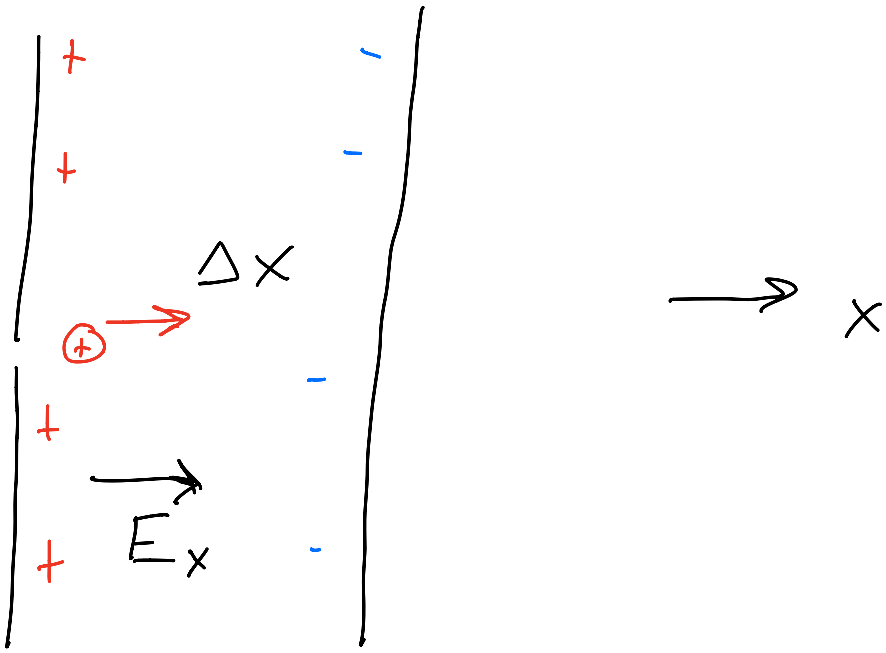
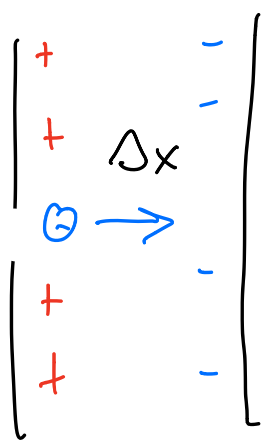


$$\Delta U = -W = -F_x \Delta x = -eE_x \Delta x$$



$$\Delta U = -W = -F_x \Delta x = (-e)E_x \Delta x$$
$$= eE_x \Delta x$$

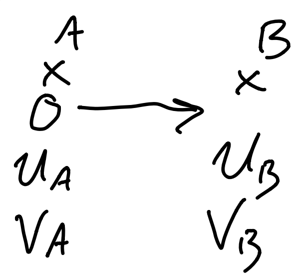
$$\Delta U_{proton} = (+e)(-E_x \Delta x)$$
$$\Delta U_{electron} = (-e)(-E_x \Delta x)$$

Electric Potential

$$-E_x \Delta x = \Delta V$$

$$\Delta U = q\Delta V, \text{ so } \Delta V = \frac{\Delta U}{q}$$

$$\vec{E} = \frac{\vec{F}}{q} \Rightarrow \Delta V = \frac{\Delta U}{q}$$



$$\Delta V = V_B - V_A$$
$$\Delta U = U_B - U_A$$

units of ΔV? $\frac{\Delta U}{q} = \frac{J}{C} \equiv$ Volts $= V$

Generalize

If $\vec{E}$ and $\Delta\vec{r}$ both point in $\hat{x}$

then $\Delta V = -E_x \Delta r_x = -E_x \Delta x$

What if $\vec{E} = \langle E_x, E_y, E_z \rangle$

$\Delta\vec{r} = \langle \Delta x, \Delta y, \Delta z \rangle$?

$$\Delta V = -E_x \Delta x - E_y \Delta y - E_z \Delta z$$

$$\Delta V = -(E_x \Delta x + E_y \Delta y + E_z \Delta z)$$

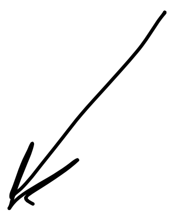
$$\boxed{\Delta V = -\vec{E} \cdot \Delta\vec{r}}$$

uniform field

Example

$\times$ B $= \langle 0, 0, 0 \rangle$ m

$\vec{E} = \langle -200, -300, 0 \rangle \frac{N}{C}$



$\times$ C $= \langle 0, -2, 0 \rangle$ m

ΔV from B to C?

