*Science in China Series E: Technological Sciences*

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# Topological relations between directed lines and simple geometries

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**Directed lines are fundamental geometric elements to represent directed linear entities. The representations of their topological relations are so different from those of simple lines that they cannot be solved exactly with normal methods. In this paper, a new model based on point-set topology is defined to represent the topological relations between directed lines and simple geometries. Through the intersections between the start-points, end-points, and interiors of the directed lines and the interiors, boundaries, and exteriors of the simple geometries, this model identifies 5 cases of topological relations between directed lines and points, 39 cases of simple lines, and 26 cases of simple polygons. Another 4 cases of simple lines and one case of simple polygons are distinguished if considering the exteriors of the directed lines. All possible cases are furthermore grouped into an exclusive and complete set containing 11 named predicts. And the conceptual neighborhood graph is set up to illustrate their relationship and similarity. This model can provide a basis for natural language description and spatial query language to present the dynamic semantics of directed lines relative to the background features.**

topological relation, directed line, simple geometry, intersection model

## 1 Introduction

In the context of Geographic Information Sciences, spatial relations play an important role in geospatial knowledge representation and reasoning. Geospatial knowledge includes three development stages, i.e. landmark knowledge, route knowledge and survey knowledge, among which route knowledge is based on travel processes that connect sequent landmarks[1]. The route is actually shaped by spatial relations, especially topological relations, between the path and the landmarks. An example of a route can be described in natural language as “start from Peking

Received December 3, 2007; accepted January 5, 2008
doi: 10.1007/s11431-008-5010-9
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Supported by the National Natural Science Foundation of China (Grant Nos. 40701134 and 40771171) and the National Hi-Tech Research and Development Program of China (Grant No. 2007AA12Z216)

University, go along the Baiyi road, cross the Third Ring road, and arrive at Friendship Hotel finally". This route is represented as a directed trajectory, which is topologically related to landmarks at certain positions. These topological relations are described by some action predicts with landmarks as objects, such as start, along, cross and arrive, etc. In general, the route is simply abstracted as a directed line with a start-point, an end-point and the direction; the landmarks are represented by simple geometries; then the relations between them are mapped to the topological relations between directed lines and simple geometries.

Directed lines are fundamental geometric elements to represent directed linear entities[2], such as roads, rivers, power lines, airlines and the like. They are always used to represent dynamic features or phenomena, especially moving objects and their trajectories[3]. They are also extended to arrow symbols in some diagrams[4]. Directed linear features are dynamic relatively to static features in background as referenced landmarks, so the topological relations between them are important to the dynamic properties of directed lines. This kind of relations, however, is different from those between simple features because directions and motion semantics are essential.

The formal representations of topological relations are core issues in geospatial reasoning research, in which the Region Connection Calculus (RCC) [5] and the 9-Intersection Model (9IM) [6] are the most important two. But they are both proposed for simple geometries, the former for regions, and the latter for points, lines, and polygons. So directed lines, which are extended from simple lines, cannot employ them directly. Renz[7] extended Allen's Interval Algebra[8] with a spatial interpretation to the Directed Intervals Algebra (DIA) for qualitative spatial representation and reasoning about directed intervals, which consisted of 26 jointly exhaustive and pairwise disjoint base relations. Wang et al.[9] extended DIA further to RNDIA for road network moving objects. Based on 9IM, Kurata and Egenhofer[2] proposed the head-body-tail intersection model to capture spatial relations between pairs of directed line segments. Through the intersections of the segments' heads, bodies, and tails, 68 classes of topological relations are identified, and another 12 classes are distinguished if considering segments' exteriors as well. These models are applicable to topological relations between directed intervals or segments, but not suitable to the more complicated relations between directed lines and simple geometries.

In this paper, a new model based on point-set topology is developed to represent the topological relations between directed lines and simple geometries. Through the intersections between different parts of directed lines and simple points, lines, and polygons, the main cases of the topological relations are identified. And a predict set of the relations is proposed furthermore to support natural language description and spatial query language to present dynamic semantics.

## 2 Topological relations modeling of directed lines

### 2.1 Topological relations representation

According to Egenhofer's point-set topology and 9-intersection model[6,10], topological relations are calculated based on the existence/non-existence of the pair-wise intersections between the interiors, boundaries and exteriors of two geometries and are classified based on the entries in the resulting intersection matrix. For a simple geometry $O$ in $\boldsymbol{R}^2$ (i.e. point, line, or polygon), its interior, boundary and exterior are denoted as $O^\circ$, $\partial O$ and $O^-$, respectively.

Directed lines are different from simple lines for directions, so the topological relations between directed lines and simple geometries are also different for directional and dynamic semantics. In Figure 1, a simple line is just intersected with a polygon (Figure 1(a)), but it is not enough for the directed line in the same layout because the semantics of get-out (Figure 1(b)) and enter (Figure 1(c)) are involved. Moreover, Figure 1 (a) and (d) both express the intersection relation and have 9***I***=(*TTTTTTTTT*), but these two cases should be distinguished for directed lines (Figure 1(b) to (e), Figure 1(c) to (f)) because Figure 1(e) and (f) express the cross relation compared to get-out (Figure 1(b)) and enter (Figure 1(c)). These differences are determined by the relations between the two ordered nodes of the directed line and the geometries, which cannot be represented by 9IM.

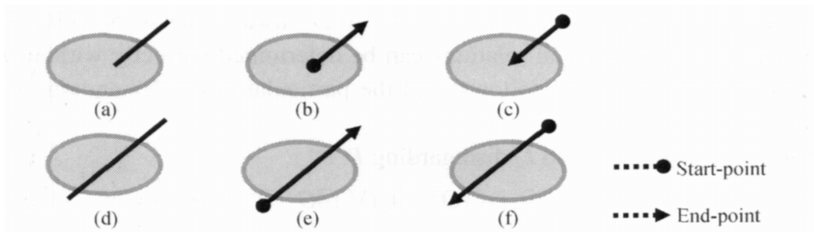



**Figure 1** Comparison of topological relations of directed lines and simple lines. (a) (d) Two kinds of intersect relation for a simple line; (b) (c) intersect relations for a directed line with get-out and enter semantics; (e) (f) cross relations for a directed line with two different directions.

