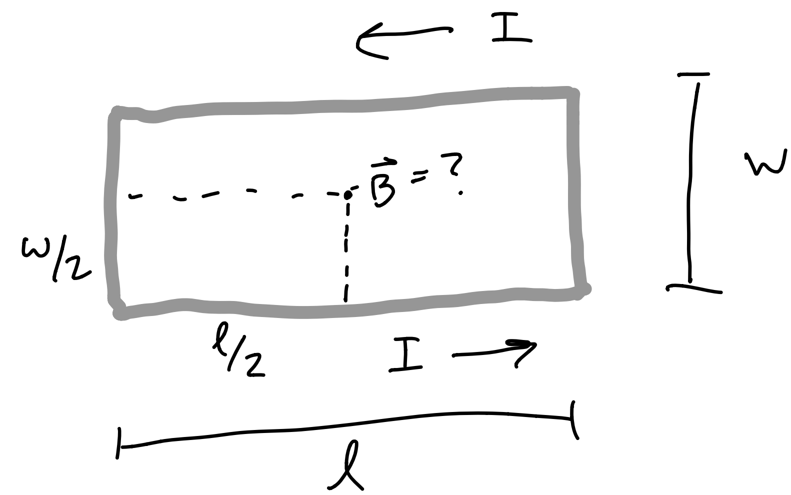
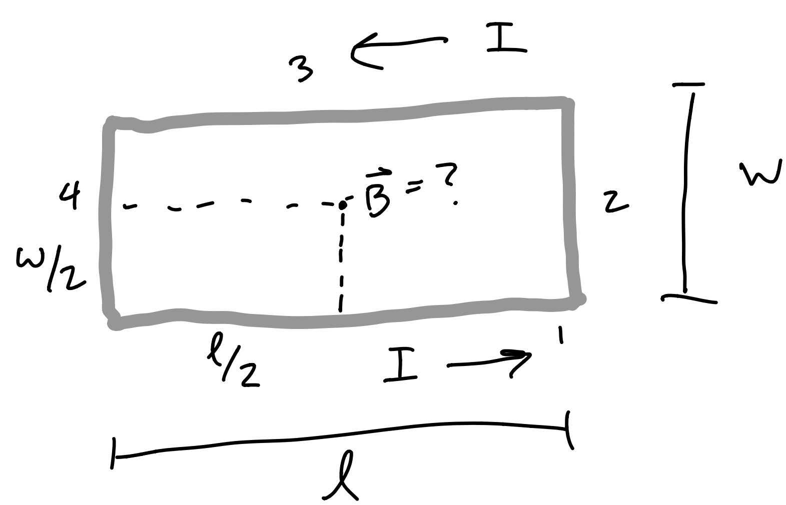
$$|\vec{B}| = \frac{\mu_0}{4\pi} \frac{LI}{r\sqrt{r^2 + (L/2)^2}}$$

Direction? Circling the current.

Example:



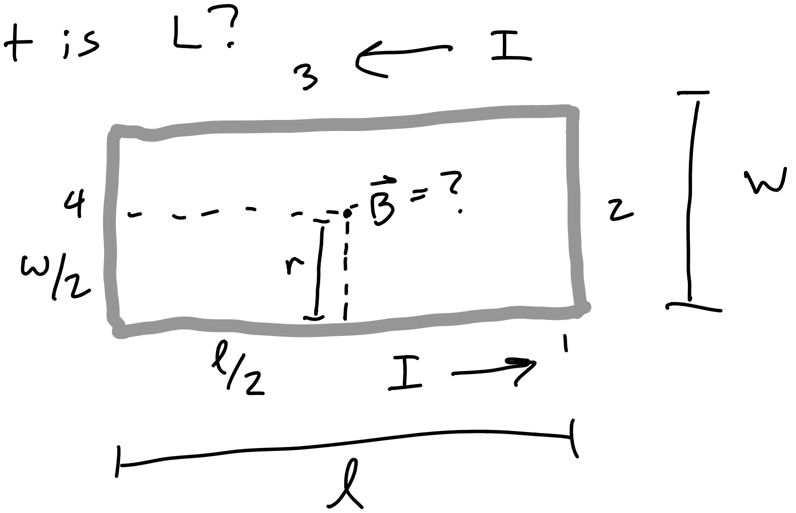
Superposition of 4 wires



$\vec{B}_1$: $|\vec{B}| = \frac{\mu_0}{4\pi} \frac{L I}{r\sqrt{r^2 + (L/2)^2}}$

What is r?

