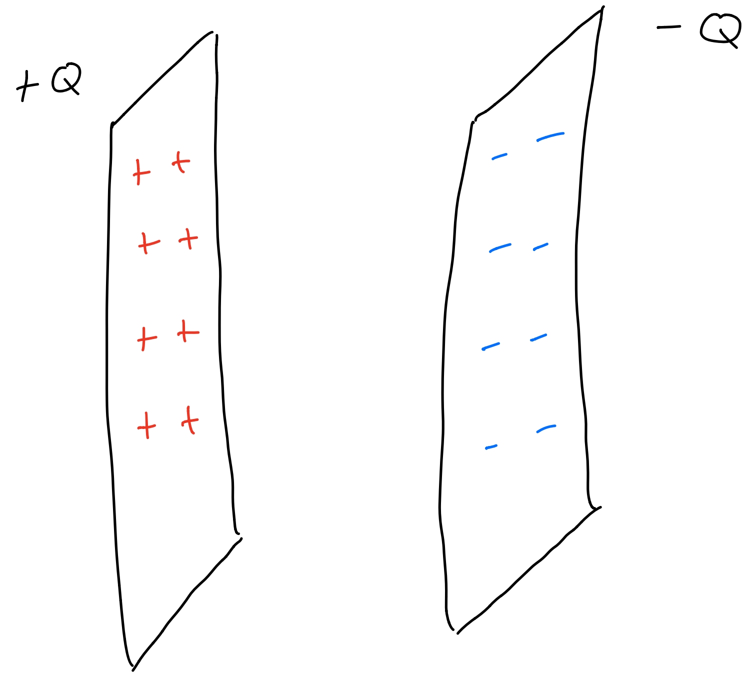
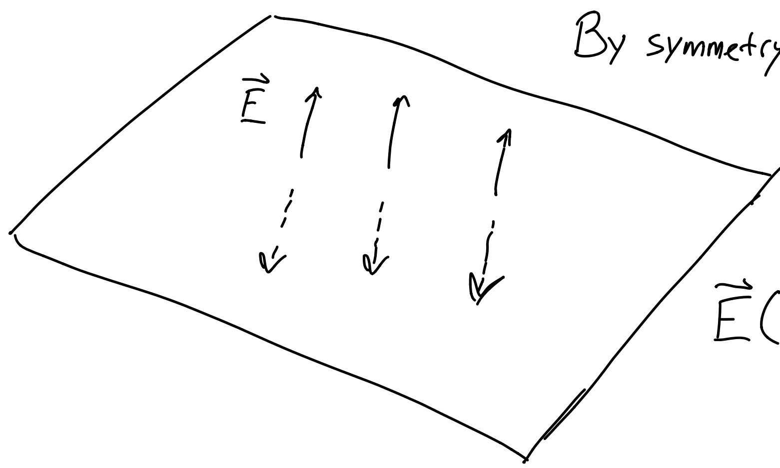
The parallel plate capacitor



What does the field look like?

Approximation: infinite plates w/ $\sigma = \frac{Q}{A}$



By symmetry, we know what $\vec{E}$ looks like

$$\vec{E}(x,y,z) = E(z)\hat{z}$$

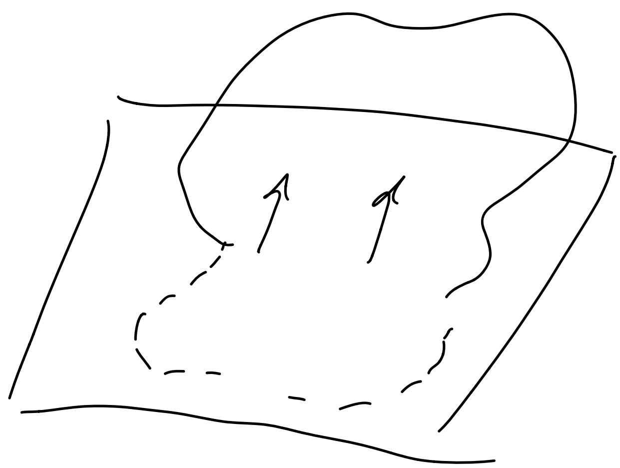
Use Gauss' Law

(Ask)

