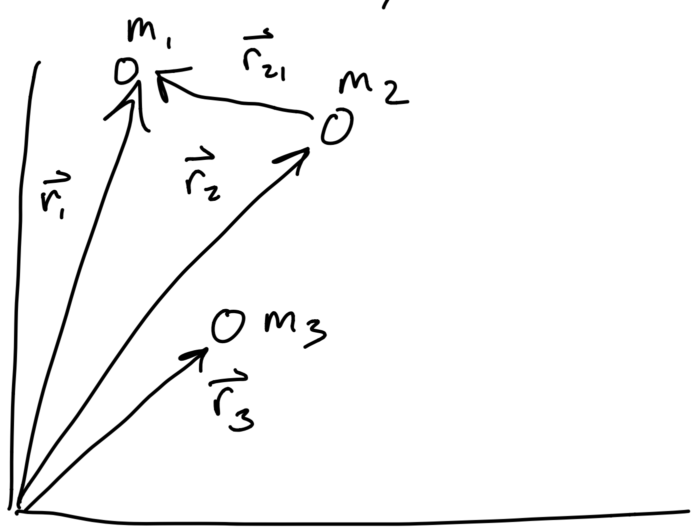
The 3-body problem



$$\vec{F}_{net,1} = \vec{F}_{21} + \vec{F}_{31}$$

$$\vec{F}_{21} = \frac{-G m_2 m_1}{r_{21}^2} \hat{r}_{21}$$

$$\vec{r}_{21} = (x_1 - x_2)\hat{x} + (y_1 - y_2)\hat{y}$$

$$\vec{F}_{21} = \frac{-G m_2 m_1}{\left[(x_1-x_2)^2 + (y_1-y_2)^2\right]^{3/2}} \left[(x_1-x_2)\hat{x} + (y_1-y_2)\hat{y}\right]$$

$$\vec{F}_{ij} = \frac{-G m_i m_j}{\left[(x_i-x_j)^2 + (y_i-y_j)^2\right]^{3/2}} \left[(x_i-x_j)\hat{x} + (y_i-y_j)\hat{y}\right]$$

$$F_{1,x} = F_{21,x} + F_{31,x}$$

$$F_{1x} = -\frac{Gm_2m_1\,(x_1 - x_2)}{\left[(x_1-x_2)^2+(y_1-y_2)^2\right]^{3/2}} - \frac{Gm_3m_1\,(x_1 - x_3)}{\left[(x_1-x_3)^2+(y_1-y_3)^2\right]^{3/2}}$$

$$F_{1y} = \frac{-Gm_2m_1\,(y_1 - y_2)}{\left[(x_1-x_2)^2+(y_1-y_2)^2\right]^{3/2}} \; \frac{-Gm_3m_1\,(y_1 - y_3)}{\left[(x_1-x_3)^2+(y_1-y_3)^2\right]^{3/2}}$$

$$\frac{dV_{1x}}{dt} = \frac{-Gm_2\,(x_1 - x_2)}{\left[(x_1-x_2)^2+(y_1-y_2)^2\right]^{3/2}} - \frac{Gm_3\,(x_1 - x_3)}{\left[(x_1-x_3)^2+(y_1-y_3)^2\right]^{3/2}}$$

$$\frac{dV_{1y}}{dt} = \frac{-Gm_2\,(y_1 - y_2)}{\left[(x_1-x_2)^2+(y_1-y_2)^2\right]^{3/2}} - \frac{Gm_3\,(y_1 - y_3)}{\left[(x_1-x_3)^2+(y_1-y_3)^2\right]^{3/2}}$$

