Chapter 8 ESTIMATION

SECTION 8.2 POINT ESTIMATOR OF A POPULATION MEAN

PROBLEMS

2. NO. The estimate would not be unbiased since this sample will not be a random sample.

4. Let X = number of hours per day spent watching TV by pre-schoolers in the given neighborhood.

   Then, $\overline{X}$ = 2.5 hours.

6. n = $(4)^2$(1000) =16,000.

8. Let X = number of minutes patient waited to see a physician at a medical clinic.

   Then, $\overline{X}$ =44.0833 minutes.

10. (a) SE($\overline{X}$) = σ / √n

    (b) SE($\overline{X}$) = (√2/3)[σ / √n] = 0.8165[σ / √n].

    Comparing the standard errors for X in (a) and (b), we see that the standard error in (a) is larger. Thus, the data in (b) will yield a more precise estimator for μ.

    (c) Let n_1 be the sample size in part (a) and n_2 be associated with part (b) Then, we want $\sigma / \sqrt{n_1} = (\sqrt{2})[\sigma / \sqrt{n_2}] = \sigma / \sqrt{(n_2 / 2)}$. That is, we want $n_1 = n_2/2$, from which $n_2 = 2n_1$. Thus, to achieve the same precision in (b) of the estimator in (a), we would need twice of the sample size in (a).

SECTION 8.3 POINT ESTIMATOR OF A POPULATION PROPORTION

PROBLEMS

2. Let X = number of Americans in the sample who felt that the communist party will win a free election in the Soviet Union (in 1985).

n = 1325, X = 510, $\hat{p}$ = X/n = 510/1325 - 0.3849. Thus, the estimate for

SE($\hat{p}$)= √((03849)(l-0.3849)/1325) -= .0134.

4. Let X = number of solitaire games won in the 20 games played.

n=20, X=7

(a) $\hat{p}$ = X/n = 7/20 = 0.35.

(b) SE($\hat{p}$)= √(0.35)(l-0.35) 720 =0.1067.

6. Let X = number of parents who are in favor of raising the driving age to eighteen in the sample.

n=100, X = 64. (a) $\hat{p}$ = X/n = 64/100 = 0.64.

(b) SE($\hat{p}$)= √((0.64)(l-0.64)/100) =0.048.

8. Let X = number of female architects in the sample, n = 500, X = 104.

(a) $\hat{p}$ = X/n = 104/500 = 0.208.

(b) SE($\hat{p}$) - √((0.208)(1-0.208)/500) = 0.0182.

10. (a) $\hat{p} = 0.0233$; $SE(\hat{p}) = \sqrt{((0.0233)(1 - 0.0233)/1200)} = 0.0044$.

(b) $\hat{p} = 0.0375$; $SE(\hat{p}) = \sqrt{((0.0375)(1-0.0375)/1200)} = 0.0055$.

(c) $\hat{p} = 0.0867$; $SE(\hat{p}) = \sqrt{((0.0867)(1-0.0867)/1200)} = 0.0081$.

12. If n is the sample size, we need $1/(2\sqrt{n}) = 0.1$, from which $n = 25$.

14. Let X = number of full time African American law enforcement officers in the sample who were employed in Chicago in 1990.

$n = 600$, $X = 87$.

(a) $\hat{p} = X/n = 87/600 = 0.145$.

(b) $SE(\hat{p}) = \sqrt{[12048(0.145)(1 - 0.145)]} = 38.6$.

SECTION 8.3.1 ESTIMATING THE PROBABILITY OF A SENSITIVE EVENT

PROBLEMS

2. The estimate of p, $\hat{p}$ = [1 - 2(10/50)] = 0.6.

SECTION 8.4 ESTIMATING A POPULATION VARIANCE

PROBLEMS

2. Let X = width of a slot.

Estimate of the population mean = $\overline{X}$ = 8.7509 inches.

Estimate of the population standard deviation = S = 0.0057 inches.

4. Let X = size (in inches) of a bounce.

With n = 30, then $S^2 = \dfrac{\sum_{i=1}^{30}(X_i - \overline{X})^2}{30-1} = \dfrac{\sum_{i=1}^{30}(X_i)^2 - 30\overline{X}^2}{30-1}$

$= [136.2 - 30(52.1/30)^2]/29 = 1.5765.$

Hence the estimate for the population standard deviation of the size of a bounce is S = $\sqrt{1.5765}$=1.2556.

6. Let X = weight of a runner in the Boston marathon in 1990.

Estimate of the population mean = $\overline{X}$ = 105.70 pounds.

Estimate of the population variance S^2 = 30.6778.

8. Let X = weight of a runner in the Boston marathon in 1990.

Given μ= 106 pounds, then the estimate for the population variance is given

by $\dfrac{\sum_{i=1}^{10}(X_i - 106)^2}{10}$ = 227/10 = 27.7. Thus, the estimate for the standard deviation

is $\sqrt{27.7}$ = 5.2631 pounds.

10. Let X = burn time for the chair.

(a) Estimate of the population mean = X = 464.14 (°F).

(b) Estimate of the population standard deviation = S = 19.32 (°F).

12. Let X = systolic blood pressure for a worker in the mining industry.
(a) Estimate of the population mean = X = 132.23.

(b) Estimate of the population standard deviation = S = 10.49.

(c) $P\{X > 150\} \approx P\{Z > (150\text{-}132.23)/10.49\} = P\{Z > 1.69\}$

$= 1 - P\{Z < 1.69\} = 1 - 0.9545 = 0.0455.$

14. Let X = weight of a fish weighed in the particular scale.

$\overline{X} = 5.8750$ grams and S = 0.2301 grams. Now,

S^2 = Var(data) = Var(X) = 0.0530, and Var(error) = 0.01.

