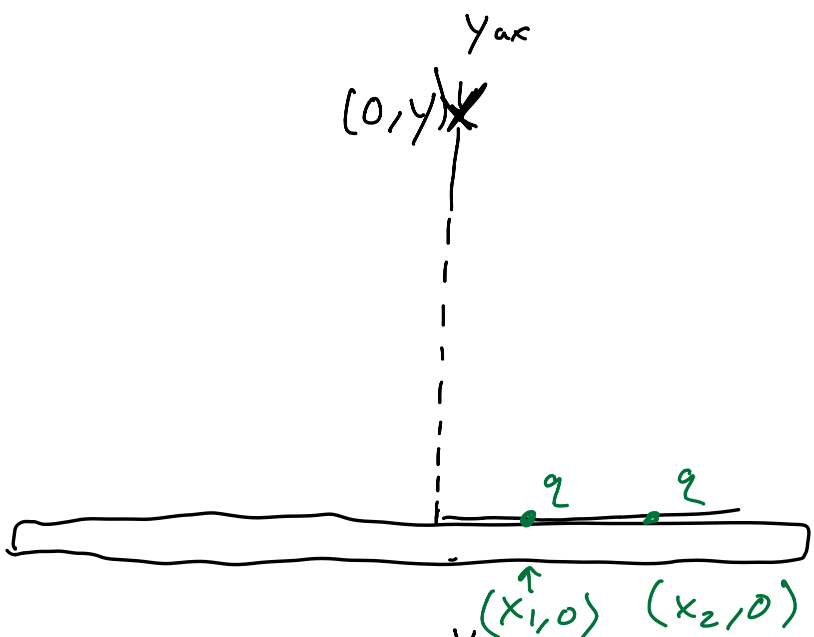
- In lab, we spent a lot of time charging rods.
    - To estimate the force on the induced dipole, we assumed the rod was a point charge
    - That's not actually accurate
- Today we want to get a more accurate estimate of the electric field due to charges on a rod.
- First let's start with a simple example:

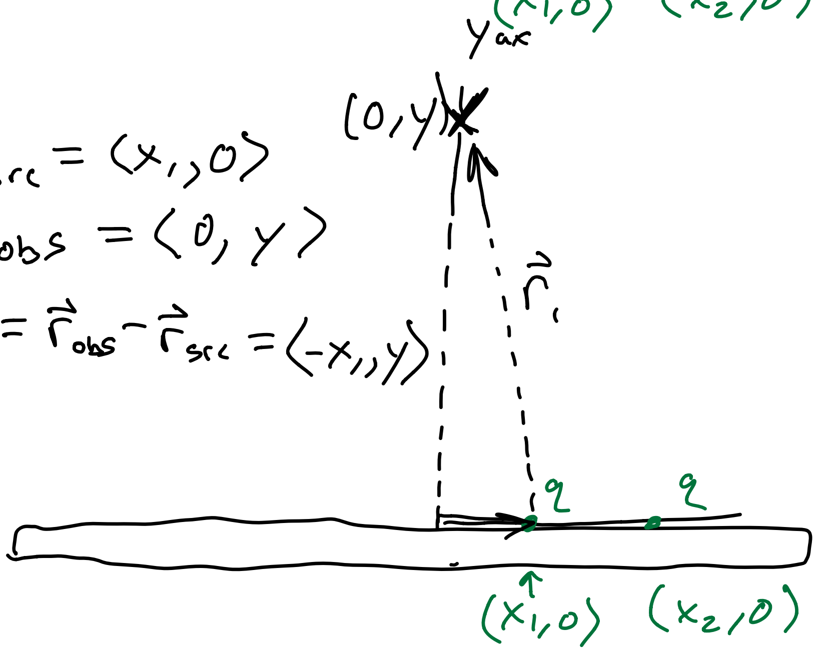


1)

