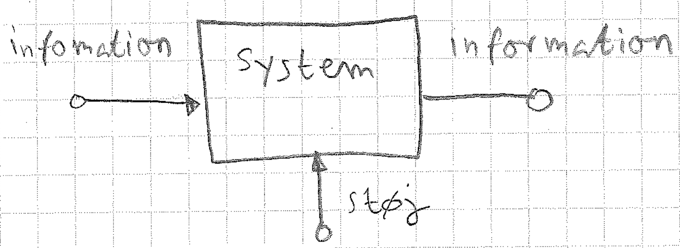
Noise Theory - MM1

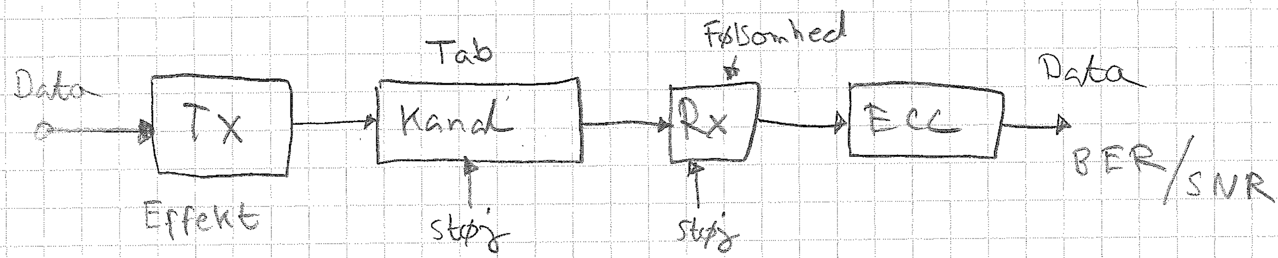
Kommunikationssystemer



BER
- Bit error rate

ECC
- Error control coding

Parametre i spil



Bedre navn end BER:

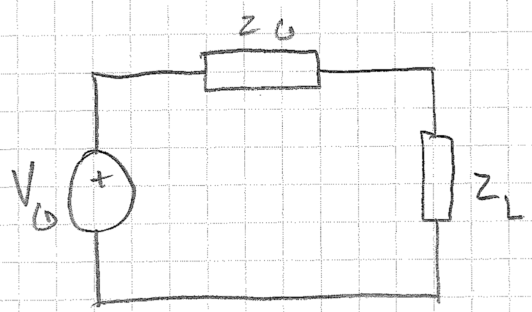
BEP : Bit Error Probability (BER)

SEP : Symbol —— " ——

FEP : Frame —— " ——

PEP : Packet —— " ——

Power calculations



For conjugate match

$(Z_G = Z_L^*)$

$$P_{ZL} = \frac{|V_G|^2}{4 \cdot RE[Z_L]} \ [W]$$

For real impedances ($Z_L = Z_G = R\,[\Omega]$) we set:

$$P_A = \frac{V_G^2}{4 \cdot R}\ [W] \quad \left(\text{or } \frac{V^2}{\Omega}\right)$$

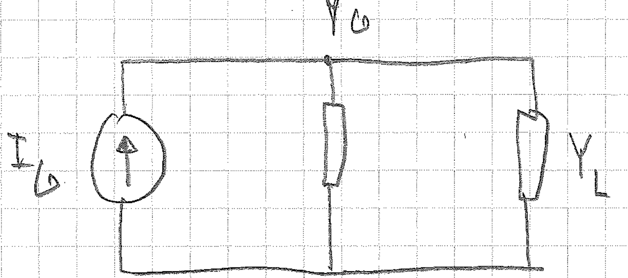
=avalible

and also

$$V_G^2 = 4 \cdot P_A \cdot R\ [V^2] \quad (\text{or } W \cdot \Omega)$$

$\boxed{\text{Watts in } 1\Omega}$

For currents we see:

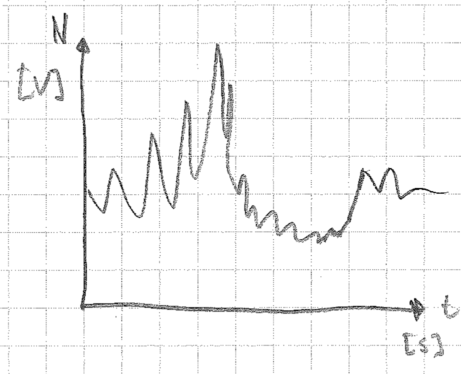


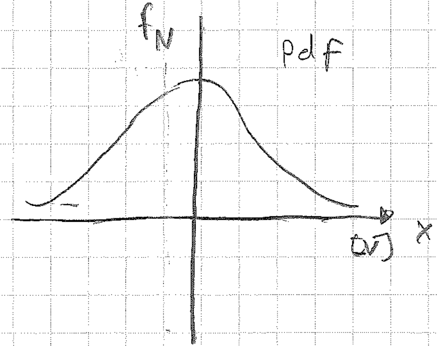
$$I_G^2 = 4 \cdot P_A \cdot G\ [A^2]$$

(or $W \cdot S$)