Since, Var(data) = Var(true weight) + Var(error), then

0.0529 = Var(true weight) + 0.01, from which

Var(true weight) = 0.0430 or SD(true weight) = 0.2073.

SECTION 8.5 INTERVAL ESTIMATORS OF THE MEAN OF A NORMAL POPULATION WITH KNOWN POPULATION VARIANCE

PROBLEMS

2. That is, we can assert with "90% confidence" that the ***average*** birth weights of all boys born at that certain hospital will lie between 6.6 pounds and 7.2 pounds.

4. Let X = PCB level of a fish caught in Lake Michigan.

$n = 40$, $X = 11.480$ ppm, $\sigma = 0.86$ ppm, $\alpha = 0.05$, $\alpha/2 = 0.025$, $Z_{0.025}=1.96$.

Since the $(1 - \alpha)$ 100% confidence interval for μ (mean PCB levels) is given by $X \pm Z_{\alpha/2}\,\sigma/\sqrt{n}$, then we have $11.480 \pm (1.96)(0.86/\sqrt{40}\,)$. Thus

the 95% confidence interval for μ is (11.2123, 11.7477).

6. Let X = length of time for a component to function, $n = 9$, $\bar{X} = 10.80$

hours, $\sigma = 3.4$ hours.

(a) $\alpha = 0.05$, $\alpha/2 = 0.025$, $Z_{0.025} = 1.96$. Since the $(1 - \alpha)100\%$ confidence interval for μ (mean life) is given by $\bar{X} \pm Z_{\alpha/2}\,\sigma/\sqrt{n}$, then

we have $10.80 \pm (1.96)(3.4/\sqrt{9}\,)$. Thus the 95% confidence interval for μ is (8.5787, 13.0213).

(b) $\alpha = 0.01$, $\alpha/2 = 0.005$, $Z_{0.005} = 2.575$ (using the standard normal probability table). Since the $(1 - \alpha)$ 100% confidence interval for μ (mean life) is given by $\bar{X} \pm Z_{\alpha/2}\,\sigma/\sqrt{n}$, then we have $10.80 \pm (2.575)(3.4/\sqrt{9}\,)$. Thus the 99% confidence interval for μ is (7.8817, 13.7183).

8. Let X = test score on a certain achievement test.

$n = 324$, $\bar{X} = 74.6$, $\sigma = 11.3$, $\alpha - 0.1$, $\alpha/2 = 0.05$, $Z_{0.05} = 1.645$.

Since the (1 - α) 100% confidence interval for μ, (mean test score) is given by $\overline{X} \pm Z_{\alpha/2}\, \sigma/\sqrt{n}$ =, then we have 74.6 ± (1.645)(11.3/√324). Thus

the 90% confidence interval for μ is (73.5673, 75.6327).

10. Let X=life of the tire.

n = 10, $\overline{X}$ = 28,400 miles, σ = 3,300 miles, α = 0.05, α/2 = 0.025,

Since the (1 - α) 100% confidence interval for μ, (mean test score) is given by $\overline{X} \pm Z_{\alpha/2}\, \sigma/\sqrt{n}$, then we have 28,400 ± (1.96)(3,300/√10).

Thus the 95% confidence interval for μ is (26,354.6388, 30,445.3612).

12. α = 0.1,α/2 = 0.05,$Z_{0.05}$ = 1.645, σ = 180, b = 40. Since $n \geq \left(\frac{2Z_{\alpha/2}\sigma}{b}\right)^2$ = [(2)(1.645)(180)/40]^2 = 219.188. Thus you would need a sample size of at least 220.

14. α = 0.05, α/2 = 0.025, $Z_{0.025}$ = 1.96, σ = 70, b = 4. Since n ≥ $n \geq \left(\frac{2Z_{\alpha/2}\sigma}{b}\right)^2$ = [(2)(1.96)(70)/4]^2 = 4705.96 ≈ 4706. Thus you would need a sample size of at least 4706.

16. (a) We need an Upper Confidence Bound (UCB).

n = 10, $\overline{X}$ = 9.8, α = 0.05, $Z_{0.05}$ = 1.645, σ = 3. Since the (1- α)100% UCB for μ is given by $\overline{X} + Z_\alpha \frac{\sigma}{\sqrt{n}}$, then the required UCB is [9.8 + (1.645)(3)/√10] = 11.3606.

(b) We need a Lower Confidence Bound (LCB).

n = 10, $\overline{X}$ = 9.8, α = 0.01, $Z_{0.01}$ = 2.575, σ = 3. Since the (1- α)100% UCB for μ is given by $\overline{X} + Z_\alpha \frac{\sigma}{\sqrt{n}}$, then the required LCB is [9.8 – (2.575)(3)/√10] = 8.2394.

SECTION 8.6 INTER VAL ESTIMA TORS OF THE MEAN OF A NORMAL POPULATION WITH UNKNOWN POPULATION VARIANCE

PROBLEMS

2. Let X = number of days it takes for California customers to receive their orders.

   (a) n = 12, $\overline{X}$ =12.25, S = 3.67, α = 0.1, α/2 = 0.05, $t_{11,\,0.05}$ = 1.796.

