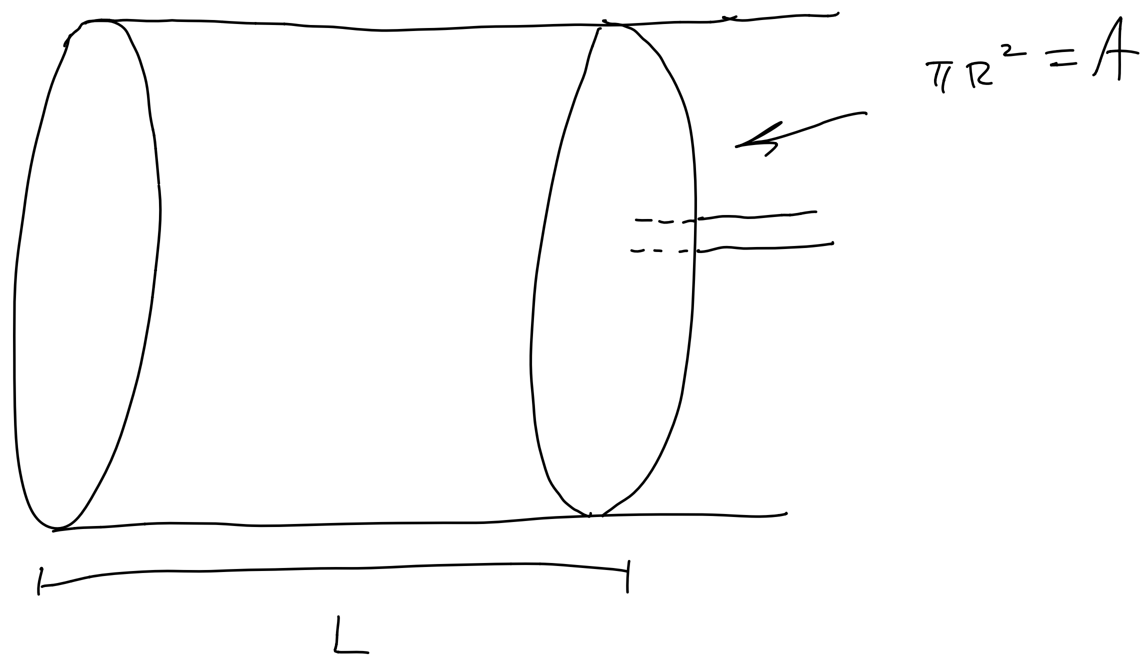
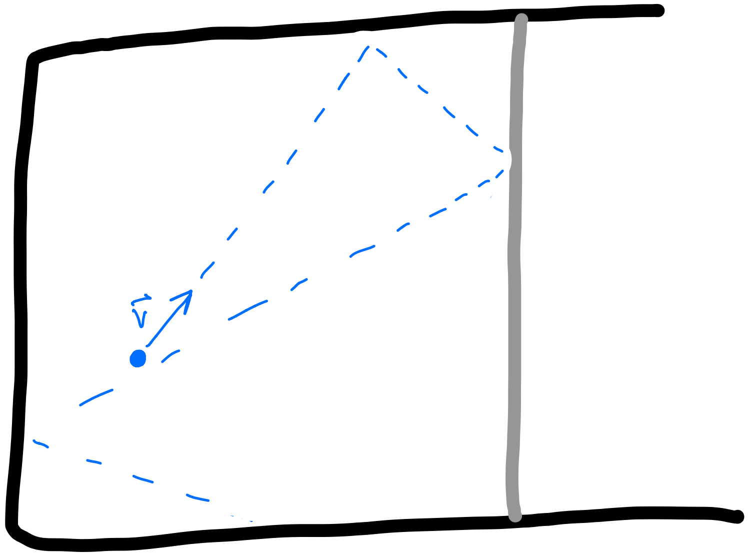
Thermal energy

