

# Group 3 Final Report

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## Introduction

The Industrial Revolution has been a boon to humanity, leading to longer life spans, increased wealth, easy access to clean water, it gave us Netflix...but with every benefit there comes a cost, and the main cost with expansion of human flourishing has been what we now call climate change. This term encapsulates a host of changes to our environment such as rising

sea levels, more extreme weather events, reduction in biodiversity, etc. The climate changing isn't particularly new, the climate is always changing. What's unique about this period in Earth's history is the rate at which everything is changing. And the consensus is that rate of change is driven by human activity dumping CO2 emissions into the atmosphere, driving up CO2 concentration in the atmosphere, causing the global temperature to rise and leading to these rapid ecosystem alterations.

## Project Problem/Statement (might need to generalize this since CO2 emissions came about through testing different predictors)

This project was inspired by a desire to get involved in understanding the scope of the problem and hopefully finding a way to solve or mitigate it. Specifically we wanted to find a way to accurately isolate the impact of CO2 emissions on the increasing temperature trend, and by extension an individual's (defined as nation, state, city, organization etc) impact on the warming trend. The belief is that by quantifying impact, people will be better equipped to take action on changing their behavior and upgrading systems in ways that slow or reverse the warming trends.

## Data Overview

```
df <- read_csv("data/global_temp.csv")  
  
dim(df)
```

```
[1] 2109    12
```

```
str(df)
```

```
spec_tbl_ [2,109 x 12] (S3: spec_tbl_df/tbl_df/tbl/data.frame)  
 $ Year           : num [1:2109] 1850 1850 1850 1850 1850 1850 1850 1850 1850 1850 ...  
 $ Month          : num [1:2109] 1 2 3 4 5 6 7 8 9 10 ...  
 $ Monthly_Anomaly : num [1:2109] -0.469 -0.66 -0.427 -0.81 -0.375 -0.362 -0.331 -0.256 -0.2 ...  
 $ Monthly_Unc     : num [1:2109] 0.662 0.456 0.611 0.347 0.44 0.384 0.416 0.311 0.225 0.2 ...  
 $ Annual_Anomaly  : num [1:2109] NA NA NA NA NA -0.427 -0.424 -0.396 -0.389 -0.384 ...  
 $ Annual_Unc      : num [1:2109] NA NA NA NA NA 0.232 0.233 0.218 0.205 0.206 ...  
 $ FiveYear_Anomaly : num [1:2109] NA NA NA NA NA NA NA NA NA NA NA ...  
 $ FiveYear_Unc    : num [1:2109] NA NA NA NA NA NA NA NA NA NA NA ...
```

```

$ TenYear_Anomaly    : num [1:2109] NA NA NA NA NA NA NA NA NA NA NA ...
$ TenYear_Unc        : num [1:2109] NA NA NA NA NA NA NA NA NA NA NA ...
$ TwentyYear_Anomaly: num [1:2109] NA NA NA NA NA NA NA NA NA NA NA ...
$ TwentyYear_Unc     : num [1:2109] NA NA NA NA NA NA NA NA NA NA NA ...
- attr(*, "spec")=
.. cols(
..   Year = col_double(),
..   Month = col_double(),
..   Monthly_Anomaly = col_double(),
..   Monthly_Unc = col_double(),
..   Annual_Anomaly = col_double(),
..   Annual_Unc = col_double(),
..   FiveYear_Anomaly = col_double(),
..   FiveYear_Unc = col_double(),
..   TenYear_Anomaly = col_double(),
..   TenYear_Unc = col_double(),
..   TwentyYear_Anomaly = col_double(),
..   TwentyYear_Unc = col_double()
.. )
- attr(*, "problems")=<externalptr>

```

```
summary(df[3:12])
```

Monthly_Anomaly	Monthly_Unc	Annual_Anomaly	Annual_Unc
Min. : -0.813000	Min. : 0.0200	Min. : -0.564000	Min. : 0.0150
1st Qu.: -0.285000	1st Qu.: 0.0690	1st Qu.: -0.278000	1st Qu.: 0.0370
Median : -0.087000	Median : 0.1560	Median : -0.106000	Median : 0.0960
Mean : 0.006927	Mean : 0.1819	Mean : 0.005231	Mean : 0.1099
3rd Qu.: 0.199000	3rd Qu.: 0.2620	3rd Qu.: 0.179750	3rd Qu.: 0.1680
Max. : 1.449000	Max. : 0.7700	Max. : 1.305000	Max. : 0.3600
		NA's : 11	NA's : 11
FiveYear_Anomaly	FiveYear_Unc	TenYear_Anomaly	TenYear_Unc
Min. : -0.476000	Min. : 0.01100	Min. : -0.43600	Min. : 0.00900
1st Qu.: -0.291000	1st Qu.: 0.02700	1st Qu.: -0.28700	1st Qu.: 0.02200
Median : -0.090000	Median : 0.08200	Median : -0.06150	Median : 0.08000
Mean : -0.003982	Mean : 0.09376	Mean : -0.01355	Mean : 0.08783
3rd Qu.: 0.190750	3rd Qu.: 0.15600	3rd Qu.: 0.13250	3rd Qu.: 0.14300
Max. : 1.059000	Max. : 0.24100	Max. : 1.00400	Max. : 0.20000
NA's : 59	NA's : 59	NA's : 119	NA's : 119
TwentyYear_Anomaly	TwentyYear_Unc		
Min. : -0.36800	Min. : 0.00800		
1st Qu.: -0.29100	1st Qu.: 0.01500		

Median	:-0.04350	Median	:0.08300
Mean	:-0.02976	Mean	:0.07891
3rd Qu.	: 0.07475	3rd Qu.	:0.11700
Max.	: 0.83500	Max.	:0.17900
NA's	:239	NA's	:239

The data from **Berkely Earth** (insert ref) comes in the form of monthly anomalies. They use a baseline period (1950-1980) and record temperature observations as differences from the average temp (by month) of this period. They include the uncertainties, and 5, 10, and 20 year lag periods. For our purposes, we're only concerned with the Monthly\_Anomaly column

## Convert to Tsibble and Temperatures

```
temp_data <- df |> mutate(
  Month = month.abb[Month],
  Date = yearmonth(paste(Month, Year)),
  Monthly_Anomaly = replace_na(Monthly_Anomaly, mean(Monthly_Anomaly, na.rm = T))) |>
  select(c(Date, Monthly_Anomaly)) |>
  as_tsibble(index = Date)

baseline_vector <- c(
  '1' = 12.30, '2' = 12.50, '3' = 13.13, '4' = 14.06,
  '5' = 15.00, '6' = 15.66, '7' = 15.90, '8' = 15.75,
  '9' = 15.18, '10' = 14.27, '11' = 13.28, '12' = 12.57
)

converted_df <- temp_data |>
  mutate(
    month_char = as.character(month(Date)),
    baseline_temp = baseline_vector[month_char],
    actual_temp = baseline_temp + Monthly_Anomaly
  ) |>
  select(c(Date, actual_temp))
```

Reporting in monthly anomalies is common for publications given that the main interest is usually in the overall trend of warming. However for our purposes, since we are attempting to isolate the human effect on the warming trend, we also need to be able to accurately model temperature separate of human impact. The seasonality of global temperatures is a big part of that and using only the monthly anomaly measurements largely strips out that component. Both sets of data will be explored and modeled however to see which one is most useful and it may turn out that each is valuable in different capacities.

## Exploratory Data Analysis of Global Anomalies

```
climate_raw <- read_csv(  
  "data/global_temp.csv",  
  #skip = 1 # skip "Land-Ocean Temperature Index (C)"  
)
```

```
climate_ts <- climate_raw %>%  
  mutate(date = yearmonth(paste(Year, Month, sep = "-"))) %>%  
  as_tsibble(index = date) %>%  
  select(date, Monthly_Anomaly)
```

```
glimpse(climate_raw)
```

```
Rows: 2,109  
Columns: 12  
$ Year          <dbl> 1850, 1850, 1850, 1850, 1850, 1850, 1850, 1850, 185~  
$ Month         <dbl> 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 1, 2, 3, 4, ~  
$ Monthly_Anomaly <dbl> -0.469, -0.660, -0.427, -0.810, -0.375, -0.362, -0.~  
$ Monthly_Unc    <dbl> 0.662, 0.456, 0.611, 0.347, 0.440, 0.384, 0.416, 0.~  
$ Annual_Anomaly <dbl> NA, NA, NA, NA, NA, NA, -0.427, -0.424, -0.396, -0.389,~  
$ Annual_Unc     <dbl> NA, NA, NA, NA, NA, NA, 0.232, 0.233, 0.218, 0.205, 0.2~  
$ FiveYear_Anomaly <dbl> NA, NA, NA, NA, NA, NA, NA, NA, NA, NA, NA, NA, NA, NA,~  
$ FiveYear_Unc    <dbl> NA, NA, NA, NA, NA, NA, NA, NA, NA, NA, NA, NA, NA, NA,~  
$ TenYear_Anomaly <dbl> NA, NA, NA, NA, NA, NA, NA, NA, NA, NA, NA, NA, NA, NA,~  
$ TenYear_Unc     <dbl> NA, NA, NA, NA, NA, NA, NA, NA, NA, NA, NA, NA, NA, NA,~  
$ TwentyYear_Anomaly <dbl> NA, NA, NA, NA, NA, NA, NA, NA, NA, NA, NA, NA, NA, NA,~  
$ TwentyYear_Unc  <dbl> NA, NA, NA, NA, NA, NA, NA, NA, NA, NA, NA, NA, NA, NA,~
```

```
names(climate_raw)
```

```
[1] "Year"          "Month"          "Monthly_Anomaly"  
[4] "Monthly_Unc"   "Annual_Anomaly" "Annual_Unc"  
[7] "FiveYear_Anomaly" "FiveYear_Unc"   "TenYear_Anomaly"  
[10] "TenYear_Unc"   "TwentyYear_Anomaly" "TwentyYear_Unc"
```

This dataset contains 2,109 monthly observations from 1850–2025, with global temperature anomalies and uncertainties, where **Monthly\_Anomaly** is the primary usable series and most multi-year anomaly fields contain NA values except at the end of their smoothing windows.

*Do we want to add a visual here of the time series? Otherwise it's not really clear that there are year-to-year fluctuations or that the LOWESS-smoothed line clarifies the upward trend (I deleted it out)*

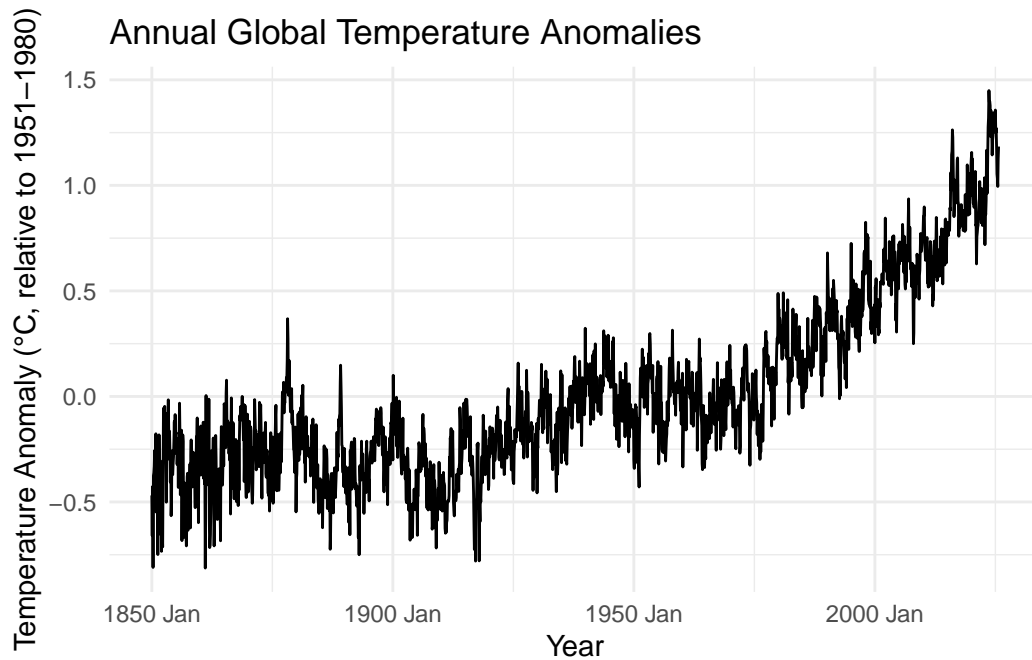
```
climate_ts <- climate_raw |>
  mutate(
    date = yearmonth(paste(Year, Month, sep = "-"))
  ) |>
  as_tsibble(index = date)

climate_ts
```

```
# A tsibble: 2,109 x 13 [1M]
  Year Month Monthly_Anomaly Monthly_Unc Annual_Anomaly Annual_Unc
  <dbl> <dbl>          <dbl>         <dbl>         <dbl>      <dbl>
1  1850     1        -0.469         0.662          NA         NA
2  1850     2        -0.66         0.456          NA         NA
3  1850     3       -0.427         0.611          NA         NA
4  1850     4       -0.81         0.347          NA         NA
5  1850     5       -0.375         0.44           NA         NA
6  1850     6       -0.362         0.384        -0.427      0.232
7  1850     7       -0.331         0.416        -0.424      0.233
8  1850     8       -0.256         0.311        -0.396      0.218
9  1850     9       -0.33         0.225        -0.389      0.205
10 1850    10       -0.376         0.281        -0.384      0.206
# i 2,099 more rows
# i 7 more variables: FiveYear_Anomaly <dbl>, FiveYear_Unc <dbl>,
#   TenYear_Anomaly <dbl>, TenYear_Unc <dbl>, TwentyYear_Anomaly <dbl>,
#   TwentyYear_Unc <dbl>, date <mth>
```

The 1880s exhibit consistently negative temperature anomalies, with both raw and smoothed values showing a clear cool period centered around 1885–1887.

```
climate_ts |>
  autoplot(Monthly_Anomaly) +
  labs(
    title = "Annual Global Temperature Anomalies",
    x = "Year",
    y = "Temperature Anomaly (°C, relative to 1951-1980)"
  ) +
  theme_minimal()
```



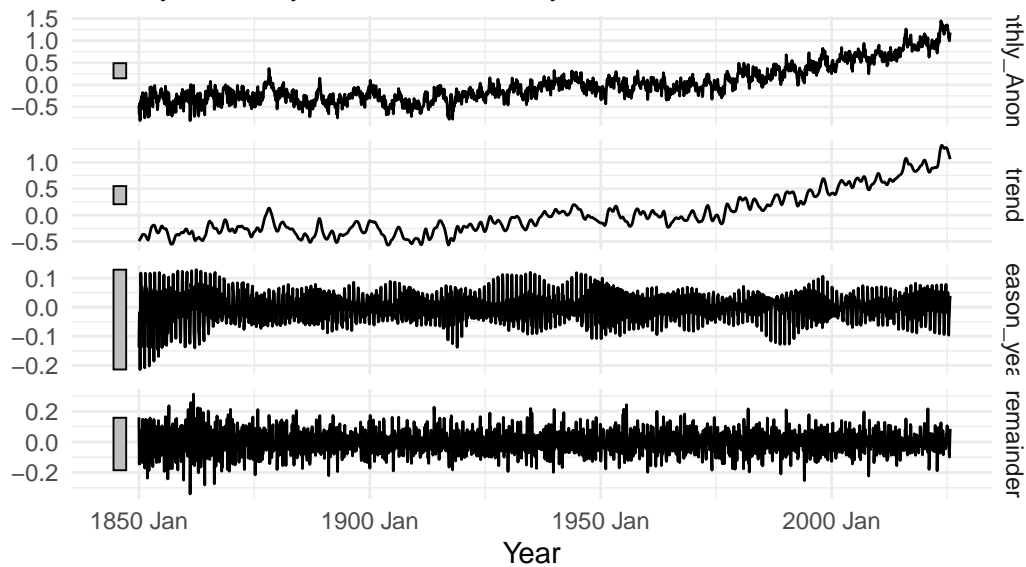
Temperatures fluctuate annually but show a clear upward trend, with modern anomalies far exceeding those of the early record.

```
climate_stl <- climate_ts |>
  model(
    STL(Monthly_Anomaly ~ trend(window = 15))
  ) |>
  components()

autoplot(climate_stl) +
  labs(
    title = "STL Decomposition of Annual Temperature Anomalies",
    x = "Year"
  ) +
  theme_minimal()
```