   Since the (1 - α) 100% confidence interval for μ (mean number of days) is given by $\overline{X} \pm t_{n-1,\alpha/\sqrt{n}} \dfrac{S}{\sqrt{n}}$, then we have 12.25 ± (1.796)(3.67/√12). Thus the 90% confidence interval for μ is (10.3472, 14.1528). That is, we can be 90% confident that the average number of days it takes for the California customers to receive their orders is between 10.35 and 14.15 days. (Answer rounded to two decimal places).

   (b) n = 12, $\overline{X}$ =12.25, S = 3.67, α = 0.05, α /2 = 0.025, $t_{11,\,0.025}$ = 2.201.

   Since the (1 - α) 100% confidence interval for μ (mean number of days) is given by $\overline{X} \pm t_{n-1,\alpha/\sqrt{n}} \dfrac{S}{\sqrt{n}}$, then we have 12.25 ± (2.201)(3.67/√12). Thus the 95% confidence interval for μ is (9.9182, 14.5818). That is, we can be 95% confident that the average number of days it takes for the California customers to receive their orders is between 9.92 and 14.58 days. (Answer rounded to two decimal places).

4. Let $\overline{X}$ = number of daily inter-city bus riders,

   (a) $\overline{X}$ = 54.42(b) S = 7.24
   (c) n = 12, α = 0.05, α/2 = 0.025, $t_{11,\,0.025}$ = 2.201.

   Since the (1 - α) 100% confidence interval for μ (mean number of days) is given by $\overline{X} \pm t_{n-1,\alpha/\sqrt{n}} \dfrac{S}{\sqrt{n}}$, then we have 54.42 ± (2.201)(7.24/√12). Thus the 95% confidence interval for μ is (49.82, 59.02). That is, we can be 95% confident that the average number of daily inter-city bus riders is between 49.82 and 59.02.

6. Let $\overline{X}$ = lifetime of a General Electric transistor.

$n = 30$, $\overline{X} = 1210$ hours, $S = 92$ hours.

(a) $\alpha = 0.1$, $\alpha/2 - 0.05$, $t_{29,0.05} = 1 \text{-}699$.

Since the (1 - α) 100% confidence interval for μ, (mean lifetime) is given by $\overline{X} \pm t_{n-1,\alpha/\sqrt{n}} \dfrac{S}{\sqrt{n}}$, then we have $1210 \pm (1.699)(92/\sqrt{30})$. Thus the 90% confidence interval for μ is (1181.4622, 1238.5378). That is, we can be 90% confident that the average lifetimes for the transistors is between 1181.4622 and 1238.5378 hours.

(b) $n = 30$, $\alpha = 0.05$, $\alpha/2 = 0.025$, $t_{29;\,0.025} = 2.045$.

Since the (1 - α) 100% confidence interval for μ, (mean lifetime) is given by $\overline{X} \pm t_{n-1,\alpha/\sqrt{n}} \dfrac{S}{\sqrt{n}}$, then we have $1210 \pm (2.045)(92/\sqrt{30})$. Thus the 95% confidence interval for μ is (1 175.6505, 1244.3495). That is, we can be 95% confident that the average lifetime for the transistors is between 1175.6505 and 1244.3495 hours.

(c) $n = 30$, $\alpha = 0.01$, $\alpha/2 = 0.005$, $t_{29,0.005} = 2.756$.

Since the (1 - α) 100% confidence interval for μ (mean lifetime) is given by $\overline{X} \pm t_{n-1,\alpha/\sqrt{n}} \dfrac{S}{\sqrt{n}}$, then we have $1210 \pm (2.756)(92/\sqrt{30})$. Thus the 99% confidence interval for μ is (1 163.708, 1256.292). That is, we can be 99% confident that the average lifetime for the transistors is between 1163.708 and 1256.292 hours.

8. Let X = losing score in a Super Bowl football game.

$n = 7$, $\overline{X} = 18.14$, $S = 6.57$, $\alpha = 0.05$, $\alpha/2 = 0.025$, $t_{6,\,0.025} = 2.447$.

Since the (1 - α) 100% confidence interval for μ (mean losing score) is given by $\overline{X} \pm t_{n-1,\alpha/\sqrt{n}} \dfrac{S}{\sqrt{n}}$, then we have $18.14 \pm (2.447)(6.57/\sqrt{7}\,)$. Thus the 95% confidence interval for μ is (12.0635, 24.2165). That is, we can be 95% confident that the average losing score in Super Bowl games is between 12.0635 and 24.2165 points.

10. Let X = time it takes to perform the task.

$n = 20$, $\overline{X} = 12.4$, $S = 3.3$, $\alpha = 0.05$, $\alpha/2 = 0.025$, $t_{19,0.025} = 2.093$.

Since the (1 - α) 100% confidence interval for μ (mean time) is given by $\overline{X} \pm t_{n-1,\alpha/\sqrt{n}} \frac{S}{\sqrt{n}}$,

then we have 12.4 ± (2.093)(3.3/√20).Thus the 95% confidence interval for μ is (10.8556, 13.9444). That is, we can be 95% confident that the average time to complete the task is between 10.8556 and 13.9444 minutes.

12. Let X = height of a male from the certain tribe.

(a) n = 64, $\overline{X}$ = 72.4, S = 2.2, α = 0.05, α/2 = 0.025, $t_{63,\,0.025} \approx 2$.

Since the (1 - α) 100% confidence interval for μ, (mean height) is given by $\overline{X} \pm t_{n-1,\alpha/\sqrt{n}} \frac{S}{\sqrt{n}}$, then we have 72.4 ± (2)(2.2/√64). Thus the 95% confidence interval for μ is (71.85, 72.95). That is, we can be 95% confident that the average heights of the males in this certain tribe is between 71.85 and 72.95 inches.

