CHAPTER 23

AMPERE'S LAW & ELECTROMAGNETIC RADIATION

MAXWELL'S EQUATIONS

* Incomplete!

Equation	Name	Explanation
$\oint \vec{E} \cdot \hat{n} dA = \frac{1}{\epsilon_0} \sum q_{\text{inside}}$	Gauss's Law for Electricity	‣ How charges produce electric fields ‣ Used to derive Coulomb's Law
$\oint \vec{B} \cdot \hat{n} dA = 0$	Gauss's Law for Magnetism	‣ No magnetic monopoles ‣ Constrains shape of magnetic field ("curly")
$\oint \vec{E} \cdot d\vec{l} = -\int \vec{B} \cdot \hat{n} dA$	Faraday's Law	‣ Curly electric field produced by time-varying magnetic field
$\oint \vec{B} \cdot d\vec{l} = \mu_0 \sum I_{\text{inside}}$	Ampere's Law*	‣ How currents produce magnetic fields ‣ Used to derive Biot-Savart Law

FIXING AMPERE'S LAW

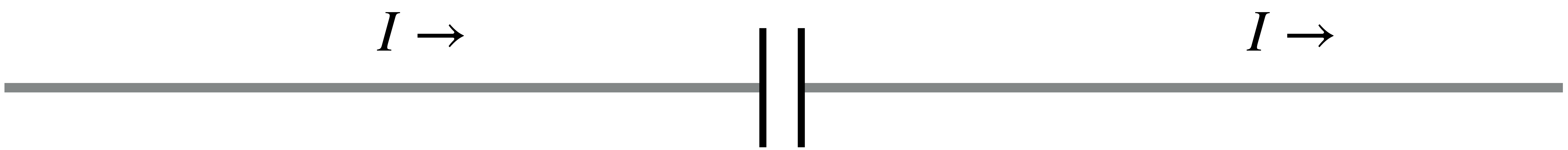
- Faraday's Law: time-varying $\overrightarrow{B}$ field $\rightarrow$ $\overrightarrow{E}$ field

FIXING AMPERE'S LAW

- Faraday's Law: time-varying $\overrightarrow{B}$ field $\rightarrow$ $\overrightarrow{E}$ field
- Does a time-varying $\overrightarrow{E}$ field produce a $\overrightarrow{B}$ field?

CONSIDER

- Long, current carrying wire with a capacitor



MAXWELL'S EQUATIONS (COMPLETE)

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$\oint \vec{E} \cdot \hat{n} dA = \frac{1}{\epsilon_0} \sum q_{\text{inside}}$	Gauss's Law for Electricity	‣ How charges produce electric fields ‣ Used to derive Coulomb's Law
$\oint \vec{B} \cdot \hat{n} dA = 0$	Gauss's Law for Magnetism	‣ No magnetic monopoles ‣ Constrains shape of magnetic field ("curly")
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$\oint \vec{B} \cdot d\vec{l} = \mu_0 \sum I_{\text{inside}} + \epsilon_0 \frac{d}{dt} \int \vec{E} \cdot \hat{n} dA$	Ampere-Maxwell Law	‣ How currents (and electric fields!) produce magnetic fields ‣ Used to derive Biot-Savart Law

ALL OF ELECTROMAGNETISM IN ONE SLIDE

How The Fields are Produced

$$\oint \vec{E} \cdot \hat{n} dA = \frac{1}{\epsilon_0} \sum q_{\text{inside}}$$

$$\oint \vec{B} \cdot \hat{n} dA = 0$$

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \int \vec{B} \cdot \hat{n} dA$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \left(\sum I_{\text{inside}} + \epsilon_0 \frac{d}{dt} \int \vec{E} \cdot \hat{n} dA \right)$$

How the fields effect matter

$$\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

MAXWELL'S EQUATIONS (DIFFERENTIAL FORM)

$$\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = \frac{1}{\epsilon_0}\rho$$

$$\frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} = 0$$

$$\left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right)\hat{x} + \left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right)\hat{y} + \left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right)\hat{z} = -\frac{\partial \vec{B}}{\partial t}$$

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SOLUTIONS TO MAXWELL'S EQUATIONS

- Typically, we specify a charge/current distribution and apply Maxwell's equations to find $\overrightarrow{E}$ and $\overrightarrow{B}$

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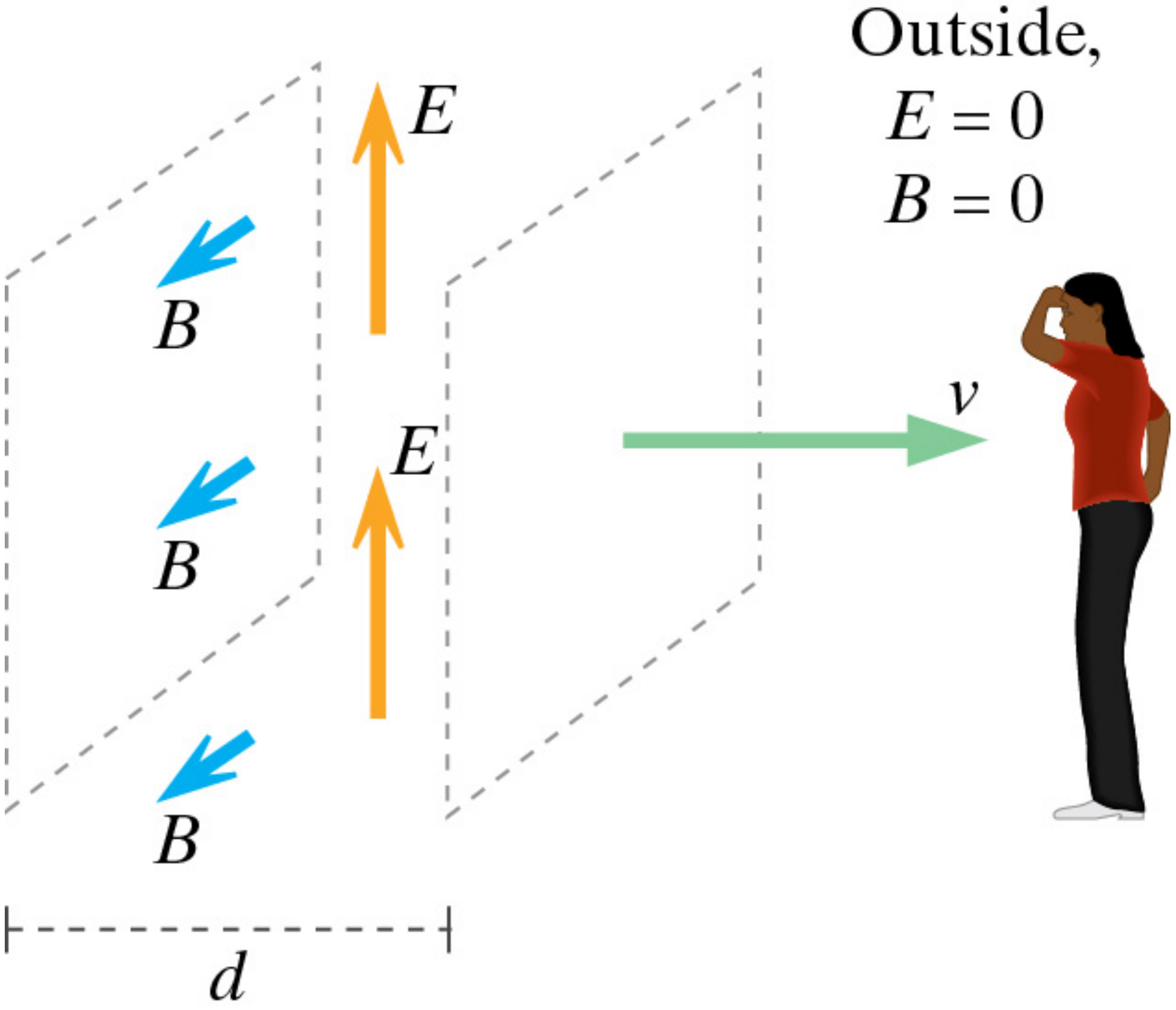
SOLUTIONS TO MAXWELL'S EQUATIONS

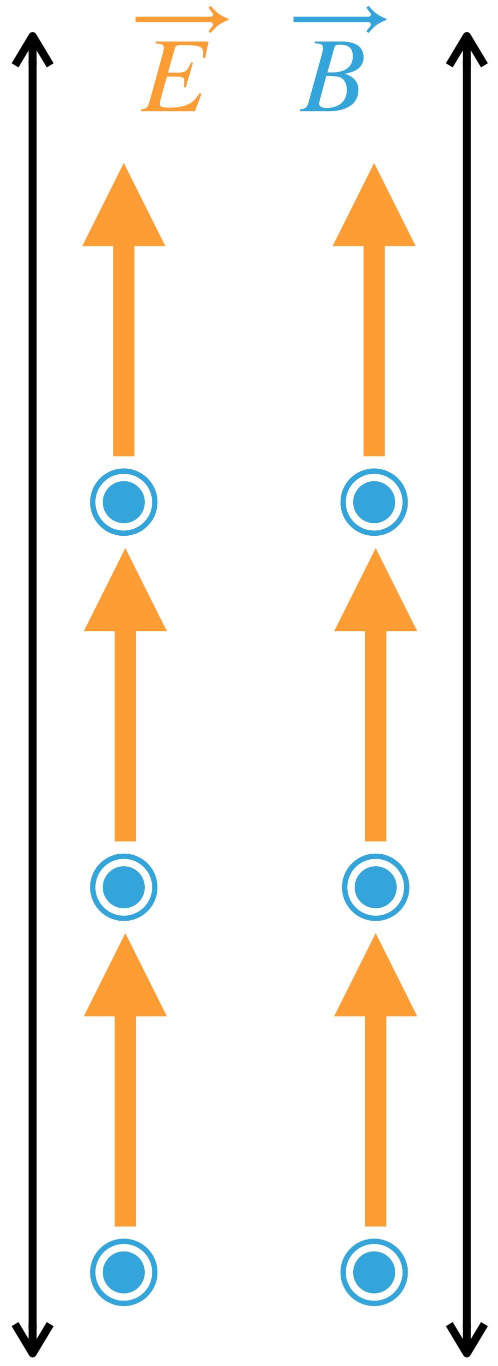
- Typically, we specify a charge/current distribution and apply Maxwell's equations to find $\overrightarrow{E}$ and $\overrightarrow{B}$
- Special case: can there be nonzero $\overrightarrow{E}$ and $\overrightarrow{B}$ fields without charge or current?
  - Yes
  - We will not solve Maxwell's equations
  - I will suggest a solution, and we will show that it *satisfies* Maxwell's equations

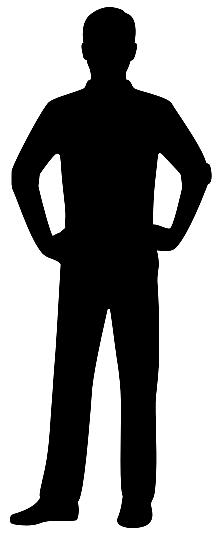
CONSIDER

- Inside slab: uniform $\vec{E}$ pointing up, uniform $\vec{B}$ pointing out
- Outside slab: $\vec{E} = \vec{B} = 0$
- Slab moving to right with speed v



