

## A mixed integer non-linear programming model for tactical value chain optimization of a wood biomass power plant

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### HIGHLIGHTS

- Developed a mathematical model to optimize the supply chain of a forest biomass power plant.
- Considered supply, storage, production and ash management.
- The model provided more profit compared to the actual profit of the company.
- Biomass purchase cost had the highest share in total cost.
- Investing in a new ash recovery system has environmental and economic benefits.

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### ABSTRACT

Forest biomass is one of the renewable sources of energy that has been used for generating electricity. The feasibility and cost of producing electricity from forest biomass depend on long term availability of biomass, its cost and quality, and the cost of collecting, pre-processing, handling, transportation, and storage of forest biomass, in addition to the operating and maintenance costs of the conversion facility. To improve the cost competitiveness of forest biomass for electricity generation, mathematical programming models can be used to manage and optimize its supply chain. In this paper, the supply chain configuration of a typical forest biomass power plant is presented and a dynamic optimization model is developed to maximize the overall value of the supply chain. The model considers biomass procurement, storage, energy production and ash management in an integrated framework at the tactical level. The developed model is a nonlinear mixed integer programming which is solved using the outer approximation algorithm provided in AIMMS software package. It is then applied to optimize the supply chain of a real biomass power plant in Canada. The optimum solution provides more profit compared to the actual profit of the power plant. Different scenarios for maximum available supply and also investment in a new ash recovery system were evaluated and the results were analyzed. The model in particular shows that investment in a new ash recovery system has economic as well as environmental benefits for the power plant.

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### 1. Introduction

Using wood, or forest biomass, as an energy source is not a new idea. Before the 20th century, wood was one of the main sources of world's energy, but it has been substituted partly by coal, oil and natural gas during the past century [1]. Today, the growing demand for energy, environmental issues and the instability of the main energy producer countries make it necessary to use alternative energy sources, such as forest biomass. Forest biomass com-

bustion is considered as a carbon neutral process if managed (produced, transported and used) in a sustainable manner since the combustion releases the CO<sub>2</sub> that trees captured during the photosynthesis process. Utilizing biomass for energy generation can decrease the gap between the actual emission levels and the international protocol targets such as those in the Kyoto and Copenhagen Accords [2]. Biomass is a flexible energy source, capable of generating electricity, heat, biofuels or a combination of them. It is one of the few renewable energy sources that can be stored and used to generate energy on-demand. It can also provide economic value, job opportunity and sustainable energy for communities [3]. In Canada, after hydro, the highest share of renewable energy production is contributed by biomass (2.9% in 2008 according to [4]).

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Despite the advantages of using forest biomass for power generation, there are several barriers to its efficient utilization, such as feedstock availability, cost and quality, conversion efficiency, transportation cost, and the efficiency of supply logistics system. Generally, the main concern in using biomass for energy generation is to design and manage its supply chain effectively in which it can compete with other options in the energy market [1]. The costs associated with different activities in generating power from forest biomass are highly case specific and depend on the availability of feedstock, distance of the suppliers from the plant, technology, and other factors. Generally, it is difficult to collect, transport, handle and store low-density materials. The quality of raw material (such as moisture content and heat value) also plays an important role in the performance of the production process [5]. Moreover, the effect of a decision made on one part of the supply, logistics, production and distribution would propagate to the other parts. Therefore, the whole system or “supply chain” should be considered and managed in an integrated manner.

Supply chains can generally be defined as distributed organizations where materials and information flow in many directions within and across organizational boundaries through complex business networks of suppliers, manufacturers, distributors, and final customers [6,7]. A supply chain model can be developed to help decision makers in their decisions and manage the supply chain more efficiently. Operations research and mathematical modeling have been used in supply chain planning and management including forestry related production/distribution design and management problems. There are also studies on design and/or management of forest biomass supply chain for energy generation. Most of these studies were done on biofuels, heating or combined heating and electricity plants. There are some studies which used simulation for modeling the biomass supply chain design or management such as [8–10]. Since the focus of our research is on optimization of forest biomass supply chain, we provide brief review of previous studies using optimization techniques in bio-energy supply chain design and management.

Several authors developed optimization model to determine the optimal material flow, transportation, storage and chipping location of energy systems, mainly heating plants [11–15]. Eriksson and Björheden [11] developed a model with decision variables related to storage and the chipping location for a heating plant. Gunnarsson et al. [12] developed a mixed integer programming model for tactical-strategic supply chain management of forest fuel used in a heating plant in Sweden by focusing on supply procurement decisions rather than the production process. In [12], the raw material were kept separated in the storage and multiple time steps were considered in model. The developed model was used to solve six generated problems rather than being applied to a real case study and the results of using different solution methods (LP and IP, and IP heuristic) to solve the problems were compared. The model developed by Kanzian et al. [13] included 16 combined heat and power plants and eight terminal storages in Austria. Freppaz et al. [14] used geographic information system (GIS) along with mathematical modeling to develop a decision support system for energy (thermal and electricity) production from biomass with a case study in Italy. In another study, done by Van Dyken et al. [15], a linear mixed-integer model was developed for biomass supply chain with transportation, storage and processing operations over 12 weekly time steps considering supply, constant demand, three different biomass products and two demand loads for chips and heat. This study focused on operational supply chain planning and the developed model was not applied to a real case study. A truck scheduling optimization model was developed in [16] for transportation of four types of forest biomass to energy plants in Oregon, US. This study only considered the transportation part of the supply chain.

