Adjoint-based data assimilation of sea surface height for the Baltic Sea

Tuomas Kärnä[1], Joe Wallwork[2], Stephan Kramer[2]
[1]Finnish Meteorological Institute
[2]Imperial College London

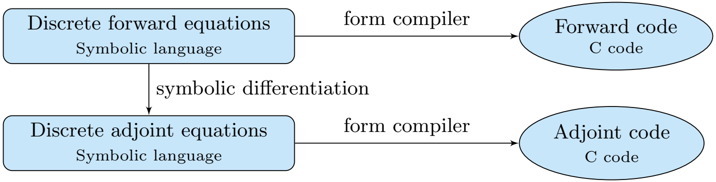
Ocean Sciences Meeting, March 2, 2022

ILMATIETEEN LAITOS
METEOROLOGISKA INSTITUTET
FINNISH METEOROLOGICAL INSTITUTE

Adjoint-based inverse modeling

1. Automatic derivation of the adjoint model
2. Application to parameter estimation:
   Optimizing bottom friction coefficient in a Baltic Sea simulation

Automated generation of an adjoint model



- Domain Specific Language modeling frameworks
- Model equations defined with a symbolic language (high level of abstraction)
- Automated code generator generates efficient C code at run time (low level)
- Adjoint model derived by differentiating the symbolic equations and using the same code generator
- \+ Exact discrete adjoint model
- \+ Computationally efficient implementation
- Better than traditional *automatic differentiation* that operates on source code level

Farrell et. al (2013). Automated Derivation of the Adjoint of High-Level Transient Finite Element Programs. SIAM J. Sci. Comput., 35(4), C369–C393. https://doi.org/10.1137/120873558.

Firedrake framework/ Thetis model

Firedrake

firedrakeproject.org

- Generic finite element modeling package
- Python embedded domain-specific language
- Automated adjoint capability
- `pyadjoint` package: tape etc.
- Efficient: Gradient evaluation cost ~4x forward model cost

THETIS Thetis ocean model

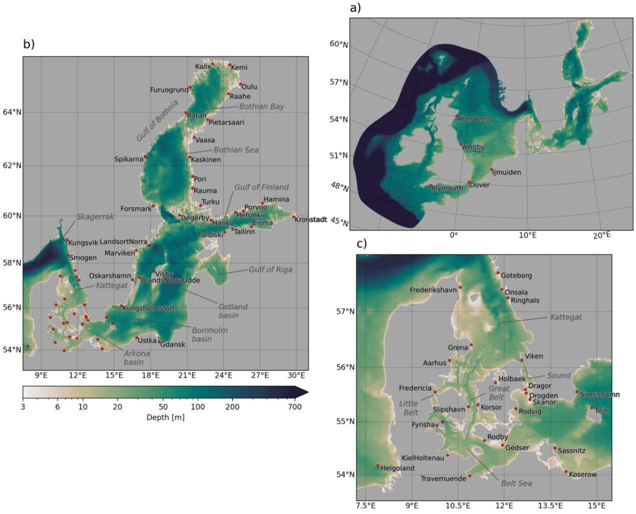
thetisproject.org

- Unstructured mesh ocean model
- Implemented on Firedrake
- Discontinuous Galerkin FEM
- Both 2D and 3D versions

Kärnä et al. (2018). Thetis coastal ocean model: discontinuous Galerkin discretization for the three-dimensional hydrostatic equations. Geosci. Model Dev., 11, 4359–4382. https://doi.org/10.5194/gmd-11-4359-2018.

Application: Baltic Sea water elevation model

- Thetis 2D shallow water model
- Covers North Sea and Baltic Sea
- Fully implicit solver
- Efficiency: 1800x real time
  1 day in 48 s; 1 month in 24 mins
- North Sea: Tidally-dominated
- Baltic Sea: No tides, seiche oscillations, slow mean level variability
- SSH observations for 60+ tide gauges



ILMATIETEEN LAITOS
METEOROLOGISKA INSTITUTET
FINNISH METEOROLOGICAL INSTITUTE

Optimizing bottom friction

- Cost function:

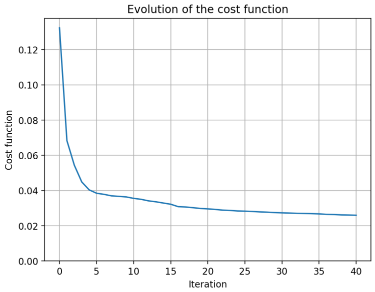
$$J(\mu) = \frac{1}{N_j N_i} \sum_{i,j} \frac{1}{Var(o_j)} \left((m_{j,i} - \overline{m_j}) - (o_{j,i} - \overline{o_j})\right)^2 + J_{reg}$$

- Mean-Square-Deviation, Bias removed, Scaled by observation variance
  - $o_{j,i}$, $m_{j,i}$: observation/model time series, j stations, i time steps
  - $\overline{m_j}$ time average
- J_{reg} additional regularization term
- Control variable: Manning friction coefficient μ
- Adjoint model provides the gradient $dJ/d\mu$
  $\Rightarrow$ can use any gradient-based optimization method

Optimization results

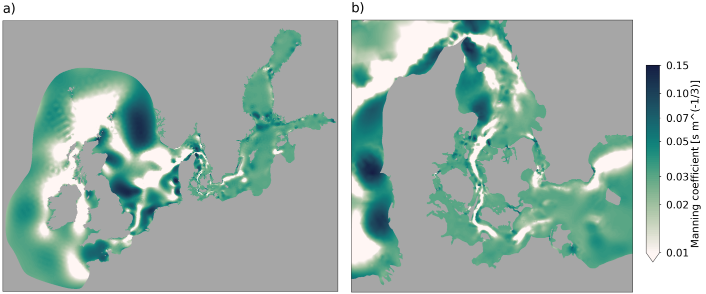
Optimized with a Quasi-Newton iteration (L-BGFS)

- Period: June 1 – June 17, 2019
- Initial Manning coeff. = 0.03
- 40 iterations



Optimized Manning field

- Large variability in the North-Sea; ⇝ Propagation of tides
- Lower friction in the Danish Straits; ⇝ Volume flux to Baltic Sea
- Higher friction in the Archipelago Sea; ⇝ Unresolved archpelago



