Productivity Growth, Technical Progress, and Efficiency Change in Industrialized Countries

By Rolf Färe, Shawna Grosskopf, Mary Norris, and Zhongyang Zhang*

This paper analyzes productivity growth in 17 OECD countries over the period 1979–1988. A nonparametric programming method (activity analysis) is used to compute Malmquist productivity indexes. These are decomposed into two component measures, namely, technical change and efficiency change. We find that U.S. productivity growth is slightly higher than average, all of which is due to technical change. Japan's productivity growth is the highest in the sample, with almost half due to efficiency change. (*JEL* C43, D24)

In this paper we apply recently developed techniques to the analysis of productivity growth for a sample of OECD countries. The technique we use allows us to decompose productivity growth into two mutually exclusive and exhaustive components: changes in technical efficiency over time and shifts in technology over time. These components lend themselves in a natural way to the identification of catching up and the identification of innovation, respectively.

Our measure of productivity growth is a geometric mean of two Malmquist productivity indexes. The Malmquist index was introduced by Douglas W. Caves et al. (1982b) as a theoretical index which they showed was equivalent, under certain conditions,[1] to the easily computable Törnqvist index. Unlike Caves et al., we calculate the Malmquist index directly by exploiting the fact that the distance functions on which the Malmquist index is based can be calculated by exploiting their relationship to the technical-efficiency measures developed by Michael J. Farrell (1957). This also leads to our decomposition of productivity into changes in efficiency (catching up) and changes in technology (innovation). We argue that the Malmquist productivity-change index is more general than the Törnqvist index advocated by Caves et al.: it allows for inefficient performance and does not presume an underlying functional form for technology.

We calculate the component distance functions of the Malmquist index using nonparametric programming methods. These are very closely related to the nonparametric methods used in Jean-Paul Chavas and Thomas L. Cox (1990), which are also based on linear-programming problems. Our technique constructs a "grand" or world frontier based on the data from all of the countries

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[1]These include that technology is translog, second-order terms are constant over time, and firms are cost-minimizers and revenue-maximizers.

in the sample. Each country is compared to that frontier. How much closer a country gets to the world frontier is what we call "catching up"; how much the world frontier shifts at each country's observed input mix is what we call "technical change" or "innovation." The product of these two components yields a frontier version of productivity change.

We apply our methods to a sample of OECD countries over the period 1979–1988. We find that U.S. overall performance is close to the average for the sample; however, the United States is above average in terms of technical change. In fact, the United States consistently shifts the frontier over the entire sample period. Productivity growth in Japan is well above average, due in large part to "catching up" to the frontier rather than due to technical change (shifts in the frontier).

The paper is organized as follows. We begin with a brief stylized summary of some of the recent work related to the convergence hypothesis in Section I. This is followed by a discussion of the Malmquist productivity index, the distance functions from which it is constructed, and how we propose to calculate them. Section III contains a discussion of our data and results.

I. Background

Several issues related to productivity growth have received recent attention. Most of these issues are related to or were motivated by the much-documented and discussed productivity slowdown observed in the United States and other industrialized countries during the 1960's and 1970's. The implications of this relative slowdown for the competitive position of the United States, especially relative to Japan, have become a matter of public debate. Some scholarly debate has been devoted to determining whether this relative slowdown is part of a natural longer-term pattern of convergence. That is, these productivity trends are viewed as a natural process of convergence, as countries with low initial levels of productivity exploit the public-goods aspects of technology advances. In this case the relatively slow productivity gains of the United States relative to Japan, for example, may be due to a natural catching-up process.

The convergence view has been articulated by many, including Moses Abramovitz (1986, 1990), William J. Baumol (1986), and Baumol et al. (1989). Using data collected by Angus Maddison (1982, 1989), these authors provide evidence that incomes have been converging over a fairly long period. For example, Baumol (1986) finds a high inverse correlation between a country's productivity level (as proxied by GDP per work-hour) in 1870 and its productivity growth in terms of GDP per work-hour over the next 110 years. While these results have been shown to be very sensitive to the sample of countries selected (see J. Bradford De Long, 1988), there remains evidence that convergence has occurred among an *ex ante* chosen subset of OECD countries (Baumol and Edwin J. Wolff, 1988; Baumol et al., 1989). We note that the partial measure of productivity used in these studies, namely, labor productivity, may also have influenced their results. The goal here is to measure explicitly total factor productivity.