$\vec{r}_{src} = \langle x_1, 0\rangle$

$\vec{r}_{obs} = \langle 0, y\rangle$

$\vec{r}_1 = \vec{r}_{obs} - \vec{r}_{src} = \langle -x_1, y\rangle$



$$\vec{E}_1 = \frac{kq}{|\vec{r}_1|^2}\hat{r}_1$$

$$|\vec{r}_1| = \sqrt{x_1^2+y^2}$$

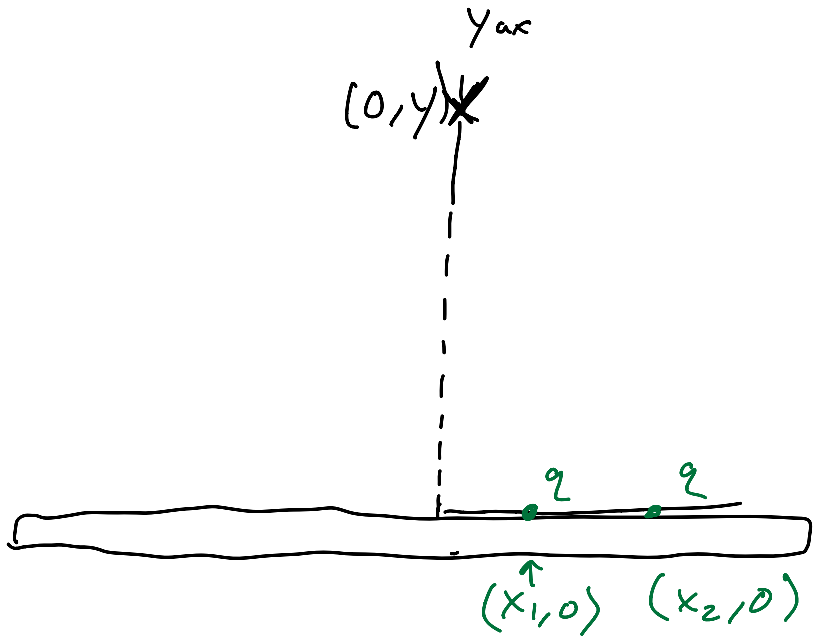
$$\hat{r}_1 = \frac{1}{\sqrt{x_1^2+y^2}}\langle -x_1, y\rangle$$

$$\vec{E}_1 = \frac{kq}{(x_1^2+y^2)}\frac{1}{\sqrt{x_1^2+y^2}}\langle -x_1, y\rangle$$

$$= a \cdot a^{\frac{1}{2}} = a^{\frac{3}{2}}$$

$$\vec{E}_1 = \frac{kq}{(x_1^2+y^2)^{\frac{3}{2}}}\langle -x_1, y\rangle$$

E_2



$\vec{E}_2$ $\quad \vec{r}_{src} = \langle x_2, 0 \rangle$

$$\vec{r}_{obs} = \langle 0, y \rangle$$

$$\vec{r}_2 = \langle -x_2, y \rangle$$

q is same just change

$$x_1 \rightarrow x_2$$

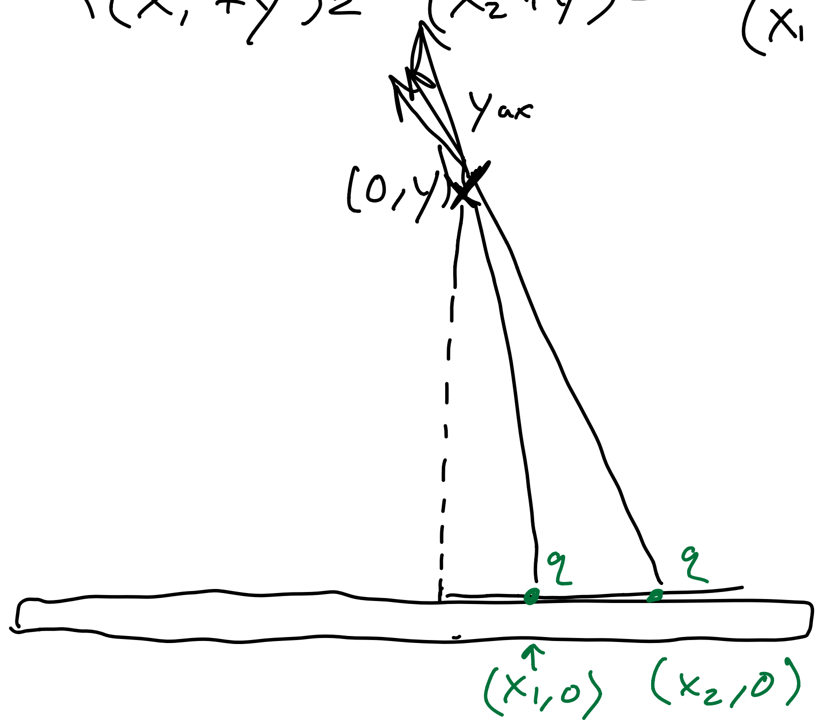
$$\vec{E}_2 = \frac{kq}{(x_2^2 + y^2)^{\frac{3}{2}}} \langle -x_2, y \rangle$$

