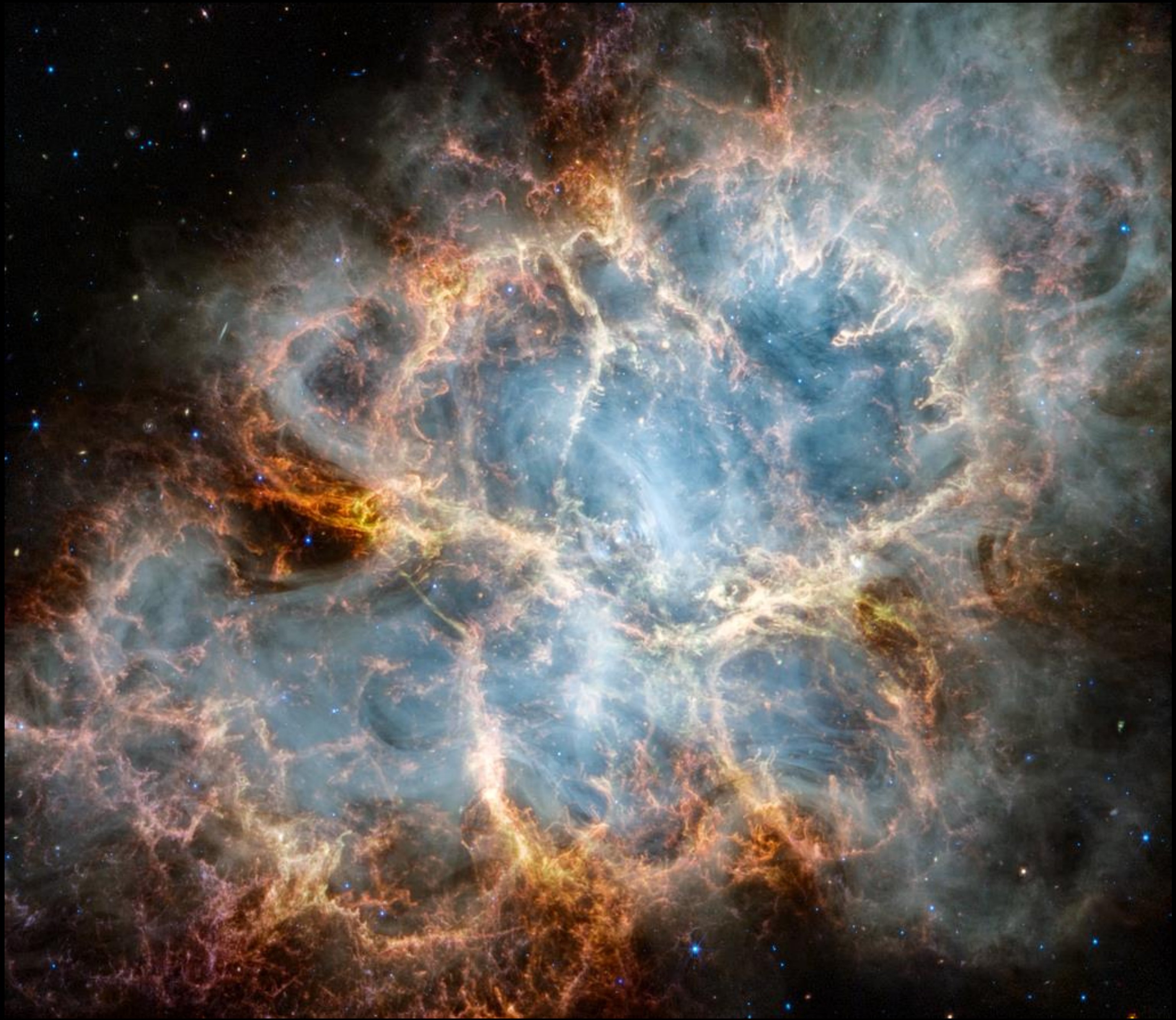
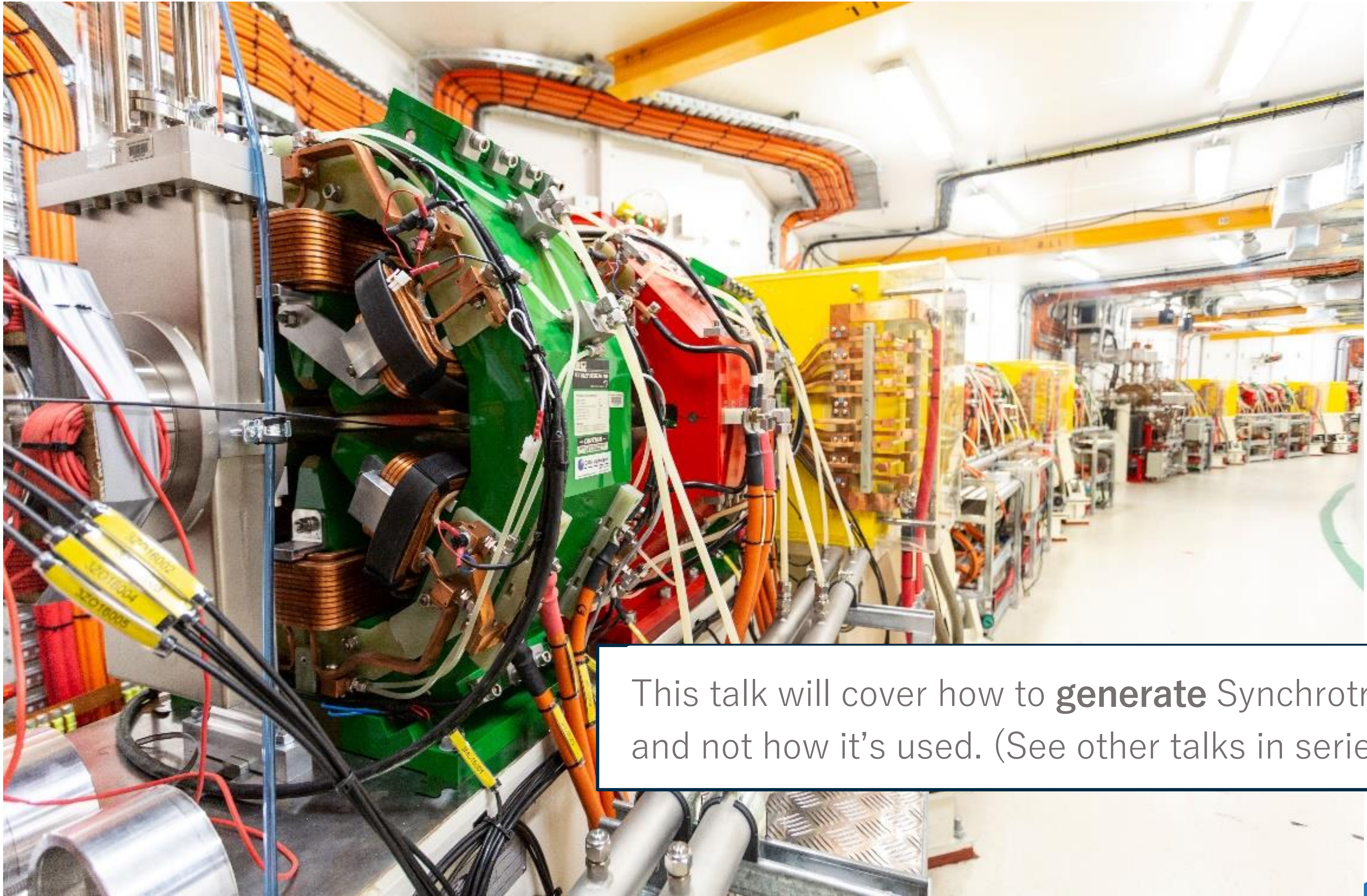
Basics of Light Sources

Tessa Charles
Australian Synchrotron

Science. Ingenuity. Sustainability.


Credit: NASA, ESA, CSA, STScI, T. Temim (Princeton University)



This talk will cover how to **generate** Synchrotron Radiation and not how it's used. (See other talks in series for that).

First Observation of Synchrotron Radiation on Earth

General Electric Research Laboratory in New York, in April 1947.

They didn't call it synchrotron radiation. Instead, they called it **"Schwinger radiation"**, after J. Schwinger who developed much of the theory of classical radiation from accelerated electrons.

> *"If the 100-MeV betatron had been built with a transparent glass vacuum tube, as was a 70-MeV synchrotron in 1946, synchrotron radiation today would be called **Betatron radiation**."*

- Herbert H. C. Pollock, "The Discovery of Synchrotron Radiation" *Am. J. Phys.* 51, No. 3 (1983) 278

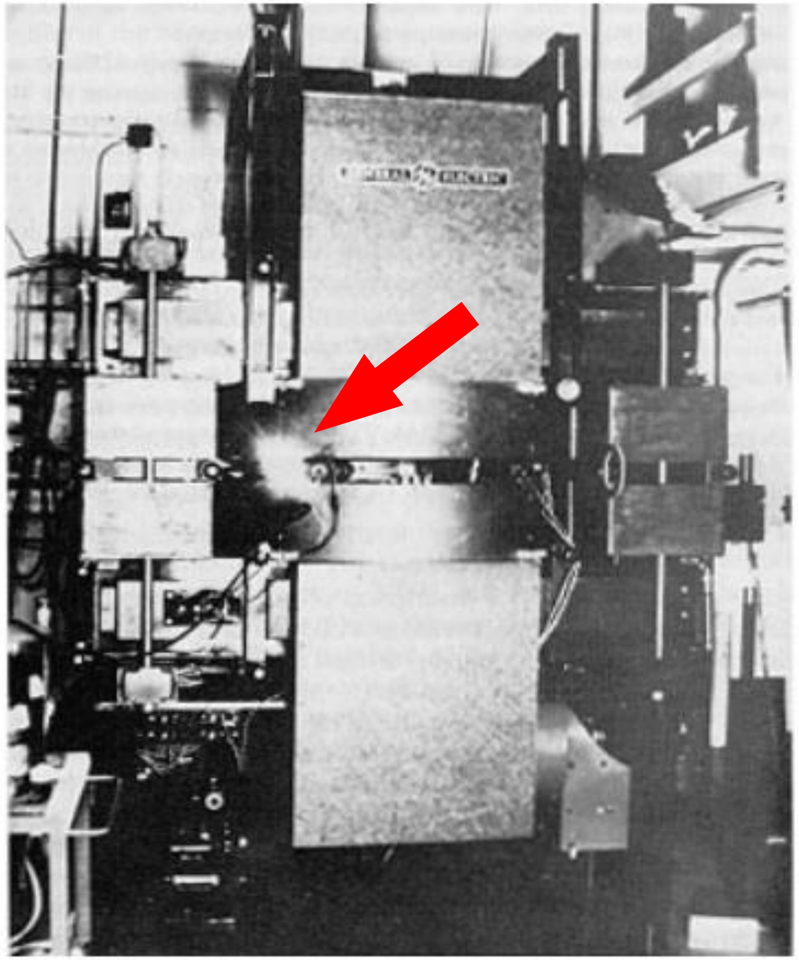


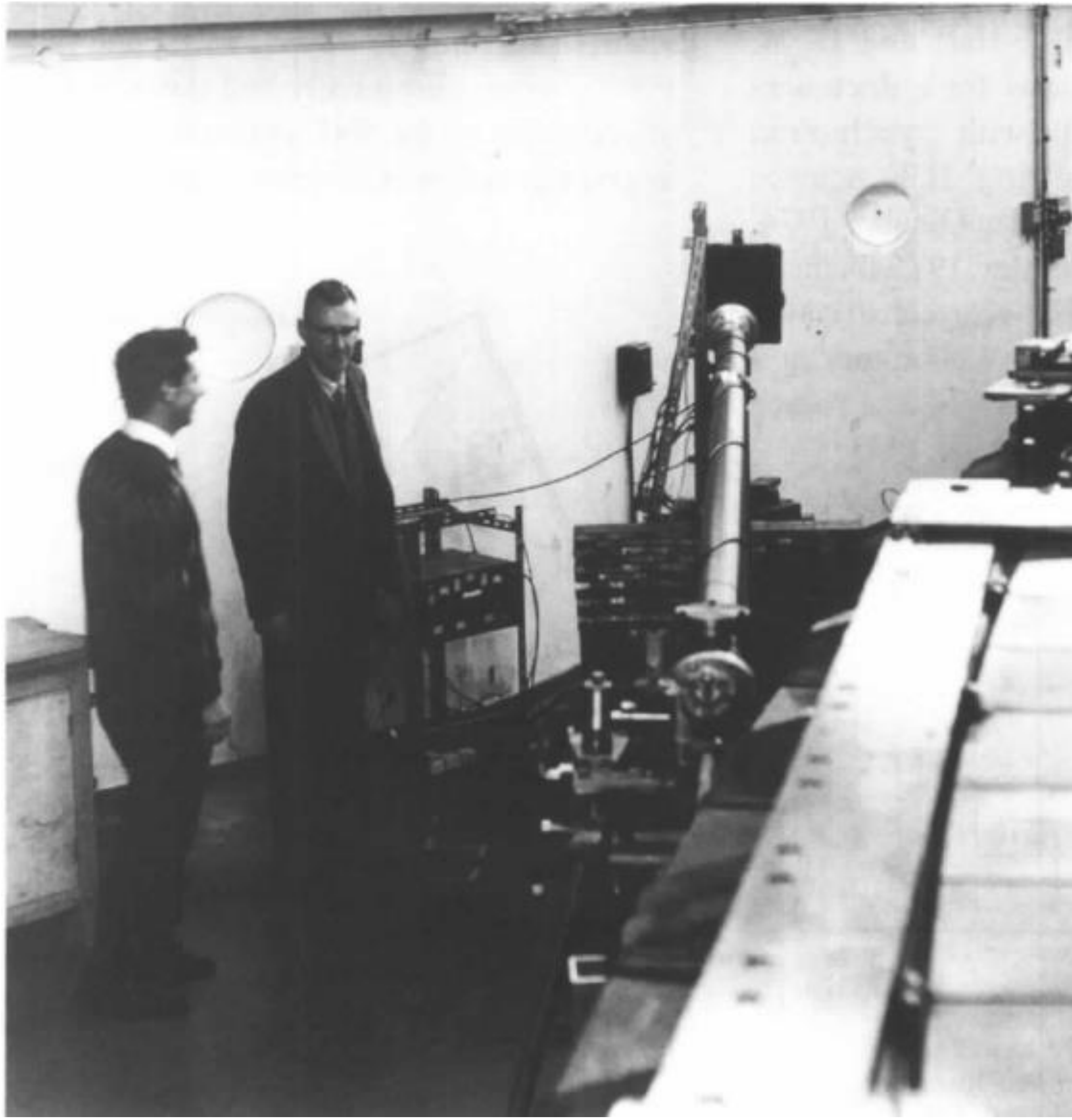
Fig. 1. 70-MeV synchrotron with optical radiation from the electron beam visible through the glass wall of the vacuum "donut" tangent to the beam orbit.

First Observation of Synchrotron Radiation

- The radiation was initially considered a nuisance.
  Energy loss due to SR $\propto E^4$

- “Parasitic” beam lines were set up to use the radiation. And these first Users were referred to as “parasites” or “pirates”.

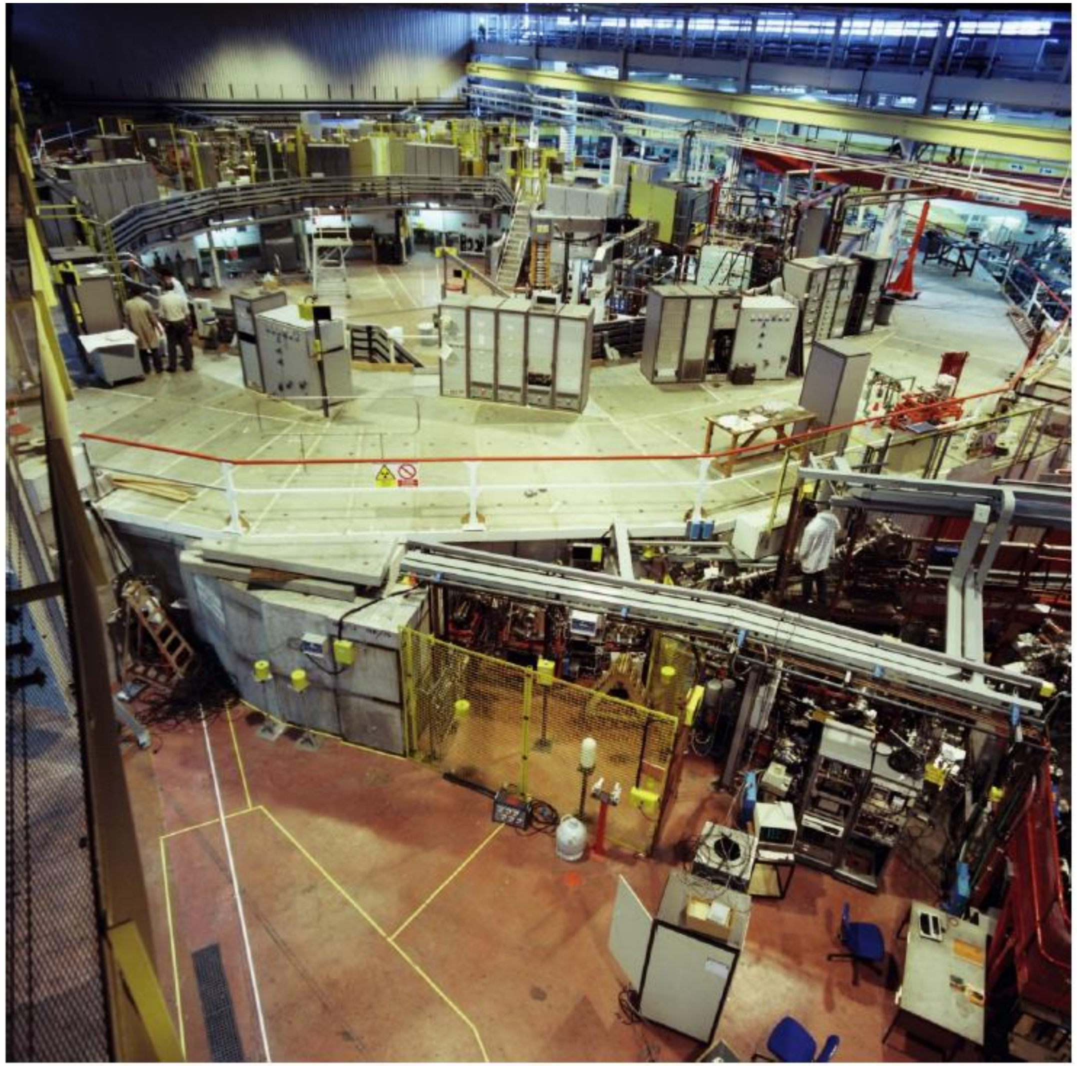
- Competition priorities:
  - Colliders typically want **maximum ρ** to minimise SR power
  - Early light sources want **minimum ρ** to maximise SR power



The first synchrotron radiation beamline on NINA (c.1967)

Image: I. H. Munro J. Synchrotron Rad. (1997). 4, 344-358

Second Generation Light Sources



The Synchrotron Radiation Source (SRS) at the Daresbury Laboratory in the UK was the first, dedicated light source facility. Making it the first second generation light source.