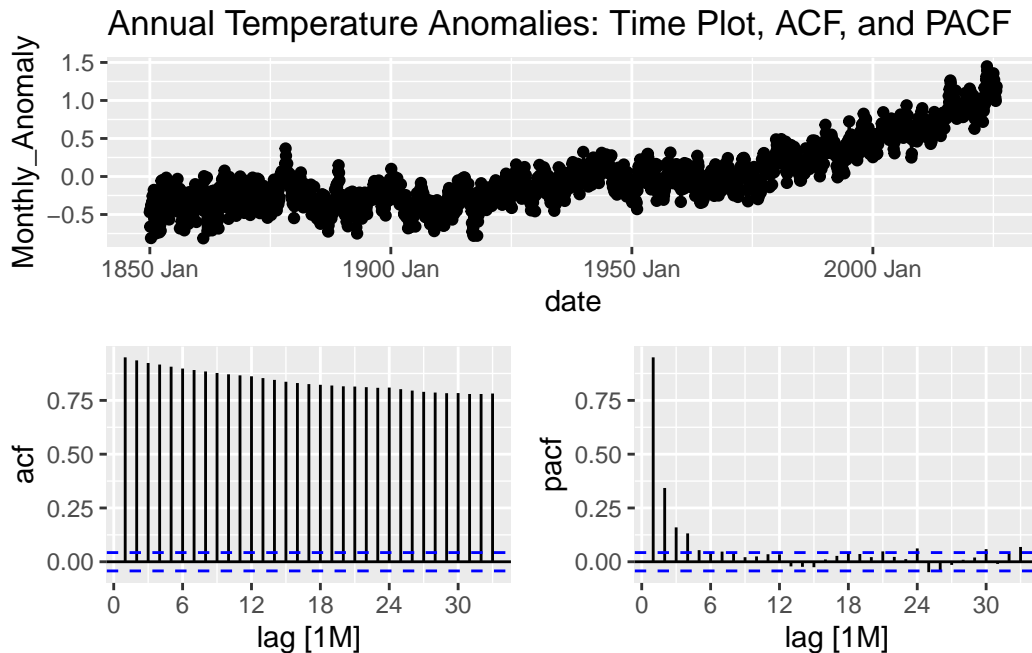
## STL Decomposition of Annual Temperature Anomalies

Monthly\_Anomaly = trend + season\_year + remainder



The STL results highlight how the underlying warming signal has grown stronger over time, while the remainder shows noisy but relatively modest deviations from the trend. This reinforces that annual variability exists but is dwarfed by the long-term increase in global temperatures.

```
climate_ts |>
  gg_tsdisplay(Monthly_Anomaly, plot_type = "partial") +
  labs(
    title = "Annual Temperature Anomalies: Time Plot, ACF, and PACF"
  )
```



The climate series exhibits a persistent warming trend, and the ACF's slow decay underscores how each year's temperature is tightly linked to previous years. The PACF's immediate drop-off reinforces that this warming signal creates strong year-to-year inertia in global temperatures.

```
# KPSS
climate_ts |>
  features(Monthly_Anomaly, unitroot_kpss)
```

```
# A tibble: 1 x 2
  kpss_stat kpss_pvalue
  <dbl>      <dbl>
1    17.4         0.01
```

The KPSS test confirms what the plots already suggest: the anomaly series isn't stationary, with the low p-value indicating that the warming trend dominates over random fluctuations.

```
# Non-seasonal differencing needed?
climate_ts |>
  features(Monthly_Anomaly, unitroot_ndiffs)
```

```
# A tibble: 1 x 1
```

```
ndiffs
<int>
1      1
```

```
climate_ts |>
  features(Monthly_Anomaly, guerrero)
```

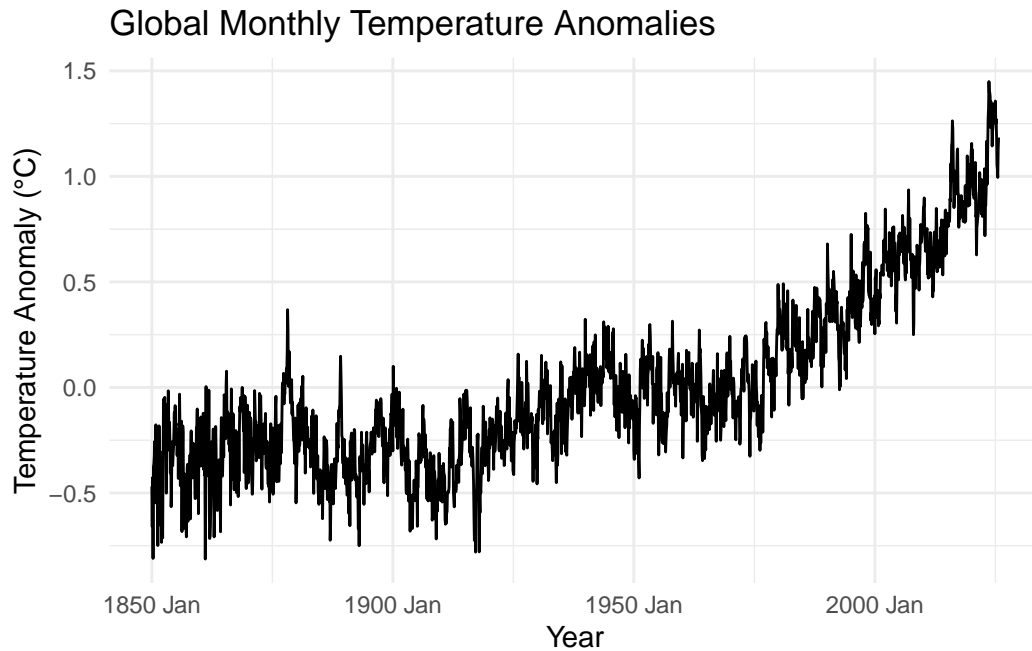
```
# A tibble: 1 x 1
  lambda_guerrero
      <dbl>
1          0.966
```

1.16 → little to no Box–Cox transformation needed; variance is already stable. Unitroot test suggests differencing may be needed

```
temp_ts <- climate_ts|>
  mutate(
    Month = yearmonth(paste(Year, Month, sep = "-"))
  )|>
  # rename Monthly_Anomaly -> Anomaly so the rest of the code works
  rename(Anomaly = Monthly_Anomaly)|>
  as_tsibble(index = Month)|>
  arrange(Month)

train <- temp_ts|> filter(Month < yearmonth("2020 Jan"))
test  <- temp_ts|> filter(Month >= yearmonth("2020 Jan"))

autoplot(temp_ts, Anomaly) +
  labs(title = "Global Monthly Temperature Anomalies",
       x = "Year", y = "Temperature Anomaly (°C)") +
  theme_minimal()
```



The plot shows that what was once mostly cooler-than-average (average as defined as the period between 1950 - 1980) global temperatures has transitioned into a new normal of sustained warming, particularly over the last 40 years. The sharp upward trend in recent decades signals the accelerating pace of climate change.

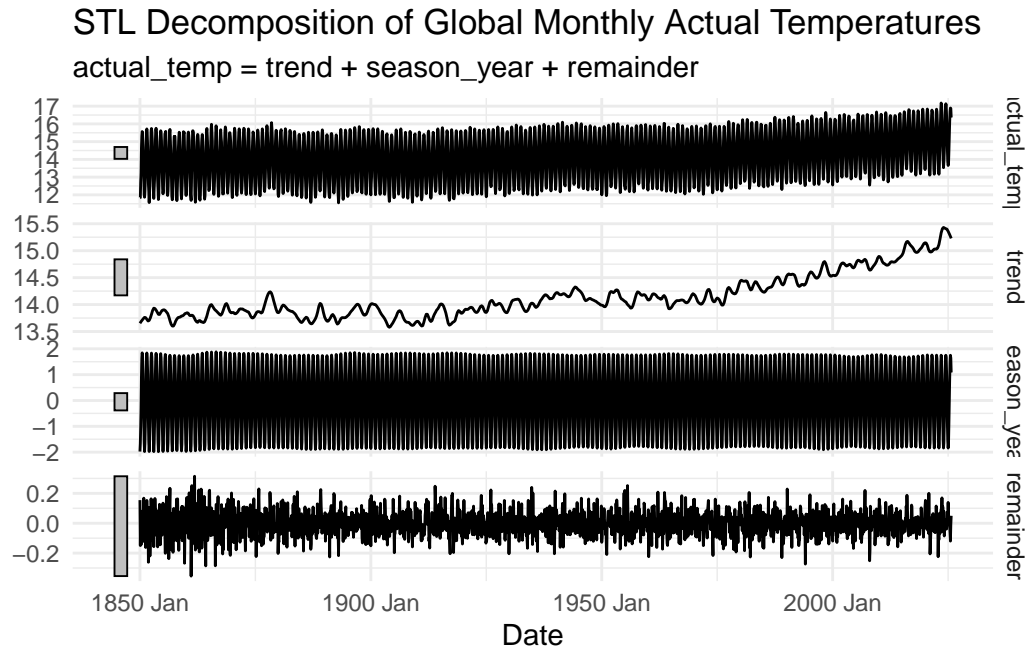
## Exploratory Data Analysis of Global Temperatures

```
summary(converted_df[2])
```

```
actual_temp
Min.      :11.55
1st Qu.:12.89
Median   :14.17
Mean     :14.14
3rd Qu.:15.40
Max.     :17.18
```

```
converted_df |>
  model(STL(actual_temp)) |>
  components() |>
```

```
autoplot() +
  labs(title = "STL Decomposition of Global Monthly Actual Temperatures") +
  theme_minimal()
```



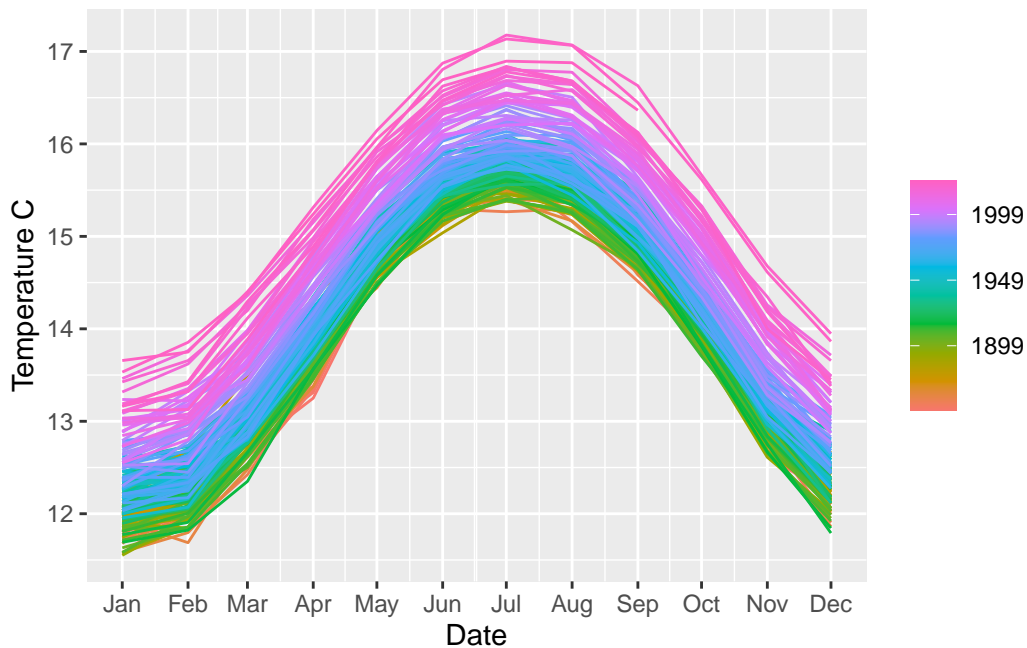
Decomposition of global temperatures show seasonality to mainly be constant with some compression in later years (could be a result of more precise measurements, could also be a result of warming trends flattening temperature swings each season). The main driver of the changing level is the trend which stayed fairly constant until 1920 when the trend starts to increase, then really takes off after 1980

```
converted_df |>
  ggtime::gg_season() +
  labs(y = "Temperature C")
```

Registered S3 methods overwritten by 'ggtime':

method	from
+.gg_tsensemble	feasts
autolayer.tbl_ts	fabletools
autoplot.dcmp_ts	fabletools
autoplot.tbl_ts	fabletools
grid.draw.gg_tsensemble	feasts
print.gg_tsensemble	feasts

Plot variable not specified, automatically selected `y = actual\_temp`



```
theme_minimal()
```

```
<theme> List of 144
```

```
$ line                                     : <ggplot2::element_line>
  ..@ colour      : chr "black"
  ..@ linewidth   : num 0.5
  ..@ linetype    : num 1
  ..@ lineend     : chr "butt"
  ..@ linejoin    : chr "round"
  ..@ arrow       : logi FALSE
  ..@ arrow.fill  : chr "black"
  ..@ inherit.blank: logi TRUE
$ rect                                     : <ggplot2::element_rect>
  ..@ fill        : chr "white"
  ..@ colour      : chr "black"
  ..@ linewidth   : num 0.5
  ..@ linetype    : num 1
  ..@ linejoin    : chr "round"
  ..@ inherit.blank: logi TRUE
$ text                                    : <ggplot2::element_text>
```

```

..@ family      : chr ""
..@ face        : chr "plain"
..@ italic      : chr NA
..@ fontweight  : num NA
..@ fontwidth   : num NA
..@ colour      : chr "black"
..@ size        : num 11
..@ hjust       : num 0.5
..@ vjust       : num 0.5
..@ angle       : num 0
..@ lineheight  : num 0.9
..@ margin      : <ggplot2::margin> num [1:4] 0 0 0 0
..@ debug       : logi FALSE
..@ inherit.blank: logi TRUE
$ title                                     : <ggplot2::element_text>
..@ family      : NULL
..@ face        : NULL
..@ italic      : chr NA
..@ fontweight  : num NA
..@ fontwidth   : num NA
..@ colour      : NULL
..@ size        : NULL
..@ hjust       : NULL
..@ vjust       : NULL
..@ angle       : NULL
..@ lineheight  : NULL
..@ margin      : NULL
..@ debug       : NULL
..@ inherit.blank: logi TRUE
$ point                                     : <ggplot2::element_point>
..@ colour      : chr "black"
..@ shape       : num 19
..@ size        : num 1.5
..@ fill        : chr "white"
..@ stroke      : num 0.5
..@ inherit.blank: logi TRUE
$ polygon                                   : <ggplot2::element_polygon>
..@ fill        : chr "white"
..@ colour      : chr "black"
..@ linewidth   : num 0.5
..@ linetype    : num 1
..@ linejoin    : chr "round"
..@ inherit.blank: logi TRUE

```

```

$ geom : <ggplot2::element_geom>
  ..@ ink : chr "black"
  ..@ paper : chr "white"
  ..@ accent : chr "#3366FF"
  ..@ linewidth : num 0.5
  ..@ borderwidth: num 0.5
  ..@ linetype : int 1
  ..@ bordertype : int 1
  ..@ family : chr ""
  ..@ fontsize : num 3.87
  ..@ pointsize : num 1.5
  ..@ pointshape : num 19
  ..@ colour : NULL
  ..@ fill : NULL
$ spacing : 'simpleUnit' num 5.5points
  ..- attr(*, "unit")= int 8
$ margins : <ggplot2::margin> num [1:4] 5.5 5.5 5.5 5.5
$ aspect.ratio : NULL
$ axis.title : NULL
$ axis.title.x : <ggplot2::element_text>
  ..@ family : NULL
  ..@ face : NULL
  ..@ italic : chr NA
  ..@ fontweight : num NA
  ..@ fontwidth : num NA
  ..@ colour : NULL
  ..@ size : NULL
  ..@ hjust : NULL
  ..@ vjust : num 1
  ..@ angle : NULL
  ..@ lineheight : NULL
  ..@ margin : <ggplot2::margin> num [1:4] 2.75 0 0 0
  ..@ debug : NULL
  ..@ inherit.blank: logi TRUE
$ axis.title.x.top : <ggplot2::element_text>
  ..@ family : NULL
  ..@ face : NULL
  ..@ italic : chr NA
  ..@ fontweight : num NA
  ..@ fontwidth : num NA
  ..@ colour : NULL
  ..@ size : NULL
  ..@ hjust : NULL

