Radio Propagation at Frequencies Above 30 Megacycles*

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Summary—Radio propagation is affected by many factors, including the frequency, distance, antenna heights, curvature of the earth, atmospheric conditions, and the presence of hills and buildings. The influence of each of these factors at frequencies above about 30 megacycles is discussed, with most of the quantitative data being presented in a series of nomograms. By means of three or four of these charts, an estimate of the received power and the received field intensity for a given point-to-point radio transmission path ordinarily can be obtained in a minute or less.

The theory of propagation over a smooth spherical earth is presented in a simplified form that is made possible by restricting the frequency range to above about 30 megacycles, where variations in the electrical constants of the earth have only a secondary effect. The empirical methods used in estimating the effects of hills and buildings and of atmospheric refraction are compared with experimental data on shadow losses and on fading ranges.

I. Introduction

METHODS of calculating ground-wave propagation over a smooth spherical earth have been given by Burrows and Gray and by Norton for all values of distance, frequency, antenna height, and ground constants.[1,2] These two methods are different in form but they give essentially the same results. Both methods are relatively simple to use at the lower frequencies where grounded antennas are in common use, but their complexity increases as the frequency increases. At frequencies above 30 or 40 megacycles, elevated antennas are in common use, and the radio path loss between two horizontal antennas tends to be equal to the loss between two vertical antennas. In addition, both types of transmission tend to be independent of the electrical constants of the ground, so that considerable simplification is possible. This paper presents a series of nomograms which have been found useful in solving radio propagation problems in the very-high-frequency range and above. These charts are arranged so that radio transmission can be expressed in terms of either the received field intensity or the received power delivered to a matched receiver. The field-intensity concept may be more familiar, but the power-transfer concept becomes more convenient as the frequency is increased.

In addition to the smooth-earth theory, an approximate method is included for estimating the effects of hills and other obstructions in the radio path. The phenomena of atmospheric refraction (bending away from straight-line propagation), atmospheric ducts (tropospheric propagation), and atmospheric absorption are discussed briefly, but the principal purpose is to provide simplified charts for predicting radio propagation under average weather conditions. It is expected that, normally, the nomograms will provide the desired answer directly without any additional computation, except the addition of the decibel values obtained from three or four individual charts. The basic formulas are presented as an aid to understanding the principles involved and as a more accurate method, should one be required. This paper does not consider sky-wave propagation, although ionospheric reflections may occur at frequencies above 30 megacycles and may cause occasional long-distance interference between systems operating on the same frequency.[3]

A convenient starting point for the theory of radio propagation is the condition of two antennas in free space, which is discussed in terms of both received field intensity and received power. Since most radio paths cannot be considered to be free-space paths, the next step is to determine the effect of a perfectly flat earth, and this is followed by the effect of the curvature of the earth. After the basic smooth-earth theory is completed, there is a discussion of the variations in received power caused by atmospheric conditions and by irregularities on the earth surface, but the methods used in predicting these factors are necessarily less exact than the data for a smooth spherical earth in a uniform atmosphere.

II. Free-Space Field

A free-space transmission path is a straight-line path in a vacuum or in an ideal atmosphere, and sufficiently removed from all objects that might absorb or reflect radio energy. The free-space field intensity E_0 at a distance d meters from the transmitting antenna is given by

$$E_0 = \frac{\sqrt{30 g_1 P_1}}{d} \text{ volts per meter} \tag{1}$$

where P_1 is the radiated power in watts and g_1 is the power-gain ratio of the transmitting antenna. The subscript $_1$ refers to the transmitter and the subscript $_2$ will refer to the receiver. For an ideal (isotropic) antenna that radiates power uniformly in all directions, $g=1$. For any balanced antenna in free space (or located more than a quarter-wavelength above the ground), g is the power-gain ratio of the antenna relative to the isotropic antenna. A small doublet or dipole whose over-all physical length is short compared with a half-wavelength has a directivity gain of $g=1.5$ (1.76 decibels) and a half-

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[1] C. R. Burrows and M. C. Gray, "The effect of earth's curvature on ground-wave propagation," Proc. I.R.E., vol. 29, pp. 16–24; January, 1941.

[2] K. A. Norton, "The calculation of ground-wave field intensities over a finitely conducting spherical earth," Proc. I.R.E., vol. 29, pp. 623–639; December, 1941.

[3] E. W. Allen, "Very-high-frequency and ultra-high-frequency signal ranges as limited by noise and co-channel interference," Proc. I.R.E., vol. 35, pp. 128–136; February, 1947.

wave dipole has a gain of $g=1.64$ (2.15 decibels) in the direction of maximum radiation. In other directions of transmission the field is reduced in accordance with the free-space antenna pattern obtained from theory or measurement. Consequently, the free-space field intensity in a direction perpendicular to a half-wave dipole is

$$E_0 = \sqrt{\frac{30 \times 1.64 P_1}{d}} \sim 7\sqrt{\frac{P_1}{d}}. \quad (2)$$

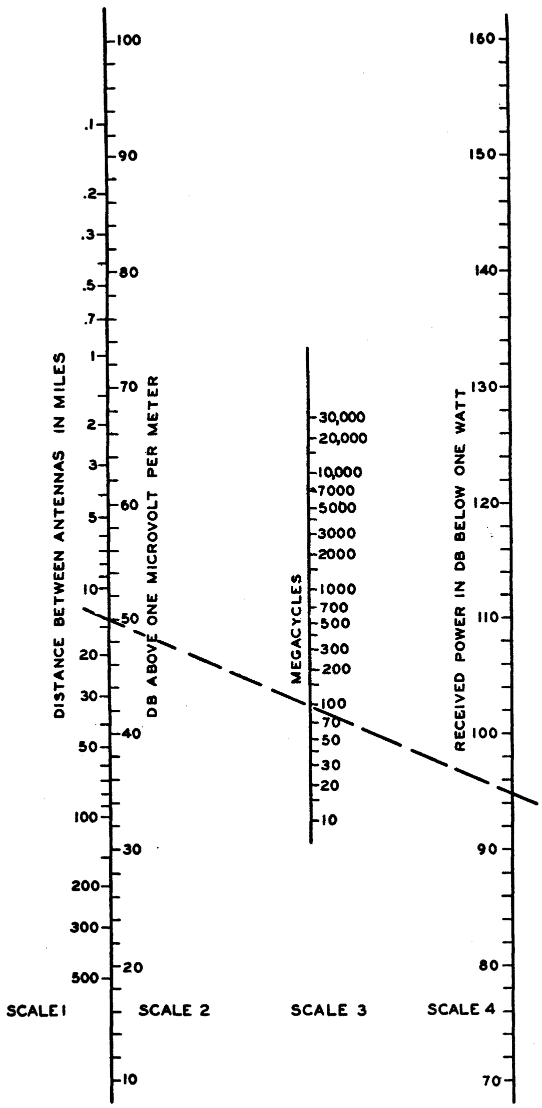

Fig. 1—Free-space field intensity and received power between half-wave dipoles, 1 watt radiated.

This field intensity in microvolts per meter for 1 watt of radiated power is shown on scale 2 of Fig. 1 as a function of the distance in miles shown on scale 1. For radiated power of P watts, the correction factor to apply to the field intensity or power is 10 log P decibels. For example, the free-space field intensity at 100 miles from a half-wave dipole radiating 1 watt is 33 decibels above 1 microvolt per meter (about 45 microvolts per meter). When the radiated power is 50 watts (17 decibels above 1 watt), the received field intensity is 33+17 =50 decibels above 1 microvolt per meter (about 315 microvolts per meter). It will be noted that the field intensity is related to the energy density of the radio wave at the receiving antenna, but is independent of the type of the receiving antenna.

The directivity gain of an array of n dipoles (sum of driven and parasitic elements) of optimum design is approximately equal to n times the gain of one dipole, although some allowance should be made for antenna power losses. The theoretical power-gain ratio of a horn, paraboloid, or lens antenna whose aperture has an area of B square meters is $g=4\pi B/\lambda^2$; however, the effective area is frequently taken as one-half to two-thirds of the actual area of the aperture to account for antenna inefficiencies.

III. Relation Between the Received Power and the Radiated Power

Before discussing the modifications in the free-space field that result from the presence of the earth, it is convenient to show the relation between the received field intensity (which is not necessarily equal to the free-space field intensity) and the power that is available to the receiver. The maximum useful power P_2 that can be delivered to a matched receiver is given by

$$P_2 = \left(\frac{E\lambda}{2\pi}\right)^2 \frac{g_2}{120} \text{ watts} \quad (3)$$

where

E=received field intensity in volts per meter
λ=wavelength in meters$=300/F$
F=frequency in megacycles
g_2=power-gain ratio of the receiving antenna.

This relation between received power and the received field intensity is shown by scales 2, 3, and 4 in Fig. 1 for a half-wave dipole. For example, the maximum useful power at 100 megacycles that can be picked up by a half-wave dipole in a field of 50 decibels above 1 microvolt per meter is 95 decibels below 1 watt.

A general relation for the ratio of the received power to the radiated power obtained from (1) and (3) is

$$\frac{P_2}{P_1} = \left(\frac{\lambda}{4\pi d}\right)^2 g_1 g_2 \left(\frac{E}{E_0}\right)^2. \quad (4)$$

When both antennas are half-wave dipoles, the power-transfer ratio is

$$\frac{P_2}{P_1} = \left(\frac{1.64\lambda}{4\pi d}\right)^2 \left(\frac{E}{E_0}\right)^2 = \left(\frac{0.13\lambda}{d}\right)^2 \left(\frac{E}{E_0}\right)^2 \quad (4a)$$

and is shown on Fig. 1 for free-space transmission ($E/E_0=1$).