Strategic decisions such as plant size and location were studied in [17–19]. The most profitable configuration (plant size) of a multi-source biomass district heating plant in Italy was considered in [17]. Optimum locations of bio-energy plants were studied in [18] with a case study in Austria. In [19], the authors used GIS to determine the optimal locations, sizes and number of bio-energy facilities (pellet plants) in Alberta, Canada while optimizing the transportation cost. Some studies evaluated the conversion technology and the possibility of co-generation in the district heating system supply chain design using mathematical programming such as [20–25]. Some previous studies considered biomass supply chain management for generating biofuels. An optimal configuration of a biofuel supply chain was studied in [26] with a case study in Italy. Ekşioğlu et al. [27] developed a model for cellulosic ethanol bio-refinery supply chain to determine the number, size and location of bio-refinery plants. The considered raw material was agricultural and woody biomass and the case study was located in Mississippi, US. Another study [28] used optimization methods to determine the optimal location, biomass supply area, and the size of a power plant that used olive tree pruning residues for energy generation. The trade-off between economic and environmental objectives in an optimal planning of a bio-refinery in Mexico was evaluated by Santibáñez-Aguilar et al. [29]. In this study, different types of raw material including agricultural biomass, wood chips, sawdust, and commercial wood were considered to be used for producing ethanol, hydrogen, and biodiesel (generated only from agricultural biomass). The authors developed a multi-objective optimization model to make decisions about feedstock, processing technology, and products in a bio-refinery supply chain. Kim et al. [30] developed an optimization model for supply chain design of bio-gasoline and biodiesel production from six forestry resources (logging residuals, thinnings, prunings, inter-cropped grasses, and chips/shavings). Their case study was based on an industrial database related to a case in the Southeastern United States. The same authors [31] also performed a global sensitivity analysis of biomass supply chain networks for biofuels under uncertainty and identified the most effective uncertain parameter on profit. The authors then developed an optimal model to maximize the expected profit of all the defined scenarios. Chen and Fan [32] developed a mixed integer stochastic programming model to incorporate uncertainty in strategic planning of bio-energy supply chain systems and optimal feedstock resource allocation. Their case study was located in California, US. They considered bioethanol production, feedstock procurement, and fuel delivery in an integrated model. Zhang et al. [33] designed an optimal switchgrass-based bioethanol supply chain using a mixed integer linear programming model with a case study in North Dakota, US. The objective function of their model was to minimize the total annualized cost which included marginal land rental cost, switchgrass cultivation, harvest, storage cost, different transportation costs, preprocessing and operational cost, and annualized fixed cost of preprocessing facilities and biorefineries. They also evaluated the effect of changes in bioethanol demand and harvest methods, bio-refinery locations, switchgrass yield on the results using sensitivity analysis.

None of the above mentioned studies considered the supply chain of forest biomass for electricity generation. The only study that considered a biomass power plant in an optimization framework is [34]. In this study, a multi-objective optimization model was developed to minimize the procurement and transportation costs and maximize the biomass quality (moisture content) for a biomass power plant. The first priority was given to procurement; the second priority was given to distance of procurement, and the third priority was given to biomass moisture content. Their single time-step model optimized the amount of each individual biomass type, including harvesting residues and poplar trees from different harvesting zones. Their case study was a 50 MW h biomass power

plant in Ontario, Canada. The authors used goal programming to solve this 3-objective optimization problem. They did not consider multiple planning periods (time steps) in their model.

In our paper, an optimization model is developed to maximize the overall value of a forest biomass power plant supply chain. We applied the model to a real case study. The model has a one-year planning horizon with monthly time steps. It includes all parts of the supply chain from procurement, to storage, production and ash management. The effects of the quality (moisture content, energy value, ash content) of purchased biomass mixing different types of biomass with different quality in the storage area on the amount of generated ash, cost of ash handling, total production cost and total amount of generated electricity are considered in our model. The results of the model are analyzed and compared to the current situation of the power plant. Moreover, scenario and sensitivity analyses are performed in this study.

## 2. The power plant supply chain

The supply chain of a power plant, including procurement of different raw materials from different suppliers, storage, production and ash management, is considered in this research. Fig. 1 shows the supply chain components of the considered power plant. Details of each component are explained below.

**Raw material:** The forest biomass can be supplied from forest residues, by-products of forest product mills, or fast growing crops grown specifically for energy purposes [35]. Forest residues include branches and tops left in the harvest areas after the logs have been transported to wood manufacturing facilities, as well as small diameter and infected trees not suitable for lumber production. By-products of forest product mills include wood chips, sawdust, bark and shavings [35]. Poplar and willow are examples of fast growing crops grown specifically for energy purposes [36].

In addition to the long term availability of biomass, its quality is an important factor in economic feasibility of bio-energy projects and the amount of energy generated from it. The quality of biomass depends on a variety of factors such as Higher Heating Value (HHV), moisture content, physical, chemical and thermal properties. HHV is the amount of heat released from complete combustion of dry biomass under standard conditions. Different types of biomass (e.g. bark, sawdust, shaving, etc.) and different species have different HHV. The moisture content affects the biomass heat content since energy has to be used to evaporate water at the beginning of the combustion process. Density, porosity, size and shape of biomass are other important physical properties of biomass impacting its utilization as fuel. Different energy conversion technologies require different particle size ranges. For combustion, particle size of biomass should be between 0.6 cm [37] and 10 cm. [Personal communication with power plant managers.]. Size and shape can be modified and improved through pre-processing operations such as chipping which can take place at the forest, a sort yard or at the power plant. Bark, sawdust and shavings usually need minor screening and chipping, while larger sized raw materials, such as roadside logging debris, need to be chipped before they can be used in direct combustion. Biomass chemical properties in-

clude the amount of chemical elements, such as carbon, hydrogen, oxygen, nitrogen, in biomass and its structural components, which is the amount of cellulose, lignin and hemicelluloses. These properties are different in different species and impact the HHV of the biomass as mentioned earlier. Important thermal properties are specific heat, thermal conductivity, and emissivity and vary with the moisture content and temperature [37–40].

