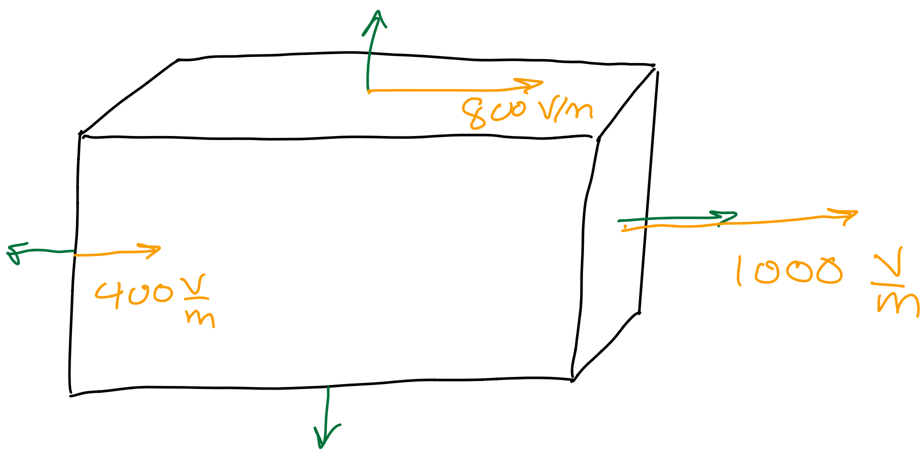
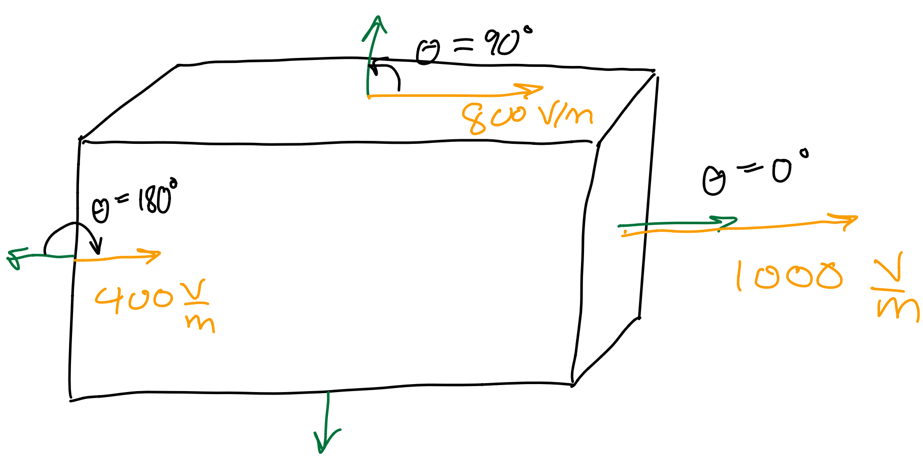
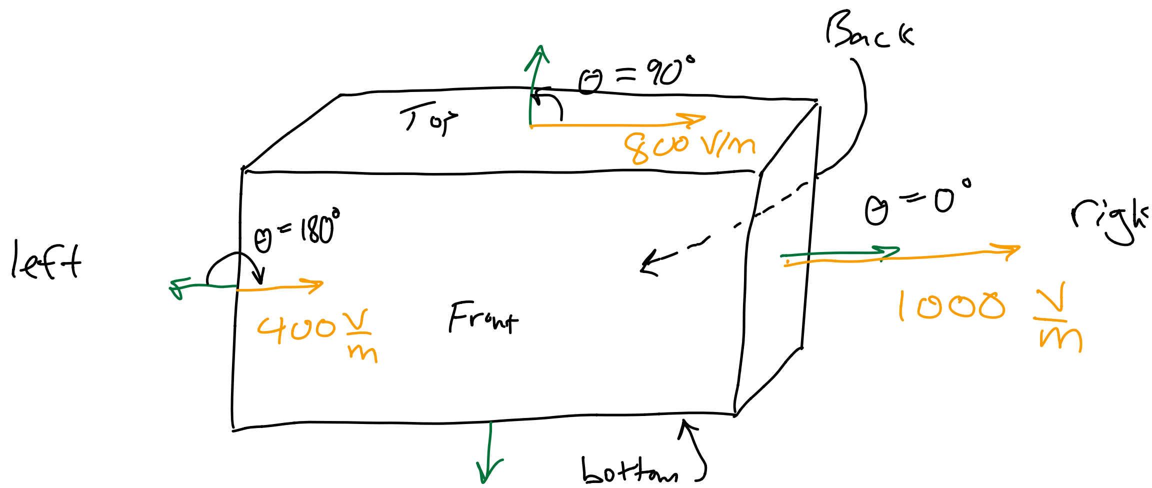
Q 21.2c





