

Lab 3 – RSA Public Key Encryption and Signature(Seed Labs)

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Abstract—The goal of the lab is to allow the user to gain a greater understanding RSA Public key encryption and the bignums library. This lab demonstrates the use of secure and non-secure was to generate randomness. This lab also has the user demonstrate signing a certificate and manually verifying a X.509 certificate from a certificate authority. This lab gives the user a hands on understanding of the importance of RSA encryption in web security.

I. LAB DEFINITION

THE goal of this lab is to give students the opportunity to learn about how RSA works. It also gives the student the tools necessary to understand how to utilize the bignums library. This lab shows several popular libraries used in pseudo random number generation and ask the student to perform task with theses libraries. In task 1, the student is asked to get the private key given p , q and e . Tasks 2 and 3 has the student encrypt and decrypt a message using RSA encryption and the Bignums library. Task 4 has the student sign a message, to ensure the integrity of the message through RSA encryption. The student is then asked to modify the signed message as a proof that RSA will preserve the integrity of the original message. Task 5 provides the student with a signature and ask that the student verify the authentication of a message signed with the private key of the provided signature's owner. Task 6 goes into a “real-world” scenario and has the student verify a certification authorities signature for a website certificate on a website of their choosing.

II. STEP-BY-STEP INSTRUCTIONS - LAB TASK

A. Task A – Setting up the Virtual Machine

In order to begin this lab, I chose to utilize VirtualBox along with the pre-built VM provided at <https://seedsecuritylabs.org/labsetup.html> [10]. I had previously configured this VM from lab one, below are the steps I took to set up this VM This VM utilizes Ubuntu 20.04. After downloading the SEED-Ubuntu20.04.vdi file, I proceeded to create a new virtual machine to utilize the image with. I followed the setup instructions provided in the above URL to complete the setup of the lab. I have frequently utilized VirtualBox in the past, so this setup task was no problem at all.