What is L?



$r = w/2$ $\quad L = l$

$$|\vec{B}_1| = \frac{\mu_0}{4\pi} \frac{l I}{\frac{w}{2}\sqrt{(\frac{w}{2})^2 + (\frac{l}{2})^2}}$$

$\hat{B}_1$? $= \odot$ (RHR)

$\vec{B}_2$?

$r = l/2$ $\quad L = w$

$$|\vec{B}_2| = \frac{\mu_0}{4\pi} \frac{w I}{\frac{l}{2}\sqrt{(\frac{l}{2})^2 + (\frac{w}{2})^2}}$$

Dir $= \odot$

B_3

$r = \frac{w}{2} \quad L = \ell$

$|\vec{B}_3| = |\vec{B}_1|$

$\hat{B}_3 = \odot$

$$|\vec{B}_3| = \frac{\mu_0}{4\pi} \frac{\ell I}{\frac{w}{2}\sqrt{\left(\frac{w}{2}\right)^2 + \left(\frac{\ell}{2}\right)^2}}$$

$\vec{B}_4$:

$r = \ell/2, \quad L = w$

$|\vec{B}_4| = |\vec{B}_2|$

$$|\vec{B}_4| = |\vec{B}_2| = \frac{\mu_0}{4\pi} \frac{w I}{\frac{\ell}{2}\sqrt{\left(\frac{\ell}{2}\right)^2 + \left(\frac{w}{2}\right)^2}}$$

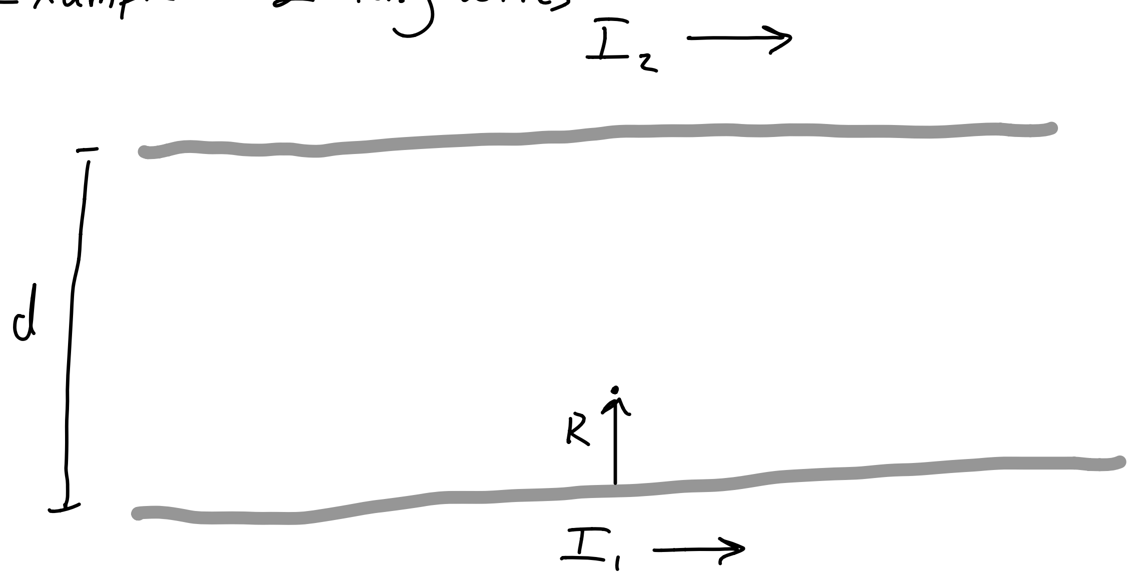
Dir $= \odot$

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{\ell I}{\frac{w}{2}\sqrt{\left(\frac{w}{2}\right)^2 + \left(\frac{\ell}{2}\right)^2}} + \frac{\mu_0}{4\pi} \frac{w I}{\frac{\ell}{2}\sqrt{\left(\frac{\ell}{2}\right)^2 + \left(\frac{w}{2}\right)^2}}$$

$$+ \frac{\mu_0}{4\pi} \frac{\ell I}{\frac{w}{2}\sqrt{\left(\frac{w}{2}\right)^2 + \left(\frac{\ell}{2}\right)^2}} + \frac{\mu_0}{4\pi} \frac{w I}{\frac{\ell}{2}\sqrt{\left(\frac{\ell}{2}\right)^2 + \left(\frac{w}{2}\right)^2}} \quad \odot$$

$$\vec{B} = \frac{2\mu_0 I}{4\pi}\left[\frac{2\ell}{w} + \frac{2w}{\ell}\right]$$

