A modified shallow water model for investigating convective-scale data assimilation

Tom Kent*, Onno Bokhove, Steve Tobias
School of Mathematics, University of Leeds, UK. *Corresponding email: mmtk@leeds.ac.uk

1. Background

The goal of **data assimilation** (DA) is to update a forecast with observations of the modelled system in a dynamically consistent and optimal way, given **estimates** of their respective **errors** [1]. DA techniques need to evolve in order to keep up with advances in **high-resolution** Numerical Weather Prediction (NWP) as it approaches the scale at which convection is explicitly resolved. To investigate the potential of high-resolution DA schemes, **idealised models** can be employed that capture some fundamental features of **convective-scale dynamics** while remaining computationally inexpensive.

2. Approach

1. Introduce a **physically plausible idealised model** and implement numerically.
   - based on the shallow water equations (SWEs), often used in the meteorological community for modelling atmospheric circulation.
   - compare dynamics of the modified model to those of the classical shallow water theory, e.g., the Rossby geostrophic adjustment and flow over topography.
2. Idealised **convective-scale data assimilation** experiments: Ensemble Kalman Filter (EnKF) with perturbed observations.
   - 'truth' trajectory is determined by very high resolution numerical solution and pseudo-observations are generated by randomly perturbing this 'truth'.
   - ensembles are generated at a coarser 'forecast' resolution in which small-scale features are not fully resolved.

3. Model: SWEs with 'rain'

We outline an idealised NWP model (after [2]) which is designed to represent an **idealised atmosphere with moist convection**. Moisture is incorporated into the traditional SWEs by the introduction of a **'rain mass fraction'** variable, r, which acts on the momentum equation (1b) via the geopotential, and is allowed to precipitate via an additional conservation equation (1d).

$$\partial_t h + \partial_x(hu) = 0, \quad \text{(1a)}$$

$$\partial_t(hu) + \partial_x(hu^2 + p(h)) + gh\partial_x b + hc_0^2\partial_x r - fhv = 0, \quad \text{(1b)}$$

$$\partial_t(hv) + \partial_x(huv) + fhu = 0, \quad \text{(1c)}$$

$$\partial_t(hr) + \partial_x(hur) + h\widetilde{\beta}\partial_x u + \alpha hr = 0, \quad \text{(1d)}$$

$$\partial_t b = 0, \quad \text{(1e)}$$

where $p = p(h)$ is 'effective' pressure defined by:

$$p(h) = \begin{cases} \frac{1}{2}gH_c^2, & \text{for } h + b > H_c, \\ \frac{1}{2}gh^2, & \text{otherwise,} \end{cases}$$

and:

$$\widetilde{\beta}(h, \partial_x u) = \begin{cases} \beta, & \text{for } h + b > H_r \text{ and } \partial_x u < 0, \\ 0, & \text{otherwise.} \end{cases}$$

(Black - standard SWEs; red - modifications.)

- h = fluid depth, (u, v) = velocities, b = bottom topography, r = rain mass fraction; all as a function of (x, t).
- H_c, H_r = threshold heights, above which convection and 'rain' processes occur.
- $c_0^2 = gH_r$, α = rate of 'rain' removal, β controls 'rain' formation, f = Coriolis frequency, g = gravity.

4. Why SWEs?

The **traditional** SWEs:

- admit **gravity waves** and have **scale interactions** (fast/slow modes).
- admit discontinuous solutions, akin to the **propagation of atmospheric fronts**.

The **modified** SWEs:

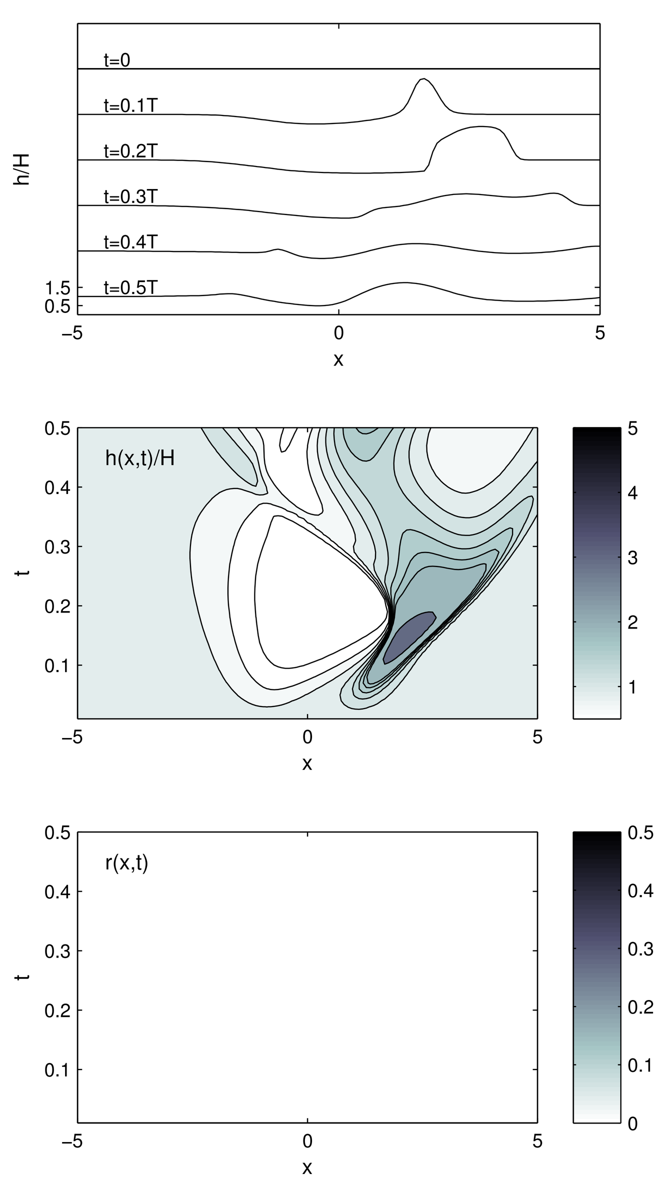
- artificially mimic **conditional instability** (positive buoyancy) and include idealised transport of **moisture**.
- contain **switches** for the onset of convection and precipitation - realistic features of operational NWP models.

5. Dynamics: classical and modified

Rossby geostrophic adjustment in a periodic domain

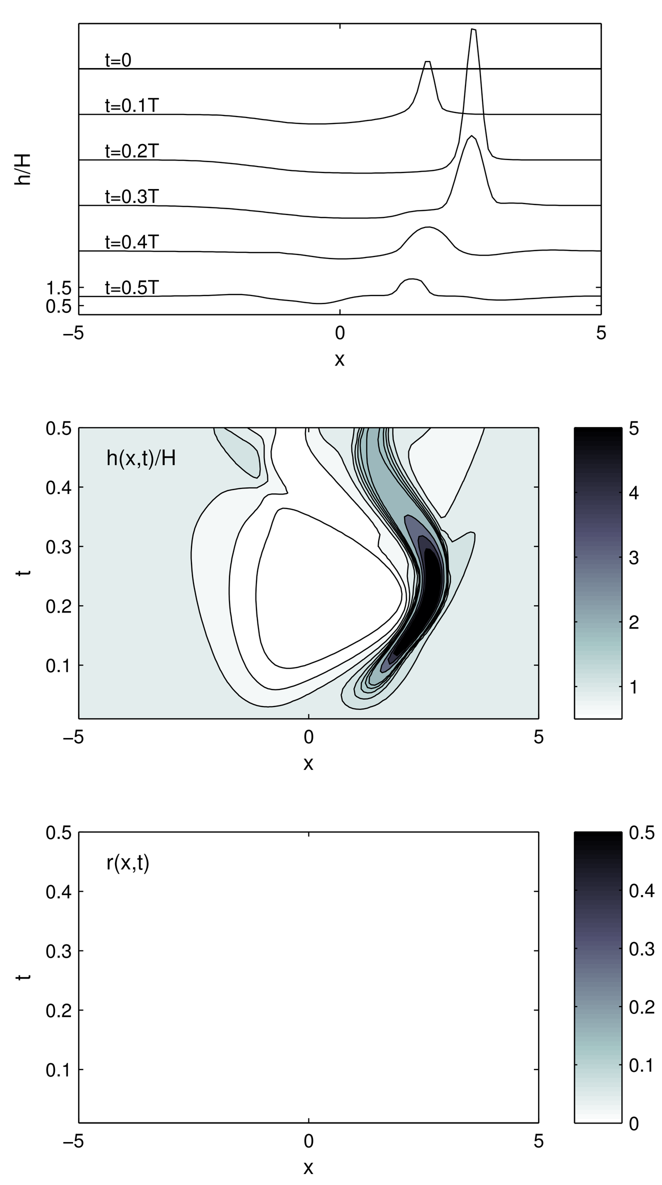
- describes the evolution of the free surface height h when disturbed from its rest state by a transverse jet.
- to adjust to this initial momentum imbalance, the height field evolves rapidly, emitting inertia gravity waves which propagate from the jet and eventually occupy the whole domain.
- periodic boundary conditions mean that waves travelling out from the jet core interact at a later time (not shown here).
- solved numerically using a shock-capturing finite volume/element method [3] which deals robustly with the high nonlinearity of the switches and dynamics.

