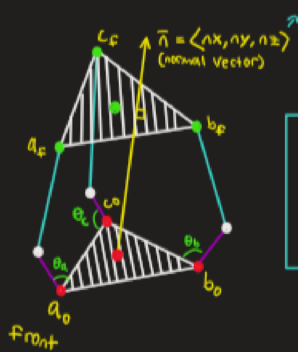
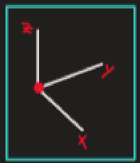
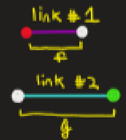
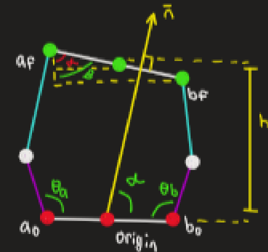
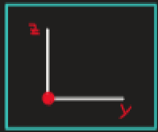
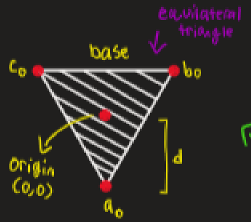


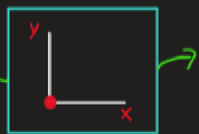


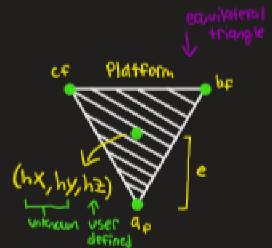












$a_0 = (0, -d, 0)$

$b_0 = (d\cos 30^\circ, d\sin 30^\circ, 0) = \left(\frac{d\sqrt3}{2}, \frac d2, 0\right)$

$c_0 = (-d\cos 30^\circ, -d\sin 30^\circ, 0) = \left(-\frac{d\sqrt3}{2}, \frac d2, 0\right)$

Point (x,y,z) lies on Platform Plane P

$x n_x + y n_y + z n_z = h_x n_x + h_y n_y + h_z n_z$

af_z in terms of h_z

$\bar n \cdot \langle 0,1,0\rangle = \cos\alpha$

$\cos^{-1}(n_y) = \alpha$

$af_z = h_z + e\cdot\cos\alpha$

$af_z = h_z + e n_y$

$a_f = (a_{fx}, a_{fy}, a_{fz}) = (0, a_{fy}, h_z + e n_y)$

$b_f = (b_{fx}, b_{fy}, b_{fz}) = \left(\sqrt3\, b_{fy}, b_{fy}, h_z - \frac e2(\sqrt3 n_x + n_y)\right)$

$c_f = (c_{fx}, c_{fy}, c_{fz}) = \left(-\sqrt3\, c_{fy}, c_{fy}, h_z - \frac e2(-\sqrt3 n_x + n_y)\right)$

$\overline{bf - af} \times \overline{cf - af} = 3e^2 \sin 60^\circ \bar n = \frac{3\sqrt3}{2} e^2 \bar n = w\bar n$

$$= \begin{vmatrix} \hat i & \hat j & \hat k \\ \sqrt3\, b_{fy} & (b_{fy} - a_{fy}) & t \\ -\sqrt3\, c_{fy} & (c_{fy} - a_{fy}) & v \end{vmatrix}$$

$w = \frac{3\sqrt3 e^2}{2}$

$t = bf_z - af_z$

$v = cf_z - af_z$

$\hat i$: $v(b_{fy} - a_{fy}) - t(c_{fy} - a_{fy}) = w n_x$

$v b_{fy} - t c_{fy} + a_{fy}(t - v) = w n_x$

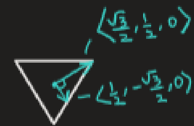
$\hat j$: $-\sqrt3 v b_{fy} - \sqrt3 t c_{fy} = w n_y$

$v b_{fy} + t c_{fy} = \frac{w n_y}{-\sqrt3}$

$c_{fy} = \dfrac{\frac{w n_y}{-\sqrt3} - v b_{fy}}{t}$

$\hat k$. $\sqrt3\, b_{fy}(c_{fy} - a_{fy}) + \sqrt3\, c_{fy}(b_{fy} - a_{fy}) = w n_z$

bf_x in terms of bf_y



$\frac12 bf_x - \frac{\sqrt3}{2} bf_y = 0$

$bf_x = \sqrt3\, bf_y$

bf_z, cf_z in terms of h_z

$\bar n \cdot (\overline{b_0 - a_0})_u = \cos\beta$

$\dfrac{\langle \frac{\sqrt3}{2}, \frac32, 0\rangle}{\sqrt{\frac34 + \frac94}} = \langle \frac12, \frac{\sqrt3}{2}, 0\rangle$

