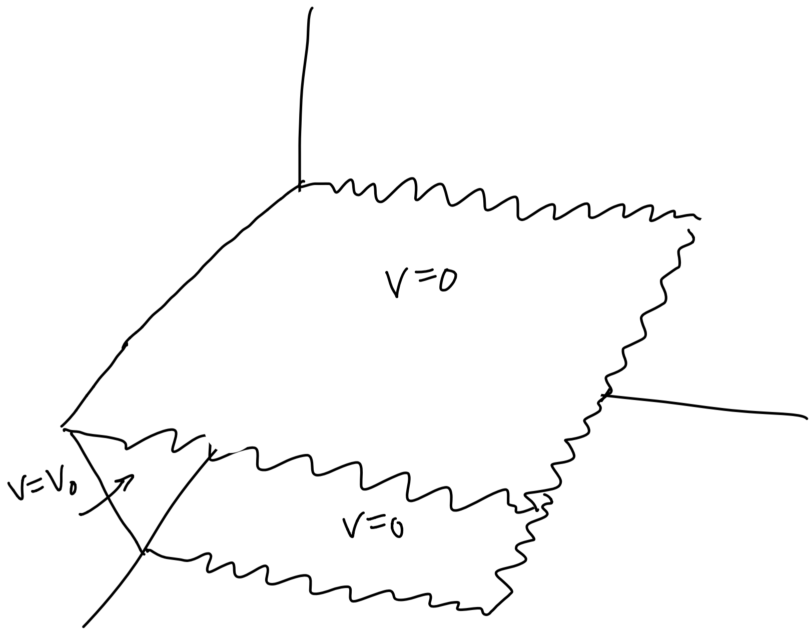
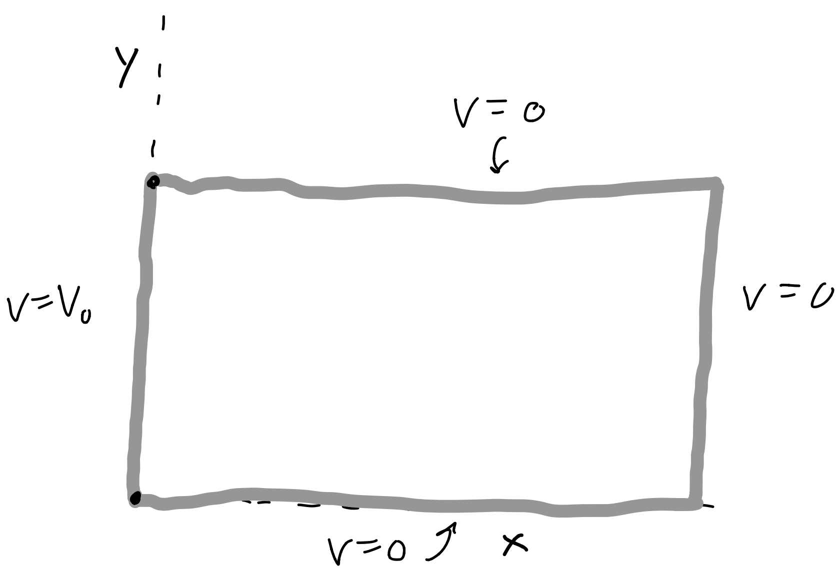
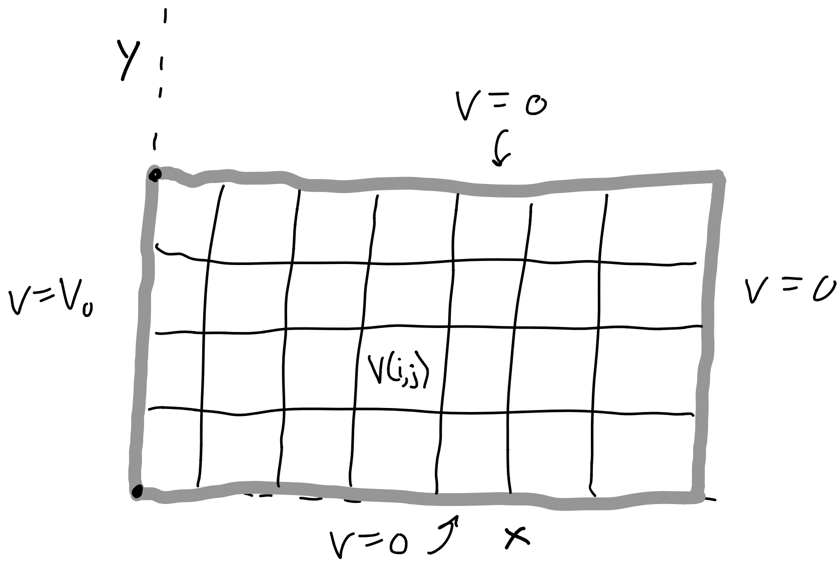
Jacobi in 2D