$$\vec{E}_1 = \frac{kq}{(x_1^2+y^2)^{\frac{3}{2}}} \langle -x_1, y \rangle$$

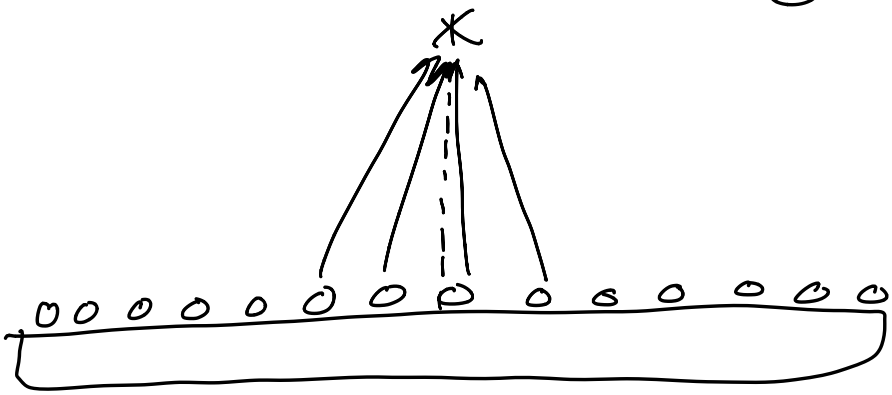
$$\vec{E}_2 = \frac{kq}{(x_2^2+y^2)^{\frac{3}{2}}} \langle -x_2, y \rangle$$

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

$$= \left\langle \frac{-kqx_1}{(x_1^2+y^2)^{\frac{3}{2}}} - \frac{kqx_2}{(x_2^2+y^2)^{\frac{3}{2}}}, \frac{kqy}{(x_1^2+y^2)^{\frac{3}{2}}} + \frac{kqy}{(x_2^2+y^2)^{\frac{3}{2}}} \right\rangle$$



We could keep adding q



$\vec{E} = \sum \vec{E}_i$

Chg Rod: $Q = {}^{-}10\,nC$

How many point charges?

$10\,nC = 10^{-8}C$,

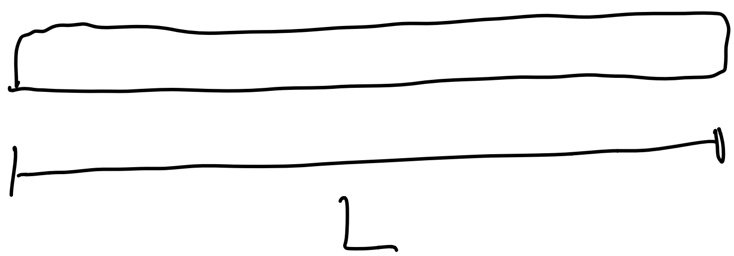
$1e = 1.6\times10^{-19}C \Rightarrow 1C = 6\times10^{18}e$

$10\,nC = 10^{-8}C = 10^{-8}\times6\times10^{18}e$

$= 6\times10^{10}e$

~60 billion electrons

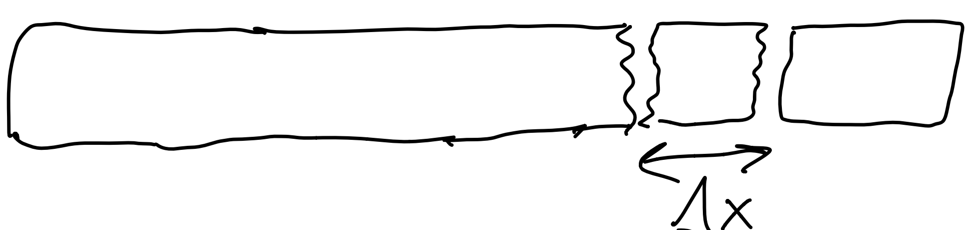
- More practical to deal with charge *density*



