11. $V_A = V_B$

$T_A > T_B$

$P_A = P_B$

this must be true, or else a net force would push air from one room to another until pressure equalizes

$PV = NkT$

$$N_A = \frac{P_A V_A}{k T_A} \qquad N_B = \frac{P_B V_B}{k T_B}$$

$$\frac{N_A}{N_B} = \frac{P_A V_A}{k T_A} \frac{k T_B}{P_B V_B} = \frac{T_B}{T_A} < 1$$

$N_A < N_B$

$\boxed{\text{Room B has greater air mass}}$

12. $PV = NkT$

$$V = \frac{NkT}{P}, \quad \frac{V}{N} = \frac{kT}{P}$$

$$T = 25^\circ C = 298\ K$$

$$P = 10^5\ Pa$$

$$k = 1.38 \times 10^{-23}\ \frac{J}{K}$$

$$\frac{V}{N} = 4 \times 10^{-26}\ m^3$$

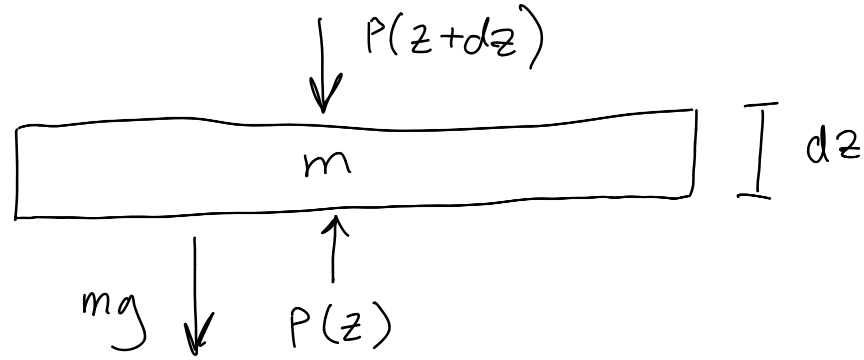
$$d = \left(\frac{V}{N}\right)^{1/3} = 3.5 \times 10^{-9}\ m = 3.5\ nm$$

size of $N_2$ is $\sim 0.3$ nm

distance btwn molecules $\approx 10 \cdot$ size of molecules

this is why the ideal gas approximation more or less holds up

13.




a)

$$F_{net} = P(z) \cdot A - mg - P(z+dz)A$$

$$F_{net} = 0$$

$$P(z)A - P(z+dz)A = mg$$

$$m = \rho_{air} \cdot \underbrace{A \cdot dz}_{V}$$

$$P(z)A - P(z+dz)A = \rho_{air} A dz\, g$$

$$\frac{P(z) - P(z+dz)}{dz} = \rho_{air} \cdot g$$

$$-\frac{dP}{dz} = g\rho_{air} \Rightarrow \boxed{\frac{dP}{dz} = -\rho_{air} g}$$

b) $PV = NkT$

$$P = \frac{N}{V} kT$$

$$\rho_{air} = \frac{Nm}{V}$$

$$P = \frac{\rho_{air}}{m} kT$$

$$\rho_{air} = \frac{Pm}{kT}$$

$$\frac{dP}{dz} = -\rho_{air} g = -\frac{Pm}{kT} g$$

$$\boxed{\frac{dP}{dz} = \frac{-mg}{kT} P}$$

c) $$\frac{dP}{P} = \frac{-mg}{kT} dz$$

integrate

$$\ln(P) = \frac{-mg}{kT} z + \text{const}$$

$$P = e^{\frac{-mg}{kT} z} e^{\text{const}} = A e^{-\frac{mg}{kT} z} \Rightarrow \boxed{P = P(0) e^{\frac{-mg}{kT} z}}$$

$\rho_{air} = \frac{m}{kT} P$

18.
$$V_{rms} = \sqrt{\frac{3kT}{m}}$$

m of $N_2$?

