CHAPTER 19

MACROSCOPIC CIRCUIT ANALYSIS

OVERVIEW

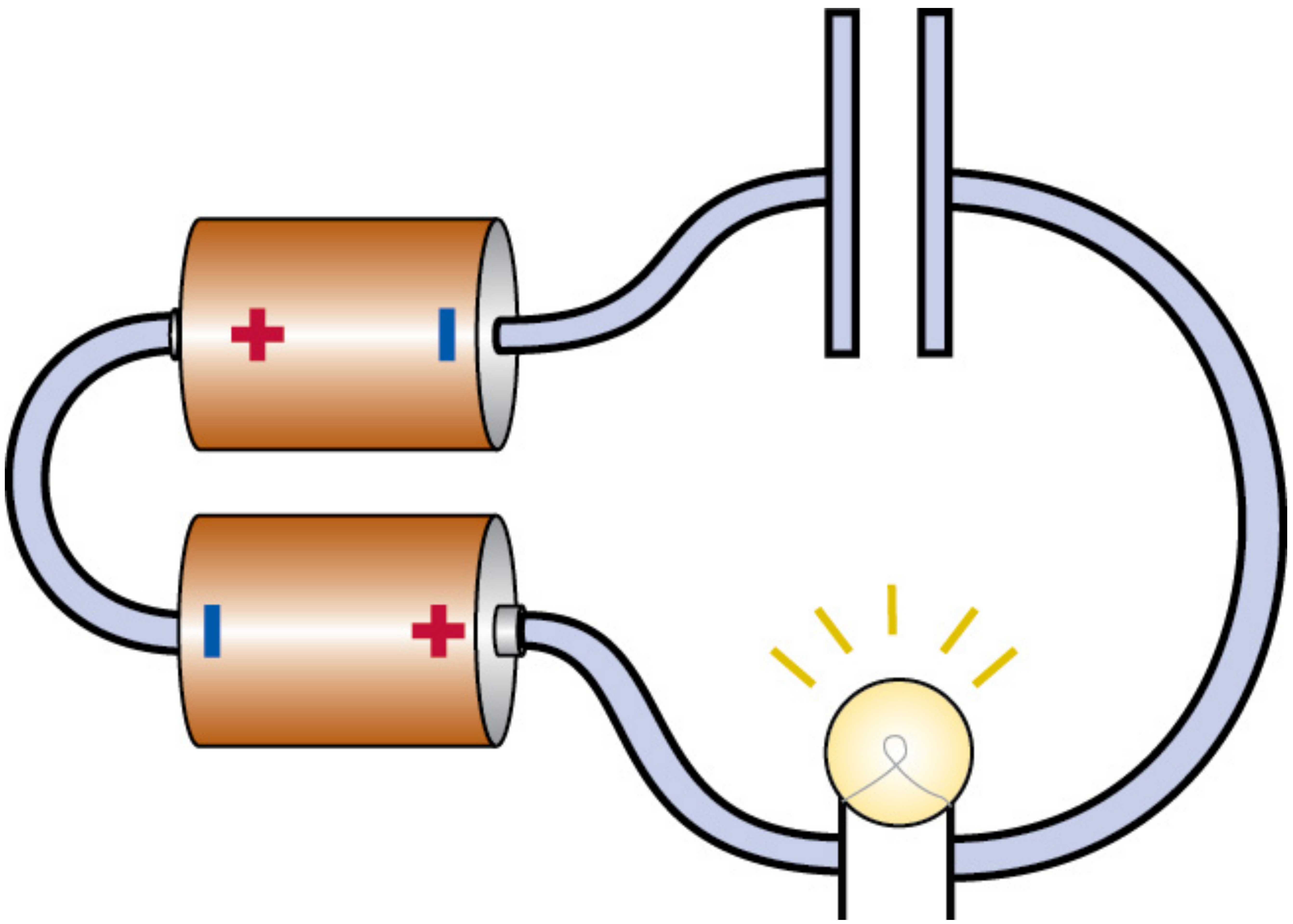
- Last chapter, we developed a qualitative sense of the microscopic behavior of a circuit in terms of fundamental principles
  - Where does the field come from?
  - What is the function of a battery?
  - What is a resistor?
  - Charge & energy conservation

OVERVIEW

- In this chapter, we will apply this understanding to understand circuits *macroscopically*
  - How do circuit elements behave within a circuit?
    - Capacitors & resistors
  - Calculate ΔV across different circuit elements
  - Calculate conventional current I in every part of a circuit

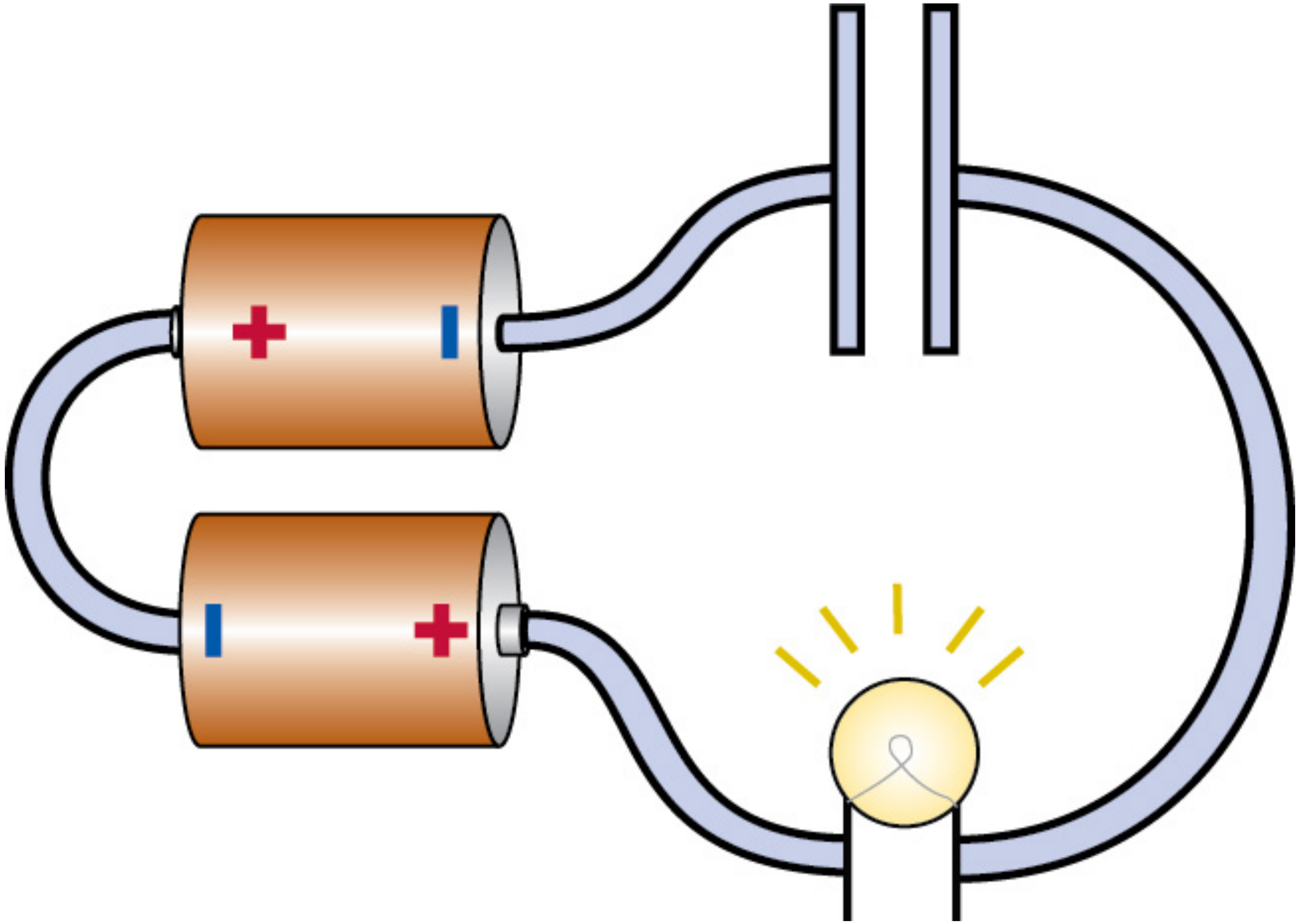
CONSIDER THIS CIRCUIT

- Assume it is initially disconnected



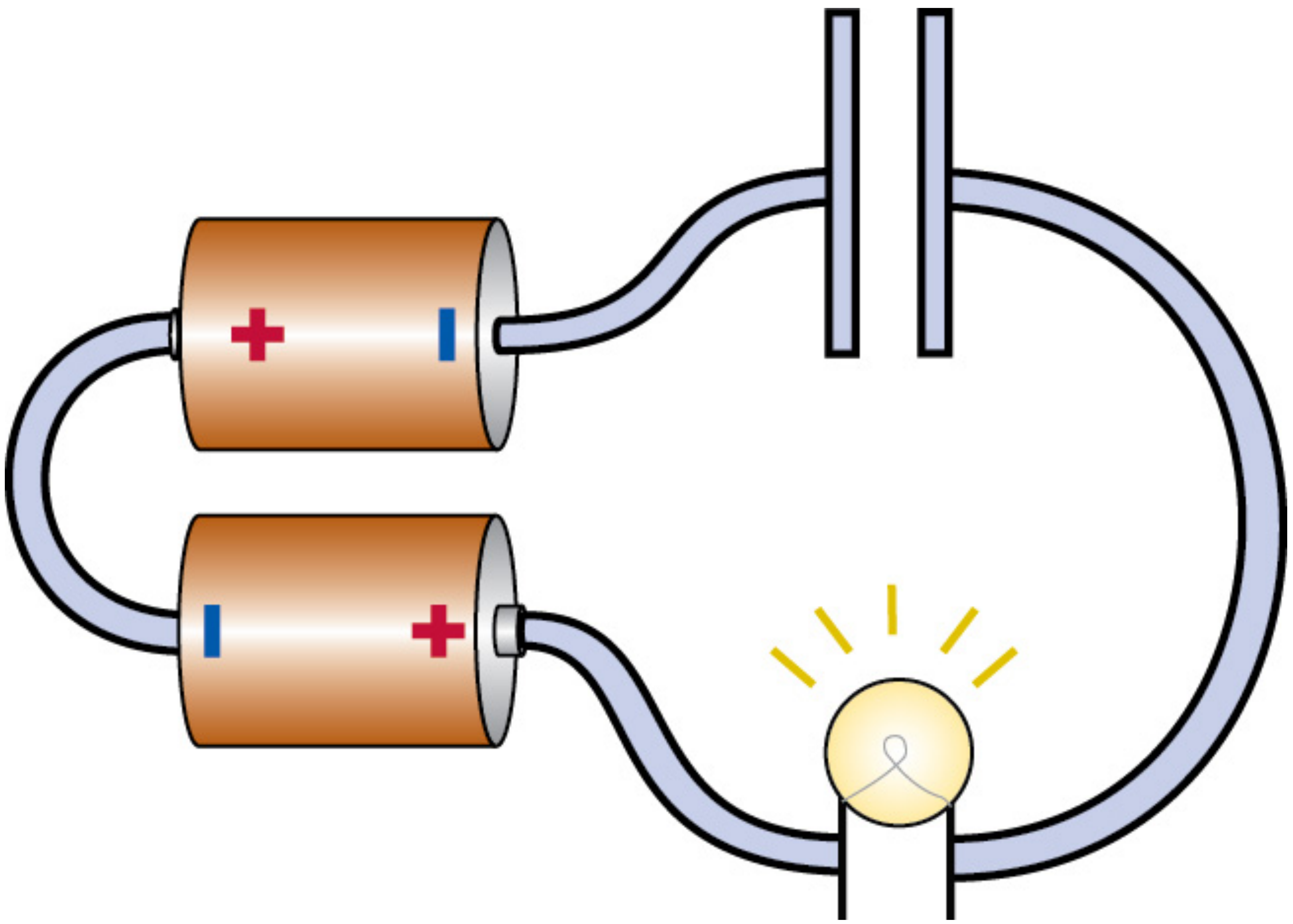
CONSIDER THIS CIRCUIT

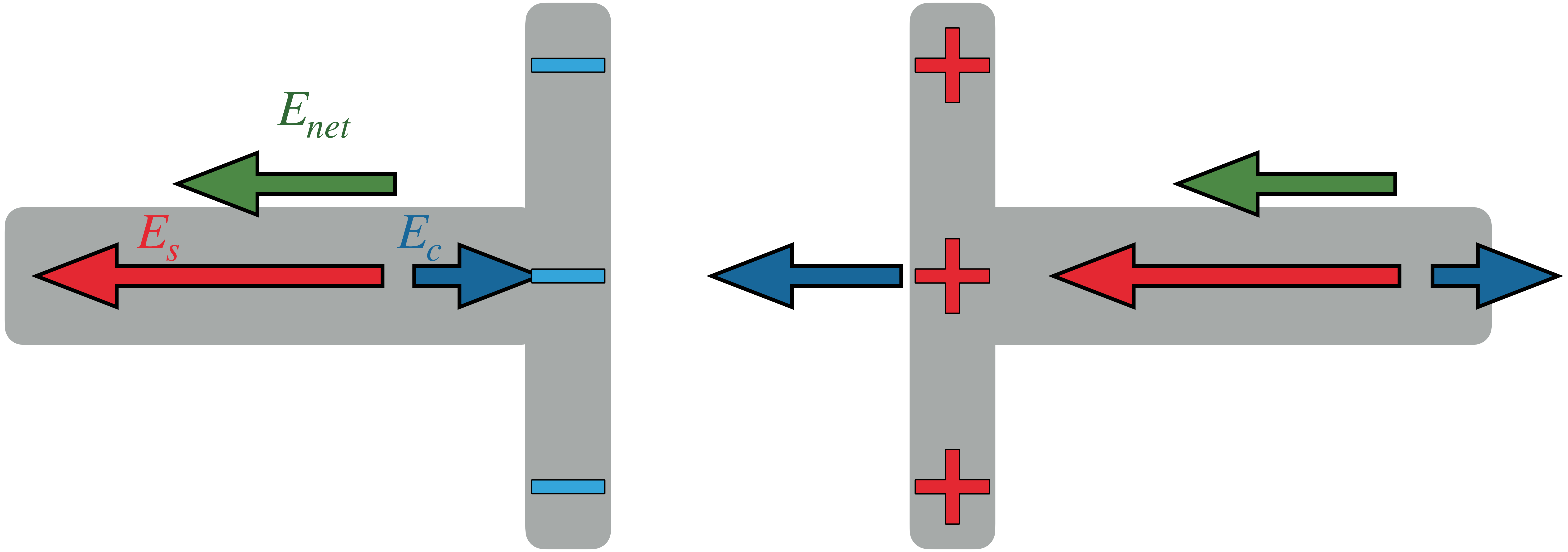
- Assume it is initially disconnected
- What happens when I connect the circuit?



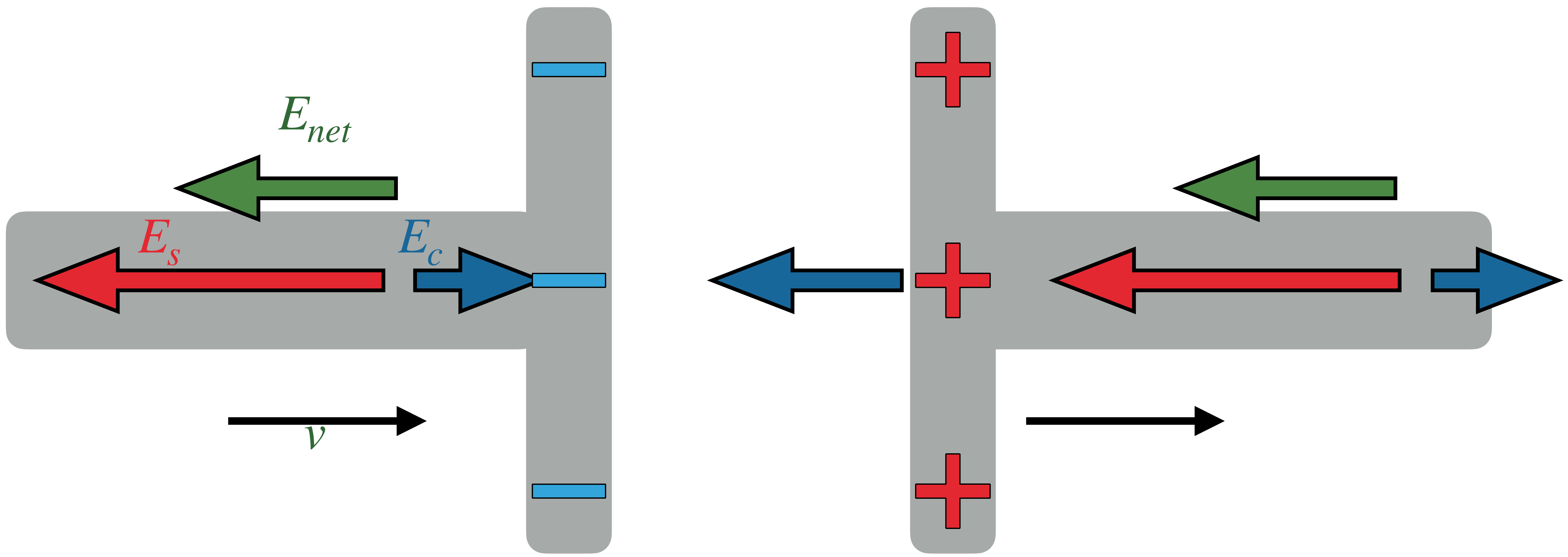
CONSIDER THIS CIRCUIT

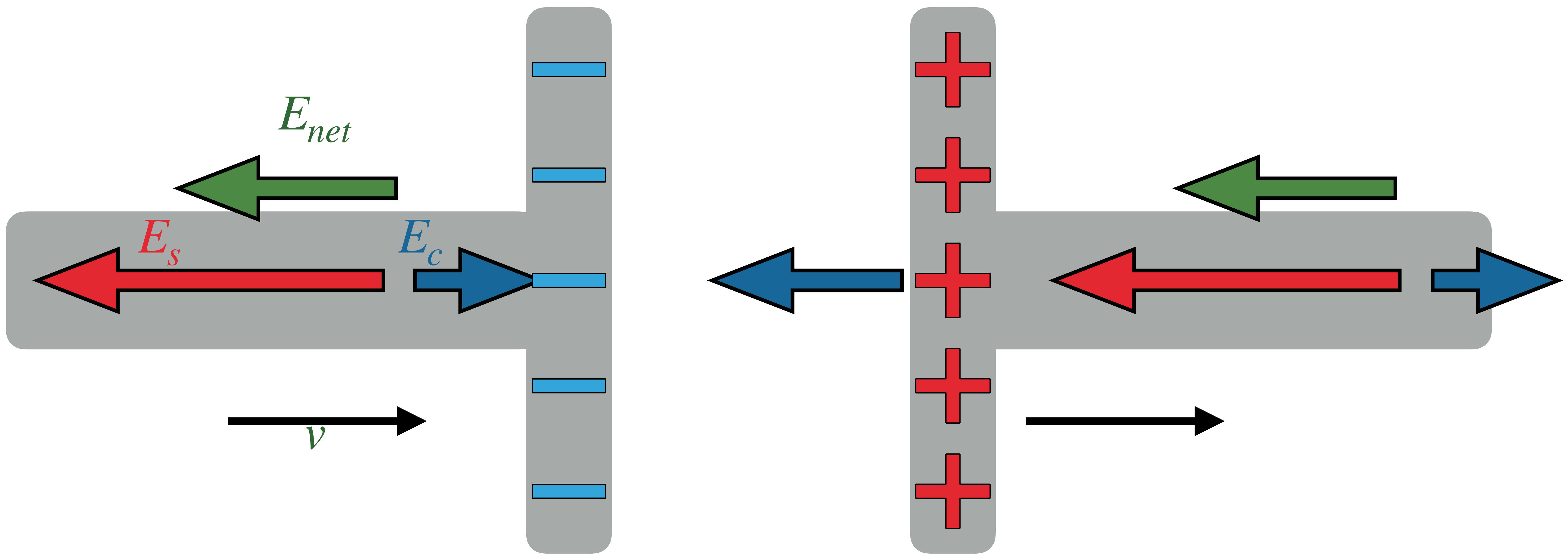
- Assume it is initially disconnected
- What happens when I connect the circuit?
- Why?



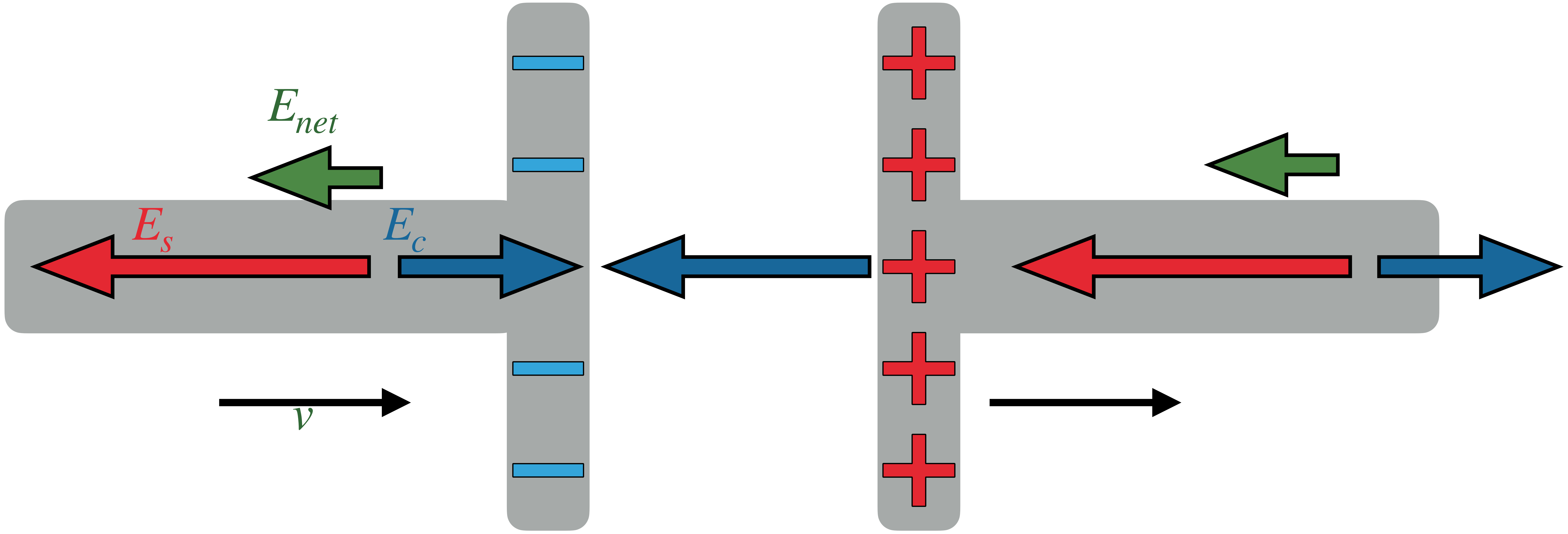


E_{net}
E_s
E_c

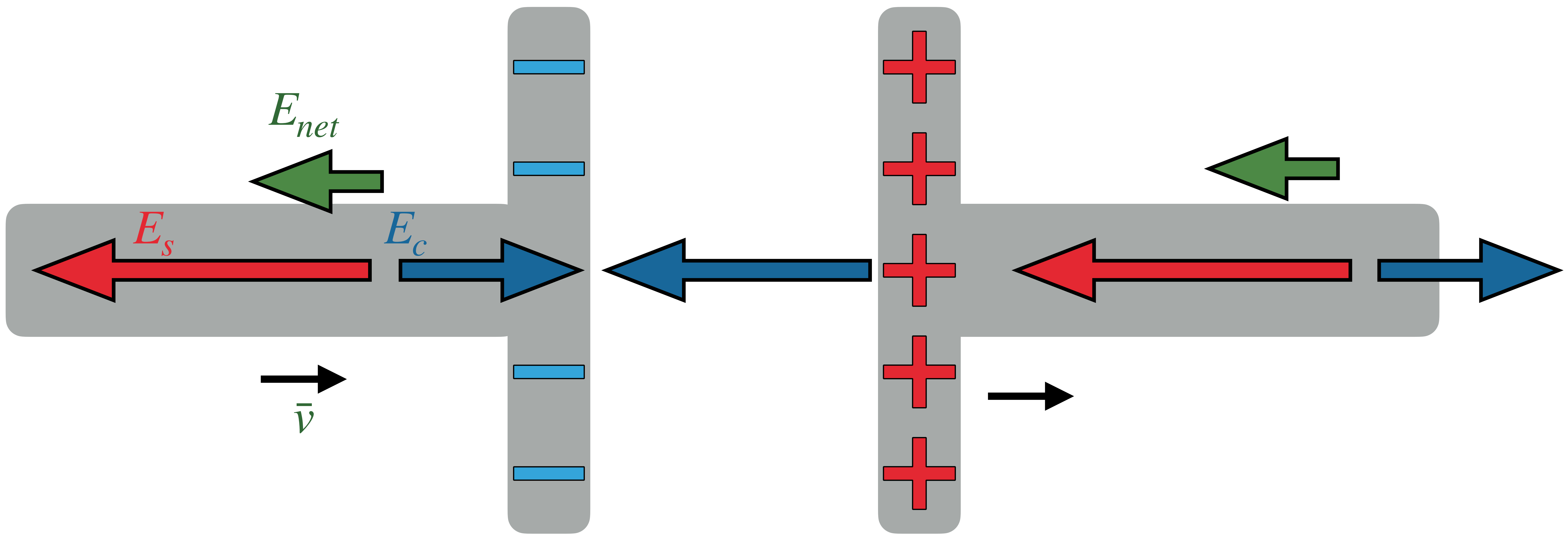

E_{net}
E_s
E_c
v



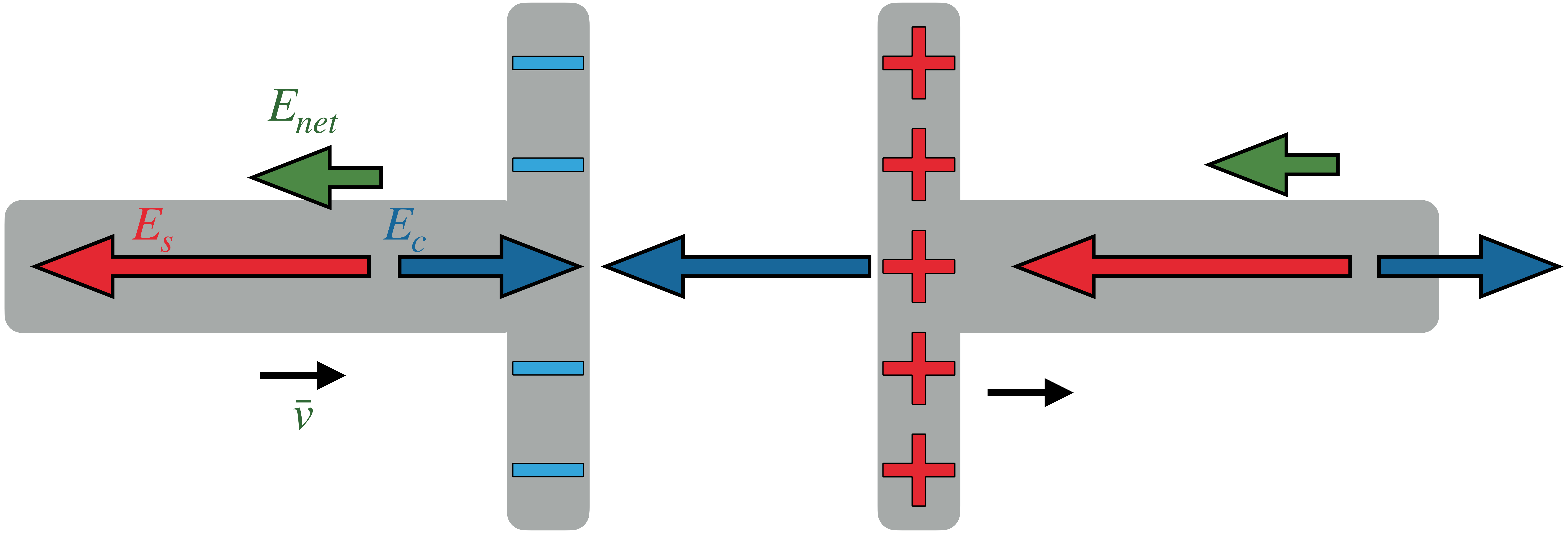
E_{net}
E_s
E_c
v



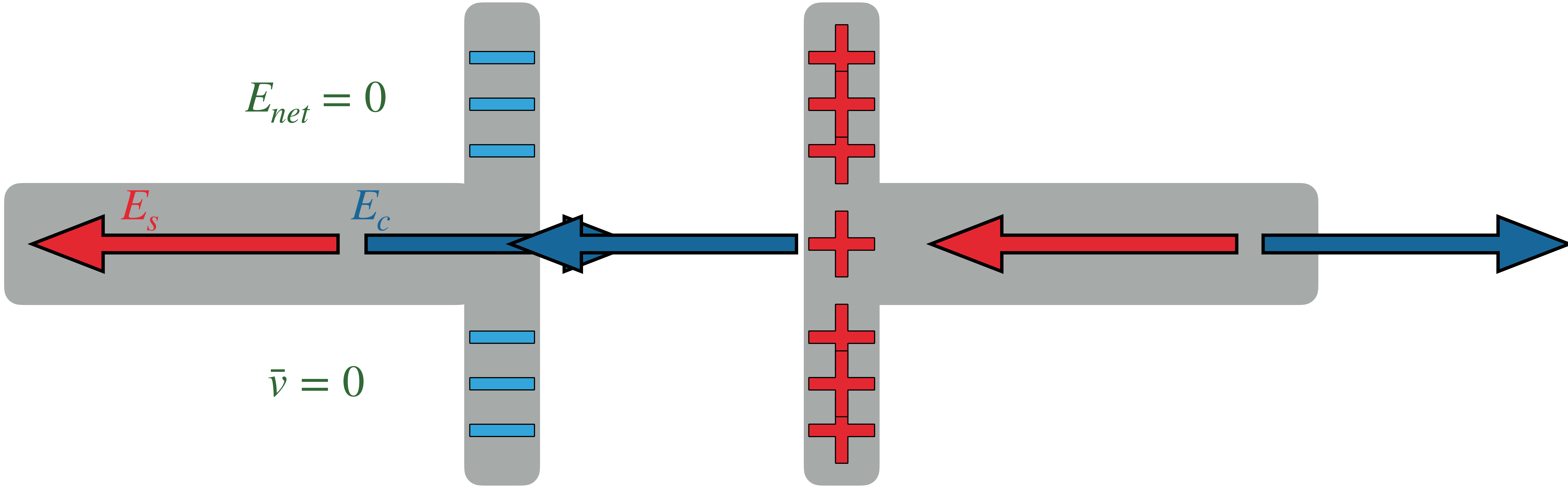
E_{net}
E_s
E_c
v


E_{net}
E_s
E_c
$\bar{v}$

- Capacitor continues charging until ______?