Add flux of each surface (6 of them)

left, right, top, bottom, front, back



Left:

$$\Phi_{left} = \vec{E} \cdot \hat{n}\, \Delta A$$

$$= E \Delta A \cos(180)$$

$$= -(400)(0.03)(0.02)$$

$$= -0.24 \ V \cdot m$$

Right

$$\Phi_{right} = \vec{E} \cdot \hat{n}\, \Delta A$$

$$= (1000)\cos(0)(0.03)(0.02)$$

$$= 0.6 \ V \cdot m$$

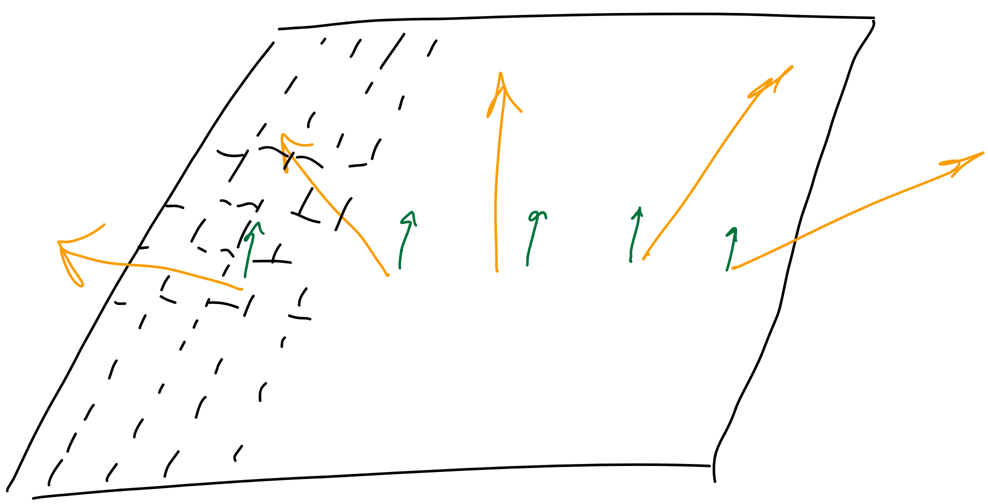
Top: $\vec{E} \cdot \hat{n}\, \Delta A$

$E \Delta A \cos(90)$

$$\Phi_E = 0.36\ \mathrm{V \cdot m}$$

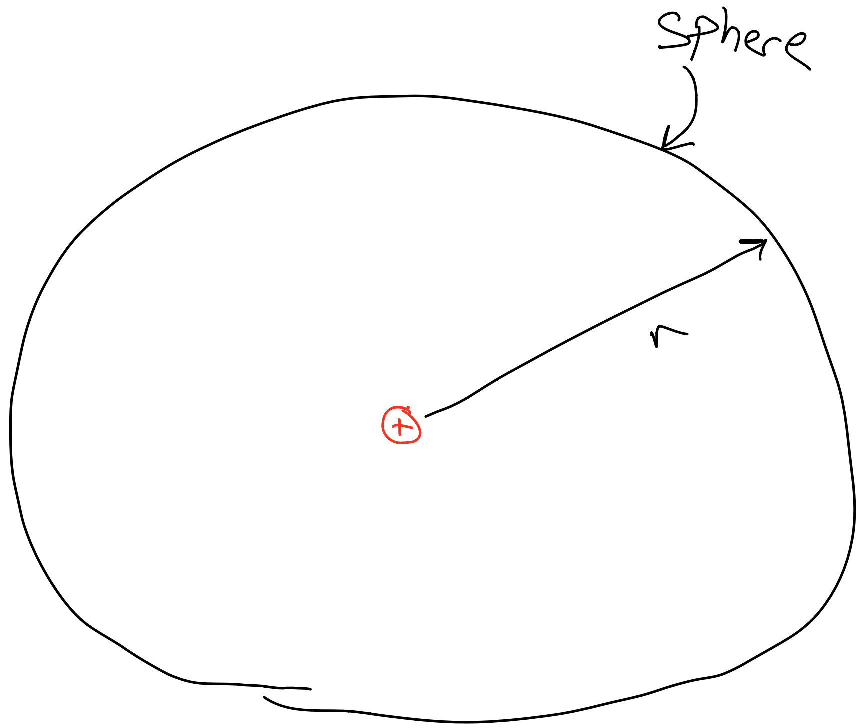
