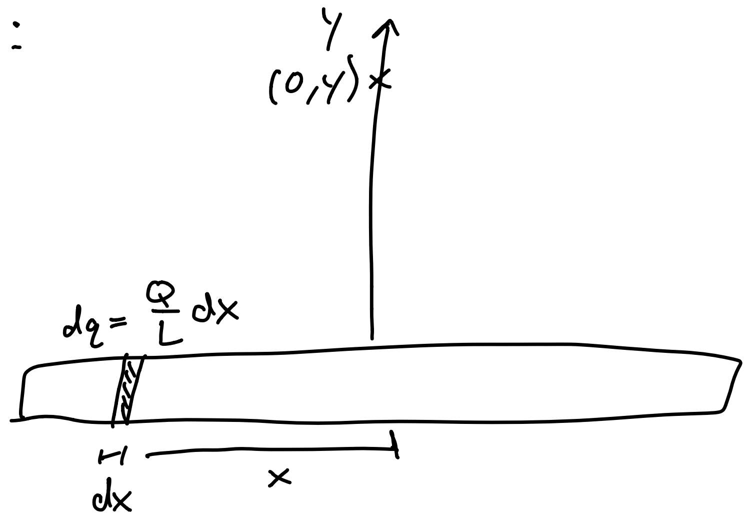
Friday:



$$\vec{E} = \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{KQ}{L(x^2+y^2)^{\frac{3}{2}}} \langle -x, y \rangle \, dx$$

$$\vec{E} = \langle E_x, E_y \rangle$$

$$E_x = \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{-KQx\,dx}{L(x^2+y^2)^{\frac{3}{2}}}$$

$$E_y = \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{KQy\,dx}{L(x^2+y^2)^{\frac{3}{2}}}$$

solve E_x

$$E_x = \frac{-KQ}{L} \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{x \, dx}{(x^2+y^2)^{\frac{3}{2}}}$$

$$u = x^2 + y^2$$

$$du = 2x\,dx \quad \Rightarrow \quad x = \frac{1}{2} \frac{du}{dx}$$

$$E_x = \frac{-KQ}{L} \int \frac{\frac{1}{2} \frac{du}{dx} dx}{u^{3/2}} = -\frac{1}{2} \frac{KQ}{L} \int \frac{du}{u^{3/2}}$$

Limits?

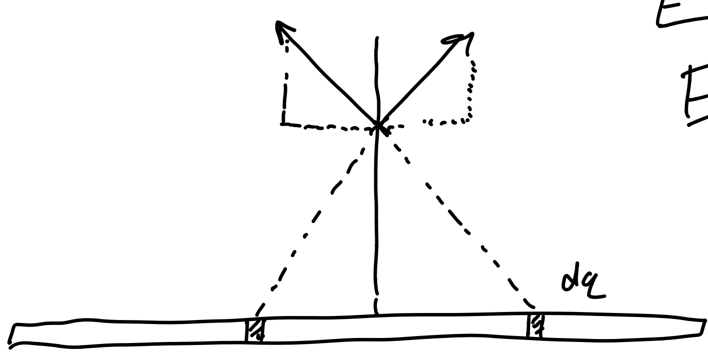
$$u_0 = x_0^2 + y^2 = \left(-\frac{L}{2}\right)^2 + y^2$$

$$u_1 = x_1^2 + y^2 = \left(\frac{L}{2}\right)^2 + y^2$$

$$E_x = -\frac{1}{2} \frac{KQ}{L} \int_{\frac{L^2}{4}+y^2}^{\frac{L^2}{4}+y^2} \frac{du}{u^{3/2}}$$

$$E_x = 0$$

Does this make sense?



