A liter of air at room temp + standard pressure is slowly allowed to expand to a volume of 5 L. (so slow that T doesn't change).

How much heat enters the gas during this process?

$$\Delta U = Q + W = 0$$

$$Q = -W$$

$$W = NkT \ln\left(\frac{V_i}{V_f}\right)$$

$$Q = NkT \ln\left(\frac{V_f}{V_i}\right)$$

$$V_f/V_i = 5$$

$$T = 300\ K$$

$$N = ?$$

$$PV = NkT, \quad N = \frac{P_i V_i}{kT}$$

$$P_i = 1\ atm = 1.01 \times 10^5\ Pa$$

$$V_i = 1\,L = 10^{-3}\,m^3$$

$$N = \frac{(1.01 \times 10^5)(10^{-3})}{(1.38 \times 10^{-23})(300)}$$

$$\frac{Pa \cdot m^3}{\frac{J}{K}\,K} = \frac{\frac{N}{m^2}\,m^3}{\frac{J}{K}\,K} = \frac{Nm}{J} = \phi$$

$$N = 2.4 \times 10^{22}$$

$$Q = NKT \ln\left(\frac{V_f}{V_i}\right)$$

$$Q \cong 160\ J$$

On Monday

Equipartition: $U = N \cdot f \cdot \frac{1}{2} kT$

1st Law : $\Delta U = Q + W$

Work done by compression

$$W = -\int_{V_i}^{V_f} P\,dV$$

Ideal Gas

- Isothermal
- Adiabatic

- Isothermal : so slow that gas equilibrates with surrounding and $\Delta T = 0$

if $\Delta T = 0$, $\Delta U = 0$,

bc $\Delta U = \frac{1}{2} NfkT$

$$W = NkT \ln\left(\frac{V_i}{V_f}\right) = -Q$$

Adiabatic Compression

- Gas compressed so quickly that ≈ 0 heat has time to flow

$$Q = 0$$

How much work is done if we move from $V_i \rightarrow V_f$?

$$W = -\int P(V)\, dV$$

$$PV = NkT$$

$$P = \frac{NkT}{V}$$

$$W = -\int \frac{NkT}{V} dV$$

Not helpful, since $T = T(V)$

Need to relate T & V

Energy

$$U = \frac{1}{2} N f k T$$

small change in T dT

$$dU = \frac{1}{2} N f k\, dT$$

also know: $\Delta U = Q + W$, $Q = 0$

$$\Delta U = W$$

$$dU = -PdV$$

$$\frac{1}{2} N f k dT = -PdV$$

$$P = \frac{NkT}{V}$$

$$\frac{1}{2} N f k dT = -\frac{NkT}{V} dV$$

$$\frac{1}{2} f \frac{dT}{T} = -\frac{dV}{V}$$

$$\frac{1}{2} f \ln\left(\frac{T_f}{T_i}\right) = -\ln\left(\frac{V_f}{V_i}\right)$$

$$\ln\left(\left(\frac{T_f}{T_i}\right)^{f/2}\right) = \ln\left(\frac{V_i}{V_f}\right)$$

$$\left(\frac{T_f}{T_i}\right)^{\frac{f}{2}} = \frac{V_i}{V_f}$$

$$V_f T_f^{\frac{f}{2}} = V_i T_i^{\frac{f}{2}}$$

$$V T^{\frac{f}{2}} = \text{const}$$

$$PV = NkT$$

$$T = \frac{PV}{Nk}$$

$$V\left(\frac{PV}{Nk}\right)^{\frac{f}{2}} = \text{const}$$

$$\left(V\left(\frac{PV}{Nk}\right)^{\frac{f}{2}}\right)^{\frac{2}{f}} = \left(\text{const}\right)^{2/f}$$

$$V^{2/f+1}P = \text{const}$$

$$\frac{2}{f}+1 = \frac{2+f}{f} \equiv \gamma$$

$$\boxed{PV^{\gamma} = \text{const}}$$

$$W = -\int P(V)\,dV$$

$$PV^{\gamma} = P_i V_i^{\gamma}$$

$$P = \frac{P_i V_i^{\gamma}}{V^{\gamma}}$$

$$W = -P_i V_i^{\gamma}\int_{V_i}^{V_f}\frac{1}{V^{\gamma}}\,dV$$

$$\gamma = \frac{2+f}{f},\ f \geq 1$$

$$\gamma \geq 3$$

$$\int V^{-\gamma}\, dV = \frac{1}{1-\gamma} V^{1-\gamma}$$

$$W = -P_i V_i^{\gamma} \left(\frac{1}{1-\gamma}\right)\left(\frac{1}{V_f^{\gamma-1}} - \frac{1}{V_i^{\gamma-1}}\right)$$

$$= \frac{1}{\gamma-1}\left(\frac{P_i V_i^{\gamma}}{V_f^{\gamma-1}} - \frac{P_i V_i^{\gamma}}{V_i^{\gamma-1}}\right)$$

$$P_i V_i^{\gamma} = P_f V_f^{\gamma}$$

$$W = \frac{1}{\gamma-1}\left(\frac{P_f V_f^{\gamma}}{V_f^{\gamma-1}} - \frac{P_i V_i^{\gamma}}{V_i^{\gamma-1}}\right)$$

$$\boxed{W = \frac{1}{\gamma-1}\left(P_f V_f - P_i V_i\right)}$$

$$\Delta U = W = \frac{1}{2} N f k \Delta T$$

Ex

Ex: Gasoline vapor injected into a cylinder

The vapor is initially @

$$T = 20^\circ C$$
$$P = 10^5 \ Pa$$
$$V = 240 \ cm^3$$

The mixture is adiabatically compressed down to $40 \ cm^3$

1) What is the temperature after compression

2) How much work is done on the gas?