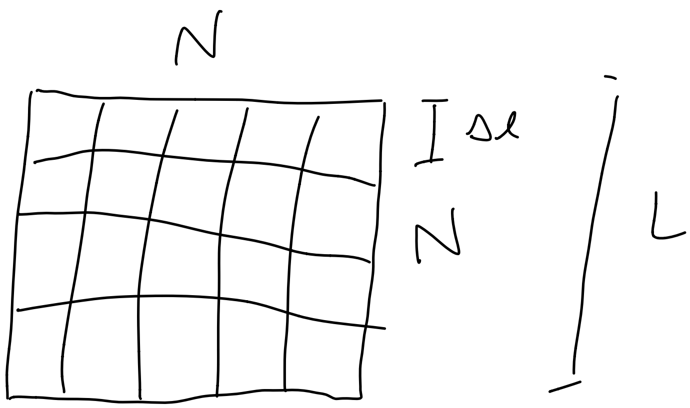
1	0	0	0	0	0	0
1	0	0	0	0	0	0
1	0	0	0	0	0	0
1	0	0	0	0	0	0
1	0	0	0	0	0	0
1	0	0	0	0	0	0

1	0	0	0	0	0	0
1	0.5	0.25	0	0	0	0
1	0.5	0.25	0	0	0	0
1	0.5	0.25	0	0	0	0
1	0.5	0.25	0	0	0	0
1	0	0	0	0	0	0

$$\frac{1+.5+.5+.25}{4}$$

1	0	0	0	0	0	0
1	0.5	0.25	0	0	0	0
1		0.25	0	0	0	0
1	0.5	0.25	0	0	0	0
1	0.5	0.25	0	0	0	0
1	0	0	0	0	0	0

Dramatically slower for large 2D systems.



Consider evaluating V on an $N \times N$ grid, where $N = \frac{L}{\Delta l}$

# of iterations to converge $n \sim N^2$

Each iteration performs N^2 operations

Computational time $\tau \sim N^4$ (!)

if we want $\Delta l \rightarrow \frac{1}{2}\Delta l$, $N \rightarrow 2N$

$16\times$ the computation!

