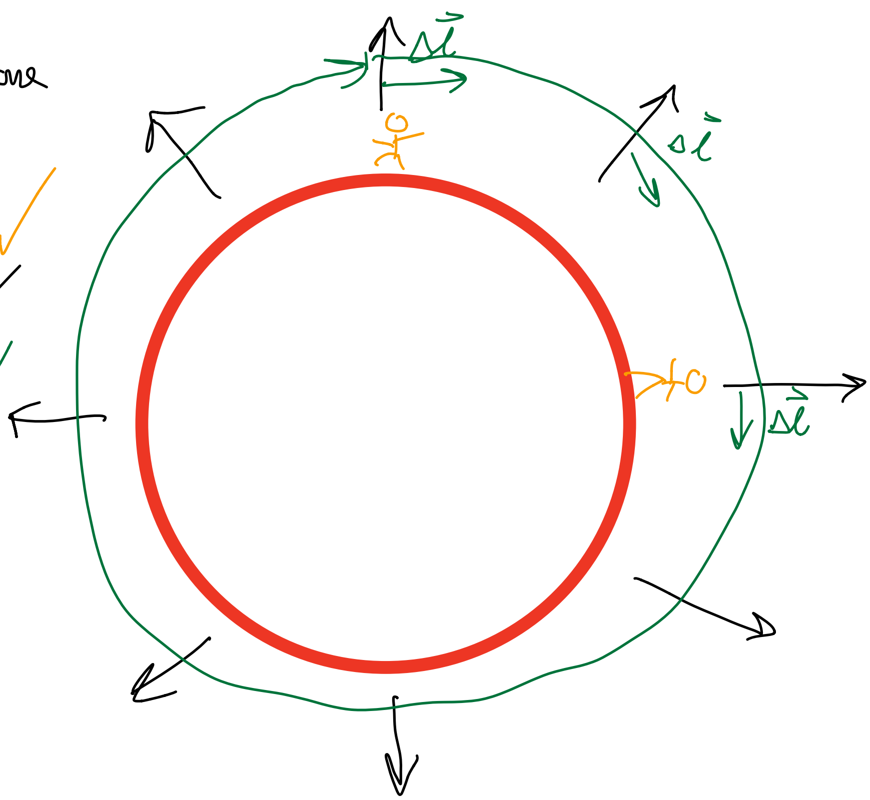


only have one option:

Symmetry? ✓

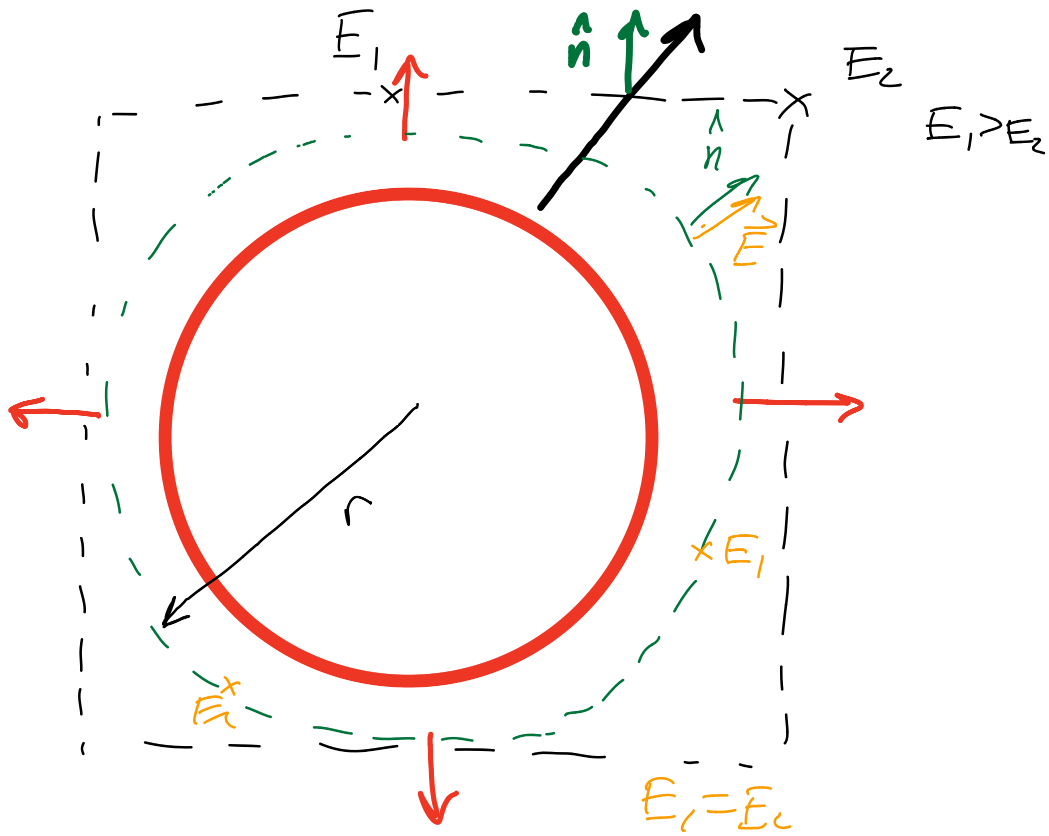
$\Delta V_{rt} = 0$? ✓

$\vec{E} \cdot \Delta \vec{l} = 0$ ✓



what shape?

$\oint \vec{E}\cdot\hat{n}\,dA$



Everywhere, $\vec{E}\cdot\hat{n} = E$

E is constant on the surface

$$\oint \vec{E}\cdot\hat{n}\,dA = E\int dA = E(4\pi r^2)$$

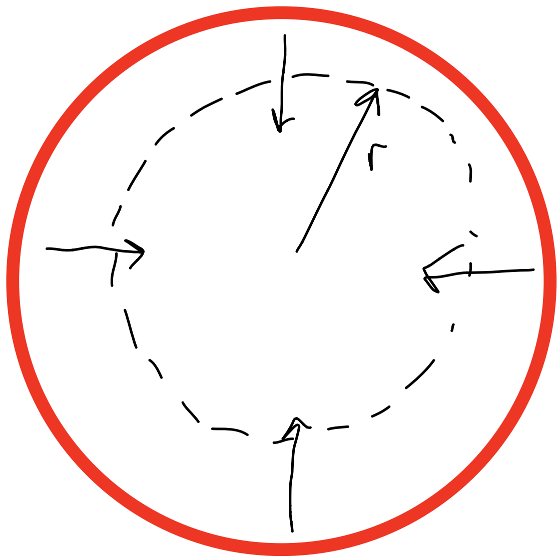
q inside? $= Q$

$$E(4\pi r^2) = \frac{1}{\epsilon_0} Q$$

$$E = \frac{Q}{4\pi\epsilon_0 r^2} \quad \checkmark \quad r > R$$

Inside:

Symmetry arguments are the same



$$\oint \vec{E}\cdot\hat{n}\,dA = E(4\pi r^2) = ?$$

$$q_{inside} = 0$$

$$E(4\pi r^2) = 0 \Rightarrow E = 0$$

Gauss' Law

1) What does the field look like? (symmetry)
2) What surface can I use to simplify

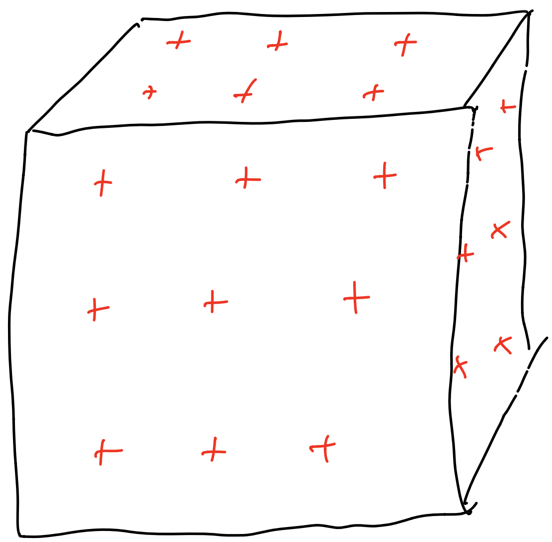
$$\oint \vec{E}\cdot\hat{n}\,dA \;?$$

- Choose a surface so that

$\hat{n}$ is always $\parallel$ or $\perp$ to $\vec{E}$, $\vec{E}$ is constant on surface