Along these lines, Steven Dowrick and Duc-Tho Nguyen (1989) have added further evidence for convergence based on the postwar period for a sample of OECD countries. They argue that one needs to distinguish between catch-up or convergence of income (or income per capita or income per work hour) and total factor productivity (TFP) catch-up. Following Baumol (1986) and Abramovitz (1986, 1990), Dowrick and Nguyen posit that TFP catch-up is inversely related to a country's initial level of relative labor productivity. Unlike earlier convergence studies, they use trend growth rates of GDP as their dependent variable. Their regression results show a highly significant inverse relationship between growth of GDP and a country's initial relative productivity. This result was even more pronounced when growth of capital and employment were added as explanatory variables. Because they control for capital and employment growth, they interpret the coefficient on the initial productivity variable as a measure of TFP catch-up. They find that "TFP catching up has been a dominant and stable feature of the pattern of growth in the OECD since 1950" (Dowrick and Nguyen, 1989 p. 1024). Moreover, their results are relatively insensitive to the sample selection of countries, time periods considered, and model specification.

Steven Dowrick (1989) extended the Dowrick and Nguyen results by allowing for sectoral change. He found evidence that "GDP growth since 1950 has been systematically higher in those OECD countries which have been able to reallocate a greater proportion of their labour force out of agriculture" (p. 335). These sectoral results, however, did not change the basic result that there are strong patterns of TFP convergence for this sample of OECD countries.

The purpose of our paper is to provide evidence concerning patterns of total factor productivity growth (including TFP catching up) using an alternative measure of TFP (the Malmquist index of total factor productivity growth) which allows us to isolate catching up to the frontier from shifts in the frontier. Although others have proposed such a decomposition of productivity growth, including Mieko Nishimizu and John M. Page (1982) and Paul W. Bauer (1990), they require specification of a functional form for technology, whereas our approach is nonparametric.

II. The Productivity Index

In this study we calculate productivity change as the geometric mean of two Malmquist productivity indexes. The Malmquist index was introduced by Caves et al. (1982a, b) who dubbed it the (output-based) Malmquist productivity index after Sten Malmquist, who earlier proposed constructing quantity indexes as ratios of distance functions (see Malmquist, 1953). Distance functions are function representations of multiple-output, multiple-input technology which require data only on input and output quantities. Consequently, our Malmquist index is a "primal" index of productivity change that, in contrast to the Törnqvist index, does not require cost or revenue shares to aggregate inputs and outputs, yet is capable of measuring total factor productivity growth in a multiple-output setting.

In their papers, Caves et al. (1982a, b) (hereafter, CCD) show that under certain circumstances, the Törnqvist index (which is the discrete counterpart of the Divisia index) is equivalent to the geometric mean of two Malmquist output productivity indexes.[2] Moreover, they show that the Törnqvist index is "exact" for technology that is translog (i.e., one can compute a nonparametric [in the sense that one need not estimate the parameters of technology] productivity index that is "exactly" consistent with the translog form). Furthermore, since the translog is flexible, the Törnqvist index is "superlative" in the terminology coined by W. Erwin Diewert (see e.g., Diewert, 1976).

In its original form, the Törnqvist index does not allow for the decomposition of productivity growth into changes in performance and changes in (frontier) technology, since the Törnqvist index presumes that production is always efficient. The same interpretation carries over to the growth-accounting approach to measurement of total factor productivity (i.e., it is implicitly assumed that observed production is efficient).

To define the output-based Malmquist index of productivity change, we assume that for each time period $t=1,\ldots,T$, the production technology $\mathbf{S}^t$ models the transformation of inputs, $\mathbf{x}^t \in \mathbb{R}_+^N$, into outputs, $\mathbf{y}^t \in \mathbb{R}_+^M$,

$$(1)\quad \mathbf{S}^t = \{(\mathbf{x}^t, \mathbf{y}^t) : \mathbf{x}^t \text{ can produce } \mathbf{y}^t\}$$

[2]The conditions include technical efficiency, allocative efficiency, that the underlying technology must be translog, and that all second-order terms must be identical over time. In contrast, the Malmquist index does not require any assumptions with respect to efficiency or functional form. Our specification of productivity change as the geometric mean of two Malmquist indexes stems from CCD.

(i.e., the technology consists of the set of all feasible input/output vectors). We assume that $\mathbf{S}^t$ satisfies certain axioms which suffice to define meaningful output distance functions (see Ronald W. Shephard [1970] or Färe [1988] for such axioms).

