2.41

First card: 52 possibilities

Second : 51

3^{rd} : 50

4^{th} : 49

5^{th} : 48

There are $5!$ many ways to organize the cards into a hand

$$\Omega = \frac{52!}{5!(52-5)!}$$

$$\Omega = 2.59896 \times 10^6$$

$$\Omega \approx 2.6 \times 10^6$$

There is only one way to get a "royal flus" in a given suit, and there are 4 suits

so $P(\text{royal flush}) = \dfrac{4}{2.6 \times 10^6} = 1.5 \times 10^{-6}$

2.6)

$$\Omega(N,q) = \frac{(q+N-1)!}{q!\,(N-1)!}$$

$$\Omega(30,30) = \frac{(30+30-1)!}{30!\,(30-1)!} = 5.9 \times 10^{16}$$

2.8)

$$N_A = N_B = 10 := N$$

$$q = 20 = q_A + q_B$$

a) Macro states

$q_A = 0, 1, \ldots 19, 20$

\# macro states $= 21$

b) \# of ways to divide 20 energy units among 20 oscillators

$$\Omega_{all} = \frac{(20+20-1)!}{20!\,(20-1)!} = 6.9 \times 10^{10}$$

c) $\Omega(q_A = 20) = \Omega_A(q_A = 20)\,\Omega_B(q_B = 0)$

$$= \frac{(20+10-1)!}{20!\,(10-1)!}\,\frac{(0+10-1)!}{0!\,(10-1)!} = 1.00 \times 10^{7}$$

$$P(q_A=20) = \frac{1.00\times10^7}{6.9\times10^{10}} = 1.45\times10^{-4}$$

$$\frac{\Omega(q_A=20)}{\Omega(\text{all})}$$

d) $$\Omega(q_A=10) = \frac{(10+10-1)!}{10!\,(10-1)!} \times \frac{(10+10-1)!}{10!\,(10-1)!}$$

$$= 8.5\times10^9$$

$$P(q_A=10) = \frac{8.5\times10^9}{6.9\times10^{10}}$$

$$P = 0.12$$

e) The evolution from macrostate $q_A \sim 0$ to $q_A \sim 10$ is near certain, since $q_A = 10$ is ~ 1000 times more likely than $q_A = 0$. The reverse (evolution from $q_A \sim 10 \rightarrow \sim 0$) will almost never happen.

$\boxed{2.13}$

a) $e^{a \ln b} = e^{\ln b^a} = b^a$

b) $\ln(a+b) = \ln\left(a\left(1+\frac{b}{a}\right)\right)$

$= \ln(a) + \ln(1 + b/a)$

$|b/a| << 1$

Taylor Expansion

$$f(1+\epsilon) \approx f(1) + \epsilon f'(1) + \frac{1}{2}\epsilon^2 f''(1) + \dots$$

$$\ln(1+\epsilon) \approx \ln(1) + \epsilon \left.\frac{d}{dx}\ln(x)\right|_{x=1} + \dots$$

1

$\epsilon^2 \approx 0$

$\ln(1+\epsilon) \approx 0 + \epsilon$

$\ln(1+b/a) \approx b/a$

$\ln(a+b) = \ln(a) + \ln\left(1+\frac{b}{a}\right) \approx \ln(a) + b/a$

2.17

Start at EQN 18, before Schroeder uses the $q >> N$ approx

$$\ln \Omega = (q+N)\ln(q+N) - q\ln q - N\ln N$$

if $N >> q$

$$\ln(q+N) \approx \ln(N) + \frac{q}{N}$$

$$\ln \Omega = (q+N)\left(\ln N + \frac{q}{N}\right) - q\ln q - N\ln N$$

$$= q\ln N + \frac{q^2}{N} + \cancel{N\ln N} + q - q\ln q - \cancel{N\ln N}$$

$$= q\ln N - q\ln q + q + \frac{q^2}{N}$$

$$= q\ln\left(\frac{N}{q}\right) + q + \frac{q^2}{N}$$

Assume

$q \sim 10^{23}$

$N \sim 10^{23}$, while $N >> q$

$q\ln(N/q) \sim q$ (Large)

$q \sim q$

$q^2/N \sim q\left(\frac{q}{N}\right)$

$\frac{q}{N}$ is small!

$\frac{q^2}{N} \approx 0$ compared to other terms

$$\ln \Omega = q \ln \frac{N}{q} + q$$

$$\Omega = e^{q\ln\frac{N}{q} + q}$$

$$= \left(\frac{N}{q}\right)^q e^q$$

$$\Omega = \left(\frac{eN}{q}\right)^q$$

2.26

Volume of energy hypersurface

$$w = \int_{H \le E} d^{2N} r \, d^{2N} p$$

$$H = \frac{1}{2m} \left(p_{x1}^2 + p_{y1}^2 + p_{x2}^2 + p_{y2}^2 + p_{x3}^2 + p_{y3}^2 + \dots \right) = E$$

$$w = \int d^{2N} r \int_{H \le E} d^{2N} p$$

$$\int dx_1 dy_1 \int dx_2 dy_2 \dots \int dx_N dy_N$$

$$A \cdot A \cdot A \dots \cdot A$$

$$= A^N$$

$$w = A^N \int_{H < E} d^{2N} p$$

$$W = A^N \underbrace{\int_{H<E} d^{2N} p}_{\text{volume of a 2N dim hypersphere}}$$

