Comparison of hamiltonians and dissipation strength

Let a be the lowering operator of the harmonic oscillator and $a^\dagger$ it's hermitian transpose. Let $H_0 = \hbar\omega(a^\dagger a + \frac{1}{2})$ be the harmonic oscillator, and let $\mathbb{x} = \frac{1}{\sqrt{2}}(a + a^\dagger)$ be the position operator. Here we will compare the time evolution of systems with hamiltonians $H = H_0$, $H = H_0 + \lambda\mathbb{x}$, and $H = H_0 + \lambda\mathbb{x}^4$ and differing damping strengths, κ. The evolution of each system is governed by

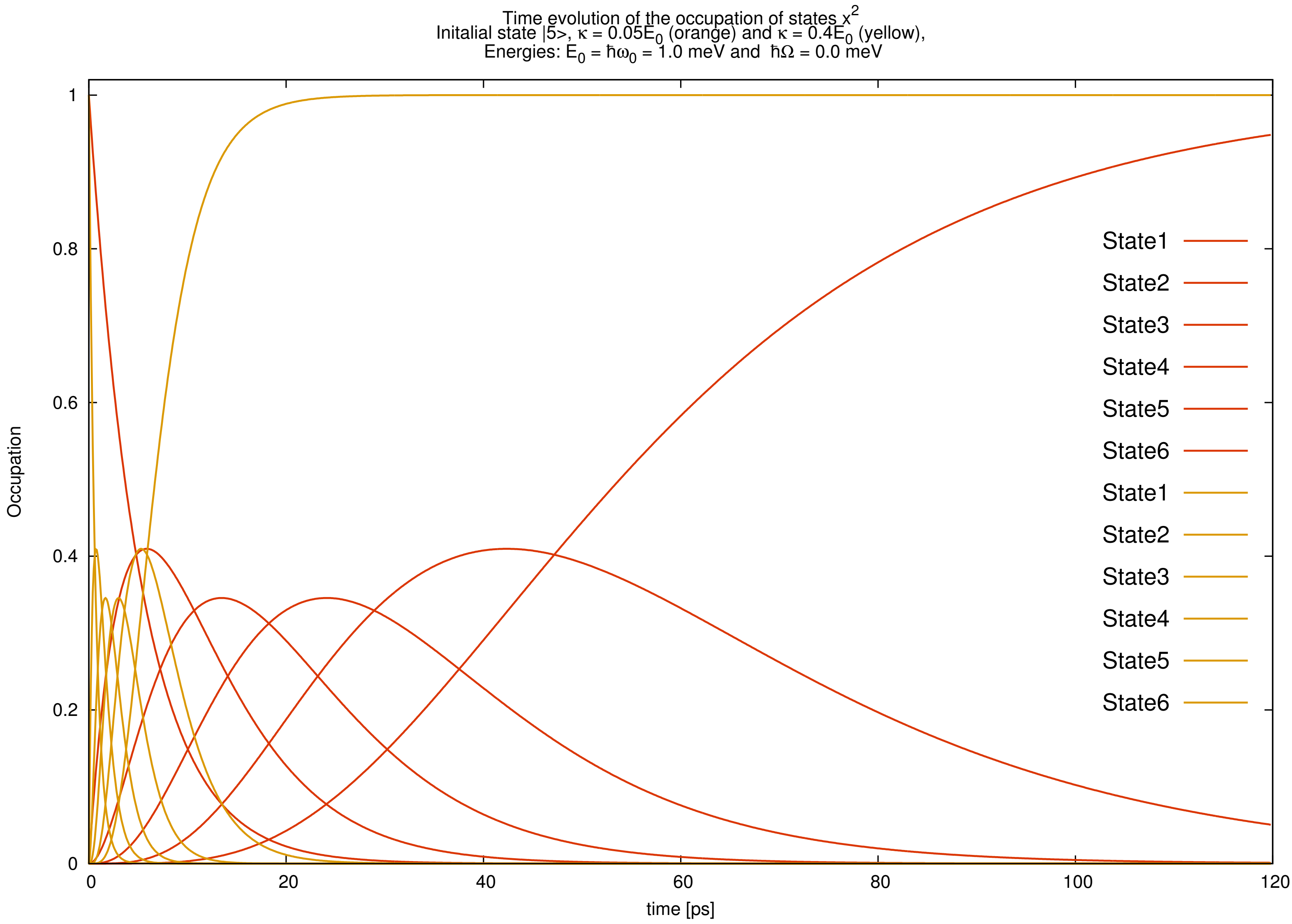
$$i\hbar\dot{\rho} = [H, \rho] + i\tfrac{\kappa}{2}\{[a, \rho a^\dagger] + [a\rho, a^{\rceil}]\}$$