```

```

..@ vjust      : num 0
..@ angle      : NULL
..@ lineheight : NULL
..@ margin     : <ggplot2::margin> num [1:4] 0 0 2.75 0
..@ debug      : NULL
..@ inherit.blank: logi TRUE
$ axis.title.x.bottom      : NULL
$ axis.title.y             : <ggplot2::element_text>
..@ family                : NULL
..@ face                  : NULL
..@ italic                : chr NA
..@ fontweight            : num NA
..@ fontwidth            : num NA
..@ colour                : NULL
..@ size                  : NULL
..@ hjust                : NULL
..@ vjust                : num 1
..@ angle                 : num 90
..@ lineheight           : NULL
..@ margin               : <ggplot2::margin> num [1:4] 0 2.75 0 0
..@ debug                : NULL
..@ inherit.blank        : logi TRUE
$ axis.title.y.left        : NULL
$ axis.title.y.right       : <ggplot2::element_text>
..@ family                : NULL
..@ face                  : NULL
..@ italic                : chr NA
..@ fontweight            : num NA
..@ fontwidth            : num NA
..@ colour                : NULL
..@ size                  : NULL
..@ hjust                : NULL
..@ vjust                : num 1
..@ angle                 : num -90
..@ lineheight           : NULL
..@ margin               : <ggplot2::margin> num [1:4] 0 0 0 2.75
..@ debug                : NULL
..@ inherit.blank        : logi TRUE
$ axis.text                : <ggplot2::element_text>
..@ family                : NULL
..@ face                  : NULL
..@ italic                : chr NA
..@ fontweight            : num NA

```

```

..@ fontwidth      : num NA
..@ colour         : chr "#4D4D4DFF"
..@ size           : 'rel' num 0.8
..@ hjust          : NULL
..@ vjust          : NULL
..@ angle          : NULL
..@ lineheight     : NULL
..@ margin         : NULL
..@ debug          : NULL
..@ inherit.blank: logi TRUE
$ axis.text.x      : <ggplot2::element_text>
..@ family         : NULL
..@ face           : NULL
..@ italic         : chr NA
..@ fontweight     : num NA
..@ fontwidth      : num NA
..@ colour         : NULL
..@ size           : NULL
..@ hjust          : NULL
..@ vjust          : num 1
..@ angle          : NULL
..@ lineheight     : NULL
..@ margin         : <ggplot2::margin> num [1:4] 2.2 0 0 0
..@ debug          : NULL
..@ inherit.blank: logi TRUE
$ axis.text.x.top  : <ggplot2::element_text>
..@ family         : NULL
..@ face           : NULL
..@ italic         : chr NA
..@ fontweight     : num NA
..@ fontwidth      : num NA
..@ colour         : NULL
..@ size           : NULL
..@ hjust          : NULL
..@ vjust          : NULL
..@ angle          : NULL
..@ lineheight     : NULL
..@ margin         : <ggplot2::margin> num [1:4] 0 0 4.95 0
..@ debug          : NULL
..@ inherit.blank: logi TRUE
$ axis.text.x.bottom : <ggplot2::element_text>
..@ family         : NULL
..@ face           : NULL

```

```

..@ italic      : chr NA
..@ fontweight  : num NA
..@ fontwidth   : num NA
..@ colour      : NULL
..@ size        : NULL
..@ hjust       : NULL
..@ vjust       : NULL
..@ angle       : NULL
..@ lineheight  : NULL
..@ margin      : <ggplot2::margin> num [1:4] 4.95 0 0 0
..@ debug       : NULL
..@ inherit.blank: logi TRUE
$ axis.text.y           : <ggplot2::element_text>
..@ family        : NULL
..@ face          : NULL
..@ italic        : chr NA
..@ fontweight    : num NA
..@ fontwidth     : num NA
..@ colour        : NULL
..@ size          : NULL
..@ hjust         : num 1
..@ vjust         : NULL
..@ angle         : NULL
..@ lineheight    : NULL
..@ margin        : <ggplot2::margin> num [1:4] 0 2.2 0 0
..@ debug         : NULL
..@ inherit.blank: logi TRUE
$ axis.text.y.left      : <ggplot2::element_text>
..@ family        : NULL
..@ face          : NULL
..@ italic        : chr NA
..@ fontweight    : num NA
..@ fontwidth     : num NA
..@ colour        : NULL
..@ size          : NULL
..@ hjust         : NULL
..@ vjust         : NULL
..@ angle         : NULL
..@ lineheight    : NULL
..@ margin        : <ggplot2::margin> num [1:4] 0 4.95 0 0
..@ debug         : NULL
..@ inherit.blank: logi TRUE
$ axis.text.y.right     : <ggplot2::element_text>

```

```

..@ family      : NULL
..@ face        : NULL
..@ italic      : chr NA
..@ fontweight  : num NA
..@ fontwidth   : num NA
..@ colour      : NULL
..@ size        : NULL
..@ hjust       : NULL
..@ vjust       : NULL
..@ angle       : NULL
..@ lineheight  : NULL
..@ margin      : <ggplot2::margin> num [1:4] 0 0 0 4.95
..@ debug       : NULL
..@ inherit.blank: logi TRUE
$ axis.text.theta : NULL
$ axis.text.r     : <ggplot2::element_text>
..@ family      : NULL
..@ face        : NULL
..@ italic      : chr NA
..@ fontweight  : num NA
..@ fontwidth   : num NA
..@ colour      : NULL
..@ size        : NULL
..@ hjust       : num 0.5
..@ vjust       : NULL
..@ angle       : NULL
..@ lineheight  : NULL
..@ margin      : <ggplot2::margin> num [1:4] 0 2.2 0 2.2
..@ debug       : NULL
..@ inherit.blank: logi TRUE
$ axis.ticks      : <ggplot2::element_blank>
$ axis.ticks.x    : NULL
$ axis.ticks.x.top : NULL
$ axis.ticks.x.bottom : NULL
$ axis.ticks.y    : NULL
$ axis.ticks.y.left : NULL
$ axis.ticks.y.right : NULL
$ axis.ticks.theta : NULL
$ axis.ticks.r    : NULL
$ axis.minor.ticks.x.top : NULL
$ axis.minor.ticks.x.bottom : NULL
$ axis.minor.ticks.y.left : NULL
$ axis.minor.ticks.y.right : NULL

```

```

$ axis.minor.ticks.theta      : NULL
$ axis.minor.ticks.r          : NULL
$ axis.ticks.length          : 'rel' num 0.5
$ axis.ticks.length.x         : NULL
$ axis.ticks.length.x.top     : NULL
$ axis.ticks.length.x.bottom : NULL
$ axis.ticks.length.y         : NULL
$ axis.ticks.length.y.left    : NULL
$ axis.ticks.length.y.right   : NULL
$ axis.ticks.length.theta     : NULL
$ axis.ticks.length.r         : NULL
$ axis.minor.ticks.length     : 'rel' num 0.75
$ axis.minor.ticks.length.x   : NULL
$ axis.minor.ticks.length.x.top : NULL
$ axis.minor.ticks.length.x.bottom: NULL
$ axis.minor.ticks.length.y   : NULL
$ axis.minor.ticks.length.y.left : NULL
$ axis.minor.ticks.length.y.right : NULL
$ axis.minor.ticks.length.theta : NULL
$ axis.minor.ticks.length.r   : NULL
$ axis.line                   : <ggplot2::element_blank>
$ axis.line.x                 : NULL
$ axis.line.x.top             : NULL
$ axis.line.x.bottom          : NULL
$ axis.line.y                 : NULL
$ axis.line.y.left            : NULL
$ axis.line.y.right           : NULL
$ axis.line.theta             : NULL
$ axis.line.r                  : NULL
$ legend.background           : <ggplot2::element_blank>
$ legend.margin               : NULL
$ legend.spacing              : 'rel' num 2
$ legend.spacing.x            : NULL
$ legend.spacing.y            : NULL
$ legend.key                   : <ggplot2::element_blank>
$ legend.key.size              : 'simpleUnit' num 1.2lines
..- attr(*, "unit")= int 3
$ legend.key.height           : NULL
$ legend.key.width            : NULL
$ legend.key.spacing          : NULL
$ legend.key.spacing.x        : NULL
$ legend.key.spacing.y        : NULL
$ legend.key.justification    : NULL

```

```

$ legend.frame           : NULL
$ legend.ticks           : NULL
$ legend.ticks.length    : 'rel' num 0.2
$ legend.axis.line       : NULL
$ legend.text            : <ggplot2::element_text>
  ..@ family             : NULL
  ..@ face               : NULL
  ..@ italic             : chr NA
  ..@ fontweight         : num NA
  ..@ fontwidth          : num NA
  ..@ colour             : NULL
  ..@ size               : 'rel' num 0.8
  ..@ hjust              : NULL
  ..@ vjust              : NULL
  ..@ angle              : NULL
  ..@ lineheight         : NULL
  ..@ margin             : NULL
  ..@ debug              : NULL
  ..@ inherit.blank: logi TRUE
$ legend.text.position    : NULL
$ legend.title           : <ggplot2::element_text>
  ..@ family             : NULL
  ..@ face               : NULL
  ..@ italic             : chr NA
  ..@ fontweight         : num NA
  ..@ fontwidth          : num NA
  ..@ colour             : NULL
  ..@ size               : NULL
  ..@ hjust              : num 0
  ..@ vjust              : NULL
  ..@ angle              : NULL
  ..@ lineheight         : NULL
  ..@ margin             : NULL
  ..@ debug              : NULL
  ..@ inherit.blank: logi TRUE
$ legend.title.position   : NULL
$ legend.position         : chr "right"
$ legend.position.inside  : NULL
$ legend.direction       : NULL
$ legend.byrow           : NULL
$ legend.justification    : chr "center"
$ legend.justification.top : NULL
$ legend.justification.bottom : NULL

```

```

$ legend.justification.left      : NULL
$ legend.justification.right     : NULL
$ legend.justification.inside    : NULL
[list output truncated]
@ complete: logi TRUE
@ validate: logi TRUE

```

Clearer picture of the increasing trend over time, occurring across all periods

```
features(converted_df, actual_temp, guerrero)
```

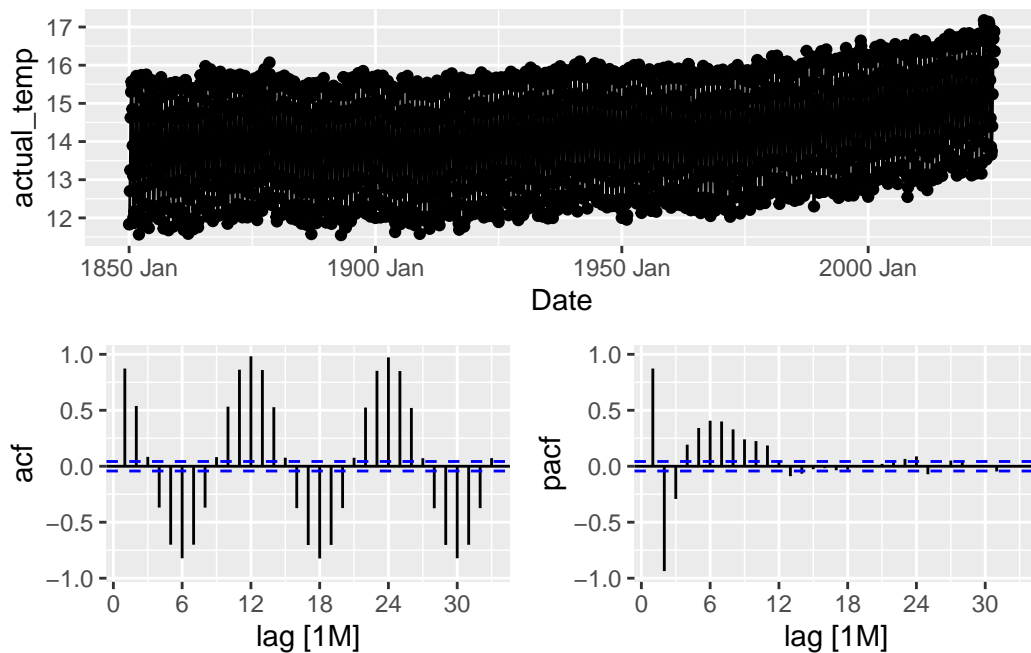
```

# A tibble: 1 x 1
  lambda_guerrero
      <dbl>
1             1.44

```

1.44, so a power transformation may be helpful

```
gg_tsdisplay(converted_df, actual_temp, plot_type = 'partial')
```



```
features(converted_df, actual_temp, c(unitroot_kpss, unitroot_ndiffs, unitroot_nsdiffs))
```

```
# A tibble: 1 x 4
  kpss_stat kpss_pvalue ndiffs nsdiffe
  <dbl>      <dbl>   <int>   <int>
1      8.88      0.01     1       1
```

Unitroot tests suggest at least one difference and one seasonal difference. Using `gg_tsdisplay`, the seasonal autocorrelation is obvious, with the summer and winter months grouping together on opposite sides of the line. The `pacf` shows that the seasonal effects are strong. All of this shows the data is not stationary

## Modeling / Forecasting

### Anomaly Modeling

```
# Fit ETS on the training data

fit_ets <- train |>
model(ETS(Anomaly))

# Print model details

report(fit_ets)
```

```
Series: Anomaly
Model: ETS(A,N,A)
Smoothing parameters:
  alpha = 0.4616505
  gamma = 0.0001007547

Initial states:
  l[0]      s[0]      s[-1]      s[-2]      s[-3]      s[-4]      s[-5]
-0.504211 -0.01987071 -0.01378345 0.03587174 -0.01243867 0.01035226 0.02029624
  s[-6]      s[-7]      s[-8]      s[-9]      s[-10]     s[-11]
0.03041389 0.01181375 -0.004461374 -0.01081024 -0.0249877 -0.02239573

sigma^2: 0.013
```

	AIC	AICc	BIC
	6708.789	6709.026	6793.099

```
fc_ets <- fit_ets |>
  forecast(new_data = test)

fc_ets_zoom <- fc_ets|>
filter_index("2020 Jan" ~ "2025 Dec")

autoplot(
  fc_ets_zoom,
  temp_ts|> filter_index("2020 Jan" ~ "2025 Dec")
) +
labs(
  title = "Auto-ETS Forecast vs Actuals (2020–2025)",
  subtitle = "Automatically selected ETS model",
  x = "Year",
  y = "Temperature Anomaly (°C)"
)
```

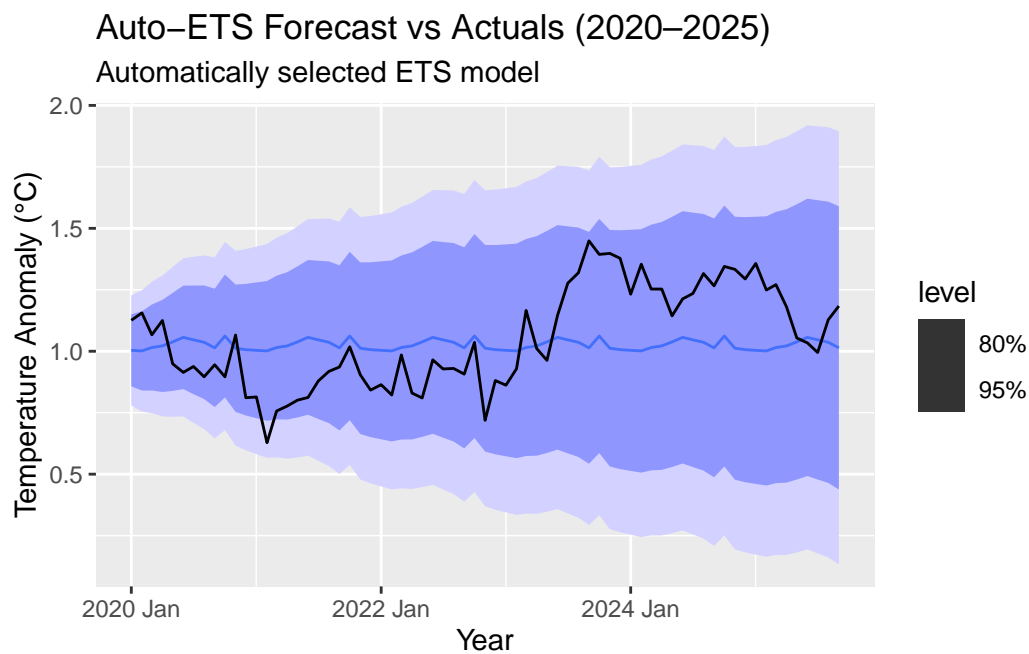


Figure 1: Auto-ETS forecast vs actuals (2020–2025).

