Introduction to Structural Equation Modelling (using the 'lavaan' package in R)

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   Confirmatory factor analysis (CFA)

3. Models with structural and measurement components

   Uses CFA to account for measurement error
   Yet, models directional ('causal') relationships

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$\longrightarrow$ directional ("*causal*") relationship

$\leftrightarrow$ non-directional relationship (covariances for unstandardized solutions or correlations for standardized ones)

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  - Correlation $\neq$ Causation

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    - For outcomes (observed or latent factors): residual is variance unexplained by their predictors

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1. Goal 1: to understand patterns of covariances among observed variables
2. Goal 2: to explain as much variation as possible with model

How Does SEM Differ from Regression?

Estimated Covariance

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Regression Estimate

$$\beta = rr_{XY}(SD_Y / SD_X)$$

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- ζ is a vector of disturbance terms; $cov(\zeta) = \Psi$ is their covariance matrix

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- Ideal 20:1 (e.g., if $q = 10$, then need sample size of at least 200
- Minimal 10:1 ratio (just like regression)

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  1. Specify model but without directionality between key variables (i.e., no causal paths)
  2. Specify and test alternative models with different causal directionalities (with similar results, no statistical method can identify which is correct)
  3. And/or include reciprocal effects to cover both possibilities (but can create problems of identification)

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  Total number of variances (4) and covariances (6) in the data matrix (fewer can be estimated, but the max is 10)

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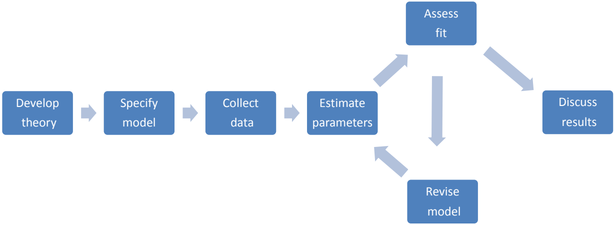
6. Rinse and repeat
7. Interpret the results

SEM Flowchart



Practical Example: Support for Social Welfare Spending

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4. **Pro-Social Orientations**
   - Humanitarianism (*value* helping those in need)
   - Empathy (*ability* to understand/feel what another being is experiencing)

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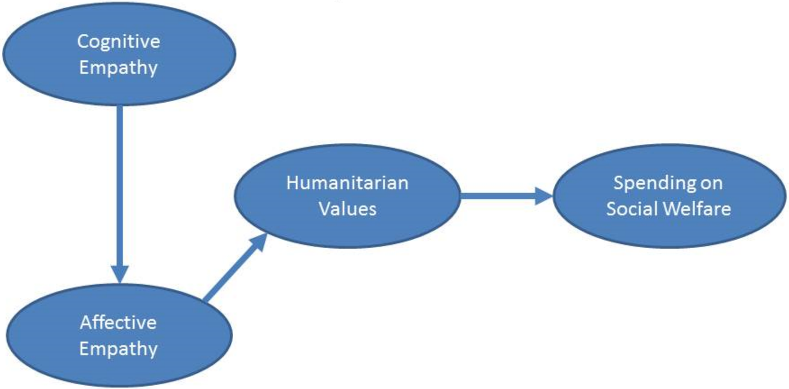


Practical Example: Support for Social Welfare Spending

- American National Election Study 2008 - 2009 Panel Study
  - Monthly surveys with representative Internet panel
  - 1,420 to 2,665 completed interviews per wave
  - Social Spending toward Social Security, Aid to the Poor, Job Retraining, and Public Schools
  - 8 Humanitarianism Items
    - 'It is important to help one another so that the community in general is a better place.'
  - 21 Empathy Items
  - Other Demographic Controls