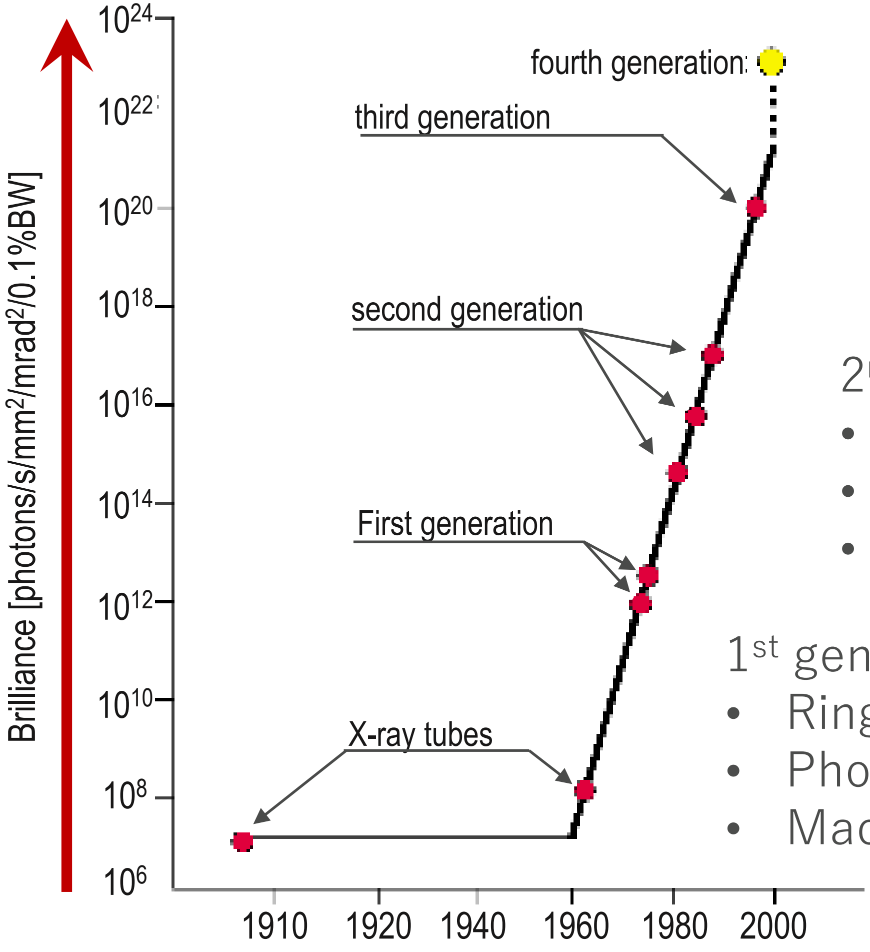
First light June 1980 [1].

Dipole Henge

[1] D.J. Holder , P.D. Quinn, N.G. Wyles, *"The SRS at Daresbury Laboratory: A Eulogy to the World's First Dedicated High-Energy Synchrotron Radiation Source"*, Proceedings of EPAC08, Genoa, Italy WEPC062

Generations of Light Sources

A new generation requires a few orders of magnitude increase in brightness, which requires new technology.



5th generation ??

4th generation

- Ultra-low emittance. Approaching the diffraction limit

3rd generation

- Insertion devices

2nd generation

- Dedicated machines
- Bending magnet radiation
- Large emittance and lower energy x-rays

1st generation

- Rings build as colliders
- Photon beamlines operated parasitically
- Machines built and run primarily for High Energy Physics

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Spectral Brightness

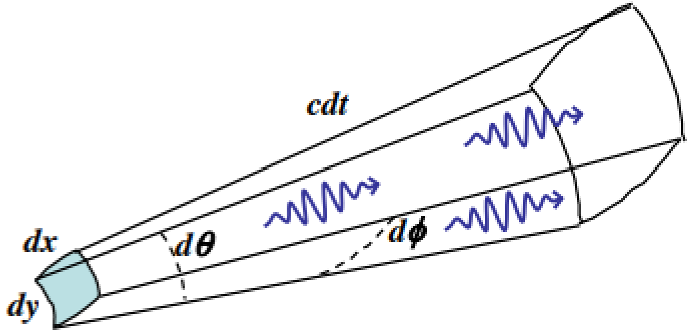
An important figure of merit for synchrotron light sources is brightness.

Flux = no. of photons in $\Delta\omega/\omega$ per unit time:

$$F = \frac{\dot{N}}{(\Delta\omega/\omega)}$$

Spectral Brightness = flux per solid angle and unit area:

$$B(\lambda) = \frac{F}{4\,\pi^2 \Delta x\, \Delta y\, \Delta\theta \Delta\phi}$$



Spectral Brightness

We often write brightness in terms of e⁻ beam and photon beam sizes:

$$B(\lambda) = \frac{F}{4\,\pi^2 \Sigma_x \Sigma_y \; \Sigma_{x\prime} \Sigma_{y\prime}}$$

Unit source area   Unit solid angle

where

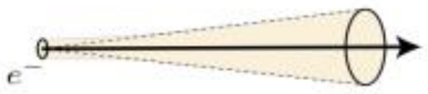
$$\Sigma_{x,y} = \sqrt{\sigma_{x,y}^2 + \sigma_r^2}$$

$$\Sigma_{x\prime,y\prime} = \sqrt{\sigma_{x\prime,y\prime}^2 + \sigma_{r\prime}^2}$$

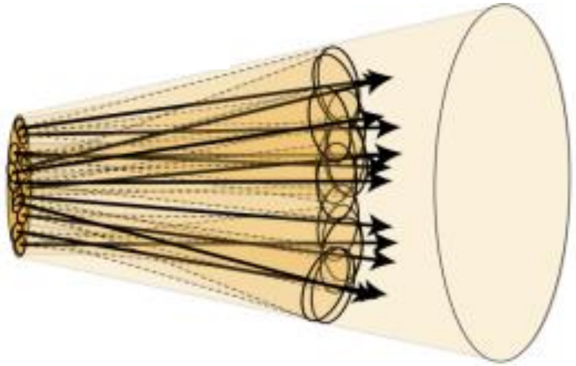
σ_x = e⁻ beam size
$\sigma_{x\prime}$ = e⁻ beam divergence
σ_r = photon source size
$\sigma_{r\prime}$ = photon divergence

$$\sigma_r = \frac{\sqrt{\lambda L}}{4\pi}$$

$$\sigma_{r\prime} = \frac{\sqrt{\lambda}}{L}$$



Single electron



3rd generation light source

4th generation light source

Spectral Brightness

We often write brightness in terms of e⁻ beam and photon beam sizes:

$$B(\lambda) = \frac{F}{4\,\pi^2 \Sigma_x \Sigma_y \;\; \Sigma_{x'} \Sigma_{y'}}$$

Unit source area   Unit solid angle

where

$$\Sigma_{\mathrm{x,y}} = \sqrt{\sigma_{x,y}^2 + \sigma_r^2}$$

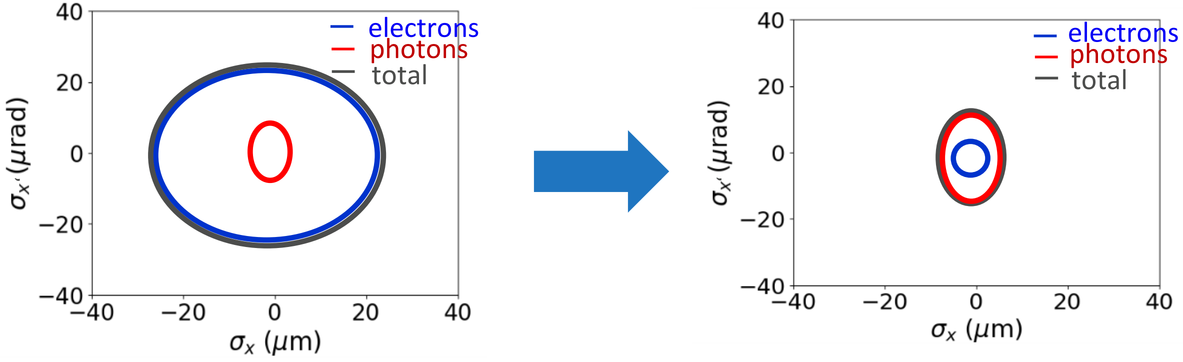
$$\Sigma_{\mathrm{x',y'}} = \sqrt{\sigma_{x',y'}^2 + \sigma_{r'}^2}$$

σ_x=e⁻ beam size
$\sigma_{x'}$= e⁻ beam divergence
σ_r=photon source size
$\sigma_{r'}$= photon divergence

$$\sigma_r = \frac{\sqrt{\lambda L}}{4\pi}$$

$$\sigma_{r'} = \frac{\sqrt{\lambda}}{L}$$

Typical 3rd gen. light source   Typical 4th gen. light source



When the contribution from the e- beam and the photon beam are comparable, we refer the SR as being **"diffraction-limited"**.

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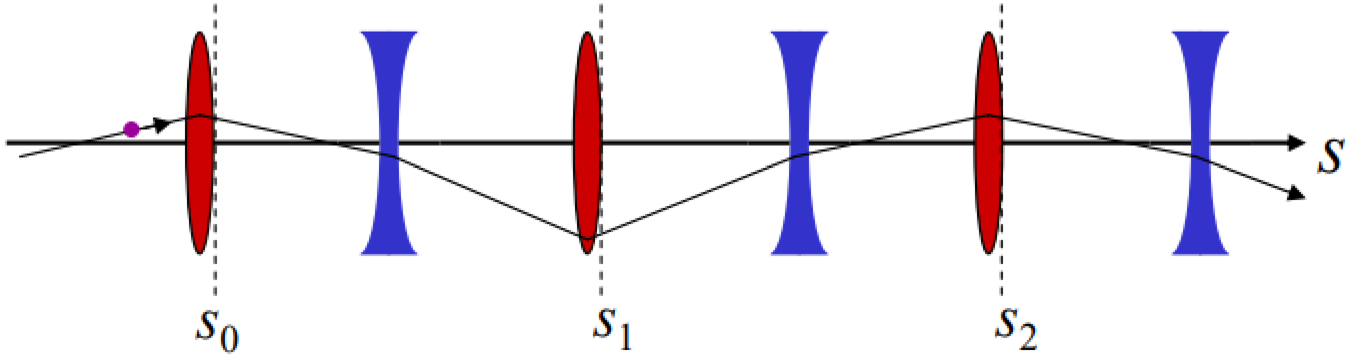
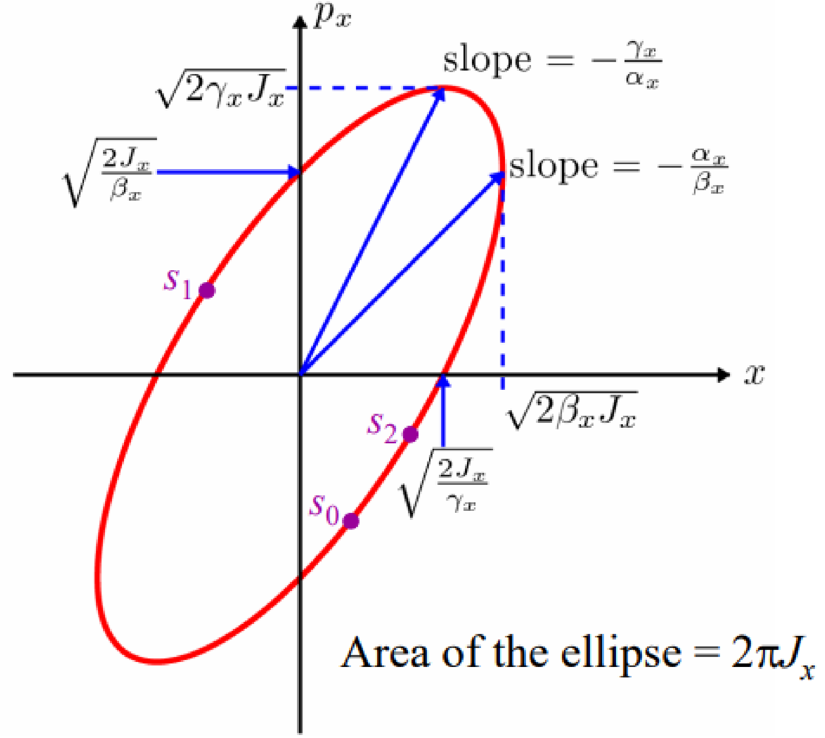
Emittance

Emittance, also referred to as “beam quality”, is a key parameter that governs the overall performance of an accelerator.





Particles trace out ellipses in phase space. The shape of the ellipse defines the Twiss parameters at the observation point.

Figures: A. Wolski, “Low Emittance Machines” CAS September 2007

Emittance

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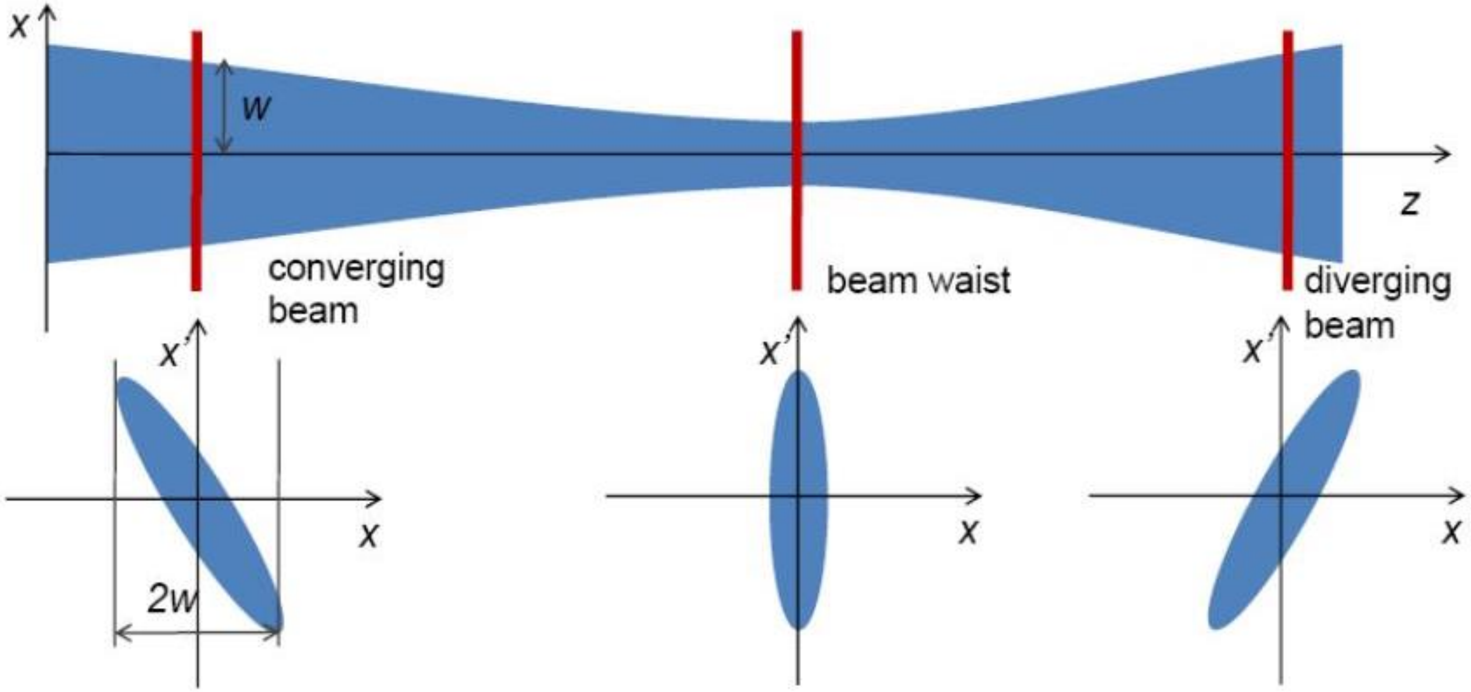

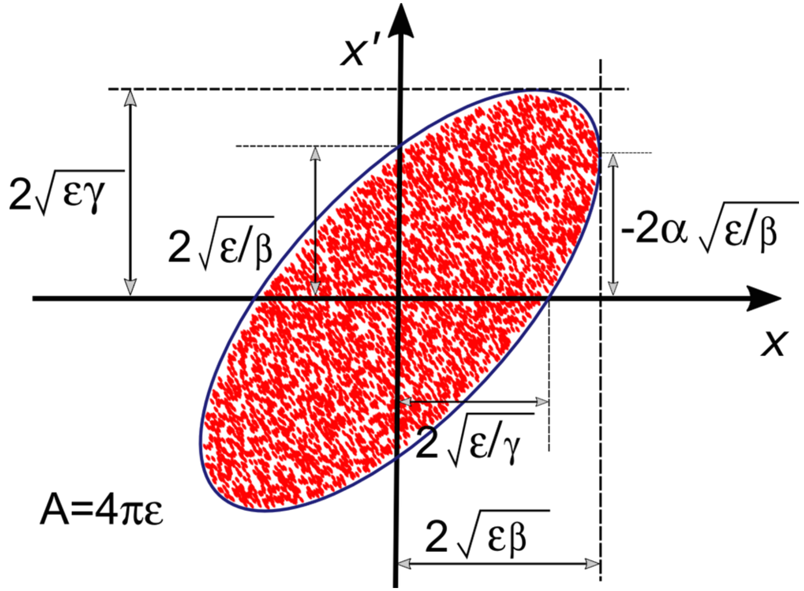

Around the ring, the shape of the ellipse in x-x’ space will change, but importantly, the area will remain constant*.