$$V_d(R) = \frac{\pi^{d/2}}{\frac{d}{2}\,\Gamma\left(\frac{d}{2}\right)} R^d$$

$R = \sqrt{2mE}$

$d = 2N$

$$V_{2N}(\sqrt{2mE}) = \frac{\pi^N}{N\,\Gamma(N)}(2mE)^N$$

$\Gamma(N) = N!$

$$W = A^N \frac{\pi^N}{N\,\Gamma(N)}(2mE)^N$$

$$\sigma = \frac{\partial w}{\partial E} = A^N \frac{\pi^N (2m)^N \cancel{N} E^{N-1}}{\cancel{N}(N!)}$$

$$E^{N-1} \approx E^N$$

$$\sigma = \frac{A^N \pi^N (2mE)^N}{N!}$$

$$\Omega = \frac{\sigma}{\hbar^{2N} N!}$$

$$\boxed{\Omega = \frac{A^N \pi^N (2mE)^N}{\hbar^{2N} (N!)^2}}$$

2.27

Since $\Omega \propto V^N$

$$P \approx \frac{\Omega(V=0.99)}{\Omega(V=1)} = 0.99^N$$

$N = 100$, $P \approx 0.37$

$N = 10^4$, $P \approx 2.2 \times 10^{-44}$

$N = 10^{23}$, $P \approx 0.99^{10^{23}} \longrightarrow$ small.

2.30

a) From 22 b

$$\Omega = \frac{2^{4N}}{\sqrt{8\pi N}}, \quad \frac{S}{k} = \ln \Omega =$$

$$\frac{S}{k} = 4N \ln 2 - \ln\left(\sqrt{8\pi N}\right)$$

if $N = 10^{23}$

$$\frac{S}{k} = 2.77 \times 10^{23} - 28.1$$

$$\frac{S}{k} = 2.77 \times 10^{23}$$

b) from 22c, $\Omega = \frac{2^{4N}}{4\pi N}$

$$\frac{S}{k} = 4N \ln 2 - \ln(4\pi N)$$

$$\frac{S}{k} = 2.77 \times 10^{23} - 55.5$$

c) ΔS between (a) & (b) is negligible, so no

d) again ΔS is negligible. Though I probably will lose sleep over it.

$\boxed{2.34}$

$$Q = NkT \ln \frac{V_i}{V_f} \qquad (1.30)$$

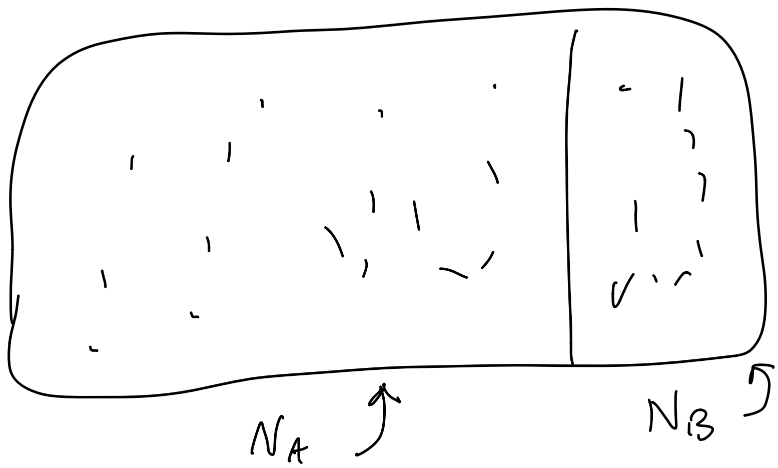
From (2.49)

$$S = Nk \ln(\text{const} \cdot V) + \frac{5}{2} Nk$$

$$\Delta S = S_f - S_i = Nk \ln \frac{V_f}{V_i} = \frac{Q}{T}$$

2.37

$\Delta S_{tot} = \Delta S_A + \Delta S_B$



$V_{tot} = \frac{NkT}{P}$

B expands from $\frac{N_B kT}{P}$ to $\frac{NkT}{P}$

So $\frac{V_f}{V_i} = \frac{N}{N_B} = \frac{1}{x}$

$\Delta S_B = N_B k \ln \frac{1}{x}$ $\qquad \frac{N_B}{N} = x$

$\Delta S_B = -xNk \ln x$

For A, $\frac{V_f}{V_i} = \frac{N}{N_A} = \frac{N}{N-N_B} = \frac{1}{1-x}$

$$\Delta S_A = N_A k \ln \frac{1}{1-x}$$

$$= -Nk(1-x)\ln(1-x)$$

$$N_A = N - N_B$$

$$= N(1-x)$$

$$\Delta S = \Delta S_A + \Delta S_B = -Nk\left[(1-x)\ln(1-x) - x\ln x\right]$$

$$x = \frac{1}{2}$$

$$\Delta S = -Nk\left(\frac{1}{2}\ln\frac{1}{2} - \frac{1}{2}\ln\frac{1}{2}\right)$$

$$\Delta S = Nk\ln 2$$

here N is total #

in text 2N is