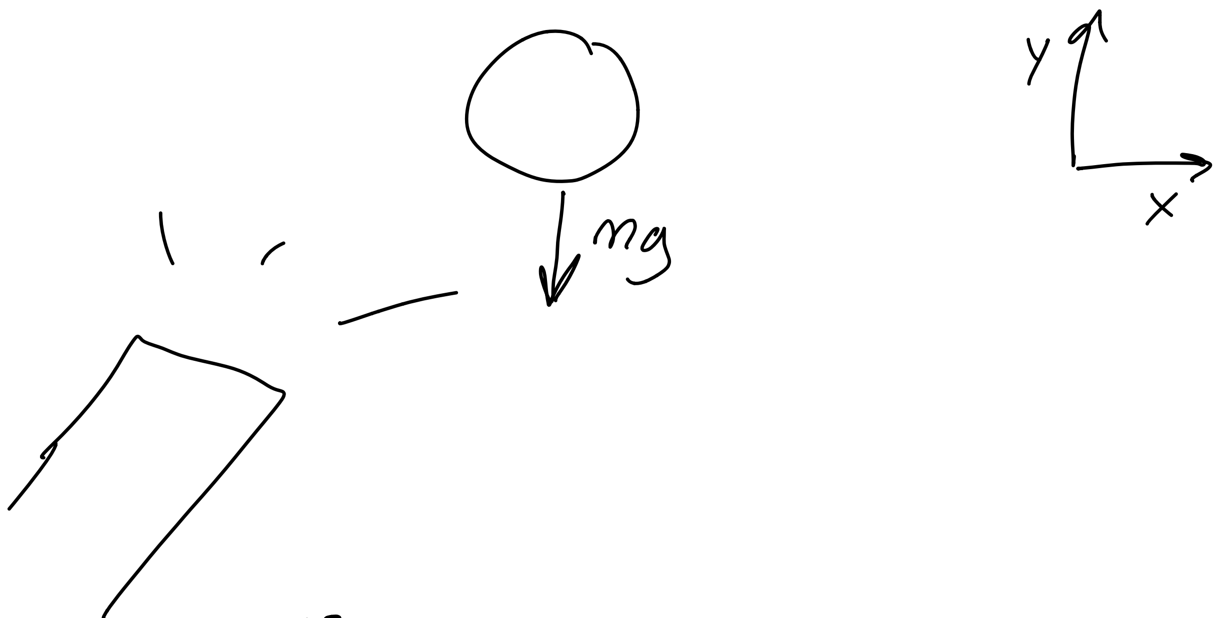
- Review last lecture
- We learned:
    - How to use Euler to solve second-order equations
    - How to apply Euler in more than one dimension



$$\frac{d^2}{dt^2}x = 0$$

$$\frac{d^2}{dt^2}y = -g$$

$$\frac{d^2}{dt^2}(r) \longrightarrow \begin{cases} \dfrac{dv_r}{dt} = F/m \\ \dfrac{dr}{dt} = v_r \end{cases}$$

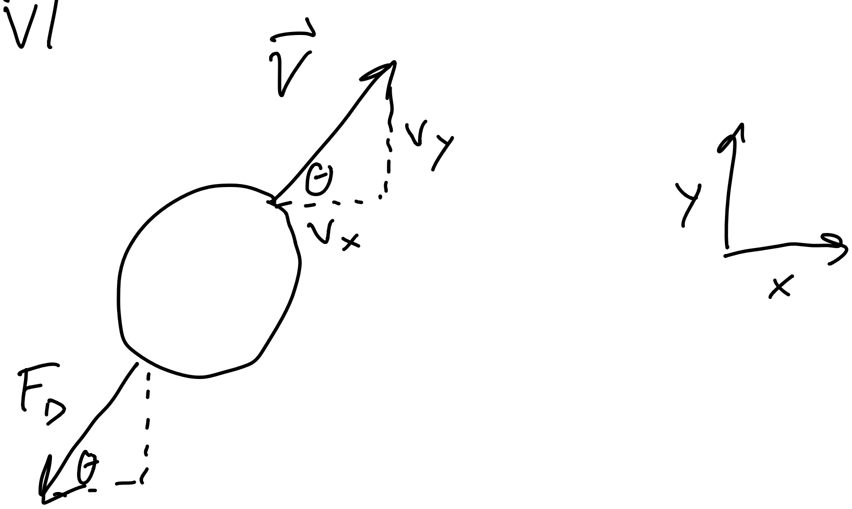
$$\frac{dv_x}{dt} = 0 \qquad \frac{dx}{dt} = v_x$$