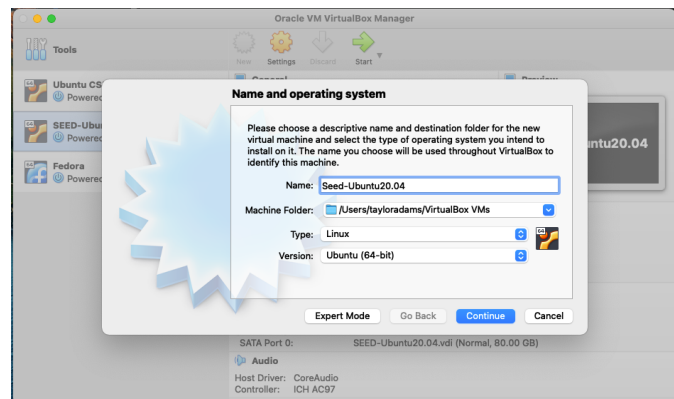


Fig. 1. Creating the Virtual Machine

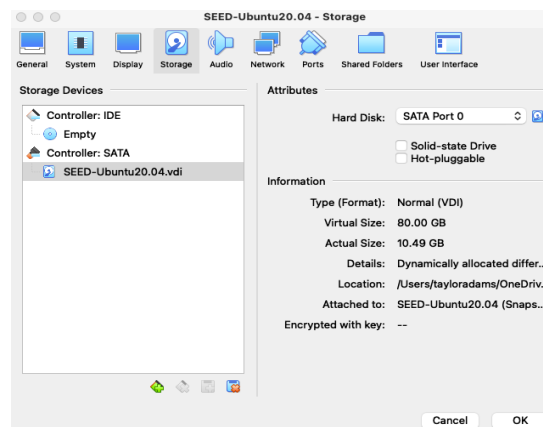


Fig. 1a. Pre-Built VDI file placed in the virtual disk

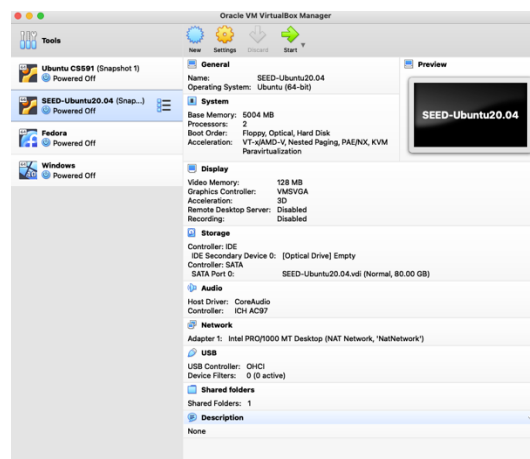


Fig. 1b. SEED Virtual Machine Complete Settings

I also installed the bignum library on my host machine (through openssl). This took around an hour to get set up. I did learn a lot from installing the bignum library on my host machine as opposed to having it come preinstalled on a vm. After installing the bignums library, I created a makefile to run on my host machine which takes in the necessary flags for the bignums library.

```
Lab 3 files > Makefile
1
2 CC = clang
3 CFLAGS = -Wall -L/usr/local/opt/openssl@3/lib -L/usr/lib -lssl -lcrypto
4
5 all:
6     $(CC) $(CFLAGS) Lab3.c -o Lab3
7
8 exec:
9     ./Lab3
10
11 clean:
12     rm -rf Lab3
13
14 test:
15     @$(CC) $(CFLAGS) Lab1.c -o Lab3
16     @echo "\nRunning Program...\n"
17     @./Lab3
```

Fig. 1c. Makefile with necessary flags to utilize bignums on my host machine.

Running “make test” in the terminal compiles and runs the program. (Note: To run this program on other machines the CFLAGS locations will likely need to be modified to the machine its run on.)

III. BY-STEP INSTRUCTIONS - LAB TASK

A. Task 1: Deriving the Private Key

For task 1, I was given three prime numbers :

- $p = F7E75FDC469067FFDC4E847C51F452DF$
- $q = E85CED54AF57E53E092113E62F436F4F$
- $e = 0D88C3$

I then let (e, n) be the public key where $n = p * q$.

```
void task1()
{
    BIGNUM *p = BN_new();
    BIGNUM *q = BN_new();
    BIGNUM *e = BN_new();

    // Assign the first large prime
    BN_hex2bn(&p, "F7E75FDC469067FFDC4E847C51F452DF");

    // Assign the second large prime
    BN_hex2bn(&q, "E85CED54AF57E53E092113E62F436F4F");

    // Assign the Exponent
    BN_hex2bn(&e, "0D88C3");

    BIGNUM* priv_key1 = get_private_key(p, q, e);
    printBN("the private key for task1 is:", priv_key1);

    return;
}
```

Fig. 1. Task 1 with setting up provided initial values

Task1() then passes in the given values to the below `get_private_key` function to compute the key.

```
69 BIGNUM* get_private_key(BIGNUM* p, BIGNUM* q, BIGNUM* e)
70 {
71     //Initialize big number variables
72     BIGNUM* p_one = BN_new();
73     BIGNUM* q_one = BN_new();
74     BIGNUM* number_one = BN_new();
75     BIGNUM* tt = BN_new();
76     BIGNUM* result = BN_new();
77
78     BN_dec2bn(&one, "1");
79     //BN_sub() subtracts b from a and places the result in r ("r=a-b")
80     BN_sub(p_one, p, number_one);
81     BN_sub(q_one, q, number_one);
82
83     //create ctx object required by BN
84     BN_CTX *ctx = BN_CTX_new();
85
86     //BN_mul() multiplies a and b and places the result in r ("r=a*b").
87     //r may be the same BIGNUM as a or b. For multiplication by powers of 2, use BN_lshift(3).
88     BN_mul(tt, p_one, q_one, ctx);
89
90     //BN_mod_inverse() computes the inverse of a modulo n places
91     //the result in r ("a*r%n=1"). If r is NULL, a new BIGNUM is created.
92     BN_mod_inverse(result, e, tt, ctx);
93
94     // ctx is a previously allocated BN_CTX used for temporary variables. r may be the same BIGNUM as a or n.
95     BN_CTX_free(ctx);
96     return res;
97 }
98
```

Fig. 2. Getprivatekey function called by task 1 after initial values entered

At the beginning of the `get_private_key` function it initializes the BIGNUM variables to be utilized. Standard arithmetic can't be used on big numbers since it is not a native data type to the c language. This requires additional variables be created such as: one, p-one, and q-one, rather than being able to do: $p = p - 1$. After initialization one is subtracted from both p and q using the `BN_sub` function. A CTX object is then created for the following calculations. P and Q are multiplied and then the inverse is taken between e and the result of the multiplication between p and q . The returning result is the private key.

the private key for task1 is:

```
3587A24598E5F2A21DB007D89D18CC50ABA5075BA19A33890FE7C28
A9B496AEB
```