Following Shephard (1970) or Färe (1988), the output distance function is defined at t as

$$(2)\quad D_o^t(\mathbf{x}^t,\mathbf{y}^t) = \inf\{\theta\colon (\mathbf{x}^t,\mathbf{y}^t/\theta)\in \mathbf{S}^t\} = \left(\sup\{\theta\colon (\mathbf{x}^t,\theta\mathbf{y}^t)\in \mathbf{S}^t\}\right)^{-1}.$$

This function is defined as the reciprocal of the "maximum" proportional expansion of the output vector $\mathbf{y}^t$, given inputs $\mathbf{x}^t$.[3] It completely characterizes the technology. In particular, note that $D_o^t(\mathbf{x}^t,\mathbf{y}^t)\leq 1$ if and only if $(\mathbf{x}^t,\mathbf{y}^t)\in \mathbf{S}^t$. In addition, $D_o^t(\mathbf{x}^t,\mathbf{y}^t)=1$ if and only if $(\mathbf{x}^t,\mathbf{y}^t)$ is on the boundary or frontier of technology. In the terminology of Farrell (1957), that occurs when production is technically efficient.[4] This is illustrated in Figure 1. In this figure, scalar input is used to produce scalar output. In our figure, observed production at t is interior to the boundary of technology at t; that is, we say that (x^t,y^t) is not technically efficient. The distance function seeks the reciprocal of the greatest proportional increase in output(s), given input(s), such that output is still feasible. In our diagram, maximum feasible production, given x^t, is at (y^t/θ^*). The value of the distance function for our observation in terms of distances on the y-axis is $0a/0b$, which is less than 1. More generally, we may write the value of the distance function for observation $(\mathbf{x}^t,\mathbf{y}^t)$ as $\|\mathbf{y}^t\|/\|\mathbf{y}^t/\theta^*\|$.

Note that under constant returns to scale, maximum feasible output is achieved when average productivity, y/x, is maximized. In our simple single-output, single-input example, that is also the maximum observed total factor average product (productivity). In our empirical work, that maximum is the "best practice" or highest productivity observed in our sample of countries and is determined using programming techniques; this is explained in more detail in the next section.

It follows from the definition of the distance function that it is homogeneous of degree $+1$ in outputs. In addition, it is the reciprocal of Farrell's (1957) measure of output technical efficiency, which calculates "how far" an observation is from the frontier of technology: in Figure 1, Farrell output technical efficiency of $(x^t,y^t)=0b/0a$. The distance function can also readily model multiple-output technology in contrast to the production function.[5]

To define the Malmquist index we need to define distance functions with respect to two different time periods such as

$$(3)\quad D_o^t(\mathbf{x}^{t+1},\mathbf{y}^{t+1}) = \inf\{\theta\colon (\mathbf{x}^{t+1},\mathbf{y}^{t+1}/\theta)\in \mathbf{S}^t\}.$$

This distance function measures the maximal proportional change in outputs required to make $(\mathbf{x}^{t+1},\mathbf{y}^{t+1})$ feasible in relation to the technology at t. This is illustrated in Figure 1. Note that production (x^{t+1},y^{t+1}) occurs outside the set of feasible production in period t (i.e., technical

[3] The input distance function is defined similarly: $D_i^t(\mathbf{x}^t,\mathbf{y}^t)=\sup\{\lambda\colon (\mathbf{x}^t/\lambda,\mathbf{y}^t)\in \mathbf{S}^t\}$. Under constant returns to scale, $D_o(\mathbf{x},\mathbf{y})=(D_i(\mathbf{x},\mathbf{y}))^{-1}$. See, for example, Angus Deaton (1979) for applications of the input distance function.

[4] In his empirical work, Farrell defines technical efficiency as the maximal proportional contraction of inputs. One important implication of this interpretation is that cost could be reduced by the same proportion. He also indicates that under constant returns to scale this may be interpreted as the percentage by which output could be increased using the same inputs (see Farrell, 1957 p. 254). The interpretation of Farrell's measures of technical efficiency as reciprocals of distance functions follows the convention in Färe et al. (1985).

[5] In the scalar case, we can show the relationship between the distance and production functions. Let the production technology be described by $S^t=\{(\mathbf{x}^t,y^t)\colon y^t\leq f(\mathbf{x}^t)\}$, which is equivalent to $D_o^t(\mathbf{x}^t,y^t)=y^t/f(\mathbf{x}^t)$, or the ratio of observed to maximum potential output.

