Postdoc Active Sensing and Control for Hydrofoil Craft under Uncertain Loading

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1 Introduction

The goal of the project is clearly stated in the job ad which I quote it here again:
"The goal of the project is to leverage sensor fusion techniques to predict the state of hydrofoil craft and assign control inputs to optimize its response to random wave excitation. Using the HEARP hydrofoil education and research platform at TU Delft as a case study, the Postdoc will design optimal sensor layouts on the platform to robustly quantify uncertain parameters and excitation factors and will exploit these results to obtain robust optimal controllers. Aided by the expertise of both research groups, she(he) will develop control algorithms that will be tested in physical experiments on the hydrofoil platform to quantify controller performance and hydrofoil reliability."
This project is complex in nature, However to shape it in a structured way, we start from simple models and manageable assumptions. We then continue by adding further complexities after in-depth understanding of the dynamics of the system and analysing the possibilities of validating the results using experiments.

2 State-Space Model

The idea is to make a simple model in this section and add complexities step by step. This helps to understand the underlying dynamics for further model developments. To start with a simple model, several assumptions are made:

1. The operation mode of foilborne is considered at a constant speed of U.
2. Added mass is not considered
3. Lift and drag forces are assumed to be only a function of the angle of attack
4. External disturbances such as waves and wind effects are not modeled

5. A 2 degrees of freedom (DOF) model considering only pitch and heave motion will be made for control of the hydrofoil (The other motions can be controlled by a single-input-single-output (SISO) controllers included in the current HEARP program)

6. Drag forces are present

7. The model is linearized at the operating condition

8. The lever arm of Lift and Drag forces are considered constant in the calculation of moments.

9. A body-fixed coordinate system is located on the center of mass

10. The hydrofoil craft is symmetric about the xz plane (If physically not symmetric, the center of mass and moments of inertia can be adjusted by adding out-of-axis weights)

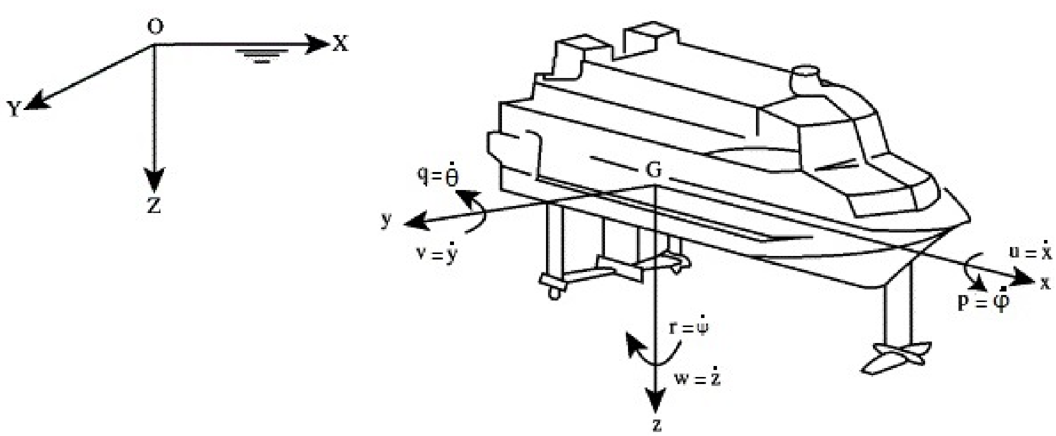

Figure 1: Body-fixed coordinate system

A body-fixed coordinate system is chosen to be located on the center of mass of the hydrofoil craft. The equations of motion of the hydrofoil will be:

$$\begin{aligned} m\dot{w} - mqU(0) &= \sum_i F_{z_i} \\ I_{yy}\dot{q} &= \sum_i M_{y_i} \end{aligned} \tag{1}$$

From Equation (1) and Figure 2:

$$
\begin{aligned}
m\dot{w} - mqU(0) \quad &= mg\cos(\theta) \\
&\quad - \frac{1}{2}\rho AU^2 C_{L_f}(\theta, \delta_f, \dot{\theta}, \dot{z})\cos(\alpha_{rf}) \\
&\quad - \frac{1}{2}\rho AU^2 C_{L_a}(\theta, \delta_a, \dot{\theta}, \dot{z})\cos(\alpha_{ra}) \\
&\quad - \frac{1}{2}\rho AU^2 C_{D_f}(\theta, \delta_f, \dot{\theta}, \dot{z})\sin(\alpha_{rf}) \\
&\quad - \frac{1}{2}\rho AU^2 C_{D_a}(\theta, \delta_a, \dot{\theta}, \dot{z})\sin(\alpha_{ra}) \\
I_{yy}\dot{q} &= \\
&\frac{1}{2}\rho AU^2 C_{L_f}(\theta, \delta_f, \dot{\theta}, \dot{z})d_1 \\
&\quad - \frac{1}{2}\rho AU^2 C_{L_a}(\theta, \delta_a, \dot{\theta}, \dot{z})d_2 \\
&\quad - \frac{1}{2}\rho AU^2 C_{D_f}(\theta, \delta_f, \dot{\theta}, \dot{z})d_3 \\
&\quad - \frac{1}{2}\rho AU^2 C_{D_a}(\theta, \delta_a, \dot{\theta}, \dot{z})d_4
\end{aligned}
\tag{2}
$$

in which lift and drag depend on the angle of attack which in turn is a function of geometrical angles θ, δ_a, and the relative flow direction $\dot{\theta}, \dot{z}$. A proper definition of angle of attack with respect to the geometrical angles and the relative flow direction will provide a good accuracy in modeling. Moreover, it makes it possible to incorporate uncertainties such as waves in the formulations in a structured fashion.