**Transportation:** Different transportation modes, e.g. trucks, railcars, vessels and barge, can be used for transporting biomass to the power plant. However, forest biomass power plants are usually supplied from local suppliers with relatively short distance and therefore road transport is more likely to be used among other possible methods. As forest biomass density is relatively low (400 and 900 kg/m<sup>3</sup> [35]), its transportation to the power plant requires a large number of trucks which increases the delivered cost of biomass and the complexity of its logistics system. Biomass can be transported directly to the power plants, or stored at a satellite storage and used later. The transportation cost of biomass depends on the power plant size, raw material availability, average transportation distance, biomass density, carrying capacity, and the traveling speed. Transportation and handling costs usually represent a significant proportion of the total biomass delivered cost (as high as 50% in some cases [41]).

**Storage:** Storage is an important issue in a forest biomass supply chain. The storage site can be located either in the forest, at the power station or at an intermediate point. Usually, when forest biomass is kept in pile, it generates internal heat over time which is the result of respiration of living cells of wood [42]. The internal heat generated during storage makes the biomass more homogeneous and warmer hence it is easier to be burnt. Thus to improve the quality of biomass, it is better to keep it in storage for a period of time (1–2 months) [Personal communication with power plant managers.], and do not let the storage to drop below a certain level. The storage level has to be kept within certain limits, which depend on the power plant size. On the other hand, if the storage amount is more than a certain level, its handling cost increases incrementally since there is a need for an extra operator and a material handling equipment. The risk of fire and also biomass deterioration are higher when it is kept in huge piles [42].

**Combustion:** Direct combustion is a way to convert the energy originated from sun and stored in biomass through photosynthesis into other forms of energy, mainly heat and then electricity [35]. Combustion is defined as a series of chemical reactions resulting carbon and hydrogen deoxidization. Biomass elements, moisture content, and air are critical components of wood combustion. The products of these reactions include CO, hydrocarbons HC, oxides of nitrogen, sulfur and inorganic species such as the alkali chlorides, sulfates, carbonates and silicates. When wood is combusted, a mass loss process takes place which is related to the combustion temperature. This can be depicted in a plot, called the burning profile. The first pick in the burning profile is related to the release of moisture content and small amount of other absorbed gases. Then, as the temperature increases to 175–225°, other volatiles start to be released and ignite. Then, the rate of mass loss increases drastically up to temperatures between 325° and 425°, when the mass loss process starts to decrease. This decline will continue and finally the weight will be almost constant. These temperatures are different for different types of biomass [38].

**Production:** The electricity production process depends on the technology used and the layout of the power plant [5]. There are different biomass combustion technologies for energy generation such as fixed bed combustion, fluidized bed combustion and pulverized bed combustion [38]. The scale of forest biomass energy conversion plants can vary from very small scale (for domestic heating) up to a scale in the range of 100 MWe. The main restriction on the power plant size is the availability of the local

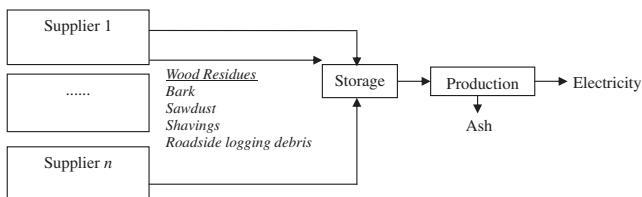


Fig. 1. Schematic of supply chain configuration of a wood biomass power plant.

feedstock, which makes it difficult to have biomass power plants larger than 25 MW. If dedicated feedstock supplies are available, larger power plants can be built producing 50–75 MW. The net electrical efficiency depends on the scale of the power plant and ranges from 20% to 40%. Small sized biomass power plants have low efficiencies, while the large sized biomass power plants can be as efficient as fossil fuel systems, however, access to high volume of biomass throughout the year and high cost of transporting low density material are the issues with these types of plants [35].

In this study, the overall configuration of the forest biomass power plant is based on a conventional power cycle used in typical thermal utility plants. The power plant uses forest biomass which is transferred by conveyors to a boiler where it is burned to generate heat for steam production. The steam is then transferred into a turbine, where the thermal energy is converted to electrical power. The exhaust steam is converted to water by a condenser and used in the system again. The water is re-used for almost seven cycles and then it is discharged to the sewer network. Other equipment pieces might include a high voltage step up transformer, a solid fuel handling system, an ash removal/handling system and other steam cycle auxiliary pieces of equipment, multiple cyclones and an electrostatic precipitator.

**Ash management:** Ash management is an important challenge for a direct combustion power plant. Generally, wood ash properties are related to several factors such as: species of tree or shrub, part of the tree or shrub (bark, wood, and leaves), type of waste (wood, pulp, or paper residue), combination with other fuel sources, type and quality of soil, weather conditions and combustion process [38]. Forest biomass ash generally contains calcium (Ca) and potassium (K) [38]. Two kinds of ash are generated in the production process: one is resulted from soil and rock contamination (called bottom ash) and the other one is resulted from the minerals in the foliage or wood (called fly ash). Ash disposal is a challenge for most of the power plants and has economic, environmental and social costs for the power plant. High ash content biomass is less desirable to be used as fuel [37]. For example, sawdust has lower ash content than bark and logging residues [39].

