

For the ideal gas

$$U = \frac{1}{2} N f k T$$

$$C_v = \left(\frac{\partial U}{\partial T} \right)_v = \frac{1}{2} N f k$$

$$\begin{aligned} C_p &= \left(\frac{\partial U}{\partial T} \right)_p + P \left(\frac{\partial V}{\partial T} \right)_p \\ &= C_v + P \left(\frac{\partial V}{\partial T} \right)_p \end{aligned}$$

$$P V = N k T \Rightarrow V = \frac{N k T}{P}$$

$$C_p = C_v + P \left(\frac{\partial}{\partial T} \frac{N k T}{P} \right)_p$$

$$C_p = C_v + N k$$

$$\frac{C_p}{C_v} = \frac{\frac{1}{2} N f k + N k}{\frac{1}{2} N f k} = \frac{\frac{1}{2} f + 1}{\frac{1}{2} f} = \frac{2+f}{f} = \gamma$$

Ex: How much ^{heat} energy to bring 1.5 L of water to boiling?

$$Q = C_v \Delta T$$

$$c_v = \frac{C_v}{m}, \quad C_v = m c_v$$

$$\rho_w = 10^3 \frac{\text{kg}}{\text{m}^3} \times 1.5 \text{ L} \times \frac{1 \text{ m}^3}{1000 \text{ L}} = 1.5 \text{ kg}$$

$$c_v = \frac{4.2 \text{ J}}{\text{g} \cdot \text{K}} \quad \swarrow \text{1 cal}$$

$$C_v = \frac{4.2 \text{ J}}{\text{g} \cdot \text{K}} \times 1500 \text{ g} = 6300 \frac{\text{J}}{\text{K}}$$

$$Q = 630 \Delta T$$

↑

Room temp ($\sim 30^\circ\text{C}$) to boiling (100°C)

$$Q = (630)(70)$$

$$Q = 4.41 \times 10^5 \text{ J}$$

$$\sim 0.12 \text{ kWh}$$

$$\text{avg} \sim 12 \text{¢ / kWh}$$

$$\sim 1-2 \text{ ¢}$$

OK, now we have 1.5 L
of boiling water

- Remove from heat, & throw in
340 g of pasta

What happens to T of water?

Heat flow from water to pasta

Heat out of water

=

Heat into pasta

$$C_{\text{pasta}} = 1.8 \frac{\text{J}}{\text{g} \cdot \text{K}}$$

$$C_w \Delta T_w = -C_p \Delta T_p$$

$$m_w C_w (T_f - T_w) = -m_p C_p (T_f - T_p)$$

$$m_w C_w (T_f - T_w) = m_p C_p (T_p - T_f)$$

$$m_w C_w T_f - m_w C_w T_w = m_p C_p T_p - m_p C_p T_f$$

$$T_f (m_w C_w + m_p C_p) = m_w C_w T_w + m_p C_p T_p$$

$$T_f = \frac{m_w C_w T_w + m_p C_p T_p}{m_w C_w + m_p C_p}$$

$$= \frac{(1500 \text{ g})(4.2 \frac{\text{J}}{\text{g} \cdot \text{K}})(373) + (340 \text{ g})(1.8 \frac{\text{J}}{\text{g} \cdot \text{K}})(303 \text{ K})}{(1500 \text{ g})(4.2 \frac{\text{J}}{\text{g} \cdot \text{K}}) + (340 \text{ g})(1.8 \frac{\text{J}}{\text{g} \cdot \text{K}})}$$

$$T_f = 366.8 \text{ K} = 93.8^\circ \text{C}$$

(water cools by
~6°C)
use more water!

E_x

0.5 L of water @ $T = 25^\circ\text{C}$

Toss in 50 g of ice ($T = -15^\circ\text{C}$)

Final temp of water?

- Heat leaves water & enters ice
- Raises temp of ice & melts it
- Raises temp of melted ice

