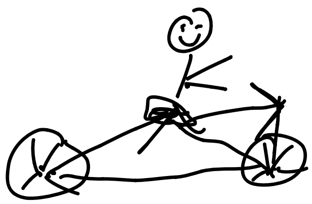
- Intro to chapter 2
- We want to use the tools we've developed to investigate some more interesting problems
- In this chapter we'll investigate realistic projectile motion: including the effects of air resistance, changing atmospheric density, spin, and more
- Let's begin with a simple example and then we'll build up from there
- Motion of a bicyclist
    - Force difficult to analyze
    - Assume constant power exertion



$$\frac{dv}{dt} = \frac{F}{m} \qquad F(t) = ?$$

$$\frac{dE}{dt} = P \Rightarrow \frac{d}{dt}\left(\frac{1}{2}mv^2\right) = P$$

$$\frac{1}{2}m\left(2v\frac{dv}{dt}\right) = P$$

$$\boxed{\frac{dv}{dt} = \frac{P}{m}\frac{1}{v}}$$

- Explain this term
    - 1/v: With constant energy input, we get diminishing returns on velocity increase
        - Takes more and more energy to keep increasing velocity by the same amount
    - P/m: Exerting more energy per unit time will increase the velocity faster
        - More mass takes more energy to accelerate
- We can solve this analytically:

$$\frac{dv}{dt} = \frac{P}{m}\frac{1}{v}$$

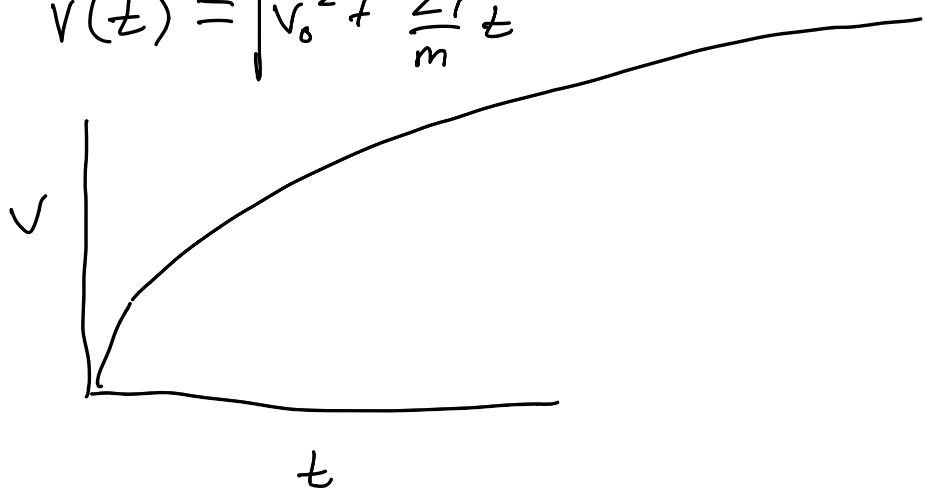
$$v\,dv = \frac{P}{m}dt$$

$$\frac{1}{2}v^2 = \frac{P}{m}t + C$$

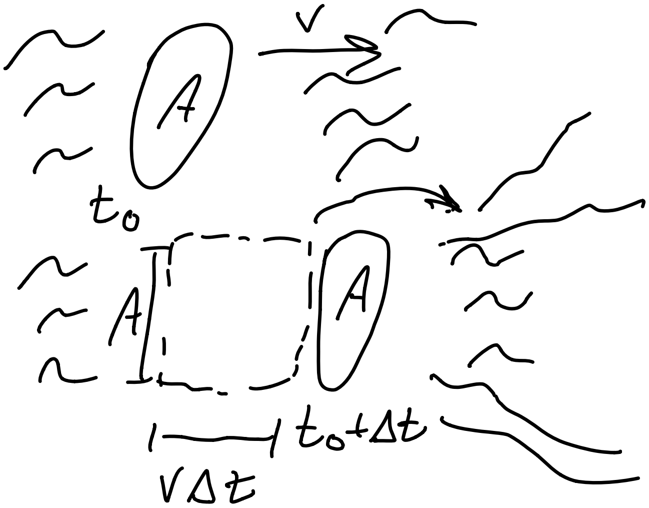
$$v^2 = \frac{2P}{m}t + C_1$$

$$v^2(0) = C_1 = v_0^2$$

$$v(t) = \sqrt{v_0^2 + \frac{2P}{m}t}$$



- Obvious problem
  - v increases indefinitely!
  - For P=400 W, m=70kg, we are going ~60 mph after a single minute of pedaling
  - What are we forgetting?
  - Friction
  - Main source of friction here is air resistance
  - Let's try and estimate the force due to air resistance



