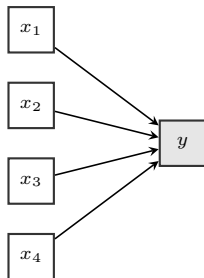

Using the lavaan R package

May 5, 2021

a simple regression analysis in R



```
# read in your data
myData <- read.csv("c:/temp/myData.csv")

# fit model using lm
fit <- lm(formula = y ~ x1 + x2 + x3 + x4,
          data    = myData)

# show results
summary(fit)
```

The standard linear model:

- $$y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \beta_3 x_{i3} + \beta_4 x_{i4} + \epsilon_i \quad (i = 1, 2, \dots, n)$$

formula type	operator	mnemonic
latent variable definition	=~	is measured by
regression	~	is regressed on
(residual) (co)variance	~~	is correlated with
intercept	~ 1	intercept

lm() output artificial data (N=100)

Call:

```
lm(formula = y ~ x1 + x2 + x3 + x4, data = myData)
```

Residuals:

	Min	1Q	Median	3Q	Max
	-102.372	-29.458	-3.658	27.275	148.404

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	97.7210	4.7200	20.704	<2e-16 ***
x1	5.7733	0.5238	11.022	<2e-16 ***
x2	-1.3214	0.4917	-2.688	0.0085 **
x3	1.1350	0.4575	2.481	0.0149 *
x4	0.2707	0.4779	0.566	0.5724

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

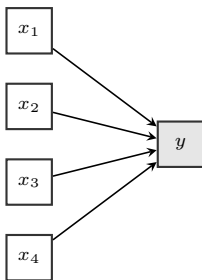
Residual standard error: 46.74 on 95 degrees of freedom

Multiple R-squared: 0.5911, Adjusted R-squared: 0.5738

F-statistic: 34.33 on 4 and 95 DF, p-value: < 2.2e-16

```
# create artificial data
set.seed(1)
x1 <- rnorm(100) * 10; x2 <- rnorm(100) * 10
x3 <- rnorm(100) * 10; x4 <- rnorm(100) * 10
y <- 100 + 5*x1 + (-2)*x2 + 1*x3 + 0.1*x4 + rnorm(100, sd=40)
myData <- data.frame(y,x1,x2,x3,x4)
```

the lavaan model syntax – a simple regression



```
library(lavaan)
myData <- read.csv("c:/temp/myData.csv")

myModel <- ' y ~ x1 + x2 + x3 + x4 '

# fit model
fit <- sem(model = myModel,
           data = myData)

# show results
summary(fit)
```

- to ‘see’ the intercept, use either

```
fit <- sem(model=myModel, data=myData, meanstructure=TRUE)
```

or include it explicitly in the syntax:

```
myModel <- ' y ~ 1 + x1 + x2 + x3 + x4 '
```

output (artificial data, N=100)

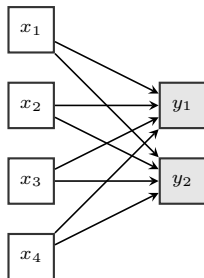
lavaan (0.5-13) converged normally after 1 iterations

Number of observations	100
Estimator	ML
Minimum Function Test Statistic	0.000
Degrees of freedom	0
P-value (Chi-square)	1.000

Parameter estimates:

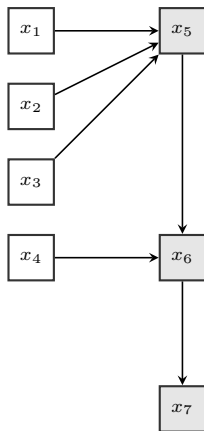
Information				Expected
Standard Errors				Standard
	Estimate	Std.err	Z-value	P(> z)
Regressions:				
y ~				
x1	5.773	0.511	11.309	0.000
x2	-1.321	0.479	-2.757	0.006
x3	1.135	0.446	2.545	0.011
x4	0.271	0.466	0.581	0.561
Variances:				
y	2075.100	293.463		

the lavaan model syntax – multivariate regression



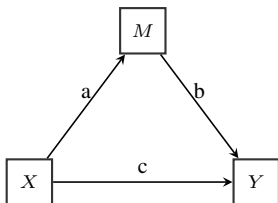
```
myModel <- ' y1 ~ x1 + x2 + x3 + x4  
             y2 ~ x1 + x2 + x3 + x4 '
```

the lavaan model syntax – path analysis



```
myModel <- ' x5 ~ x1 + x2 + x3  
             x6 ~ x4 + x5  
             x7 ~ x6 '
```

the lavaan model syntax – mediation analysis



```
model <- '  
    Y ~ b*M + c*X  
    M ~ a*X  
  
    indirect := a*b  
    total    := c + (a*b)  
,  
  
fit <- sem(model,  
    data = myData,  
    se = "bootstrap")  
summary(fit)
```

output

...

