Matrix Norms

A "matrix norm" is a way of assigning a numerical measurement to a matrix. There are different types of matrix norms, each useful in their own context.

The *Frobenius norm* is defined as:

$$||A||_F = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{m}(A_{i,j})^2}$$

The function for the *Frobenius norm* written in R:

```r
F.norm <- function(A) {
  sum_squares = 0
  for (i in 1:nrow(A)) {
    for (j in 1:ncol(A)) {
      sum_squares = sum_squares + A[i,j]^2
    }
  }
  return(sqrt(sum_squares))
}
```

The *1-norm* is defined as:

$$||A||_1 = \max_{1 \le j \le m} (\sum_{i=1}^{n}|A_{i,j}|)$$

The function for the *1-norm* written in R:

```r
one.norm <- function(A) {
  n = nrow(A)
  m = ncol(A)
  col_sums = vector(length=m)
  for (j in 1:m) {
    sum = 0
    for (i in 1:n) {
      sum = sum + abs(A[i,j])
    }
    col_sums[j] = sum
  }
  return(max(col_sums))
}
```

The ∞-*norm* is defined as:

$$||A||_\infty = \max_{1 \le i \le n} \left(\sum_{j=1}^{n} |A_{i,j}| \right)$$

The function for the ∞-*norm* written in R:

```r
inf.norm <- function(A) {
  n = nrow(A)
  m = ncol(A)
  row_sums = vector(length=n)
  for (i in 1:n) {
    sum = 0
    for (j in 1:m) {
      sum = sum + abs(A[i,j])
    }
    row_sums[i] = sum
  }
  return(max(row_sums))
}
```

The Matrix Norm function to select the type of norm is as follows:

```r
mat.norm <- function(A, type=c("one", "inf", "F")) {
  switch (type,
    "one" = one.norm(A),
    "inf" = inf.norm(A),
    "F" = F.norm(A)
  )
}
```

File: MatrixNorm.R

Homework 5 for MATH 366: *Applied Mathematical Computation*

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