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Components of a synchrotron light source

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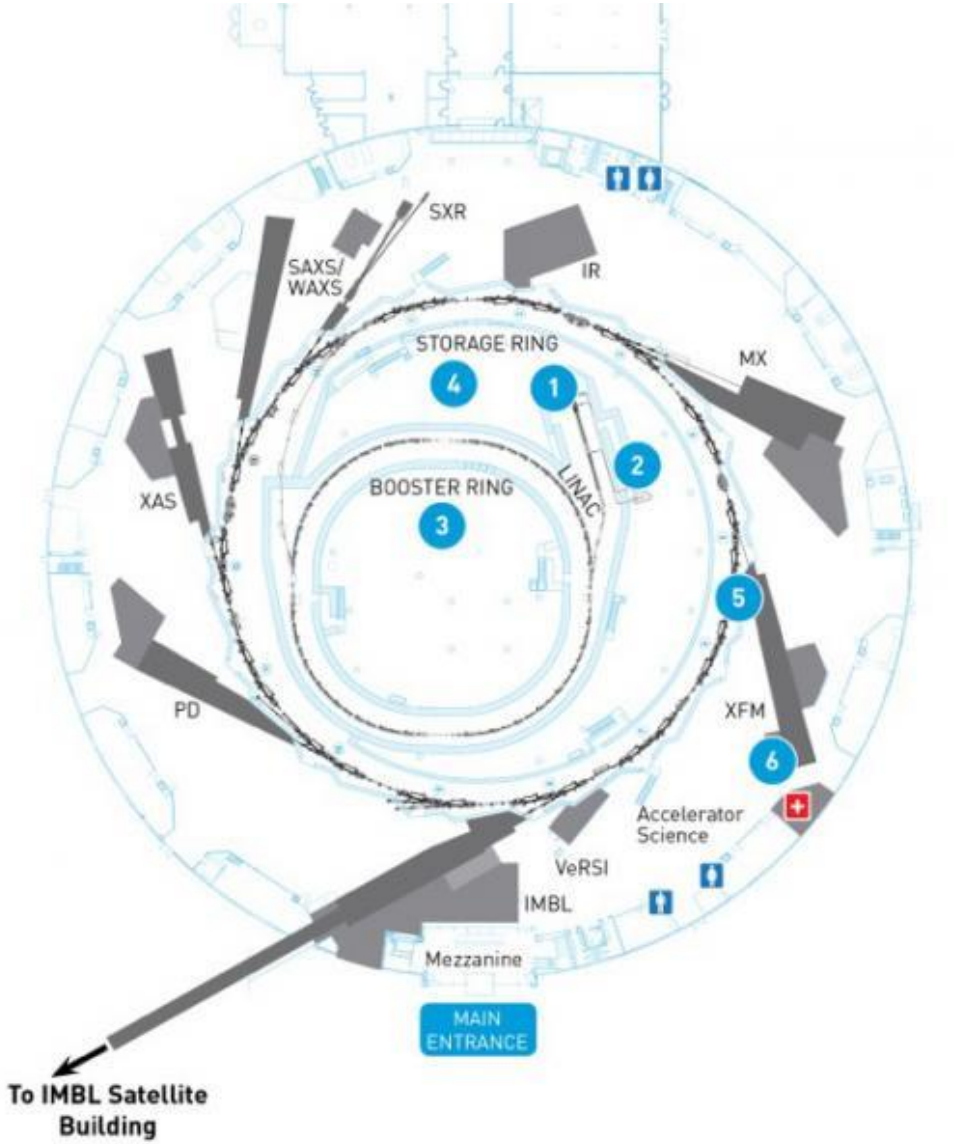
Australian Synchrotron

We have 3 accelerators:

- Linear Accelerator (Linac),
- Booster Ring and
- Storage Ring



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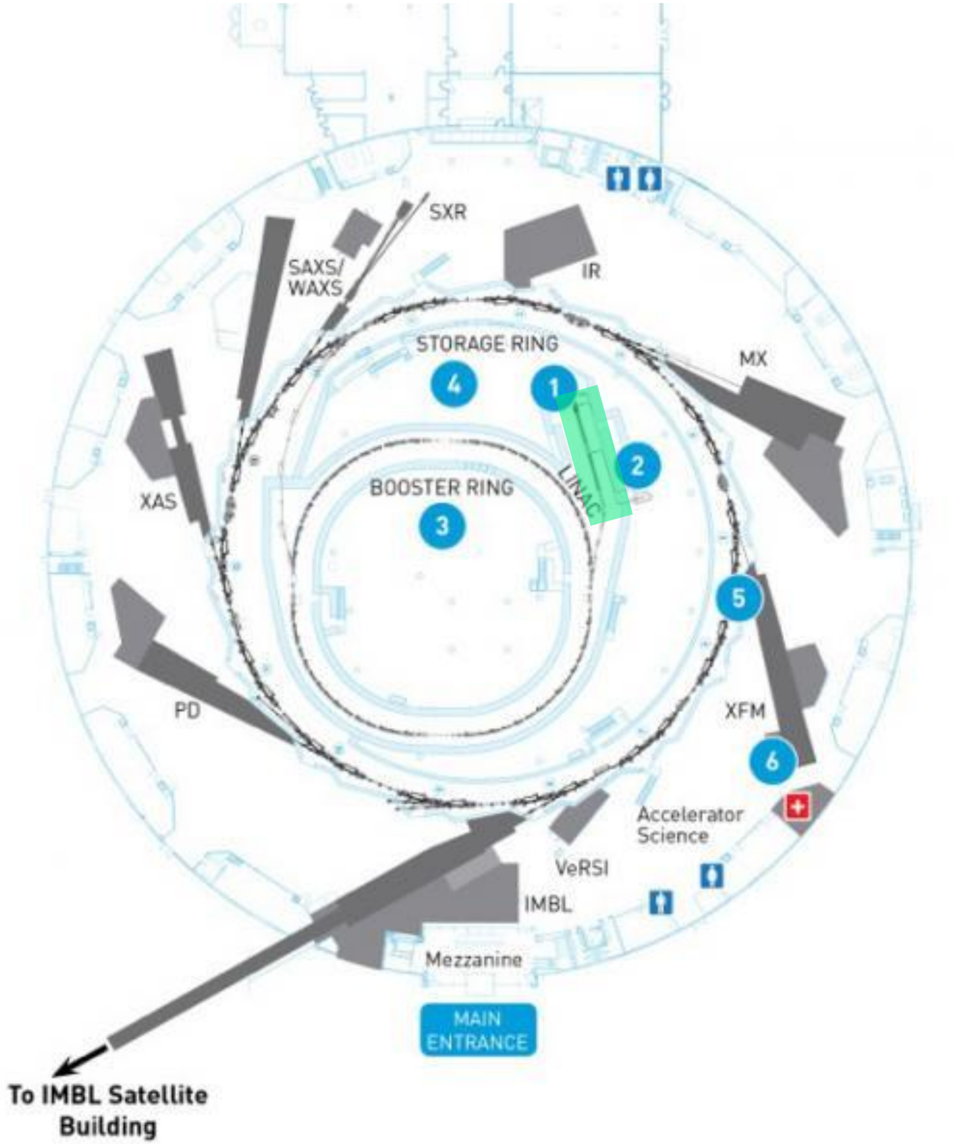

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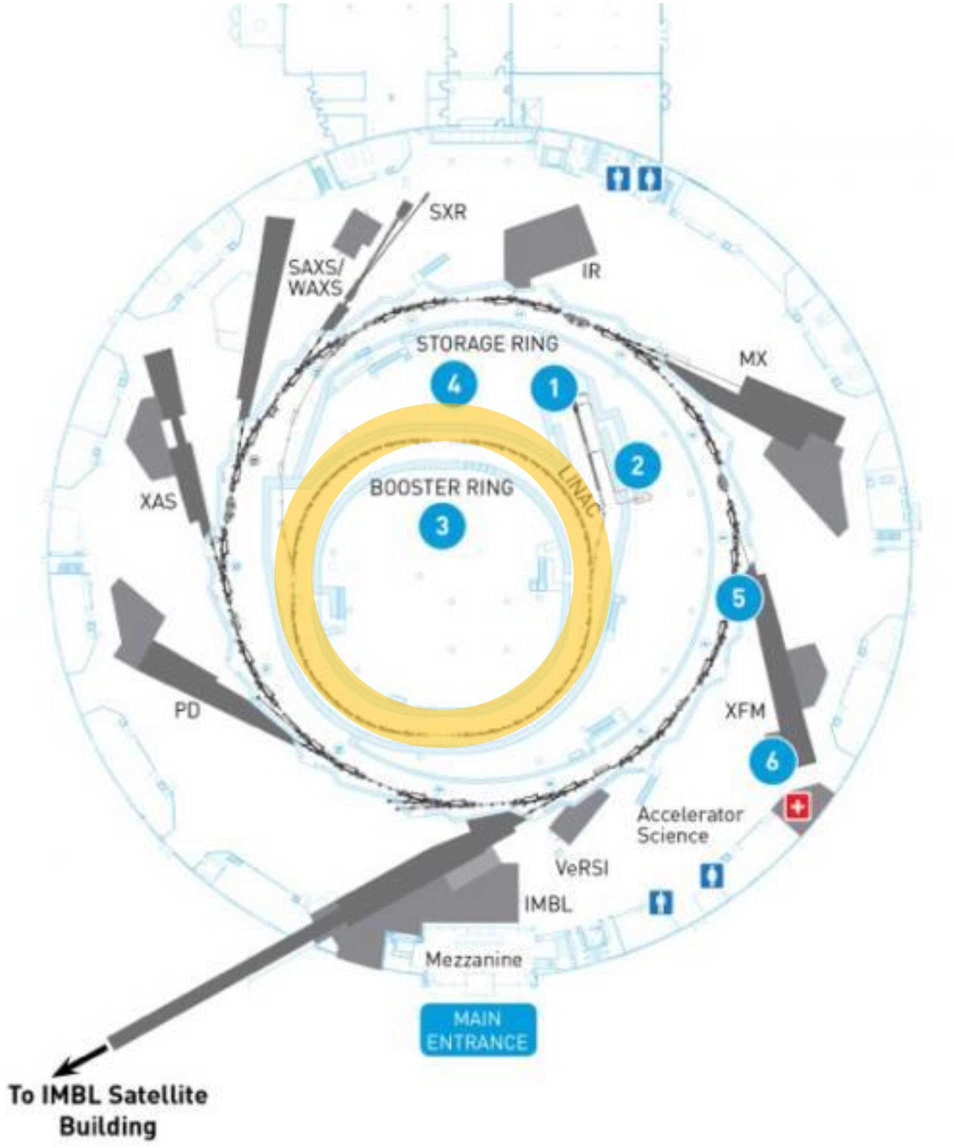

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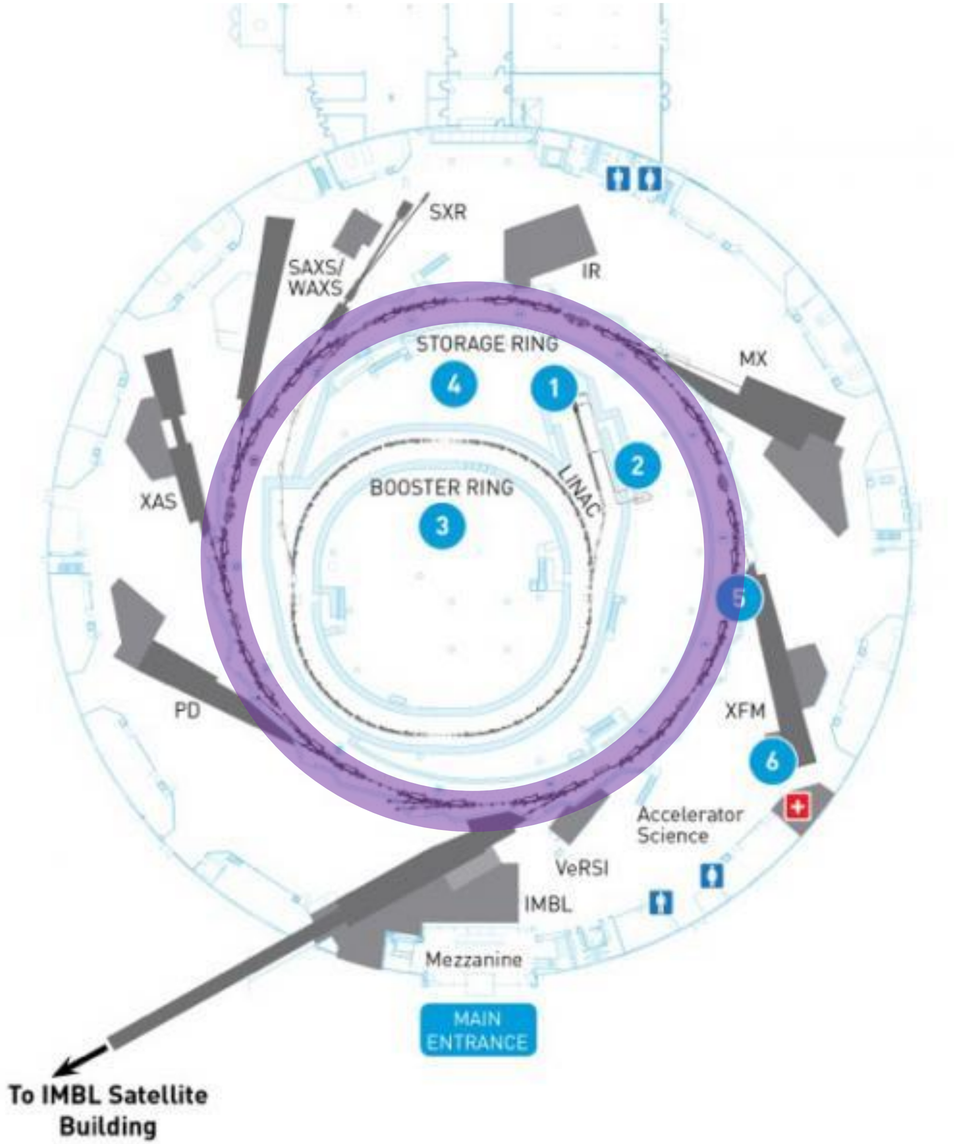
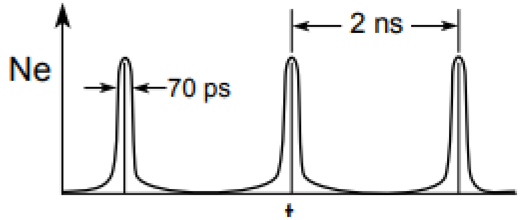
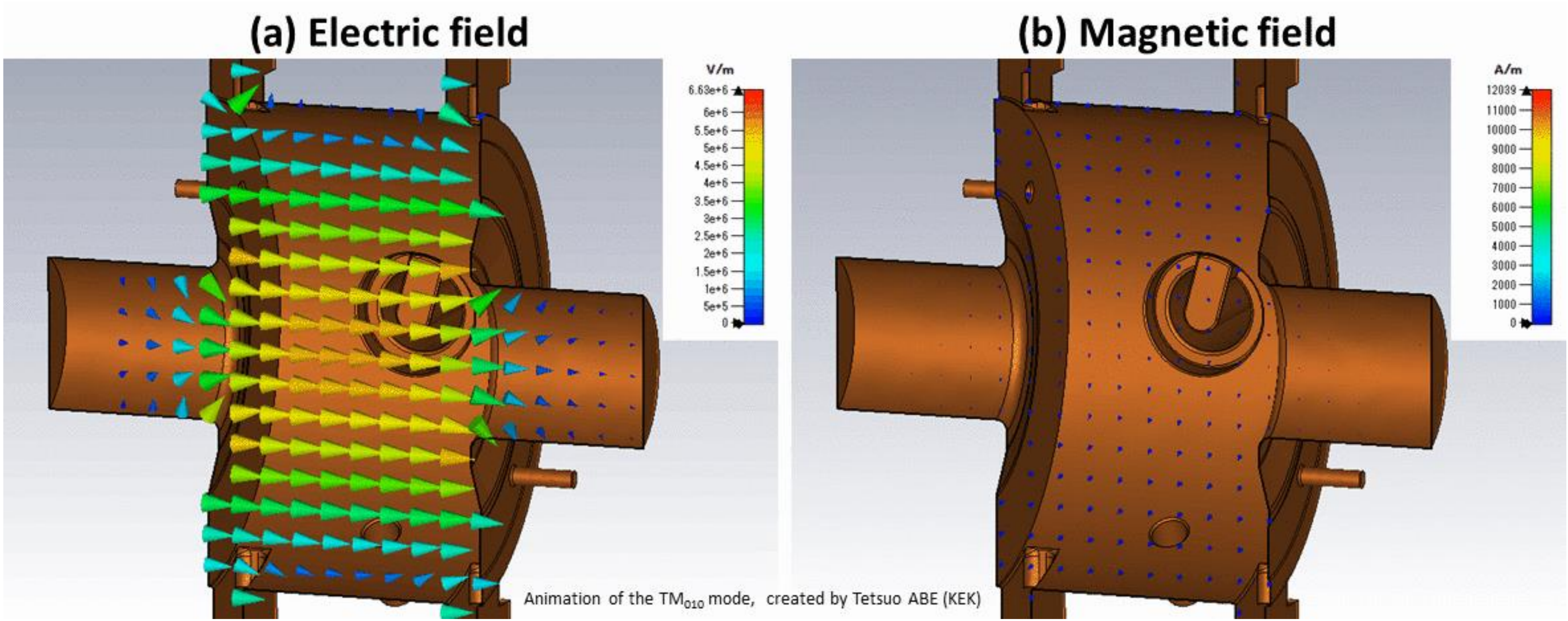


Linac

- Accelerates electrons to 100 MeV
- Electrons exit the linac travelling at 0.999987c
- Radio frequency (RF) of 3 GHz is used to provide the energy for electron acceleration