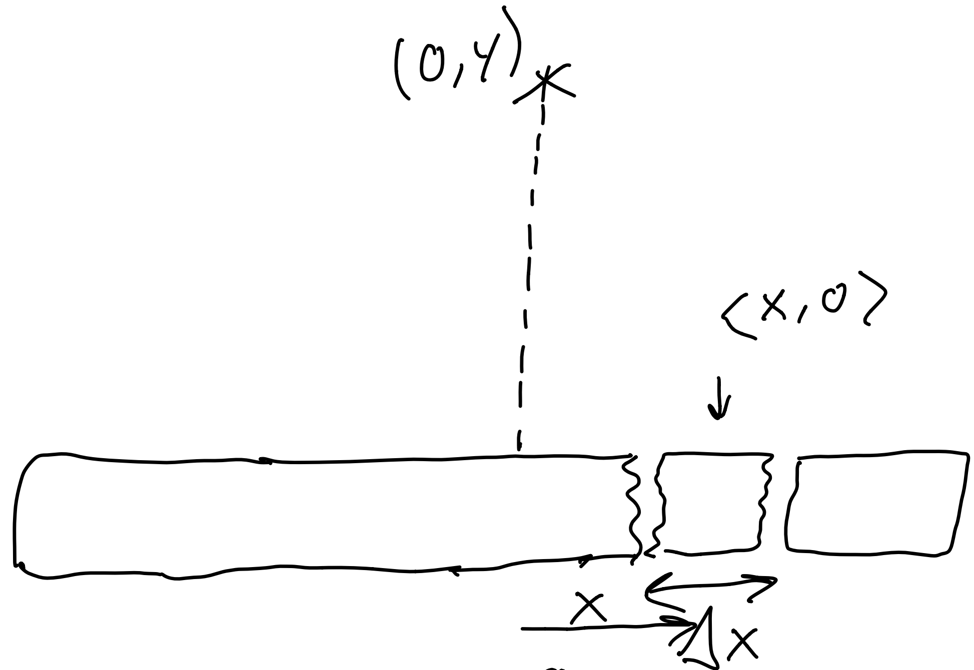
Total charge Q
uniformly distributed

Charge density (charge per length) $= \frac{Q}{L}$

Think air density
$\rho \sim 1.2 \frac{kg}{m^3}$



$$\Delta Q = \frac{Q}{L} \Delta x$$



$$\Delta Q = \frac{Q}{L} \Delta x$$

if Δx is very small, ΔQ is just a point charge

What is $\Delta \vec{E}$?

