Info retrieval sheet 9

Exercise 1

A = U * S * V
$\Leftrightarrow U^T * A = U^T * (U * S * V) \overset{assoz.}{=} (U^T * U) * (S * V) = S * V$, weil $U^T * U = Id_r$

$\Leftrightarrow S^{-1} * U^T * A = S^{-1} * (S * V) = (S^{-1} * S) * V = V$, weil $S^{-1} * S = Id_x$

Exercise 2

1.

$$A * A^T = \begin{pmatrix} 388 & 384 & 0 \\ 384 & 612 & 0 \\ 0 & 0 & 400 \end{pmatrix}$$

2.

$$1.Eigenvector : x_1 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \ A * A^T * \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 400 \end{pmatrix} = 400 * x_1$$

$$2.Eigenvector : x_2 = \begin{pmatrix} -4/5 \\ 3/5 \\ 0 \end{pmatrix}, \ A * A^T * \begin{pmatrix} -4/5 \\ 3/5 \\ 0 \end{pmatrix} = \begin{pmatrix} -400/5 \\ 300/5 \\ 0 \end{pmatrix} = 100 * x_2$$

$$2.Eigenvector : x_2 = \begin{pmatrix} 3/5 \\ 4/5 \\ 0 \end{pmatrix}, \ A * A^T * \begin{pmatrix} 3/5 \\ 4/5 \\ 0 \end{pmatrix} = \begin{pmatrix} 540 \\ 720 \\ 0 \end{pmatrix} = 900 * x_3$$

3.

$$U = \begin{pmatrix} 0 & -4/5 & 3/5 \\ 0 & 3/5 & 4/5 \\ 1 & 0 & 0 \end{pmatrix}$$

$$U^T * U = \begin{pmatrix} 0 & 0 & 1 \\ -4/5 & 3/5 & 0 \\ 3/5 & 4/5 & 0 \end{pmatrix} * \begin{pmatrix} 0 & -4/5 & 3/5 \\ 0 & 3/5 & 4/5 \\ 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$S = \begin{pmatrix} 20 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 30 \end{pmatrix}$$, becouse of $A * A^T = U * S^s * U^T$ and becouse of slide 13.

4.

$V = S^{-1} * U^T * A$
$S * S^{-1} = Id_3$

$$\Leftrightarrow S^{-1} = \begin{pmatrix} 1/20 & 0 & 0 \\ 0 & 1/10 & 0 \\ 0 & 0 & 1/30 \end{pmatrix}$$

$$V = S^{-1} * U^T * A = \begin{pmatrix} 0 & 0 & 1/20 \\ -2/25 & 3/50 & 0 \\ 1/50 & 4/150 & 0 \end{pmatrix} * A = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ -0.5 & 0.5 & 0.5 & 0 & -0.5 \\ 0.5 & 0.5 & 0.5 & 0 & 0.5 \end{pmatrix}$$

5.

$$\begin{pmatrix} 0 & -4/5 & 3/5 \\ 0 & 3/5 & 4/5 \\ 1 & 0 & 0 \end{pmatrix} * \begin{pmatrix} 20 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 30 \end{pmatrix} * \begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ -0.5 & 0.5 & 0.5 & 0 & -0.5 \\ 0.5 & 0.5 & 0.5 & 0 & 0.5 \end{pmatrix} = A$$