- Review of last class
  - Wrote down drag force and solved for bicycle motion
  - Today we want perform the same analysis on a moving projectile

$$\frac{d^2}{dt^2}x = 0$$

$$\frac{d^2}{dt^2}y = -g$$

$$\frac{dV_x}{dt} = 0$$

$$\frac{dx}{dt} = V_x$$

$$\frac{dV_y}{dt} = -g$$

$$\frac{dy}{dt} = V_y$$

$$x_i = x_{i-1} + v_{x,i-1}\,\Delta t$$

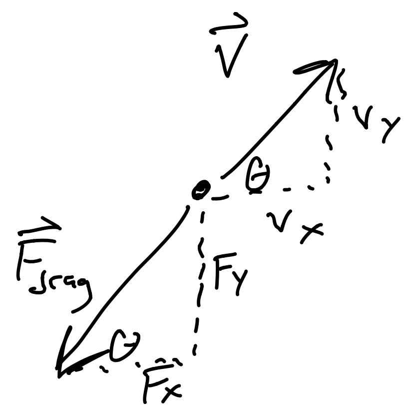
$$v_{x,i} = v_{i-1}$$

$$y_i = y_{i-1} + v_{y,i-1}\,\Delta t$$

$$v_{y,i} = v_{y,i-1} - g\,\Delta t$$

- Step through jupyter program
- Now let's add a drag force

$$\text{Let } F_{drag} = B_2 v^2$$



$$|\vec{v}| = \sqrt{v_x^2 + v_y^2}$$

$$F_x = F_{drag}\cos\theta = F\frac{v_x}{|\vec{v}|} = B_2\,|\vec{v}|^2\,\frac{v_x}{|\vec{v}|}$$

$$\cos\theta = \frac{v_x}{|\vec{v}|} \qquad = -B_2\,|\vec{v}|\,v_x$$

$$F_y = F\sin\theta = -B_2 |\vec{v}| v_y$$

$$F_x = m\frac{dv_x}{dt} = -B_2\, v\, v_x$$

$$\frac{dv_x}{dt} = \frac{-B_2}{m} v \cdot v_x$$

$$\frac{dv_y}{dt} = -g - \frac{B_2}{m} v \cdot v_y$$

Normalization (optional)

$\bar{v} = \frac{v}{v_0}$, $\bar{t} = \frac{t}{t_0}$

$$\frac{dv_y}{dt} = -g - \frac{B_2}{m} v v_y$$

$$\frac{v_0}{t_0}\frac{d\bar{v}_y}{d\bar{t}} = \text{" "}$$

$$\frac{d\bar{v}_y}{d\bar{t}} = -\frac{t_0}{v_0} g - \frac{t_0}{v_0} \frac{B_2}{m} v_0^2 \bar{v} \bar{v}_y$$

$$\frac{d\bar{v}_y}{d\bar{t}} = -\frac{t_0}{v_0} g - v_0 t_0 \frac{B_2}{m} \bar{v} \bar{v}_y$$

$v_0 = v_i \qquad \bar{v}(0) = 1$

$$\frac{v_0}{t_0} = g \Rightarrow t_0 = \frac{v_i}{g}$$

$$v_0 t_0 = v_i^2 / g$$

$$\frac{d\bar{v}_y}{d\bar{t}} = -1 - \frac{v_i^2}{g} \frac{B_2}{m} \bar{v} \bar{v}_y$$

$$\left[\frac{B_2}{m}\right] = \frac{1}{L}$$

$$\frac{v^2}{g} \frac{1}{L} = \frac{L^2}{T^2} \frac{T^2}{L} \frac{1}{L} = \phi$$

units:

$$\bar{v} = \frac{v}{v_i} \qquad \bar{t} = \frac{t}{v_i/g} \; ; \; \frac{v_i}{g} \text{ time for gravity to decelerate}$$