We linearize the equations of motion around the operating point of $(z_0, \theta_0, \dot{z}_0, \dot{\theta}_0)$ and control inputs of $(\delta_{f0}, \delta_{a0})$.

$$
\begin{aligned}
m\dot{w} - mqU(0) &= mg\cos(\theta) \\
&\quad - \frac{1}{2}\rho A U^2 \Bigg(C_{L_f}(\theta_0, \delta_{f0}, \dot{\theta}_0, \dot{z}_0) + \frac{\partial C_{L_f}}{\partial \theta}(\theta_0, \delta_{f0}, \dot{\theta}_0, \dot{z}_0)(\theta - \theta_0) \\
&\quad + \frac{\partial C_{L_f}}{\partial \dot{\theta}}(\theta_0, \delta_{f0}, \dot{\theta}_0, \dot{z}_0)(\dot{\theta} - \dot{\theta}_0) + \frac{\partial C_{L_f}}{\partial \dot{z}}(\theta_0, \delta_{f0}, \dot{\theta}_0, \dot{z}_0)(\dot{z} - \dot{z}_0) \\
&\quad + \Big(\frac{\partial C_{L_f}}{\partial \delta_f}(\theta_0, \delta_{f0}, \dot{\theta}_0, \dot{z}_0)\cos(\delta_{f_0}) - C_{L_f}(\theta_0, \delta_{f0}, \dot{\theta}_0, \dot{z}_0)\sin(\delta_{f0}) \Big)(\delta_f - \delta_{f0}) \Bigg) \\
&\quad - \frac{1}{2}\rho A U^2 \Bigg(C_{L_a}(\theta_0, \delta_{a0}, \dot{\theta}_0, \dot{z}_0) + \frac{\partial C_{L_a}}{\partial \theta}(\theta_0, \delta_{a0}, \dot{\theta}_0, \dot{z}_0)(\theta - \theta_0) \\
&\quad + \frac{\partial C_{L_a}}{\partial \dot{\theta}}(\theta_0, \delta_{a0}, \dot{\theta}_0, \dot{z}_0)(\dot{\theta} - \dot{\theta}_0) + \frac{\partial C_{L_a}}{\partial \dot{z}}(\theta_0, \delta_{a0}, \dot{\theta}_0, \dot{z}_0)(\dot{z} - \dot{z}_0) \\
&\quad + \Big(\frac{\partial C_{L_a}}{\partial \delta_a}(\theta_0, \delta_{a0}, \dot{\theta}_0, \dot{z}_0)\cos(\delta_{a_0}) - C_{L_a}(\theta_0, \delta_{a0}, \dot{\theta}_0, \dot{z}_0)\sin(\delta_{f0}) \Big)(\delta_a - \delta_{a0}) \Bigg) \\
&\quad - \frac{1}{2}\rho A U^2 \Bigg(C_{D_f}(\theta_0, \delta_{f0}, \dot{\theta}_0, \dot{z}_0) + \frac{\partial C_{D_f}}{\partial \theta}(\theta_0, \delta_{f0}, \dot{\theta}_0, \dot{z}_0)(\theta - \theta_0) \\
&\quad + \frac{\partial C_{D_f}}{\partial \dot{\theta}}(\theta_0, \delta_{f0}, \dot{\theta}_0, \dot{z}_0)(\dot{\theta} - \dot{\theta}_0) + \frac{\partial C_{D_f}}{\partial \dot{z}}(\theta_0, \delta_{f0}, \dot{\theta}_0, \dot{z}_0)(\dot{z} - \dot{z}_0) \\
&\quad + \Big(\frac{\partial C_{D_f}}{\partial \delta_f}(\theta_0, \delta_{f0}, \dot{\theta}_0, \dot{z}_0)\cos(\delta_{f_0}) - C_{D_f}(\theta_0, \delta_{f0}, \dot{\theta}_0, \dot{z}_0)\sin(\delta_{f0}) \Big)(\delta_f - \delta_{f0}) \Bigg) \\
&\quad - \frac{1}{2}\rho A U^2 \Bigg(C_{D_a}(\theta_0, \delta_{a0}, \dot{\theta}_0, \dot{z}_0) + \frac{\partial C_{D_a}}{\partial \theta}(\theta_0, \delta_{a0}, \dot{\theta}_0, \dot{z}_0)(\theta - \theta_0) \\
&\quad + \frac{\partial C_{D_a}}{\partial \dot{\theta}}(\theta_0, \delta_{a0}, \dot{\theta}_0, \dot{z}_0)(\dot{\theta} - \dot{\theta}_0) + \frac{\partial C_{D_a}}{\partial \dot{z}}(\theta_0, \delta_{a0}, \dot{\theta}_0, \dot{z}_0)(\dot{z} - \dot{z}_0) \\
&\quad + \Big(\frac{\partial C_{D_a}}{\partial \delta_a}(\theta_0, \delta_{a0}, \dot{\theta}_0, \dot{z}_0)\cos(\delta_{a_0}) - C_{D_a}(\theta_0, \delta_{a0}, \dot{\theta}_0, \dot{z}_0)\sin(\delta_{a0}) \Big)(\delta_a - \delta_{a0}) \Bigg)
\end{aligned} \tag{3}
$$