$$28 \frac{g}{mol} \times \frac{1 mol}{6 \times 10^{23}} \times \frac{1 kg}{10^3 g} = 4.7 \times 10^{-26} kg$$

$T = 298 K$

$V_{rms} \approx 512 \frac{m}{s}$

23. He is mono atomic so $f = 3$

$$U = \frac{1}{2} f NkT = \frac{3}{2} NkT$$

$$PV = NkT$$

$$U = \frac{3}{2} PV = \left(\frac{3}{2}\right)(10^5 Pa)(10^{-3} m^3)$$

$U = 150 J$ for He

air is mostly $N_2$ & $O_2$ (diatomic)

so $f = 5$

$$U = \frac{5}{2} NkT = \frac{5}{2} PV = \frac{5}{3}(150\text{ J})$$

$$U = 250\text{ J} \quad \text{for air}$$

28. Let's say we have $\approx 200$ g of water

$$Q = C_V \Delta T$$

$$= m c_V \Delta T$$

$c_V = 4.2 \frac{\text{J}}{\text{gK}}$ $\qquad \Delta T = 100°C - 25°C = 75°C$

$$Q = (200)(4.2)(75) = 6.3 \times 10^4\text{ J}$$

$$\frac{Q}{\Delta t} = 600\text{ W}$$

$$\Delta t = \frac{Q}{600} = 105\text{ s}$$

36.

a) $PV^{\gamma} = \text{const}$

$P_0 = 10^5 \text{ Pa}$ , $V_0 = 10^{-3} \text{ m}^3$

$P_f = 7 \times 10^5 \text{ Pa}$

$P_0 V_0^{\gamma} = P_f V_f^{\gamma}$

$V_f^{\gamma} = \dfrac{P_0 V_0^{\gamma}}{P_f}$

$V_f = V_0 \left( \dfrac{P_0}{P_f} \right)^{1/\gamma}$

in air , $f = 5$ so $\gamma = \dfrac{2+f}{f} = \dfrac{7}{5}$

$V_f = 10^{-3} \text{ m}^3 \left( \dfrac{1}{7} \right)^{5/7} = 0.25 \times 10^{-3} \text{ m}^3$

$V_f = 0.25 \text{ L}$

b) $W = -\int_{V_i}^{V_f} P\,dV$

$$PV^{\gamma} = P_i V_i^{\gamma}$$

$$P = P_i V_i^{\gamma} \frac{1}{V^{\gamma}}$$

$$W = -P_i V_i^{\gamma} \int_{V_i}^{V_f} \frac{1}{V^{\gamma}}\,dV$$

$$W = 188\ \mathrm{J}$$

c) $P_i V_i^{\gamma} = P_f V_f^{\gamma}$

$$PV = NKT$$

$$P = \frac{NKT}{V}$$

$$NKT_i V_i^{\gamma-1} = NKT_f V_f^{\gamma-1}$$

$$T_i V_i^{\gamma-1} = T_f V_f^{\gamma-1}$$

$$T_f = T_i \left( \frac{V_i}{V_f} \right)^{\gamma - 1}$$

$$= 300\text{ K} \left( \frac{1\text{ L}}{.25\text{ L}} \right)^{\frac{7}{5} - 1}$$

$$T_f = 522\text{ K}$$

37. Use above equation

$$T_f = T_i \left( \frac{V_i}{V_f} \right)^{\gamma - 1}$$

$$\gamma = \frac{7}{5}$$

$$T_f = 300 (20)^{\frac{7}{5} - 1}$$

$$T_f \approx 994\text{ K} \quad (720^\circ\text{C})$$

41.

a) $Q_{water} = m_{water} c_v \Delta T$

$$= (250\,g)\left(4.2\,\frac{J}{gK}\right)(24° - 20°)$$

$$Q_{water} = 4200\ J$$

b) $Q_{metal} = -Q_{water}$

$$= -4200\ J$$

c) $Q = C_v \Delta T$

$$C_v = \frac{Q}{\Delta T} = \frac{-4200}{(24° - 100°)} = 55\ \frac{J}{K}$$

d) $c_v = \frac{C_v}{m} = \frac{55\ \frac{J}{K}}{100\ g} = 0.55\ \frac{J}{gK}$

45.