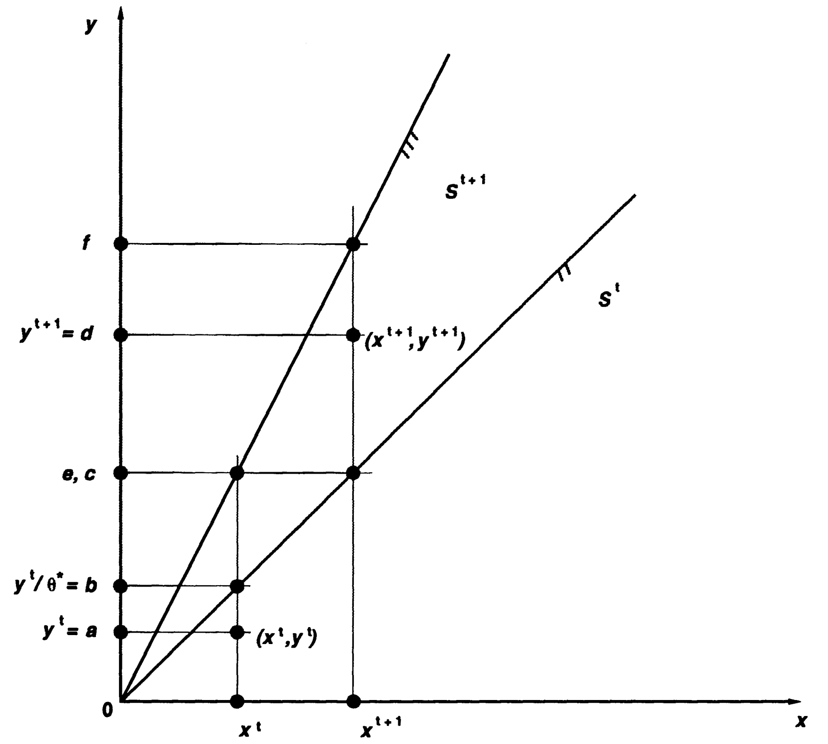

FIGURE 1. THE MALMQUIST OUTPUT-BASED INDEX OF TOTAL FACTOR PRODUCTIVITY AND OUTPUT DISTANCE FUNCTIONS

change has occurred). The value of the distance function evaluating (x^{t+1}, y^{t+1}) relative to technology in period t is $0d/0e$, which is greater than 1.

Similarly, one may define a distance function that measures the maximal proportional change in output required to make $(\mathbf{x}^t, \mathbf{y}^t)$ feasible in relation to the technology at $t+1$ which we call $D_o^{t+1}(\mathbf{x}^t, \mathbf{y}^t)$.

CCD define the Malmquist productivity index as

$$(4) \qquad M_{\mathrm{CCD}}^{t} = \frac{D_o^t(\mathbf{x}^{t+1}, \mathbf{y}^{t+1})}{D_o^t(\mathbf{x}^t, \mathbf{y}^t)}.$$

In this formulation, technology in period t is the reference technology. Alternatively, one could define a period-$(t+1)$-based Malmquist index as

$$(5) \qquad M_{\mathrm{CCD}}^{t+1} = \frac{D_o^{t+1}(\mathbf{x}^{t+1}, \mathbf{y}^{t+1})}{D_o^{t+1}(\mathbf{x}^t, \mathbf{y}^t)}.$$

In order to avoid choosing an arbitrary benchmark, we specify the output-based Malmquist productivity change index as the geometric mean of two CCD-type Malmquist productivity indexes:[6]

$$(6) \quad M_o(\mathbf{x}^{t+1}, \mathbf{y}^{t+1}, \mathbf{x}^t, \mathbf{y}^t) = \left[\left(\frac{D_o^t(\mathbf{x}^{t+1}, \mathbf{y}^{t+1})}{D_o^t(\mathbf{x}^t, \mathbf{y}^t)}\right)\left(\frac{D_o^{t+1}(\mathbf{x}^{t+1}, \mathbf{y}^{t+1})}{D_o^{t+1}(\mathbf{x}^t, \mathbf{y}^t)}\right)\right]^{1/2}.$$

[6]This form is typical of Fisher ideal indexes. This is also the form which CCD use to prove that the Törnqvist index is exact.

Following Färe et al. (1989, 1992) an equivalent way of writing this index is

$$(7)\quad M_o(\mathbf{x}^{t+1},\mathbf{y}^{t+1},\mathbf{x}^t,\mathbf{y}^t) = \frac{D_o^{t+1}(\mathbf{x}^{t+1},\mathbf{y}^{t+1})}{D_o^t(\mathbf{x}^t,\mathbf{y}^t)} \times\left[\left(\frac{D_o^t(\mathbf{x}^{t+1},\mathbf{y}^{t+1})}{D_o^{t+1}(\mathbf{x}^{t+1},\mathbf{y}^{t+1})}\right)\left(\frac{D_o^t(\mathbf{x}^t,\mathbf{y}^t)}{D_o^{t+1}(\mathbf{x}^t,\mathbf{y}^t)}\right)\right]^{1/2}$$

where the ratio outside the brackets measures the change in relative efficiency (i.e., the change in how far observed production is from maximum potential production) between years t and $t+1$. The geometric mean of the two ratios inside the brackets captures the shift in technology between the two periods evaluated at $\mathbf{x}^t$ and $\mathbf{x}^{t+1}$, that is,