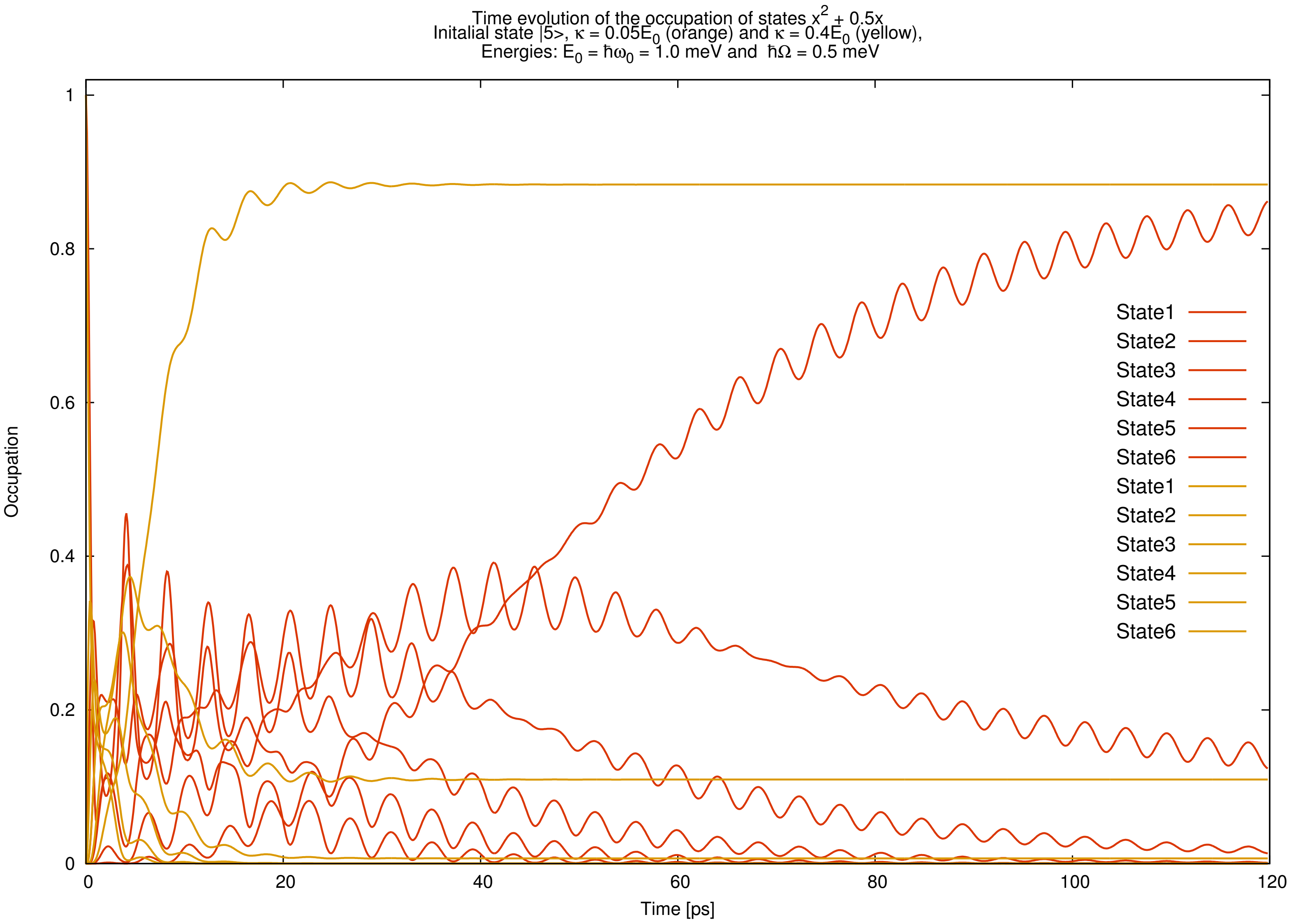
and shown below for a couple of values of λ and κ. In all the plots the initial state of the system is the following (using a matrix representation in the basis of the harmonic oscillator):