Jacobi not used much in practice

A slightly different algo: Gauss-Seidel

Jacobi

$(1, 0, 0, 0, \ldots, 0, 0, 0, -1)$

$(1, 0.5, 0, 0, \ldots, 0, 0, -.5, -1)$

$(1, 0.5, 0.25, 0, 0, \ldots, -0.25, -.5, -1)$

$(1, 0.625, 0.25, 0.125, \ldots, -0.125, -0.25, -0.5, -1)$

Use numbers as they become available

$(1, 0, 0, 0, 0, -1)$

$(1, 0.5, 0.25, 0.125, -0.4375, -1)$

$(1, 0.625, 0.375, -0.03125, -0.516\ldots, -1)$

$$V_n(i) = V_{n-1}(i+1) + \underline{\underline{V_n(i-1)}}$$

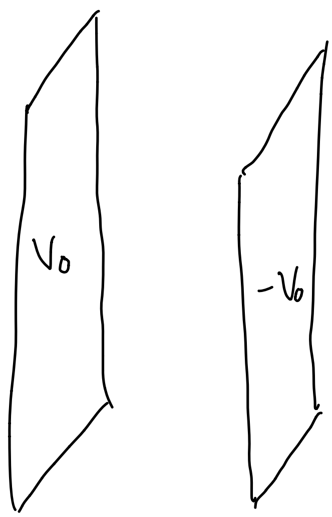
$\sim$ Factor of 2 efficeincy boost

||

capac

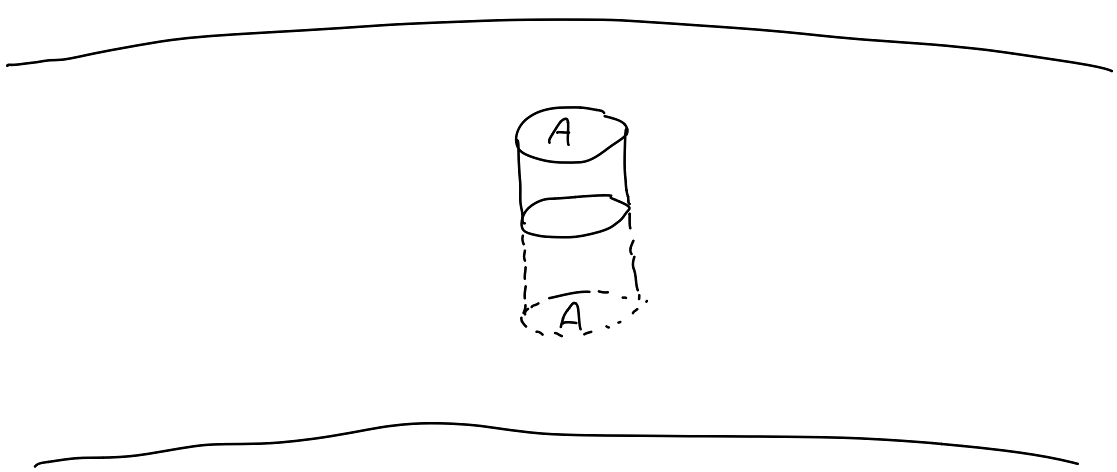
Example

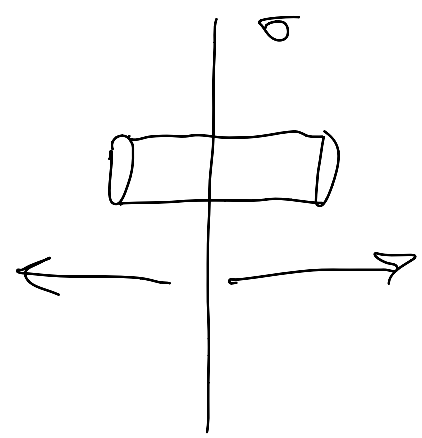
Parallel Plate Capacitor



Analytically: (infinite sheets)