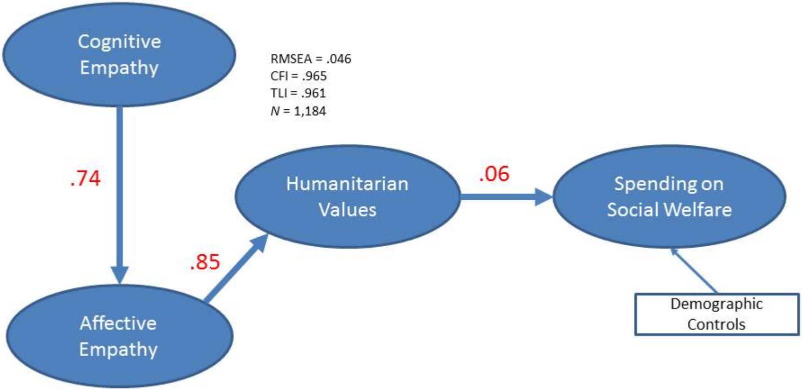
Empathy

- Interpersonal Reactivity Index (Davis (1980, 1983)
  - **Empathic Perspective-Taking**
  - **Empathic Concern**
  - Personal Distress
  - Fantasy

2-Factor CFA Model of Empathy

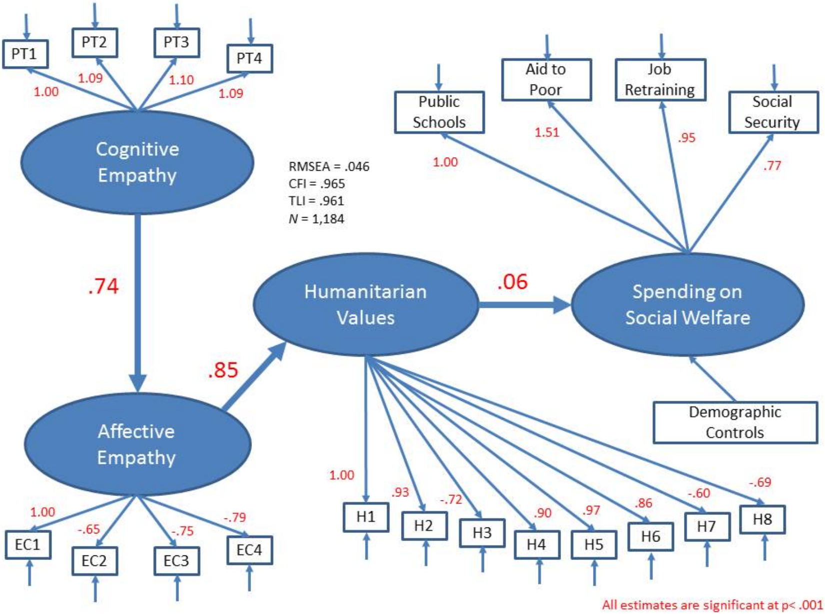
Survey Item	Cognitive Empathy	Affective Empathy
I try to look at everybody's side of a disagreement before I make a decision.	**.64**	.36
I sometimes try to understand my friends better by imagining how things look from their perspective.	**.73**	.41
I believe that there are two sides to every question and try to look at them both.	**.73**	.40
Before criticizing somebody, I try to imagine how I would feel if I were in their place.	**.69**	.38
I often have tender, concerned feelings for people less fortunate than me.	.39	**.70**
Sometimes I don't feel very sorry for other people when they are having problems. (R)	-.33	**-.59**
When I see someone being treated unfairly, I sometimes don't feel very much pity for them. (R)	-.37	**-.67**
Other people's misfortunes do not usually disturb me a great deal. (R)	-.38	**-.69**

Structural Equation Model Results



All estimates are significant at p< .001

Structural Equation Model Results



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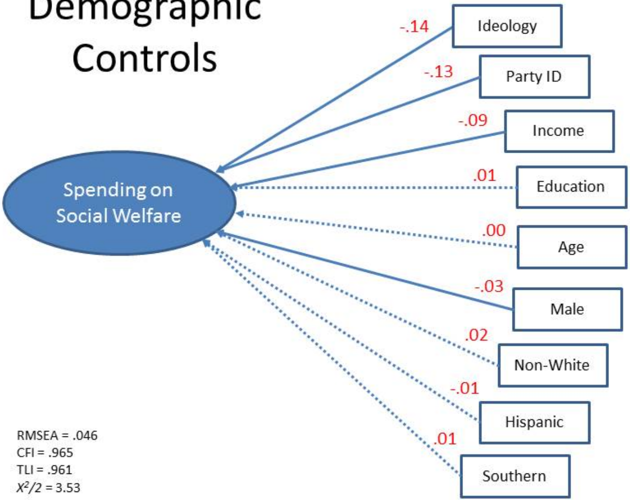
Demographic Controls



