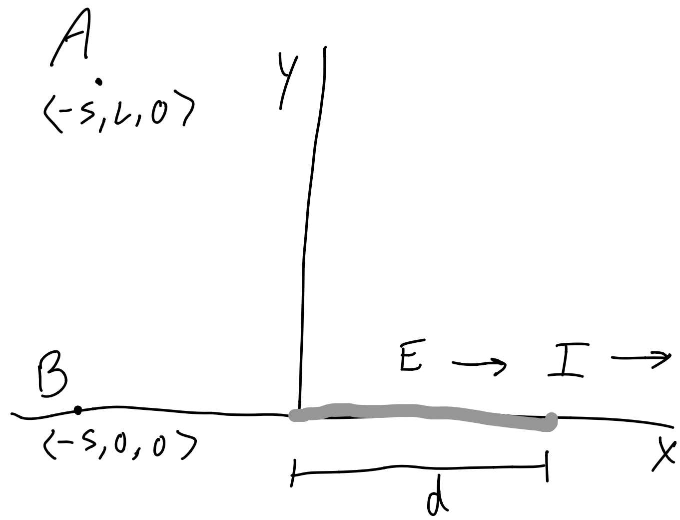
1.



a) $\vec{B} = \frac{\mu_0}{4\pi} \int \frac{I \, d\vec{l} \times \hat{r}}{r^2}$

$i = nAuE$

$I = enAuE$

$d\vec{l} = dx \, \hat{x}$

$\vec{r} = \vec{r}_{obs} - \vec{r}_{src}$

$= \langle -s, L, 0 \rangle - \langle x, 0, 0 \rangle$

$\vec{r} = \langle -(s+x), L, 0 \rangle$

$r = \sqrt{(s+x)^2 + L^2}$

$\hat{r} = \frac{\vec{r}}{r}$

$$\vec{B} = \frac{\mu_0 I}{4\pi} \int_0^d \frac{dx\,\hat{x}}{\left[(s+x)^2 + L^2\right]^{3/2}} \times \langle -(s+x), L, 0 \rangle$$

$$dx\,\hat{x} \times \langle -(s+x), L, 0 \rangle$$

$$= dx\,\hat{x} \times \left[-(s+x)\hat{x} + L\hat{y} \right]$$

$$= L\,dx\,\hat{z}$$

$$\vec{B} = \frac{\mu_0}{4\pi} \int_0^d \frac{I\,L\,dx}{\left[(s+x)^2 + L^2\right]^{3/2}}\hat{z} \quad , \quad I = enAuE$$

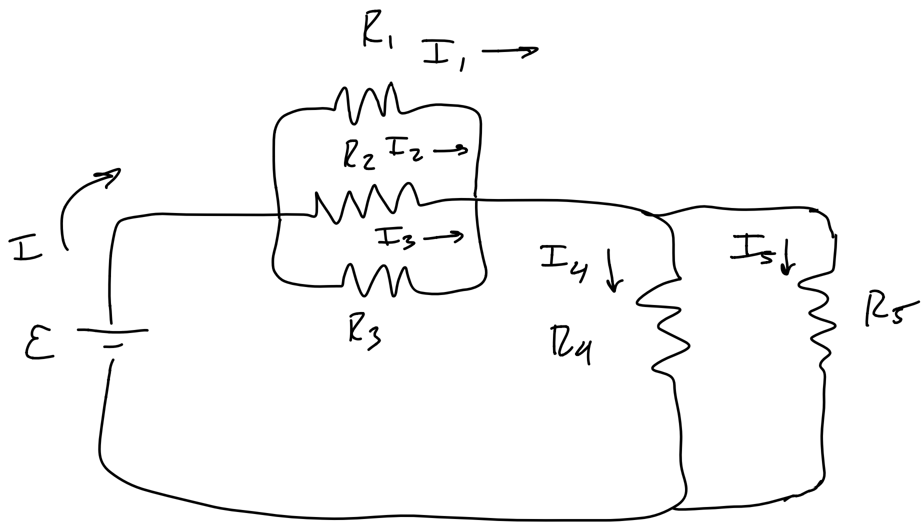
$$\boxed{\vec{B} = \frac{\mu_0}{4\pi} enAuEL \int_0^d \frac{dx}{\left[(s+x)^2 + L^2\right]^{3/2}}\hat{z}}$$