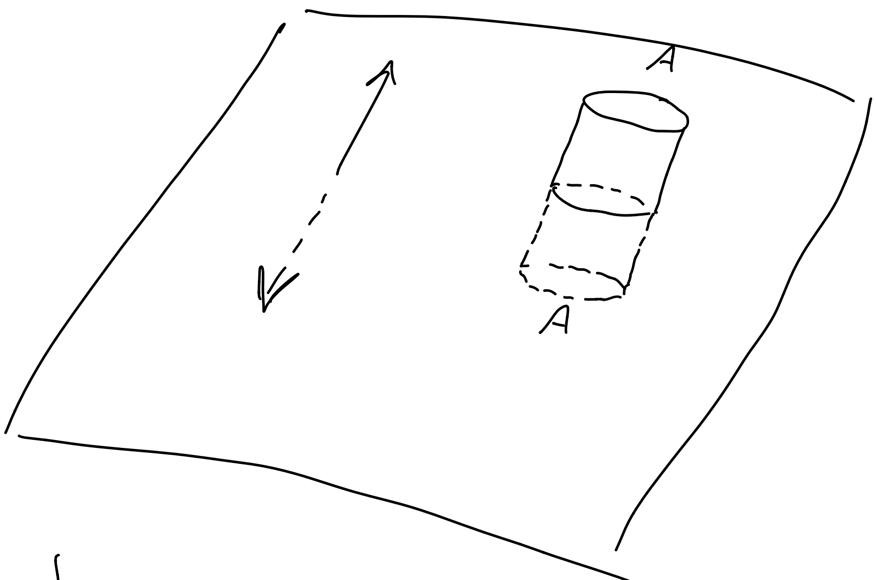
Gauss' Law

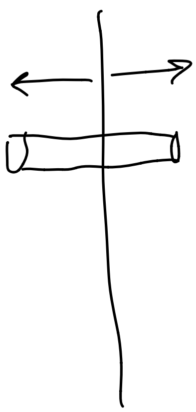
$$\int \vec{E} \cdot d\vec{a} = \frac{Q_{enc}}{\epsilon_0}$$

True for ANY surface



$\int \vec{E} \cdot d\vec{a}$ is hard/impossible for most surfaces





Because $\hat{E} = \hat{z}$, $\int \vec{E}\cdot d\vec{a}$ thru cylinder body is zero

Only need $\int \vec{E}\cdot d\vec{a}$ thru end caps

$$\int \vec{E}\cdot d\vec{a} = \int E(z)\,\hat{z}\cdot da\,\hat{z} = \int E(z)\,da$$

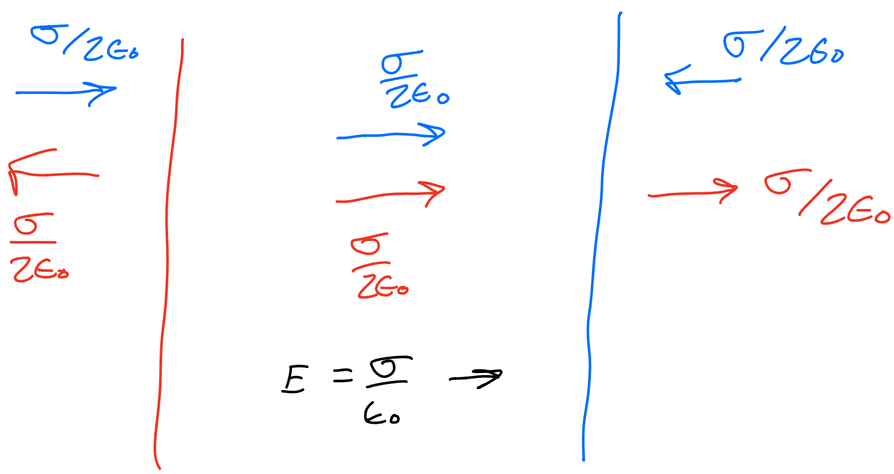
z is constant $\quad E(z)\int da = 2E(z)A$