Fig. 3. Resulting Generated key

B. Task 2: Encrypting a Message

For Task 2 I was asked to encrypt a message using RSA. The message I was told to encrypt was my name + my cwid. I began by writing the c code to convert the string to hexadecimal format rather than use the python command.

```
//converts a string to hex using c instead of python
void string2hexString(char* input, char* output)
{
    int loop;
    int i;

    i=0;
    loop=0;

    while(input[loop] != '\0')
    {
        sprintf((char*)(output+i),"%02X", input[loop]);
        loop+=1;
        i+=2;
    }

    //insert NULL at the end of the output string
    output[i++] = '\0';
}
```

Fig. 4. Convert string to hex[2]

The plaintext value in hex for my string was:

```
5461796C6F724164616D732B3131373032353530
```

I was then given the following variables to use in the encryption:

- modulus_n
 - DCBFFE3E51F62E09CE7032E2677A78946A849DC4CDDE3A4D0CB81629242FB1A5
- public_key_e
 - 010001
- Private_key_d
 - 74D806F9F3A62BAE331FFE3F0A68AFE35B3D2E4794148AACBC26AA381CD7D30D

I then initialized the BIGNUM variables. I assigned those variables the values provided earlier. Following these two actions I created a BIGNUM ciphertext variable to store the generated ciphertext. I utilized the BN_mod_exp function to generate the ciphertext using the provided values. BN_mod_exp(ciphertext, plaintext_M, public_key_e, modulus_n, ctx);

The resulting ciphertext was:

```
ciphertext:
8A908419D68FB1E587773550F2BEE558BF7262F3BB5
7F2B7BD3A6225FCBD1A65
```

Fig. 5. Task 2 resulting ciphertext

```
void task2(){
    printf("\nTask 2 begins\n");

    //initialize strings
    char ascii_str[] = "TaylorAdams+11702550";
    int len = strlen(ascii_str);
    char hex_str[(len*2)+1];

    //convert string to hex
    string2hexString(ascii_str, hex_str);

    //Print the hex string generated
    printf("Hex string:\t %s", hex_str);

    BN_CTX *ctx = BN_CTX_new();
    BIGNUM *private_key_d = BN_new();
    BIGNUM *modulus_n = BN_new();
    BIGNUM *public_key_e = BN_new();
    BIGNUM *plaintext_M = BN_new();
    // converts the plaintext to be encrypted from hex to binary and stores in a big number
    BN_hex2bn(&plaintext_M, hex_str);
    printBN("\nBIGNUM plaintext before encryption:\t\t", plaintext_M);

    BN_hex2bn(&modulus_n, "DCBFFE3E51F62E09CE7032E2677A78946A849DC4CDDE3A4D0CB81629242FB1A5");
    BN_dec2bn(&public_key_e, "65537");
    BN_hex2bn(&private_key_d, "74D806F9F3A62BAE331FFE3F0A68AFE35B3D2E4794148AACBC26AA381CD7D30D");

    BIGNUM *ciphertext = BN_new();
    BN_mod_exp(ciphertext, plaintext_M, public_key_e, modulus_n, ctx);
    BN_CTX_free(ctx);
    printBN("ciphertext:\t", ciphertext);

    task3(ciphertext, private_key_d, public_key_e, modulus_n);
}
```

Fig. 6. Task 2 code

```
Task 2 begins
Hex string: 5461796C6F724164616D73283131373032353530
BIGNUM plaintext before encryption: 5461796C6F724164616D73283131373032353530
ciphertext: 8A908419D68FB1E587773550F2BEE558BF7262F3BB57F2B7BD3A6225FCBD1A65
```

Fig. 7. Task 2 resulting outputs

C. Task 3: RSA decryption

For this task I was required to decrypt a ciphertext hex string and convert it back to ascii to reveal the message. The public and private keys were the same ones as in task 2 so task 3 was called from task 2 passing in the appropriate values and BIGNUM variables, so that I would not need to recreate them.

A new ciphertext was however provided by the lab to decrypt. I replaced the BIGNUM value of ciphertext with the ciphertext provided by the lab:

```
C = 8C0F971DF2F3672B28811407E2DABB
E1DA0FEBBBDFC7DCB67396567EA1E2493F
```

Fig. 8. Ciphertext provided by the lab

After changing the ciphertext value I ran the same function as before BN_mod_exp();

```
BN_mod_exp(decrypted_ciphertext, ciphertext, private_key_d,
modulus_n, ctx);
```

Fig. 9. Task3 BN_mod_exp() call;

The above function take in an empty BIGNUM decrypted_ciphertext as the output, the ciphertext itself, the private key, the modulus, and the BIGNUM CTX var.