In order to design and manage an efficient supply chain for biomass power plants, all the mentioned components of the supply chain should be considered. It is important that the power plant receives the required amount of biomass at the right time with a competitive price to meet the electricity demand and maximize the profit. This can be determined by an optimization model. An optimum configuration of all processes in the supply chain can help decision makers have a better operation. The optimality is usually related to cost/profit, however other factors, such as the fuel consumption, greenhouse gas emissions and customer satisfaction, are also important in the supply chain management. Usually there is a trade-off between these different objectives which brings more challenge into the system.

### 3. The optimization model

The model presented here considers all the mentioned components including multiple procurement sources, several storage options, different types of forest fuel, and several time periods. The overall objective of the optimization model developed in this paper is to provide estimates of the amount of biomass to be purchased, stored and consumed in each month during a One-year planning horizon. Here, the main component of the model, decision variables, constraints and the objective function are defined. The index  $t$  corresponds to the model time steps (month) in one year time horizon ( $t \in \{\text{January, February, ..., December}\}$ ). The index  $s$  is related to different suppliers and the index  $p$  stands for different

**Table 1**  
List of parameters used in the optimization model.

Parameter	Definition
AshC	Average ash content of biomass, percentage of a unit of biomass converted to ash after combustion (%)
AshHCost	Cost of ash handling (\$/green ton)
ChemicalCost	Incremental cost of chemical used for power production (\$/MW h)
D( $t$ )	Electricity demand from client in month $t$ (MW h)
Efficiency	System's overall efficiency (%)
EPrice( $t$ )	Electricity price in month $t$ (\$/MW h)
EV( $s,p,t$ )	Energy value of biomass type $p$ purchased from supplier $s$ in month $t$ (MW h/green ton)
FCost( $s,p$ )	Price of biomass type $p$ supplied from supplier $s$ (\$/green ton)
HHV( $s,p,t$ )	Higher heating value of biomass type $p$ purchased from supplier $s$ in month $t$ (MW h/dry ton)
LowerLimitStorage	The storage lower limit below which the quality of biomass decreases (green ton)
MaxF( $s,t$ )	Maximum available biomass from supplier $s$ in month $t$ (green ton)
MaxS	Maximum absolute storage capacity (green ton)
MC( $s,p,t$ )	Moisture content of biomass type $p$ purchased from supplier $s$ in month $t$ (%)
PenaltyStorageCost	Penalty cost if storage is above storage upper limit (\$)
ProductRatio( $s,p,t$ )	Ratio of biomass type $p$ produced by supplier $s$ in month $t$ (%)
QualityReductionFactor	Reduction in biomass quality if storage is less than storage lower limit (%)
SewerCost	Incremental cost of sewer used for power production (\$/MW h)
TargetS	Target storage for the last month (green ton)
TCost( $s,t$ )	Transportation cost for supplier (\$/green ton)
UpperLimitStorage	The storage upper limit above which extra cost of storage incur (green ton)
WaterCost	Incremental cost of water used for power production (\$/MW h)

raw materials. Table 1 shows the definition of different parameters used in the developed model.

The model's decision variables are:

$$\begin{aligned}
 F(s,t) & \text{ Amount of biomass purchased from supplier } s \text{ in month } t \\
 & \text{ (green ton)} \\
 S(t) & \text{ Amount of mixed biomass stored in month } t \text{ (green ton)} \\
 C(t) & \text{ Amount of mixed biomass consumed in month } t \text{ to produce} \\
 & \text{ electricity (green ton)} \\
 E(t) & \text{ Amount of electricity generated in month } t \text{ (MW h)}
 \end{aligned}$$

There are also two sets of binary variables controlling the storage level in each month.

$$\begin{aligned}
 Y(t) & 1 \text{ if storage is higher than storage upper limit in month } t \\
 & \text{ (binary)} \\
 Z(t) & 1 \text{ if storage is less than the storage lower limit in month } t \\
 & \text{ (binary)}
 \end{aligned}$$

The objective function is to maximize the profit (Eq. (1)).

$$\text{Profit} = \text{Revenue from selling electricity}$$

$$\begin{aligned}
 & - (\text{biomass procurement cost} + \text{ash handling cost} \\
 & + \text{storage penalty cost} + \text{production cost})
 \end{aligned} \quad (1)$$

Revenue from selling electricity to customer is calculated by multiplying electricity unit price by the amount of electricity produced by the power plant (Eq. (2)).

$$\text{Revenue} = \sum_t \text{EPrice}(t) \times E(t) \quad (2)$$

Biomass procurement cost includes biomass purchase cost and its transportation cost (Eq. (3)). Biomass purchase cost is calculated by multiplying biomass price by the ratio of each biomass type produced in each supplier and the amount of biomass purchased. The transportation cost depends on the distance between suppliers and the power plant, and the amount of purchased biomass. It is assumed that no chipping is done in this power plant. If this is not the case, the model can easily be modified to include that.