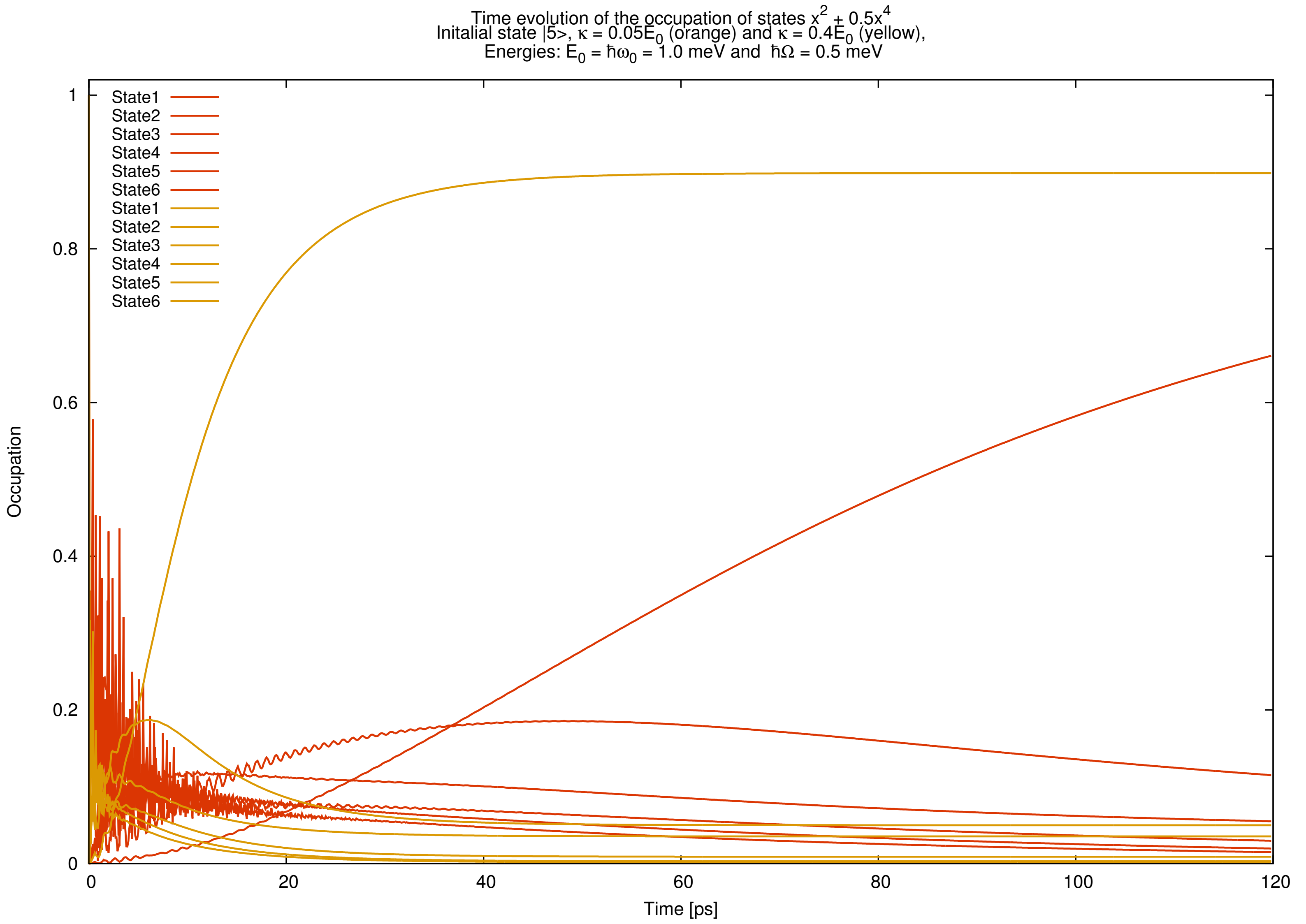
$$\rho = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 0 & 0 & 0 & 0 & 1 & 0 & \dots \\ 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