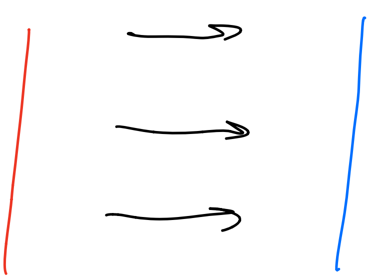
$$2EA = \frac{Q_{enc}}{\epsilon_0}$$

$$Q_{enc} = \sigma A$$

$$2EA = \frac{\sigma A}{\epsilon_0}$$

$$E = \frac{\sigma}{2\epsilon_0}$$





A real capacitor has finite size plates

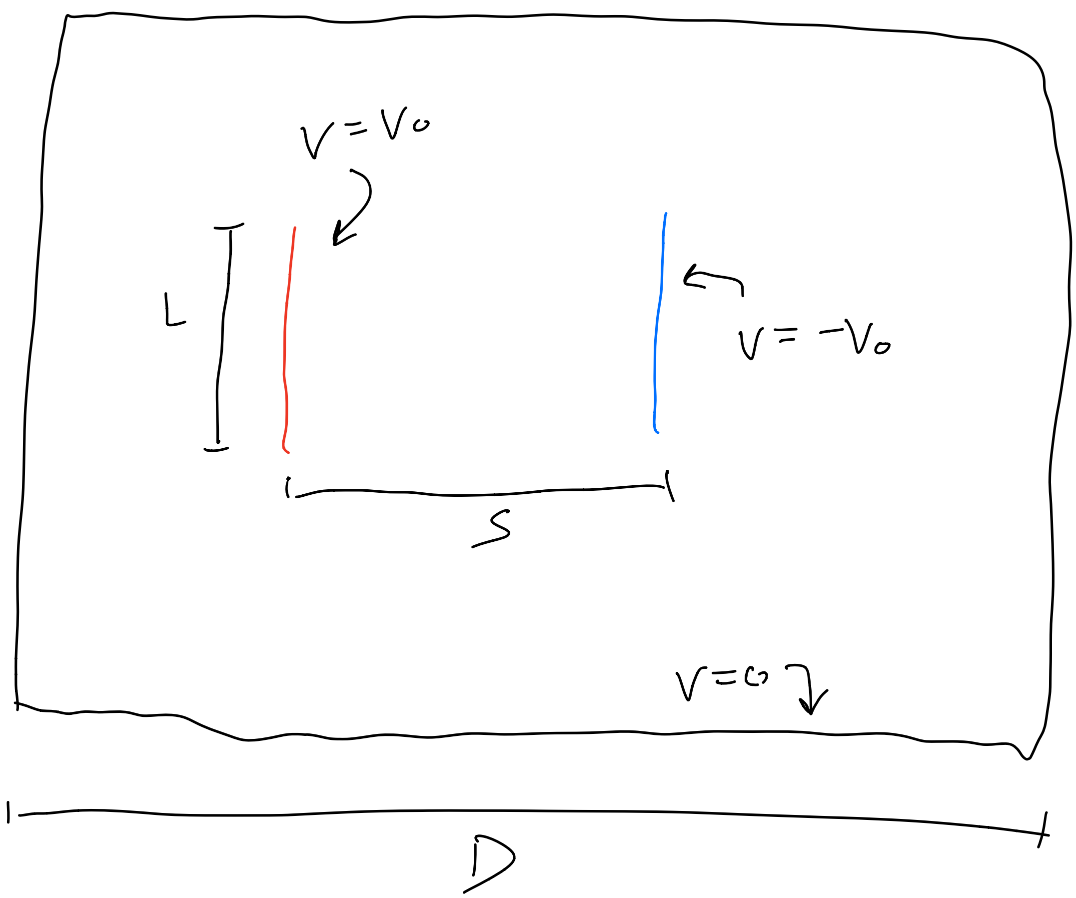
Model: Two plates held at equal & opposite potential