$$
\begin{aligned}
&I_{yy}\dot{q} = \\
&\quad \frac{1}{2}\rho A U^2 \Bigg(C_{L_f}(\theta_0, \delta_{f0}, \dot{\theta}_0, \dot{z_0}) + \frac{\partial C_{L_f}}{\partial \theta}(\theta_0, \delta_{f0}, \dot{\theta}_0, \dot{z_0})(\theta - \theta_0) \\
&\quad + \frac{\partial C_{L_f}}{\partial \dot{\theta}}(\theta_0, \delta_{f0}, \dot{\theta}_0, \dot{z_0})(\dot{\theta} - \dot{\theta}_0) + \frac{\partial C_{L_f}}{\partial \dot{z}}(\theta_0, \delta_{f0}, \dot{\theta}_0, \dot{z_0})(\dot{z} - \dot{z}_0) \\
&\quad + \Big(\frac{\partial C_{L_f}}{\partial \delta_f}(\theta_0, \delta_{f0}, \dot{\theta}_0, \dot{z_0}) \cos(\delta_{f_0}) - C_{L_f}(\theta_0, \delta_{f0}, \dot{\theta}_0, \dot{z_0}) \sin(\delta_{f0}) \Big)(\delta_f - \delta_{f0}) \Bigg) d_1 \\
&\quad - \frac{1}{2}\rho A U^2 \Bigg(C_{L_a}(\theta_0, \delta_{a0}, \dot{\theta}_0, \dot{z_0}) + \frac{\partial C_{L_a}}{\partial \theta}(\theta_0, \delta_{a0}, \dot{\theta}_0, \dot{z_0})(\theta - \theta_0) \\
&\quad + \frac{\partial C_{L_a}}{\partial \dot{\theta}}(\theta_0, \delta_{a0}, \dot{\theta}_0, \dot{z_0})(\dot{\theta} - \dot{\theta}_0) + \frac{\partial C_{L_a}}{\partial \dot{z}}(\theta_0, \delta_{a0}, \dot{\theta}_0, \dot{z_0})(\dot{z} - \dot{z}_0) \\
&\quad + \Big(\frac{\partial C_{L_a}}{\partial \delta_a}(\theta_0, \delta_{a0}, \dot{\theta}_0, \dot{z_0}) \cos(\delta_{a_0}) - C_{L_a}(\theta_0, \delta_{a0}, \dot{\theta}_0, \dot{z_0}) \sin(\delta_{f0}) \Big)(\delta_a - \delta_{a0}) \Bigg) d_2 \\
&\quad - \frac{1}{2}\rho A U^2 \Bigg(C_{L_f}(\theta_0, \delta_{f0}, \dot{\theta}_0, \dot{z_0}) + \frac{\partial C_{D_f}}{\partial \theta}(\theta_0, \delta_{f0}, \dot{\theta}_0, \dot{z_0})(\theta - \theta_0) \\
&\quad + \frac{\partial C_{D_f}}{\partial \dot{\theta}}(\theta_0, \delta_{f0}, \dot{\theta}_0, \dot{z_0})(\dot{\theta} - \dot{\theta}_0) + \frac{\partial C_{D_f}}{\partial \dot{z}}(\theta_0, \delta_{f0}, \dot{\theta}_0, \dot{z_0})(\dot{z} - \dot{z}_0) \\
&\quad + \Big(\frac{\partial C_{D_f}}{\partial \delta_f}(\theta_0, \delta_{f0}, \dot{\theta}_0, \dot{z_0}) \cos(\delta_{f_0}) - C_{D_f}(\theta_0, \delta_{f0}, \dot{\theta}_0, \dot{z_0}) \sin(\delta_{f0}) \Big)(\delta_f - \delta_{f0}) \Bigg) d_3 \\
&\quad - \frac{1}{2}\rho A U^2 \Bigg(C_{D_a}(\theta_0, \delta_{a0}, \dot{\theta}_0, \dot{z_0}) + \frac{\partial C_{D_a}}{\partial \theta}(\theta_0, \delta_{a0}, \dot{\theta}_0, \dot{z_0})(\theta - \theta_0) \\
&\quad + \frac{\partial C_{D_a}}{\partial \dot{\theta}}(\theta_0, \delta_{a0}, \dot{\theta}_0, \dot{z_0})(\dot{\theta} - \dot{\theta}_0) + \frac{\partial C_{D_a}}{\partial \dot{z}}(\theta_0, \delta_{a0}, \dot{\theta}_0, \dot{z_0})(\dot{z} - \dot{z}_0) \\
&\quad + \Big(\frac{\partial C_{D_a}}{\partial \delta_a}(\theta_0, \delta_{a0}, \dot{\theta}_0, \dot{z_0}) \cos(\delta_{a_0}) - C_{D_a}(\theta_0, \delta_{a0}, \dot{\theta}_0, \dot{z_0}) \sin(\delta_{f0}) \Big)(\delta_a - \delta_{a0}) \Bigg) d_4
\end{aligned}
\tag{4}
$$

in which:

$$
\begin{aligned}
\alpha_f &= \alpha_0 + \delta_f + \theta + \frac{d_f\dot{\theta} + \dot{z}}{U} \\
\alpha_a &= \alpha_0 + \delta_a + \theta + \frac{d_a\dot{\theta} + \dot{z}}{U}
\end{aligned}
\tag{5}
$$

$$
\begin{aligned}
\alpha_{rf} &= \alpha_f - \alpha_0 \\
\alpha_{ra} &= \alpha_a - \alpha_0
\end{aligned}
\tag{6}
$$

and the distances d_1 to d_4 are functions of θ and δ. These distances can be calculated using geometrical constants of d_a, d_f, d_h, L_a, L_h. In the first steps, for simplicity, we assume that d_1 to d_4 are constants. By striking the equilibrium forces and moments (steady-state operating condition) and replacing operating states $(z_0, \theta_0, \dot{z_0}, \dot{\theta_0}) = (0, 0, 0, 0)$ and control actions $(\delta_{f0}, \delta_{a0}) = (0, 0)$ the state-space model of the system can be made in matrix form.

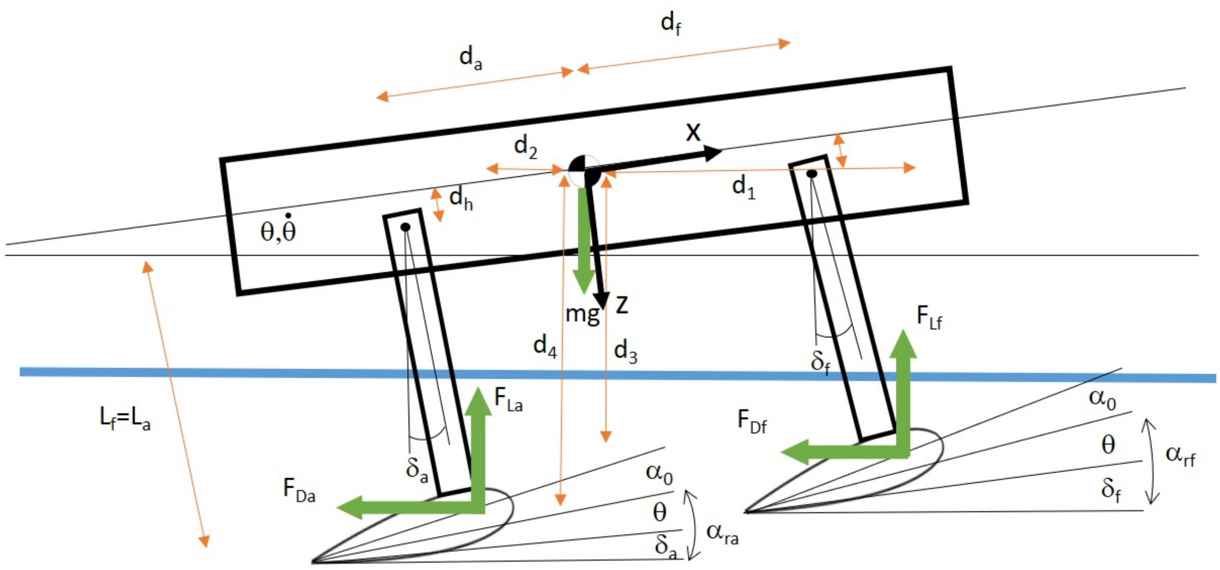

Figure 2: Hydrofoil schematic

The states are selected as heave, pitch, heave velocity and pitch angular velocity. The input of the system is δ_f and δ_a and the outputs are height from the mean water-level and pitch angle. The state space model is expressed as:

$$\begin{aligned} \dot{\boldsymbol{x}}(t) &= \mathbf{A}\boldsymbol{x}(t) + \mathbf{B}\boldsymbol{u}(t) \\ \boldsymbol{y}(t) &= \mathbf{C}\boldsymbol{x}(t) + \mathbf{D}\boldsymbol{u}(t) \end{aligned} \tag{7}$$

where:

$$\dot{\boldsymbol{x}} = \begin{bmatrix} \dot{z} \\ \dot{\theta} \\ \ddot{z} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} w \\ q \\ \dot{w} \\ \dot{q} \end{bmatrix} \qquad \boldsymbol{u} = \begin{bmatrix} \delta_f \\ \delta_a \end{bmatrix} \tag{8}$$

$$\begin{bmatrix} \dot{z} \\ \dot{\theta} \\ \ddot{z} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} w \\ q \\ \dot{w} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & A_{32} & A_{33} & A_{34} \\ 0 & A_{43} & A_{43} & A_{44} \end{bmatrix} \begin{bmatrix} z \\ \theta \\ \dot{z} \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ B_{31} & B_{32} \\ B_{41} & B_{42} \end{bmatrix} \begin{bmatrix} \delta_f \\ \delta_a \end{bmatrix} \tag{9}$$

in which the elements of $\mathbf{A}$ and $\mathbf{B}$ tensors are found from Equation (2): (See Details in Appendix A.

$$\begin{aligned} A_{32} = -\frac{1}{2m}\rho A U^2 \Bigg(& \frac{\partial C_{L_f}}{\partial \theta}(\theta_0, \delta_{f0}, \dot{\theta}_0, \dot{z}_0) + \frac{\partial C_{L_a}}{\partial \theta}(\theta_0, \delta_{a0}, \dot{\theta}_0, \dot{z}_0) \\ & + \frac{\partial C_{D_f}}{\partial \theta}(\theta_0, \delta_{f0}, \dot{\theta}_0, \dot{z}_0) + \frac{\partial C_{D_a}}{\partial \theta}(\theta_0, \delta_{a0}, \dot{\theta}_0, \dot{z}_0) \Bigg) \end{aligned} \tag{10}$$

$$\begin{aligned}A_{33} =& -\frac{1}{2m}\rho AU^2\Bigg(\frac{\partial C_{L_f}}{\partial \dot{z}}(\theta_0,\delta_{f0},\dot{\theta}_0,\dot{z}_0)+\frac{\partial C_{L_a}}{\partial \dot{z}}(\theta_0,\delta_{a0},\dot{\theta}_0,\dot{z}_0)\\ &+\frac{\partial C_{D_f}}{\partial \dot{z}}(\theta_0,\delta_{f0},\dot{\theta}_0,\dot{z}_0)+\frac{\partial C_{D_a}}{\partial \dot{z}}(\theta_0,\delta_{a0},\dot{\theta}_0,\dot{z}_0)\Bigg)\end{aligned} \tag{11}$$

$$\begin{aligned}A_{34} =& -\frac{1}{2m}\rho AU^2\Bigg(\frac{\partial C_{L_f}}{\partial \dot{\theta}}(\theta_0,\delta_{f0},\dot{\theta}_0,\dot{z}_0)+\frac{\partial C_{L_a}}{\partial \dot{\theta}}(\theta_0,\delta_{a0},\dot{\theta}_0,\dot{z}_0)\\ &+\frac{\partial C_{D_f}}{\partial \dot{\theta}}(\theta_0,\delta_{f0},\dot{\theta}_0,\dot{z}_0)+\frac{\partial C_{D_a}}{\partial \dot{\theta}}(\theta_0,\delta_{a0},\dot{\theta}_0,\dot{z}_0)\Bigg)\end{aligned} \tag{12}$$

$$\begin{aligned}A_{42} =& \frac{1}{2I_{yy}}\rho AU^2\Bigg(\frac{\partial C_{L_f}}{\partial \theta}(\theta_0,\delta_{f0},\dot{\theta}_0,\dot{z}_0)d_1-\frac{\partial C_{L_a}}{\partial \theta}(\theta_0,\delta_{a0},\dot{\theta}_0,\dot{z}_0)d_2\\ &-\frac{\partial C_{D_f}}{\partial \theta}(\theta_0,\delta_{f0},\dot{\theta}_0,\dot{z}_0)d_3-\frac{\partial C_{D_a}}{\partial \theta}(\theta_0,\delta_{a0},\dot{\theta}_0,\dot{z}_0)d_4\Bigg)\end{aligned} \tag{13}$$

$$\begin{aligned}A_{43} =& \frac{1}{2I_{yy}}\rho AU^2\Bigg(\frac{\partial C_{L_f}}{\partial \dot{z}}(\theta_0,\delta_{f0},\dot{\theta}_0,\dot{z}_0)d_1-\frac{\partial C_{L_a}}{\partial \dot{z}}(\theta_0,\delta_{a0},\dot{\theta}_0,\dot{z}_0)d_2\\ &-\frac{\partial C_{D_f}}{\partial \dot{z}}(\theta_0,\delta_{f0},\dot{\theta}_0,\dot{z}_0)d_3-\frac{\partial C_{D_a}}{\partial \dot{z}}(\theta_0,\delta_{a0},\dot{\theta}_0,\dot{z}_0)d_4\Bigg)\end{aligned} \tag{14}$$

$$\begin{aligned}A_{44} =& \frac{1}{2I_{yy}}\rho AU^2\Bigg(\frac{\partial C_{L_f}}{\partial \dot{\theta}}(\theta_0,\delta_{f0},\dot{\theta}_0,\dot{z}_0)d_1-\frac{\partial C_{L_a}}{\partial \dot{\theta}}(\theta_0,\delta_{a0},\dot{\theta}_0,\dot{z}_0)d_2\\ &-\frac{\partial C_{D_f}}{\partial \dot{\theta}}(\theta_0,\delta_{f0},\dot{\theta}_0,\dot{z}_0)d_3-\frac{\partial C_{D_a}}{\partial \dot{\theta}}(\theta_0,\delta_{a0},\dot{\theta}_0,\dot{z}_0)d_4\Bigg)\end{aligned} \tag{15}$$

$$B_{31} = -\frac{1}{2m}\rho AU^2\Bigg(\frac{\partial C_{L_f}}{\partial \delta_f}(\theta_0,\delta_{f0},\dot{\theta}_0,\dot{z}_0)+\frac{\partial C_{D_f}}{\partial \delta_f}(\theta_0,\delta_{a0},\dot{\theta}_0,\dot{z}_0)\Bigg) \tag{16}$$

$$B_{32} = -\frac{1}{2m}\rho AU^2\Bigg(\frac{\partial C_{L_a}}{\partial \delta_a}(\theta_0,\delta_{f0},\dot{\theta}_0,\dot{z}_0)+\frac{\partial C_{D_a}}{\partial \delta_a}(\theta_0,\delta_{a0},\dot{\theta}_0,\dot{z}_0)\Bigg) \tag{17}$$

$$B_{41} = \frac{1}{2I_{yy}}\rho AU^2\Bigg(\frac{\partial C_{L_f}}{\partial \delta_f}(\theta_0,\delta_{f0},\dot{\theta}_0,\dot{z}_0)d_1-\frac{\partial C_{D_f}}{\partial \delta_f}(\theta_0,\delta_{a0},\dot{\theta}_0,\dot{z}_0)d_3\Bigg) \tag{18}$$

$$B_{42} = -\frac{1}{2I_{yy}}\rho AU^2\Bigg(\frac{\partial C_{L_a}}{\partial \delta_a}(\theta_0,\delta_{f0},\dot{\theta}_0,\dot{z}_0)d_2+\frac{\partial C_{D_a}}{\partial \delta_a}(\theta_0,\delta_{a0},\dot{\theta}_0,\dot{z}_0)d_4\Bigg) \tag{19}$$