$$V = A \cdot v \Delta t$$

$$m_{air} = \rho_{air} V = \rho_{air} A v \Delta t$$

if $m_{air} << m_{obj}$

then $v_{air} \approx v$

$$\Delta KE_{air} = \frac{1}{2} m_{air} v_{air}^2$$

$$= \frac{1}{2}(\rho_{air} A v \Delta t) v^2$$

$$\Delta KE = \frac{1}{2}\rho_{air} A \Delta t v^3$$

$$\Delta KE = W = F\Delta x = F v \Delta t$$

$$F v \Delta t = \frac{1}{2}\rho_{air} A \Delta t v^3$$

$$F_{drag} = \frac{1}{2} \rho_{air} A v^2$$

$$\boxed{F_{drag} = \frac{1}{2} C \rho_{air} A v^2}$$

Actually

$$F \approx B_1 v + B_2 v^2$$

For typical v, v^2 dominates

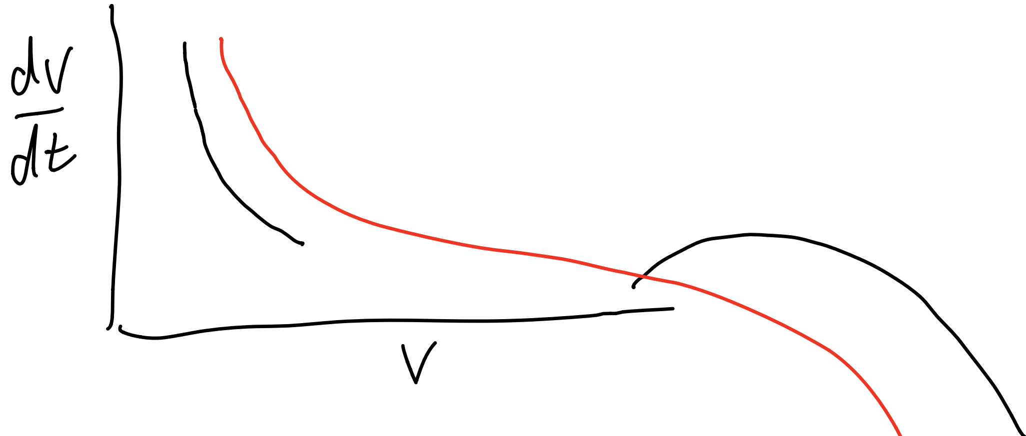
and $F \approx B_2 v^2$

$$B_2 = \frac{1}{2} C \rho_{air} A v^2$$

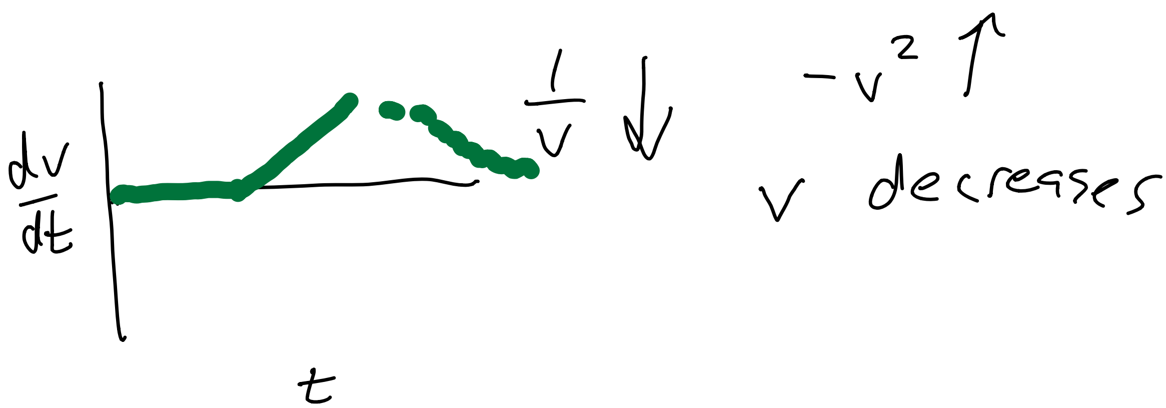
- Now we can add this term into our dv/dt equation