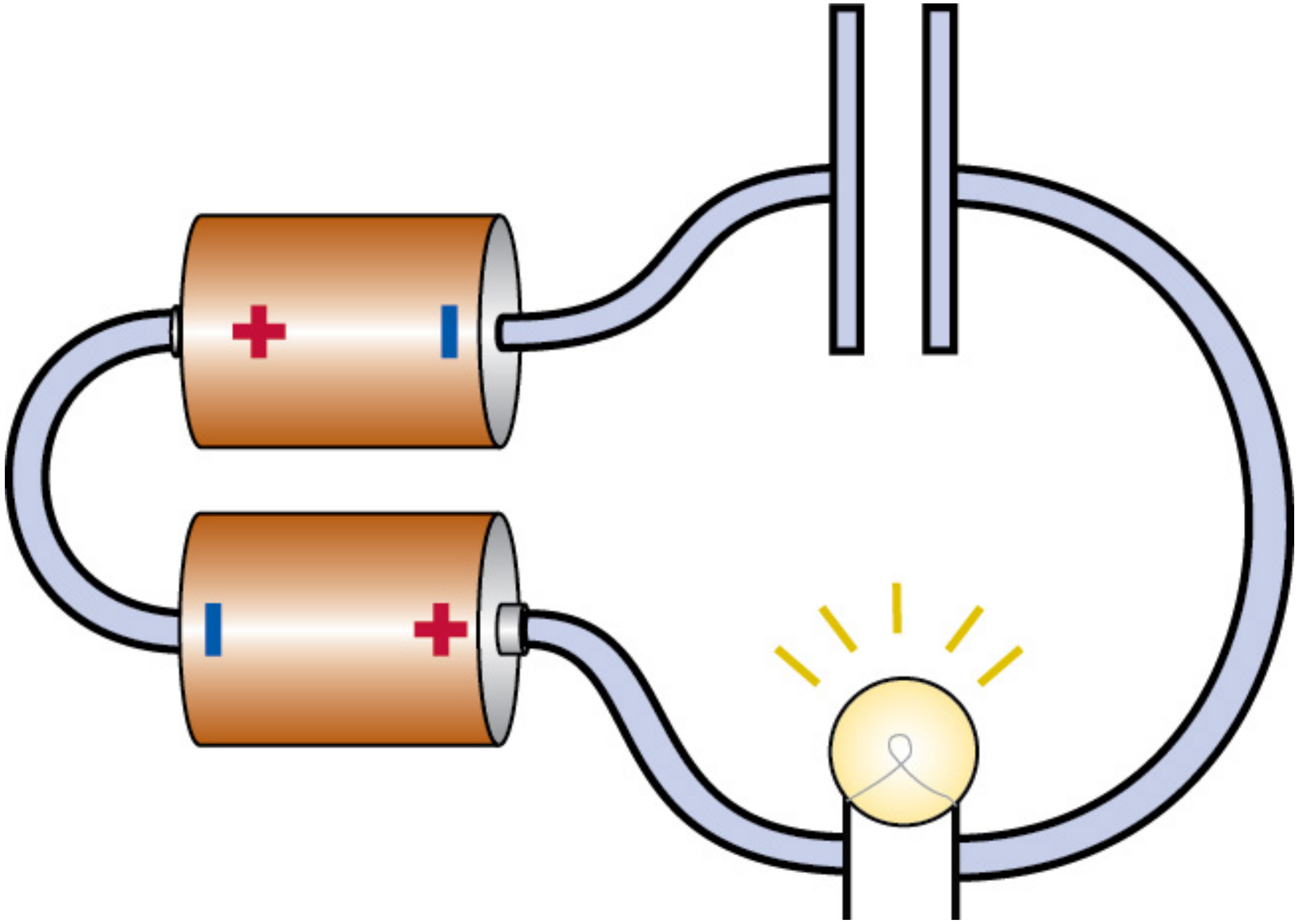


- Capacitor continues charging until $E_{net} = 0$



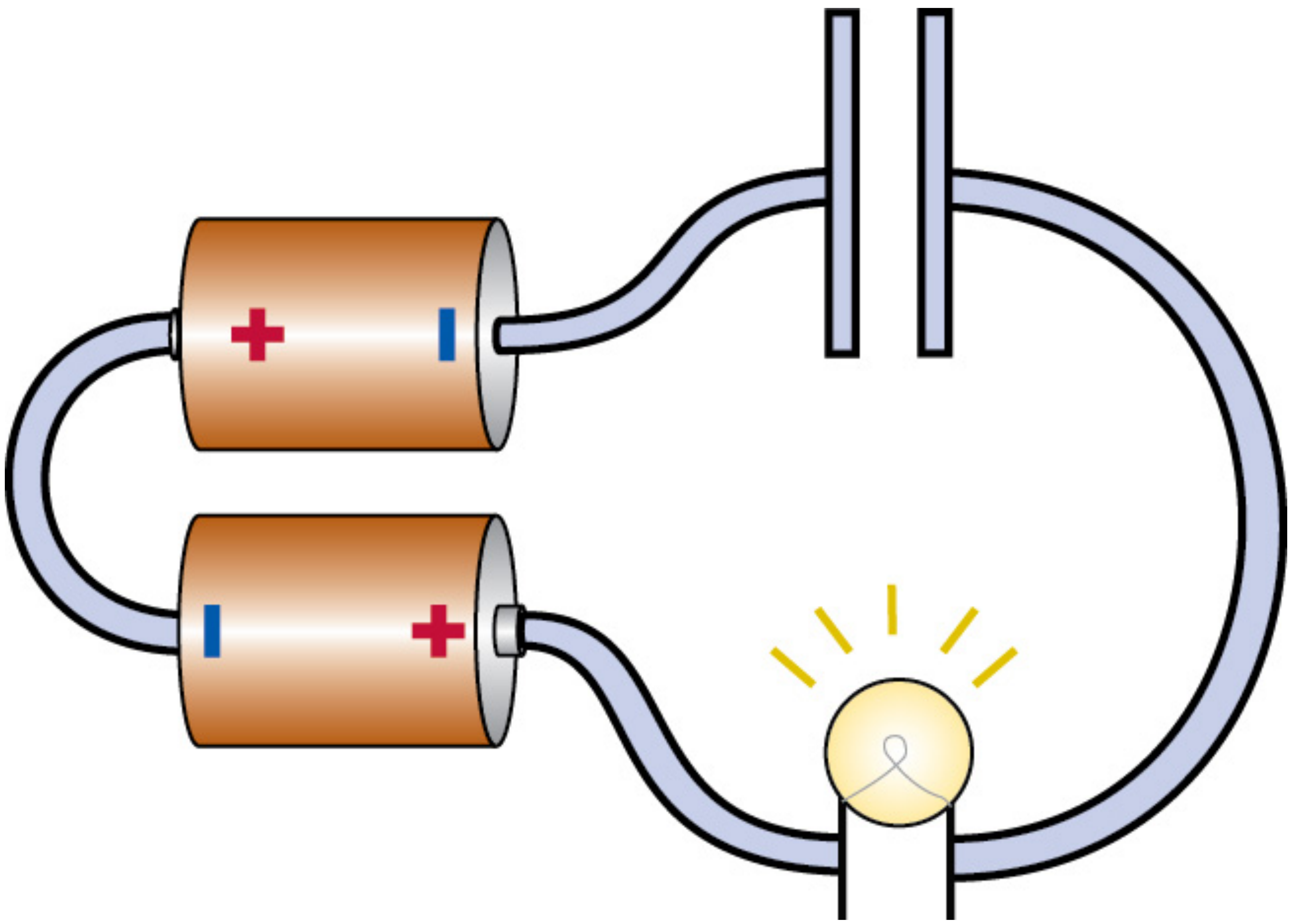
CAPACITOR CHARGING CIRCUIT

- Capacitor is initially "ignored"
- Charge accumulates on the capacitor, decreases current
    - Battery power diverted to capacitor, bulb dims



CAPACITOR CHARGING CIRCUIT

- Charge continues to accumulate until fringe field of capacitor cancels field in wire
- Current stops flowing
    - Bulb is completely dark



CAPACITORS IN CIRCUITS

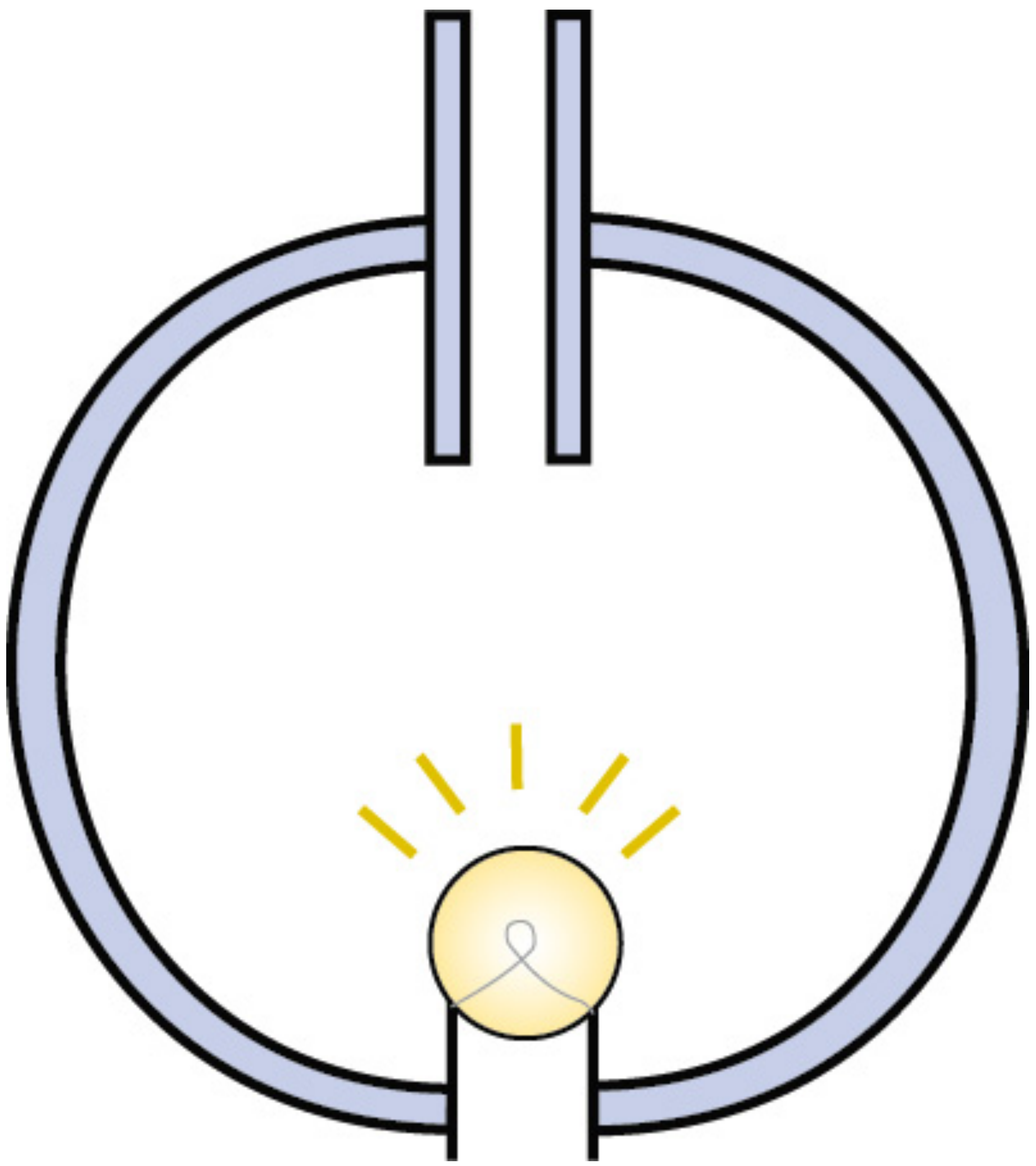
Charge on the capacitor: $Q = C\Delta V_c$

C is the **capacitance** of the capacitor (how much charge can it hold?)

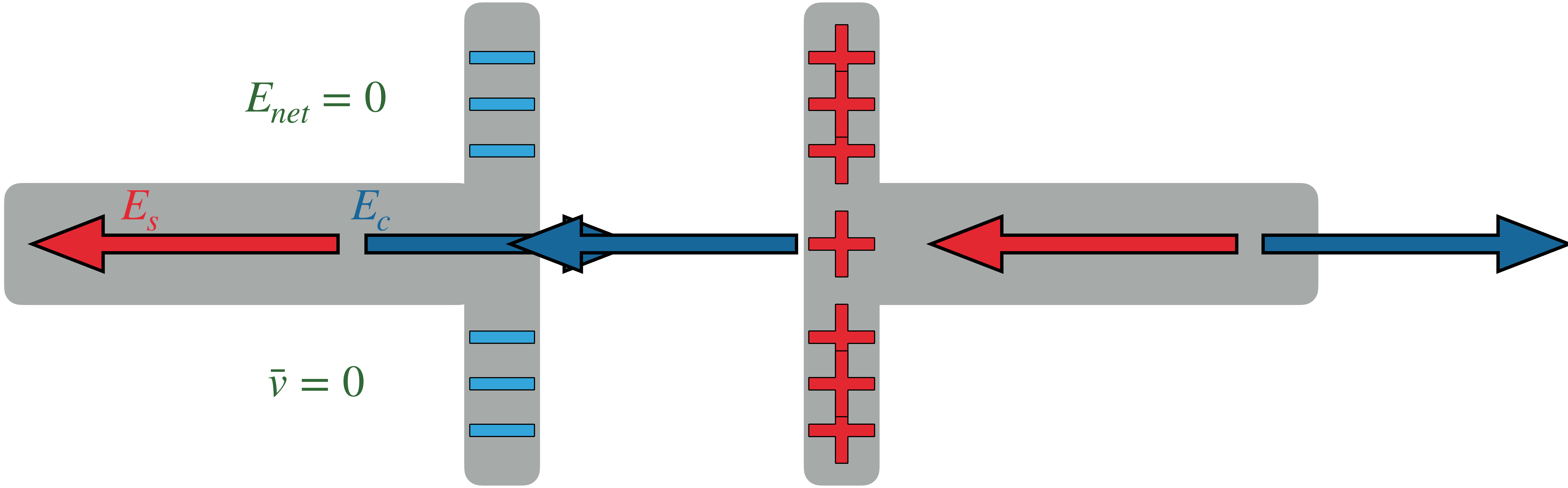
- For same ΔV_c , higher C means more charge stored on the capacitor

CAPACITORS IN CIRCUITS

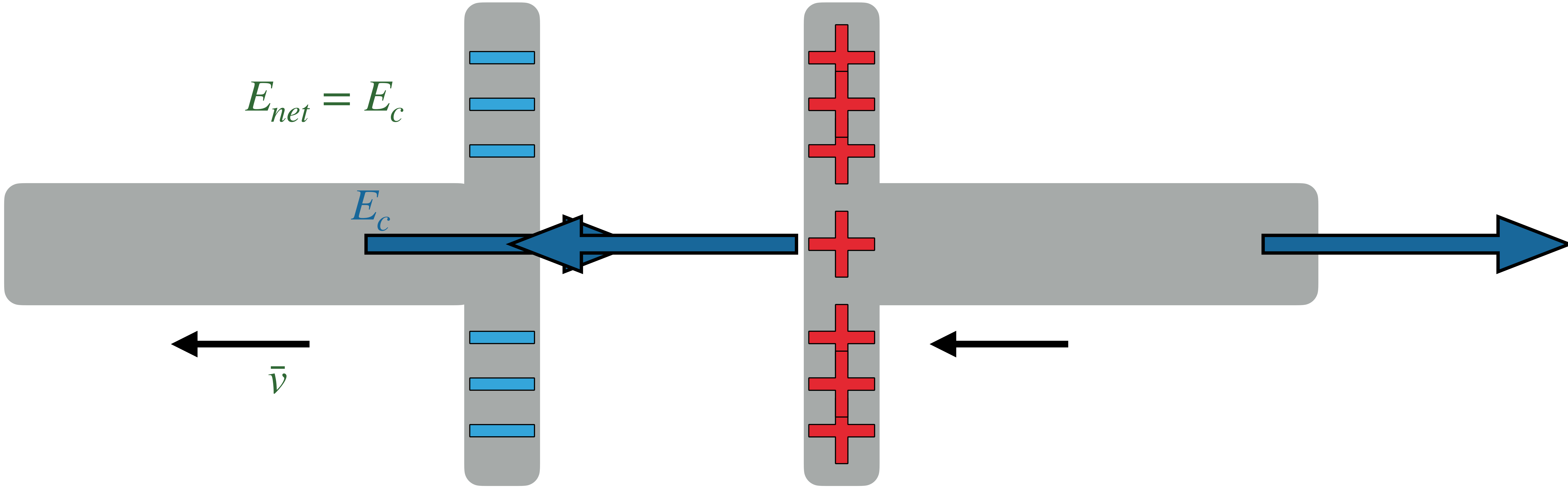
Full charged capacitor, remove the battery



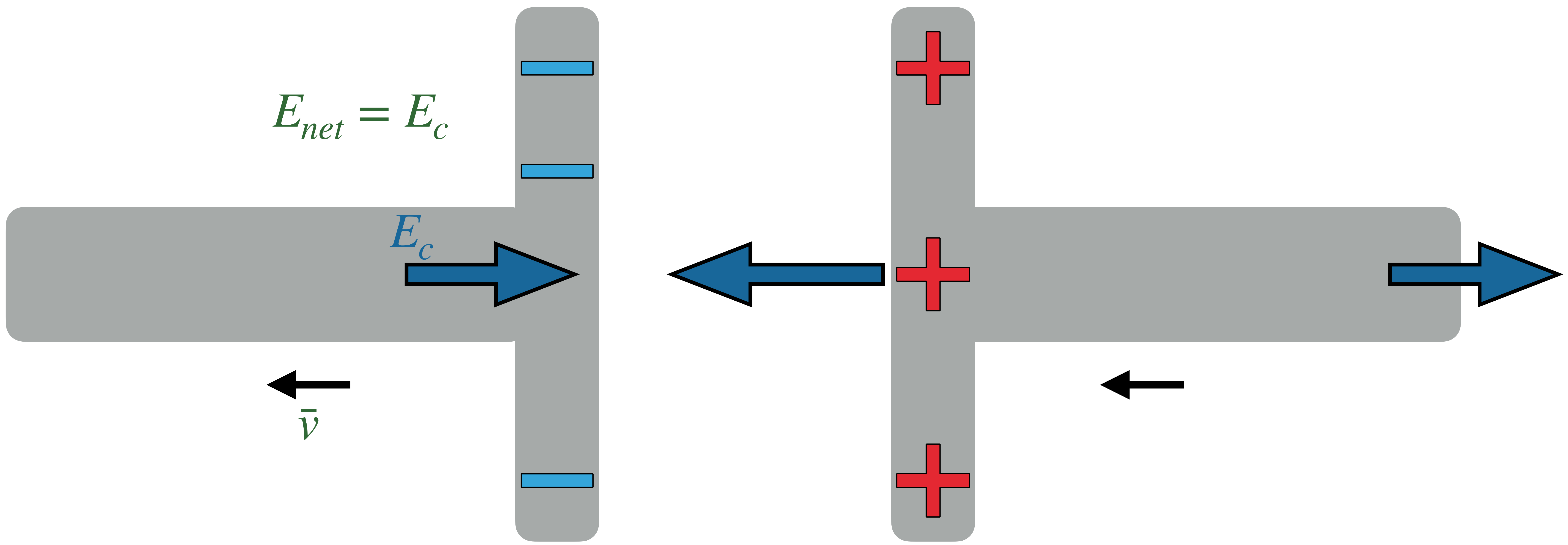
- Fully charged capacitor with battery connected



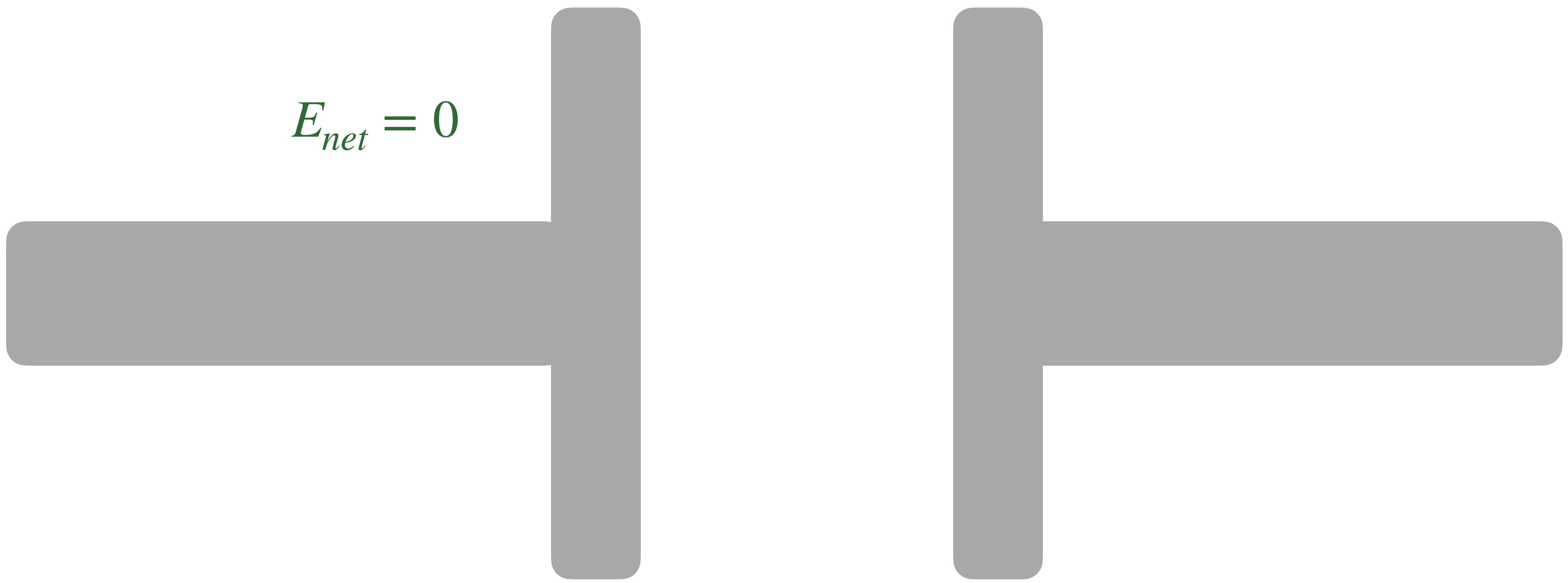
- Now remove the battery
- Charge leaves capacitor (current flows)
- Bulb lights up brightly



- Charge leaves capacitor –> E_c weakens
- Current weakens, bulb dims



- Eventually, capacitor is completely discharged
- $E_{net} = 0$, no more current
    - Bulb is off



CAPACITORS IN CIRCUITS

<u>Uncharged capacitor connected to a battery:</u>

- Initially acts like a piece of wire (no effect on the circuit)
- Builds up charge until it completely stops current flow (acts like an open wire)
- Fully charged capacitor: $Q = C\Delta V_c = C\varepsilon$

