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## Production, Manufacturing and Logistics

# Supply chain modelling of forest fuel

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### Abstract

We study the problem of deciding when and where forest residues are to be converted into forest fuel, and how the residues are to be transported and stored in order to satisfy demand at heating plants. Decisions also include whether or not additional harvest areas and saw-mills are to be contracted. In addition, we consider the flow of products from saw-mills and import harbors, and address the question about which terminals to use. The planning horizon is one year and monthly time periods are considered. The supply chain problem is formulated as a large mixed integer linear programming model. In order to obtain solutions within reasonable time we have developed a heuristic solution approach. Computational results from a large Swedish supplying entrepreneur are reported.

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**Keywords:** Supply chain management; Integer programming; Inventory; Transportation

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### 1. Introduction

In recent years, the use and importance of bioenergy fuel have increased. In Sweden, its share of the total energy supply increased from 15% to 18% during the first half of the 1990s, see Brunberg et al. [3]. Considering the environmental regulations and taxation on, for example, CO<sub>2</sub> emissions, this share is expected to increase even more in the future. Bioenergy fuel is often used by heating plants, which are normally operated by local communities to provide energy for the cities. The number of such heating plants is steadily increasing. The increased demand of bioenergy fuel has led to an increased demand for decision support tools which can help the complex planning of supplying the heating plant with bioenergy fuel. Since it is important to find high quality plans, there is a need to integrate optimisation models and solution procedures in the decision support tools.

Bioenergy fuel consists of several assortments. One important type is wood fuel which can be further divided into forest fuel, energy forest fuel and recycled wood fuel, see Filipsson [7]. The difference between forest fuel and energy forest fuel is that the latter consists of trees planted in order to be used as fuel. Other

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## Nomenclature

### Variables

$r_{ipt}^F$	volume of product $p$ that is forwarded at source $i$ in time period, $\in I_H \cup I_{HC}$
$r_{ipt}^C$	volume of product $p$ that is chipped at source $i$ in time period $t$ , $i \in I_H \cup I_{HC}$
$x_{ijpt}^N$	volume of non-chipped product that is transported from source $i$ to terminal $j$ in time period $t$ , $i \in I_H \cup I_{HC}$
$x_{ijpt}^C$	volume of chipped product $p$ that is transported from source $i$ to terminal $j$ in time period $t$
$z_{ikpt}$	volume of chipped product $p$ that is transported from source $i$ to heating plant $k$ in time period $t$
$y_{jkpt}$	volume of chipped product $p$ that is transported from terminal $j$ to heating plant $k$ in time period $t$
$L_{ipt}^F$	volume of non-chipped product $p$ that is stored at source $i$ at the end of time period $t$ , $i \in I_H \cup I_{HC}$
$L_{jpt}^N$	volume of non-chipped product $p$ that is stored at terminal $j$ at the end of period $t$
$L_{jpt}^C$	volume of chipped product $p$ that is stored at terminal $j$ at the end of time period $t$
$q_{jpt}$	volume of product $p$ that are chipped at terminal $j$ in time period $t$
$v_{it}^F$	$\begin{cases} 1, & \text{if the forest residues at source } i \text{ is forwarded in period } t \\ 0, & \text{otherwise} \end{cases}$ $\begin{cases} 1, & \text{if the forest residues at source } i \text{ is chipped in period } t \\ 0, & \text{otherwise} \end{cases}$
$v_{it}^C$	$\begin{cases} 1, & \text{if } i \in I_{SC} \text{ and the saw-mill } i \text{ is contracted} \\ 0, & \text{otherwise} \end{cases}$
$w_j$	$\begin{cases} 1, & \text{if the terminal } j \text{ is used} \\ 0, & \text{otherwise} \end{cases}$

### Constants

$s_{ip}$	the volume of product $p$ available at source $i$
$s_{ipt}$	the volume of byproduct $p$ available at saw-mill $i$ in time period $t$
$e_t^F$	the total forwarding capacity in time period $t$
$e_{jt}^C$	the chipping capacity at terminal $j$
$e_t^C$	the total capacity of mobile chipping in time period $t$
$b_j^C$	the storage capacity of chipped products at terminal $j$
$b_j$	the total storage capacity of chipped and non-chipped products at terminal $j$
$d_{kt}$	the demand at heating plant $k$ in time period $t$
$a_{pt}^z$	the energy value of one volume unit of product $p$ transported from an harvest area or saw-mill in time period $t$
$a_{pt}^y$	the corresponding energy value for product $p$ transported from terminals in time period $t$
$A_k^b$	the maximal proportion of byproducts at heating plant $k$

$c_{ijpt}^N$	the transportation cost per volume unit of a non-chipped product $p$ from supply source $i$ to terminal $j$ in time period $t$
$c_{ijpt}^C$	the corresponding transportation cost for a chipped product $p$
$c_{ikpt}^z$	the transportation cost per volume unit of a chipped product $p$ from supply source $i$ to heating plant $k$
$c_{jkpt}^y$	the corresponding transportation cost from terminal $j$ to heating plant $k$
$c_p^F$	the chipping cost per volume unit of product $p$ at a harvest area
$c_p^q$	the corresponding chipping cost at a terminal
$c_i^P$	the purchasing cost per volume unit for supply source $i$
$g_j$	the fixed cost of keeping terminal $j$ open during the planning period
$h_p^F$	the cost per volume unit of storing a non-chipped product $p$ at an harvest area
$h_{jp}^N$	the corresponding cost of a non-chipped product $p$ at terminal $j$
$h_{jp}^C$	the cost per volume unit of storing a chipped product $p$ at terminal $j$
<i>Sets</i>	
$I$	the set of sources
$I_H$	the set of self-owned harvest areas
$I_{HC}$	the set of harvest areas with a potential to be contracted
$I_S$	the set of self-owned saw-mills
$I_{SC}$	the set of saw-mills with a potential to be contracted
$J_C$	the set of terminals with permanent chipping capacity
$J_{NC}$	the set of terminals without chipping capacity
$J$	the set of terminals
$K$	the set of heating plants
$K_b$	the set of heating plants requiring a maximal proportion of byproducts in the fuel
$P$	the set of products
$P_b$	the set of byproducts
$T$	the set of time periods

types of bioenergy fuel are reed fuel, straw fuel and waste paper. The heating plants can use several of these bioenergy fuel types, but in this paper we focus only on forest fuel, and on the problem of satisfying a given demand of forest fuel at a number of heating plants.

The supply of forest fuel is provided by companies which are obliged by contract to deliver a certain amount of bioenergy (forest fuel), specified in MW h, for each time period (normally a month) during the time of the contract. In most contracts there is also a clause that makes it possible for the heating plant to reduce or increase the demanded amount of energy up to 10–15%, incurring a penalty cost for the heating plant. The main reason for including such a clause is for the heating plant to have the possibility to adapt to unexpected cold or warm weather.

The problem we consider in this paper is the problem of the supplying company, which is to minimise the total cost for satisfying the demand given by the contract.

Forest fuel is mainly obtained from forest residues in harvest areas or from byproducts from saw-mills. Both harvest areas and saw-mills can be either owned by the company or available to the company by long-term contracts. Forest residues are branches and tops left in the harvest areas after the logs have been transported to, for example, saw-mills or pulp-mills. The forest residues have to be chipped (converted into small pieces) before they can be used as fuel by the heating plants, and the chipping can be made either

directly at the harvest area or at a terminal, before transported to a heating plant. Byproducts from sawmills consist of bark and sawdust, and they can either be transported directly to the heating plants, or to a terminal for storage and use in a forthcoming period. There is also a possibility to import different types of forest fuel, mainly by boat from Russia and the Baltic states.