E_x cancels

E_y adds

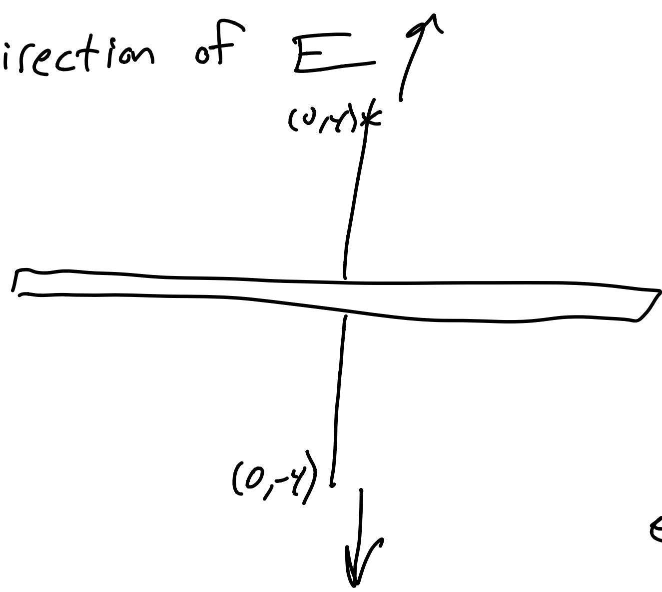
$$\vec{E} = \langle 0, E_y \rangle$$

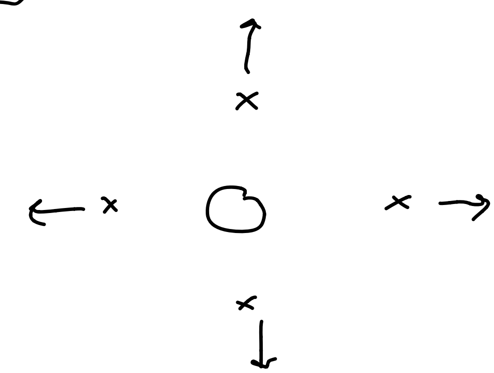
$$E_y = \frac{KQy}{L}\int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{dx}{(x^2+y^2)^{3/2}}$$

$$E_y = KQ\,\frac{1}{y\sqrt{\left(\frac{L}{2}\right)^2 + y}}$$

$$\frac{KQ}{r\sqrt{r^2}} \longrightarrow \frac{KQ}{r^2} \quad ✓$$

Direction of E





$$y \longrightarrow r$$

$$\vec{E} = KQ\,\frac{1}{r\sqrt{r^2+\left(\frac{L}{2}\right)^2}}$$

$$\vec{E} = \frac{kQ \quad 1}{r\sqrt{r^2 + \left(\frac{L}{2}\right)^2}} \hat{r}$$

Check: $L << r$

$L \longrightarrow 0$

$$\vec{E} = \frac{kQ}{r\sqrt{r^2}} \hat{r} = \frac{kQ}{r^2} \hat{r}$$ point charge

Opposite: $r << L$

$$\frac{1}{r\sqrt{r^2 + \frac{L^2}{4}}} = \frac{1}{r\sqrt{\frac{L^2}{4}}} = \frac{1}{r\, L/2}$$

$$= \frac{2}{Lr}$$

$$\vec{E} \approx \frac{2kQ}{Lr} = \frac{Q}{L} \frac{2k}{r}$$

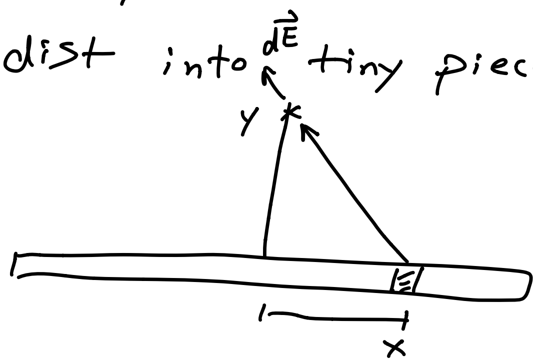
$$Q/L = \lambda$$

$\vec{E} \sim \frac{1}{r}$ very close to rod

Summarize

summary

1) Cut dist into tiny pieces & draw $\Delta\vec{E}$



2) Write $\Delta\vec{E}$ for a single piece

Find ΔQ density × length

$$\Delta\vec{E} = \frac{k\Delta Q}{|\vec{r}|}\hat{r}, \quad \vec{r} = \vec{r}_{obs} - \vec{r}_{src}$$

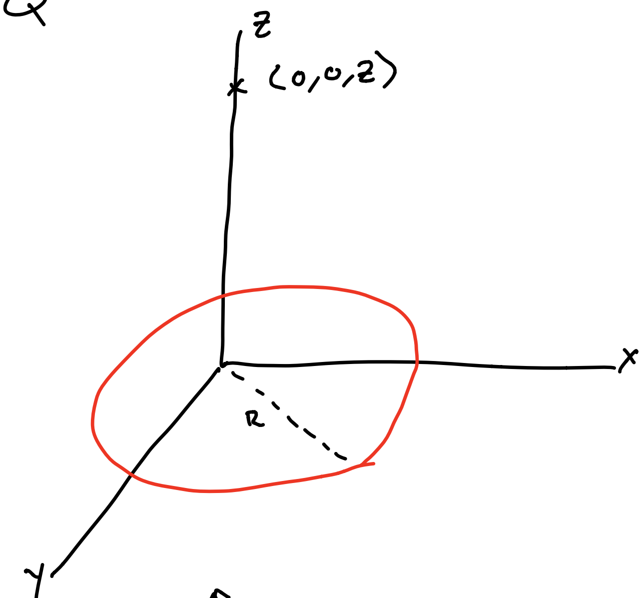
$$\vec{r} = \langle 0, y\rangle - \langle x, 0\rangle$$
$$= \langle -x, y\rangle$$

3) Add up all pieces. Sum/integral over x.

