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## 2.4 PRIORITY QUEUES

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- ▶ *API and elementary implementations*
- ▶ *binary heaps*
- ▶ *heapsort*
- ▶ *event-driven simulation*

# Algorithms

ROBERT SEDGEWICK | KEVIN WAYNE

<http://algs4.cs.princeton.edu>

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# Priority queue

---

Collections. Insert and delete items. Which item to delete?

Stack. Remove the item most recently added.

Queue. Remove the item least recently added.

Randomized queue. Remove a random item.

Priority queue. Remove the **largest** (or **smallest**) item.

| <i>operation</i>  | <i>argument</i> | <i>return value</i> |
|-------------------|-----------------|---------------------|
| <i>insert</i>     | P               |                     |
| <i>insert</i>     | Q               |                     |
| <i>insert</i>     | E               |                     |
| <i>remove max</i> |                 | Q                   |
| <i>insert</i>     | X               |                     |
| <i>insert</i>     | A               |                     |
| <i>insert</i>     | M               |                     |
| <i>remove max</i> |                 | X                   |
| <i>insert</i>     | P               |                     |
| <i>insert</i>     | L               |                     |
| <i>insert</i>     | E               |                     |
| <i>remove max</i> |                 | P                   |

# Priority queue API

(Stack / Queue & Priority Queue)

Requirement. Generic items are Comparable.

|   |  |
|---|--|
| public class MaxPQ<Key extends Comparable<Key>> | Key must be Comparable<br>(bounded type parameter) |
| MaxPQ()   | (Amongst)<br>create an empty priority queue        |
| MaxPQ(Key[] a)                                  | create a priority queue with given keys            |
| void insert(Key v)                              | insert a key into the priority queue               |
| Key delMax()                                    | return and remove the largest key                  |
| boolean isEmpty()                               | is the priority queue empty?                       |
| Key max()                                       | return the largest key                             |
| int size()                                      | number of entries in the priority queue            |

# Priority queue applications

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- Event-driven simulation. [customers in a line, colliding particles]
- Numerical computation. [reducing roundoff error]
- Data compression. [Huffman codes]
- Graph searching. [Dijkstra's algorithm, Prim's algorithm]
- Number theory. [sum of powers]
- Artificial intelligence. [A\* search]
- Statistics. [maintain largest M values in a sequence]
- Operating systems. [load balancing, interrupt handling]
- Discrete optimization. [bin packing, scheduling]
- Spam filtering. [Bayesian spam filter]

Generalizes: stack, queue, randomized queue.

# Priority queue client example

**Challenge.** Find the largest  $M$  items in a stream of  $N$  items.

- Fraud detection: isolate \$\$ transactions.
- File maintenance: find biggest files or directories.

$N$  huge,  $M$  large

**Constraint.** Not enough memory to store  $N$  items.

```
% more tinyBatch.txt
Turing      6/17/1990   644.08
vonNeumann  3/26/2002   4121.85
Dijkstra    8/22/2007   2678.40
vonNeumann  1/11/1999   4409.74
Dijkstra    11/18/1995   837.42
Hoare       5/10/1993   3229.27
vonNeumann  2/12/1994   4732.35
Hoare       8/18/1992   4381.21
Turing      1/11/2002   66.10
Thompson    2/27/2000   4747.08
Turing      2/11/1991   2156.86
Hoare       8/12/2003   1025.70
vonNeumann  10/13/1993  2520.97
Dijkstra    9/10/2000   708.95
Turing      10/12/1993  3532.36
Hoare       2/10/2005   4050.20
```

```
% java TopM 5 < tinyBatch.txt
Thompson    2/27/2000   4747.08
vonNeumann  2/12/1994   4732.35
vonNeumann  1/11/1999   4409.74
Hoare       8/18/1992   4381.21
vonNeumann  3/26/2002   4121.85
```

sort key

# Priority queue client example

**Challenge.** Find the largest  $M$  items in a stream of  $N$  items.

- Fraud detection: isolate \$\$ transactions.
- File maintenance: find biggest files or directories.

$N$  huge,  $M$  large

**Constraint.** Not enough memory to store  $N$  items.

```
use a min-oriented pq      MinPQ<Transaction> pq = new MinPQ<Transaction>();  
                          ↑  
while (StdIn.hasNextLine())  
{  
    String line = StdIn.readLine();  
    Transaction item = new Transaction(line);  
    pq.insert(item);  
    if (pq.size() > M) ← pq contains  
        pq.delMin();      largest M items  
    }  
                          ↑  
Transaction data  
type is Comparable  
(ordered by $$)
```

# Priority queue client example

---

**Challenge.** Find the largest  $M$  items in a stream of  $N$  items.

order of growth of finding the largest  $M$  in a stream of  $N$  items

| implementation | time       | space | $(N \gg M)$ |
|----------------|------------|-------|-------------|
| sort           | $N \log N$ | $N$   |             |
| elementary PQ  | $M N$      | $M$   |             |
| binary heap    | $N \log M$ | $M$   |             |
| best in theory | $N$        | $M$   |             |

# Priority queue: unordered and ordered array implementation

| <i>operation</i>                             | <i>argument</i> | <i>return value</i> | <i>size</i> | <i>contents<br/>(unordered)</i> | <i>del at</i> | <i>Max</i> | <i>return</i> | <i>contents<br/>(ordered)</i> | <i>insert at</i> | <i>ordering</i> |
|--|-----------------|---------------------|-------------|---------------------------------|---------------|------------|---------------|-------------------------------|------------------|-----------------|
| <i>insert</i>                                | P               | 1                   |             | P                               |               |            |               | P                             |                  |                 |
| <i>insert</i>                                | Q               | 2                   |             | P Q                             |               |            |               | P Q                           |                  |                 |
| <i>insert</i>                                | E               | 3                   |             | P Q E                           |               |            |               | E P Q                         |                  |                 |
| <i>remove max</i>                            |                 | Q                   | 2           | P E                             |               |            |               | E P                           |                  |                 |
| <i>insert</i>                                | X               | 3                   |             | P E X                           |               |            |               | E P X                         |                  |                 |
| <i>insert</i>                                | A               | 4                   |             | P E X A                         |               |            |               | A E P X                       |                  |                 |
| <i>insert</i>                                | M               | 5                   |             | P E X A M                       |               |            |               | A E M P X                     |                  |                 |
| <i>remove max</i>                            |                 | X                   | 4           | P E M A                         |               |            |               | A E M P                       |                  |                 |
| <i>insert</i>                                | P               | 5                   |             | P E M A P                       |               |            |               | A E M P P                     |                  |                 |
| <i>insert</i>                                | L               | 6                   |             | P E M A P L                     |               |            |               | A E L M P P                   |                  |                 |
| <i>insert</i>                                | E               | 7                   |             | P E M A P L E                   |               |            |               | A E E L M P P                 |                  |                 |
| <i>remove max</i>                            |                 | P                   | 6           | E M A P L E                     |               |            |               | A E E L M P                   |                  |                 |
| A sequence of operations on a priority queue |                 |                     |             |                                 |               |            |               |                               |                  |                 |

# Priority queue: unordered array implementation

```
public class UnorderedMaxPQ<Key extends Comparable<Key>>
{
    private Key[] pq;      // pq[i] = ith element on pq
    private int N;         // number of elements on pq

    public UnorderedMaxPQ(int capacity)
    {   pq = (Key[]) new Comparable[capacity]; }

    public boolean isEmpty()
    {   return N == 0; }

    public void insert(Key x)
    {   pq[N++] = x; }

    public Key delMax()
    {
        int max = 0;
        for (int i = 1; i < N; i++)
            if (less(max, i)) max = i;
        exch(max, N-1);
        return pq[--N];
    }
}
```

no generic array creation

less() and exch()  
similar to sorting methods

null out entry  
to prevent loitering

# Priority queue elementary implementations

---

Challenge. Implement **all** operations efficiently.

**order of growth of running time for priority queue with N items**

| implementation  | insert   | del max  | max      |
|-----------------|----------|----------|----------|
| unordered array | 1        | N        | N        |
| ordered array   | N        | 1        | 1        |
| goal            | $\log N$ | $\log N$ | $\log N$ |

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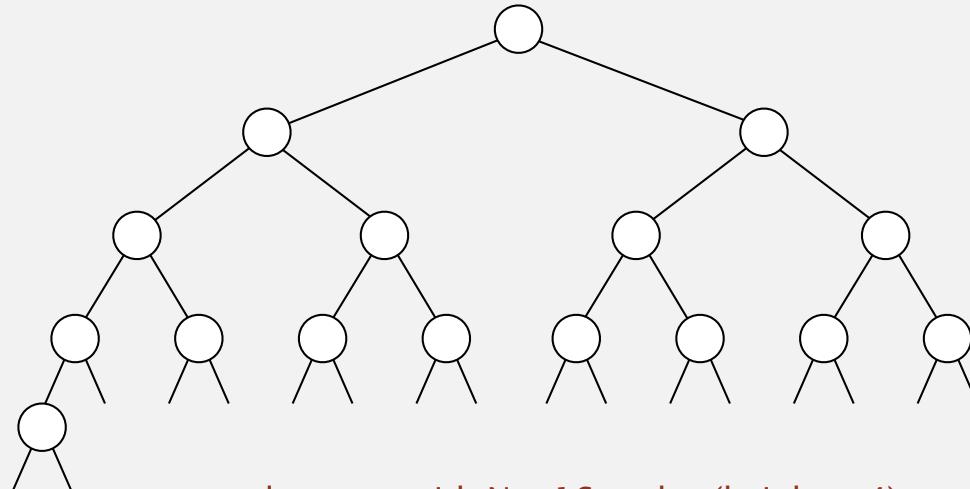
- ▶ *API and elementary implementations*
- ▶ ***binary heaps***
- ▶ *heapsort*
- ▶ *event-driven simulation*

# Complete binary tree

Binary tree. Empty or node with links to left and right binary trees.

Complete tree. Perfectly balanced, except for bottom level.

(bottom 레벨 까지는 같은 수의 노드)



complete tree with  $N = 16$  nodes (height = 4)

Property. Height of complete tree with  $N$  nodes is  $\lfloor \lg N \rfloor$ .

Pf. Height only increases when  $N$  is a power of 2.

(height은  $N = 2^n$  일 때만 증가)

# A complete binary tree in nature

---



# Binary heap representations

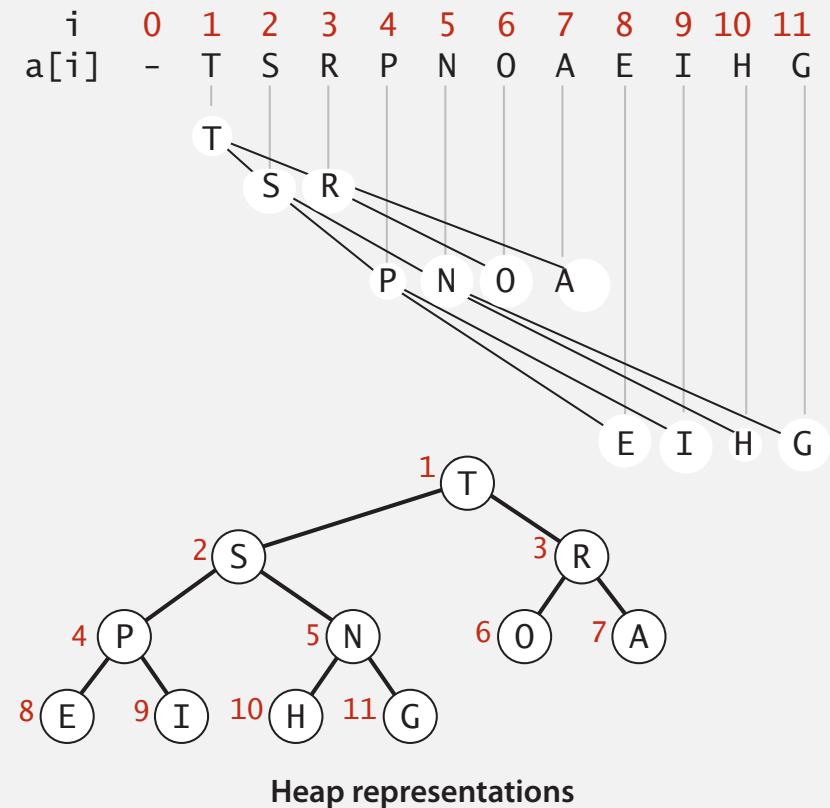
Binary heap. Array representation of a heap-ordered complete binary tree.

Heap-ordered binary tree.

- Keys in nodes.
- Parent's key no smaller than children's keys.

Array representation.

- Indices start at 1.
- Take nodes in **level** order.
- No explicit links needed!