$$\frac{dv_y}{dt} = -g \qquad \frac{dy}{dt} = v_y$$

- So we've split one second order diffEQ into two first order ones
  - We've done this in two dimensions
- Then we considered drag force

$$F_D = B_2 |\vec{v}|^2$$

$$\hat{F}_D = -\hat{v}$$



$$F_{D,x} = B_2 v^2 \frac{v_x}{v} = B_2 v v_x$$

$$F_{D,y} = B_2 v v_y, \qquad v = |\vec{v}| = \sqrt{v_x^2 + v_y^2}$$

$$\frac{dv_x}{dt} = \frac{F}{m} = -\frac{B_2 v v_x}{m}$$

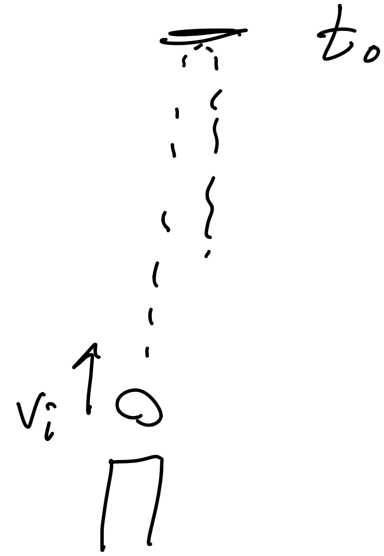
$$\frac{dv_y}{dt} = -g - \frac{B_2}{m} v v_y$$

<u>Units</u>

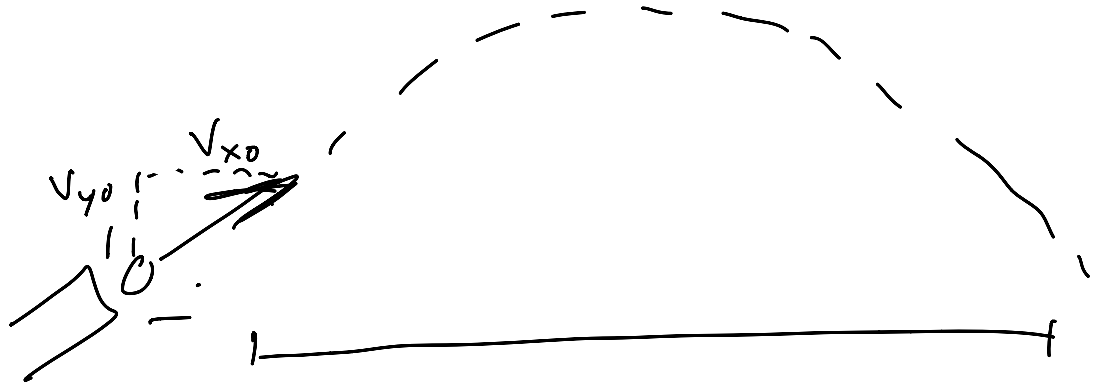
$$\bar{v} = \frac{v}{v_0}, \quad \bar{t} = \frac{t}{t_0}, \quad \bar{x} = \frac{x}{r_0}, \quad \bar{y} = \frac{y}{r_0}$$