Radio Frequency Cavities

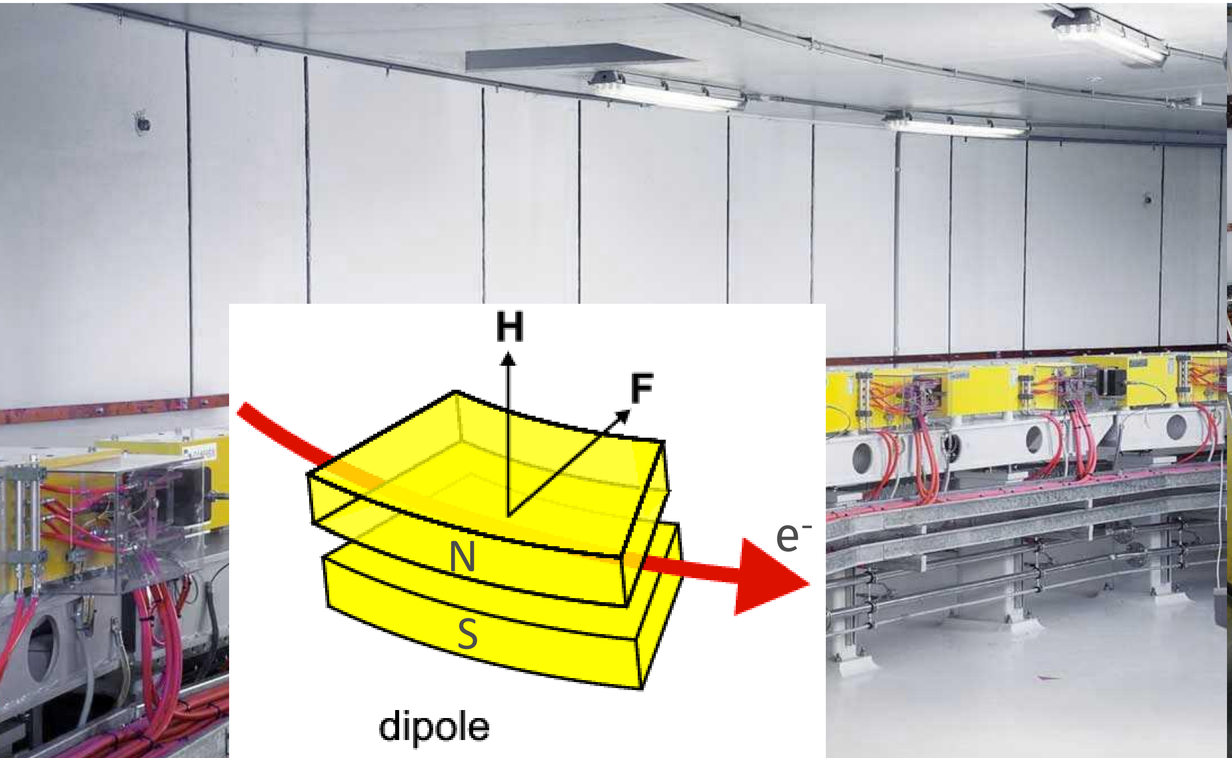




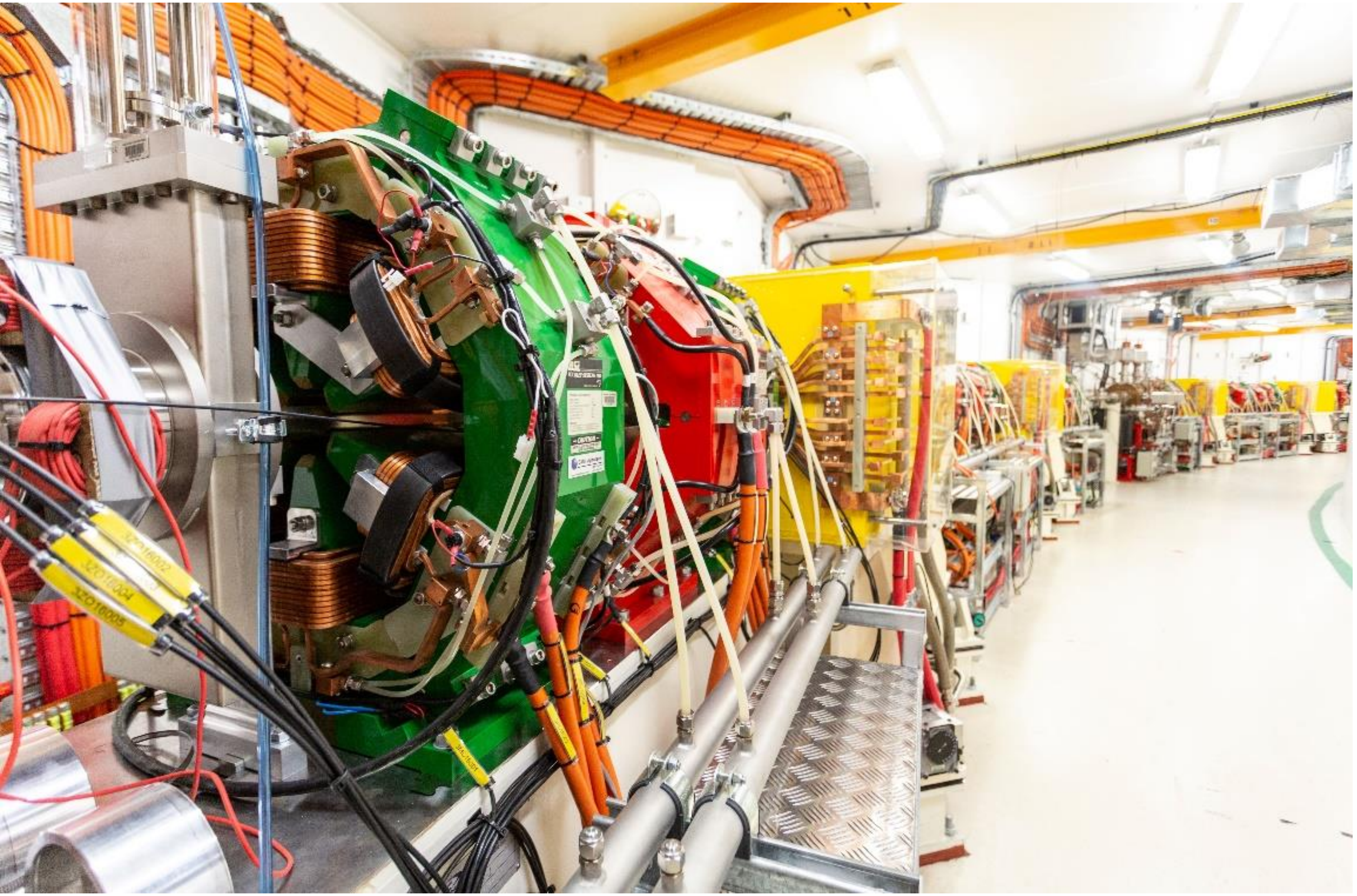
Animation of the TM_{010} mode, created by Tetsuo ABE (KEK)

Booster

- Circumference = 134 m
- Electron energy is increased 30x (to 3 GeV) in 600 ms
- Electrons complete ~1.38 million laps of the booster ring

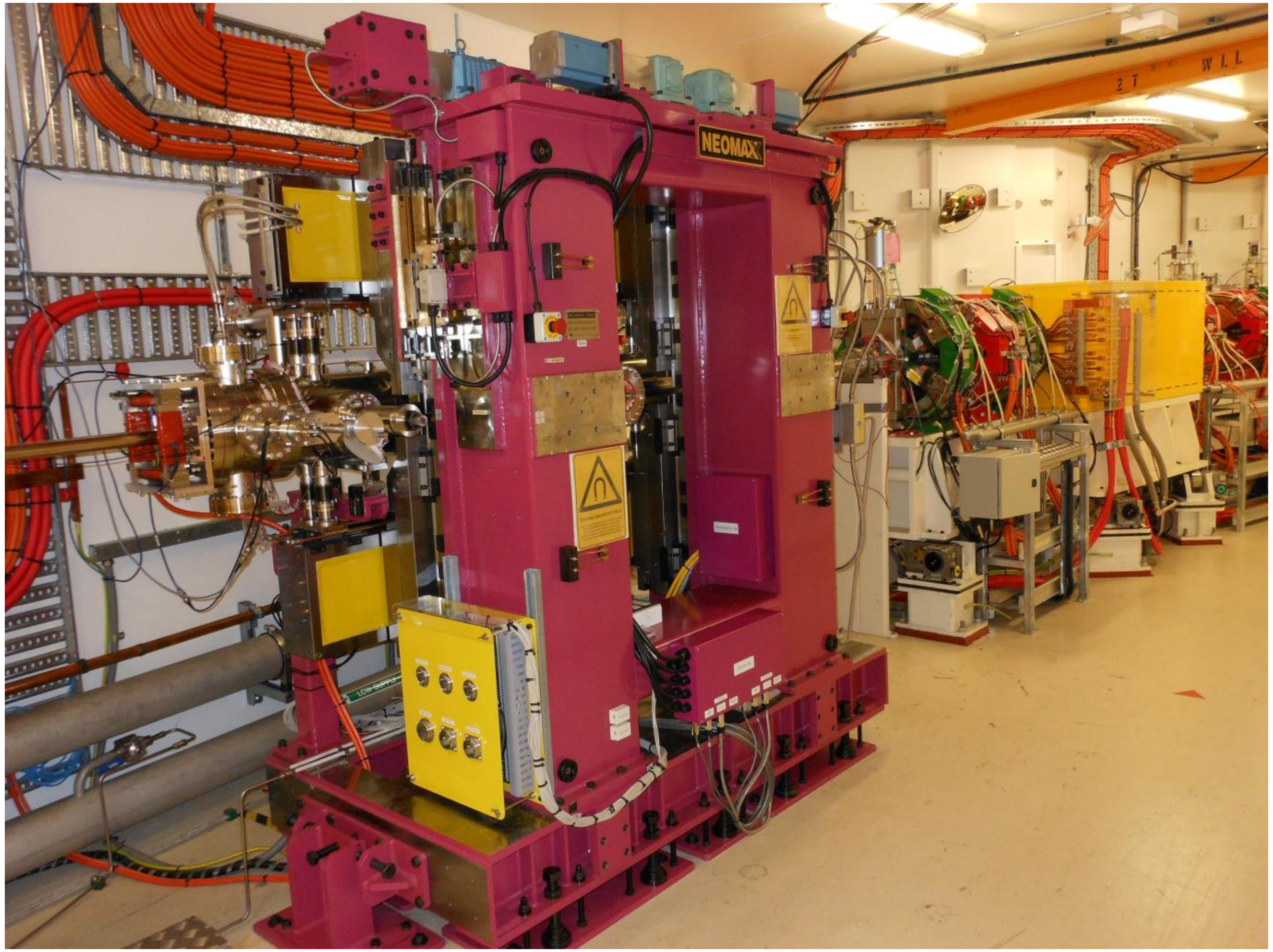


Storage Ring

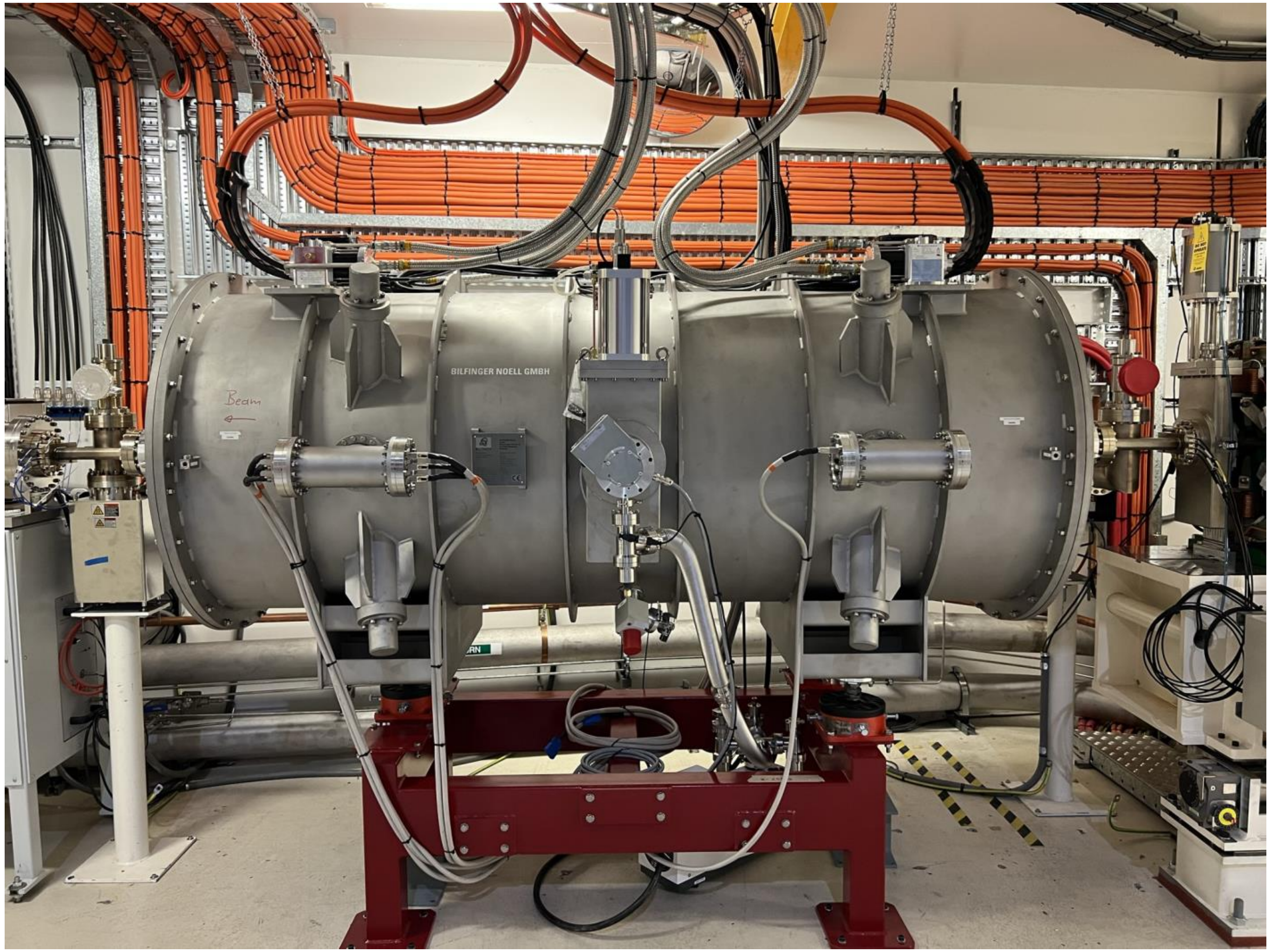


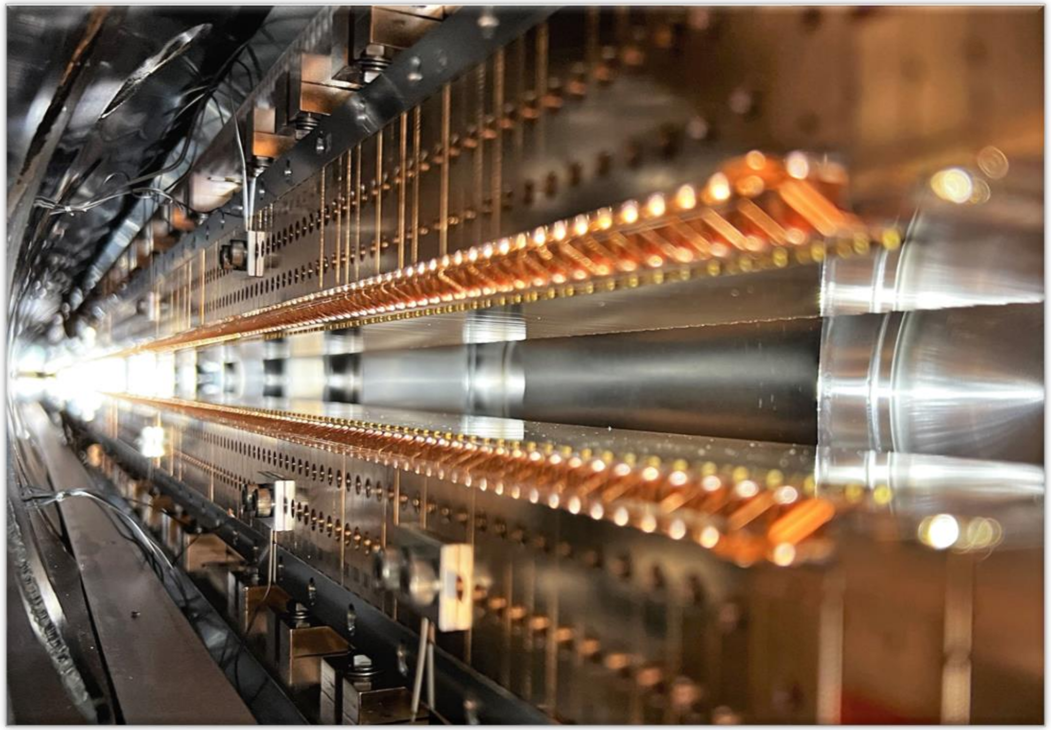
- Circumference = 216 m
- Electrons are injected, adding to the stored beam every 5 minutes ("top-up mode")
  - Average lifetime of electrons is 20-30 hours
- RF cavities resupply the energy that the beam loses as synchrotron radiation
  - ~200 kW of power lost
  - 1.4 million revolutions per second
- The Storage Ring is not circular, it's a 28-sided polygon.

Insertion Devices



Insertion Devices





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Generation of synchrotron radiation

Some useful formulas from Special Relativity

Length contraction:

$$L = \frac{L_0}{\gamma}$$

Lorentz factor:

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{1}{\sqrt{1 - \beta^2}}, \qquad \beta = \frac{v}{c}$$

Some example γ values:

	E (GeV)	γ
Australian Synchrotron	3	5871
Spring 8	8	15656
Advanced Light Source (ALS)	1.9	3718

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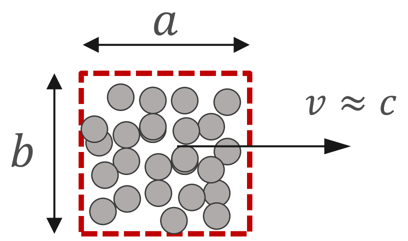
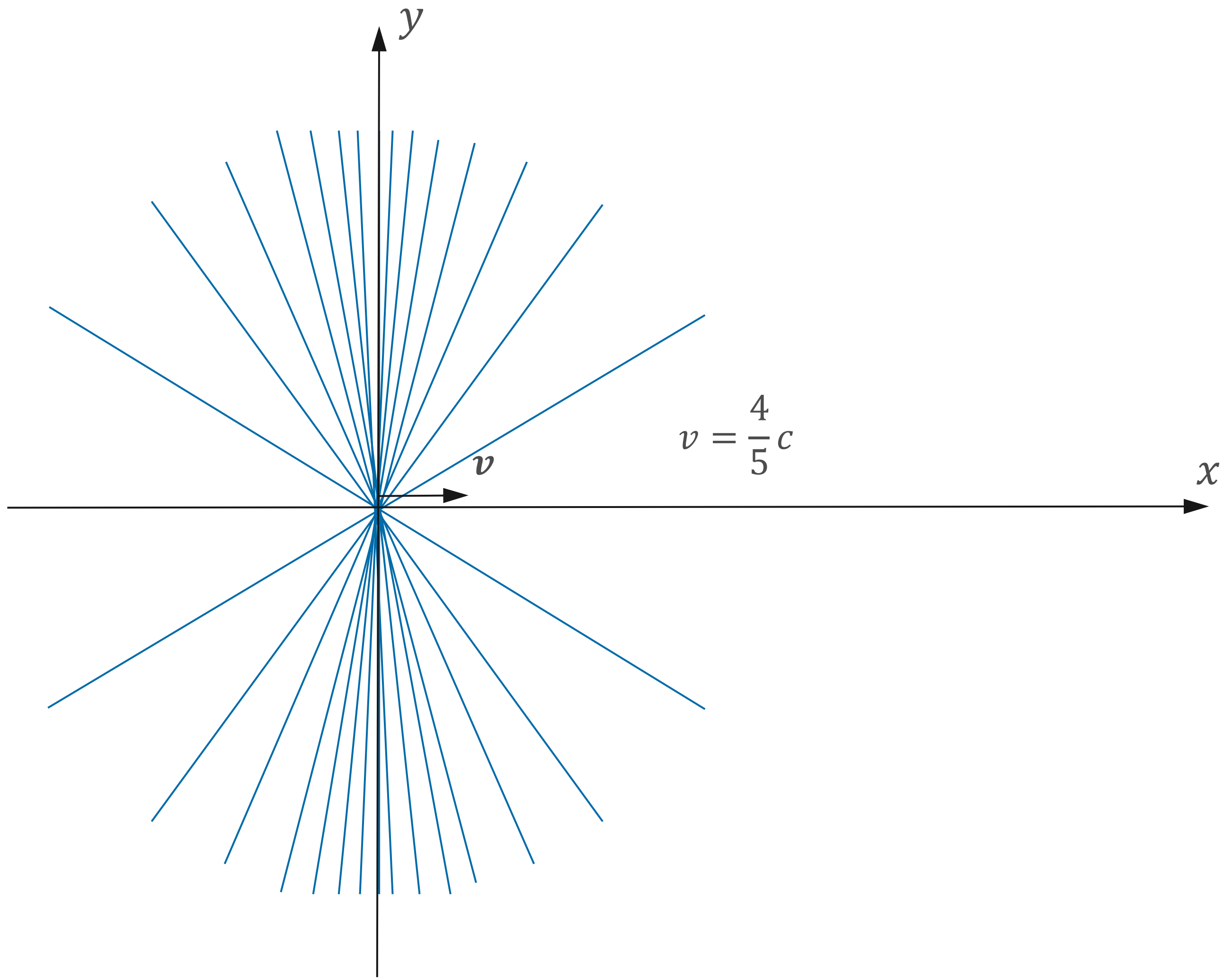
Synchrotron radiation production



