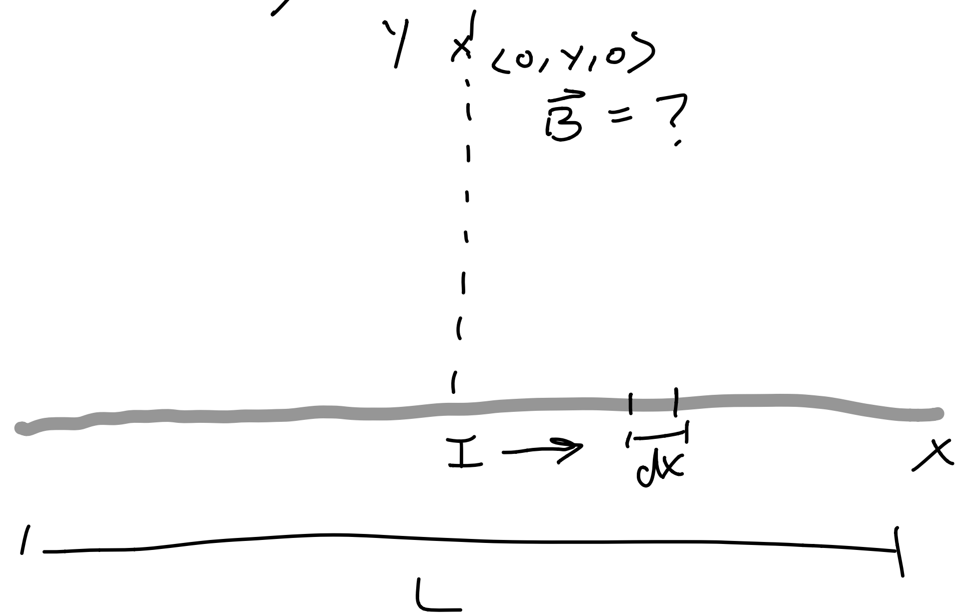
On Wednesday:



1) cut into pieces

2) Pick a piece find $d\vec{l}$

$$d\vec{l} = dx\,\hat{x} = dx\langle 1,0,0\rangle$$

3) Write $d\vec{B}$ of the piece

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{l}\times\hat{r}}{r^2}$$

$$\vec{r} = \vec{r}_{obs} - \vec{r}_{src} = \langle 0, y, 0\rangle - \langle x, 0, 0\rangle$$

$$\vec{r} = \langle -x, y, 0\rangle$$

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,dx\,\hat{x}}{(x^2+y^2)^{3/2}} \times \langle -x, y\rangle$$

4) Eval cross product

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I}{(x^2+y^2)^{3/2}} dx\,\hat{x} \times (-x\hat{x} + y\hat{y})$$

$$= \left(\frac{\mu_0}{4\pi}\frac{I}{(x^2+y^2)^{3/2}} dx\right)\left(-x\,\hat{x}\times\hat{x} + y\,\hat{x}\times\hat{y}\right)$$

Note on X prod:

$$a\hat{x} \times b\hat{y} = ab(\hat{x}\times\hat{y})$$
$$= ba(\hat{x}\times\hat{y})$$
$$= -ba(\hat{y}\times\hat{x})$$

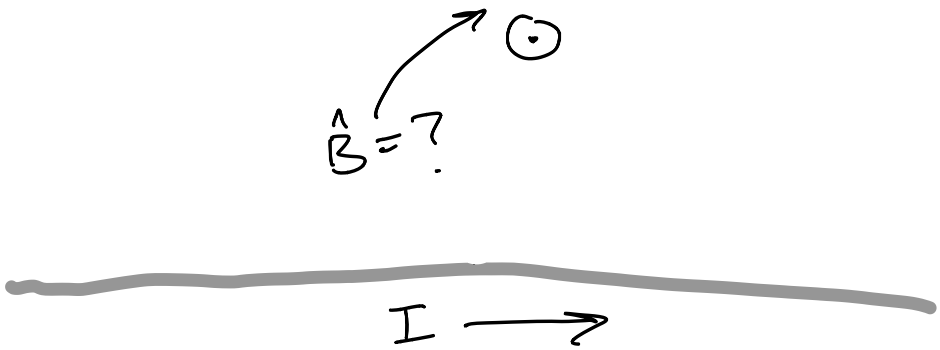
$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,y\,dx}{(x^2+y^2)^{3/2}}\hat{z}$$

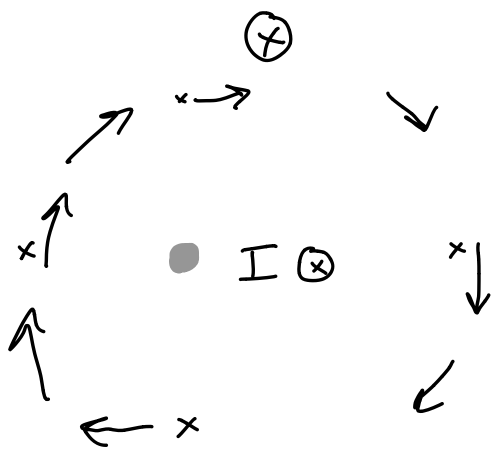
5) Write an integral

$$\vec{B} = \int_{-L/2}^{L/2} \frac{\mu_0 I}{4\pi}\frac{y}{(x^2+y^2)^{3/2}}\hat{z}$$

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{LI}{y\sqrt{y^2+(L/2)^2}}\hat{z}$$

$\downarrow$





Azimuthal

Mag field circles the conventional current. Another RHR.

