

1 | Worksheet

1.1 Friis

The reason why there is a loss through free space, is that the signal density gets lower, as it spread over a larger area, which happens when the distance that the signal has travelled gets longer [Ian Poole, 2016].

For example, when using an isotropic antenna the signal density is equal all the way around the antenna, forming a sphere around the antenna. The power of the signal is in total always the same. Therefore the density of the signal power is only depended on the surface area of the sphere. As the signal travels longer away from the antenna the sphere gets bigger as do the surface, which means that the signal density gets lower. So the free space loss, is the factor describing the signal that is not going in the direction of the receiving antenna.

Friis transmission equation is used to calculate the power, that is received at the receiving antenna, out from the gains in the antenna, the power of the transmitted signal and the free space loss [Prof. Amy Connolly's group, 2013].

$$P_r = P_t G_t G_r \left(\frac{\lambda}{4\pi d} \right)^2 \quad (1.1)$$

Where:

P_r	is the received power at the receiving antenna	[W]
P_t	is the transmitted power at the transmitting antenna	[W]
G_t	is the gain in the transmitting antenna	[1]
G_r	is the gain in the receiving antenna	[1]
λ	is the wavelength of the transmitted signal	[m]
d	is the distance between the transmitting antenna and the receiving antenna	[m]

It is used to calculate the loss through the free space and only accounts for the direct wave, the free space loss is equal to $\left(\frac{\lambda}{4\pi d}\right)^{-2}$. To find the power received the power transmitted is multiplied by the gains in both antennas and divided with the free space path loss.

This is the simple form of friis formulae and it is only correct, if these conditions is met [Wikipedia, 2016]:

- d is much greater than λ . If d is smaller than λ , there will be gain in power through the transmission between the antennas, which is a violation of the law of conservation of energy.
- The transmission goes through freespace, with no multipath. So no obstacle in the transmission line or around it (See worksheet about Line of sight (LOS)).
- The antennas are aligned and have the same polarization.
- The bandwidth is narrow enough, so that a single wavelength can be specified.
- P_r and P_t is the available power at the antennas, and do not take into account the loss through the cable running from antennas. Furthermore, the power will only be fully delivered and received, if the antennas and transmission lines are conjugate matched.

When the antennas are not aligned and/or do not have the same polarization, the simple version of the equation cannot be used. Another problem, is if the impedances are mismatched, which gives a reflection at the antennas, which is another loss in the system. Also there is loss through the air, where the air absorbs some of the power from the signal. With these losses, the equation is expanded to:

$$P_r = P_t G_t(\theta_t, \phi_t) G_r(\theta_r, \phi_r) \left(\frac{\lambda}{4\pi d} \right)^2 (1 - |\Gamma_t|^2) (1 - |\Gamma_r|^2) |a_t \cdot a_r^*|^2 e^{-\alpha d} \quad (1.2)$$

Where:

P_r	is the received power at the receiving antenna	[W]
P_t	is the transmitted power at the transmitting antenna	[W]
$G_t(\theta_t, \phi_t)$	is the gain in the transmitting antenna	[1]
$G_r(\theta_r, \phi_r)$	is the gain in the receiving antenna	[1]
λ	is the wavelength of the transmitted signal	[m]
d	is the distance between the transmitting antenna and the receiving antenna	[m]
Γ_t	is the reflection constant at the transmitting antenna	[1]
Γ_r	is the reflection constant at the receiving antenna	[1]
a_t	is the polarization vector of the transmitting antenna	[1]

a_r	is the polarization vector of the receiving antenna	[1]
α	is the medium of transportations absorption coefficient	[1]

The new terms comes from different losses in the system, when the system is not ideal Prof. Amy Connolly's group [2013].

1.2 Two Ray Plane Earth

In contrast to the Friss pathloss model, the approximated two-ray-ground-reflection path loss model (ATRPL) [Tom Henderson, 2011], considers both the direct wave and the reflected ground wave. Also the ATRPL does not depend on the frequency, as the Friss path loss model does. The received power depending on the distance is given in (1.3).

$$P_r(d) = \frac{P_t G_t G_r}{L} \left(\frac{h_t h_r}{d^2} \right)^2 \quad (1.3)$$

Where h_t and h_r are the heights of the transmitter and receiver antennas respectively. And L is the system loss. A illustration of a scenario of when to use the ATRPL, can be seen on the following Figure.

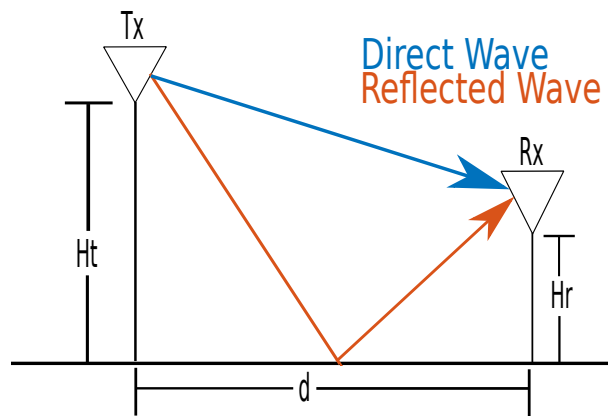


Figure 1.1: Illustration of a scenario of when to use the ATRPL model

The ATRPL is used if the distance d is greater than a critical point d_c given in (1.4) if the condition is false then Friss path loss model can be used.

$$d > d_c \quad (1.4)$$

with d_c given as:

$$d_c = \frac{4\pi \cdot h_t h_r}{\lambda} \quad (1.5)$$

If the condition is false then interference occurs which looks like ripples; this is caused by the constructive and destructive combination of the two rays. The ATRPL does not account for this.