Parameter estimates:

Information	Observed
Standard Errors	Bootstrap
Number of requested bootstrap draws	1000
Number of successful bootstrap draws	1000

		Estimate	Std.err	Z-value	P(> z)
Regressions:					
Y ~					
M	(b)	0.597	0.098	6.068	0.000
X	(c)	2.594	1.210	2.145	0.032
M ~					
X	(a)	2.739	0.999	2.741	0.006

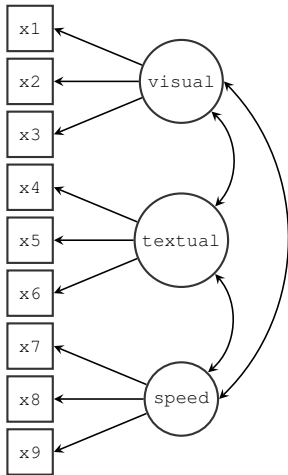
Variances:

Y	108.700	17.747
M	105.408	16.556

Defined parameters:

indirect	1.636	0.645	2.535	0.011
total	4.230	1.383	3.059	0.002

the lavaan model syntax – using `cfa()` or `sem()`



```
HS.model <- ' visual  =~ x1 + x2 + x3
              textual =~ x4 + x5 + x6
              speed   =~ x7 + x8 + x9
              '
```

```
fit <- cfa(model = HS.model,
           data  = HolzingerSwineford1939)
```

```
summary(fit, fit.measures = TRUE,
        standardized = TRUE)
```

output

lavaan (0.5-13) converged normally after 35 iterations

Number of observations	301
Estimator	ML
Minimum Function Test Statistic	85.306
Degrees of freedom	24
P-value (Chi-square)	0.000

Model test baseline model:

Minimum Function Test Statistic	918.852
Degrees of freedom	36
P-value	0.000

Full model versus baseline model:

Comparative Fit Index (CFI)	0.931
Tucker-Lewis Index (TLI)	0.896

Loglikelihood and Information Criteria:

Loglikelihood user model (H0)	-3737.745
Loglikelihood unrestricted model (H1)	-3695.092
Number of free parameters	21

Akaike (AIC)	7517.490
Bayesian (BIC)	7595.339
Sample-size adjusted Bayesian (BIC)	7528.739

Root Mean Square Error of Approximation:

RMSEA	0.092
90 Percent Confidence Interval	0.071 0.114
P-value RMSEA <= 0.05	0.001

Standardized Root Mean Square Residual:

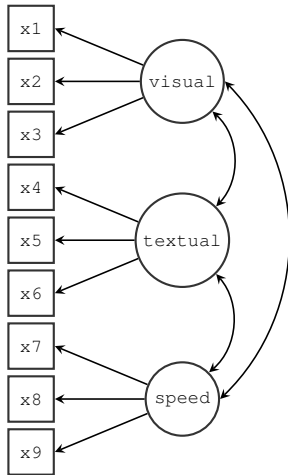
SRMR	0.065
------	-------

Parameter estimates:

Information				Expected		
Standard Errors				Standard		
	Estimate	Std.err	Z-value	P(> z)	Std.lv	Std.all
Latent variables:						
visual =~						
x1	1.000				0.900	0.772
x2	0.554	0.100	5.554	0.000	0.498	0.424
x3	0.729	0.109	6.685	0.000	0.656	0.581
textual =~						
x4	1.000				0.990	0.852
x5	1.113	0.065	17.014	0.000	1.102	0.855

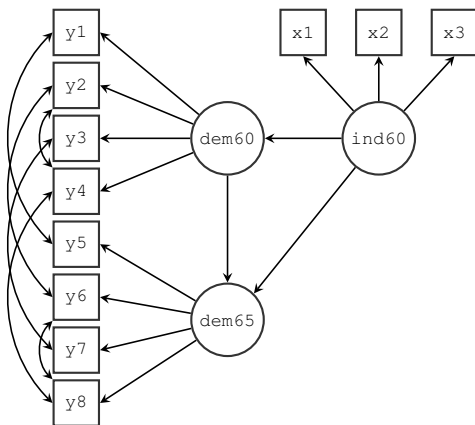
x6	0.926	0.055	16.703	0.000	0.917	0.838
speed =~						
x7	1.000				0.619	0.570
x8	1.180	0.165	7.152	0.000	0.731	0.723
x9	1.082	0.151	7.155	0.000	0.670	0.665
Covariances:						
visual ~~						
textual	0.408	0.074	5.552	0.000	0.459	0.459
speed	0.262	0.056	4.660	0.000	0.471	0.471
textual ~~						
speed	0.173	0.049	3.518	0.000	0.283	0.283
Variances:						
x1	0.549	0.114			0.549	0.404
x2	1.134	0.102			1.134	0.821
x3	0.844	0.091			0.844	0.662
x4	0.371	0.048			0.371	0.275
x5	0.446	0.058			0.446	0.269
x6	0.356	0.043			0.356	0.298
x7	0.799	0.081			0.799	0.676
x8	0.488	0.074			0.488	0.477
x9	0.566	0.071			0.566	0.558
visual	0.809	0.145			1.000	1.000
textual	0.979	0.112			1.000	1.000
speed	0.384	0.086			1.000	1.000