When the antennas are horns, paraboloids, or multielement arrays, a more convenient expression for the ratio of the received power to the radiated power is given by

$$\frac{P_2}{P_1} = \frac{B_1 B_2}{(\lambda d)^2} \left(\frac{E}{E_0}\right)^2 \quad (4b)$$

where B_1 and B_2 are the effective areas of the transmitting and receiving antennas, respectively. This relation is obtained from (4) by substituting $g = 4\pi B/\lambda^2$, and is shown on Fig. 2 for free-space transmission when $B_1 = B_2$. For example, the free-space loss at 4000 megacycles between two antennas of 10 square feet effective area is about 72 decibels for a distance of 30 miles.

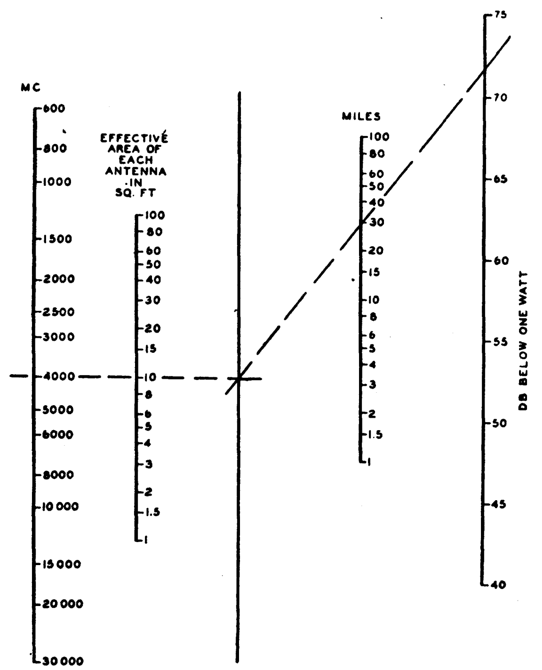

Fig. 2—Received power in free space between two antennas of equal effective areas, 1 watt radiated.

IV. Transmission Over Plane Earth

The presence of the ground modifies the generation and the propagation of the radio waves so that the received field intensity is ordinarily less than would be expected in free space. The ground acts as a partial reflector and as a partial absorber, and both of these properties affect the distribution of energy in the region above the earth. The principal effect of plane earth on the propagation of radio waves is indicated by the following equation:[4,5]

$$E = E_0[\underbrace{1}_{\text{Direct Wave}} + \underbrace{Re^{j\Delta}}_{\text{Reflected Wave}} + \underbrace{(1 - R)Ae^{j\Delta}}_{\text{Surface Wave}} + \underbrace{\cdots}_{\text{Induction Field and Secondary Effects of the Ground}}]. \tag{5}$$

R is the reflection coefficient of the ground and is approximately equal to -1 when the angle θ between the reflected ray and the ground is small. The commonly used concept of a *perfectly conducting* earth, for which the reflection coefficient for vertical polarization is $+1$ for any angle of incidence, may cause some misunderstanding at this point. In practice, the principal interest is in low angles, and as the angle θ approaches zero the reflection coefficient approaches -1 for any finite value for the conductivity of the earth, even if it were made of solid copper. The magnitude and phase of the reflection coefficient can be computed from the following equation:[6]

$$R = \frac{\sin\theta - z}{\sin\theta + z} \tag{6}$$

where

$z = \sqrt{\epsilon_0 - \cos^2\theta}/\epsilon_0$ for vertical polarization

$z = \sqrt{\epsilon_0 - \cos^2\theta}$ for horizontal polarization

$\epsilon_0 = \epsilon - j60\sigma\lambda$

ϵ = dielectric constant of the ground relative to unity in free space

σ = conductivity of the ground in mhos per meter

λ = wavelength in meters

$j = \sqrt{-1}$

$e^{j\Delta} = \cos\Delta + j\sin\Delta$.

The quantity A is the surface-wave attenuation factor which depends upon the frequency, ground constants, and type of polarization. It is never greater than unity and decreases with increasing distance and frequency, as indicated by the following approximate equation:[7]

$$A \simeq \frac{-1}{1 + j\dfrac{2\pi d}{\lambda}(\sin\theta + z)^2}. \tag{7}$$

[4] Charles R. Burrows, "Radio propagation over plane earth-field strength curves," *Bell Sys. Tech. Jour.*, vol. 16, pp. 45–75; January, 1947.

[5] Kenneth A. Norton, "The propagation of radio waves over the surface of the earth and in the upper atmosphere, part II," Proc. I.R.E., vol. 25, pp. 1203–1236; September, 1937.

[6] It will be noted that for vertical polarization this expression agrees with the data given by Burrows and subsequently included in Terman's "Radio Engineer's Handbook," p. 699, first edition, but for horizontal polarization it is the negative of that given in these references. This change was necessary in order to make equations (5) and (6) independent of polarization. The pseudo-Brewster angle frequently mentioned in the literature occurs when the reflection coefficient is a minimum and is approximately equal to the value of θ for which $\sin\theta = |z|$; this occurs with vertical polarization only, since $z > 1$ for horizontal polarization. The reflection coefficient is sometimes modified by a divergence factor to give a first approximation of the effect of the curvature of the earth, but this additional complication does not seem essential here. The effect of the curvature of the earth is discussed in the next section, and for conditions of frequency and antenna height where some interpolation is required, the possible variations due to atmospheric conditions are usually greater than the error introduced by the omission of the divergence factor. The measured data on the plane-earth reflection coefficient agrees reasonably well with the theoretical values at frequencies below about 1000 megacycles. At higher frequencies the magnitude of the reflection coefficient is sometimes less than 1, presumably due to multiple reflections from the irregularities on the earth's surface. Measured values as low as -0.2 at 10,000 megacycles over rolling country have been reported by W. M. Sharpless. The low value of reflection coefficient is not expected to be important for ground-to-ground transmission, but it tends to smooth the lobes that occur in high-angle radiation and, hence, may be important in air-to-ground transmission.

[7] This approximate expression is sufficiently accurate as long as $A < 0.1$, and it gives the magnitude of A within about 2 decibels for all values of A. However, as A approaches unity, the error in phase approaches 180 degrees. More accurate values are given by Norton, where in his nomenclature $A = f(P, B)e^{i\phi}$.

The angle Δ used in (5) is the phase difference in radians resulting from the difference in the length of the direct and reflected rays. It is equal to $4\pi h_1 h_2/\lambda d$ radians, when the distance d between antennas is greater than about five times the sum of the two antenna heights h_1 and h_2.

The effect of the ground shown in (5) indicates that ground-wave propagation may be considered to be the sum of three principal terms; namely, the direct wave, reflected wave, and surface wave. The first two types correspond to our common experience with visible light, but the surface wave is less familiar. Since the earth is not a perfect reflector, some energy is transmitted into the ground and is absorbed. As this energy enters the ground, it sets up ground currents, which is another way of saying that the distribution of the electromagnetic field in the region near the surface of the earth is distorted relative to what it would have been over an ideal perfectly reflecting surface. The surface wave is defined as the vertical electric field for vertical polarization, or the horizontal electric field for horizontal polarization, that is associated with these ground currents.[8] The practical importance of the surface wave is limited to a region above the ground of about 1 wavelength over land or 5 to 10 wavelengths over sea water, since for greater heights the sum of the direct and reflected waves is larger in magnitude. Thus the surface wave is the principal component of the total ground wave at frequencies less than a few megacycles, but it is of secondary importance in the very-high-frequency range (30 to 300 megacycles) and it usually can be neglected at frequencies above 300 megacycles.

A physical picture of the various components of the ground wave can be obtained from (5), but an equivalent expression which is more convenient for this discussion is

$$\frac{E}{E_0} = 2 \sin \frac{\Delta}{2} + j[(1 + R) + (1 - R)A]e^{j(\Delta/2)}. \quad (8)$$

When the angle $\Delta = 4\pi h_1 h_2/\lambda d$ is greater than about 0.5 radian, the terms inside the brackets (which include the surface wave) are usually negligible, and a sufficiently accurate approximation is given by

$$\frac{E}{E_0} = 2 \sin \frac{2\pi h_1 h_2}{\lambda d} \cdot \quad (8a)$$

In this case the principal effect of the ground is to produce interference fringes or lobes so that the field intensity, at a given distance and for a given frequency, oscillates around the free-space field as either antenna height is increased.

When the angle Δ is less than about 0.5 radian, the receiving antenna is below the maximum of the first lobe and the surface wave may be important. A sufficiently accurate approximation for this condition is[9]

$$\left|\frac{E}{E_0}\right| = \left|\frac{4\pi h_1' h_2'}{\lambda d}\right|. \quad (8b)$$

In this equation $h' = h + jh_0$ where h is the actual antenna height and $h_0 = \lambda/2\pi z$ has been designated as the minimum effective antenna height. The magnitude of the minimum effective height $|h_0|$ is shown in Fig. 3 for sea water and for "good" and "poor" soil. "Good" soil corresponds roughly to clay, loam, marsh, or swamp, while "poor" soil means rocky or sandy ground.

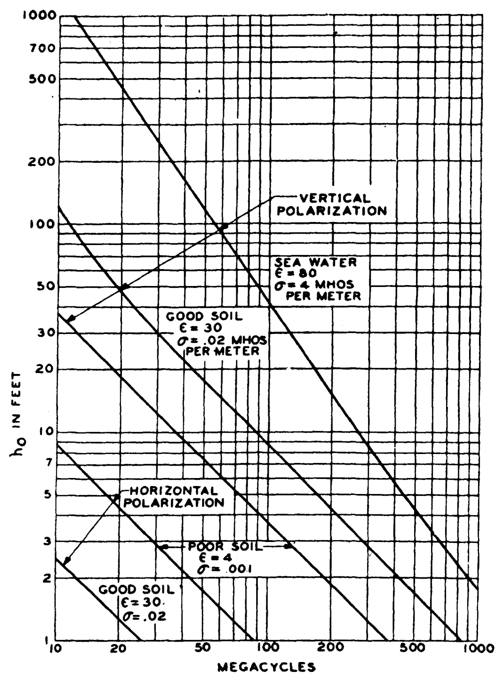

Fig. 3—Minimum effective height.

