Training Computers to Build LEGO Models using Reinforcement Learning with Deep Q-Network

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Abstract

This project shows a way to train a computer (agent) to build a LEGO model in a simulated LEGO environment according to a given LEGO model or a picture. Deep Q-learning [MKS⁺15, Dee] is an algorithm which combines deep learning and reinforcement learning and uses artificial neural networks to train the agent in a specific simulated environment. This project shows a way to build up a LEGO environment along with a suitable neural network. Then, by expanding the environment and upgrading the algorithm, the agent can finally build a 3D LEGO model according to a real-world picture.

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1 Introduction

1.1 Overview

As computer hardware has become much more powerful than that in the last decade, the high performance of computers gave rise to the popularity of machine learning, especially its uses in artificial intelligence. A group of scientists from DeepMind suggested a new way, which combined deep learning and reinforcement learning, to train an agent to learn to play Atari games [SB98] in 2015. After [SB98] had been published, many people tried to train computers to play different video games, such as games on Atari, Flappy Bird [Lau16, Lin16], etc., using this algorithm. However I believed that this algorithm had some other application, so I tried to create an agent to learn to build LEGO models.

This project shows a way to train an agent to learn to rig up a LEGO model in a simulated environment using as input only the rendered image of the model which the agent has currently built and a value given by a pre-defined reward function which evaluates the gap between current model and the target model. Initially, the agent is offered the exact bricks required to build the target model. This is in a similar way where people purchase a box of a LEGO model from a store, where the bricks in the box are exactly the bricks of which the model consists, and pour the bricks onto a table and then assemble them piece by piece. The agent may choose a brick to move one step at each time.

In the later stage, the agent was upgraded and was able to pick a best brick from a large pool of bricks with different colours and put it to the best place at each step. Compared to the previous procedure, this is more similar to the process where a LEGO expert who has enough LEGO bricks and uses them to build a model from scratch.

I also added some simple physics into the LEGO environment, and observed if the agent can also learn to rig up models in an environment which is closer to the real world.

Here follows a rough outline of how the agent works:

- 1. Initially, we have an untrained Q-network, a target model, a current model and a rendered image for each model.
- 2. The agent chooses the best action according to the Q values given by the Q-network.
- The agent inputs the chosen action to the LEGO environment, and get a model after that action along with the rendered image of this model.
- 4. The agent compares the model we get and the target model and gets a reward value from the reward function.
- 5. Put the reward value and other useful information into the replay pool.
- 6. If there are enough replays in the replay pool, update the Q-network using some randomly picked replays from the replay pool.
- 7. Repeat step 2 to 6 until the agent is able to build the target model or a pre-set maximum number of steps has been reached.

1.2 Purpose

The purpose of this project is to design an artificial intelligence and a simulated LEGO environment, improve the reward function and upgrade the LEGO environment so that it is more similar to the way where humans play with LEGO.

Everyone may easily get a real-world picture of an object, but it's hard for a person who is lacking designing skills to design a LEGO model using some software such as LEGO DIGITAL DESIGNER. The project also aims to find such a way that the agent would be able to build the LEGO model according to a real world picture of an object. In order to build a 3D model, the picture might be captured by a 3D camera (which gives the picture a depth channel), or the system might require a set of multi-view pictures as its input. There are still some imperfections in the system, which can be improved in the future.

1.3 Project Road Map

This project aims to build up a complete system consisting of main codes, a neural network, a simulated LEGO environment and a rendering program. As the goal of this project is very ambitious and the whole system is complicated, I built the system part by part and upgraded some modules to improve the performance.

I will now explain the whole system in the following order.

Firstly, in the 'Background' section, I will define some symbols which will be used in the rest part of the report I will also introduce some essential background knowledge of reinforcement learning as it is not fully covered in the machine learning course. Then, I will explain deep reinforcement learning, the most important algorithm in the whole system.

In the 'Design' section, I will give a graphical overview of the entire system which reflects the inter-communication between each module and the outline of the algorithm. Then I will explain how the whole system works, how the neural network has been constructed, and how the individual modules work, the roles they play and how do they interact with each other. Moreover, if some modules have been upgraded, I will explain the reason why I upgraded them and the improvement after the upgrade.