Note: If you use $t_{63,\,0.025} \approx Z_{0.025} = 1.96$, then the confidence interval will be (71.86, 72.94).

(b) n = 64, $\overline{X}$ = 72.4, S = 2.2, α = 0.01, α/2 = 0.005, $t_{63,\,0.025} \approx 2.66$.

Since the (1 - α) 100% confidence interval for μ, (mean height) is given by $\overline{X} \pm t_{n-1,\alpha/\sqrt{n}} \frac{S}{\sqrt{n}}$, then we have 72.4 ± (2.66)(2.2/√64). Thus the 99% confidence interval for μ is (71.6685, 73.1315). That is, we can be 99% confident that the average heights of the males in this certain tribe is between 71.6685 and 73.1315 inches.

Note: If you use $t_{63,\,0.005} \approx Z_{0.005} = 2.576$, then the confidence interval will be (71.6916, 73.1084).

14. Let X = melting point of lead.

(a) n = 20, X = 330.2, S = 15.4, α = 0.05, α /2 = 0.025, $t_{19,\,0.025} = 2.093$.

Since the (1 - α) 100% confidence interval for μ (mean melting point) is given by $\overline{X} \pm t_{n-1,\alpha/\sqrt{n}} \frac{S}{\sqrt{n}}$, then we have 330.2 ± (2.093)(15.4/√20). Thus the 95% confidence interval for μ is (332.9927, 337.4073). That is, we can be 95% confident that the average melting point for lead is between 332.9927 and 337.4073 degrees centigrade.

(b) n = 20, X = 330.2, S = 15.4, α = 0.01, α/2 = 0.005, $t_{19,\,0.005} = 2.861$.

Since the (1 - α) 100% confidence interval for μ (mean melting point) is given by $\bar{X} \pm t_{n-1,\alpha/\sqrt{n}} \frac{S}{\sqrt{n}}$, then we have 330.2 ± (2.861)(15.4/√20). Thus the 99% confidence interval for μ is (320.3480, 340.0520). That is, we can be 99% confident that the average melting point for lead is between 320.3480 and 340.0520 degrees centigrade.

16. Let X = length of time for an officer on the Chicago police force.

n = 46, X = 14.8 years, S = 8.2 years.

(a) α = 0.1, α/2 = 0.05, $t_{45,\,0.05} \approx 1.684$.

Since the (1 - α) 100% confidence interval for μ (mean time) is given by $\bar{X} \pm t_{n-1,\alpha/\sqrt{n}} \frac{S}{\sqrt{n}}$, then we have 14.8 ± (1.684)(8.2/√46). Thus the 90% confidence interval for μ is (12.764, 16.7925). That is, we can be 90% confident that the average time for the officers on the Chicago police force is between 12.764 and 16.7925 years.

(b) α = 0.05, α/2 = 0.025, $t_{45,\,0.025} \approx 2.021$.

Since the (1 - α) 100% confidence interval for μ (mean time) is given by $\bar{X} \pm t_{n-1,\alpha/\sqrt{n}} \frac{S}{\sqrt{n}}$, then we have 14.8 ± (2.021)(8.2/√46). Thus the 95% confidence interval for μ is (12.3566, 17.2434). That is, we can be 95% confident that the average time for the officers on the Chicago police force is between 12.3566 and 17.2434 years.

(c) α = 0.01, α/2 = 0.005, $t_{45,\,0.005} \approx 2.704$.

Since the (1 - α) 100% confidence interval for μ (mean time) is given by $\bar{X} \pm t_{n-1,\alpha/\sqrt{n}} \frac{S}{\sqrt{n}}$, then we have 14.8 ± (2.704)(8.2/√46). Thus the 99% confidence interval for μ is (11.5308, 18.0692). That is, we can be 99% confident that the average times for the officers on the Chicago police force is between 11.5308 and 18.0692 years.

18. Let X = price of the crude oil stock.

n = 20, $\bar{X}$ = 17.465, S = 0.4252, α = 0.05, α/2 = 0.025, $t_{19,\,0.025} = 2.093$.

Since the (1 - α) 100% confidence interval for μ (mean price) is given by $\bar{X} \pm t_{n-1,\alpha/\sqrt{n}} \frac{S}{\sqrt{n}}$, then we have 17.465 ± (2.093)(0.4252/√20). Thus the 95% confidence interval for μ is

(17.2660, 17.6640). That is, we can be 95% confident that the average price of the crude oil stock is between 17.266 and 17.664.

20. Let X = number of days of school missed by a student.

(a) n = 50, $\overline{X}$ = 8.4, S = 5.1, α = 0.05, α/2 - 0.025, $t_{49,\,0.025} \approx 2.01$.

Since the (1 - α) 100% confidence interval for μ (mean missed days) is given by $\overline{X} \pm t_{n-1,\alpha/\sqrt{n}} \dfrac{S}{\sqrt{n}}$, then we have 8.4 ± (2.01)(5.1/√50). Thus the 95% confidence interval for μ is (6.9503, 9.8497). That is, we can be 95% confident that the average number of school days missed by the students is between 6.9503 and 9.8497.

(b) We need to compute the upper confidence bound (UCB) for X. Since the (1 – α)100% UCB is given by $\overline{X} \pm t_{n-1,\alpha/\sqrt{n}} \dfrac{S}{\sqrt{n}}$, then we have 8.4 + (1.675)(5.1/√50). Thus the 95% UCB for X is 9.6081. Thus "with 95% confidence I can state that the average number of days misses is less than 9.6081."