- measured by temperature
- related to avg kinetic energy

Let's explore further

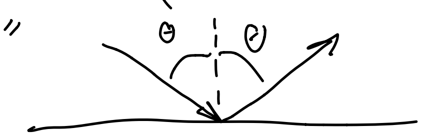


volume $V = AL$

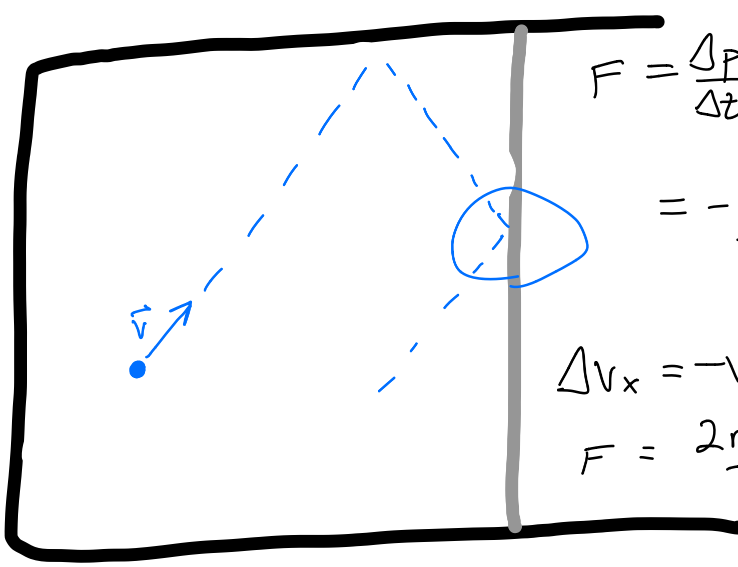


1) all collisions are elastic ($|\vec{v}|$ is const)

2) cylinder is "smooth"



What is the average Pressure on the piston?

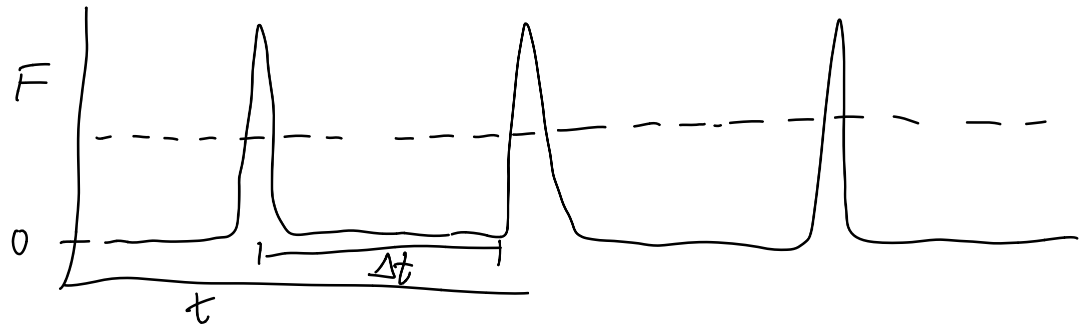


$$F = \frac{\Delta p}{\Delta t} = \frac{-\Delta p_{molecule}}{\Delta t}$$

$$= \frac{-m\Delta v_x}{\Delta t}$$

$$\Delta v_x = -v_x - v_x = -2v_x$$

$$F = \frac{2mv_x}{\Delta t}$$



$$\Delta t = \frac{2L}{V_x}$$

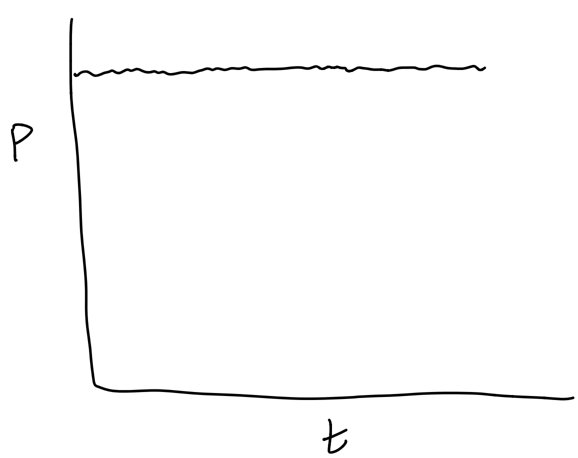
$$\bar{F} = \frac{2mV_x}{(2L/V_x)} = \frac{mV_x^2}{L}$$

$$\bar{P} = \frac{\bar{F}}{A} = \frac{mV_x^2}{AL} = \frac{mV_x^2}{V}$$

V_x^2: Faster v_x means a) more frequent collisions
b) more forceful collisions

In reality, there are many molecules (N many)

$$N \sim 10^{23}$$



- Random positions & directions

- Ignore inter molecular interactions (~OK for low density)

$$\overline{P}V = mv_x^2$$

For N-many

$$\overline{P}V = mV_{x_1}^2 + mV_{x_2}^2 + mV_{x_3}^2 + \ldots$$

$$\overline{P}V = \sum_{i=1}^{N} mV_{x_i}^2$$

Drop $\overline{P}$ (P is ~ continuous)

