ODE's

y, $\frac{dy}{dt} = f(t)$

$$\frac{dx}{dt} = -gt$$

$$\frac{dy}{dt} = f(t,y)$$

$$\frac{dN}{dt} = -\frac{1}{\tau} N$$

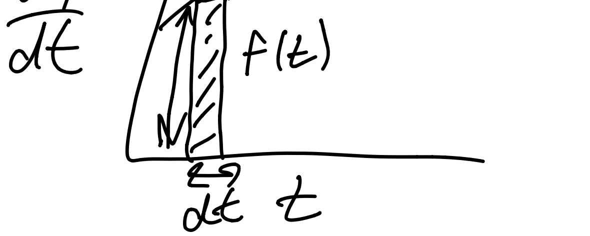
$$\frac{d^2x}{dt^2} = -\frac{GM}{x^2}$$

$$\frac{dx}{dt} = -gt$$

$$dx = -gt\,dt$$

$$x = -\frac{gt^2}{2} + C$$

$$\frac{dy}{dt} \sim \text{complicated}(t)$$



$$\frac{dy}{dt} = f(t, y)$$

$$\frac{dy}{dt} \longrightarrow y \longrightarrow \frac{dy}{dt}$$

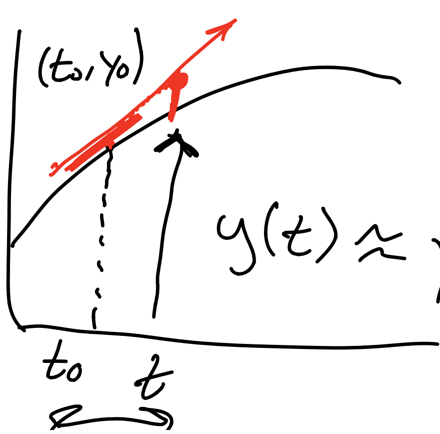
$$\frac{dN}{dt} = -\frac{1}{\tau} N$$

$$N(t_0) = N_0$$

$$\frac{dy}{dt} = f(t, y)$$

$$\frac{dy_0}{dt} = f(t_0, y_0)$$

$$y(t) = y(t_0) + \frac{dy}{dt}(t_0, y_0)\,(t - t_0) + \frac{1}{2}\frac{d^2y}{dt^2}(t_0, y_0)\,(t - t_0)^2 + \frac{1}{6}\frac{d^3y}{dt^3} \cdots (\Delta t)^3$$



$$y(t) \approx y(t_0) + \frac{dy}{dt}(t_0)\,\Delta t$$

Δt

$$\Delta t \qquad t_n = t_0 + n\Delta t$$

$$y_n = y(t_n)$$

$$y(t_0) = y_0$$

$$y_1 \approx y_0 + \frac{dy_0}{dt}\Delta t \qquad y(t_1)$$

$$y_2 \approx y_1 + \frac{dy_1}{dt}\Delta t$$

$$y_n = y_{n-1} + \frac{dy_{n-1}}{dt}\Delta t$$

$$\frac{dy}{dt} = -\frac{1}{\tau} y$$

$$y(0) = 5$$

$$\frac{dy_0}{dt} = -\frac{1}{\tau} y_0 = -\frac{1}{\tau} 5$$

$$y_1 = y_0 + \frac{dy_0}{dt}\Delta t$$

$$y_1 = 5 + -\frac{1}{\tau} 5 \Delta t = \#$$

$$y_n = y_{n-1} + \frac{dy_{n-1}}{dt}\Delta t + \cancel{\frac{1}{2}\frac{dy^2_{n-1}}{dt}}\Delta t^2$$

@ each Step

we incur an error

$$E \sim \Delta t^2$$

$$E = \mathcal{O}(\Delta t^2)$$

<u>Truncation Error</u>

$$E(y_n - y_{n-1}) \sim \underline{\underline{\Delta t^2}}$$

Error at n^{th} pt

$E_n = ?$ $\quad E_n = E(t)$

Δt^2 $\quad N = \frac{t - t_0}{\Delta t}$ $\quad t - t_0 \sim 1$

$$N \sim \frac{1}{\Delta t}$$

$$E_n = \Delta t^2 \cdot N = \Delta t^2 \frac{1}{\Delta t} = \Delta t$$

$$E_n \sim \Delta t$$

$$E_n \sim 10^{-6} \longrightarrow \Delta t \sim 10^{-6}$$