the lavaan model syntax – using lavaan()



```
HS.model <- '  
  # latent variables  
  visual  =~ 1*x1 + x2 + x3  
  textual =~ 1*x4 + x5 + x6  
  speed   =~ 1*x7 + x8 + x9  
  
  # factor (co)variances  
  visual ~~ visual; visual ~~ textual  
  visual ~~ speed; textual ~~ textual  
  textual ~~ speed; speed   ~~ speed  
  
  # residual variances  
  x1 ~~ x1; x2 ~~ x2; x3 ~~ x3  
  x4 ~~ x4; x5 ~~ x5; x6 ~~ x6  
  x7 ~~ x7; x8 ~~ x8; x9 ~~ x9  
,  
  
fit <- lavaan(model = HS.model,  
              data = HolzingerSwineford1939)
```

lavaan model syntax: full sem



```
myModel <- '  
  
# latent variable definitions  
ind60 =~ x1 + x2 + x3  
dem60 =~ y1 + a*y2 + b*y3 + c*y4  
dem65 =~ y5 + a*y6 + b*y7 + c*y8  
  
# regressions  
dem60 ~ ind60  
dem65 ~ ind60 + dem60  
  
# residual covariances  
y1 ~~ y5  
y2 ~~ y4  
y2 ~~ y6  
y3 ~~ y7  
y4 ~~ y8  
y6 ~~ y8  
  
,  
  
fit <- sem(model = myModel,  
           data = ...)
```

output

lavaan (0.5-13) converged normally after 61 iterations

Number of observations	75
Estimator	ML
Minimum Function Test Statistic	40.179
Degrees of freedom	38
P-value (Chi-square)	0.374

Parameter estimates:

Information				Expected
Standard Errors				Standard
	Estimate	Std.err	Z-value	P(> z)
Latent variables:				
ind60 =~				
x1	1.000			
x2	2.180	0.138	15.751	0.000
x3	1.818	0.152	11.971	0.000
dem60 =~				
y1	1.000			
y2 (a)	1.191	0.139	8.551	0.000
y3 (b)	1.175	0.120	9.755	0.000
y4 (c)	1.251	0.117	10.712	0.000
dem65 =~				

y5		1.000			
y6	(a)	1.191	0.139	8.551	0.000
y7	(b)	1.175	0.120	9.755	0.000
y8	(c)	1.251	0.117	10.712	0.000
Regressions:					
dem60 ~					
ind60		1.471	0.392	3.750	0.000
dem65 ~					
ind60		0.600	0.226	2.660	0.008
dem60		0.865	0.075	11.554	0.000
Covariances:					
y1 ~~					
y5		0.583	0.356	1.637	0.102
y2 ~~					
y4		1.440	0.689	2.092	0.036
y6		2.183	0.737	2.960	0.003
y3 ~~					
y7		0.712	0.611	1.165	0.244
y4 ~~					
y8		0.363	0.444	0.817	0.414
y6 ~~					
y8		1.372	0.577	2.378	0.017
Variances:					
x1		0.081	0.019		
x2		0.120	0.070		

x3	0.467	0.090
y1	1.855	0.433
y2	7.581	1.366
y3	4.956	0.956
y4	3.225	0.723
y5	2.313	0.479
y6	4.968	0.921
y7	3.560	0.710
y8	3.308	0.704
ind60	0.449	0.087
dem60	3.875	0.866
dem65	0.164	0.227

shortcut: robust standard errors and scaled test statistic

```
> fit <- cfa(HS.model,  
            data = HolzingerSwineford1939,  
            estimator = "MLM")  
> summary(fit, fit.measures=TRUE)  
lavaan (0.5-13) converged normally after 35 iterations
```

Number of observations	301	
Estimator	ML	Robust
Minimum Function Test Statistic	85.306	80.872
Degrees of freedom	24	24
P-value (Chi-square)	0.000	0.000
Scaling correction factor for the Satorra-Bentler correction		1.055