Having explained the whole system, I will show some testing results and analyse the results to show the actual improvement of the system. Finally, I will draw conclusions from the results and the analysis that I have done, and then reveal some imperfections which can be further improved.

2 Background

2.1 Definition

(Here I will include some symbols which I use in the report. Not finished.) action space

action
state space
state
value in rl
optimal value in rl
policy
reward
reward for an action
overall reward
Q values

2.2 Reinforcement Learning

In machine learning, reinforcement learning is an area inspired by behaviourist psychology [Tho98] and neuroscience [SDM97]. It concerns how agents may take actions in an environment corresponding to the responses of the environment so as to maximise the total reward [SB98]. The whole process mimics how animals learn to react to an environment.

A reinforcement learning model consists of: [SB98]

- 1. a set of environment states S;
- 2. a set of actions A;
- 3. rules of transitioning between states;
- 4. rules that determine the scalar immediate reward of a transition; and
- 5. rules that describe what the agent observes.

Assume the problem is episodic. An episode starts when the problem starts in an initial state and ends when some terminal states is reached. We also assume each episode is finite. In other words, whichever the actions are taken by the agent, the problem always terminates at some point. We define a policy as a mapping to some probability distribution over all possible actions. Hence, the expectation of the total reward for any policy is well-defined.

The expected return ρ^{π} to policy π is:

$$\rho^{\pi} = \mathbb{E}[R|\pi]$$

where the return value R is defined by:

$$R = \sum_{t=0}^{N-1} r_{t+1}$$

where r_{t+1} is the immediate reward after the t-th transition. Here, N denotes the time step when a terminal state is reached. [SB98] Sometimes, in order to control the number of steps, the reward at each step can be multiplied by a discount factor γ , so R can also be defined by:

$$R = \sum_{t=0}^{N-1} \gamma r_{t+1}$$

There are many algorithms for reinforcement learning [SB98]. In this project, Value Function approaches is related to Deep Q-Learning algorithm.

Value function approaches aim to find a policy which maximises the overall reward value.

Let V^* be the optimality and it is defined as:

$$V^*(s) = \max_{\pi} V^{\pi}(s)$$

where V^{π} denotes the value of a policy π :

$$V^{\pi}(s) = \mathbb{E}[R|s,\pi]$$

where R is the random return with respect to the policy π from the initial state s.

Moreover, the state-values can be updated to action-values, and the algorithm becomes Q-Learning. [Wat89] And the Q-values are defined as: [WD92]

$$Q^{\pi}(s, a) = r_s(a) + \gamma \sum_{a'} \mathbb{E}[s, a' | \pi] V^{\pi}(a').$$

2.3 Deep Learning

Deep Learning is a class of machine learning in which artificial neural networks (also known as neural networks) are used to extract and transform features from raw data. A neural networks is a flow of many layers, and each layer consists a lot of non-linear processing units which are also called as nodes or neurons. Each neural network has an input layer, an output layer and one or more hidden layers. [DY14, LBH15, Sch]

In a neural network, neurons in the input layer receive the input data and output to next layer. Neurons in a hidden layer receive the output from the previous layer, and output values after some linear or non-linear transformations, according to parameters of neurons, to the next layer. And finally, the output layer receives values and output to the user. [LBH15, Sch15, Sch, DY14, Ben09]

In this project, a deep neural network is used to simulate a function which returns Q values [MKS+15], so the network is also called Q-network.

2.4 Deep Reinforcement Learning, Deep Q-Learning and Deep Q-Networks

Deep Reinforcement Learning (also known as Deep Q-learning) is an machine learning algorithm which was posted by DeepMind on their website in 2013. [Sil16] DeepMind then published an article about deep reinforcement learning in Nature in 2015. [MKS⁺15] In this article, they described an algorithm called Deep Q-network implemented deep reinforcement learning.