$$\text{efficiency change} = \frac{D_o^{t+1}(\mathbf{x}^{t+1},\mathbf{y}^{t+1})}{D_o^t(\mathbf{x}^t,\mathbf{y}^t)}$$

$$\text{technical change} = \left[\left(\frac{D_o^t(\mathbf{x}^{t+1},\mathbf{y}^{t+1})}{D_o^{t+1}(\mathbf{x}^{t+1},\mathbf{y}^{t+1})}\right) \times\left(\frac{D_o^t(\mathbf{x}^t,\mathbf{y}^t)}{D_o^{t+1}(\mathbf{x}^t,\mathbf{y}^t)}\right)\right]^{1/2}.$$

Note that if $\mathbf{x}^t=\mathbf{x}^{t+1}$ and $\mathbf{y}^t=\mathbf{y}^{t+1}$ (i.e., there has been no change in inputs and output between the periods), the productivity index (7) signals no change: $M_o(\cdot)=1$. In this case the component measures of efficiency change and technical change are reciprocals, but not necessarily equal to 1.

The decomposition is illustrated in Figure 1 for constant-returns-to-scale technology, where we provide an illustration in which technical advance has occurred in the sense that $\mathbf{S}^t\subset\mathbf{S}^{t+1}$. Note that $(x^t,y^t)\in\mathbf{S}^t$ and $(x^{t+1},y^{t+1})\in\mathbf{S}^{t+1}$; however, (x^{t+1},y^{t+1}) does not belong to $\mathbf{S}^t$ (i.e., technical progress has occurred). In terms of the distances along the y-axis, the index (7) becomes

$$(8)\quad M_o(x^{t+1},y^{t+1},x^t,y^t) = \left(\frac{0d}{0f}\right)\left(\frac{0b}{0a}\right)\left[\left(\frac{0d/0e}{0d/0f}\right)\left(\frac{0a/0b}{0a/0c}\right)\right]^{1/2} = \left(\frac{0d}{0f}\right)\left(\frac{0b}{0a}\right)\left[\left(\frac{0f}{0e}\right)\left(\frac{0c}{0b}\right)\right]^{1/2}.$$

The last expression shows that the ratios inside the brackets measure shifts in technology at input levels x^t and x^{t+1}, respectively; thus technical change is measured as the geometric mean of those two shifts.[7] The terms outside the brackets measure relative technical efficiency at t and $t+1$, capturing changes in relative efficiency over time, that is, whether production is getting closer (catching up) or farther from the frontier. We would expect this component to capture diffusion of technology. In our application, which employs aggregate macro data, we would also expect to see variation in capacity utilization and differences in the structure of the economy (i.e., whether highly regulated or competitive) to be reflected in changes in efficiency as well. This follows from the fact that observations are compared to the best-practice frontier. In our empirical work, individual countries will be compared to the world frontier.

Improvements in productivity yield Malmquist indexes greater than unity. Deterioration in performance over time is associated with a Malmquist index less than unity. In addition, improvements in any of the components of the Malmquist index are also associated with values greater than unity of those components, and deterioration is associated with values less than unity. Note that while the product of the efficiency-change and technical-change components must by definition equal the Malmquist in-

[7]This form of index as the geometric mean of two ratios is the same form as the Fisher ideal index. Here, however, each component is the multi-output generalization of the technical change index defined by Diewert (1980 p. 268).

dex, those components may be moving in opposite directions. For example, a Malmquist index greater than unity, say, 1.25 (which signals a productivity gain) could have an efficiency-change component less than 1 (e.g., 0.5) and a technical-change component greater than 1 (e.g., 2.5).

To sum up, we define productivity growth as the product of efficiency change and technical change. We interpret our components of productivity growth as follows: improvements in the efficiency-change component are considered to be evidence of catching up (to the frontier),[8] while improvements in the technical-change component are considered to be evidence of innovation. This decomposition thus provides an alternative way of testing for convergence of productivity growth, as well as allowing identification of innovation, a distinction which was not made in earlier studies of productivity growth in OECD countries.