$$\Delta \vec{E} = \frac{K \Delta Q}{|\vec{r}|^2} \hat{r}$$

$$\vec{r}_{src} = \langle x, 0 \rangle$$

$$\vec{r}_{obs} = \langle 0, y \rangle$$

$$\vec{r} = \vec{r}_{obs} - \vec{r}_{src} = \langle -x, y \rangle$$

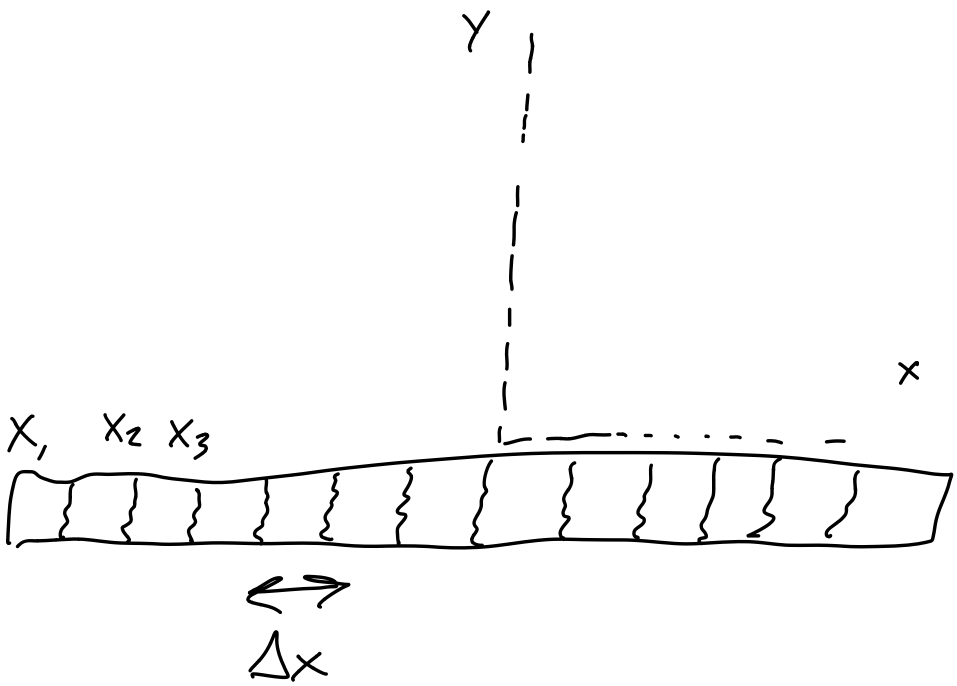
$$|\vec{r}| = \sqrt{x^2+y^2}$$

$$\hat{r} = \frac{1}{\sqrt{x^2+y^2}} \langle -x, y \rangle$$

$$\Delta \vec{E} = \frac{K \Delta Q}{(x^2+y^2)^{\frac{3}{2}}} \langle -x, y \rangle$$

$$\Delta Q = \frac{Q}{L} \Delta x$$

$$\Delta \vec{E} = \frac{K \frac{Q}{L} \Delta x}{(x^2+y^2)^{\frac{3}{2}}} \langle -x, y \rangle \quad \checkmark$$



$$\Delta Q = \frac{Q}{L}\Delta x$$

$$\vec{E}_1 = \frac{k\,\Delta Q}{(x_1^2+y^2)^{\frac{3}{2}}}\langle -x_1, y\rangle$$

$$\vec{E}_2 = \frac{k\,\Delta Q}{(x_2^2+y^2)^{\frac{3}{2}}}\langle -x_2, y\rangle$$

$$\vec{E}_3 = \ldots$$

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \ldots$$

$$\vec{E} = \sum_i \vec{E}_i$$

$$\vec{E} = \sum_i \frac{K \Delta Q}{(x_i^2 + y^2)^{\frac{3}{2}}} \langle -x_i, y \rangle$$

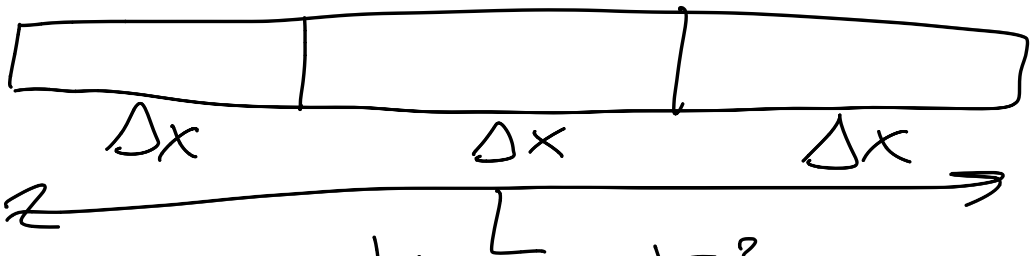
$$= \frac{K \Delta Q}{(x_1^2 + y^2)^{\frac{3}{2}}} \langle -x_1, y \rangle$$

$$+ \frac{K \Delta Q}{(x_2^2 + y^2)^{\frac{3}{2}}} \langle -x_2, y \rangle$$

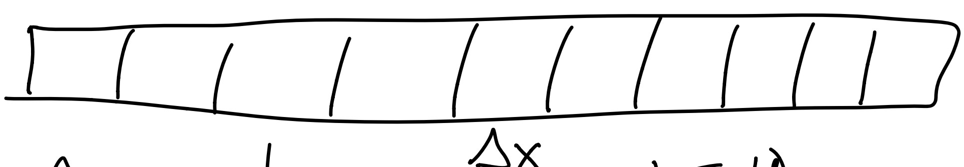
$$\Delta Q = \frac{Q}{L} \Delta x$$

$$\vec{E} = \sum_i \frac{K \frac{Q}{L} \Delta x}{(x_i^2 + y^2)^{\frac{3}{2}}} \langle -x_i, y \rangle$$

Point charge approximation gets better with smaller Δx



$\Delta x = \frac{1}{3}L$ $\quad N=3$



$\Delta x = \frac{1}{10}L$ $\quad N=10$

$\Delta x = \frac{1}{100}L$? $\quad \frac{1}{1000}L$

over

$$\vec{E} = \sum_i \frac{K\frac{Q}{L}\Delta x}{(x_i^2+y^2)^{\frac{3}{2}}} \langle -x_i, y \rangle$$