$Y_G = Y_L = G$

Thermal noise description





Model

$N \to X$

Sthocastic process

$$f_N(x) = \frac{1}{\sqrt{2\pi\sigma_n^2}} \cdot e^{-\frac{1}{2} \cdot \frac{(x-\mu_N)^2}{\sigma_N^2}}$$

statistic moments:

$$E[N] = \int_{-\infty}^{\infty} x \, f_N(x) \, dx = \mu_N = 0 \; [V]$$

(mean or average)

$$Var[N] = E[(N-\mu_N^2)] = E[N^2] = \sigma_N^2 \; [V^2]$$

(mean squared)

$$S_N = \sqrt{Var[N]} = \sigma_N \; [V]$$

(Standard deviation) (root of the mean of the squared)

Signal description of random signal

μ_N : DC- value , μ_N^2 : DC- Power

σ_N : AC- value , σ_N^2 : AC- power

$$\underset{\text{(AC-power)}}{Var[N]} = \underset{\text{(Total power)}}{E[N^2]} - \underset{\text{(DC-power)}}{\mu_N^2}$$

$$\underset{\text{Total signal power}}{E[N^2]} = \underset{\text{AC-power}}{Var[N]} + \underset{\text{DC-power}}{\mu_N^2}$$

Thermal noise

Johnson/Nyquist found that thermal noise is Gaussian with zero mean, and has available power:

$$\Downarrow$$

$$N = kTB \; [W] \quad \text{or} \quad \left(\frac{V^2}{\Omega}\right)$$

where:

$k =$ Boltzmanns constant $[J/K]$

$T =$ Temprature $[K]$

$B =$ Bandwidth $[Hz]$

Standard temprature
(IRE 1962)

$$T_0 = 290K \; (\approx 17^\circ C) \quad (0K = -273.16^\circ C)$$

Noise power density

$$N' = \frac{N}{B} = kT \left[\frac{W}{Hz}\right]$$

Noise floor

$$N_0 = kT_0 = 4E\text{-}21 \left[\frac{W}{Hz}\right]$$

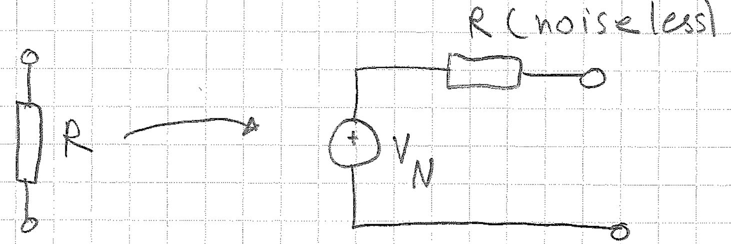
$$= -174 \, [dBm/Hz]$$

$$= -204 \left[\frac{dBW}{Hz}\right]$$

Noise quantities

a. Equivalent noise generator.
b. —— " —— resistor.
c. Noise temprature.

