Three-body within a solar system

Circular orbit?

$$F_{sun} = F_{cf}$$

$$\frac{Mm_1}{r^2} = \frac{m_1 V_\phi^2}{r}$$

$$\frac{M}{V_\phi^2} = r \quad , \quad M \gg m_1, m_2$$

$$M = 1$$

$$r = \frac{1}{V_\phi^2}$$

Resonance

$$\tau_{outer} = n\,\tau_{inner}$$

$$\tau^2 \propto r^3$$

$$\tau_{outer}^2 = n^2\,\tau_{inner}^2$$

$$r_{outer}^3 = n^2\,r_{inner}^3$$

$$r_{inner} = \frac{1}{n^{2/3}}\,r_{outer}$$

$$n = 2$$

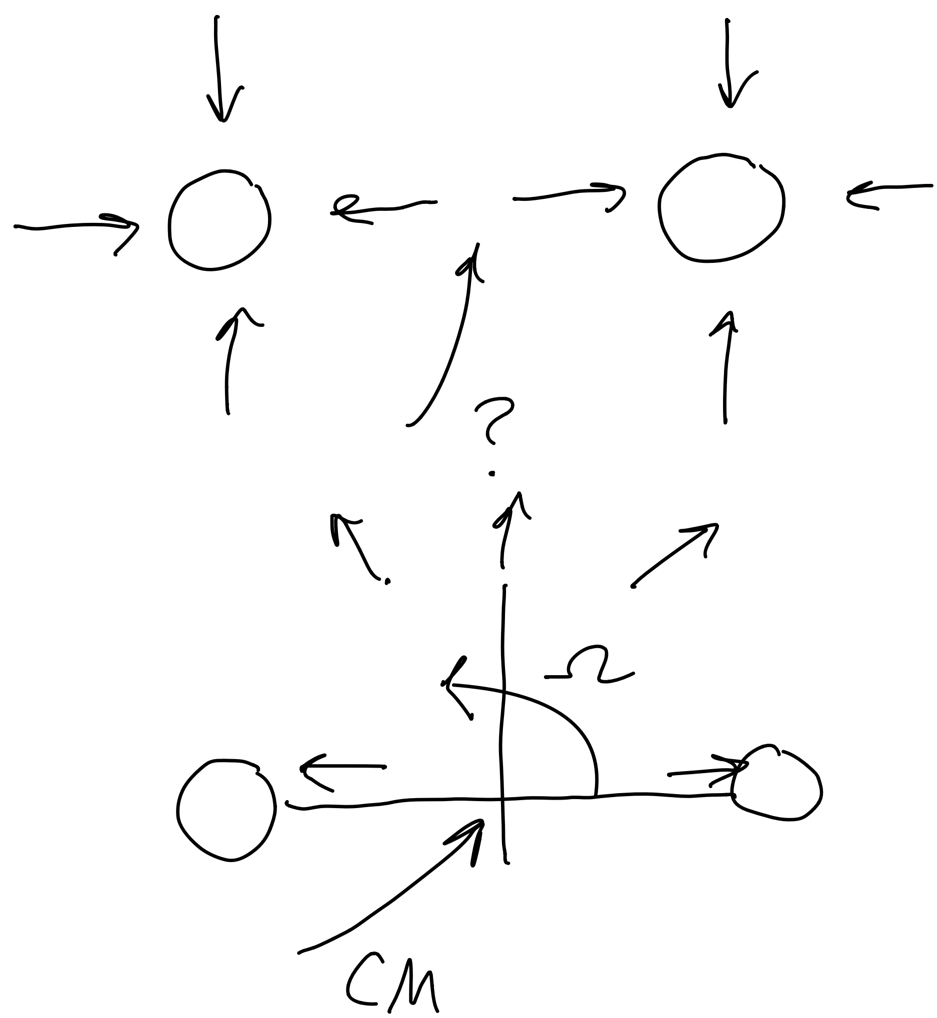
$$r_{inner} = \frac{1}{4^{1/3}}\,r_{outer}$$

$$V_\varphi = \frac{1}{\sqrt{r}}$$

For a single body, we can find an equilibrium point where $\vec{F}_{grav} - \vec{F}_{cf} = 0$.

$$r_0 = \frac{1}{GM}\left(\frac{\ell}{\mu}\right)^2$$

What about two bodies?