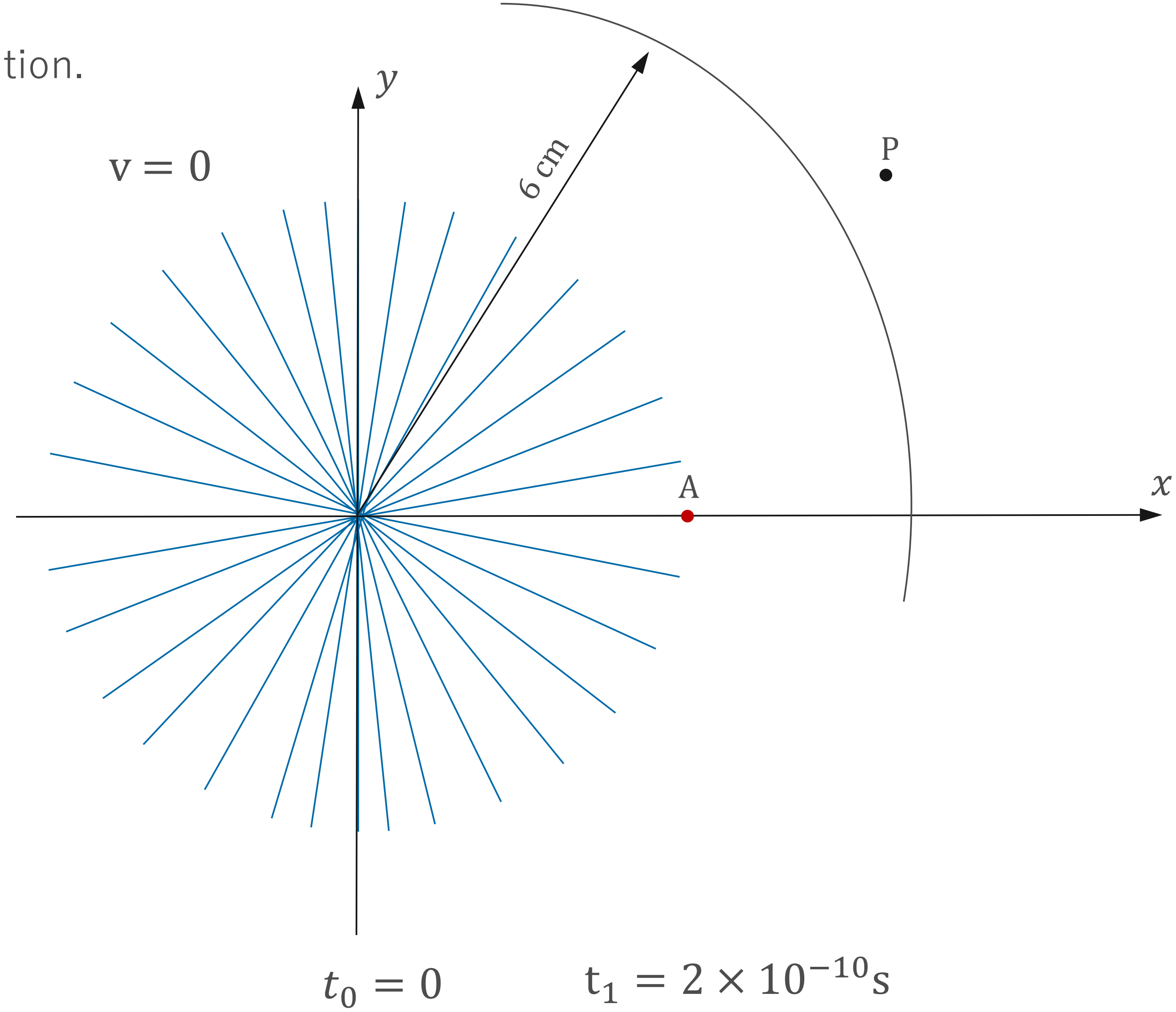
Due to length contraction, at relativistic velocities, the electric field from an electron will flatten and become stronger perpendicular to the direction of motion.



Synchrotron radiation production

Now, let's consider acceleration.

Consider an electron at rest, that then rapidly accelerates at t =0 and then moves at a constant velocity of $v = \frac{4}{5}c$ moving in the positive x-direction.



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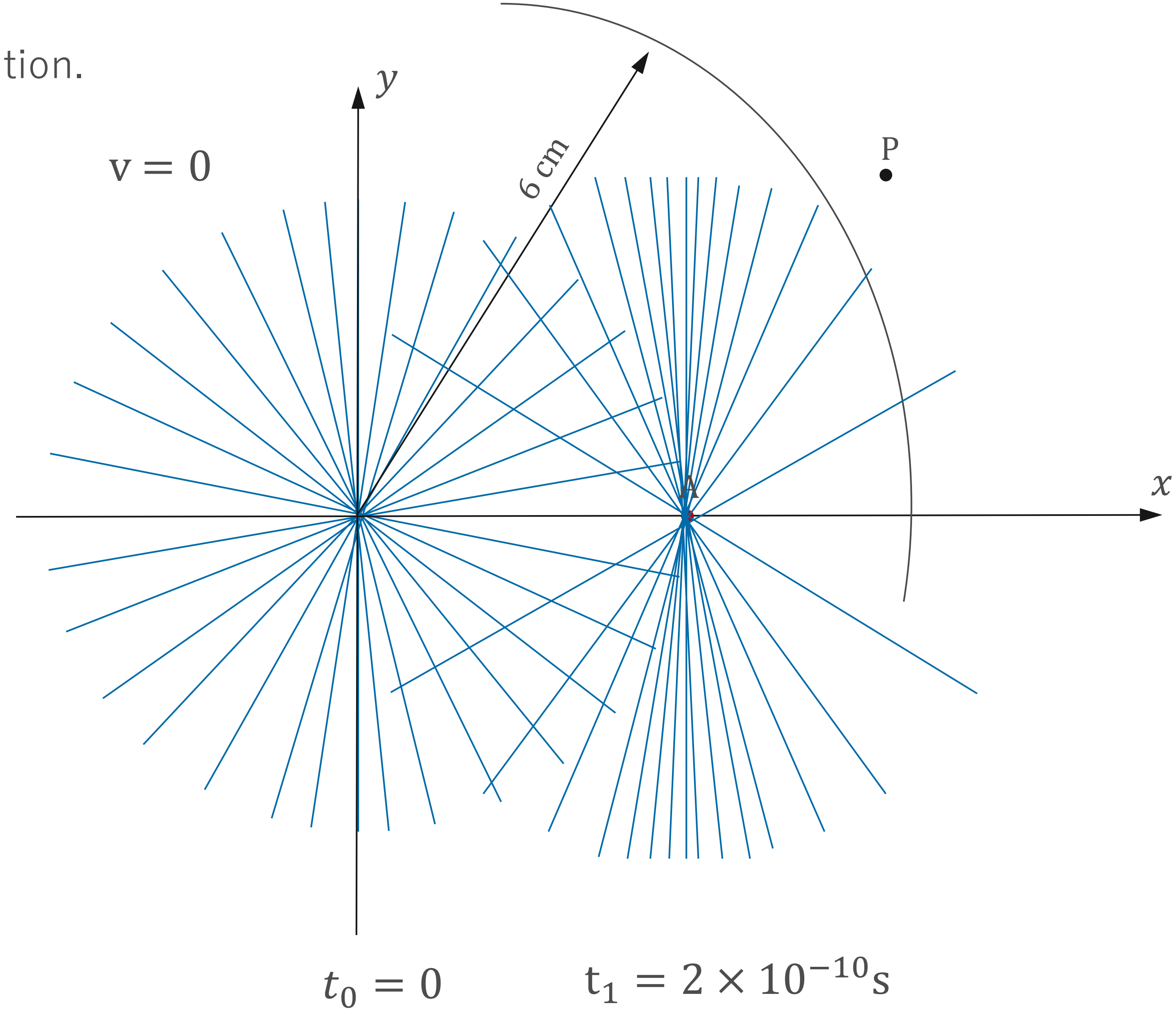

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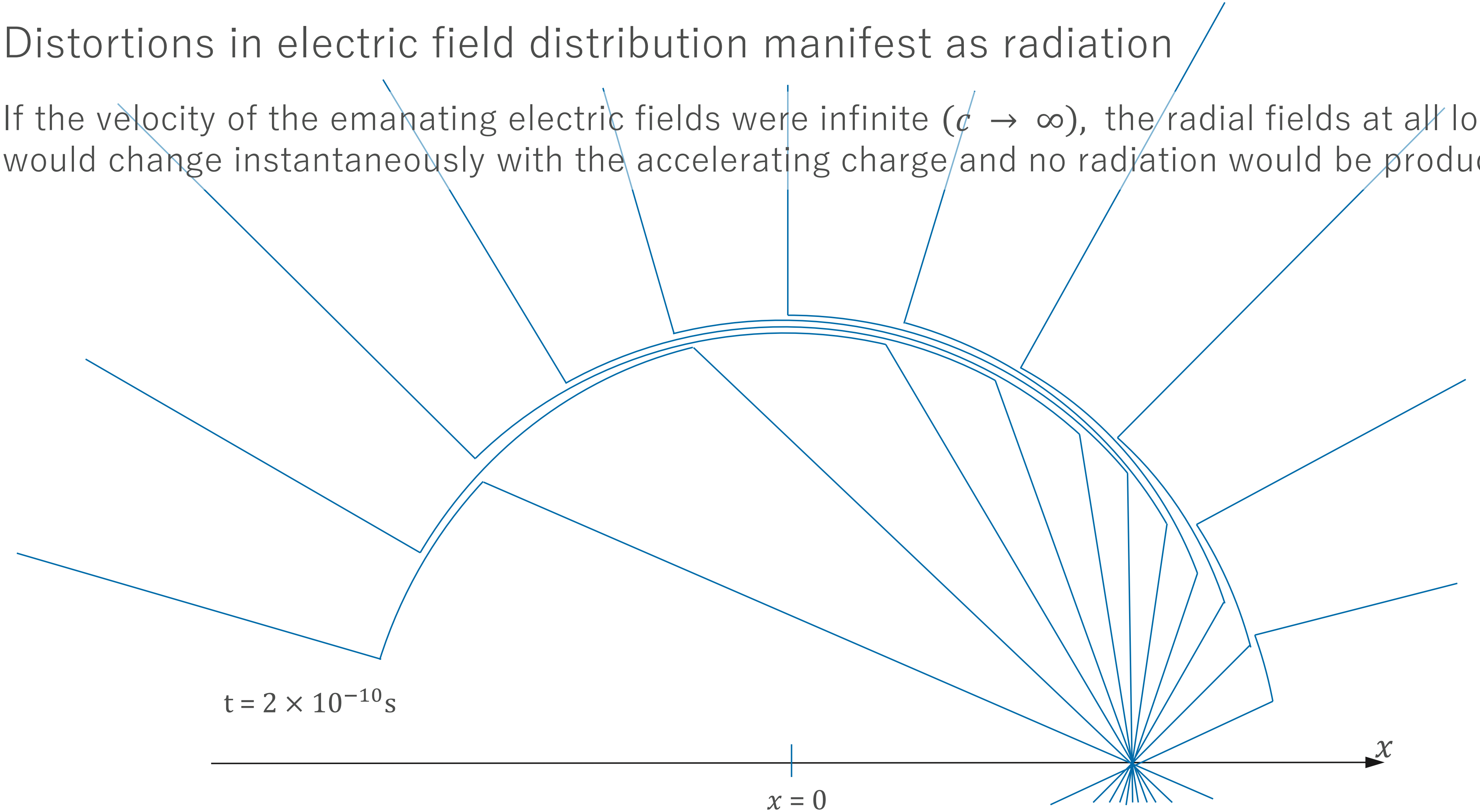
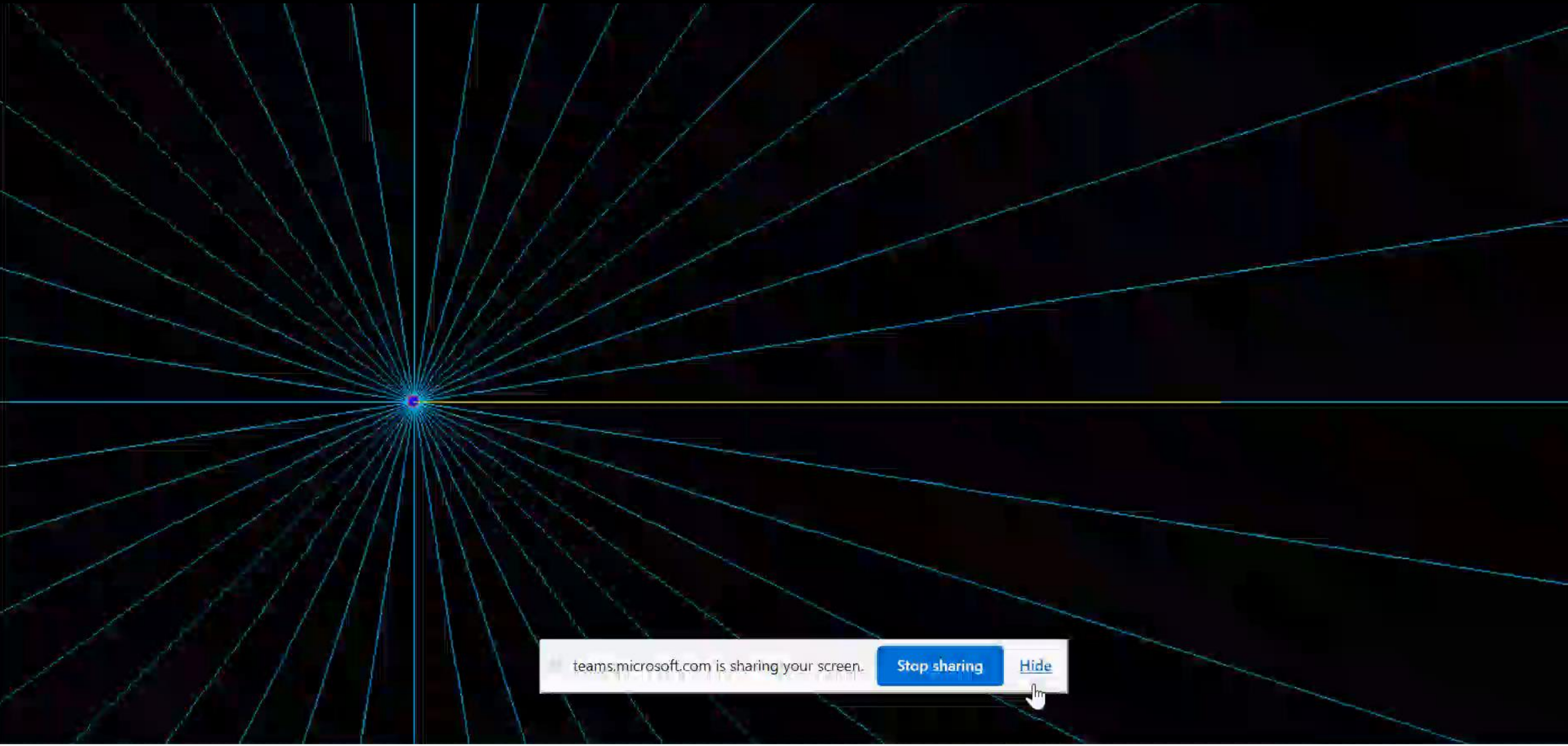
Synchrotron radiation production

Distortions in electric field distribution manifest as radiation

If the velocity of the emanating electric fields were infinite ($c \rightarrow \infty$), the radial fields at all locations would change instantaneously with the accelerating charge and no radiation would be produced.

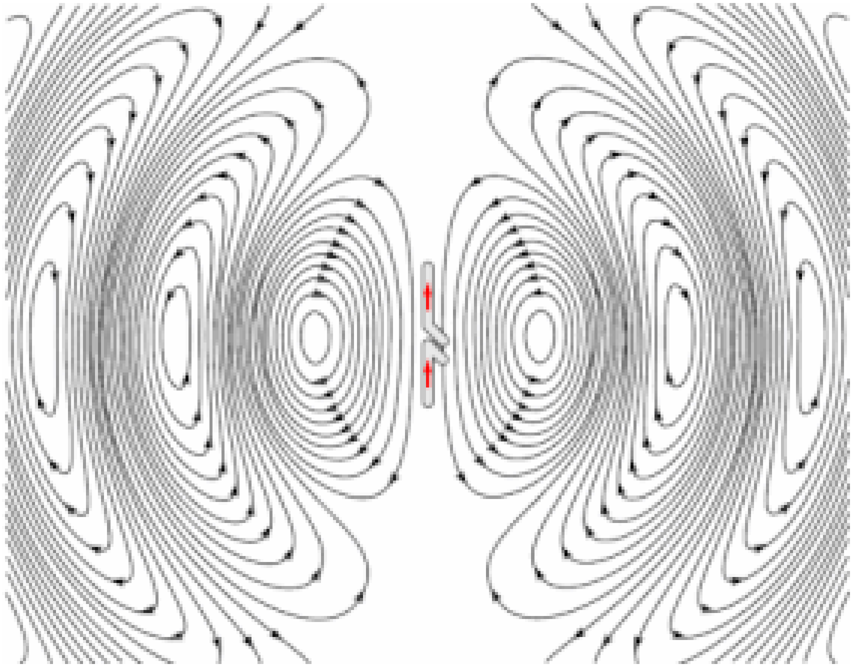




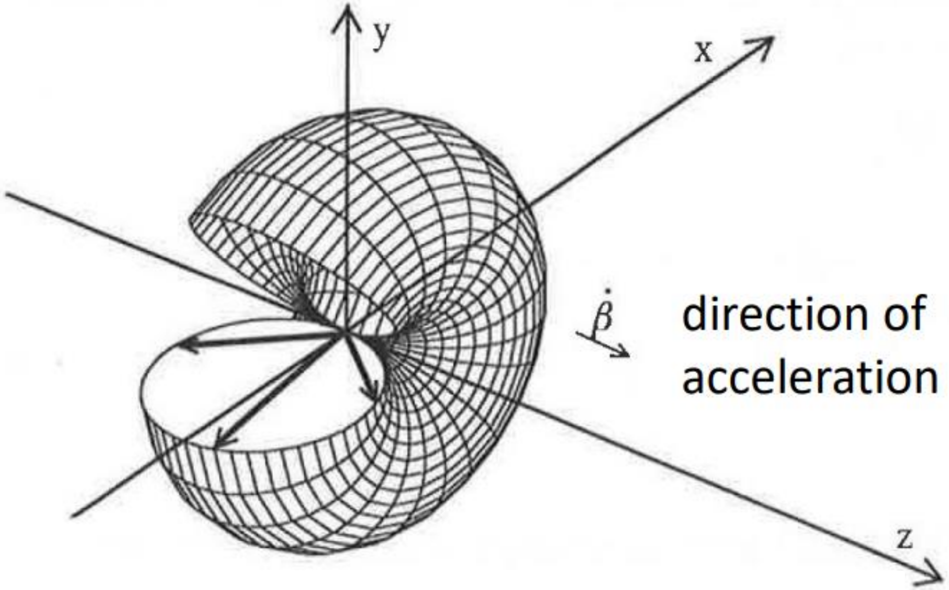
T. Shintake “Real-time animation of synchrotron radiation”, 507 (2003) 89–92
Software - Radiation2D | OIST Groups https://groups.oist.jp/qwmu/software

Spatial Distribution of Radiation

In an electron's rest frame the field is distributed in $\sin^2\theta$ doughnut pattern, like a radio antenna.