The following plots are in the order

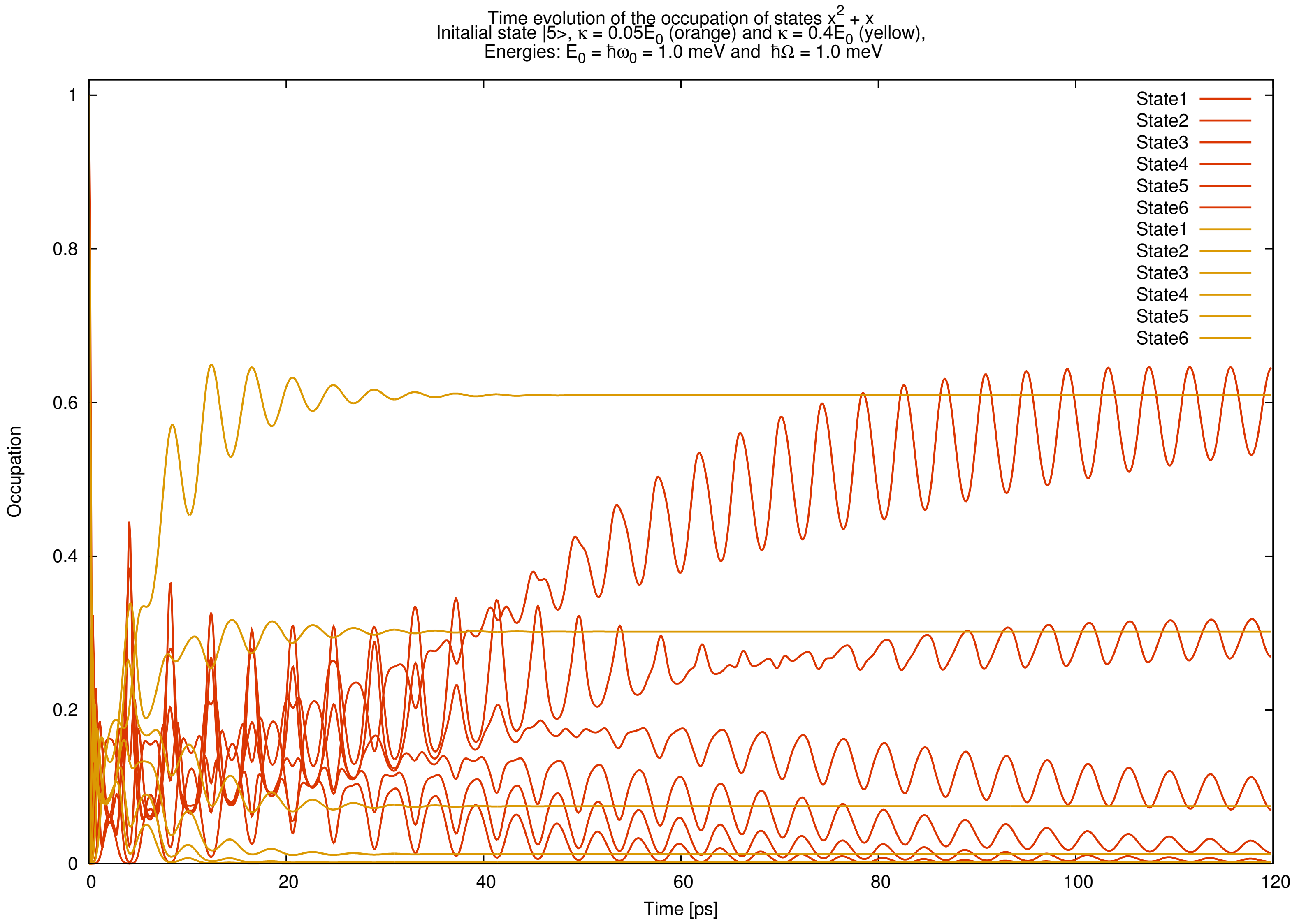
1. Simple quantum harmonic oscillator, H_0
2. $H_0 + 0.5\mathbb{x}$
3. $H_0 + 0.5\mathbb{x}^4$
4. $H_0 + \mathbb{x}$
5. $H_0 + \mathbb{x}^4$