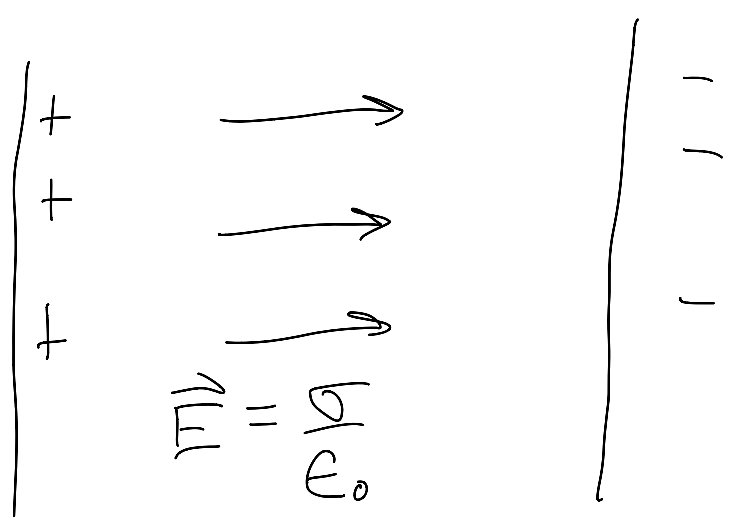
$Q/A = \sigma$





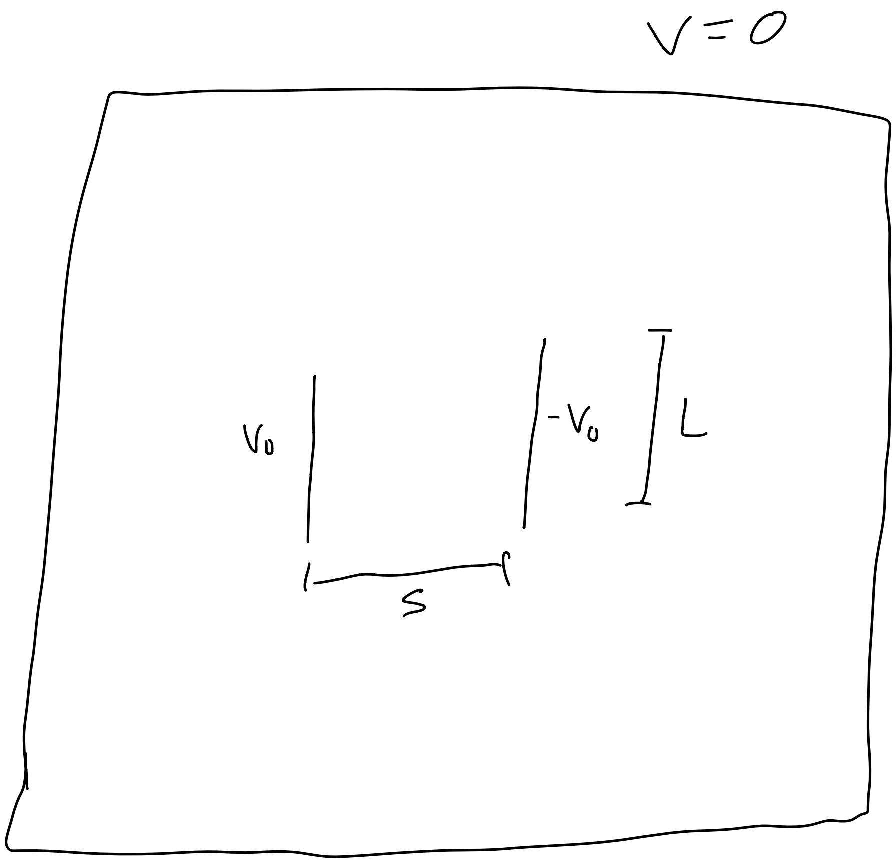
$$\int \vec{E} \cdot d\vec{a} = 2EA = \frac{Q_{enc}}{\epsilon_0} = \frac{\sigma A}{\epsilon_0}$$

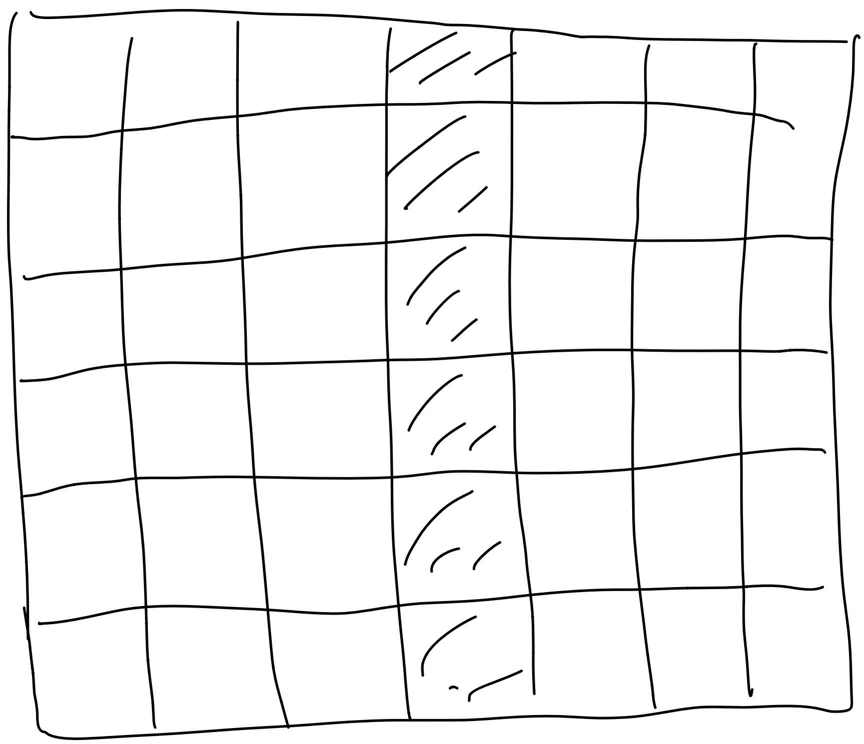
$$E = \frac{1}{2} \frac{\sigma}{\epsilon_0}$$