1) $\Delta \vec{r} = \vec{r}_{Final} - \vec{r}_{initial} = \langle 0, -2, 0 \rangle - \langle 0, 0, 0 \rangle = \langle 0, -2, 0 \rangle$

$$\Delta V = -\vec{E} \cdot \Delta \vec{r}$$

$$= -\langle -200, -300, 0 \rangle \cdot \langle 0, -2, 0 \rangle$$

$$= 0 - 600 + 0 = \boxed{-600\ V}$$

ΔV from C to B? $\boxed{600\ V}$

Ex:

$\times A \ \langle 3,2,0\rangle m$ $\qquad \times B \ \langle 5,2,0\rangle m$

$$\vec{E} = \langle 0,-400,0\rangle \ \frac{N}{C}$$

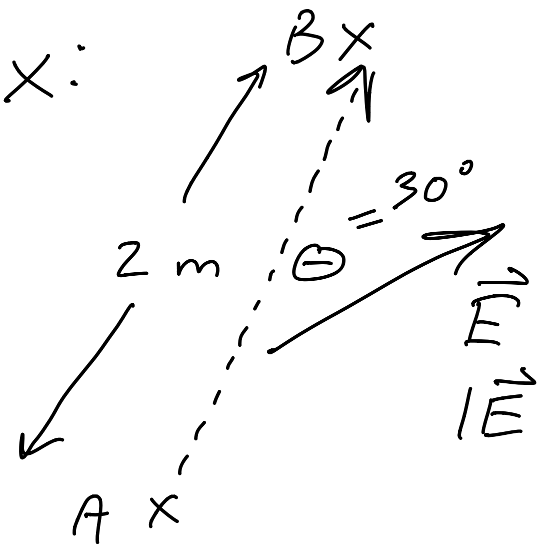
ΔV A to B?

1) $\Delta\vec{r} = \langle 5,2,0\rangle - \langle 3,2,0\rangle$
$= \langle 2,0,0\rangle$

$$\Delta V = -\vec{E}\cdot\Delta\vec{r} = \langle 0,-400,0\rangle \cdot \langle 2,0,0\rangle$$

$$\Delta V = 0$$

Ex:



$|\vec{E}| = 100\ \frac{N}{C}$

$$\Delta V = -\vec{E} \cdot d\vec{r}$$

$$= -|\vec{E}||\Delta\vec{r}|\cos\theta$$

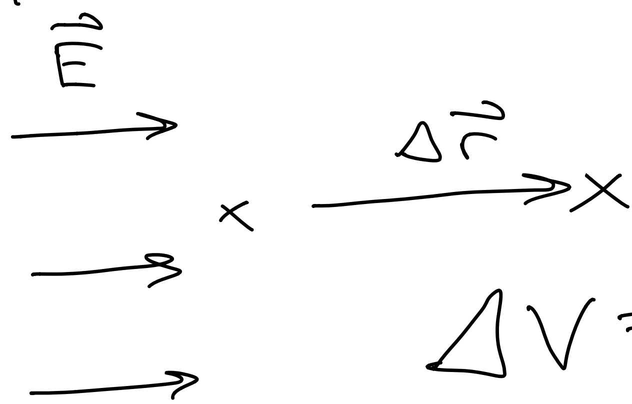
$$= -\left(100\tfrac{N}{C}\right)(2m)\cos(30)$$