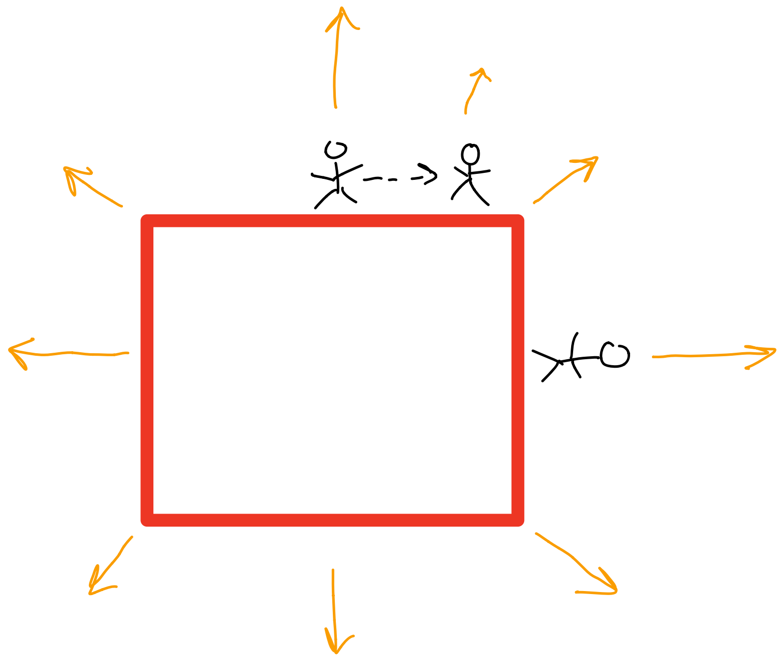
3) what is q_{inside} in the surface?

