

# Matrix Norms

A “matrix norm” is a way of assigning a numerical measurement to a matrix. There are different types of matrix norms, each useful in their own context.

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The *Frobenius norm* is defined as:

$$\|A\|_F = \sqrt{\sum_{i=1}^n \sum_{j=1}^m (A_{i,j})^2}$$

The function for the *Frobenius norm* written in R:

```
F.norm <- function(A) {  
  sum_squares = 0  
  for (i in 1:nrow(A)) {  
    for (j in 1:ncol(A)) {  
      sum_squares = sum_squares + A[i,j]^2  
    }  
  }  
  return(sqrt(sum_squares))  
}
```

The *1-norm* is defined as:

$$\|A\|_1 = \max_{1 \leq j \leq m} \left( \sum_{i=1}^n |A_{i,j}| \right)$$

The function for the *1-norm* written in R:

```
one.norm <- function(A) {  
  n = nrow(A)  
  m = ncol(A)  
  col_sums = vector(length=m)  
  for (j in 1:m) {  
    sum = 0  
    for (i in 1:n) {  
      sum = sum + abs(A[i,j])  
    }  
    col_sums[j] = sum  
  }  
  return(max(col_sums))  
}
```

The  $\infty$ -norm is defined as:

$$\|A\|_{\infty} = \max_{1 \leq i \leq n} \left( \sum_{j=1}^n |A_{i,j}| \right)$$

The function for the  $\infty$ -norm written in R:

```
inf.norm <- function(A) {  
  n = nrow(A)  
  m = ncol(A)  
  row_sums = vector(length=n)  
  for (i in 1:n) {  
    sum = 0  
    for (j in 1:m) {  
      sum = sum + abs(A[i,j])  
    }  
    row_sums[i] = sum  
  }  
  return(max(row_sums))  
}
```

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The main Matrix Norm function is as follows:

```
mat.norm <- function(A, type=c("one", "inf", "F")) {  
  switch (type,  
    "one" = one.norm(A),  
    "inf" = inf.norm(A),  
    "F" = F.norm(A)  
  )  
}
```

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Homework 5 for MATH 366: *Applied Mathematical Computation*