1 | Groundwave

When transmitting from one radio to another the signal travel as electromagnetic waves. These waves can reach the receiver taking multiple paths. It could be a direct path from transmitter to receiver, or a reflected wave of a surface such as the ground this can be seen in Figure 1.1.

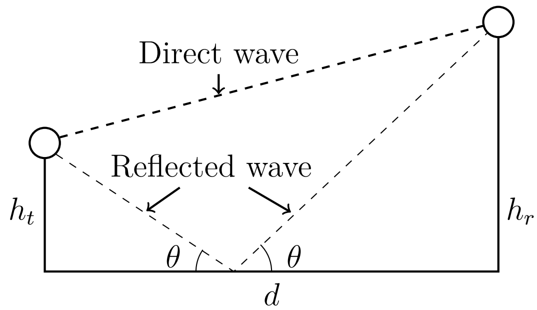

Figure 1.1: Direct and reflected path from transmitter to receiver.

This worksheet will focus on the paths close to the ground, and primarily on the surface wave component since the direct and reflected wave are described in further detail in ??. The ground acts both as a reflector and also as an absorber [Bullington, 1947]. This means that a received wave can be contributed from three main factors. In 1947 Burlington wrote that *"The principal effect of plane earth on the propagation of radio waves is indicated by the following equation"*[Bullington, 1947]. The surface part comes from the grounds absorption of the electromagnetic wave, when the energy enters the ground it sets up ground currents. These currents are a representation of the imperfect reflection.

$$E = E_0 \left[\underbrace{1}_{direct} + \underbrace{Re^{j\Delta}}_{reflected} + \underbrace{(1-R)Ae^{j\Delta}}_{surface} \right] \tag{1.1}$$

Where:

E	is the recieved field intensity	$\left[\frac{V}{m}\right]$
E_0	is the recieved field intensity in free space	$\left[\frac{V}{m}\right]$
R	is the reflection coefficient of the ground	[1]
Δ	is the phase difference between direct and reflected wave	[rad]
A	is the surface-wave attenuation factor	[1]

The phase difference Δ, is the difference in phase resulting only from the difference in length between the direct wave and the reflected wave, L, which can be found using geometry. The phase difference can be found as:

$$\Delta = 2\pi \frac{L}{\lambda} \tag{1.2}$$

Where:

L	is the difference in length between the direct wave and the reflected wave	[m]
λ	is wavelenght	[m]

For a plane earth case meaning that there are no bumps or curvatures on the earth, its just plane then L is found as:

$$L = \sqrt{(h_t + h_r)^2 + d^2} - d \tag{1.3}$$

Where:

d	is the distance between the transmitter and receiver	[m]
h_t	is the height of the transmitter	[m]
h_r	is the height of the receiver	[m]

For cases where $d > 5(h_t + h_r)$ Equation 1.3 can be approximated as [Bullington, 1947]:

$$L = \frac{2h_t h_r}{d} \tag{1.4}$$

$$\Delta = 4\pi \frac{h_t h_r}{d\lambda} \tag{1.5}$$

The reflection coeficient R is dependent on the incoming angle of the signal, θ, as well as the electromagnetic properties of the ground, ϵ and σ. The relation can be written as[Bullington, 1947]:

$$R = \frac{\sin(\theta) - z}{\sin(\theta) + z} \tag{1.6}$$

Where:

z	$\frac{\sqrt{\epsilon_0 - \cos^2(\theta)}}{\epsilon_0}$ for vertical polerization	[1]
z	$\sqrt{\epsilon_0 - \cos^2(\theta)}$ for horizontal polerization	[1]
ϵ_0	is the complex relative permittivity $= \epsilon - j60\sigma\lambda$	[1]
ϵ	is the dielectric constant of the ground relative to unity in free space	[1]

σ	is the conductivity of the ground in mhos per meter	$\left[\frac{mhos}{m}\right]$
θ	is reflection angle	[rad]

By looking at this equation it can be seen that as θ goes towards 0, R goes towards -1. Which also implies that for sufficiently low receiver- and transmitter heights, the direct and reflected wave is completely out of phase since $Re^{j\Delta} = -1$. This leaves only the surface wave part of the equation.