All the partial derivatives of Lift and drag coefficients with respect to various parameters can be obtained using the chain rule and equation (6).

For $\mathbf{C}$:

$$\mathbf{C} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{20}$$

2.1 LQR Controller design

To begin with a simple controller design we start using a Linear–quadratic regulator (LQR). the feedback control law is in form of:

$$u = -Kx \tag{21}$$

where K minimized the quadratic cost function of form:

$$J = \int_0^\infty (x^T Q x + u^T R u) dt \tag{22}$$

in which Q and R penalize motion and control actions respectively. The feedback control law that minimizes the above -mentioned cost function is defined by:

$$K = R^{-1} B^T P \tag{23}$$

where P is the solution of continuous time algebraic Riccati equation:

$$A^T P + PA - PBR^{-1} B^T P + Q = 0 \tag{24}$$

3 Uncertainty sources

Initial assumptions regarding uncertainty:

- **Location of applied forces (Lift and drag)**
  The lift force is the component of the total force which is perpendicular to the relative flow over the hydrofoil and the drag force is parallel to the relative flow over the hydrofoil. These forces are applied on the center of pressure. A common convention is to use a point specified at the airfoil quarter chord, the point which is located one quarter of the chord from the leading edge. For a symmetric airfoil as angle of attack and lift coefficient change, the center of pressure does not move. However, for an asymmetric case, the center of pressure is not necessarily close to the quarter chord point and this affects the dynamics of the system.

- **Center of mass**
  the center of mass moves due to the actuation of the foils. As those masses are far from the body the effect can be significant. This can be modeled in the dynamics of the system.

- **Presence of the waves**
  The wave models in the literature will be studied and will be presented.

- Wind effects

- All the measured quantities will be evaluated as they are prone to uncertainty.

3.1 Waves

The lift and drag forces applied on the foil depend on the angle of attack which is varied by the orbital motion of the waves, free surface effect, foil depth, foil motion, etc. [1]

To describe the kinematics of the waves, we start from the continuity equation [2]:

$$\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z} = 0 \tag{25}$$

by defining $\phi = \phi(x, y, z, t)$ as the velocity potential function such that:

$$\begin{aligned} \frac{\partial \phi}{\partial x} &= u_x \\ \frac{\partial \phi}{\partial y} &= u_y \\ \frac{\partial \phi}{\partial z} &= u_z \end{aligned} \tag{26}$$

we obtain Laplace equation as follows:

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0 \tag{27}$$

The boundary conditions are defined at the surface and at the bottom of the wave. At the surface the particles are not supposed to leave the wave and at the bottom the particles may not penetrate the bottom surface:

$$u_z = \frac{\partial \eta}{\partial t} = \frac{\partial \phi}{\partial z} \qquad at\ z = 0 \tag{28}$$

$$u_z = \frac{\partial \phi}{\partial x} = 0 \qquad at\ z = -d \tag{29}$$

in which η is the surface elevation, measured from the mean level of the wave as shown in figure 3. One of the analytical solutions of the Laplace equation with the boundary conditions

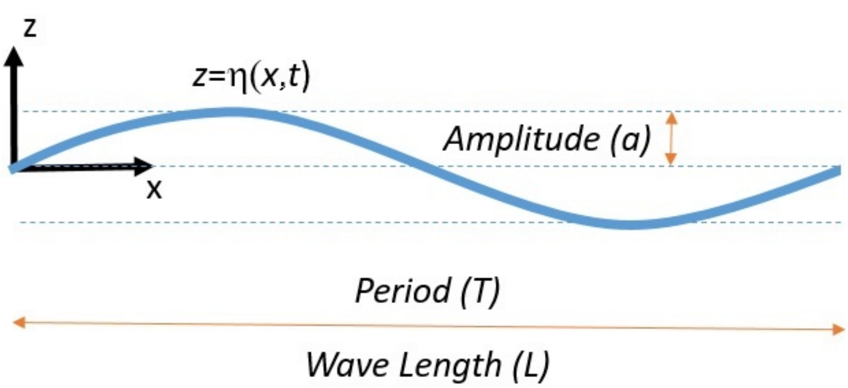

Figure 3: Propagation of harmonic wave

stated in equations (28) and (29) is:

$$\eta(x,t) = a\sin(\omega t - kx) \tag{30}$$

Where ω is the wave frequency, k is the wave number. This solution is referred to as long-crested harmonic wave with the following potential function:

$$\phi(x,t) = \hat{\phi}\cos(\omega t - kx) \tag{31}$$

in which:

$$\hat{\phi} = \frac{\omega a}{k}\frac{\cosh[k(d+z)]}{\sinh(kd)} \tag{32}$$

this solution is based on small-amplitude approximation and the amplitude of the wave must be smaller compared with the wave length and the water depth. From Equations (26) and (27) the velocities can be derived:

$$u_x = \omega a\frac{\cosh[k(d+z)]}{\sinh(kd)}\sin(\omega t - kx) \tag{33}$$

$$u_z = \omega a\frac{\sinh[k(d+z)]}{\sinh(kd)}\cos(\omega t - kx) \tag{34}$$

in deep water $kd \to \infty$ and he expressions of velocity reduce to:

$$u_x = \omega a e^{kz}\sin(\omega t - kx) \tag{35}$$

$$u_z = \omega a e^{kz}\cos(\omega t - kx) \tag{36}$$

The frequency in equations 38 and 39 is formulated in an earth-fixed reference frame as shown in figure 3. In a body-fixed reference frame on the hydrofoil the encounter frequency is used to describe the orbital motion of the waves [3]:

$$\omega_e = \omega + kU\cos(\beta) \tag{37}$$

in which , U is the vessel speed and β is the angle between the velocity of the foil and the direction of wave propagation. With this definition, $\beta = 0$ represents the head waves. considering that the body-fixed z axis is downwards and replacing the intrinsic wave frequency with the encounter frequency we obtain:

$$u_x = u_w = \omega a e^{-kh}\sin(\omega_e t - kx) \tag{38}$$

$$u_z = v_w = \omega a e^{-kh}\cos(\omega_e t - kx) \tag{39}$$

where, h defines the depth from the mean line of the wave. The orbital motion of the waves is one of the most important phenomena that affects the foil lift and angle of attack. The equations of motion in Equation (2) can be rewritten as:

$$
\begin{aligned}
m\dot{w} - mqU(0) &= mg\cos(\theta) \\
&\quad - \frac{1}{2}\rho AU^2 C_{L_f}(\theta, \delta_f, \dot{\theta}, \dot{z}, u_w, v_w)\cos(\alpha_{rf}) \\
&\quad - \frac{1}{2}\rho AU^2 C_{L_a}(\theta, \delta_a, \dot{\theta}, \dot{z}, u_w, v_w)\cos(\alpha_{ra}) \\
&\quad - \frac{1}{2}\rho AU^2 C_{D_f}(\theta, \delta_f, \dot{\theta}, \dot{z}, u_w, v_w)\sin(\alpha_{rf}) \\
&\quad - \frac{1}{2}\rho AU^2 C_{D_a}(\theta, \delta_a, \dot{\theta}, \dot{z}, u_w, v_w)\sin(\alpha_{ra}) \\
I_{yy}\dot{q} &= \\
&\quad \frac{1}{2}\rho AU^2 C_{L_f}(\theta, \delta_f, \dot{\theta}, \dot{z}, u_w, v_w)d_1 \\
&\quad - \frac{1}{2}\rho AU^2 C_{L_a}(\theta, \delta_f, \dot{\theta}, \dot{z}, u_w, v_w)d_2 \\
&\quad - \frac{1}{2}\rho AU^2 C_{D_f}(\theta, \delta_f, \dot{\theta}, \dot{z}, u_w, v_w)d_3 \\
&\quad - \frac{1}{2}\rho AU^2 C_{D_a}(\theta, \delta_f, \dot{\theta}, \dot{z}, u_w, v_w)d_4
\end{aligned}
\tag{40}
$$

Figure 4 schematically shows the motion of the hydrofoil in head waves. Recalling from

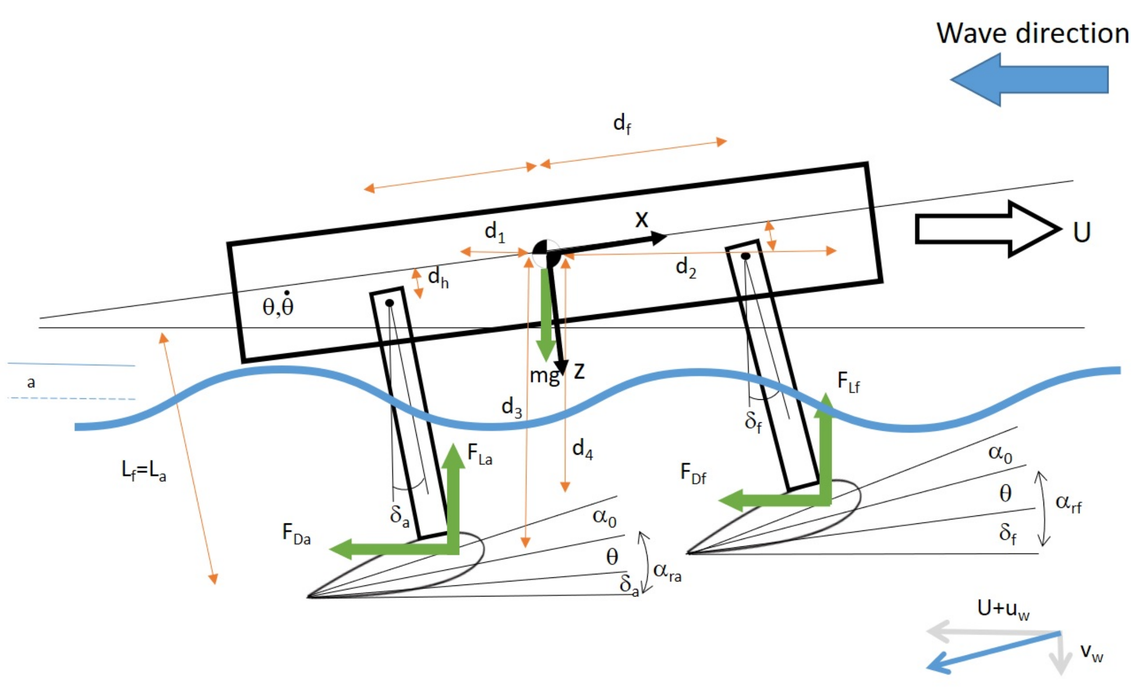

Figure 4: Hydrofoil motion in head waves

Equation (2), lift and drag depend on the angle of attack which in turn is a function of geometrical angles $\theta, \delta_a, \delta_f$ and the relative flow direction $\dot{\theta}, \dot{z}$. The contribution of wave orbital motion to the change of the angle of attack is considered by modifying equation 5 such that:

$$\begin{aligned}\alpha_f &= \alpha_0 + \delta_f + \theta + \frac{d_f\dot{\theta} + \dot{z}}{U} + \frac{v_{wf}}{U + u_{wf}} \\ \alpha_a &= \alpha_0 + \delta_a + \theta + \frac{d_a\dot{\theta} + \dot{z}}{U} + \frac{v_{wa}}{U + u_{wa}}\end{aligned} \tag{41}$$

where

$$\begin{aligned}\alpha_{rf} &= \alpha_f - \alpha_0 \\ \alpha_{ra} &= \alpha_a - \alpha_0\end{aligned} \tag{42}$$

where, u_{wf} and v_{wf} are function of (h, x, t) or in other words $u_{wf} = f(h) * g(x, t)$ and $v_{wf} = f(h) * g(x, t)$.