Non-relativistic case

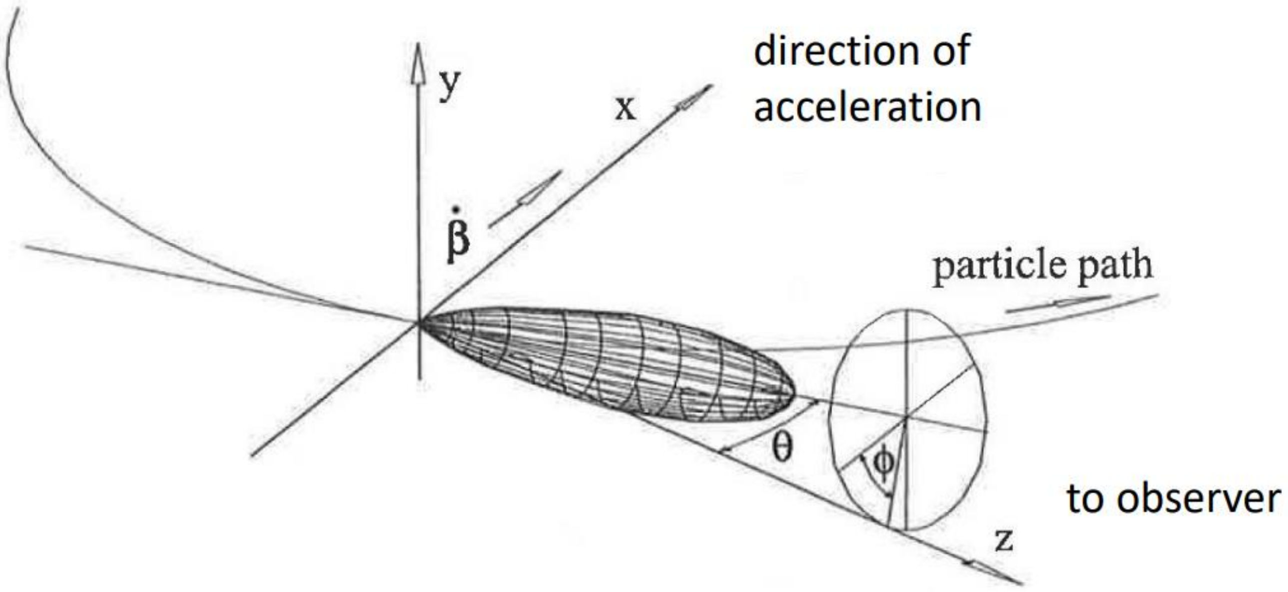


Images: left: https://en.wikipedia.org/wiki/Omnidirectional_antenna
Right: H. Wiedemann "Particle Accelerator Physics" Springer

Spatial Distribution of Radiation

In the lab frame, the radiation emitted by highly-relativistic particles is Lorentz boosted into **a narrow cone** $\theta \sim \frac{1}{\gamma}$

Relativistic case



Lorentz transformation effects the angle with the observer views the source.

$$\tan(\theta) = \frac{\sin\theta'}{\gamma(\cos\theta' - \beta)}$$

At the peak emission is at 90deg to the direction of the electron acceleration, and the opening angle is $\theta \sim \pm\frac{1}{\gamma}$

Image: H. Wiedemann "Particle Accelerator Physics" Springer

Spatial Distribution of Radiation

In the lab frame, the radiation emitted by highly-relativistic particles is Lorentz boosted into **a narrow cone** $\theta \sim \frac{1}{\gamma}$

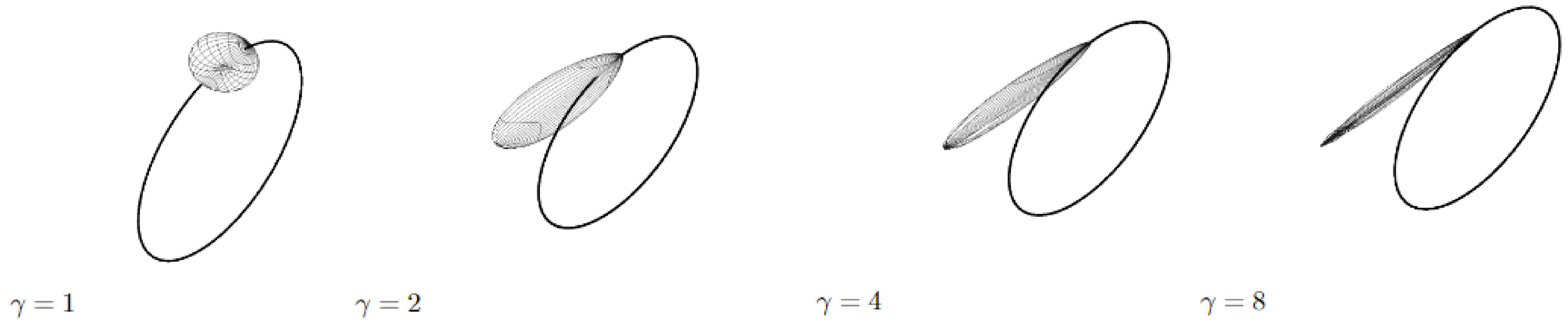


Image: "The Science and Technology of Particle Accelerators"

Radiated Power

Making a Lorentz transformation of the Lamor formula gives the total power emittance in the relativistic case (the Liénard formula):

$$P = \frac{e^2}{6\pi\epsilon_0 c}\gamma^6\left[\dot{\boldsymbol{\beta}}^2 - \left(\boldsymbol{\beta}\times\dot{\boldsymbol{\beta}}\right)^2\right]$$

Consider two cases:

1. Linear accelerator, where **v** is *parallel* to **a**. $(\boldsymbol{\beta}\times\dot{\boldsymbol{\beta}} = 0)$
2. Circular accelerator, where **v** is *perpendicular* to **a**.
   $(\dot{\boldsymbol{\beta}}^2 - \left(\boldsymbol{\beta}\times\dot{\boldsymbol{\beta}}\right)^2 = |\dot{\boldsymbol{\beta}}^2|(1-\beta^2) = |\dot{\boldsymbol{\beta}}^2|/\gamma^2)$

$$P_\parallel = \frac{e^2}{6\pi\epsilon_0 c}\gamma^6\dot{\boldsymbol{\beta}}_\parallel^{\,2} = \frac{e^2 c}{6\pi\epsilon_0}\frac{1}{(mc^2)^2}\left(\frac{d\boldsymbol{p}}{dt}\right)^2$$

$$P_\perp = \frac{e^2}{6\pi\epsilon_0 c}\gamma^4\dot{\boldsymbol{\beta}}_\perp^{\,2} = \frac{e^2 c}{6\pi\epsilon_0}\frac{\gamma^2}{(mc^2)^2}\left(\frac{d\boldsymbol{p}}{dt}\right)^2$$

$$\boxed{P_\perp = \gamma^2 P_\parallel}$$

where $\boldsymbol{p} = \gamma\boldsymbol{\beta}mc$.

Radiated Power

$$P_\perp = \frac{e^2}{6\,\pi\epsilon_0 c}\gamma^4 \dot{\boldsymbol{\beta}}_\perp{}^2 = \frac{e^2 c}{6\,\pi\epsilon_0}\frac{\gamma^2}{(mc^2)^2}\left(\frac{d\boldsymbol{p}}{dt}\right)^2$$

In the case of circular motion with bend radius ρ, we have $\frac{\mathrm{d}p}{dt} = \frac{\gamma m c^2 \beta^2}{\rho}$,

The instantaneous power radiated by a single electron is:

$$P_\perp = \frac{e^2 c \gamma^4}{6\,\pi\epsilon_0}\left(\frac{\beta^4}{\rho^2}\right)$$

$$\boxed{P \propto \frac{\gamma^4}{\rho}}$$

More conveniently,

$$P_\perp = \frac{e^2}{6\,\pi\epsilon_0}\frac{\beta^4}{m^4 c^5}E^2 B^2$$

$$\boxed{P \propto \frac{1}{m^4}}$$

Why do we accelerate electrons? $\left(\frac{m_p}{m_e}\right)^4 \approx 10^{13}$

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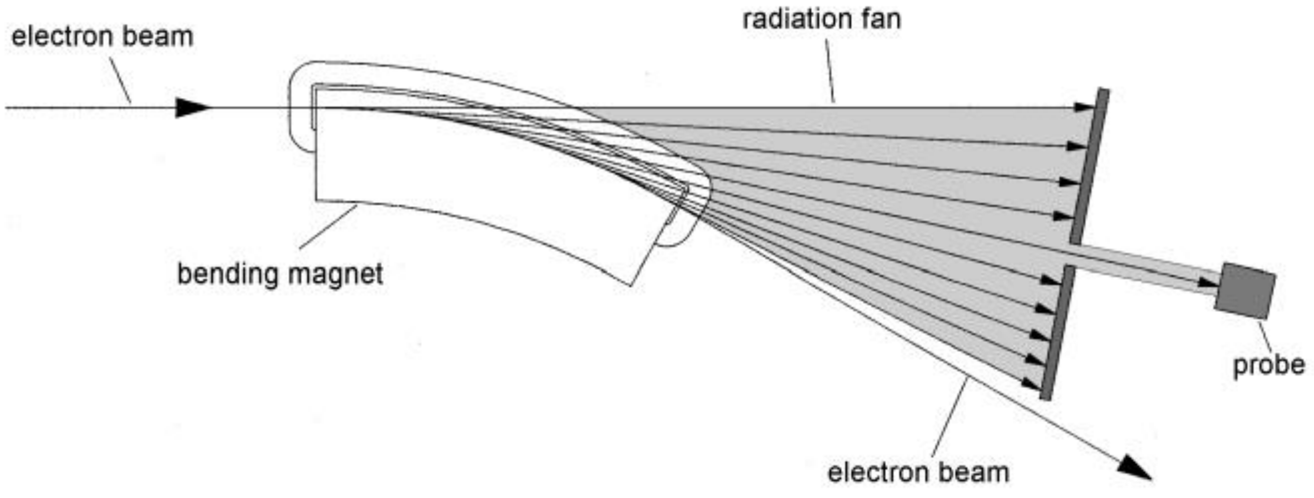
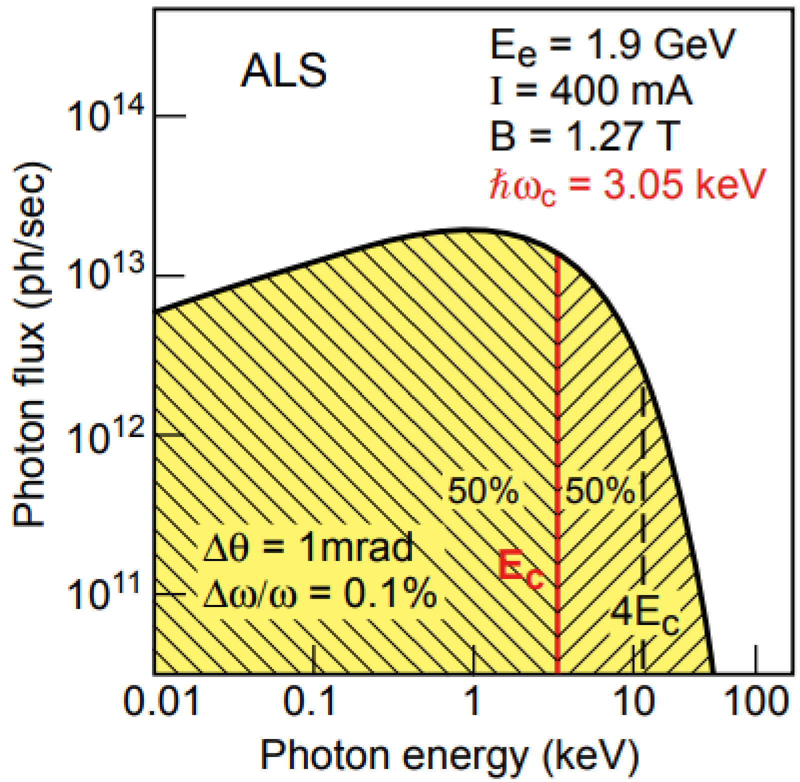
Radiation from a Bending Magnet





We can characterize this spectrum with the **critical frequency,** ω_c.

Half the SR power is emitted above ω_c and half below.

$$\boxed{\omega_c = \frac{3c\gamma^3}{2\rho}} \qquad \lambda_c = \frac{2\pi c}{\omega_c}, \qquad E_c = \hbar\omega_c = \frac{3hc\gamma^3}{4\pi\rho}$$

Figure: (right) David Attwood

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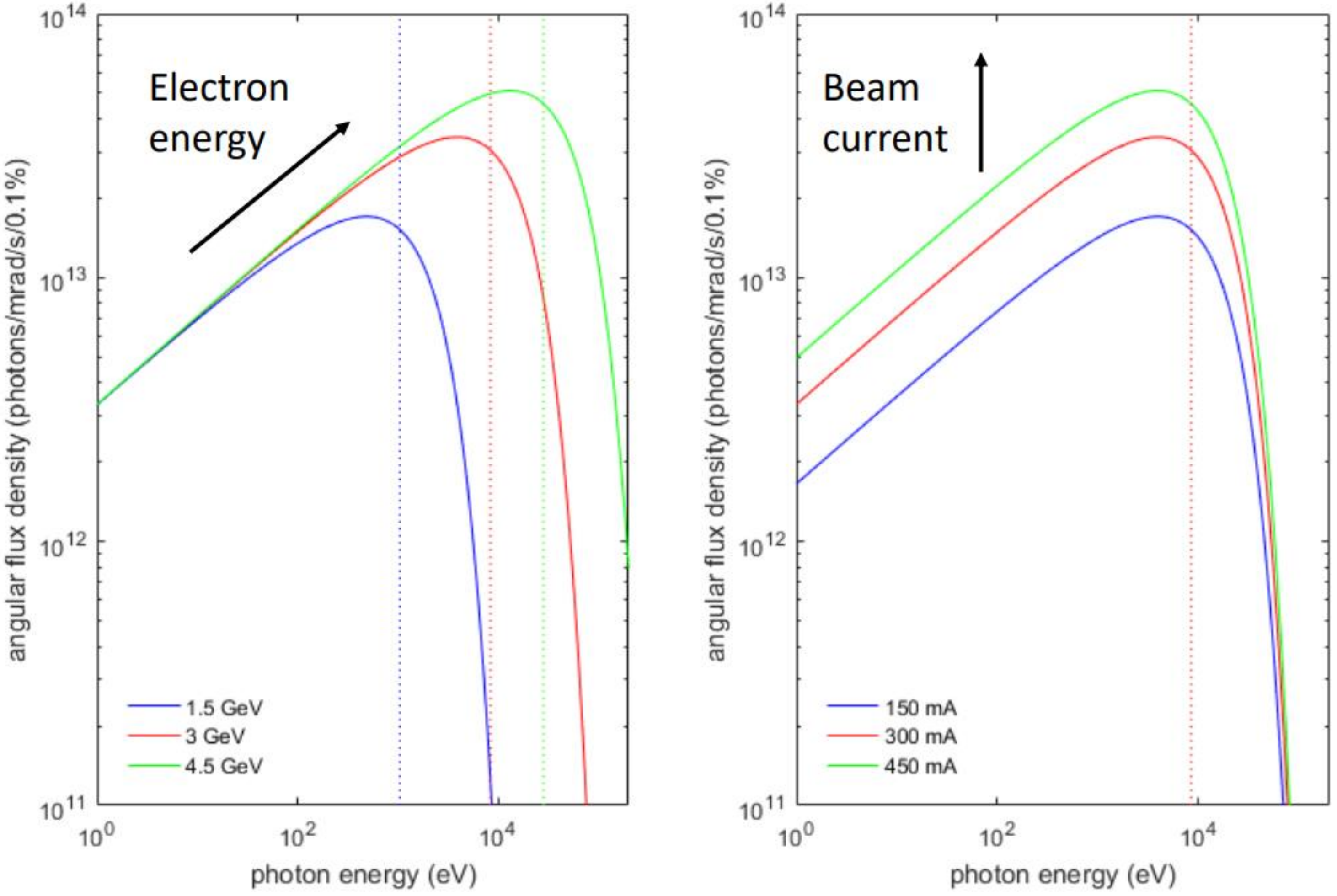
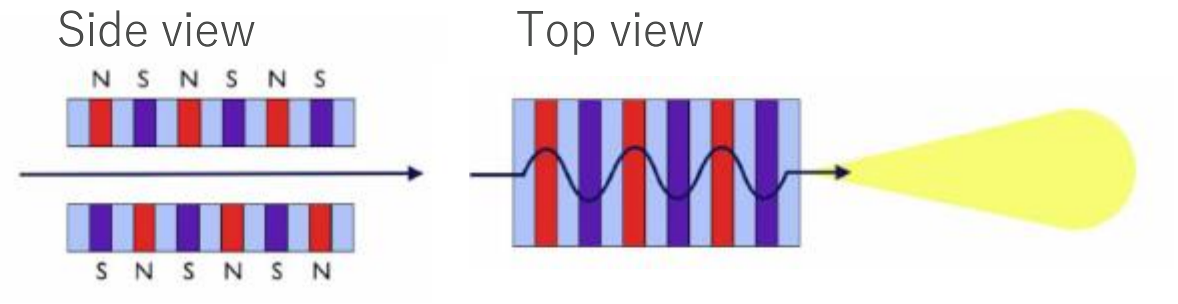
Bending Magnet Radiation Scaling