Example: 2 long wires



$$\vec{B} = \vec{B}_1 + \vec{B}_2$$

$$|\vec{B}| = \frac{\mu_0}{2\pi}\frac{I}{r}$$

$$|\vec{B}_1| = \frac{\mu_0}{2\pi}\frac{I_1}{R}$$

$$\vec{B}_1 = \frac{\mu_0}{2\pi}\frac{I_1}{R}\odot$$

$$|\vec{B}_2| = \frac{\mu_0}{2\pi}\frac{I_2}{d-R}$$

$$\vec{B}_2 = \frac{\mu_0}{2\pi}\frac{I_2}{d-R}\otimes$$

$$\vec{B} = \frac{\mu_0}{2\pi}\frac{I_1}{R}\odot + \frac{\mu_0}{2\pi}\frac{I_2}{d-R}\otimes$$

$$\vec{B} = \frac{\mu_0}{2\pi}\frac{I_1}{R}\odot - \frac{\mu_0}{2\pi}\frac{I_2}{d-R}\odot$$

$$\vec{B} = \frac{\mu_0}{2\pi}\left(\frac{I_1}{R} - \frac{I_2}{d-R}\right)$$

$$\vec{B} = 0 \Rightarrow \frac{I_1}{R} - \frac{I_2}{d-R} = 0$$

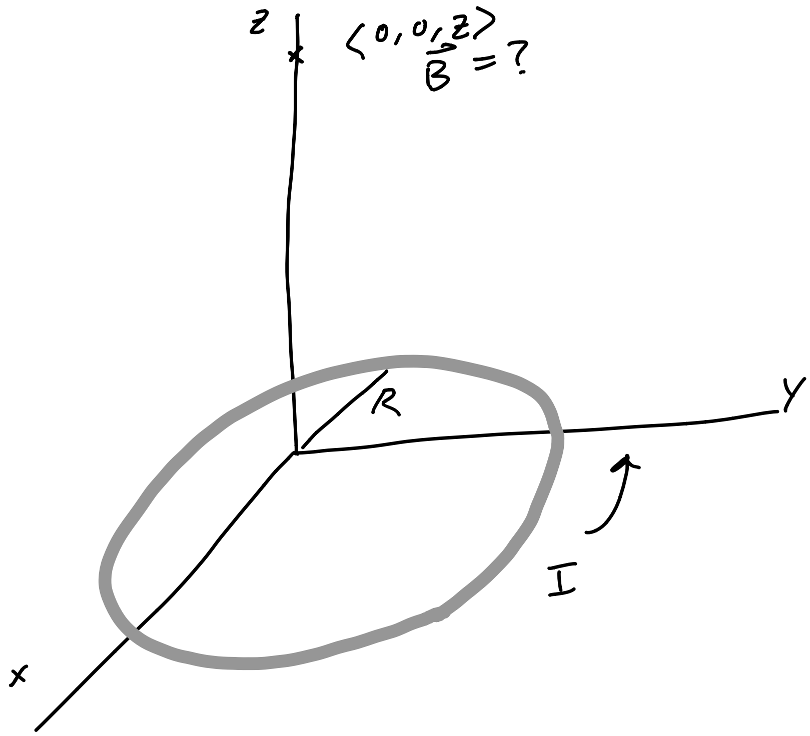
$$\frac{I_1(d-R)}{R(d-R)} - \frac{I_2 R}{R(d-R)} = 0$$