a)

$$w(x,z) = xy = x\left(\frac{x}{z}\right) = \frac{x^2}{z}$$

$$w(y,z) = xy = (yz)y = y^2 z$$

b)

$$\left(\frac{\partial w}{\partial x}\right)_y = \left(\frac{\partial}{\partial x} xy\right)_y = y$$

$$\left(\frac{\partial w}{\partial x}\right)_z = \left(\frac{\partial}{\partial x} \frac{x^2}{z}\right)_z = \frac{2x}{z}$$

$y = \frac{x}{z} \neq \frac{2x}{z}$, so the derivatives are <u>not</u> equal c)

$$\left(\frac{\partial w}{\partial y}\right)_z \stackrel{?}{=} \left(\frac{\partial w}{\partial y}\right)_x$$

$$\left(\frac{\partial}{\partial y} y^2 z\right)_z \stackrel{?}{=} \left(\frac{\partial}{\partial y} xy\right)_x$$

$$2yz \stackrel{?}{=} y$$

$$z \neq \frac{1}{2}$$

NOT EQUAL

$$\left(\frac{\partial w}{\partial z}\right)_x \stackrel{?}{=} \left(\frac{\partial w}{\partial z}\right)_y$$

$$\left(\frac{\partial}{\partial z} \frac{x^2}{z}\right)_x \stackrel{?}{=} \left(\frac{\partial}{\partial z} y^2 z\right)_y$$

$$\frac{-x^2}{z^2} \stackrel{?}{=} y^2 \qquad \left(y = \frac{x}{z}\right)$$

NOT EQUAL

47.

Heat lost by tea = heat gained by ice

$Q_{tea} = m_t C_t \Delta T_t = (200g)(4.2 \frac{J}{gK})(65-100)$

$= -29400 \text{ J}$

$Q_{ice} = ?$

1) bring ice from $-15$ to $0$

2) melt ice

3) bring melted ice up to $65°C$

$Q_{ice} = m C_{ice}(0--15) + m L_i + m C_{water}(65-0)$

$L_i$ = latent heat $= 333 \frac{J}{g}$

$C_{ice} = 2 \frac{J}{g}$

$Q_{ice} = (15)(2)m + 333m + (4.2)(65)m$

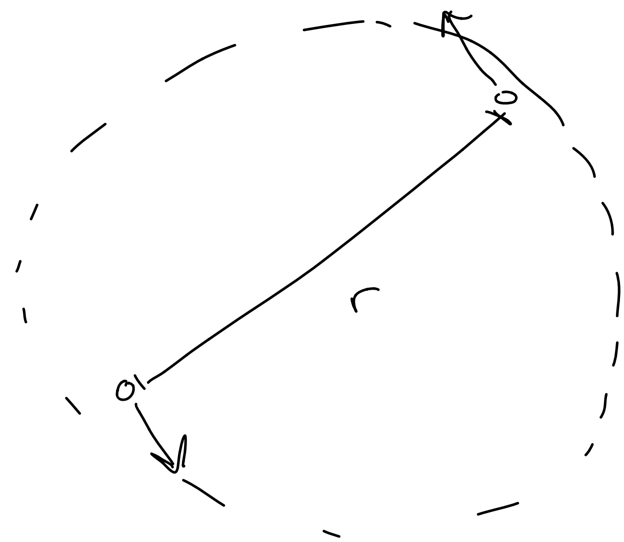
$= 636m$

$Q_{ice} = -Q_{tea}$

$636m = 29400 \Rightarrow \boxed{m = 46\text{ g}}$

5.5.

a)