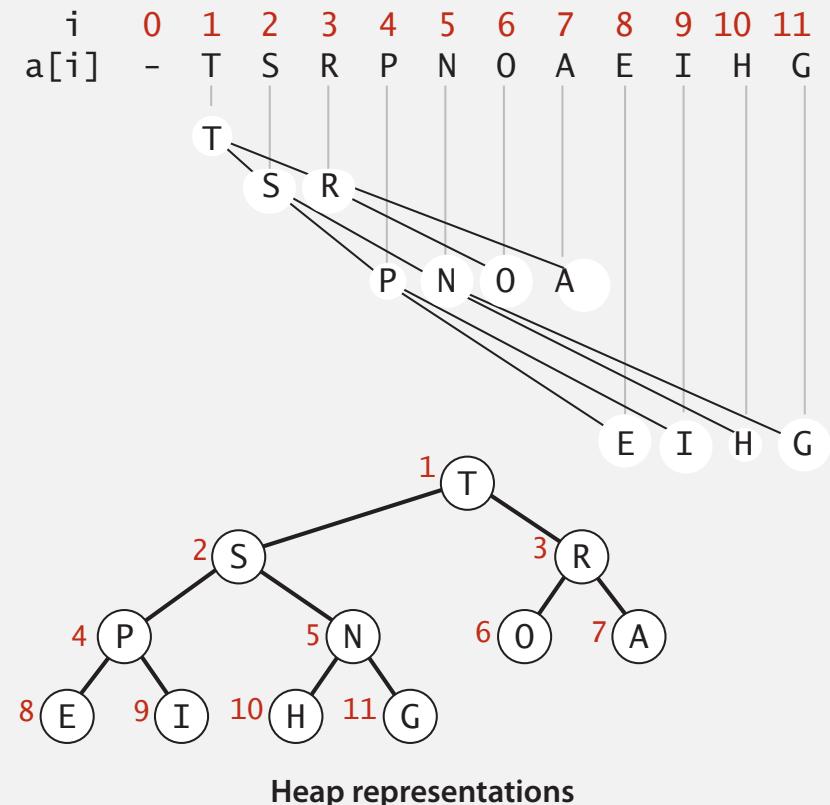
# Binary heap properties

---

Proposition. Largest key is  $a[1]$ , which is root of binary tree.

Proposition. Can use array indices to move through tree.

- Parent of node at  $k$  is at  $k/2$ .
- Children of node at  $k$  are at  $2k$  and  $2k+1$ .



# Promotion in a heap

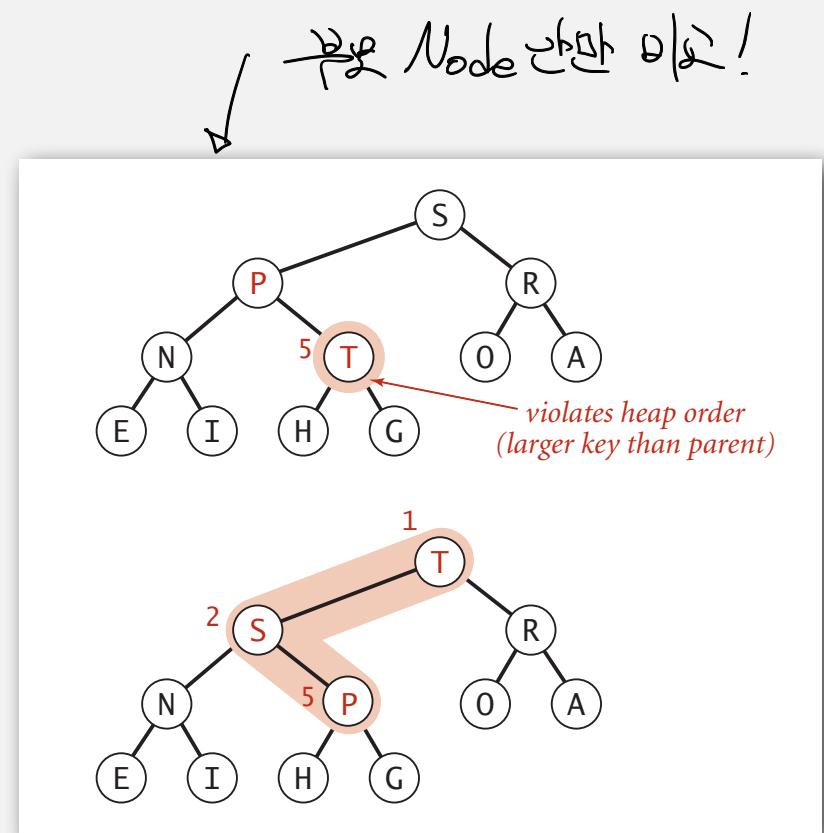
Scenario. Child's key becomes **larger** key than its parent's key.

To eliminate the violation:

- Exchange key in child with key in parent.
- Repeat until heap order restored.

```
private void swim(int k)
{
    while (k > 1 && less(k/2, k))
    {
        exch(k, k/2);
        k = k/2;
    }
}
```

parent of node at k is at k/2



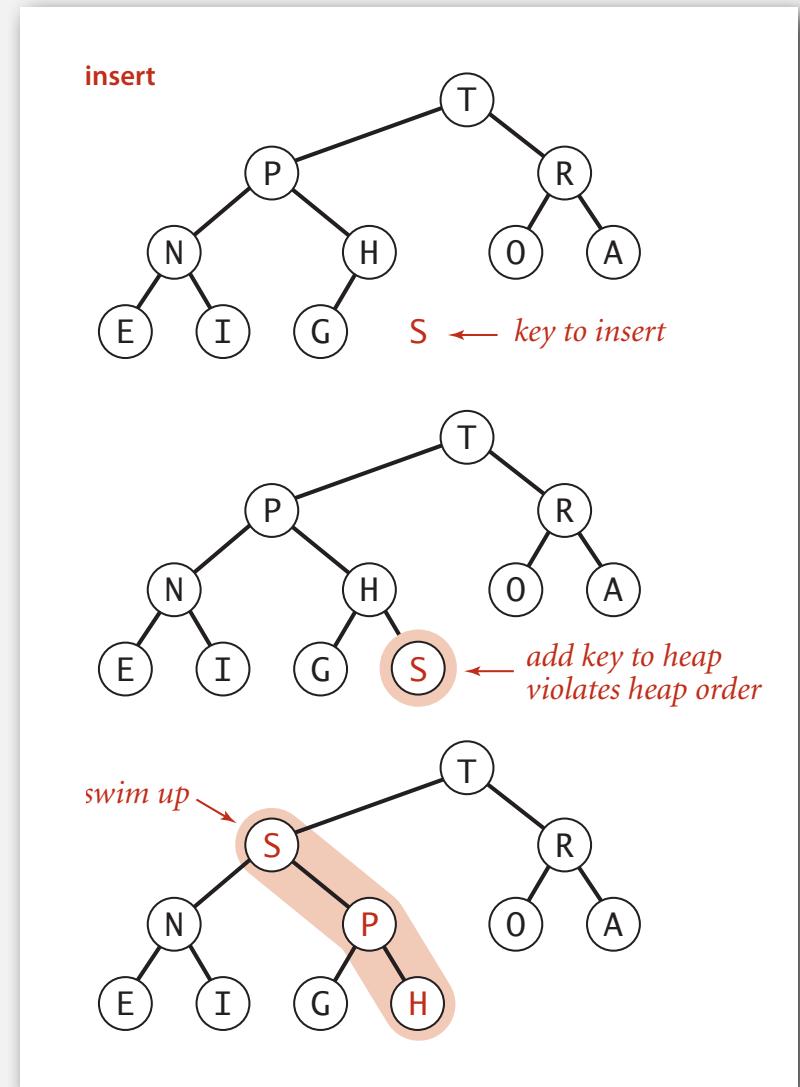
Peter principle. Node promoted to level of incompetence.

# Insertion in a heap

Insert. Add node at end, then swim it up.

Cost. At most  $1 + \lg N$  compares.

```
public void insert(Key x)
{
    pq[++N] = x; tree의 끝에 추가
    swim(N);
}
```



# Demotion in a heap

Scenario. Parent's key becomes **smaller** than one (or both) of its children's.

To eliminate the violation:

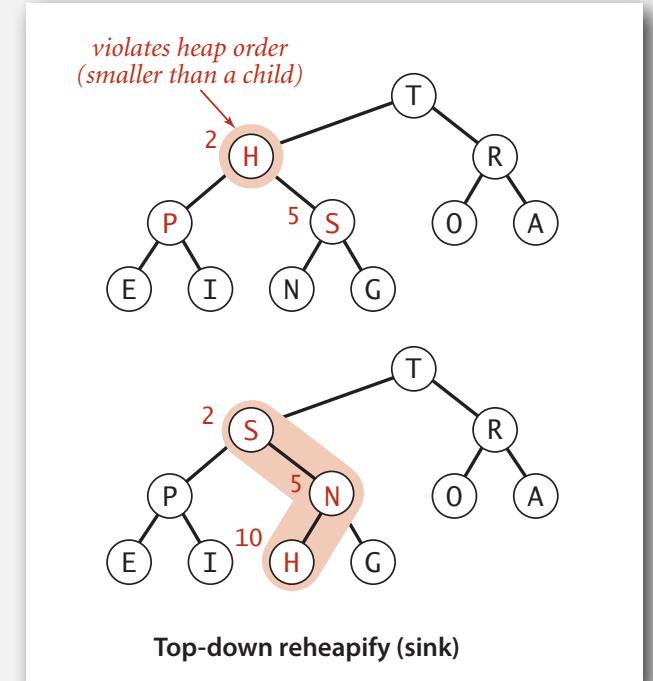
- Exchange key in parent with key in larger child.
- Repeat until heap order restored.

why not smaller child?

```
private void sink(int k)
{
    while (2*k <= N)
    {
        int j = 2*k;
        if (j < N && less(j, j+1)) j++;
        if (!less(k, j)) break;
        exch(k, j);
        k = j;
    }
}
```

children of node at k  
are  $2k$  and  $2k+1$

children



Top-down reheapify (sink)

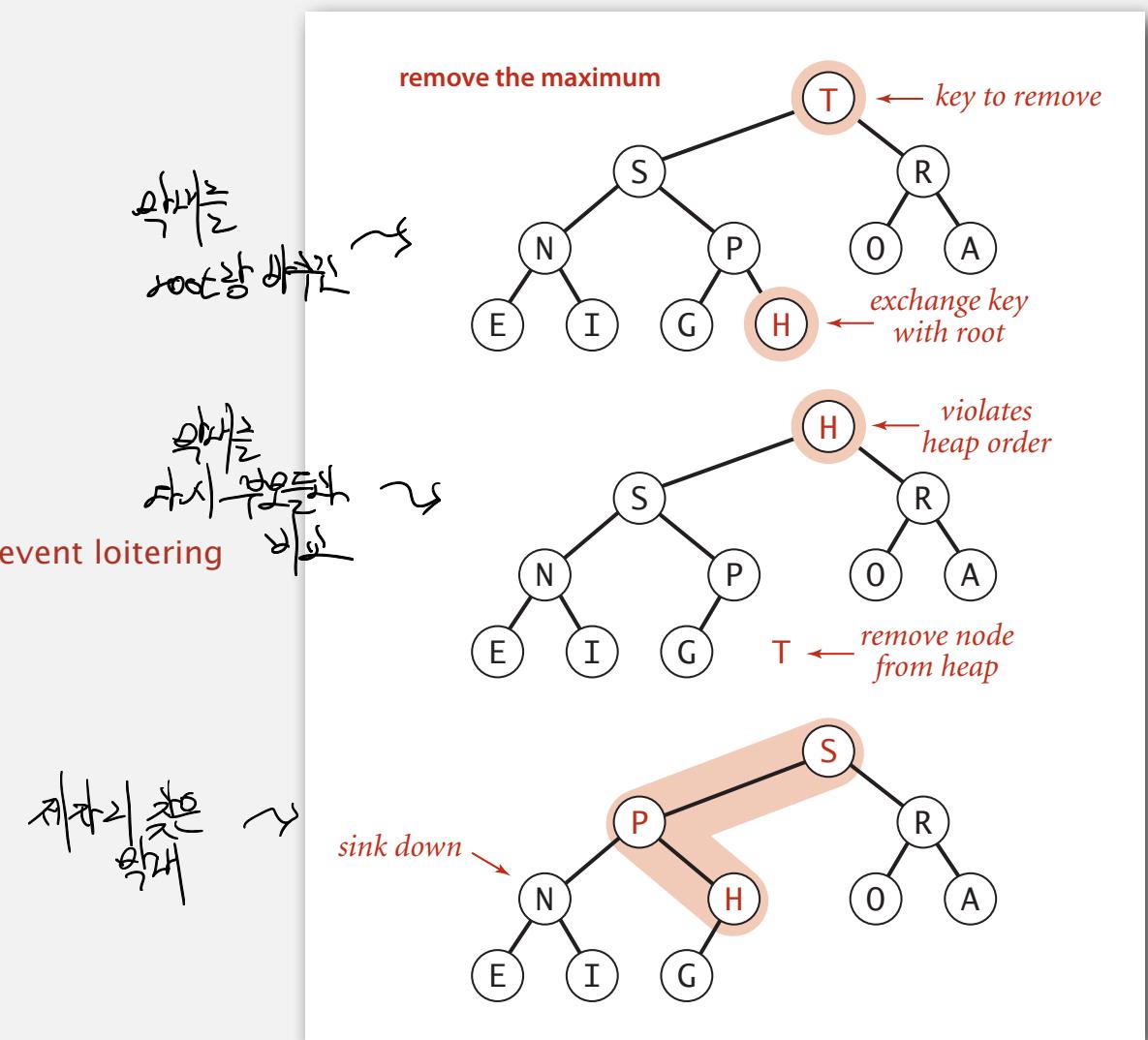
Power struggle. Better subordinate promoted.