22. Let X = number of days it takes for California customers to receive their orders.

n = 12, $\overline{X}$ = 12.25, S = 3.67, α = 0.05, $t_{11,\,0.05} = 1.796$.

Since the (1 – α)100% UCB is given by $\overline{X} \pm t_{n-1,\alpha/\sqrt{n}} \dfrac{S}{\sqrt{n}}$, then we have 12.25 + (1.796)(3.67/√12). Thus the 95% UCB for X is 14.1528. So, "with 95% confidence we can state that the average number of days it takes for the California customers to receive their orders is less than 14.1528 days."

24. Let X = amount of carbon monoxide measured.

n = 7, $\overline{X}$ = 100.7, S = 5.982, α = 0.01, $t_{6,\,0.01} = 3.143$.

Since the (1 – α)100% UCB is given by $\overline{X} \pm t_{n-1,\alpha/\sqrt{n}} \dfrac{S}{\sqrt{n}}$, then we have 100.7 + (3.143)(5.982/√7). Thus the 99% UCB for X is 107.8063. Thus with 99% confidence the inspector can state that the average amount of carbon monoxide measured less than 107.8065 parts per million. (Hopefully, this will calm the concerns of the group).

SECTION 8.7 INTERVAL ESTIMATORS OF A POPULATION PROPORTION

PROBLEMS

2. Let X = number of people that suffer an additional heart attack within one year of their first.

   (a) $n = 300$, $X = 46$, $\hat{p} = 46/300 = 0.1533$, $\alpha = 0.05$, $\alpha/2 = 0.025$, $Z_{0.025} = 1.96$.

   Since the (1 - α) 100% confidence interval for p (proportion of second heart attack within one year) is given by $\hat{p} \pm Z_{\alpha/2}\sqrt{\hat{p}(1-\hat{p})/n}$, then we have $0.1533 \pm (1.96)(0.0208)$.

   Thus the 95% confidence interval for p is (0.1125, 0.1941). That is, we can be 95% confident that the proportion of second heart attacks within one year is between 11.25% and 19.41%.

   (b) Let X = number of people that suffer an additional heart attack within one year of their first.

   $n = 300$, $X - 92$, $\hat{p} = 92/300 = 0.3067$, $\alpha = 0.05$, $\alpha/2 = 0.025$, $Z_{0.025} = 1.96$.

   Since the (1 - α) 100% confidence interval for p (proportion of second heart attack within one year) is given by $\hat{p} \pm Z_{\alpha/}\sqrt{\hat{p}(1-\hat{p})/n}$, then we have 0.3067 $\pm (1.96)(0.0266)$. Thus the 95% confidence interval for p is (0.2546, 0.3588). That is, we can be 95% confident that the proportion of second heart attacks within one year is between 25.46% and 35.88%.

4. $n = 1200$, $\hat{p} = 0.57$, $\alpha = 0.01$, $\alpha/2 = 0.005$, $Z_{0.005} = 2.575$.

   Since the (1 - α) 100% confidence interval for p (proportion of the population that favored Reagan at the time of the poll) is given by $\hat{p} \pm Z_{\alpha/}\sqrt{\hat{p}(1-\hat{p})/n}$, then we have $0.57 \pm (2.75)(0.0143)$ the 99% confidence interval for p is (0.5332, 0.6068). That is, we can be 99% confident that the proportion of the population that favored Reagan at the time of the 1980 poll was between 53.32% and 60.68%.

6. Let X = number of recent science PhD's that are optimistic.

   (a) $n = 100$, $X = 42$, $\hat{p} = 42/100 = 0.42$, $\alpha = 0.1$, $\alpha/2 = 0.05$, $Z_{0.05} = 1.645$.

Since the (1 - α) 100% confidence interval for p (proportion of recent science PhD's who are optimistic) is given by $\hat{p} \pm Z_{\alpha/}\sqrt{\hat{p}(1-\hat{p})/n}$, then we have 0.42 ± (1.645)(0.0494). Thus the 90% confidence interval for p is (0.3387, 0.5013). That is, we can be 90% confident that the proportion of recent science PhD's who are optimistic is between 33.87% and 50.13%.

(b) n = 100, X = 42, $\hat{p}$ = 42/100 = 0.42, α = 0.01, α/2 = 0.005, $Z_{0.005}$=2.575.

Since the (1 - α) 100% confidence interval for p (proportion of recent science PhD's who are optimistic) is given by $\hat{p} \pm Z_{\alpha/}\sqrt{\hat{p}(1-\hat{p})/n}$, then we have 0.42 ± (2.575)(0.0494). Thus the 99% confidence interval for p is (0.2928, 0.5472). That is, we can be 99% confident that the proportion of recent science PhD's who are optimistic is between 29.28% and 54.72%.

8. Let X = number of solitaire games won.

n = 20, X = 7, $\hat{p}$ = 7/20 = 0.35, α = 0.1, α/2 = 0.05, $Z_{0.005}$ = 1.645.

Since the (1 - α) 100% confidence interval for p (proportion of solitaire games won) is given by $\hat{p} \pm Z_{\alpha/2}\sqrt{\hat{p}(1-\hat{p})/n}$, then we have 0.35 ± (1.645)(0.1067). Thus the 90% confidence interval for p is (0.1745, 0.5255). That is, we can be 90% confident that the probability of winning at solitaire will be between 17.45% and 52.55%.

10. Let X = number of cups of coffee that had less than the specified amount.

n = 100, X = 9, $\hat{p}$ = 9/100 = 0.09, α = 0.1, α/2 = 0.05, $Z_{0.05}$ = 1.645.