Example: Uniformly charged ~~bx~~ box



Gauss' Law still true, but not useful to us

What does E look like

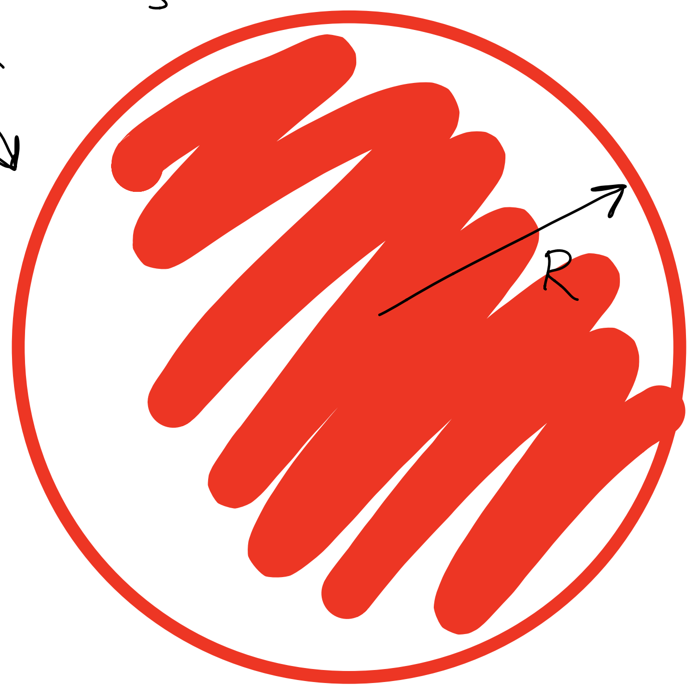


Only have symmetry for 45° rotations

Ex: Uniformly charged, solid

Chg density: $\frac{Q}{V} = \frac{Q}{4/3 \pi R^3} = \rho$

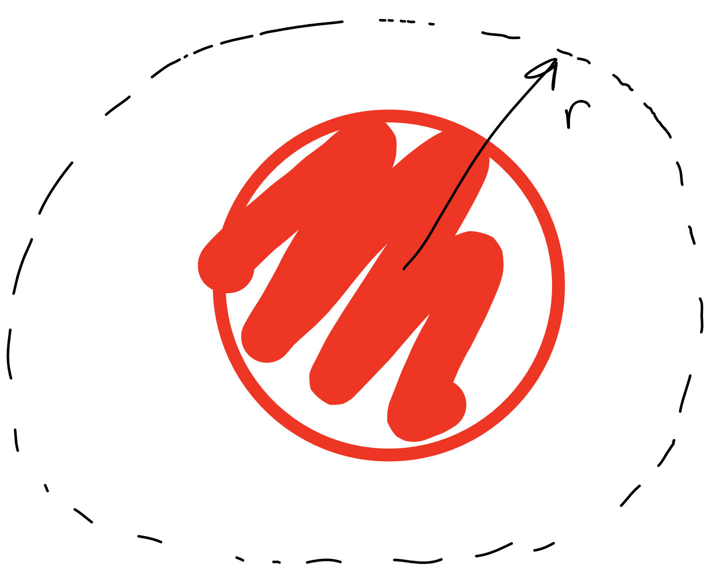
+Q

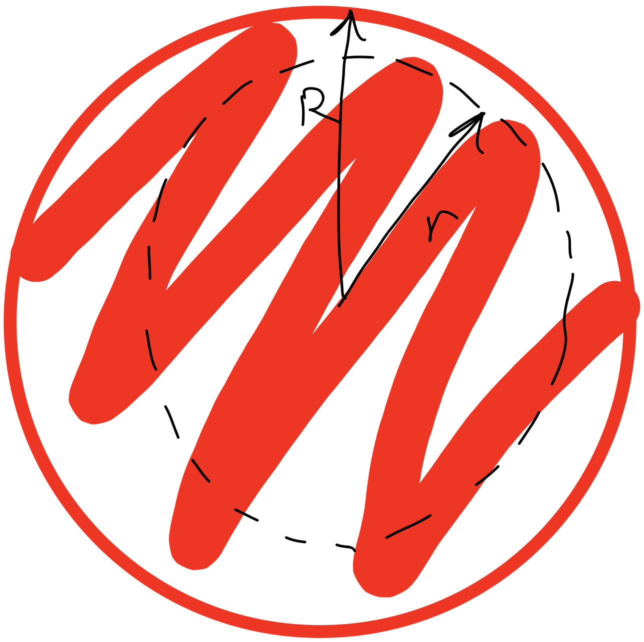


<u>outside</u>

$$E \, 4\pi r^2 = \frac{Q}{\epsilon_0}$$

$$E = \frac{Q}{4\pi\epsilon_0 r^2}$$





$$\oint \vec{E}\cdot\hat{n}\,dA = E(4\pi r^2)$$

q_{inside}? $=$ chg density $\times$ $V_{surface}$

$$= \rho\,\frac{4}{3}\pi r^3$$

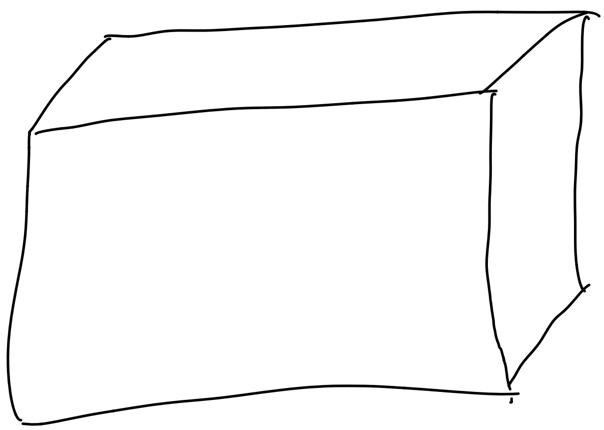
$$E(4\pi r^2) = \frac{\rho}{\epsilon_0}\left(\frac{4}{3}\pi r^3\right)$$

$$E(\cancel{4\pi r^2}) = \frac{\rho}{\epsilon_0}\left(\frac{\cancel{4}}{3}\cancel{\pi} r^{\cancel{3}}\right)$$