Electric Flux Through a surface

$$\Phi_E = \sum_{\text{surface}} \vec{E} \cdot \hat{n}\, \Delta A$$



$\Delta A \longrightarrow$ small

$$\boxed{\Phi_E = \oint \vec{E} \cdot \hat{n}\, dA}$$



$$\Phi_E = \oint \vec{E} \cdot \hat{n} \, dA$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r}$$

$$\hat{n} = \hat{r}$$

$$\Phi_E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \int dA$$

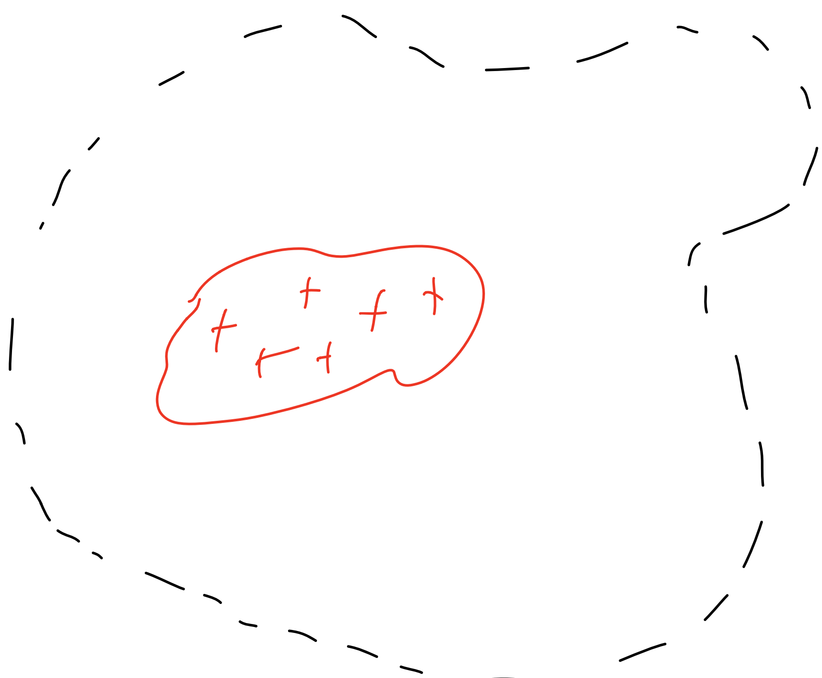
$$= \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} 4\pi r^2 = \frac{q}{\epsilon_0}$$