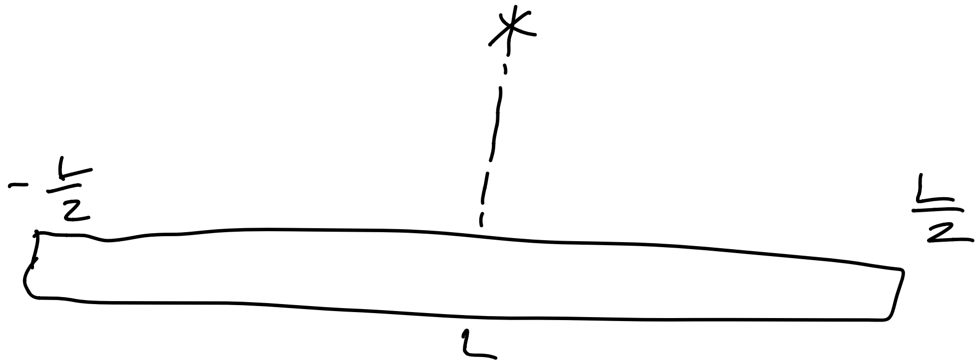
$$\Delta x \longrightarrow 0, \quad N \longrightarrow \infty$$

$$\lim_{\Delta x \to 0} \sum_i \frac{K\frac{Q}{L}\Delta x}{(x_i^2+y^2)^{\frac{3}{2}}} \langle -x_i, y \rangle$$

$$\sum \longrightarrow \int$$

$$\Delta x \longrightarrow dx$$

$$\vec{E} = \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{K\frac{Q}{L}dx}{(x^2+y^2)^{\frac{3}{2}}} \langle -x, y \rangle$$



$$\vec{E} = \langle E_x , E_y \rangle$$

$$E_x = \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{-\frac{KQ}{L} x\, dx}{(x^2+y^2)^{\frac{3}{2}}}$$

$$E_y = \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{\frac{KQ}{L} y\, dx}{(x^2+y^2)^{\frac{3}{2}}}$$

$$E_x = \frac{-KQ}{L} \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{x \quad dx}{(x^2+y^2)^{\frac{3}{2}}}$$

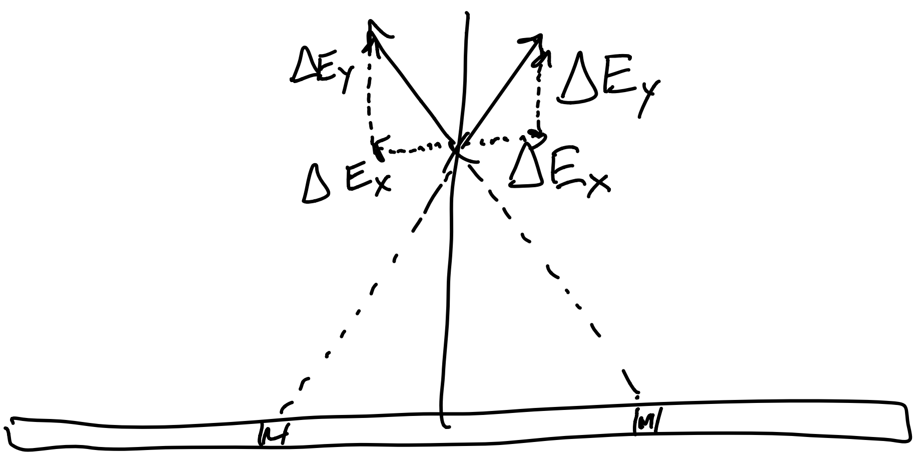
$$u = x^2+y^2 \qquad x = \frac{1}{2}\frac{du}{dx}$$
$$du = 2x\,dx$$

$$= \frac{-KQ}{L} \int \frac{\frac{1}{2}\frac{du}{\cancel{dx}} \cancel{dx}}{u^{3/2}}$$

$$u_0 = \left(-\frac{L}{2}\right)^2 + y^2 = \frac{L^2}{4} + y^2$$
$$u_1 = \left(\frac{L}{2}\right)^2 + y^2 = \frac{L^2}{4} + y^2$$

$$= -\frac{kQ}{2L}\int_{\frac{L^2}{4}+y^2}^{\frac{L^2}{4}+y^2} \frac{du}{u^{3/2}} = 0$$

