3.7

$$S = \frac{8\pi^2 G M^2 k}{hc}$$

$$U = Mc^2 \rightarrow M = \frac{U}{c^2} \rightarrow M^2 = \frac{U^2}{c^4}$$

$$S = \frac{8\pi^2 G k U^2}{hc^5}$$

$$\frac{1}{T} = \frac{\partial S}{\partial U} = \frac{16\pi^2 G k U}{hc^5} = \frac{16\pi^2 G k M c^2}{hc^5}$$

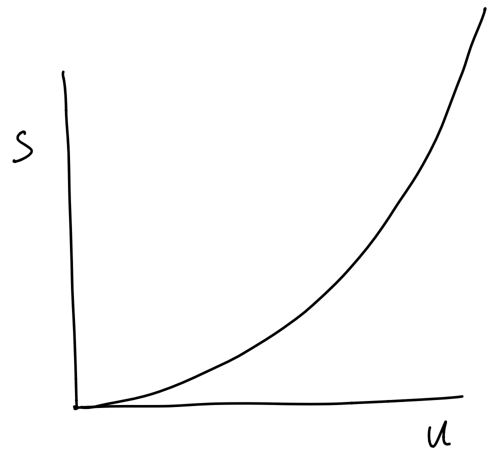
$$\frac{1}{T} = \frac{16\pi^2 G k M}{hc^3}$$

$$T = \frac{hc^3}{16\pi^2 G k M}$$

$$= \frac{(6.63\times10^{-34}\,\mathrm{J\cdot s})(3\times10^{8}\,\mathrm{m/s})^3}{16\pi^2 (6.67\times10^{-11}\,\frac{\mathrm{Nm^2}}{\mathrm{kg^2}})(2\times10^{30}\,\mathrm{kg})(1.38\times10^{-23}\,\frac{\mathrm{J}}{\mathrm{K}})}$$

$$T = 6.1\times10^{-8}\,\mathrm{K}$$ cold!

S vs U is a parabola



$\frac{\partial S}{\partial U}$ is increasing with U

T decreases with U

$U \propto \frac{1}{T}$

Heat capacity is negative!

$\boxed{3.12}$

$$\Delta S = \Delta S_{house} + \Delta S_{outside}$$

$$T_{house} = 298\ K$$

$$T_{outside} = 273\ K$$

Q leaves the house and enters the outside

$$\Delta S_{house} = \frac{-Q}{T_{house}}$$

$$\Delta S_{out} = \frac{Q}{T_{out}}$$

$$\Delta S = Q\left(\frac{1}{T_{out}} - \frac{1}{T_{house}}\right) > 0$$

3.13

a) $\Delta S = \Delta S_{sun} + \Delta S_{earth}$

$$Q = 1000 \frac{J}{s} \times 3\times10^7 s = 3\times10^{10} J$$

$$\Delta S = \frac{-3\times10^{10} J}{6\times10^3 K} + \frac{3\times10^{10} J}{300 K}$$

$$\Delta S = 9.5\times10^7 \frac{J}{K}$$

$$\frac{\Omega_{new}}{\Omega_{old}} = e^{10^7} \approx 10^{5000000}$$

$10^{5\times10^7}$ more ways to organiz energy on Earth!

b) $1 m^2$ of grass

$$\rho_{grass} \approx 1500 \frac{kg}{m^3}$$

grass grows 1m high

1500 kg of grass

Assume it's all carbon $n_{carbon} = \frac{1500 kg}{0.012 kg} = 125{,}000$ moles

$$N \approx 10^5 \times 10^{23} = 10^{28}$$

$$S \approx Nk \approx 10^5 \ \frac{J}{K}$$

– So the sunshine increases entropy by $\sim 10^8 \frac{J}{K}$

– grass decreases it by $\sim 10^5 \frac{J}{K}$

Net increase

3.16 Since we erased, we do not know the previous state of the system.

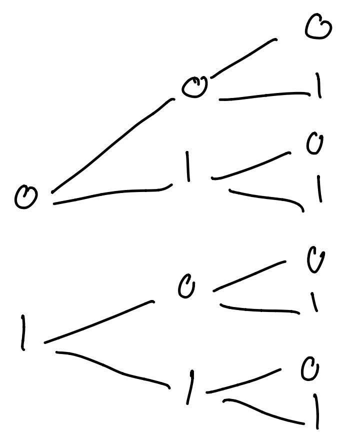
How many ways are there to erase 1 GB?

memory = [0, 1, 1, 0, 1, 0, ... , 0, 1]

each bit can be 0 or 1

(2 outcomes)