1.2.1 Critical point calculation

For the measurements done, the condition given in (1.4) is tested:

Calculation example for both transmitter and receiver antennas heights at 2 m, at 858MHz

By inserting the heights and wavelength into (1.5) the critical distance can be found to:

$$d_c = \frac{4\pi \cdot 2m \cdot 2m}{0.3494m} = 143.86m \quad (1.6)$$

So in this case all distances between 1m to 30m, from transmitter to receiver, does not fulfill the condition and thus Friis should be used.

1.2.2 Critical point test for 858Mhz

For $h_r = 0.01m$ set, and $h_t = 0.01m, 0.08m, 0.34m, 2m$, $d = 1m, 2m, 4m, 8m, 15m, 30m$

In the following a Table 1.1 is made to illustrate if the condition stated in 1.4, is met for all distances, 1m,2m,4m,8m,15m and 30m, between the transmitter and receiver for the frequency 858MHz with 0.01m set as transmitter height h_t while the receiver height positions are $h_r = 0.01m, 0.08m, 0.34m, 2m$.

h_t, h_r	Not met	Met
0.01m, 0.01m, $d_c = 0.0036m$		At all distances
0.01m, 0.08m, $d_c = 0.028m$		At all distances
0.01m, 0.34m, $d_c = 0.12m$		At all distances
0.01m, 2m, $d_c = 0.72m$		At all distances

Table 1.1: Critical distance for $h_t = 0.01m$

From the above Table 1.1 it can be concluded that the ATRPL can be used.

For $h_t = 0.08m$ set, and $h_r = 0.08m, 0.34m, 2m$, $d = 1m, 2m, 4m, 8m, 15m, 30m$

h_t, h_r	Not met	Met
0.08m, 0.08m, $d_c = 0.23\text{m}$		At all distances
0.08m, 0.34, $d_c = 0.97\text{m}$		At all distances
0.08m, 2m, $d_c = 5.76\text{m}$	1,2 and 4m	At 8,15 and 30m

Table 1.2: Critical distance for $h_t = 0.08\text{m}$

For $h_t=0.08\text{m}$ and $h_r=2\text{m}$ the condition $d < d_c$ for distances of 1m, 2m and 4m, is met. This means that the Friss, can be used. While the rest of the time ATPL shall be used.

For $h_t = 0.34\text{m}$ set, and $h_r = 0.34\text{m}, 2\text{m}, d = 1\text{m}, 2\text{m}, 4\text{m}, 8\text{m}, 15\text{m}, 30\text{m}$

h_t, h_r	Not met	Met
0.34m, 0.34m, $d_c = 4.16\text{m}$	1,2,4	At 8, 15,30m
0.34m, 2m, $d_c = 24.48\text{m}$	1,2,4,8,15	At 30

Table 1.3: Critical distance for $h_t = 0.34\text{m}$

For $h_t = 2m$ set, and $h_r = 2m$, $d = 1m, 2m, 4m, 8m, 15m, 30m$

h_t, h_r	Not met	Met
2m, 2m. $d_c = 144m$	At all distances	

Table 1.4: Critical distance for $h_t = 2m$

1.2.3 Critical point test for 2.58Ghz

For $h_r = 0.01m$ set, and $h_t = 0.01m, 0.08m, 0.34m, 2m$, $d = 1m, 2m, 4m, 8m, 15m, 30m$

h_t, h_r	Met	Not met
0.01m, 0.01m $d_c = 0.010$		At all distances
0.01m, 0.08m $d_c = 0.0865$		At all distances
0.01m, 0.34m $d_c = 0.36$		At all distances
0.01m, 2m $d_c = 2.16$	1m and 2m	At 4, 8, 15, 30m

Table 1.5: Critical distance for $h_t = 0.01m$

For $h_t = 0.08m$ set, and $h_r = 0.08m, 0.34m, 2m$, $d = 1m, 2m, 4m, 8m, 15m, 30m$

h_t, h_r	Not met	Met
0.08m, 0.08m $d_c = 0.69$		At all distances
0.08m, 0.34m $d_c = 2.94$	1, 2m	At, 4, 8, 15 and 15m
0.08m, 2m $d_c = 17.31$	1, 2, 4, 8 and 15m	At 30m

Table 1.6: Critical distance for $h_t = 0.08m$

For $h_t = 0.34m$ set, and $h_r = 0.34m, 2m$, $d = 1m, 2m, 4m, 8m, 15m, 30m$

h_t, h_r	Not met	Met
0.34m, 0.34m $d_c = 12.51$	At 1, 2, 4 and 8m	At 15, 30m
0.34m, 2m $d_c = 73.6$	At all distances	

Table 1.7: Critical distance for $h_t = 0.34\text{m}$

For $h_t = 2\text{m}$ set, and $h_r = 2\text{m}$, $d = 1\text{m}, 2\text{m}, 4\text{m}, 8\text{m}, 15\text{m}, 30\text{m}$

h_t, h_r	Not met	Met
2m, 2m $d_c = 432.9$	At all distances	

Table 1.8: Critical distance for $h_t = 2\text{m}$

[Poole, 2016]

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