CAPACITORS IN CIRCUITS

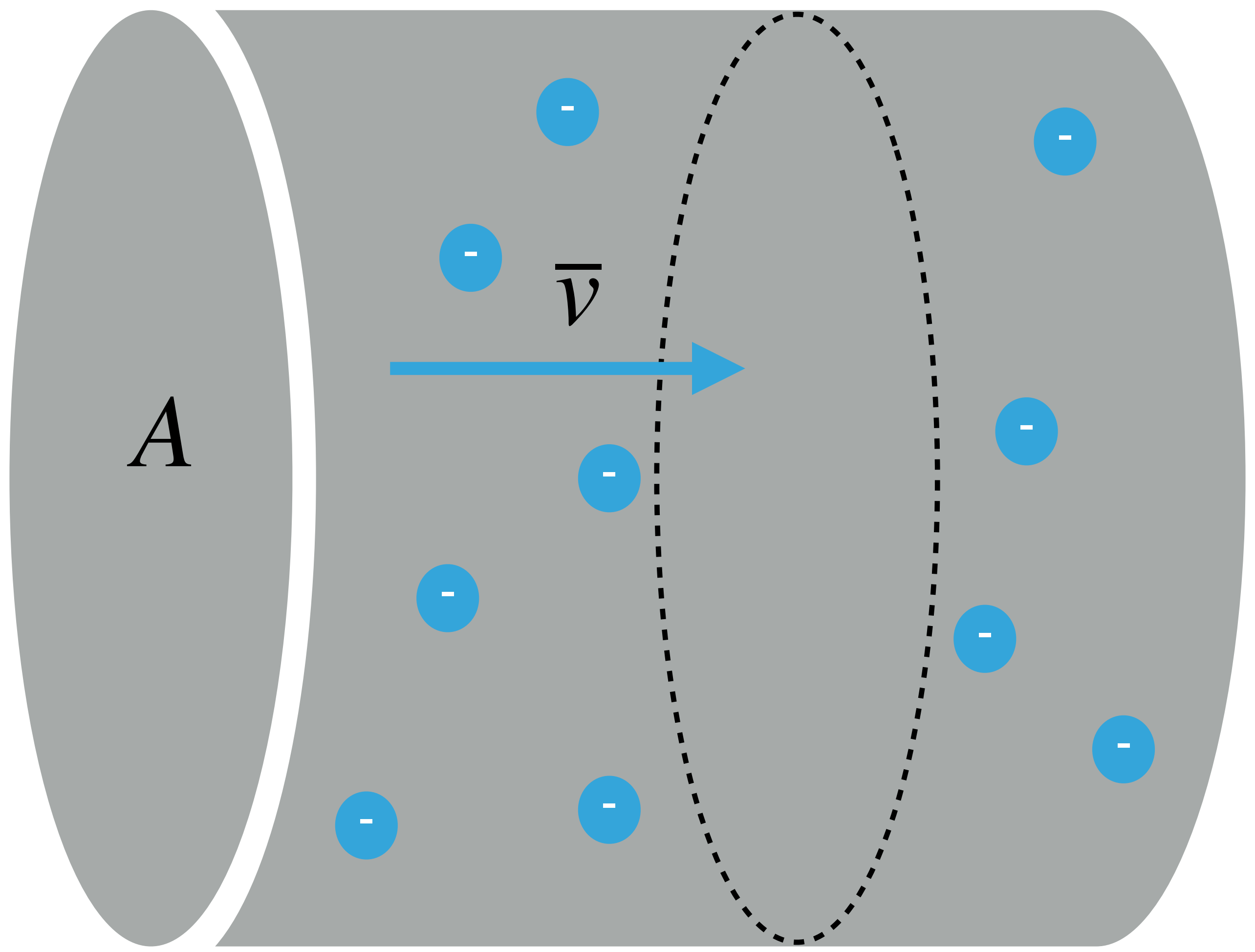
Fully capacitor disconnected from battery

- Initially acts like a battery with "emf" of $\Delta V = Q/C$
- ΔV drives current through circuit
- Capacitor slow discharges, decreasing ΔV, until current flow stops

MICRO VS MACRO

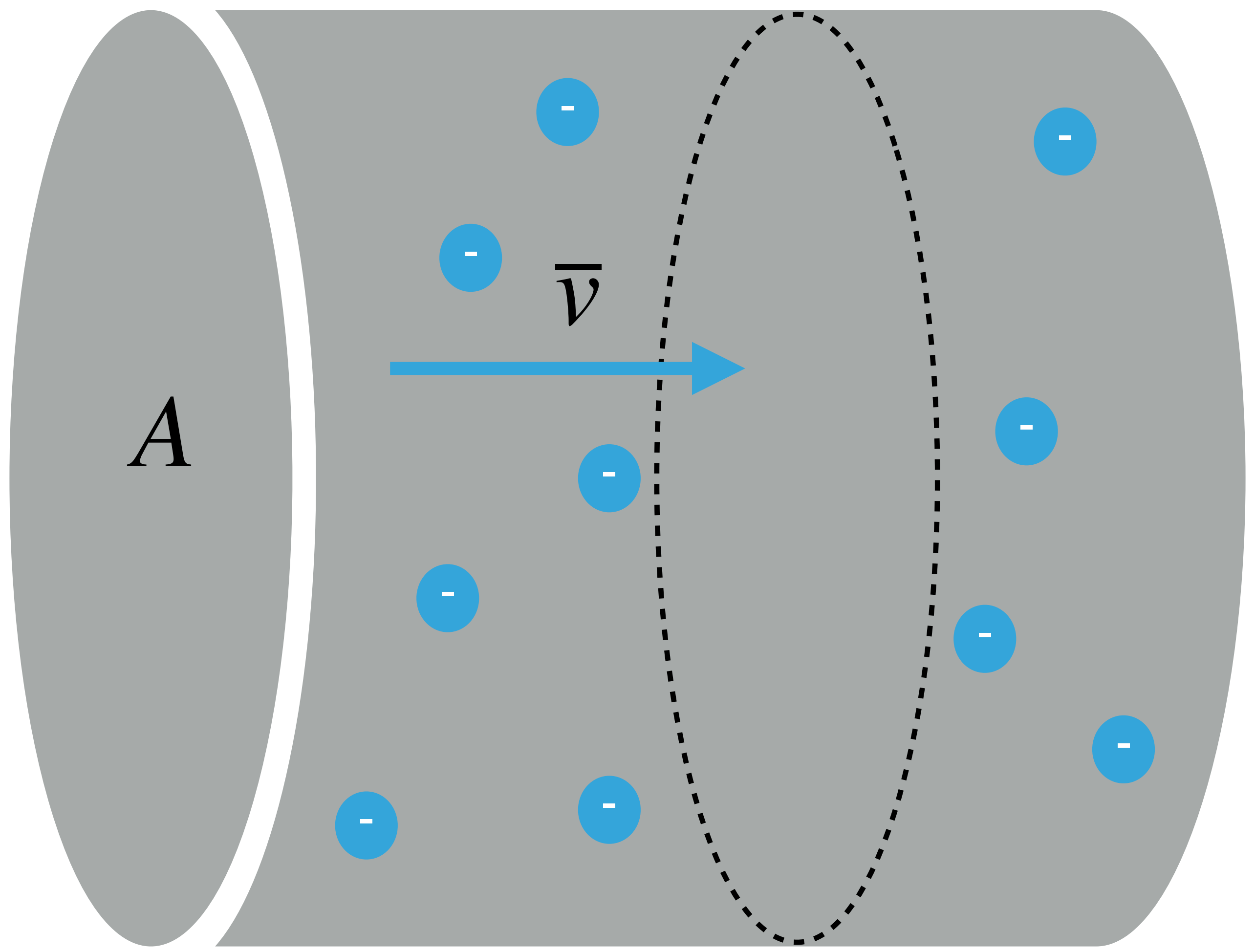
- In chapter 18, we learned how to describe circuits in terms of their atomic-level properties: i, n, u, E, etc
- These quantities are more “fundamental”, but hard to work with in practice (hard to measure!)
- Easier to work with conventional current I, potential difference ΔV, etc

MICRO VS MACRO



MICRO VS MACRO

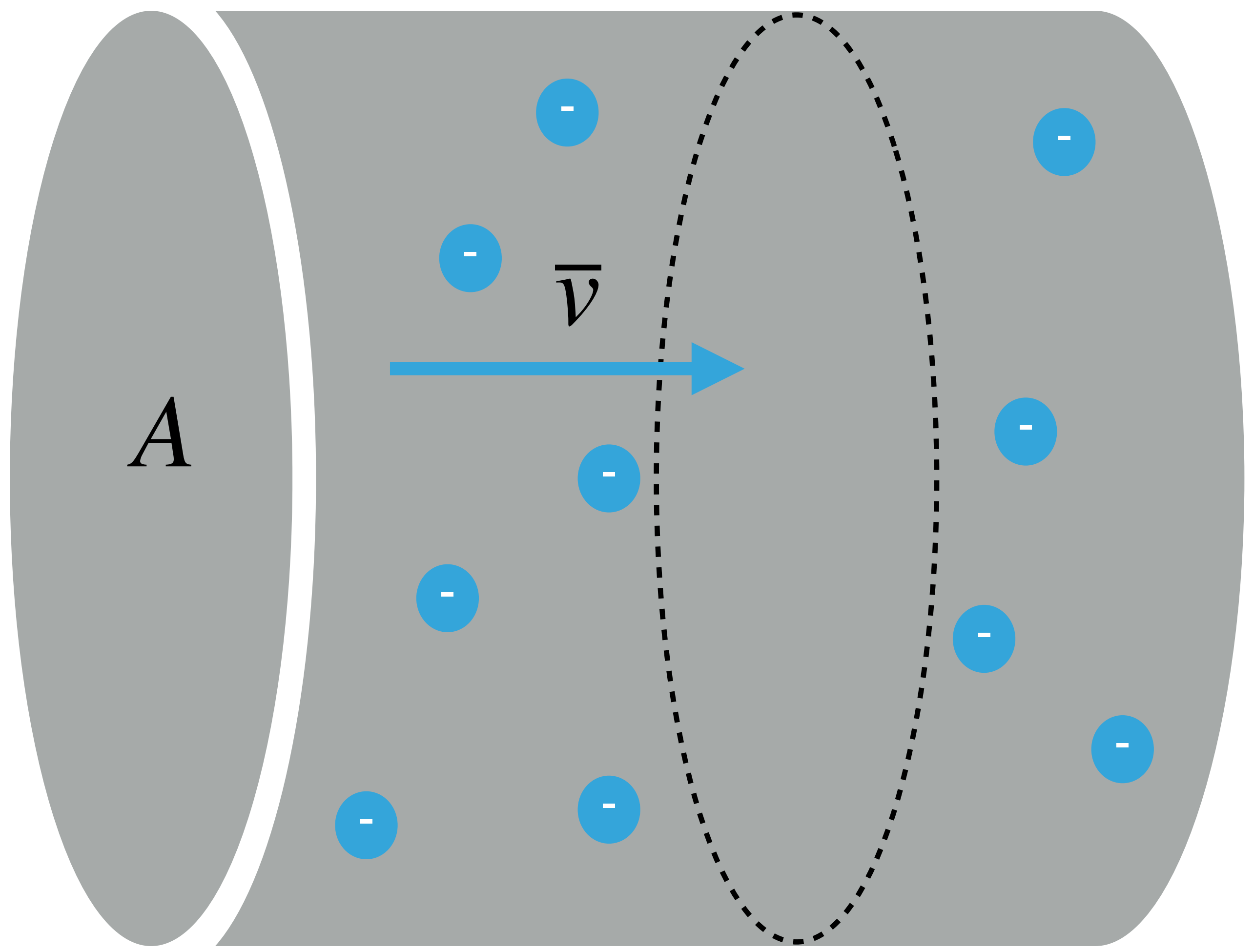
$$J = |q| nuE$$



MICRO VS MACRO

$$J = |q| nuE$$

	$\lvert q \rvert$	n	u	$\lvert q \rvert nu$
Copper	1.6×10^{-19} C	8.5×10^{28} m^{-3}	$4.5 \times 10^{-3} \dfrac{\text{m/s}}{\text{N/C}}$	$6.1 \times 10^{7} \dfrac{\text{C}^2}{\text{m}^2\text{Ns}}$
Tungsten	1.6×10^{-19} C	6×10^{28} m^{-3}	$1.8 \times 10^{-3} \dfrac{\text{m/s}}{\text{N/C}}$	$1.7 \times 10^{7} \dfrac{\text{C}^2}{\text{m}^2\text{Ns}}$
etc				



ELECTRICAL CONDUCTIVITY

$$\sigma = \left| q \right| nu$$

- σ is called the "electrical conductivity"
    - Lumps together all relevant properties of the material

ELECTRICAL CONDUCTIVITY

$$\sigma = |q|\, nu$$

- σ is called the "electrical conductivity"
    - Lumps together all relevant properties of the material
- Higher σ $\implies$ less field needed for same current

CONSIDER

Length L of wire with cross-sectional area A and conductivity σ

What is $\left|\Delta V\right|$ across the wire?

ELECTRICAL RESISTANCE

"Resistance" $R = \dfrac{L}{\sigma A}$

- Combines geometry and inherent conductivity
- Units: The "ohm" (Ω)

OHM'S LAW

$$I = \frac{\Delta V}{R}$$

- Electric potential difference causes charges to move (current)
- Higher $\Delta V \implies$ more charge motion $\implies$ higher current
- Higher R (low conductivity, fewer free electrons in wire, more collisions with atomic nuclei, etc) $\implies$ lower current

EXAMPLE

A resistor is made of nichrome wire (made of nickel, iron, and chromium) which has a cross-sectional area of $80\ \mu\text{m}^2$.

- How long of a wire do you need to construct a $220\ \Omega$ resistor?
- What is the current through this resistor when connected to a $9\ \text{V}$ battery?

$$\sigma_{\text{Nichrome}} = 0.10 \times 10^7\ \Omega^{-1}\text{m}^{-1}$$

EXAMPLE

The resistor is connected to the battery via a length of thick copper wire (cross-sectional area of $0.3\ \text{mm}^2$)

- How much copper wire do you need to use to match the resistance of the Nichrome resistor?
- $\sigma_{\text{Copper}} = 6.0 \times 10^7\ \Omega^{-1}\text{m}^{-1}$

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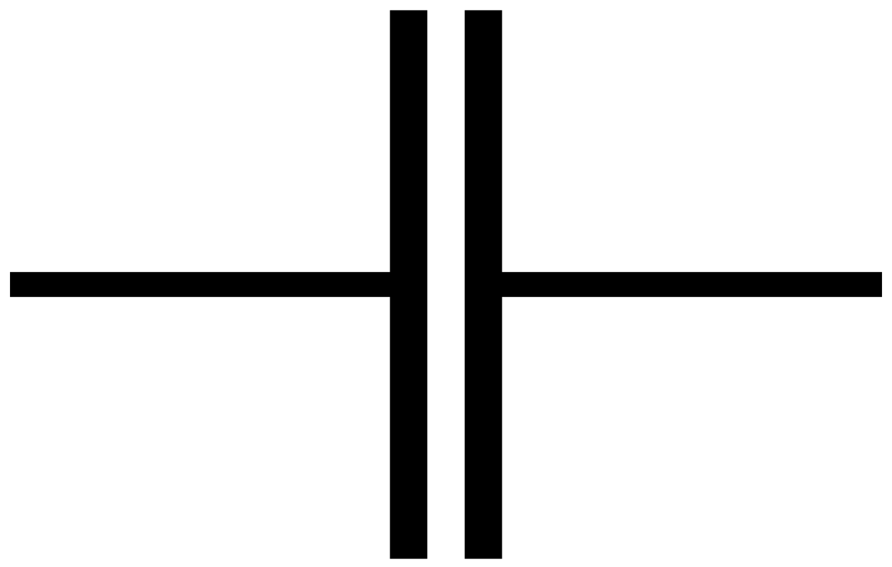
CONCLUSION? WE CAN SAFELY IGNORE CONNECTING WIRES IN CIRCUITS

CIRCUIT ELEMENTS

- For our purposes, a “circuit element” is structure placed in the circuit which uses (or supplies) significant energy (ΔV)
    - Capacitors
    - Resistors
    - Batteries

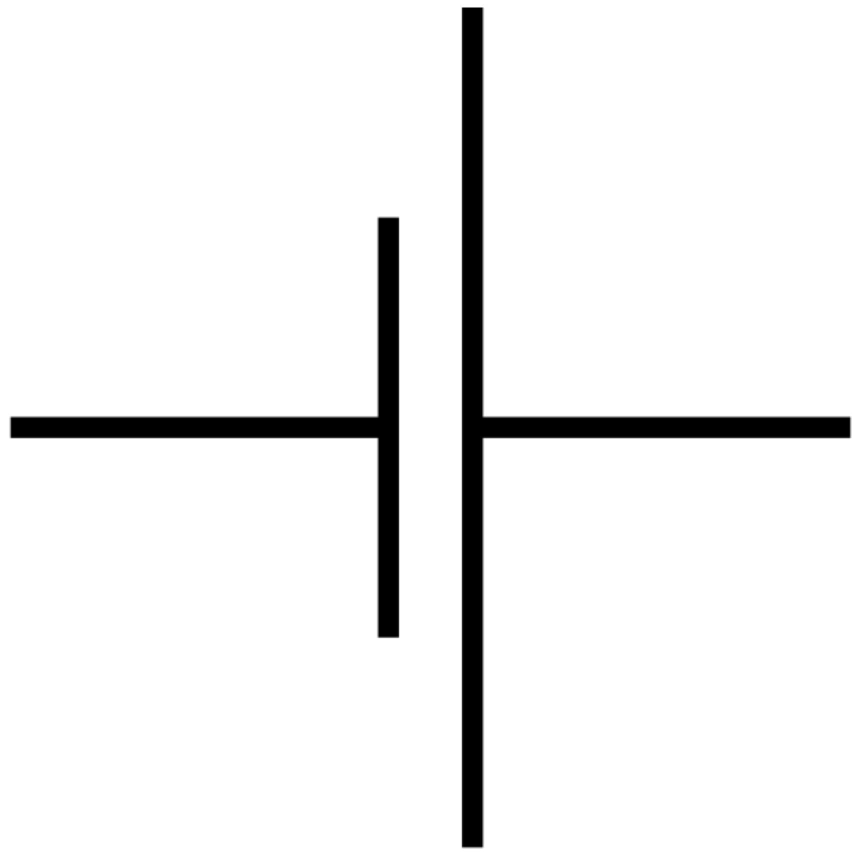
CIRCUIT DIAGRAMS

Capacitor



Resistor

Battery



CIRCUIT DIAGRAMS

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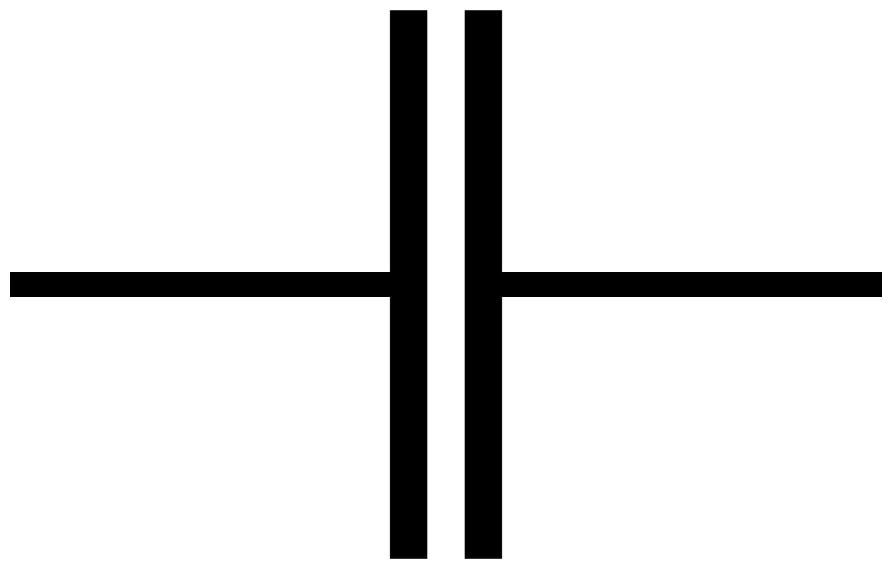
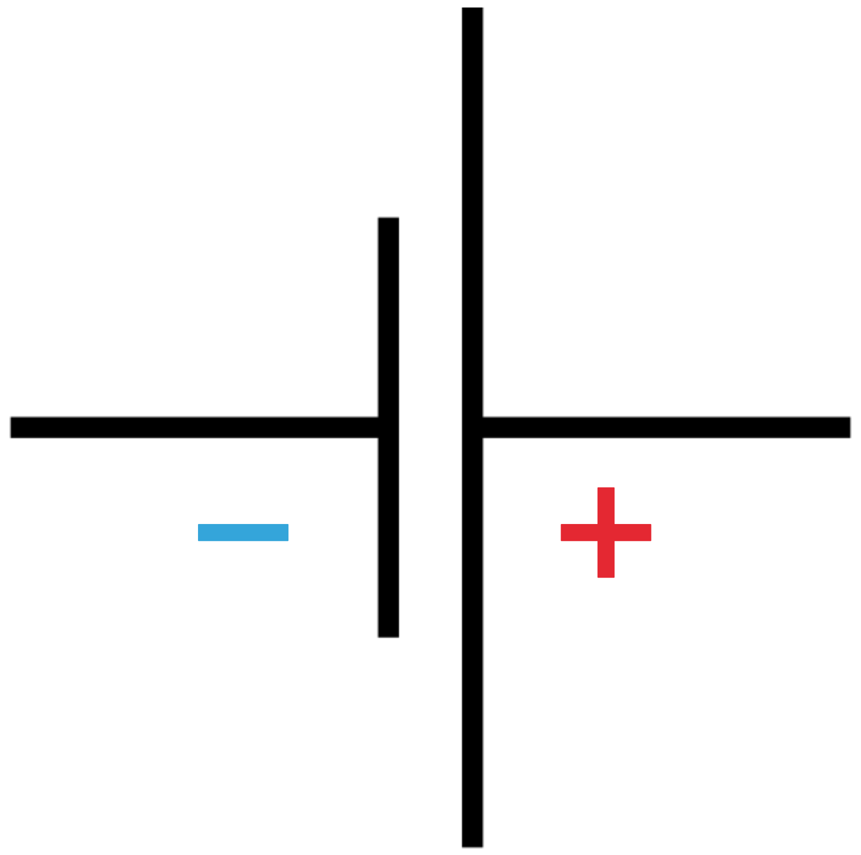


Resistor

Battery



THE MICRO-MACRO CONNECTION

Microscopic View

$$\overline{v} = uE$$

$$i = nA\overline{v} = nAuE$$

Macroscopic View

$$J = \sigma E$$

$$I = \left| q \right| nA\overline{v} = \frac{1}{R}\Delta V$$

THE MICRO-MACRO CONNECTION

Microscopic View

Node Rule:

$$\sum_{\text{node}} i_{\text{in}} = \sum_{\text{node}} i_{\text{out}}$$

Loop Rule:

$$\sum_{\text{loop}} \Delta V = 0$$

Macroscopic View

Node Rule:

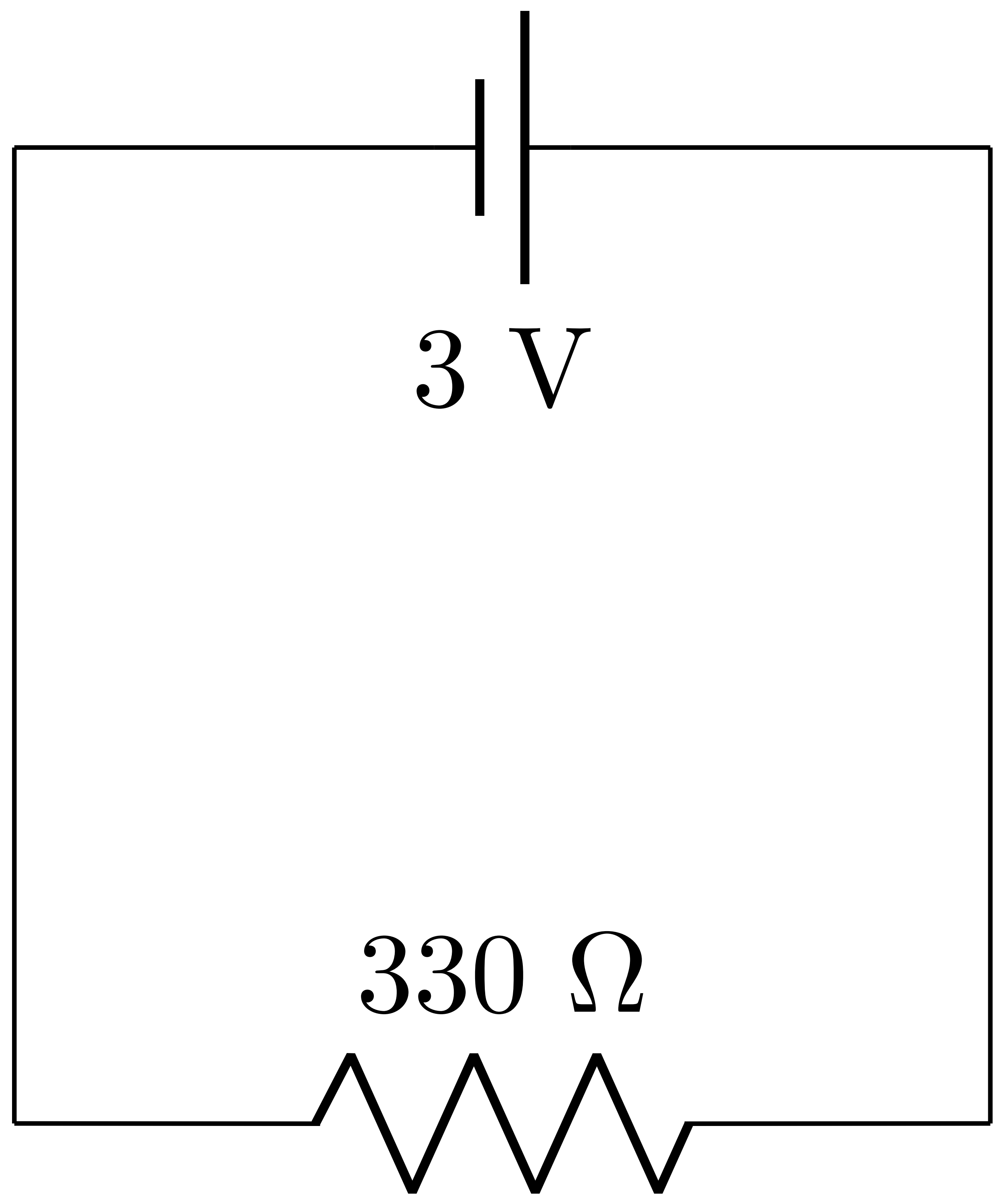
$$\sum_{\text{node}} I_{\text{in}} = \sum_{\text{node}} I_{\text{out}}$$

Loop Rule:

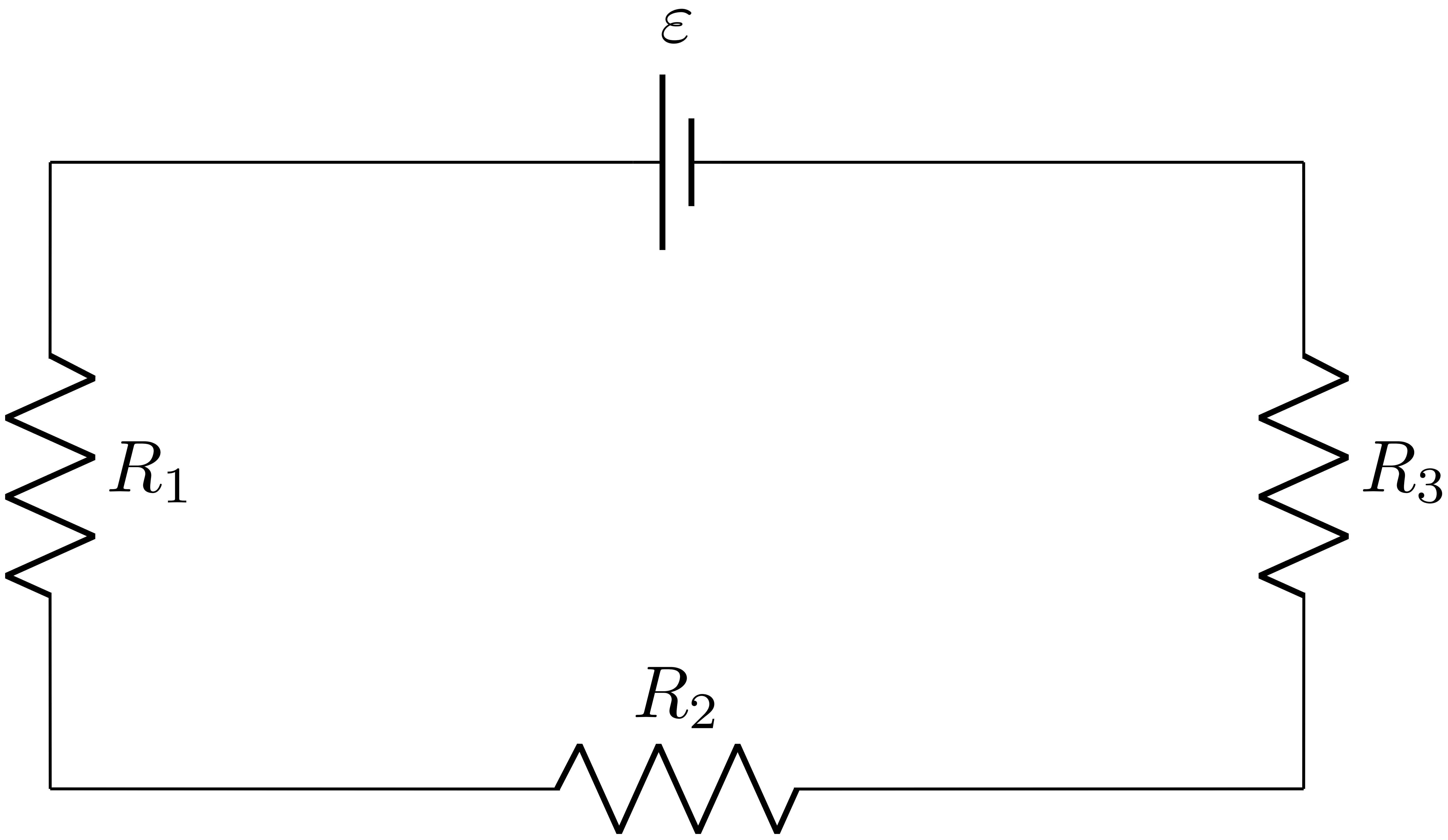
$$\sum_{\text{loop}} \Delta V = 0$$

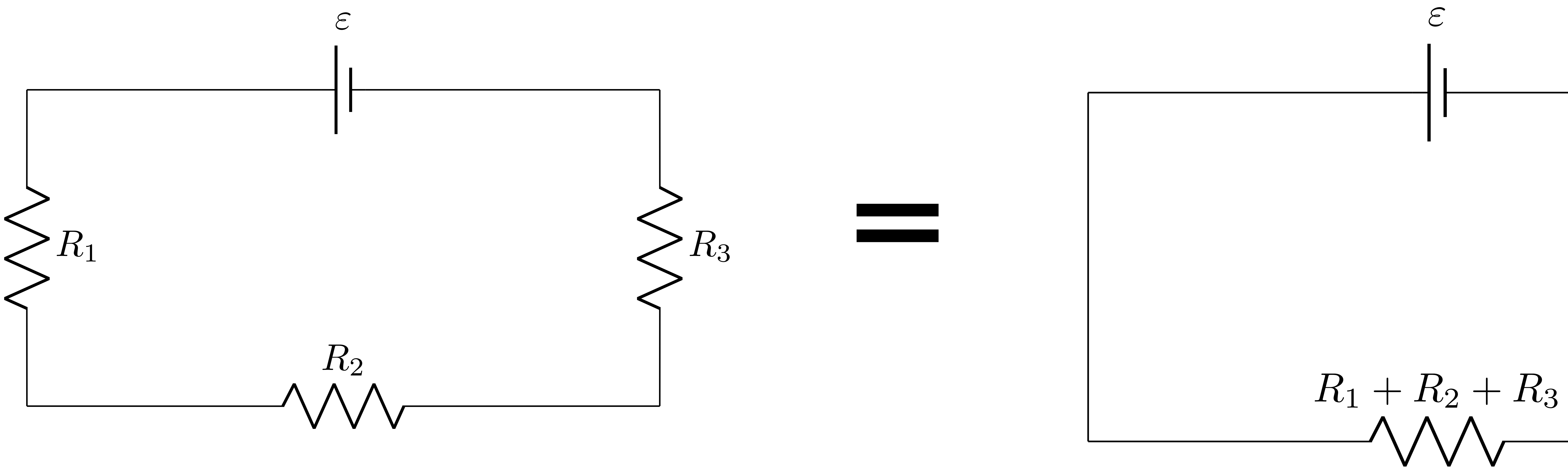
EXAMPLE

Current through and voltage across the resistor?