Figures: Ian Martin (Diamond Light Source)

Wigglers and Undulators



The electron trajectory angular deflection is:

$$\dot{x}(s) = \frac{B_0 e}{\gamma m_0 c}\frac{\lambda_u}{2\pi}\cos\left(\frac{2\pi s}{\lambda_u}\right)$$

Maximum angular deflection

The **deflection parameter** is defined as:

$$\boxed{K = \frac{B_0 e}{m_0 c}\frac{\lambda_u}{2\pi}}$$

The **maximum angular deflection** is: $\frac{K}{\gamma}$

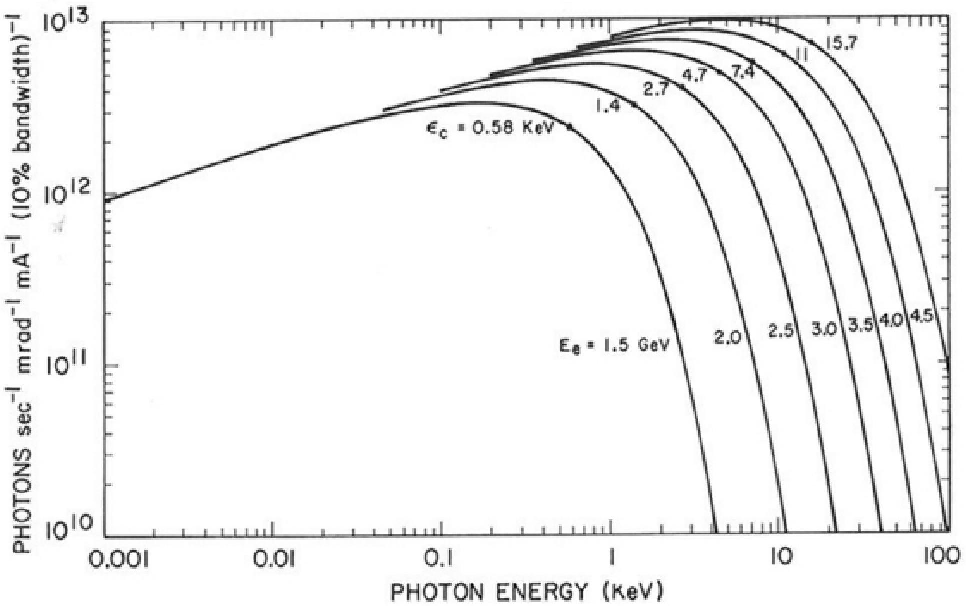
The **maximum transverse displacement** is: $\frac{K}{\gamma}\frac{\lambda_u}{2\pi}$

As an example: For a 3 GeV e-beam through an undulator with a period of $\lambda_u = 50$ mm and K =1, the maximum amplitude of oscillation is only 0.7 μm.

Wigglers

Very high power densities can be achieved!

Wigglers harden the spectrum, by deceasing the bending radius ρ and increasing the flux through having many magnetic poles.



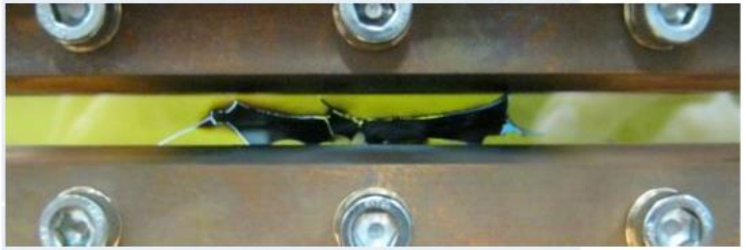


Figure: (left) J.D. Jackson "Classical Electrodynamics"
Photos: Brad Mountford, Australian Synchrotron

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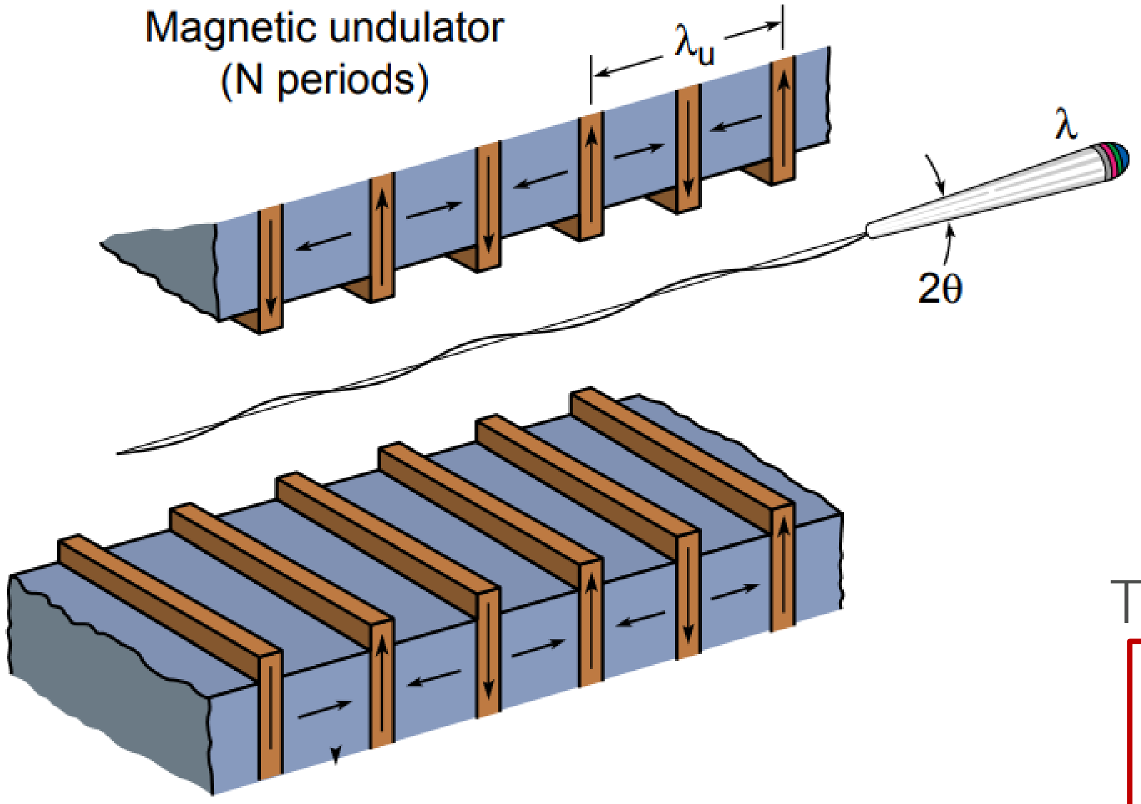
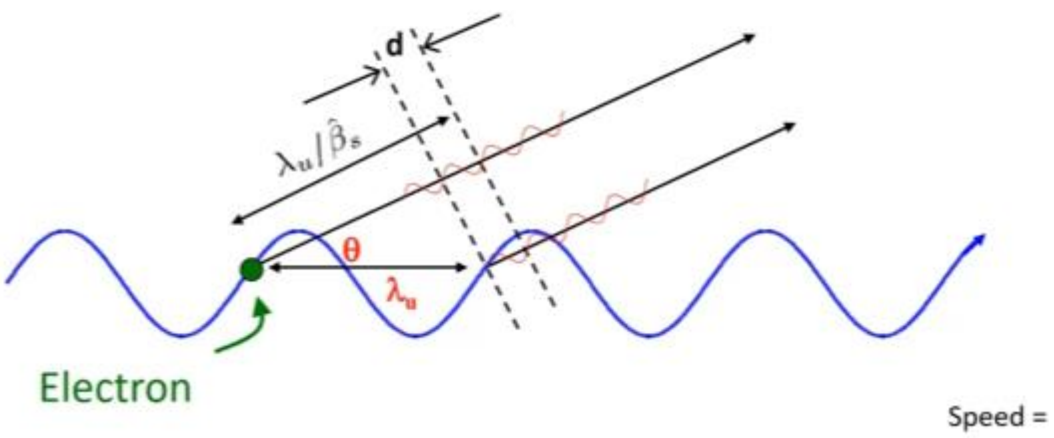
Undulator Radiation

$$K = \frac{B_0 e}{m_0 c} \frac{\lambda_u}{2\pi}$$

The key difference between a wiggler and an undulator, is that an that the undulator output is due to the **interference of the light** emitted by an electron.





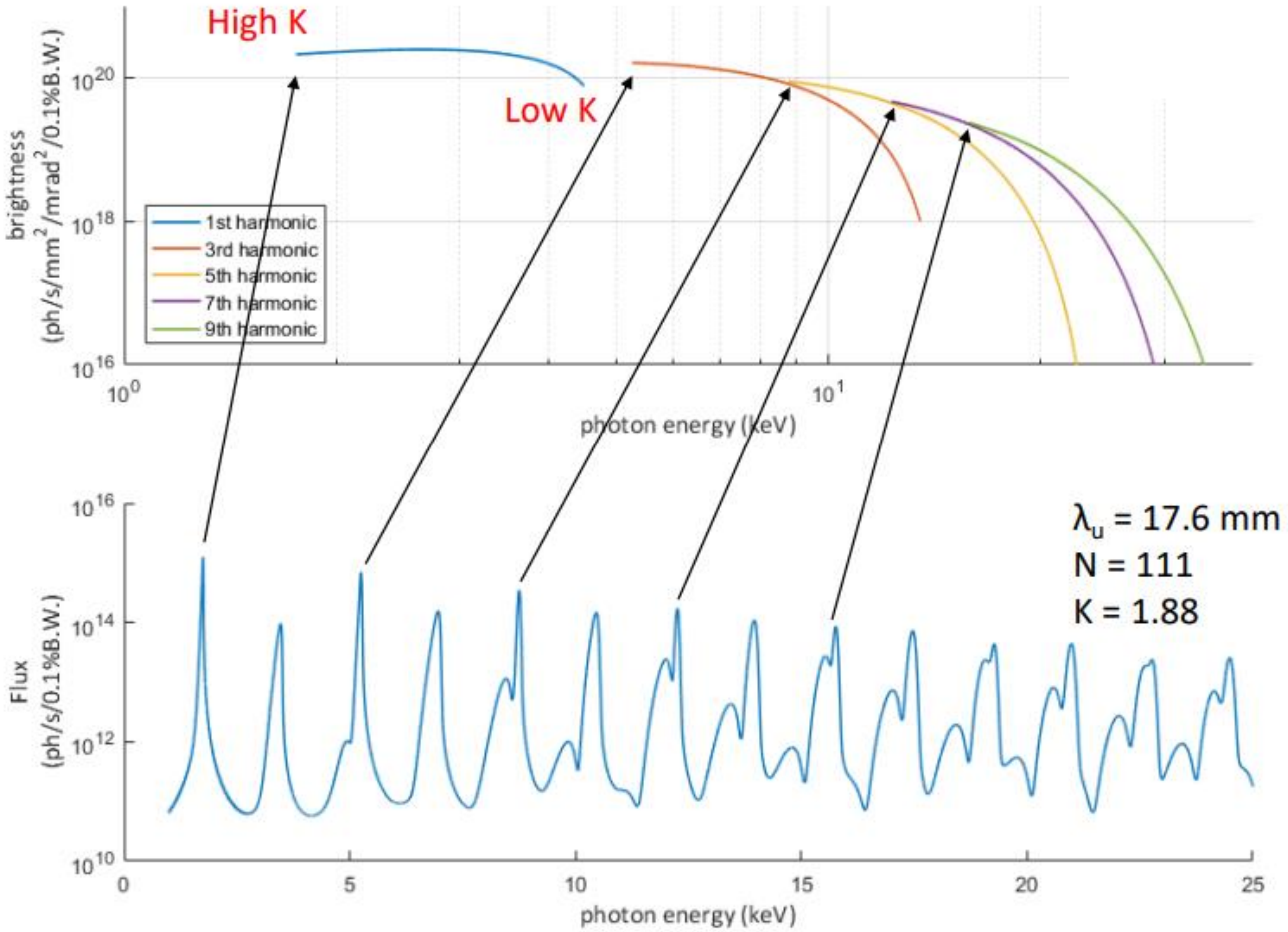
The undulator equation:

$$\lambda = \frac{\lambda_u}{2n\gamma^2}\left(1 + \frac{K^2}{2} + \theta^2\gamma^2\right)$$

Figure: (left) David Attwood

Undulator Radiation

$$K = \frac{B_0 e}{m_0 c} \frac{\lambda_u}{2\pi}$$



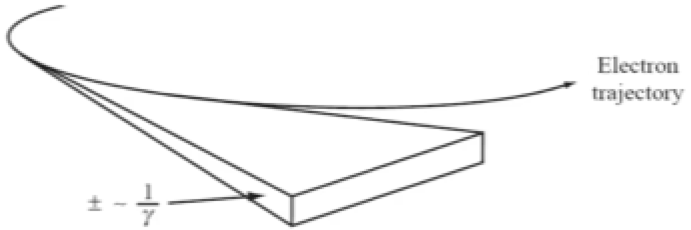
The photon energy can be varied by changing the K parameter (the B-field)

Figure: Ian Martin, Diamond Light Source

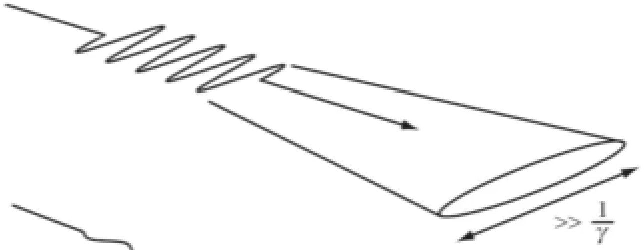
Insertion Devices

$$K = \frac{B_0 e}{m_0 c} \frac{\lambda_u}{2\pi}$$

Bending magnet



Wiggler

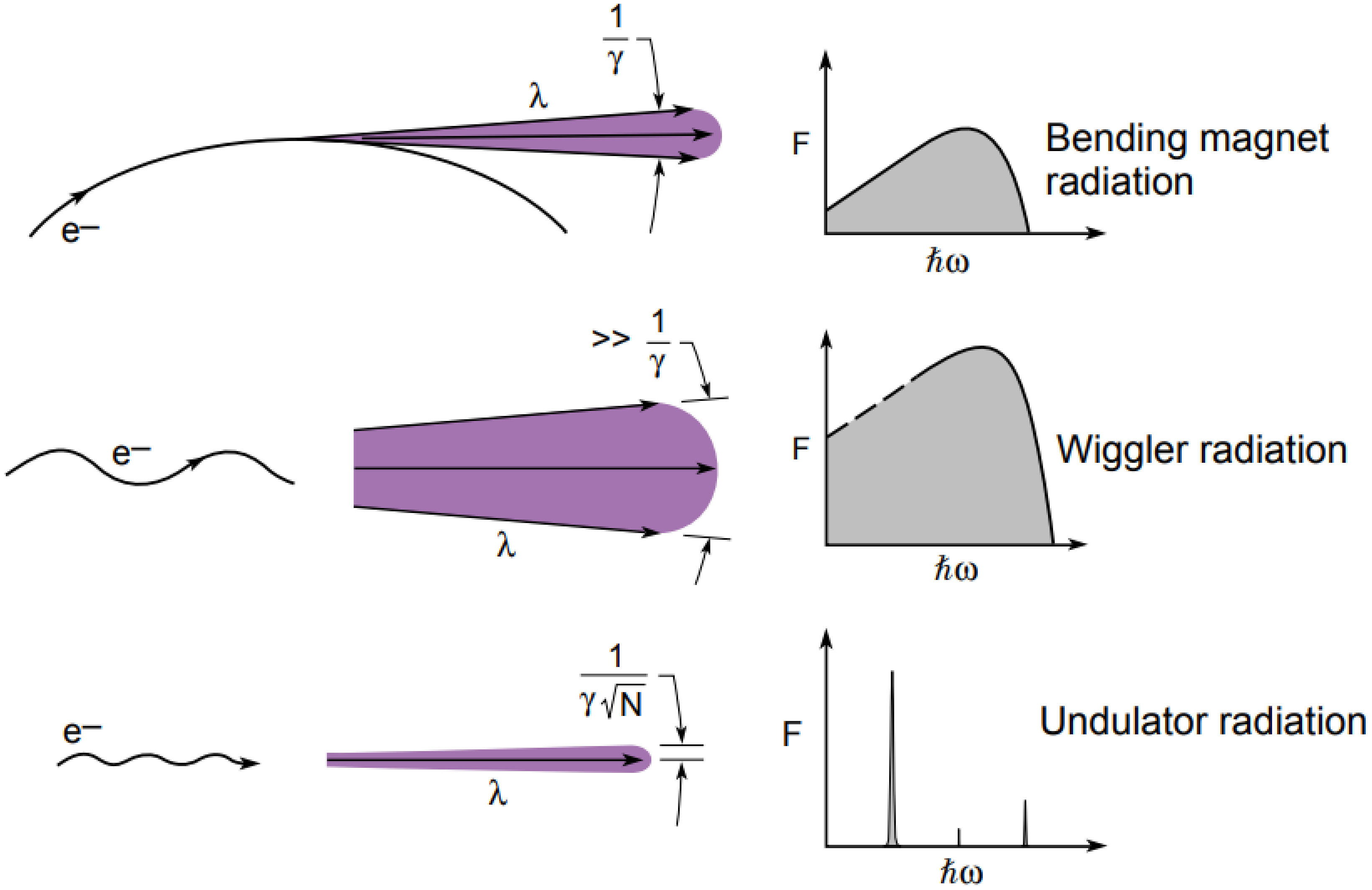


If $K \gg 1$ then there is little overlap between the e- trajectory and the emitted SR, and source each source point is effectively independent.