Biomass procurement cost

$$= \sum_t \sum_s \left( \sum_p F\text{Cost}(s,p,t) \times \text{ProductRatio}(s,p,t) + T\text{Cost}(s,t) \right) \times F(s,t) \quad (3)$$

Ash handling cost is equal to the average ash content of all biomass times the biomass consumed for power production times the cost of handling ash (Eq. (4)). It should be noted that the average ash content is considered in this equation which can be substituted by more detailed parameter such as ash content of each type of biomass if enough data were available. Usually different types of wood are burnt together, therefore, the ash content of the mix of biomass is available.

$$\text{Ash handling cost} = \sum_t \text{AshHCost} \times \text{AshC} \times C(t) \quad (4)$$

The penalty storage cost is calculated by multiplying a fixed penalty cost by a binary variable if the decision variable related to storage is more than a certain level (Eq. (5)).

$$\text{Storage penalty cost} = \sum_t \text{Penalty Storage Cost} \times Y(t) \quad (5)$$

The production cost contains water, sewer and chemical costs multiplied by the amount of power produced by the power plant (Eq. (6)).

$$\begin{aligned} \text{Production cost} &= (\text{WaterCost} + \text{ChemicalCost} + \text{SewerCost}) \\ &\times \sum_t E(t) \end{aligned} \quad (6)$$

Constraints of the model are listed below:

In month  $t$ , biomass purchased from supplier  $s$  has to be less than or equal to the maximum biomass produced by that supplier.

$$F(s,t) \leq \text{Max}F(s,t) \quad [\text{For all } s,t] \quad (7)$$

Storage level in each month has to be less than or equal to the absolute maximum storage levels.

$$S(t) \leq \text{Max}S \quad [\text{For all } t] \quad (8)$$

Mass balance equation is considered in Eq. (9), which indicates that storage in month  $t$  is equal to storage in month  $(t-1)$  plus the total biomass purchased from all suppliers minus biomass consumption in month  $t$ .

$$S(t) = S(t-1) + \sum_t F(s,t) - C(t) \quad [\text{For all } t] \quad (9)$$

Eq. (10) shows that the power generated by the power plant in each month has to be equal to the customer's monthly demand.

$$E(t) = D(t) \quad [\text{For all } t] \quad (10)$$

Eq. (11) implies the storage in the last month has to be equal to a target storage level. This constraint guarantees that initial storage condition is near optimal for the next year. The target storage level can be set by managers or can be determined by the optimization model as will be discussed in the result section.

$$S(T) = \text{Target}S \quad (11)$$

Eq. (12) relates the amount of electricity produced in each month to the energy value of biomass utilized in that month, efficiency of the system and the reduction in biomass quality if the storage level is less than the storage lower limit ( $1 - \text{QualityReductionFactor} \times Z(t)$ ).

$$\begin{aligned} E(t) &= C(t) \times \text{AveEV}(t) \times \text{efficiency} \times (1 \\ &- \text{QualityReductionFactor} \times Z(t)) \quad [\text{For all } t] \end{aligned} \quad (12)$$

The average energy value of biomass in storage in month  $t$  is calculated based on the weighted average of energy values of biomass purchased from suppliers in that month and the average energy value of stored biomass in month  $(t-1)$  as shown in the following equation::

$$\begin{aligned} \text{AveEV}(t) &= \left( \sum_s F(s,t) \times \sum_p \text{ProductRatio}(s,p,t) \times \text{EV}(s,p,t) \right. \\ &\quad \left. + S(t-1) \times \text{AveEV}(t-1) \right) / \left( \sum_s F(s,t) + S(t-1) \right) \quad [\text{For all } t] \end{aligned} \quad (13)$$

$\text{AveEV}(t)$  is a decision variable since it is calculated based on variables  $F(s,t)$  and  $S(t-1)$ .  $\text{EV}(s,p,t)$  is calculated based on higher heating value  $\text{HHV}(s,p,t)$  and the corresponding moisture content  $\text{MC}(s,p,t)$  of biomass as implied in following equation [1]:

$$\text{EV}(s,p,t) = \text{HHV}(s,p,t) \times (1 - \text{MC}(s,p,t)) \quad [\text{For all } s,p,t] \quad (14)$$

And finally, all the continuous variables are non-negative as shown in the following equation:

$$F(s,t), S(t), C(t), E(t), \text{AveEV}(t) \geq 0 \quad [\text{For all } t] \quad (15)$$

It can be seen that the model is a non-linear mixed integer programming (MINLP) model since Eqs. (12) and (13) contain non-linear terms and the model contains both continuous and binary variables.

MINLP is a combination of two theoretically difficult to solve categories of problems particularly for large scale problems: mixed integer programs (MIP) and non-linear programs (NLP). Therefore, it is not straightforward to solve MINLP problems and get precise results [43]. However, several methods have been developed and improved in the past few years which make it possible to solve MINLP problems more precisely under special circumstances (convexity, etc.), i.e. terminate with a guaranteed optimal solution or prove that no such solution exists [44]. The solution methods include branch-and-bound method [44], Benders decomposition [44] and Outer Approximation (OA) algorithm [44].

In this paper, the Outer Approximation (OA) algorithm was used since it was provided in the AIMMS software package [45]. The idea in this algorithm is to switch between solving the non-linear programming sub-problems and the relaxed versions of a mixed integer linear master program for a finite sequence. The main assumption in the Outer Approximation algorithm is the convexity of the non-linear sub-model. The average energy value in each month is considered as a variable as explained in Eq. (13) and the model provides its optimal value.

#### 4. Case study

The above developed model was applied to a real case study located in Canada to help them maximize their profit and manage their supply chain efficiently. The supply chain is the same as what was described in Section 2. Most of the biomass used by the power plant was supplied by residues from local sawmills at a low cost. These inexpensive sources of raw material started to decline in recent years due to the economic downturn that resulted in the closure of some of the mills in the area, and the competition to

**Table 2**  
Characteristics of the case study.

Number of suppliers with fixed (long term) contract	6
Number of suppliers without long term contract	More than 50 suppliers
Average ash content	8%
Average Moisture Content (MC)	32.4% (range: 10.2–46.7)
Average Higher Heating Value (HHV)	5.02 MW h/ton (range: 3.68–5.34) 8565 BTU/lb (range: 6275–9110)
Efficiency	30%
Range of available biomass from suppliers with fixed contract	0–14,177 (green ton per month)
Range of available biomass from suppliers without fixed contract	0–20,800 (green ton per month)

access biomass from pellet mills. Therefore, biomass prices have increased and the power plant was forced to use other sources of biomass such as logging debris. Biomass from different sources has different quality and cost which affect the costs and efficiency of the operation. Table 2 shows some of the characteristics of the power plant.