Terminals are needed in order to balance the seasonal fluctuation in demand at the heating plants. At terminals we can store non-chipped and chipped forest residues as well as byproducts from saw-mills.

The problem is a true supply chain problem as there are multiple sources (harvest areas, saw-mills and import harbors), several intermediate terminals, several demand nodes (heating plants), different types of forest fuel and several time periods. The supply chain problem of the company contains decisions concerning which type of fuel to use, the timing of forwarding and chipping, the location of chipping, the storage at terminals, and the design of transportation pattern. Main decisions are also whether or not a harvest area or saw-mill should be contracted and if a specific terminal is to be used. In addition we have to consider restrictions on capacities of chipping, forwarding and storage at terminals. An illustration of the possible flows is given in Fig. 1.

Supply chain modelling and supply chain management have received a lot of attention among companies in recent years. It provides a tool for integrated planning of several interrelated planning situations. A driving force to the development of supply chain management systems has been the development of company wide databases for data collection and efficient optimisers to solve the resulting, often large, optimisation models. A basic description of supply chain modelling is found in Shapiro [10]. A more detailed description of industrial cases can be found in Stadtler and Kilger [11]. Examples include a case of computer assembly by Kilger [8] and a case about food and beverages by Wagner and Meyr [12].

With respect to forestry planning we can mention Carlsson and Rönnqvist [4] who give an overview of supply chain modelling in the Swedish forestry industry. The decisions made for the forest fuel in our case are similar to the decisions made by harvest planners in their case. They describe a need for an annual planning where scheduling of harvest areas is made. At the same time there is a need to come up with efficient transportation and storage planning on a monthly basis. One difference is that the supply of timber is unknown to a greater extent. This is so because no detailed information regarding the timber volume at

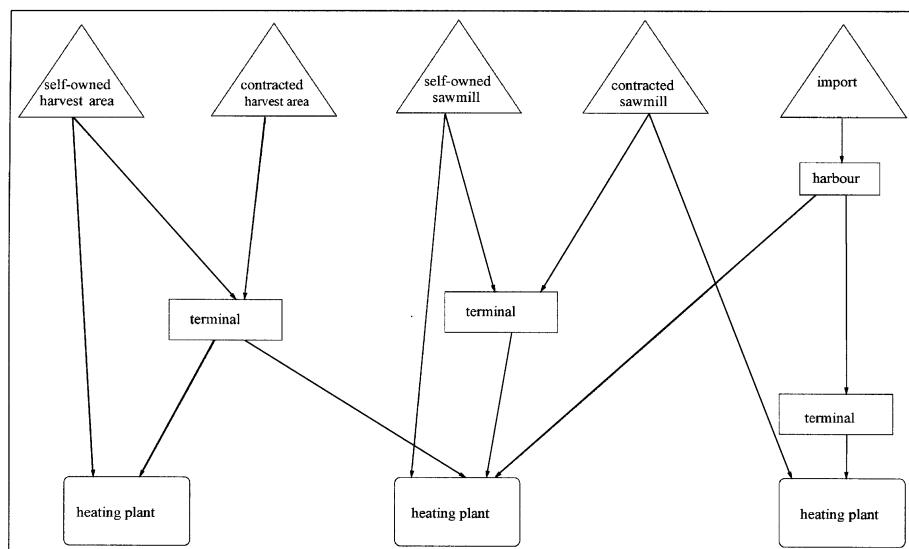


Fig. 1. An illustration of the possible transportation flows in the model.

the planned harvest areas, is collected (and if there is, it often includes faulty estimates) and several products or assortment are involved. Storage is also more difficult, as the value of logs decreases as it gets older. The demand at saw-, pulp- and paper-mills are also known to a less extent. There are, for example, no contracts stating the actual amount.

We formulate an optimisation model describing the planning problem for the supplying company. The model is a mixed integer programming model and we show that the problem can be solved in reasonable time using a heuristic solution procedure. The model can be used both as a tool for tactical planning, and as a strategic tool to analyse the effects on the current planning in various situations.

Typical strategic planning situations are, for example, when

- the company is competing on a new contract and needs to submit competitive contract prices;
- the company is analyzing the sensitivity of a solution to variations in demand (within the contract specification), for example an increase in demand during a particular month due to cold weather;
- a new terminal is available, or the capacity of an existing terminal may be changed, and the potential impact is to be found;
- chipping or transportation capacity is changed;
- chipping technology is changed, hence also the chipping cost;
- a haulage contractor is negotiating the transportation costs.

We have found very few examples dealing with fuel transportation in the literature. In Eriksson and Björheden [6], a linear programming model for solving a fuel transportation problem is presented. One conclusion in that paper is that the transportation costs constitute the most essential part of the total costs. It is also concluded that, contrary to practice, the optimal solution often included the use of flows with mobile chippers and direct transportation to the heating plants. A drawback with the model is that it contains no binary variables. Decisions regarding whether or not to contract a harvest area or saw-mill, and decisions about the time period for forwarding and chipping, cannot be analysed with the model. These decisions are essential in our problem. Our model also includes capacity constraints on storage.

Our mathematical model includes several components from traditional optimisation models. The basis of the model can be considered as a version of a two level facility location problem. Here, the upper level is represented by the harvest areas or saw-mills, the middle level by the terminals, and the lower level by the heating plants. In our application, the heating plants can be viewed as customers defining the demand and the harvest areas or saw-mills as facilities to be opened or not. Furthermore, the problem can be given a natural network representation for the different flows. Finally, we also have a time expanded model as we deal with a multi-period problem.

The outline of the paper is as follows. In Section 2 we describe the supply chain problem from harvest areas to heating plants. Then, in Section 3, we formulate the mathematical model for the problem. In Section 4 we describe the solution method and present computational results using data from a real-life case study. The case is obtained from Sydved Energileveranser AB, which is one of the largest Swedish companies delivering forest fuel to heating plants, and the results include an analysis of a number of different scenarios. Finally, in Section 5, we make some concluding remarks.

## 2. Problem description

### 2.1. Supply of forest fuel

The supplying company obtains forest fuel from several sources, namely harvest areas, saw-mills, and import sources.

The harvest areas can be classified into two groups; self-owned harvest areas and contracted harvest areas. At harvest areas that are owned by the company, the forest residues have to be removed during the planning period, but the products are available without extra cost. The forest residues in contracted harvest areas can be made available by entering into a contract with a supplier of residues. The contract specifies the price of the residues. An important question for the company is whether or not a contract should be entered.

For each harvest area, the volumes of different types of residues (products) can be estimated. As the supply of residues in the forest is based on the harvest of logs from the previous year it is relatively easy to get a nearly exact estimate. The reason is that most harvesting is done during winter and the residues are dried during the summer. As all logs are carefully measured at saw-, pulp- and paper-mills we also get as a side-product the required estimates for residues. The saw-mills are normally in full production during the whole year (besides planned vacations, etc.) and the supply from these mills are constant.

The three products we consider are forest residues from softwood, forest residues from hardwood, and decay damaged wood. Once the residues have dried, they are forwarded by special vehicles and collected into piles in the harvesting areas. The forwarding has to be made in the same time period for all products in each area, and a main decision is when to forward. After forwarding, the forest residues can be either chipped in the forest by mobile chippers or transported to a terminal for later chipping. If the forest residues are chipped in the forest, they can be transported directly to the heating plants. There is a trade-off in cost between chipping at harvest areas and chipping at terminals. It is cheaper to chip at terminals, but the transportation of non-chipped forest residues is more expensive, as compared to chipped products. After chipping in the forest, the transportation from the harvesting area has to be done immediately, since wood chips cannot be stored in the forest. Non-chipped forest residues can however be stored at the harvest areas for later chipping and transportation.