Kinetic

$$T = \cancel{\tfrac{1}{2}\mu \dot{r}^2} + \tfrac{1}{2}\mu r^2 \dot{\phi}^2$$

$$T = \frac{1}{2}\mu r^2 \dot{\phi}^2, \quad \mu = \frac{m_1 m_2}{m_1 + m_2}$$

$$U = -\frac{G m_1 m_2}{r}$$

$$l = \mu r^2 \dot{\phi} = \text{const} \qquad \dot{\phi}^2 = \frac{l^2}{\mu^2 r^4}$$

$$T = \frac{1}{2}\mu r^2 \frac{l^2}{\mu^2 r^4} = \frac{1}{2}\frac{l^2}{\mu r^2}$$

$$U = -\frac{G\mu M}{r}$$

if we're in a circle, $\ddot{r} = 0$,

which means $r = \frac{1}{GM}\left(\frac{l}{\mu}\right)^2$

Then $U = -GM\mu \cdot GM\left(\frac{\mu}{l}\right)^2$

$$U = -\mu(GM)^2\left(\frac{l}{\mu}\right)^2$$

$$T = \frac{1}{2}\frac{l^2}{\mu r^2}$$

$$= \frac{1}{2}\frac{l^2}{\mu}(GM)^2\left(\frac{\mu}{l}\right)^4$$

$$T = \frac{1}{2}(GM)^2\mu\left(\frac{l}{\mu}\right)^2$$

$$U = -\mu(GM)^2\left(\frac{l}{\mu}\right)^2$$

$$T = \frac{1}{2}\mu(GM)^2\left(\frac{l}{\mu}\right)^2$$

$$U = -2T$$

b)

$$E = T + U$$

$$= T - 2T$$

$$E = -T$$

So increasing $E$ <u>decreases</u> $T$

c) I now realize that using $T$ for kinetic energy is bad. Let's change kinetic energy to $K$!

$$K = \frac{3}{2} N k_b T$$

$$E = -K = -\frac{3}{2} N k_b T$$

$$C = \frac{dE}{dT} = \frac{-3}{2} N K$$

d)

$$U = U(G, m, R)$$

$$U = G^a m^b R^c$$

$$[U] = \frac{m L^2}{T^2}$$

$$[G] = \frac{[F] L^2}{m^2} = \frac{m L L^2}{T^2 m^2}$$

$$= \frac{L^3}{T^2 m}$$

$$\frac{mL^2}{T^2} = \left(\frac{L^3}{T^2 m}\right)^a (m)^b (L)^c$$

$$\frac{mL^2}{T^2} = m^{b-a} \quad L^{3a+c} \quad T^{-2a}$$

$$T^{-2} = T^{-2a} \Rightarrow a = 1$$

$$m^1 = m^{b-a} \Rightarrow b - a = 1$$

$$b = 2$$

$$L^2 = L^{3a+c} \Rightarrow 3a + c = 2$$

$$3 + c = 2$$

$$c = -1$$

$a = 1$, $b = 2$, $c = -1$

$$U = G^a M^b R^c$$

$$= G M^2 R^{-1}$$

$$\boxed{U = -\frac{GM^2}{R}}$$

e) $K = \frac{3}{2} N k T = -\frac{1}{2} U$

$$= +\frac{1}{2} \frac{GM^2}{R}$$

$$T = \frac{GM^2}{3NkR}$$

$M = (\# \text{ of protons})(\text{mass of proton})$

$M = N_p m_p$

$N = 2 N_p$ (assume $\#_e = \#_p$)

$M = \frac{1}{2} N m_p$

$$N = \frac{2M}{m_p} = \frac{2(2 \times 10^{30})}{1.7 \times 10^{-27}}$$

$$N = 2.4 \times 10^{57}$$

$$T = \frac{GM^2}{3NkR}$$

$$T = \frac{(6.7 \times 10^{-11})(2 \times 10^{30})^2}{(3)(2.4 \times 10^{57})(1.4 \times 10^{-23})(7 \times 10^{8})}$$

$T \simeq 4 \times 10^6$ K