Computer Vision I

Homework 6

Given Nov. 13, 2018 Due: Nov. 20, 2018

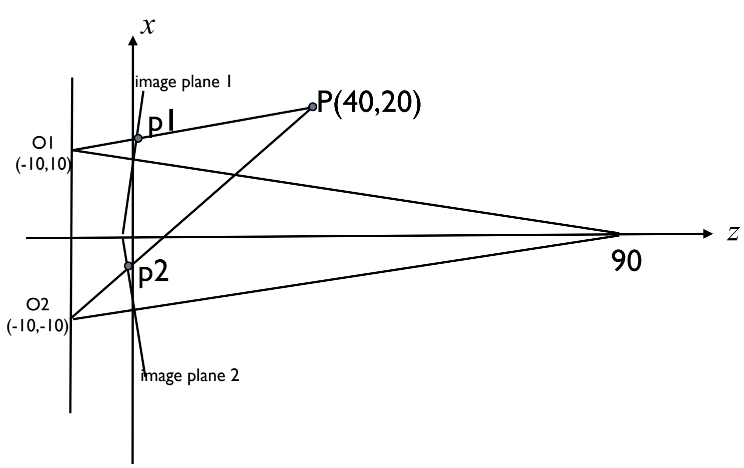
Homographies, Stereo and Motion

1. Explain the differences between a planar homography between two images, the Essential and the Fundamental matrices in stereo.

2. Where are the epipoles in the case when the two cameras have parallel optical axes (the "canonical" configuration)?

3. Show how the projection of a point in a planar scene at world coordinates (X, Y) to pixel coordinates (u, v) in an image plane can be represented using a *planar affine camera model*. Under what conditions is the use of an affine transformations appropriate when viewing a planar scene? How many degrees of freedom are there in the model and what is the minimum number of calibration points needed to estimate the transformation? What effects can a planar affine transformation have on parallel lines?

4. Consider the convergent binocular imaging system shown below. The cameras and all the points are in the $y = 0$ plane. The image planes are perpendicular to their respective camera axes. Find the disparity corresponding to the point P. *Hint*: The perpendicular distance between any point (x_o, y_o) an a line given by $ax + by + c = 0$ is $(ax_o + by_o + c)/\sqrt{a^2 + b^2}$.



5. Determine the matrices H_l and H_r needed to normalize the entries of the fundamental matrix before estimating the Fundamental matrix using the Eight Point Algorithm. Hint: given a set of points

$\mathbf{p}_i = [x_i, y_i, 1]^T$ with $i = 1, \ldots, n$ define $\bar{x} = 1/n \sum_i x_i$, $\bar{y} = 1/n \sum_i y_i$ and

$$\bar{d} = \frac{\sum_i \sqrt{(x_i - \bar{x})^2 + (y_i - \bar{y})^2}}{n\sqrt{2}}$$

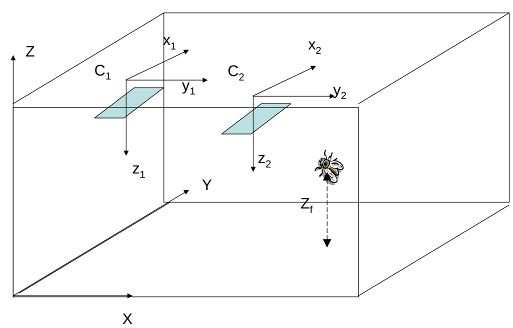
Then find a 3×3 matrix H such that

$$H\mathbf{p}_i = \hat{\mathbf{p}}_i$$

with $\hat{\mathbf{p}}_i = [(x_i - \bar{x}_i)/d, (y_i - \bar{y}_i)/d, 1]^T$ with $i = 1, \ldots, n$

6. Use the method of least squares to derive a linear system of equations to estimate the affine transformation that maps a set of points (x_i, y_i) into new points (x_i', y_i'). Show that it is not necessary to solve a 6×6 system all at once, since the problem can be decomposed into two smaller sets of equations.

7. (Old Exam) Two identical security cameras are mounted in a room as shown in the figure below. The world coordinate system $\mathcal{W}$ is at one corner of the room, and each camera has its own coordinate system $\mathcal{C}_1$ and $\mathcal{C}_2$. In the following, P^w, P^1 and P^2 represent the coordinates of a point P with respect to the world coordinate system $\mathcal{W}$, the camera 1 coordinate system $\mathcal{C}_1$ and the camera 2 coordinate system $\mathcal{C}_2$, respectively. The ***world coordinates*** of the centers of projection C_1^w and C_2^w are $(2, 2, 4)$ and $(4, 3, 3)$, respectively. The focal length of the cameras is 1 and their image planes are located at $z^i = 1$, $i = 1, 2$, respectively.



   (a) Let E_1 and E_2 be the epipoles in camera 1 and 2 respectively. Find the camera coordinates E_1^1 and E_2^2 of the epipoles expressed in their respective camera systems.

   (b) The cameras capture images of a fly in the room. Let f_1 and f_2 be the images of the fly in the first and second camera, respectively. The camera 1 coordinates of the image in the first camera are $f_1^1 = (0, 2, 1)$. Find the equation of the epipolar plane containing the fly, expressed in the camera 2 coordinate system. **Hint:** find f_1^2 first.

   (c) The fly flies following a straight line with constant velocity ***with respect to the world coordinate system***$(-3, -2, -1)$. Find the ***camera coordinates*** of the FOE **in camera 1**.

8. (Old Exam) Consider the Hankel matrix:

$$H = \begin{bmatrix} 1 & 2 & 5 & 12 & 29 \\ 2 & 5 & 12 & x & 70 \\ 5 & 12 & 29 & 70 & y \end{bmatrix}$$

   (a) What is the complexity of the underlying dynamics?

   (b) Find a regressor of the form $x_k = \sum_{i=1}^{n} a_i x_{k-i}$ explaining the data in the given matrix.

   (c) Find the values of x and y.