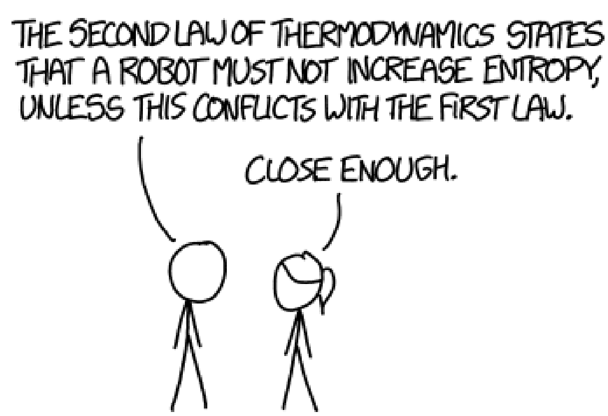
PHYS 4410 Exam I

Wednesday, March 10, 2021

Instructions: *You will have ample time to complete this exam. Take a deep breath and relax! Read each question carefully, and let me know if anything is unclear. Partial credit may be awarded, so you are encouraged to clearly and legibly show your work for each problem. Extra paper is available at the front of the room if you need it. Write your name on every extra sheet you use, and clearly label what problem you are working on. Staple this to the back of your exam when you turn it in. You may use any information contained within this exam, as well as a calculator.*
Good luck!

Name: ______________________________



Potentially useful information

Unit analysis

Power	Prefix	Name
10^{12}	T	tera
10^{9}	G	giga
10^{6}	M	mega
10^{3}	k	kilo
10^{0}	—	—
10^{-3}	m	milli
10^{-6}	μ	micro
10^{-9}	n	nano

Constants

Name	Symbol	Value
Boltzmann Constant	k_B	1.381×10^{-23} J/K
Red. Planck Constant	$\hbar$	1.055×10^{-34} J·s
Avogadro's Number	N_A	6.022×10^{23}
Newton's Constant	G	$6.673 \times 10^{-11} \mathrm{N} \cdot \mathrm{m}^2/\mathrm{kg}^2$

Useful equations

Mono-atomic Ideal Gas

Multiplicity $\Omega = \frac{1}{\hbar^{3N}} \frac{V^N \pi^{\frac{3N}{2}} (2mU)^{\frac{3N}{2}}}{N! \cdot \Gamma(\frac{3N}{2})}$

Entropy $S = N k_b \left[\ln \left(\frac{V}{N} \left(\frac{4\pi m U}{3N\hbar^2} \right)^{3/2} \right) + \frac{5}{2} \right]$

Einstein Solids

Multiplicity $\Omega = \frac{(q+N-1)!}{q!(N-1)!}$

$\Omega(q >> N)$ $\Omega \approx \left(\frac{eq}{N} \right)^N$

Unit Conversions

1 atm $= 1.013 \times 10^5$ N/m^2

T [$^\circ$ C] =T[K] - 273.15

1 cal =4.186 J

1. Two Einstein solids A and B are in contact such that they can freely share energy. Each solid has $N = 10^{23}$ oscillators, and they share a number of energy units q which is much greater than the number of oscillators (the solids are both at high temperature).

   (a) (10 points) Show that the most likely macrostate corresponds to the equal sharing of energy: $q_A = q_B = \frac{q}{2}$

   (b) (15 points) What is the approximate probability of a 0.1% deviation from the most likely macrostate? (i.e. $q_A = 0.501q$, $q_B = 0.499q$). There is no need to find the *exact* probability, just find it relative to the most likely macrostate, which is overwhelmingly more likely.

2. 15 L of mono-atomic ideal gas at atmospheric pressure and a temperature of 300 K is adiabatically compressed to 1/5 of its initial volume ($1\ \mathrm{L} = 10^{-3}\ \mathrm{m}^3$).
   (a) (5 points) How much heat enters the gas during this process?
   (b) (10 points) How much work is done on the gas during this process?
   (c) (10 points) What is the final temperature of the gas?
   (d) (10 points) What is the change in entropy of the gas for this process?

3. (20 points) A 250 g object of unknown material is initially in thermal equilibrium with a large pot of boiling water. The object is then rapidly removed and placed inside of 600 g of cool water ($T=20^\circ$ C). The temperature of the water rises, eventually stopping when it reaches 26° C. What is the specific heat capacity of the unknown object? The specific heat of water is 4.186 J/(g° C). Assume the 600 g of water is thermally isolated from everything except the object (no energy leaves the water and enters the surroundings).

4. (a) (5 points) In terms of microstates and entropy, explain why a gas is almost certain to eventually fill the volume of its container (it will never spontaneously move to one side or occupy a smaller volume).

   (b) (15 points) Currently, our room is at a temperature of 300 K, pressure of 1 atm, and occupies a volume of 300 cubic meters. What is the probability that the gas in this room spontaneously contracts to 90% of its original volume? (You should treat the gas as mono-atomic, even though it isn't).

Question	Points	Score
1	25	
2	35	
3	20	
4	20	
Total:	100	