TO A STATIONARY OBSERVER





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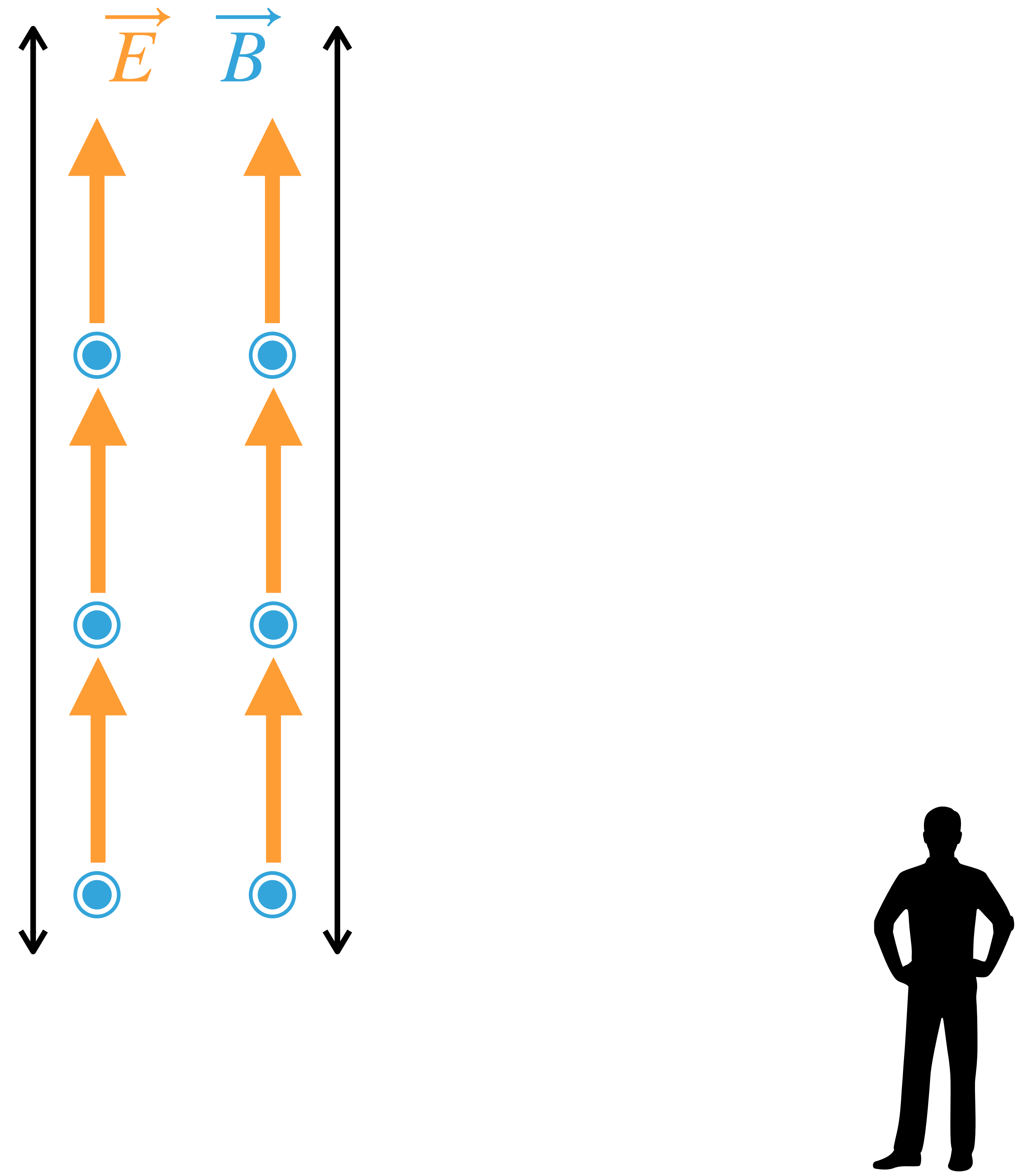

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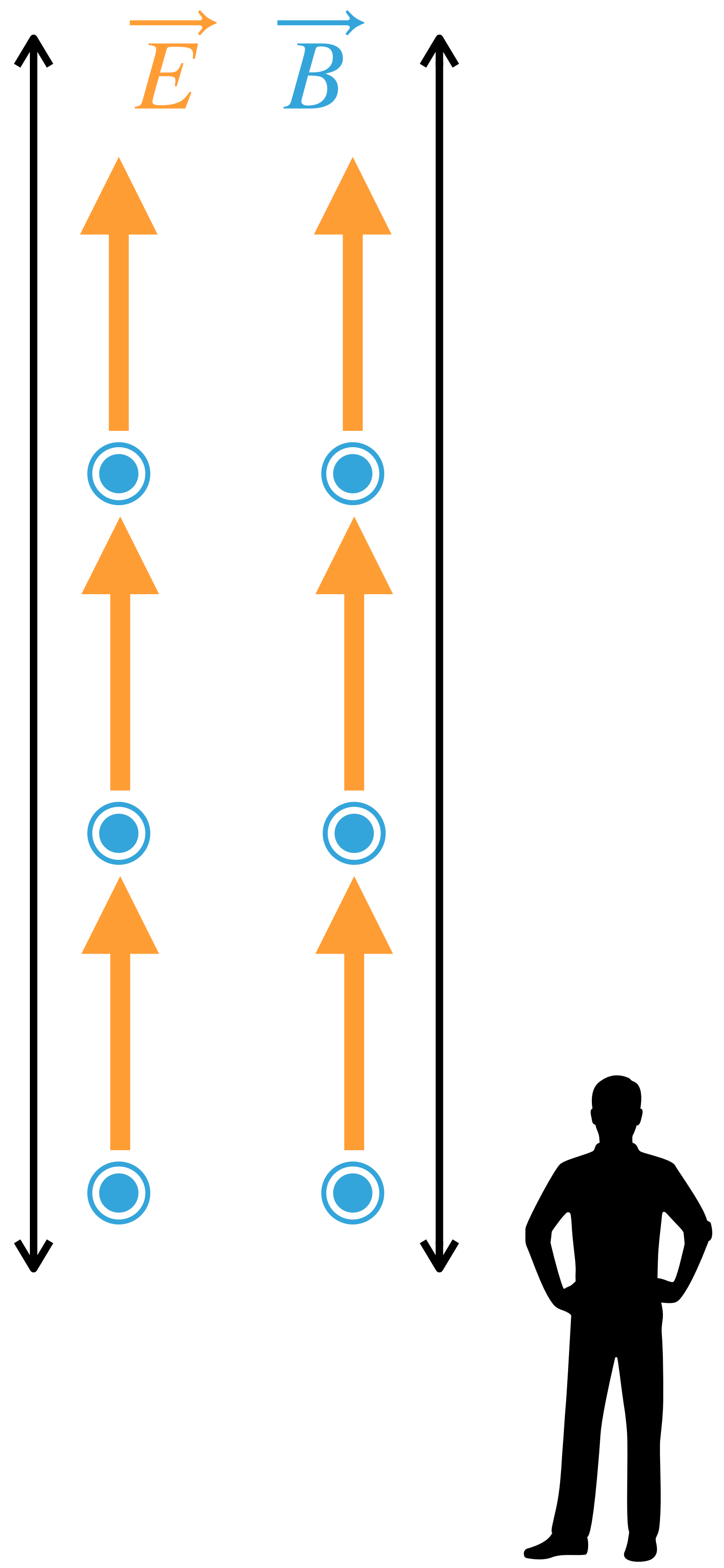

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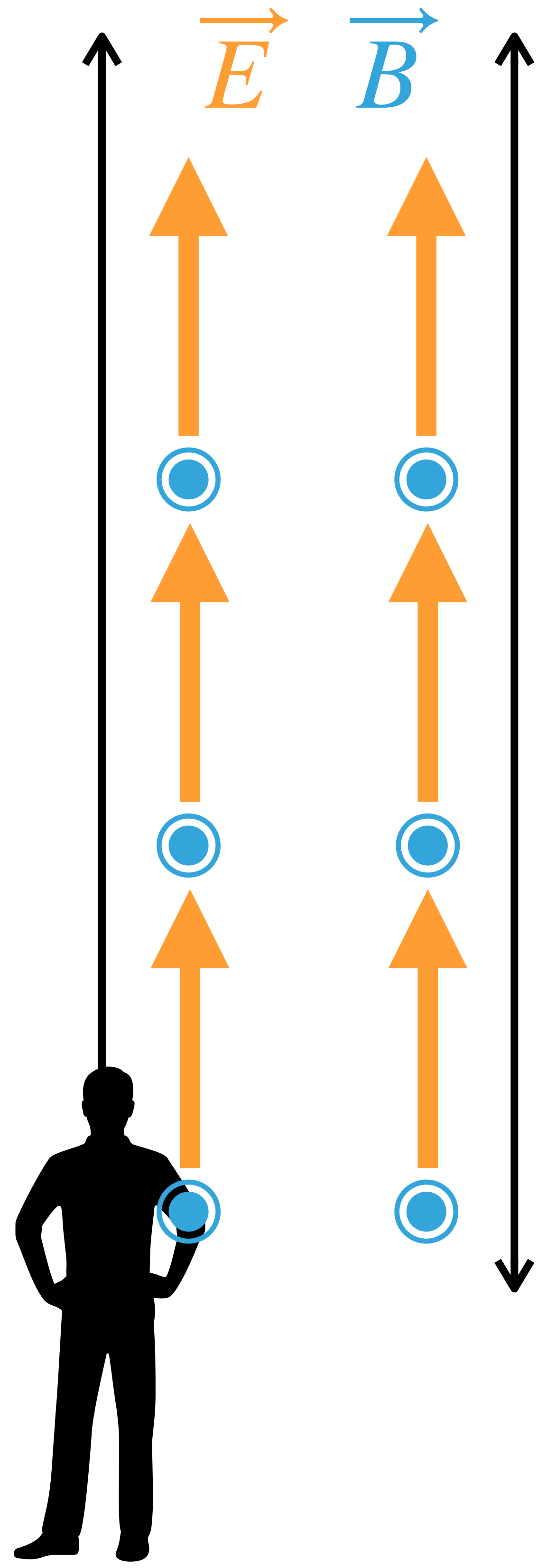