$$r_0 = \frac{v_i^2}{g} = v_i \cdot t_0 = v_i \cdot \frac{v_i}{g}$$

$$\bar{x} = \frac{x}{r_0} \qquad \bar{y} = \frac{y}{r_0}$$

$$\alpha = \frac{v_i^2}{g} \frac{B_2}{m} = \frac{v_i^2}{g} \Big/ \frac{m}{B_2}$$

$$\frac{d\bar{v}_y}{d\bar{t}} = -1 - \alpha \bar{v} \bar{v}_y$$

$$\frac{d\bar{v}_x}{d\bar{t}} = -\alpha \bar{v} \bar{v}_y$$

$$\frac{d\bar{x}}{d\bar{t}} = \bar{V}_x$$

$$\frac{d\bar{y}}{d\bar{t}} = \bar{V}_y$$

$$V_{i-1} = \sqrt{V_{x,i-1}^2 + V_{y,i-1}^2}$$

$$V_{y,i} = V_{y,i-1} - \left(1 + \alpha V_{i-1} V_{y,i-1}\right) \Delta t$$

$$V_{x,i} = V_{x,i-1} - \alpha V_{i-1} V_{y,i-1} \Delta t$$

$$X_i = X_{i-1} + V_{x,i-1} \Delta t$$

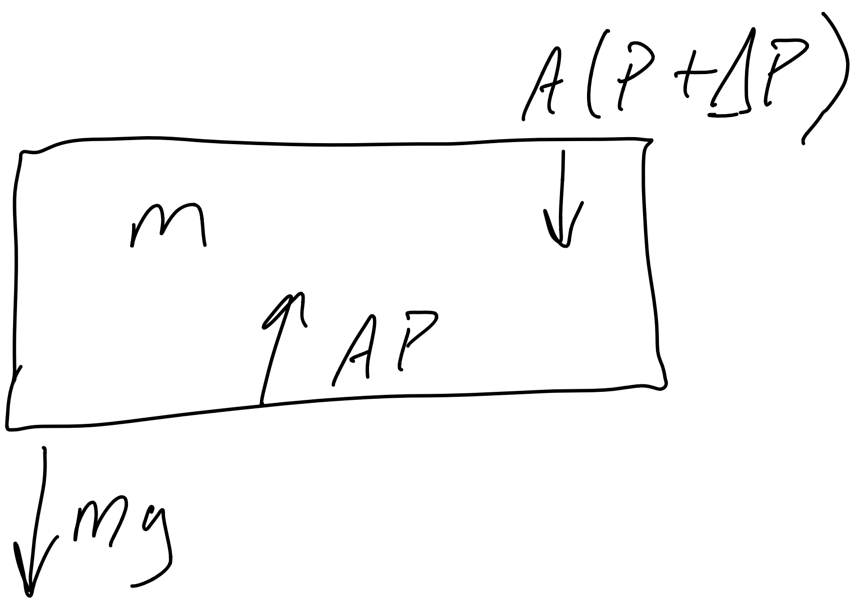
$$y_i = y_{i-1} + V_{y,i-1} \Delta t$$

jupyter

Here, B_z is constant

In reality, is altitude dependent

How so?



$$-mg + AP - A(P+\Delta P) = 0$$

$$A\Delta P = -mg$$

$$\Delta P = -\frac{mg}{A}$$

$$m = \rho A \Delta y$$

$$\Delta P = -\rho g \Delta y$$

$$\frac{\Delta P}{\Delta y} = -\rho g$$

$$\rho = \frac{Nm}{\bar{V}}$$

$$P\bar{V} = NkT$$

$$\bar{V} = \frac{NkT}{P}$$

$$\rho = \frac{Nm}{\frac{NKT}{P}}$$

$$\rho = \frac{Pm}{KT}$$

$$\frac{\Delta P}{\Delta y} = -\rho g = -\frac{Pm}{KT} g$$

$$\frac{\Delta P}{\Delta y} \longrightarrow \frac{dP}{dy} \quad (\Delta y \to 0)$$

$$\frac{dP}{dy} = \frac{-mg}{KT} P$$

$$\frac{dP}{P} = \frac{-mg}{kT}\,dy$$

$$\ln(P) = \frac{-mg}{kT}\,y + c$$

$$P = e^{\frac{-mg}{kT}y + c}$$

$$= e^{\frac{-mg}{kT}y}\,e^{c}$$

$$P = P_0\,e^{-\frac{mg}{kT}y}$$

$$P = \frac{N}{V}\,kT$$

$$P = \frac{\rho}{m}\,kT$$

if $\frac{dT}{dy} = 0$

then

$$\rho \propto P$$

$$\rho = \rho_0 e^{\left(-\frac{mg}{kT} y\right)}$$

$$F_{drag} \longrightarrow \frac{\rho}{\rho_0} F_{drag}$$

$$\frac{mg}{kT} = y_0$$

$$F_{drag}^{*} = F_{drag} e^{\left(-\frac{y}{y_0}\right)}$$

$$B_2 \longrightarrow B_2 e^{-\frac{y}{y_0}}$$

$$\alpha \longrightarrow \alpha e^{-y/y_0}$$

$$y_0 \approx 10^4 \, m$$

$$\bar{y}_0 = \frac{g}{V_i^2} y_0$$

need actual values for g & V_i to get $\bar{y}_0$