assume diatomic ($f = 5$)

$$\gamma = \frac{2+f}{f} = \frac{7}{5}$$

$$P_i V_i^\gamma = P_f V_f^\gamma$$

$$P_f = P_i \left(\frac{V_i}{V_f}\right)^\gamma = 10^5 \left(\frac{240}{40}\right)^{7/5} = 10^5 (6)^{7/5}$$
$$= 1.23 \times 10^6 \ Pa$$

$$P_f = 1.23 \times 10^6 \ \frac{N}{m^2}$$

$$P_f V_f = NKT_f \ , \ P_i V_i = NkT_i \Rightarrow N = \frac{P_i V_i}{kT_i}$$

$$T_f = \frac{P_f V_f}{N k} = \frac{P_f V_f}{\left(P_iV_i/kT_i\right) k} = \frac{P_f V_f}{P_i V_i} T_i$$

$$T_f = \frac{(1.23\times10^{6})(40\times10^{-6})}{(10^{5})(240\times10^{-6})}(293)$$

$$\boxed{T_f = 601\ \mathrm{K} = 328\,^{\circ}\mathrm{C}}$$

$$W = \frac{1}{\gamma-1}\left(P_f V_f - P_i V_i\right)$$

$$= \frac{1}{\frac{7}{5}-1}\left[(1.23\times10^{6})(40\times10^{-6}) - (10^{5})(240\times10^{-6})\right]$$

$$= \frac{5}{2}\left[49.2 - 24\right]$$

$$\boxed{W = 63\ \mathrm{J}}$$

$$\Delta U = W = \tfrac{1}{2} N f k \Delta T$$

$$\Delta T = \frac{2W}{N f k}, \quad N = \frac{P_i V_i}{k T_i}$$

$$\Delta T = \frac{2W}{P_i V_i f} T_i = 308\ \mathrm{K} \qquad T_i + \Delta T = 293 + 308 = 601\ \mathrm{K}$$

Iso:

$$P \sim \frac{1}{V}$$

$$\Delta T = 0$$

$$\Delta U = 0$$

Ad:

$$P \sim \frac{1}{V^{\gamma}}$$

$$Q = 0$$

$$\Delta U = W$$

$$PV^{\gamma} = \text{const}, \quad \gamma = \frac{2+f}{f}$$

Break question:

- About how many air molecules are in this room?

- What is the total energy of the air in this room?

$$U = \frac{1}{2} N f k T$$

$$f = 5$$

Question: Have a gas in a confined space (it can't expand) + thermally insulated (no heat flow)

I briefly breach the insulation to add heat to the gas.

How much heat does it take to raise the temperature by 10°C?

$$\Delta U = W + Q$$

$$\Delta V = 0 \longrightarrow W = 0$$

$$\Delta U = \boxed{\frac{1}{2} N f k \Delta T = Q}$$

More generally, take $\dfrac{Q}{\Delta T} = \dfrac{1}{2} N f k$

We call this the objects

<u>heat capacity</u> $C = \dfrac{Q}{\Delta T}$

- How much energy is required to change T by a given amount?

Notes:

- Property of the object

  each substance has a heat capacity

- Depends on amount of material

  more energy to change T of a lake than a cup of tea

  - we see this from the N in the eqn

  - Sometimes we use the "specific heat capacity"

$$c = \frac{C}{m}$$

- For clarity our example considered ideal gas at const volume

in general, consider

$$C = \frac{Q}{\Delta T}$$

In general this is ambiguous

since $Q = \Delta U - W$, + W is not specified

One workaround: the heat cap at constant volume

$$\Delta V = 0 \Rightarrow W = -P\Delta V = 0$$

$$\Delta U = Q$$

$$\frac{Q}{\Delta T} = \frac{\Delta U}{\Delta T}$$

write like this:

$$C_V = \left(\frac{\Delta U}{\Delta T}\right)_V$$ to indicate V is constant

if $\Delta T \longrightarrow 0$

$$C_V = \left(\frac{\partial U}{\partial T}\right)_V$$

For 1g of water, $C_V \approx 4.2$ J/°C

most objects expand as they heat

Another possibility:

heat capacity at constant pressure

if P is const

$$Q = \Delta U - W = \Delta U - (-P\Delta V)$$

$$= \Delta U + P\Delta V$$

$$C_P = \left(\frac{\Delta U + P\Delta V}{\Delta T} \right)_P$$

$$C_P = \left(\frac{\partial U}{\partial T} \right)_P + P\left(\frac{\partial V}{\partial T} \right)_P$$

For the ideal gas

$$U = \frac{1}{2} N f k T$$

$$C_v = \left(\frac{\partial U}{\partial T}\right)_v = \frac{1}{2} N f k$$

$$C_p = \left(\frac{\partial U}{\partial T}\right)_p + P\left(\frac{\partial V}{\partial T}\right)_P$$

$$= C_v + P\left(\frac{\partial V}{\partial T}\right)_P$$

$$PV = NkT \Rightarrow V = \frac{NkT}{P}$$

$$C_p = C_v + P\left(\frac{\partial}{\partial T}\frac{NkT}{P}\right)_P$$

$$C_p = C_v + Nk$$

$$\frac{C_p}{C_v} = \frac{\frac{1}{2}Nfk + Nk}{\frac{1}{2}Nfk} = \frac{\frac{1}{2}f + 1}{\frac{1}{2}f} = \frac{2+f}{f} = \gamma$$