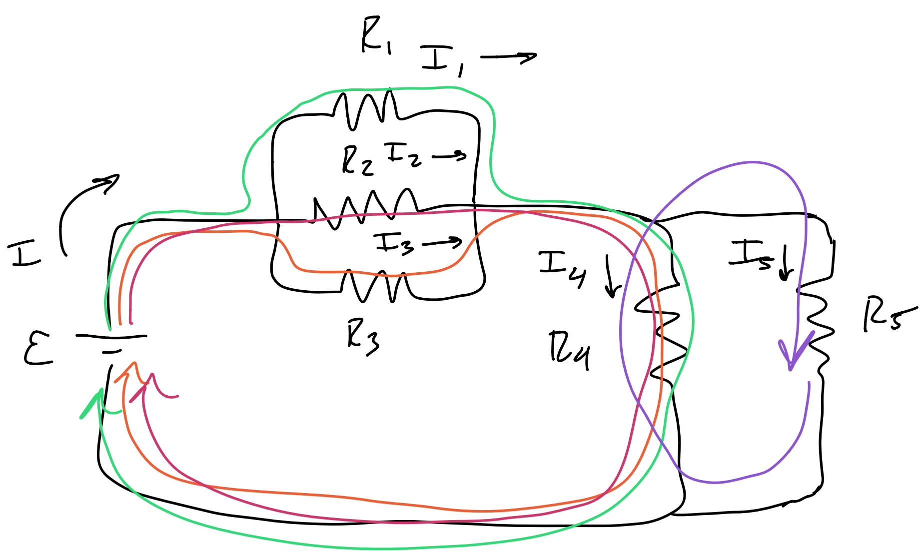
b) $d\vec{l} = dx\,\hat{x}$

$\vec{r} = -(s+x)\hat{x}$

$\boxed{\vec{B} = 0}$ because $d\vec{l} \times \hat{r} = 0$

2.





$$\mathcal{E} - I_3R_3 - I_4R_4 = 0 \quad (1)$$

$$\mathcal{E} - I_2R_2 - I_4R_4 = 0 \quad (2)$$

$$\mathcal{E} - I_1R_1 - I_4R_4 = 0 \quad (3)$$

$$-I_5R_5 + I_4R_4 = 0 \quad (4)$$

$$I_1 + I_2 + I_3 = I_4 + I_5 \quad (5)$$

(1) $$I_3 = \frac{\mathcal{E} - I_4 R_4}{R_3}$$

(2) $$I_2 = \frac{\mathcal{E} - I_4 R_4}{R_2}$$

(3) $$I_1 = \frac{\mathcal{E} - I_4 R_4}{R_1}$$

(4) $$I_5 = I_4 \frac{R_4}{R_5}$$

(5) $$(\mathcal{E} - I_4 R_4)\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right] = I_4\left(1 + \frac{R_4}{R_5}\right)$$

$$\mathcal{E}\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) = I_4\left[1 + \frac{R_4}{R_5} + R_4\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)\right]$$

$R_1 = 40\,\Omega$ $\qquad \mathcal{E} = 9\text{V}$

$R_2 = 60\,\Omega$

$R_3 = 20\,\Omega$

$R_4 = 30\,\Omega$

$R_5 = 60\,\Omega$

$$9\left(\frac{11}{120}\right) = I_4\left(1 + \frac{1}{2} + 30\left(\frac{11}{120}\right)\right)$$

$$\frac{33}{40} = \frac{17}{4} I_4$$

$$\boxed{I_4 = 33/170\text{ A} \approx 0.19\text{A}}$$

$$I_4 R_4 = \frac{33}{170}(30) = \frac{99}{17} V$$

$$I_3 = \frac{\mathcal{E} - I_4 R_4}{R_3} = \frac{9 - \frac{99}{17}}{R_3} = \frac{54}{17} \frac{1}{R_3}$$