I used the BN_bn2hex method call on the decrypted_ciphertext variable to convert the var to a hex format for converting to an ascii string.

I then found some C code online to convert from hex to ascii characters. This function is called hex_to_string(). The resulting output after conversion to ascii was:

Password is dees

```
Task 3 begins
Decrypted Ciphertext: 50617373776F72642069732064656573
The decrypted hex is: Password is dees
```

Fig. 10. Task 3 resulting plaintext from the encrypted ciphertext

```
void task3(BIGNUM* ciphertext, BIGNUM* private_key_d, BIGNUM* public_key_e, BIGNUM* modulus_n){
    printf("\nTask 3 begins\n");

    BN_CTX *ctx = BN_CTX_new();
    BIGNUM *decrypted_ciphertext = BN_new();
    BN_hex2bn(&ciphertext, "8C0F971DF2F3672B28811407E2DABBE1DA0FEBBBDFC7DCB67396567EA1E2493F");
    //decrypt the ciphertext
    BN_mod_exp(decrypted_ciphertext, ciphertext, private_key_d, modulus_n, ctx);
    //print decrypted ciphertext in hex format
    printBN("Decrypted Ciphertext: ", decrypted_ciphertext);
    BN_CTX_free(ctx);
    //generate vars for hex conversion
    uint8_t *hex = BN_bn2hex(decrypted_ciphertext);
    int len = strlen(hex);
    uint8_t *ascii[100];
    int ascii_len = 100;
    hex_to_string(ascii, len, hex, len);

    return;
}
```

Fig. 11. Task 3 code

D. Task 4: Signing a Message

In task 4, I was asked to sign a message using RSA. The public and private keys were the same from task 2. But the message to sign was "Taylor Adams owes you \$2000." I was also asked to sign another string as well "Taylor Adams owes you \$3000." and compare the resulting signed values.

To begin this task I declared my two strings, got the length and passed them into the method used earlier to convert them to hexadecimal format (string2hexString()). Once I had my hex strings I created BIGNUM variables for both of them. I then Initialized BIGNUM variables for the ctx, private key, modulus, and public key. I then ran BN_mod_exp on both of the stings and compared the digital signatures.

The first message “Taylor Adams owes you \$2000.” Became the signed message:

```
Signed Message: 94A8906B5E808337E676738AFAC7152F076007F67051342132234803A5003054
```

Fig. 12. Signed message from string “Taylor Adams owes you \$2000.”

The second message “Taylor Adams owes you \$3000” Became the signed message:

```
Signed Message Modified: 7588A5030ED158F7B987FB48DE7D54B1A3D7D6097A93C290621E6D9177FE0D1D
```

Fig. 13. Signed message from string “Taylor Adams owes you \$3000.”

It is clearly visible that both of this hex strings are completely different with just one char modified. This displays the non-repudiation where the sender cannot deny they sent a message when it is decrypted with their private key. This also shows how an attacker would need to have access to a victims private key in order to sign a fake message and make it believable because an attacker would not be able change the value of a character in the signed message to change the resulting decrypted message from \$2000 to \$3000.

```
void task4(){
    printf("\n\nTask 4 begins!\n");

    //Initialize strings and convert to hex
    char string[] = "Taylor Adams owes you $2000";
    char string1[] = "Taylor Adams owes you $3000";
    int len = strlen(string);
    char string_hex[(len*2)+1];
    char string1_hex[(len*2)+1];
    string2hexString(string, string_hex);
    string2hexString(string1, string1_hex);

    //Initialize bignum variables
    BIGNUM *string_M = BN_new();
    BIGNUM *string1_M = BN_new();
    BN_CTX *ctx = BN_CTX_new();
    BIGNUM *private_key_d = BN_new();
    BIGNUM *modulus_n = BN_new();
    BIGNUM *public_key_e = BN_new();

    //convert hex to BIGNUM data type
    BN_hex2bn(&string_M, string_hex);
    BN_hex2bn(&string1_M, string1_hex);

    //declare the BIGNUM variables
    BN_hex2bn(&modulus_n, "DCBF3E351F62E99CE7832E677A78946A849DC4CDE3A40CB81629242F81A5");
    BN_dec2bn(&public_key_e, "65537");
    BN_hex2bn(&private_key_d, "74D886F9F3A62BAE331FFE3F0A68AFE35B3D2E4794148AACBC26AA381CD7D300");

    //create output BIGNUMs to pass to BN_mod_exp
    BIGNUM *signed_Message = BN_new();
    BIGNUM *signed_Message1 = BN_new();
    /*m*d mod n
    In order to sign a message you need to perform the operation m*d mod n where m = the message; d = private key; and n =
    */
    BN_mod_exp(signed_Message, string_M, private_key_d, modulus_n, ctx);
    BN_mod_exp(signed_Message1, string1_M, private_key_d, modulus_n, ctx);

    printf("\nSigned Message:\t\t", signed_Message);
    printf("\nSigned Message Modified:\t", signed_Message1);
    // BN_mod_exp(signed_Message, signed_Message, public_key_e, modulus_n, ctx);
    // printf("\nSigned Message:\t", signed_Message);
    BN_CTX_free(ctx);
}
```