The surface wave is controlling for antenna heights less than the minimum effective height, and in this region the received field or power is not affected appreciably by changes in the antenna height. For antenna heights that are greater than the minimum effective height, the received field or power is increased approximately 6 decibels every time the antenna height is doubled until free-space transmission is reached. It is ordinarily sufficiently accurate to assume that h' is equal to the actual antenna height or the minimum effective antenna height, whichever is the larger.

The ratio of the received power to the radiated power

[8] Another component of the electric field associated with the ground currents is in the direction of propagation. It accounts for the success of the wave antenna at lower frequencies, but it is always smaller in magnitude than the surface wave as defined above. The components of the electric vector in three mutually perpendicular co-ordinates are given by Norton.

[9] This approximate expression is obtained from (8) by assuming:

$$\sin \theta = \frac{h_1 + h_2}{d} \ll z \quad (1)$$

$$\sin \frac{2\pi h_1 h_2}{\lambda d} = \frac{2\pi h_1 h_2}{\lambda d} \quad (2)$$

$$A = -\frac{\lambda}{j2\pi d z^2} \cdot \quad (3)$$

for transmission over plane earth is obtained by substituting (8b) into (4), resulting in

$$\frac{P_2}{P_1} = \left(\frac{\lambda}{4\pi d}\right)^2 g_1g_2\left(\frac{4\pi h_1'h_2'}{\lambda d}\right)^2 = \left(\frac{h_1'h_2'}{d^2}\right)^2 g_1g_2. \tag{9}$$

This relation is independent of frequency, and is shown on Fig. 4 for half-wave dipoles ($g=1.64$). A line through the two scales of antenna height determines a point on the unlabeled scale between them, and a second line through this point and the distance scale determines the

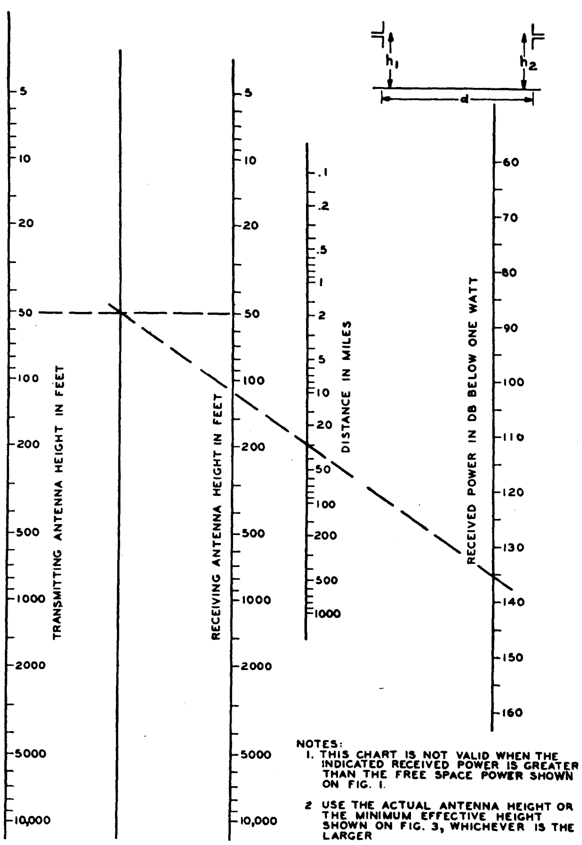

Fig. 4—Received power over plane earth between half-wave dipoles, 1 watt radiated.

received power for 1 watt radiated. When the received field intensity is desired, the power indicated on Fig. 4 can be transferred to scale 4 of Fig. 1, and a line through the frequency on scale 3 indicates the received field intensity on scale 2. The results shown on Fig. 4 are valid as long as the value of received power indicated is lower than shown on Fig. 1 for free-space transmission. When this condition is not met, it means that the angle Δ is too large for (8b) to be accurate and that the received field intensity or power oscillates around the free-space value as indicated by (8a).

As an example, consider a 250-watt, 30-megacycle transmitter with both transmitting and receiving dipoles mounted 50 feet above the ground and separated by a distance of 30 miles over *plane* earth. The transmission loss is shown on Fig. 4 to be 135.5 decibels. Since 250 watts is 24 decibels above 1 watt, the received power is $135.5-24=111.5$ decibels below 1 watt. (The free-space power transfer shown on Fig. 1 indicates a received power of $91-24=67$ decibels below 1 watt, so Fig. 4 is controlling.) The received field intensity can be obtained from Fig. 1, which shows that a received power of 111.5 decibels below 1 watt corresponds to a received field intensity of about 23 decibels above 1 microvolt per meter at a frequency of 30 megacycles. Should one antenna be only 10 feet above "good" soil, rather than 50 feet, the minimum effective height of 30 feet shown on Fig. 3 should be used on one of the height scales on Fig. 4 in determining the transmission loss. It will be noted that this example assumes a perfectly flat earth. The curvature of the earth introduces an additional loss of about 4 decibels, as discussed in the next section.

In addition to the effect of plane earth on the propagation of radio waves, the presence of the ground may affect the impedance of an antenna and thereby may have an effect on the generation and reception of radio waves. This effect usually can be neglected at frequencies above 30 megacycles, except where whip antennas are used. The impedance in the presence of the ground oscillates around the free-space value, but the variations are unimportant as long as the center of the antenna is more than a quarter-wavelength above the ground. A convenient method of showing the effect of the change in impedance of a balanced antenna near the ground is to replace the directivity gain g in the preceding formulas by the arbitrary factor of $g'=g/r$ where r is the ratio of the input resistance in the presence of the ground to the input resistance of the *same* antenna in free space. This assumes an impedance match between the antenna and the transmitting equipment with proper tuning to balance out any reactance.

For horizontal dipoles less than a quarter-wavelength above the ground, the ratio r is less than unity. It approaches zero as the antenna approaches a perfectly conducting earth, but in practice it does not reach zero at zero height because of the finite conductivity of the earth. The wave antenna and the top-loaded antenna frequently used at lower frequencies are sometimes called horizontal antennas, but since they are used to radiate or receive vertically polarized waves they are not horizontal antennas in the sense used here.

For vertical half-wave dipoles the factor r is approximately equal to unity, since the height of the center of the antenna can never be less than a quarter-wavelength above the ground. For very short vertical dipoles, however, the ratio r is greater than unity and it approaches a value of $r=2$ for antennas very near to the ground. This means that, whereas a short vertical dipole whose total length $2l$ is small compared with the wavelength has an input radiation resistance of $80(\pi l/\lambda)^2$ ohms in free space, it has a resistance of $160\ (\pi l/\lambda)^2$ ohms near the ground.

Correct results for a vertical whip antenna working against a perfect counterpoise are obtained by using

$r=2$. This means that a vertical whip antenna of length l is 3 decibels less efficient than a dipole of length $2l$ (located more than a quarter-wavelength above the ground) for either transmitting or receiving. The poorer efficiency at the receiver is not important when external noise is controlling.

V. Diffraction Around the Curvature of the Earth

Radio waves are bent around the earth by the phenomenon of diffraction, with the ease of bending decreasing as the frequency increases. Diffraction is a fundamental property of wave motion, and in optics it is the correction to apply to geometrical optics (ray theory) to obtain the more accurate wave optics. In other words, all shadows are somewhat "fuzzy" on the edges and the transition from "light" to "dark" areas is gradual, rather than infinitely sharp. Our common experience is that light travels in straight lines and that shadows are sharp, but this is only because the diffraction effects for these very short wavelengths are too small to be noticed without the aid of special laboratory equipment. The order of magnitude of the diffraction at radio frequencies may be obtained by recalling that a 1000-megacycle radio wave has about the same wavelength as a 1000-cycle sound wave in air, so that these two types of waves may be expected to bend around obstacles with approximately equal facility.

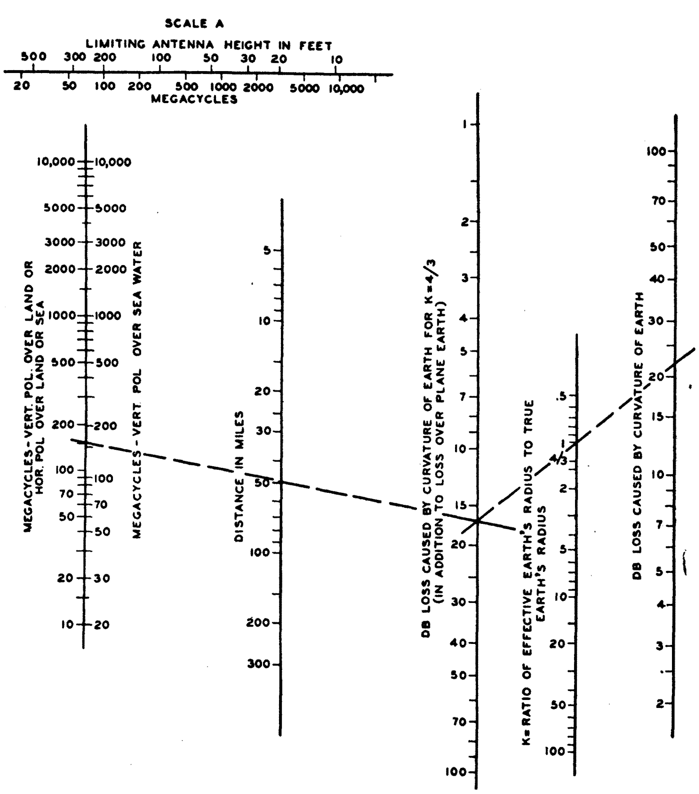

Fig. 5—Diffraction loss caused by curvature of the earth, assuming neither antenna height is higher than shown on scale A.