$$V_0 = V_{initial} \implies \bar{v}(0) = 1$$

$$t_0 = \frac{V_i}{g}$$



$$r_0 = v_0 t_0 = v_i t_0 = \frac{v_i^2}{g}$$

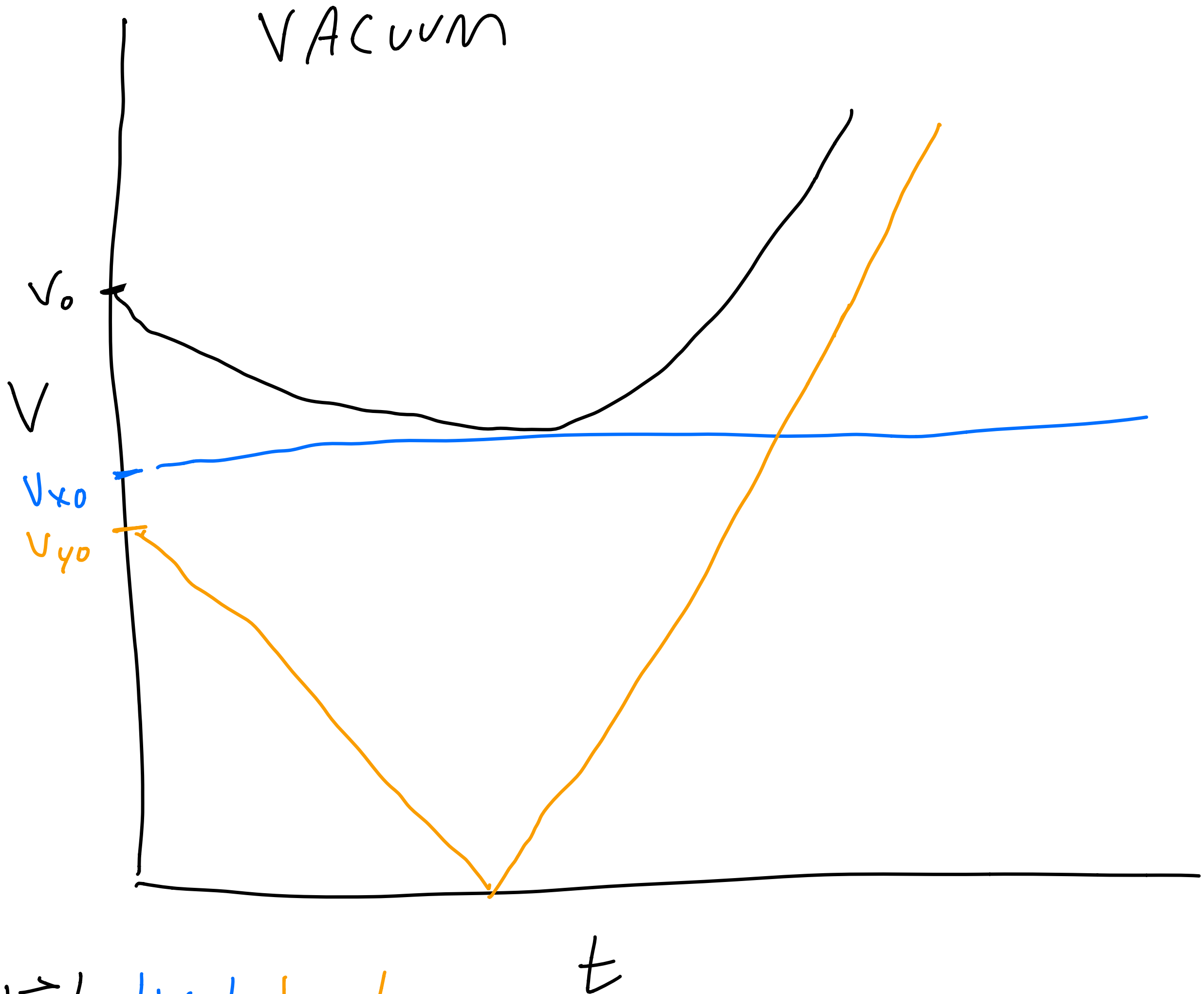


$$\Delta t = \frac{V_{y0}}{g} + \frac{V_{y0}}{g} = \frac{2V_{y0}}{g}$$

$$\Delta x = V_{x0}\,\Delta t = \frac{2V_{x0}V_{y0}}{g}$$

$$\Delta x \sim \frac{v^2}{g}$$

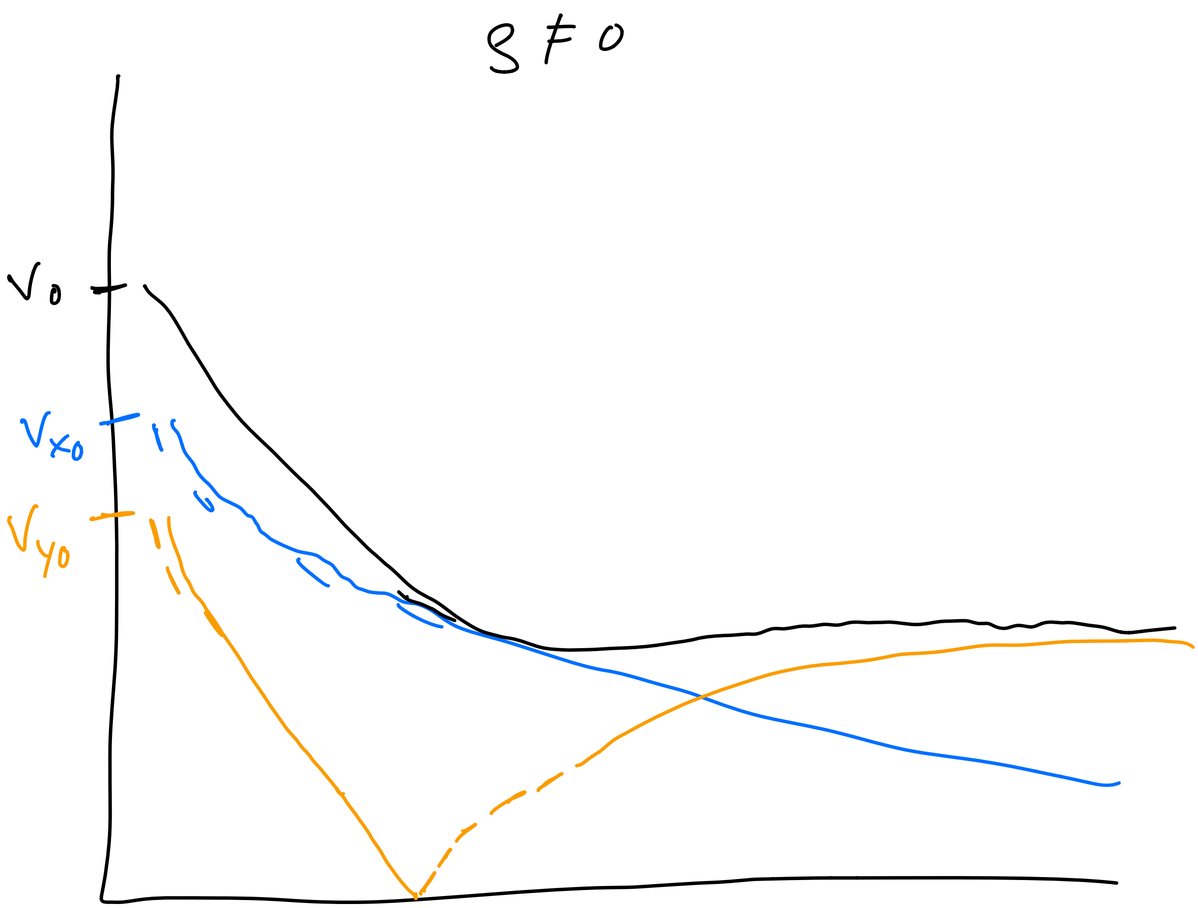
- Go to jupyter and explain importance of units
    - First do theta=90 (time of flight = 2*time unit, length unit=2*ymax)
    - Then theta=60 (time of flight ~ 1.5*time unit, length unit~xrange~yrange)
    - Vary initial velocity and show that units adjust accordingly
    - Turn friction on and show units are still reasonable
    - When we normalize, all dynamic variables are of order 1
- We haven't looked at velocities yet? How do we expect v to behave?
- First: In a vacuum



$|\vec{v}|$, $|v_x|$ $|v_y|$

$v_y \propto t$

$$v = \sqrt{v_x^2 + v_y^2}$$
$$= \sqrt{K + at^2}$$



- VX: No acceleration term, always decreasing but at an ever decreasing rate (because smaller velocity-->smaller drag force)
  - Eventually --> 0
- VY: Initially decelerated by BOTH gravity and drag
  - Gravity is constant, drag force decreases
  - Eventually: VY=0
  - Only for an instant (gravity still working!)
  - Switches direction and speeds up
  - As it does so, drag force gets stronger
  - Reaches terminal velocity eventually
- V:
  - Harder to analyze but know three things:
  - Initially decreasing
  - V=Vx when Vy=0
  - V-->Vy-->Vterm

