CS7646 Project 1: Martingale

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Abstract—This project serves as an introduction to Python programming along with computing risk, probability, and betting using a simple toy example of a simple betting strategy for a simple probability-based game.

1 EXPERIMENT 1

1.1 Question 1

The probability of winning \$80 within 1000 sequential bets, given the condition that a player may go into infinite debt, is 1. In this first experiment, out of 10 runs, no run resulted in the player failing to win \$80. This betting strategy starts with a bet amount of \$1 and the bet amount doubles whenever a bet is lost. The player can also go into infinite debt and continue to play, but stops immediately after reaching \$80 in winnings. This means that a player has to win only a few bets in the 1000 opportunities given in order to reach \$80. $2^n - \sum_{x=1}^{n-1} = 2$ so once bets become large enough, the player can reach the \$80 with just two bets. E.g. the player can lose every bet up until bet #7 (bet amount of \$64) and win any two bets afterwards and reach the \$80 threshold.

1.2 Question 2

The estimated expected value after 1000 sequential bets is \$80. Every simulation has terminated with winnings of \$80, so the expected value cannot be anything else. The winnings are exactly \$80 because of the strategy condition that says that once winnings are greater than or equal to \$80, \$80 should be used to fill forward all of the data.

1.3 Question 3

The upper and lower lines representing mean winnings +/- standard deviation show the spread of winnings at each particular bet number. These lines diverge as the bet amount increases exponentially (doubling) until all sample runs reach the termination threshold of \$80. There, the standard deviation lines converge to \$80, which is equal to the mean winnings, which are all exactly \$80. At that point, standard deviation is \$0, as each run has terminated and bets are \$0 and the value of winnings no longer changes.

These lines reach maximum values, however these maximums are sensitive to probability, as any runs that have not yet terminated will continue to drive up the standard deviation until they terminate themselves.

2 EXPERIMENT 2

2.1 Question 4

Out of the 1000 runs in this experiment, 649 runs resulted in winnings reaching the \$80 threshold while the other 351 runs failed to do so. This gives an estimated probability of winning \$80 to be 64.9%.

2.2 Question 5

The mean of the winnings at the end of each of the 1000 experimental runs is -\$37.94. This gives us the expected value of our winnings after 1000 sequential bets. It should be noted that the probability of winning \$80 is over 50%, so an expected value might be “expected” to be greater than \$0, however, since winnings are capped at a maximum of \$80 but are allowed to fall as low as \$256, the losses that make up just 35.1% runs pull down the expected value of winnings.

2.3 Question 6

As before, these lines representing mean +/- standard deviation increase with the number of sequential bets and reach a maximum when all runs have terminated. Then, the lines converge to the mean value, as standard deviation becomes \$0. The lines converge to the mean value of winnings and do not move thereafter, as at that point, all runs have terminated and winnings no longer change.