The effect of diffraction around the earth's curvature is to make possible transmission beyond the line-of-sight, but it introduces an additional loss. The magnitude of this loss increases as either the distance or the frequency is increased and it depends to some extent on the antenna height. The loss resulting from the curvature of the earth is indicated by Fig. 5 as long as neither antenna is higher than the limiting value shown at the top of the chart. This loss is in addition to the transmission loss over plane earth obtained from Fig. 4.

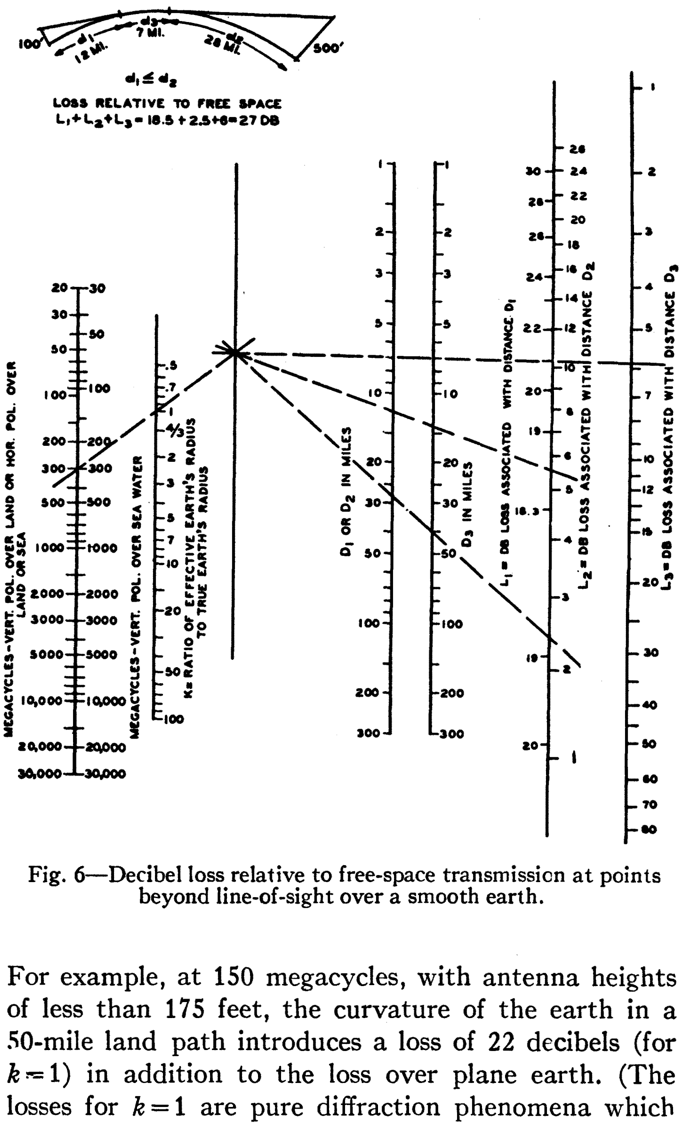

Fig. 6—Decibel loss relative to free-space transmission at points beyond line-of-sight over a smooth earth.

For example, at 150 megacycles, with antenna heights of less than 175 feet, the curvature of the earth in a 50-mile land path introduces a loss of 22 decibels (for $k=1$) in addition to the loss over plane earth. (The losses for $k=1$ are pure diffraction phenomena which would occur even in a vacuum. In the next section it will be shown that atmospheric refraction may cause a similar effect, although the cause is different. The parameter k is introduced to account for refraction in the earth's atmosphere, so that Fig. 5 shows the combined effects of diffraction and refraction.)

When either antenna is as much as twice as high as the limiting value shown on Fig. 5, this method of correcting for the curvature of the earth indicates a loss that is too great by about 2 decibels, with the error increasing as the antenna height increases. An alternate method of determining the effect of the earth's curvature is given by Fig. 6. This method is derived in the Appendix and is approximately correct for any antenna height, but it is theoretically limited in distance to points at or beyond the line-of-sight, assuming that the curved earth is the only obstruction. Fig. 6 gives the loss relative to free-space transmission (and hence is used with Figs. 1 or 2) as a function of three distances: d_1 is the distance to the horizon from the *lower* antenna, d_2 is the distance to the horizon from the higher antenna, and d_3 is the distance beyond the line-of-sight. In other words, the total distance between antennas $d=d_1+d_2+d_3$. The distance to the horizon is shown in Fig. 7 for various values of k and antenna height. As an example,

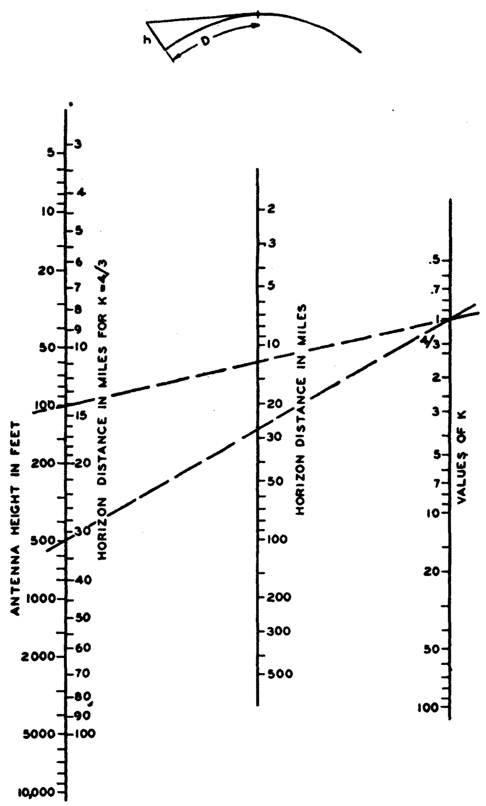

Fig. 7—Distance to the horizon.

consider a radio system operating at 300 megacycles over smooth earth with antenna heights of 500 and 100 feet. Fig. 7 indicates that $d_1=12$ miles and $d_2=28$ miles for a value of $k=1$. Table I shows the loss relative to free-space transmission, as obtained from Fig. 6, for various distances at and beyond the line-of-sight.

In Table I, the estimated received power at line-of-sight is 21 decibels less than would be expected in free space, and it decreases about 0.8 decibels for each mile beyond line-of-sight. At distances well within line-of-sight, the earth can be considered flat. There is no accurate method of joining the curve for points well within line-of-sight to the curve for points beyond line-of-sight, but one or both of the following empirical methods may be helpful. The data for flat earth, given in Fig. 4, can be used at distances near grazing, provided that each antenna height is interpreted as the height above an imaginary plane drawn tangent to the earth at the point of reflection. An alternate method is to use Fig. 6 at points near grazing by considering the loss L_3 to be negative whenever d_3 is negative. In either case, the estimated received power should not be greater than would be obtained over flat earth.

For more accurate results at and beyond the line-of-sight, the field intensity or received power computed by means of Fig. 6 should be increased by

$$10 \log \left[\frac{1}{\sqrt{2}}\left(1+\frac{d_1}{d_2}+\frac{d_3}{d_2}\right)\right]$$

decibels as long as one antenna height is higher than the value shown on the top of Fig. 5. In the region of antenna heights where Fig. 5 is applicable, it is easier to use and more accurate than the data on Fig. 6.

TABLE I
SAMPLE COMPUTATIONS BEYOND LINE-OF-SIGHT
300 Megacycles—Antenna Heights, 500 and 100 Feet

Miles				Decibel Losses			Decibel Loss in Addition to the Free-Space Loss
d	d_1	d_2	d_3	L_1	L_2	L_3	
40	12	28	0	18.5	2.5	0	21
45	12	28	5	18.5	2.5	4	25
50	12	28	10	18.5	2.5	8.5	29.5
55	12	28	15	18.5	2.5	13.5	34.5
60	12	28	20	18.5	2.5	19	40
65	12	28	25	18.5	2.5	24	45

VI. ATMOSPHERIC REFRACTION AND ABSORPTION

Thus far it has been assumed that the radio waves are traveling through a vacuum, or through an ideal atmosphere which has a dielectric constant of unity at all points and has zero absorption. Actually, the dielectric constant of the air is slightly greater than 1 and is variable. It depends on the pressure and temperature of the air and on the amount of water vapor present, so that it varies with weather conditions and with the height above the ground. The change in dielectric constant in several thousand feet is never more than a few parts in ten thousand, but this variation is sufficient to have an appreciable effect on radio propagation.

Whenever the dielectric constant varies with the height above the ground, the path of a radio wave deviates from a straight line. This change in direction is called refraction. A general solution of the problem which would allow any possible distribution of dielectric constant with the height above the ground at any point along the radio path is virtually impossible because of a large number of variables involved, so some simplifying assumptions are needed in order to obtain an engineering solution. The first assumption usually made is that of horizontal stratification, which means that for any given height the dielectric constant has the same value at all distances along the radio path. Typical solutions based on this assumption have been worked out, but the problem is still too complex for most applications. A simple engineering solution can be obtained by making the additional assumption that the dielectric constant is a linear function of the height. On this basis, the effect of atmospheric refraction can be included in the expression of diffraction around the smooth earth (without discarding the useful concept of straight-line propagation) by multiplying the actual earth's radius by

$$k = \frac{1}{1 + \frac{a}{2} \frac{\Delta\epsilon}{\Delta h}}$$

where a is the radius of the earth and $\Delta\epsilon$ is the change in dielectric constant in going from height h to $h+\Delta h$. Physically, the phenomenon of refraction is entirely separate from the concept of diffraction discussed in the preceding section, although for computational purposes the two effects are combined by introducing the parameter k in Figs. 5 and 6.