Saw-mills are also classified into self-owned saw-mills and contracted saw-mills. In the latter group the products become available after entering into a contract with the saw-mill covering the whole planning period and specifying the price for the byproducts. For each saw-mill, there is an estimated volume of each type of byproduct (bark, sawdust and dry chips) that is produced in each time period. The byproducts must be removed on a continuous basis, often several times in every time period, since there is no storage capacity at the saw-mills. The products can either be transported to terminals, or transported directly to heating plants.

It is also possible to import non-chipped and chipped forest residues, and even bark, sawdust and dry chips. For each option, volume, product type, delivery time, and price are specified. In the case of import, the products are first stored at a terminal adjacent to the harbor and later, depending on the form of the products, transported to terminals or directly to heating plants.

## *2.2. Terminals*

Terminals are used to balance seasonal variation of supply and demand and also to offer more chipping possibilities. At terminals, both non-chipped and chipped forest residues can be stored. Some terminals have permanent chipping capacity, others can only chip when a mobile chipper is located at the terminal. Each terminal has a specified total storage capacity. Moreover, there is a separate capacity for chipped products, the reason is that chipped products must be stored on a hard (for example concrete) surface and the size of this surface is limited. Forest residues, on the other hand, can be stored on any surface. There are different costs of storage depending on the type of products, one reason is that chipped products have to be protected against rain. Another reason is that the storage of forest residues deteriorates the energy value. As regarding decay damaged wood, the situation is quite the opposite. The storing of decay damaged wood improves the energy value, see Brunberg et al. [3].

Terminals adjacent to harbors are used only for imported forest products and they have no chipping possibility. Terminals located at heating plants have no storage capacity, and the chipping and further use are made in the same time period as the residues arrive.



Fig. 2. Demand profile during one year.

### 2.3. Heating plants

Due to the climate in Sweden, there is large fluctuation in demand for energy during a year. An example of an annual demand for a Swedish supplying entrepreneur is given in Fig. 2. During the summer, there is essentially no demand; during the winter, however, it increases considerably. The demand at the heating plants is specified in MWh for each time period and the volume of chipped products used to satisfy the demand must therefore be converted into energy values. The energy value depends on several factors, such as the kind of products and their moisture content. Bark has the highest moisture content (50–60%) and chipped decay damaged wood has the lowest moisture content (40%), see Arlinger et al. [1].

Some of the heating plants require a certain maximal proportion of bark and sawdust in the fuel composition. The reason is that too much byproducts can damage the heating furnaces. A few heating plants also have a limited capacity to receive non-chipped products.

### 2.4. Transportation

The transportation is done by haulier companies using different types of trucks. There are, for example, different types of trucks that carry the forest residues as compared to those carrying the chipped products. The key question for the transportation planning is whether chipped products should be first transported to terminals or transported directly to heating plants. Forest residues that are not chipped at harvest areas

Table 1  
The product types and their possible origins and destinations

Type	From	To
Bark, sawdust, dry chips	Saw-mill	Terminal/plant
Forest residue from soft or hardwood	Forest	Terminal
Decay damaged wood	Forest	Terminal
Chips from soft or hardwood	Forest/terminal	Terminal/plant
Bark, sawdust, dry chips	Harbor	Terminal/plant
Chips from softwood or hardwood	Harbor	Terminal/plant
Forest residue from soft or hardwood	Harbor	Terminal

must be transported to terminals. If heating plants with a possibility to receive non-chipped forest residues exist, this situation can be treated by defining terminals adjacent to the heating plants. The cost varies with distance and type of products. There may be a limited total transport capacity to consider for each time period. The various flows of different types/products are given in Table 1.

### 3. Mathematical model

In this section we present the mathematical model of the forest fuel supply chain problem. We first describe the sets of variables, then follows the constraints and the objective function. The full model is given in the Appendix A.

Let  $I$  be the set of supply sources,  $J$  the set of terminals,  $P$  the set of products,  $K$  the set of heating plants and  $T$  the set of time periods. The set of supply sources contains subsets for self-owned harvest areas ( $I_H$ ), harvest areas with a potential to be contracted ( $I_{HC}$ ), self-owned saw-mills ( $I_S$ ) and saw-mills with a potential to be contracted ( $I_{SC}$ ). The set of terminals contains subsets for terminals with permanent chipping capacity ( $J_C$ ) and for terminals without chipping capacity ( $J_{NC}$ ). We will use index  $i$  for sources,  $j$  for terminals,  $k$  for heating plants,  $p$  for products and  $t$  for time periods. Unless otherwise stated we assume that definitions using for example index  $i$  is valid for all  $i \in I$ .

#### 3.1. Variables

First we define variables representing the supply of forest residues in each time period  $t \in T$ . The volumes of forwarding and chipping at a harvest area can be defined as

$$\begin{aligned} r_{ipt}^F & \text{ volume of product } p \text{ that is forwarded at source } i \text{ in time period } t, i \in I_H \cup I_{HC}, \\ r_{ipt}^C & \text{ volume of product } p \text{ that is chipped at source } i \text{ in time period } t, i \in I_H \cup I_{HC}. \end{aligned}$$

We also need variables representing the transportation flows of products from sources to terminals, from sources to heating plants, and from terminals to heating plants. We define

$$\begin{aligned} x_{ijpt}^N & \text{ volume of non-chipped product } p \text{ that is transported from source } i \text{ to terminal } j \text{ in time period } t, \\ & i \in I_H \cup I_{HC}, \\ x_{ijpt}^C & \text{ volume of chipped product } p \text{ that is transported from source } i \text{ to terminal } j \text{ in time period } t, \\ z_{ikpt} & \text{ volume of chipped product } p \text{ that is transported from source } i \text{ to heating plant } k \text{ in time period } t, \\ y_{jkpt} & \text{ volume of chipped product } p \text{ that is transported from terminal } j \text{ to heating plant } k \text{ in time period } t. \end{aligned}$$

Variables related to storing in harvest areas and at terminals can be defined as

$$\begin{aligned} L_{ipt}^F & \text{ volume of non-chipped product } p \text{ that is stored at source } i \text{ at the end of time period } t, i \in I_H \cup I_{HC}, \\ L_{jpt}^N & \text{ volume of non-chipped product } p \text{ that is stored at terminal } j \text{ at the end of period } t, \\ L_{jpt}^C & \text{ volume of chipped product } p \text{ that is stored at terminal } j \text{ at the end of time period } t, \end{aligned}$$

where the volumes for  $t = 0$  define the initial conditions.

We also need variables representing chipping at terminals, and we define

$$q_{jpt} \quad \text{volume of product } p \text{ that is chipped at terminal } j \text{ in time period } t.$$

All variables defined so far are continuous variables, and they can be interpreted as network flow variables in a multi-commodity network describing the possible flows of products from harvest areas and saw-mills to the heating plants. In Fig. 3 we describe the subnetwork for one harvest area and one heating plant. For a saw-mill the corresponding network is given in Fig. 4, where  $s_{ipt}$  denotes the given volume of byproduct  $p$  that has to be picked up in period  $t$  at sources  $i$ ,  $i \in I_S \cup I_{SC}$ . Since byproducts from saw-mills only exists in the form of chipped products, the network structure is much simpler.