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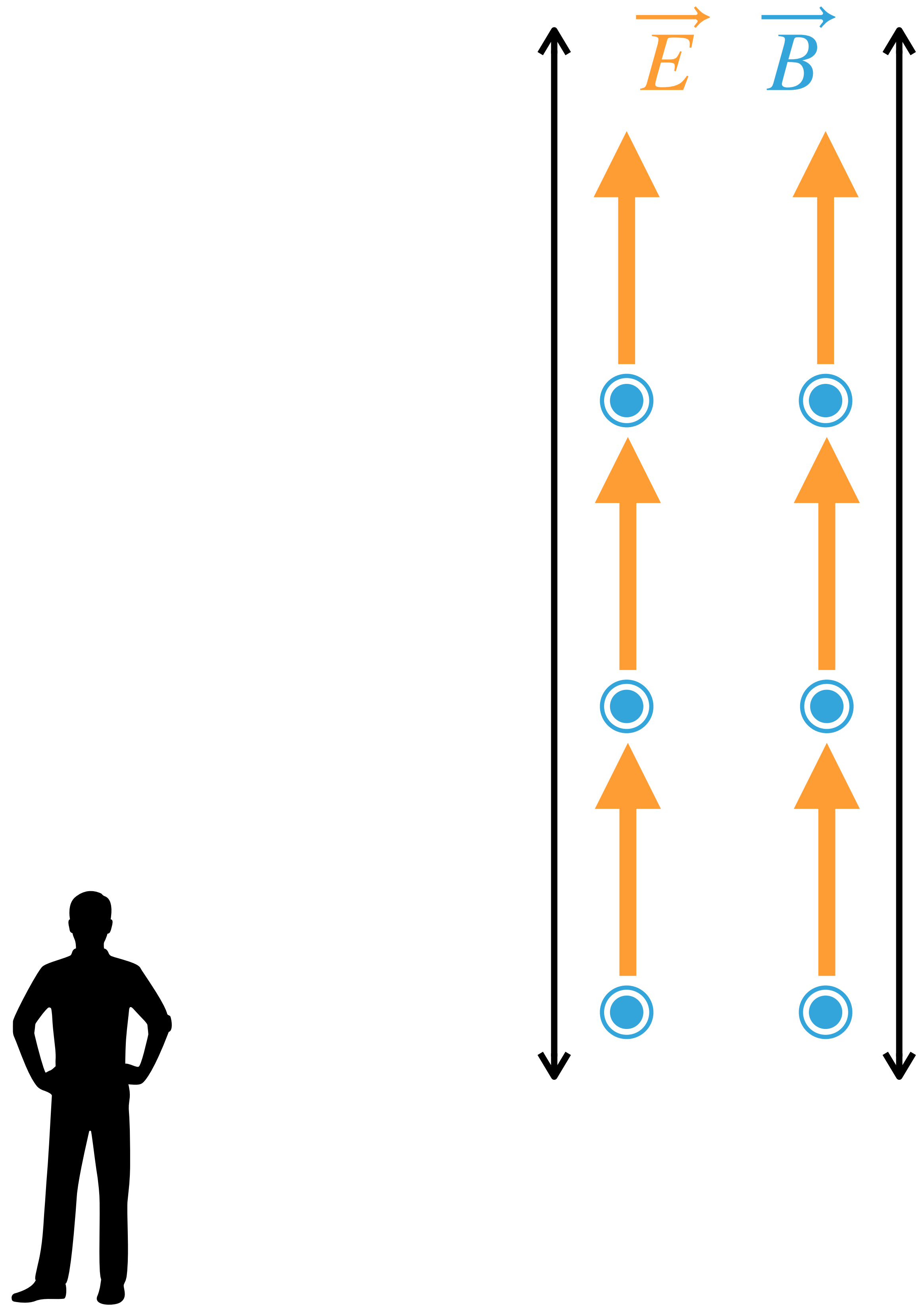

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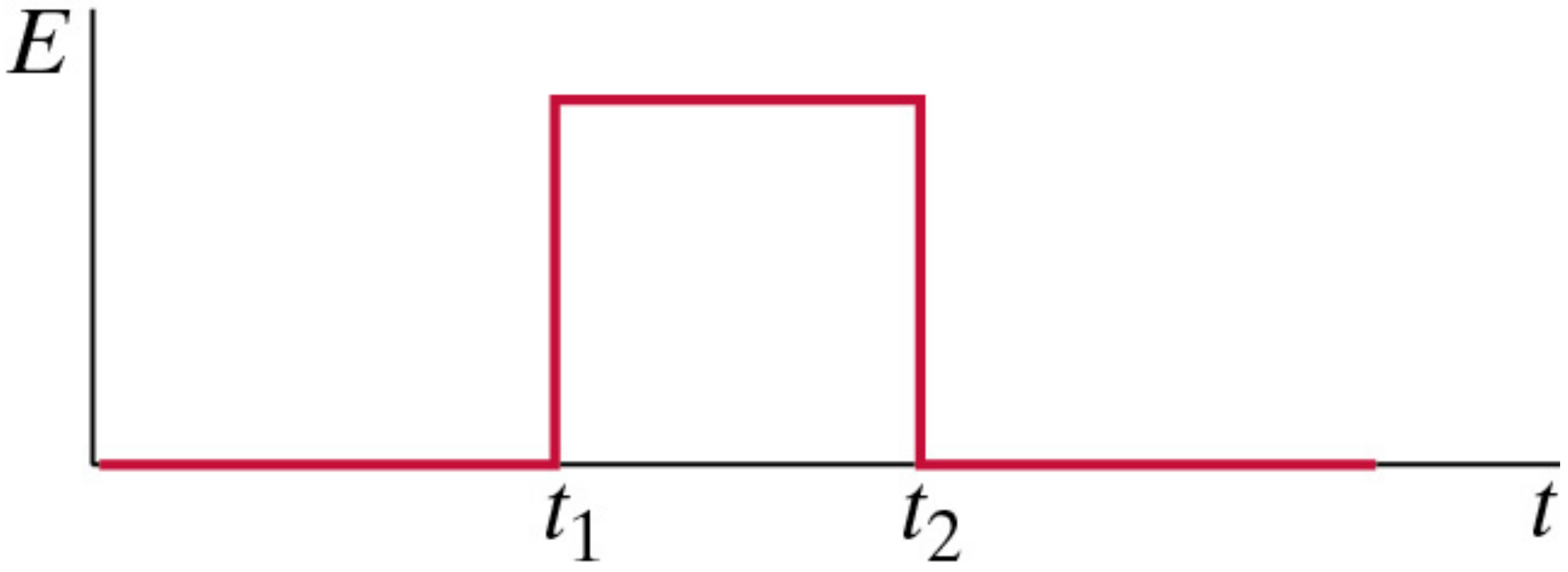
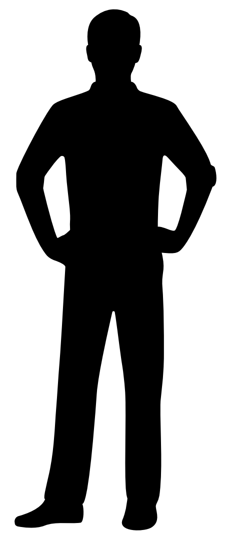
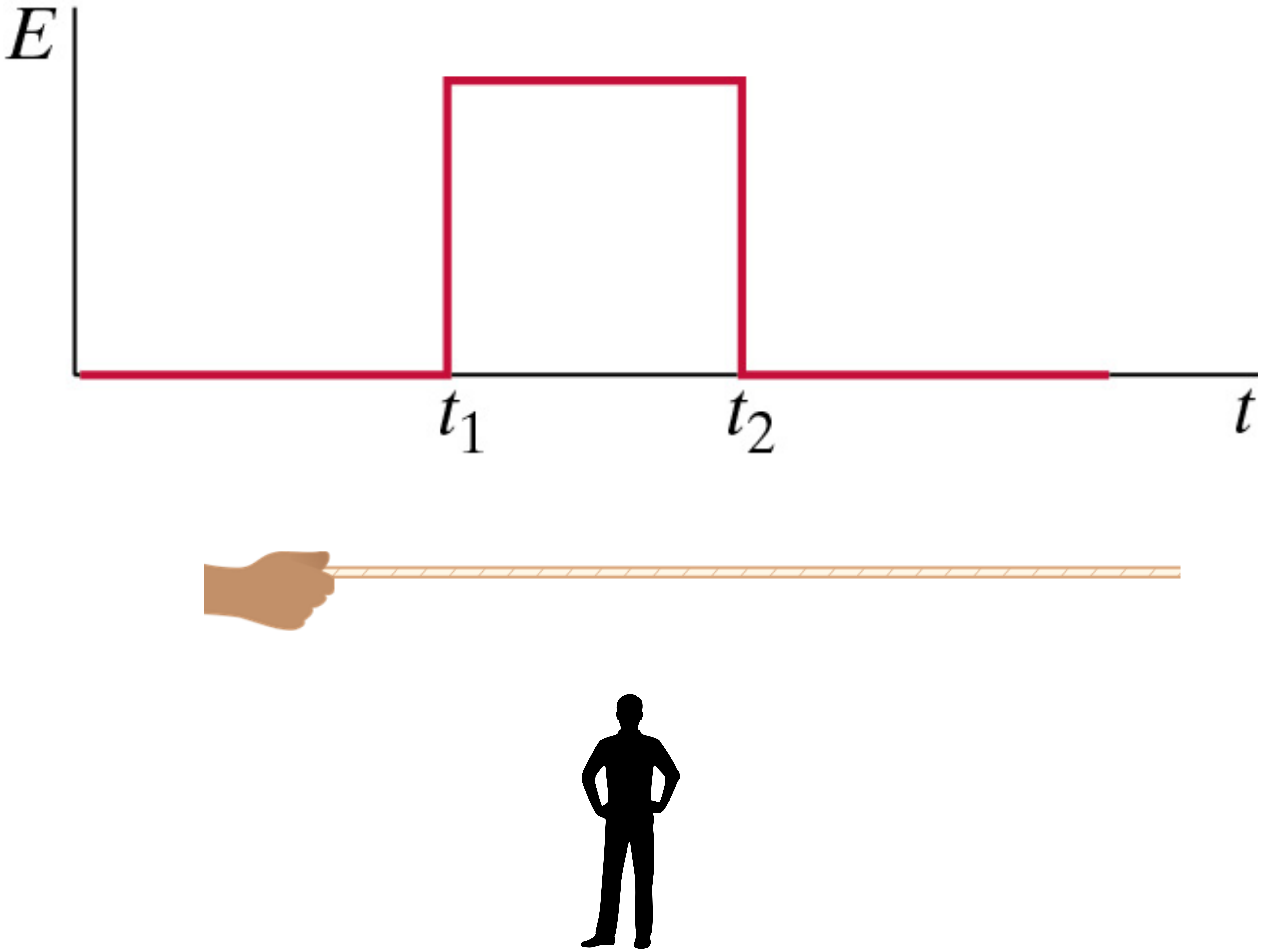




AN ELECTRO-MAGNETIC WAVE



MAXWELL'S EQUATIONS

$$\oint \vec{E} \cdot \hat{n} dA = \frac{1}{\epsilon_0} \sum q_{\text{inside}}$$

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$$\oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \int \vec{B} \cdot \hat{n} dA$$