The dielectric constant normally decreases with increasing height ($k>1$) and the radio waves are bent toward the earth. However, under certain atmospheric conditions the dielectric constant may increase ($0<k<1$) over a reasonable height, thereby causing the radio waves in this region to bend away from the earth. Since the earth's radius is about 2.1×10^7 feet, a decrease in dielectric constant of only 2.4×10^{-8} per foot of height results in a value of $k=4/3$, which is commonly assumed to be a good average value. When the dielectric constant decreases about four times as rapidly (or by about 10^{-7} per foot of height), the value of $k=\infty$. This means that, as far as radio propagation is concerned, the earth can be considered flat, since any ray that starts parallel to the earth will remain parallel.

When the dielectric constant decreases more rapidly than 10^{-7} per foot of height, radio waves that are radiated parallel to or at an angle above the earth's surface may be bent downward sufficiently to be reflected from the earth. After reflection the ray is again bent toward the earth, and the path of a typical ray is similar to the path of a bouncing tennis ball. The radio energy appears to be trapped in a duct or wave guide between the earth and the maximum height of the radio path. This phenomenon is variously known as trapping, duct transmission, anomalous propagation, or guided propagation. It will be noted that in this case the path of a typical guided wave is similar in form to the path of sky waves, which are lower-frequency waves trapped between the earth and the ionosphere. However, there is little or no similarity between the virtual heights, the frequencies, or the causes of refraction in the two cases.

In addition to the simple form of a duct where the earth is the lower boundary, trapping may also occur in an elevated duct. For example, assume an ideal case where the curve of the dielectric constant versus the height above the ground can be represented by three straight lines. The lower segment corresponds to the height interval of 0 to 100 feet above the ground and in this region the dielectric constant decreases very slowly, or it may even increase. The middle segment corresponds to a height interval of 100 to 150 feet and in this region the dielectric constant decreases more rapidly than 10^{-7} per foot. The third section corresponds to heights greater than 150 feet and the dielectric constant decreases at a rate less than 10^{-7} per foot. In this ideal case it can be shown that most of the radio energy (at frequencies above about 300 megacycles) is trapped within a height interval of about 50 to 150 feet above the ground, and that the actual path for any given ray is approximately a sine wave whose axis is 100 feet above the ground.

The phenomenon of trapping is of considerable interest, but quantitative data on radio propagation in a duct are beyond the scope of this paper. The concept of an effective earth's radius used in Figs. 5 and 6 fails in this case because the parameter k is negative, and negative values are contrary to the original assumptions in diffraction theory. However, experience indicates that the received field intensity or received power is seldom greater than would be expected for plane earth ($k=\infty$), so this limitation is not expected to be serious.

Meteorological measurements indicate that the actual curve of dielectric constant versus the height above the ground is frequently a curved line which may have one or more sharp bends, rather than a straight line as required in using the concept of an effective earth's radius.[10] Theoretical considerations indicate that this curve can be approximated with reasonable accuracy by a series of straight lines as long as each individual line corresponds to a change in height of not more than 20 to 50 wavelengths. At 30 megacycles, for example, the actual curve of dielectric constant versus height can be approximated by a number of straight lines, each of which has a slope corresponding to the average change in dielectric constant over a height interval of 600 to 1500 feet. Since most of the radio energy transmitted between two ground stations travels in the first of these height intervals, the concept of effective earth radius is a useful one and is sufficiently accurate at 30 megacycles. As the frequency increases, however, more than

[10] The investigators in this field usually use M units rather than the dielectric constant. The M unit is defined as

$$M = \left(\sqrt{\epsilon} - 1 + \frac{h}{a}\right) 10^6$$

where ϵ is the dielectric constant at height h, and a is the radius of the earth. The M unit provides a number of convenient size (usually 200 to 500) and is modified by the term h/a for use on a flat earth diagram. The relation between the parameter k and M units is given by

$$k = \frac{10^6}{M'a} = \frac{.048}{M'},$$

where M' is the change in M units per foot of height.

one segment must be considered, since the straight-line approximation is valid over smaller and smaller height intervals. At 3000 megacycles, for example, this interval is only 6 to 15 feet, and the concept of effective earth radius becomes inadequate for analytical use. Considerable progress has been made toward formulating a method of correcting for atmospheric refraction that avoids these limitations, but the complexity of the problem and the amount of basic data required indicate that ordinarily a statistical study of fading ranges may be preferred to an analytical solution.

The experimental data on fading in the 30- to 150-megacycle range can be correlated reasonably well with the concept of effective earth's radius, if it be assumed that the value of k is rarely less than 0.8 and is seldom greater than 2 or 3. At higher frequencies the received signal at distances beyond the line-of-sight is seldom less than would be expected for $k=0.8$ and may be equal to or above the free-space value.[11] This means that the fading range at 3000 megacycles over a path 10 miles beyond the optical line-of-sight may be as much as 60 or 70 decibels. The per cent of time that the signal is either extremely high or extremely low depends on meteorological conditions, which, in turn, are functions of the geography and season of the year as well as the daily weather variations. Within the line-of-sight, the received power at frequencies above 2000 or 3000 megacycles may vary from several decibels above to 15 decibels or more below the free-space value, even over a good optical path. A good optical path is defined as one with full first-Fresnel-zone clearance, as discussed in the next section. The fading appears to be more severe over sea water than over land, but whether this difference results primarily from a difference in atmospheric conditions or from a difference in the reflecting properties of sea water and land has not been clearly established.

The atmosphere not only affects the direction of a radio wave but also it may introduce some absorption in the transmission path. The presence of rain, snow, or fog introduces an additional attenuation which depends on the amount of moisture and on the frequency. During a rain of cloudburst proportions the attenuation at 10,000 megacycles (3 centimeters) may reach 5 decibels per mile, and at 30,000 megacycles (1 centimeter) it may be in excess of 25 decibels per mile.[12,13] In addition to the effect of moisture, some selective absorption may result from the oxygen, nitrogen, and other components of the atmosphere. The first absorption peak due to water vapor occurs at a wavelength of about 1.25 centimeters and the first peak for oxygen occurs at about 0.5 centimeters. These absorption bands are well known at frequencies of infrared and visible light, and are to be expected as the radio spectrum is extended to higher frequencies.

VII. Transmission Over Sharp Ridges

The preceding discussion assumes that the earth is a perfectly smooth sphere. The modification in these results caused by the presence of hills, trees, and buildings is difficult or impossible to compute, but the order of magnitude of these effects may be obtained from a consideration of the other extreme case, which is propagation over a perfectly absorbing knife edge.

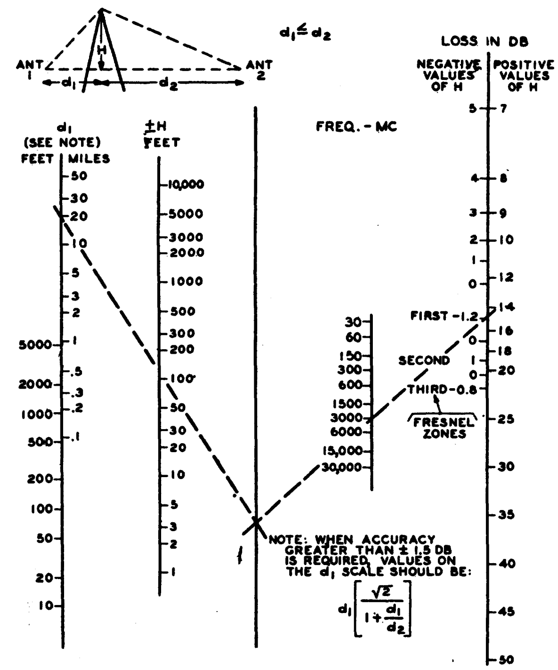

Fig. 8—Shadow loss relative to free space.

The diffraction of plane waves over a knife edge or screen causes a shadow loss whose magnitude is shown on Fig. 8.[14] The height of the obstruction H is measured from the line joining the two antennas to the top of the ridge. It will be noted that the shadow loss approaches 6 decibels as H approaches 0 (grazing incidence), and that it increases with increasing positive values of H. When the direct ray clears the obstruction, H is negative, and the shadow loss approaches 0 decibels in an oscillatory manner as the clearance is increased. In other words, a substantial clearance is required over line-of-sight paths in order to obtain "free-space" transmission.

There is an optimum clearance, called the first-Fresnel-zone clearance, for which the transmission is theoretically 1.2 decibels better than in free space. Phys-

[11] The theory of trapping would indicate that the received signal may be considerably higher than indicated by free-space transmission. Instantaneous peaks of 18 to 20 decibels above free space have been reported, but the average signal is greater than 3 or 4 decibels above the free-space value for only a small percentage of the total time.

[12] S. D. Robertson and A. P. King, "The effect of rain upon the propagation of waves in the 1- and 3-centimeter regions," Proc. I.R.E., vol. 34, pp. 178–180; April, 1946.

[13] G. E. Mueller, "Propagation of 6-millimeter waves," Proc. I.R.E., vol. 34, pp. 181–183; April, 1946.

[14] The theory of diffraction over a knife edge is discussed in several textbooks including J. C. Slater and N. H. Frank, "Introduction to Theoretical Physics," McGraw-Hill Book Co., New York, N. Y. 1933, pp. 315–323.

ically, this clearance is of such magnitude that the phase shift along a line from the antenna to the top of the obstruction and from there to the second antenna is about $\frac{1}{2}$ wavelength greater than the phase shift of the direct path between antennas. When this phase difference is 1 wavelength, the path clears the first two Fresnel zones, and there is theoretically a loss of about 1 decibel relative to free space. Similarly, when the phase difference is 3/2 wavelengths the path clears the first three Fresnel zones, and this is a gain of about 0.8 decibel relative to free space. The locations of the first three Fresnel zones are indicated on the right-hand scale on Fig. 8, and by means of this chart the required clearances can be obtained. At 3000 megacycles, for example, the direct ray should clear all obstructions in the center of a 40-mile path by about 120 feet to obtain full first-zone clearance. The corresponding clearance for a ridge 100 feet in front of either antenna is 4 feet. Should the ridge project above the direct path by 4 feet, the shadow loss is about 15 decibels. It will be noted that the effective clearance obtained on a particular path will vary with the weather conditions, since the effect of atmospheric refraction is neglected in Fig. 8.

