Last lecture:

$$l = \mu r^2 \dot{\phi}$$

$$\ddot{r} = \left(\frac{l}{\mu}\right)^2 \frac{1}{r^3} - \frac{1}{\mu}\frac{\partial U}{\partial r}$$

$$U = -\frac{G m_1 m_2}{r}$$

$$\ddot{r} = \left(\frac{l}{\mu}\right)^2 \frac{1}{r^3} - \frac{GM}{r^2}$$

$$r_0 = \frac{1}{GM}\left(\frac{l}{\mu}\right)^2$$

$$\ddot{r}(r) = \left(\frac{l}{\mu}\right)^2 \frac{1}{r^3} - \frac{GM}{r^2}$$

$$\ddot{r}(r_0+\Delta r) = \left(\frac{l}{\mu}\right)^2 \frac{1}{(r+\Delta r)^3} - \frac{GM}{(r+\Delta r)^2}$$

Let $\Delta r << r$

$$(r_0 + \Delta r)^{-\alpha} = r_0^{-\alpha}\left(1 + \frac{\Delta r}{r_0}\right)^{-\alpha}$$

$$\left(1 + \frac{\Delta r}{r_0}\right)^{-\alpha} \approx 1 - \alpha\frac{\Delta r}{r_0}$$

$$(r_0 + \Delta r)^{-\alpha} \approx \left(1 - \alpha\frac{\Delta r}{r_0}\right) r_0^{-\alpha}$$

$$\ddot{r}(r_0 + \Delta r) \approx \left(\frac{l}{\mu}\right)^2 \left(1 - 3\frac{\Delta r}{r_0}\right)\frac{1}{r_0^3} - GM\left(1 - 2\frac{\Delta r}{r_0}\right)\frac{1}{r_0^2}$$

$$\ddot{r} \approx \underbrace{\left(\frac{l}{\mu}\right)^2 \frac{1}{r_0^3} - \frac{GM}{r_0^2}}_{=0} - 3\left(\frac{l}{\mu}\right)^2 \frac{\Delta r}{r_0^4} + \frac{2GM}{r_0^3}\Delta r$$

$$\ddot{r} \approx 2GM\frac{\Delta r}{r_0^3} - 3\left(\frac{l}{\mu}\right)^2 \frac{\Delta r}{r_0^4}$$

$$\ddot{r} \approx \left[\frac{2GM}{r_0^3} - 3\left(\frac{l}{\mu}\right)^2 \frac{1}{r_0^4}\right]\Delta r$$

$$r_0 = \frac{1}{GM}\left(\frac{l}{\mu}\right)^2$$

$$\frac{2GM}{r_0^3} = 2GM(GM)^3\left(\frac{\mu}{l}\right)^6 = 2(GM)^4\left(\frac{\mu}{l}\right)^6$$

$$3\left(\frac{l}{\mu}\right)^2 \frac{1}{r_0^4} = 3\left(\frac{l}{\mu}\right)^2 (GM)^4 \left(\frac{\mu}{l}\right)^8 = 3(GM)^4 \left(\frac{\mu}{l}\right)^6$$

$$\ddot{r} \approx -(GM)^4 \left(\frac{\mu}{l}\right)^6 \Delta r$$

oscillation with $\omega^2 = (GM)^4 \left(\frac{\mu}{l}\right)^6$

$$\omega^2 = (GM)^4 \left(\frac{\mu}{l}\right)^6 = \frac{GM}{r_0^3}$$

$$\boxed{\omega^2 = \frac{GM}{r_0^3}}$$

Equivalently...