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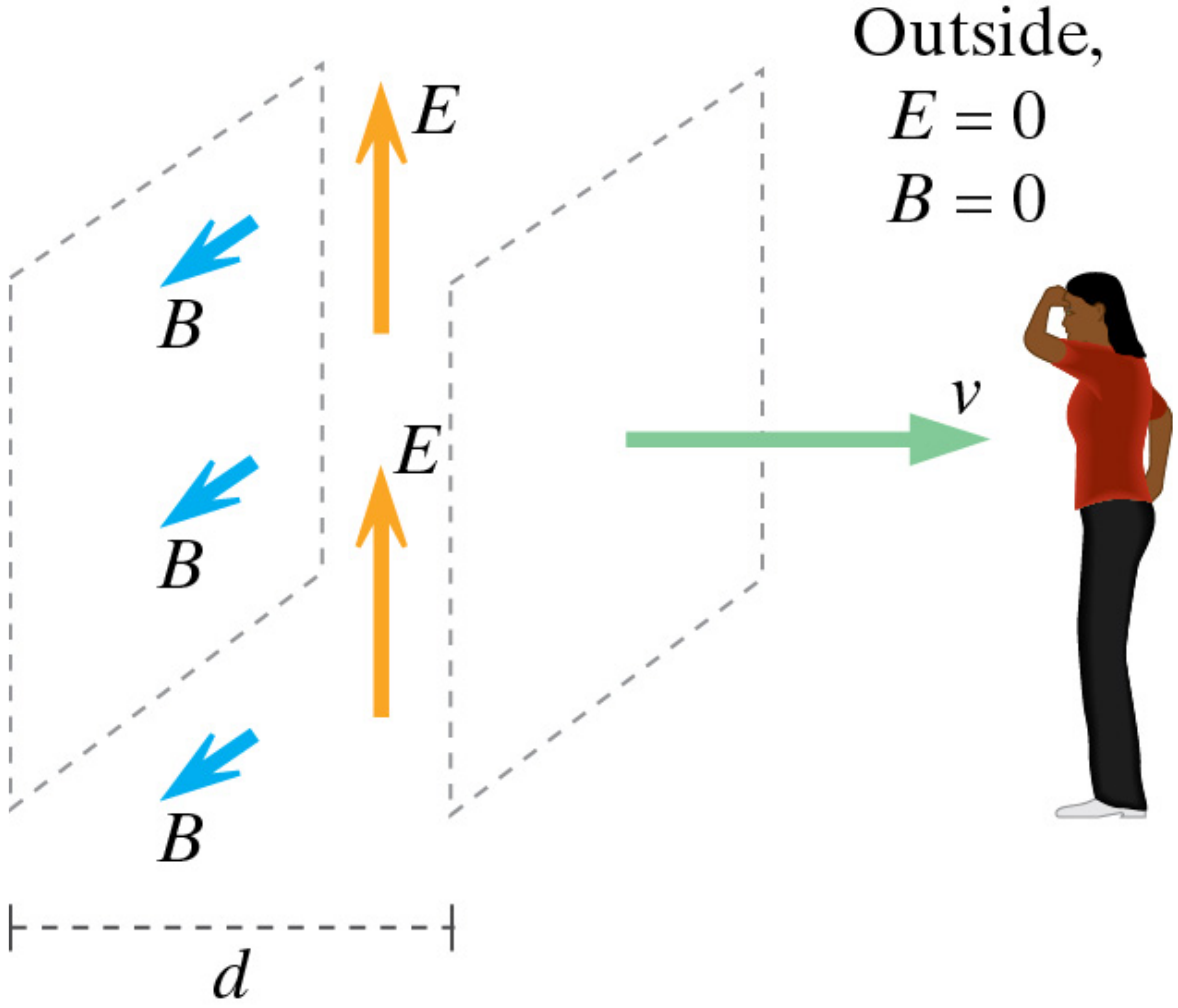
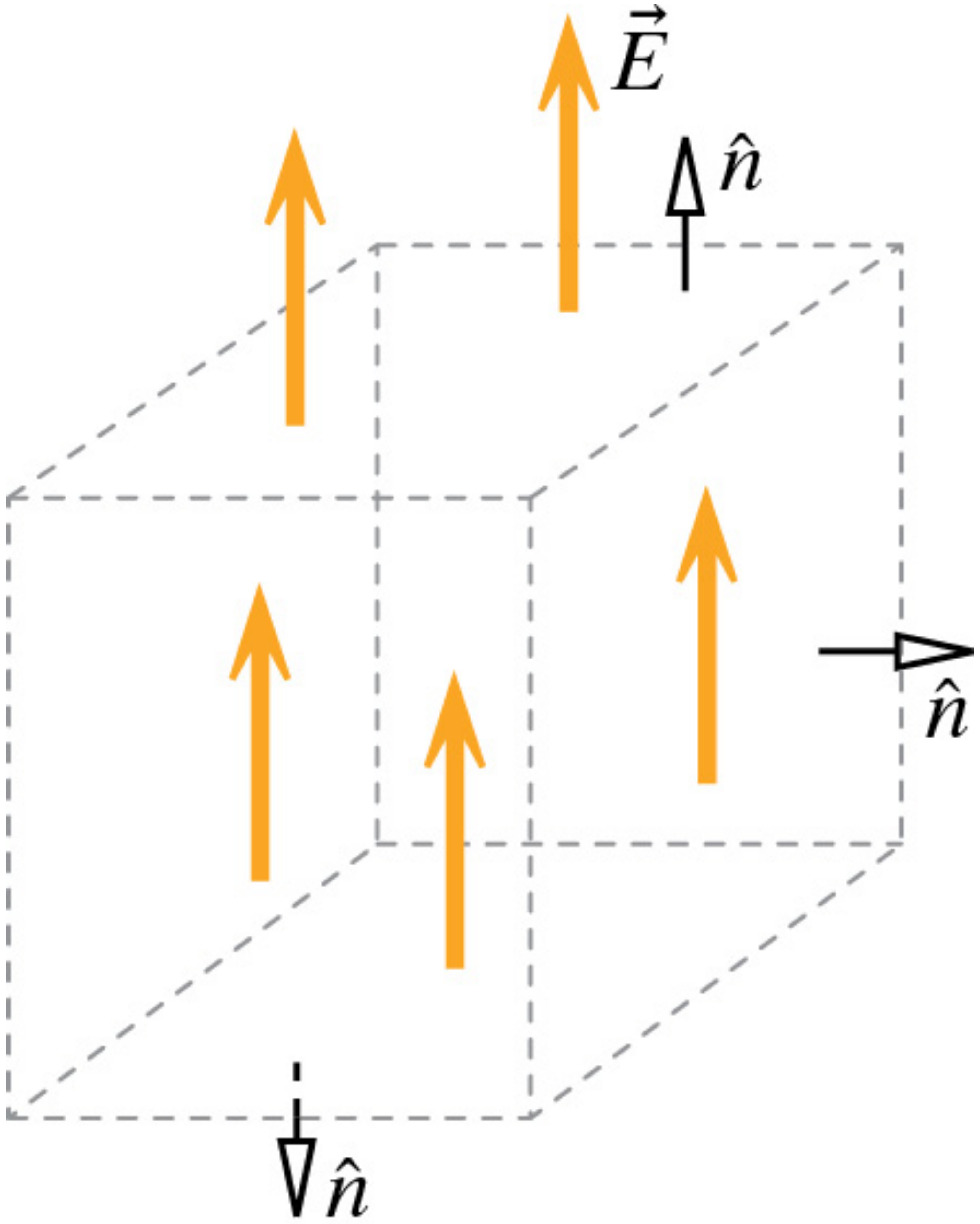


$$\oint \overrightarrow{E} \cdot \hat{n} dA \stackrel{?}{=} \frac{1}{\epsilon_0} \sum q_{\text{inside}}$$

$$0 = 0$$



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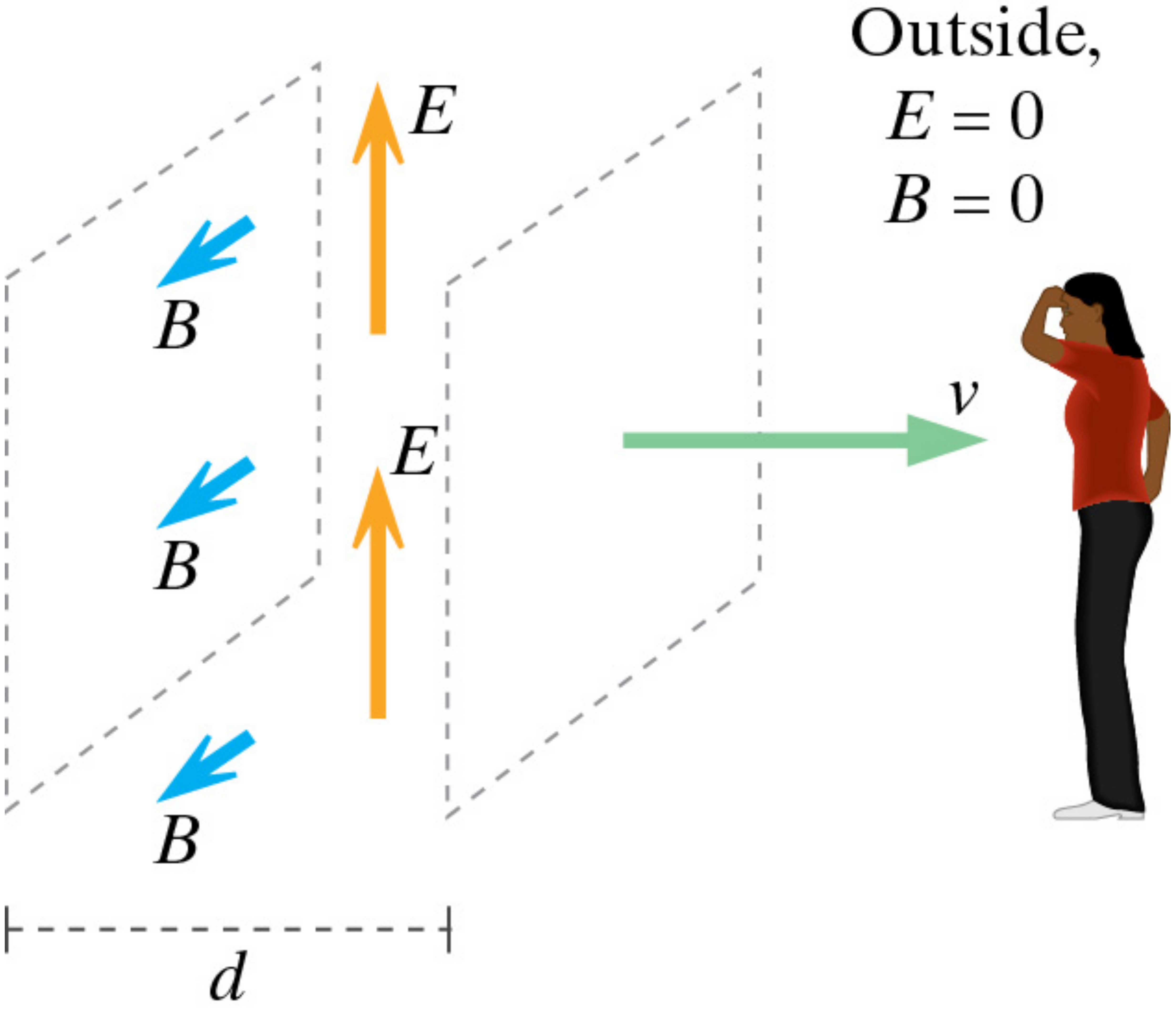