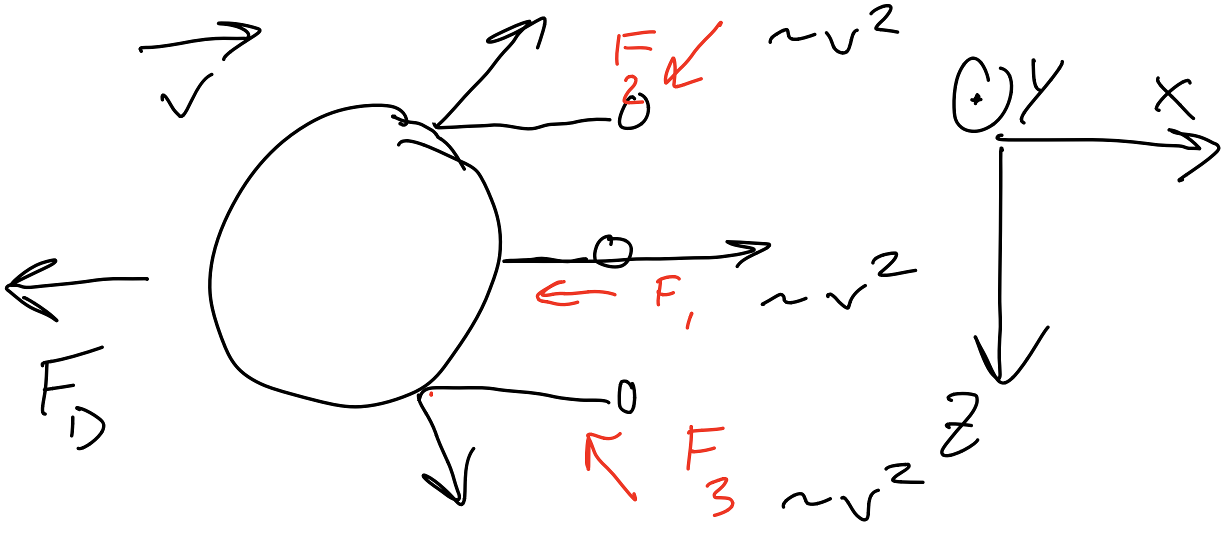
V_{term}: $\frac{dv_y}{dt} = 0$

$$-g + \frac{B_2}{m} v^2 = 0$$

$$v^2 = \frac{mg}{B_2}$$

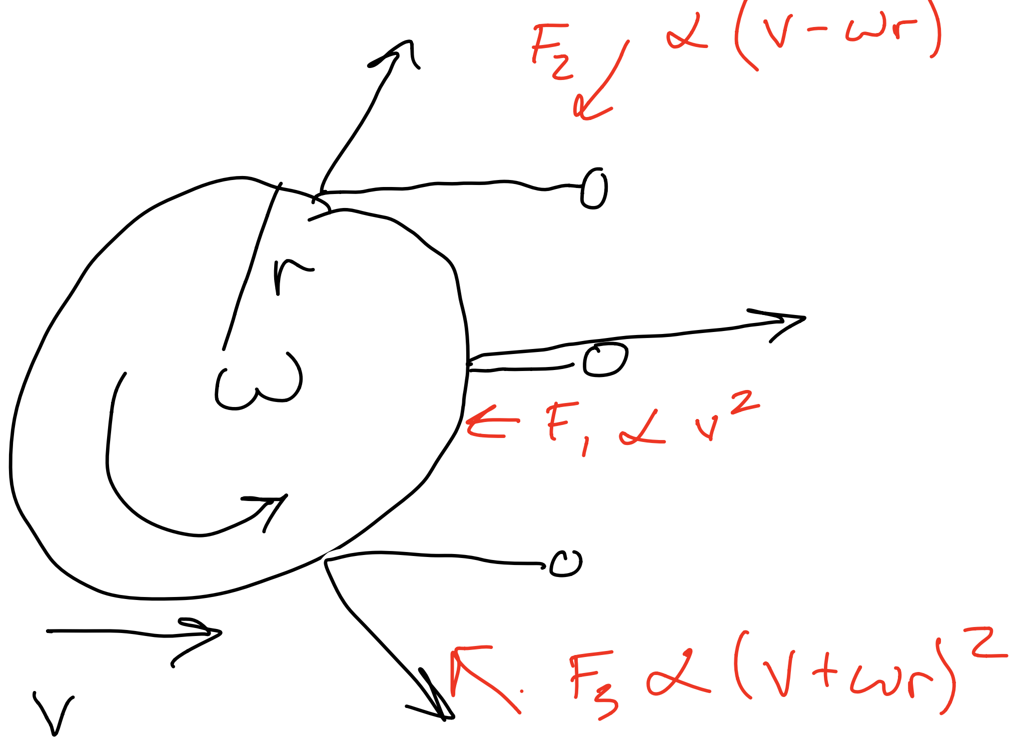
$$V_{term} = \sqrt{\frac{mg}{B_2}}$$

Magnus



$$\vec{F}_D = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$$

$$\hat{F}_D = -\hat{v}$$



$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$$

$$= \vec{F}_D + ?$$

Non-cancelling $\hat{z}$ terms

$$\vec{F}_m \propto (v-\omega r)^2 \hat{z} - (v+\omega r)^2 \hat{z}$$

$$\propto (v^2 + \omega^2 r^2 - 2v\omega r$$
$$- v^2 - \omega^2 r^2 - 2v\omega r)$$

$$\propto -4v\omega r \hat{z}$$

$$\vec{F}_m = -S_0 v\omega \hat{z}$$

$$\hat{F} = -\hat{z}$$

$$\hat{v} = \hat{x}$$

