Quiz 4

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1 Question 1

A pharmaceutical company is interested in testing a potential blood pressure lowering medication. Their first examination considers only subjects that received the medication at baseline then two weeks later. The data are as follows (SBP in mmHg)

Subject	Baseline	Week 2
1	140	132
2	138	135
3	150	151
4	148	146
5	135	130

Consider testing the hypothesis that there was a mean reduction in blood pressure? Give the P-value for the associated two sided T test.

(Hint, consider that the observations are paired.)

1.1 Answer

Here, for a mean difference between the followup and baseline of μ_d, the null hypothesis is $H_0 : \mu_d = 0$ which says that there is no difference in the blood pressure while the alternative hypothesis is $H_a : \mu_d \neq 0$ (there is a difference). So, the p-value for the associated two-sided t-test for these paired observations is as follows:

```r
baseline <- c(140, 138, 150, 148, 135)
followup <- c(132, 135, 151, 146, 130)

t.test(followup, baseline, alternative = "two.sided", paired = TRUE)$p.value
```

```
## [1] 0.08652278
```

2 Question 2

A sample of 9 men yielded a sample average brain volume of 1,100cc and a standard deviation of 30cc. What is the complete set of values of μ_0 that a test of $H_0 : \mu = \mu_0$ would fail to reject the null hypothesis in a two sided 5% Students t-test?

2.1 Answer

```r
1100 + c(-1, 1) * qt(0.975, 8) * 30/sqrt(9)
```

```
## [1] 1076.94 1123.06
```

3 Question 3

Researchers conducted a blind taste test of Coke versus Pepsi. Each of four people was asked which of two blinded drinks given in random order that they preferred. The data was such that 3 of the 4 people chose

Coke. Assuming that this sample is representative, report a P-value for a test of the hypothesis that Coke is preferred to Pepsi using a one sided exact test.

3.1 Answer

Let p be the proportion of people who prefer Coke, such that the null hypothesis is $H_0 : p = 0.5$ versus the alternative hypothesis $H_a : p > 0.5$.

Let X be the number of out of 4 that prefer Coke and assume $X \sim Binomial(p, 0.5)$.

With this, the p-value is calculated as

$$p - value = P(X \geq 3) = \binom{4}{3} 0.5^3 0.5^1 + \binom{4}{4} 0.5^4 0.5^0$$
$$= \boldsymbol{\binom{4}{3} 0.5^4 + \binom{4}{3} 0.5^4}$$

This is computed in R using the binomial distribution as shown below:

```
pbinom(2, size = 4, prob = 0.5, lower.tail = FALSE)
```

```
## [1] 0.3125
```

Alternatively, the equation above can also be used to calculate this p-value:

```
choose(4, 3) * 0.5^4 + choose(4, 4) * 0.5^4
```

```
## [1] 0.3125
```

4 Question 4

Infection rates at a hospital above 1 infection per 100 person days at risk are believed to be too high and are used as a benchmark. A hospital that had previously been above the benchmark recently had 10 infections over the last 1,787 person days at risk. About what is the one sided P-value for the relevant test of whether the hospital is *below* the standard?

4.1 Answer

The null and alternative hypotheses are $H_0 : \lambda = 0.01$ versus $H_a : \lambda < 0.01$ with $X = 11$ and $t = 1787$, and assuming $X \sim Poisson(0.01 \times t)$, the one-sided p-value is

```
ppois(10, lambda = 0.01 * 1787)
```

```
## [1] 0.03237153
```

5 Question 5

Suppose that 18 obese subjects were randomized, 9 each, to a new diet pill and a placebo. Subjects' body mass indices (BMIs) were measured at a baseline and again after having received the treatment or placebo for four weeks. The average difference from follow-up to the baseline (followup - baseline) was -3 kg/m2 for the treated group and 1 kg/m2 for the placebo group. The corresponding standard deviations of the differences was 1.5 kg/m2 for the treatment group and 1.8 kg/m2 for the placebo group. Does the change in BMI appear to differ between the treated and placebo groups? Assuming normality of the underlying data and a common population variance, give a pvalue for a two sided t test.

5.1 Answer

The null hypothesis is $H_0 : \mu_{difference,treated} = \mu_{difference,placebo}$

```
n_t <- n_p <- 9

# treated
mu_t <- -3
s_t <- 1.5

# placebo
mu_p <- 1
s_p <- 1.8

s_pool <- sqrt(((n_t - 1) * s_t^2 + (n_p - 1) * s_p^2) / (n_t + n_p - 2))

ts <- (mu_t - mu_p) / (s_pool * sqrt(1/n_t + 1/n_p))
2 * pt(ts, n_t + n_p - 2)
```

```
## [1] 0.0001025174
```

Therefore, the p-value for a two-sided t-test is **less than 0.01**.