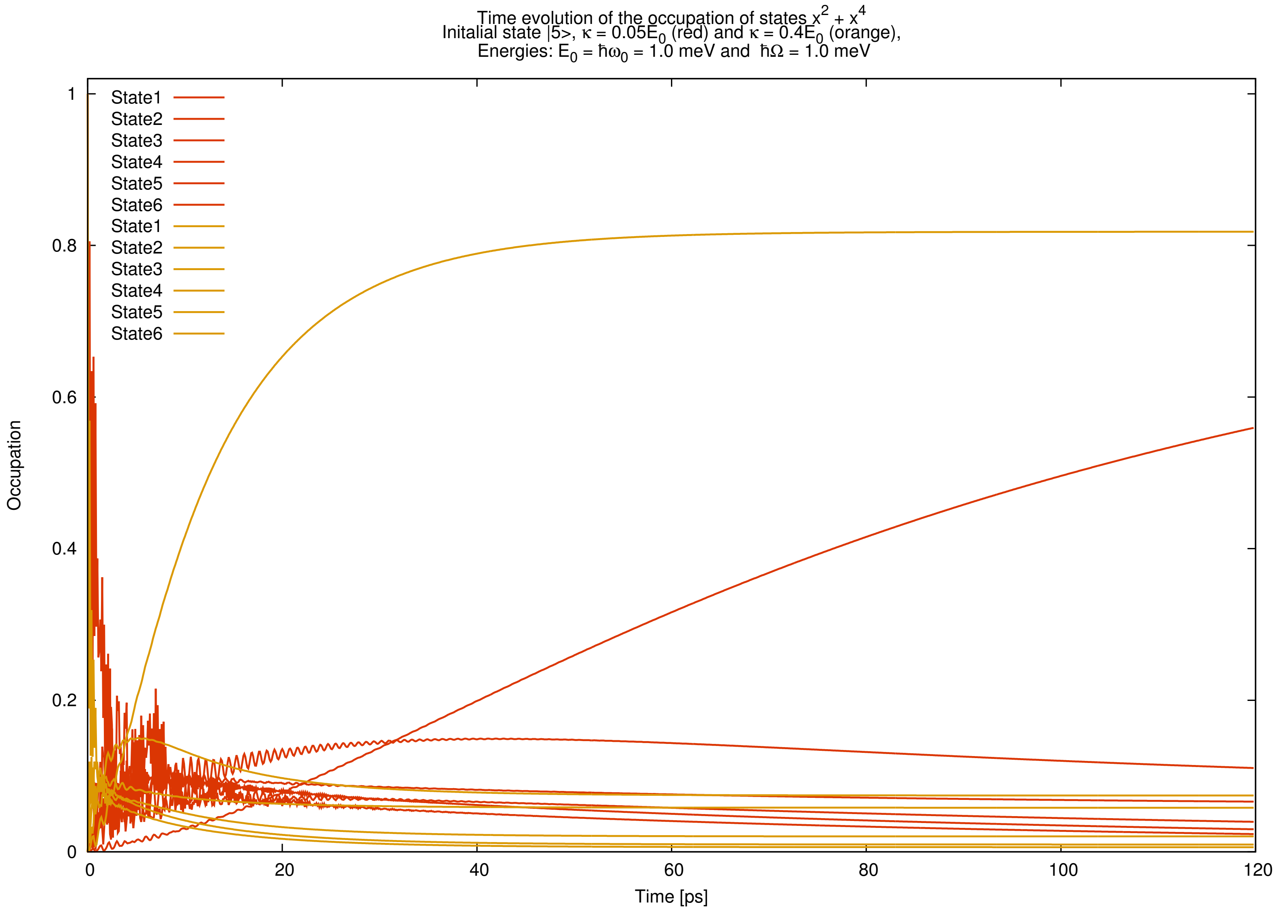

Time evolution of the occupation of states x^2
Initalial state |5>, κ = 0.05E0 (orange) and κ = 0.4E0 (yellow),
Energies: E0 = ħω0 = 1.0 meV and ħΩ = 0.0 meV
Occupation
time [ps]
0
20
40
60
80
100
120
0
0.2
0.4
0.6
0.8
1
State1
State2
State3
State4
State5
State6
State1
State2
State3
State4
State5
State6


Time evolution of the occupation of states x^2 + 0.5x
Initalial state |5>, κ = 0.05E_0 (orange) and κ = 0.4E_0 (yellow),
Energies: E_0 = ħω_0 = 1.0 meV and ħΩ = 0.5 meV
Occupation
Time [ps]
0
0.2
0.4
0.6
0.8
1
20
40
60
80
100
120
State1
State2
State3
State4
State5
State6
State1
State2
State3
State4
State5
State6



Time evolution of the occupation of states $x^2 + 0.5x^4$
Initalial state |5>, $\kappa = 0.05E_0$ (orange) and $\kappa = 0.4E_0$ (yellow),
Energies: $E_0 = \hbar\omega_0 = 1.0$ meV and $\hbar\Omega = 0.5$ meV
State1
State2
State3
State4
State5
State6
State1
State2
State3
State4
State5
State6
Occupation
Time [ps]
0
0.2
0.4
0.6
0.8
1
0
20
40
60
80
100
120


Time evolution of the occupation of states x^2 + x
Initalial state |5>, κ = 0.05E0 (orange) and κ = 0.4E0 (yellow),
Energies: E0 = ħω0 = 1.0 meV and ħΩ = 1.0 meV
Occupation
Time [ps]
0
0.2
0.4
0.6
0.8
1
20
40
60
80
100
120
State1
State2
State3
State4
State5
State6
State1
State2
State3
State4
State5
State6


Time evolution of the occupation of states $x^2 + x^4$
Initalial state |5>, $\kappa = 0.05E_0$ (red) and $\kappa = 0.4E_0$ (orange),
Energies: $E_0 = \hbar\omega_0 = 1.0$ meV and $\hbar\Omega = 1.0$ meV
State1
State2
State3
State4
State5
State6
State1
State2
State3
State4
State5
State6
Occupation
Time [ps]
0
0.2
0.4
0.6
0.8
1
0
20
40
60
80
100
120