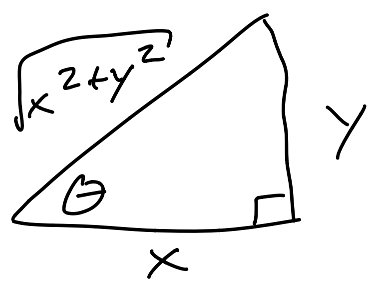
$E_x = 0$? Does this make sense?



E_y

$$E_y = \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{\frac{KQ}{L}\, y\, dx}{(x^2+y^2)^{\frac{3}{2}}}$$

$$E_y = \frac{KQy}{L} \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{dx}{(x^2+y^2)^{3/2}}$$



Optional

$$\frac{y}{\sqrt{x^2+y^2}} = \sin\theta$$

$$\frac{y^2}{x^2+y^2} = \sin^2\theta$$

$$\sin^3\theta = \frac{y^3}{(x^2+y^2)^{3/2}}$$

$$\frac{1}{(x^2+y^2)^{\frac{3}{2}}} = \frac{\sin^3\theta}{y^3}$$

$$\frac{x}{y} = \cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\frac{dx}{y} = -\csc^2(\theta)\, d\theta$$

$$= \frac{-1}{\sin^2(\theta)}\, d\theta$$

$$dx = \frac{-y}{\sin^2\theta}\, d\theta$$

$$E_y = \frac{kQy}{L} \int -\frac{\sin^3\theta}{y^3} \frac{y}{\sin^2\theta}\, d\theta$$

$$= \frac{-kQ}{Ly} \int_{\theta_0}^{\theta_1} \sin\theta\, d\theta$$

$$\theta_0 = \arccos\left[\frac{-\frac{L}{2}}{\sqrt{\frac{L^2}{4}+y^2}}\right]$$

$$\theta_1 = \arccos\left[\frac{L/2}{\sqrt{\frac{L^2}{4}+y^2}}\right]$$

$$E_y = \frac{+KQ}{Ly} \cos\theta \Big|_{\theta_0}^{\theta_1}$$

$$\cos(\arccos(x)) = x$$

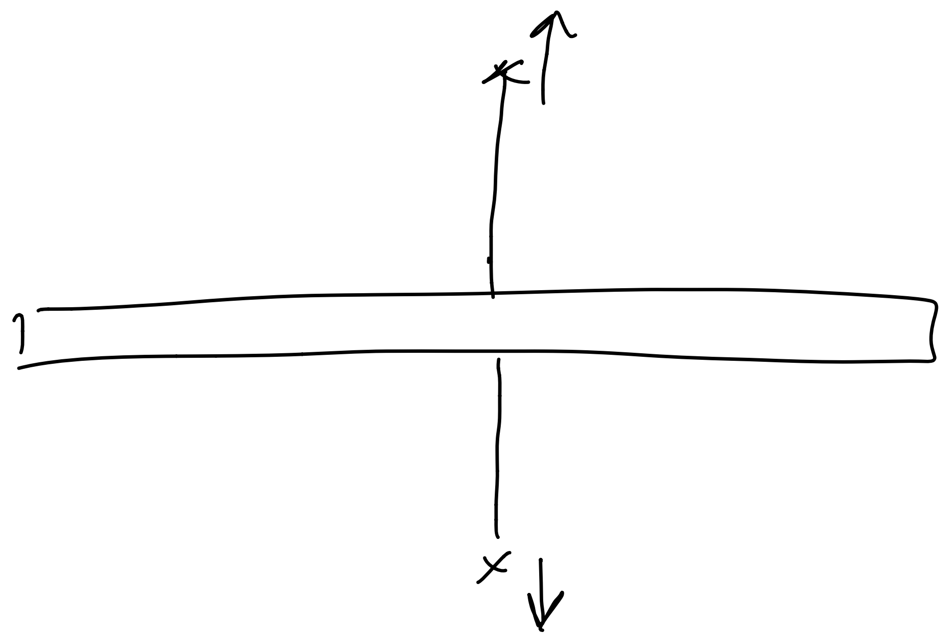
$$= \frac{KQ}{Ly} \frac{L}{\sqrt{\frac{L^2}{4} + y^2}}$$