Some of the specifications of the case study supply chain are presented below.

#### 4.1. Raw material

The feedstock used in the power plant include bark, sawdust, shavings and roadside logging debris (RLD) ( $p \in \{\text{Bark, Sawdust, Shaving, Roadside Logging Debris (RLD)}\}$ ). All types of biomass are transported to the power plant by trucks and then kept in storage and mixed together before combustion. There is a long list of possible suppliers for the power plant with different contracts, terms, conditions and prices. For some suppliers, if they are in the operation, the power plant has to buy their biomass. These mills are located close to the power plant and their biomass is relatively inexpensive because of the short transportation distance and long term contracts. However, if these mills decide not to operate, there will be raw material shortage for the power plant. Therefore, the amount of biomass that can be purchased from these suppliers is not exactly known in advance. There are other suppliers which can be considered as ad hoc suppliers which have no contract with the power plant and usually provide more expensive biomass. Therefore, Eq. (7) can be broken into two sets of constraints:

$$F(s,t) = \text{Max}F(s,t) \quad [\text{For all } s \text{ in suppliers with fixed contract}, t] \quad (16)$$

$$F(s,t) \leq \text{Max}F(s,t) \quad [\text{For all } s \text{ in suppliers without fixed contract}, t] \quad (17)$$

#### 4.2. Storage

The capacity of the storage for biomass is limited and known. Having storage more than a certain level forces the power plant to hire an additional operator and use additional material handling equipment. In addition, there is another upper storage limit, above which additional operator and additional handling material equipment are needed, moreover the risk of fire in the storage increases. Therefore, in the model if storage is more than the first upper level in month  $t$ , a binary variable called  $Y_1(t)$  becomes 1 and if it is more than the second upper storage level, another binary variable called  $Y_2(t)$  becomes also 1. Then, the storage penalty cost will be:

$$\begin{aligned} \text{Storage penalty cost} = & \sum_t \text{PenaltyStorageCost} \times (Y_1(t) \\ & + Y_2(t)) \end{aligned} \quad (18)$$

If the level of the storage decreases below a certain level called the minimum storage level, the quality of biomass is decreased by a certain amount (6% in this case) and also there is a risk of biomass shortage (Eq. (12)).

#### 4.3. Demand

The power plant has a long term contract with a costumer to provide a fixed amount of electricity per year, called the firm load. The rest of the production, named the surplus load, can be produced and sold to the same customer whenever it is profitable. The firm load demand has to be met at all the times, while the power plant has the option not to produce the surplus load. There is also a total amount for firm load demand in a year. Usually, the power plant decides whether or not to produce the surplus load in the beginning of the year and informs the customer about its decision. If the power plant decides to produce the firm load, the first fixed amount of production in each hour is considered as the firm load and the rest is the surplus load. When the production covers the firm demand, the rest of the production is considered as the surplus demand. If the power plant decides not to produce the surplus load, the total production in each hour is considered as the firm load and the power plant will be shut down for the rest of the year after meeting the total firm demand.

To model different demand types, several parameters and decision variables have to be added as listed and explained in Table 3.

Then, the following constraint (Eq. (19)) needs to be added to the model. This constraint implies that the electricity produced in each month has two forms. If the surplus load is being produced, it is equal to the total firm and surplus demand in one year distributed monthly based on the ratio of working hour of that month to the total working hour in one year. If the surplus load is not being produced, the electricity in each month is equal to the firm load generated in each month.

$$\begin{aligned} E(t) = & (\text{FirmDemand} + \text{SurplusDemand}) \\ & \times (\text{WorkingHour}(t) / \sum_t \text{WorkingHour}(t)) \\ & \times \text{SurplusBinary} + \text{FirmProduction}(t) \times (1 \\ & - \text{SurplusBinary}) \end{aligned} \quad (19)$$

The objective function also needs to be modified to contain Eq. (19) since the revenue from selling electricity to the customer is equal to the firm load demand times the power price for firm electricity plus surplus load demand times the surplus energy price times the binary variable if the surplus power is being produced.

**Table 3**  
Variables and parameters for the case study.

Symbol	Type	Definition
$E(t)$	Continuous variable	Total electricity generated in each month (MW h)
SurplusBinary	Binary variable	Binary variable, 1 if surplus electricity is produced in a year, 0 otherwise
FirmDemand	Parameter	Total firm demand in one year (MW h)
SurplusDemand	Parameter	Total surplus demand in one year (MW h)
FirmProduction( $t$ )	Parameter	Amount of firm electricity generated in each month if the surplus is not being produced (MW h)
WorkingHour	Parameter	Number of working hours in each month
FirmPrice	Parameter	Energy price for firm load (\$/MW h)
SurplusPrice	Parameter	Energy price for surplus load (\$/MW h)

$$\begin{aligned} \text{Revenue} = & \text{FirmPrice} \times \text{FirmDemand} + \text{SurplusPrice} \\ & \times \text{SurplusDemand} \times \text{SurplusBinary} \end{aligned} \quad (20)$$

The optimization model for this case study includes (Eqs. (8)–(17)) and the objective function is to maximize (20 – 3 – 4 – 6 – 18).