MULTIPLE RESISTORS IN SERIES





ε
R_1
R_3
R_2
=
ε
$R_1 + R_2 + R_3$

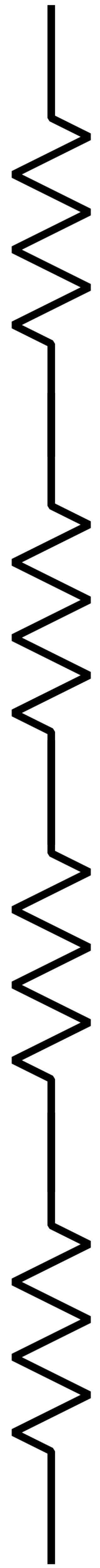
CIRCUIT ELEMENTS CONNECTED IN SERIES

$$I_1 = I_2 = I_3 = \ldots = I_n = I$$

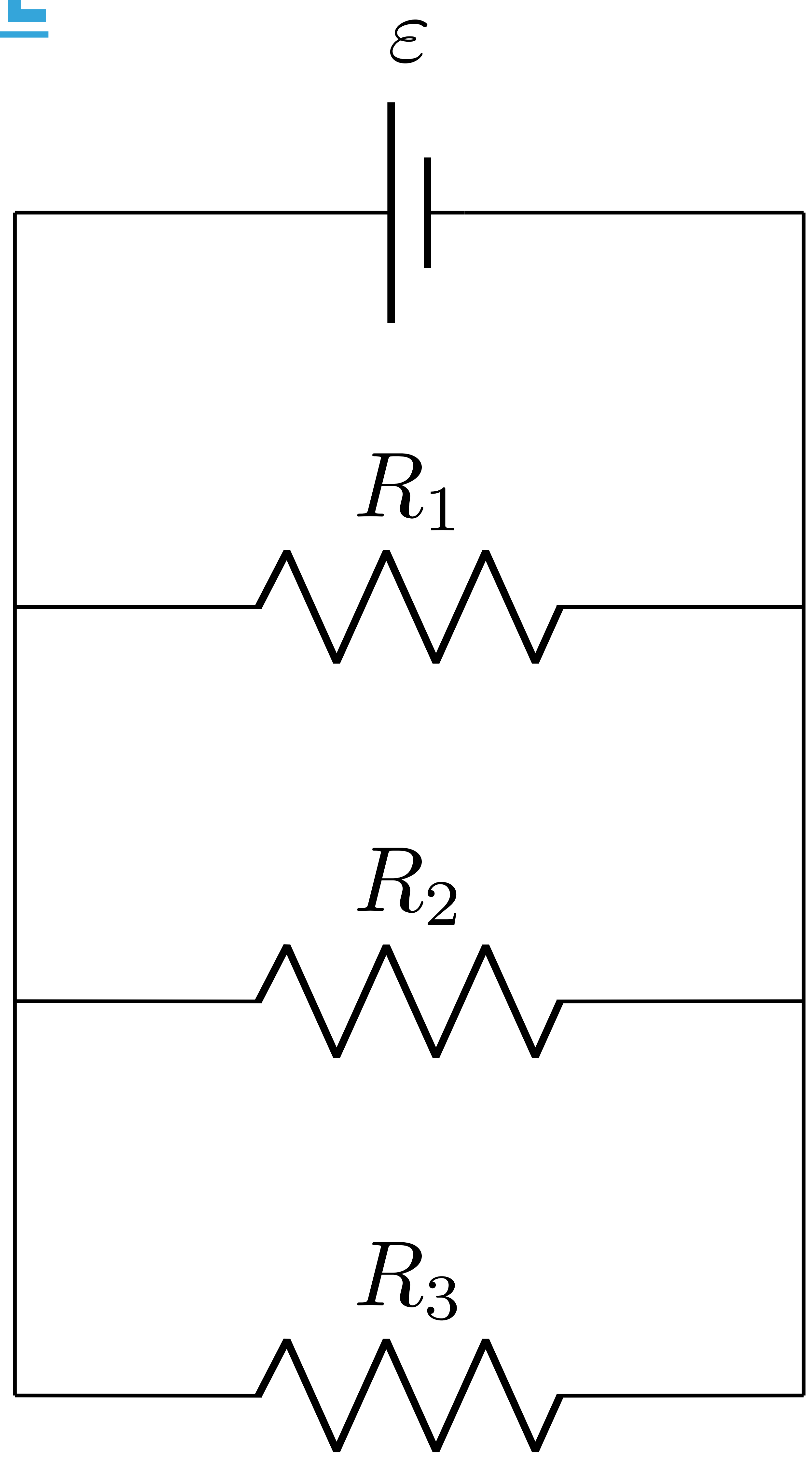
$$\Delta V = \Delta V_1 + \Delta V_2 + \Delta V_3 + \ldots + \Delta V_n$$

$$I = \frac{\Delta V}{R_1 + R_2 + R_3 + \ldots + R_n} = \frac{\Delta V}{R_{\mathrm{eqiv}}}$$

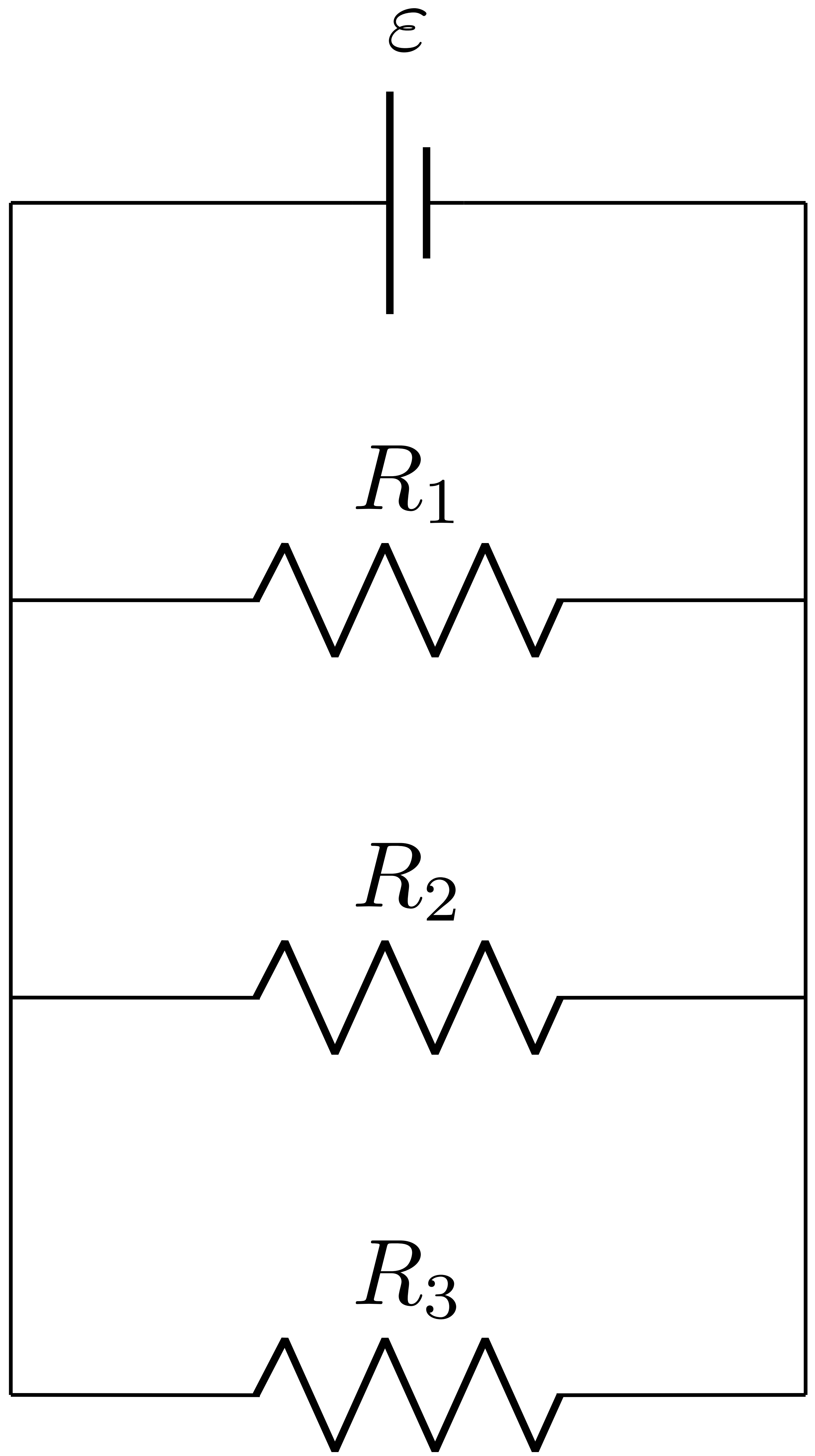
$$R_{\mathrm{equiv}} = R_1 + R_2 + R_3 + \ldots + R_n$$



MULTIPLE RESISTORS IN PARALLEL



MULTIPLE RESISTORS IN PARALLEL



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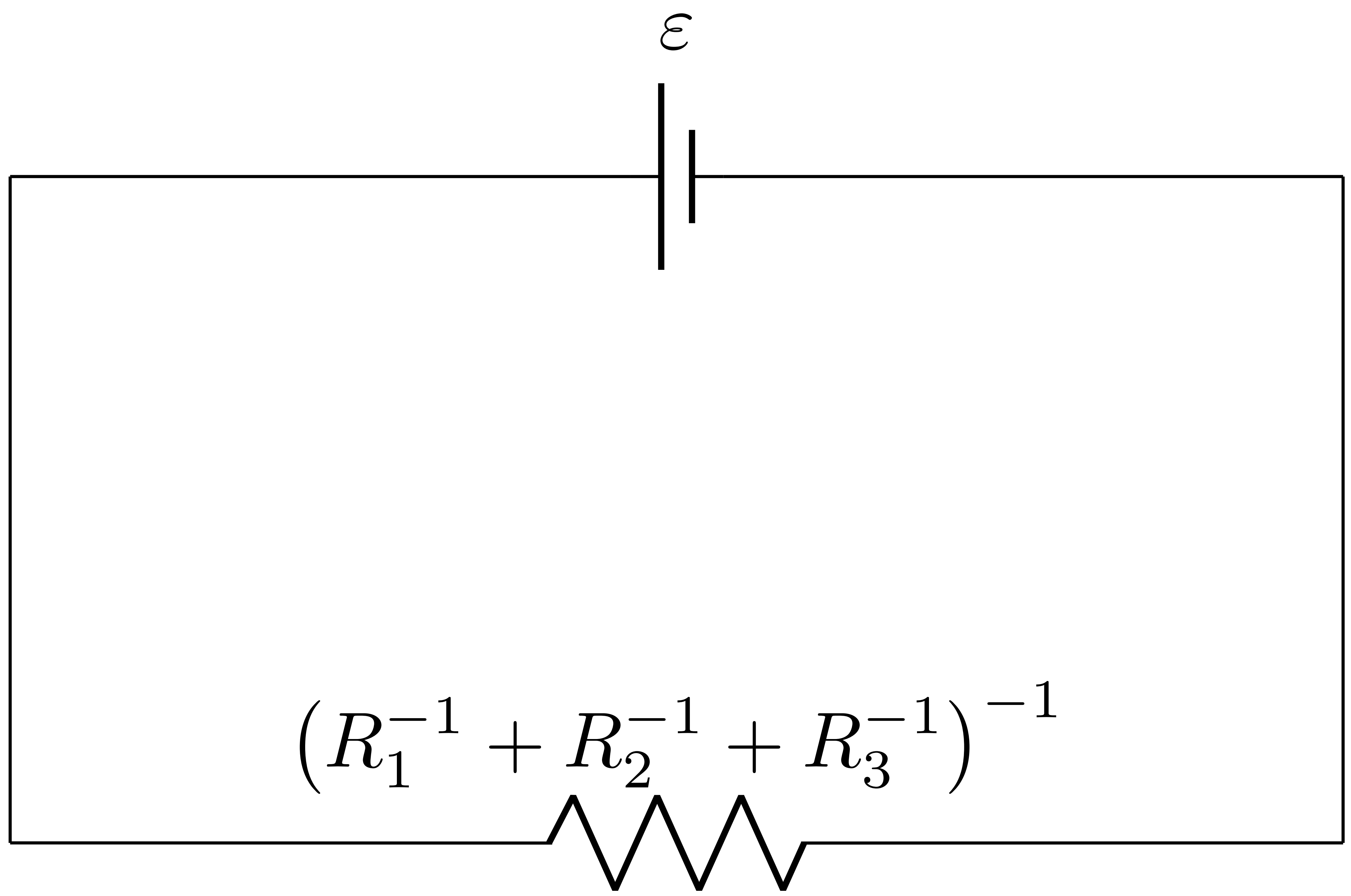


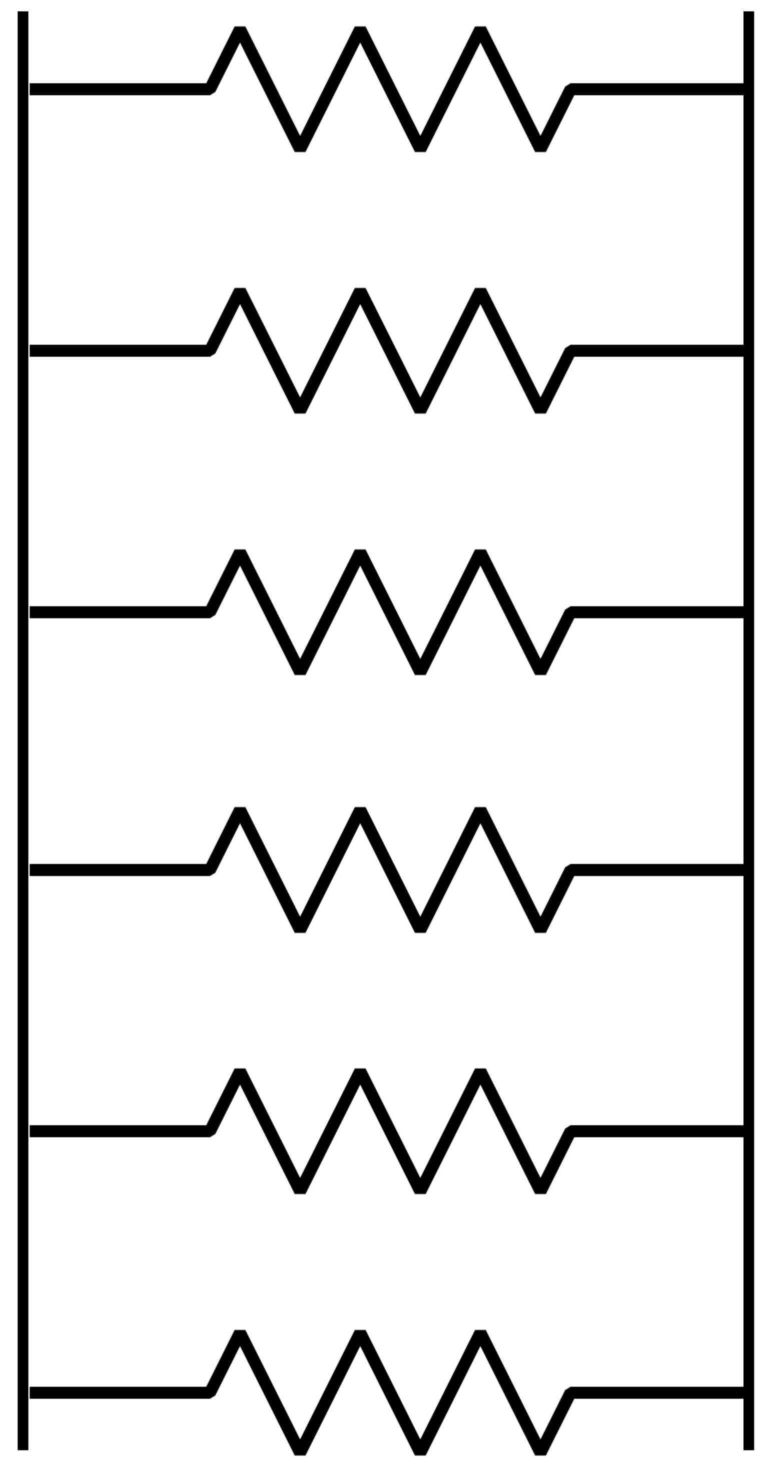
CIRCUIT ELEMENTS CONNECTED IN PARALLEL

$$I = I_1 + I_2 + I_3 + \ldots + I_n$$

$$\Delta V = \Delta V_1 = \Delta V_2 = \Delta V_3 = \ldots = \Delta V_n$$

$$I = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots + \frac{1}{R_n} \right) \varepsilon = \frac{\varepsilon}{R_{\text{equiv}}}$$

$$\frac{1}{R_{\text{equiv}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots + \frac{1}{R_n}$$



VOLTAGE ACROSS PARALLEL RESISTORS

How can the voltage be the same across every resistor in parallel? Doesn’t that violate energy conservation?

ENERGY USED UP IN A RESISTOR

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Electrical energy from the battery is converted to thermal energy in the resistor at a rate:

$$\frac{\Delta E}{\Delta t} = I\Delta V$$

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$$\frac{\Delta E}{\Delta t} = I\Delta V$$

Power = $P = I\Delta V$

ENERGY USED UP IN A RESISTOR

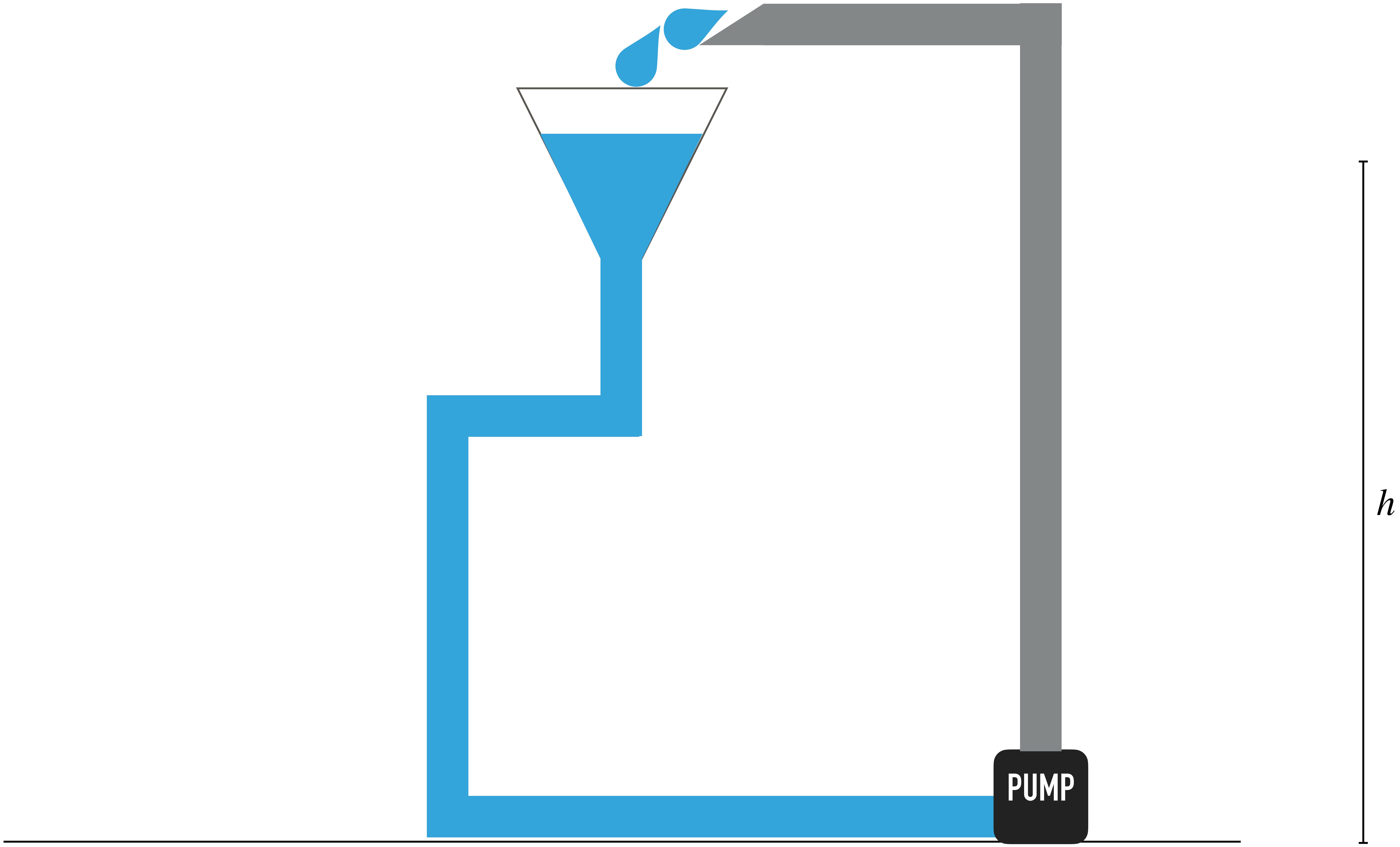
- Power: change in energy per unit time
- Units?

