LIST OF PUBLICATIONS AND PREPRINTS

THOMAS YAHL

The following is a current list of my publications and preprints, presented in reverse chronological order.

1. *Computing Galois groups of finite Fano problems.* 2022. arXiv:2209.07010. In preparation.

Abstract. A Fano problem consists of enumerating linear spaces of a fixed dimension on a variety, generalizing the classical problem of 27 lines on a cubic surface. Those Fano problems with finitely many linear spaces have an associated Galois group that acts on these linear spaces and controls the complexity of computing them in coordinates via radicals. Galois groups of Fano problems were first studied by Jordan, who considered the Galois group of the problem of 27 lines on a cubic surface. Recently, Hashimoto and Kadets nearly classified all Galois groups of Fano problems by determining them in a special case and by showing that all other Fano problems have Galois group containing the alternating group. We use computational tools to prove that several Fano problems of moderate size have Galois group equal to the symmetric group, each of which were previously unknown.

2. *Real solutions to systems of polynomial equations in Macaulay2.* 2022. arXiv:2208.05576. Submitted.

Abstract. The *Macaulay2* package `RealRoots` provides symbolic methods to study real solutions to systems of polynomial equations. It updates and expands an earlier package developed by Grayson and Sottile in 1999. We provide mathematical background and descriptions of the `RealRoots` package, giving examples which illustrate some of its implemented methods. We also prove a general version of Sylvester's Theorem whose statement and proof we could not find in the literature.

3. *Galois groups in enumerative geometry and applications.* 2021. arXiv:2108.07905. Being revised for publication.

Abstract. As Jordan observed in 1870, just as univariate polynomials have Galois groups, so do problems in enumerative geometry. Despite this pedigree, the study of Galois groups in enumerative geometry was dormant for a century, with a systematic study only occuring in the past 15 years. We discuss the current directions of this study, including open problems and conjectures.

4. *Polyhedral homotopies in Cox coordinates.* 2020. arXiv:2012.04255. Submitted.

Abstract. We introduce the Cox homotopy algorithm for solving a sparse system of polynomial equations on a compact toric variety X_Σ. The algorithm lends its name from a construction, described by Cox, of X_Σ as a GIT quotient $X_\Sigma = (\mathbb{C}^k \setminus Z)//G$ of a quasi-affine variety by the action of a reductive group. Our algorithm tracks paths in the total coordinate space $\mathbb{C}^k$ of X_Σ and can be seen as a homogeneous version of the standard polyhedral homotopy, which works on the dense torus of X_Σ. It furthermore generalizes the commonly used path tracking algorithms in (multi)projective spaces in that it tracks a set of homogeneous coordinates contained in the G-orbit corresponding to each solution. The Cox homotopy combines the advantages of polyhedral homotopies and (multi)homogeneous homotopies, tracking only mixed volume many solutions and providing an elegant way to deal with solutions on or near the special divisors of X_Σ. In addition, the strategy may help to understand the deficiency of the root count for certain families of systems with respect to the BKK bound.

5. *Decomposable sparse polynomial systems.* 2020. arXiv:2006.03154. Journal of Software for Algebra and Geometry Vol. 11 (2021), 53-59.

Abstract. The *Macaulay2* package `DecomposableSparseSystems` implements methods for studying and numerically solving decomposable sparse polynomial systems. We describe the structure of decomposable sparse systems and explain how the methods in this package may be used to exploit this structure, with examples.

6. *Solving decomposable sparse systems.* 2020. arXiv:2001.04228. Numerical Algorithms **88**, 453-474 (2021).

Abstract. Améndola et al. proposed a method for solving systems of polynomial equations lying in a family which exploits a recursive decomposition into smaller systems. A family of systems admits such a decomposition if and only if the corresponding Galois group is imprimitive. When the Galois group is imprimitive we consider the problem of computing an explicit decomposition. A consequence of Esterov's classification of sparse polynomial systems with imprimitive Galois groups is that this decomposition is obtained by inspection. This leads to a recursive algorithm to solve decomposable sparse systems, which we present and give evidence for its efficiency.

T. Yahl, Department of Mathematics, Texas A&M University, College Station, Texas 77843, USA

Email address: `thomasjyahl@math.tamu.edu`

URL: `http://www.math.tamu.edu/~thomasjyahl`