3.2 Stochastic Model Predictive Control

We model the dynamics of the uncertain system by:

$$\begin{aligned}\dot{\boldsymbol{x}}(t) &= \mathbf{A}\boldsymbol{x}(t) + \mathbf{B}\boldsymbol{u}(t) + \mathbf{G}\boldsymbol{w}(t) \\ \boldsymbol{y}(t) &= \mathbf{C}\boldsymbol{x}(t) + \mathbf{D}\boldsymbol{u}(t)\end{aligned} \tag{43}$$

where:

$$\dot{\boldsymbol{x}} = \begin{bmatrix}\dot{z} \\ \dot{\theta} \\ \ddot{z} \\ \ddot{\theta}\end{bmatrix} = \begin{bmatrix}w \\ q \\ \dot{w} \\ \dot{q}\end{bmatrix} \qquad \boldsymbol{u} = \begin{bmatrix}\delta_f \\ \delta_a\end{bmatrix} \qquad \boldsymbol{w} = \begin{bmatrix}u_w \\ v_w\end{bmatrix} \tag{44}$$

$$\begin{bmatrix}\dot{z} \\ \dot{\theta} \\ \ddot{z} \\ \ddot{\theta}\end{bmatrix} = \begin{bmatrix}w \\ q \\ \dot{w} \\ \dot{q}\end{bmatrix} = \begin{bmatrix}0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & A_{32} & A_{33} & A_{34} \\ 0 & A_{43} & A_{43} & A_{44}\end{bmatrix} \begin{bmatrix}z \\ \theta \\ \dot{z} \\ \dot{\theta}\end{bmatrix} + \begin{bmatrix}0 & 0 \\ 0 & 0 \\ B_{31} & B_{32} \\ B_{41} & B_{42}\end{bmatrix} \begin{bmatrix}\delta_f \\ \delta_a\end{bmatrix} + \begin{bmatrix}0 & 0 \\ 0 & 0 \\ G_{31} & G_{32} \\ G_{41} & G_{42}\end{bmatrix} \begin{bmatrix}u_w \\ v_w\end{bmatrix} \tag{45}$$

in which the constants of the $\mathbf{A}$, $\mathbf{B}$ and and $\mathbf{G}$ tensors are found from Equation (40).

References

[1] S.-H. Kim and H. Yamato, "An experimental study of the longitudinal motion control of a fully submerged hydrofoil model in following seas," *Ocean Engineering*, vol. 31, no. 5, pp. 523–537, 2004.

[2] L. H. Holthuijsen, *Waves in Oceanic and Coastal Waters.* Cambridge University Press, 2007.

[3] O. M. Faltinsen, *Hydrodynamics of High-Speed Marine Vehicles.* Cambridge University Press, 2006.

Appendices

A State-space model details

By striking the equilibrium forces and moments (steady-state operating condition) and replacing operating states $(z_0, \theta_0, \dot{z_0}, \dot{\theta_0}) = (0,0,0,0)$ and control actions $(\delta_{f0}, \delta_{a0}) = (0,0)$

$$
\begin{aligned}
m\dot{w} - mqU(0) &= \cancel{mg\cos(\theta)} \\
&- \frac{1}{2}\rho A U^2 \Bigg(\cancel{C_{L_f}(\theta_0, \delta_{f0}, \dot{\theta}_0, \dot{z}_0)} + \frac{\partial C_{L_f}}{\partial \theta}(\theta_0, \delta_{f0}, \dot{\theta}_0, \dot{z}_0)(\theta - \cancel{\theta_0}) \\
&+ \frac{\partial C_{L_f}}{\partial \dot{\theta}}(\theta_0, \delta_{f0}, \dot{\theta}_0, \dot{z}_0)(\dot{\theta} - \cancel{\dot{\theta}_0}) + \frac{\partial C_{L_f}}{\partial \dot{z}}(\theta_0, \delta_{f0}, \dot{\theta}_0, \dot{z}_0)(\dot{z} - \cancel{\dot{z}_0}) \\
&+ \Big(\frac{\partial C_{L_f}}{\partial \delta_f}(\theta_0, \delta_{f0}, \dot{\theta}_0, \dot{z}_0)\cancelto{1}{\cos(\delta_{f0})} - \cancel{C_{L_f}(\theta_0, \delta_{f0}, \dot{\theta}_0, \dot{z}_0)\sin(\delta_{f0})} \Big)(\delta_f - \cancel{\delta_{f0}}) \Bigg) \\
&- \frac{1}{2}\rho A U^2 \Bigg(\cancel{C_{L_a}(\theta_0, \delta_{a0}, \dot{\theta}_0, \dot{z}_0)} + \frac{\partial C_{L_a}}{\partial \theta}(\theta_0, \delta_{a0}, \dot{\theta}_0, \dot{z}_0)(\theta - \cancel{\theta_0}) \\
&+ \frac{\partial C_{L_a}}{\partial \dot{\theta}}(\theta_0, \delta_{a0}, \dot{\theta}_0, \dot{z}_0)(\dot{\theta} - \cancel{\dot{\theta}_0}) + \frac{\partial C_{L_a}}{\partial \dot{z}}(\theta_0, \delta_{a0}, \dot{\theta}_0, \dot{z}_0)(\dot{z} - \cancel{\dot{z}_0}) \\
&+ \Big(\frac{\partial C_{L_a}}{\partial \delta_a}(\theta_0, \delta_{a0}, \dot{\theta}_0, \dot{z}_0)\cancelto{1}{\cos(\delta_{a0})} - \cancel{C_{L_a}(\theta_0, \delta_{a0}, \dot{\theta}_0, \dot{z}_0)\sin(\delta_{f0})} \Big)(\delta_a - \cancel{\delta_{a0}}) \Bigg) \\
&- \frac{1}{2}\rho A U^2 \Bigg(\cancel{C_{D_f}(\theta_0, \delta_{f0}, \dot{\theta}_0, \dot{z}_0)} + \frac{\partial C_{D_f}}{\partial \theta}(\theta_0, \delta_{f0}, \dot{\theta}_0, \dot{z}_0)(\theta - \cancel{\theta_0}) \\
&+ \frac{\partial C_{D_f}}{\partial \dot{\theta}}(\theta_0, \delta_{f0}, \dot{\theta}_0, \dot{z}_0)(\dot{\theta} - \cancel{\dot{\theta}_0}) + \frac{\partial C_{D_f}}{\partial \dot{z}}(\theta_0, \delta_{f0}, \dot{\theta}_0, \dot{z}_0)(\dot{z} - \cancel{\dot{z}_0}) \\
&+ \Big(\frac{\partial C_{D_f}}{\partial \delta_f}(\theta_0, \delta_{f0}, \dot{\theta}_0, \dot{z}_0)\cancelto{1}{\cos(\delta_{f0})} - \cancel{C_{D_f}(\theta_0, \delta_{f0}, \dot{\theta}_0, \dot{z}_0)\sin(\delta_{f0})} \Big)(\delta_f - \cancel{\delta_{f0}}) \Bigg) \\
&- \frac{1}{2}\rho A U^2 \Bigg(\cancel{C_{D_a}(\theta_0, \delta_{a0}, \dot{\theta}_0, \dot{z}_0)} + \frac{\partial C_{D_a}}{\partial \theta}(\theta_0, \delta_{a0}, \dot{\theta}_0, \dot{z}_0)(\theta - \cancel{\theta_0}) \\
&+ \frac{\partial C_{D_a}}{\partial \dot{\theta}}(\theta_0, \delta_{a0}, \dot{\theta}_0, \dot{z}_0)(\dot{\theta} - \cancel{\dot{\theta}_0}) + \frac{\partial C_{D_a}}{\partial \dot{z}}(\theta_0, \delta_{a0}, \dot{\theta}_0, \dot{z}_0)(\dot{z} - \cancel{\dot{z}_0}) \\
&+ \Big(\frac{\partial C_{D_a}}{\partial \delta_a}(\theta_0, \delta_{a0}, \dot{\theta}_0, \dot{z}_0)\cancelto{1}{\cos(\delta_{a0})} - \cancel{C_{D_a}(\theta_0, \delta_{a0}, \dot{\theta}_0, \dot{z}_0)\sin(\delta_{a0})} \Big)(\delta_a - \cancel{\delta_{a0}}) \Bigg)
\end{aligned} \tag{46}
$$

