1 | Complex Relative Permittivity

In the ground wave theory the complex relative permittivity is needed. This can be found in different ways[Kim and Narayanan, 2002]. One of the easier ways to do this is the method described in [Kim and Narayanan, 2002]. This method sets far less restrictions for the antennas compared to other methods[Kim and Narayanan, 2002].

The theory behind the method comes from the the two-ray path loss model. A generic illustration of a two-ray setup can be seen in Figure 1.1.

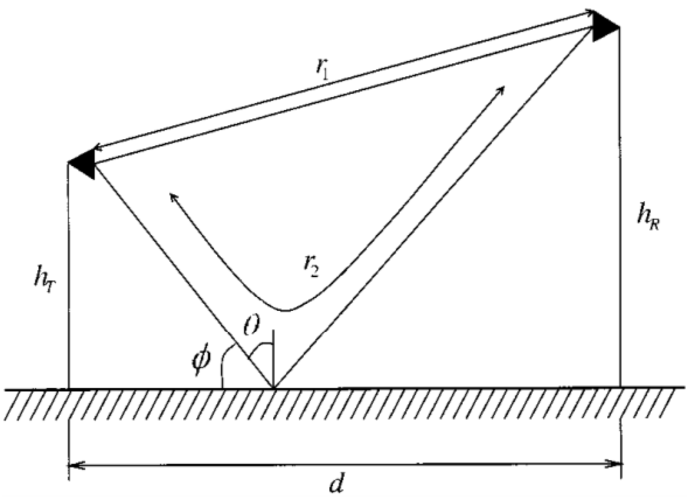

Figure 1.1: Illustration of setup and symbol meaning. Figure taken from [Kim and Narayanan, 2002].

The two-ray model can be written as[Kim and Narayanan, 2002]:

$$P_r = \frac{P_t G_t G_t}{L} \left(\frac{\lambda}{4\pi}\right)^2 \cdot \left| \frac{1}{r_1} \mathrm{e}^{-jkr_1} + \sqrt{\alpha_t}\sqrt{\alpha_r}\rho_{h,v} \frac{1}{r_2} \mathrm{e}^{-jkr_2} \right|^2 \tag{1.1}$$

Where:

P_r	is the received power	[W]
P_t	is the transmitted power	[W]
G_t	is the transmitters antenna gain	[1]
G_r	is the receiver antennas gain	[1]
L	is loss factor for the cables and connectors	[1]
λ	is the wavelength	[m]
r_1	is the direct path length	[m]
k	is the wavenumber	$[m^{-1}]$

α_t	is the magnitude ratios of power along the reflected to the direct path directions for the transmit antenna	[1]
α_r	is the magnitude ratios of power along the reflected to the direct path directions for the receiver antenna	[1]
$\rho_{h,v}$	is the reflection coefficient in for either horizontal or vertical polarization	[1]
r_2	is the reflected path length	[m]

Do note that if the signal measured was horizontal polarized the horizontal reflection coefficient is used, if vertical polarized then the vertical reflection coefficient is used.

Since the reflection coefficient is complex it can be utilized that

$$\rho_{h,v} = \Gamma_{h,v} + j\zeta_{h,v} \tag{1.2}$$

Where:

$\Gamma_{h,v}$	is the real part of the permittivity	[1]
ζ	is the imaginary part of the permittivity	[1]

By using Equation 1.2 in Equation 1.1 and rearranging, it can be seen that this draws circles on the complex reflection coefficient plane:

$$\left(\Gamma_{h,v} + \frac{r_r \cos(kr_d)}{\sqrt{\alpha_r}\sqrt{\alpha_t}}\right)^2 + \left(\zeta_{h,v} + \frac{r_r \sin(kr_d)}{\sqrt{\alpha_r}\sqrt{\alpha_t}}\right)^2 = \left(\frac{4\pi r_2 \sqrt{P_d}}{\lambda\sqrt{\alpha_r}\sqrt{\alpha_t}}\right)^2 \tag{1.3}$$

Where:

r_r	is $\frac{r_2}{r_1}$	[1]
r_d	$r_2 - r_1$	[m]
P_d	$\frac{P_r L}{P_t G_t G_r}$	[1]

Since this is a circle three separate measurements are needed to get a unique intersection point. Because the complex relative permittivity vary with frequency and incidence angle, these must be fixed for the measurements. This is illustrated in Figure 1.2.

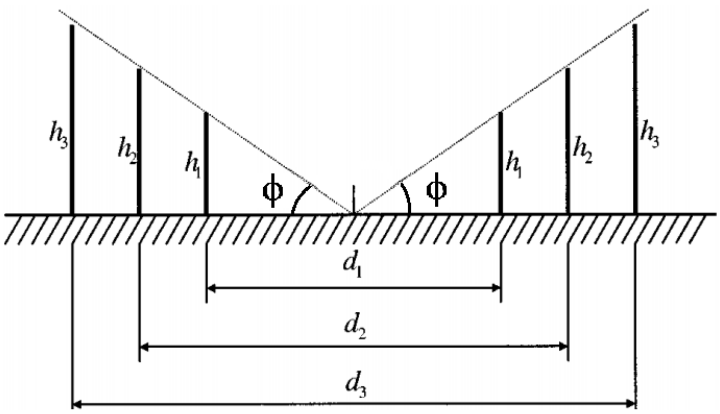

Figure 1.2: Illustration of the three distances and heights needed. Figure taken from [Kim and Narayanan, 2002].

When the intersection point has been determined[1] for both horizontal and vertical polarized waves the complex relative permittivity can be found as[Kim and Narayanan, 2002]

$$\epsilon_0 = \epsilon - j60\sigma\lambda = \frac{(1+\rho_v)(1-\rho_h)}{1+\rho_h)(1-\rho_v)} \tag{1.4}$$

Where:

ϵ_0	is the complex relative permittivity	[1]
ϵ	is the dielectric constant of the ground relative to unity in free space	[1]
σ	is the conductivity of the ground in mhos per meter	$\left[\frac{mhos}{m}\right]$

A stepwise procedure could then be:

1. Decide frequency and incidence angle
2. Decide measurement setup, including P_t heights and distances between transmitter and receiver for all three measurements points
3. Determine G_t, G_r, L, α_t, α_r for both horizontal and vertical cases
4. Calculate λ, k, r_1, r_2
5. Measure P_r in all six cases

[1] In practice there might not be a unique solution, due to measurement inaccuracies, There will then be three intersections that are close, and a sufficient estimate can be found by averaging these [Kim and Narayanan, 2002].

6. Find intersection point using Equation 1.3 for both horizontal and vertical polarization
7. Calculate the complex relative permittivity using Equation 1.4

Bibliography

Kim, H.-S. and Narayanan, R. M. (2002). A New Measurement Technique for Obtaining the Complex Relative Permittivity of Terrain Surfaces. *IEEE Transactions on Geoscience and Remote Sensing*.