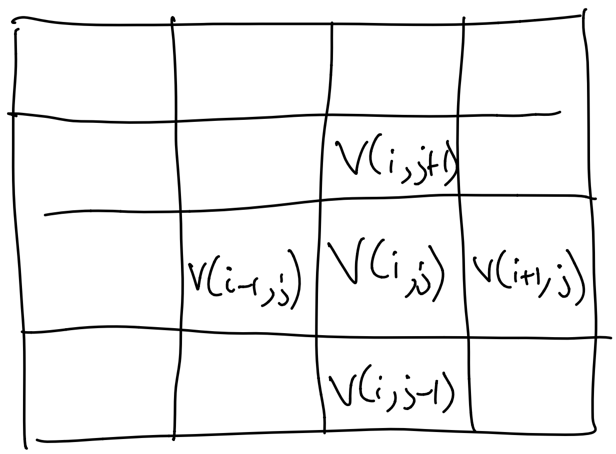
$$V(i,j,k) = \frac{1}{6}\Big[V(i+1,j,k) + V(i-1,j,k) + V(i,j+1,k) + V(i,j-1,k) + V(i,j,k+1) + V(i,j,k-1)\Big]$$



How to solve?

Iteratively

- Start with some initial guess $V_0(i,j,k)$
  must satisfy BC's.

- Then calculate $V_1(i,j,k)$

$$V_1(i,j,k) = \frac{1}{6}\Big[V_0(i+1,j,k) + V_0(i-1,j,k) + V_0(i,j+1,k) + V_0(i,j-1,k) + V_0(i,j,k+1) + V_0(i,j,k-1)\Big]$$

Then calculate V_2:

$$V_2(i,j,k) = \frac{1}{6}\Big[V_1(i+1,j,k) \;\ldots\;\Big]$$

$$V_n(i,j,k) = \frac{1}{6}\Big[V_{n-1}(i+1,j,k) + V_{n-1}(i-1,j,k) + V_{n-1}(i,j+1,k) + V_{n-1}(i,j-1,k) + V_{n-1}(i,j,k+1) + V_{n-1}(i,j,k-1)\Big]$$

How to code: (1 dimension)

- Need 2 arrays

  Vold, Vnew

Vold = (anything)

Vold = zeros

vold[0] = V(x=a)

Vold[-1] = V(x=b)

Vnew = Vold

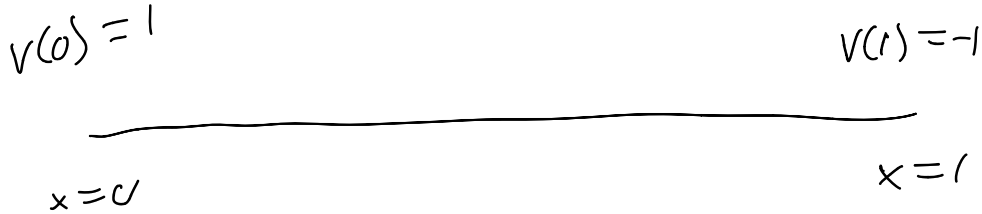
for i from 1 to N-1

    Vnew[i] = 0.5*(Vold[i-1] + Vold[i+1])

Vold = Vnew

  repeat...

Example:



$$V_0 = [1, 0, 0, 0, \ldots, 0, 0, -1]$$

$$V_1 = [1, 0.5, 0, 0, \ldots, 0, -0.5, -1]$$

$$V_2 = [1, 0.5, 0.25, \ldots, -0.25, -0.5, -1]$$

$$V_3 = [1, 0.625, 0.125, 0, 0 \ldots, -0.125, -0.625, -1]$$

$$\vdots$$

Analytical Solution?

$$V = C_1 x + C_2$$

$$v(0) = C_2 = 1$$

$$v(1) = C_1 + C_2 = -1, \quad C_1 + 1 = -1, \quad C_1 = -2$$

$$V(x) = -2x + 1$$

When to stop iterating?

Consider change between

V_n + V_{n-1}

$$\Delta V_{avg} = \frac{1}{N} \sum_{i=0}^{N} \left| V_n(i) - V_{n-1}(i) \right|$$

Keep looping until ΔV_{avg} is small compared to $|V|$ at boundary