The MINLP model was solved using the AIMMS software [45] and the OA algorithm. The model had 260 variables (73 binary variables and 187 continuous variables) and 333 constraints. It took less than a minute to solve the problem using a 2.80 GHz CPU. The model was implemented with real data and validated by the power plant managers. It is notable that this model can be used for monthly decision making. It means that the model can be used at the beginning of the planning horizon (January) to indicate the optimal decision variables. However, if some parameters change later, the model can be updated using the new information, the previous month variables has to be fixed based on the real values and the model can be run again for the rest of the months.

## 5. Results and discussion

The results in Table 4 are based on 2011 data from the case study. The profit from the optimization model is higher than the actual profit of the power plant in 2011. The actual total biomass procurement cost of the power plant in 2011 was \$11 million which was 15% higher than the optimum total biomass procurement cost from the model. Table 4 also shows the contribution of different costs in the total cost of the power plant. The biomass purchase cost accounts for more than 63% of the total cost. Transportation cost encompass 33% of the total cost which is relatively low compared to other studies [41] due to power plant's access to biomass from local suppliers.

Fig. 2 shows the firm and surplus production profile in different months. The production in May is lower than other months due to maintenance days scheduled in that month.

Fig. 3 shows the optimum decision variables for the amount of biomass purchased, stored and consumed in each month based on the 2011 data. The initial storage level was 80,000 green ton in 2011. The optimum storage profile declines till April, then increases in May due to the maintenance shut down in the power plant.

**Table 4**  
Results of cost, revenue and profit for optimization model (in Million\$).

	Biomass purchase cost	Transportation cost	Ash handling cost	Firm revenue	Surplus revenue	Total profit
Model results	6.08	3.18	0.32	21.5	4.72	
15.03						

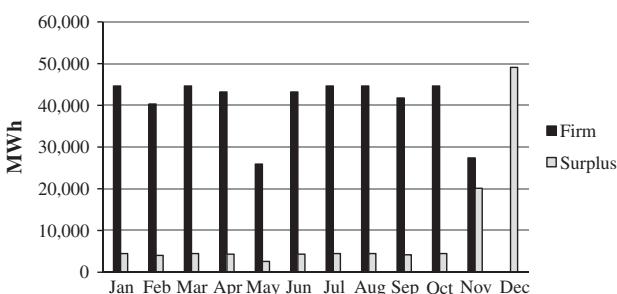


Fig. 2. The amount of firm and surplus production in each month.

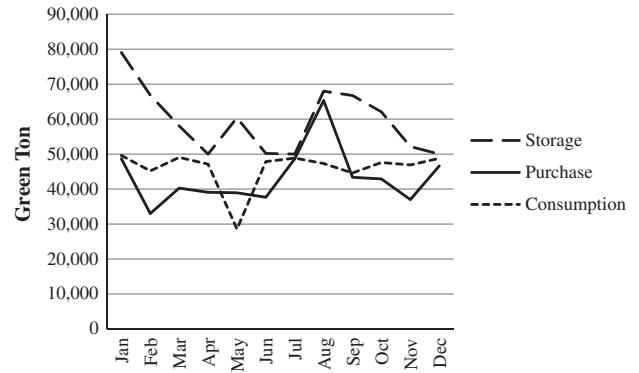


Fig. 3. Optimum amount of biomass stored, purchased and consumed in each month based on 2011 data.

### 5.1. Scenario analysis

In addition to the base case scenario considered in the previous section, we analyzed three other scenarios here. These scenarios were important for the power plant managers.

- Scenario 1: One of the suppliers with fixed contract shuts down its operation which is possible due to the economic situation (pessimistic scenario).
- Scenario 2: One of the previous suppliers with fixed contract which was closed in the past couple of years resumes its operation (optimistic scenario).
- Scenario 3: The power plant makes an investment in a new piece of equipment for ash recovery. Having a new ash recovery system with a capital cost of \$2 million, the ash generated in the power plant is screened and the unburnt parts are separated and sent back to the operation process. It has been estimated that this system will collect 20% of the ash and use it as a biomass source. It is assumed that the energy value of the unburnt part of the ash is the same as the average energy value of the biomass consumed in that month. Therefore, it has the potential benefit of providing more biomass and less ash for the power plant which reduces the ash disposal costs.

Table 5 shows the total profit and the total purchase cost of biomass for different scenarios. The results show that Scenario 1 has a lower profit (24.48%) than the base case scenario. Scenario 2 has a higher profit than the base case scenario since the power plant has access to cheaper biomass from a supplier with fixed contract. The amount of increase in profit is \$3.27 million (21.76%) which is the result of reduction in biomass purchase cost. Scenario 3, investment in a new ash recovery system, has a higher profit than the base case scenario (\$160,000, 1.06%). This means that investing in a new ash recovery system has economic and environmental benefits for the power plant. The increase in profit is partly from the reduction in the biomass purchase cost (3.46%), and partly from reduction in the ash handling cost (20.75%). The capital cost of the ash recovery equipment was converted to the yearly cost using 2% interest rate and 10 years service life, and then the yearly cost was subtracted from the profit.

### 5.2. Sensitivity analysis

To evaluate the effect of variations in model parameters on profit, sensitivity analysis was performed. The results of variation in profit are depicted in Fig. 4. It can be seen that the impact of variation in some of the parameters, such as electricity price, higher heating value and maximum available biomass from suppliers

**Table 5**

Total profit and biomass procurement cost for four different scenarios.