# Delete the maximum in a heap

Delete max. Exchange root with node at end, then sink it down.

Cost. At most  $2 \lg N$  compares.

```
public Key delMax()
{
    Key max = pq[1];
    exch(1, N--);
    sink(1);
    pq[N+1] = null; ← prevent loitering
    return max;
}
```



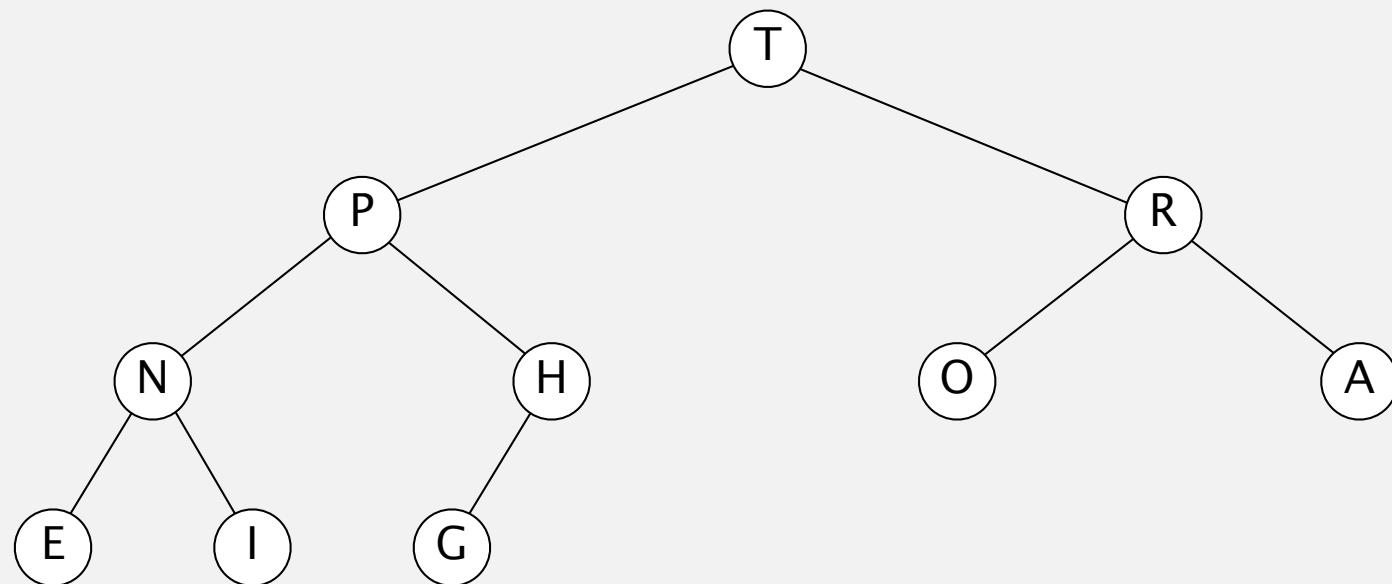
# Binary heap demo

---

**Insert.** Add node at end, then swim it up.

**Remove the maximum.** Exchange root with node at end, then sink it down.

heap ordered



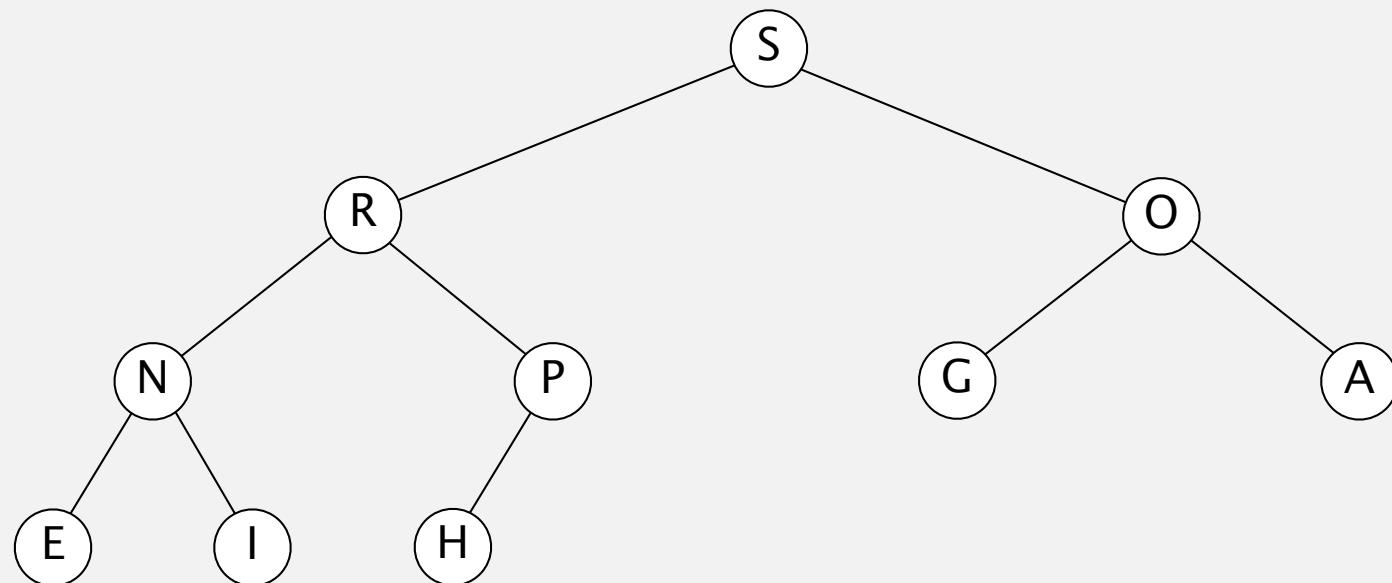
# Binary heap demo

---

**Insert.** Add node at end, then swim it up.

**Remove the maximum.** Exchange root with node at end, then sink it down.

heap ordered



# Binary heap: Java implementation

```
public class MaxPQ<Key extends Comparable<Key>>
{
    private Key[] pq;
    private int N;

    public MaxPQ(int capacity)
    {   pq = (Key[]) new Comparable[capacity+1]; }

    public boolean isEmpty()
    {   return N == 0;   }
    public void insert(Key key)
    public Key delMax()
    {   /* see previous code */ }

    private void swim(int k)
    private void sink(int k)
    {   /* see previous code */ }

    private boolean less(int i, int j)
    {   return pq[i].compareTo(pq[j]) < 0;   }
    private void exch(int i, int j)
    {   Key t = pq[i]; pq[i] = pq[j]; pq[j] = t;   }
}
```

fixed capacity  
(for simplicity)

PQ ops

heap helper functions

array helper functions

# Priority queues implementation cost summary

order-of-growth of running time for priority queue with N items

| implementation  | insert     | del max      | max |
|-----------------|------------|--------------|-----|
| unordered array | 1          | N            | N   |
| ordered array   | N          | 1            | 1   |
| binary heap     | $\log N$   | $\log N$     | 1   |
| d-ary heap      | $\log_d N$ | $d \log_d N$ | 1   |
| Fibonacci       | 1          | $\log N$ †   | 1   |
| impossible      | 1          | 1            | 1   |

† amortized

← why impossible?  
Data  $\xrightarrow{\text{insert / delete}}$  Constant  $\xrightarrow{\text{get}}$

# Binary heap considerations

---

## Immutability of keys.

- Assumption: client does not change keys while they're on the PQ.
- Best practice: **use immutable keys.**

## Underflow and overflow.

- Underflow: throw exception if deleting from empty PQ.
- Overflow: add no-arg constructor and use resizing array.

leads to log N  
amortized time per op  
(how to make worst case?)

## Minimum-oriented priority queue.

- Replace `less()` with `greater()`.
- Implement `greater()`.

## Other operations.

- Remove an arbitrary item.
- Change the priority of an item.

can implement with `sink()` and `swim()` [stay tuned]

# Immutability: implementing in Java

**Data type.** Set of values and operations on those values.

**Immutable data type.** Can't change the data type value once created.

```
public final class Vector {  
    private final int N;  
    private final double[] data; | ← can't override instance methods  
  
    public Vector(double[] data) {  
        this.N = data.length;  
        this.data = new double[N];  
        for (int i = 0; i < N; i++) ← all instance variables private and final  
            this.data[i] = data[i];  
    }  
    ...  
}
```

defensive copy of mutable instance variables

instance methods don't change instance variables

**Immutable.** String, Integer, Double, Color, Vector, Transaction, Point2D.

**Mutable.** StringBuilder, Stack, Counter, Java array.

# Immutability: properties

**Data type.** Set of values and operations on those values.

**Immutable data type.** Can't change the data type value once created.

**Advantages.**

- Simplifies debugging.
- Safer in presence of hostile code.
- Simplifies concurrent programming.
- Safe to use as key in priority queue or symbol table.

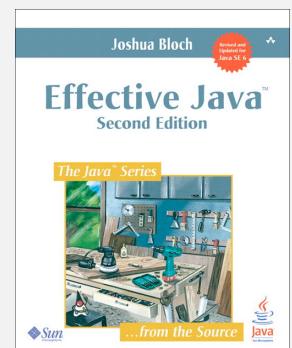


**Disadvantage.** Must create new object for each data type value.

( সুজি কোট্টা )

*“Classes should be immutable unless there's a very good reason to make them mutable.... If a class cannot be made immutable, you should still limit its mutability as much as possible.”*

— Joshua Bloch (Java architect)



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- ▶ *heapsort*
- ▶ *event-driven simulation*

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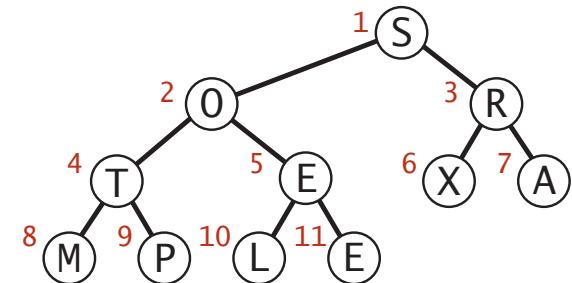
- ▶ *API and elementary implementations*
- ▶ *binary heaps*
- ▶ *heapsort*
- ▶ *event-driven simulation*

# Heapsort

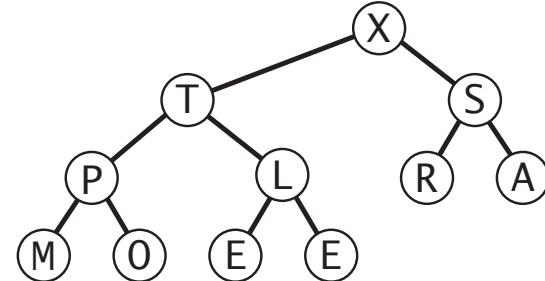
Basic plan for in-place sort.

- Create max-heap with all  $N$  keys.
- Repeatedly remove the maximum key.