In order to provide some intuition concerning the relationship of the Malmquist productivity index to traditional measures of productivity growth using aggregate production functions, suppose that technology can be represented by a Cobb-Douglas production function

$$(9)\quad y^t = A(t)\prod_{n=1}^{N}\left(x_n^t\right)^{\alpha_n} \qquad \alpha_n > 0.$$

In this case the output distance function at t becomes

$$(10)\quad D_o^t(x^t, y^t)$$
$$= \inf\left\{\theta\colon y^t/\theta \leqq A(t)\prod_{n=1}^{N}\left(x_n^t\right)^{\alpha_n}\right\}$$
$$= \inf\left\{\theta\colon y^t\Big/\left(A(t)\prod_{n=1}^{N}\left(x_n^t\right)^{\alpha_n}\right) \leqq \theta\right\}$$
$$= y^t\Big/\left(A(t)\prod_{n=1}^{N}\left(x_n^t\right)^{\alpha_n}\right).$$

Inserting (10) and the other Cobb-Douglas distance functions into the Malmquist index in (6) yields

$$(11)\quad M_o(\mathbf{x}^{t+1}, y^{t+1}, \mathbf{x}^t, y^t)$$
$$= \left(\frac{y^{t+1}}{\prod_{n=1}^{N}\left(x_n^{t+1}\right)^{\alpha_n}}\right)\left(\frac{\prod_{n=1}^{N}\left(x_n^t\right)^{\alpha_n}}{y^t}\right).$$

From (9) we find that the index can be written as

$$(12)\quad M_o(\mathbf{x}^{t+1}, y^{t+1}, \mathbf{x}^t, y^t) = A(t+1)/A(t)$$

(i.e., as the ratio of the efficiency parameters of the Cobb-Douglas production function).

The formulation in (12) is actually equivalent to the more general formulation by Robert Solow (1957), which is the basis for the growth-accounting approach to measurement of total factor productivity. In that approach, $A(t+1)/A(t)$ is calculated by taking the time derivatives of (9), dividing through by y, and using observed factor shares as proxies for the α_n, that is,

$$(13)\quad \dot{A}/A = \dot{y}/y - \sum_{n=1}^{N}\alpha_n\dot{x}_n/x_n$$

where dots refer to time derivatives, and y and x would be in natural logs for the Cobb-Douglas case.[9] In this approach observed output is assumed to be equivalent to frontier output, and this growth-accounting index of total factor productivity would be interpreted as capturing shifts in the technology (i.e., technical change). In the presence of inefficiency, this approach would give a biased estimate of technical change.[10]

[8]This is not the same notion of catching up discussed by Abramovitz (1986, 1990) and the authors mentioned earlier. Their notion of catching up is based on an inverse correlation between low initial levels of TFP or income and TFP growth.

[9]Notice that if $\dot{y}/y$ is approximated by $\ln y^{t+1} - \ln y^t$ and similarly for inputs and shares, (11) becomes a Törnqvist index.

[10]Note that there are two possible sources of inefficiency: the first is technical inefficiency (i.e., production below the frontier), and the second is allocative

In terms of Figure 1, ignoring technical inefficiency means that the frontier of technology is assumed to go through the observed points (x^t, y^t) and (x^{t+1}, y^{t+1}) in periods t and $t+1$, respectively. Productivity would then be assumed to be synonymous with technical change, and technical change would be measured as change in observed performance (adjusting for changes in input use).

One may calculate the Malmquist index in several ways. In their 1982 *Econometrica* paper, CCD showed that, if the distance functions are of translog form with identical second-order terms, then (6) can be computed as the quotient of Törnqvist indexes.[11] Bert Balk (1993) generalized conditions developed by Färe and Grosskopf (1990) under which the Malmquist index may be calculated as a quotient of Fisher ideal indexes.[12] Here we follow Färe et al. (1989) and calculate the distance functions that make up the Malmquist index by applying the linear-programming approach outlined by Färe et al. (1985). One could also calculate the component distance functions using the Dennis Aigner and S. F. Chu (1968) parametric linear-programming approach as well as frontier econometric approaches.[13]

In our empirical work, we calculate the Malmquist productivity index using nonparametric programming techniques. We assume that there are $k = 1, \ldots, K$ countries using $n = 1, \ldots, N$ inputs $x_n^{k,t}$ at each time period $t = 1, \ldots, T$. These inputs are used to produce $m = 1, \ldots, M$ outputs $y_m^{k,t}$. In our data set, each observation of inputs and outputs is strictly positive, and the number of observations remains constant over all years.[14]

The reference (or frontier) technology in period t is constructed from the data as

$$\text{(14)} \quad \mathbf{S}^t = \left\{ (\mathbf{x}^t, \mathbf{y}^t) : y_m^t \leqq \sum_{k=1}^{K} z^{k,t} y_m^{k,t} \qquad m = 1, \ldots, M; \right.$$
$$\sum_{k=1}^{K} z^{k,t} x_n^{k,t} \leqq x_n^t \qquad n = 1, \ldots, N;$$
$$\left. z^{k,t} \geqq 0 \qquad k = 1, \ldots, K \right\}$$

which exhibits constant returns to scale and strong disposability of inputs and outputs (see Färe et al. [1985] for details). Following Sidney Afriat (1972), the assumption of constant returns to scale may be relaxed to allow nonincreasing returns to scale by adding the following restriction:

$$\text{(15)} \qquad \sum_{k=1}^{K} z^{k,t} \leqq 1$$

where $z^{k,t}$ is an intensity variable indicating at what intensity a particular activity (in our case, each country is an activity) may be employed in production.[15] Again, following Afriat (1972), one may also allow for variable returns to scale (i.e., increasing, constant, or decreasing returns to scale) by changing the inequality in (15) to an equality. It is also important to note that the technology and, consequently, the associ-

inefficiency. Allocative inefficiency would be reflected in the shares used to aggregate inputs.