POWER

- Power: change in energy per unit time
- Units?
    - SI Unit: $1 \text{ Watt} = 1 \ W = 1 \ \frac{\text{J}}{\text{s}}$

ENERGY CONSERVATION

- Total power consumed by all circuit elements **must** be equal to the power output of the battery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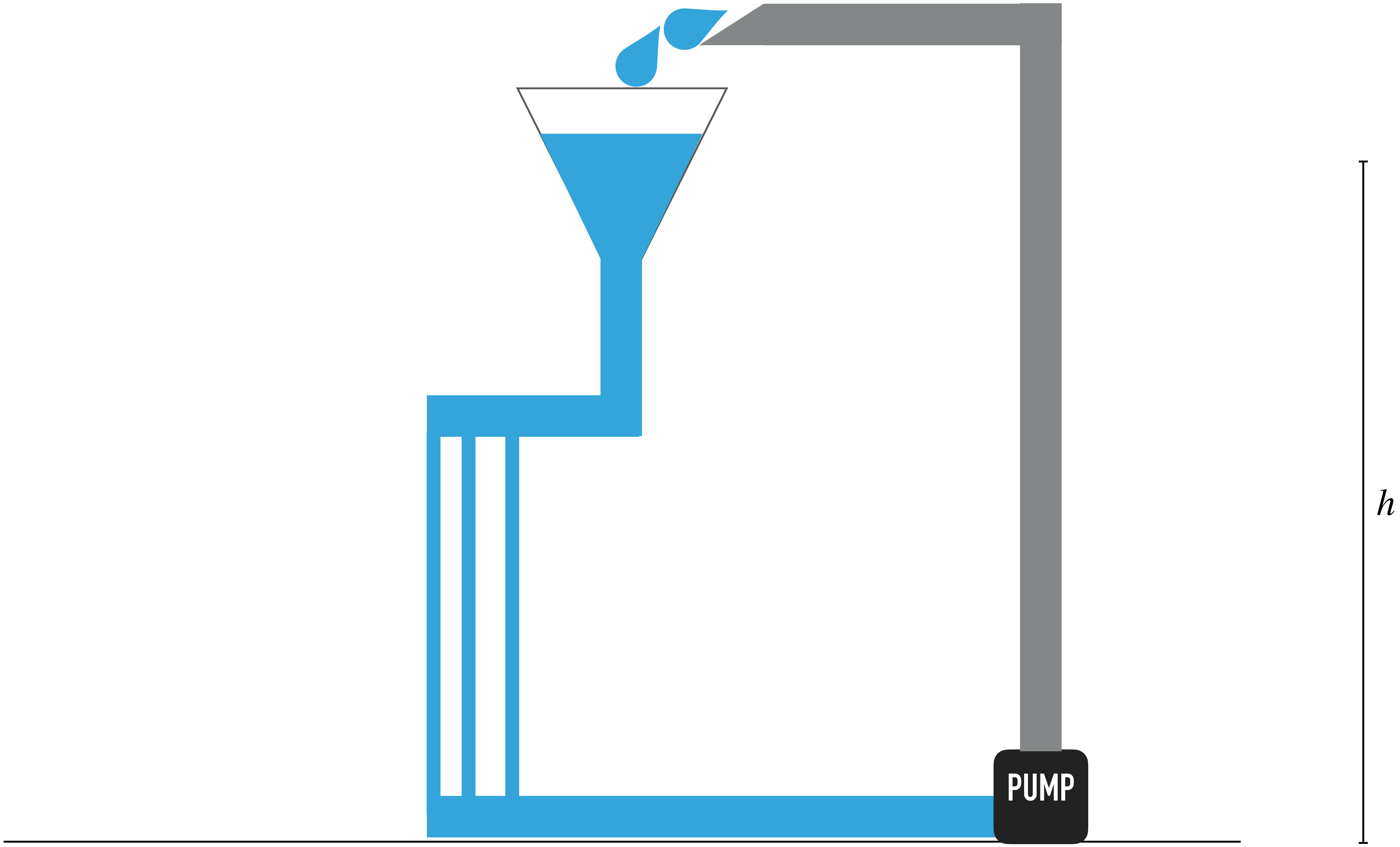
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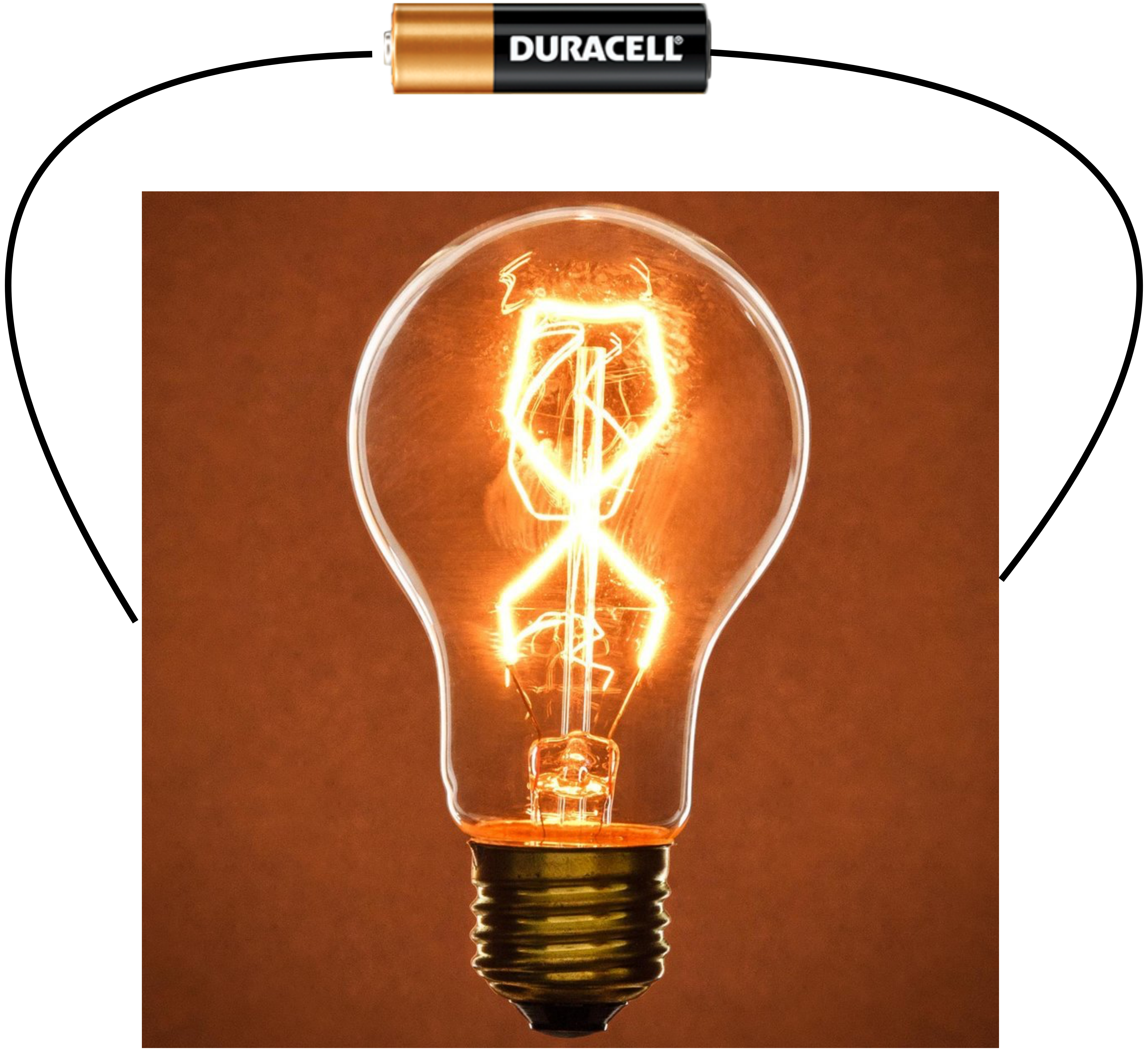
A 60-WATT LIGHTBULB

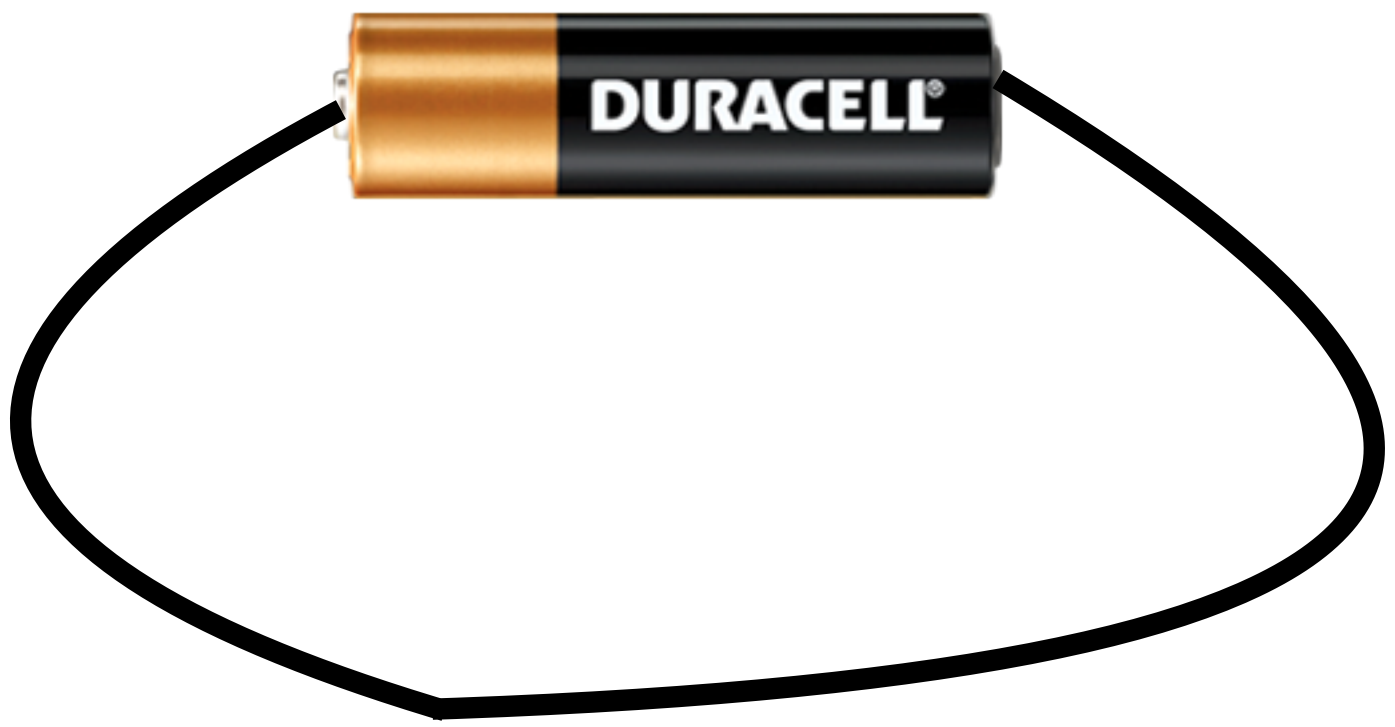
A 60-WATT LIGHTBULB

1.5 V



SHORT CIRCUIT

- Circuit components connected only by wires are said to be "shorted" to one another
- A circuit formed only of wires (with not circuit elements) is called a "short circuit"



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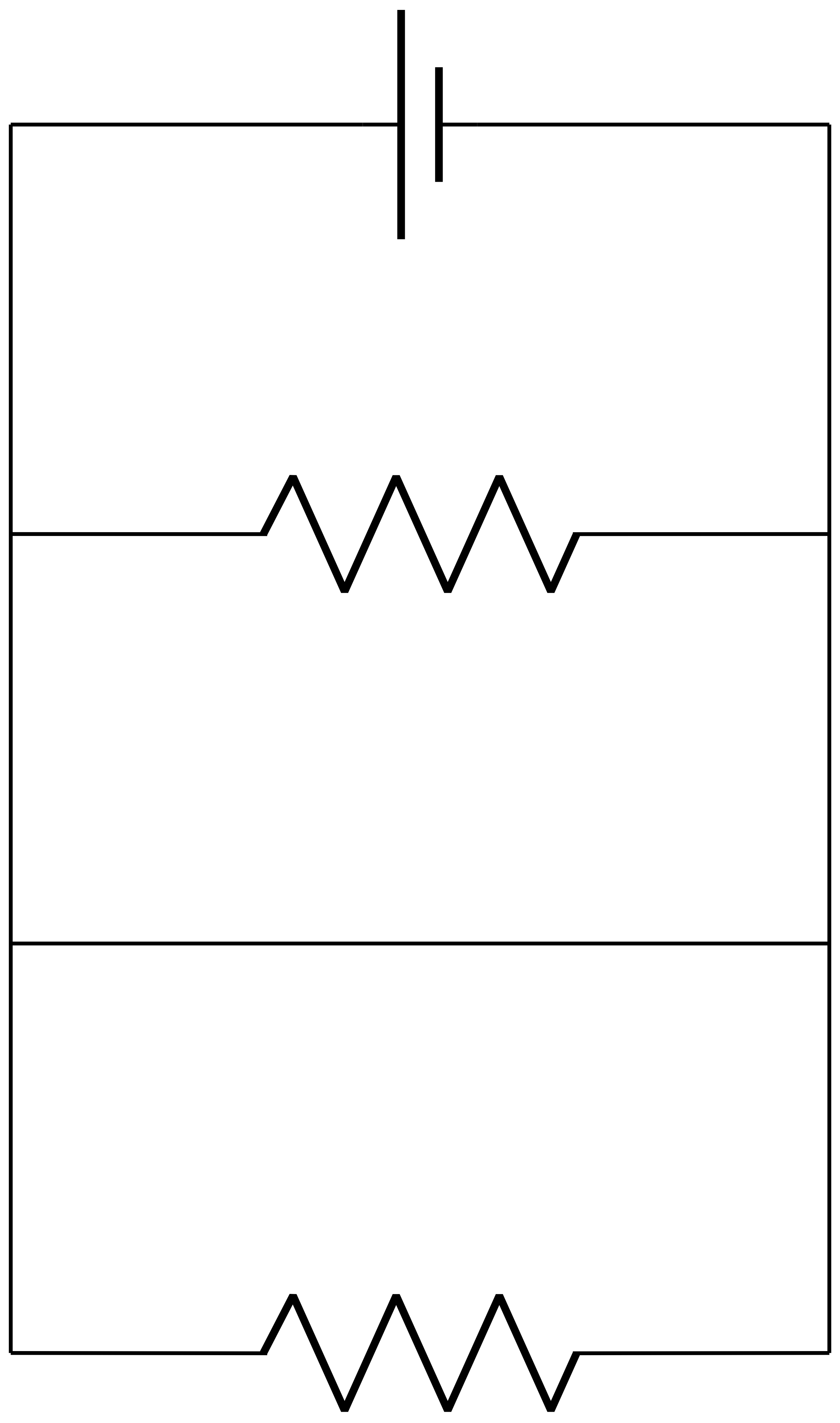
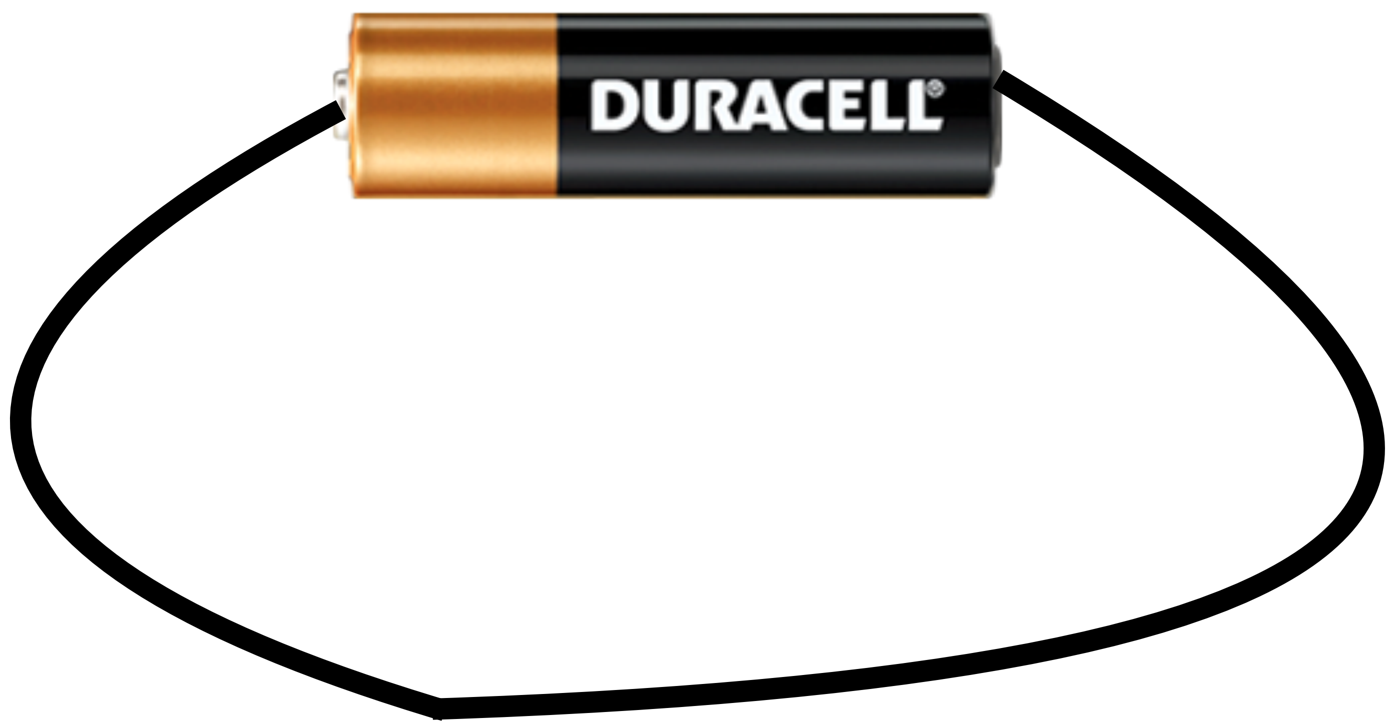


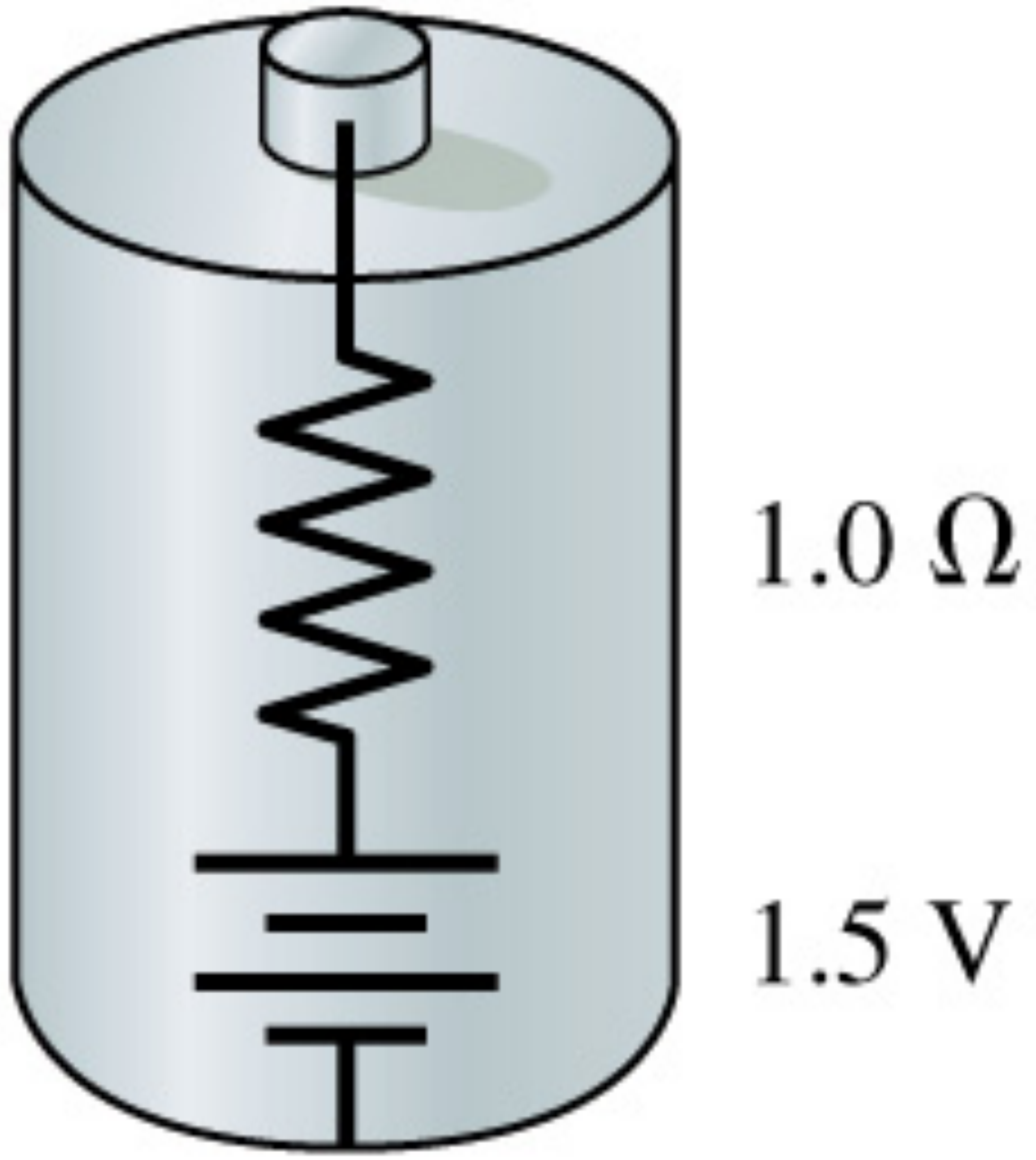
SHORT CIRCUIT

- How much current is drawn during a short circuit?



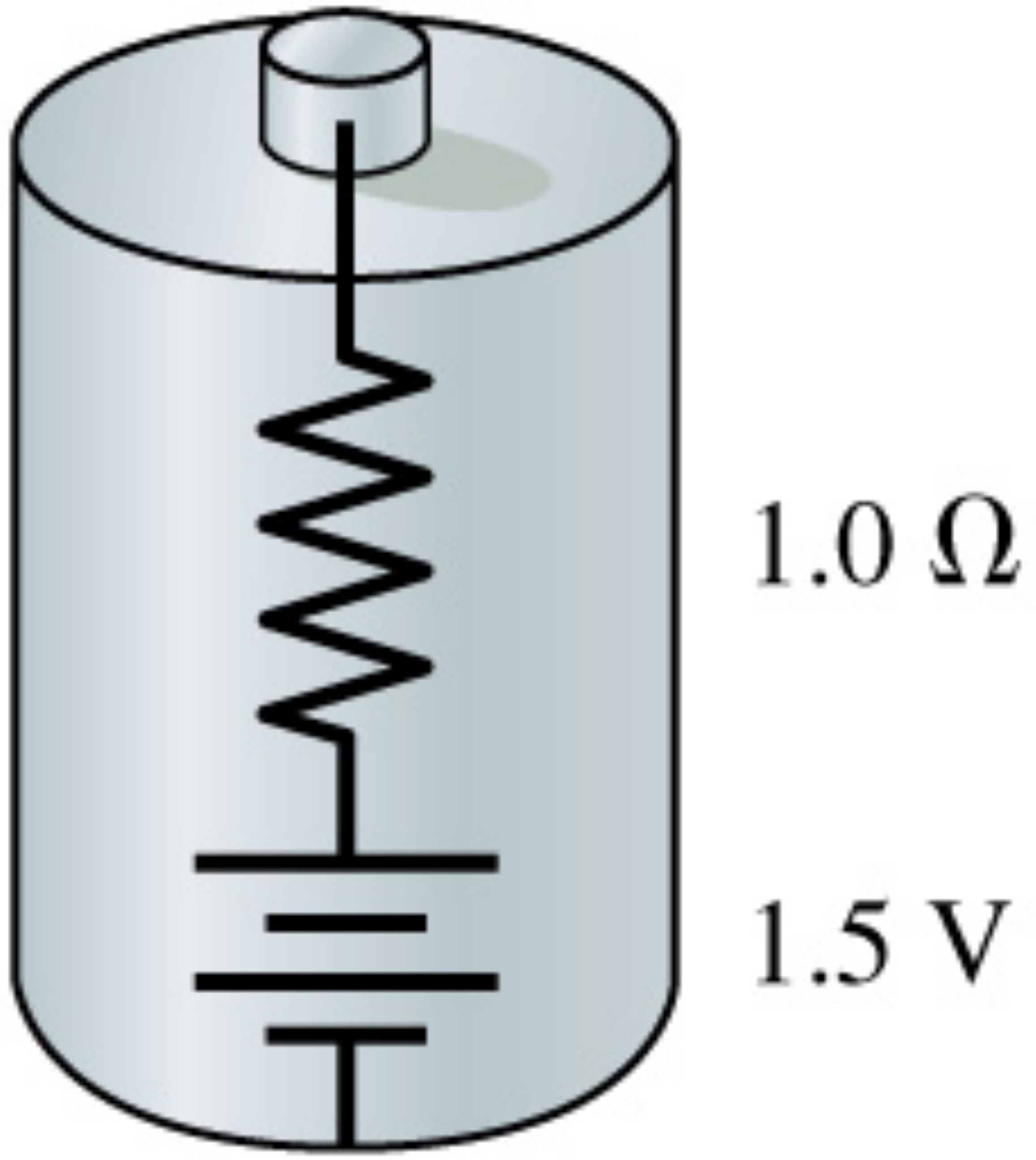
"REAL" BATTERIES

- We haven't been accounting for the fact that a small amount of energy is lost within the battery itself
- Model the battery as an ideal battery in series with a tiny resistor
  - "Internal resistance"



“REAL” BATTERIES

- “Ideal battery”
  - No internal resistance
  - $\Delta V_{\text{battery}} = \varepsilon$
- “Real battery”
  - Small but non-zero internal resistance
  - $\Delta V_{\text{battery}} = \varepsilon - Ir_{\text{int}}$



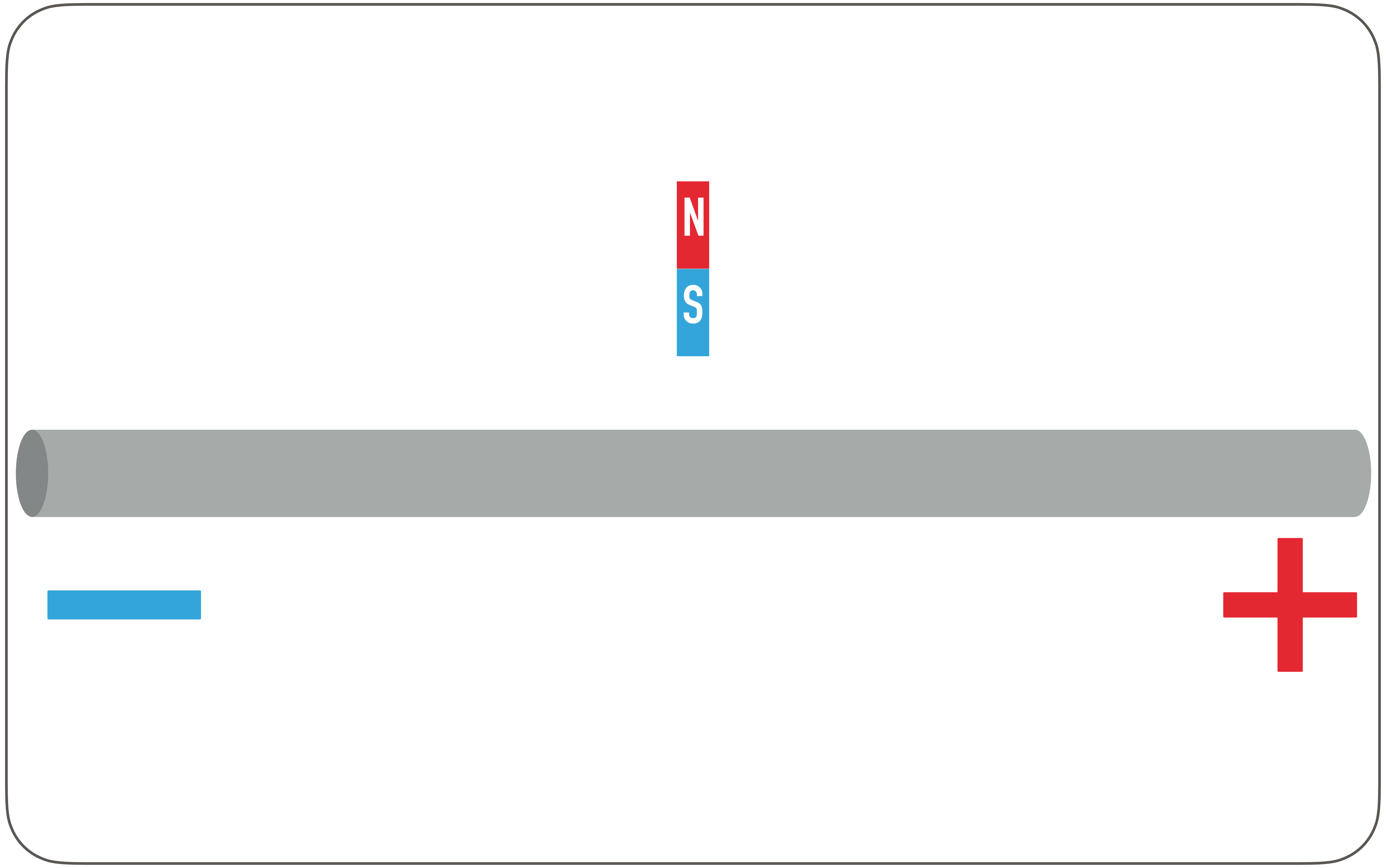
MEASURING CURRENT AND VOLTAGE

- We have used both voltmeters and ammeters in labs this semester. How do these instruments work?

