Error Control Coding - MM7

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Linear systematic block code

A code word:

An (n,k) code

$$\underbrace{\underbrace{\boxed{\text{Data}}}_{k\text{-bit}}\underbrace{\boxed{\text{Check}}}_{r\text{-bit}}}_{n\text{-bit code}}$$

$$n = k + r$$

A code is a mapping

	Data $\longrightarrow$	Code
#	2^k	2^k
	k bit	n bit

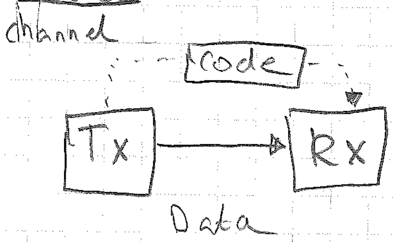
2^k legal code words

2^n possible words

$(2^n - 2^k)$ illegal words

(we want to detect these)

Model:



The check bits are calculated by the ②

parity equations:

Binit numbering:

$c_1\, c_2\, c_3 \ldots c_n = d_1\, d_2 \ldots d_k\, r_1 \ldots r_2$

$r_1 = c_{k+1} = P_{11} \cdot d_1 + P_{21}\, d_2 + \ldots + P_{k1} \cdot d_k$

$r_2 = c_{k+2} = P_{12} \cdot d_1 + P_{22}\, d_2 + \ldots + P_k \cdot d_k$

$r_r = c_{k+r} = P_{1r} \cdot d_1 + \ldots + P_{kr}\, d_k$

and also

$c_1 = d_1$

$c_2 = d_2$

$c_k = d_k$

Legend:

$\left.\begin{array}{l} d\text{: data bit} \\ c\text{: code bits} \\ r\text{: check bits} \\ p\text{: parity constants} \end{array}\right\}$ Binary numbers modulo-2

Modulo-2 calculations

A	B	A + B	A · B
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1
		XOR	AND

Matrix notation:

$$\bar{C} = \bar{D} \cdot \bar{\bar{G}}$$

$\bar{C}$: n-bit row vector

$\bar{D}$: k-bit row vector

$\bar{\bar{G}}$: k×n matrix. Defines the code

$$\underset{1\times n}{[c_1\, c_2 \ldots c_n]} = \underset{1\times k}{[d_1\, d_2 \ldots d_n]} = \underbrace{\left[\begin{array}{ccccc|cccc} \multicolumn{5}{c|}{\overbrace{}^{k}} & \multicolumn{4}{c}{\overbrace{\phantom{p_{11}\,p_{12}\,\cdots\,p_{1r}}}^{r}} \\ 1 & 0 & 0 & \cdots & 0 & p_{11} & p_{12} & \cdots & p_{1r} \\ 0 & 1 & 0 & \cdots & 0 & p_{21} & p_{22} & \cdots & p_{2r} \\ 0 & 0 & 1 & \cdots & 0 & \vdots & & & \\ & & & \ddots & & & & & \\ & & & & 1 & p_{k1} & \cdots & & p_{kr} \end{array}\right]}_{n}_{k\times n}$$

Ex 1

A (6,3) code, linear systematic block code

n=6, k=3, r = n−k = 3 (check-bits)

$$\bar{\bar{G}} = \underset{3\times 6}{\begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}}$$

$\bar{D}_1 = 100 \Rightarrow C_1 = 100.011$

$\bar{D}_2 = 010 \Rightarrow C_2 = 010.101$

$\bar{D}_3 = 001 \Rightarrow C_3 = 110\ 100$
$(C_1 + C_2)$

Hamming metric

Hamming weight: The number of ones in the word

Hamming distance: The number of places the two words differ

Eks 2

$\bar{c}_1 = 100.011$

$\bar{c}_2 = 010.101$

$w(\bar{c}_1) = \underline{3}$ - number of ones

$d(\bar{c}_1, \bar{c}_2) = \underline{4}$ - nr places the two differ

Theorem

$$\boxed{w(c_1 + c_2) = d(c_1, c_2)}$$

Eks 3

$\bar{c}_1 + \bar{c}_2 = 110110 \quad w = \underline{4}$

$d(\bar{c}_1, \bar{c}_2) = \underline{4}$

Minimum distance δ (or dmin)

The minimum distance that can be found between any two code words in a code.

Theorem:

δ is equal to the weight of the "lightest" code word in a linear code apart from the all-zero word

Detection and correction

Detection : $(\delta - 1)$ errors

Correction : $[\delta - 1/2]$ errors
integer part

Receiving code word

We receive the word :

$$\overline{R} = \overline{C} + \overline{E}$$

Ex 4

$$\overline{C} = 100.011$$
$$\overline{E} = 001.000$$
$$\overline{R} \quad 101.011$$

Parity check matrix :

$$\overline{\overline{G}} \cdot \overline{\overline{H}}^T = \overline{0}$$

It can be seen that

$$\overline{C} \cdot \overline{\overline{H}}^T = (\overline{D} \cdot \overline{\overline{G}}) \cdot \overline{\overline{H}}^T = \overline{0}$$

For a linear systematic code $\overline{\overline{H}}$ can be found this way :

$$\underset{k \times n}{\overline{\overline{G}}} = [I_k \quad P_{k \times r}]$$

$$\overline{\overline{H}}_{r \times n} = \left[P^T_{r \times k} \quad I_r \right]$$

$$\overline{\overline{H}}^T_{n \times r} = \begin{bmatrix} P_{k \times r} \\ I_r \end{bmatrix}$$

Syndrome calculation

The receiver calculates the syndrome

$$\underset{[1 \times r}{\overline{S}} = \underset{1 \times n}{\overline{R}} \cdot \underset{n \times r]}{\overline{\overline{H}}^T}$$ (an r-bit vector)

It can be seen:

$$\overline{S} = \overline{R} \cdot \overline{\overline{H}}^T = (\overline{C} + \overline{E}) \overline{\overline{H}}^T$$
$$= \overline{C} \cdot \overline{\overline{H}}^T + \overline{E} \cdot \overline{\overline{H}}^T$$
$$= \overline{E} \cdot \overline{\overline{H}}^T$$

and also:

$$\overline{E} = \overline{0} \Rightarrow \overline{S} = \overline{0}$$

The receiver uses:

$\overline{S} = \overline{0} \longrightarrow$ Probabily no errors!