We also need some sets of binary variables in the model formulation. For the harvest areas we define

$$v_{it}^F = \begin{cases} 1, & \text{if the forest residues at source } i \text{ is forwarded in period } t, i \in I_H \cup I_{HC}, \\ 0, & \text{otherwise,} \end{cases}$$

and

$$v_{it}^C = \begin{cases} 1, & \text{if the forest residues at source } i \text{ is chipped in period } t, i \in I_H \cup I_{HC}, \\ 0, & \text{otherwise.} \end{cases}$$

The variables regarding forwarding is also used to represent the decision whether or nor a harvest area should be contracted. The possibility to contract saw-mills is given by the variables

$$u_i = \begin{cases} 1, & \text{if } i \in I_{SC} \text{ and the saw-mill } i \text{ is contracted,} \\ 0, & \text{otherwise,} \end{cases}$$

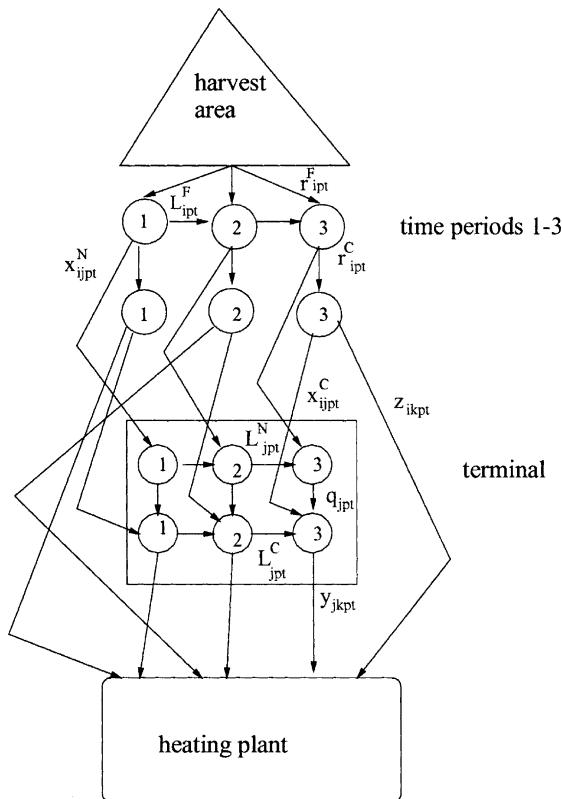


Fig. 3. Possible flows from a harvest area to a heating plant.

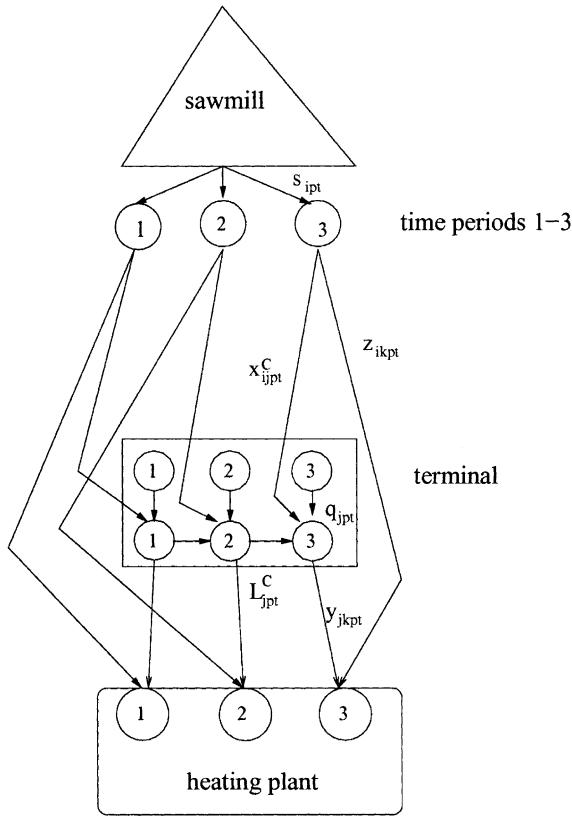


Fig. 4. Possible flows from a saw-mill to a heating plant.

and the set of variables related to the use of terminals is defined as

$$w_j = \begin{cases} 1, & \text{if the terminal } j \text{ is used,} \\ 0, & \text{otherwise.} \end{cases}$$

### 3.2. Constraints

To describe the need for forwarding at different harvest areas we have the constraints

$$\sum_{t \in T} v_{it}^F = 1, \quad \forall i \in I_H, \quad (1)$$

and

$$\sum_{t \in T} v_{it}^F \leq 1, \quad \forall i \in I_{HC}. \quad (2)$$

Constraints (1) specify that each self-owned harvest area has to be forwarded exactly once (in exactly one time period) during the planning period, and constraints (2) specify that each potential harvest area to be contracted has to be forwarded at most once during the planning period. If a constraint in (2) is satisfied with strict inequality, no forwarding takes place in any of the time periods in that harvest area, which is interpreted as not contracting the harvest area. In such a case, there is no supply from the harvest area.

The constraints

$$r_{ipt}^F = s_{ip}v_{it}^F, \quad \forall i \in I_H \cup I_{HC}, \quad \forall p \in P, \quad \forall t \in T, \quad (3)$$

where  $s_{ip}$  is the volume of product  $p$  available at source  $i$ , ensure that the forwarded volume of a product in a period is equal to the available volume of forest residues.

To assure that all the forest residues at harvest area  $i$  are chipped in the same time period, we have the constraints

$$\sum_{t \in T} v_{it}^C \leq 1, \quad \forall i \in I_H \cup I_{HC}, \quad (4)$$

and

$$r_{ipt}^C = s_{ip}v_{it}^C, \quad \forall i \in I_H \cup I_{HC}, \quad \forall p \in P, \quad \forall t \in T. \quad (5)$$

These constraints also assure that the chipped quantity is equal to the available quantity. If a constraint in (4) is satisfied with strict inequality, we get from (5) that nothing is chipped in the corresponding harvest area.

The network structure described in Figs. 3 and 4 indicate that we need some sets of flow balancing constraints. For non-chipped residues at the harvest areas, the balancing constraints can be formulated as

$$L_{ip,t-1}^F + r_{ipt}^F = L_{ipt}^F + r_{ipt}^C + \sum_{j \in J} x_{ijpt}^N, \quad \forall i \in I_H \cup I_{HC}, \quad \forall p \in P, \quad \forall t \in T, \quad (6)$$

and for chipped products at the harvest areas the constraints become

$$r_{ipt}^C = \sum_{j \in J} x_{ijpt}^C + \sum_{k \in K} z_{ikpt} \quad \forall i \in I_H \cup I_{HC}, \quad \forall p \in P, \quad \forall t \in T. \quad (7)$$

Constraints (7) ensure that the volume of chipped products in every time period has to be further transported in the same period, either directly to a heating plant or to a terminal.

Since byproducts from saw-mills have to be transported in the same time period they become available, either directly to a heating plant or to a terminal for later transportation, the flow balancing constraints become

$$\sum_{j \in J} x_{ijpt}^C + \sum_{k \in K} z_{ikpt} = s_{ipt}, \quad \forall i \in I_S, \quad \forall p \in P, \quad \forall t \in T, \quad (8)$$

and

$$\sum_{j \in J} x_{ijpt}^C + \sum_{k \in K} z_{ikpt} = s_{ipt}u_i, \quad \forall i \in I_{SC}, \quad \forall p \in P, \quad \forall t \in T. \quad (9)$$

Constraints (8) regard self-owned saw-mills and constraints (9) regard saw-mills with potential to be contracted.