$$\Delta V = -173\ V$$

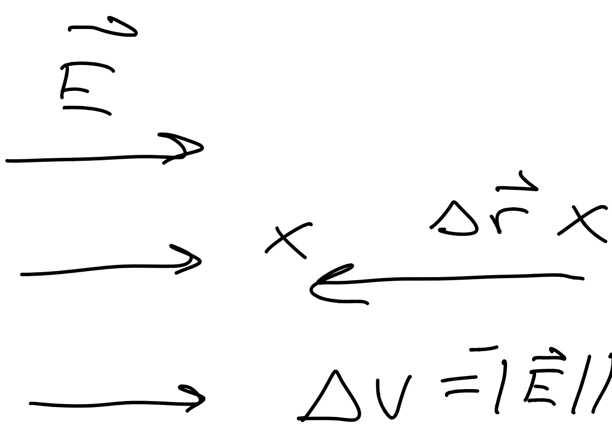
Sign of ΔV

The Sign of ΔV

ΔV decreases if your path moves with $\vec{E}$

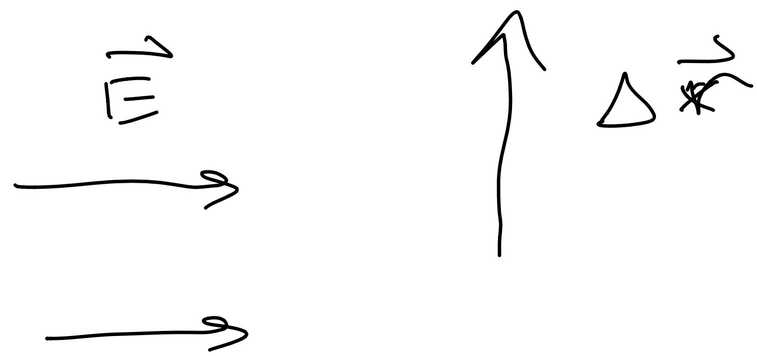


$$\Delta V = -\vec{E}\cdot\Delta\vec{x}$$
$$= |\vec{E}||\Delta x|\cos\theta$$
$$\cos(0) = 1$$
$$\Delta V = |\vec{E}||\Delta\vec{x}|$$



$$\Delta V = -|\vec{E}||\Delta\vec{x}|\cos(180)$$
$$= -|\vec{E}||\Delta\vec{x}|(-1)$$
$$= |\vec{E}||\Delta\vec{x}|$$

Path Perpendicular to Field?



$$\Delta V = -|\vec{E}|\,|\Delta \vec{r}| \cos(90)$$
$$= 0$$

No Force

PATH IN DIRECTION OF $\vec{E}$

Potential decrease $\Delta V < 0$

PATH OPPOSITE TO $\vec{E}$

Potential increase $\Delta V > 0$

PATH PERP TO $\vec{E}$: $\Delta V = 0$

Going back:

$$\vec{E} = \langle E_x, 0, 0 \rangle$$

$$\Delta \vec{r} = \langle \Delta x, 0, 0 \rangle$$

$$\Delta V = -E_x \Delta x$$

$$E_x = -\frac{\Delta V}{\Delta x}$$

$$\lim_{\Delta x \to 0} -\frac{\Delta V}{\Delta x} = -\frac{dV}{dx}$$

$$E_x = -\frac{dV}{dx}, \quad E_y = -\frac{dV}{dy}, \quad E_z = -\frac{dV}{dz}$$

$$\vec{E} = -\left\langle \frac{d}{dx}V, \frac{d}{dy}V, \frac{d}{dz}V \right\rangle$$

Can Find $\vec{E}$, a vector, from a scalar!