3 CHARTS

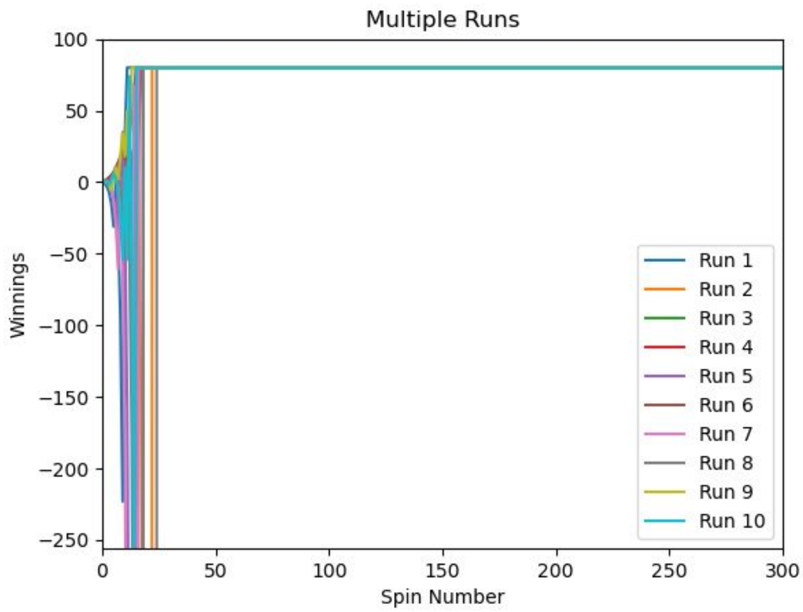

Figure 1—Winnings from 10 runs conducted in Experiment 1.

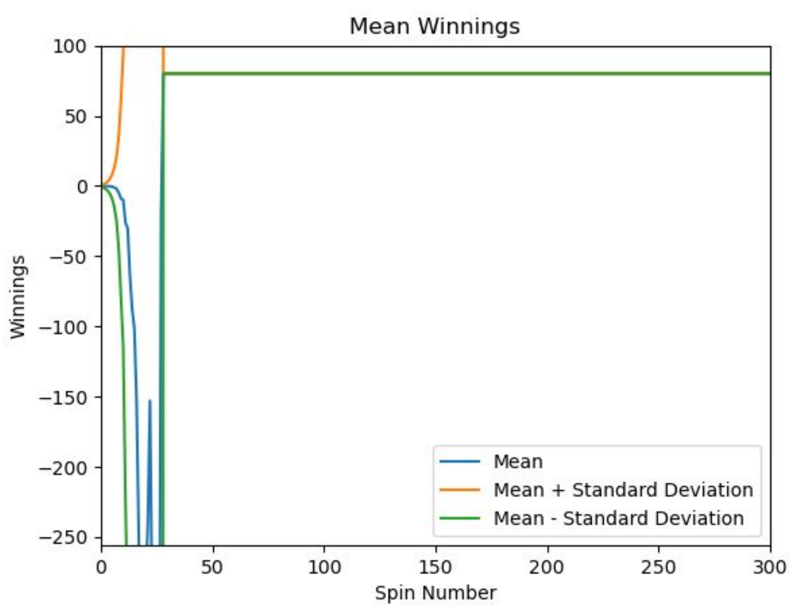

Figure 2—Mean winnings and standard deviation from 1000 runs conducted in Experiment 1.

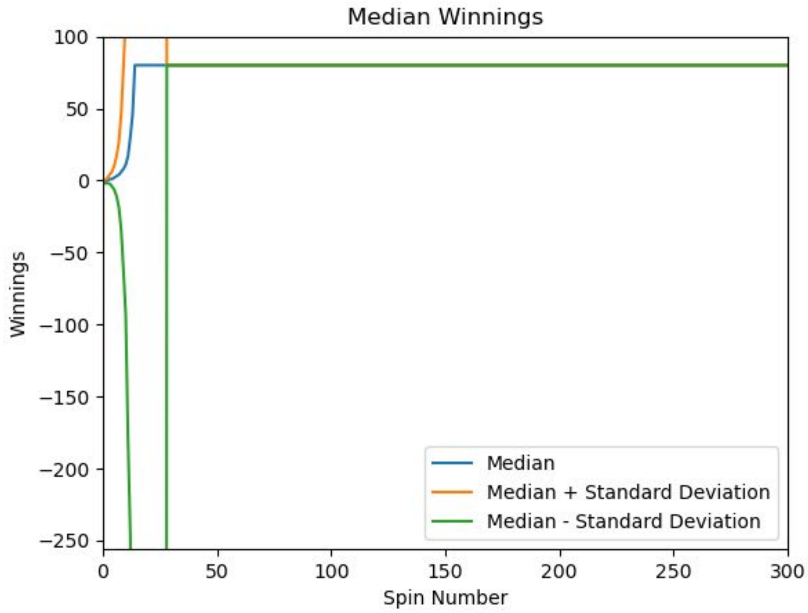

Figure 3—Median winnings and standard deviation from 1000 runs conducted in Experiment 1.