A directed line is defined as a simple line[11] connected with a start-point and an end-point, and its direction is from the start-point to the end-point, i.e.

**Definition 1.** A directed line is a continuous linear mapping $f$: $[0,1]\rightarrow \boldsymbol{R}^2$. Its start-point is denoted as $\partial_s D=f(0)$, and its end-point is $\partial_e D=f(1)$.

Given a directed line $D$, its boundary is denoted as $\partial D=\{f(0), f(1)\}=\{\partial_s D, \partial_e D\}$, and its interior is $D°=f((0,1))=D-\partial D$.

**Definition 2.** A directed line is simple if $\forall x, y \in (0, 1),\ x \neq y \Rightarrow f(x) \neq f(y)$ and $f(0)\neq f(x) \wedge f(1) \neq f(x)$. A simple directed line is intersection-free.

A directed line is closed if $f(0)=f(1)$. A closed directed line is still simple. Comparatively a closed line is not simple and its boundary is empty. The discrimination of the start-point and the end-point makes this difference. In this paper, only the simple directed lines are discussed.

The start-point and the end-point determine the direction of the directed line, and their relations with other features also determine the relations between the directed line and the features. The boundary of a directed line is substituted by the start-point and the end-point. Then the topological relations are calculated by the intersections between the directed line's start-point, end-point, interior and the simple geometry's interior, boundary, exterior.

**Definition 3.** Given a directed line $D$ and a simple geometory $O$, the topological relations between them are calculated by the intersection matrix $\boldsymbol{M}_d$, i.e.

$$\boldsymbol{M}_d(D,O)=\begin{pmatrix} \partial_s D\cap O° & \partial_s D\cap \partial O & \partial_s D\cap O^- \\ D°\cap O° & D°\cap \partial O & D°\cap O^- \\ \partial_e D\cap O° & \partial_e D\cap \partial O & \partial_e D\cap O^- \\ D^-\cap O° & D^-\cap \partial O & D^-\cap O^- \end{pmatrix}.$$

In this 3×4 matrix, each intersection is empty ($\phi$) or non-empty ($\neg\phi$), so $2^{12}$=4096 configurations are given. But not all of them exist due to constraints among each other. The basic conditions constraining $9\boldsymbol{I}$[6] are still valid to $\boldsymbol{M}_d$. Furthermore, $D$' start-point or end-point cannot intersect with more than one part of $O$ separately because they are both points, which is the basic conditions constraining $\boldsymbol{M}_d$, i.e.

**Condition 1.** $D$' start-point intersects with one and only one part of $O$.

**Condition 2.** $D$' end-point intersects with one and only one part of $O$.

So, there is one and only one non-empty entry in row 2 or 3 in $\boldsymbol{M}_d$. In addition, because $O$ is the point, line or polygon type, there are more conditions for each type. These conditions will be discussed in detail in the coming subsections.

A directed line $D$ is a 1-dimensional entity in $\boldsymbol{R}^2$, so $D^-$ always intersects with $\partial O$, $O^\circ$, and $O^-$ except few cases and the topological relations can be determined correctly without $D^-$ involved. Then $\boldsymbol{M}_d$ can be simplified to $\boldsymbol{I}_d$ as follows, and the particular cases demanding $D^-$ will be discussed separately.

**Definition 4.** $\boldsymbol{M}_d$ is simplified to $\boldsymbol{I}_d$ disregarding $D^-$:

$$\boldsymbol{I}_d(D,O)=\begin{pmatrix} \partial_s D\bigcap O^\circ & \partial_s D\bigcap \partial O & \partial_s D\bigcap O^- \\ D^\circ\bigcap O^\circ & D^\circ\bigcap \partial O & D^\circ\bigcap O^- \\ \partial_e D\bigcap O^\circ & \partial_e D\bigcap \partial O & \partial_e D\bigcap O^- \end{pmatrix}.$$

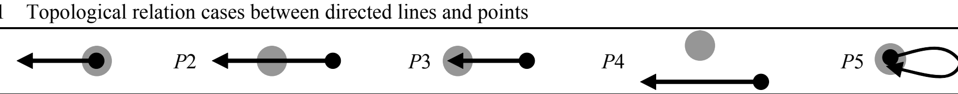
Thus $\boldsymbol{I}_d$ has $2^9$=512 configurations, however not all of which are valid because the above conditions should be satisfied as well.

$\boldsymbol{M}_d$ (or $\boldsymbol{I}_d$) is not symmetrical as $9\boldsymbol{I}$ because $D$ and $O$ are different types. $\boldsymbol{M}_d\,(O,D)$ must be converted to $\boldsymbol{M}_d\,(D,O)$ before its is calculated.

### 2.2 Topological relations between directed lines and points

A point is a 0-dimensional geometry and represents a single location in coordinate space. The boundary of a point is an empty set. The topological relations between a directed line and a point are relatively simple, having only 5 cases (Table 1), in which $P5$ exists only when the directed line is closed. The exterior of a directed line is not necessary in this condition.

**Table 1** Topological relation cases between directed lines and points



| $P1$ | $P2$ | $P3$ | $P4$ | $P5$ |
|---|---|---|---|---|
| $\begin{pmatrix} \neg\phi & \phi & \phi \\ \phi & \phi & \neg\phi \\ \phi & \phi & \neg\phi \end{pmatrix}$ | $\begin{pmatrix} \phi & \phi & \neg\phi \\ \neg\phi & \phi & \neg\phi \\ \phi & \phi & \neg\phi \end{pmatrix}$ | $\begin{pmatrix} \phi & \phi & \neg\phi \\ \phi & \phi & \neg\phi \\ \neg\phi & \phi & \phi \end{pmatrix}$ | $\begin{pmatrix} \phi & \phi & \neg\phi \\ \phi & \phi & \neg\phi \\ \phi & \phi & \neg\phi \end{pmatrix}$ | $\begin{pmatrix} \neg\phi & \phi & \phi \\ \phi & \phi & \neg\phi \\ \neg\phi & \phi & \phi \end{pmatrix}$ |

### 2.3 Topological relations between directed lines and simple lines

A simple line is a 1-dimensional geometry with two 0-dimensional nodes as the boundary. Condition 1 and 2 apply, and additional constraints must hold as follows.

