Project 3

Quantum Tunneling
Due: Thursday, December 17

Overview

In this project you will examine the behavior of a particle incident upon a potential barrier. Specifically, you will record the transmission probability as a function of potential energy of the barrier. You will start with a Gaussian moving with momentum $p = \hbar k$:

$$\Psi_0(x,0) = \left(\frac{1}{\pi\sigma^2}\right)^{1/4} e^{-\frac{1}{2}\frac{(x-x_0)^2}{\sigma^2}} e^{ik(x-x_0)} \tag{1}$$

The potential barrier is characterized by the so-called "height" V_0 and the width d.
You will simulate this scenario several times while varying V_0.

Details

Use the following parameters for your program:

Grid parameters:

- x runs from 0 to 1.5 in steps of $\Delta x = 5 \times 10^{-4}$
- t runs from 0 to 2×10^{-3} in steps of $\Delta t = 10^{-7}$

Ψ_0:

- $x_0 = 0.3$
- $\sigma = 0.05$
- $k = 350$

Potential barrier

- $x_0 = 0.6$
- $d = 5 \times 10^{-3}$
- V_0: ranging from 0 to $2E$, where E is the initial energy of the particle $E = \frac{1}{2}k^2$.

With wavenumber k, the particle has a momentum $p = k$ (in our units, $\hbar = m = 1$) and total energy $E = p^2/2m = k^2/2$. You will run the simulation 11 times (stepping from $V_0 = 0$ to $V_0 = 2E$ in steps of $0.2E$).

In addition to the 11 values of transmission probability, you should turn in a plot of the wave function at the final time step for any one of your simulations, as well as all of your code.