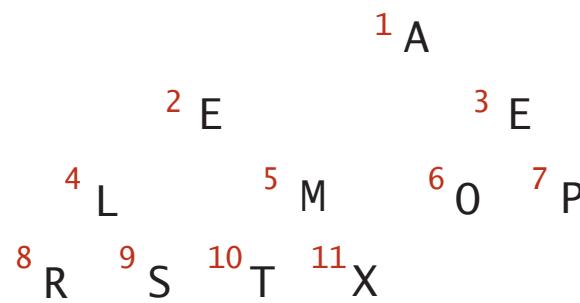
start with array of keys  
in arbitrary order



build a max-heap  
(in place)



sorted result  
(in place)



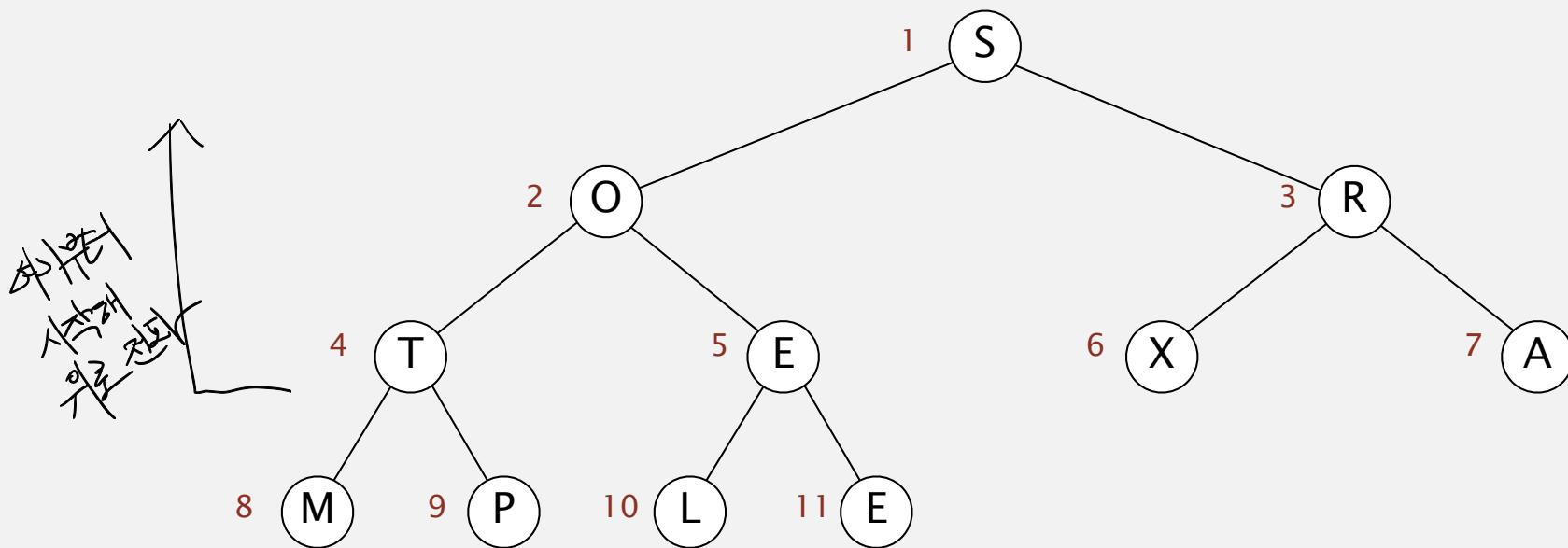
# Heapsort demo

Heap construction. Build max heap using **bottom-up** method.



we assume array entries are indexed 1 to N

array in arbitrary order



tree of "Heap Order" ↗ (최선 root 을 del, (tree가 끊어지거나))

|   |   |   |   |   |   |   |   |   |    |    |
|---|---|---|---|---|---|---|---|---|----|----|
| S | O | R | T | E | X | A | M | P | L  | E  |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |



# Heapsort demo

---

**Sortdown.** Repeatedly delete the largest remaining item.

array in sorted order

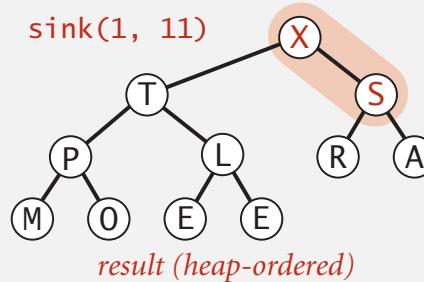
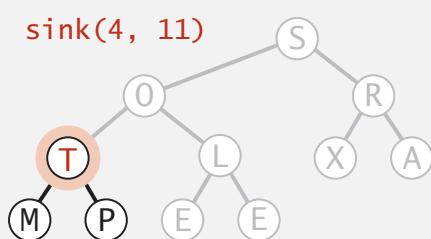
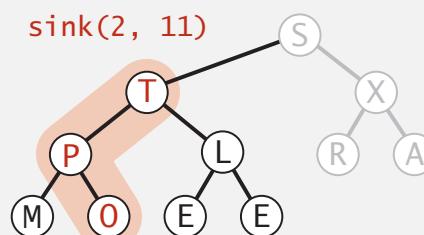
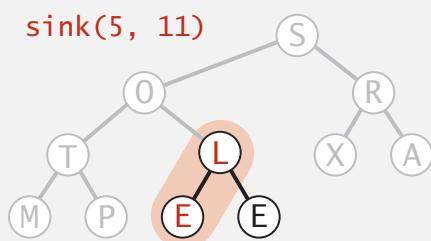
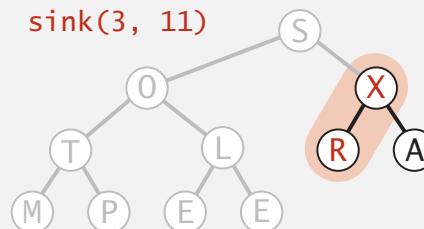
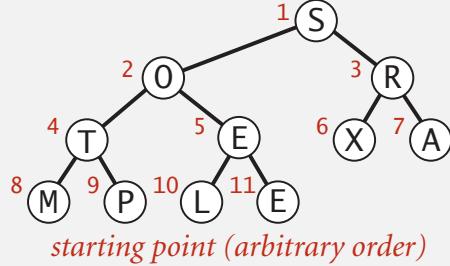


# Heapsort: heap construction

First pass. Build heap using bottom-up method.

→ 마지막으로 시작.

```
for (int k = N/2; k >= 1; k--)  
    sink(a, k, N);
```

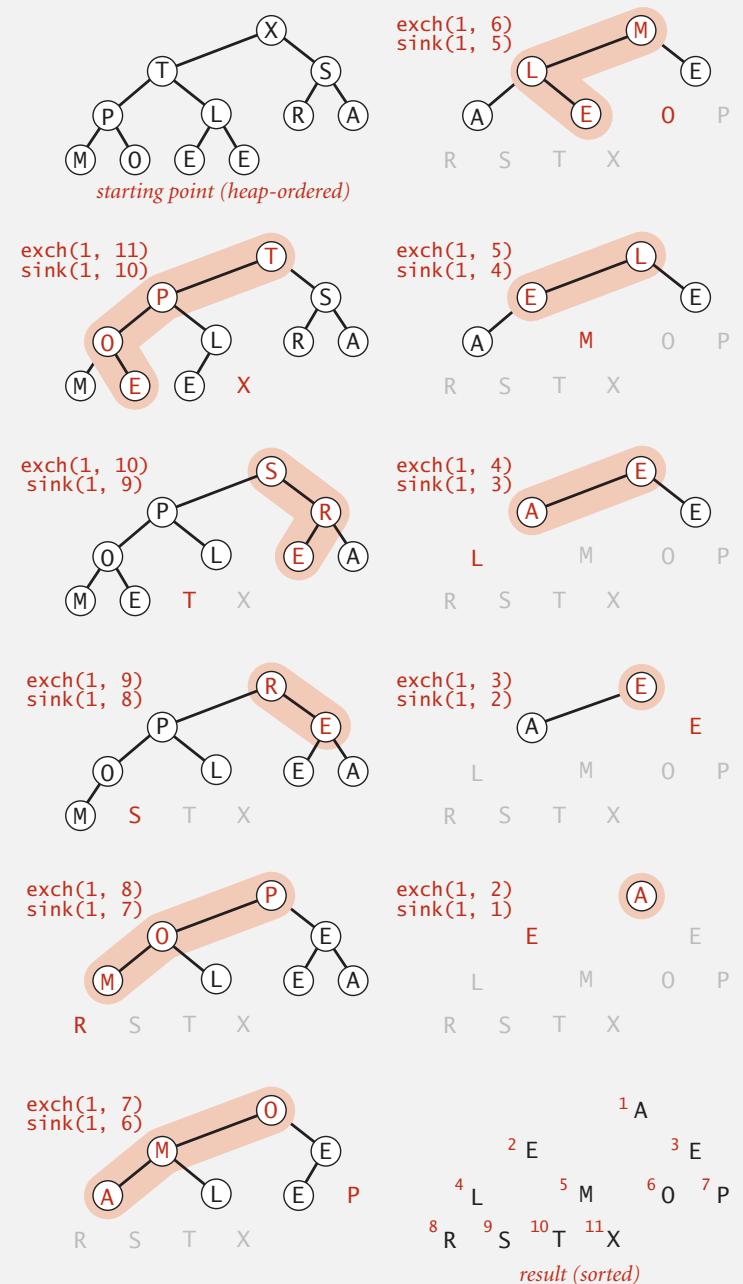


# Heapsort: sortdown

Second pass.

- Remove the maximum, one at a time.
- Leave in array, instead of nulling out.

```
while (N > 1)
{
    exch(a, 1, N--);
    sink(a, 1, N);
}
```



# Heapsort: Java implementation

```
public class Heap
{
    public static void sort(Comparable[] a)
    {
        int N = a.length;
        for (int k = N/2; k >= 1; k--)
            sink(a, k, N);
        while (N > 1)
        {
            exch(a, 1, N);
            sink(a, 1, --N);
        }
    }

    private static void sink(Comparable[] a, int k, int N)
    { /* as before */ }

    private static boolean less(Comparable[] a, int i, int j)
    { /* as before */ }

    private static void exch(Comparable[] a, int i, int j)
    { /* as before */ }
}
```

but convert from  
1-based indexing to  
0-base indexing

# Heapsort: trace

---

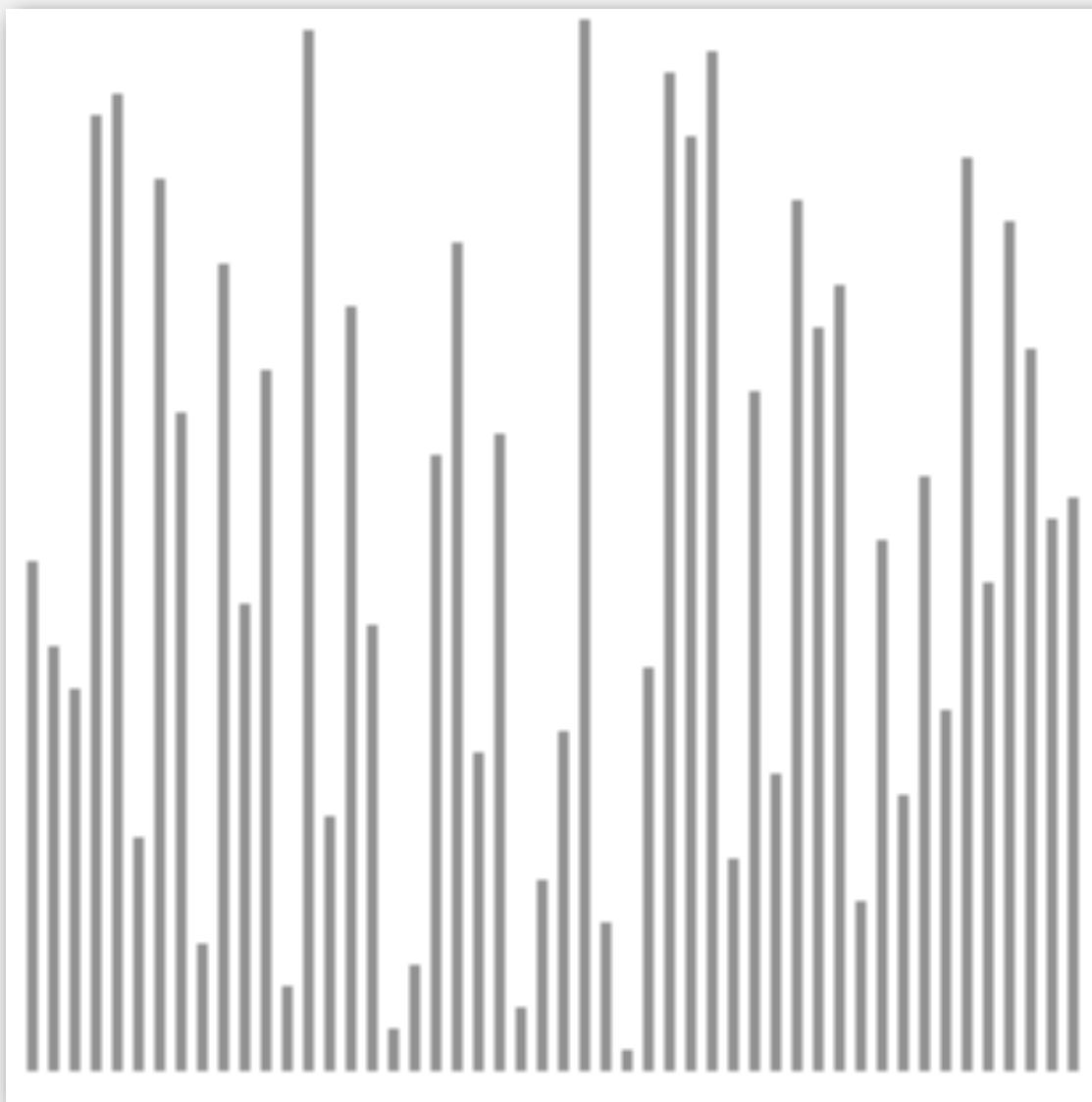
| N                     | k | a[i] |   |   |   |   |   |   |   |   |   |    |    |
|-----------------------|---|------|---|---|---|---|---|---|---|---|---|----|----|
|                       |   | 0    | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| <i>initial values</i> |   | S    | O | R | T | E | X | A | M | P | L | E  |    |
| 11                    | 5 | S    | O | R | T | L | X | A | M | P | E | E  |    |
| 11                    | 4 | S    | O | R | T | L | X | A | M | P | E | E  |    |
| 11                    | 3 | S    | O | X | T | L | R | A | M | P | E | E  |    |
| 11                    | 2 | S    | T | X | P | L | R | A | M | O | E | E  |    |
| 11                    | 1 | X    | T | S | P | L | R | A | M | O | E | E  |    |
| <i>heap-ordered</i>   |   | X    | T | S | P | L | R | A | M | O | E | E  |    |
| 10                    | 1 | T    | P | S | O | L | R | A | M | E | E | X  |    |
| 9                     | 1 | S    | P | R | O | L | E | A | M | E | T | X  |    |
| 8                     | 1 | R    | P | E | O | L | E | A | M | S | T | X  |    |
| 7                     | 1 | P    | O | E | M | L | E | A | R | S | T | X  |    |
| 6                     | 1 | O    | M | E | A | L | E | P | R | S | T | X  |    |
| 5                     | 1 | M    | L | E | A | E | O | P | R | S | T | X  |    |
| 4                     | 1 | L    | E | E | A | M | O | P | R | S | T | X  |    |
| 3                     | 1 | E    | A | E | L | M | O | P | R | S | T | X  |    |
| 2                     | 1 | E    | A | E | L | M | O | P | R | S | T | X  |    |
| 1                     | 1 | A    | E | E | L | M | O | P | R | S | T | X  |    |
| <i>sorted result</i>  |   | A    | E | E | L | M | O | P | R | S | T | X  |    |