$$I_3 = \frac{54}{17(20)} = \frac{27}{170}$$

$$\boxed{I_3 = \frac{27}{170} A \approx 0.16 A}$$

$$I_2 = \frac{\mathcal{E} - I_4 R_4}{R_2}$$

$$= \frac{54}{17(60)} = \frac{9}{170}$$

$$\boxed{I_2 = \frac{9}{170} A \approx 0.05 A}$$

$$I_1 = \frac{\mathcal{E} - I_4 R_4}{R_1}$$

$$= \frac{54}{17(40)} = \frac{27}{340}$$

$$\boxed{I_1 = \frac{27}{340}\,A \approx 0.08A}$$

$$I_5 = I_4 \frac{R_4}{R_5} = \frac{1}{2} I_4$$

$$= \frac{33}{340}$$

$$\boxed{I_5 = \frac{33}{340}\,A \approx 0.10\,A}$$

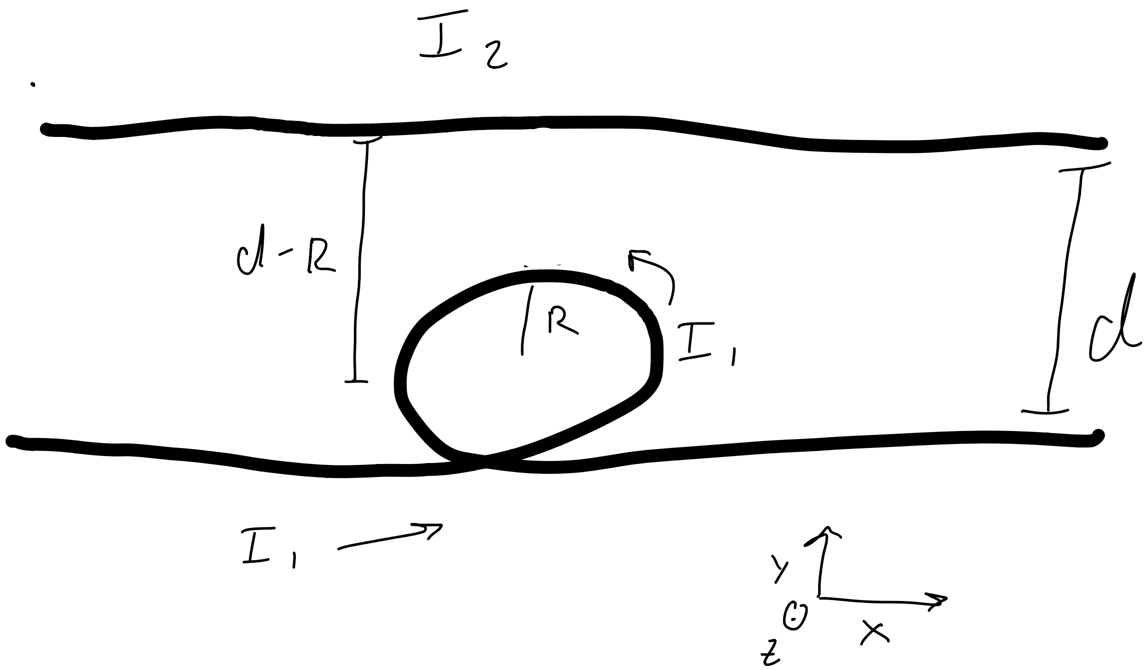
$$\boxed{\begin{aligned} I_1 &= \frac{27}{340}\,A \approx 0.08\,A \\ I_2 &= \frac{18}{340}\,A \approx 0.05\,A \\ I_3 &= \frac{54}{340}\,A \approx 0.16\,A \\ I_4 &= \frac{66}{340}\,A \approx 0.19\,A \\ I_5 &= \frac{33}{340}\,A \approx 0.10\,A \end{aligned}}$$

Check:

$$I_1 + I_2 + I_3 = I_4 + I_5$$

$$\frac{99}{340} = \frac{99}{340} \quad \checkmark$$

3.



$$\vec{B}_1 = \frac{\mu_0 I_1}{2\pi R}\hat{z} + \frac{\mu_0 I_1}{2R}\hat{z}$$

$$\vec{B}_2 = -\frac{\mu_0 I_2}{2\pi(d-R)}\hat{z}$$

I_2 must run left to right

$$B = \frac{\mu_0 I_1}{2R}\left(\frac{1}{\pi} + 1\right) - \frac{\mu_0 I_2}{2\pi(d-R)}$$

$$B = 0 = \frac{I_1}{R}(1+\pi) - \frac{I_2}{d-R}$$

$$I_2 = (1+\pi)\frac{d-R}{R} I_1 \quad , \quad \boxed{I_2 = 1.74\text{ A} \text{ left to right}}$$