Since the (1 - α) 100% confidence interval for p (proportion of cups of coffee that were under-fill) is given by $\hat{p} \pm Z_{\alpha/2}\sqrt{\hat{p}(1-\hat{p})/n}$, then we have 0.09 ± (1.645)(0.0286). Thus the 90% confidence interval for p is (0.0430, 0.1370). That is, we can be 90% confident that the proportion of cups of coffee that had less than the specified amount will be between 4.3% and 13.7%.

12. Let X = number of male authors.

n = 300, X=117, $\hat{p}$ = 117/300 = 0.39, α = 0,05, α/2=0.025,

$Z_{0.025}$ = 1.96.

Since the (1 - α) 100% confidence interval for p (proportion of male authors) is given by $\hat{p} \pm Z_{\alpha/2}\sqrt{\hat{p}(1-\hat{p})/n}$, then we have 0.39 ± (1.96)(0.0282). Thus the 95% confidence interval for p is (0.3347, 0.4453). That is, we can be 95% confident that the proportion of male authors will be between 33.47% and 44.53%.

14. Let X = number of male psychologists.

n = 1000, X = 457, $\hat{p}$ = 457/1000 = 0.457, α = 0.05, $\alpha/2$ = 0.025, $Z_{0.025}$ = 1.96.

Since the (1 - α) 100% confidence interval for p (proportion of male authors) is given by $\hat{p} \pm Z_{\alpha/2}\sqrt{\hat{p}(1-\hat{p})/n}$, then we have 0.457 ± (1.96)(0.0158). Thus the 95% confidence interval for p is (0.4260, 0.4880). That is, we can be 95% confident that the proportion of male psychologists will be between 42.6% and 48.8%.

16. Let X = number of people that were in favor of the war against Iraq on January 22, 1991.

(a) n = 600, X = 450, $\hat{p}$ = 450/600 = 0.75, α = 0.1, α /2 = 0.05, $Z_{0.05}$= 1.645.

Since the (1 - α) 100% confidence interval for p (proportion of people that favored the war) is given by $\hat{p} \pm Z_{\alpha/2}\sqrt{\hat{p}(1-\hat{p})/n}$, then we have 0.75 ± (1.645)(0.0177). Thus the 90% confidence interval for p is (0.7210, 0.7791). That is, we can be 90% confident that the proportion of the population who favored the war against Iraq on January 22, 1991 was between 72.10% and 77.91%.

(b) n = 600, X = 450, $\hat{p}$ = 450/600 = 0.75, α = 0.05, α /2 = 0.025, $Z_{0.025}$ = 1.96

Since the (1 - α) 100% confidence interval for p (proportion of people that favored the war) is given by $\hat{p} \pm Z_{\alpha/2}\sqrt{\hat{p}(1-\hat{p})/n}$, then we have 0.75 ± (1.96)(0.0177). Thus the 95% confidence interval for p is (0.7153, 0.7847). That is, we can be 95% confident that the proportion of the population who favored the war against Iraq on January 22, 1991 was between 71.53% and 78.47%.

(c) n = 600, X = 450, $\hat{p}$ =450/600 = 0.75, α = 0.01, α /2 = 0.005, $Z_{0.005}$ = 2.575.

Since the (1 - α) 100% confidence interval for p (proportion of people that favored the war) is given by $\hat{p} \pm Z_{\alpha/2}\sqrt{\hat{p}(1-\hat{p})/n}$, then we have 0.75 ± (2.575)(0.0177). Thus the 99% confidence interval for p is (0.7044, 0.7956). That is, we can be 99% confident that the proportion of the population who favored the war against Iraq on January 22, 1991 was between 70.44% and 79.56%.

18. NO. The error of ±4% if applied to both estimates will be 47% ± 4% and 53% ± 4%. That is, we have intervals of (43%, 51%) and (49%, 57%). Since these intervals overlap, we cannot say with certainty that candidate A is the current choice.

20. $\alpha = 0.05$, $\alpha/2 = 0.025$, $Z_{0.025} = 1.96$, $b = 2(0.01) = 0.02$.

Since $n > \left(\frac{Z_{\alpha/2}}{b}\right)^2$, then $n > (1.96/0.02)^2 = 9{,}604$. That is, a sample size of at least 9,604 is needed if we want to estimate the required proportion to within 0.01 with 95% confidence.

22. Let X = number of male psychologists.

$n = 1000$, $X = 457$, $\hat{p} = 457/1000 = 0.457$, $\alpha = 0.05$, $Z_{0.05} = 1.645$.
Since the $(1 - \alpha)$ 100% lower confidence bound for p (proportion of male authors) is given by $\hat{p} \pm Z_{\alpha/2}\sqrt{\hat{p}(1-\hat{p})/n}$, then we have
0.457 - (1.645)(0.0158). Thus the 95% lower confidence bound for p is 0.4310. That is, we can be 95% confident that the proportion of male psychologists will be more than 43.1%.

24. We need to compute a lower confidence bound.

Let X = number of consumers who are satisfied with the product.

$n = 500$, $\hat{p} = 0.92$, $\alpha = 0.05$, $Z_{0.05} = 1.645$.

Since the $(1 - \alpha)$ 100% lower confidence bound for p (proportion of satisfied customers) is given by $\hat{p} \pm Z_{\alpha/2}\sqrt{\hat{p}(1-\hat{p})/n}$, then we have 0.92 - (1.645)(0.0121). Thus the 95% lower confidence bound for p is 0.9001. That is, we can be 95% confident that the proportion of satisfied customers will be more than 90.01%. Thus, let x be equal to 90.01%.