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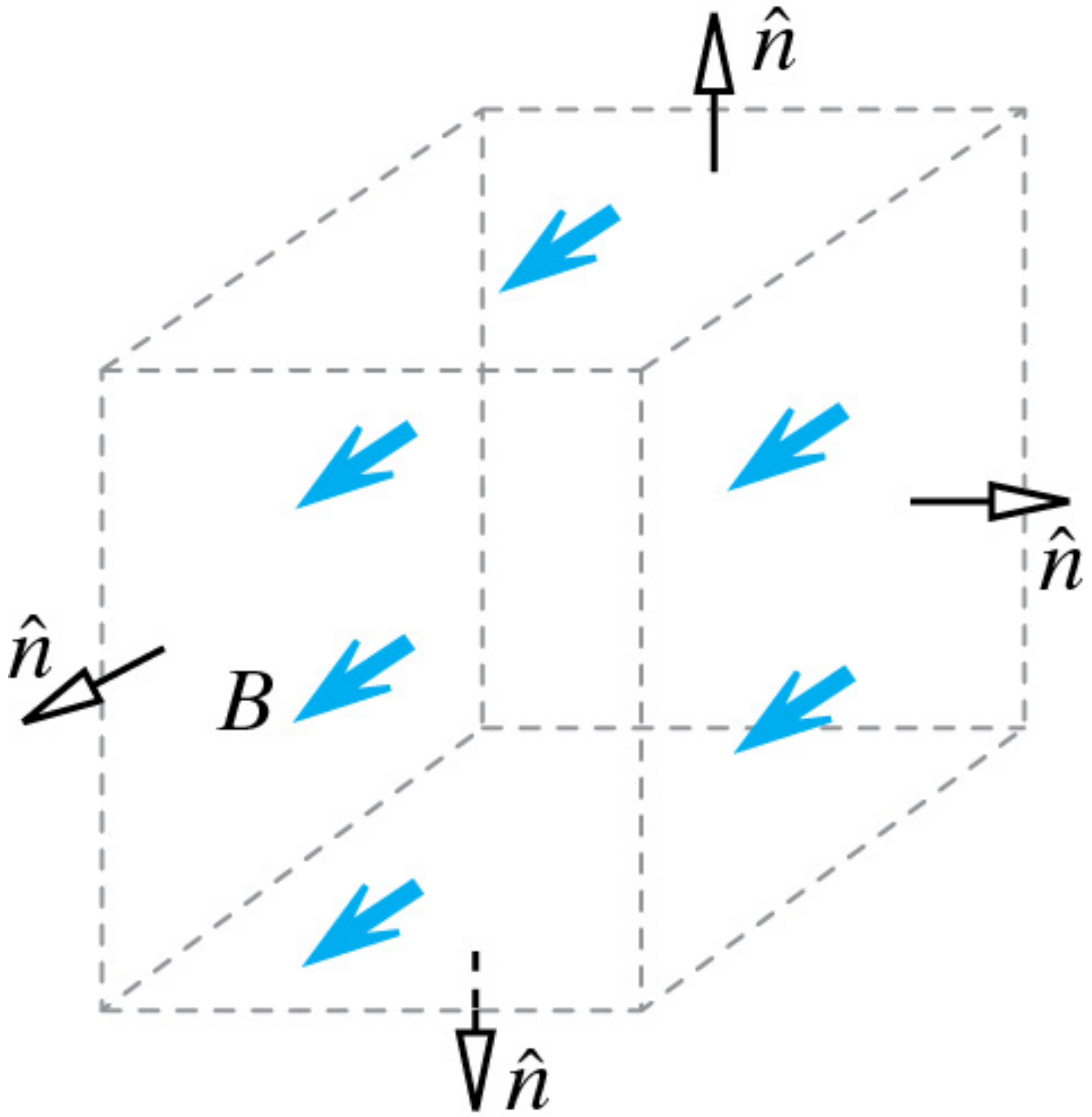

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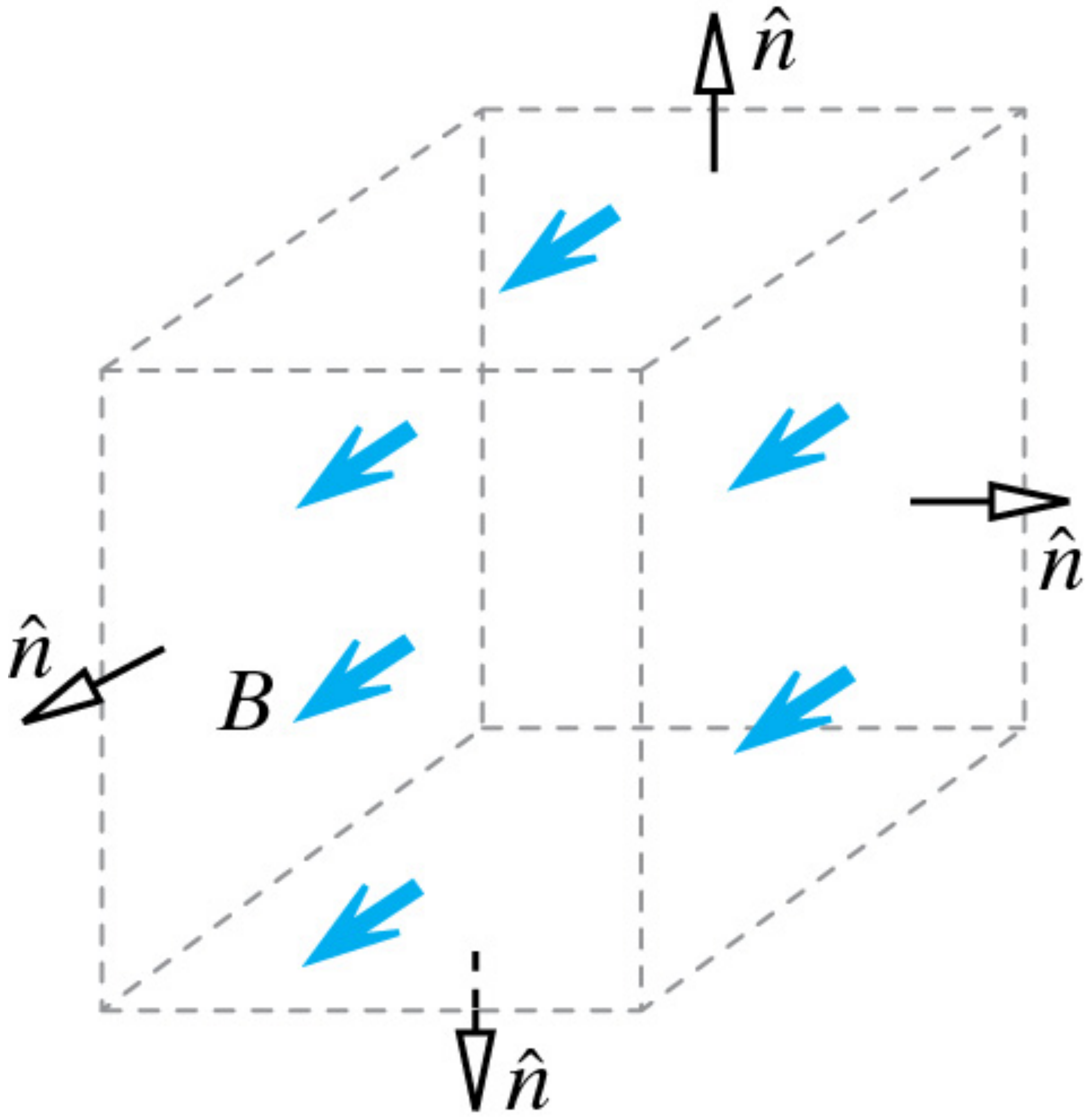

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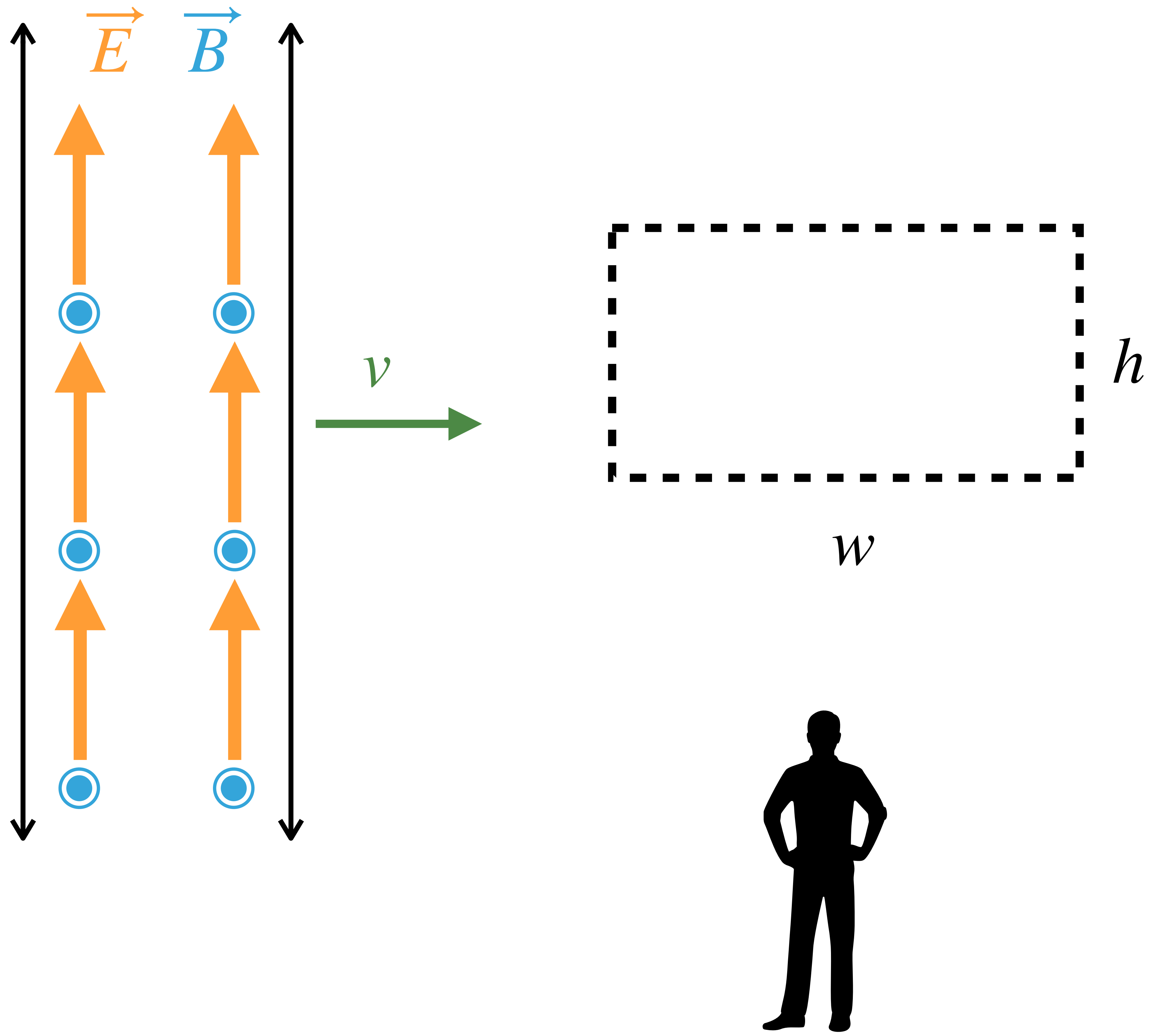