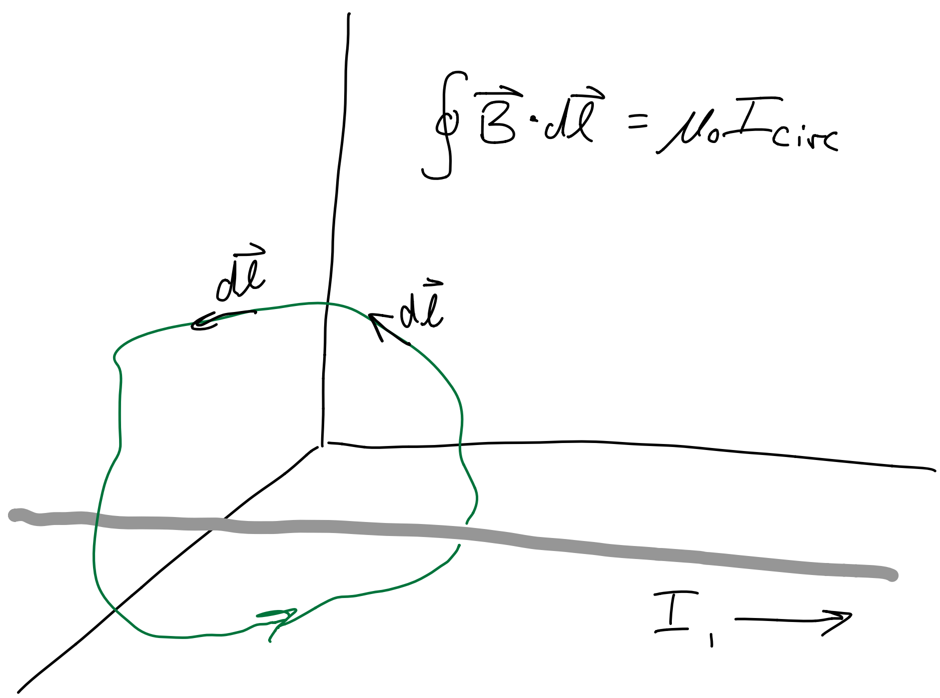
$$E = \frac{1}{3}\rho r$$

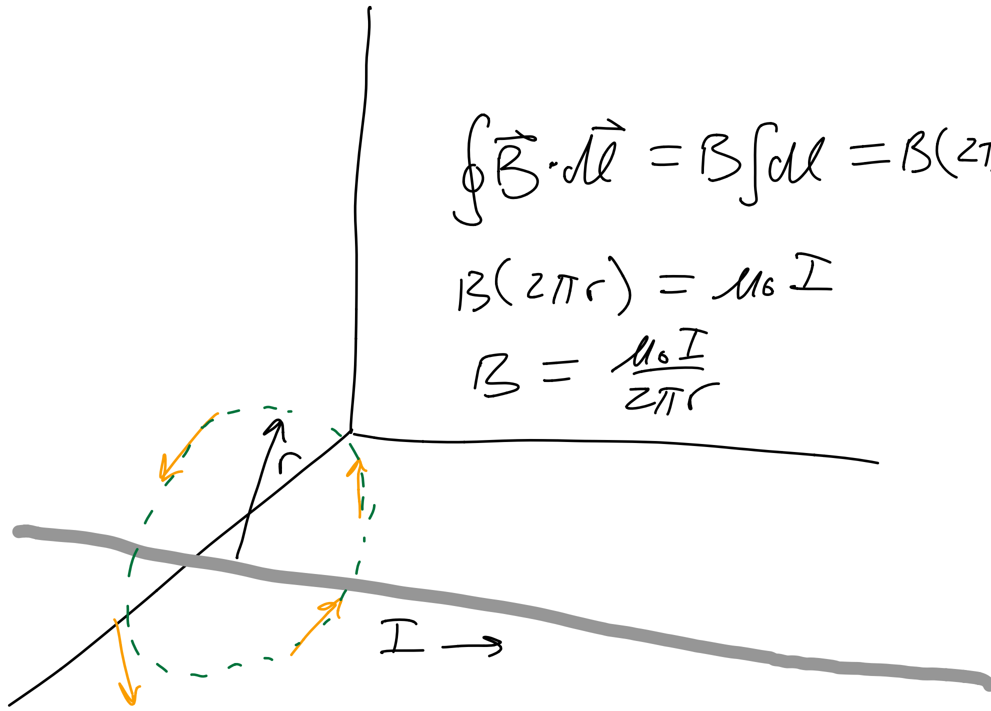
$$E = \frac{1}{3\epsilon_0}\frac{Q}{4/3\,\pi R^3}\,r \quad , \quad E = \frac{Q}{4\pi R^3\epsilon_0}\,r \quad , \quad r < R$$

For magnetism



$$\oint \vec{B} \cdot \hat{n}\, dA = 0$$





$$\oint \vec{B} \cdot d\vec{l} = B \int dl = B(2\pi r)$$

$$B(2\pi r) = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r}$$