Deep Reinforcement Learning is a combination of Deep Learning and Reinforcement Learning, which integrates both their features. DeepMind wanted to use Deep Reinforcement Learning to create artificial agents which achieve a similar level of performance and generality. The Agents learn by themselves to achieve successful strategies leading to a greatest long-term rewards (feature of reinforcement learning) and learn their own knowledge directly from raw inputs, such as vision (feature of deep learning). [Dee]

In Deep Q-learning, we are not concerning immediate reward values for each transition, but we are trying to maximise long-term reward values. There is also a set of Q-values for each state-action pair. Therefore a Deep Q-learning model consists of:

- 1. a set of environment states S;
- 2. a set of actions A;
- 3. a set of Q-values Q;
- 4. rules of transitioning between states:
- 5. rules that determine the scalar immediate reward of a state from the nearest final state; and
- 6. rules that describe what the agent observes.

The agent is able to do some action in the environment; the agent can perceive any feedback from the environment; and most importantly, the agent has a Q-network, which takes the feedback as an input and outputs values for different actions. The agent can then take the best action accordingly.

The whole process simulates humans (or other animals). When humans are in an environment, humans are able to do some action in the environment; humans can perceive any feedback from the environment by means of senses of sight, hearing, touch (maybe taste and smell); and humans have brains which collect the information from sense organs and tell bodies to take some reactions.

With this in mind, we may consider the problem in which the agent interacts with the environment. The agent receives an observation from the environment, selects an action, receives a reward from the environment, then receives the next observation and keeps doing these. The goal of the agent is to select actions at each state so that it can maximise cumulative future reward. [MKS⁺15] We achieve this by using the Q-network. The Q-network is a deep convolutional neural network which approximates the optimal action-value function

$$Q^*(s, a) = \max_{\pi} \mathbb{E}[r_t + \gamma r_{t+1} + \gamma^2 r_{t+2} + ... | s_t = s, a_t = a, \pi]$$

which maximises the overall rewards discounted by γ at each time step t. This is achieved by a behaviour policy $\pi = P(a|s)$, after an observation s and taking an action a.[MKS⁺15]

The agent has a limited memory to store replays (experiences). Each replay is a tuple $e_t = (s_t, a_t, r_t, s_{t+1})$ where s_t is the state at time t, a_t is the action taken after s_t , r_t is the reward after a_t had been taken, and s_{t+1} is the state after a_t had been taken from s_t . All the replays e_1 to e_t are stored in a replay pool D_t . When the number of replays in the pool reaches a threshold, the agent can uniformly pick one or more of these replays, wrap them as a mini-batch and train the Q-network with the mini-batch.

During learning, Q-learning updates are applied on the mini-batch. For each iteration i the following loss function is used to update the Q-network:

$$L_{i}(\theta_{i}) = \mathbb{E}_{(s,a,r,s')}[(r + \gamma \max_{a'} Q(s',a';\theta_{i}^{-}) - Q(s,a;\theta_{i}))^{2}]$$

where γ is the discount factor mentioned before, θ_i are the parameters of the Q-network at iteration i and θ_i^- are the parameters used to compute the target at iteration i. [MKS⁺15]

The overall DQN algorithm is shown by Algorithm 1.

```
Algorithm 1 Deep Q-learning with experience replay (Quoted from [MKS+15])
```

```
1: Initialise replay memory D to capacity N
 2: Initialise action-value function Q with random weights \theta
 3: Initialise target action-value function \hat{Q} with weights \theta^- = \theta
    for episode = 1, M do
          Initialise sequence s_1 = x_1 and preprocessed sequence \phi_1 = \phi(s_1)
 5:
          for t = 1, T do
 6:
              With probability \epsilon select a random action a_t
 7:
              otherwise select a_t = \operatorname{argmax}_a Q(\phi(s_t), a; \theta)
 8:
              Execute action a_t in emulator and observe reward r_t and image x_{t+1}
 9:
10:
              Set s_{t+1} = s_t, a_t, x_{t+1} and preprocess \phi_{t+1} = \phi(s_{t+1})
              Store transition (\phi_t, a_t, r_t, \phi_{t+1}) in D
11:
              Sample random mini-batch of transitions (\phi_j, a_j, r_j, \phi_{j+1}) from D
12:
              Set y_j = \begin{cases} r_j & \text{episode terminates at step} \\ j+1 \\ r_j + \gamma \max_{a'} \hat{Q}(\phi_{j+1}, a'; \theta^-) & \text{otherwise} \end{cases}
Perform a gradient descent step on (y_j - Q(\phi_j, a_j; \theta))^2 with respect
                                                                       episode terminates at step
13:
14:
              to the network parameters \theta
              Every C steps reset \hat{Q} = Q
15:
          end for
16:
17: end for
```

2.5 LEGO Environment

The LEGO Group offers a virtual building software, LEGO Digital Designer[ldd]. The software supports a virtual way to design, build and observe LEGO models, and store the LEGO models into <code>.lxfml</code> or <code>.lxf</code> format files. This format is essentially an xml format.