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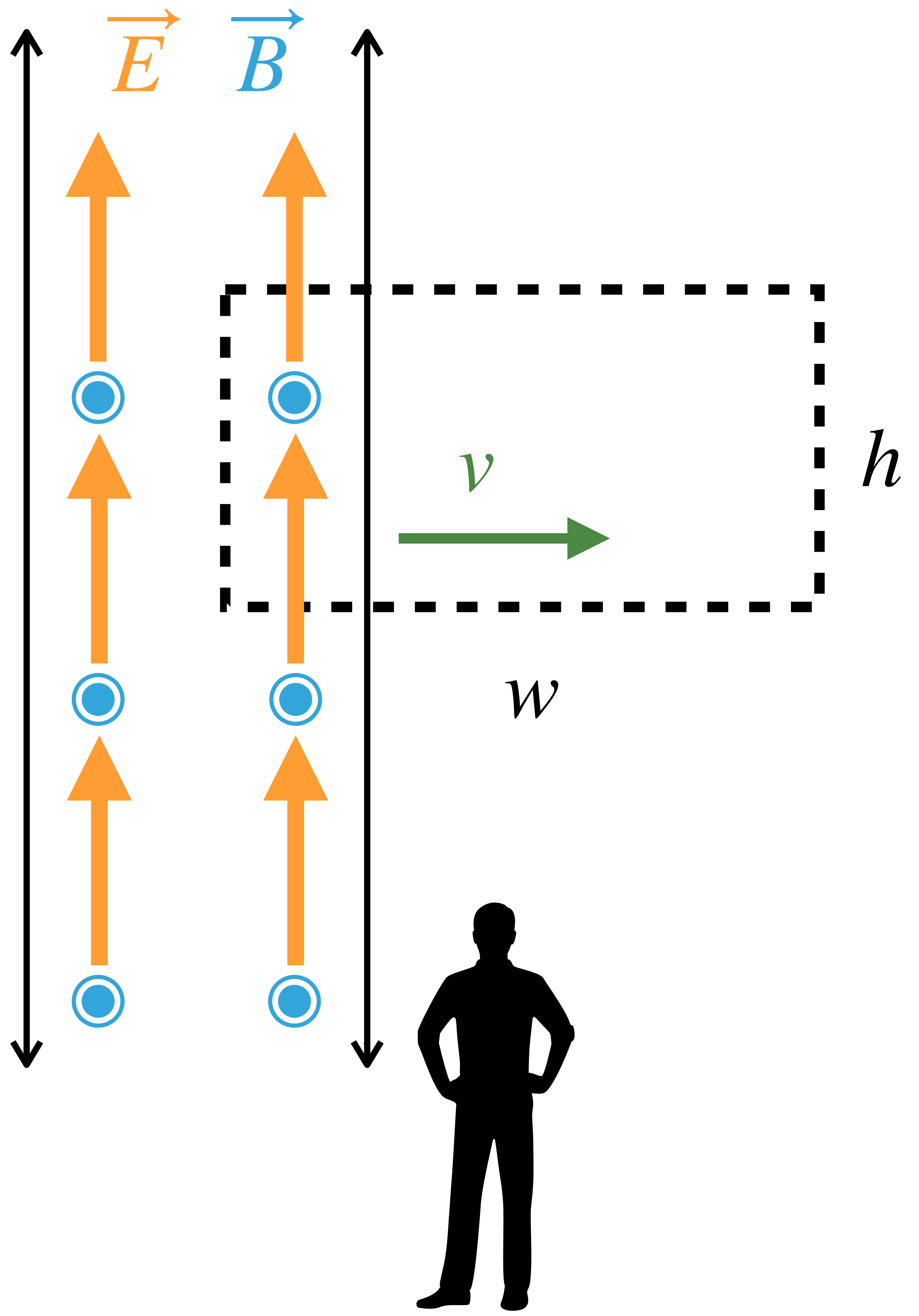

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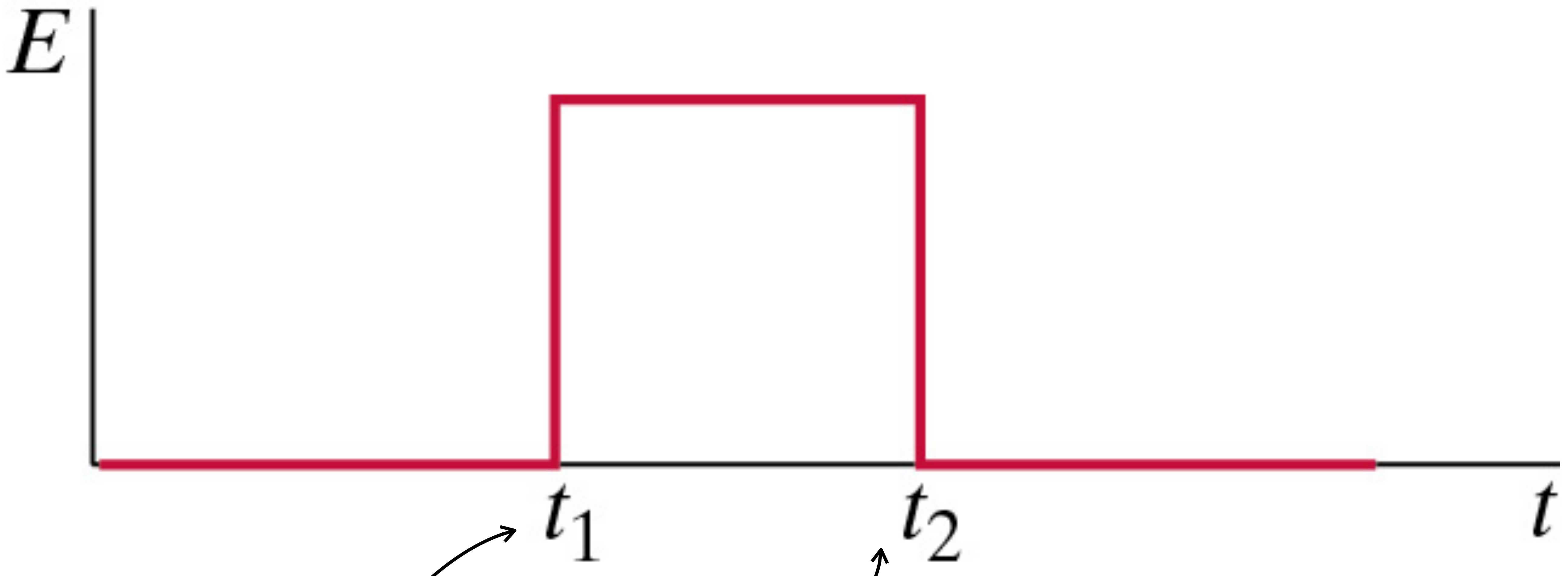
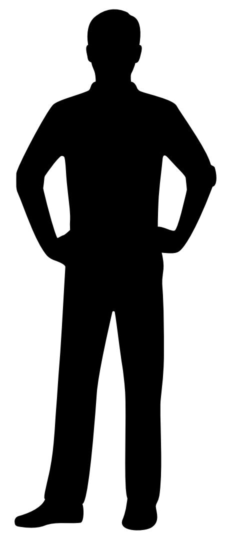


$$\frac{d\vec{E}}{dt} \neq 0$$



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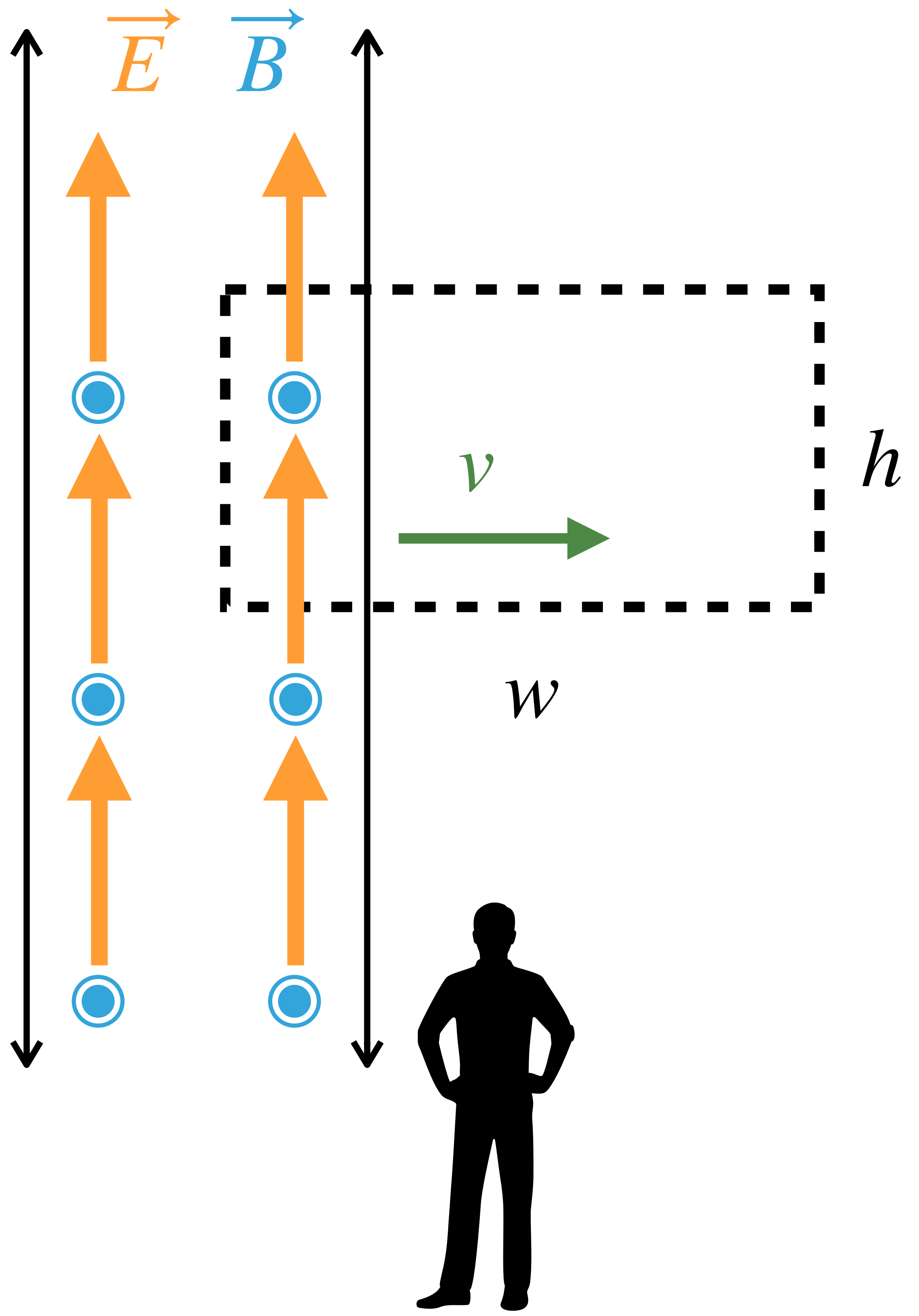