Below H_c and H_r:



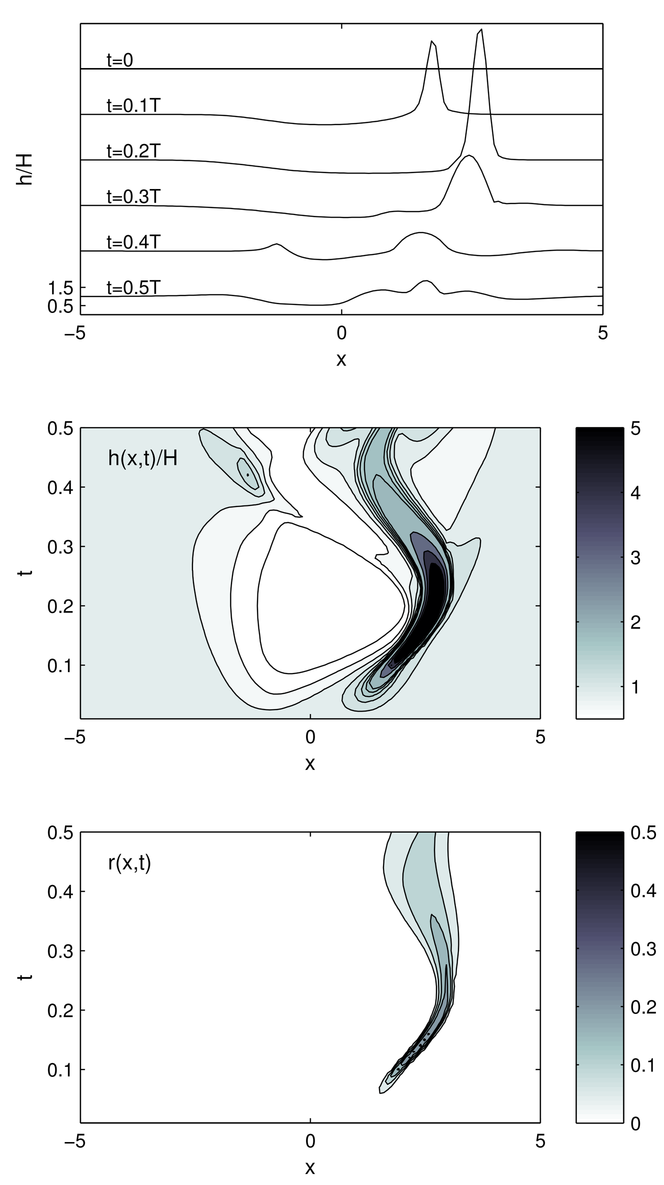
The model reduces to the classical rotating shallow water model when the fluid does not exceed the threshold heights H_c and H_r.

Above H_c but below H_r:



Exceedence of H_c triggers positive buoyancy leading to a convective updraft, but no 'rain' is produced as H_r is not exceeded.

Above H_c and H_r:



Given H_r exceedence and convergence ($\partial_x u < 0$), 'rain' is produced and then slowly precipitates, providing a downdraft to suppress convection.

6. Current and future steps

1. Set up meaningful experiments for the EnKF, relevant for convective-scale NWP:
   - choose initial ensemble to have sufficiently **fast perturbation growth**, providing a **representation of forecast error**.
   - "tune" the observing system by varying observation density, frequency, and noise.
2. Implement a **variational** DA algorithm and compare methods.

References

[1] Kalnay, E., 2003: *Atmospheric Modeling, Data Assimilation, and Predictability*. Cambridge University Press.
[2] Würsch, M., and Craig, G.C., 2014: A simple dynamical model of cumulus convection for data assimilation research. *Meteorologische Zeitschrift*. 23(5), 483-490.
[3] Rhebergen, S., Bokhove, O., and Van der Vegt, J., 2008: Discontinuous Galerkin finite element methods for hyperbolic nonconservative partial differential equations. *J. Comp. Phys.*, 227(3), 1887-1922.

Acknowledgements

This research is supported by the EPSRC and the Met Office through a CASE award.