$d = 0.12\text{ m}$ $\quad R = 0.05$ $\quad I_1 = 0.3\text{A}$

4. $\mathcal{E} - Ir = 0$

$$r = \frac{\mathcal{E}}{I} = \frac{1}{2}\,\Omega$$

$$\mathcal{E} - Ir - \frac{1}{C}Q = 0$$

$$Q = C\left[\mathcal{E} - Ir\right]$$

$$I = \frac{\mathcal{E}}{r} e^{-t/rC}$$

$$Q = C\left[\mathcal{E} - \mathcal{E} e^{-t/rC}\right]$$

$$Q = C\mathcal{E}\left[1 - e^{-t/rC}\right]$$

$$Q_{final} = C\mathcal{E}$$

$$Q = 50\% = 0.5\,C\mathcal{E}$$

$$0.5\,C\mathcal{E} = C\mathcal{E}\left[1 - e^{-t/RC}\right]$$

$$0.5 = 1 - e^{-t/RC}$$

$$e^{-t/RC} = 0.5$$

$$-\frac{t}{RC} = \ln(.5)$$

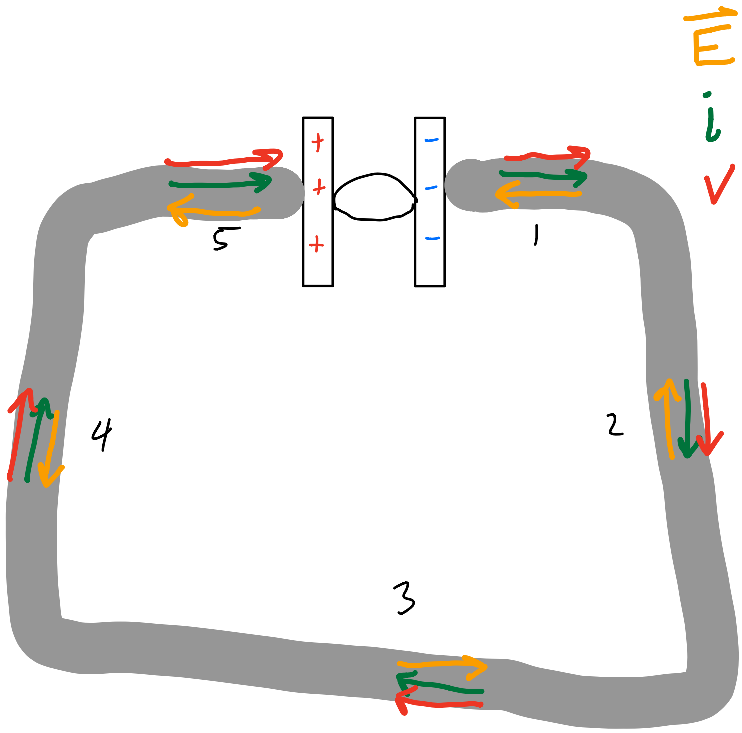
$$t = -RC\ln(.5)$$

$$t = -\left(\frac{1}{2}\right)\left(500\times10^{-6}\right)\ln(.5)$$

$$\boxed{t = 1.73\times10^{-4}\ s = 173\ \mu s}$$

5.

a)



b) $i_2 = i_3 = i_4 = i_5 = i_1$

c) $v_2 = v_3 = v_4 = v_5 = v_1$

d) $E_2 = E_3 = E_4 = E_5 = E_1$

e) (d) is <u>not</u> consistent with a dipole. This is because the field does not come from the battery alone, but also from surface charges built up on the wire.

(f) $v = uE$

$$\varepsilon - EL = 0 \Rightarrow E = \varepsilon / L$$

$$v = u\frac{\varepsilon}{L} = 4.5\times10^{-3}\left(\frac{3}{0.1}\right)$$

$$\boxed{v = 0.135\ m/s}$$