When $\alpha = 0.1$, $Z_{0.1} = 1.28$, from which x = 90.45%.

26. (a) Let X = number who favor the war at the time of the poll (see #16).

$n = 600$, $X = 450$, $\hat{p} = 450/600 = 0.75$, $\alpha = 0.05$, $Z_{0.05} = 1.645$.

Since the $(1 - \alpha)$ 100% upper confidence bound for p (proportion of the population who favored the war) is given by $\hat{p} \pm Z_{\alpha/}\sqrt{\hat{p}(1-\hat{p})/n}$, then we have 0.75 + (1.645)(0.0177). Thus the 95% upper confidence bound for p is 0.7791. That is, we can be 95% confident that the proportion of the population who favored the war will be less than 77.91%.

(b) $n = 600$, $X = 450$, $\hat{p} = 450/600 = 0.75$, $\alpha = 0.05$, $Z_{0.05} = 1.645$.

Since the (1 - α) 100% lower confidence bound for p (proportion of the population who favored the war) is given by $\hat{p} \pm Z_{\alpha/}\sqrt{\hat{p}(1-\hat{p})/n}$, then we have 0.75 - (1.645)(0.0177). Thus the 95% lower confidence bound for p is 0.7209. That is, we can be 95% confident that the proportion of the population who favored the war will be more than 72.09%.

28. We need to compute the lower and upper confidence bounds.

(a) Let X = number of LA residents who favor strict gun control legislation.

$n = 100$, $X = 64$, $\hat{p} = 64/100 = 0.64$, $\alpha = 0.05$, $Z_{0.05} = 1.645$.

Since the (1 - α) 100% lower confidence bound for p (proportion of LA population who favored the legislation) is given by $\hat{p} \pm Z_{\alpha/}\sqrt{\hat{p}(1-\hat{p})/n}$, then we have 0.64 - (1.645)(0.048). Thus the 95% lower confidence bound for p is 0.5610. That is, "With 95% confidence, more than 56.10% of all Los Angeles residents favor gun control."

(b) Let X = number of LA residents who favor strict gun control legislation.

$n=100$, $X = 64$, $\hat{p} = 64/100 = 0.64$, $\alpha = 0.05$, $Z_{0.05} = 1.645$.

Since the (1 - α) 100% upper confidence bound for p (proportion of LA population who favored the legislation) is given by $\hat{p} \pm Z_{\alpha/}\sqrt{\hat{p}(1-\hat{p})/n}$, then we have 0.64 + (1.645)(0.048). Thus the 95% upper confidence bound for p is 0.7190. That is, "With 95% confidence, less than 71.90% of all Los Angeles residents favor gun control."

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2. Let X = weight of a ball bearing.

(a) $\sigma = 0.5$, $\alpha = 0.05$, $\alpha/2 = 0.025$, $Z_{0.025} = 1.96$, b = 2(0.1)= 0.2.

Since, $n \geq \left(\frac{2Z_{\alpha/2}\sigma}{b}\right)^2$, then n ≥ $(2x1.96x0.5/0.2)^2 = 96.04 \approx 97$.

(b) $\sigma = 0.5$, $\alpha = 0.05$, $\alpha/2 = 0.025$, $Z_{0.025} = 1.96$, b = 2(0.01) = 0.02.

Since, $n \geq \left(\frac{2Z_{\alpha/2}\sigma}{b}\right)^2$, then n ≥ $(2x\ 1.96x0.5/0.02)^2 = 9604$.

(c) n = 8, $\overline{X} = 3.938$, S = 0.403, $\alpha = 0.05$, $\alpha/2 = 0.025$, $t_{7, 0.025} = 2.365$.

Since the (1 - α) 100% confidence interval for μ (mean weight) is given by $\overline{X} \pm t_{n-1,\alpha/\sqrt{n}} \frac{S}{\sqrt{n}}$, then we have 3.938 ± (2.365)(0.403/√8). Thus the 95% confidence interval for μ is (3.601, 4.275). That is, we can be 95% confident that the average weight of the ball bearings is between 3.601 (units) and 4.275 (units).

4. The 90% confidence intervals (CI) for the variables *All, Male* and *Female* are given below in the table.

Variable	n	$\overline{X}$	S	Std. Error	90% CI
All	30	195.57	12.13	2.21	(191.92, 199.21)
Male	15	191.00	12.80	3.30	(185.18, 196.82)
Female	15	200.13	9.81	2.53	(195.67, 204.60)

Thus, we are 90% confident that the average blood cholesterol levels for both the males and females lie between the intervals given in the table.

6. $\alpha = 0.1$, $\alpha/2 = 0.05$, $Z_{0.05} = 1.645$, $b = 2(0.02) = 0.04$.

Since $n > \left(\frac{Z_{\alpha/2}}{b}\right)^2$, then $n > (1.645/0.04)^2 = 1691.2656$. That is, a sample size of at least 1692 is needed if we want to estimate the required proportion to within 0.02 (2%) with 90% confidence.

8. Let X = length (in minutes) of a rapid eye movement (REM) interval.

$n = 7$, $X = 41.43$, $S - 7.32$, $\alpha = 0.01$, $\alpha/2 = 0.005$, $t_{6,\,0.005} = 3.707$.

Since the (1 - α) 100% confidence interval for μ (mean length of an interval) is given by $\bar{X} \pm t_{n-1,\alpha/\sqrt{n}} \frac{S}{\sqrt{n}}$, then we have $41.43 \pm (3.707)(7.32/\sqrt{7})$. Thus the 99% confidence interval for μ is (31.1738, 51.6862). That is, we can be 99% confident that the average length of the REM interval is between 31.1738 minutes and 51.6862 minutes.