Heapsort trace (array contents just after each sink)

# Heapsort animation

---

50 random items



- ▲ algorithm position
- in order
- not in order

<http://www.sorting-algorithms.com/heap-sort>

## Heapsort: mathematical analysis

---

Proposition. Heap construction uses  $\leq 2N$  compares and exchanges.

Proposition. Heapsort uses  $\leq 2N \lg N$  compares and exchanges.

Significance. In-place sorting algorithm with  $N \log N$  worst-case.

- Mergesort: no, linear extra space. ← in-place merge possible, not practical
- Quicksort: no, quadratic time in worst case. ←  $N \log N$  worst-case quicksort possible, not practical
- Heapsort: yes!

Bottom line. Heapsort is optimal for both time and space, **but**:

- Inner loop longer than quicksort's.
- Makes poor use of cache memory.
- Not stable.

# Sorting algorithms: summary

---

|             | inplace? | stable? | worst      | average    | best      | remarks   |
|-------------|----------|---------|------------|------------|-----------|---|
| selection   | x        |         | $N^2 / 2$  | $N^2 / 2$  | $N^2 / 2$ | $N$ exchanges   |
| insertion   | x        | x       | $N^2 / 2$  | $N^2 / 4$  | N         | use for small N or partially ordered                      |
| shell       | x        |         | ?          | ?          | N         | tight code, subquadratic                                  |
| quick       | x        |         | $N^2 / 2$  | $2N \ln N$ | $N \lg N$ | $N \log N$ probabilistic guarantee<br>fastest in practice |
| 3-way quick | x        |         | $N^2 / 2$  | $2N \ln N$ | N         | improves quicksort in presence<br>of duplicate keys       |
| merge       |          | x       | $N \lg N$  | $N \lg N$  | $N \lg N$ | $N \log N$ guarantee, stable                              |
| heap        | x        |         | $2N \lg N$ | $2N \lg N$ | $N \lg N$ | $N \log N$ guarantee, in-place                            |
| ???         | x        | x       | $N \lg N$  | $N \lg N$  | $N \lg N$ | holy sorting grail  |

# Algorithms

ROBERT SEDGEWICK | KEVIN WAYNE

<http://algs4.cs.princeton.edu>

## 2.4 PRIORITY QUEUES

---

- ▶ *API and elementary implementations*
- ▶ *binary heaps*
- ▶ *heapsort*
- ▶ *event-driven simulation*

# Algorithms

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## 2.4 PRIORITY QUEUES

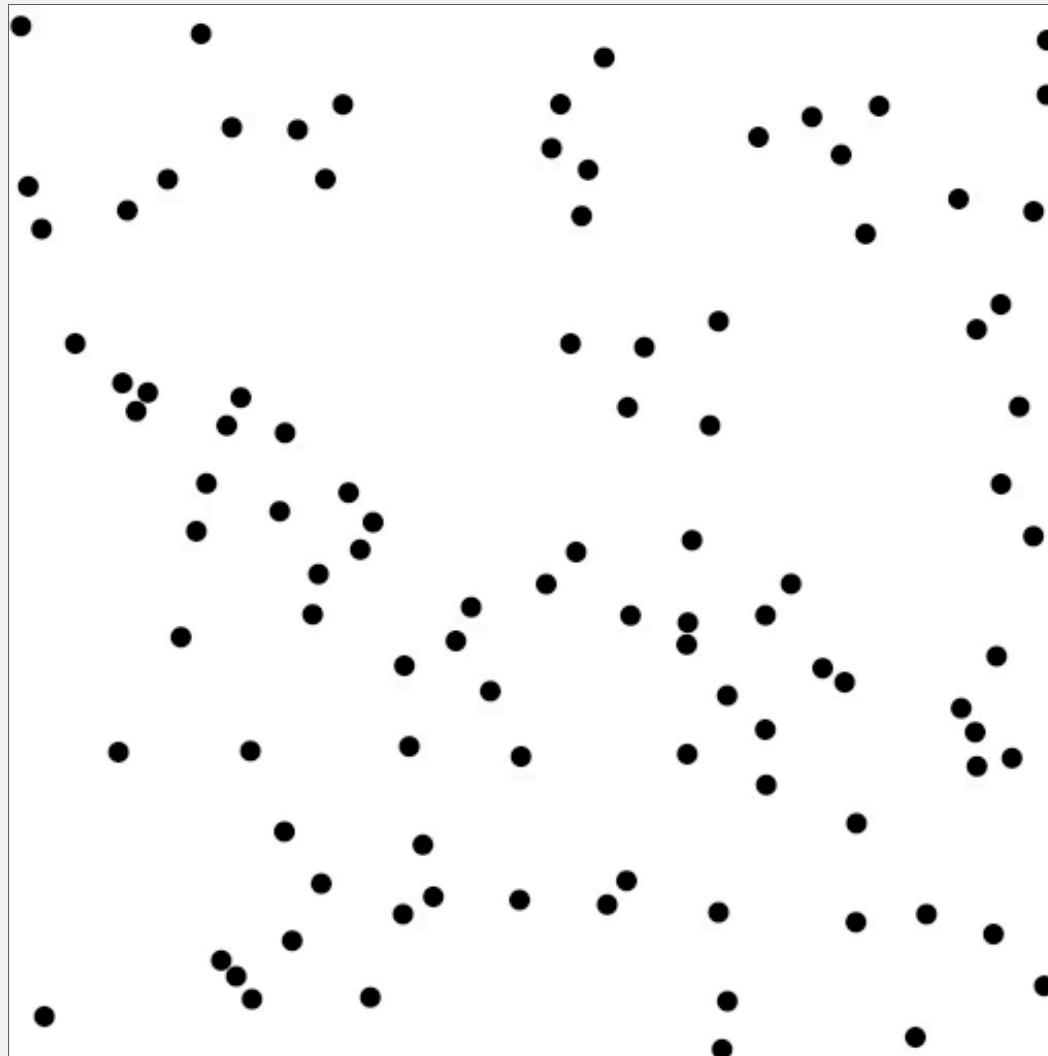
---

- ▶ *API and elementary implementations*
- ▶ *binary heaps*
- ▶ *heapsort*
- ▶ *event-driven simulation*

# Molecular dynamics simulation of hard discs

---

**Goal.** Simulate the motion of  $N$  moving particles that behave according to the laws of elastic collision.



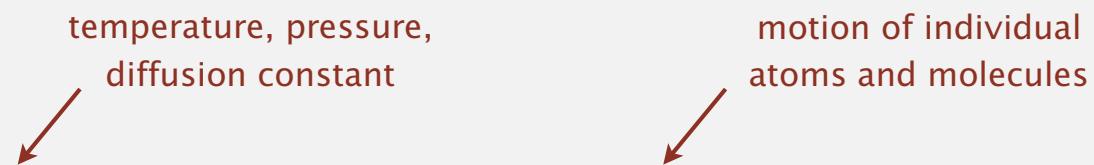
# Molecular dynamics simulation of hard discs

---

**Goal.** Simulate the motion of  $N$  moving particles that behave according to the laws of elastic collision.

## Hard disc model.

- Moving particles interact via elastic collisions with each other and walls.
- Each particle is a disc with known position, velocity, mass, and radius.
- No other forces.



**Significance.** Relates macroscopic observables to microscopic dynamics.

- Maxwell-Boltzmann: distribution of speeds as a function of temperature.
- Einstein: explain Brownian motion of pollen grains.

# Warmup: bouncing balls

Time-driven simulation.  $N$  bouncing balls in the unit square.

```
public class BouncingBalls
{
    public static void main(String[] args)
    {
        int N = Integer.parseInt(args[0]);
        Ball[] balls = new Ball[N];
        for (int i = 0; i < N; i++)
            balls[i] = new Ball();
        while(true)
        {
            StdDraw.clear();
            for (int i = 0; i < N; i++)
            {
                balls[i].move(0.5);
                balls[i].draw();
            }
            StdDraw.show(50);
        }
    }
}
```

```
% java BouncingBalls 100
```



main simulation loop

## Warmup: bouncing balls

```
public class Ball
{
    private double rx, ry;          // position
    private double vx, vy;          // velocity
    private final double radius;    // radius
    public Ball(...)
    { /* initialize position and velocity */ }

    public void move(double dt)
    {
        if ((rx + vx*dt < radius) || (rx + vx*dt > 1.0 - radius)) { vx = -vx; }
        if ((ry + vy*dt < radius) || (ry + vy*dt > 1.0 - radius)) { vy = -vy; }
        rx = rx + vx*dt;
        ry = ry + vy*dt;
    }

    public void draw()
    { StdDraw.filledCircle(rx, ry, radius); }
}
```

check for collision with walls



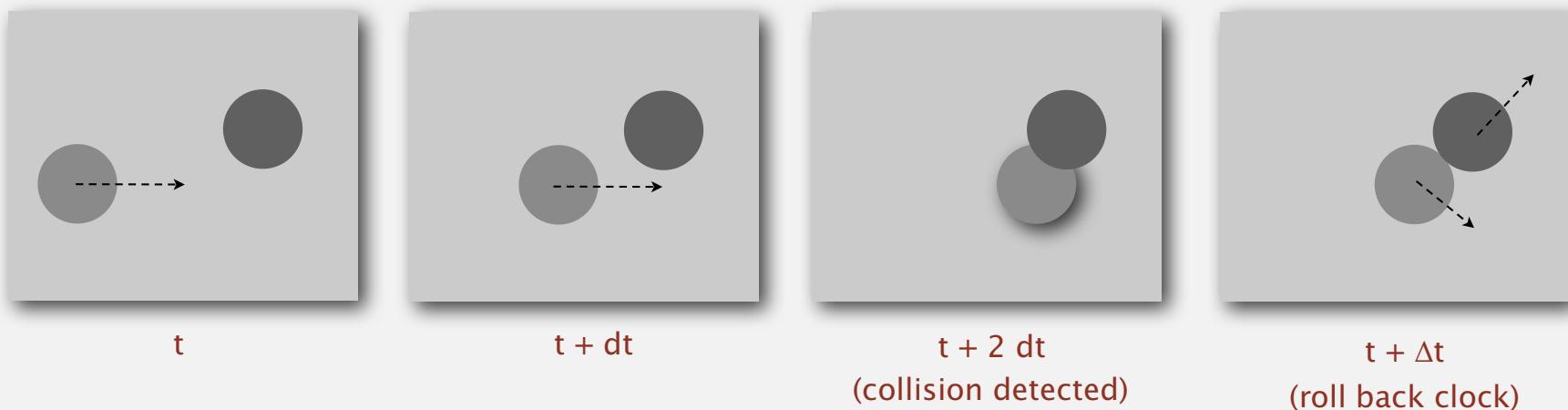
Missing. Check for balls colliding with each other.