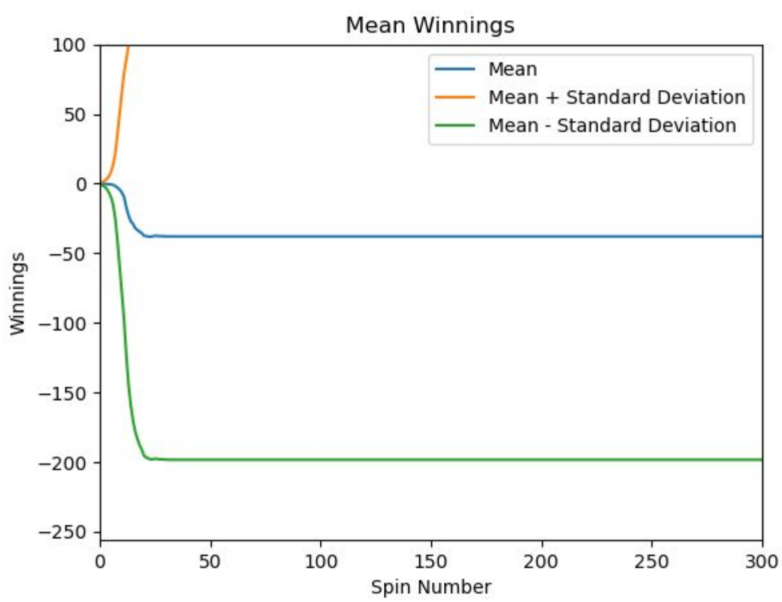

Figure 4—Mean winnings and standard deviation from 1000 runs conducted in Experiment 2.

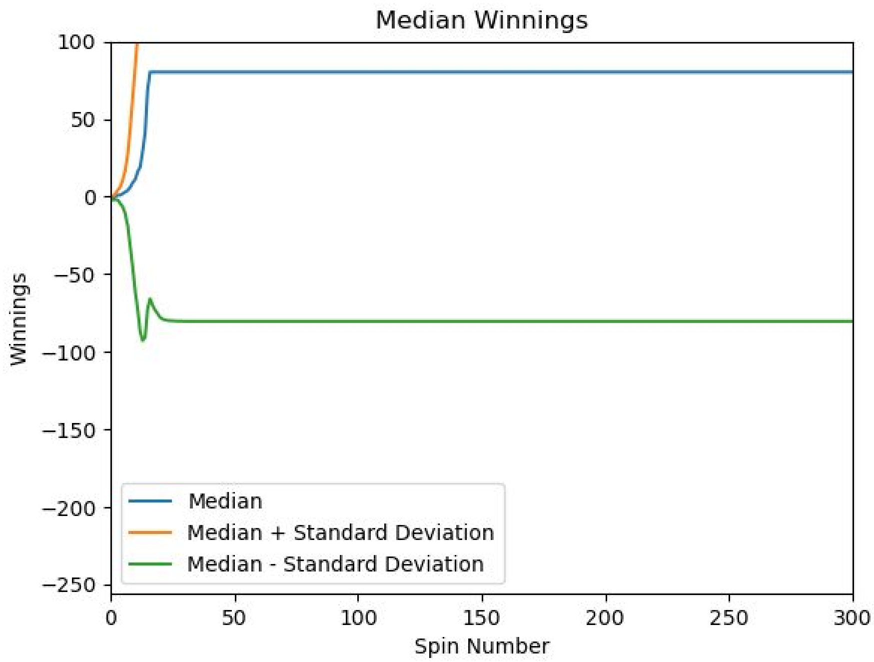

Figure 5—Mean winnings and standard deviation from 1000 runs conducted in Experiment 2.