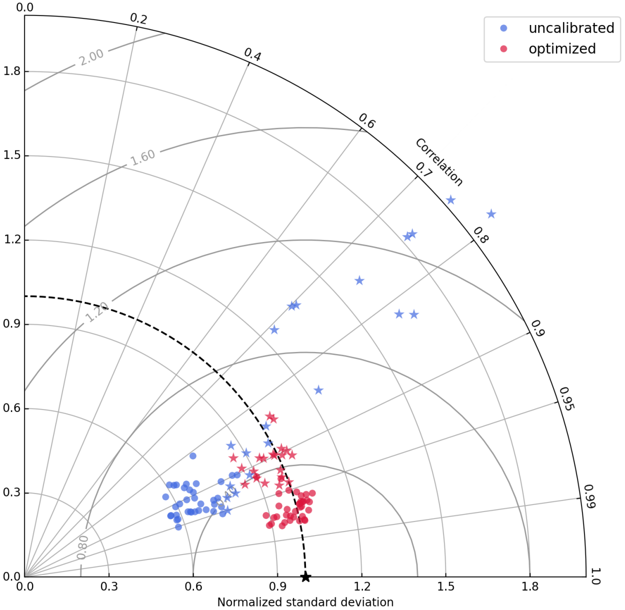
Validation

Optimization improves skill significantly

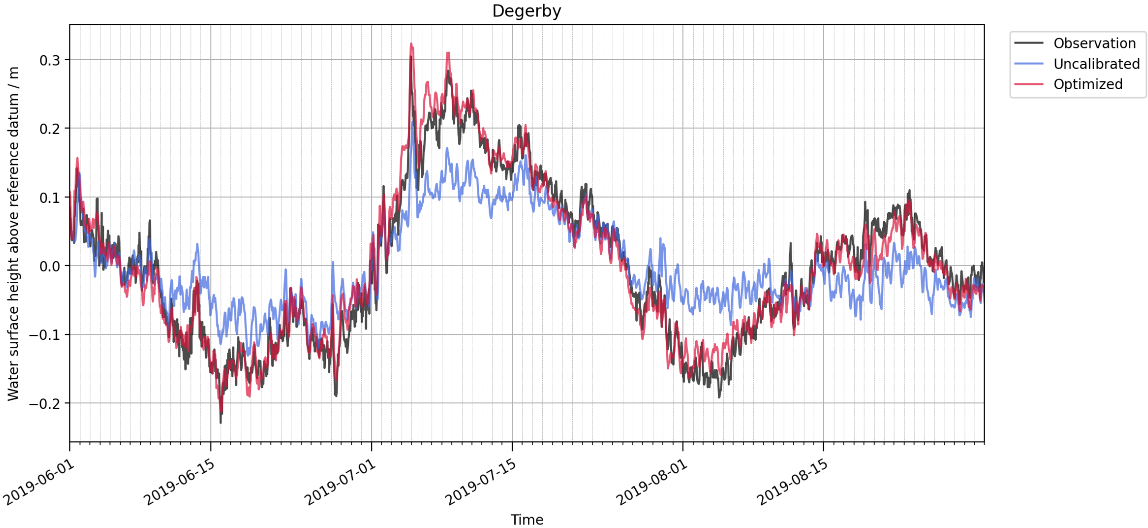
3 month simulation:
June – August, 2019.

- Centralized Root Mean Square Deviation (CRMSD)
- Baltic Sea
  - < 6 cm
  - typ. ~ 3 cm
- Danish Straits (DS + Kat, $\star$)
  - < 8 cm
- Standard deviation (STDDEV) close to observation (black markers)

Overall performance comparable to best forecast models



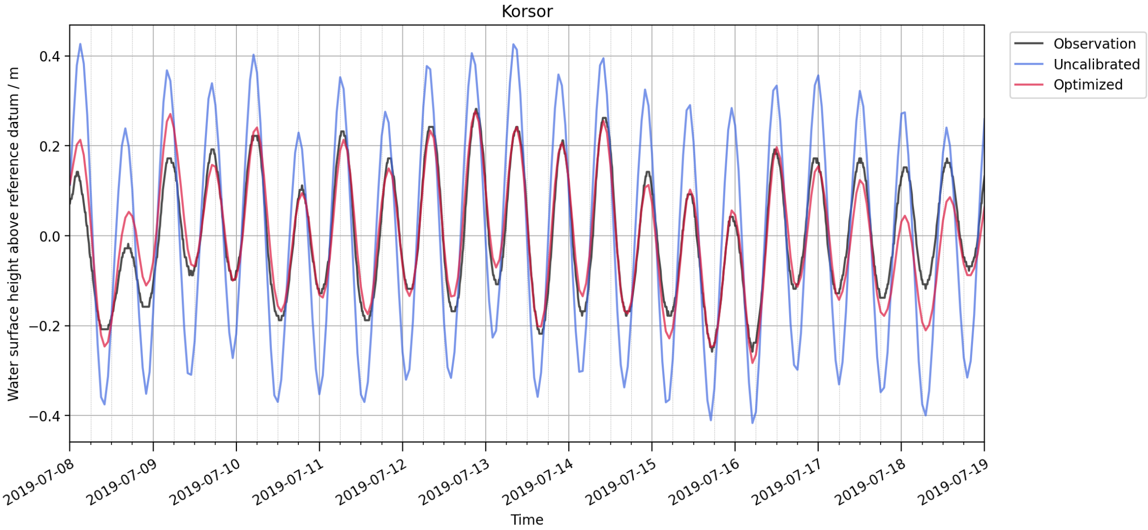
Validation: time series examples



Correct wind-driven volume changes in the Baltic Sea.

ILMATIETEEN LAITOS
METEOROLOGISKA INSTITUTET
FINNISH METEOROLOGICAL INSTITUTE

Validation: time series examples



Correct tidal amplitude in the Danish Straits.

Take-home messages

1. Automatic derivation of the adjoint model
   - Domain specific language modeling frameworks offer flexibility and computational efficiency
   - The adjoint model can be derived automatically
2. Application to parameter estimation:
   Optimizing bottom friction coefficient in a Baltic Sea simulation
   - Adjoint model is accurate and efficient
   - Optimized model delivers excellent performance