[11]They also assume that $D_0^t(\mathbf{x}^t, \mathbf{y}^t)$ and $D_0^{t+1}(\mathbf{x}^{t+1}, \mathbf{y}^{t+1})$ are each equal to unity (i.e., they assume no technical or allocative inefficiency as defined by Farrell [1957]).

[12]Balk (1993) shows that if there is no allocative efficiency, the two are approximately equal. Strict equality requires a strong form of neutrality as well. The Fisher ideal index also requires no allocative efficiency. It also requires information on both prices (or shares) and quantities of inputs and outputs, whereas the Malmquist does not require share data.

[13]For an example of the stochastic-frontier approach see Fabienne Fecher and Sergio Perelman (1989). They apply the Nishimizu and Page (1982) decomposition using a stochastic-frontier production function. That requires assuming a specific functional form, which our approach does not.

[14]In contrast to the Törnqvist index, our approach admits zero values of (some) inputs and outputs. One may also use an unbalanced panel, although the index will be undefined for missing observations.

[15]Imposing nonincreasing returns to scale (when inputs and outputs are strictly positive) is sufficient to guarantee that solutions exist to the output-oriented linear-programming problems used to calculate the mixed-period distance functions. Under variable returns to scale, if technical progress occurs, observations in period t may not be feasible in period $t+1$.

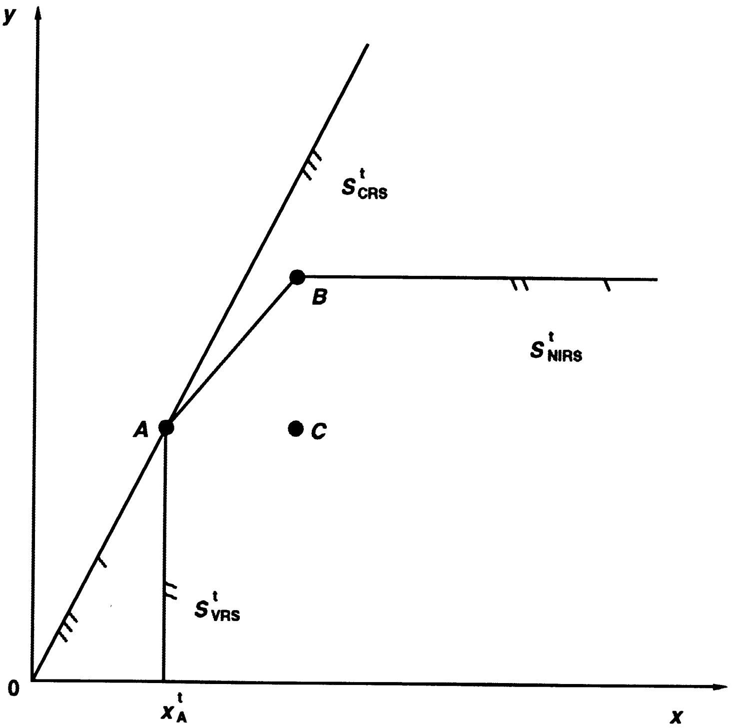

FIGURE 2. CONSTRUCTION OF REFERENCE TECHNOLOGY S^t

ated distance functions are independent of the units of measurement.

The construction of technology based on (14) is illustrated in Figure 2, which illustrates construction of technology for scalar input and output for one period t. Suppose there are three observations or countries, A, B, and C. If we restrict the intensity variables to sum to less than or equal to 1, (i.e., allow for nonincreasing returns to scale), technology will be bounded by $0AB$ and the horizontal extension from B. The intensity variables allow us to take convex combinations of observed data; the inequalities allow for disposability of inputs and outputs (the horizontal and vertical extensions of the data and its convex combinations, respectively). If we impose constant returns by allowing the elements of $\mathbf{z}$ to take any nonnegative values, technology becomes a cone. Finally, in the variable-returns-to-scale case the technology is bounded by x_A^t, A, B, and the horizontal extension from B.

