Wireless PHY – MAC

Multiple Access Capacity

Aim

- Capacity comparison of different multiple access methods – TDMA, FDMA, CDMA – in comparison to (optimum) Superposition Coding
  - Broadcast AWNG
  - Multiple Access AWGN

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Rate and Capacity regions

- With multiple users there is an infinite number of ways how to divide the resources – consequently, we need to consider capacity/rate *regions*
  - each point in this R^N (for N users) space represents a vector of achievable rates that can be maintained by all users simultaneously
  - the union of all achievable rate vectors is the *capacity region* of the multiuser system

Shannon-Hartley theorem

$$C_i = B\log_2\left(1+\frac{S}{n_i B}\right)$$

effective noise

$$n_i = \tilde{n}_i / g_i$$

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Summary results

- Some capacity ($\mathcal{C}$) results
  - Broadcast channel:
    - with flexible bandwidth and power assignment $\mathcal{C}$_FDMA = $\mathcal{C}$ _TDMA > $\mathcal{C}$_orth. CDMA
    - … with fixed power assignment $\mathcal{C}$ _TDMA < $\mathcal{C}$_FDMA (optimal freq. and pow. allocation is important for very disparate channel quality)
    - with equal bandwidth and flexible power allocation $\mathcal{C}$ _orth. CDMA = $\mathcal{C}$_FDMA = $\mathcal{C}$ _TDMA
    - $\mathcal{C}$ _nonorth. CDMA (convex region) < C_TDMA <= C_FDMA (concave region)
    - Superposition coding (with intereference cancellation) is superior to all other schemes
  - Multiple Access channel:
    - Same relative results as for broadcast, except FDMA can achieve one point on the superposition capacity region

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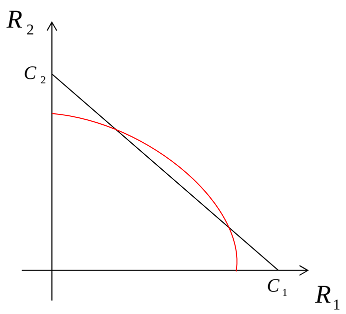
Time division

- **Equal power**
  - **fixed power, over fraction of time**

$$\left\{\bigcup R_1 = \tau C_1, R_2 = (1-\tau)C_2; 0 \le \tau \le 1\right\}$$

  - **average power is τS for user 1**



- **Variable power**
  - **unequal power, over fraction of time**

$$\left\{\bigcup R_1 = \tau B\log_2\left(1+\frac{S_1}{n_1 B}\right), R_2 = (1-\tau)B\log_2\left(1+\frac{S_2}{n_2 B}\right); \tau S_1 + (1-\tau)S_2 = S; 0 \le \tau \le 1\right\}$$

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TD - FD duality

- **Generally**

$\gamma_i = B_i / B; \alpha_i = S_i / S; \lambda_i = \tau_i S_i / S = \tau_i \alpha_i$

then $\tau = (\tau_1, \tau_2), \alpha = (\alpha_1, \alpha_2), \gamma = (\gamma_1, \gamma_2)$ and $\lambda = (\lambda_1, \lambda_2)$ are points in

$\Psi_2 = \{(s_1, s_2); s_i \ge 0, s_1 + s_2 = 1\}$

- **Consider rewriting TD VP**

$$\left\{\bigcup R_1 = \tau_1 B\log_2\left(1+\frac{S_1}{n_1 B}\right), R_2 = \tau_2 B\log_2\left(1+\frac{S_2}{n_2 B}\right); \tau \in \Psi_2\right\}$$

  - **where S_1 is allocated to user 1 for duration τ_1 and S_2 to user 2 for duration τ_2, out of a time period of *T* where**

$$\tau_1 T S_1 + \tau_2 T S_2 = TS$$

  we get

$$\left\{\bigcup R_1 = \tau_1 B\log_2\left(1+\frac{\lambda_1 S}{n_1 \tau_1 B}\right), R_2 = \tau_2 B\log_2\left(1+\frac{\lambda_2 S}{n_2 \tau_2 B}\right); \tau, \lambda \in \Psi_2\right\}$$

$$\xrightarrow{\text{duality}}$$

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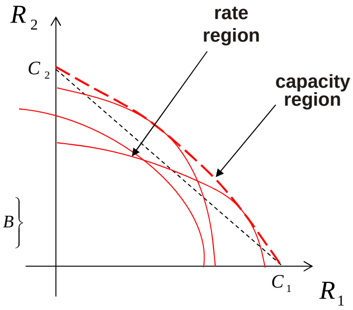
Frequency division