Model test baseline model:

Minimum Function Test Statistic	918.852	789.298
Degrees of freedom	36	36
P-value	0.000	0.000

Full model versus baseline model:

Comparative Fit Index (CFI)	0.931	0.925
Tucker-Lewis Index (TLI)	0.896	0.887

...

binary and ordered categorical data

```
# binary version of Holzinger & Swineford
HS9 <- HolzingerSwineford1939[,c("x1", "x2", "x3", "x4", "x5",
                                "x6", "x7", "x8", "x9")]
HSbinary <- as.data.frame( lapply(HS9, cut, 2, labels=FALSE) )

# single factor model
model <- ' visual  =~ x1 + x2 + x3
          textual  =~ x4 + x5 + x6
          speed    =~ x7 + x8 + x9 '

# binary CFA
fit <- cfa(model, data=HSbinary, ordered=names(HSbinary))

# summary
summary(fit, fit.measures=TRUE)
```

output

lavaan (0.5-13) converged normally after 36 iterations

Number of observations	301	
Estimator	DWLS	Robust
Minimum Function Test Statistic	30.918	38.546
Degrees of freedom	24	24
P-value (Chi-square)	0.156	0.030
Scaling correction factor		0.866
Shift parameter		2.861
for simple second-order correction (Mplus variant)		

Model test baseline model:

Minimum Function Test Statistic	582.533	469.769
Degrees of freedom	36	36
P-value	0.000	0.000

Full model versus baseline model:

Comparative Fit Index (CFI)	0.987	0.966
Tucker-Lewis Index (TLI)	0.981	0.950

Root Mean Square Error of Approximation:

RMSEA	0.031	0.045
-------	-------	-------

90 Percent Confidence Interval	0.000	0.059	0.014	0.070
P-value RMSEA <= 0.05		0.848	0.598	

Parameter estimates:

	Information Standard Errors		Expected Robust.sem	
	Estimate	Std.err	Z-value	P(> z)
Latent variables:				
visual =~				
x1	1.000			
x2	0.900	0.188	4.788	0.000
x3	0.939	0.197	4.766	0.000
textual =~				
x4	1.000			
x5	0.976	0.118	8.241	0.000
x6	1.078	0.125	8.601	0.000
speed =~				
x7	1.000			
x8	1.569	0.461	3.403	0.001
x9	1.449	0.409	3.541	0.000
Covariances:				
visual ~~				
textual	0.303	0.061	4.981	0.000
speed	0.132	0.049	2.700	0.007
textual ~~				

speed	0.076	0.046	1.656	0.098
-------	-------	-------	-------	-------

Intercepts:

visual	0.000
textual	0.000
speed	0.000

Thresholds:

x1 t1	-0.388	0.074	-5.223	0.000
x2 t1	-0.054	0.072	-0.748	0.454
x3 t1	0.318	0.074	4.309	0.000
x4 t1	0.180	0.073	2.473	0.013
x5 t1	-0.257	0.073	-3.506	0.000
x6 t1	1.024	0.088	11.641	0.000
x7 t1	0.231	0.073	3.162	0.002
x8 t1	1.128	0.092	12.284	0.000
x9 t1	0.626	0.078	8.047	0.000

Variances:

x1	0.592
x2	0.670
x3	0.640
x4	0.303
x5	0.336
x6	0.191
x7	0.778
x8	0.453
x9	0.534

visual	0.408	0.112
textual	0.697	0.101
speed	0.222	0.094

```
> inspect(fit, "sampstat")
```

```
$cov
```

	x1	x2	x3	x4	x5	x6	x7	x8	x9
x1	1.000								
x2	0.284	1.000							
x3	0.415	0.389	1.000						
x4	0.364	0.328	0.232	1.000					
x5	0.319	0.268	0.138	0.688	1.000				
x6	0.422	0.322	0.206	0.720	0.761	1.000			
x7	-0.048	0.061	0.041	0.200	0.023	-0.029	1.000		
x8	0.159	0.105	0.439	-0.029	-0.059	0.183	0.464	1.000	
x9	0.165	0.210	0.258	0.146	0.183	0.230	0.335	0.403	1.000

```
$mean
```

x1	x2	x3	x4	x5	x6	x7	x8	x9
0	0	0	0	0	0	0	0	0

```
$th
```

x1 t1	x2 t1	x3 t1	x4 t1	x5 t1	x6 t1	x7 t1	x8 t1	x9 t1
-0.388	-0.054	0.318	0.180	-0.257	1.024	0.231	1.128	0.626