However, this software can't be running on command line or any Linux system. As a result, I should design a virtual LEGO environment by myself. Thanks to the format, we can parse the xml tree from the <code>.lxfml</code> or <code>.lxf</code> files and extract some other useful data from the software. The <code>.lxfml</code> or <code>.lxf</code> files store the information of every brick, including a type (shape) of the brick, a texture of the brick and a linear translation matrix of the brick. With all those information, I built up a LEGO environment in LUA, which offers all the command needed in the project and finally encapsulated in a class so the agent can communicate with the environment.

Even though the agent cannot communicate directly with LEGO Digital Designer, we can still use the software to open .lxfml or .lxf files, so that we can observe models constructed by the agent and do some analysis.

I will explain in detail how I designed the environment in the Design section.

2.6 Others

I will also introduce some other tools or packages which I used in the project. (All the tools and packages are free and open-source.)

2.6.1 Blender Python

Blender Python (also known as bpy) is a package for Python. The package includes most tools in Blender. Thanks to bpy, we can use Blender on command line by using Python scripts.

In this project, I used bpy to render models into images and depth maps.

2.6.2 Optical Flow

Optical flow algorithm is commonly used in computer vision. It measures the movements of objects in two images. I used optical flow algorithm as a part of the reward function since it can measure the movements of bricks and also give a scalar magnitude for each movement. I will explain this in detail about how I used optical flow algorithm to evaluate the similarity of two images and finally return a reward value.

Although there are several different optical flow algorithms, I chose one particular algorithm, TV-L1 [PMLF13], as it worked well in early tests. This algorithm is available in the OpenCV library.

As this algorithm is not an important part in my project, I will not discuss the principle of the algorithm in this report.

3 Requirements

4 Design

In this section, I will explain how the whole system works, how each module (or class) works and how the modules (or classes) interact with others. I will also explain how I expanded each module and upgraded the whole algorithm.

There are several stages in my project. Initially the environment only supported 2D, so the agent could only move bricks in a 2D plane. Then I expanded the renderer so that the environment supported 3D, and correspondingly, the agent might move bricks in a 3D space. I also upgraded the environment so that some simple physical laws are implemented. After that, I tried different reward functions. With some function based on image comparison, we could input a real-world image to the system as a target, rather than a LEGO model. Then I upgraded the whole system and the algorithm, so that the agent could play with more bricks.

In the end, the system can train an agent to build a LEGO model according to a real-life picture. I will show you a final achievement in the 'Results' section.

4.1 Overall

Figure 1 shows the overall system. As the same as most artificial intelligent systems, the whole system consists of two parts: an agent and an environment.

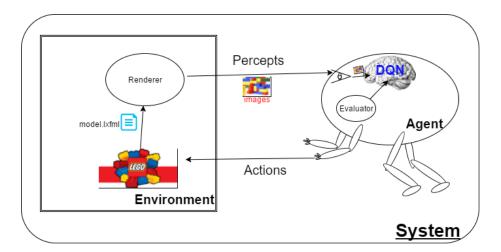


Figure 1: Overall System.

The environment consists of a simulated LEGO environment and a renderer. The simulated LEGO environment offers the agent actions. According to the action commands sent from the agent, LEGO models can be rigged up in the simulated LEGO environment. The simulated LEGO environment can also import a model from a <code>.lxfml</code> or <code>.lxf</code> file, or export the current model to a <code>.lxfml</code> or <code>.lxf</code> file. The renderer can then input a <code>.lxfml</code> or <code>.lxf</code> file, and then render the model into an image which can be perceived by the agent.

The agent consists of a reward function, a deep Q-network and a replay (experience) pool. The reward function evaluates the difference between the

model which the agent has already rigged up and the target model. The deep Q-network is the core of the whole algorithm, which tells the agent Q-values of each action at each state. Replays (experiences) in the pool can be randomly picked to train the network.