- Physics problems: when? what effect?
- CS problems: which object does the check? too many checks?

# Time-driven simulation

---

- Discretize time in quanta of size  $dt$ .
- Update the position of each particle after every  $dt$  units of time, and check for overlaps.
- If overlap, roll back the clock to the time of the collision, update the velocities of the colliding particles, and continue the simulation.

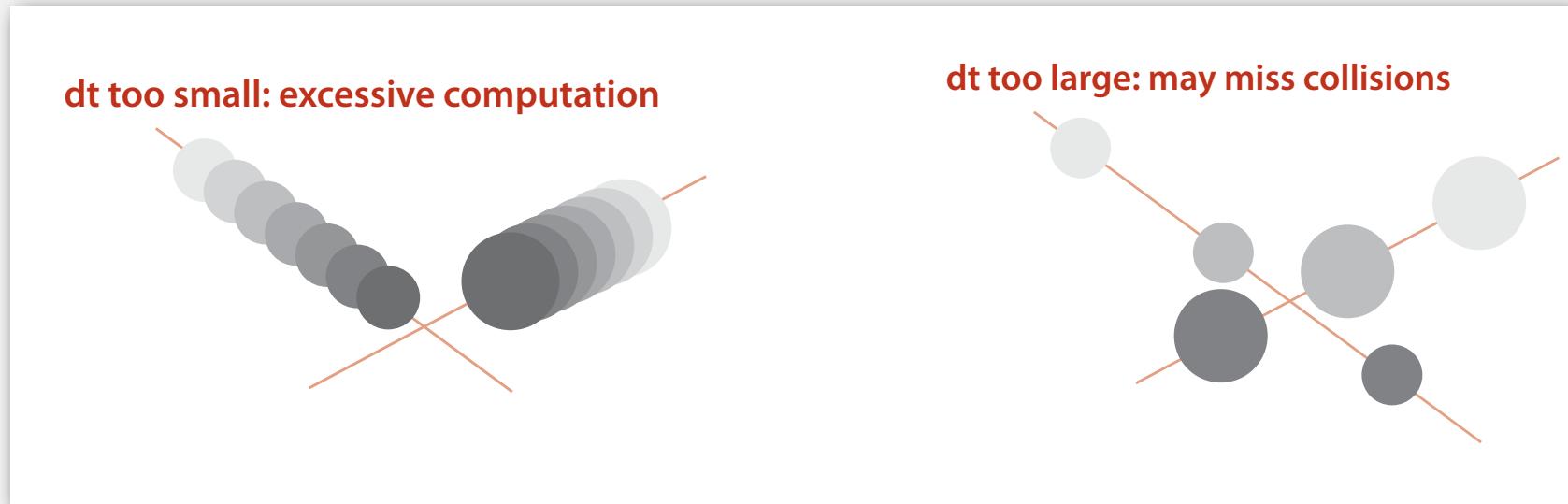


## Time-driven simulation

~ 시간 단위로 계산하는 시뮬레이션

### Main drawbacks.

- $\sim N^2 / 2$  overlap checks per time quantum.
- Simulation is too slow if  $dt$  is very small.
- May miss collisions if  $dt$  is too large.  
(if colliding particles fail to overlap when we are looking)



## Event-driven simulation

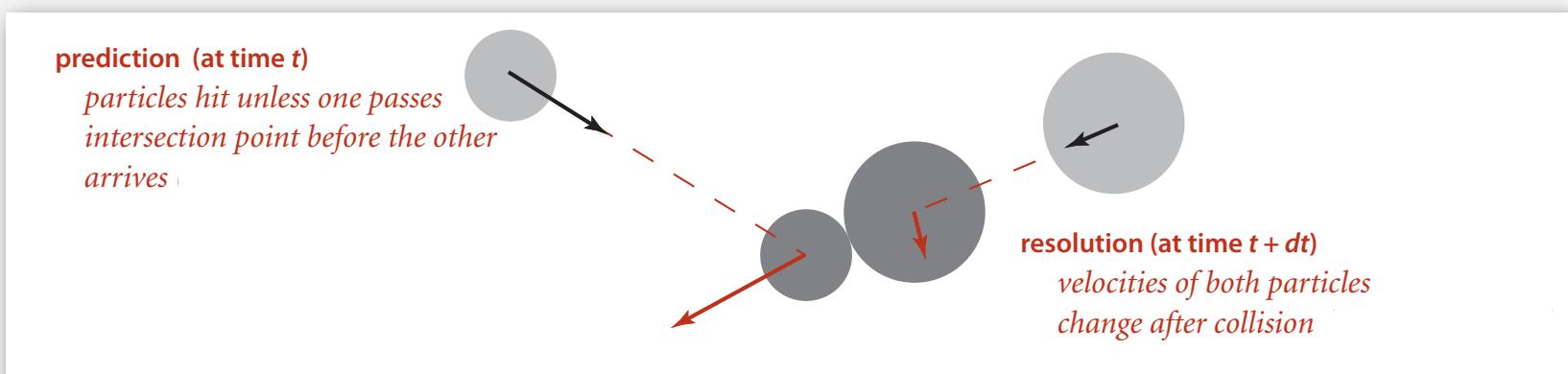
→ 시계열에 일어난 충돌 PQ에 기록

Change state only when something happens.

- Between collisions, particles move in straight-line trajectories.
- Focus only on times when collisions occur.
- Maintain **PQ** of collision events, prioritized by time.
- Remove the min = get next collision.

**Collision prediction.** Given position, velocity, and radius of a particle, when will it collide next with a wall or another particle?

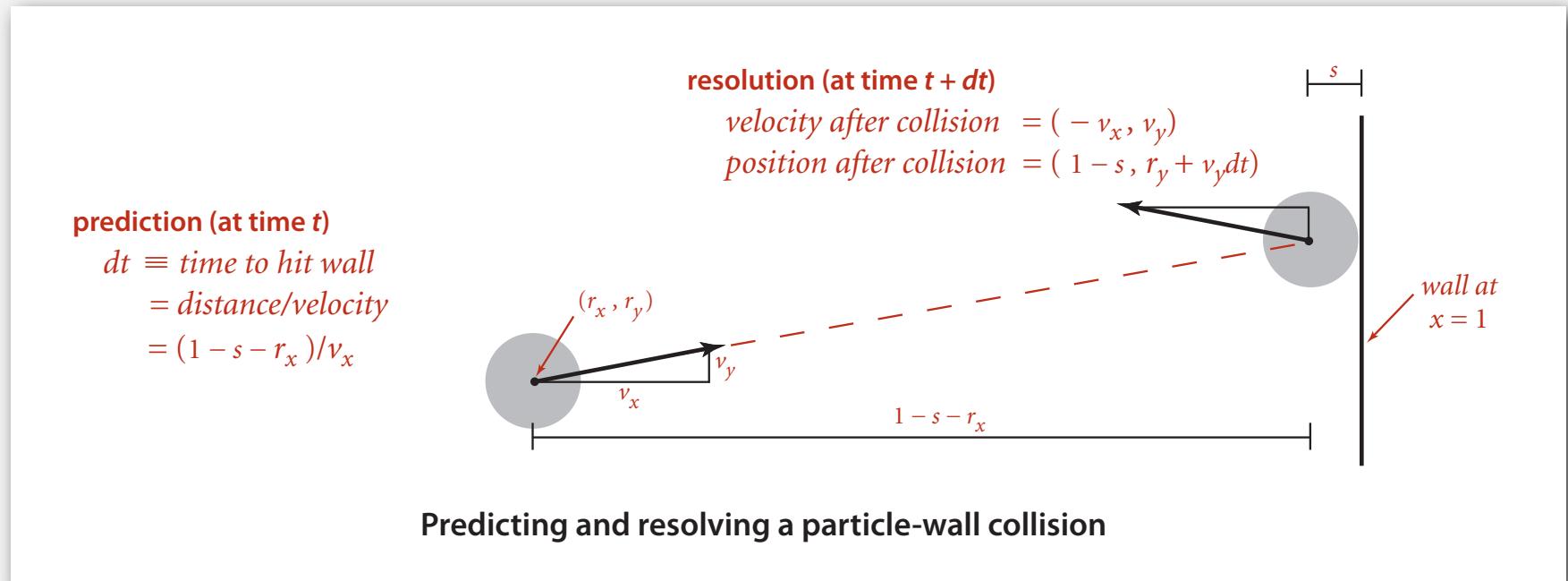
**Collision resolution.** If collision occurs, update colliding particle(s) according to laws of elastic collisions.



# Particle-wall collision

## Collision prediction and resolution.

- Particle of radius  $s$  at position  $(rx, ry)$ .
- Particle moving in unit box with velocity  $(vx, vy)$ .
- Will it collide with a vertical wall? If so, when?

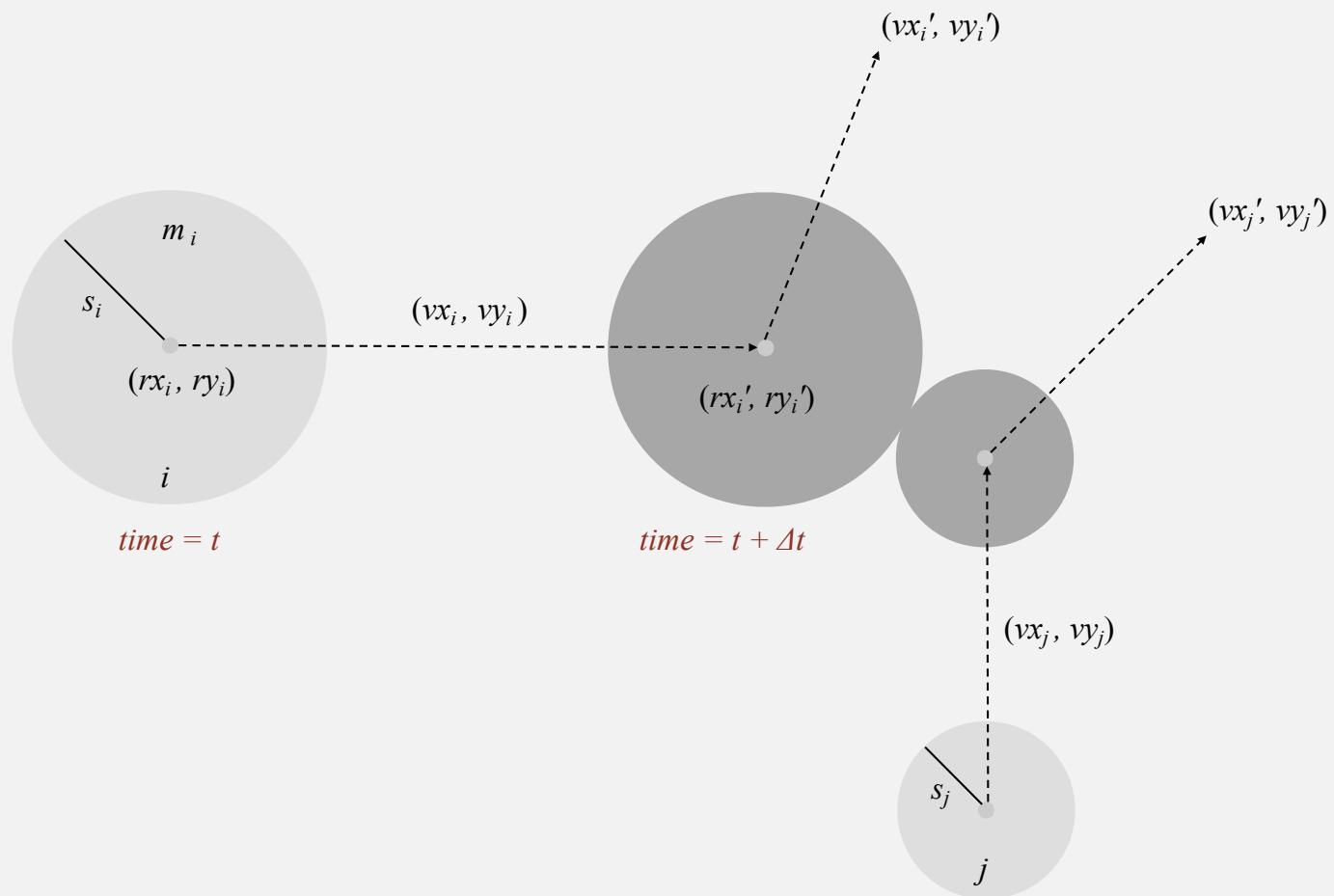


# Particle-particle collision prediction

---

## Collision prediction.

- Particle  $i$ : radius  $s_i$ , position  $(rx_i, ry_i)$ , velocity  $(vx_i, vy_i)$ .
- Particle  $j$ : radius  $s_j$ , position  $(rx_j, ry_j)$ , velocity  $(vx_j, vy_j)$ .
- Will particles  $i$  and  $j$  collide? If so, when?



