Some questions about MI

Jialin

July 2022

1 how should I get h(f,r)

The equation in paper [1] of mutual information I:

Estimations for the marginal and joint image intensity distributions $p_{F,\alpha(f)}$, $p_{R,\alpha}(r)$, and $p_{FR,\alpha}(f,r)$ are obtained by normalization of $h_\alpha(f,r)$

$$p_{FR,\alpha}(f,r) = \frac{h_\alpha(f,r)}{\sum_{f,r} h_\alpha(f,r)} \tag{9}$$

$$p_{F,\alpha}(f) = \sum_r p_{FR,\alpha}(f,r) \tag{10}$$

$$p_{R,\alpha}(r) = \sum_f p_{FR,\alpha}(f,r). \tag{11}$$

The MI registration criterion $I(\alpha)$ is then evaluated by

$$I(\alpha) = \sum_{f,r} p_{FR,\alpha}(f,r) \log_2 \frac{p_{FR,\alpha}(f,r)}{p_{F,\alpha}(f)\, p_{R,\alpha}(r)} \tag{12}$$

and the optimal registration parameter α^* is found from

$$\alpha^* = \arg \max_\alpha I(\alpha). \tag{13}$$

NN

Figure 1: equations of MI

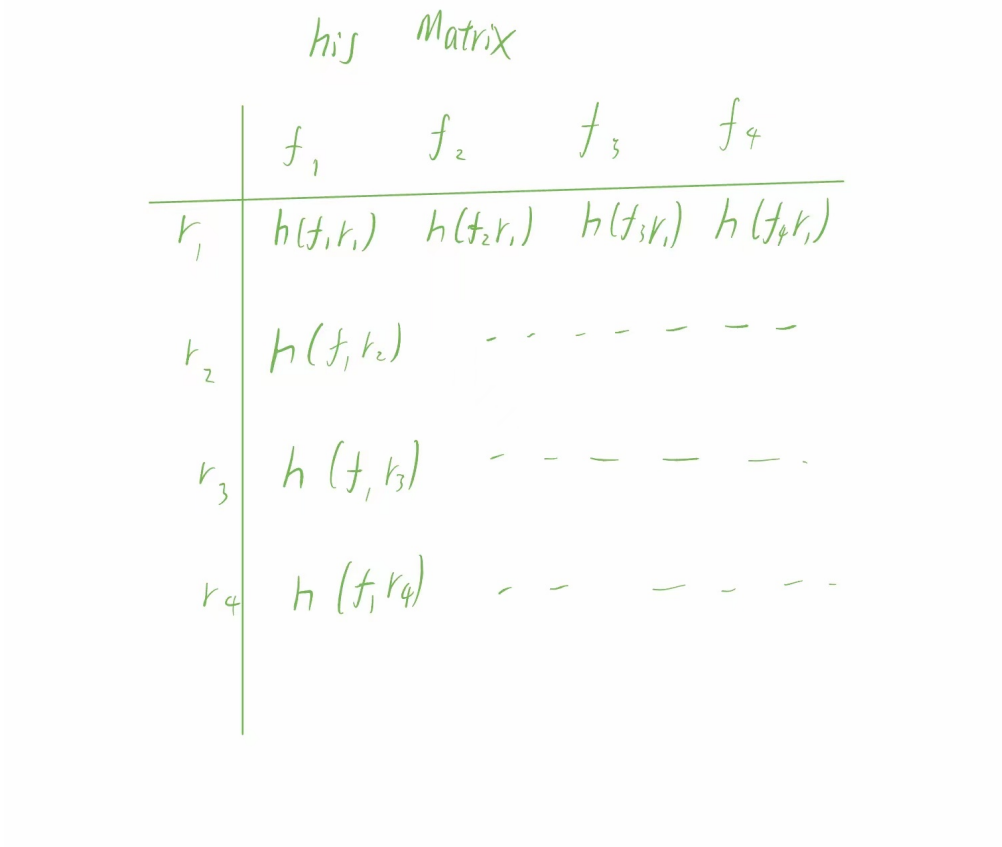

Figure 2: his matrix

Since the last meeting, I understand the way I construct "his matrix" is wrong. I wrote down what Professor Simon said and tried to link them up:

- " you should build this his graph from lots of pre-segmented images"
- "The performance of semantic segmentation network is precisely P_{FR}, P_F, P_R
- "$P_F R(f, r)$ is what's the probability that the same pixel will give you f in pixel F, and r in pixel R."
- "the his matrix, specify the performance of the classifier over a large amount of data. You are trying to come up with a model of how well the classifier works. "
- "it's all about what the classifier does over a large number of images. And for this particular image we are interested in, we then say, given this particular case, I got this pixel value here and that pixel value there, what's the probability I would observe this?"
- "build up his matrix from kitti 360 data"
- "Take a semantic segmentation network, run kitti360 through that network"
- "What's the probability that it will classify the same object to the same class twice, what's the probability its classification is correct"
- "do we care about the error in classification, or do we just care about whether the classification is the same"

I understand most of these, but I just don't know how should I get $h(f,r)$ values from the performance of semantic segmentation network. "his matrix" is composed of h(f,r) values.

2 Equation 12 may be wrong

Some properties of Mutual Information

TABLE I
SOME PROPERTIES OF MUTUAL INFORMATION

Non-negativity:	$I(A,B) \geq 0$		
Independence:	$I(A,B) = 0 \Leftrightarrow p_{AB}(a,b) = p_A(a).p_B(b)$		
Symmetry:	$I(A,B) = I(B,A)$		
Self information:	$I(A,A) = H(A)$		
Boundedness:	$I(A,B)$	$\leq$	$\min(H(A), H(B))$
		$\leq$	$(H(A) + H(B))/2$
		$\leq$	$\max(H(A), H(B))$
		$\leq$	$H(A,B)$
		$\leq$	$H(A) + H(B)$
Data processing:	$I(A,B)$	$\geq$	$I(A, T(B))$

When we apply $I \geq 0$ to euqation 12, it would give us

$$log \frac{P_{FR,\alpha}(f,r)}{P_{F,\alpha}(f) P_{R,\alpha}(r)} \geq 0$$

$$P_{FR,\alpha}(f,r) \geq P_{F,\alpha}(f) P_{R,\alpha}(r)$$

Substitute in equation 9, 10, 11 ($N = \sum_{f,r} h_\alpha(f,r)$)

$$\frac{h_\alpha(f,r)}{N} \geq \frac{\sum_r h_\alpha(f,r)}{N} \frac{\sum_f h_\alpha(f,r)}{N}$$

let $h(r) = \sum_r h_\alpha(f,r)$, $h(f) = \sum_f h_\alpha(f,r)$,
$h(r) \geq h(f,r) \geq 0$, $h(f) \geq h(f,r) \geq 0$,

$$\frac{h(f,r)}{N} \geq \frac{h(f)}{N} \frac{h(r)}{N} \tag{1}$$

$$h(f,r)N \geq h(f)h(r) \tag{2}$$

$$\tag{3}$$

we know $N \geq h(f) + h(r) - h(f,r)$, let's make $N = h(f) + h(r) - h(f,r) + C$, $C \geq 0$

$$h(f,r)(h(f) + h(r) - h(f,r) + C) \geq h(f)h(r) \tag{4}$$

$$h(f,r)h(f) + h(f,r)(h(r) - h(f,r)) + h(f,r)C \geq h(f)h(r) \tag{5}$$

$$h(f,r)(h(r) - h(f,r)) + h(f,r)C \geq h(f)(h(r) - h(f,r)) \tag{6}$$

$$\tag{7}$$

in case that $C = 0$

$$h(f,r)(h(r) - h(f,r)) \geq h(f)(h(r) - h(f,r)) \tag{8}$$
$$h(f,r) \geq h(f) \tag{9}$$
$$\tag{10}$$

This is obviously in contradictory with the condition $h(f) \geq h(f,r)$. The only solution is $h(f) = h(f,r)$. In this case, $N = h(r)$. This would then make $h(f) = h(f,r) = 0$. However, $log(0)$ would give us an invalid solution. Hence, I think equation 12 may be wrong. The truth is, when I apply this algorithm on semantically segmented images, I got positive values, negative values and "inf" for I. I still couldn't understand why would $I = inf$. This would only happen when $P_F(f)P_R(r) = 0$, but that's impossible.

References

[1] Frederik Maes, Andre Collignon, Dirk Vandermeulen, Guy Marchal, and Paul Suetens. Multimodality image registration by maximization of mutual information. *IEEE transactions on Medical Imaging*, 16(2):187–198, 1997.