The balancing constraints for non-chipped products at terminals are expressed as

$$L_{jp,t-1}^N + \sum_{i \in I_H \cup I_{HC}} x_{ijpt}^N = L_{jp,t}^N + q_{jpt}, \quad \forall j \in J, \quad \forall p \in P, \quad \forall t \in T, \quad (10)$$

and in a similar way the balancing constraints for chipped products at terminals are expressed as

$$L_{jp,t-1}^C + \sum_{i \in I} x_{ijpt}^C + q_{jpt} = L_{jp,t}^C + \sum_{k \in K} y_{jkpt}, \quad \forall j \in J, \quad \forall p \in P, \quad \forall t \in T. \quad (11)$$

Note that the products originating from the saw-mills are included in the last set of constraints.

We also have to consider a number of capacity restrictions regarding forwarding at harvest areas, permanent and mobile chipping and storing of products at terminals. Let the total forwarding capacity in period  $t$  be denoted by  $e_t^F$ . Then, the constraints

$$\sum_{i \in I_H \cup I_{HC}} \sum_{p \in P} r_{ipt}^F \leq e_t^F, \quad \forall t \in T, \quad (12)$$

make sure that this capacity is not exceeded, and they restrict the possibility to forward forest residues in too many harvest areas in the same period.

At terminals with permanent chipping equipment there is a restriction on the volume that can be chipped each time period. Let  $e_{jt}^C$  denote the chipping capacity at terminal  $j$ . Also, the chipping capacity of the mobile chippers working at terminals and at harvest areas gives a restriction on the total volume that can be chipped in each time period. Let  $e_t^C$  denote the total capacity of mobile chipping in time period  $t$ . The capacity constraints on chipping can then be formulated as

$$\sum_{p \in P} q_{jpt} \leq e_{jt}^C, \quad \forall j \in J_C, \quad \forall t \in T, \quad (13)$$

and

$$\sum_{i \in I_H \cup I_{HC}} \sum_{p \in P} r_{ipt}^C + \sum_{j \in J_{NC}} \sum_{p \in P} q_{jpt} \leq e_t^C, \quad \forall t \in T. \quad (14)$$

The terminals have a limited storage capacity of products. Let  $b_j^C$  denote the storage capacity of chipped products at terminal  $j$ , and let  $b_j$  denote the total storage capacity of chipped and non-chipped products at terminal  $j$ . To ensure that the volumes stored at a terminal never exceeds the stated storage capacities we formulate the constraints

$$\sum_{p \in P} L_{jpt}^C + \sum_{p \in P} \sum_{k \in K} y_{jkpt} \leq b_j^C, \quad \forall j \in J, \quad \forall t \in T, \quad (15)$$

and

$$\sum_{p \in P} L_{jpt}^C + \sum_{p \in P} \sum_{k \in K} y_{jkpt} + \sum_{p \in P} L_{jpt}^N \leq b_j w_j, \quad \forall j \in J, \quad \forall t \in T. \quad (16)$$

In constraints (15) the capacity is defined as the sum of the volume of chipped products stored at the end of the period and the volume transported from the terminal during the period. This volume is then an upper bound on the volume stored at any point in time during the period. In constraints (16) the volume of non-chipped products is added to define the total volume stored during period  $t$ . Constraints (16) also ensure that nothing can be transported to or from a terminal which is not opened ( $w_j = 0$ ).

Finally, we have to express constraints ensuring that the demand at the heating plants is satisfied. The demand at heating plant  $k$  in time period  $t$  is denoted by  $d_{kt}$ . This demand is however expressed in terms of energy (MW h), but all products transported to the heating plant from harvest areas, saw-mills, or terminals are expressed in volume units. We therefore need to introduce conversion factors from volume to energy. Let  $a_{pt}^z$  denote the energy value of one volume unit of product  $p$  transported from an harvest area or saw-mill in time period  $t$ , and let  $a_{pt}^y$  denote the corresponding energy value for products transported from a terminal. Note that the energy value can be different from one time period to another. The demand constraints can now be expressed as

$$\sum_{i \in I} \sum_{p \in P} a_{pt}^z z_{ikpt} + \sum_{j \in J} \sum_{p \in P} a_{pt}^y y_{jkpt} = d_{kt}, \quad \forall k \in K, \quad \forall t \in T. \quad (17)$$

These constraints can also be interpreted as flow balancing constraints for the heating plant nodes in the networks expressed in Figs. 3 and 4.

Byproducts from saw-mills (i.e. bark, sawdust and dry chips) are less attractive to use as fuel, and some heating plants therefore require a maximal proportion of byproducts in the volumes received. There may also exist similar restrictions regarding bark, sawdust and dry chips separately. Let the maximal proportion of byproducts at heating plant  $k$  be denoted by  $A_k^b$ . We can then express this restriction as

$$\sum_{i \in I} \sum_{p \in P_b} z_{ikpt} + \sum_{j \in J} \sum_{p \in P_b} y_{jkpt} \leq A_k^b \left( \sum_{i \in I} \sum_{p \in P} z_{ikpt} + \sum_{j \in J} \sum_{p \in P} y_{jkpt} \right), \quad \forall k \in K_b, \quad \forall t \in T, \quad (18)$$

where  $P_b$  denotes the set of byproducts and  $K_b$  denotes the set of heating plants requiring a maximal proportion of byproducts in the fuel.

### 3.3. Objective function

The objective for the supplying company is to minimise the total cost for satisfying the contracted demand at the heating plants. The total cost can be expressed as

$$z = C^{\text{tran}} + C^{\text{chip}} + C^{\text{purc}} + C^{\text{term}} + C^{\text{stor}},$$

where  $C^{\text{tran}}$  = transportation cost,  $C^{\text{chip}}$  = chipping cost,  $C^{\text{purc}}$  = purchasing cost,  $C^{\text{term}}$  = terminal cost, and  $C^{\text{stor}}$  = storage cost.

Let  $c_{ijpt}^N$  be the transportation cost per volume unit of a non-chipped product  $p$  from supply source  $i$  to terminal  $j$  in time period  $t$ , and let  $c_{ijpt}^C$  be the corresponding transportation cost for a chipped product  $p$ . Let  $c_{ikpt}^z$  be the transportation cost per volume unit of a chipped product  $p$  from supply source  $i$  to heating plant  $k$ , and let  $c_{jkpt}^y$  be the corresponding transportation cost from terminal  $j$  to heating plant  $k$ . The total transportation cost can now be expressed as

$$C^{\text{tran}} = \sum_{i \in I_H \cup I_{HC}} \sum_{j \in J} \sum_{p \in P} \sum_{t \in T} c_{ijpt}^N x_{ijpt}^N + \sum_{i \in I} \sum_{j \in J} \sum_{p \in P} \sum_{t \in T} c_{ijpt}^C x_{ijpt}^C + \sum_{i \in I} \sum_{k \in K} \sum_{p \in P} \sum_{t \in T} c_{ikpt}^z z_{ikpt} + \sum_{j \in J} \sum_{k \in K} \sum_{p \in P} \sum_{t \in T} c_{jkpt}^y y_{jkpt}, \quad (19)$$

where the four terms express transportation of non-chipped products from supply sources to terminals, transportation of chipped products from supply sources to the terminals, transportation of chipped products from supply sources to heating plants, and transportation from terminals to heating plants, respectively.