# Particle-particle collision prediction

## Collision prediction.

- Particle  $i$ : radius  $s_i$ , position  $(rx_i, ry_i)$ , velocity  $(vx_i, vy_i)$ .
- Particle  $j$ : radius  $s_j$ , position  $(rx_j, ry_j)$ , velocity  $(vx_j, vy_j)$ .
- Will particles  $i$  and  $j$  collide? If so, when?

$$\Delta t = \begin{cases} \infty & \text{if } \Delta v \cdot \Delta r \geq 0 \\ \infty & \text{if } d < 0 \\ -\frac{\Delta v \cdot \Delta r + \sqrt{d}}{\Delta v \cdot \Delta v} & \text{otherwise} \end{cases}$$

$$d = (\Delta v \cdot \Delta r)^2 - (\Delta v \cdot \Delta v) (\Delta r \cdot \Delta r - \sigma^2) \quad \sigma = \sigma_i + \sigma_j$$

$$\Delta v = (\Delta vx, \Delta vy) = (vx_i - vx_j, vy_i - vy_j)$$

$$\Delta r = (\Delta rx, \Delta ry) = (rx_i - rx_j, ry_i - ry_j)$$

$$\Delta v \cdot \Delta v = (\Delta vx)^2 + (\Delta vy)^2$$

$$\Delta r \cdot \Delta r = (\Delta rx)^2 + (\Delta ry)^2$$

$$\Delta v \cdot \Delta r = (\Delta vx)(\Delta rx) + (\Delta vy)(\Delta ry)$$

**Important note:** This is high-school physics, so we won't be testing you on it!

# Particle-particle collision resolution

**Collision resolution.** When two particles collide, how does velocity change?

$$\begin{aligned} vx_i' &= vx_i + Jx / m_i \\ vy_i' &= vy_i + Jy / m_i \\ vx_j' &= vx_j - Jx / m_j \\ vy_j' &= vy_j - Jy / m_j \end{aligned}$$

Newton's second law  
(momentum form)

$$Jx = \frac{J \Delta rx}{\sigma}, \quad Jy = \frac{J \Delta ry}{\sigma}, \quad J = \frac{2m_i m_j (\Delta v \cdot \Delta r)}{\sigma(m_i + m_j)}$$

impulse due to normal force

(conservation of energy, conservation of momentum)

**Important note:** This is high-school physics, so we won't be testing you on it!

# Particle data type skeleton

```
public class Particle
{
    private double rx, ry;          // position
    private double vx, vy;          // velocity
    private final double radius;    // radius
    private final double mass;      // mass
    private int count;              // number of collisions

    public Particle(...) { }

    public void move(double dt) { }
    public void draw() { }

    public double timeToHit(Particle that) { }
    public double timeToHitVerticalWall() { }
    public double timeToHitHorizontalWall() { }

    public void bounceOff(Particle that) { }
    public void bounceOffVerticalWall() { }
    public void bounceOffHorizontalWall() { }

}
```

predict collision  
with particle or wall

resolve collision  
with particle or wall

# Particle-particle collision and resolution implementation

```
public double timeToHit(Particle that)
{
    if (this == that) return INFINITY;
    double dx = that.rx - this.rx, dy = that.ry - this.ry;
    double dvx = that.vx - this.vx; dvy = that.vy - this.vy;
    double dvdr = dx*dvx + dy*dvy;
    if( dvdr > 0) return INFINITY; ← no collision
    double dvdv = dvx*dvx + dvy*dvy;
    double drdr = dx*dx + dy*dy;
    double sigma = this.radius + that.radius;
    double d = (dvdr*dvdr) - dvdv * (drdr - sigma*sigma);
    if (d < 0) return INFINITY;
    return -(dvdr + Math.sqrt(d)) / dvdv;
}
```

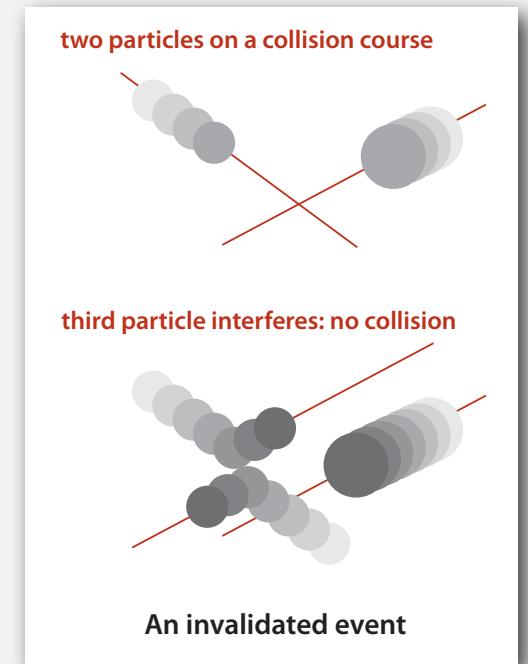
```
public void bounceOff(Particle that)
{
    double dx = that.rx - this.rx, dy = that.ry - this.ry;
    double dvx = that.vx - this.vx, dvy = that.vy - this.vy;
    double dvdr = dx*dvx + dy*dvy;
    double dist = this.radius + that.radius;
    double J = 2 * this.mass * that.mass * dvdr / ((this.mass + that.mass) * dist);
    double Jx = J * dx / dist;
    double Jy = J * dy / dist;
    this.vx += Jx / this.mass;
    this.vy += Jy / this.mass;
    that.vx -= Jx / that.mass;
    that.vy -= Jy / that.mass;
    this.count++;
    that.count++;     Important note: This is high-school physics, so we won't be testing you on it!
}
```

# Collision system: event-driven simulation main loop

## Initialization.

- Fill PQ with all potential particle-wall collisions.
- Fill PQ with all potential particle-particle collisions.

“potential” since collision may not happen if some other collision intervenes



## Main loop.

- Delete the impending event from PQ (min priority =  $t$ ).
- If the event has been invalidated, ignore it.
- Advance all particles to time  $t$ , on a straight-line trajectory.
- Update the velocities of the colliding particle(s).
- Predict future particle-wall and particle-particle collisions involving the colliding particle(s) and insert events onto PQ.

# Event data type

## Conventions.

- Neither particle null  $\Rightarrow$  particle-particle collision.
- One particle null  $\Rightarrow$  particle-wall collision.
- Both particles null  $\Rightarrow$  redraw event.

code 양호한 법.

```
private class Event implements Comparable<Event>
{
    private double time;          // time of event
    private Particle a, b;        // particles involved in event
    private int countA, countB;   // collision counts for a and b

    public Event(double t, Particle a, Particle b) { }           ← create event

    public int compareTo(Event that)
    {   return this.time - that.time; }                            ← ordered by time

    public boolean isValid()
    {   }
}
```

invalid if  
intervening  
collision

## Collision system implementation: skeleton

```
public class CollisionSystem
{
    private MinPQ<Event> pq;           // the priority queue
    private double t = 0.0;              // simulation clock time
    private Particle[] particles;       // the array of particles

    public CollisionSystem(Particle[] particles) { }

    private void predict(Particle a)      add to PQ all particle-wall and particle-
    {                                     -particle collisions involving this particle
        if (a == null) return;
        for (int i = 0; i < N; i++)
        {
            double dt = a.timeToHit(particles[i]);
            pq.insert(new Event(t + dt, a, particles[i]));
        }
        pq.insert(new Event(t + a.timeToHitVerticalWall() , a, null));
        pq.insert(new Event(t + a.timeToHitHorizontalWall(), null, a));
    }

    private void redraw() { }

    public void simulate() { /* see next slide */ }
}
```

# Collision system implementation: main event-driven simulation loop

```
public void simulate()
{
    pq = new MinPQ<Event>();
    for(int i = 0; i < N; i++) predict(particles[i]);
    pq.insert(new Event(0, null, null));
```

initialize PQ with collision events and redraw event

```
while(!pq.isEmpty())
{
    Event event = pq.delMin();
    if(!event.isValid()) continue;
    Particle a = event.a;
    Particle b = event.b;
```

get next event

```
    for(int i = 0; i < N; i++)
        particles[i].move(event.time - t);
    t = event.time;
```

update positions and time

```
    if      (a != null && b != null) a.bounceOff(b);
    else if (a != null && b == null) a.bounceOffVerticalWall()
    else if (a == null && b != null) b.bounceOffHorizontalWall();
    else if (a == null && b == null) redraw();
```

process event

```
    predict(a);
    predict(b);
```

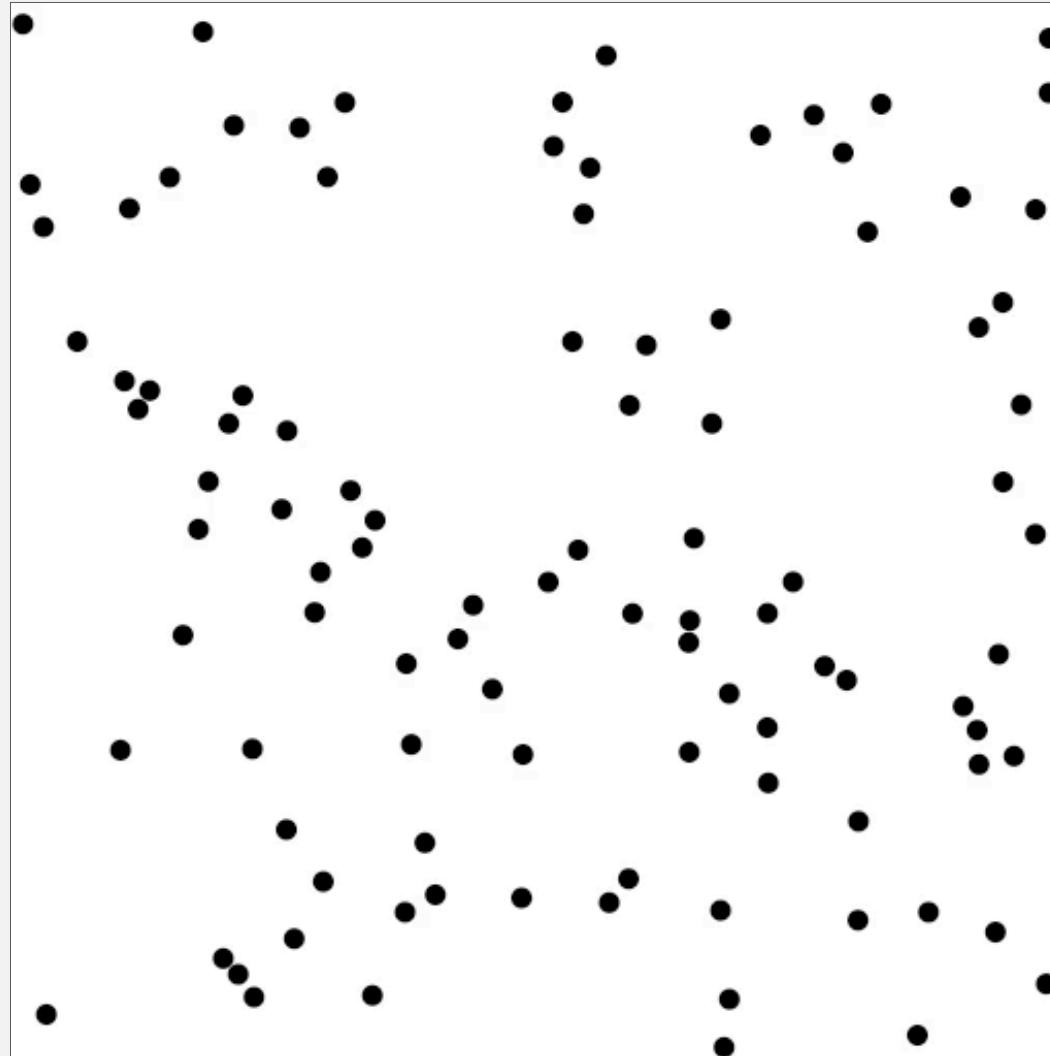
predict new events based on changes

```
}
```