Solve $\nabla^2 V = 0$

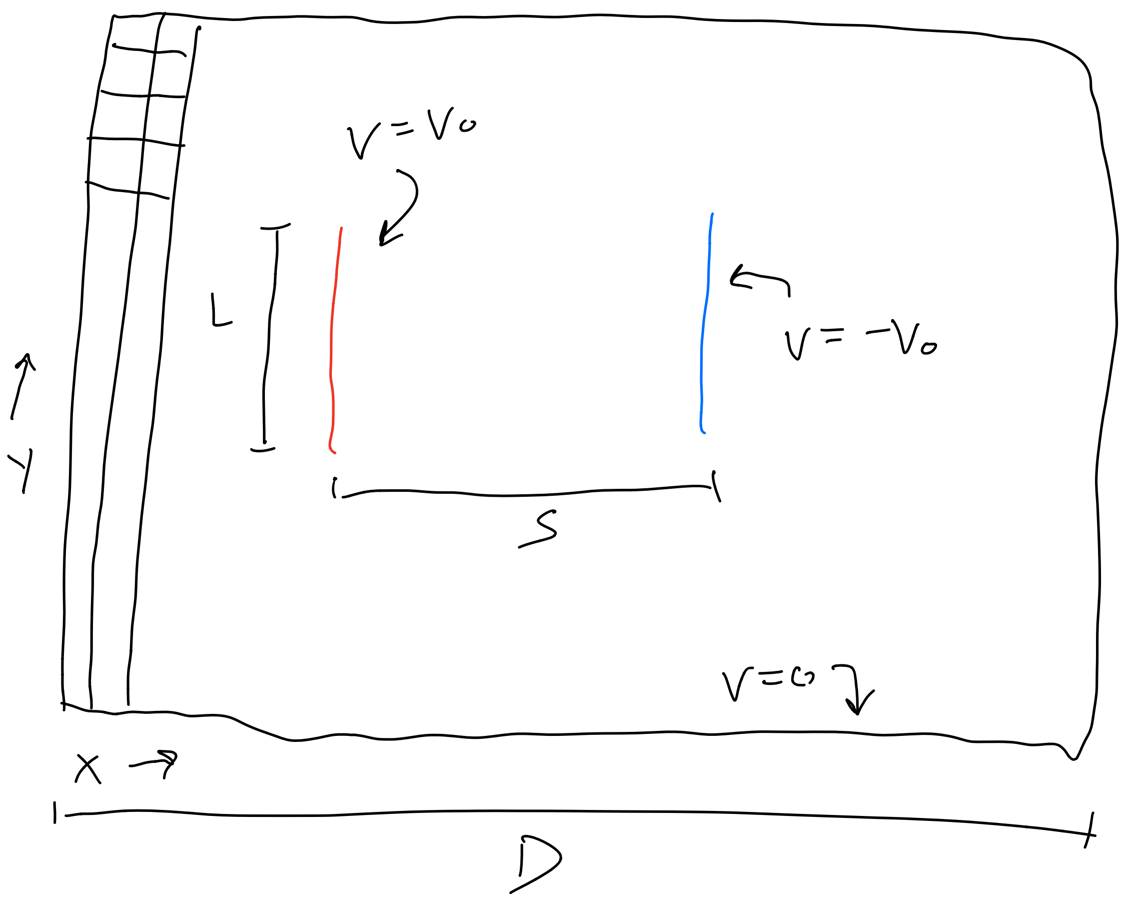
Analytically, $V(\infty = 0)$

Impractical for computers

Sketch



COPY!