The surface wave attenuation factor can be approximated as[Bullington, 1947]

$$A \approx \frac{-1}{1 + j\frac{2\pi d}{\lambda}\left(\sin(\theta) + z\right)^2} \tag{1.7}$$

This approximation holds for $A < 0.1$, however for A approaching unity the phase approaches 180 degree [Bullington, 1947].

When θ approaches zero meaning that $d >> h_r, h_t$ only the surface wave component remains and it is thus sufficient to look at the magnitude of it [Chong and Kim, 2013]

$$|(1-R)A| \approx \left|2\frac{-1}{1 + j\frac{2\pi d}{\lambda}\left(\sin(\theta) + z\right)^2}\right| \tag{1.8}$$

$$|(1-R)A| \approx \frac{2}{\frac{2\pi d}{\lambda}z^2} \tag{1.9}$$

By introducing h_0, the minimum effective antenna height, as:

$$h_0 = \left|\frac{\lambda}{2\pi z}\right| \tag{1.10}$$

Where:

h_0	is the minimum effective antenna height	[m]

By utilizing this Equation 1.9, can be written as:

$$|(1-R)A| \approx \frac{4\pi h_0^2}{\lambda d} \tag{1.11}$$

The received power can then be found using [Chong and Kim, 2013]

$$P_r = P_0 \left(\frac{E}{E_0}\right)^2 \tag{1.12}$$

$$P_r = P_t G_r G_t \frac{\lambda^2}{(4\pi)^2 d^2} \left(\frac{E}{E_0}\right)^2 \tag{1.13}$$

$$P_r = P_t G_r G_t \left(\frac{h_0}{d}\right)^4 \tag{1.14}$$

Where:

P_r	is the received power at the receiver antenna	[W]
P_0	is the predicted power by the free space model (see free space model worksheet)	[W]
P_t	is the transmitted power from the transmitter	[W]
G_r	is the gain of the receiver antenna	[1]
G_t	is the gain of the transmitter antenna	[1]

2 | Complex Relative Permittivity

In the ground wave theory the complex relative permittivity is needed. This can be found in different ways[Kim and Narayanan, 2002]. One of the easier ways to do this is the method described in [Kim and Narayanan, 2002]. This method sets far less restrictions for the antennas compared to other methods[Kim and Narayanan, 2002].

The theory behind the method comes from the the two-ray path loss model. A generic illustration of a two-ray setup can be seen in Figure 2.1.

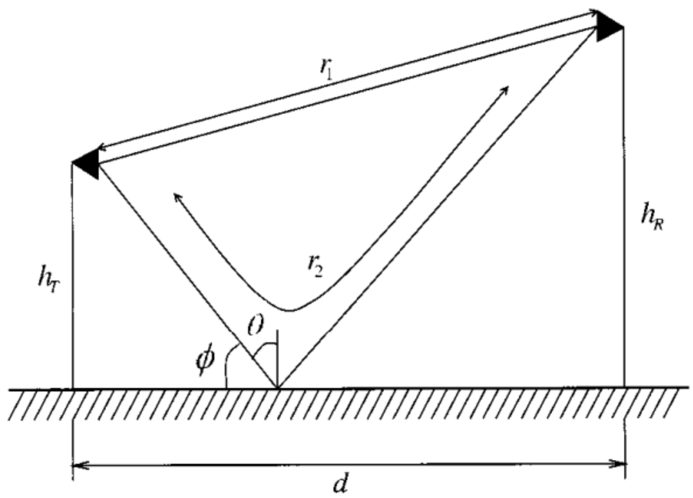

Figure 2.1: Illustration of setup and symbol meaning. Figure taken from [Kim and Narayanan, 2002].

The two-ray model can be written as[Kim and Narayanan, 2002]:

$$P_r = \frac{P_t G_t G_t}{L}\left(\frac{\lambda}{4\pi}\right)^2 \cdot \left|\frac{1}{r_1}\mathrm{e}^{-jkr_1} + \sqrt{\alpha_t}\sqrt{\alpha_r}\rho_{h,v}\frac{1}{r_2}\mathrm{e}^{-jkr_2}\right|^2 \tag{2.1}$$

Where:

P_r	is the received power	[W]
P_t	is the transmitted power	[W]
G_t	is the transmitters antenna gain	[1]
G_r	is the receiver antennas gain	[1]
L	is loss factor for the cables and connectors	[1]
λ	is the wavelength	[m]
r_1	is the direct path length	[m]
k	is the wavenumber	$[m^{-1}]$

α_t	is the magnitude ratios of power along the reflected to the direct path directions for the transmit antenna	[1]
α_r	is the magnitude ratios of power along the reflected to the direct path directions for the receiver antenna	[1]
$\rho_{h,v}$	is the reflection coefficient in for either horizontal or vertical polarization	[1]
r_2	is the reflected path length	[m]

Do note that if the signal measured was horizontal polarized the horizontal reflection coefficient is used, if vertical polarized then the vertical reflection coefficient is used.