Given a directed line $D$ and a simple line $L$, then

**Condition 3.** No more than two parts of $D$ intersect $L$'s boundary, i.e.

$$\boldsymbol{I}_d(D,L)\neq\begin{pmatrix} - & \neg\phi & - \\ - & \neg\phi & - \\ - & \neg\phi & - \end{pmatrix}.$$

As a result, there are no more than two non-empty entries in column 2 of $\boldsymbol{I}_d$ (or $\boldsymbol{M}_d$).

**Condition 4.** If $D$'s interior intersects with $L$'s boundary, then it must also intersect with $L$'s exterior, i.e.

$$\boldsymbol{I}_d(D,L) \neq \begin{pmatrix} - & - & - \\ - & \neg\phi & \phi \\ - & - & - \end{pmatrix}.$$

**Condition 5**. If $D$'s boundary, its start-point or its end-point, intersects with $L$'s exterior (interior), then $D$'s interior must also intersect with $L$'s exterior (interior), i.e.

$$\boldsymbol{I}_d(D,L) \neq \begin{pmatrix} - & - & \neg\phi \\ - & - & \phi \\ - & - & - \end{pmatrix} \vee \begin{pmatrix} - & - & - \\ - & - & \phi \\ - & - & \neg\phi \end{pmatrix} \vee \begin{pmatrix} \neg\phi & - & - \\ \phi & - & - \\ - & - & - \end{pmatrix} \vee \begin{pmatrix} - & - & - \\ \phi & - & - \\ \neg\phi & - & - \end{pmatrix}.$$

Constrained by the above conditions, the topological relations between directed lines and simple lines contain 39 cases (Table 2), in which $L32$ and $L38$ exist only when the directed line is closed.

The exterior of the directed line, $D^-$, should be employed to refine the configurations only when the condition is satisfied:

$$D^\circ \cap L^\circ = \neg\phi \wedge D^\circ \cap \partial L = \neg\phi \wedge D^\circ \cap L^- = \neg\phi \wedge \partial D \cap L^- = \neg\phi.$$

Otherwise, $D^-$ intersects with $\partial L$, $L^\circ$ and $L^-$ definitely. As a result, another 4 cases are distinguished, such as $L23$-1, $L23$-2, $L24$-1 and $L37$-1 in Table 3 corresponding to $L23$, $L24$ and $L37$ in Table 2 separately.

### 2.4 Topological relations between directed lines and simple polygons

A simple polygon is a 2-dimensional geometry with one 1-dimensional exterior boundary and no holes. Condition 1 and 2 apply, and additional constraints must hold as follows.

Given a directed line $D$ and a simple polygon $G$, then

**Condition 6.** If $D$'s interior intersects only with $G$'s boundary, then $D$'s start-point and end-point must both intersect with $G$'s boundary, i.e.

$$\boldsymbol{I}_d(D,G) \neq \begin{pmatrix} - & \phi & - \\ \phi & \neg\phi & \phi \\ - & \phi & - \end{pmatrix} \vee \begin{pmatrix} - & \neg\phi & - \\ \phi & \neg\phi & \phi \\ - & \phi & - \end{pmatrix} \vee \begin{pmatrix} - & \phi & - \\ \phi & \neg\phi & \phi \\ - & \neg\phi & - \end{pmatrix}.$$

**Condiyion 7.** If $D$'s boundary, its start-point or its end-point, intersects with $L$'s exterior (interior), then $D$'s interior must also intersect with $L$'s exterior (interior), i.e.

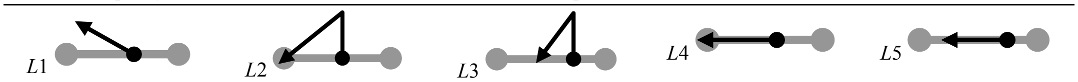
$$\boldsymbol{I}_d(D,G) \neq \begin{pmatrix} - & - & \neg\phi \\ - & - & \phi \\ - & - & - \end{pmatrix} \vee \begin{pmatrix} - & - & - \\ - & - & \phi \\ - & - & \neg\phi \end{pmatrix} \vee \begin{pmatrix} \neg\phi & - & - \\ \phi & - & - \\ - & - & - \end{pmatrix} \vee \begin{pmatrix} - & - & - \\ \phi & - & - \\ \neg\phi & - & - \end{pmatrix}.$$

**Condition 8.** If $D$'s start-point intersects with $L$'s interior and its end-point intersects with $L$'s exterior, or $D$'s start-point intersects with $L$'s exterior and its end-point intersects with $L$'s interior, then $D$'s interior must intersect with all $L$'s exterior, boundary and interior, i.e.

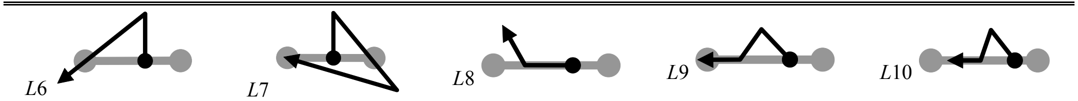
$$\begin{pmatrix} \neg\phi & - & - \\ - & - & - \\ - & - & \neg\phi \end{pmatrix} \subseteq \begin{pmatrix} \neg\phi & - & - \\ \neg\phi & \neg\phi & \neg\phi \\ - & - & \neg\phi \end{pmatrix} \text{ and } \begin{pmatrix} - & - & \neg\phi \\ - & - & - \\ \neg\phi & - & - \end{pmatrix} \subseteq \begin{pmatrix} - & - & \neg\phi \\ \neg\phi & \neg\phi & \neg\phi \\ \neg\phi & - & - \end{pmatrix}.$$

Constrained by these conditions, the topological relations between directed lines and simple polygons contain 26 cases (Table 4: $G1-G26$).