$$\Phi_E = \frac{q}{\epsilon_0}$$

Gauss' Law

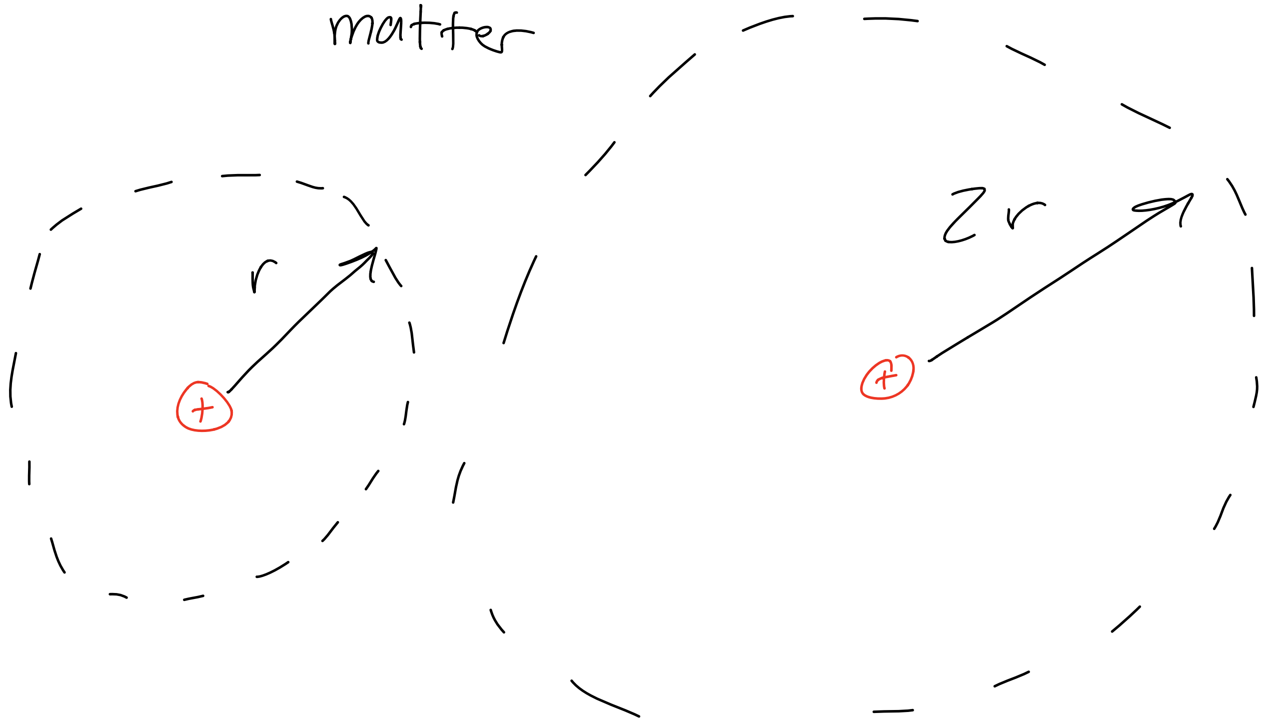
For ANY closed surface

$$\boxed{\Phi_E = \oint \vec{E} \cdot \hat{n}\, da = \frac{q_{inside}}{\epsilon_0}}$$



Some notes

- Size of the surface doesn't matter



$$E_1 \cdot A_1 = \frac{q}{4\pi\epsilon_0} \frac{1}{r^2} \cdot 4\pi r^2 = \frac{q}{\epsilon_0}$$

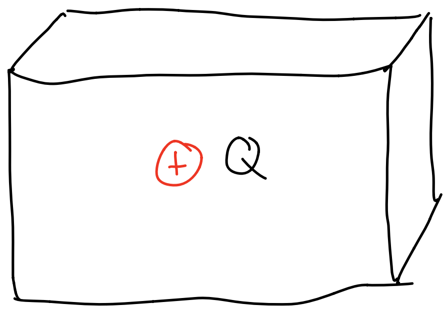
$$E_2 \cdot A_2 = \frac{q}{4\pi\epsilon_0} \frac{1}{(2r)^2} \cdot 4\pi (2r)^2 = \frac{q}{\epsilon_0}$$