Since the reflection coefficient is complex it can be utilized that

$$\rho_{h,v} = \Gamma_{h,v} + j\zeta_{h,v} \tag{2.2}$$

Where:

$\Gamma_{h,v}$	is the real part of the permittivity	[1]
ζ	is the imaginary part of the permittivity	[1]

By using Equation 2.2 in Equation 2.1 and rearranging, it can be seen that this draws circles on the complex reflection coefficient plane:

$$\left(\Gamma_{h,v} + \frac{r_r \cos(kr_d)}{\sqrt{\alpha_r}\sqrt{\alpha_t}}\right)^2 + \left(\zeta_{h,v} + \frac{r_r \sin(kr_d)}{\sqrt{\alpha_r}\sqrt{\alpha_t}}\right)^2 = \left(\frac{4\pi r_2 \sqrt{P_d}}{\lambda\sqrt{\alpha_r}\sqrt{\alpha_t}}\right)^2 \tag{2.3}$$

Where:

r_r	is $\frac{r_2}{r_1}$	[1]
r_d	$r_2 - r_1$	[m]
P_d	$\frac{P_r L}{P_t G_t G_r}$	[1]

Since this is a circle three separate measurements is needed to get a unique intersection point. Because the complex relative permittivity vary with frequency and incidence angle, these must be fixed for the measurements. This is illustrated in Figure 2.2.

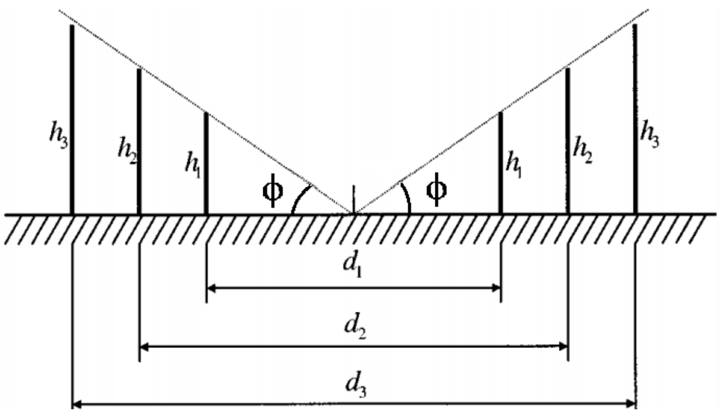

Figure 2.2: Illustration of the three distances and heights needed. Figure taken from [Kim and Narayanan, 2002].

When the intersection point has been determined[1] for both horizontal and vertical polarized waves the complex relative permittivity can be found as[Kim and Narayanan, 2002]

$$\epsilon_0 = \epsilon - j60\sigma\lambda = \frac{(1+\rho_v)(1-\rho_h)}{1+\rho_h)(1-\rho_v)} \tag{2.4}$$

Where:

ϵ_0	is the complex relative permittivity	[1]
ϵ	is the dielectric constant of the ground relative to unity in free space	[1]
σ	is the conductivity of the ground in mhos per meter	$\left[\frac{mhos}{m}\right]$

A stepwise procedure could then be:

1. Decide frequency and incidence angle
2. Decide measurement setup, including P_t heights and distances between transmitter and receiver for all three measurements points
3. Determine G_t, G_r, L, α_t, α_r for both horizontal and vertical cases
4. Calculate λ, k, r_1, r_2
5. Measure P_r in all six cases

[1]In practice there might not be a unique solution, due to measurement inaccuracies, There will then be three intersections that are close, and a sufficient estimate can be found by averaging these [Kim and Narayanan, 2002].

6. Find intersection point using Equation 2.3 for both horizontal and vertical polarization
7. Calculate the complex relative permittivity using Equation 2.4

Bibliography

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