U_{eff} ↘

$$E = T + U$$

$$E = \frac{1}{2}\mu \dot{r}^2 + \frac{1}{2}\mu r^2 \dot{\phi}^2 - \frac{GM\mu}{r}$$

$$\dot{\phi}^2 = \left(\frac{l}{\mu}\right)^2 \frac{1}{r^4}$$

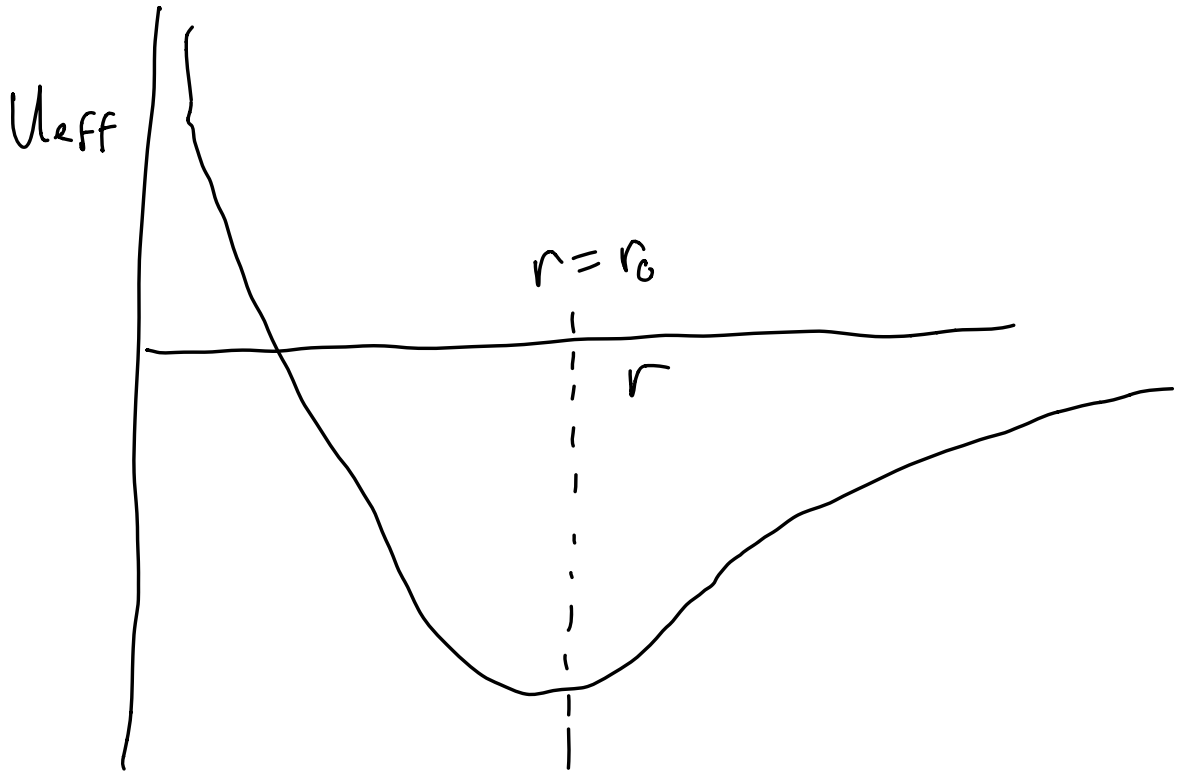
$$E = \frac{1}{2}\mu \dot{r}^2 + \frac{1}{2}\mu r^2 \left(\frac{l}{\mu}\right)^2 \frac{1}{r^4} - \frac{GM\mu}{r}$$