MEASURING CURRENT AND VOLTAGE

- We have used both voltmeters and ammeters in labs this semester. How do these instruments work?
- Ammeter: a device to measure current (“amp” meter)
- Voltmeter: no one knows

A PRIMITIVE AMMETER



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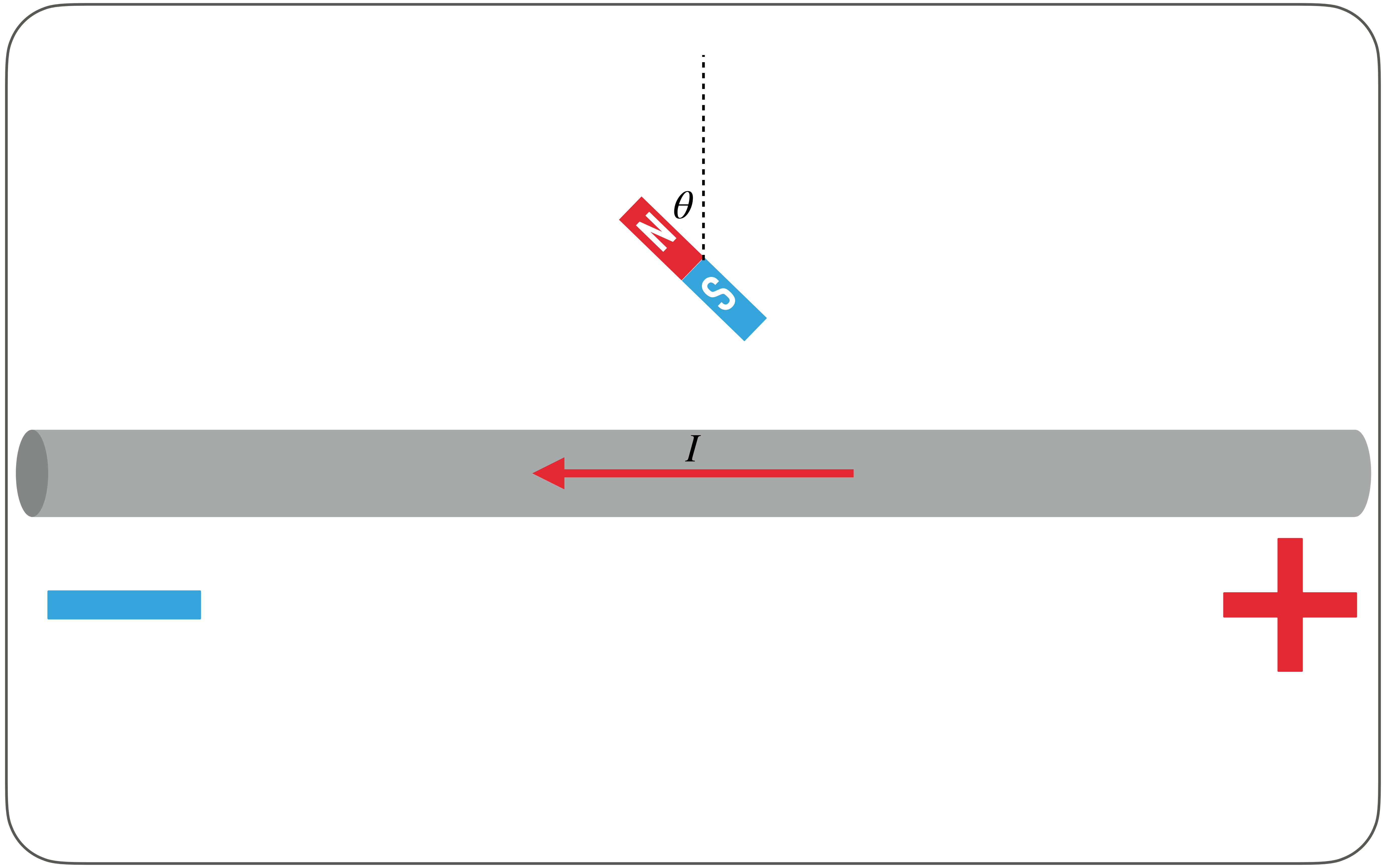

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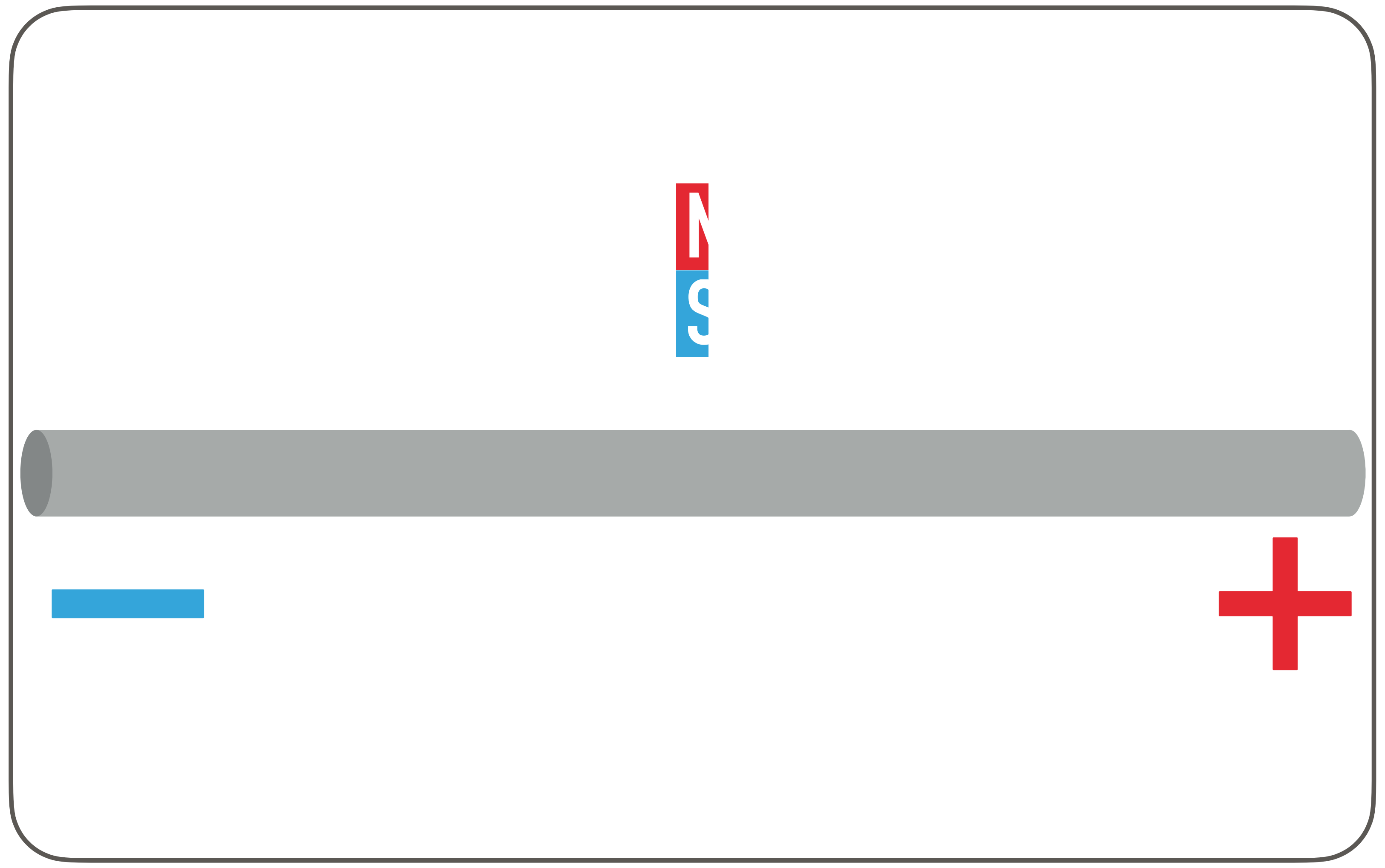
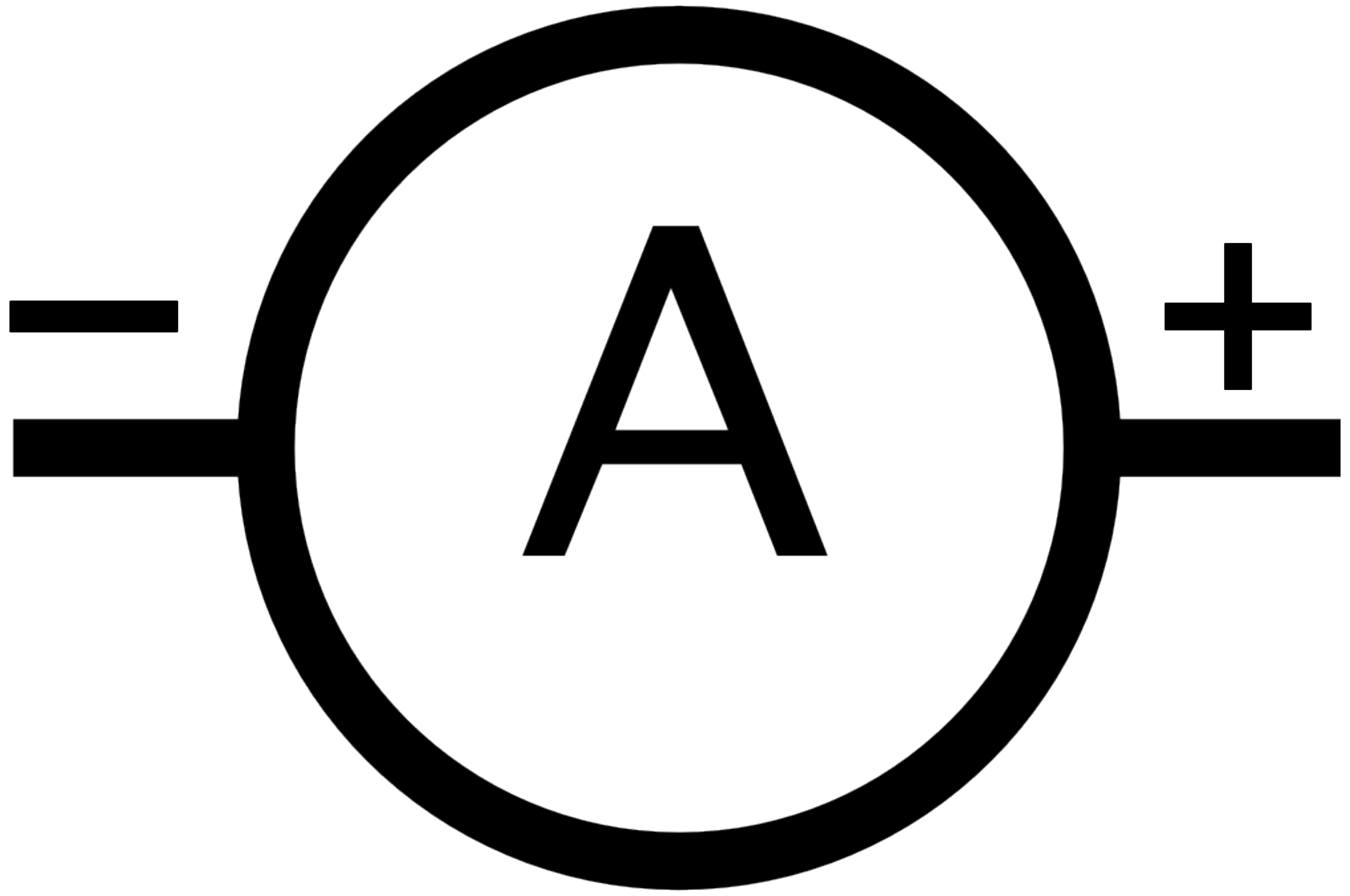
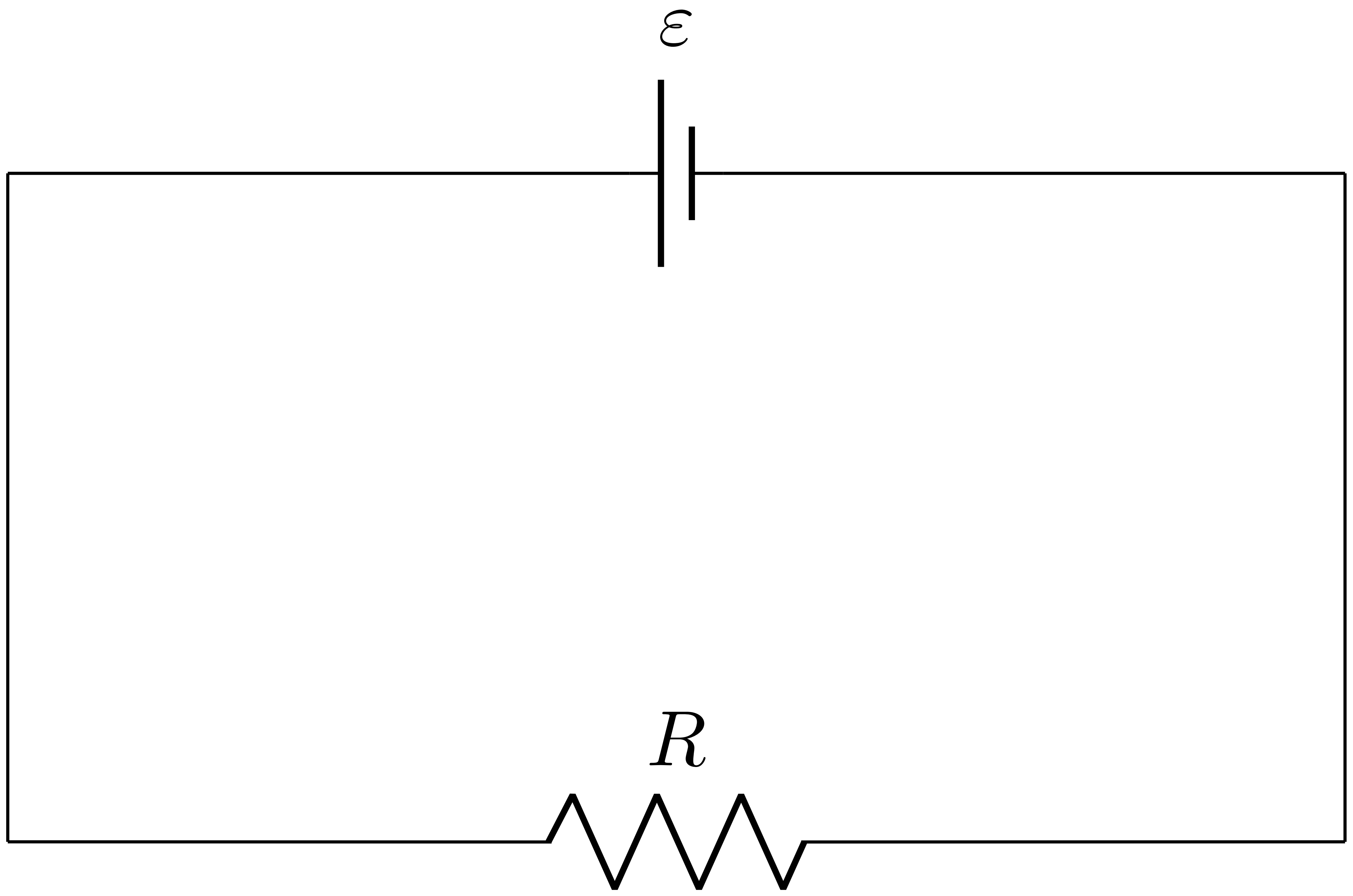


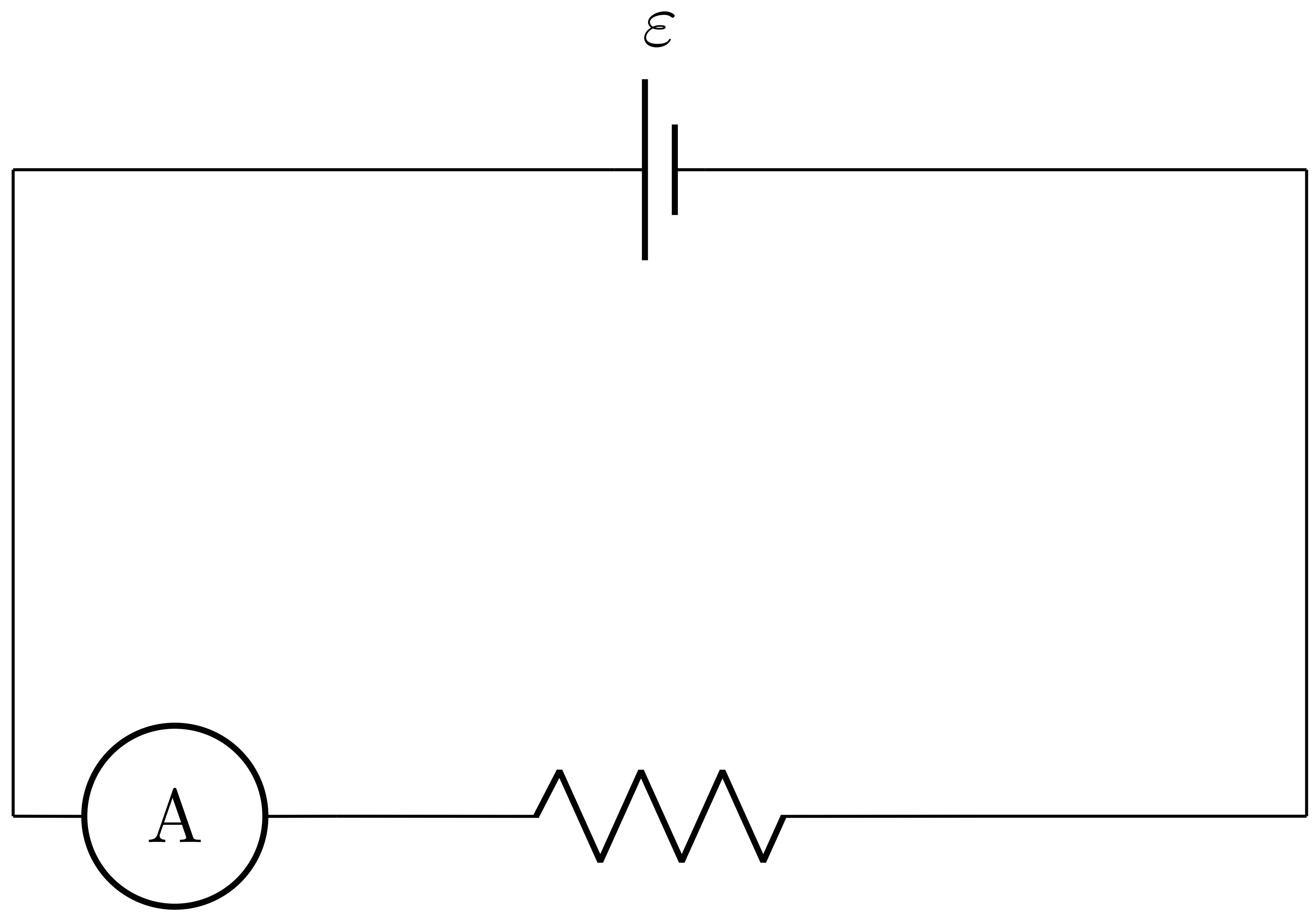


USING AN AMMETER

HOW TO MEASURE CURRENT THROUGH RESISTOR?



USING AN AMMETER



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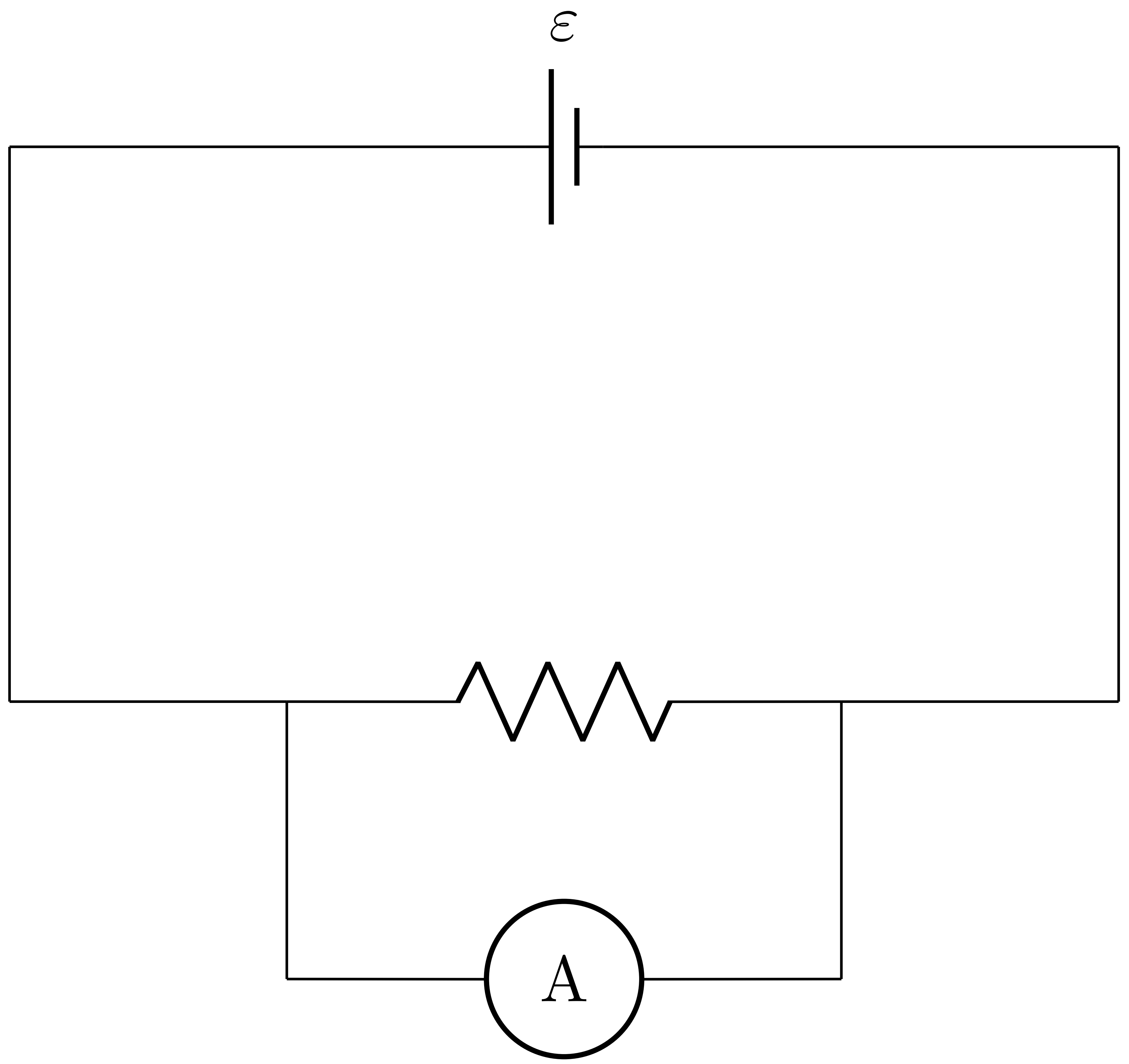
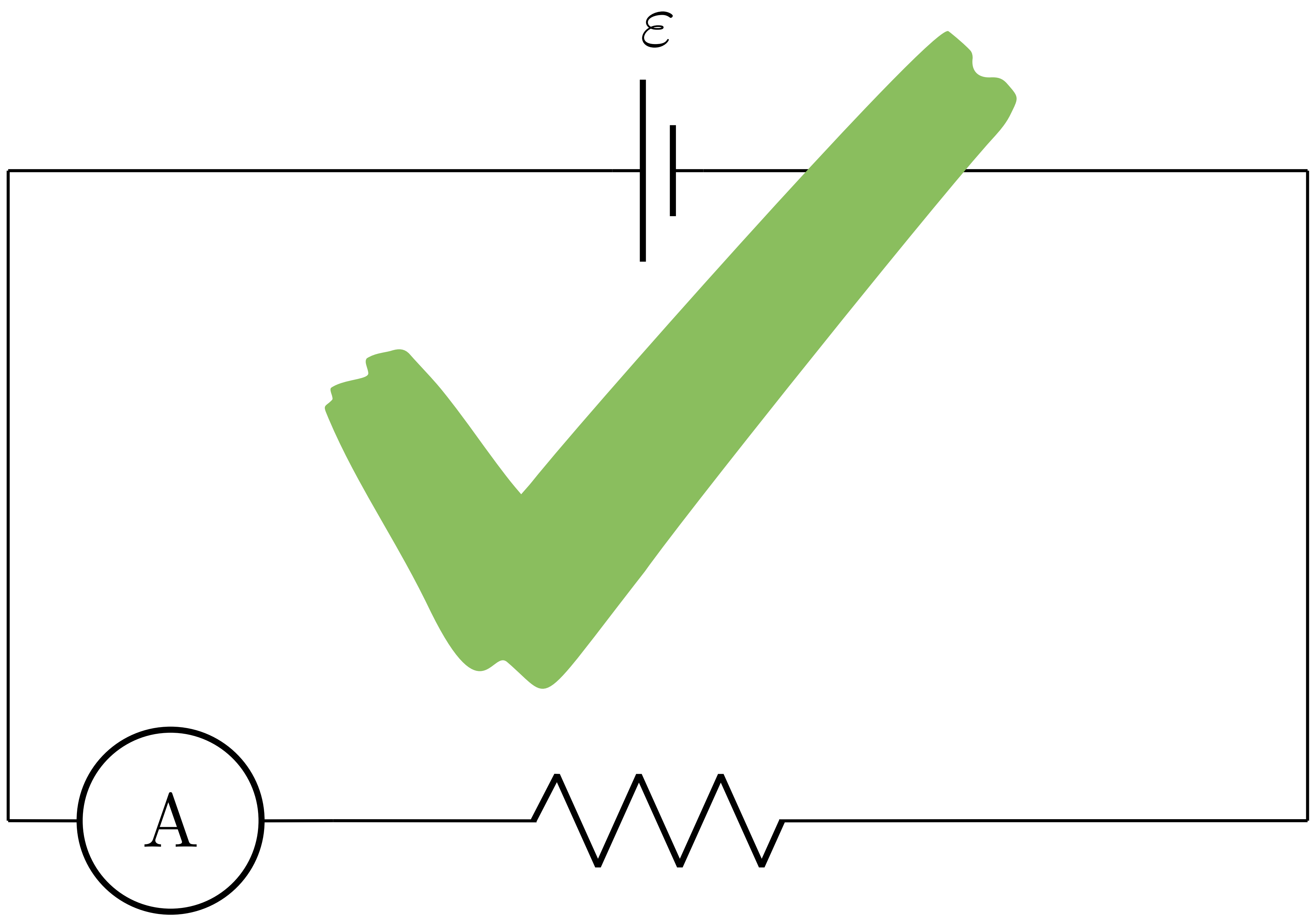


USING AN AMMETER



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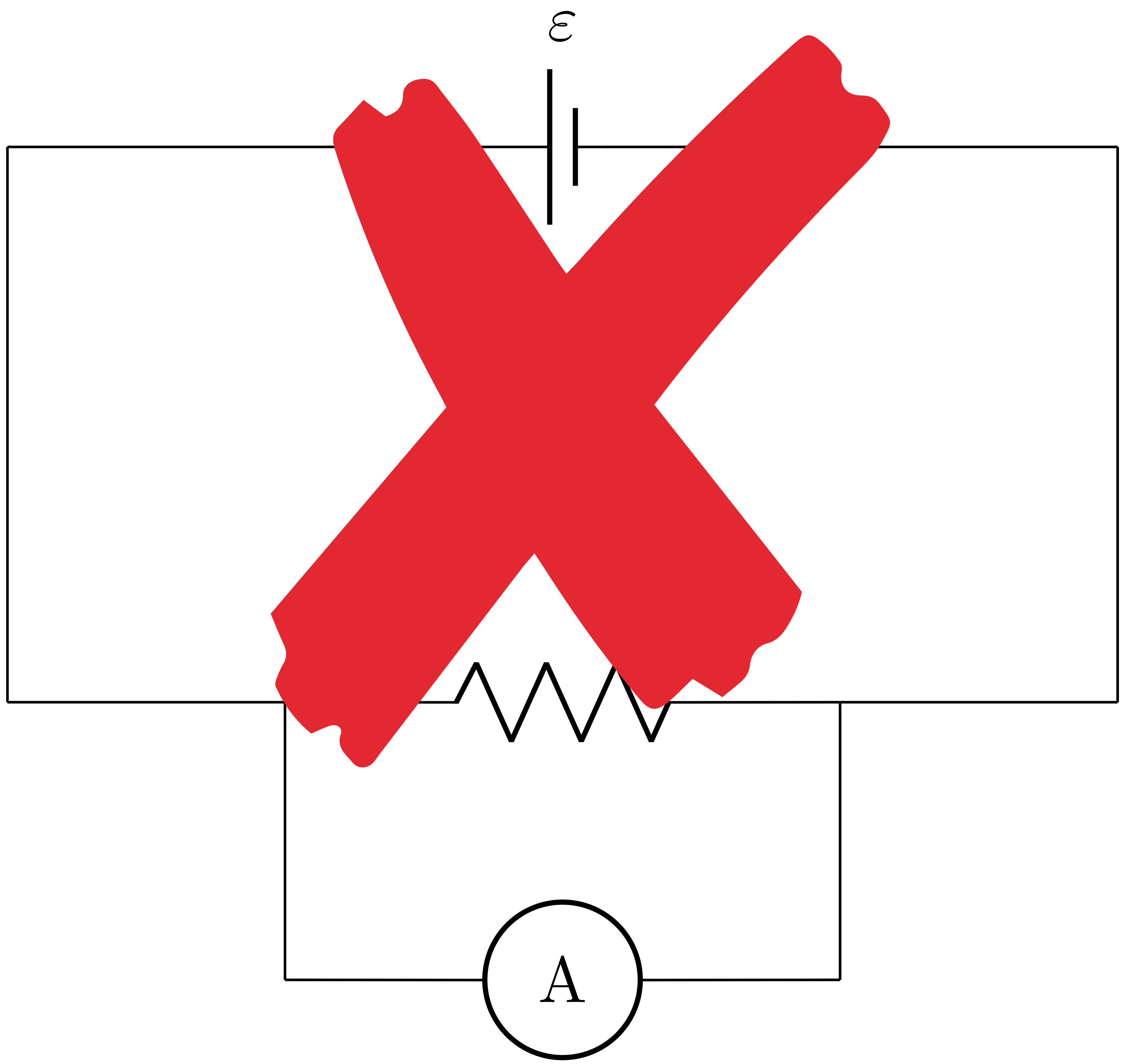


USING AN AMMETER

- To measure the current through a circuit element, connect it in *series* with the ammeter
    - Node rule guarantees that current is the same through both components

CONSTRUCTING AN AMMETER

- What kind of wire should you use to conduct current through the ammeter?