Normalize

$$E = \frac{1}{2}m_1V_1^2 + \frac{1}{2}m_2V_2^2 + \frac{1}{2}m_3V_3^2$$

$$-\frac{Gm_1m_2}{r_{12}} - \frac{Gm_1m_3}{r_{13}} - \frac{Gm_2m_3}{r_{23}}$$

$$\bar{E} = \frac{1}{E_0}\left[\frac{1}{2}m_1\bar{V}_1^2V_0^2 + \frac{1}{2}m_2\bar{V}_2^2V_0^2 + \frac{1}{2}m_3\bar{V}_3^2V_0^2\right]$$

$$+\frac{1}{E_0}\left[-\frac{Gm_1m_2}{r_0\bar{r}_{12}} - \frac{Gm_1m_3}{r_0r_{13}} - \frac{Gm_2m_3}{r_0r_{23}}\right]$$

$$M v_0^2 = E_0$$

$$E_0 r_0 = G M^2$$

$$r_0 = \frac{G M^2}{E_0}$$

$$\left(\frac{r_0}{t_0}\right)^2 = \frac{E_0}{M} \Rightarrow t_0^2 = \frac{M}{E_0} r_0^2$$

$$= \frac{M}{E_0} \frac{G^2 M^4}{E_0^2}$$

$$t_0^2 = \frac{M^5 G^2}{E_0^3} = \left(\frac{M^2 G}{E_0}\right)^2 \frac{M}{E_0}$$

$$\vec{F}_{ij} = \frac{-G m_i m_j}{\left[(x_i - x_j)^2 + (y_i - y_j)^2\right]^{3/2}} \left[(x_i - x_j)\hat{x} + (y_i - y_j)\hat{y}\right]$$

$$m_i \frac{d^2 x_i}{dt^2} = \frac{-G m_i m_j (x_i - x_j)}{\left[(x_i - x_j)^2 + (y_i - y_j)^2\right]^{3/2}}$$

$$\frac{d^2 x_i}{dt^2} = \frac{-G m_j (x_i - x_j)}{\left[(x_i - x_j)^2 + (y_i - y_j)^2\right]^{3/2}}$$

$$\frac{r_0}{t_0^2}\frac{d^2\bar{x}_i}{dt^2} = \frac{1}{r_0^2}\frac{-Gm_j(\bar{x}_i - \bar{x}_j)}{\left[(\bar{x}_i-\bar{x}_j)^2+(\bar{y}_i-\bar{y}_j)^2\right]^{3/2}}$$

$$\frac{d^2\bar{x}_i}{d\bar{t}^2} = -\frac{t_0^2}{r_0^3}\,Gm_j\,\frac{(\bar{x}_i - \bar{x}_j)}{\left((\bar{x}_i-\bar{x}_j)^2+(\bar{y}_i-\bar{y}_j)^2\right)^{3/2}}$$

$$\frac{t_0^2}{r_0^3} = \frac{M}{E_0}\frac{1}{r_0} = \frac{M}{E_0}\frac{E_0}{GM^2} = \frac{1}{GM}$$

$$\frac{d^2\bar{x}_i}{d\bar{t}^2} = -\frac{m_j}{M}\frac{(\bar{x}_i - \bar{x}_j)}{\left((\bar{x}_i-\bar{x}_j)^2+(\bar{y}_i-\bar{y}_j)^2\right)^{3/2}}$$

$$= -\bar{m}_j\frac{(\bar{x}_i - \bar{x}_j)}{\left((\bar{x}_i-\bar{x}_j)^2+(\bar{y}_i-\bar{y}_j)^2\right)^{3/2}}$$

$$\frac{dV_{1x}}{dt} = \frac{-Gm_2}{\left[(x_1-x_2)^2+(y_1-y_2)^2\right]^{3/2}}(x_1-x_2) - \frac{Gm_3}{\left[(x_1-x_3)^2+(y_1-y_3)^2\right]^{3/2}}(x_1-x_3)$$

$$\frac{dV_{1y}}{dt} = \frac{-Gm_2}{\left[(x_1-x_2)^2+(y_1-y_2)^2\right]^{3/2}}(y_1-y_2) - \frac{Gm_3}{\left[(x_1-x_3)^2+(y_1-y_3)^2\right]^{3/2}}(y_1-y_3)$$