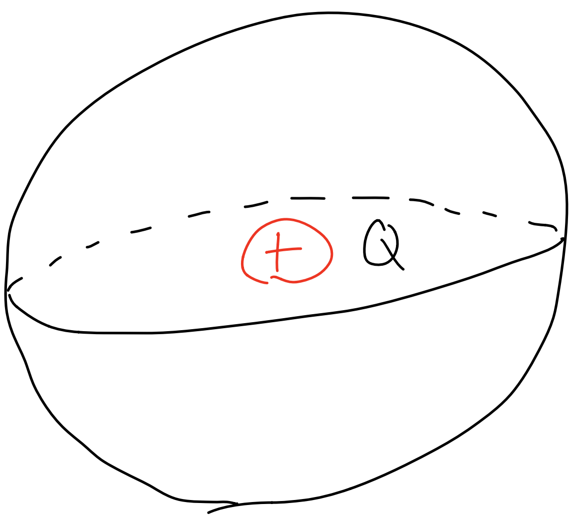
$A \propto r^2$

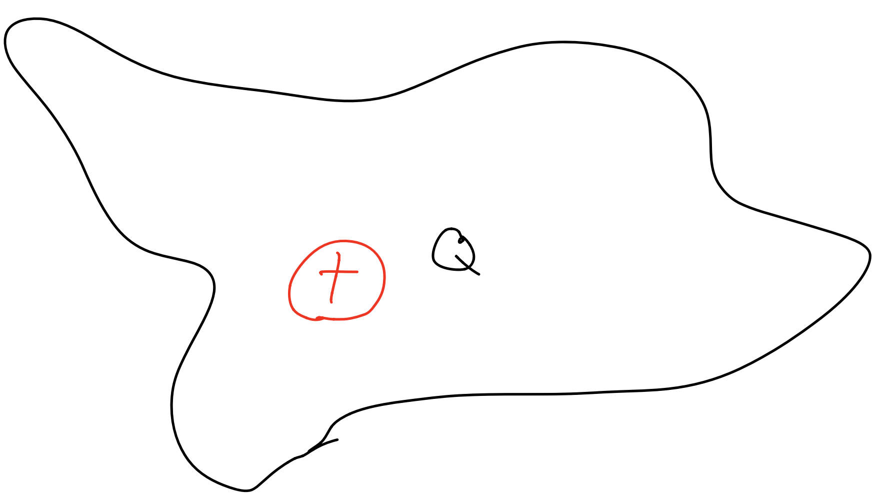
$E \propto \frac{1}{r^2}$

$E \cdot A = \text{const}$

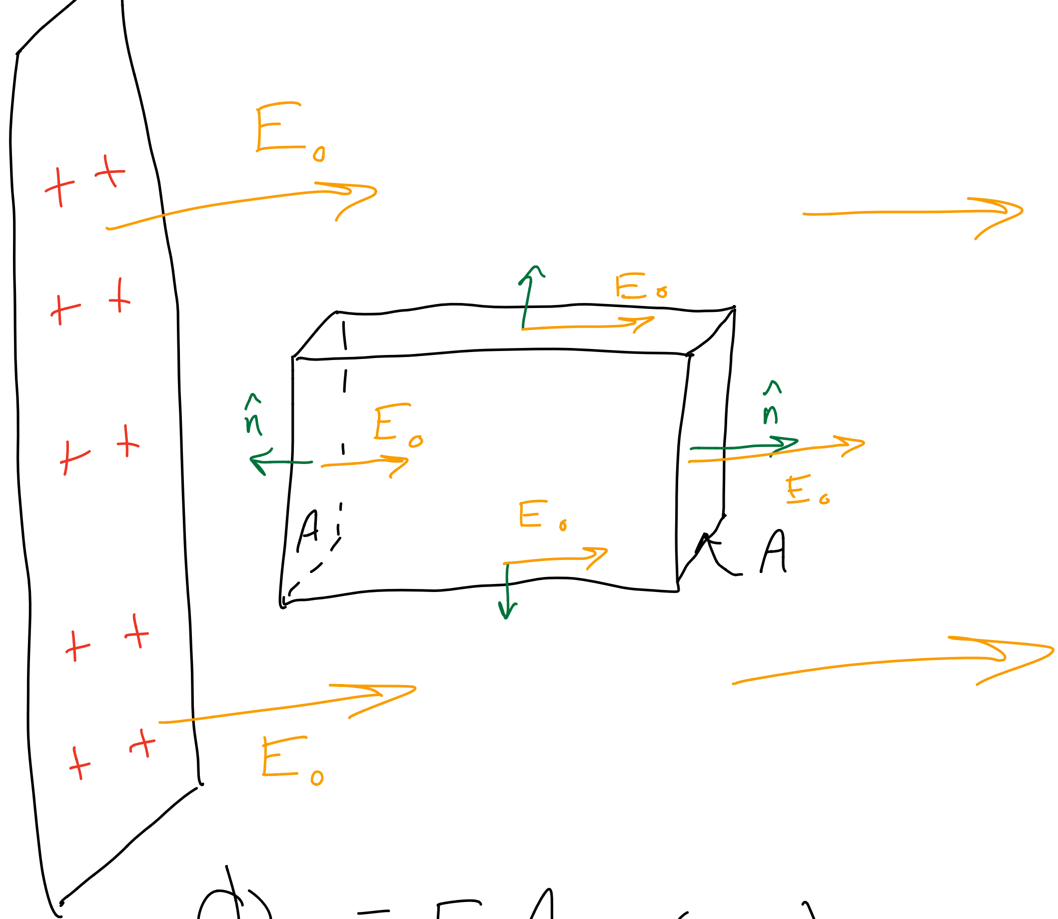
- Shape of surface doesn't matter





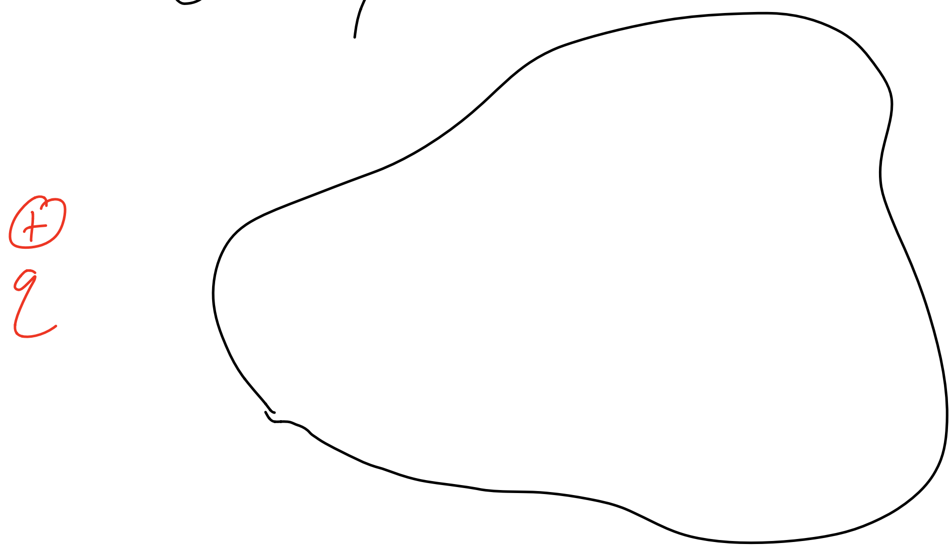


- Charges outside the surface contributes 0 flux



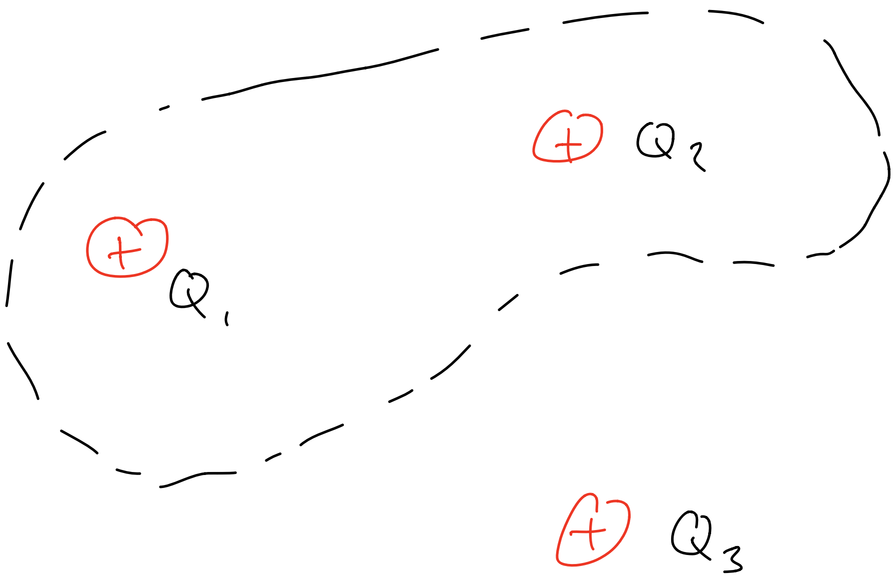
$$\Phi_E = E_0 A \cos(180) + E_0 A \cos(0) = 0$$

More generally:



$$\oint \vec{E} \cdot \hat{n}\, dA = 0 = \frac{q_{inside}}{\epsilon_0}$$

This applies for any number of charges



$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$$

$$Q_{inside} = Q_1 + Q_2$$

$$\oint \vec{E} \cdot \hat{n}\, dA = \frac{Q_{inside}}{\epsilon_0}$$

$$\oint (\vec{E}_1 + \vec{E}_2 + \vec{E}_3) \cdot \hat{n}\, dA = \frac{Q_1 + Q_2}{\epsilon_0}$$

E: net field from all charges, inside OR out

Q_{inside}: Just the charge inside the surface