The ETS model anticipates a relatively stable anomaly level, but the actual values frequently climb toward the upper edge of the 80% and even 95% prediction intervals. This pattern indicates that recent warming spikes are occurring faster and more intensely than the model expects based on historical structure. When observations consistently press against the top of the interval bands, it suggests the model may be underestimating both the trend strength and the volatility of contemporary climate behavior.

## Temperature Modeling

```
cv_data <- converted_df |>
  stretch_tsibble(.init = 1506, .step = 60)

cv_trn <- cv_data |>
  group_by(.id) |>
  slice(1:(n() - 60)) |>
  ungroup()

cv_valid <- cv_data |>
  group_by(.id) |>
  slice_tail(n = 60) |>
  ungroup()
```

Modeling on all temp data with cross validation sets created from the first roughly 125 years of data (1850 - 1930) and rolling forward by 5 years (60 months). This creates 11 cv splits covering the range of available observations

## ETS

```
ets_fit <- cv_trn |>
  model(
    ets_auto = ETS(),
    additive = ETS(actual_temp ~ error("A") + trend("A") + season("A")),
    multiplicative = ETS(actual_temp ~ error("M") + trend("A") + season("M")),
    damped = ETS(actual_temp ~ error("A") + trend("Ad") + season("A")),
    mult_damp = ETS(actual_temp ~ error("M") + trend("Ad") + season("M")),
    log_auto = ETS(log(actual_temp)),
    box_cox_auto = ETS(box_cox(actual_temp, lambda = 1.5))
  )
```

```
ets_fit |>
  accuracy() |>
  group_by(.model) |>
  summarise(across(.cols = c(RMSE, MAE, MAPE), .fns = mean)) |>
  arrange(RMSE) |>
  kable(digits = 4)
```

.model	RMSE	MAE	MAPE
box_cox_auto	0.1146	0.0893	0.6523
ets_auto	0.1158	0.0907	0.6619
damped	0.1158	0.0909	0.6635
additive	0.1166	0.0917	0.6688
mult_damp	0.1195	0.0942	0.6856
log_auto	0.1198	0.0947	0.6889
multiplicative	0.1249	0.0988	0.7186

```
ets_fit |>
  tidy() |>
  group_by(.model, term) |>
  summarise(across(.cols = estimate, .fns = mean)) |>
  pivot_wider(names_from = term, values_from = estimate) |>
  kable(digits = 4)
```

.model	alpha	b[0]	beta	gamma	s[-10]	s[-11]	s[-1]	s[-2]	s[-3]	s[-4]	s[-5]	s[-6]	s[-7]	s[-8]	s[-9]	s[0]	phi
additive	0.4026	0.0007	0.0001	0.2853	0.7083	-	-	0.1590	0.0365	0.6367	0.8145	0.5752	0.9159	-	-	-	NA
				1.6602	0.9380	0.8587							0.0674	0.0208	0.5920		
box_cox_auto	0.4386	0.0348	0.0006	0.0003	0.2388	-	-	0.5292	0.8393	0.1387	0.7915	0.8887	0.2558	-	-	-	0.9727
				6.1130	0.8453	0.3186							0.4163	0.8855	0.8637		
damped	0.4325	0.0077	0.0002	0.0178	0.6748	-	-	0.1648	0.0571	0.6385	0.8022	0.5750	0.8824	-	-	-	0.9787
				1.6682	0.8870	0.8484							0.1060	0.0177	0.5932		
ets_auto	0.4300	0.0058	0.0003	0.0108	0.6493	-	-	0.1700	0.0487	0.6345	0.7965	0.5720	0.8852	-	-	-	0.9800
				1.6639	0.8788	0.8536							0.0972	0.0242	0.5893		
log_auto	0.4139	0.0001	0.0001	0.0342	0.6085	-	-	0.0165	0.0770	0.1160	0.1261	0.1122	0.0669	-	-	-	0.9754
				0.1236	0.1426	0.0574							0.0036	0.0720	0.1169		
mult_damp	0.4469	0.0110	0.0001	0.0295	0.6062	0.8806	0.8640	0.9393	0.0123	0.0753	0.1160	0.1293	0.1133	0.0640	0.9919	0.9270	0.8860
multiplicative	0.4281	0.0045	0.0057	0.0755	0.6459	0.8780	0.8610	0.9399	0.0103	0.0797	0.1197	0.1311	0.1165	0.0657	0.9876	0.9240	0.8847

```
ets_fit |>
  filter(.id == 10) |>
  select(box_cox_auto) |>
  report()
```

```
Series: actual_temp
Model: ETS(A,Ad,A)
Transformation: box_cox(actual_temp, lambda = 1.5)
Smoothing parameters:
  alpha = 0.463544
  beta  = 0.0003877467
  gamma = 0.0001066393
  phi   = 0.9742304

Initial states:
  l[0]      b[0]      s[0]      s[-1]      s[-2]      s[-3]      s[-4]      s[-5]
33.19618 0.03777566 -5.881403 -3.326745 0.5289844 3.854027 6.1683 6.788088
  s[-6]      s[-7]      s[-8]      s[-9]      s[-10]      s[-11]
5.898173 3.25394 -0.389869 -3.896 -6.13688 -6.860616

sigma^2: 0.1778

      AIC      AICc      BIC
11670.18 11670.53 11770.87
```

All versions of ETS performed well on the training data with `box_cox_auto` performing the best. Auto selection chose an additive model with a damped trend. Looking the parameters, the `alpha` parameter values ranged from .2812 to .4469. This shows that the level is fairly stable and perhaps indicates that the recent warming trend is being masked by the long tail of relatively stable temperatures. This idea is bolstered by the the really small, almost 0 values of the `beta` parameter which indicates a stable trend. The `gamma` parameters are all very small as showing, suggesting that the ETS models are correctly reading that the seasonal pattern is relatively stable.

```
ets_fc <- ets_fit |>
  forecast(new_data = cv_valid)

ets_fc |>
  accuracy(cv_valid) |>
  group_by(.model) |>
  summarise(across(.cols = c(RMSE, MAE, MAPE), .fns = mean)) |>
```

```

arrange(RMSE) |>
kable(digits = 4)

```

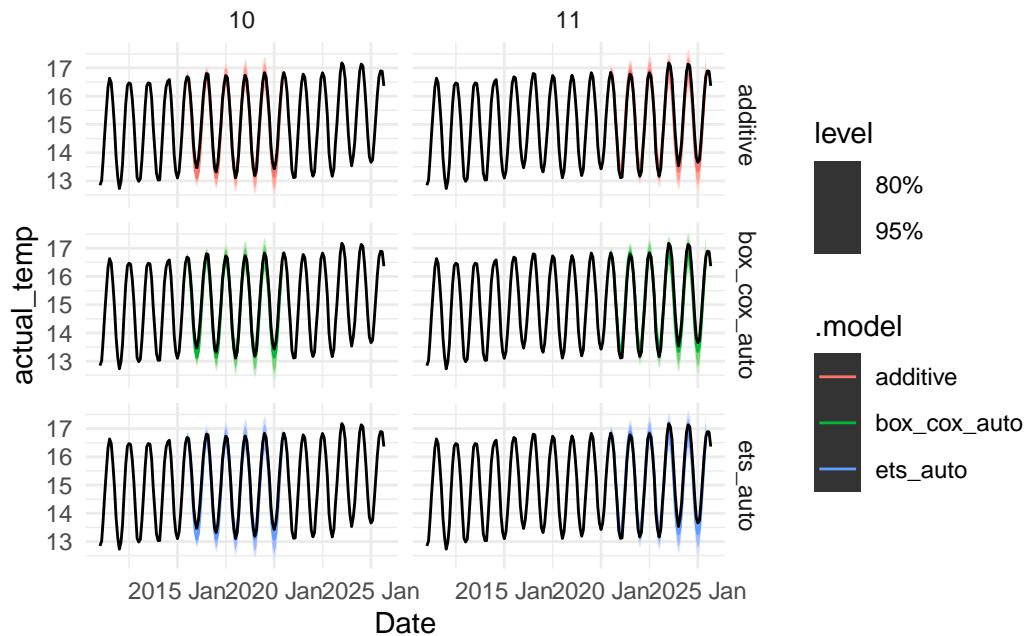
.model	RMSE	MAE	MAPE
additive	0.1724	0.1412	0.9862
ets_auto	0.1726	0.1409	0.9823
box_cox_auto	0.1730	0.1409	0.9843
damped	0.1735	0.1419	0.9900
log_auto	0.1758	0.1441	1.0026
mult_damp	0.1763	0.1442	1.0052
multiplicative	0.1919	0.1627	1.1320

The additive model performs the best on the validation data but the out performance is negligible to ets\_auto and box\_cox\_auto

```

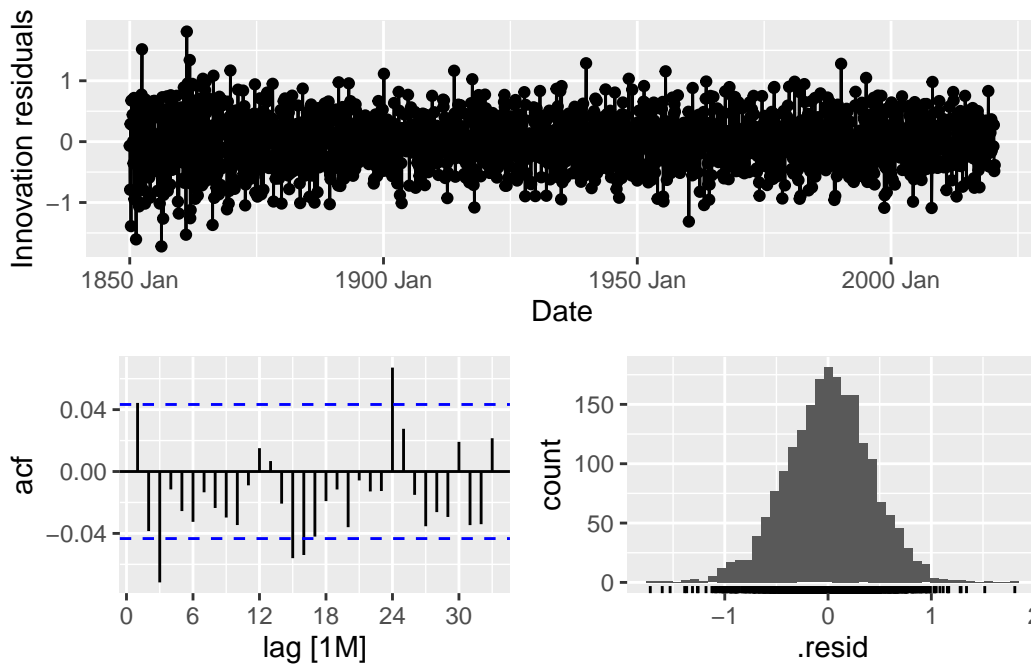
ets_fc |>
  filter(.id %in% c(10, 11), .model %in% c('additive', 'box_cox_auto', 'ets_auto')) |>
  autoplot() +
  autolayer(converted_df |> filter(year(Date) > 2010), actual_temp) +
  facet_grid(.model ~ .id) +
  theme_minimal()

```



Visualizing the top 3 models illustrates how ETS models do just fine at predicting the fall and spring months, but consistently undershoot the seasonal peaks and exhibit wide prediction intervals around those areas. ETS models have trouble capturing the increasing trend

```
ets_fit |>
  select(box_cox_auto) |>
  tail(n = 1) |>
  ggtime::gg_tsresiduals()
```



Using box cox and checking the residuals of the model fitted on the most data, the residuals show a mostly normal distribution. Slight skew to the right. There are autocorrelations at lag 3 and lag 24. We can confirm that the residuals aren't white noise with a Ljung-Box test. There are certainly factors that influence temperature that ETS isn't capturing

```
ets_fit |>
  augment() |>
  filter(.model == "box_cox_auto") |>
  features(.innov, ljung_box, lag = 24) |>
  summarize(across(.cols = lb_pvalue, .fns = mean))
```

```
# A tibble: 1 x 1
  lb_pvalue
  <dbl>
1 0.00413
```

## TSLM

```
tslm_fit <- cv_trn |>
  model(
    tslm_auto = TSLM(actual_temp),
    tslm_trend = TSLM(actual_temp ~ trend()),
    tslm_trend_season = TSLM(actual_temp ~ trend() + season()),
    tslm_fourier = TSLM(actual_temp ~ trend() + fourier(K = 2)),
    tslm_log = TSLM(log(actual_temp) ~ trend() + season()),
    tslm_box_cox = TSLM(box_cox(actual_temp, lambda = 1.5) ~ trend() + season()),
    tslm_piecewise = TSLM(actual_temp ~ trend(knots = c(1920, 1975)) + season())
  )
```

Testing Fourier terms for a simpler model as opposed to using season dummy variables, and piece-wise to capture changes in the trend at those points

```
tslm_fit |>
  accuracy() |>
  group_by(.model) |>
  summarise(across(.cols = c(RMSE, MAE, MAPE), .fns = mean)) |>
  arrange(RMSE) |>
  kable(digits = 4)
```

.model	RMSE	MAE	MAPE
tslm_piecewise	0.1861	0.1492	1.0841
tslm_trend_season	0.1861	0.1492	1.0841
tslm_box_cox	0.1861	0.1493	1.0843
tslm_log	0.1874	0.1501	1.0902
tslm_fourier	0.1880	0.1508	1.0949
tslm_trend	1.3262	1.1887	8.6156
tslm_auto	1.3409	1.1979	8.6844

```
tslm_fit |>
  select(tslm_trend_season) |>
  tail(n = 1) |>
  report()
```

Series: actual\_temp  
Model: TSLM

Residuals:

	Min	1Q	Median	3Q	Max
	-0.63061	-0.15153	-0.01237	0.14543	0.79855

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	1.172e+01	1.870e-02	626.999	<2e-16 ***
trend()	5.171e-04	8.187e-06	63.157	<2e-16 ***
season()year2	2.096e-01	2.365e-02	8.861	<2e-16 ***
season()year3	8.370e-01	2.365e-02	35.387	<2e-16 ***
season()year4	1.779e+00	2.365e-02	75.227	<2e-16 ***
season()year5	2.735e+00	2.365e-02	115.630	<2e-16 ***
season()year6	3.411e+00	2.365e-02	144.184	<2e-16 ***
season()year7	3.636e+00	2.369e-02	153.494	<2e-16 ***
season()year8	3.479e+00	2.369e-02	146.845	<2e-16 ***
season()year9	2.890e+00	2.369e-02	122.000	<2e-16 ***
season()year10	2.024e+00	2.369e-02	85.447	<2e-16 ***
season()year11	9.840e-01	2.369e-02	41.538	<2e-16 ***
season()year12	2.759e-01	2.369e-02	11.645	<2e-16 ***

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.2187 on 2033 degrees of freedom

Multiple R-squared: 0.9743, Adjusted R-squared: 0.9742

F-statistic: 6435 on 12 and 2033 DF, p-value: < 2.22e-16

The trend\_season, box\_cox and piecewise models all perform the same on the training data. Looking at the model report for the trend\_season model we can see that all parameters are highly significant