Let  $c_p^F$  be the chipping cost per volume unit of product  $p$  at a harvest area, and let  $c_p^q$  be the corresponding chipping cost at a terminal. The total chipping costs can then be expressed as

$$C^{\text{chip}} = \sum_{i \in I} \sum_{p \in P} \sum_{t \in T} c_p^F r_{ipt}^C + \sum_{j \in J} \sum_{p \in P} \sum_{t \in T} c_p^q q_{jpt}, \quad (20)$$

where the first term expresses the chipping costs at harvest areas and the second term expresses the chipping costs at terminals.

Let  $c_i^P$  be the fixed purchasing cost for supply source  $i$ , which can be either a harvest area or a saw-mill. The total purchasing cost can then be expressed as

$$C^{\text{purc}} = \sum_{i \in I_{HC}} c_i^P \left( \sum_{t \in T} v_{it}^F \right) + \sum_{i \in I_{SC}} c_i^P u_i, \quad (21)$$

where the first term expresses the purchase cost of harvest areas and the second term expresses the purchase cost of saw-mills.

Let  $g_j$  be the fixed cost of keeping terminal  $j$  open during the planning period. The total terminal costs can then be expressed as

$$C^{\text{term}} = \sum_{j \in J} g_j w_j. \quad (22)$$

Finally, we have to express the storage cost at terminals and harvest areas. Let  $h_p^F$  be the cost per volume unit of storing a non-chipped product  $p$  at an harvest area, and let  $h_{jp}^N$  be the corresponding cost of a non-chipped product  $p$  at terminal  $j$ . Let  $h_{jp}^C$  be the cost per volume unit of storing a chipped product  $p$  at terminal  $j$ . We can then express the total storage costs as

$$C^{\text{stor}} = \sum_{i \in I_H \cup I_{HC}} \sum_{p \in P} \sum_{t \in T} h_p^F L_{ipt}^F + \sum_{j \in J} \sum_{p \in P} \sum_{t \in T} h_{jp}^N L_{jpt}^N + \sum_{j \in J} \sum_{p \in P} \sum_{t \in T} h_{jp}^C L_{jpt}^C, \quad (23)$$

where the first term expresses the cost of storing at harvest areas, and where the second and the third term express the costs of storing non-chipped and chipped products, respectively, at terminals.

### *3.4. Including import in the model*

We mentioned in Section 2 that import is an alternative supply source but import is not explicitly included in the mathematical model. In fact, all considered products can be imported, also non-chipped forest residues. There are some special considerations with respect to import. Firstly, if an import alternative is chosen the volume delivered is fixed. Secondly, once the product is delivered to the harbor it must be further transported as soon as possible to some terminal, since the harbor can not to be used as a storage location. It is possible to include the above aspects into the model by introducing new sets of variables and constraints. The variables then represent the flows of products and/or non-chipped forest residues to the terminals. The constraints specify the flow to the delivered volumes and enforces transportation to terminals in the same time period. However, we can note that an import alternative is very similar to a supply from a contracted saw-mill. The major differences from a modelling point of view are: if a saw-mill is contracted then it generates a pre-specified volume of different products for several time periods, whereas if an import alternative is chosen the volume occurs in only one time period; moreover, a saw-mill do not supply non-chipped forest residues. The first aspect makes no difference from a modelling point of view since the import alternative can be interpreted as supply and a simplification of the saw-mill covering only one time period. To allow transportation of non-chipped forest residues we can easily modify the saw-mill alternative. In summary, instead of introducing a new sets of variables and constraints to describe the possibility to utilise import, we model this alternative by simply making some minor adjustments to the contracted saw-mill supply alternative.

## **4. Solution methods and computational results**

In this section we present a solution procedure for the proposed supply chain model, and we show that the model can be solved in reasonable time using data from a real-life case.

The test problem is given from the Swedish entrepreneur of Sydved Energileveranser AB. Sydved is one of the largest suppliers of bioenergy fuel in Sweden with an annual turnover of about \$US 37 million. The company is fully owned by two major forest companies, StoraEnso and Munksjö, and it has therefore a large number of harvest areas which can be considered as self-owned. The company is divided into four geographical regions, and the case data is taken from region North, which includes the counties of Värmland, Närke, Södermanland, Stockholm, Uppland and Västmanland, see Fig. 5. All the harvest areas and saw-mills in this test case are self-owned.

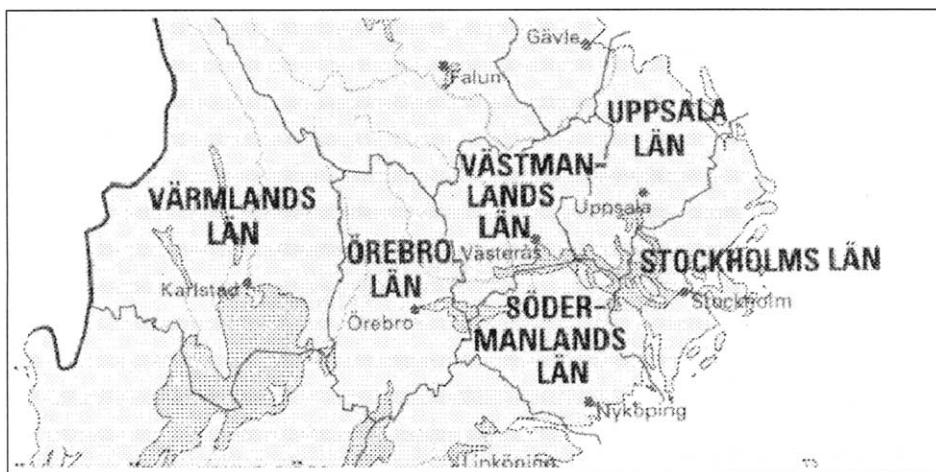


Fig. 5. The case study area in Sweden.

Information regarding the size of the test problem is given in Table 2. It can be noticed that the number of variables and constraints in the problem is very large. The demand at the included heating plants is given in Table 3.

#### 4.1. Solution methods

We have tested two approaches to solve the problem. The first approach is to use the integer programming solver CPLEX 6.5 [5] directly, and the second approach is to use a heuristic. The main steps of the heuristic can be described as follows.

Table 2  
The size of the test problem

Number of harvest areas	426
Number of saw-mills	3
Number of terminals	4
Number of heating plants	7
Number of time periods	12
Total number of variables	505 112
Number of binary variables	10 201
Number of constraints	63 122

Table 3  
Demand at the heating plants

Heating plant	Demand in MWh
Brista	25 000
Eskilstuna	100 000
Gruvön	30 300
Heden	215 000
Högdalen	50 000
Munkfors	50 000
Ösmo	16 000

1. Solve the linear relaxation of the problem.
2. If there are no variables with fractional values in the solution, then stop.
3. Select a fractional variable according to the following criterion:
  - (i) select the earliest time period for which an integer variable is fractional, and
  - (ii) within the time period obtained from (i), select a variable representing as early activity in the supply chain as possible (forwarding, chipping).
4. Set the value of the selected fractional variable to 1. Go to step 1.

This sequential procedure is repeated until all relaxed binary variables have obtained integer values. The procedure can be interpreted as a depth first search in a restricted Branch & Bound scheme for integer programming. Similar ideas are described in Barnhart et al. [2] and used in e.g. airline schedule planning.

The modelling language AMPL [9] has been used to model the problem and to implement the heuristic. The problem was solved using a Sun10 workstation. As LP-solver we have used CPLEX. To solve the integer problem directly using CPLEX we used the default settings, but we introduced some additional termination criteria (time and number of nodes in the search tree). The tolerance from the optimal integer solution was set to 1%.