The inter-communication between modules can be illustrated by Figure 2.

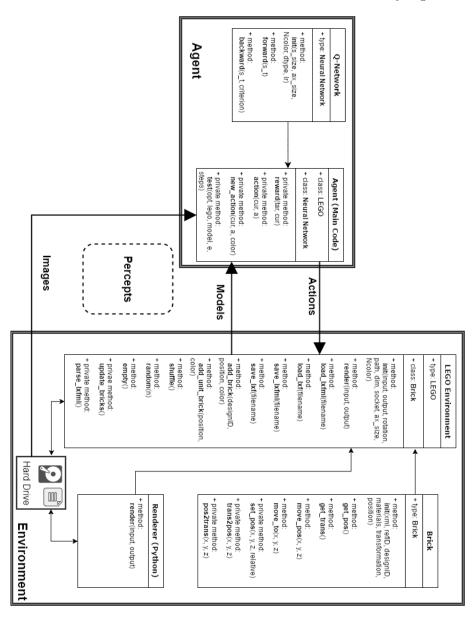


Figure 2: Inter-Communication between modules.

The communication between the agent and the environment is based on both memory and hard drive. Model files (.lxfml and .lxf files) and rendered images are stored on the hard drive.

4.2 Agent

end for

4.2.1 Early Stage

In the early stage of the project, the LEGO environment was only in 2D. The agent could only move bricks in a 2D plane. All the interrelated images in this stage have 3 channels (RGB).

The main code of the agent is shown by Algorithm 2 and the corresponding testing procedure is shown by Algorithm 3.

```
Algorithm 2 LEGO with deep Q-learning (at the early stage)
  Require hyper-parameters
  Initialise replay memory D to capacity N
  Initialise action-value function Q with random weights \theta
  Initialise target action-value function \hat{Q} with weights \theta^- = \theta
  Input a target model M_{target} to the agent and the environment
  Initialise a neural network Net with random parameters
  Initialise Q-values for each state-action pair
  for episode = 1, N\_episode do
      Generate an initial model M_1 in the environment
      The agent gets the initial model M_1 from the environment
      Acquire an image x_1 of M_1 from the environment
      Initialise sequence s_1 = M_1
      for t = 1, N do
          Pass x_t to Net to get Q-values Q
          With probability \epsilon select a random action a_t
          otherwise select a_t = \operatorname{argmax}_a Q(\phi(s_t), a; \theta)
          Translate the selected action a_t to a command (brick, axis, direction)
          Pass the command (brick, axis, direction) to the environment
          Acquire the next model M_{t+1} and a corresponding image x_{t+1}
          Pass M_{target} and M_{t+1} to the reward function
          Get a reward r_t and a terminating flag terminate_{t+1}
          Store transition (x_t, a_t, r_t, x_{t+1}) in D
          Sample random mini-batch of transitions (x_t, a_t, r_t, x_{t+1}) from D
         Set y_j = \begin{cases} r_j & \text{episode to} \\ j+1 \\ r_j + \gamma \max_{a'} \hat{Q}(x_{j+1}, a'; \theta^-) & \text{otherwise} \end{cases}
                                                        episode terminates at step
          Perform a gradient descent step on (y_j - Q(x_j, a_j; \theta))^2 with respect
          to the network parameters \theta
          Every C steps reset \hat{Q} \leftarrow Q
          Store statistics
          if terminate == 0 then
             Break the loop
          end if
      end for
      Every N\_test episodes, \mathbf{Test}(M_{target}, Net)
      Store statistics
```

Initially, a target model is required from the user. Then $N_{-}episode$ episodes

Algorithm 3 Testing procedure (at the early stage)

```
procedure Test(M_{target}, Net)
   Pass the target model M_{target} to the environment
   Generate an initial model M_1 in the environment
   The agent gets the initial model M_1 from the environment
   Acquire an image x_1 of M_1 from the environment
   for t = 1, N do
       Pass x_t to Net to get Q-values Q
       Select a_t = \operatorname{argmax}_a Q(\phi(s_t), a; \theta)
       Translate the selected action a_t to a command (brick, axis, direction)
       Passes the command (brick, axis, direction) to the environment
       Acquire the next model M_{t+1} and a corresponding image x_{t+1}
       Pass M_t arget and M_{t+1} to the reward function
       Get a reward r_t and a terminating flag terminate
       Store the statistics
       if terminate == 0 then
          Break the loop
       end if
   end for
end procedure
```

will be processed.

