1.

a) $\Omega_A = \left(\frac{e q_A}{N}\right)^N$

$\Omega_B = \left(\frac{e q_B}{N}\right)^N$

$\Omega_{tot} = \left(\frac{e}{N}\right)^{2N} (q_A q_B)^N$

$q_B = q - q_A$

$\Omega_{tot} = \left(\frac{e}{N}\right)^{2N} [q_A(q-q_A)]^N$

max Ω @ $\frac{\partial \Omega_{tot}}{\partial q_a} = 0$

$$\frac{\partial \Omega}{\partial q_a} = \left(\frac{e}{N}\right)^{2N} \left[q_A^N\left(-N(q-q_A)^{N-1}\right) + N q_A^{N-1}(q-q_A)^N \right]$$

$$= \left(\frac{e}{N}\right)^{2N} N q_A^{N-1}(q-q_A)^{N-1}\left[-q_A + (q-q_A)\right]$$

$$0 = \left(\frac{e}{N}\right)^{2N} N q_A^{N-1}(q-q_A)^{N-1}(q-2q_A)$$

$$q_A = 0,\ q,\ \frac{q}{2}$$

$$\Omega_{tot}(q_A = 0) = 0$$

$$\Omega_{tot}(q_A = q) = 0$$

$$\Omega_{tot}\left(q_A = \frac{q}{2}\right) = \left(\frac{e}{N}\right)^{2N}\left[\frac{q}{2}\left(q - \frac{q}{2}\right)\right]^N$$

$$= \left(\frac{e}{N}\right)^{2N}\left(\frac{q^2}{4}\right)^N$$

$$\Omega_{max} = = \left(\frac{e}{N}\right)^{2N}\left(\frac{q}{2}\right)^{2N}$$

So Ω is max @ $q_A = \frac{1}{2}q$

b) $$P(q_a = xq) = \frac{\cancel{\left(\frac{e}{N}\right)^{2N}}\left[fq(q - fq)\right]^N}{\cancel{\left(\frac{e}{N}\right)^{2N}}\left(\frac{q}{2}\right)^{2N}}$$

$$= \left(\frac{fq(q - fq)}{\frac{q^N q^N}{4^N}}\right)^N = \left[4f(1-f)\right]^N$$

$$P = \left[4(0.501)(1 - 0.501)\right]^{10^{23}} = 0.999996^{10^{23}} \cong 0$$

2.

a) $Q = 0$ (adiabat)

b) $PV^{\gamma} = \text{const}$

$$PV^{\gamma} = PVV^{\gamma-1} = NkTV^{\gamma-1}$$

$$T_i V_i^{\gamma-1} = T_f V_f^{\gamma-1}$$

$$T_f = \left(\frac{V_i}{V_f}\right)^{\gamma-1} T_i$$

$$\gamma = \frac{f+2}{f} = \frac{3+2}{3} = \frac{5}{3}$$

$$T_f = (5)^{\frac{2}{3}}(300) = 877.2 \text{ K}$$

$$\Delta U = \frac{3}{2} Nk \Delta T$$

$$\Delta T = (877 - 300)$$

$$Nk = \frac{P_i V_i}{T_i}$$

$$\Delta U = \frac{3}{2}\left(\frac{P_i V_i}{T_i}\right)(T_f - T_i)$$

$$= \frac{3}{2}\left(\frac{10^5 \cdot 15 \times 10^{-3}}{300}\right)(877 - 300)$$

$$\Delta U = 4385.8 \text{ J}$$

$$\Delta U = Q + W = W = 4385 \text{ J}$$

c) $T_f = 877 \text{ K}$

d) $$S = N k_b \left[\ln \left(\frac{V}{N} \left(\frac{4\pi m U}{3N\hbar^2} \right)^{3/2} \right) + \frac{5}{2} \right]$$

$$\Delta N = 0$$

$$S(V,U) = Nk \left[\ln(\text{const}) + \ln\left(V U^{\frac{3}{2}}\right) + \frac{5}{2} \right]$$

$$\Delta S = Nk \ln \left(\frac{V_f U_f^{3/2}}{V_i U_i^{3/2}} \right)$$

But $T V^{\gamma - 1} = \text{const}$

$$\gamma - 1 = \frac{f+2}{f} - \frac{f}{f} = \frac{2}{f}$$

$$T V^{\frac{2}{f}} = \text{const}$$

$$T^{\frac{f}{2}} V = \text{const}$$

$$U^{\frac{3}{2}} V = \text{const}$$

so $\Delta S = 0$

3. $Q_{water} = m_{water} c_w \Delta T$

$$= (600\ g)\left(4.186\ \frac{J}{g\,°C}\right)(6) = 15069.6\ J$$

$$Q_{water} = -Q_{object}$$

$$C_v = \frac{Q_{obj}}{\Delta T} = \frac{-15069.6}{26 - 100} =$$

$$C_v = 203.6\ \frac{J}{K}$$

$$\text{specific } c_v = \frac{203.6\ J/K}{250\ g} = \boxed{0.81\ \frac{J}{g\,°C}}$$

4. If all microstates are equally likely, then macrostates w/ more microstates are favored.

For an ideal gas, the number of microstates increases w/ volume as V^N, & with $N \sim 10^{23}$, even small deviations are nearly impossible. This is the gist of the 2nd Law: multiplicity tends to increase.

$$\Omega \propto V^N$$

$$\frac{\Omega(0.9V)}{\Omega(V)} = 0.9^N$$

$$N = \frac{PV}{kT} = \frac{(10^5)(300)}{(1.4\times10^{-23})(300)} = 7\times10^{27} \rightarrow 10^{28}$$

$$0.9^{10^{28}} = 0$$