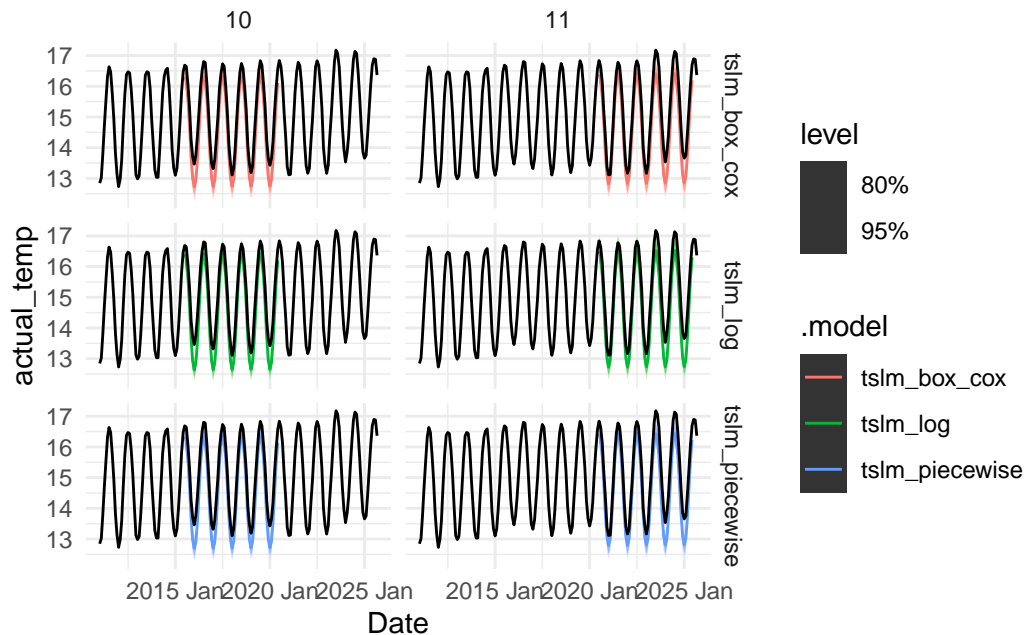
```
tslm_fc <- tslm_fit |>
  forecast(new_data = cv_valid)

tslm_fc |>
  accuracy(cv_valid) |>
  group_by(.model) |>
  summarise(across(.cols = c(RMSE, MAE, MAPE), .fns = mean)) |>
  arrange(RMSE) |>
  kable(digits = 4)
```

.model	RMSE	MAE	MAPE
tslm_log	0.3198	0.2881	1.9929
tslm_piecewise	0.3212	0.2933	2.0116
tslm_trend_season	0.3212	0.2933	2.0116
tslm_box_cox	0.3212	0.2940	2.0089
tslm_fourier	0.3222	0.2939	2.0162
tslm_trend	1.3361	1.1945	8.1337
tslm_auto	1.4553	1.2584	8.3874

TSLM log performs the best on the validation data but significantly under-perform ETS models

```
tslm_fc |>
  filter(.id %in% c(10, 11), .model %in% c('tslm_log', 'tslm_piecewise', 'tslm_box_cox')) |>
  autoplot() +
  autolayer(converted_df |> filter(year(Date) > 2010), actual_temp) +
  facet_grid(.model ~ .id) +
  theme_minimal()
```



It's pretty clear from the visuals that TSLM models are failing even harder than the ETS models at capturing the warming trend.

## ARIMA

```
trn_data <- converted_df |>
  slice(1:(n() - 60))

valid_data <- converted_df |>
  slice_tail(n = 60)
```

ARIMA models take too long to converge on CV splits and often time out, so testing them first on normal splits

```
arima_fit <- trn_data |>
  model(
    arima_auto = ARIMA(actual_temp),
    arima_box = ARIMA(box_cox(actual_temp, lambda = 1.5)),
    arima_log = ARIMA(log(actual_temp))
  )

arima_fit |>
  accuracy() |>
  select(.model, ME:MAPE) |>
  arrange(RMSE) |>
  kable(digits = 4)
```

.model	ME	RMSE	MAE	MPE	MAPE
arima_box	-4e-04	0.1191	0.0931	-0.0108	0.6772
arima_auto	0e+00	0.1192	0.0932	-0.0077	0.6780
arima_log	2e-04	0.1318	0.1027	-0.0035	0.7477

ARIMA box\_cox and auto performed the same on the training data

```
arima_fit |>
  select(arima_box) |>
  report()
```

```
Series: actual_temp
Model: ARIMA(1,0,0)(0,1,1)[12] w/ drift
Transformation: box_cox(actual_temp, lambda = 1.5)
```

Coefficients:

	ar1	sma1	constant
	0.7088	-0.8825	0.0079
s.e.	0.0176	0.0126	0.0012

sigma<sup>2</sup> estimated as 0.1955: log likelihood=-1235.64  
AIC=2479.28 AICc=2479.3 BIC=2501.76

```
arima_fit |>  
  select(arima_auto) |>  
  report()
```

Series: actual\_temp

Model: ARIMA(1,0,0)(0,1,1)[12] w/ drift

Coefficients:

	ar1	sma1	constant
	0.7053	-0.8840	0.0021
s.e.	0.0176	0.0126	0.0003

sigma<sup>2</sup> estimated as 0.01431: log likelihood=1426.97  
AIC=-2845.94 AICc=-2845.92 BIC=-2823.47

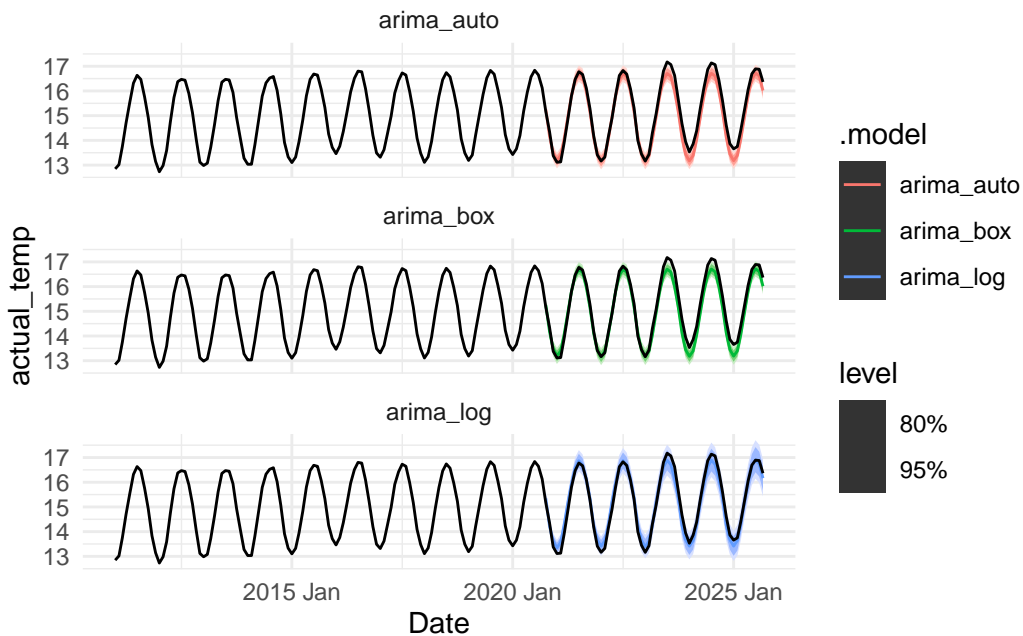
Surprisingly the ARIMA models only selected a seasonal differencing instead of both a seasonal and non-seasonal. 1 non-seasonal AR component was selected and 1 seasonal moving average. So the auto selected models are using the previous observation and the previous seasons forecast error to make its predictions.

```
arima_fc <- arima_fit |>  
  forecast(new_data = valid_data)  
  
arima_fc |>  
  accuracy(valid_data) |>  
  group_by(.model) |>  
  summarise(across(.cols = c(RMSE, MAE, MAPE), .fns = mean)) |>  
  arrange(RMSE) |>  
  kable(digits = 4)
```

.model	RMSE	MAE	MAPE
arima_log	0.2174	0.1823	1.2263
arima_box	0.2929	0.2365	1.5458
arima_auto	0.2929	0.2357	1.5416

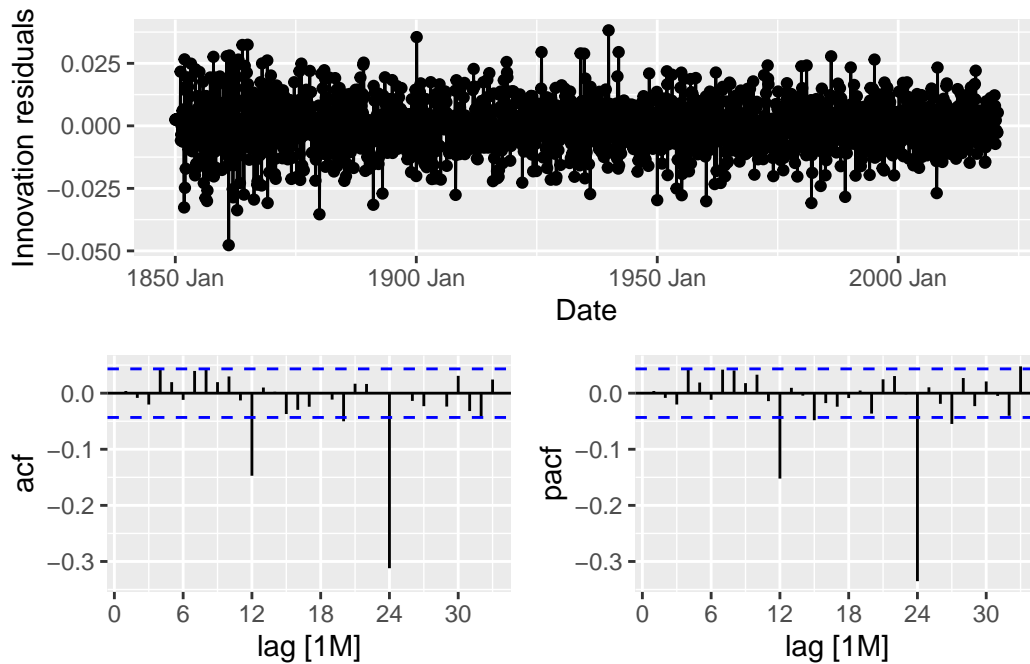
Arima log does best on the validation data

```
arima_fc |>
  autoplot() +
  autolayer(converted_df |> filter(year(Date) > 2010), actual_temp) +
  facet_wrap(~ .model, ncol = 1) +
  theme_minimal()
```



We see that the ARIMA log model does slightly better at capturing the peaks than the other two, but all still don't quite capture the increasing trend

```
arima_fit |>
  select(arima_log) |>
  ggtime::gg_tsresiduals(plot_type = 'partial')
```



```

arima_fit |>
  augment() |>
  filter(.model == "arima_box") |>
  features(.innov, ljung_box, lag = 24)

```

```

# A tibble: 1 x 3
  .model    lb_stat lb_pvalue
  <chr>      <dbl>    <dbl>
1 arima_box  230.         0

```

Residuals aren't even close to stationary. With ARIMA we can experiment with other parameter values

```

arima_fit <- trn_data |>
  model(
    arima_diff = ARIMA(actual_temp ~ pdq(d = 1)),
    arima_box_diff = ARIMA(box_cox(actual_temp, lambda = 1.5) ~ pdq(d = 1)),
    arima_box_ar_two = ARIMA(box_cox(actual_temp, lambda = 1.5) ~ pdq(p = 2)),
    arima_custom = ARIMA(actual_temp ~ 0 + pdq(3, 1, 2) + PDQ(1, 1, 2))
  )

arima_fc <- arima_fit |>

```

```

forecast(new_data = valid_data)

arima_fc |>
  accuracy(valid_data) |>
  group_by(.model) |>
  summarise(across(.cols = c(RMSE, MAE, MAPE), .fns = mean)) |>
  arrange(RMSE) |>
  kable(digits = 4)

```

.model	RMSE	MAE	MAPE
arima_custom	0.2110	0.1714	1.1450
arima_diff	0.2169	0.1814	1.2346
arima_box_diff	0.2183	0.1818	1.2387
arima_box_ar_two	0.2212	0.1851	1.2447

Adding non-seasonal differencing, another AR term and a seasonal MA term to deal with the autocorrelation at lag 24 helped improved model results on the validation data

```

arima_fit |>
  augment() |>
  filter(.model == "arima_custom") |>
  features(.innov, ljung_box, lag = 24)

```

```

# A tibble: 1 x 3
  .model      lb_stat lb_pvalue
  <chr>      <dbl>    <dbl>
1 arima_custom 21.1      0.634

```

A high p-value tells us that the residuals are now resembling white noise

## Compare Best Models

```

compare_fit <- cv_trn |>
  model(
    ets = ETS(actual_temp ~ error("A") + trend("A") + season("A")),
    tslm = TSLM(log(actual_temp) ~ trend() + season()),
    arima = ARIMA(actual_temp ~ 0 + pdq(3, 1, 2) + PDQ(1, 1, 2))
  )

```

Warning in log(s2): NaNs produced  
Warning in log(s2): NaNs produced

Warning in sqrt(diag(best\$var.coef)): NaNs produced  
Warning in sqrt(diag(best\$var.coef)): NaNs produced

Warning in log(s2): NaNs produced

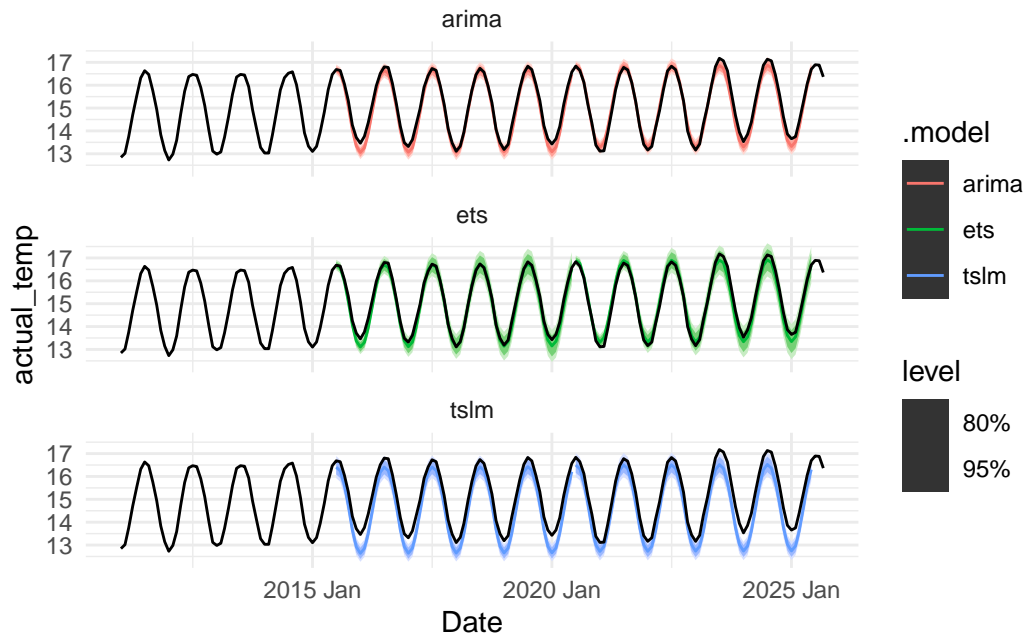
Warning in wrap\_arima(y, order = c(p, d, q), seasonal = list(order = c(P, :  
possible convergence problem: optim gave code = 1

Warning: 1 error encountered for arima  
[1] non-finite finite-difference value [1]

```
compare_fc <- compare_fit |>  
  forecast(new_data = cv_valid)  
  
compare_fc |>  
  accuracy(cv_valid) |>  
  filter(.id != 6) |>  
  group_by(.model) |>  
  summarise(across(.cols = c(RMSE, MAE, MAPE), .fns = mean)) |>  
  arrange(RMSE) |>  
  kable(digits = 4)
```

.model	RMSE	MAE	MAPE
arima	0.1634	0.1323	0.9250
ets	0.1748	0.1429	0.9995
tslm	0.3191	0.2880	1.9909

```
compare_fc |>  
  filter(.id %in% c(10, 11)) |>  
  autoplot() +  
  autolayer(converted_df |> filter(year(Date) > 2010), actual_temp) +  
  facet_wrap(~ .model, ncol = 1) +  
  theme_minimal()
```



The metrics based on RMSE show that ARIMA does slightly better than ETA and both do much better than TSLM. Comparing ARIMA to ETS visually, we see much more uncertainty in the ETS predictions. Both models still seem to under shoot the peaks which is a strong indicator that other factors are at play that will need to be considered

### TSLM External Predictors

How far back accurate monthly data is available for the following predictors varies. However for the main drivers of temperature, GHG's, the most accurate monthly estimates began in 1958. So that will be the cutoff date for our training data.