In each episode, the environment first generates a initial model M_1 which consists of every brick in the target model in random positions, and passes it to the agent. The agent then calls the **render** method and reads an rendered image x_1 of M_1 from the hard drive.

Then the agent will repeat the following processes in N steps or the agent finishes to build the model.

At each time step t, the agent first passes the rendered image x_t to the network, and gets Q-values Q. It then select the action with the highest Q-value in the action space, and translates it to a command (brick, axis, direction). Each command stands for 'moving a brick along either x-axis or y-axis in a direction by a unit distance'. The action space A has a cardinality of $|Bricks| \times 2 \times 2$. This command will be inputted into the environment by the $\mathbf{move_pos}$ method. The environment changes the model to M_{t+1} correspondingly, and the agent can get the model M_{t+1} and the rendered image x_{t+1} . The agent passes the target model M_{target} and the M_{t+1} to the reward function and get a reward r_t and a terminating flag $terminate_{t+1}$.

The reward function here calculates the difference in distance for each identical brick in the target model M_{target} and the next model M_{t+1} , and returns the negative sum of distances as the reward value r_t and a terminating flag $terminate_{t+1}$. The reward value is negative because we want the reward value becomes negatively large if the difference in distances is large.

$$reward(M_{target}, M_{t+1}) = -\sum_{brick \in M_{target}} ||coor_{M_{target}}(brick) - coor'_{M_{t+1}}(brick)||$$

$$terminate_{t+1} = (M_{target} == M_{t+1})$$

Then the agent stores the experience (x_t, a_t, r_t, x_{t+1}) in D. If there are enough experiences in D, the agent uniformly draws some experiences and performs a gradient descent step to the network.

Then the loop will continue if the agent was unable to build the target model. If the agent managed to do so, the next episode will start.

For every N_{test} episodes, the agent may test itself using the currently trained network. In order to increase time efficiency, I set N_{test} to a high value.

4.2.2 3D

After the early stage, I expand the render so a 3D environment is supported. The main code doesn't change too much except that the action space A is expanded and the cardinality of A becomes $|Bricks| \times 3 \times 2$, since the agent may move a brick along the z-axis. The related images are expanded to 4 channels (RGB and a depth channel), so the agent can detect the relative depth of each pixel in an image.

4.2.3 Reward Function

The initial reward function was limited as it takes two models as arguments. The whole system was therefore restricted so that the agent could only imitate a LEGO model. In order to broke through, I tried some different reward functions which were based on image comparison. The final reward function I used was TV-L1[PMLF13].

TV-L1 is an optical flow algorithm which detects movements of objects in two images. I found it working well in finding the movements of bricks. The algorithm returns a pair of matrices (magnitude, angle), where magnitude is a matrix which, for each pixel, gives a magnitude of the movement between 0 and 1, and angle is a matrix which, for each pixel, gives a vector representing the direction of movement. The reward function sums up the matrix magnitude and return the sum as the reward value.

Now the agent has the ability of comparing a real-world image and a rendered image of a LEGO model.

Even though this algorithm works well, we still need to explore and find some other algorithms as the reward function which works better than this algorithm, as finding a good reward function is a major task in reinforcement learning. Due to the time limitation of this project, I couldn't try many algorithms. However, I have some ideas in mind and I will show them in the 'Future Works' section.

4.3 Deep Q-Network

In the first stage of the project, the Q-network is much simpler than it in the later stage. The network takes an image with 4 channels as an input, and outputs Q-values for each action. Between the input layer and the output layer, there are 5 hidden layers.

In the later stage of the project, in addition to the image, the network also takes an one-hot encoded value, representing the index of the colour of a brick to be added into the model. For example, if the agent tries to add a red brick where the index of red is 2, the input is an image of current state and a one-hot encoded value 010...0. The output of the network also changes. This time, there are three output layers. Each layer represents the coordinate on an axis, so the three layers together give the coordinates which a brick of the colour should be put on.