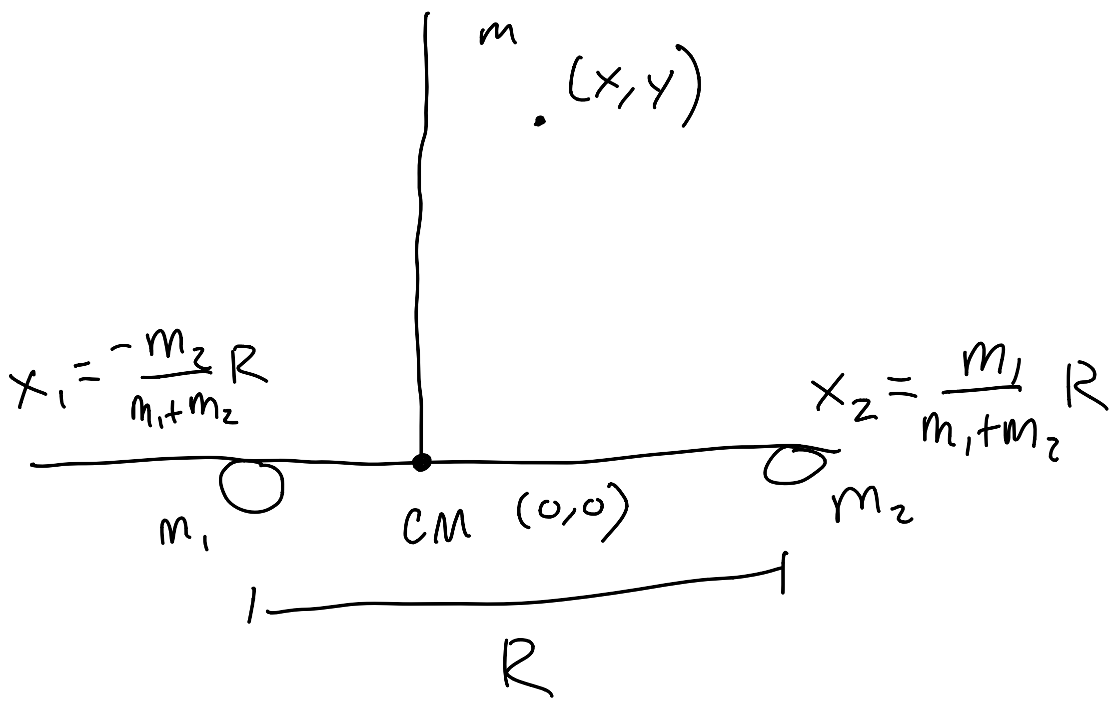


$$\vec{F}_{grav} = -\frac{m}{r^2}\hat{r}$$

$$\vec{F}_{cf} = m\Omega^2 \vec{r}_{cm}$$

$$\vec{F}_{net} = \vec{F}_{grav} + \vec{F}_{cf}$$

At a point x, y



$$\vec{F}_{grav} = \frac{-mm_1}{[(x_1-x)^2+(y_1-y)^2]^{3/2}}\left[(x_1-x)\hat{x}+(y_1-y)\hat{y}\right]$$

$$\frac{-mm_2}{[(x_2-x)^2+(y_2-y)^2]^{3/2}}\left[(x_2-x)\hat{x}+(y_2-y\ \hat{y})\right]$$

$$\vec{F}_{cf} = m\Omega^2(x\hat{x}+y\hat{y})$$

$$\vec{F}_{net} = \vec{F}_{grav} + \vec{F}_{cf}$$