- **Variable**
  - **fractions of power, over fraction of bandwidth**
  - **benefit in disparate channel conditions**



$$\left\{ \bigcup R_1 = B_1 \log_2\left(1+\frac{S_1}{n_1 B_1}\right), R_2 = B_2 \log_2\left(1+\frac{S_2}{n_2 B_2}\right); S_1+S_2=S; B_1+B_2=B \right\}$$

$$\Updownarrow$$

$$\left\{ \bigcup R_1 = \gamma_1 B \log_2\left(1+\frac{\alpha_1 S}{n_1 \gamma_1 B}\right), R_2 = \gamma_2 B \log_2\left(1+\frac{\alpha_2 S}{n_2 \gamma_2 B}\right); \alpha,\gamma \in \Psi_2 \right\}$$

- **…viz. TDMA (from previous page)**

$$\left\{ \bigcup R_1 = \tau_1 B \log_2\left(1+\frac{\lambda_1 S}{n_1 \tau_1 B}\right), R_2 = \tau_2 B \log_2\left(1+\frac{\lambda_2 S}{n_2 \tau_2 B}\right); \tau,\lambda \in \Psi_2 \right\}$$

Time division is dual to frequency division only if time variations in input power is allowed, just as variations of power are allowed over frequency (bands)

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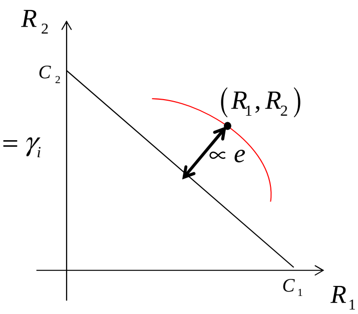
Equalised channels

- **Define** $e = \frac{R_1}{C_1} + \frac{R_2}{C_2} - 1$

- **When power is allocated in proportion to bandwidth, i.e.** $\alpha_i = \gamma_i$



$$e = \frac{R_1(\gamma_1,\gamma_1)}{C_1} + \frac{R_2(\gamma_2,\gamma_2)}{C_2} - 1$$

$$\Updownarrow$$

$$e = \frac{\gamma_1 B \log_2\left(1+\frac{\gamma_1 S}{n_1 \gamma_1 B}\right)}{B \log_2\left(1+\frac{S}{n_1 B}\right)} + \frac{\gamma_2 B \log_2\left(1+\frac{\gamma_2 S}{n_2 \gamma 2 B}\right)}{B \log_2\left(1+\frac{S}{n_2 B}\right)} - 1$$

$$\Updownarrow$$

$$e = \gamma_1 + \gamma_2 - 1 = 0 \qquad \Rightarrow$$

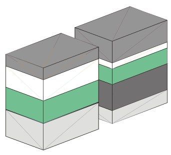
we need disparate channels!

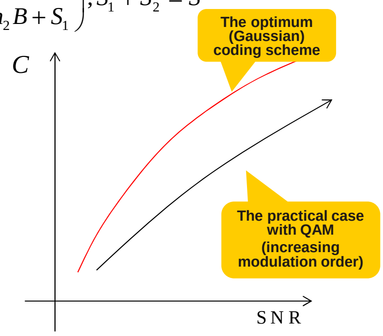
- **Same result can be proved when channels are the same, i.e.** $n_1 = n_2$

Superposition coding (SC)

- **Basic version**

$$R_1 = B\log_2\left(1+\frac{S_1}{n_1 B + S_2}\right), R_2 = B\log_2\left(1+\frac{S_2}{n_2 B + S_1}\right); S_1 + S_2 = S$$





- **.. the "elaborate"**

$n_1 < n_2$:

$$R_1 = B\log_2\left(1+\frac{S_1}{n_1 B}\right), R_2 = B\log_2\left(1+\frac{S_2}{n_2 B + S_1}\right); S_1 + S_2 = S$$