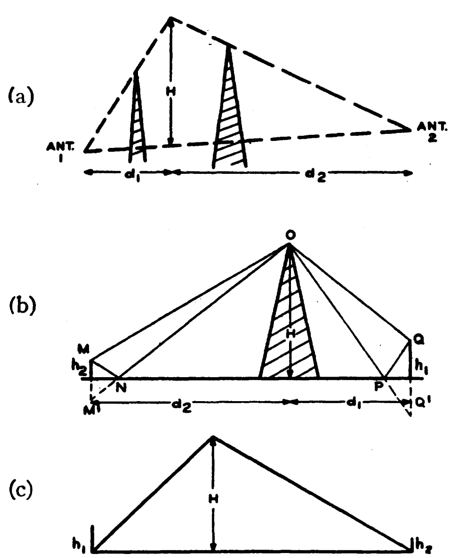

Fig. 9—Ideal profiles used in developing theory of diffraction over hills.

The problem of two or more knife-edge obstructions between the transmitting and receiving antennas, such as is shown in Fig. 9(a), has not been solved rigorously. However, graphical integration indicates that the shadow loss for this case is equivalent within 2 or 3 decibels to the shadow loss for the knife edge represented by the height of the triangle composed of a line joining the two antennas and a line from each antenna through the top of the peak that blocks the line-of-sight from that antenna.

Thus far it has been assumed that the transmission between the two antennas would be approximately the same as in free space, if the obstacles could be removed. This assumption is usually valid only at centimeter wavelengths, and at lower frequencies it is necessary to include the effects of waves reflected from the ground. This results in four paths, namely, MOQ, MOQ', $M'OQ$, and $M'OQ'$, shown on Fig. 9(b) for a single obstruction. Each of these paths is similar in form to the single path illustrated by Fig. 9(a). The sum of the field intensities over these four paths, considering both magnitude and phase, is given by the following equation:

$$\left|\frac{E}{E_0}\right| = S_1\left[1 - \frac{S_2}{S_1}e^{-j(\Delta+b)} - \frac{S_3}{S_1}e^{-j(\Delta+c)} + \frac{S_4}{S_1}e^{-j(b+c)}\right] \quad (12)$$

where

E = received field intensity
E_0 = free-space field intensity
S_1 = magnitude of the shadow loss over path MOQ
S_2 = magnitude of the shadow loss over path MOQ'
S_3 = magnitude of the shadow loss over path $M'OQ$
S_4 = magnitude of the shadow loss over path $M'OQ'$
$\Delta = 4\pi h_1 h_2/\lambda(d_1+d_2)$ radians
b is approximately equal to $4\pi h_2 H/\lambda d_2$ radians
c is approximately equal to $4\pi h_1 H/\lambda d_1$ radians.

This equation assumes that the reflection coefficient is -1 and that the actual antenna heights are greater than the minimum effective antenna height h_0. This means that the surface wave is neglected, and the equation fails when either antenna height approaches zero. The angles b and c are phase angles associated with the diffraction phenomena, and the approximate values given above assume that H is greater than h_1 or h_2. This assumption permits the shadow losses to be averaged so that $S_1 = S_2 = S_3 = S_4 = S$. After several algebraic manipulations, (12) can be reduced to

$$\left|\frac{E}{E_0}\right| = 2(2S)\left|\sin\frac{\Delta}{2}\cos\frac{b-c}{2} + j\left(\sin^2\frac{b+c}{4} - \sin^2\frac{b-c}{4}\right)e^{j(\Delta/2)}\right| \quad (13)$$

where S is the average shadow loss for the four paths. This means that the shadow triangle should be drawn from a point midway between the location of the actual antenna and the image antenna, as shown in Fig. 9(c). For small values of H this equation is approximately equal to

$$\left|\frac{E}{E_0}\right| = 2\,(2S)\sin\frac{\Delta}{2}. \quad (14)$$

Since the field intensity over plane earth (assuming that the antenna heights are greater than the minimum effective height h_0) is $2E_0 \sin \Delta/2$, the first-order effect of the hill is to add a loss of 20 log 2S decibels, which is shown by the nomogram on Fig. 10.

The complete expression given in (12) indicates that, under favorable conditions, the field intensity behind sharp ridges may be greater than over plane earth. This result has been found experimentally in a few cases, but the correlation between theory and experiment is not complete. In general, the field intensity predicted by either (12) or (13) tends to be too high; that is, shadow losses rather than gains occur on most of the paths on which measured data are available. The less-approximate expression given in (14) agrees more closely with experimental data, is more conservative, and is easier to use. Consequently, it is usually assumed that the effect of obstructions to line-of-sight transmission (at least in the 30- to 150-megacycle range), is to introduce the loss shown on Fig. 10 in addition to the normal loss over smooth earth for the antenna heights and distances involved. Measured results on a large number of paths in the 30- to 150-megacycle range indicate that about 50 per cent of the paths are within 5 or 6 decibels of the values predicted on this basis. The correlation on 10 per cent of the paths is no better than 10 to 12 decibels, and an occasional path may differ by 20 decibels.

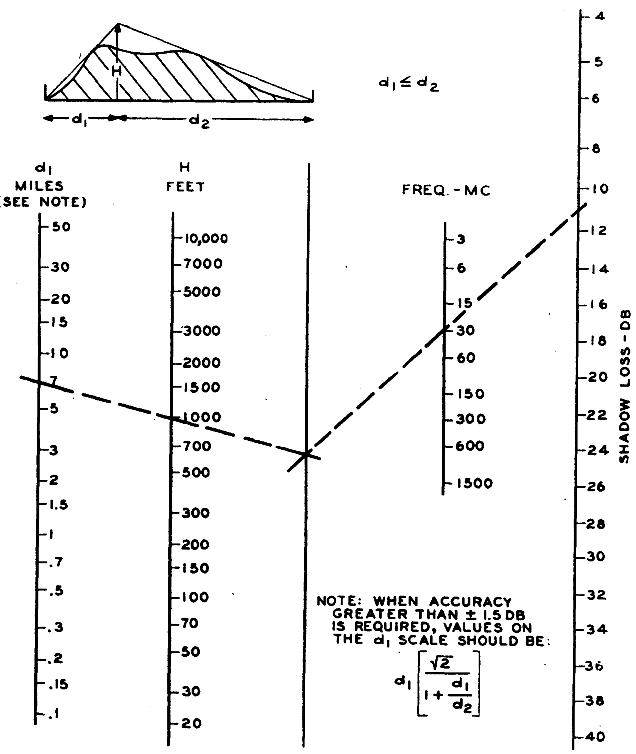

Fig. 10—Shadow loss relative to smooth earth.

VIII. Miscellaneous Factors Affecting Propagation

The height of an antenna located over plane earth is the height of the center of the antenna above ground level. Locating an antenna on a hill which slopes downward toward the distant terminal usually increases the received power. The magnitude of this improvement can be estimated by assuming that the effective antenna height (for use on Fig. 4) is the difference in elevation between the antenna and the bottom of the hill, *providing* that first-Fresnel-zone clearance is obtained over the immediate foreground. At 30 megacycles this means a clearance of about 20 feet at a distance of 20 feet, 40 feet at a distance of 100 feet, 90 feet at a distance of 500 feet, etc. The required clearance decreases as the square root of the wavelength, and may be obtained from Fig. 8. When these clearances are not met, it is convenient to assume that the effective antenna height is the difference in elevation between the antenna and the point where the actual profile intercepts the curve of required clearance (first Fresnel zone).

Built-up areas have little effect on radio transmission at frequencies below a few megacycles, since the size of any obstruction is usually small compared with the wavelength, and the shadows caused by steel buildings and bridges are not noticeable except immediately behind these obstructions. However, at 30 megacycles and above, the absorption of a radio wave in going through an obstruction and the shadow loss in going over it are not negligible, and both types of losses tend to increase as the frequency increases. The attenuation through a brick wall, for example, may vary from 2 to 5 decibels at 30 megacycles and from 10 to 40 decibels at 3000 megacycles, depending on whether the wall is dry or wet. Consequently most buildings are rather opaque at frequencies of the order of thousands of megacycles. Shadow losses at street level in the downtown area of large cities may be of the order of 30 decibels or more at frequencies in the 30- to 150-megacycle range, and the received power may vary 15 to 20 decibels within a few feet because of wave interference caused by multiple-path transmission. As the frequency increases the number of possible multiple paths also increase, so that there is some tendency to fill in the deep shadow regions. This means that the average shadow loss at street level may not increase as rapidly with frequency as the shadow loss behind an isolated ridge, and this is one reason for limiting the distance scale on Fig. 10 to values greater than 0.1 mile.

When an antenna is surrounded by moderately thick trees and below tree-top level, the average loss at 30 megacycles resulting from the trees is usually 2 or 3 decibels for vertical polarization and is negligible with horizontal polarization.[15] However, large and rapid variations in the received field intensity may exist within a small area, resulting from the standing-wave pattern set up by reflections from trees located at a distance of as much as 100 feet or more from the antenna. Consequently, several near-by locations should be inves-

[15] National Defense Research Council Report, Division 13, "Effect of hills and trees as obstructions to radio propagation," C. M. Jansky and S. L. Bailey; November, 1943.

tigated for best results. At 100 megacycles the average loss from surrounding trees may be 5 to 10 decibels for vertical polarization and 2 or 3 decibels for horizontal polarization. The tree losses continue to increase as the frequency increases, and above 300 to 500 megacycles they tend to be independent of the type of polarization. Above 1000 megacycles trees that are thick enough to block vision present an almost solid obstruction, and the diffraction loss over or around these obstructions can be obtained from Fig. 8.