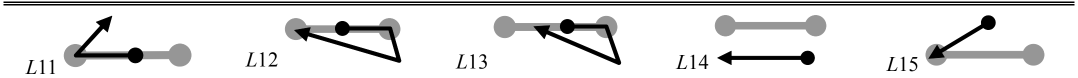
**Table 2** Topological relation cases between directed lines and simple lines



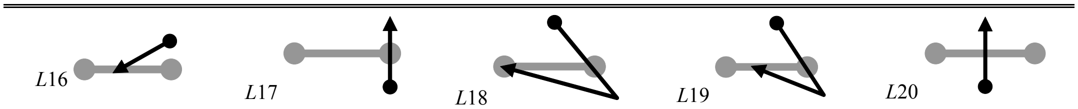
| | | | | |
|---|---|---|---|---|
| L1 | L2 | L3 | L4 | L5 |
| $\begin{pmatrix}\neg\phi & \phi & \phi\\ \phi & \phi & \neg\phi\\ \phi & \phi & \neg\phi\end{pmatrix}$ | $\begin{pmatrix}\neg\phi & \phi & \phi\\ \phi & \phi & \neg\phi\\ \phi & \neg\phi & \phi\end{pmatrix}$ | $\begin{pmatrix}\neg\phi & \phi & \phi\\ \phi & \phi & \neg\phi\\ \neg\phi & \phi & \phi\end{pmatrix}$ | $\begin{pmatrix}\neg\phi & \phi & \phi\\ \neg\phi & \phi & \phi\\ \phi & \neg\phi & \phi\end{pmatrix}$ | $\begin{pmatrix}\neg\phi & \phi & \phi\\ \neg\phi & \phi & \phi\\ \neg\phi & \phi & \phi\end{pmatrix}$ |



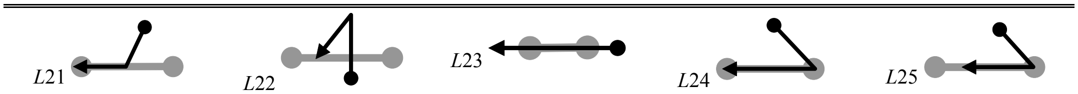
| | | | | |
|---|---|---|---|---|
| L6 | L7 | L8 | L9 | L10 |
| $\begin{pmatrix}\neg\phi & \phi & \phi\\ \phi & \neg\phi & \neg\phi\\ \phi & \phi & \neg\phi\end{pmatrix}$ | $\begin{pmatrix}\neg\phi & \phi & \phi\\ \phi & \neg\phi & \neg\phi\\ \phi & \neg\phi & \phi\end{pmatrix}$ | $\begin{pmatrix}\neg\phi & \phi & \phi\\ \neg\phi & \phi & \neg\phi\\ \phi & \phi & \neg\phi\end{pmatrix}$ | $\begin{pmatrix}\neg\phi & \phi & \phi\\ \neg\phi & \phi & \neg\phi\\ \phi & \neg\phi & \phi\end{pmatrix}$ | $\begin{pmatrix}\neg\phi & \phi & \phi\\ \neg\phi & \phi & \neg\phi\\ \neg\phi & \phi & \phi\end{pmatrix}$ |



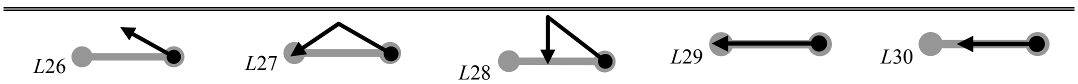
| | | | | |
|---|---|---|---|---|
| L11 | L12 | L13 | L14 | L15 |
| $\begin{pmatrix}\neg\phi & \phi & \phi\\ \neg\phi & \neg\phi & \neg\phi\\ \phi & \phi & \neg\phi\end{pmatrix}$ | $\begin{pmatrix}\neg\phi & \phi & \phi\\ \neg\phi & \neg\phi & \neg\phi\\ \phi & \neg\phi & \phi\end{pmatrix}$ | $\begin{pmatrix}\neg\phi & \phi & \phi\\ \neg\phi & \neg\phi & \neg\phi\\ \neg\phi & \phi & \phi\end{pmatrix}$ | $\begin{pmatrix}\phi & \phi & \neg\phi\\ \phi & \phi & \neg\phi\\ \phi & \phi & \neg\phi\end{pmatrix}$ | $\begin{pmatrix}\phi & \phi & \neg\phi\\ \phi & \phi & \neg\phi\\ \phi & \neg\phi & \phi\end{pmatrix}$ |



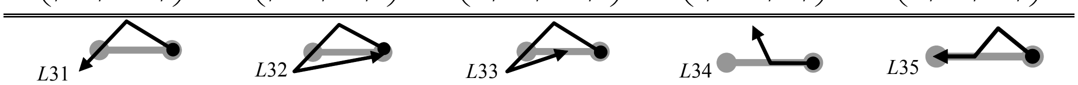
| | | | | |
|---|---|---|---|---|
| L16 | L17 | L18 | L19 | L20 |
| $\begin{pmatrix}\phi & \phi & \neg\phi\\ \phi & \phi & \neg\phi\\ \neg\phi & \phi & \phi\end{pmatrix}$ | $\begin{pmatrix}\phi & \phi & \neg\phi\\ \phi & \neg\phi & \neg\phi\\ \phi & \phi & \neg\phi\end{pmatrix}$ | $\begin{pmatrix}\phi & \phi & \neg\phi\\ \phi & \neg\phi & \neg\phi\\ \phi & \neg\phi & \phi\end{pmatrix}$ | $\begin{pmatrix}\phi & \phi & \neg\phi\\ \phi & \neg\phi & \neg\phi\\ \neg\phi & \phi & \phi\end{pmatrix}$ | $\begin{pmatrix}\phi & \phi & \neg\phi\\ \neg\phi & \phi & \neg\phi\\ \phi & \phi & \neg\phi\end{pmatrix}$ |