6 Question 6

Brain volumes for 9 men yielded a 90% confidence interval of 1,077 cc to 1,123 cc. Would you reject in a two sided 5% hypothesis test of $H_0 : \mu = 1,078$?

6.1 Answer

No, you wouldn't reject. The 95% interval would be wider than the 90% interval. Since 1,078 is in the narrower 90% interval, it would also be in the wider 95% interval. Thus, in either case it's in the interval and so you would fail to reject.

7 Question 7

Researchers would like to conduct a study of 100 healthy adults to detect a four year mean brain volume loss of $.01mm^3$. Assume that the standard deviation of four year volume loss in this population is $.04mm^3$. About what would be the power of the study for a 5%, percent one sided test versus a null hypothesis of no volume loss?

7.1 Answer

The hypothesis is $H_0 : \mu_\Delta = 0$ versus $H_a : \mu_\Delta > 0$ where μ_Δ is volume loss (defined as Baseline - Four Weeks). The test statistic is $10\frac{\bar{X}_\Delta}{0.04}$ which is rejected if it is larger than $Z_{0.95} = 1.645$.

We want to calculate

$$\begin{aligned}
P\left(\frac{\bar{X}_\Delta}{\sigma_\Delta/10} > 1.645 | \mu_{\Delta=0.01}\right) &= P\left(\frac{\bar{X}_\Delta - 0.01}{0.004} > 1.645 - \frac{0.01}{0.004} | \mu_{\Delta=0.01}\right) \\
&= P(Z > -0.855) \\
&= 0.80
\end{aligned}$$

Or note that $\bar{X}_\Delta$ is $N(0.01, 0.004)$ under the alternative and we want $P(\bar{X}_\Delta > 1.645 \times 0.004)$ under H_a.

```r
pnorm(1.645 * 0.004, mean = 0.01, sd = 0.004, lower.tail = FALSE)
```

```
## [1] 0.8037244
```

Alternatively, the power can be calculated using `power.t.test` in R for a similar result:

```r
power.t.test(n = 100, delta = 0.01 - 0, sd = 0.04, type = "one.sample", alt = "one.sided")$power
```

```
## [1] 0.7989855
```

8 Question 8

Researchers would like to conduct a study of n healthy adults to detect a four year mean brain volume loss of $.01mm^3$. Assume that the standard deviation of four year volume loss in this population is $.04mm^3$. About what would be the value of n needed for 90%, percent power of type one error rate of 5%, percent one sided test versus a null hypothesis of no volume loss?

8.1 Answer

The hypothesis is $H_0 : \mu_\Delta = 0$ versus $H_a : \mu_\Delta > 0$ where μ_Δ is volume loss (defined as Baseline - Four Weeks). The test statistic is $10\frac{\bar{X}_\Delta}{0.04/\sqrt{n}}$ which is rejected if it is larger than $Z_{0.95} = 1.645$.

We want to calculate

$$\begin{aligned}
P\left(\frac{\bar{X}_\Delta}{\sigma_\Delta/\sqrt{n}} > 1.645|\mu_{\Delta=0.01}\right) &= P\left(\frac{\bar{X}_\Delta - 0.01}{0.04/\sqrt{n}} > 1.645 - \frac{0.01}{0.04/\sqrt{n}}|\mu_{\Delta=0.01}\right) \\
&= P(Z > 1.645 - \sqrt{n}/4) \\
&= 0.90
\end{aligned}$$

So we need $1.645 - \sqrt{n}/4 = Z_{0.10} = -1.282$ and thus

$$n = [4 \times (1.645 + 1.282)]^2$$

```r
ceiling( (4 * ( qnorm(0.95) - qnorm(0.1) )) ^ 2 )
```

```
## [1] 138
```

Alternatively, `power.t.test` in R can be used to get the sample size:

```r
power.t.test(power = 0.9, delta = 0.01 - 0, sd = 0.04, type = "one.sample", alt = "one.sided")$n
```

```
## [1] 138.3856
```

9 Question 9

As you increase the type one error rate, α, what happens to power?

9.1 Answer

You will get larger power because as you require less evidence to reject (your α rate goes up), you will have larger power.