$$I_1(d-R) - I_2 R = 0$$

$$R(-I_1 - I_2) + I_1 d = 0$$

$$R = \frac{I_1}{I_1 + I_2} d$$

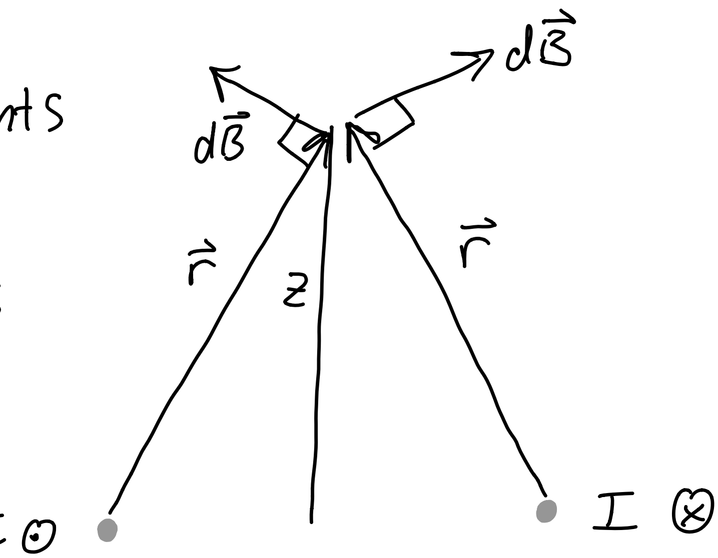
RING

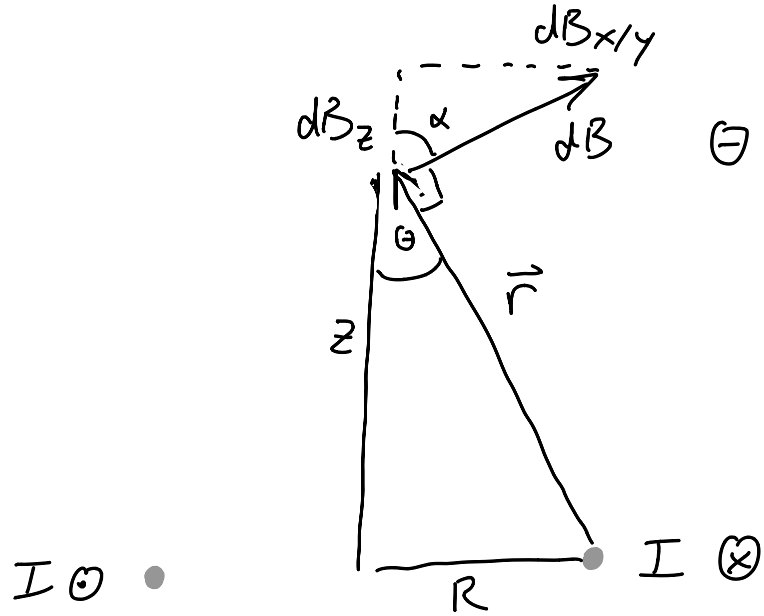


Direction of $\vec{B}$ at $\langle 0,0,z\rangle$?

X + Y components cancel

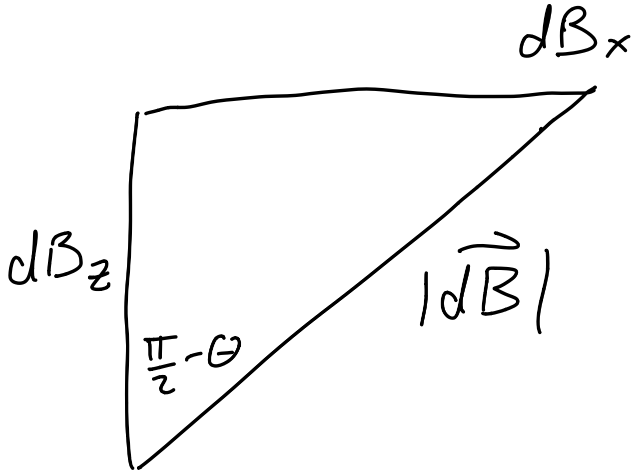
$\vec{B}$ is $\hat{z}$





$$\theta + \frac{\pi}{2} + \alpha = \pi$$

$$\alpha = \frac{\pi}{2} - \theta$$



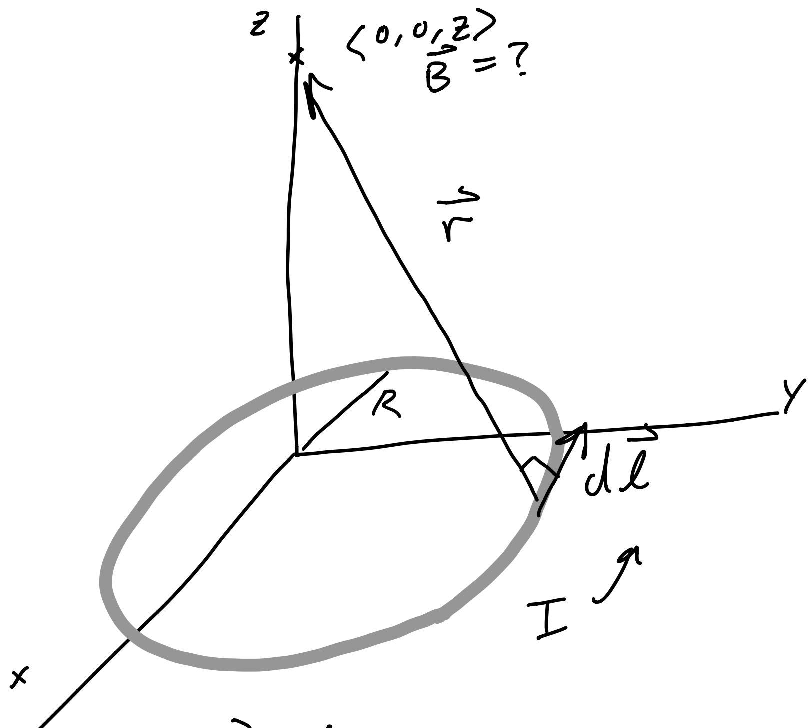
$$dB_z = |d\vec{B}| \cos\left(\frac{\pi}{2} - \theta\right)$$