# Particle collision simulation example 1

---

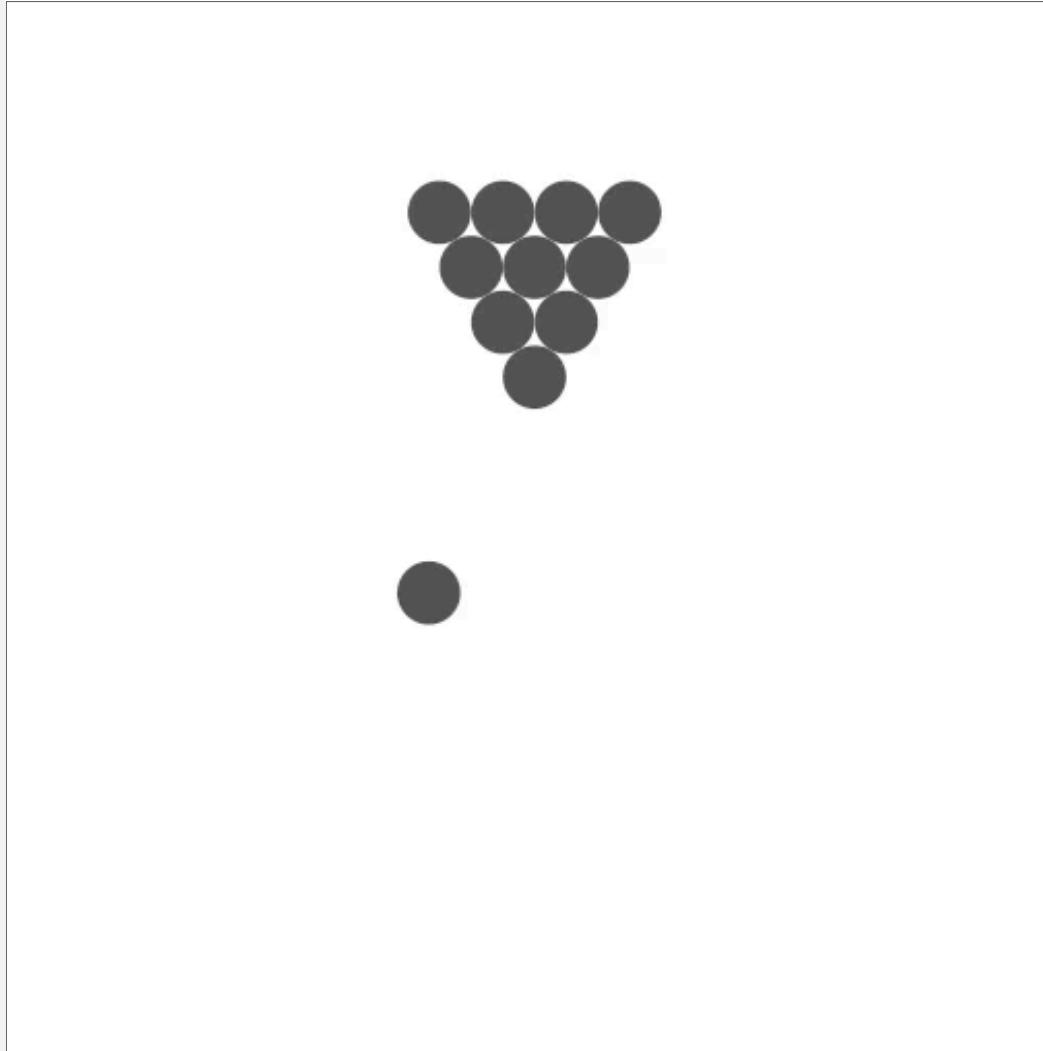
```
% java CollisionSystem 100
```



## Particle collision simulation example 2

---

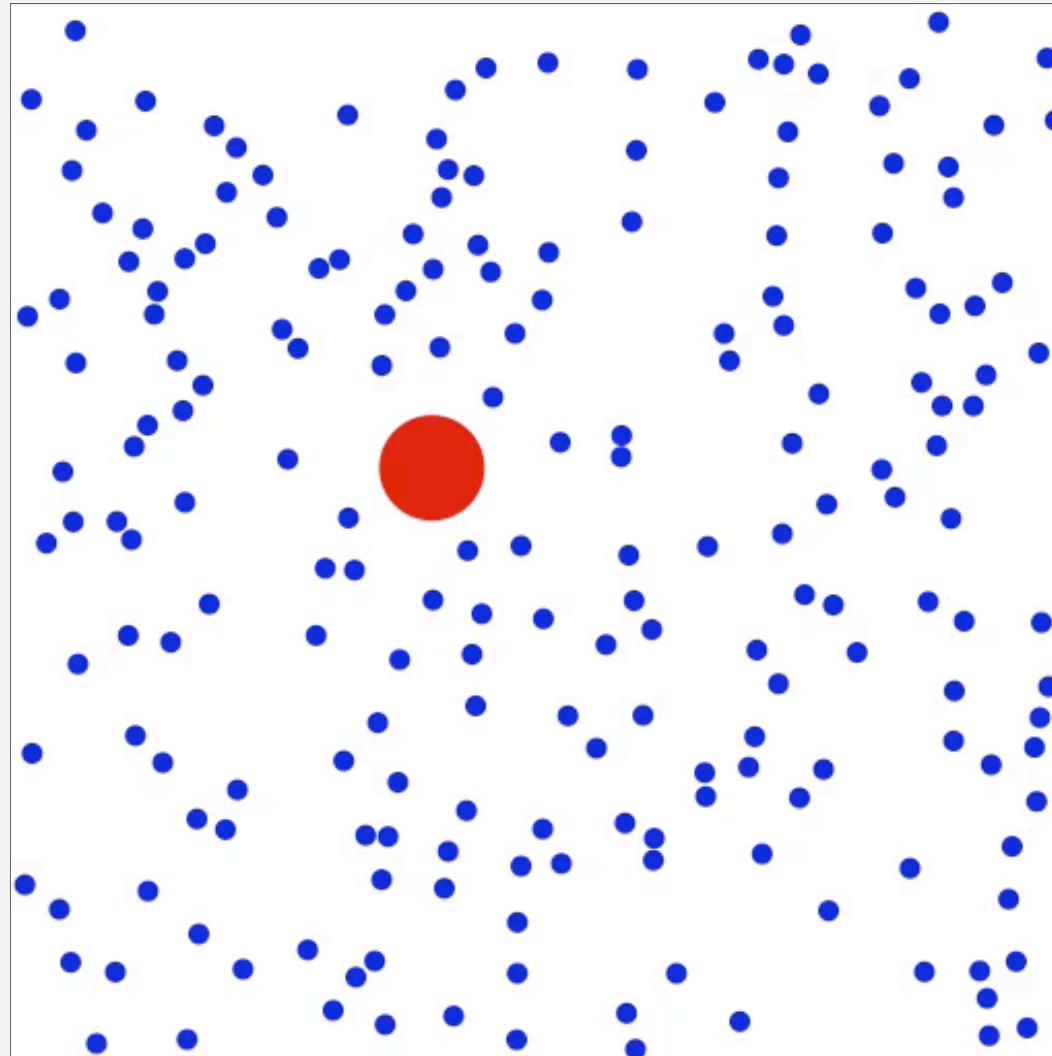
```
% java CollisionSystem < billiards.txt
```



## Particle collision simulation example 3

---

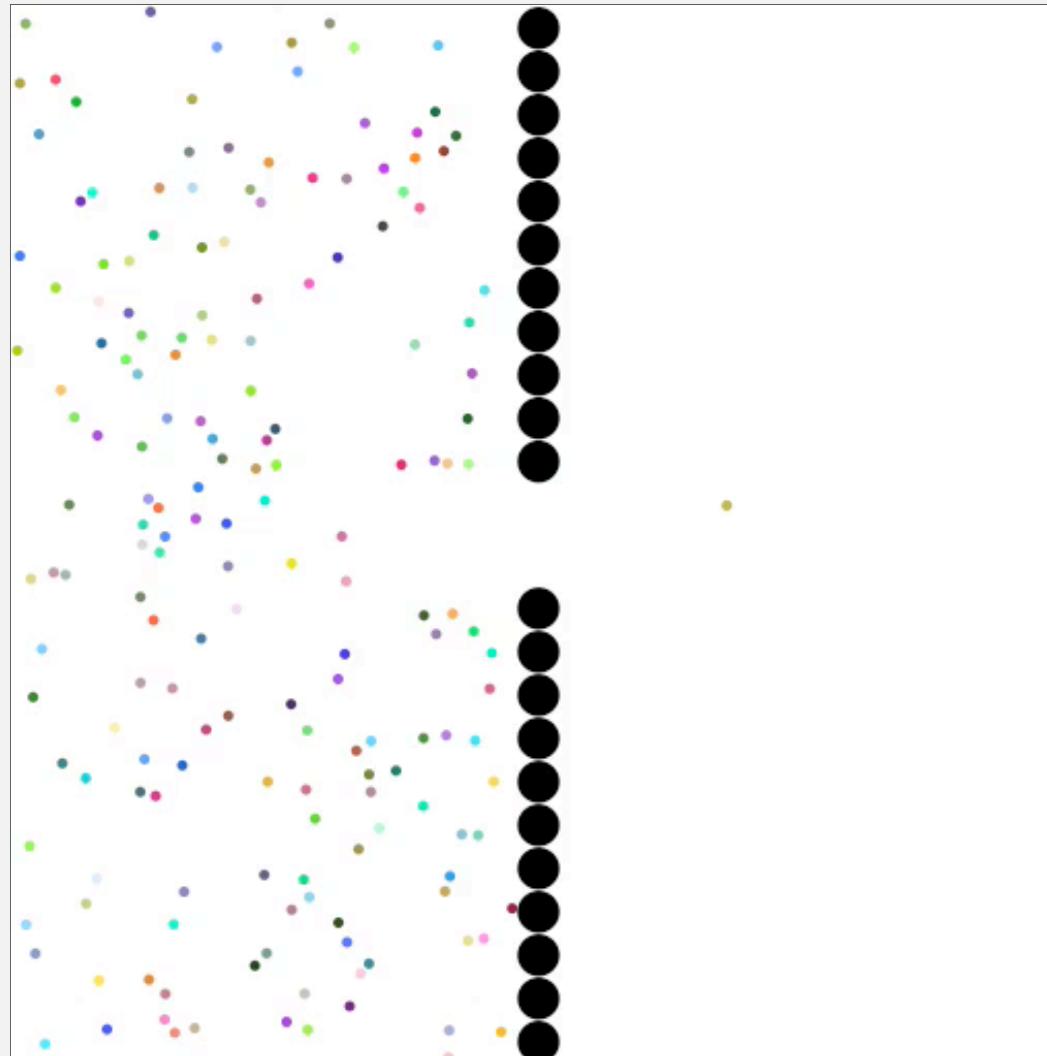
```
% java CollisionSystem < brownian.txt
```



# Particle collision simulation example 4

---

```
% java CollisionSystem < diffusion.txt
```



# Algorithms

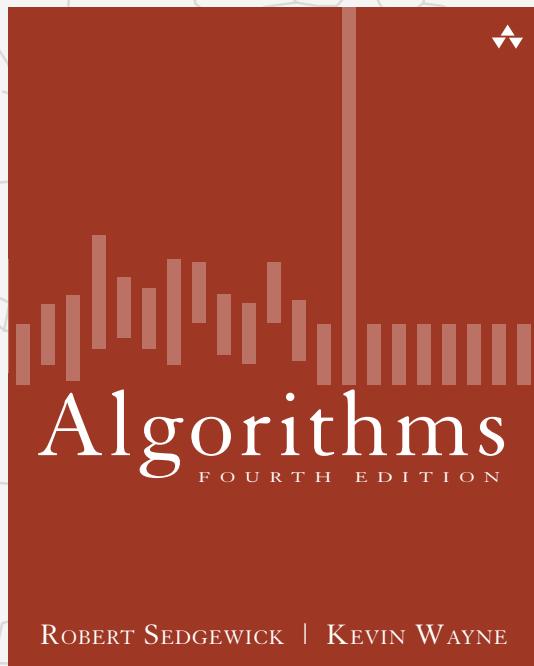
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- ▶ *binary heaps*
- ▶ *heapsort*
- ▶ *event-driven simulation*



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- ▶ *heapsort*
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