| | | | | |
|---|---|---|---|---|
| L21 | L22 | L23 | L24 | L25 |
| $\begin{pmatrix}\phi & \phi & \neg\phi\\ \neg\phi & \phi & \neg\phi\\ \phi & \neg\phi & \phi\end{pmatrix}$ | $\begin{pmatrix}\phi & \phi & \neg\phi\\ \neg\phi & \phi & \neg\phi\\ \neg\phi & \phi & \phi\end{pmatrix}$ | $\begin{pmatrix}\phi & \phi & \neg\phi\\ \neg\phi & \neg\phi & \neg\phi\\ \phi & \phi & \neg\phi\end{pmatrix}$ | $\begin{pmatrix}\phi & \phi & \neg\phi\\ \neg\phi & \neg\phi & \neg\phi\\ \phi & \neg\phi & \phi\end{pmatrix}$ | $\begin{pmatrix}\phi & \phi & \neg\phi\\ \neg\phi & \neg\phi & \neg\phi\\ \neg\phi & \phi & \phi\end{pmatrix}$ |



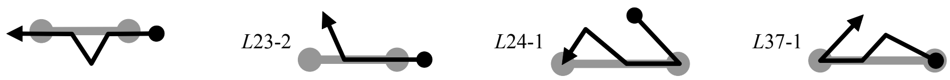
| | | | | |
|---|---|---|---|---|
| L26 | L27 | L28 | L29 | L30 |
| $\begin{pmatrix}\phi & \neg\phi & \phi\\ \phi & \phi & \neg\phi\\ \phi & \phi & \neg\phi\end{pmatrix}$ | $\begin{pmatrix}\phi & \neg\phi & \phi\\ \phi & \phi & \neg\phi\\ \phi & \neg\phi & \phi\end{pmatrix}$ | $\begin{pmatrix}\phi & \neg\phi & \phi\\ \phi & \phi & \neg\phi\\ \neg\phi & \phi & \phi\end{pmatrix}$ | $\begin{pmatrix}\phi & \neg\phi & \phi\\ \neg\phi & \phi & \phi\\ \phi & \neg\phi & \phi\end{pmatrix}$ | $\begin{pmatrix}\phi & \neg\phi & \phi\\ \neg\phi & \phi & \phi\\ \neg\phi & \phi & \phi\end{pmatrix}$ |



| | | | | |
|---|---|---|---|---|
| L31 | L32 | L33 | L34 | L35 |
| $\begin{pmatrix}\phi & \neg\phi & \phi\\ \phi & \neg\phi & \neg\phi\\ \phi & \phi & \neg\phi\end{pmatrix}$ | $\begin{pmatrix}\phi & \neg\phi & \phi\\ \phi & \neg\phi & \neg\phi\\ \phi & \neg\phi & \phi\end{pmatrix}$ | $\begin{pmatrix}\phi & \neg\phi & \phi\\ \phi & \neg\phi & \neg\phi\\ \neg\phi & \phi & \phi\end{pmatrix}$ | $\begin{pmatrix}\phi & \neg\phi & \phi\\ \neg\phi & \phi & \neg\phi\\ \phi & \phi & \neg\phi\end{pmatrix}$ | $\begin{pmatrix}\phi & \neg\phi & \phi\\ \neg\phi & \phi & \neg\phi\\ \phi & \neg\phi & \phi\end{pmatrix}$ |

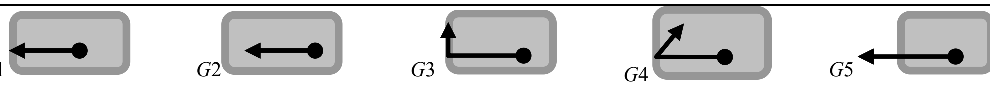
| | | | |
|---|---|---|---|
| L36 | L37 | L38 | L39 |
| $\begin{pmatrix}\phi & \phi & \neg\phi\\ \neg\phi & \phi & \neg\phi\\ \neg\phi & \phi & \phi\end{pmatrix}$ | $\begin{pmatrix}\phi & \neg\phi & \phi\\ \neg\phi & \neg\phi & \neg\phi\\ \phi & \phi & \neg\phi\end{pmatrix}$ | $\begin{pmatrix}\phi & \neg\phi & \phi\\ \neg\phi & \neg\phi & \neg\phi\\ \phi & \neg\phi & \phi\end{pmatrix}$ | $\begin{pmatrix}\phi & \neg\phi & \phi\\ \neg\phi & \neg\phi & \neg\phi\\ \neg\phi & \phi & \phi\end{pmatrix}$ |

**Table 3** Additional topological relation cases between directed lines and simple lines considering $D^{-}$

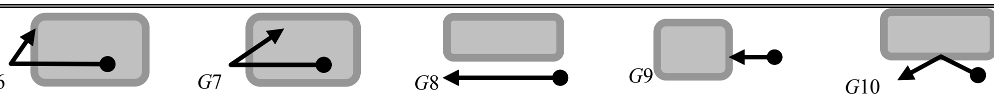


| $L$23-1 | $L$23-2 | $L$24-1 | $L$37-1 |
|---|---|---|---|
| $\begin{pmatrix} \phi & \phi & \neg\phi \\ \neg\phi & \neg\phi & \neg\phi \\ \phi & \phi & \neg\phi \\ \neg\phi & \phi & \neg\phi \end{pmatrix}$ | $\begin{pmatrix} \phi & \phi & \neg\phi \\ \neg\phi & \neg\phi & \neg\phi \\ \phi & \phi & \neg\phi \\ \neg\phi & \neg\phi & \neg\phi \end{pmatrix}$ | $\begin{pmatrix} \phi & \phi & \neg\phi \\ \neg\phi & \neg\phi & \neg\phi \\ \phi & \neg\phi & \phi \\ \neg\phi & \phi & \neg\phi \end{pmatrix}$ | $\begin{pmatrix} \phi & \neg\phi & \phi \\ \neg\phi & \neg\phi & \neg\phi \\ \phi & \phi & \neg\phi \\ \neg\phi & \phi & \neg\phi \end{pmatrix}$ |