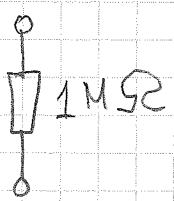
Equiv. noise generator



$$V_N^2 = 4 \cdot P_A \cdot R = 4 \cdot N \cdot R$$
$$= 4 \cdot kTBR$$

$$V_N = \sqrt{4kTBR} \; [V]$$

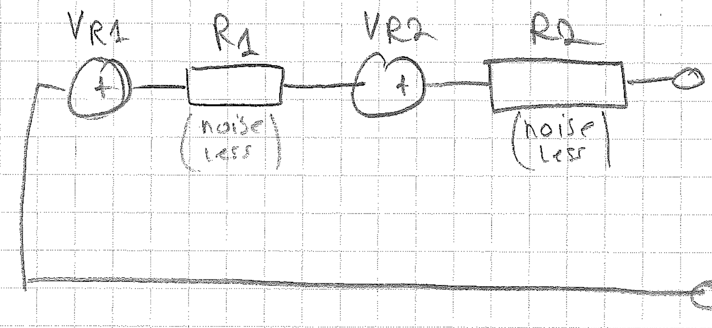
Eks. 1



290K $\Rightarrow V_N = 18\,\mu V$

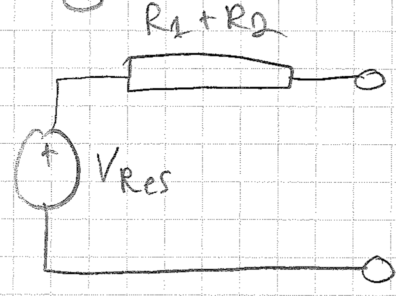
20kHz

Combining noise generator



Current instead of voltage when parallel

$\Rightarrow$



$$R_{Res} = R_1 + R_2 \quad [\Omega]$$

$$V_{NRES}^2 = V_{N1}^2 + V_{N2}^2$$

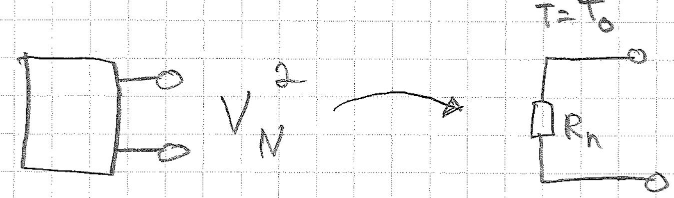
$$V_{NRES} = \sqrt{V_{N1}^2 + V_{N2}^2} \quad [V]$$

Voltage cannot be added due to randomness could be 1mV then 100V

Power does not vary with time

Equiv. noise resistor

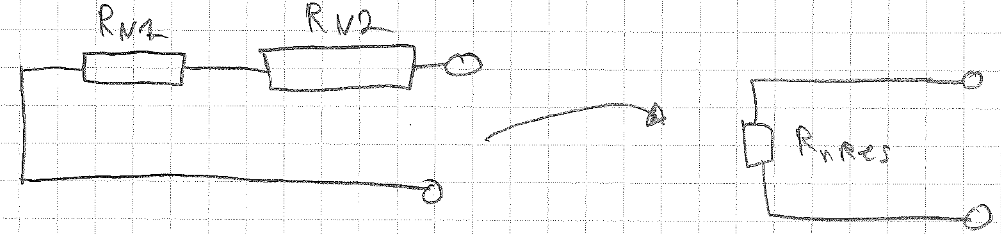
$R_V = G$ when current



$$V_N^2 = 4kTBR = 4kT_0BR_N \; [V^2]$$

$$\Rightarrow R_n = \frac{V_N^2}{4kT_0BR_N} = \frac{V_N^2}{4 \cdot N_0 \cdot B} \; [\Omega]$$

Combining noise resistor

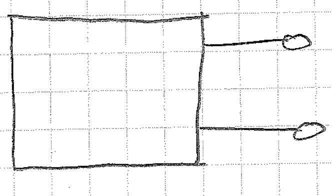


$$V_{NRes}^2 = 4kT_0BR_{N1} + 4kT_0BR_{N2}$$

$$= 4kT_0B(R_{N1} + R_{N2})$$

$$R_{NRes} = R_{N1} + R_{N2} = [\Omega]$$

Noise temprature



N

$$T = \frac{N}{kB} \ [K]$$

$$T = \frac{N'}{k} \ [K]$$

$N = kTB$ $N : kT = V_N^2 / 4kBR \ [K]$

$N' = kT$

$V_N^2 = 4kTBR$

Short-cuts

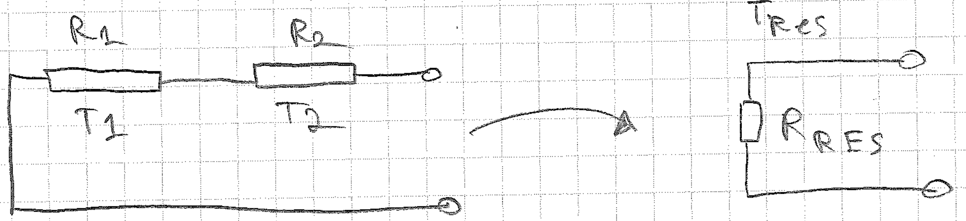
$R_V = G$
for current

$$V_N^2 = 4kTBR = 4kT_0BR_N$$

$$\Rightarrow T/T_0 = R_N/R = G_N/G \ [\cdot]$$

R_N = eq noise resistance

Combining components



$$R_{RES} = R_1 + R_2$$

$$R_{VRES} = R_{V1} + R_{V2} = R_1 \cdot \frac{T_1}{T_0} + R_2 \cdot \frac{T_2}{T_0}$$

$$= 1/T_0 \, (T_1R_1 + T_2R_2)$$

$$T_{RES} = T_0 \cdot \frac{R_{nRES}}{R_{RES}} = \frac{R_1 T_1 + R_2 T_2}{R_1 + R_2}$$

$\Rightarrow$ ↳ for current and parallel

$$= T_1 \cdot \frac{R_1}{R_1 + R_2} + T_2 \cdot \frac{R_2}{R_1 + R_2} \quad [K]$$