According to the Intergovernmental Panel on Climate Change (IPCC, 2021), the main factors that influence global temperature are greenhouse gas concentrations, natural weather and ocean-atmosphere patterns, and variations in solar irradiance. To see which drivers best serve our forecasting purpose, we can model these factors individually and compare their predictive performance. Intergovernmental Panel on Climate Change (IPCC). *Climate Change 2021: The Physical Science Basis*.

<https://www.ipcc.ch/report/ar6/wg1/>

TSI:

ENSO Index:

CO2:

RF\_CO2:

Volcanic Activity:

CH4:

N2O:

```
predictor_modeling <- read_rds("data/lagged_external_predictors.rds")
```

The earliest data that could be found on historical el nino/la nina events dated back to 1896, so the regressor data will extend from 1896 to Sept 2025

### Here go into detail about interpolating and lags

There are over 100 total combinations of external predictors, so instead of trying all of them, a handful of complementary ones are tested and reviewed to see if any other combinations are worth exploring. For example, current co2\_ppm combined with la\_nina and el\_nino events are compared against the same but with a 10yr lag on the co2\_ppm variable. If the lagged version of this model performs better, then lagged trends are worth looking further into

```
cv_data <- predictor_modeling |>
  filter(year(Date) > 1957) |>
  stretch_tsibble(.init = 573, .step = 60)

cv_trn <- cv_data |>
  group_by(.id) |>
  slice(1:(n() - 60)) |>
  ungroup()

cv_valid <- cv_data |>
  group_by(.id) |>
  slice_tail(n = 60) |>
  ungroup()

tslm_regressor_fit <- cv_trn |>
  model(
    tslm_trend_season = TSLM(actual_temp ~ trend() + season()),
    tslm_log = TSLM(log(actual_temp) ~ trend() + season()),
    tslm_box_cox = TSLM(box_cox(actual_temp, lambda = 1.5) ~ trend() + season()),
    tslm_all = TSLM(actual_temp ~ trend() + season() + el_nino + la_nina + co2_ppm + ch4_ppb),
    tslm_all_box = TSLM(box_cox(actual_temp, lambda = 1.5) ~ trend() + season() + el_nino + la_nina + co2_ppm + ch4_ppb),
    tslm_ch4_co2_nino = TSLM(actual_temp ~ trend() + season() + el_nino + la_nina + co2_ppm + ch4_ppb)
```

```

tslm_co2_nino = TSLM(actual_temp ~ trend() + season() + el_nino + la_nina + co2_ppm),
tslm_nino = TSLM(actual_temp ~ trend() + season() + el_nino + la_nina),
tslm_enso = TSLM(actual_temp ~ trend() + season() + ENSO),
tslm_enso_smooth = TSLM(actual_temp ~ trend() + season() + enso_smooth_12),
tslm_co2_nino_lag = TSLM(actual_temp ~ trend() + season() + el_nino + la_nina + co2_lag1
)

```

Training metrics show that adding predictors improve model results over baseline. This could be due to overfitting

```

tslm_regressor_fit |>
  select(tslm_all_box) |>
  tail(n = 1) |>
  report()

```

Series: actual\_temp

Model: TSLM

Transformation: box\_cox(actual\_temp, lambda = 1.5)

Residuals:

Min	1Q	Median	3Q	Max
-1.3498	-0.2808	-0.0138	0.3062	1.1927

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )	
(Intercept)	-1.829e+02	6.784e+01	-2.696	0.007188	**
trend()	-6.771e-03	1.535e-03	-4.409	1.21e-05	***
season()year2	6.992e-01	8.594e-02	8.136	2.00e-15	***
season()year3	2.899e+00	8.647e-02	33.525	< 2e-16	***
season()year4	6.328e+00	8.815e-02	71.788	< 2e-16	***
season()year5	9.818e+00	8.898e-02	110.341	< 2e-16	***
season()year6	1.257e+01	8.739e-02	143.882	< 2e-16	***
season()year7	1.360e+01	8.564e-02	158.751	< 2e-16	***
season()year8	1.322e+01	8.679e-02	152.368	< 2e-16	***
season()year9	1.096e+01	8.932e-02	122.746	< 2e-16	***
season()year10	7.592e+00	8.963e-02	84.702	< 2e-16	***
season()year11	3.581e+00	8.759e-02	40.887	< 2e-16	***
season()year12	1.017e+00	8.645e-02	11.766	< 2e-16	***
el_nino	1.346e-01	4.439e-02	3.032	0.002526	**
la_nina	-3.062e-01	4.695e-02	-6.523	1.37e-10	***
co2_ppm	7.708e-02	7.967e-03	9.675	< 2e-16	***

```

ch4_ppb          1.690e-03  5.032e-04   3.358 0.000831 ***
n2o_ppb          9.750e-03  1.197e-02   0.815 0.415562
TSI              1.334e-01  4.994e-02   2.672 0.007721 **
volcano_forcing -9.692e-02  2.823e-02  -3.434 0.000633 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

```

Residual standard error: 0.4599 on 666 degrees of freedom  
 Multiple R-squared: 0.9921, Adjusted R-squared: 0.9919  
 F-statistic: 4417 on 19 and 666 DF, p-value: < 2.22e-16

All predictors on the best performing training model show as significant except for N2O

```

tslm_fc <- tslm_regressor_fit |>
  forecast(new_data = cv_valid)

tslm_fc |>
  accuracy(cv_valid) |>
  group_by(.model) |>
  summarise(across(.cols = c(RMSE, MAE, MAPE), .fns = mean)) |>
  arrange(RMSE) |>
  kable(digits = 4)

```

.model	RMSE	MAE	MAPE
tslm_all_box	0.1471	0.1172	0.8089
tslm_co2_nino_lag	0.1476	0.1166	0.7978
tslm_all	0.1493	0.1196	0.8180
tslm_co2_nino	0.1519	0.1203	0.8227
tslm_enso_smooth	0.1519	0.1254	0.8510
tslm_ch4_co2_nino	0.1531	0.1229	0.8393
tslm_enso	0.1578	0.1297	0.8796
tslm_nino	0.1705	0.1382	0.9377
tslm_trend_season	0.1746	0.1429	0.9737
tslm_box_cox	0.1761	0.1463	0.9916
tslm_log	0.1766	0.1398	0.9629

TSLM with a box\_cox transformation and all of the predictors performed the best on the validation data. The `enso_smooth` variable also performed better than either ENSO or the nino dummy variables. These results indicate that other model combinations are worth exploring. Interesting to note that adding `ch4_ppb` actually reduced model performance compared to just using `co2_ppm`

```

tslm_second_fit <- cv_trn |>
  model(
    tslm_all_lag_box = TSLM(box_cox(actual_temp, lambda = 1.5) ~ trend() + season() + el_nino),
    tslm_nino_five_lag = TSLM(actual_temp ~ trend() + season() + el_nino + la_nina + co2_lag1),
    tslm_enso_co2_lag = TSLM(actual_temp ~ trend() + season() + ENSO + co2_lag1),
    tslm_enso_smooth_lag_all = TSLM(actual_temp ~ trend() + season() + enso_smooth_12 + co2_lag1),
    tslm_all_lag = TSLM(actual_temp ~ trend() + season() + el_nino + la_nina + co2_lag1 + n2o),
    tslm_nino_10_lag = TSLM(box_cox(actual_temp, lambda = 1.5) ~ trend() + season() + el_nino),
    tslm_enso_smooth_lag = TSLM(box_cox(actual_temp, lambda = 1.5) ~ trend() + season() + enso_smooth_12)
  )

tslm_second_fc <- tslm_second_fit |>
  forecast(new_data = cv_valid)

tslm_second_fc |>
  accuracy(cv_valid) |>
  group_by(.model) |>
  summarise(across(.cols = c(RMSE, MAE, MAPE), .fns = mean)) |>
  arrange(RMSE) |>
  kable(digits = 4)

```

.model	RMSE	MAE	MAPE
tslm_enso_smooth_lag	0.1217	0.0950	0.6554
tslm_enso_smooth_lag_all	0.1298	0.1028	0.7030
tslm_enso_co2_lag	0.1374	0.1112	0.7605
tslm_nino_10_lag	0.1413	0.1101	0.7575
tslm_nino_five_lag	0.1445	0.1133	0.7777
tslm_all_lag_box	0.1464	0.1144	0.7892
tslm_all_lag	0.1475	0.1172	0.8017

The target variable undergoing a box\_cox transformation using the smoothed enso index over 6 months and a 10 year lag in CO2 concentration performs the best out of all the models by a fair margin

```

tslm_second_fit |>
  select(tslm_enso_smooth_lag) |>
  tail(n = 1) |>
  report()

```

Series: actual\_temp

Model: TSLM

Transformation: box\_cox(actual\_temp, lambda = 1.5)

Residuals:

	Min	1Q	Median	3Q	Max
	-1.448628	-0.292156	0.003149	0.296378	1.450508

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	1.297e+01	1.246e+00	10.410	<2e-16 ***
trend()	2.270e-04	4.573e-04	0.496	0.62
season()year2	7.171e-01	8.137e-02	8.813	<2e-16 ***
season()year3	2.934e+00	8.152e-02	35.994	<2e-16 ***
season()year4	6.402e+00	8.197e-02	78.097	<2e-16 ***
season()year5	9.928e+00	8.222e-02	120.741	<2e-16 ***
season()year6	1.263e+01	8.189e-02	154.285	<2e-16 ***
season()year7	1.362e+01	8.142e-02	167.267	<2e-16 ***
season()year8	1.316e+01	8.157e-02	161.305	<2e-16 ***
season()year9	1.087e+01	8.234e-02	132.039	<2e-16 ***
season()year10	7.474e+00	8.282e-02	90.241	<2e-16 ***
season()year11	3.528e+00	8.216e-02	42.944	<2e-16 ***
season()year12	1.024e+00	8.177e-02	12.526	<2e-16 ***
enso_smooth_6	3.182e-01	2.166e-02	14.694	<2e-16 ***
co2_lag10	4.747e-02	4.146e-03	11.450	<2e-16 ***

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.4564 on 738 degrees of freedom

Multiple R-squared: 0.9921, Adjusted R-squared: 0.992

F-statistic: 6628 on 14 and 738 DF, p-value: < 2.22e-16

And we can see from the report that all the predictors used are significant except trend. It could be that smoothing out the enso index and the 10 yr lag of co2\_ppm captures most of the trend component

```
tslm_best_fit <- cv_trn |>
  model(
    tslm_enso_6 = TSLM(box_cox(actual_temp, lambda = 1.5) ~ trend() + season() + enso_smooth_6,
    tslm_enso_12 = TSLM(box_cox(actual_temp, lambda = 1.5) ~ trend() + season() + enso_smooth_12,
    tslm_no_trend = TSLM(box_cox(actual_temp, lambda = 1.5) ~ season() + enso_smooth_12 + co2_lag10,
    tslm_volcanic = TSLM(box_cox(actual_temp, lambda = 1.5) ~ season() + enso_smooth_12 + co2_lag10 + volcanic)
  )
```

```

tslm_best_fc <- tslm_best_fit |>
  forecast(new_data = cv_valid)

tslm_best_fc |>
  accuracy(cv_valid) |>
  group_by(.model) |>
  summarise(across(.cols = c(ME, RMSE, MAE, MAPE), .fns = mean)) |>
  arrange(RMSE) |>
  kable(digits = 4)

```

.model	ME	RMSE	MAE	MAPE
tslm_no_trend	-0.0032	0.1183	0.0927	0.6389
tslm_volcanic	0.0013	0.1191	0.0936	0.6432
tslm_enso_12	-0.0031	0.1211	0.0942	0.6489
tslm_enso_6	-0.0063	0.1217	0.0950	0.6554

```

tslm_best_fit |>
  select(tslm_no_trend) |>
  tail(n = 1) |>
  report()

```

Series: actual\_temp

Model: TSLM

Transformation: box\_cox(actual\_temp, lambda = 1.5)

Residuals:

Min	1Q	Median	3Q	Max
-1.628084	-0.287697	0.001792	0.303473	1.553132

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	1.233e+01	2.424e-01	50.849	<2e-16 ***
season()year2	7.143e-01	8.076e-02	8.844	<2e-16 ***
season()year3	2.923e+00	8.076e-02	36.191	<2e-16 ***
season()year4	6.380e+00	8.078e-02	78.985	<2e-16 ***
season()year5	9.896e+00	8.079e-02	122.495	<2e-16 ***
season()year6	1.260e+01	8.078e-02	155.962	<2e-16 ***
season()year7	1.358e+01	8.076e-02	168.156	<2e-16 ***
season()year8	1.312e+01	8.076e-02	162.517	<2e-16 ***

```

season()year9  1.085e+01  8.077e-02 134.317    <2e-16 ***
season()year10 7.461e+00  8.111e-02  91.985    <2e-16 ***
season()year11 3.522e+00  8.109e-02  43.432    <2e-16 ***
season()year12 1.020e+00  8.108e-02  12.581    <2e-16 ***
enso_smooth_12 3.785e-01  2.500e-02  15.142    <2e-16 ***
co2_lag10      4.966e-02  6.909e-04  71.876    <2e-16 ***
---

```

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.4532 on 739 degrees of freedom

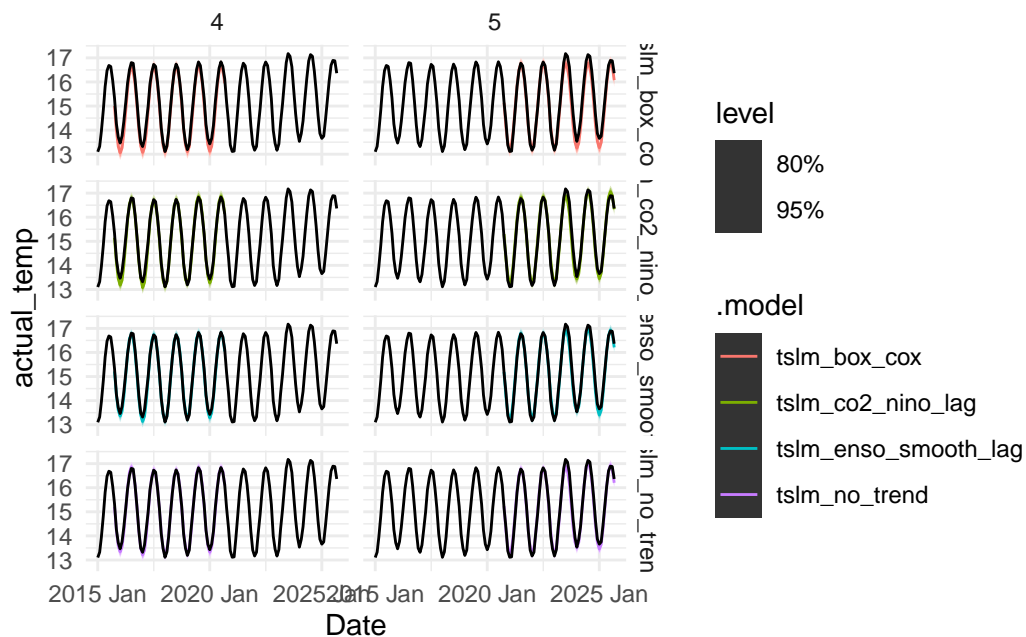
Multiple R-squared: 0.9922, Adjusted R-squared: 0.9921