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{LI}{y\sqrt{y^2+(L/2)^2}}\,\hat{z}$$

What if $y << L$? $\left(\frac{y}{L} << 1\right)$

$$y\sqrt{y^2+(L/2)^2} = y\frac{L}{2}\sqrt{\underbrace{\left(\frac{2y}{L}\right)^2}_{\approx 0}+1} \approx \frac{Ly}{2}$$

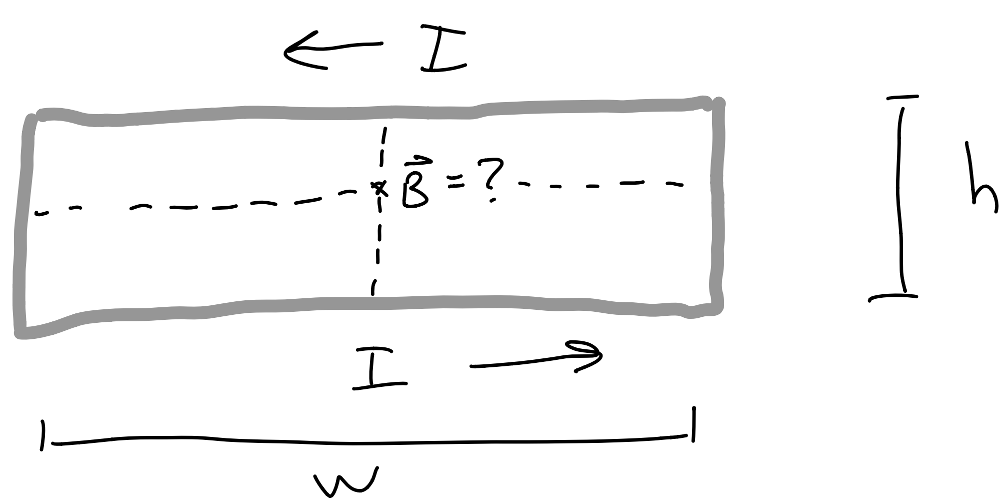
$$\vec{B}(y \ll L) \approx \frac{\mu_0}{4\pi} \frac{LI}{Ly/2} \hat{z}$$

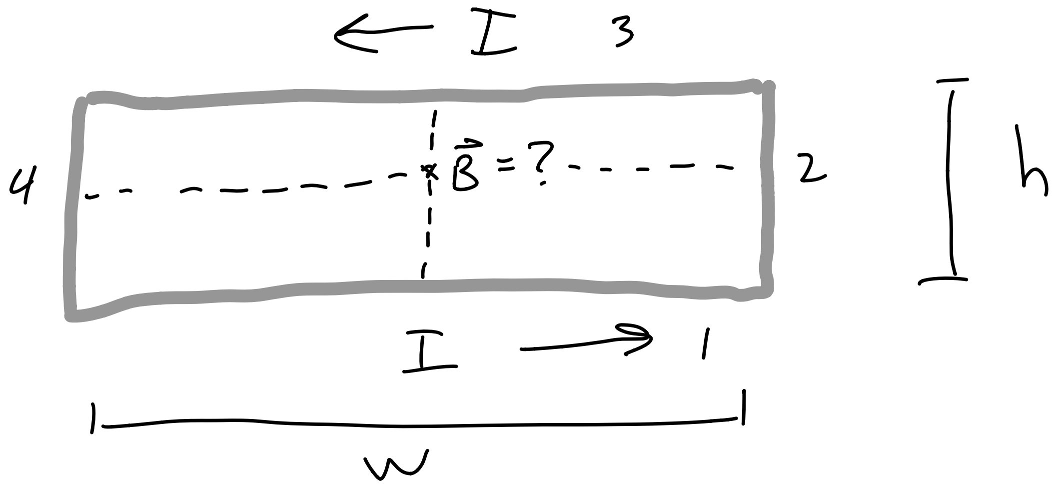
$$\vec{B}(y \ll L) \approx \frac{\mu_0}{2\pi} \frac{I}{y} \hat{z}$$

$$|\vec{B}| \approx \frac{\mu_0}{2\pi} \frac{I}{r}$$

Long, straight wire

Example:





$$\left|\vec{B}\right| = \frac{\mu_0}{4\pi}\frac{LI}{y\sqrt{y^2+(L/2)^2}}$$

$$B_1 = \frac{\mu_0}{4\pi}\frac{wI}{\frac{h}{2}\sqrt{(h/2)^2+(w/2)^2}}$$

$$B_2 = \frac{\mu_0}{4\pi}\frac{hI}{\frac{w}{2}\sqrt{(w/2)^2+(h/2)^2}}$$

$$B_3 = \frac{\mu_0}{4\pi}\frac{wI}{\frac{h}{2}\sqrt{(h/2)^2+(w/2)^2}}$$

$$B_4 = \frac{\mu_0}{4\pi}\frac{hI}{\frac{w}{2}\sqrt{(w/2)^2+(h/2)^2}}$$

$$B = \frac{\mu_0}{2\pi} \frac{I}{\sqrt{(w/2)^2 + (h/2)^2}} \left[\frac{w}{h} + \frac{h}{w} + \frac{w}{h} + \frac{h}{w} \right]$$

$$= \frac{\mu_0}{\pi} \frac{I}{\sqrt{(w/2)^2 + (h/2)^2}} \left[\frac{w}{h} + \frac{h}{w} \right]$$

$$\frac{w^2}{hw} + \frac{h^2}{hw}$$

$$B = \frac{\mu_0}{\pi} \frac{I}{\sqrt{(w/2)^2 + (h/2)^2}} \frac{1}{hw} (w^2 + h^2)$$

$\odot$