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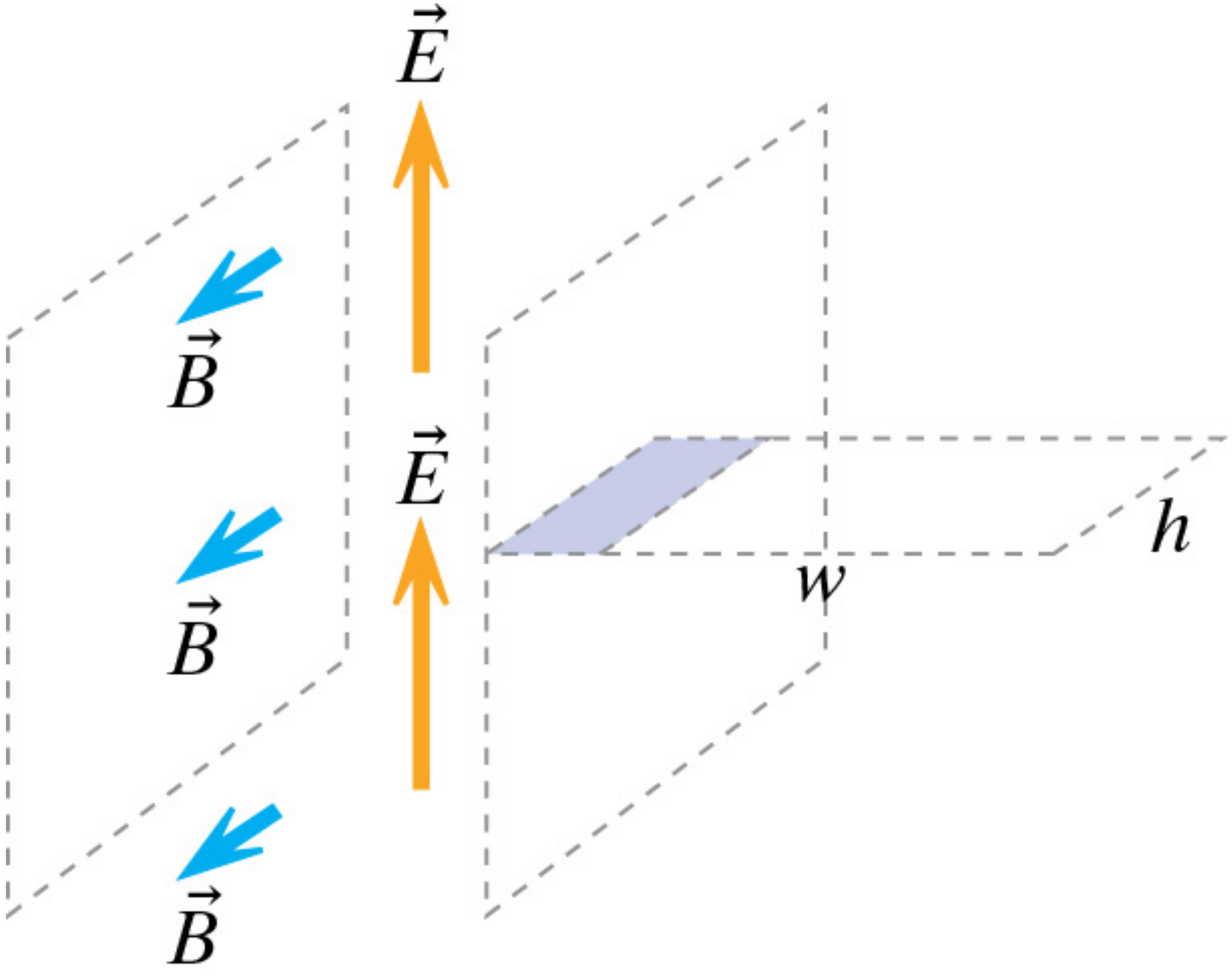

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ELECTROMAGNETIC RADIATION (LIGHT!)

Electro-magnetic radiation: a propagating disturbance in $\overrightarrow{E}$ and $\overrightarrow{B}$ fields

Light!

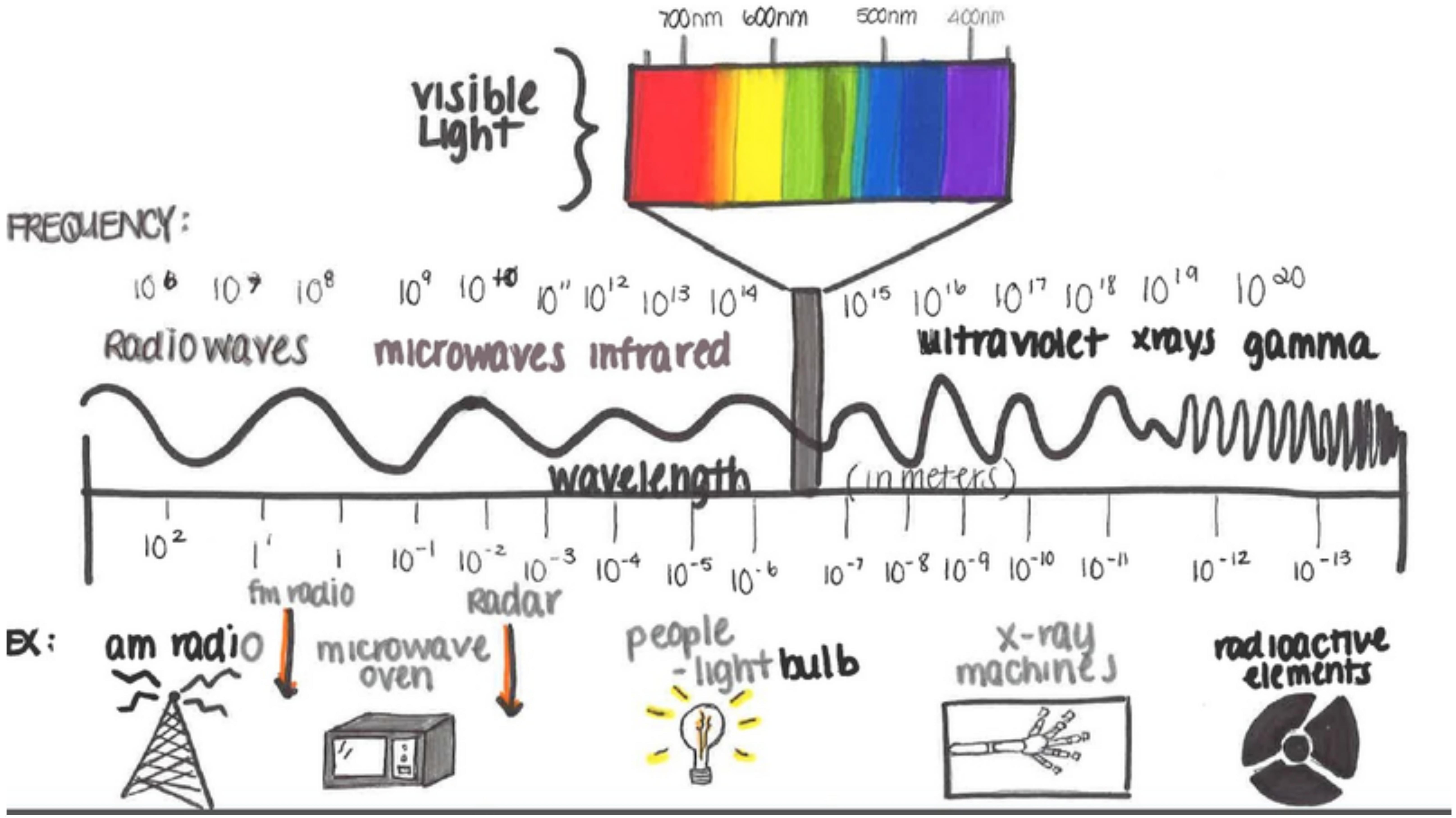
ELECTROMAGNETIC RADIATION (LIGHT!)

Electromagnetic radiation:

- $\overrightarrow{E}$ and $\overrightarrow{B}$ at right angles
- $E = cB$
- Direction of wave propagation:
  $\overrightarrow{E} \times \overrightarrow{B}$
- Speed of wave propagation:
  $c = 3 \times 10^8$ m/s

ELECTROMAGNETIC waves :

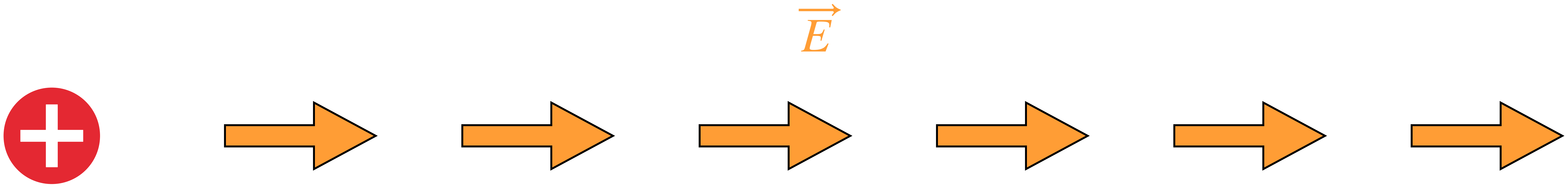
a wave produced by the acceleration of an electric charge and propagated by the periodic variation of intensities of usually perpendicular electric and magnetic fields.