F-statistic: 7239 on 13 and 739 DF, p-value: < 2.22e-16

```

tslm_fc |>
  bind_rows(tslm_second_fc) |>
  bind_rows(tslm_best_fc) |>
  filter(.id %in% c(4, 5), .model %in% c("tslm_no_trend", "tslm_enso_smooth_lag", "tslm_nino.
  autoplot() +
  autolayer(predictor_modeling |> filter(year(Date) >= 2015), actual_temp) +
  facet_grid(.model ~ .id) +
  theme_minimal()

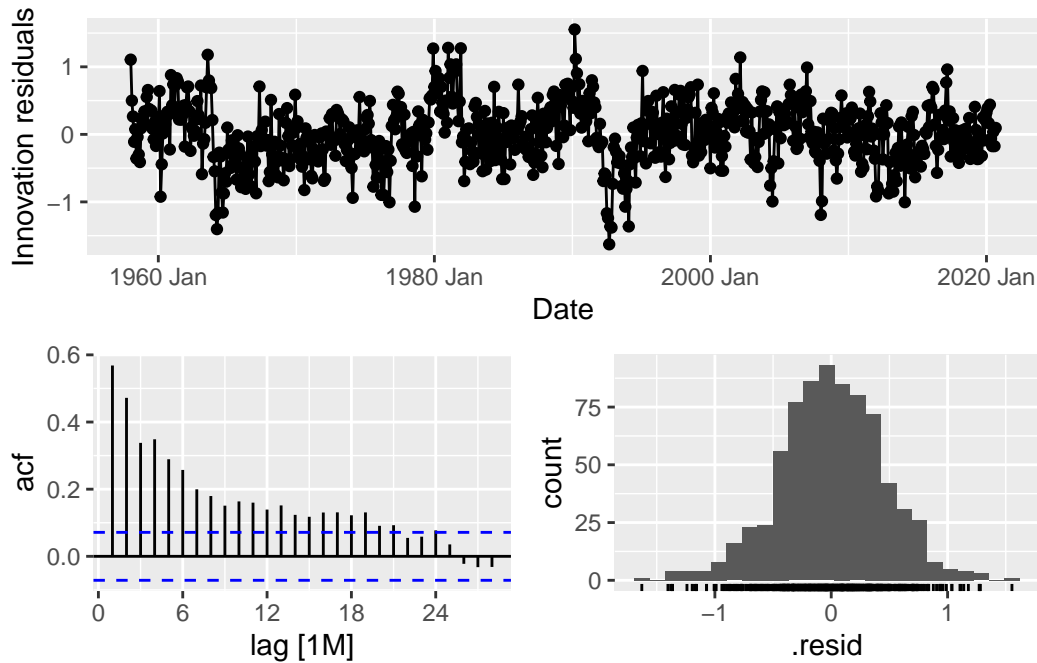
```



Comparing best models to the under-performing ones, we see that adding predictors, especially

ones that are lagged, closes the gap in the increasing trend that other models seem to miss.

```
tslm_best_fit |>
  select(tslm_no_trend) |>
  tail(n = 1) |>
  ggtime::gg_tsresiduals()
```



The biggest issue is that there is significant autocorrelation in the residuals, indicating there is some pattern the TSLM model doesn't catch. We'll see if the ARIMA models are able to accurately account for these factors

## ARIMA External Predictors

Using a non-cv split initially to speed up fit time

```
arima_trn <- predictor_modeling |>
  filter(year(Date) > 1957) |>
  slice(1:(n() - 60))

arima_valid <- predictor_modeling |>
  filter(year(Date) > 1957) |>
  slice_tail(n = 60)
```

```

arima_fit <- arima_trn |>
  model(
    arima_auto = ARIMA(actual_temp ~ co2_ppm + PDQ(D=1)),
    arima_box = ARIMA(box_cox(actual_temp, lambda = 1.5) ~ co2_ppm + PDQ(D=1)),
    arima_log = ARIMA(log(actual_temp) ~ co2_ppm + PDQ(D=1)),
    arima_all = ARIMA(actual_temp ~ co2_ppm + ch4_ppb + n2o_ppb + TSI + el_nino + la_nina + v
    arima_lag_ten = ARIMA(actual_temp ~ co2_lag1 + PDQ(D=1)),
    arima_lag_ten_nino = ARIMA(actual_temp ~ co2_lag1 + el_nino + la_nina + PDQ(D=1)),
    arima_enso = ARIMA(actual_temp ~ ENSO + PDQ(D=1)),
    arima_enso_smooth = ARIMA(actual_temp ~ enso_smooth_6 + PDQ(D=1)),
    arima_emissions = ARIMA(actual_temp ~ aggregate_emissions + PDQ(D=1)),
    arima_forcing = ARIMA(log(actual_temp) ~ rf_co2 + PDQ(D=1)),
    arima_enso_delay = ARIMA(actual_temp ~ enso_delay + PDQ(D=1)),
    arima_lag_3 = ARIMA(actual_temp ~ co2_lag3 + PDQ(D=1)),
    arima_physics_robust = ARIMA(log(actual_temp) ~ rf_co2 + volcano_linear + enso_delay + P
    arima_volcano_basic = ARIMA(log(actual_temp) ~ ENSO + co2_ppm + volcano_forcing + PDQ(D
  )

arima_fit |>
  glance() |>
  arrange(AICc) |>
  select(.model, AICc, AIC, BIC)

```

```

# A tibble: 14 x 4
  .model          AICc    AIC    BIC
  <chr>          <dbl> <dbl> <dbl>
1 arima_volcano_basic -5155. -5155. -5118.
2 arima_physics_robust -5021. -5021. -4975.
3 arima_forcing      -4925. -4926. -4893.
4 arima_log          -4925. -4925. -4888.
5 arima_enso_smooth  -1259. -1259. -1222.
6 arima_enso         -1255. -1255. -1218.
7 arima_enso_delay   -1228. -1228. -1191.
8 arima_lag_ten_nino -1138. -1138. -1092.
9 arima_auto         -1126. -1126. -1089.
10 arima_emissions   -1125. -1125. -1089.
11 arima_lag_ten     -1124. -1124. -1087.
12 arima_lag_3       -1124. -1124. -1087.
13 arima_all         -963.  -963.  -903.
14 arima_box          828.   828.   865.

```

Looking through the AICc scores, the log transformations seem to result in the best models

```

arima_fc <- arima_fit |>
  forecast(new_data = arima_valid)

arima_fc |>
  accuracy(arima_valid) |>
  select(.model, ME:MAPE) |>
  arrange(RMSE) |>
  kable(digits = 4)

```

.model	ME	RMSE	MAE	MPE	MAPE
arima_volcano_basic	0.0340	0.1444	0.1185	0.2138	0.7956
arima_physics_robust	0.0121	0.1551	0.1197	0.0434	0.8093
arima_all	-0.0221	0.1781	0.1387	-0.1978	0.9362
arima_lag_ten_nino	0.0475	0.1934	0.1549	0.2656	1.0352
arima_auto	0.0072	0.1953	0.1630	-0.0024	1.0982
arima_box	0.0069	0.1967	0.1638	-0.0069	1.1030
arima_emissions	0.0081	0.1981	0.1662	0.0027	1.1197
arima_log	-0.0205	0.1995	0.1654	-0.1885	1.1179
arima_forcing	-0.0086	0.2022	0.1675	-0.1106	1.1314
arima_lag_3	0.0328	0.2067	0.1741	0.1649	1.1692
arima_lag_ten	0.0301	0.2074	0.1742	0.1480	1.1700
arima_enso	0.1923	0.2819	0.2302	1.2342	1.5068
arima_enso_smooth	0.2114	0.2859	0.2347	1.3633	1.5325
arima_enso_delay	0.2180	0.3086	0.2519	1.3999	1.6468

The model including features for volcanic activity and non-lagged variables for CO2 and ENSO is the best overall by any metric. **include a more robust explanation of why a delay in the enso index and why co2 radiative forcing on it's own aren't strong signals**

```

# spot check models
arima_fit |>
  select(arima_volcano_basic) |>
  report()

```

Series: actual\_temp  
Model: LM w/ ARIMA(1,0,1)(0,1,2)[12] errors  
Transformation: log(actual\_temp)

Coefficients:

ar1	ma1	sma1	sma2	ENSO	co2_ppm	volcano_forcing
-----	-----	------	------	------	---------	-----------------

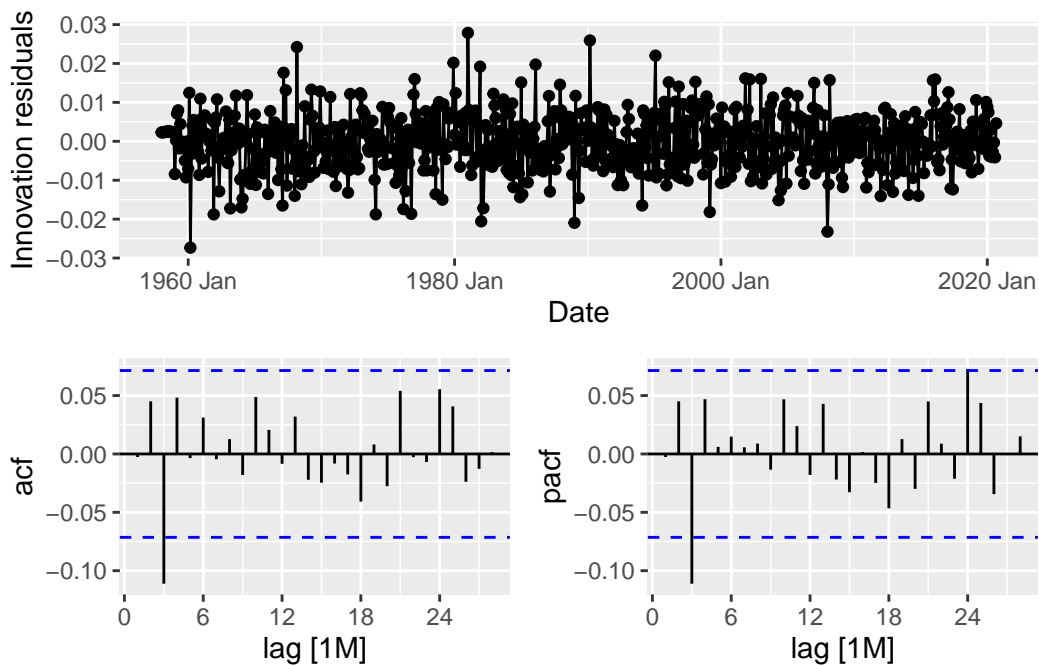
	0.8344	-0.3830	-0.9473	0.0544	0.0046	7e-04	-9e-04
s.e.	0.0326	0.0552	0.0355	0.0364	0.0007	1e-04	9e-04

sigma<sup>2</sup> estimated as 5.37e-05: log likelihood=2585.44  
AIC=-5154.88 AICc=-5154.68 BIC=-5118.02

```

arima_fit |>
  select(arima_volcano_basic) |>
  ggtime::gg_tsresiduals(plot_type = "partial")

```



The `arima_volcano_basic` model selected AR(1), MA(1) and sMA(2) with a seasonal difference. The ACF and PACF plots still contain spikes at lag 3. Interestingly, the `arima_physics` robust model contains a spike at lag 24 but not 3. There might be something in delaying the El Nino effect by a few months that explains the autocorrelation

```

# parameter check
arima_fit |>
  tidy() |>
  group_by(term) |>
  summarize(across(.cols = c(estimate:`p.value`), .fns = mean)) |>
  arrange(p.value) |>
  kable(digits = 4)

```

term	estimate	std.error	statistic	p.value
sma1	-0.9267	0.0385	-24.1588	0.0000
sar2	-0.3306	0.0350	-9.4517	0.0000
enso_smooth_6	0.0805	0.0122	6.6143	0.0000
ENSO	0.0313	0.0052	6.3122	0.0000
ar2	0.6779	0.0864	7.8968	0.0000
sar1	-0.6278	0.0347	-18.4039	0.0013
ma2	-0.2650	0.0731	-3.6212	0.0032
enso_delay	0.0158	0.0065	3.7552	0.0120
la_nina	-0.0453	0.0207	-2.1940	0.0297
ma1	0.3425	0.1148	2.8495	0.0433
el_nino	0.0333	0.0177	1.8858	0.0606
aggregate_emissions	0.0006	0.0003	1.8391	0.0663
co2_ppm	0.0126	0.0064	3.2146	0.0665
rf_co2	0.0688	0.0373	1.7833	0.0902
sma2	0.0606	0.0367	1.6484	0.1025
ar3	-0.0228	0.0852	0.1827	0.1074
co2_lag1	0.0079	0.0048	1.6714	0.1114
co2_lag3	0.0069	0.0054	1.2846	0.1993
volcano_linear	-0.0054	0.0062	-1.3169	0.2395
TSI	0.0204	0.0191	1.0671	0.2863
intercept	-0.0011	0.0011	-0.9745	0.3301
volcano_forcing	-0.0009	0.0009	-0.9703	0.3322
ar1	0.1208	0.1062	2.3740	0.4273
n2o_ppb	-0.0049	0.0123	-0.3959	0.6923
ch4_ppb	0.0001	0.0009	0.0822	0.9345

Not necessarily the best way to ascertain how important variables are, but informative nonetheless. ENSO related variables and seasonal ARIMA parameters seem to be the most important to modeling this type of data

Now we'll check the best models from above as well as a few other promising predictor combinations across 5 CV splits and see how they hold up

```

arima_second_fit <- cv_trn |>
  model(
    arima_enso_log_co2 = ARIMA(log(actual_temp) ~ ENSO + co2_ppm + PDQ(D = 1)),
    arima_auto_10 = ARIMA(log(actual_temp) ~ ENSO + co2_lag10 + PDQ(D = 1)),
    arima_physics_robust = ARIMA(log(actual_temp) ~ rf_co2 + volcano_linear + enso_delay + PDQ(D = 1)),
    arima_forcing = ARIMA(log(actual_temp) ~ rf_co2 + enso_smooth_6 + PDQ(D = 1)),
    arima_auto_10_enso_6 = ARIMA(log(actual_temp) ~ enso_smooth_6 + co2_lag10 + PDQ(D = 1)),
    arima_robust_enso_6 = ARIMA(log(actual_temp) ~ rf_co2 + volcano_linear + enso_smooth_6 + PDQ(D = 1))
  )

```

```

    arima_co2_10_volcano = ARIMA(log(actual_temp) ~ co2_lag10 + volcano_linear + PDQ(D = 1)),
    arima_co2_3_nino = ARIMA(actual_temp ~ co2_lag3 + el_nino + la_nina + PDQ(D = 1)),
    arima_co2_3_nino_tsi = ARIMA(actual_temp ~ co2_lag3 + el_nino + la_nina + TSI + PDQ(D = 1)),
    arima_physics_robust_decay = ARIMA(log(actual_temp) ~ rf_co2 + volcano_forcing + enso_decay + PDQ(D = 1)),
    arima_physics_robust_decayenso_3 = ARIMA(log(actual_temp) ~ rf_co2 + volcano_forcing + enso_smooth_3 + PDQ(D = 1)),
    arima_lag_3enso = ARIMA(log(actual_temp) ~ co2_lag3 + ENSO + PDQ(D=1)),
    arima_physicsenso_3 = ARIMA(log(actual_temp) ~ rf_co2 + enso_smooth_3 + PDQ(D=1))
  )

arima_second_fc <- arima_second_fit |>
  forecast(new_data = cv_valid)

arima_second_fc |>
  accuracy(cv_valid) |>
  select(.model, ME:MAPE) |>
  group_by(.model) |>
  summarize(across(.cols = c(ME:MAPE), .fns = mean)) |>
  arrange(RMSE) |>
  kable(digits = 4)

```

.model	ME	RMSE	MAE	MPE	MAPE
arima_auto_10enso_6	-0.0047	0.1225	0.0947	-0.0339	0.6524
arima_physics_robust_decayenso_3	0.0005	0.1233	0.0946	0.0080	0.6455
arima_forcing	-0.0005	0.1266	0.1005	-0.0087	0.6911
arima_robustenso_6	-0.0004	0.1273	0.1018	-0.0058	0.6998
arima_physicsenso_3	-0.0011	0.1304	0.1012	-0.0081	0.6936
arima_lag_3enso	0.0084	0.1322	0.1044	0.0559	0.7170
arima_auto_10	-0.0048	0.1356	0.1083	-0.0321	0.7438
arimaenso_log_co2	-0.0304	0.1358	0.1076	-0.2048	0.7358
arima_physics_robust	0.0158	0.1376	0.1065	0.1003	0.7321
arima_physics_robust_decay	0.0142	0.1379	0.1064	0.0897	0.7316
arima_co2_10_volcano	-0.0218	0.1597	0.1268	-0.1552	0.8744
arima_co2_3_nino_tsi	0.0563	0.1598	0.1275	0.3676	0.8735
arima_co2_3_nino	0.0624	0.1608	0.1290	0.4063	0.8817