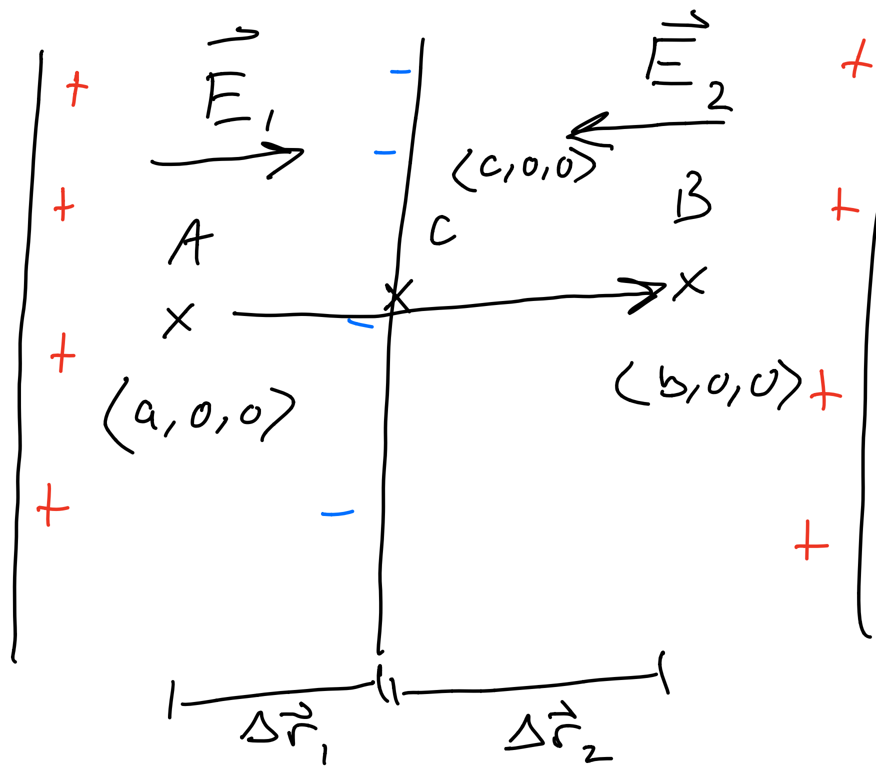
Units?

$$E_x = -\frac{\Delta V}{\Delta x} = \frac{V}{m}\ ?$$

$$V = \frac{J}{C} = \frac{Nm}{C}$$

$$\frac{V}{m} = \frac{Nm}{Cm} = \frac{N}{C}\ \checkmark$$

Example: A double capacitor



$$\Delta V = \Delta V_1 + \Delta V_2$$

$$= \Delta V_{AC} + \Delta V_{CB}$$

$$= -\vec{E}_1 \cdot \Delta\vec{r}_1 - \vec{E}_2 \cdot \Delta\vec{r}_2$$

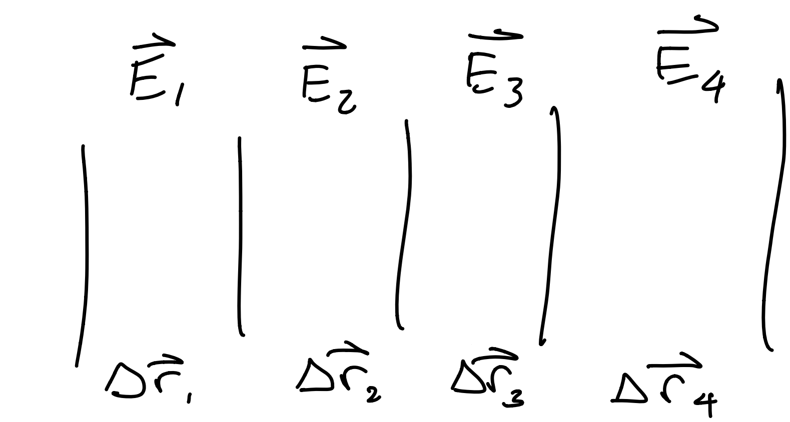
$$\Delta\vec{r}_1 = \langle c-a, 0, 0\rangle$$

$$\Delta\vec{r}_2 = \langle b-c, 0, 0\rangle$$

$$\Delta V_1 = -\vec{E}_1 \cdot \langle c-a, 0, 0 \rangle$$
$$= -\langle E_{1x}, 0, 0 \rangle \cdot \langle c-a, 0, 0 \rangle$$
$$= -E_{1x}(c-a)$$

$$\Delta V_2 = -\vec{E}_2 \cdot \langle b-c, 0, 0 \rangle$$
$$= -\langle -E_{2x}, 0, 0 \rangle \cdot \langle b-c, 0, 0 \rangle$$
$$= E_{2x}(b-c)$$

$$\Delta V = \Delta V_1 + \Delta V_2 = -E_{1x}(c-a) + E_{2x}(b-c)$$



$$\Delta V = -\vec{E}_1 \cdot \Delta\vec{r}_1 - \vec{E}_2 \cdot \Delta\vec{r}_2$$
$$- \vec{E}_3 \cdot \Delta\vec{r}_3 - \vec{E}_4 \cdot \Delta\vec{r}_4$$

$$\Delta V = \sum_i -\vec{E}_i \cdot \Delta \vec{r}_i$$