$$
\begin{aligned}
&I_{yy}\dot{q} = \\
&\quad \frac{1}{2}\rho A U^2 \Bigg(\cancel{C_{L_f}(\theta_0, \delta_{f0}, \dot{\theta}_0, \dot{z}_0)} + \frac{\partial C_{L_f}}{\partial \theta}(\theta_0, \delta_{f0}, \dot{\theta}_0, \dot{z}_0)(\theta - \cancel{\theta_0}) \\
&\quad + \frac{\partial C_{L_f}}{\partial \dot{\theta}}(\theta_0, \delta_{f0}, \dot{\theta}_0, \dot{z}_0)(\dot{\theta} - \cancel{\dot{\theta}_0}) + \frac{\partial C_{L_f}}{\partial \dot{z}}(\theta_0, \delta_{f0}, \dot{\theta}_0, \dot{z}_0)(\dot{z} - \cancel{\dot{z}_0}) \\
&\quad + \Big(\frac{\partial C_{L_f}}{\partial \delta_f}(\theta_0, \delta_{f0}, \dot{\theta}_0, \dot{z}_0)\cancelto{1}{\cos(\delta_{f_0})} - \cancel{C_{L_f}(\theta_0, \delta_{f0}, \dot{\theta}_0, \dot{z}_0)\sin(\delta_{f0})} \Big)(\delta_f - \cancel{\delta_{f0}}) \Bigg) d1 \\
&\quad - \frac{1}{2}\rho A U^2 \Bigg(\cancel{C_{L_a}(\theta_0, \delta_{a0}, \dot{\theta}_0, \dot{z}_0)} + \frac{\partial C_{L_a}}{\partial \theta}(\theta_0, \delta_{a0}, \dot{\theta}_0, \dot{z}_0)(\theta - \cancel{\theta_0}) \\
&\quad + \frac{\partial C_{L_a}}{\partial \dot{\theta}}(\theta_0, \delta_{a0}, \dot{\theta}_0, \dot{z}_0)(\dot{\theta} - \cancel{\dot{\theta}_0}) + \frac{\partial C_{L_a}}{\partial \dot{z}}(\theta_0, \delta_{a0}, \dot{\theta}_0, \dot{z}_0)(\dot{z} - \cancel{\dot{z}_0}) \\
&\quad + \Big(\frac{\partial C_{L_a}}{\partial \delta_a}(\theta_0, \delta_{a0}, \dot{\theta}_0, \dot{z}_0)\cancelto{1}{\cos(\delta_{a_0})} - \cancel{C_{L_a}(\theta_0, \delta_{a0}, \dot{\theta}_0, \dot{z}_0)\sin(\delta_{f0})} \Big)(\delta_a - \cancel{\delta_{a0}}) \Bigg) d2 \\
&\quad - \frac{1}{2}\rho A U^2 \Bigg(\cancel{C_{D_f}(\theta_0, \delta_{f0}, \dot{\theta}_0, \dot{z}_0)} + \frac{\partial C_{D_f}}{\partial \theta}(\theta_0, \delta_{f0}, \dot{\theta}_0, \dot{z}_0)(\theta - \cancel{\theta_0}) \\
&\quad + \frac{\partial C_{D_f}}{\partial \dot{\theta}}(\theta_0, \delta_{f0}, \dot{\theta}_0, \dot{z}_0)(\dot{\theta} - \cancel{\dot{\theta}_0}) + \frac{\partial C_{D_f}}{\partial \dot{z}}(\theta_0, \delta_{f0}, \dot{\theta}_0, \dot{z}_0)(\dot{z} - \cancel{\dot{z}_0}) \\
&\quad + \Big(\frac{\partial C_{D_f}}{\partial \delta_f}(\theta_0, \delta_{f0}, \dot{\theta}_0, \dot{z}_0)\cancelto{1}{\cos(\delta_{f_0})} - \cancel{C_{D_f}(\theta_0, \delta_{f0}, \dot{\theta}_0, \dot{z}_0)\sin(\delta_{f0})} \Big)(\delta_f - \cancel{\delta_{f0}}) \Bigg) d3 \\
&\quad - \frac{1}{2}\rho A U^2 \Bigg(\cancel{C_{D_a}(\theta_0, \delta_{a0}, \dot{\theta}_0, \dot{z}_0)} + \frac{\partial C_{D_a}}{\partial \theta}(\theta_0, \delta_{a0}, \dot{\theta}_0, \dot{z}_0)(\theta - \cancel{\theta_0}) \\
&\quad + \frac{\partial C_{D_a}}{\partial \dot{\theta}}(\theta_0, \delta_{a0}, \dot{\theta}_0, \dot{z}_0)(\dot{\theta} - \cancel{\dot{\theta}_0}) + \frac{\partial C_{D_a}}{\partial \dot{z}}(\theta_0, \delta_{a0}, \dot{\theta}_0, \dot{z}_0)(\dot{z} - \cancel{\dot{z}_0}) \\
&\quad + \Big(\frac{\partial C_{D_a}}{\partial \delta_a}(\theta_0, \delta_{a0}, \dot{\theta}_0, \dot{z}_0)\cancelto{1}{\cos(\delta_{a_0})} - \cancel{C_{D_a}(\theta_0, \delta_{a0}, \dot{\theta}_0, \dot{z}_0)\sin(\delta_{a0})} \Big)(\delta_a - \cancel{\delta_{a0}}) \Bigg) d4
\end{aligned}
\tag{47}
$$