$\cos^{-1}\left(\frac{n_x + \sqrt3 n_y}{2}\right) = \beta$

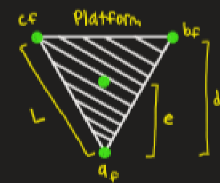
$bf_z = af_z - \sqrt3 e\cos\beta =$

$bf_z = h_z - \frac e2(\sqrt3 n_x + n_y)$

so

$cf_z = h_z - \frac e2(-\sqrt3 n_x + n_y)$

length between af, bf − af, cf − bf, cf



$d = e + e\cos 60^\circ = \frac32 e$

$L = \frac{d}{\cos 30^\circ} = \frac{3}{\cancel{2}} e \cdot \frac{\cancel{2}}{\sqrt3} = \sqrt3\, e$

2. Create vectors

$\bar a = \overline{af - a_0} = \left\langle 0, \left(\frac e2\right)\left(1 - \frac{n_x^2 + 3n_z^2 + 3n_z}{n_z + 1 - n_x^2} + \frac{n_x^4 - 3n_x^2 n_y^2}{(n_z+1)(n_z+1-n_x^2)}\right) + d, h_z + e n_y \right\rangle$

$\bar b = \overline{bf - b_0} = \left\langle \frac{\sqrt3}{2}\left(e\left(1 - \frac{n_x^2 + \sqrt3 n_x n_y}{n_z + 1}\right) - d\right), \frac{b_x}{\sqrt3}, h_z - \frac e2(\sqrt3 n_x + n_y) \right\rangle$

$\bar c = \overline{cf - c_0} = \left\langle \frac{\sqrt3}{2}\left(d - e\left(1 - \frac{n_x^2 - \sqrt3 n_x n_y}{n_z + 1}\right)\right), -\frac{c_x}{\sqrt3}, h_z - \frac e2(-\sqrt3 n_x + n_y) \right\rangle$

$\vec{c} = \overline{c_f - c_0} = \left\langle \frac{\sqrt{3}}{2}\left(d - e\left(1 - \frac{n_x^2 - \sqrt{3} n_x n_y}{n_z + 1}\right)\right), -\frac{c_x}{\sqrt{3}}, h_z - \frac{e}{2}(-\sqrt{3} n\right.$

$\hat{K}.$

$$\sqrt{3} b_{fy}(c_{fy} - a_{fy}) + \sqrt{3} c_{fy}(b_{fy} - a_{fy}) = w n_z$$

$$2 b_{fy} c_{fy} - a_{fy}(b_{fy} + c_{fy}) = \frac{w n_z}{\sqrt{3}}$$

$$a_{fy} = \frac{2 b_{fy} c_{fy} - \frac{w n_z}{\sqrt{3}}}{b_{fy} + c_{fy}}$$

1. Solve for b_{fy}

$$v b_{fy} - t c_{fy} + a_{fy}(t - v) = w n_x$$

$$v b_{fy} - t c_{fy} + \frac{2 b_{fy} c_{fy}(t - v) - \frac{w n_z}{\sqrt{3}}(t - v)}{b_{fy} + c_{fy}} = w n_x$$

$$= v b_{fy} + \frac{w n_y}{\sqrt{3}} + v b_{fy} + \frac{2 b_{fy} \frac{\frac{w n_y}{-\sqrt{3}} - v b_{fy}}{t}(t - v) - \frac{w n_z}{\sqrt{3}}(t - v)}{b_{fy} + \frac{\frac{w n_y}{-\sqrt{3}} - v b_{fy}}{t}}$$

$R = -\frac{w}{\sqrt{3}}$

$$2 v b_{fy} - R n_y + \frac{2 b_{fy} \frac{R n_y - v b_{fy}}{t}(t - v) + R n_z (t - v)}{\frac{t b_{fy} + R n_y - v b_{fy}}{t}}$$

$P = t - v$

$$2 v b_{fy} - R n_y + \frac{2 b_{fy} P (R n_y - v b_{fy}) + t R P n_z}{b_{fy} P + R n_y}$$

$$(2 v b_{fy} - R n_y)(b_{fy} P + R n_y) + 2 b_{fy} P (R n_y - v b_{fy}) + t R P n_z = w n_x (b_{fy} P + R n_y)$$

$$\cancel{2vP} b_{fy}^2 + 2 v R b_{fy} n_y - \cancel{RP b_{fy}} n_y - R^2 n_y^2 + \cancel{2} P R b_{fy} n_y - \cancel{2Pv} b_{fy}^2 + t R P n_z = w P b_{fy} n_x + R w n_y n_x$$

$$2 v R b_{fy} n_y - w P b_{fy} n_x + P R b_{fy} n_y = R^2 n_y^2 + R w n_y n_x - t R P n_z$$

$$2VR\,b_{f_y}n_y - wP\,b_{f_y}n_x + PR\,b_{f_y}n_y = R^2 n_y^2 + Rw\,n_y n_x - tRP\,n_z$$