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Code division

- **Orthogonal spreading codes**
  - **No interference, but each user can only get half the rate due to** $R = B / OSR$ **(Walsh-Hadamard codes is one implementation for this)**
  - **Corresponds to frequency division with bandwidth divided in half, hence a restricted case**
  - **The capacity region of frequency division needs more advanced (capacity achieving) codes**
- **Non-orthogonal codes**
  - **Interference from other users makes the rate region convex, and thus inferior to all the other schemes**
  - **A la basic superposition coding!**

$$R_1 = B\log_2\left(1+\frac{S_1}{n_1 B + S_2}\right), R_2 = B\log_2\left(1+\frac{S_2}{n_2 B + S_1}\right); S_1 + S_2 = S$$

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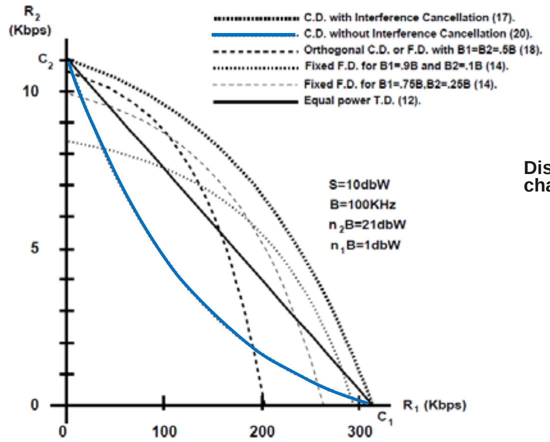
Rate regions - AWGN



Disparate channels!

Figure 14.4: Two-User Capacity Region: 20dB SNR Difference.

Goldsmith, Wireless Communications

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Rate regions - fading

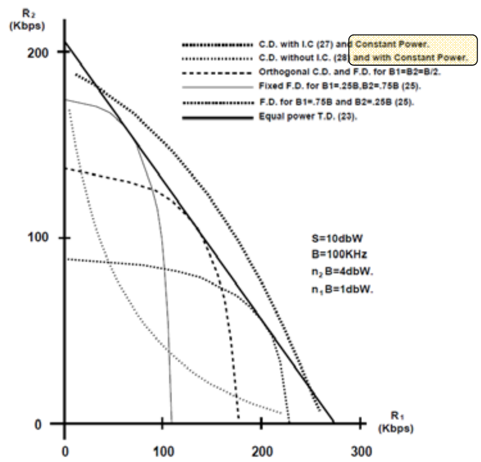

Figure 14.5: Two-User Capacity Region in Rayleigh Fading.

Goldsmith, Wireless Communications

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Multiple Access

- No power sharing
  - Superposition coding rate region (with interference cancellation)

$$R_i \leq B \log\left[1 + \frac{P_i}{nB}\right],$$
$$R_1 + R_2 \leq B \log\left[1 + \frac{P_1 + P_2}{nB}\right].$$

$$C_i = B \log\left[1 + \frac{P_i}{nB}\right],\ i = 1, 2,$$
$$C_1^* = B \log\left[1 + \frac{P_1}{nB + P_2}\right],$$
$$C_2^* = B \log\left[1 + \frac{P_2}{nB + P_1}\right].$$

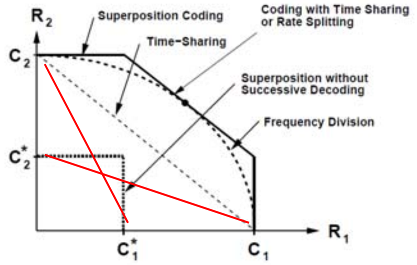

Figure 14.6: Multiaccess Channel Rate Region.

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Summary and Learnings

- Multiple Access capacity – infinity of solutions
  - Rate regions and Capacity region (2 user case)
- Relative comparison between multiple access methods?
  - CDMA < TDMA (EP) < TDMA (VP) = FDMA < SC
- AWGN and fading channels
  - same relative comparison for what concerns the average channel behaviour, but lower fading channel capacities
- Multiple Access and Broadcast
  - much the same relative behaviour although capacity regions are different