RMSEA = .046
CFI = .965
TLI = .961
$X^2/2 = 3.53$
$N = 1{,}184$

Solid lines are significant at p< .01

Lavaan Code

```
## Install Lavaan for SEM
install.packages("lavaan",repos = "http://cran.us.r-project.org/")

## Require Needed Packages
require(lavaan)
require(foreign)


## SEM for Empathy, Humanitarianism, and Social Spending
model <- ' # Latent Variables
          cempathy =~ ept2 + ept3 + ept5 + ept7
          aempathy =~ ec1 + ec2 + ec4 + epd4
          human =~ hu1 + hu2 + hu3 + hu4 + hu5 + hu6 + hu7 + hu8
          social =~ school15 + ss15 + poor15 + job15
          # Regressions
          aempathy ~ a*cempathy
          human ~ b*aempathy
          social ~ c*human
          social ~ ideology + party + male + age + hispanic
                        + nonwhite + education + income + south
          # Indirect Effect (a*b)
              ab := a*b
              bc := b*c
              abc := a*b*c
          # Residual Covariances
          # cempathy ~~ aempathy
        '

fit <- sem(model,
           data=anes,
           ordered=c("ept2","ept3",
                     "ept5","ept7",
                     "ec1", "ec2",
                     "ec4", "epd4",
                     "hu1","hu2",
                     "hu3","hu4",
                     "hu5","hu6",
                     "hu7","hu8"))

summary(fit)
parameterEstimates(fit)
fitMeasures(fit, c("cfi", "rmsea", "tli"))
```

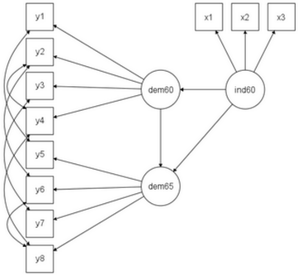
Software: lavaan Package in R: http://lavaan.ugent.be/

The official reference to the lavaan package is the following paper:

> Yves Rosseel (2012). lavaan: An R Package for Structural Equation Modeling. *Journal of Statistical Software*, 48(2), 1-36. URL http://www.jstatsoft.org/v48/i02/

First impression

To get a first impression of how lavaan works in practice, consider the following example of a SEM model. The figure below contains a graphical representation of the model that we want to fit.




```
model <- '
  # latent variables
    ind60 =~ x1 + x2 + x3
    dem60 =~ y1 + y2 + y3 + y4
    dem65 =~ y5 + y6 + y7 + y8
  # regressions
    dem60 ~ ind60
    dem65 ~ ind60 + dem60
  # residual covariances
    y1 ~~ y5
    y2 ~~ y4 + y6
    y3 ~~ y7
    y4 ~~ y8
    y6 ~~ y8
'

fit <- sem(model,
           data=PoliticalDemocracy)
summary(fit)
```

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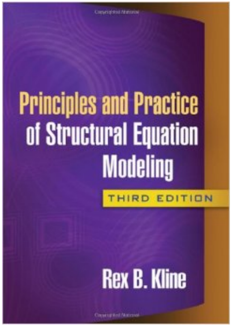
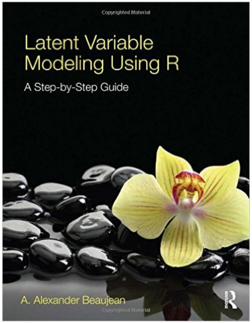
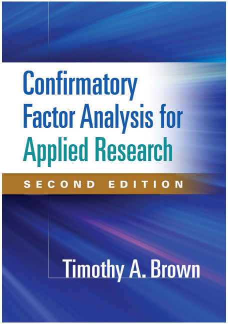
    - Binary, Categorical, and Continuous DVs

My Favourite SEM Books







Idre: http://www.ats.ucla.edu/stat/