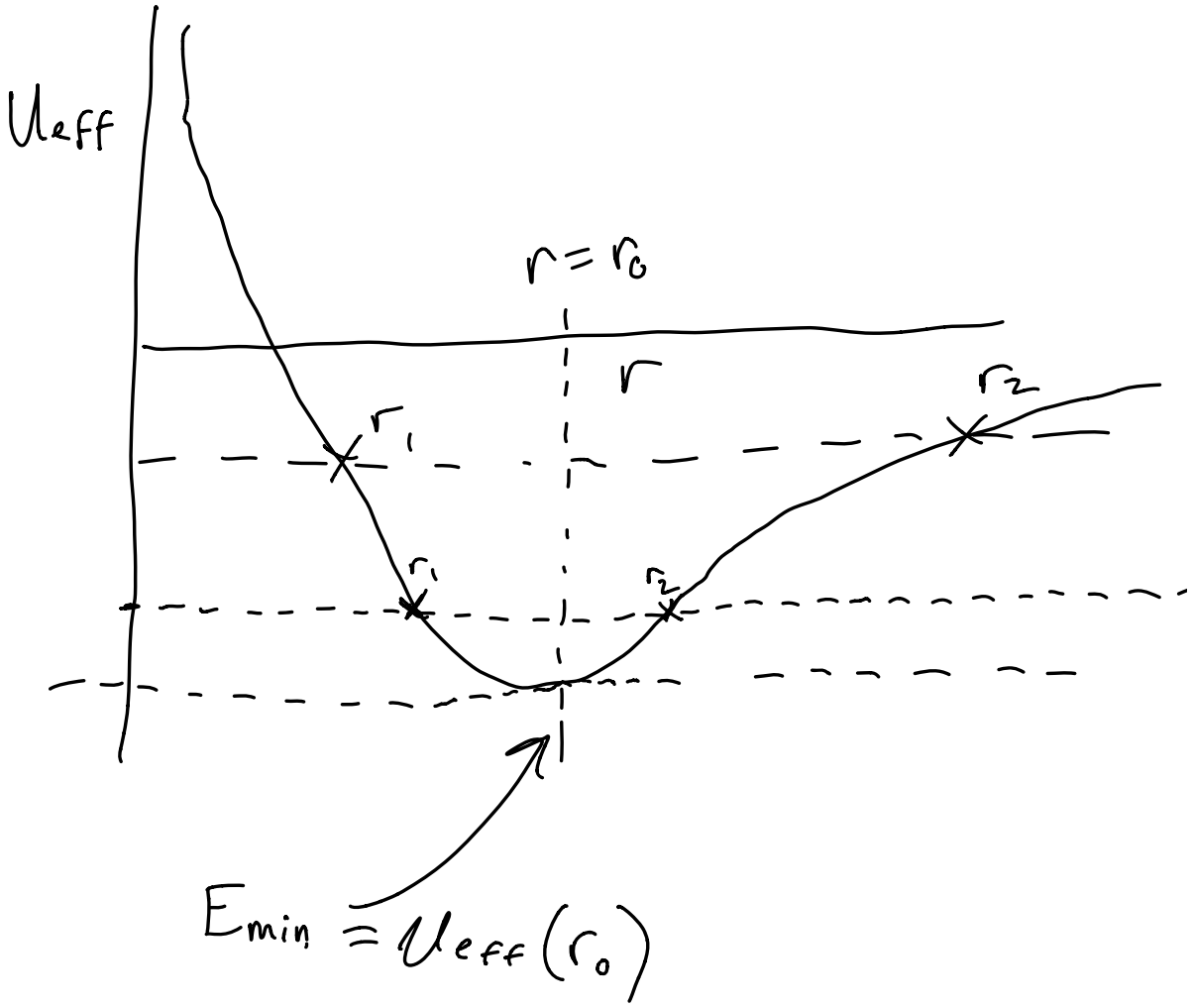
$$E = \frac{1}{2}\mu \dot{r}^2 + \frac{1}{2}\frac{l^2}{\mu r^2} - \frac{GM\mu}{r}$$

$$E = f(\dot{r}) + g(r)$$

$$U_{CF} = \frac{1}{2}\frac{l^2}{\mu r^2}$$

$$U_{eff} = \frac{1}{2}\frac{l^2}{\mu r^2} - \frac{GM\mu}{r}$$




U_{eff}
$r = r_0$
r
r_1
r_2
r_1
r_2
$E_{min} = U_{eff}(r_0)$