The proposed heuristic does not guarantee feasible solutions to any set of data. Infeasibilities might occur because, for example, the demand is not satisfied or certain capacity constraints are violated. However, in our case each fixing of a variable to one results in a situation where more supply than needed is generated. The reason is that we always fix variables in time order. This also means that we might fix a variable with a fractional value of 0.1 before a variable with value 0.9, but this latter variable relates to a later time period. The reason is again to guarantee feasibility with respect to the demand constraints. Demand should therefore always be possible to meet. The capacity constraints are normally very high and as there are costs associated with using capacity, the model does not tend to have solutions being close to capacity levels. In all our tests we have never come across a situation where we generated an infeasible solution. Moreover, in such a case it is easy to identify which constraints that are violated. Given this information, the planner can make suitable alterations to modify the data. An example would be to make sure that extra resources of storage capacity are available for planning purposes. This is a natural planning environment as the model should be used as a support tool for the planners.

#### *4.2. Computational results*

We have generated six problems to test the model and the efficiency of the solution procedures. The test problem presented above represents the basic case, and all other problems are modifications of this basic case. The six test problems are presented below. The six problems also represents various strategic analysis that can be made to test robustness using the proposed model.

*Problem P1—The basic case.*

*Problem P2—Restrictions on the storage levels.* The entering storage level in period 1 has to coincide with the leaving storage level in period  $T$ . A lower bound,  $10\,000 \text{ m}^3$ , on the total storage level at each terminal is also specified.

*Problem P3—Increased demand.* The demand at all heating plants is increased by 10% in a single time period. This problem represents a situation where we have unexpected cold weather in one period.

*Problem P4—More customers.* A new heating plant with a demand of  $100\,000 \text{ MWh}$  is included. This is an important aspect to test as the company frequently have to give offers to new customers.

Table 4  
Results of the different problems

Problem	LP-solution	Time (hours)	IP (CPLEX)	Time (hours)	IP (heuristic)	Time (hours)
P1	29 354	0.11	29 385	4.5	29 458	2.5
P2	34 299	0.18	34 420	6.4	34 392	2.6
P3	30 063	0.11	30 225	6.6	30 148	3
P4	37 385	0.15	37 490	6.6	37 452	2.5
P5	29 354	0.16	29 426	6.25	29 468	2.8
P6	27 591	0.17	27 717	6.25	27 650	3

*Problem P5—Changed chipping capacity.* The chipping capacity per time period is decreased in all time periods. This represents the situation when new chipping teams are negotiating.

*Problem P6—New terminal.* A terminal located close to an existing heating plant is introduced.

The results of solving the six problems using the two solution methods are given in Table 4. The objective function values are given in cost units, not to reveal the actual values.

From Table 4 we see that the solution time is acceptable and within practical time limits. It can also be noticed that the heuristic is better and faster in some of the problems. In P3, P4 and P6, the heuristic gave a better solution in less computational time. The quality of solutions from both procedures is very high as we get very small gaps to the optimal integer solution. The gap is of the size 0.10%–0.54% for the six problems.

The duality gap is very small. The reason for this is that the model has a strong network structure and resembles an LP-based inventory and transportation model. The fixed costs associated with the binary variables do not give rise to large gaps. The reason being that supply and demand, for our cases, is pretty much in balance, which means that essentially all harvesting areas must be forwarded and the residues chipped. The LP-relaxation might give solutions where, for example, the forwarding is spread out over several time periods. However, the sum over all fractional variables (values) still provide a fixed cost close to the true fixed cost. This is in contrast to for example the case with a facility location problem (which can have very large duality gap), where the total supply of all opened facilities vastly exceed the actual demand. The binary variables coupled with the terminals might lead to larger duality gaps, but the corresponding cost is small as compared to other costs.

We also investigated the output from the model for Problem P2. The forwarding during the planning horizon for this problem is given in Fig. 6. In Table 5, the proportion of forest residues chipped in the forest or at the terminals is given, as well as the proportions of forest fuel transported directly to heating plants or via terminals. The results presented concerns the solution procedure using CPLEX, but both our tested solution procedures give very similar results.

The fact that the supply of byproducts must be taken care of in each time period, leads to need of storage at terminals. We can also see that almost everything that is chipped in the forest is transported directly to heating plants.

Problem P2 included a restriction that the storage level at each terminal can never be allowed to fall below 10 000 m<sup>3</sup>. The obtained storage levels are given in Fig. 7. This tested restriction can imply an increase in costs. Trade-off between these extra costs and protection against variability can be considered.

It is hard to compare our solutions with manually obtained solutions, as there exists no manual solutions for this case. However, we do know how the planners work to get a solution. The solution process takes several days and is of greedy-type. In short, each heating plant will be allocated a set of harvest areas as close to the heating plant as possible. This allocation can be slightly modified during the year. The greedy approach makes the planning process rather easy but the supply chain model will choose much more flexible catchment areas for each of the heating plants. A manual solution process often works fine in the beginning of the year, but towards the end of the year there is a tendency to be “locked into a corner”. This

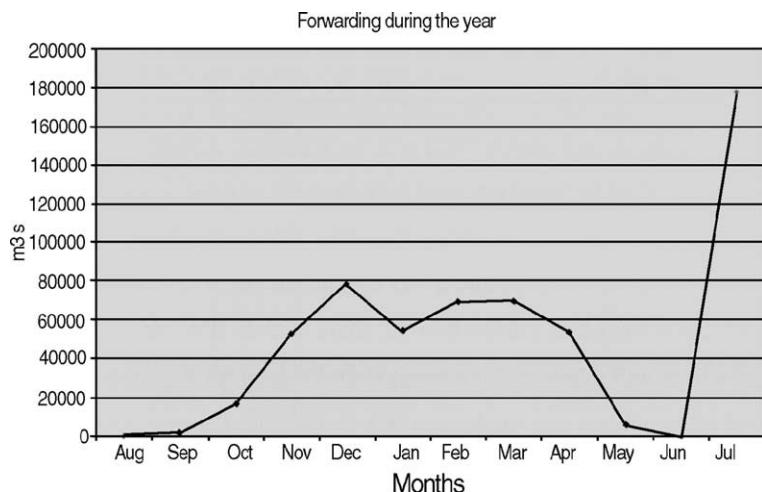


Fig. 6. Forwarding Problem P2.

Table 5  
Chipping and transportation alternatives—Problem P2

	CPLEX (%)
Forest residues chipped in the forest	50
Forest residues chipped in the forest and transported directly to heating plants	98
Forest residues chipped in the forest and transported via terminals	2
Forest residues chipped at the terminals	50
Byproducts from saw-mills transported directly to heating plants	86
Byproducts from saw-mills transported via terminals	14
Total transportation directly to heating plants	54
Total transportation via terminals	46



Fig. 7. Storage of chipped products.

often leads to the need to contract extra vehicles for the transportation and extra teams for the chipping. Using an optimisation model which cover the whole year, we can avoid these situations.

## 5. Concluding remarks

The main purpose with this paper is to present a model and solution approach that can be used as a decision support tool for strategic analysis as well as tactical planning of the supply of forest fuel. The mathematical model developed gives a detailed description of the supply chain problem considered. The resulting IP-problem will become very large. However, by using a heuristic approach based on sequential LP solving or the direct use of a commercial IP solver, i.e., CPLEX, it is possible to solve the problem within practical time limits. The heuristic approach and CPLEX provide comparable solutions, but the heuristic is about two times faster. The quality of the solutions found by both methods is very high, the objective function values are within 0.5% of the optimal value. The model is tested on a real industrial case and it has been possible to evaluate a number of strategic analyses. The present model can produce better and more flexible solutions compared to manual planning, and allows easy testing of different strategies and scenarios. In conclusion, we believe that the suggested model and solution approach can be used as an important tool in the tactical and strategic decision making by the planning staff at any entrepreneur in the considered industry.

