

Pseudocode

```
class Node{
    //false if no plank exists, true if there is a plank
    boolean up
    boolean down
    boolean left
    boolean right
    boolean visited
}

function SHORTESTPLANKPATH(mazeArr, entrance, exit)    ▷ these inputs collectively tell about the size of the pillar
grid and the plank layout
    check for invalid set size for entrance and exit    ▷ entrance and exit are tuples containing the integer coordinate
    minPath ← BFS on the current mazeArr                ▷ path can be implemented as a list of integer tuples
    toVisit ← a queue that stores the nodes to visit
    add the start node to the toVisit queue
    while toVisit is not empty do
        node.visited ← true
        for all direction not yet checked in node do
            toVisitNode ← the node that the direction points to
            if the direction does not lead to node inside the mazeArr then
                continue
            if toVisitNode.visited == true then
                continue
            if direction == false then
                mazeArrPlank ← copy the mazeArr
                mazeArrPlank ← put plank down in the direction
                plankPath ← BFS on that copy of mazeArr
                if plankPath != null and (minPath == null or length of plankPath < length of minPath ) then ▷ BFS
returns null if there is no path to the exit
                minPath ← plankPath
            else
                add toVisitNode to the toVisit queue
    return minPath
```

Proof of correctness

Loop Invariant

1. *minPath* contains the shortest path out of all the maze states using one plank formed so far (clarification: these are the copied maze with the plank put down).
 2. all of the possible maze states using one plank are not examined yet.
- conceptually, no path means that it is a path of infinite length, which is represented in this algorithm by having a null path.

Initialization

Basis

When the original maze (no plank put down) is traversed, it is the only path out of all the maze states using one plank formed so far. Thus it is trivially the shortest path.

Maintenance

Assume *minPath* contains the shortest path before the *i*th iteration.

At *i*th iteration, *minPath* is compared to *plankPath* (which is the path found in the new maze state using one plank) and the smaller one is put into *minPath*.

Termination

The loop terminates when there are no more possible maze states using one plank not yet examined.

Thus, *minPath* contains the smallest path out of all possible maze states using one plank.

Runtime analysis In the worst case, the entire maze grid has to be looked at, resulting in $O(n^2)$ for the looking at the entire grid. The worst case running time for BFS is $O(E + V)$. There are n^2 amount of vertex in a n by n square grid. Also, there are $2n^2 - 2$ amount of edges in a n by n square grid. The addition of $E+V$ is $2n^2 - 2 + n^2$, which is equal to $3n^2 - 2$. This means that the running time of BFS in a n by n square grid is $O(n^2)$. Thus, the overall worst case running time for this loop is $O(n^4)$.