Undulator

If $K < 1$ then the cone of radiation is larger than the e- trajectory, which allows it to constructively interfere and enhance certain wavelengths

Insertion Devices Summary



$F \propto N_e$

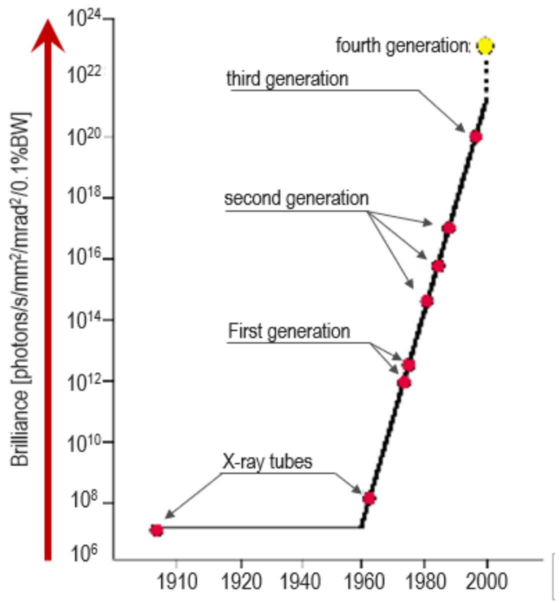
$F \propto 2\, N_e N_{period}$

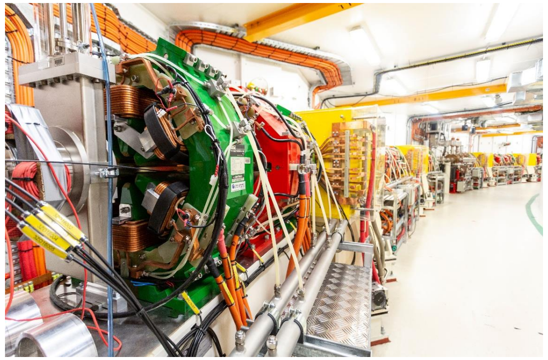
$F \propto N_e (N_{period})^2$

Figure: David Attwood

Conclusions

- Synchrotron radiation facilities are now a major research infrastructure in many countries





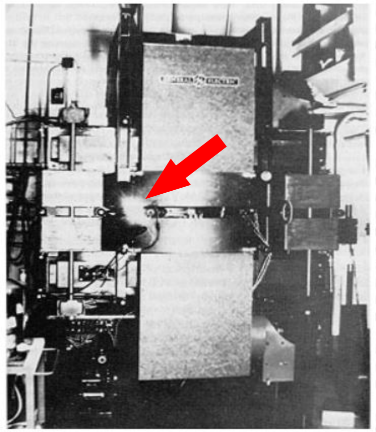
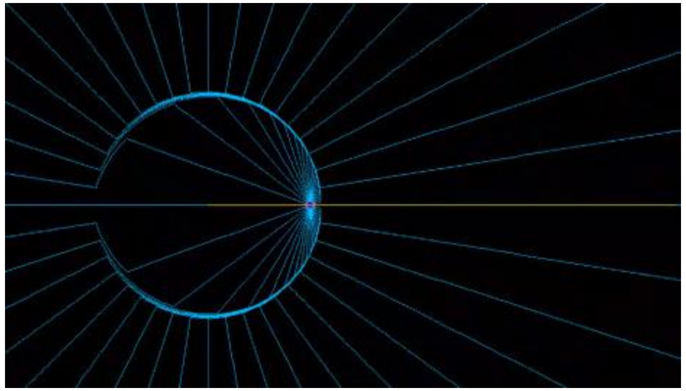
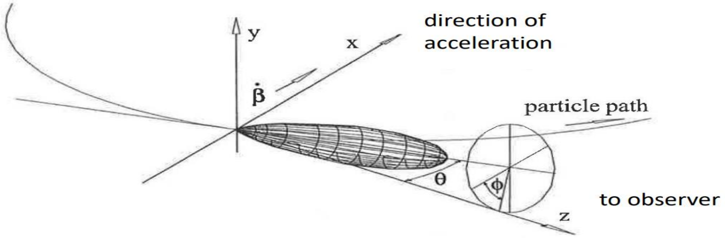


Fig. 1. 70-MeV synchrotron with optical radiation from the electron beam visible through the glass wall of the vacuum "donut" tangent to the beam orbit.

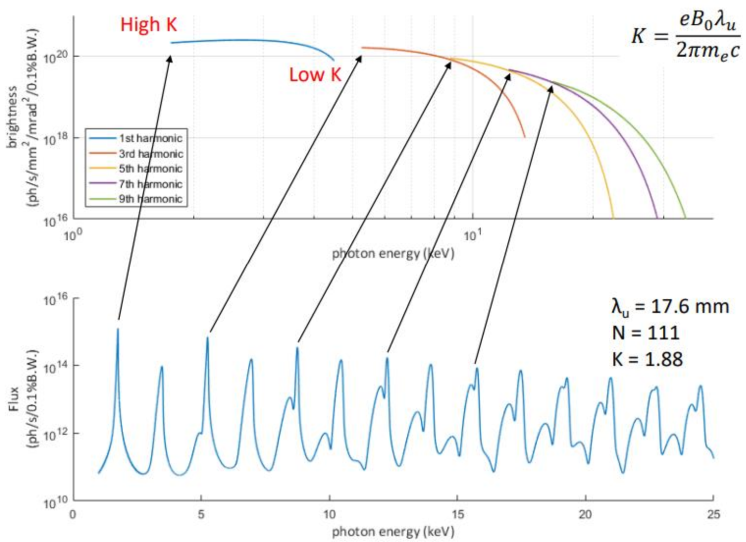


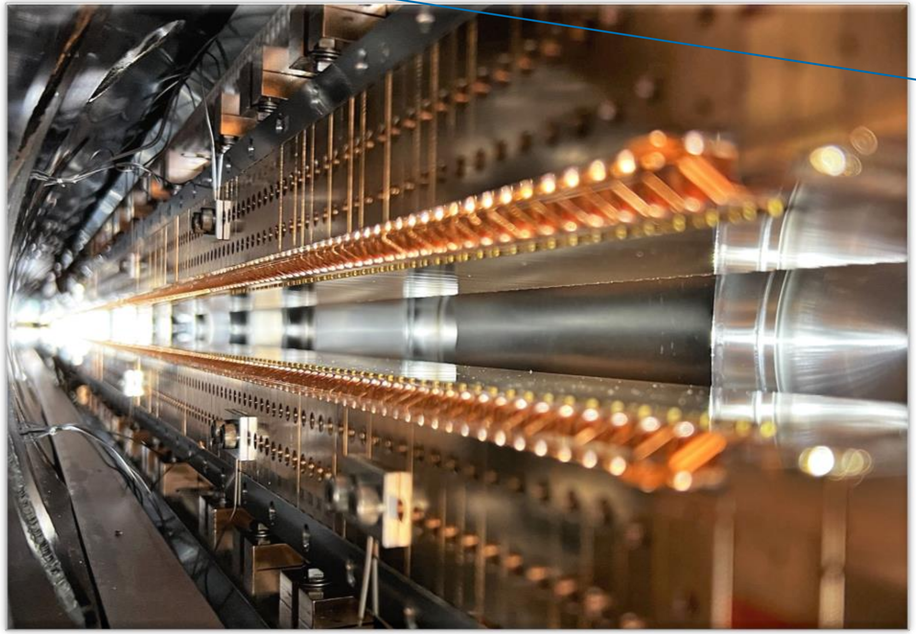
- Synchrotron radiation is emittance by relativistic accelerated charged particles.
- The combination of Lorentz contraction and Doppler shift turns cm length scale into nm wavelengths.

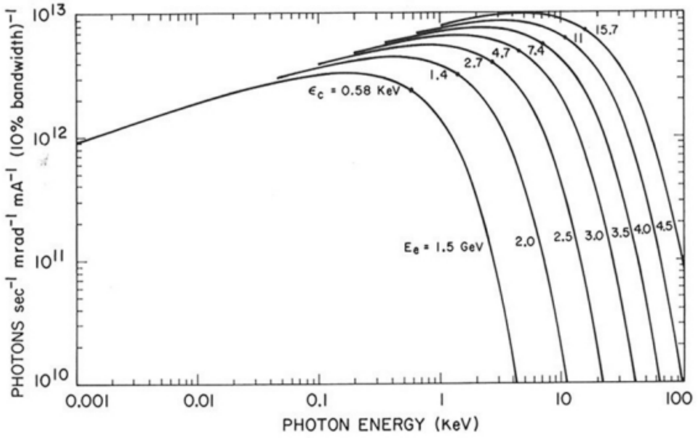


Conclusions

- Bending magnets and Insertion devices are used to provide sources of synchrotron radiation. Wigglers, and Undulators provide tuneable, high-brightness sources of synchrotron radiation.
- Undulators: $K \lesssim 1$; brightness scales with (N_{period})^2
- Wigglers: $K \gtrsim 1$; brightness scales with $2\, N_e\, N_{period}$



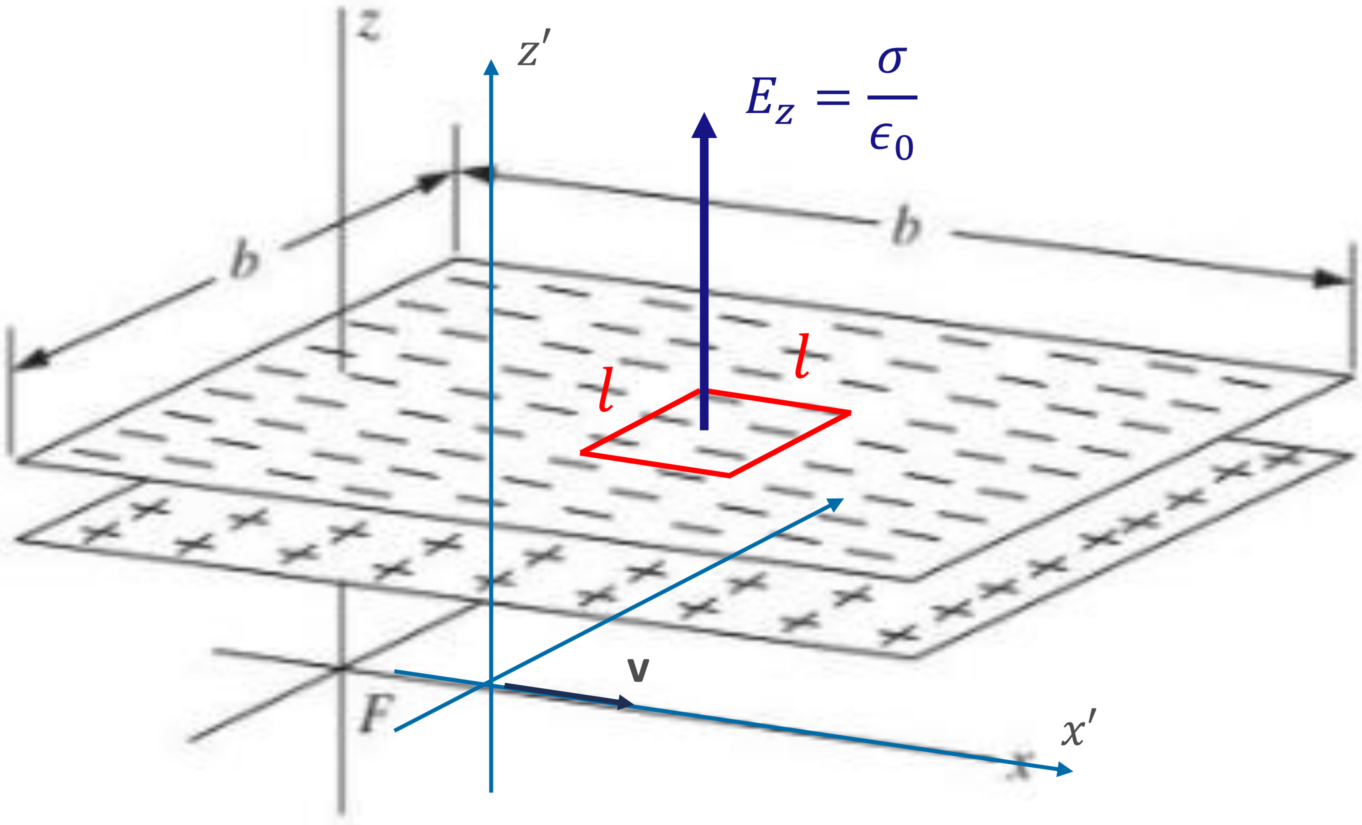




ANSTO

Electric field measured in different frames of reference

Consider a two stationary square sheets of charge of uniform density $+\sigma$ and $-\sigma$, respectively. Their separation is small compared with l, and so the field between them can be treated as uniform.



Now consider an inertial frame F' which is moving along the x-axis with velocity **v**.

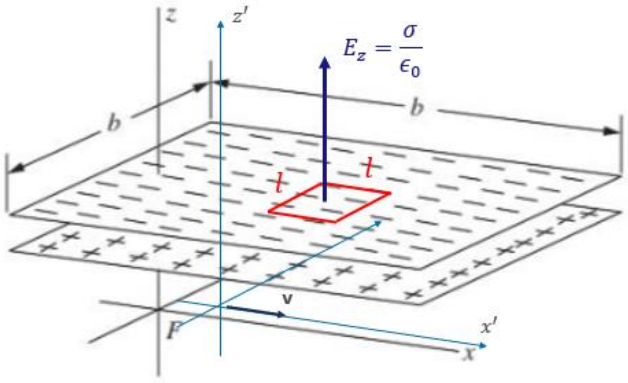
To an observer in F', the charged "squares" are no longer square. The x' dimension is contracted to become,

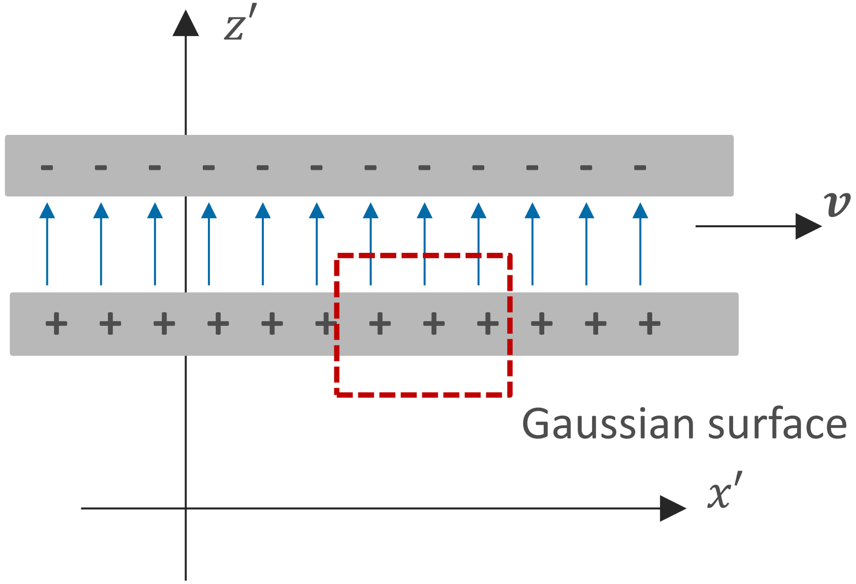
$$l' = l\sqrt{1-\beta^2} = l\gamma\,.$$

The total charge is invariant, that is, independent of reference frame, and so the charge density σ, measured in F' must be:

$$\sigma' = \frac{Q}{l\,l'}$$
$$= \frac{Q}{l^2}\gamma$$

Electric field measured in different frames of reference (cont.)





To recap, the charge density measured in F' must be greater than the charge density in F.

$$\sigma' = \sigma\gamma.$$

The electric field can be calculated by applying Gauss' law:

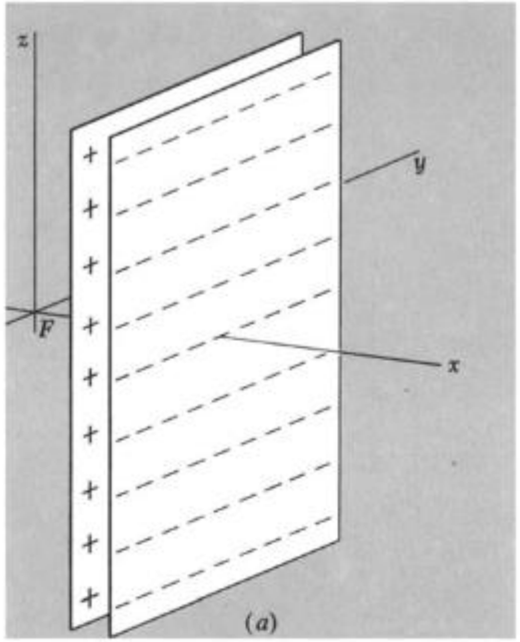
$$\oint \vec{E} \cdot \hat{n}\, dA = \frac{q_{enc}}{\epsilon_0}$$

$$E'_z = \frac{q}{2\, A\, \epsilon_0} = \frac{\sigma'}{\epsilon_0}$$

$$= \frac{\sigma}{\epsilon_0\sqrt{1+\beta^2}} = \gamma E_z$$

$$= \gamma E_z$$

Electric field measured in different frames of reference (cont.)



Now imagine a different situation: the charged sheets are oriented perpendicular to the x axis. From the perspective of the F' frame, the sheets are not contracted; only the distance between them is contracted, but that doesn't influence the strength of the field.

This time, when applying Gauss' law we find that,

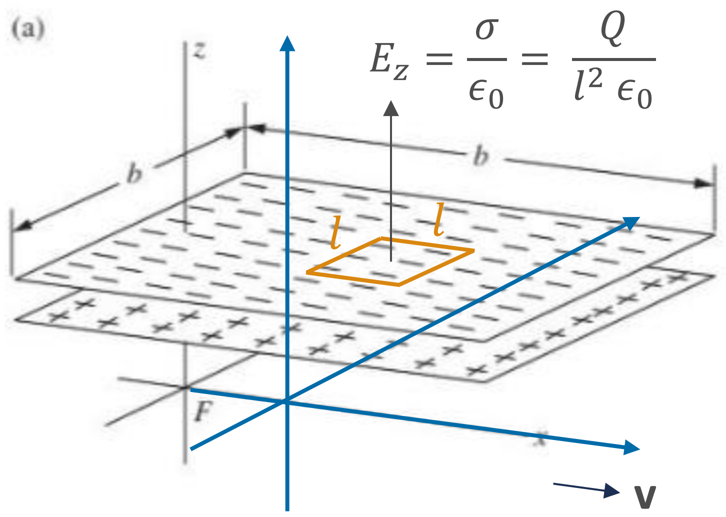
$$E'_x = \frac{\sigma}{\epsilon_0} = \frac{\sigma'}{\epsilon_0} = E_x$$

We've found that the electric field parallel and perpendicular to the velocity at which the frame F' is moving away, will be observed to be:

$$E'_{\parallel} = E_{\parallel}$$
$$E'_{\perp} = \gamma E_{\perp}$$

Electric field measured in different frames of reference

If an observer in a certain inertial frame F measures an electric field E as a certain value at a given point in space and time, what field will be measured at the same spacetime point by an observer in a different inertial frame F' moving away with a velocity **v**?



$$|E| = \frac{\sigma}{\ } = \frac{Q}{\ }$$

Consider a two stationary sheets of charge of uniform density σ and $-\sigma$, respectively.

To an observer in F' moving is in the x-direction at **v**, the $l \times l$ square is no longer square. The x' dimension is contracted to become,

$$l' = l\sqrt{1-\beta^2}.$$

The total charge is invariant, that is, independent of reference frame, and so the charge density σ, measured in F' must be

$$\sigma' = \frac{Q}{l\,l'}$$
$$= \frac{Q}{l^2\sqrt{1-\beta^2}}$$
$$= \frac{Q}{\ }\gamma$$