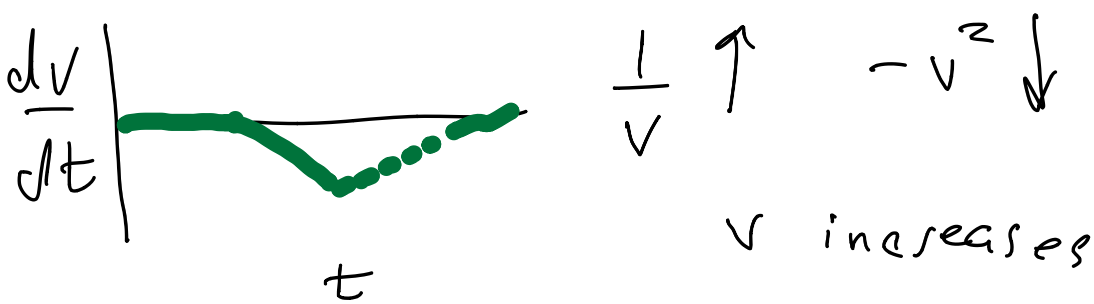
$$\frac{dv}{dt} = \frac{P}{m} \frac{1}{v} - \frac{1}{2m} C \rho_{air} A v^2$$

- First term:
  - Increases velocity, but less and less so
- Second term
  - Resistive force (hence the minus sign)
  - Increases with higher velocity



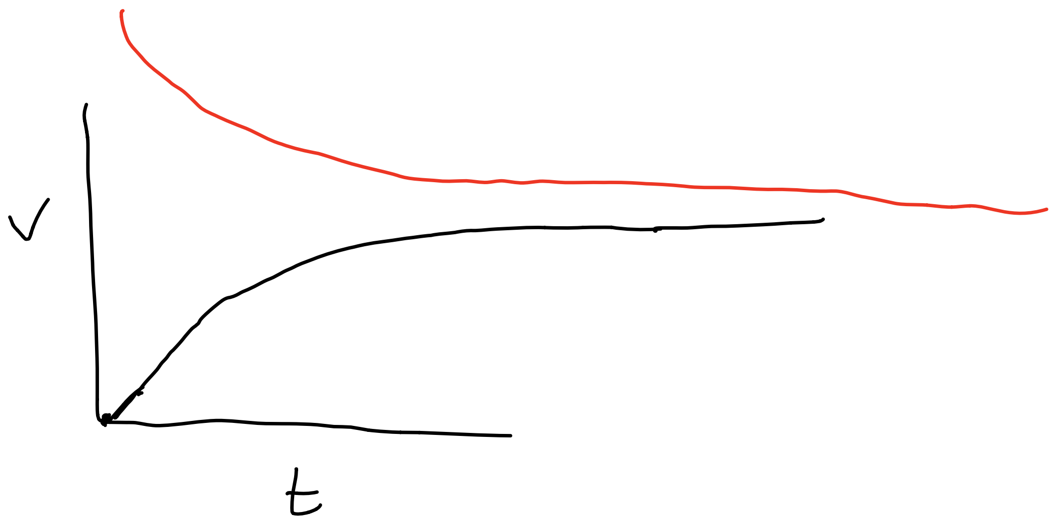
- As v increases, 1/v term gets smaller, -v^2 gets larger
    - Eventually, dv/dt = 0
    - Is this a stable equilibrium?
        - If v increases, the -v^2 term pushes it back down
        - If v decreases, the 1/v term increases it again





- System approaches an equilibrium where dv/dt = 0

Sketch



What is our equilibrium?