CONSTRUCTING AN AMMETER

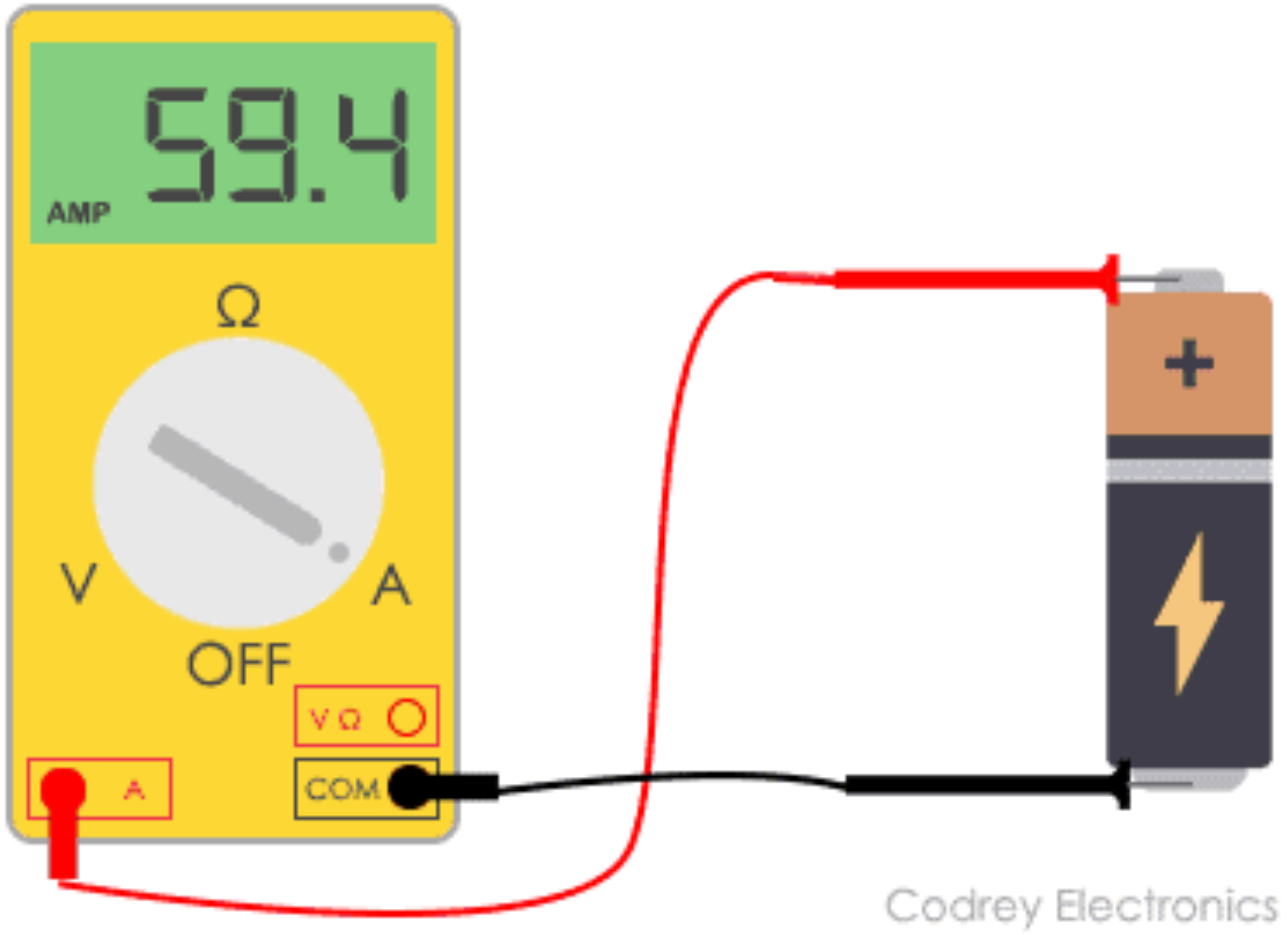
- What kind of wire should you use to conduct current through the ammeter?
    - Smallest possible R to avoid changing the circuit!

CONSTRUCTING AN AMMETER

- What kind of wire should you use to conduct current through the ammeter?
  - Smallest possible R to avoid changing the circuit!
  - Because ammeters have almost zero resistance, connecting an ammeter in *parallel* will create a short circuit!

USING AN AMMETER

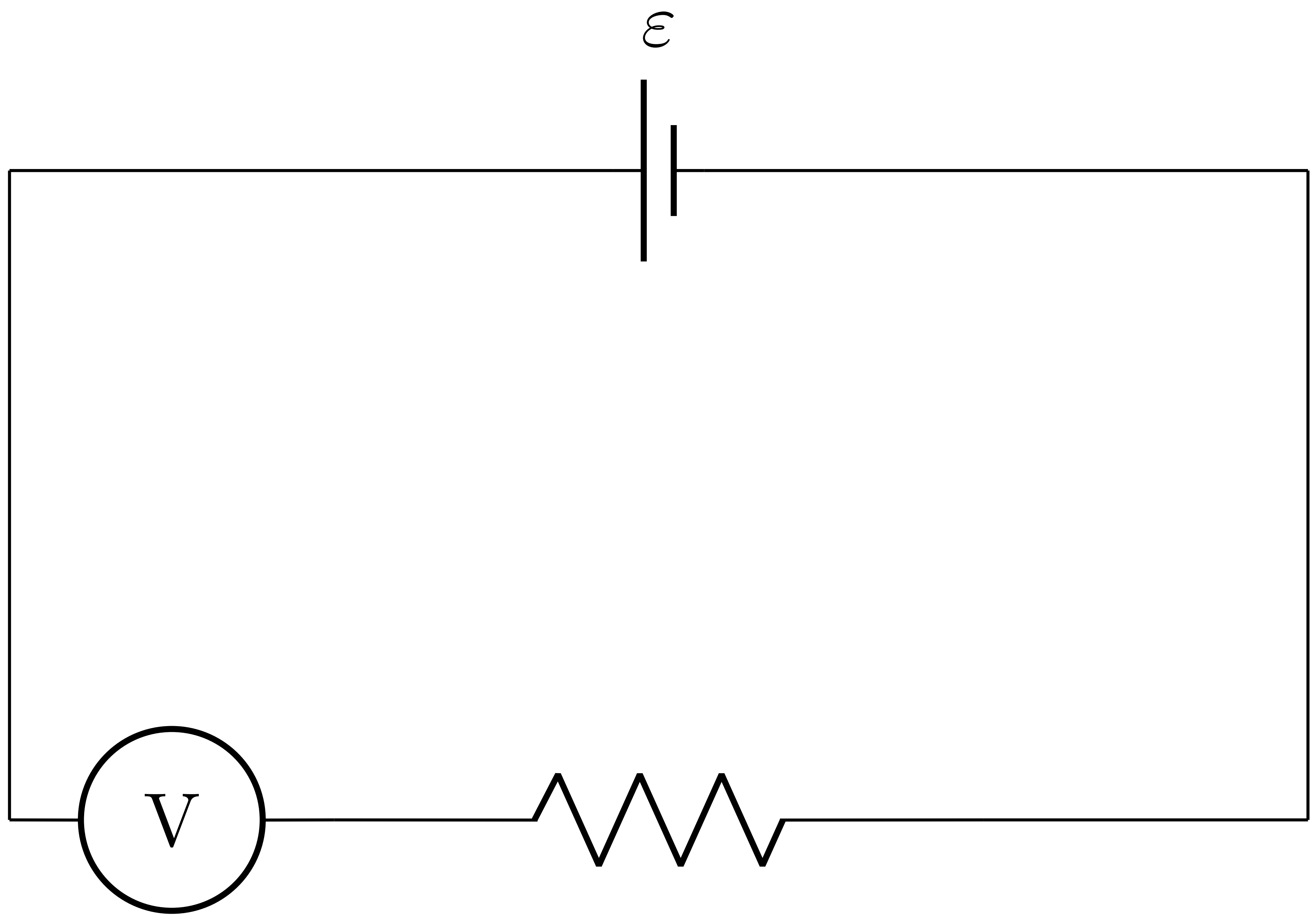
- A positive current reading occurs when conventional current flows *into* the "+" or "A" terminal and out of the "-" or "COM" terminal



A PRIMITIVE VOLTMETER

- If we know the resistance of the wire used in the ammeter, we can use Ohm's Law ($\Delta V = IR$) to calculate the potential difference across the ammeter
- This is the basic operating principle of a voltmeter

USING A VOLTMETER



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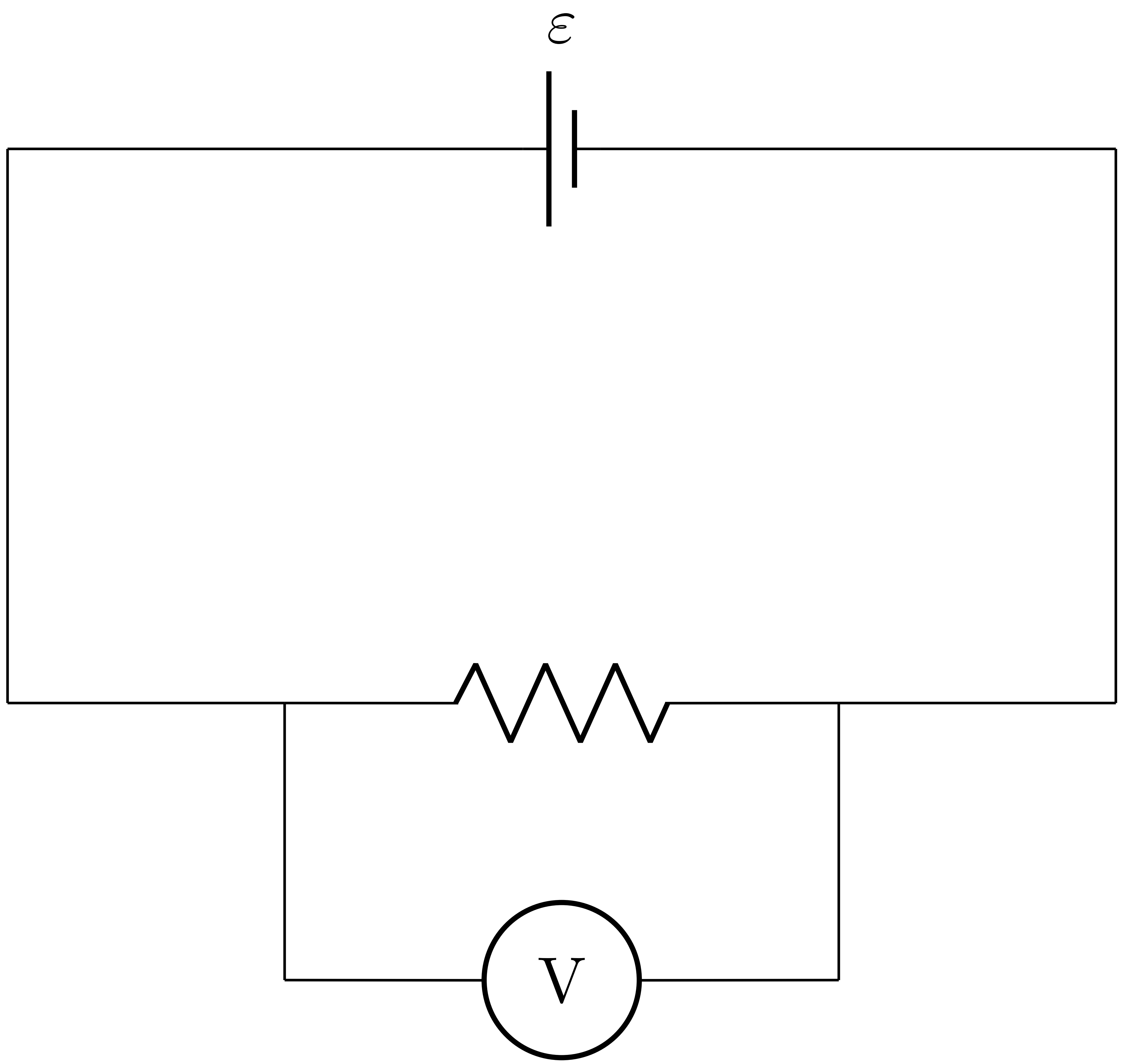
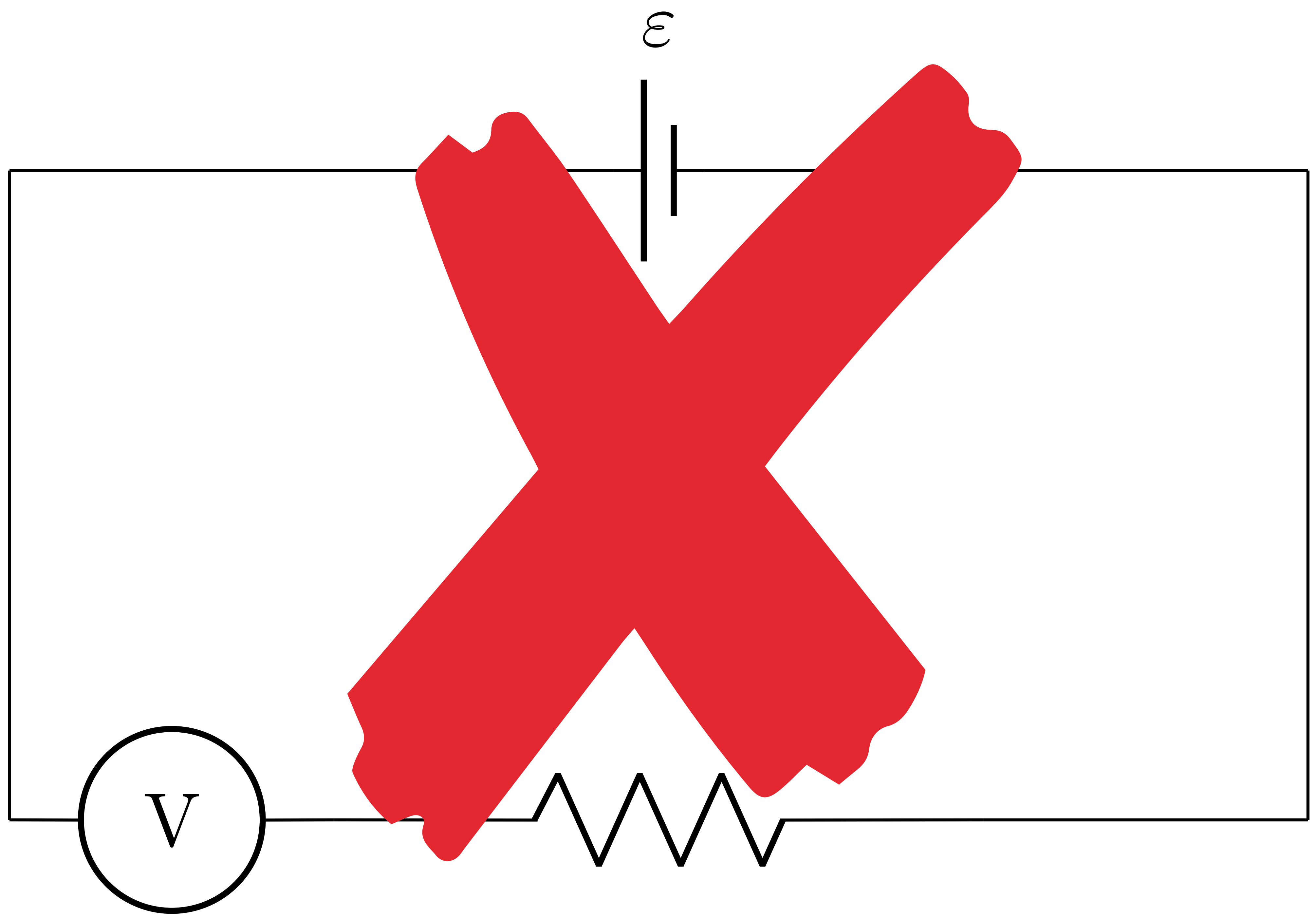


USING A VOLTMETER



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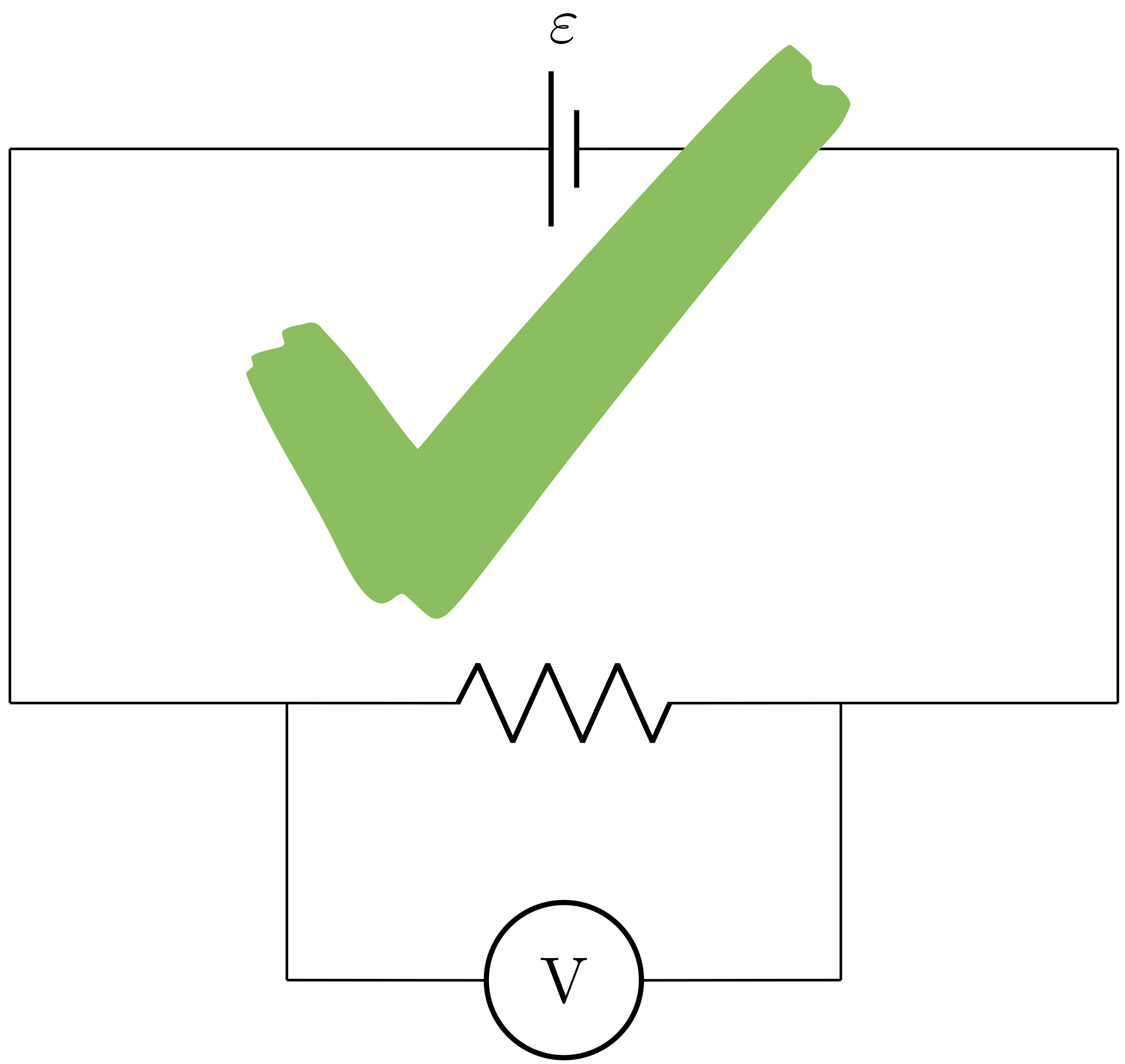
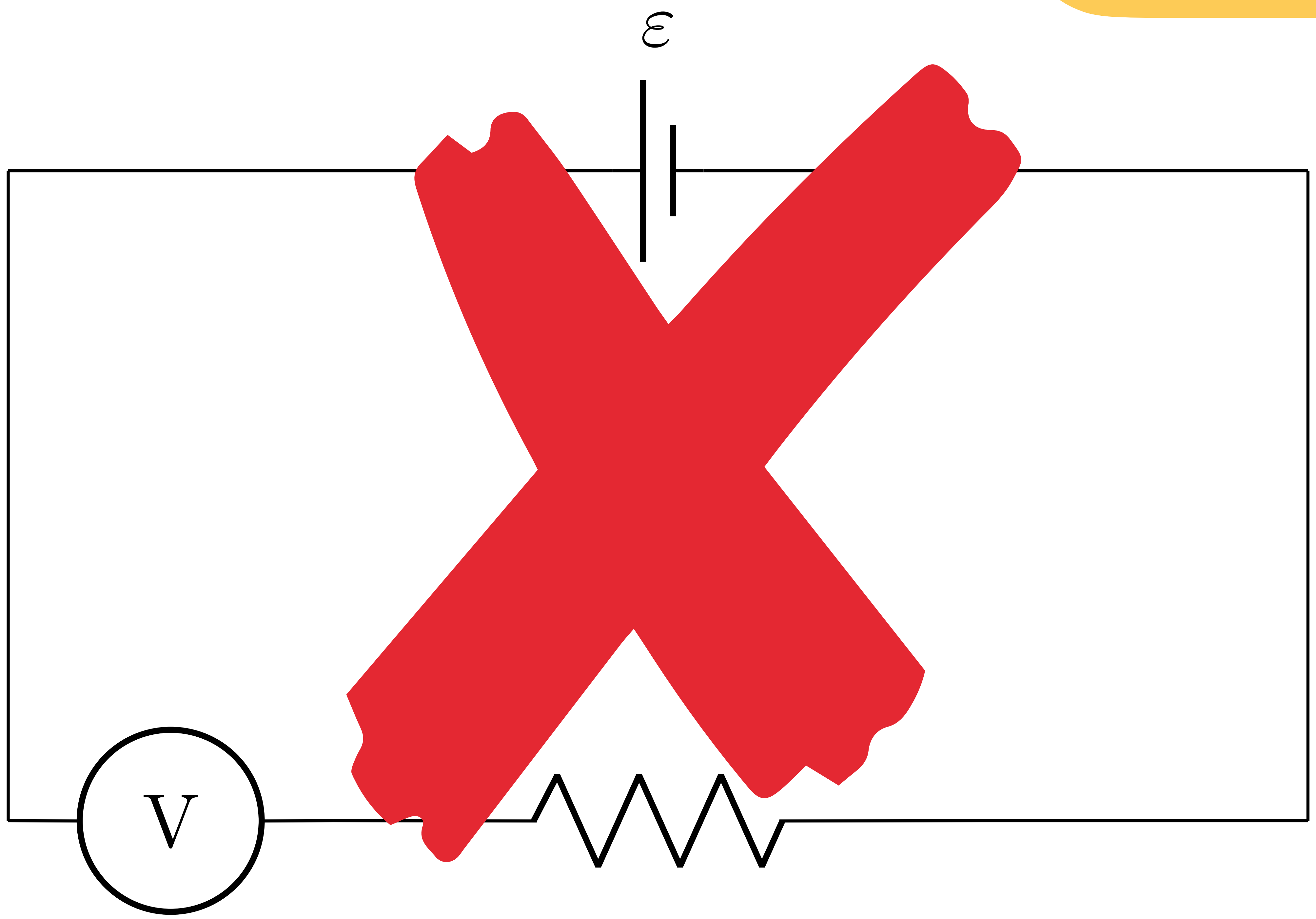