$$\hat{\omega} = \hat{y}$$

$$\hat{F} = \hat{\omega} \times \hat{v}$$

$$\vec{F}_m = -S_0\, \vec{\omega} \times \vec{V}$$

$$\vec{\omega} \times \vec{V} =$$

$$(\omega_y V_z - \omega_z V_y)\,\hat{x} \;+$$

$$(\omega_z V_x - \omega_x V_z)\,\hat{y} \;+$$

$$(\omega_x V_y - \omega_y V_x)\,\hat{z}$$

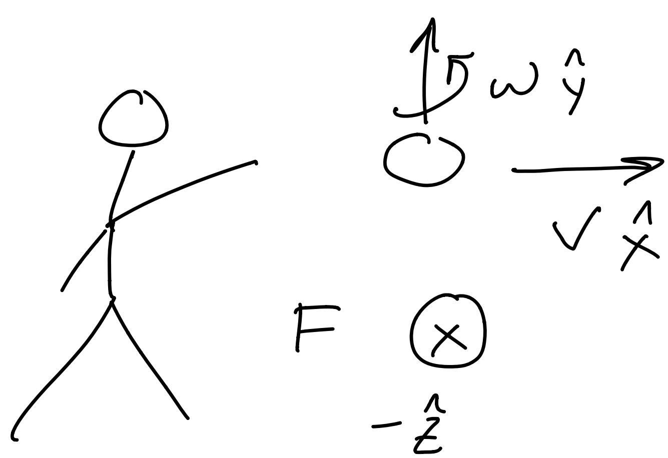
$$\frac{dv_x}{dt} = -\frac{B_2}{m} v v_x + \frac{S_0}{m}(\omega_y v_z - \omega_z v_y)$$

$$\frac{dv_y}{dt} = -g - \frac{B_2}{m} v v_y + \frac{S_0}{m}(\omega_z v_x - \omega_x v_z)$$

$$\frac{dv_z}{dt} = -\frac{B_2}{m} v v_z + \frac{S_0}{m}(\omega_x v_y - \omega_y v_x)$$

$$\frac{dx}{dt} = v_x, \quad \frac{dy}{dt} = v_y, \quad \frac{dz}{dt} = v_z$$

curveball



Golf