$$\frac{dv}{dt} = \frac{P}{m}\frac{1}{v} - \frac{1}{2m} C \rho A v^2 = 0$$

$$\frac{P}{\cancel{m}}\frac{1}{v_0} = \frac{1}{2\cancel{m}} C \rho A v_0^2$$

$$v_0^3 = \frac{2P}{C\rho A}$$

$$v_0 = \left(\frac{2P}{C\rho A}\right)^{1/3} = V_{terminal}$$

- Now let's solve numerically
    - First, normalize

$$\frac{dV}{dt} = \frac{P}{m}\frac{1}{V} - \frac{1}{2m} C \rho A v^2$$

$$V = V_0 \bar{V}, \quad t = t_0 \bar{t}$$

$$\frac{V_0}{t_0}\frac{d\bar{v}}{d\bar{t}} = \frac{P}{m}\frac{1}{V_0 \bar{V}} - \frac{C \rho A V_0^2 \bar{V}^2}{2m}$$

$$\frac{d\bar{V}}{d\bar{t}} = \frac{t_0}{V_0^2}\frac{P}{m}\frac{1}{\bar{V}} - V_0 t_0 \frac{C \rho A}{2m}\bar{V}^2$$

$$\left[\frac{P}{m}\right] = \frac{[E]}{mT} = \frac{mL^2}{T^2}\frac{1}{mT} = \frac{L^2}{T^3} = \frac{V^2}{T}$$

$$\text{Let } \frac{V_0^2}{t_0} = \frac{P}{m}$$

$$\left[\frac{C \rho A}{m}\right] = \frac{\frac{M}{L^3}L^2}{m} = \frac{1}{L}$$

$$\text{Let } V_0 t_0 = \frac{2m}{C \rho A}$$

$$\frac{v_0^2}{t_0} = \frac{P}{m} \qquad \boxed{\frac{d\bar{v}}{d\bar{t}} = \frac{1}{\bar{v}} - \bar{v}^2}$$

$$v_0 t_0 = y_0 = \frac{2m}{C\rho A}$$

$$t_0 = \frac{y_0}{v_0} = \frac{1}{v_0}\frac{2m}{C\rho A}$$

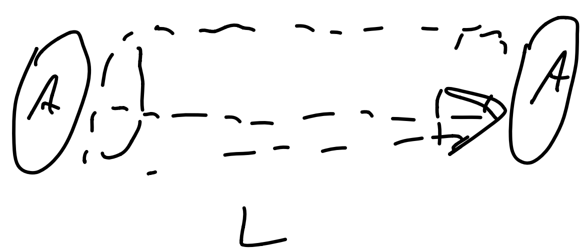
$$v_0^2 = \frac{P}{m} t_0 = \frac{P}{m}\frac{1}{v_0}\frac{2m}{C\rho A}$$

$$v_0^3 = \frac{2P}{C\rho A} \Rightarrow v_0 = \left(\frac{2P}{C\rho A}\right)^{\frac{1}{3}}$$

v_{term}!

$$t_0 = \frac{1}{v_0}\frac{2m}{C\rho A}$$

$$\left[\frac{m}{C\rho A}\right] = \frac{1}{1/L} = L$$



$$M_{air} = C\rho A L \qquad C\rho A y_0 = m$$

$$y_0 = \frac{m}{C\rho A}$$

- Y0: Length required for object w/ area A to travel to displace its own mass
  - Length required to transfer initial kinetic energy from object to air
    - "Length over which work done by drag force is significant"
  - Increases with object mass
  - Decreases with area, air density
- V0: Terminal velocity.
  - Will reach a higher terminal velocity with higher rate of energy expenditure P
  - Terminal velocity lowered by air density and object area
- T0: How quickly will we reach terminal velocity?
  - Time it takes to travel distance y0 at velocity v0
  - Time it takes to transfer kinetic energy from object to air
  - "Time over which work done by drag force is significant"
  - Increases with mass
    - Higher mass-->harder to accelerate-->longer to reach terminal velocity
  - Decreases with energy expenditure, air density, area
    - Higher energy input-->accelerate quicker-->Reach vterm faster
    - Higher air density, area-->Lower terminal velocity-->Will reach it faster
- This all explains why cyclists lean forward and draft behind one another

- Now that we understand our normalized units, let's go ahead and solve numerically

$$V_i = V_{i-1} + \frac{dV_{i-1}}{dt}\Delta t$$

$$\frac{dV_i}{dt} = \frac{1}{V_i} - V_i^2$$

- Step through jupyter code and show plots
  - Insert reasonable numbers and show the results are reasonable

- IF there is time, calculate x(t)

$$X_i = X_{i-1} + \underbrace{\frac{dX_{i-1}}{dt}}_{V_{i-1}} \Delta t$$

$$V_i = V_{i-1} + \frac{dV_{i-1}}{dt} \Delta t$$

$$X_i = X_{i-1} + V_{i-1} \Delta t$$