IX. Minimum Allowable Input Power

The effective use of the preceding data for estimating the received power requires a knowledge of the power levels needed for satisfactory operation, since the principal interest is in the signal-to-noise ratio. The signal level required at the input to the receiver depends on several factors, including the noise introduced by the receiver (called first-circuit or set noise), the type and magnitude of any external noise, the type of modulation, and the desired signal-to-noise ratio. A complete discussion of these factors is beyond the scope of this paper, but the fundamental limitations are listed below in order to show the order of magnitude. The theoretical minimum noise level is that set by the thermal agitation of the electrons, and its root-mean-square power in decibels below 1 watt is 204 decibels minus 10 log (bandwidth) where (bandwidth) is approximately equal to twice the audio (or video) bandwidth.[16] The set noise of a typical receiver may be 5 to 15 decibels higher than the theoretical minimum noise. The lower values in this range of "excess" noise are more likely to be met in the very-high-frequency range, while the higher values are more probable in the super-high-frequency range. This means that the set noise in a 3000-cycle audio band may be 151 to 161 decibels below 1 watt. Measured data indicate that the carrier power needs to be 12 to 20 decibels higher than the noise power to provide an average signal-to-noise ratio that is sufficient for moderate intelligibility. This assumes that the modulation level has been adjusted so that most of peaks of speech power can be transmitted without causing overmodulation in the transmitter. It follows that the required input power for a single-channel voice circuit is of the order of 140 decibels below 1 watt, which is roughly equivalent to 1 microvolt across a 70-ohm input resistance. This limiting input power is approximately correct (within 3 or 4 decibels) for both amplitude and frequency modulation, since the radio-frequency signal-to-noise ratio must be above that required for marginal operation before the use of frequency modulation can provide appreciable improvement in the audio signal-to-noise ratio.

The input power must be greater than 140 decibels below 1 watt when circuits of above marginal quality or greater bandwidth are desired, and when external noise rather than set noise is controlling. Man-made noise is frequently controlling at 30 megacycles, but is less serious at 150 megacycles. Above 500 megacycles, set noise is almost always controlling. For circuits requiring a high degree of reliability, a margin should also be included for the fading range to be expected during adverse weather conditions.

TABLE II
FIGURES TO USE

Type of Terrain	Both Antennas Lower in Height than Shown on Fig. 5	One or Both Antenna Heights Higher than Shown on Fig. 5	
		Within Line-of-Sight	Beyond Line-of-Sight
Plane earth	Fig. 4 or 1	Figs. 4 or 1 or 2	
Smooth earth	Figs. 4 and 5	Figs. 4 or 1 or 2	Figs. 1 or 2 and 6
Irregular terrain	Figs. 4, 5, and 10	Figs. 4 and 10 or 1 or 2 and 8	Figs. 1 or 2 6 and 10

X. Summary and Examples

In any given radio propagation problem some of the factors described above are important, while others can be neglected. Table II indicates the figures that apply to any given situation.

Whenever Fig. 4 is used, reference should be made to Figs. 1 and 3 as a check on its proper use. When Fig. 10 is used in determining the effects of hills, the profile is usually drawn on rectangular co-ordinates (neglecting the earth's curvature), and the shadow triangle is drawn to the base of the antenna (half way between the antenna and its image). Curved co-ordinates are sometimes used, but they are not necessary since the loss caused by the curvature of the earth is either negligible or has already been considered in Figs. 5 or 6. Fig. 8 is used for determining the first-zone clearance and for estimating the shadow losses, when the transmission without the obstruction is expected to be the same as in free space. In the latter case, the shadow triangle is

[16] Harald T. Friis, "Noise figures of radio receivers," Proc. I.R.E., vol. 32, pp. 419–422; July, 1944.

drawn from the actual antennas, and curved co-ordinates are useful since the curvature of the earth should be included in the profile.

Various examples of the use of these figures have been given during the discussion of each individual chart, but further examples may help to illustrate the relations between the various figures. Assume a transmitting dipole located 250 feet above the ground and a receiving dipole on a 30-foot mast. The estimated transmission losses at 30, 300, and 3000 megacycles over smooth earth are shown on Fig. 11 for $k=4/3$ for various distances between these two dipoles. The solid lines indicate values obtained from the figures and the dashed lines show the region where some interpolation is required.

The received power depends on the radiated power and the antenna-gain characteristics, as well as on the transmission loss between dipoles. A typical 30-megacycle transmitter may radiate 250 watts, so that at a

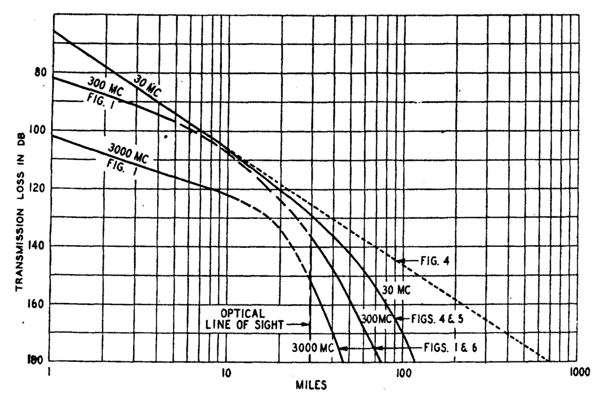

Fig. 11—Transmission over smooth earth at 30, 300, and 300 megacycles; half-wave dipoles at 250 and 30 feet.

distance of 30 miles the received power is 10 log 250 $-129=105$ decibels below 1 watt. (The value of 129 decibels is obtained from Fig. 11.) Similarly, a 300-megacycle transmitter may radiate 50 watts from a 5-decibel antenna, and when a 5-decibel receiving antenna is used the estimated received power at 30 miles is 10 log $50+5+5-137=110$ decibels below 1 watt. At 3000 megacycles the radiated power may be 0.1 watt and antenna gains of 28 decibels each are not uncommon, so the received power at 30 miles is 10 log $0.1+28+28$ $-152=106$ decibels below 1 watt. (The values of radiated power used in this example are not the maximum continuous-wave powers that can be obtained, but the downward trend with increasing frequency is a characteristic of the available tubes.)

Over irregular terrain it is assumed that the shadow loss based on knife-edge diffraction theory is to be added to the transmission loss obtained from smooth-earth theory. The computation of shadow losses for the profile shown on Fig. 12(a) is given in Table III.

The estimated transmission loss for 30 and 300 megacycles, including the shadow loss from Table III, is shown by the solid lines on Fig. 12(b), while the dashed lines are the corresponding values over smooth earth taken from Fig. 11. Superimposed on these average val-

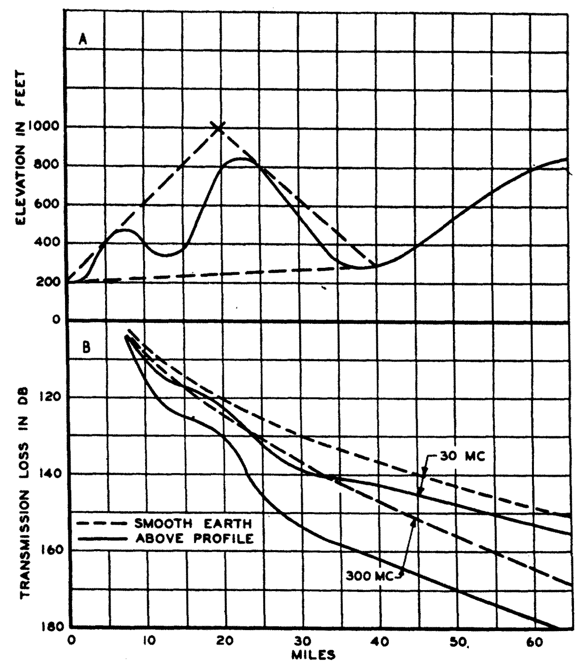

Fig. 12—Transmission loss over irregular terrain at 30 and 300 megacycles; half-wave dipoles at 250 and 30 feet.

ues will be unpredictable variations of ± 6 to 10 decibels resulting from the effects of trees and buildings and from profile irregularities that were smoothed out in drawing the profile shown in Fig. 12(a).

TABLE III

SHADOW-LOSS COMPUTATIONS

Miles from Transmitter	H (feet)	d_1 (miles)	Shadow Loss in Decibels Obtained from Fig. 10	
			30 megacycles	300 megacycles
12.5	200	4.5	3.5	9
20	70	6.5	2	4
25	310	4.5	5.5	13
35	750	14.5	7	16
40	760	20	6	14
50	610	18	5	12
60	490	16.5	4.5	11

APPENDIX

DERIVATION OF DIFFRACTION LOSS BETWEEN ELEVATED ANTENNAS

The derivation of the method shown in Fig. 6 for determining the effect of the curvature of the earth is based on the data in the appendix to the paper by Burrows and Gray,[1] and their nomenclature is used in the following discussion.

The best available equation for radio propagation over spherical earth below the line of sight is

$$\frac{E}{E_0} = (8\pi\zeta_a)^{1/2}\left|\sum_{s=0}^{\infty} f_s(h_1)f_s(h_2)\,\frac{\exp(-i\tau_s\zeta_a)}{\delta+2\tau_s}\right| \tag{15}$$

where the parameters τ_s are functions of the ground constants and the height functions $f_s(h)$ are independent of the distance between antennas. The other symbols are defined below:

$$\zeta_a = \frac{\dfrac{2\pi d}{\lambda}}{\left(\dfrac{2\pi ka}{\lambda}\right)^{2/3}} = \zeta_1+\zeta_2+\zeta_3$$

$$\delta = z^2\left(\frac{2\pi ka}{\lambda}\right)^{2/3}$$

d = distance between antennas

a = radius of the earth

λ = wavelength

$$k = \text{ratio of } \frac{\text{effective earth's radius}}{\text{true earth's radius}}$$

$$z = \frac{\sqrt{\epsilon_0 - 1}}{\epsilon_0} \quad \text{for vertical polarization}$$

$$= \sqrt{\epsilon_0 - 1} \quad \text{for horizontal polarization.}$$

The height function $f_s(h_1)$ can be represented by

$$f_s(h_1) = \frac{3}{\sqrt{2\pi\zeta_1}}\,\frac{\exp(i\tau_s\zeta_1)N_1}{\sqrt{-2\tau_s}[J_{1/3}(z_s)+J_{-1/3}(z_s)]} \tag{16}$$

where

$$\zeta_1 = \frac{\dfrac{2\pi d_1}{\lambda}}{\left(\dfrac{2\pi ka}{\lambda}\right)^{2/3}}$$

$d_1 = \sqrt{2kah_1}$ = distance to horizon from antenna height h_1

Z_s = is a function of τ_s.

