Data Structure and Algorithm HW1

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1. What if you became a DSA TA?

Proof of conjugate 1

Consider $f(n) \geq 0$ and $g(n) \geq 0$ that $\lim_{n\to\infty} \frac{f(n)}{g(n)} = c$, where $c \in R$
By difinition of limit, there exists $\epsilon > 0$, $n_0 > 0$ such that for all $n > n_0$,

$$|\frac{f(n)}{g(n)} - c| < \epsilon$$

$$-\epsilon < \frac{f(n)}{g(n)} - c < \epsilon$$

$$-\epsilon + c < \frac{f(n)}{g(n)} < \epsilon + c$$

In case that $c < \epsilon$

$$-\epsilon + c < 0 < \frac{f(n)}{g(n)} < \epsilon + c$$

In case that $c \geq \epsilon$

$$0 \leq -\epsilon + c < \frac{f(n)}{g(n)} < \epsilon + c$$

$$0 \leq f(n) < (\epsilon + c)g(n)$$

Let $(\epsilon + c) = c_1$

$$0 \leq f(n) \leq c_1 g(n)$$

which shows $f(n) = O(g(n))$, QED

1.1

Consider $c_1, c_2 > 0$
for all $n > 0$
if

$$\frac{c_1}{c_2} n \geq 1$$

then

$$c_1 n^3 \geq c_2 n^2$$

let

$$n_0 = \frac{c_2}{c_1}$$

$$\frac{c_1}{c_2}n_0 = 1$$

consider $k > 0$

$$\frac{c_1}{c_2}(n_0 + k) = 1 + \frac{c_1}{c_2}k$$

where

$$\frac{c_1}{c_2}k > 0$$

that is, for all $n \geq n_0$,

$$0 \leq c_2 n^2 \leq c_1 n^3$$

therefore, if

$$0 \leq f(x) \leq c_3 n^2$$

for all $n > n_1$ where $n_1, c_3 > 0$
then

$$0 \leq f(n) \leq c_3 n^2 \leq \frac{c_3 c_1}{c_2} n^3$$

for all $n > max(n_0, n_1)$, QED

1-2

Let $t_m = the\ number\ of\ while\ checks$, $k = the\ index\ of\ key$, $d_n = timecostofeachlinetoexecuteonce$
Since the value of m is assigned to l in the beginning of each while loop, m will start from 1 and increase by 1 when while loop is executed once, moreover, $A[m] \leq key$ will always be satisfied.
therefore,

$$t_m = k$$

in worst case, key is not in the array, in this case

$$T(n) = d_1 n + d_2(n-1) + d_3(n-1) + d_4 * 0 + d_5(n-1) + d_6 * 0 + d_7(n-1) + d_8(n-1) + d_9$$

$$T(n) = c_1 n + c_2$$

where c_1, $c_2 > 0$

$$\lim_{n \to \infty} \frac{c_1 n + c_2}{n} = c_1$$

By conjecture 1, time complexity of the algorithm is $O(n)$

1-3

$$f(n) = \Theta(n^2) \iff there\ exists\ positive\ (n_0, c_1, c_2)\ such\ that\ c_1 n^2 \leq f(n) \leq c_2 n^2\ for\ all\ n \geq n_0$$