## Acknowledgements

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## Appendix A. The complete optimisation model

$$\begin{aligned} \min & \sum_{i \in I_H \cup I_{HC}} \sum_{j \in J} \sum_{p \in P} \sum_{t \in T} c_{ijpt}^{xN} x_{ijpt}^N + \sum_{i \in I} \sum_{j \in J} \sum_{p \in P} \sum_{t \in T} c_{ijpt}^{xC} x_{ijpt}^C + \sum_{i \in I} \sum_{k \in K} \sum_{p \in P} \sum_{t \in T} c_{ikpt}^z z_{ikpt} \\ & + \sum_{j \in J} \sum_{k \in K} \sum_{p \in P} \sum_{t \in T} c_{jkpt}^y y_{jkpt} + \sum_{i \in I} \sum_{p \in P} \sum_{t \in T} c_p^F r_{ipt}^C + \sum_{j \in J} \sum_{p \in P} \sum_{t \in T} c_p^q q_{jpt} + \sum_{i \in I_{HC}} c_i^P \left( \sum_{t \in T} v_{it}^F \right) + \sum_{i \in I_{SC}} c_i^P u_i \\ & + \sum_{j \in J} g_j w_j + \sum_{i \in I_H \cup I_{HC}} \sum_{p \in P} \sum_{t \in T} h_p^F L_{ipt}^F + \sum_{j \in J} \sum_{p \in P} \sum_{t \in T} h_{jp}^N L_{jpt}^N + \sum_{j \in J} \sum_{p \in P} \sum_{t \in T} h_{jp}^C L_{jpt}^C, \end{aligned}$$

$$\sum_{t \in T} v_{it}^F = 1, \quad \forall i \in I_H,$$

$$\sum_{t \in T} v_{it}^F \leq 1, \quad \forall i \in I_{HC},$$

$$r_{ipt}^F = s_{ip} v_{it}^F, \quad \forall i \in I_H \cup I_{HC}, \quad \forall p \in P, \quad \forall t \in T,$$

$$\sum_{t \in T} v_{it}^C \leq 1, \quad \forall i \in I_H \cup I_{HC},$$

$$r_{ipt}^C = s_{ip}v_{it}^C, \quad \forall i \in I_H \cup I_{HC}, \quad \forall p \in P, \quad \forall t \in T,$$

$$L_{ip,t-1}^F + r_{ipt}^F = L_{ipt}^F + r_{ipt}^C + \sum_{j \in J} x_{ijpt}^N, \quad \forall i \in I_H \cup I_{HC}, \quad \forall p \in P, \quad \forall t \in T,$$

$$r_{ipt}^C = \sum_{j \in J} x_{ijpt}^C + \sum_{k \in K} z_{ikpt} \quad \forall i \in I_H \cup I_{HC}, \quad \forall p \in P, \quad \forall t \in T,$$

$$\sum_{j \in J} x_{ijpt}^C + \sum_{k \in K} z_{ikpt} = s_{ipt}u_i, \quad \forall i \in I_S \cup I_{SC}, \quad \forall p \in P, \quad \forall t \in T,$$

$$L_{jp,t-1}^N + \sum_{i \in I_H \cup I_{HC}} x_{ijpt}^N = L_{jpt}^N + q_{jpt}, \quad \forall j \in J, \quad \forall p \in P, \quad \forall t \in T,$$

$$L_{jp,t-1}^C + \sum_{i \in I} x_{ijpt}^C + q_{jpt} = L_{jpt}^C + \sum_{k \in K} y_{jkpt}, \quad \forall j \in J, \quad \forall p \in P, \quad \forall t \in T,$$

$$\sum_{i \in I_H \cup I_{HC}} \sum_{p \in P} r_{ipt}^F \leq e_t^F, \quad \forall t \in T,$$

$$\sum_{p \in P} q_{jpt} \leq e_{jt}^C, \quad \forall j \in J_C, \quad t \in T,$$

$$\sum_{i \in I_H \cup I_{HC}} \sum_{p \in P} r_{ipt}^C + \sum_{j \in J_{NC}} \sum_{p \in P} q_{jpt} \leq e_t^C, \quad \forall t \in T,$$

$$\sum_{p \in P} L_{jpt}^C + \sum_{p \in P} \sum_{k \in K} y_{jkpt} \leq b_j^C, \quad \forall j \in J, \quad \forall t \in T,$$

$$\sum_{p \in P} L_{jpt}^C + \sum_{p \in P} \sum_{k \in K} y_{jkpt} + \sum_{p \in P} L_{jpt}^N \leq b_j w_j, \quad \forall j \in J, \quad \forall t \in T,$$

$$\sum_{i \in I} \sum_{p \in P} a_{pt}^z z_{ikpt} + \sum_{j \in J} \sum_{p \in P} a_{pt}^y y_{jkpt} = d_{kt}, \quad \forall k \in K, \quad \forall t \in T,$$

$$\sum_{i \in I} \sum_{p \in P_b} z_{ikpt} + \sum_{j \in J} \sum_{p \in P_b} y_{jkpt} \leq A_k^b \left( \sum_{i \in I} \sum_{p \in P} z_{ikpt} + \sum_{j \in J} \sum_{p \in P} y_{jkpt} \right), \quad \forall k \in K_b, \quad \forall t \in T.$$

## References

- [1] J. Arlinger, B. Brunberg, M. Eriksson, M. Thor, Kvalitetskrav, råvarautilnyttjande och kostnader vid kraftigt ökad användning av skogsbränsle—Slutrapport för ett Optiträ-projekt, SkogForsk, Arbetsrapport nr 386 (1998) (in Swedish).
- [2] C. Barnhart, E.L. Johnson, G.L. Nemhauser, M.W.P. Savelsbergh, P.H. Vance, Branch-and-price: Column generation for solving huge integer programs, *Operation Research* 46 (3) (1998) 316–332.
- [3] B. Brunberg, G. Andersson, B. Nordén, M. Thor, Uppdragsprojekt Skogsbränsle—slutrapport, SkogForsk, Redögörelse nr 6 (1998) 1103–4580 (in Swedish).
- [4] D. Carlsson, M. Rönnqvist, Wood flow problems in Swedish forestry, in: G. Frumerie (Ed.), Report No. 1, The Forestry Research Institute of Sweden, Sweden, 1999.

- [5] <http://www.cplex.com>.
- [6] L.O. Eriksson, R. Björheden, Optimal storing, transport and processing for a forest-fuel supplier, European Journal of Operational Research 43 (1989) 26–33, in Swedish.
- [7] J. Filipsson, Trädbränsle—en kartläggning av produktion, metoder och förbrukning, SkogForsk, Arbetsrapport nr 403 (1998) (in Swedish).
- [8] C. Kilger, Computer assembly, in: H. Stadtler, C. Kilger (Eds.), Supply Chain Management and Advanced Planning, Chapter 18, Springer-Verlag, Berlin, 2000.
- [9] <http://www.modelling.com>.
- [10] F. Shapiro, Modeling the Supply Chain, Duxbury, 2001.
- [11] H. Stadtler, C. Kilger (Eds.), Supply Chain Management and Advanced Planning, Springer-Verlag, Berlin, 2000.
- [12] M. Wagner, H. Meyr, Food and beverages, in: H. Stadtler, C. Kilger (Eds.), Supply Chain Management and Advanced Planning, Chapter 19, Springer-Verlag, Berlin, 2000.