The factor N_1 is approximately equal to 1 for values of $\xi_1 > 6$, but its value for $\xi_1 < 6$ is still to be determined.

Substituting the value of $f_s(h_1)$ given in (16) and a similar one for $f_s(h_2)$ into (15) results in

$$\left|\frac{E}{E_0}\right| = \sqrt{\frac{\zeta_a}{8\pi\zeta_1\zeta_2}}\,N_1N_2\sum_{s=0}^{\infty} 4[\delta+2\tau s] \cdot \left[\frac{\exp\left(-\dfrac{i\tau_s}{2}(\zeta_a-\zeta_1-\zeta_2)\right)}{1/3\sqrt{-2\tau_s}[J_{1/3}(z_s)+J_{-1/3}(z_s)][\delta+2\tau_s]}\right]^2. \tag{17}$$

The quantity inside the second pair of brackets after the summation sign is in the same form as in (4a) in the paper by Burrows and Gray,[1] so that their solution $[L(\delta)F_L]$ can be substituted in the above equation, resulting in

$$\frac{E}{E_0} = \sqrt{\frac{\zeta_1+\zeta_2+\zeta_3}{8\pi\zeta_1\zeta_2}}\,N_1N_2[2F_{L/2}]^2[L(\delta)]^2\delta \tag{18}$$

when $\delta \gg \tau_s$.

This step involves the assumption that

$$\sum_{s=0}^{\infty} A_s^2 = \left[\sum_{s=0}^{\infty} A_s\right]^2$$

where A_s is any function of s. This assumption is not justifiable in the general case, but in this instance it affects the loss L_3 (shown on Fig. 6) only in the region near

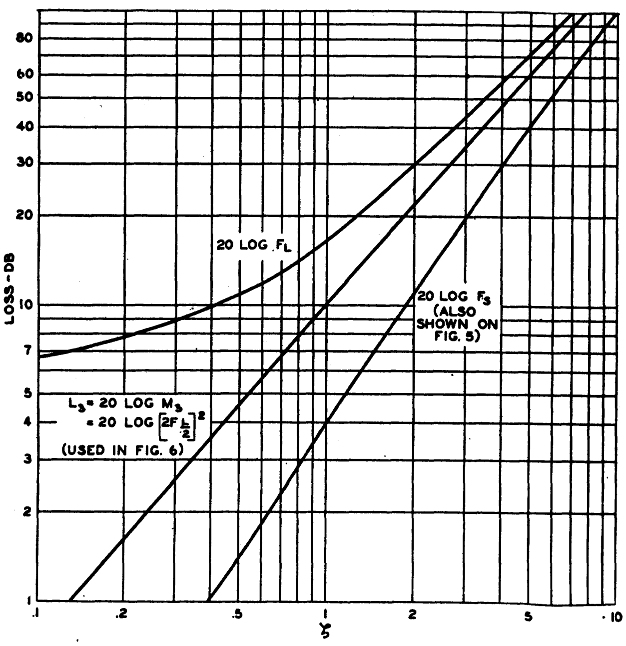

Fig. 13—Attenuation factors used in theory of diffraction over smooth earth.

the line-of-sight. When the loss L_3 is greater than about 30 decibels, the first term ($s=0$) is accurate within one or two decibels; when the loss L_3 is less than 30 decibels (which occurs near the line-of-sight), the above assumption may introduce an error of about ± 3 decibels. Since these possible errors are no greater than the effect of a small change in the assumed value of the parameter k, the over-all accuracy is not greatly impaired, and this procedure simplifies the problem considerably.

The parameter 20 log F_L (taken from Fig. 10 of the Burrows and Gray paper) is shown on Fig. 13 as a function of ξ_3, since $L=\xi_3$ for large values of δ. Also, for large values of δ, $L(\delta)=1/\sqrt{\delta}$. Consequently, (18) reduces to

$$\frac{E}{E_0} = \sqrt{\frac{1+\dfrac{d_1}{d_2}+\dfrac{d_3}{d_2}}{8\pi\zeta_1}}\,N_1N_2M_3$$

$$= \left[\frac{N_1}{\sqrt{5.656\pi\zeta_1}}\right][N_2]\,[M_3]\left[\frac{1}{\sqrt{2}}\left(1+\frac{d_1}{d_2}+\frac{d_3}{d_2}\right)\right]^{1/2} \tag{19}$$

where $M_3 = [2F_{L/2}]^2$ and 20 log M_3 is plotted in Fig. 13. The term in the first set of brackets is a function of d_1 (since N_1 depends on d_1 but is independent of d_2 and d_3); the factor in the second set of brackets is a function of d_2; and the factor in the third set of brackets is a function of d_3. The fourth term is small, *providing* that d_1 is defined as the distance to the horizon from the *lower* antenna, and it ordinarily can be neglected.

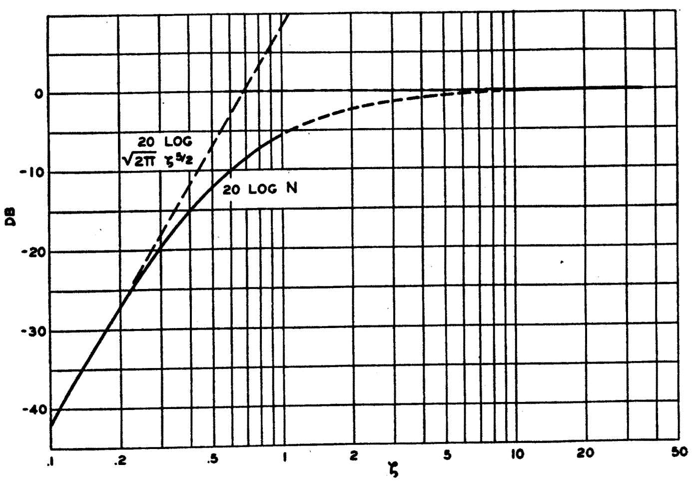

Fig. 14—Values of N.

The factor N is equal to unity when ζ is greater than about 6, but more information about the factor N is required in order that (19) can be used at lower antenna heights. When one antenna is high so that $\zeta_2 > 6$, and the other antenna low ($\zeta_1 < 1$), (19) at line-of-sight reduces to

$$\frac{E}{E_0} = \frac{N_1}{\sqrt{8\pi\zeta_1}} .$$

A solution to this same problem is given by Burrows and Gray (19) which can be shown to be equivalent to

$$\frac{E}{E_0} = \frac{\zeta_1^2}{2} .$$

By setting these two expressions equal it is found that

$$N_1 = \sqrt{2\pi}\,\zeta_1^{5/2} \tag{20}$$

for low values of ζ_1.

The asymptotic value of 20 log $\sqrt{2\pi}\ \xi_1^{5/2}$ is shown by the dashed line on the left side of Fig. 14, while the other asymptotic value of 20 log $N=0$ for high antenna heights is also shown by a dashed line. The true value of N must be a smooth curve or a family of curves joining these two asymptotic values.

A second method for determining the magnitude of N is to assume a grazing path with two antennas of equal height. This means that $\zeta_1 = \zeta_2$; $N_1 = N_2$; $\zeta_a = 2\zeta_1$. For this case (19) reduces to

$$\frac{E}{E_0} = \frac{N_1^2}{\sqrt{4\pi\zeta_1}} .$$

Another solution to this problem is well known for the case of $\xi_1 < 1$. It is

$$\frac{E}{E_0} = \frac{4\pi h_1 h_2}{\lambda d} F_s = \frac{\zeta_1^2 \zeta_2^2}{2\zeta_a} F_s$$

where F_s is the diffraction loss caused by the curvature of the earth when both antennas are near the ground. The value of 20 log F_s is shown on Fig. 13 as a function of ζ_a and is also given by the nomogram in Fig. 5. By setting these two expressions equal, it is found that

$$\frac{N_1^2}{\sqrt{4\pi\zeta_1}} = \frac{\zeta_1^4}{4\zeta_1} F_s$$

$$N_1^2 = \sqrt{\frac{\pi}{4}}\,\zeta_1^{7/2} F_s$$

or

$$20 \log N_1 = -0.5 + 35 \log \zeta_1 + 10 \log F_s \tag{21}$$

where F_s is a function of $2\zeta_1$.

The value of 20 log N_1 is shown on Fig. 14, as a function of ζ_1 (or N_2 as a function of ζ_2). The values of N for the range of $0.3 < \zeta < 1$ have been computed from (21). (The quantity ζ_1 is greater than 0.3 whenever the antenna height is greater than 50 feet at 30 megacycles, or greater than 10 feet at 300 megacycles.) The values of N in the range of $1 < \zeta < 6$ have been interpolated, but there seems little chance of serious error in this procedure.

In the nomogram on Fig. 6, the decibel loss

$$L_1 = 20 \log \frac{N_1}{\sqrt{5.656\pi\zeta_1}},$$

the decibel loss $L_2 = 20$ log N_2, and the decibel loss $L_3 = 20$ log M_3.