Across multiple folds, the forcing models and models that smooth out the enso index and/or utilize a lagged CO2 concentration variable perform the best. All the results are fairly close and would probably trade places over different subsets of the data so choosing the best one will require consideration of other factors

```

arima_second_fit |>
  glance() |>
  select(.model, AICc, AIC, BIC) |>
  group_by(.model) |>
  summarize(across(.cols = c(AICc, AIC, BIC), .fns = mean)) |>
  arrange(AICc)

```

```

# A tibble: 13 x 4
  .model                AICc    AIC    BIC
  <chr>                <dbl> <dbl> <dbl>
1 arima_physics_robust_decay_enso_3 -4300. -4300. -4257.
2 arima_auto_10          -4273. -4273. -4240.
3 arima_physics_enso_3    -4271. -4272. -4233.
4 arima_enso_log_co2     -4261. -4261. -4226.
5 arima_lag_3_enso       -4258. -4258. -4222.
6 arima_auto_10_enso_6    -4212. -4212. -4175.
7 arima_physics_robust    -4196. -4196. -4154.
8 arima_physics_robust_decay -4195. -4196. -4153.
9 arima_robust_enso_6     -4192. -4192. -4149.
10 arima_forcing          -4177. -4177. -4138.
11 arima_co2_10_volcano    -4141. -4141. -4106.
12 arima_co2_3_nino_tsi    -930.  -930.  -881.
13 arima_co2_3_nino       -918.  -918.  -874.

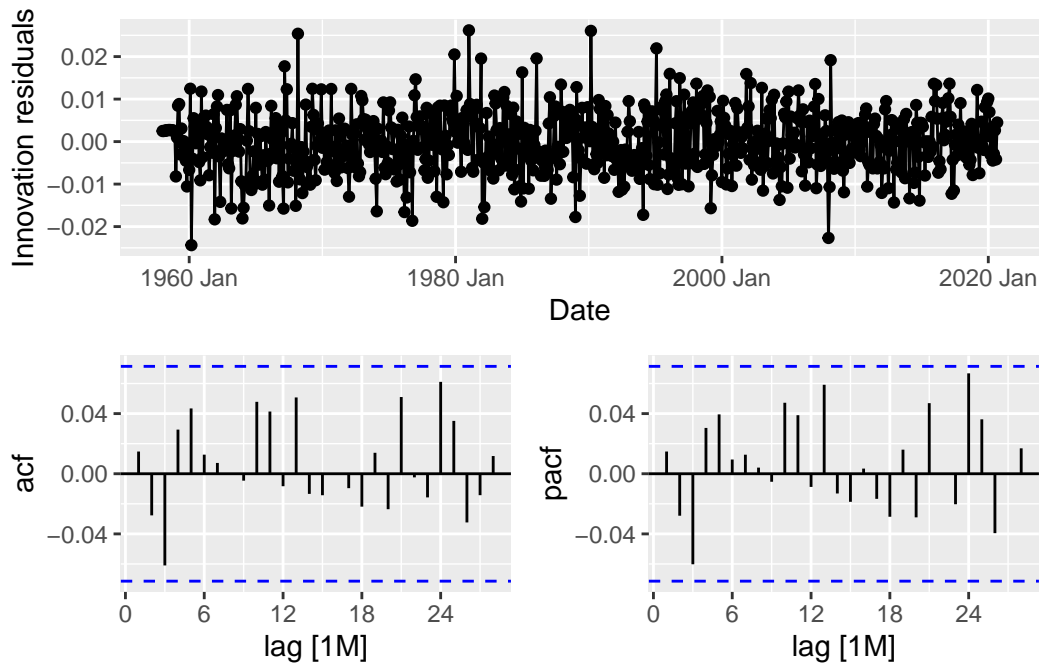
```

The top model by AICc was second best metric wise. It's also clear from the bottom models that a log transformation of the response variable drastically improves the models

```

arima_second_fit |>
  select(arima_physics_robust_decay_enso_3) |>
  tail(n = 1) |>
  ggtime::gg_tsresiduals(plot_type = "partial")

```



```

arima_second_fit |>
  select(arima_physics_robust_decay_enso_3) |>
  tail(n = 1) |>
  augment() |>
  features(.innov, ljung_box, lag = 24)

```

```

# A tibble: 1 x 4
  .id .model lb_stat lb_pvalue
<int> <chr>   <dbl>   <dbl>
1     5 arima_physics_robust_decay_enso_3 17.4    0.832

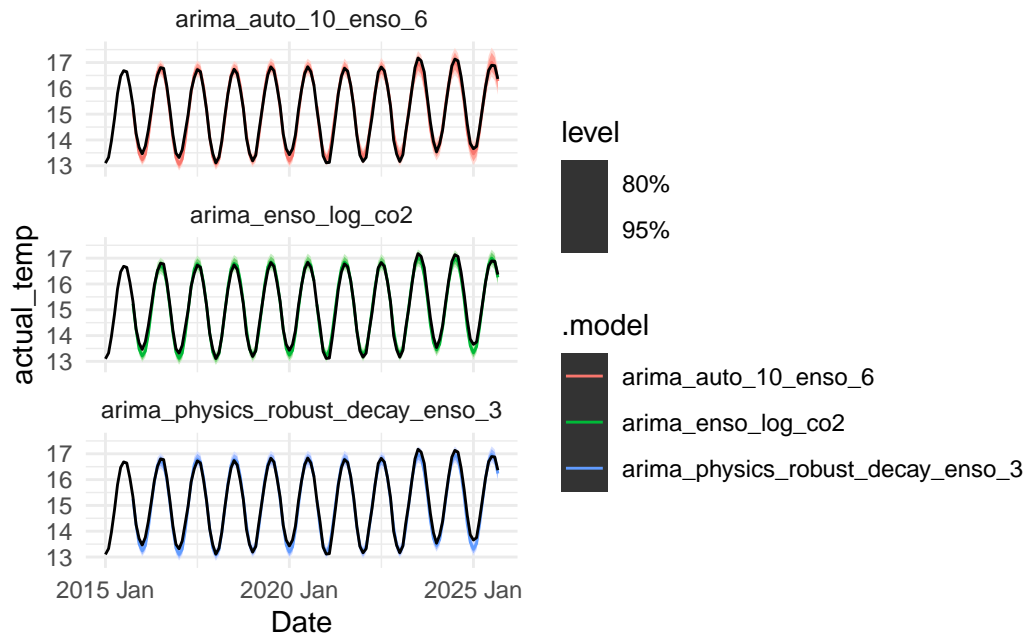
```

No autocorrelation spikes in the residuals and the Ljung-Box test confirms that the residuals resemble white noise for the `arima_physics_robust_decay_enso_3` model. Most other models have a spike at lag 3 or lag 24. The only other top model without any autocorrelation is `arima_physics_enso_3` so a 3 month smoothing of the ENSO index seems to take care of it

```

arima_second_fc |>
  filter(.id %in% c(4, 5), .model %in% c("arima_physics_robust_decay_enso_3", "arima_enso_log")) +
  autoplot() +
  autolayer(predictor_modeling |> filter(year(Date) >= 2015), actual_temp) +
  facet_wrap(~ .model, ncol = 1) +
  theme_minimal()

```



Retrain the top models on the same subset of data and compare

```
final_trn <- predictor_modeling |>
  filter(year(Date) > 1957) |>
  slice(1:(n() - 60))

final_valid <- predictor_modeling |>
  filter(year(Date) > 1957) |>
  slice_tail(n = 60)

final_fit <- cv_trn |>
  model(
    ets_auto = ETS(actual_temp),
    ets_additive = ETS(actual_temp ~ error("A") + trend("A") + season("A")),
    ets_box_auto = ETS(box_cox(actual_temp, lambda = 1.5)),
    tslm_no_trend = TSLM(box_cox(actual_temp, lambda = 1.5) ~ season() + enso_smooth_12 + co2),
    tslm_volcanic = TSLM(box_cox(actual_temp, lambda = 1.5) ~ season() + enso_smooth_12 + co2),
    tslm_enso_smooth_lag = TSLM(box_cox(actual_temp, lambda = 1.5) ~ trend() + season() + enso_smooth_lag),
    arima_auto_10 = ARIMA(log(actual_temp) ~ ENSO + co2_lag10 + PDQ(D = 1)),
    arima_physics_robust_decay_enso_3 = ARIMA(log(actual_temp) ~ rf_co2 + volcano_forcing + enso_smooth_3 + PDQ(D = 1)),
    arima_physics_enso_3 = ARIMA(log(actual_temp) ~ rf_co2 + enso_smooth_3 + PDQ(D = 1)),
    arima_enso_log_co2 = ARIMA(log(actual_temp) ~ ENSO + co2_ppm + PDQ(D = 1)),
```

```

)

final_fc <- final_fit |>
  forecast(new_data = cv_valid)

final_fc |>
  accuracy(cv_valid) |>
  select(.model, ME:MAPE) |>
  group_by(.model) |>
  summarize(across(.cols = c(ME:MAPE), .fns = mean)) |>
  arrange(RMSE) |>
  kable(digits = 4)

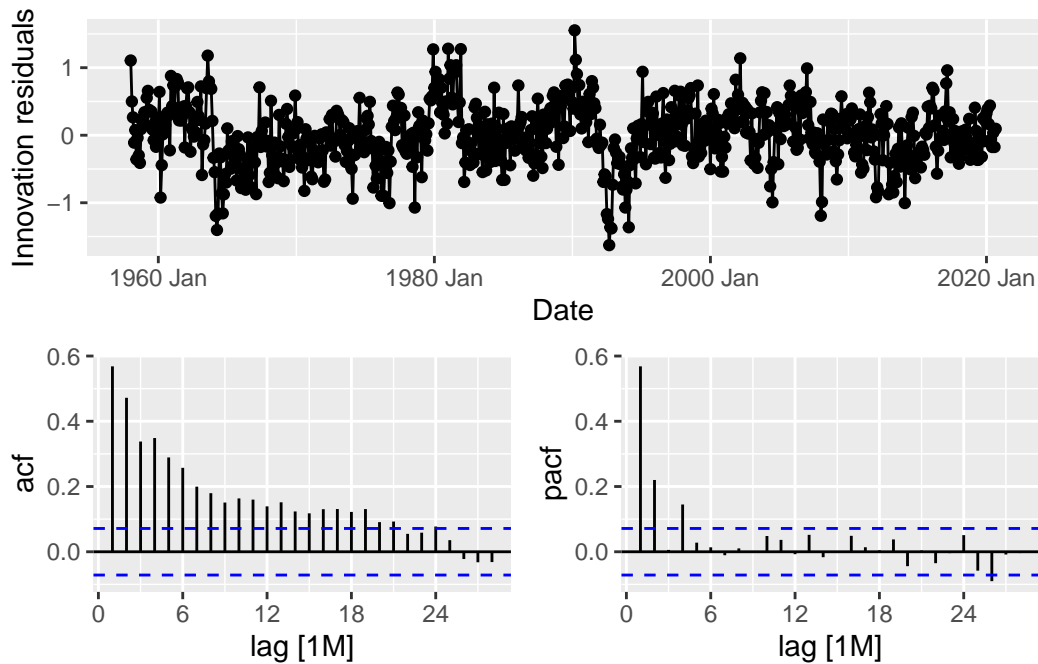
```

.model	ME	RMSE	MAE	MPE	MAPE
tslm_no_trend	-0.0032	0.1183	0.0927	-0.0388	0.6389
tslm_volcanic	0.0013	0.1191	0.0936	-0.0087	0.6432
tslm_enso_smooth_lag	-0.0063	0.1217	0.0950	-0.0599	0.6554
arima_physics_robust_decay_enso_3	0.0005	0.1233	0.0946	0.0080	0.6455
arima_physics_enso_3	-0.0011	0.1304	0.1012	-0.0081	0.6936
arima_auto_10	-0.0048	0.1356	0.1083	-0.0321	0.7438
arima_enso_log_co2	-0.0304	0.1358	0.1076	-0.2048	0.7358
ets_additive	0.0397	0.1569	0.1272	0.2595	0.8714
ets_box_auto	0.0342	0.1621	0.1331	0.2098	0.9095
ets_auto	0.0604	0.1656	0.1348	0.4006	0.9212

```

final_fit |>
  select(tslm_no_trend) |>
  tail(n = 1) |>
  ggtime::gg_tsresiduals(plot_type = "partial")

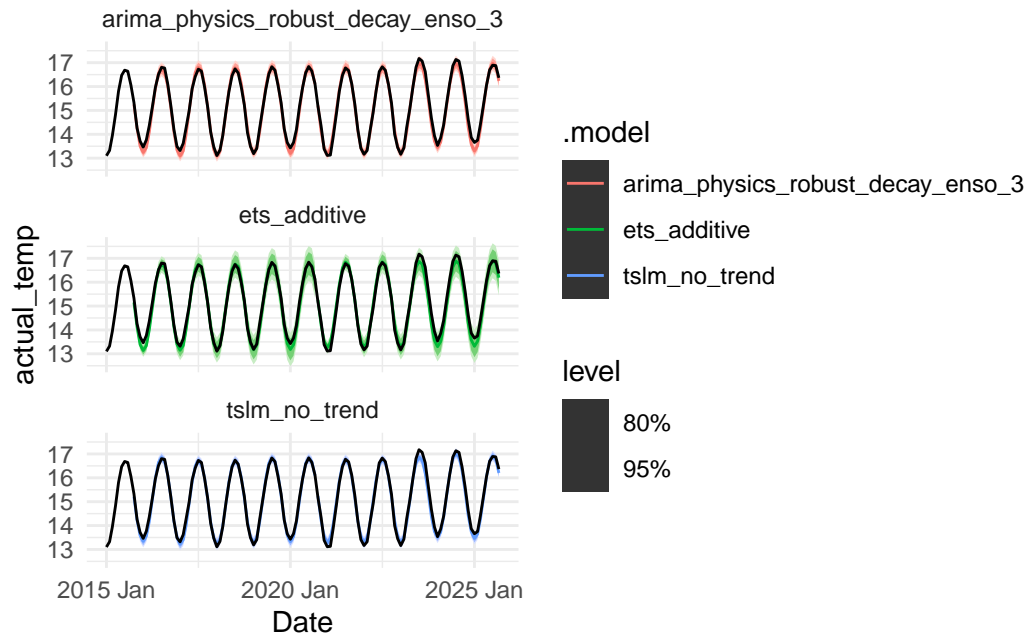
```



```
final_fit |>
  select(arima_auto_10) |>
  augment() |>
  features(.innov, ljung_box, lag = 24)
```

```
# A tibble: 5 x 4
  .id .model      lb_stat lb_pvalue
<int> <chr>      <dbl>   <dbl>
1     1 arima_auto_10  19.0    0.749
2     2 arima_auto_10  45.6    0.00501
3     3 arima_auto_10  22.8    0.530
4     4 arima_auto_10  16.8    0.856
5     5 arima_auto_10  24.7    0.424
```

```
final_fc |>
  filter(.id %in% c(4, 5), .model %in% c("tslm_no_trend", "arima_physics_robust_decay_enso_3")) +
  autoplot() +
  autolayer(predictor_modeling |> filter(year(Date) >= 2015), actual_temp) +
  facet_wrap(~ .model, ncol = 1) +
  theme_minimal()
```



```
# for the enso index, the average across the whole data set is 0.0470

low_enso <- predictor_modeling |>
  select(ENSO) |>
  slice_tail(n = 600)
# 0.0158

high_enso <- predictor_modeling |>
  select(ENSO) |>
  slice_head(n = 600)

# 0.0997

normal_enso <- predictor_modeling |>
  select(ENSO) |>
  slice(650:1250)

# 0.0469
#
# Then would need smoothed versions of these
```