- on avg, $V_x^2 = \overline{V_x^2}$ $\left(\overline{V_x^2} \neq \overline{V_x}^2\right)$

$$PV = m\overline{V_x^2} + m\overline{V_x^2} + m\overline{V_x^2} \ldots$$

$$PV = Nm\overline{V_x^2}$$

$$PV = NkT$$

$$NkT = Nm\overline{v_x^2}$$

$$kT = m\overline{v_x^2}$$

looks like KE, but isn't

$$\overline{K_{trans}} = \frac{1}{2}m\,\overline{\left(v_x^2 + v_y^2 + v_z^2\right)}$$

$$= \frac{1}{2}m\left(\overline{v_x^2} + \overline{v_y^2} + \overline{v_z^2}\right)$$

No prefered direction, so:

$$\overline{v_x^2} = \overline{v_y^2} = \overline{v_z^2}$$

$$\overline{K_{trans}} = \frac{3}{2}m v_x^2$$

$$m\overline{v_x^2} = \frac{2}{3}\overline{K_{trans}} = kT$$

$$\boxed{\overline{K_{trans}} = \frac{3}{2}kT}$$

Relation btwn Kinetic energy + temperature

$$\frac{1}{2} m \overline{v^2} = \frac{3}{2} kT$$

$$\overline{v^2} = \frac{3kT}{m}$$

$$V_{RMS} = \sqrt{\overline{v^2}} = \sqrt{\frac{3kT}{m}}$$

$V_{RMS} \neq \overline{V}$, but they are close.

To get an idea:

Room temperature $\approx 300\,K$

air $\sim$ Nitrogen (N_2)

$$N = 14 \frac{g}{mol}$$

$$N_2 = 28 \frac{g}{mol}$$

$$\text{mass of } N_2 = 28 \frac{g}{mol} \times \frac{1\ mol}{6.02 \times 10^{23}} = 4.65 \times 10^{-23}\ g$$

$$m = 4.65 \times 10^{-26}\ kg$$

$$k = 1.381 \times 10^{-23}\ J/K$$

$$V_{RMS} = \sqrt{\frac{3(1.38 \times 10^{-23})(300)}{4.65 \times 10^{-26}}} \simeq 517\ m/s$$

What have we just done?
We've related the temperature of a gas to it's translational kinetic energy.

How accurate is this? (Our derivation made some pretty simple assumptions). Actually holds up, as we'll prove in chapter 6.

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Equipartition

-As it turns out, this result <Ktrans> = 3/2kT, is a special case of a more general result
-We only considered one type of energy, Ktrans
-An object's total energy consists of many different kinds of energy

- Rotational

$$K_{rot} = \frac{1}{2} I |\vec{\omega}|^2$$

- Potential

$$U = \frac{1}{2} k x^2$$

At temp T: avg energy of any quadratic deg of freedom is $\frac{1}{2}kT$

Corollary: Energy is equally distributed among quad deg of freedom

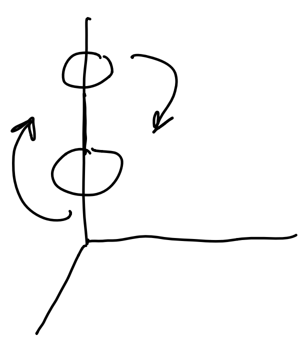
What is a deg of freedom?

"How many numbers to describe the system?"

For a single particle: 3
$(\frac{1}{2}mv_x^2, \frac{1}{2}mv_y^2, \frac{1}{2}mv_z^2)$

For 2 particles: 6
$(\frac{1}{2}mv_{1x}^2, \frac{1}{2}mv_{2x}^2, \ldots)$

Diatomic molecule:
6   3 trans + 2 rot + spring



If a system of N particles, each with f degrees of freedom, its total thermal energy is

$$U_{thermal} = N \cdot f \cdot \frac{1}{2} kT$$

For mono-atomic gas: $f = 3$

$$U_{thermal} = \frac{3}{2} NkT \quad (U_{thermal} = K_{trans})$$

Diatomic : $f = 6$

$$U = 3NkT \quad (U_{thermal} > K_{trans})$$