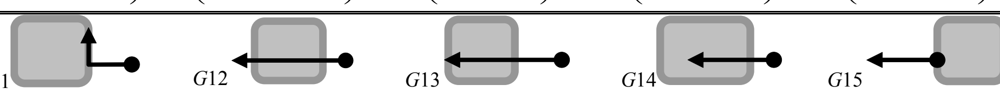
**Table 4** Topological relation cases between directed lines and simple polygons



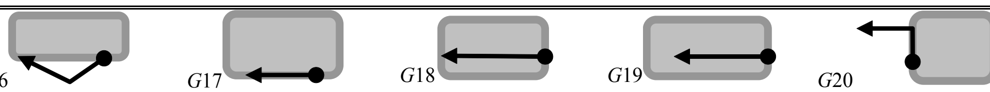
| $G$1 | $G$2 | $G$3 | $G$4 | $G$5 |
|---|---|---|---|---|
| $\begin{pmatrix} \neg\phi & \phi & \phi \\ \neg\phi & \phi & \phi \\ \phi & \neg\phi & \phi \end{pmatrix}$ | $\begin{pmatrix} \neg\phi & \phi & \phi \\ \neg\phi & \phi & \phi \\ \neg\phi & \phi & \phi \end{pmatrix}$ | $\begin{pmatrix} \neg\phi & \phi & \phi \\ \neg\phi & \neg\phi & \phi \\ \phi & \neg\phi & \phi \end{pmatrix}$ | $\begin{pmatrix} \neg\phi & \phi & \phi \\ \neg\phi & \neg\phi & \phi \\ \neg\phi & \phi & \phi \end{pmatrix}$ | $\begin{pmatrix} \neg\phi & \phi & \phi \\ \neg\phi & \neg\phi & \neg\phi \\ \phi & \phi & \neg\phi \end{pmatrix}$ |



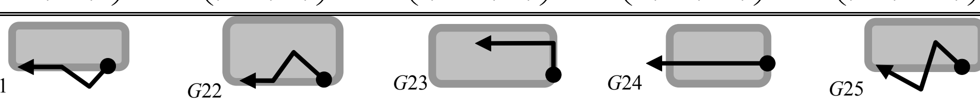
| $G$6 | $G$7 | $G$8 | $G$9 | $G$10 |
|---|---|---|---|---|
| $\begin{pmatrix} \neg\phi & \phi & \phi \\ \neg\phi & \neg\phi & \neg\phi \\ \phi & \neg\phi & \phi \end{pmatrix}$ | $\begin{pmatrix} \neg\phi & \phi & \phi \\ \neg\phi & \neg\phi & \neg\phi \\ \neg\phi & \phi & \phi \end{pmatrix}$ | $\begin{pmatrix} \phi & \phi & \neg\phi \\ \phi & \phi & \neg\phi \\ \phi & \phi & \neg\phi \end{pmatrix}$ | $\begin{pmatrix} \phi & \phi & \neg\phi \\ \phi & \phi & \neg\phi \\ \phi & \neg\phi & \phi \end{pmatrix}$ | $\begin{pmatrix} \phi & \phi & \neg\phi \\ \phi & \neg\phi & \neg\phi \\ \phi & \phi & \neg\phi \end{pmatrix}$ |



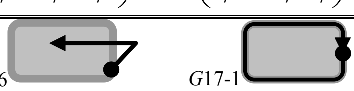
| $G$11 | $G$12 | $G$13 | $G$14 | $G$15 |
|---|---|---|---|---|
| $\begin{pmatrix} \phi & \phi & \neg\phi \\ \phi & \neg\phi & \neg\phi \\ \phi & \neg\phi & \phi \end{pmatrix}$ | $\begin{pmatrix} \phi & \phi & \neg\phi \\ \neg\phi & \neg\phi & \neg\phi \\ \phi & \phi & \neg\phi \end{pmatrix}$ | $\begin{pmatrix} \phi & \phi & \neg\phi \\ \neg\phi & \neg\phi & \neg\phi \\ \phi & \neg\phi & \phi \end{pmatrix}$ | $\begin{pmatrix} \phi & \phi & \neg\phi \\ \neg\phi & \neg\phi & \neg\phi \\ \neg\phi & \phi & \phi \end{pmatrix}$ | $\begin{pmatrix} \phi & \neg\phi & \phi \\ \phi & \phi & \neg\phi \\ \phi & \phi & \neg\phi \end{pmatrix}$ |



| $G$16 | $G$17 | $G$18 | $G$19 | $G$20 |
|---|---|---|---|---|
| $\begin{pmatrix} \phi & \neg\phi & \phi \\ \phi & \phi & \neg\phi \\ \phi & \neg\phi & \phi \end{pmatrix}$ | $\begin{pmatrix} \phi & \neg\phi & \phi \\ \phi & \neg\phi & \phi \\ \phi & \neg\phi & \phi \end{pmatrix}$ | $\begin{pmatrix} \phi & \neg\phi & \phi \\ \neg\phi & \phi & \phi \\ \phi & \neg\phi & \phi \end{pmatrix}$ | $\begin{pmatrix} \phi & \neg\phi & \phi \\ \neg\phi & \phi & \phi \\ \neg\phi & \phi & \phi \end{pmatrix}$ | $\begin{pmatrix} \phi & \neg\phi & \phi \\ \phi & \neg\phi & \neg\phi \\ \phi & \phi & \neg\phi \end{pmatrix}$ |