$$E_y = KQ \frac{1}{y\sqrt{\frac{L^2}{4} + y^2}}$$

Check:

$$\frac{1}{y\sqrt{\frac{L^2}{4}+y^2}}$$ same units as $\frac{1}{r^2}$ ✓

symmetry



$y \longrightarrow r$

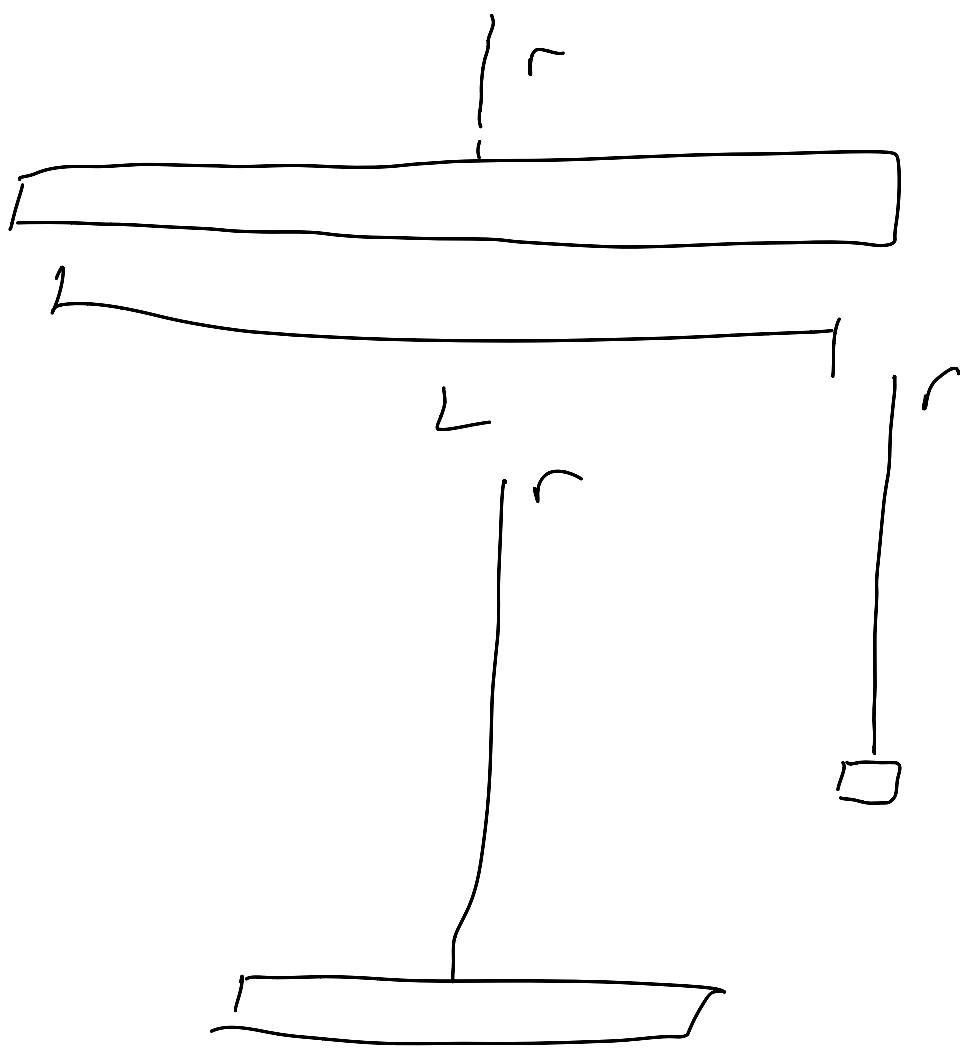
$$E_y = KQ \frac{1}{y\sqrt{\frac{L^2}{4} + y^2}}$$

$$|\vec{E}| = \frac{KQ}{r\sqrt{\frac{L^2}{4} + r^2}}$$

$\hat{E}$ is perpendicular to rod

$$E_y = kQ \frac{1}{r\sqrt{\frac{L^2}{4} + r^2}}$$

Check: $L \ll r$



$L \to 0$

$$E = \frac{kQ}{r\sqrt{r^2}} = \frac{kQ}{r^2} \checkmark$$