$$b_{f_y} = \frac{R^2 n_y^2 + Rw\,n_y n_x - tRP\,n_z}{2VR\,n_y - wP\,n_x + PR\,n_y}$$

$$= \frac{R\left(R n_y^2 + w n_y n_x - Pt\,n_z\right)}{R n_y(2V+P) - wP\,n_x}$$

$$= \frac{-\frac{w}{\sqrt{3}}\left(-\frac{w}{\sqrt{3}}n_y^2 + w n_y n_x - t(t-v)n_z\right)}{-\frac{w}{\sqrt{3}}n_y(t+V) - w n_x(t-V)}$$

$$= \frac{\frac{w}{\sqrt{3}}n_y(n_y - \sqrt{3}n_x) + t(t-v)n_z}{-n_y(t+V) - \sqrt{3}n_x(t-V)}$$

$$= \frac{\frac{3}{2}e\,n_y(n_y - \sqrt{3}n_x) - \sqrt{3}\,t\,n_x n_z}{3(1-n_z^2)}$$

$$= \frac{e n_y^2 - \sqrt{3}e\,n_x n_y + e\,n_x^2 n_z + \sqrt{3}e\,n_x n_y n_z}{2(1-n_z^2)}$$

$$= \left(\frac{e}{2}\right)\frac{n_y^2 + n_x^2 n_z + \sqrt{3}\,n_x n_y(n_z-1)}{1-n_z^2}$$

$$= \left(\frac{e}{2}\right)\left(1 + \frac{n_x^2(n_z-1) + \sqrt{3}\,n_x n_y(n_z-1)}{1-n_z^2}\right)$$

$$= \left(\frac{e}{2}\right)\left(1 + \frac{(\cancel{n_z-1})(n_x^2 + \sqrt{3}\,n_x n_y)}{\quad}\right)$$

$$w = \frac{3\sqrt{3}e^2}{2}$$

$$t = bf_z - af_z = -\frac{e}{2}(\sqrt{3}n_x + 3n_y)$$

$$v = cf_z - af_z = -\frac{e}{2}(-\sqrt{3}n_x + 3n_y)$$

$$R = -\frac{w}{\sqrt{3}}$$

$$P = t - v = -\sqrt{3}\,e\,n_x$$

$$t + v = -3e\,n_y$$

$$=\left(\frac{e}{2}\right)\left(1+\frac{(\cancel{nz-1})(nx^2+\sqrt{3}\,nx\,ny)}{-(\cancel{nz-1})(nz+1)}\right)$$

$$bfy=\left(\frac{e}{2}\right)\left(1-\frac{nx^2+\sqrt{3}\,nx\,ny}{nz+1}\right)$$

$$cfy=\left(\frac{e}{2}\right)\left(1-\frac{nx^2-\sqrt{3}\,nx\,ny}{nz+1}\right)$$

$$afy=\frac{2\,bfy\,cfy-\frac{3}{2}e^2nz}{bfy+cfy}$$

$$\frac{\frac{e^2}{2}\left(1-\frac{nx^2+\sqrt{3}\,nx\,ny}{nz+1}\right)\left(1-\frac{nx^2-\sqrt{3}\,nx\,ny}{nz+1}\right)-\frac{3}{2}e^2nz}{e-\frac{e}{2}\left(\frac{nx^2+\cancel{\sqrt{3}\,nx\,ny}+nx^2-\cancel{\sqrt{3}\,nx\,ny}}{nz+1}\right)}$$

$$\frac{\frac{e}{2}\left(1+\frac{-nx^2+\cancel{\sqrt{3}\,nx\,ny}-nx^2-\cancel{\sqrt{3}\,nx\,ny}}{nz+1}+\frac{nx^4-3nx^2ny^2}{(nz+1)^2}-3nz\right)}{1-\frac{nx^2}{nz+1}}$$

$$\left(\frac{e}{2}\right)\left(1-\frac{2nx^2}{nz+1}+\frac{nx^4-3nx^2ny^2}{(nz+1)^2}-3nz\right)\left(\frac{nz+1}{nz+1-nx^2}\right)$$

$$\left(\frac{e}{2}\right)\left(\frac{nz+1}{nz+1-nx^2}-\frac{2nx^2}{nz+1-nx^2}-\frac{3nz(nz+1)}{nz+1-nx^2}+\frac{nx^4-3nx^2ny^2}{(nz+1)(nz+1-nx^2)}\right)$$

$$afy=\left(\frac{e}{2}\right)\left(1-\frac{nx^2+3nz^2+3nz}{nz+1-nx^2}+\frac{nx^4-3nx^2ny^2}{(nz+1)(nz+1-nx^2)}\right)$$