| $G$21 | $G$22 | $G$23 | $G$24 | $G$25 |
|---|---|---|---|---|
| $\begin{pmatrix} \phi & \neg\phi & \phi \\ \phi & \neg\phi & \neg\phi \\ \phi & \neg\phi & \phi \end{pmatrix}$ | $\begin{pmatrix} \phi & \neg\phi & \phi \\ \neg\phi & \neg\phi & \phi \\ \phi & \neg\phi & \phi \end{pmatrix}$ | $\begin{pmatrix} \phi & \neg\phi & \phi \\ \neg\phi & \neg\phi & \phi \\ \neg\phi & \phi & \phi \end{pmatrix}$ | $\begin{pmatrix} \phi & \neg\phi & \phi \\ \neg\phi & \neg\phi & \neg\phi \\ \phi & \phi & \neg\phi \end{pmatrix}$ | $\begin{pmatrix} \phi & \neg\phi & \phi \\ \neg\phi & \neg\phi & \neg\phi \\ \phi & \neg\phi & \phi \end{pmatrix}$ |



| $G$26 | $G$17-1 |
|---|---|
| $\begin{pmatrix} \phi & \neg\phi & \phi \\ \neg\phi & \neg\phi & \neg\phi \\ \neg\phi & \phi & \phi \end{pmatrix}$ | $\begin{pmatrix} \phi & \neg\phi & \phi \\ \phi & \neg\phi & \phi \\ \phi & \neg\phi & \phi \\ \neg\phi & \phi & \neg\phi \end{pmatrix}$ |

The exterior of the directed line, $D^-$, should be employed to refine the configurations only when the directed line coincides with $G$'s boundary. Otherwise, $D^-$ intersects with $\partial G$, $G^\circ$ and $G^-$ definitely. As a result, another case, $G$17-1 (Table 4), is distinguished corresponding to $G$17.

## 3 Topological relation predicts for directed lines

Topological relations provide a basis for common-sense representation and reasoning of geospatial knowledge and spatial query languages in GIS and spatial databases. They are always expressed by predicts in such applications. So we group all the cases of the topological relations of directed lines into a set of named predicts which is more suitable for humans.

We set up the topological relation predict set for directed lines using the "topological relation decision" tree[12]. Every internal node in the topological relation decision tree represents a conditional expression. For a certain topological situation, if the expression evaluates to "true" then the left branch is followed, otherwise the right branch is followed. This process is repeated until a leaf node is reached which indicates the relation predict this situation belongs to. When all leaf nodes are reached, the topological relation predict set for directed lines is obtained (Figure 2).

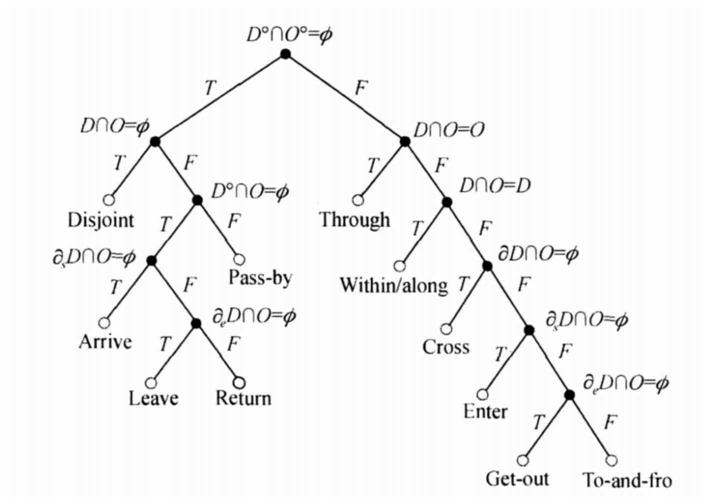



**Figure 2** The topological relation decision tree for directed lines.

**Definition 5.** The topological relation predict set for directed lines is an 11-turple: {disjoint, pass-by, arrive, leave, return, through, within/along, cross, enter, get-out, to-and-fro}.

The predict within can be assigned an alias along because along is more suitable for simple lines to represent the same relation cases.

It was proved exclusive and complete for the primary relations created by the topological relation decision tree[12], so the 11-predict set is also exclusive and complete. Each case of the topological relations of directed lines must belong to one and only one predict in the set.

**Definition 6.** Given a directed line $D$ and a simple geometry $O$, let $T$ and $F$ represent $\neg\phi$ and $\phi$ in $\boldsymbol{I}_d$ (or $\boldsymbol{M}_d$), respectively, and * represent either. Then the topological relation predicts are defined as follow:

$$D \text{ disjoint } O \Leftrightarrow D \cap O = \phi \Leftrightarrow \boldsymbol{I}_d(D,O) = (**TFFT**T),$$