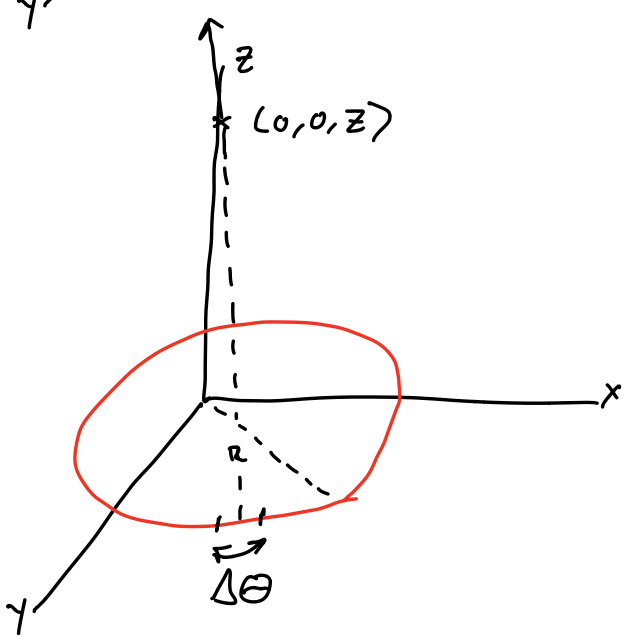
Charged ring

Uniformly charged ring

Total charge Q



1) cut into tiny pieces

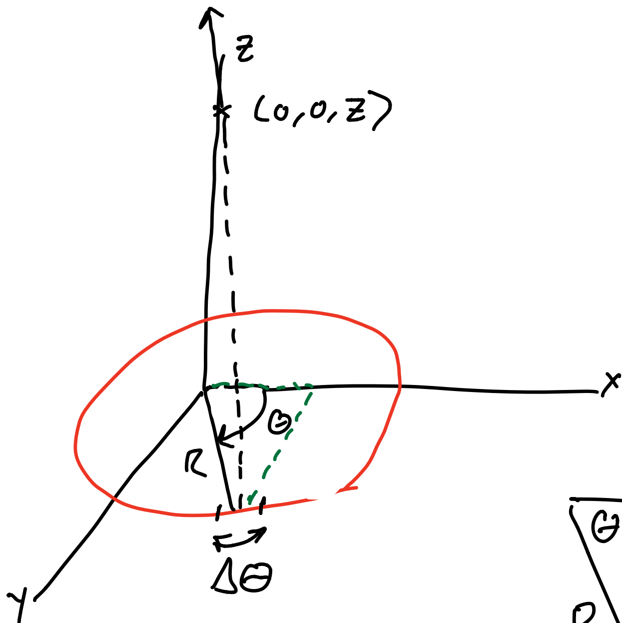


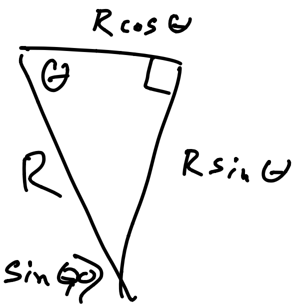
2) write $\Delta\vec{E}$

ΔQ?

$$\frac{\Delta Q}{\Delta\theta} = \frac{Q}{2\pi}$$

$$\Delta Q = \frac{Q}{2\pi}\Delta\theta$$





$$\vec{E} = \frac{KQ}{|\vec{r}|^2}\hat{r}$$

$$\vec{r}_{src} = \langle R\cos\theta, R\sin\theta \rangle$$

$$\vec{r}_{obs} = \langle 0, 0, z \rangle$$

$$\vec{r} = \langle -R\cos\theta, -R\sin\theta, z \rangle$$

$$|\vec{r}| = \sqrt{R^2\cos^2\theta + R^2\sin^2\theta + z^2}$$
$$= \sqrt{R^2 + z^2}$$

$$\Delta\vec{E} = \frac{K\Delta Q}{|\vec{r}|^2}\hat{r} = \frac{KQ}{2\pi}\Delta\theta \frac{1}{(R^2+z^2)^{\frac{3}{2}}} \langle -R\cos\theta, -R\sin\theta, z \rangle$$

3) Sum/integrate

$$\vec{E} = \int_0^{2\pi} \frac{kQ\,d\theta}{2\pi} \frac{1}{(R^2+z^2)^{3/2}} \langle -R\cos\theta, -R\sin\theta, z \rangle$$

$$E_x = \frac{-kQ}{2\pi} \frac{R}{(R^2+z^2)^{\frac{3}{2}}} \int_0^{2\pi} \cos\theta \, d\theta$$

$$\sin(2\pi) - \sin(0) = 0$$

$$E_x = E_y = 0$$

$$E_z = \frac{kQ}{2\pi} \frac{z}{(R^2+z^2)^{3/2}} \int_0^{2\pi} d\theta$$

$$E_z = \frac{kQz}{(R^2+z^2)^{3/2}}$$

$z >> R$

$$E_z = \frac{kQz}{(z^2)^{3/2}}$$

$$= \frac{kQz}{z^3}$$

$$E_z = \frac{kQ}{z^2}$$