10. Let X = number of farm workers that were in favor of unionizing.

$n = 300$, $X = 144$, $\hat{p} = 144/300 = 0.48$, $\alpha = 0.1$, $\alpha/2 = 0.05$, $Z_{0.05} = 1.645$.

Since the (1 - α) 100% confidence interval for p (proportion of farm workers who favored unionizing) is given by $\hat{p} \pm Z_{\alpha/2}\sqrt{\hat{p}(1-\hat{p})/n}$, then we have $0.48 \pm (1.645)(0.0288)$. Thus the 90% confidence interval for p is (0.4326, 0.5274). That is, we can be 90% confident that the proportion of the population of farm workers who favored unionizing is between 43 .26% and 52.74%.

12. (a) $n = 9$, $\bar{X} = 35$, $\sigma = 3$, $\alpha = 0.05$, $\alpha/2 = 0.025$, $Z_{0.025} = 1.96$.

Since the (1 – α) 100% confidence interval for μ is given by $\bar{X} \pm Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$, then we have $35 \pm (1.96)(3/\sqrt{9})$. Thus the 95% confidence interval for μ is (33.04, 36.96).

(b) $n = 9$, $\bar{X} = 35$, $\sigma = 6$, $\alpha = 0.05$, $\alpha/2 = 0.025$, $Z_{0.025} = 1.96$.

Since the (1 - α) 100% confidence interval for μ is given by $\bar{X} \pm Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$, then we have $35 \pm (1.96)(6/\sqrt{9})$. Thus the 95% confidence interval for μ is (31.08, 38.92).

(c) n = 9, $\overline{X}$ = 35, σ = 12, α = 0.05, α /2 = 0.025, $Z_{0.025}$ = 1.96.

Since the (1 - α) 100% confidence interval for μ is given by $\overline{X} \pm Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$, then we have 35±(1.96X12/√9). Thus the 95% confidence interval for μ, is (27.16, 42.84).

14. Let X = IQ scores of students at the large eastern university.

(a) n = 18, $\overline{X}$ = 133.22, S = 10.21, α = 0.1, α /2 = 0.05, $t_{17, 0.05}$ =1.74.

Since the (1 - α) 100% confidence interval for μ (mean IQ score for the students at this large eastern university) is given by $\overline{X} \pm t_{n-1, \alpha/\sqrt{n}} \frac{S}{\sqrt{n}}$, then we have 133.22 ± (1.74)(10.21/√8). Thus the 90% confidence interval for μ is (129.0327, 137.4073). That is, we can be 90% confident that the average IQ scores for these students is between 129.0327 and 137.4073.

(b) n = 18, $\overline{X}$ = 133.22, S = 10.21, α = 0.05, α /2 = 0.025, $t_{17, 0.025}$ = 2.11.

Similarly, the 95% confidence interval is (128.1422, 138.2978).

(c) n = 18, $\overline{X}$ = 133.22, S = 10.21, α = 0.01, α /2 = 0.005, $t_{17, 0.005}$ =2.898.

Similarly, the 99% confidence interval is (126.2459, 140.1941).

16. α = 0.01, α/2 = 0.005, Z_{0005} = 2.575, b = 0.03.

Since $n > \left(\frac{Z_{\alpha/2}}{b} \right)^2$, then n > $(2.575/0.03)^2$ = 7367.3611. That is, a sample size of at least 7368 is needed if we want the 99% confidence interval to have a length of at most 0.03.

18. For a given n = 1600 and α, the largest possible margin of error is

$$\pm \frac{Z_{\alpha/2}}{2\sqrt{n}} = \pm \frac{Z_{\alpha/2}}{2\sqrt{1600}} = \pm \frac{Z_{\alpha/2}}{80}$$

20. Let X = number of secondary school teachers that are female.

(a) $n = 1000$, $X= 518$, $\hat{p} = 518/1000 = 0.518$, $\alpha = 0.1$, $Z_{0.1} = 1.28$.

Since the (1 - α) 100% upper confidence bound for p (proportion of female secondary school teachers) is given by $\hat{p} \pm Z_{\alpha/2}\sqrt{\hat{p}(1-\hat{p})/n}$, then we have 0.518+ (1.28X0.0158). Thus the 90% upper confidence bound for p is 0.5382. That is, with 90% confidence, less than 53.82% of the secondary school teachers are females.

(b) $n = 1000$, $X = 518$, $\hat{p} =518/1000 = 0.518$, $\alpha = 0.05$, $Z_{0.05} = 1.645$.

Similarly, the 95% upper confidence bound is 0.5440.

(c) $n = 1000$, $X = 518$, $\hat{p} = 518/1000 = 0.518$, $\alpha = 0.01$, $Z_{0.01} =2.33$.

Similarly, the 99% upper confidence bound is 0.5548.

22. Let X = price of a house in the given city.

$n = 9$, $\overline{X} = \$222{,}000$, $S = \$12{,}000$, $\alpha = 0.05$, $t_{8,\, 0.05} = 1.86$.

Since the (1 - α) 100% upper confidence bound (UCB) is given by $\overline{X} \pm t_{n-1,\alpha/\sqrt{n}} \dfrac{S}{\sqrt{n}}$, then we have 222,000 + (1.86)(12,000/√9). Thus the 95% UCB for X is 229,440. Thus with 95% confidence the average price of all recently sold houses in the given city is less than \$229,440.

