# STAT 778: Homework 2

Tom Wallace

March 2, 2018

## 1 Introduction

This document describes a simulation study. The study consisted of generating statistics based on normal random variables and comparing the accuracy of these statistics to their theoretical expectation.

## 2 Methodology

Consider the normal random variable $X \sim N(-0.5, 2)$. For each simulation run, $n$ of such variables were randomly generated. Three different $n$ were used: $n = 50, 100, 200$. For each $n$, 1000 runs were conducted. On each run, the sample mean and sample variance were computed, as well as their respective standard errors and 95% confidence interval. These statistics were averaged over the 1000 runs conducted for each $n$. The empirical coverage probability also was computed.

## 3 Results

Simulation results are presented in Table 1. The point estimates closely match the true parameter values. The standard errors of the point estimates decrease as $n$ increases. The empirical coverage probabilities are very near 95%.

Table 1: Simulation results, average of 1000 runs

| $n$ | Parameter | True Value | Estimate | SE | 95% CI | CP(%) |
|---|---|---|---|---|---|---|
| 50 | $\mu$ | -0.5 | -0.487 | 0.199 | (-0.877, -0.097) | 95.6 |
| | $\sigma^2$ | 2 | 1.999 | 0.404 | (1.207, 2.790) | 95.0 |
| 100 | $\mu$ | -0.5 | -0.496 | 0.141 | (-0.771, -0.220) | 94.3 |
| | $\sigma^2$ | 2 | 1.990 | 0.283 | (1.435, 2.544) | 95.1 |
| 200 | $\mu$ | -0.5 | -0.498 | 0.100 | (-0.693, -0.302) | 94.9 |
| | $\sigma^2$ | 2 | 1.991 | 0.200 | (1.600, 2.382) | 94.4 |