$$d\bar{V}_{1x} = \frac{-\bar{m}_2(\bar{x}_1-\bar{x}_2)}{\left[(\bar{x}_1-\bar{x}_2)^2+(\bar{y}_1-\bar{y}_2)^2\right]^{3/2}} - \frac{\bar{m}_3(\bar{x}_1-\bar{x}_3)}{\left[(\bar{x}_1-\bar{x}_3)^2+(\bar{y}_1-\bar{y}_3)^2\right]^{3/2}}$$

$$dV_{1y} = \frac{-\bar{m}_2}{\left[(\bar{x}_1-\bar{x}_2)^2+(\bar{y}_1-\bar{y}_2)^2\right]^{3/2}}(\bar{y}_1-\bar{y}_2) - \frac{\bar{m}_3}{\left[(\bar{x}_1-\bar{x}_3)^2+(\bar{y}_1-\bar{y}_3)^2\right]^{3/2}}(\bar{y}_1-\bar{y}_3)$$

$$dV_{2x} = \frac{-\bar{m}_1(\bar{x}_2-\bar{x}_1)}{\left[(\bar{x}_2-\bar{x}_1)^2+(\bar{y}_1-\bar{y}_2)^2\right]^{3/2}} - \frac{\bar{m}_3(\bar{x}_2-\bar{x}_3)}{\left[(\bar{x}_2-\bar{x}_3)^2+(\bar{y}_2-\bar{y}_3)^2\right]^{3/2}}$$

$$dV_{2y} = \frac{-\bar{m}_1(\bar{y}_2-\bar{y}_1)}{\left[(\bar{x}_2-\bar{x}_1)^2+(\bar{y}_1-\bar{y}_2)^2\right]^{3/2}} - \frac{\bar{m}_3(\bar{y}_2-\bar{y}_3)}{\left[(\bar{x}_2-\bar{x}_3)^2+(\bar{y}_2-\bar{y}_3)^2\right]^{3/2}}$$

CODE

ARRAYS: 12

x_1 x_2 x_3 y_1 y_2 y_3

V_{x1} V_{x2} V_{x3} V_{y1} V_{y2} V_{y3}

Initial Conditions: 12

$x_1(0)$ $x_2(0)$, $x_3(0)$, $y_1(0)$ ...

$V_{x1}(0)$ $V_{y1}(0)$ ....

for i in time range:

- calculate velocity derivatives at i - 1

$$d\bar{V}_{1x} = \frac{-\bar{m}_2(\bar{x}_1-\bar{x}_2)}{\left[(\bar{x}_1-\bar{x}_2)^2+(\bar{y}_1-\bar{y}_2)^2\right]^{3/2}} - \frac{\bar{m}_3(\bar{x}_1-\bar{x}_3)}{\left[(\bar{x}_1-\bar{x}_3)^2+(\bar{y}_1-\bar{y}_3)^2\right]^{3/2}}$$

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dv1x = -m[1]*(x1[i-1] - x2[i-1])
/((x1[i-1]-x2[i-1])**2
      + (y1[i-1]-y2[i-1])**2)**(3/2)


dv1y = . . .

dv2x  . . .

v1x[i] = v1x[i-1] + dv1x * dt
v1y[i] = v1y[i-1] + dv1y * dt
        .
        :
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X1[i] = X1[i-1] + V1x[i]*dt

$$r_{eq} = \frac{1}{GM}\left(\frac{\ell}{\mu}\right)^2$$

$$\ell = \mu r^2 \dot{\phi} = \mu r v_\phi$$

$$r_{eq} = \frac{1}{GM}\left(\frac{\mu r v_\phi}{\mu}\right)^2$$

$$= \frac{1}{GM} r^2 v_\phi^2$$

$$\frac{1}{GM} \frac{r_0^4}{t_0^2} \bar{r}^2 \bar{v}_\phi^2$$

$$= r_0 \bar{r}^2 \bar{v}_\phi^2$$

$$\bar{r}_{eq} = \bar{r}^2 \bar{v}_\phi^2$$

$$1 = \bar{r} \, v_\phi^2$$

$$\bar{r} = \frac{1}{v_\phi^2}$$