4.4 LEGO Environment

4.5 Renderer

Since I am using Blender to render and Blender only supports Python, the renderer is written in Python and the other modules can call the renderer through command line and transfer files using hard drive or network socket.

In order to save memory, and for convenience, I extracted every brick's 3D model from the LEGO digital designer and saved them as .obj files on hard drive. I also stored some colour information and texture information for each kind of simple bricks into a table. When the renderer needs to render a specific brick with a specific colour or texture, it can import the corresponding .obj file from the disk and fetch the texture from the table.

- 5 Testing
- 6 Results
- 7 Conclusions
- 8 Future Works

9 Acknowledgements

References

- [Ben09] Y. Bengio. Learning deep architectures for AI. Foundations and Trends® in Machine Learning, 2(1):1–127, 2009.
- [Dee] DeepMind. Dqn. Website. https://deepmind.com/research/dqn/.
- [DY14] Li Deng and Dong Yu. Deep learning: Methods and applications. Technical report, Microsoft, May 2014. https://www.microsoft.com/en-us/research/publication/deep-learning-methods-and-applications/.
- [GBC16] Ian Goodfellow, Yoshua Bengio, and Aaron Courville. *Deep Learning*. MIT Press, 2016. http://www.deeplearningbook.org.
- [Lau16] Ben Lau. Using keras and deep q-network to play flappybird. Website, 07 2016. https://yanpanlau.github.io/2016/07/10/FlappyBird-Keras.html.
- [LBH15] Yann LeCun, Yoshua Bengio, and Geoffrey Hinton. Deep learning. *Nature*, 521(7553):436–444, may 2015.
- [ldd] Lego digital designer. Website. http://ldd.lego.com/.
- [Lin16] Yenchen Lin. Using deep q-network to learn how to play flappy bird. Website, 03 2016. https://github.com/yenchenlin/ DeepLearningFlappyBird.
- [MKS+15] Volodymyr Mnih, Koray Kavukcuoglu, David Silver, Andrei A. Rusu, Joel Veness, Marc G. Bellemare, Alex Graves, Martin Riedmiller, Andreas K. Fidjeland, Georg Ostrovski, Stig Petersen, Charles Beattie, Amir Sadik, Ioannis Antonoglou, Helen King, Dharshan Kumaran, Daan Wierstra, Shane Legg, and Demis Hassabis. Human-level control through deep reinforcement learning. Nature, 518(7540):529–533, feb 2015.
- [PMLF13] Javier Sánchez Pérez, Enric Meinhardt-Llopis, and Gabriele Facciolo. TV-l1 optical flow estimation. *Image Processing On Line*, 3:137–150, jul 2013. http://www.ipol.im/pub/art/2013/26/.

- [SB98] Richard S. Sutton and Andrew G. Barto. Reinforcement Learning: An Introduction. The MIT Press, 1998.
- [Sch] Juergen Schmidhuber. Deep learning in neural networks: An overview.
- [Sch15] Juergen Schmidhuber. Deep learning. Scholarpedia, 10(11):32832, 2015.
- [SDM97] W. Schultz, P. Dayan, and P. R. Montague. A neural substrate of prediction and reward. *Science*, 275(5306):1593–1599, mar 1997.
- [Sil16] David Silver. Deep reinforcement learning. Blog, 06 2016. https://deepmind.com/blog/deep-reinforcement-learning/.
- [Tho98] E.L. Thorndike. Animal Intelligence: An Experimental Study of the Associative Processes in Animals. Number no. 4 in Animal Intelligence: An Experimental Study of the Associative Processes in Animals. Macmillan, 1898.
- [vHGS15] Hado van Hasselt, Arthur Guez, and David Silver. Deep reinforcement learning with double q-learning. arXiv, 2015.
- [Wat89] Christopher John Cornish Hellaby Watkins. Learning from Delayed Rewards. PhD thesis, King's College, Cambridge, UK, May 1989.
- [WD92] CHRISTOPHER J.C.H. WATKINS and PETER DAYAN. Technical note:q-learning. Technical report, Kluwer Academic Publishers, 1992.