Total # of outcomes



There are 2^N ways to configure N bits

$$1\,GB \times 1024\,\frac{MB}{GB} \times 1024\,\frac{KB}{MB} \times 1024\,\frac{B}{KB} \times \frac{8b}{B}$$

$$1\,GB = \left(2^{10}\right)^3 \left(2^3\right) = 2^{33}\ \text{bits}$$

$$\Omega = 2^N = 2^{2^{33}}$$

$$S = k \ln \Omega = k \ln 2^N = Nk \ln 2$$

$$S = 2^{33} (k) \ln(2) = 8.2 \times 10^{-14}\,\frac{J}{K}$$

$$Q = T\Delta S = (300\,K)\left(8.2 \times 10^{-14}\,\frac{J}{K}\right) = 2.4 \times 10^{-11}\,J$$

$\boxed{3.32}$

a) $W = F \cdot d = (2000\,N)(10^{-3}\,m)$

$= 2\,J$

b) $Q = 0$

c) $\Delta U = Q + W$

$= 2\,J$

d) $\Delta S = \frac{1}{T}\Delta U + \frac{P}{T}\Delta V$ (assume T doesn't change)

$\Delta V = -A\Delta x = -0.01\,m^2 \cdot 0.001\,m = -10^{-5}\,m^3$

$\Delta S = \frac{1}{300K}(2J) + \frac{10^5\,N/m^2\,(-10^{-5}\,m^3)}{300\,K} = \frac{1}{300}\,\frac{J}{K}$

$Q \leq T\Delta S$

$T\Delta S > Q$ because this isn't quasistatic

$\boxed{3.33}$

$$dU = T\,dS - P\,dV + \mu\,dN$$

For C_v, $dV = dN = 0$

$$dU = T\,dS$$

$$C_v = \left(\frac{\partial U}{\partial T}\right)_V = \left(\frac{T\,\partial S}{\partial T}\right)_V = T\left(\frac{\partial S}{\partial T}\right)_V$$

$$H = U + PV$$

$$dH = dU + P\,dV \quad (dP = 0)$$

$$= T\,dS - P\,dV + P\,dV = T\,dS$$

$$dH = T\,dS$$

$$C_P = \left(\frac{\partial H}{\partial T}\right)_P = T\left(\frac{\partial S}{\partial T}\right)_P$$

$\boxed{3.37}$

a) $U = U_{kinetic} + Nmgz$

$$\mu = \left(\frac{\partial U}{\partial N}\right)_{S,V} = \left(\frac{\partial U_K}{\partial N}\right)_{S,V} + mgz$$

Eqn 3.63

$$\mu = -kT \ln\left[\frac{V}{N}\left(\frac{2\pi mkT}{h^2}\right)^{3/2}\right] + mgz$$

b) in equilibrium, $\mu(z=0) = \mu(z)$

$$-kT \ln\left[\frac{V}{N(z)}\left(\frac{2\pi mkT}{h^2}\right)^{3/2}\right] + mgz = -kT \ln\left[\frac{V}{N(0)}\left(\frac{2\pi mkT}{h^2}\right)^{3/2}\right]$$

$$mgz = -kT \ln\left(\frac{N(z)}{N(0)}\right)$$

$$N(z) = N(0)\, e^{-mgz/kT}$$

$\boxed{4.3}$

a) $e = \dfrac{W}{Q_c + W}$

$$Q_c + W = \frac{W}{e}$$

$$Q_c = W\left(\frac{1}{e} - 1\right)$$

$$= 1\,GW\left(\frac{1}{.4} - 1\right)$$

$$Q_c = 1.5\,GW$$

b) $\Delta T = \dfrac{Q_c}{C_v}$

$$C_{water} = m\left(4.2\ \frac{J}{g\,^\circ C}\right)$$

$$m = 100\,m^3 \cdot 10^3\ \frac{kg}{m^3} = 10^5\ kg = 10^8\ g$$

$$C_{water} = 4.2 \times 10^8\ \frac{J}{^\circ C}$$

$$\Delta T = \frac{1.5 \times 10^9\ J}{4.2 \times 10^8\ J/^\circ C} = 3.6^\circ C$$

c) $Q_{vap} = 2400\ \dfrac{J}{g}$

$$\text{amount evap} = \frac{Q}{Q_{vap}} = \frac{1.5 \times 10^9\ J}{2400\ \frac{J}{g}} = 625\ kg$$

must evaporate $625 \frac{kg}{s}$, $(0.625\ m^3/s)$

or 0.6% of the water

$\boxed{4.7}$ The A.C. has to dump the "waste heat" Q_h somewhere.

Since

$$\frac{Q_h}{Q_c} \geq \frac{T_h}{T_c}$$

The waste heat is greater than the removed heat.

An amount Q_c, leaves your house, an amount Q_h enters it

$$Q_{net} = Q_h - Q_c$$

$$Q_{net} \geq Q_c \left(\frac{T_h}{T_c} - 1 \right)$$

The AC will actually <u>warm</u> your home!

$\boxed{4.8}$ Same issue as previous problem

$\boxed{4.9}$ $COP \leq \dfrac{T_c}{T_h - T_c}$

T_h = outdoor temp $= 30°C$ $(86°F)$

$T_{in} = T_c = 20°C$ $(68°F)$

$$COP_{max} = \frac{303\,K}{10\,K} = 30.3$$