Heat leaving water:

$$Q = m_w C_w (T_f - T_w)$$

$$Q_{\text{ice}} = m_i C_i (273 - T_i) + m_i L_i + m_i C_w (T_f - 273)$$

$$Q_{\text{ice}} = -Q_w$$

$$-m_w C_w (T_f - T_w) =$$

$$m_i C_i (273 - T_i) + m_i L_i + m_i C_w (T_f - 273)$$

$$-m_w C_w T_f + m_w C_w T_w$$

$$= m_i C_i (273 - T_i) + m_i L_i + m_i C_w (T_f - 273)$$

$$m_w C_w T_w - m_i C_i (273 - T_i) - m_i L_i$$

$$= m_i C_w (T_f - 273) + m_w C_w T_f$$

$$= (m_i C_w + m_w C_w) T_f - m_i C_w 273$$

$$T_f = \frac{m_w C_w T_w - m_i C_i (273 - T_i) - m_i L_i + 273 m_i C_w}{m_i C_w + m_w C_w}$$

$$m_w = 0.5 \text{ L} \times \frac{1 \text{ m}^3}{100 \text{ L}} \times \frac{1000 \text{ kg}}{\text{m}^3} = 0.5 \text{ kg} = 500 \text{ g}$$

$$C_w = 4.2 \frac{\text{J}}{\text{g}^\circ\text{K}}$$

$$T_w = 293 \text{ K}$$

$$m_i = 50 \text{ g}$$

$$C_i = 2.1 \frac{\text{J}}{\text{g}^\circ\text{K}}$$

$$L_i = 333 \frac{\text{J}}{\text{g}}$$

$$m_{w2} = m_i = 50 \text{ g}$$

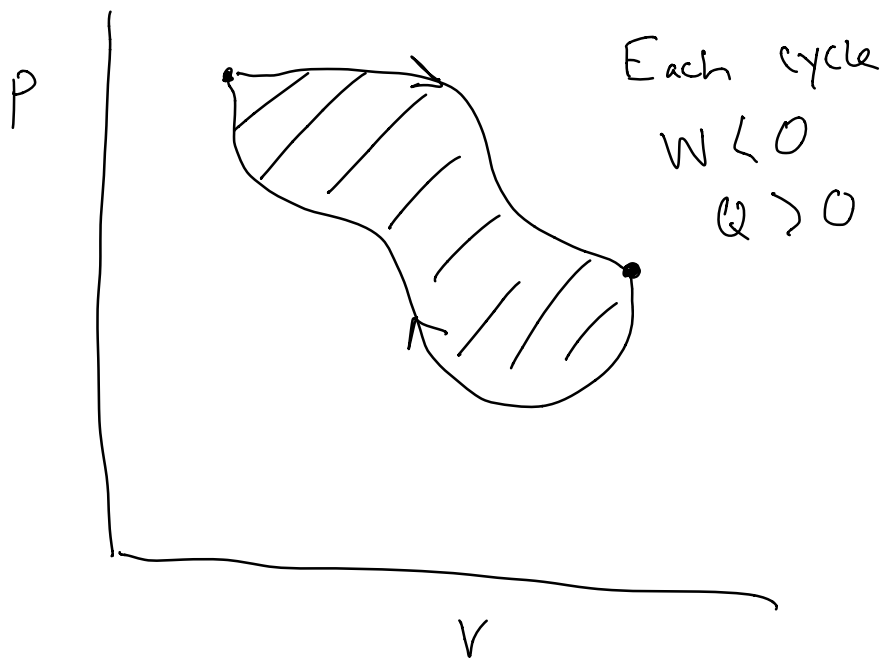
$$T_f = 282.6 \text{ K} = 9.6^\circ\text{C}$$

Ice brings water down

by $\sim 10^\circ\text{C}$

State variables

Properties of a system that don't depend on how the system came to be



U, P, V, T vs W, Q

$$\Delta U = Q + W$$

Consider a new state variable:

$$H = U + PV$$

$$\Delta H = \Delta U + \Delta(PV)$$

$$= Q + W + \Delta(PV)$$

$$\Delta H = Q - P\Delta V + \Delta(PV)$$

IF P is constant

$$\text{then } \Delta(PV) = V\Delta P + P\Delta V = P\Delta V$$

$$\Delta H = Q - P\Delta V + P\Delta V = Q$$

$$\Delta H = Q^*$$

in reality, there are other types of work

$$\Delta H = Q + W_{\text{other}}$$

How to interpret?

$$H = U + PV$$

U : internal "thermal energy"
kinetic

PV : Energy needed to create room for our system

If our system has volume V , we need to expand it against the atmosphere

- Atmosphere collapsing after system is destroyed

In practice, it's useful bc many processes occur @ constant pressure (reactions)

$$C_P = \left(\frac{\partial U}{\partial T} \right)_P + P \left(\frac{\partial V}{\partial T} \right)_P$$

$$C_P = \left(\frac{\partial H}{\partial T} \right)_P$$

$$H = U + PV$$

$$\left(\frac{\partial H}{\partial T} \right)_P = \left(\frac{\partial U}{\partial T} \right)_P + P \left(\frac{\partial V}{\partial T} \right)_P$$