	Base case scenario	Scenario 1	Scenario 2	Scenario 3
Total profit (Million\$)	15.03	11.35	18.30	15.19
Biomass procurement cost (Million\$)	9.26	12.94	6.00	8.94
Profit difference compared to the base case scenario	0	-24.48%	+21.76%	+1.06%

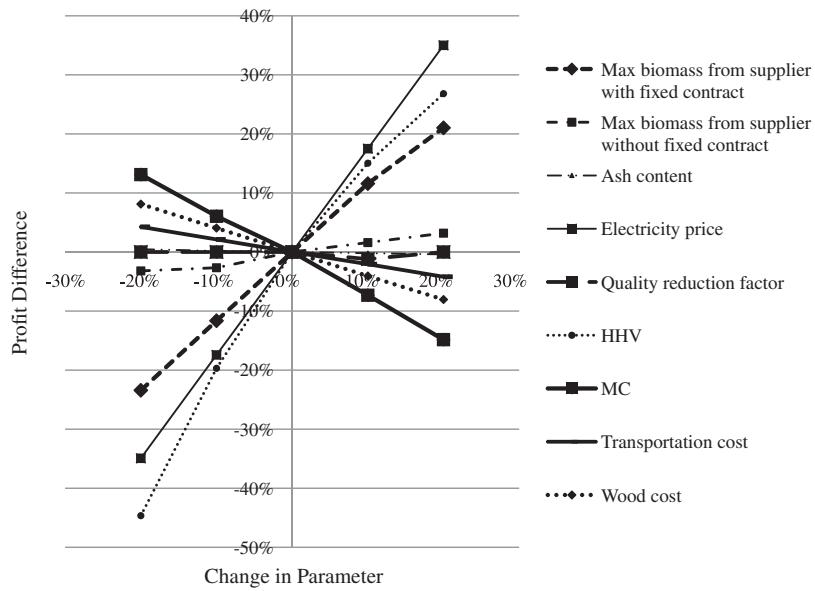


Fig. 4. Variations in profit with  $\pm 20\%$  change in different parameters.

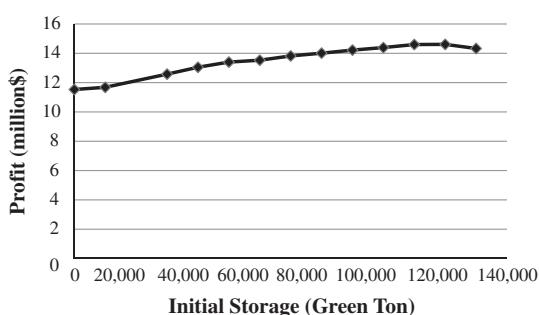


Fig. 5. Variations in profit for different initial storage levels.

with fixed contract, on the profit are high. When these parameters increase by 20%, the increase in profit is between 20 and 35%. On the other hand when they reduce by 20%, the loss is 23–45%. Other affecting parameters are moisture content, biomass cost and transportation cost which have the opposite impact on the profit. Increasing these parameters to 20% decreases the profit by about 4–15%. The effects of variation in other parameters on the overall profit are low and can be considered negligible.

Sensitivity of the results to changes in initial storage level was also determined. The initial storage level were changed from 0 to 130,000 green tons, while the target storage level was set to the initial storage level. As shown in Fig. 5, an initial storage level of 120,000 green tons generates the highest profit.

generating electricity from forest biomass is higher than most of the other sources of energy mostly due to several factors such as high transportation cost of a low bulk density material and low efficiency of the system. Optimization and mathematical modeling help in having an optimum configuration of the forest biomass supply chain and providing less expensive source of electricity.

In this paper, a mathematical optimization model was developed to find out how much biomass to buy in each month from each supplier, how much biomass to burn and store in each month, and whether or not to produce extra electricity to maximize the total profit. The developed model was a mixed integer non-linear model, which was solved by the Outer Approximation algorithm using the AIMMS software. The results for the specific case study presented here indicate that the profit from the optimization model is higher than that of the actual case in which the company's managers made decisions based on their experience. For instance, the optimum biomass procurement cost was 15% lower than the actual purchase and transportation costs of the power plant in 2011. The model provides the optimum profile for biomass storage, purchase and consumption in each month.

The current situation of the power plant was considered as the base case scenario and three other scenarios were evaluated in this study. The first one was a pessimistic scenario in which the production shutdown of a supplier with fixed contract was investigated. The second scenario, an optimistic scenario, was related to the production resume of one of the previous suppliers with fixed contract. And finally, the third scenario investigated the investment in a new ash recovery system. This system collects and reuses the unburnt parts of the produced ash. Based on the results, the profit of the first scenario is 24.48% lower than that of the base case scenario, while the profit of second and third scenarios are 21.76% and 1.06% higher than that of the base case scenario, respectively. In the third scenario, the ash handling cost has reduced by 20.75%.

## 6. Conclusions

Using forest biomass in a direct combustion power plant can help produce green energy for communities. However, the cost of

Therefore, investing in a new ash recovery system has economic benefit as well as environmental advantage for the power plant.

An important issue to consider in the supply chain of the power plant is the uncertainty in most of the parameters, such as prices, costs and maximum available biomass. Sensitivity analysis was performed to evaluate the impact of variations in different parameters on the optimum solution. The result showed that the most important parameters impacting the final solution are electricity price, higher heating value, maximum available biomass from suppliers with fixed contract and moisture content. It is important to have precise data on parameters that highly impact the results. Since in many cases it would not be possible to have precise data due to uncertainty in the system, it would be useful to develop a model which captures uncertainty and would be robust for all possible realizations of stochastic parameters. Complete modeling of uncertainty in the parameters will be done in our future work.

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