USING A VOLTMETER

TO MINIMIZE ITS EFFECT ON THE CIRCUIT, A VOLTMETER SHOULD HAVE VERY HIGH RESISTANCE



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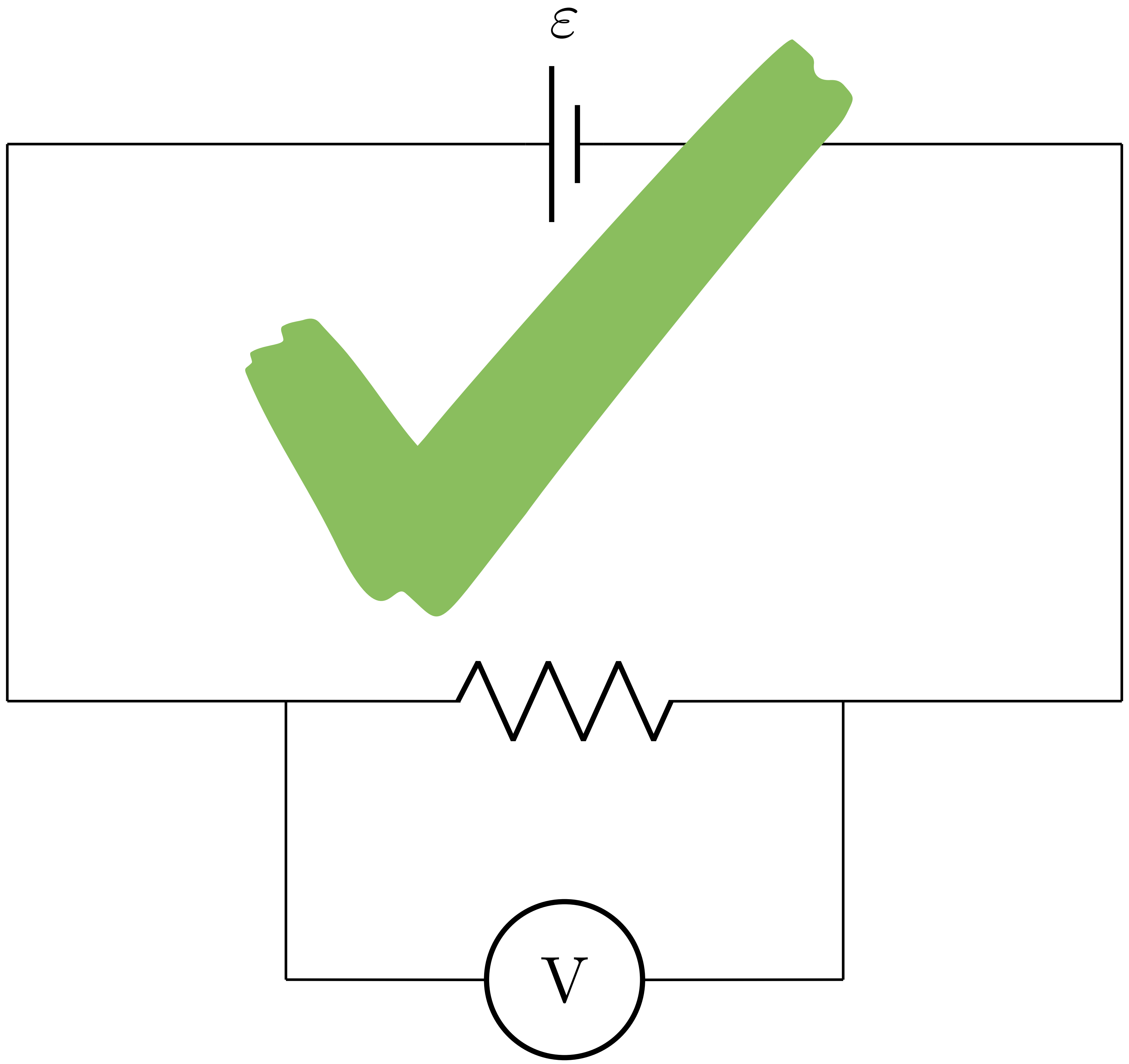
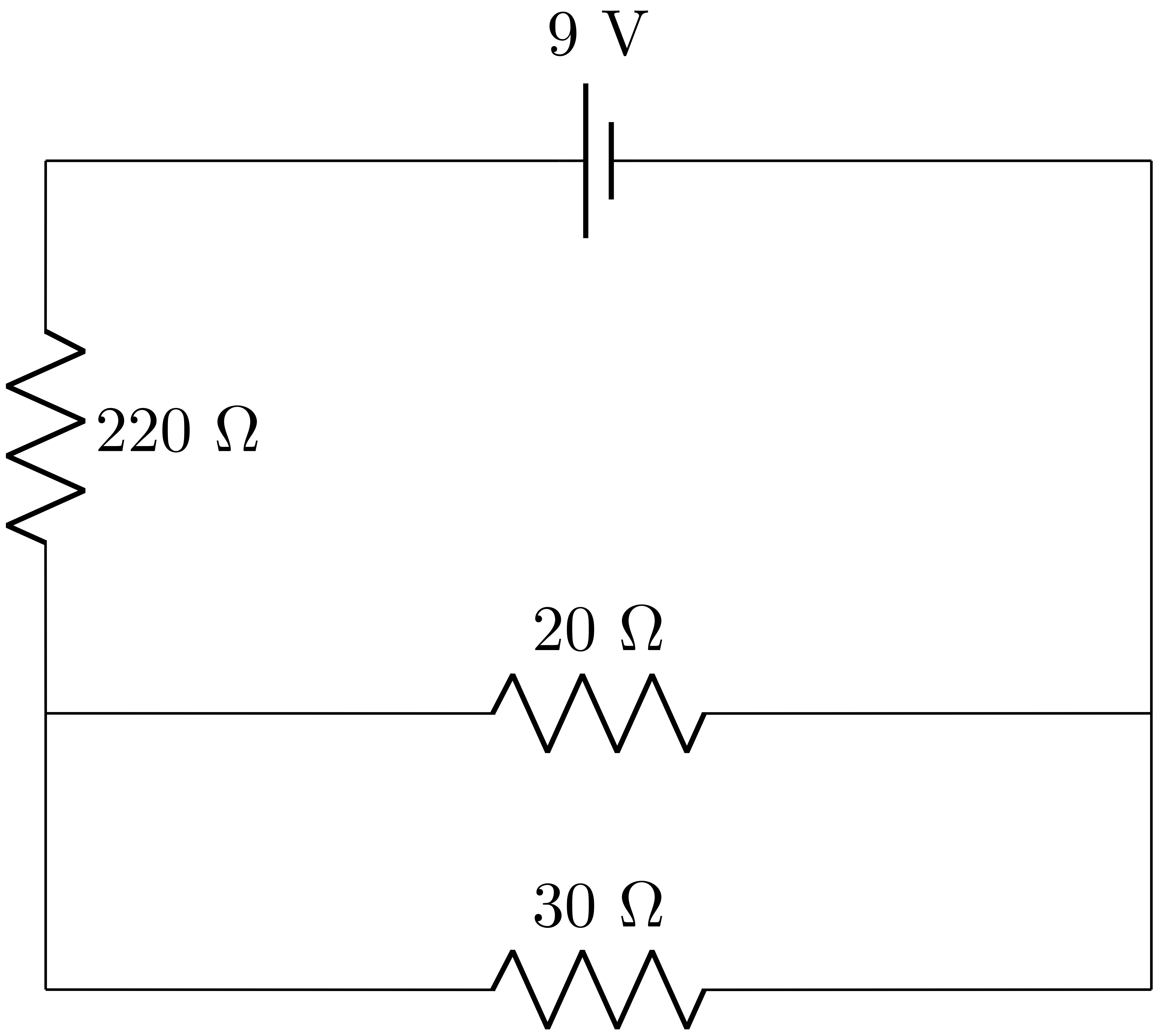


USING A VOLTMETER

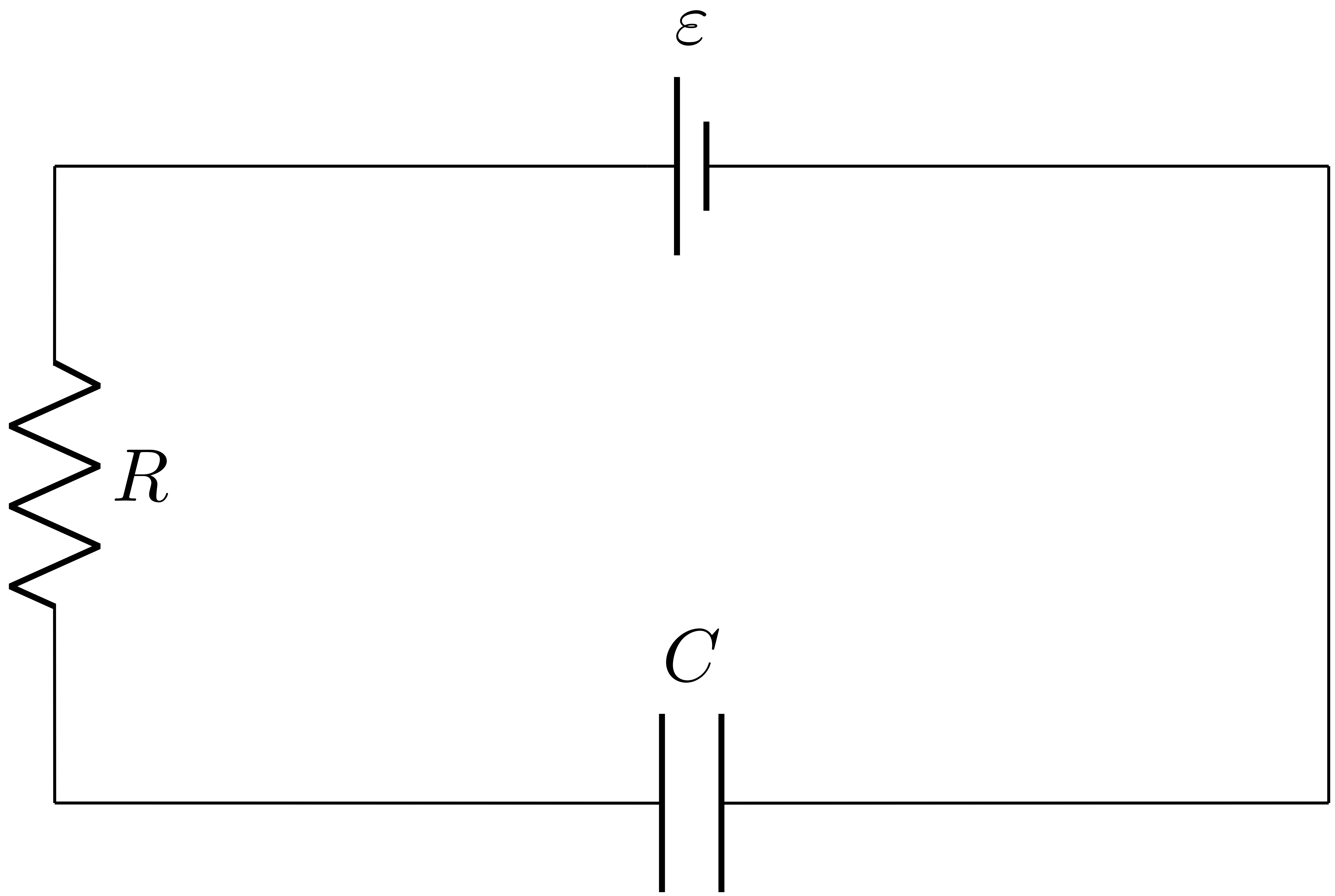
- A typical voltmeter has two terminals “+” and “-“ or maybe “V” and “COM”
- The voltage reading is $V_+ - V_-$
    - If the potential is higher at the “+” socket as compared to the “-“ socket, the reading will be **positive**

SOLVING A CIRCUIT

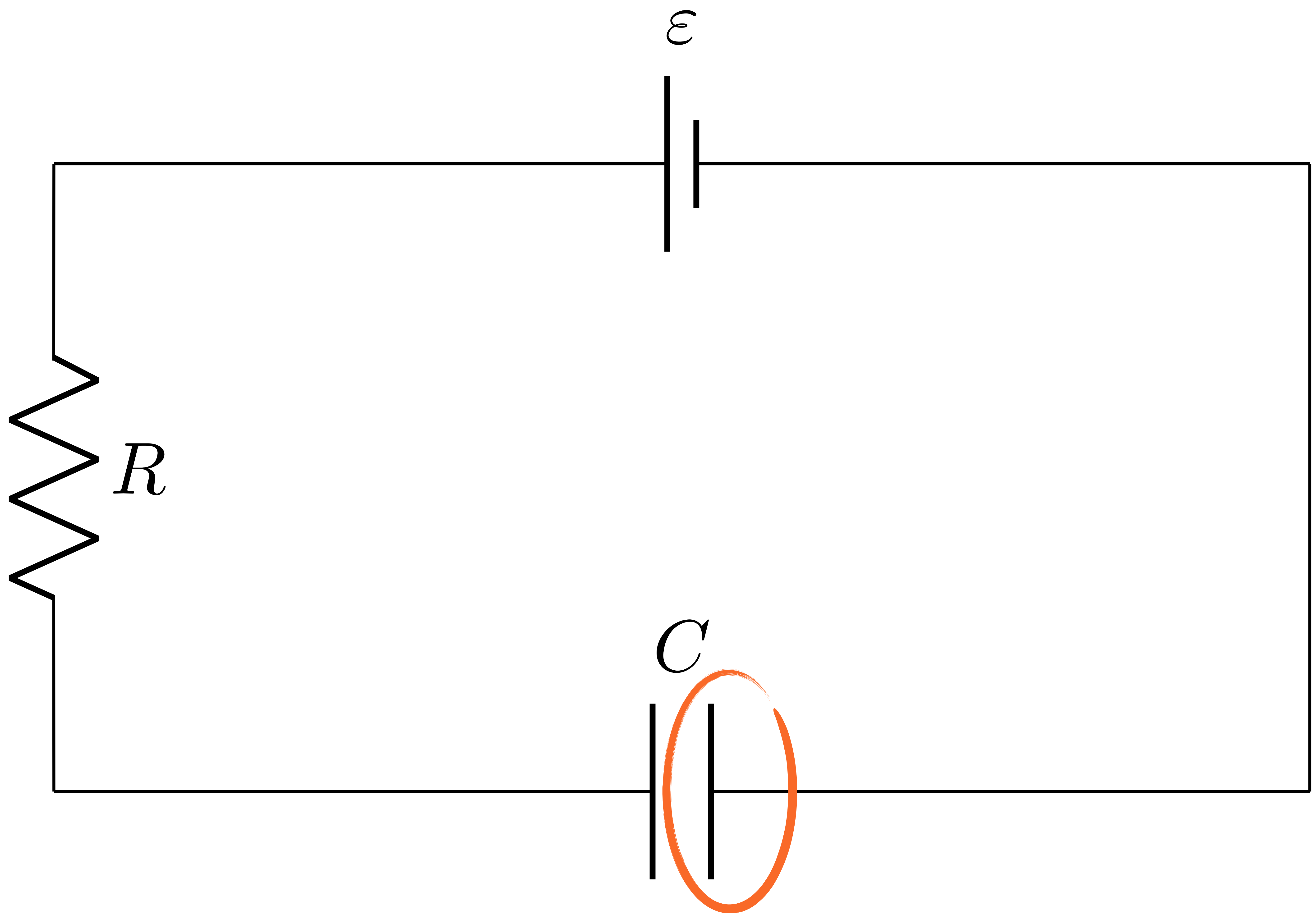
- Current through and voltage across each resistor



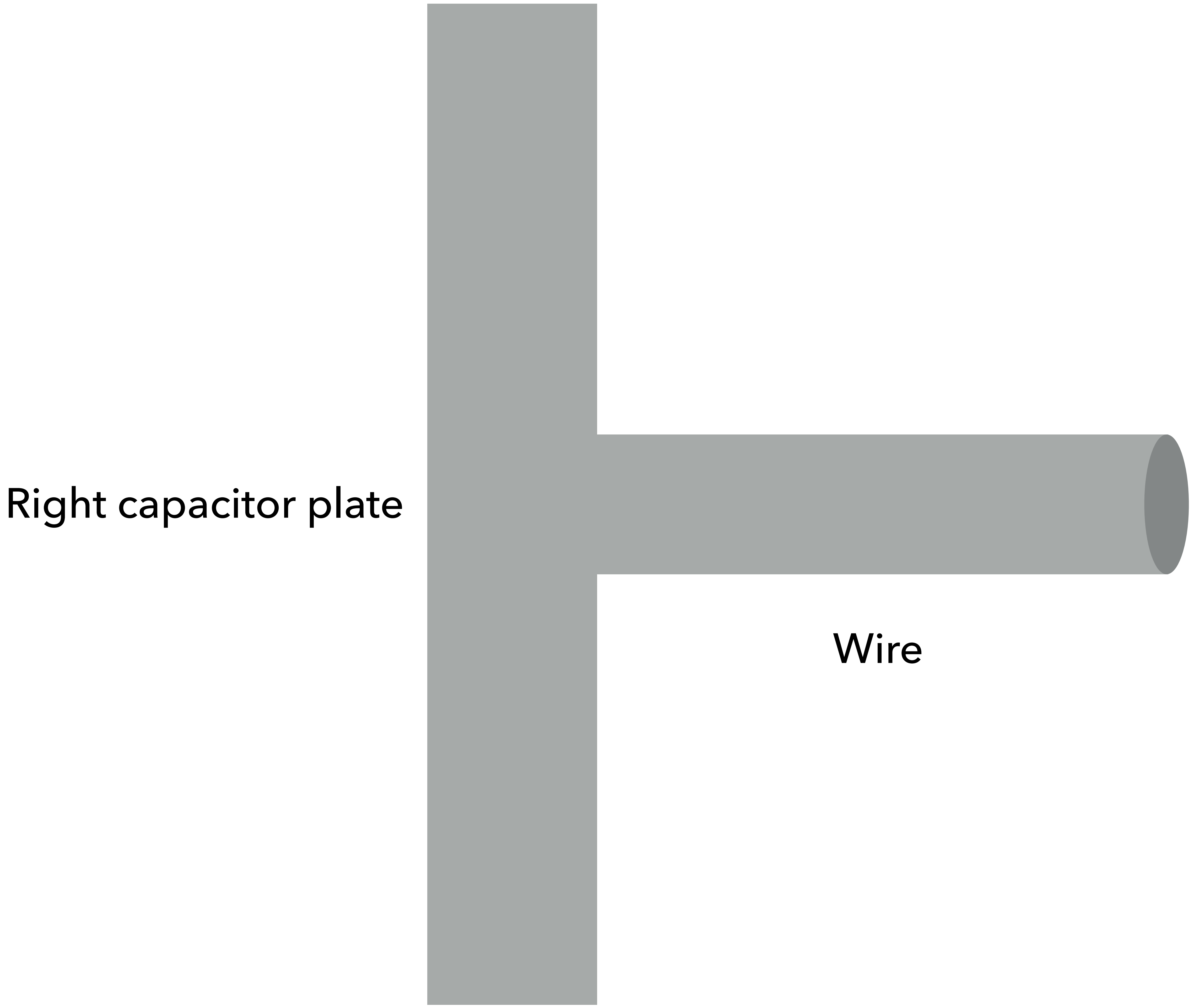
RC CIRCUIT REVISITED



RC CIRCUIT REVISITED

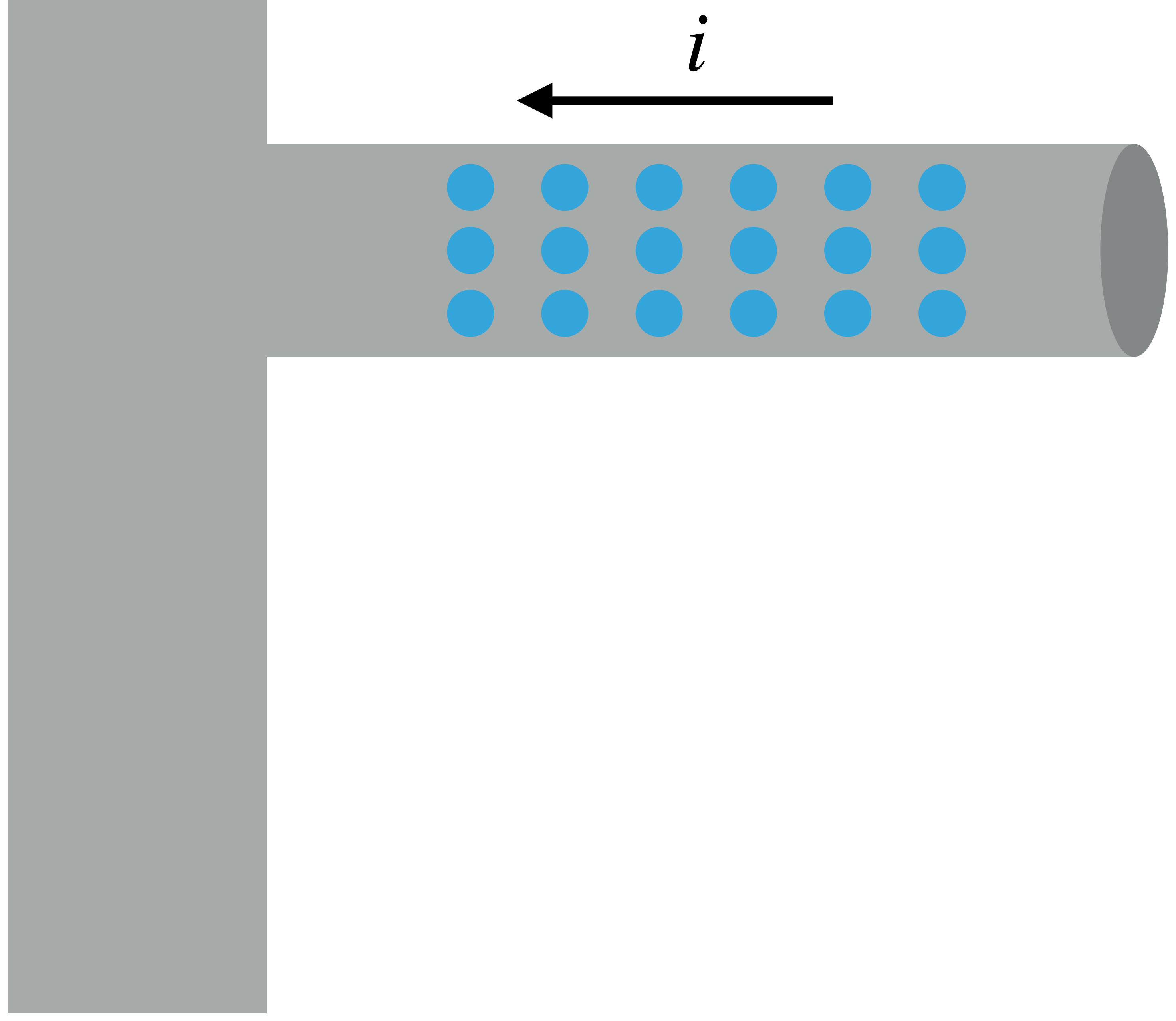


CHARGE ENTERING THE CAPACITOR



CHARGE ENTERING THE CAPACITOR

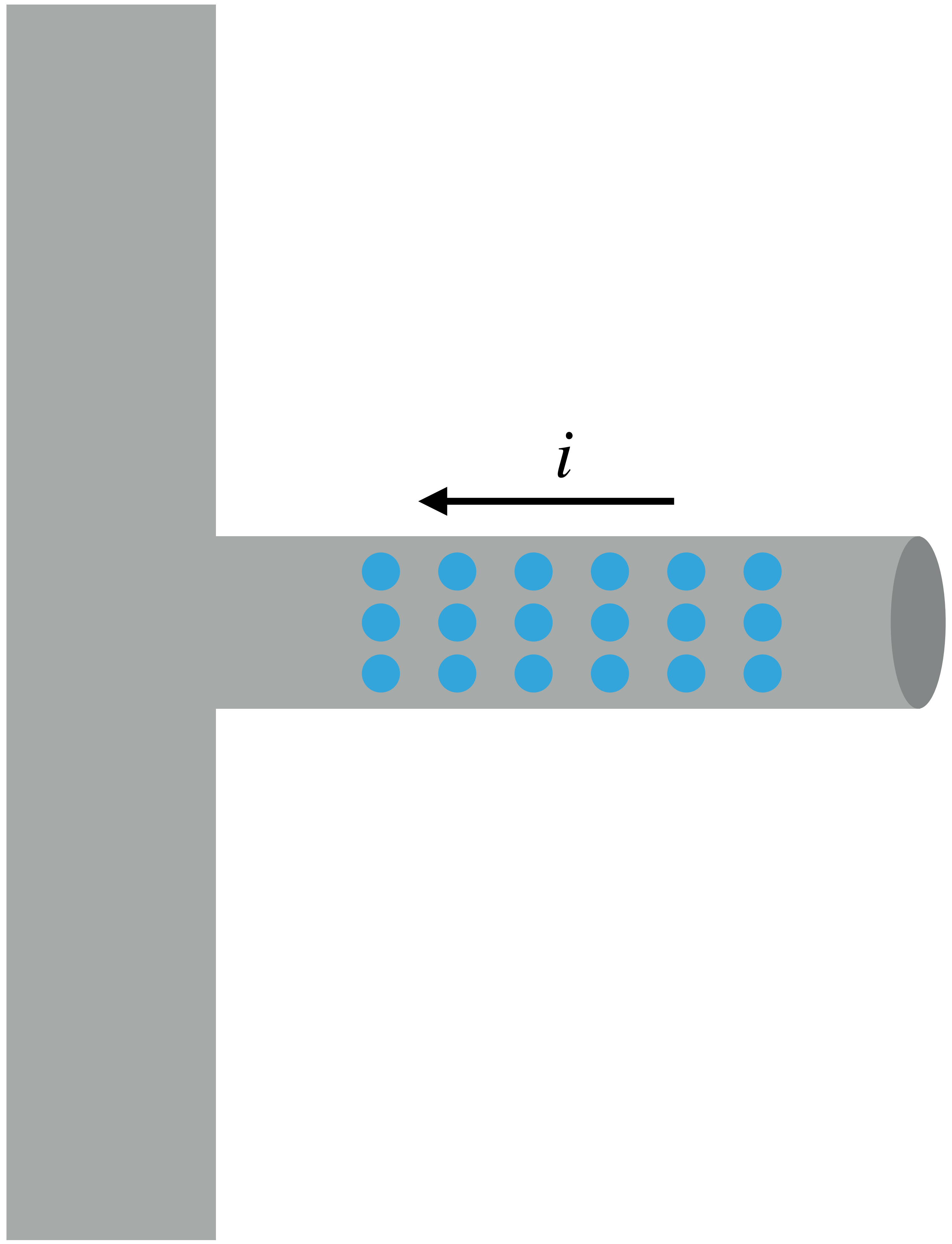
CAPACITOR IS INITIALLY UNCHARGED



CHARGE ENTERING THE CAPACITOR

After a very small time interval Δt has passed, what is the charge on the capacitor?

(Assume Δt is so small that the current is constant during this interval)

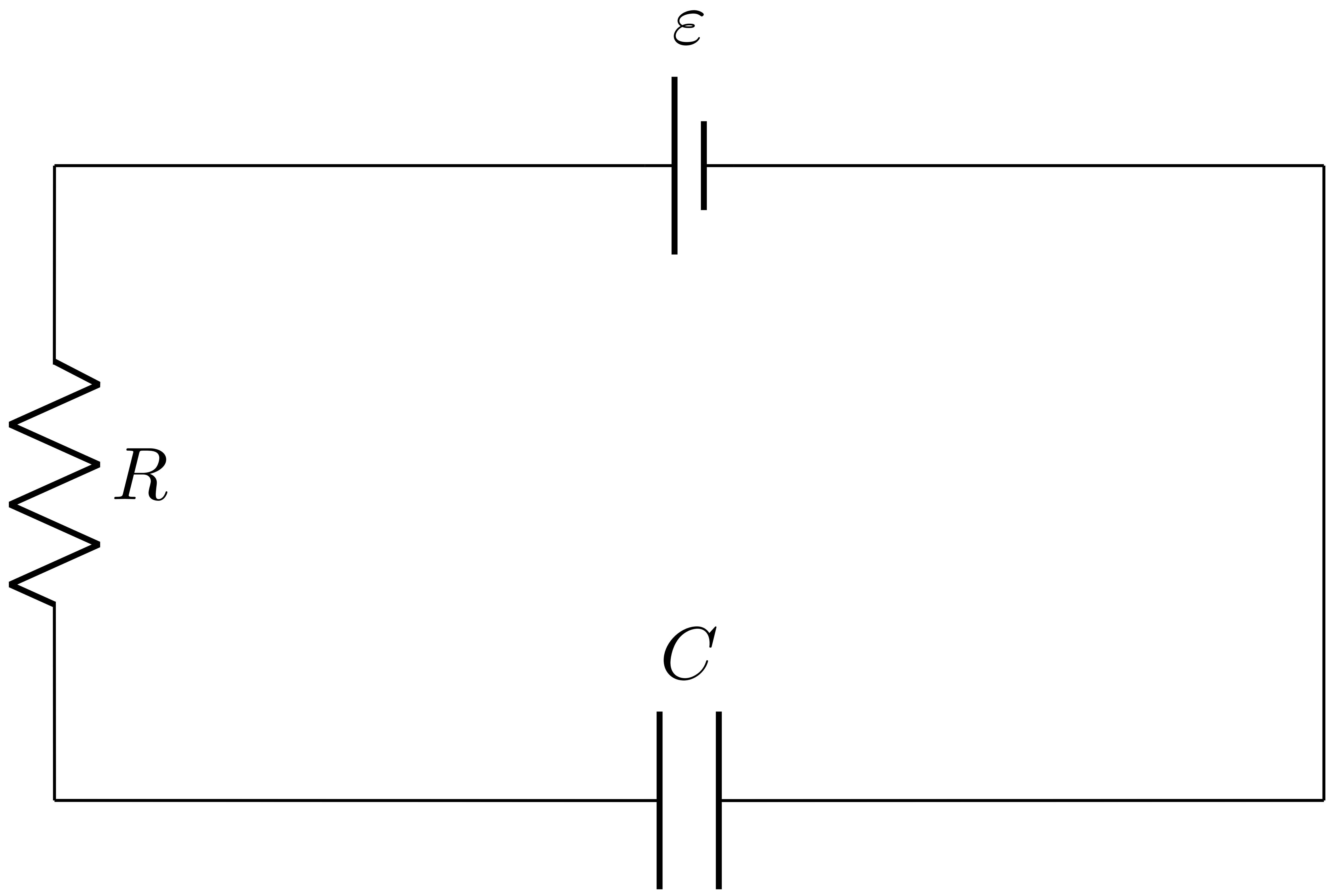


CURRENT AND CHARGE

Current is the time derivative of capacitor charge!

$$I = \frac{dQ}{dt}$$

RC CIRCUIT REVISITED



RC CIRCUIT REVISITED

Current through an RC circuit:

$$I(t) = \frac{\varepsilon}{R} e^{-\frac{t}{RC}}$$