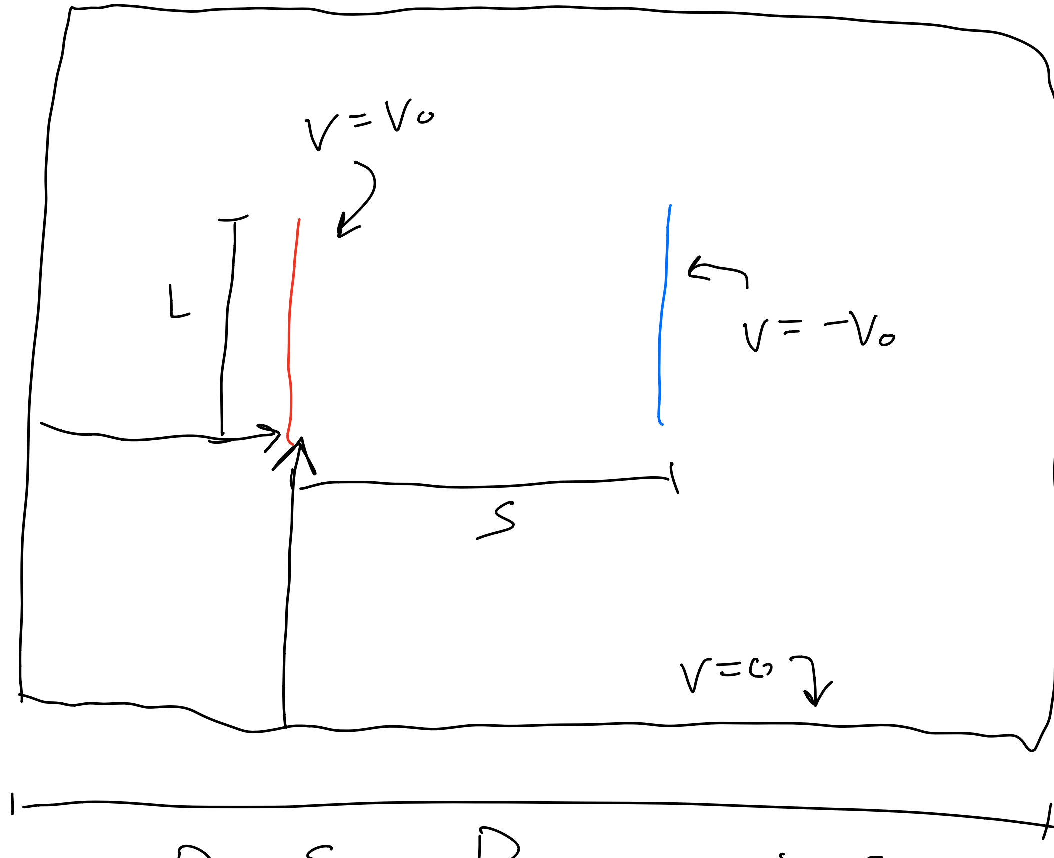
V = array of zeros

V[0,0] = 0   V[0,1] = 0   V[0,2] = 0 . . .

V[0,:] = 0

V[:,0] = 0

V[-1,:] = 0 . . .



$$x_1 = \frac{D}{2} - \frac{S}{2} \qquad x_2 = \frac{D}{2} + \frac{S}{2}$$

$$y_1 = \frac{D}{2} - \frac{L}{2} \qquad y_2 = \frac{D}{2} + \frac{L}{2}$$

$$V(x_1, y_1 < y < y_2) = V_0$$

$$V(x_2, y_1 < y < y_2) = -V_0$$

$$V[i_1, j_1 : j_2] = V_0$$

$$V[i_2, j_1 : j_2] = -V_0$$

$$i_1 = \frac{x_1}{\Delta x} \qquad i_2 = \frac{x_2}{\Delta x}$$

$$j_1 = \frac{y_1}{\Delta x} \qquad j_2 = \frac{y_2}{\Delta x}$$

Show Code

$$\vec{E} = -\vec{\nabla} V$$

$$E_x = -\frac{\partial V}{\partial x}$$

$$E_y = -\frac{\partial V}{\partial y}$$

$$E_x[i,j] = \left(V[i+1,j] - V[i-1,j]\right) / 2\Delta x$$

$$E_y[i,j] = \left(V[i,j+1] - V[i,j-1]\right) / 2\Delta y$$