$$dB_z = |d\vec{B}| \sin(\theta)$$

$$\sin\theta = \frac{R}{|\vec{r}|} = \frac{R}{\sqrt{R^2 + z^2}}$$

$$dB_z = |d\vec{B}| \frac{R}{\sqrt{R^2+z^2}}$$

What is $|d\vec{B}|$?



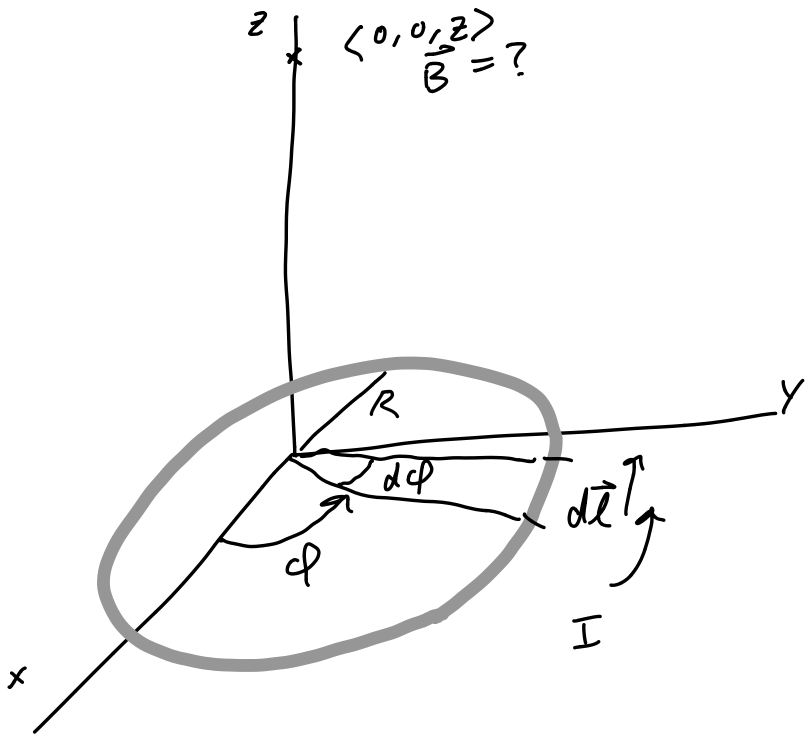
$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{\ell} \times \hat{r}}{r^2}$$

$d\vec{\ell} \perp \hat{r}$

$$|d\vec{\ell} \times \hat{r}| = |d\vec{\ell}||\hat{r}| \sin\left(\frac{\pi}{2}\right)$$
$$= |d\vec{\ell}|$$

$$|d\vec{B}| = \frac{\mu_0}{4\pi} \frac{I|d\vec{\ell}|}{r^2}$$

$|d\vec{\ell}|$? arc length



$$|d\vec{l}| = R\,d\varphi \qquad \left(\theta = \frac{s}{R}\right)$$

$$|d\vec{B}| = \frac{\mu_0}{4\pi}\,\frac{I R\,d\varphi}{R^2+z^2}$$

$$dB_z = |d\vec{B}|\frac{R}{\sqrt{R^2+z^2}}$$

$$dB_z = \frac{\mu_0}{4\pi}\,\frac{I R^2\,d\varphi}{(R^2+z^2)^{3/2}}$$

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{IR^2}{(R^2+z^2)^{3/2}} \int_0^{2\pi} d\phi \quad \hat{z}$$

$$\vec{B}_{loop} = \frac{\mu_0}{2} \frac{IR^2}{(R^2+z^2)^{3/2}} \hat{z}$$