$$D \text{ pass-by } O \Leftrightarrow D^\circ \cap O^\circ = \phi \wedge D^\circ \cap \partial O \neq \phi$$
$$\Leftrightarrow \boldsymbol{I}_d(D,O) = (***FT****),$$
$$D \text{ arrive } O \Leftrightarrow D^\circ \cap O = \phi \wedge \partial_s D \cap O = \phi \wedge \partial_e D \cap O \neq \phi$$
$$\Leftrightarrow \boldsymbol{I}_d(D,O) = (**TFFT**F),$$
$$D \text{ leave } O \Leftrightarrow D^\circ \cap O = \phi \wedge \partial_s D \cap O \neq \phi \wedge \partial_e D \cap O = \phi$$
$$\Leftrightarrow \boldsymbol{I}_d(D,O) = (**FFT***T),$$
$$D \text{ return } O \Leftrightarrow D^\circ \cap O = \phi \wedge \partial_s D \cap \partial O \neq \phi \wedge \partial_e D \cap \partial O \neq \phi$$
$$\Leftrightarrow \boldsymbol{I}_d(D,O) = (*T*FF**T*),$$
$$D \text{ through } O \Leftrightarrow D \cap O = O$$
$$\Leftrightarrow \boldsymbol{M}_d(D,O) = (F**TTTF**FFT),$$
$$D \text{ within } O \Leftrightarrow D \cap O = D \wedge D^\circ \cap O^\circ \neq \phi$$
$$\Leftrightarrow \boldsymbol{I}_d(D,O) = (T**T*FT**),$$
$$D \text{ cross } O \Leftrightarrow D^\circ \cap O^\circ \neq \phi \wedge \partial D \cap O = \phi$$
$$\Leftrightarrow \boldsymbol{I}_d(D,O) = (**TT****T),$$
$$D \text{ enter } O \Leftrightarrow D^\circ \cap O^\circ \neq \phi \wedge \partial_s D \cap O = \phi \wedge \partial_e D \cap O \neq \phi$$
$$\Leftrightarrow \boldsymbol{I}_d(D,O) = (**TT**T**),$$
$$D \text{ get-out } O \Leftrightarrow D^\circ \cap O^\circ \neq \phi \wedge \partial_s D \cap O \neq \phi \wedge \partial_e D \cap O = \phi$$
$$\Leftrightarrow \boldsymbol{I}_d(D,O) = (T**T****T),$$
$$D \text{ to-and-fro } O \Leftrightarrow D^\circ \cap O^\circ \neq \phi \wedge D \cap O \neq D \wedge \partial D \cap O \neq \phi$$
$$\Leftrightarrow \boldsymbol{I}_d(D,O) = (T**TTTT**).$$

The exterior of $D$ is not needed to examine these predicts except through. It is our conjecture that the 11-predict set is the smallest set capable of representing all relation cases and it is generally applicable to simple points, lines and polygons. In more detail, through is only suitable for simple lines; to-and-for only for simple polygon; pass-by, within/along, enter and get-out for simple lines and polygons.

## 4 Conceptual Neighborhoods of the topological relation predicts

The 11 topological relation predicts for directed lines are related each other. Their relationships, especially the similarities, can be expressed with Topological Conceptual Neighborhoods[13]. Two predicts $T_1$ and $T_2$ are topological conceptual neighbors if a continuous transformation can be performed between them without having to go through a third predict. The similarity of two predicts is computed by the topological distance which is calculated by counting the differences of the empty/non-empty entries of corresponding elements in the intersection matrix[14,15]. Then the conceptual neighborhood graph of the 11 predicts is derived, in which each predict is depicted as a node and the conceptual neighbors are the links between them (Figure 3).

What are actually calculated in this conceptual neighborhood graph are approximate average distances, because each predict groups some cases of topological relations whose matrixes may have a little differences. The dynamic and directional semantics of directed lines are also consid-

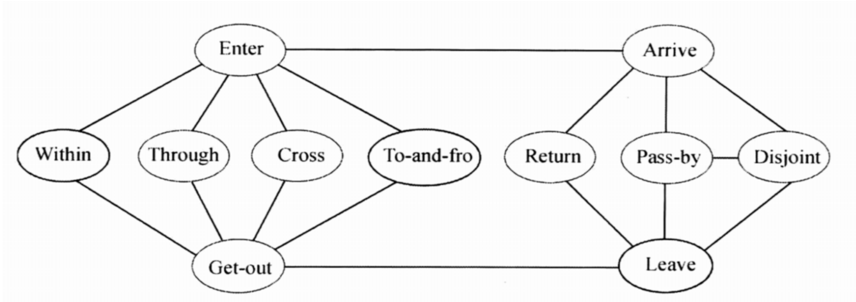



**Figure 3** The topological conceptual neighborhood graph of the relation predicts.

ered additionally. For example, the predict disjoint and arrive are considered conceptual neighbors because the transformation from disjoint to arrive expresses the process a moving object coming to the target without any intermediate relations, although they are more topologically distant in conventional computations.

The topological conceptual neighborhood can describe the successional motions when directed lines are used to model trajectories. For two sequent motion states, the path between the corresponding predicts in the conceptual neighborhood graph can express their transformation process. For example, a car's motion state sequence in Figure 4(a) is disjoint → arrive → enter → within → get-out → leave → disjoint; and in Figure 4(b) it is disjoint → arrive → pass-by → leave → disjoint. Each sequence corresponds to a path in the graph. The two paths can also be expressed separately by a single predict cross and pass-by as a whole. That means a predict describing a global state can be composed by some others describing local states. This characteristic is useful to represent the spatio-temporal states of moving objects.

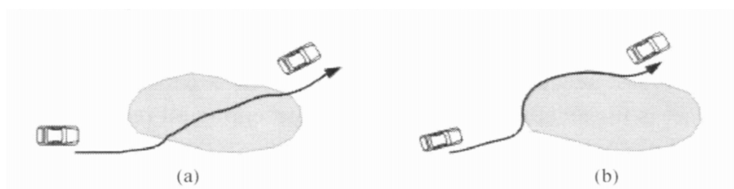



**Figure 4** Two example scenarios of trajectories described by predict sequences. (a) Disjoint → arrive → enter → within → get-out → leave → disjoint; (b) disjoint → arrive → pass-by → leave → disjoint.

## 5 Conclusions

This paper develops a new model based on point-set topology to represent the topological relations between directed lines and simple geometries. Through the intersections between different parts of directed lines and simple points, lines, and polygons, the main cases of the topological relations are identified. These cases are furthermore grouped into an exclusive and complete predict set, and the corresponding conceptual neighborhood graph are determined.

There are still shortages in this model to distinguish minor details of the topological relations. For example, if a directed line intersects with a simple line' interior, it cannot be distinguished whether the intersection is a point or a segment. And if they intersect more than one time, the order cannot be determined either. Future work will address the extension of dimension and order to improve this model.

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