Quiz 3

You may **not** use notes or your book for this quiz.

1. Starting from the definition of F: $F = U - TS$ derive expressions for:

   (a) Entropy (S)
   (b) Pressure (P)
   (c) Chemical potential (μ)

2. An electron bound to a hydrogen atom is currently in the ground state, which has an energy of -13.6 eV. There is only one possible microstate corresponding to this macrostate. At what temperature will the electron spontaneously jump to the first excited state, which has an energy of -3.4 eV, and a degeneracy of 4 (there are 4 microstates corresponding to the first excited macrostate)? You may express your answer in terms of Boltmann's constant k_B.

1. $F = U - TS$

$$dF = dU - TdS - SdT$$

$$= TdS - PdV + \mu dN - TdS - SdT$$

$$dF = -SdT - PdV + \mu dN$$

$$dF = \left(\frac{\partial F}{\partial T}\right)_{V,N} dT + \left(\frac{\partial F}{\partial V}\right)_{T,N} dV + \left(\frac{\partial F}{\partial N}\right)_{T,V} dN$$

$$S = -\left(\frac{\partial F}{\partial T}\right)_{V,N} \quad , \quad P = -\left(\frac{\partial F}{\partial V}\right)_{T,N} \quad , \quad \mu = \left(\frac{\partial F}{\partial N}\right)_{T,V}$$

2. $\Delta F = \Delta u - T\Delta S$

$$\Delta u = (-3.4\,eV) - (-13.6\,eV)$$
$$\Delta u = 10.2\,eV$$

$$\Delta S = K\ln 4 - K\ln 1$$
$$\Delta S = K\ln 4$$
$$\Delta S \cong 1.4\,K$$

$$\Delta F < 0 \implies \Delta u < T\Delta S$$

$$T > \frac{\Delta u}{\Delta S}$$

$$T > \frac{10.2\,eV}{1.4\,K}$$

$$\boxed{KT > 7.3\,eV}$$