HOW TO PRODUCE ELECTROMAGNETIC RADIATION

- The most basic way of *producing* this combination of fields is to **accelerate a charged particle**

HOW TO PRODUCE ELECTROMAGNETIC RADIATION



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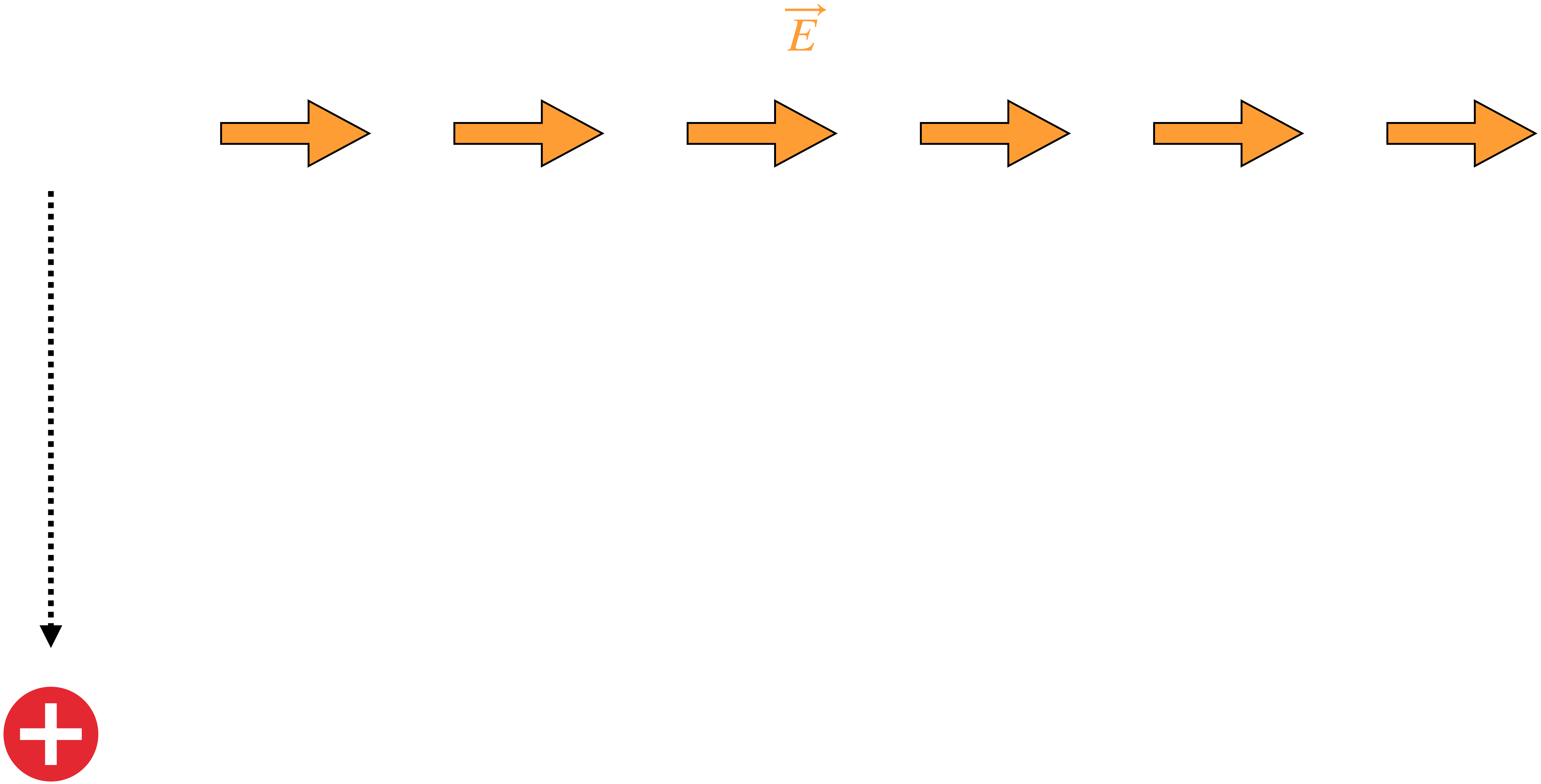

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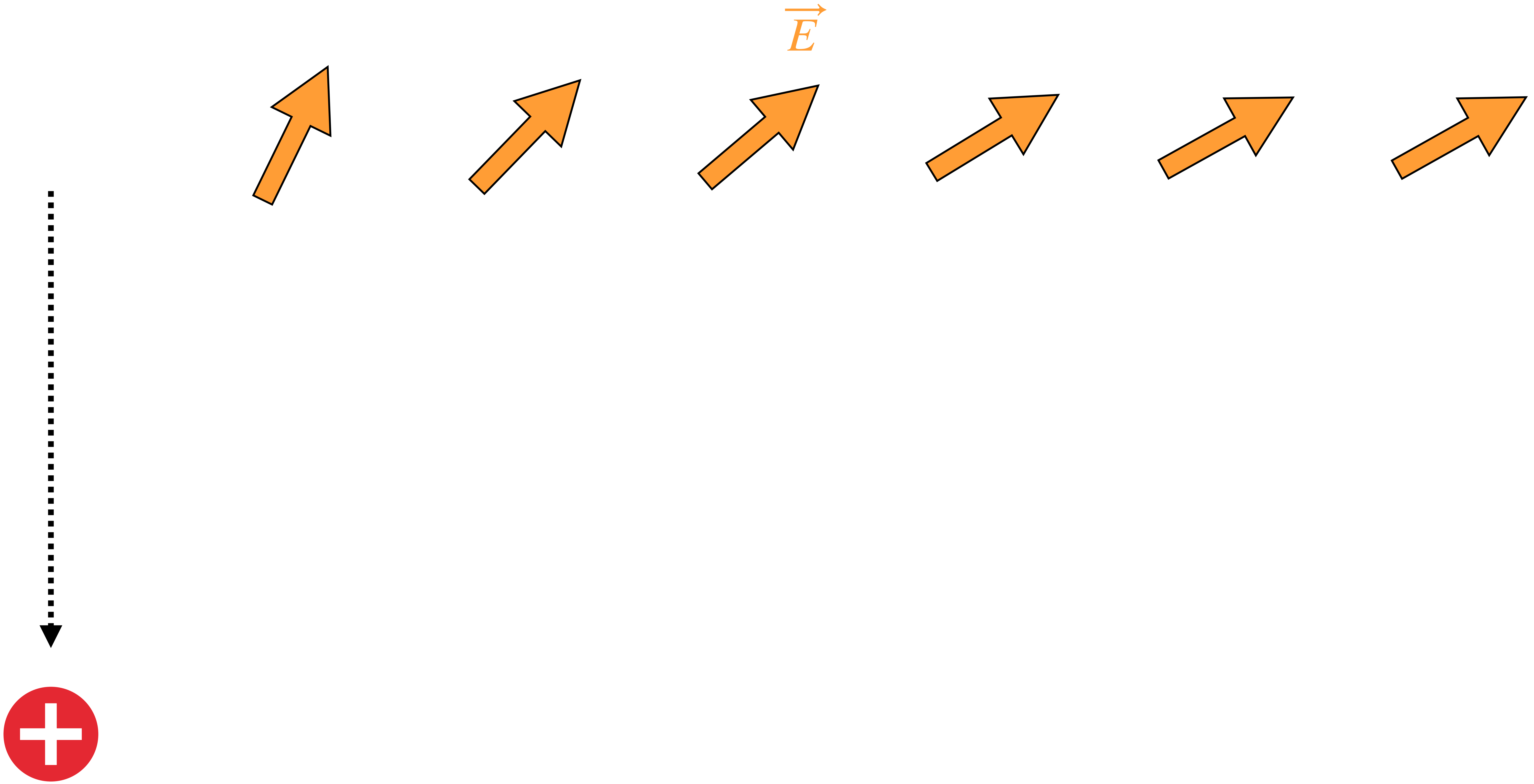

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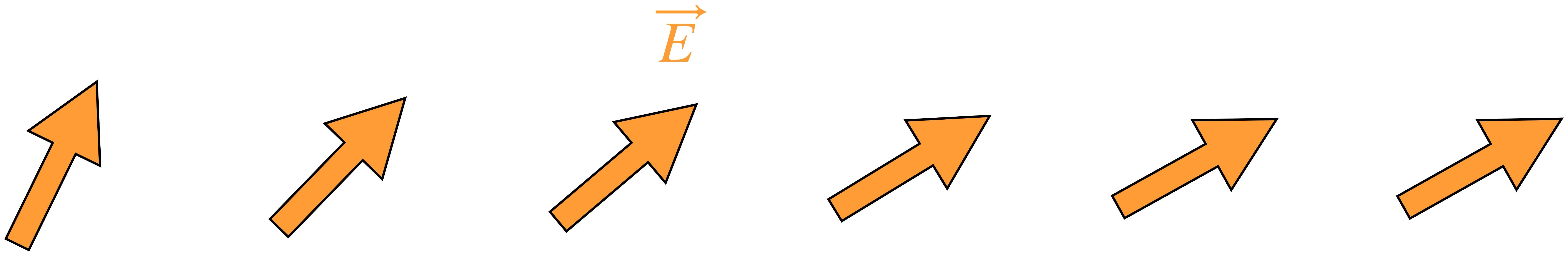
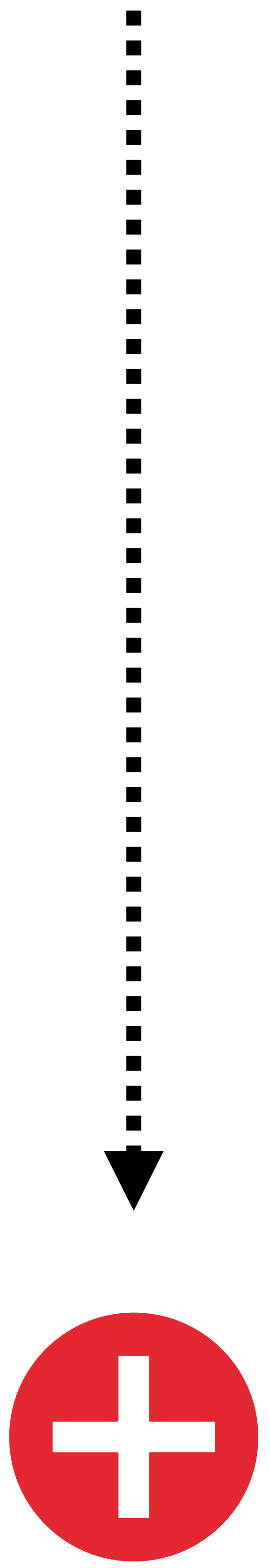
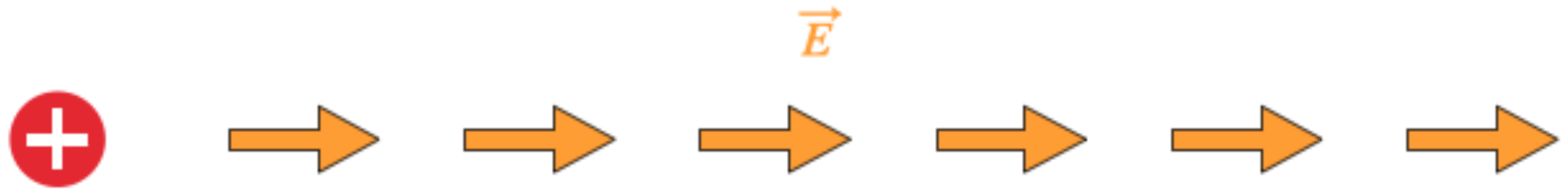




<u>Nothing</u> can travel faster than c! Electric field cannot change everywhere instantaneously!


$\vec{E}$

HOW TO PRODUCE ELECTROMAGNETIC RADIATION

FIELDS MADE BY CHARGES

- A charge a rest makes a Coulombic electric field but no magnetic field
- A charge moving with constant velocity makes a Coulombic electric field *and* a magnetic field
- An *accelerated* charge in addition makes electromagnetic radiation with both an electric and a magnetic field