Quiz 2

1. Starting with the multiplicity of a high-temperature Einstein solid ($\Omega = \left(\frac{eq}{N}\right)^N$), find an expression for energy as a function of temperature $U(T)$ (for example, for an ideal gas we have $U = \frac{3}{2}NkT$). Also find an expression for the constant volume heat capacity C_V. Recall that the total energy of an Einstein solid with q energy units is just $q\hbar\omega$.

$$S = k\ln(\Omega)$$

$$\ln\Omega = \ln\left(\frac{eq}{N}\right)^N = N\ln\left(\frac{eq}{N}\right)$$
$$= N\left(\ln\frac{q}{N} + 1\right)$$

$$U = q\hbar\omega \longrightarrow q = \frac{U}{\hbar\omega}$$

$$\ln\Omega = N\ln\left(\frac{U}{N\hbar\omega}\right) + N$$

$$S = k\ln\Omega$$

$$S = k\left[N\ln\frac{U}{N\hbar\omega} + N\right]$$

$$\frac{1}{T} = \left(\frac{\partial S}{\partial U}\right)_{V,N} = Nk\frac{\partial}{\partial U}\ln\frac{U}{N\hbar\omega} = \frac{Nk}{U}$$

$$\frac{1}{T} = \frac{Nk}{U} \longrightarrow \boxed{U = NkT}$$

$$C_V = \left(\frac{\partial U}{\partial T}\right)_V = \boxed{Nk}$$