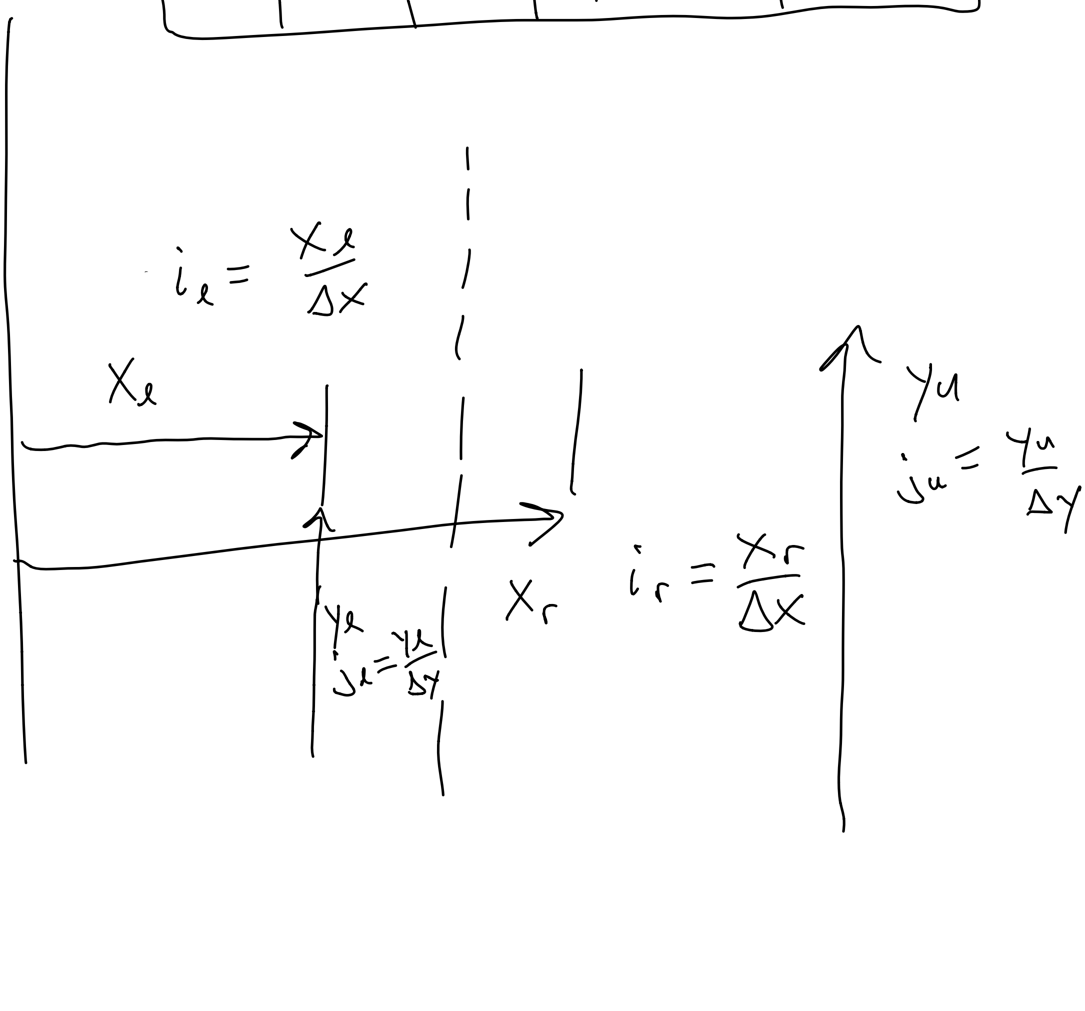


Finite sheets

Set up boundaries







$i_l = \frac{x_l}{\Delta x}$
x_l
y_u
$j_u = \frac{y_u}{\Delta y}$
y_l
$j_l = \frac{y_l}{\Delta y}$
x_r
$i_r = \frac{x_r}{\Delta x}$