Fig. 14. Task 4 code

E. Task 5: Verifying a Signature

In task 5 I was asked to verify an RSA signature. I know the resulting plaintext is “Launch a missile.” And the public key signature’s values:

- Digital Signature:
 - 643D6F34902D9C7EC90CB0B2BCA36C47FA37165C0005CAB026C0542CBDB6802F
- Public key:
 - 010001 (hex: 65537)
- Modulus:
 - AE1CD4DC432798D933779FBD46C6E1247F0CF1233595113AA51B450F18116115

This lab also asks us to corrupt the text and test what the resulting value would be.

As before, I began by initializing the BIGNUM variables for the verification. I then inserted the values provided by the lab into the BIGNUM variables. I then utilized the BN_mod_exp function as before to verify the signature of the sender.

```
BIGNUM *unsigned_Message = BN_new();
BN_mod_exp(unsigned_Message, digital_signature_S, public_key_e, modulus_n, ctx);
```

Fig. 15. Task 5 BN_mod_exp

With the uncorrupted digital signature, I got the expected output of “Launch a missile”)

```
Task 5
The decrypted hex is: Launch a missile.
```

Fig. 16. Task 5 BN_mod_exp

However, when I change the end of the digital signature from 2F to 3F and convert to ascii I get:

```
Task 5
00,00c00rm=f0:N0000
(base) tayloradams@Taylor
```

Fig. 17. Task 5 BN_mod_exp

Task 5 displays how an attacker cannot modify a digital signature as it will likely just end up corrupting itself and therefore be unreadable.

```
void task5(){
    printf("\n\nTask 5\n");

    //Initialize the BN values
    BN_CTX *ctx = BN_CTX_new();
    BIGNUM *modulus_n = BN_new();
    BIGNUM *public_key_e = BN_new();
    BIGNUM *digital_signature_S = BN_new();

    //corrupt the 2F at the end of the signature to 3F and rerun
    BN_hex2bn(&digital_signature_S, "643D6F34902D9C7EC90CB0B2BCA36C47FA37165C0005CAB026C0542CBDB6802F");
    BN_dec2bn(&public_key_e, "65537");
    BN_hex2bn(&modulus_n, "AE1CD4DC432798D933779FBD46C6E1247F0CF1233595113AA51B450F18116115");

    BIGNUM *unsigned_Message = BN_new();
    //check 5
    BN_mod_exp(unsigned_Message, digital_signature_S, public_key_e, modulus_n, ctx);

    //convert BIGNUM to hex and hex to ascii
    uint8_t *hex = BN_bn2hex(unsigned_Message);
    int len = strlen(hex);
    uint8_t *ascii[100];
    int ascii_len = 100;
    hex_to_string(ascii, ascii_len, hex, len);
}
```

Fig. 18. Task 5 code

F. Task 6: Manually Verifying an X.509 Certificate

For this task I chose to verify the X.509 certificate from www.redcross.org. I began this task by creating a separate folder for task 6 and then creating both c0.pem and c1.pem files. I then ran the command: “openssl s_client -connect www.example.org:443 -showcerts” and copied both certificates into their respective .pem files. In order to get the modulus I ran the command: “openssl x509 -in c1.pem -noout -modulus”. I then initialized a BIGNUM modulus_n in my task6 code and copied the modulus into the initialized BIGNUM.

Following the instructions I ran the command: “openssl x509 -in c1.pem -text -noout” to get the public key.