In principle, one may calculate Malmquist productivity indexes relative to any type of technology (i.e., satisfying any type of returns to scale). Here we choose to calculate the Malmquist index relative to the constant-returns-to-scale technology. We use an enhanced decomposition of the Malmquist index developed in Rolf Färe et al. (1994). This enhanced decomposition takes the efficiency-change component calculated relative to the constant-returns-to-scale technology and decomposes it into a pure efficiency-change component (calculated relative to the variable-returns technologies) and a residual scale component which captures changes in the deviation between the variable-returns and constant-returns-to-scale technology. In Figure 2, scale efficiency for observation C is the vertical distance between $\mathbf{S}_{\mathrm{VRS}}^{t}$ and $\mathbf{S}_{\mathrm{CRS}}^{t}$ evaluated at the corresponding input for observation C. Thus the scale-change component would be the ratio of scale efficiency in period t and $t+1$. This enhanced decomposition allows us to report compactly results relative to the three types of technologies illustrated in Figure 2.[16]

In order to calculate the productivity of country k' between t and $t+1$, we need to solve four different linear-programming problems: $D_{\mathrm{o}}^{t}(\mathbf{x}^{t}, \mathbf{y}^{t})$, $D_{\mathrm{o}}^{t+1}(\mathbf{x}^{t}, \mathbf{y}^{t})$, $D_{\mathrm{o}}^{t}(\mathbf{x}^{t+1},\mathbf{y}^{t+1})$, and $D_{\mathrm{o}}^{t}(\mathbf{x}^{t+1},\mathbf{y}^{t+1})$.[17] Here we make use of the fact that the output distance function is reciprocal to the output-based Farrell measure of technical efficiency and compute, for each $k'=1,\dots,K$,

$$(16) \quad \left(D_{\mathrm{o}}^{t}(\mathbf{x}^{k',t},\mathbf{y}^{k',t})\right)^{-1} = \max \theta^{k'}$$

subject to

$$\theta^{k'} y_m^{k',t} \leqq \sum_{k=1}^{K} z^{k,t} y_m^{k,t} \qquad m=1,\dots,M$$

$$\sum_{k=1}^{K} z^{k,t} x_n^{k,t} \leqq x_n^{k',t} \qquad n=1,\dots,N$$

$$z^{k,t} \geqq 0 \qquad k=1,\dots,K.$$

The computation of $D_{\mathrm{o}}^{t+1}(\mathbf{x}^{k',t+1},\mathbf{y}^{k',t+1})$ is exactly like (16), where $t+1$ is substituted for t.

Two of the distance functions used to construct the Malmquist index require information from two periods. The first of these is computed for observation k' as

$$(17) \quad \left(D_{\mathrm{o}}^{t}(\mathbf{x}^{k',t+1},\mathbf{y}^{k',t+1})\right)^{-1} = \max \theta^{k'}$$

subject to

$$\theta^{k'} y_m^{k',t+1} \leqq \sum_{k=1}^{K} z^{k,t} y_m^{k,t} \qquad m=1,\dots,M$$

$$\sum_{k=1}^{K} z^{k,t} x_n^{k,t} \leqq x_n^{k',t+1} \qquad n=1,\dots,N$$

$$z^{k,t} \geqq 0 \qquad k=1,\dots,K.$$

Note that in (17), observations from both period t and period $t+1$ are involved. The reference technology relative to which $(\mathbf{x}^{k',t+1},\mathbf{y}^{k',t+1})$ is evaluated is constructed from observations at t. Note that in (16), $(\mathbf{x}^{k',t}, \mathbf{y}^{k',t}) \in \mathbf{S}^{t}$, and therefore $D_{\mathrm{o}}^{t}(\mathbf{x}^{k',t}, \mathbf{y}^{k',t}) \leqq 1$. However, in (17), $(\mathbf{x}^{k',t+1},\mathbf{y}^{k',t+1})$ need not belong to $\mathbf{S}^{t}$, so $D_{\mathrm{o}}^{t}(\mathbf{x}^{k',t+1},\mathbf{y}^{k',t+1})$ may take values greater than 1. The last linear-programming problem we need to solve is also a mixed-period problem. It is specified as in (17), but the t and $t+1$ superscripts are transposed.

In order to calculate changes in scale efficiency, we also calculate distance functions under variable returns to scale by adding the following restriction:

$$\sum_{k=1}^{K} z^{k,t} = 1 \text{ (VRS)}.$$

Scale efficiency in each period is constructed as the ratio of the distance function satisfying constant returns to scale to the distance function restricted to satisfy variable returns to scale. The efficiency-change component is calculated as the ratio of the own-period distance functions in each period satisfying variable returns to scale. Technical change is calculated relative to the constant-returns-to-scale technology.

[16]The decomposition becomes:

$$M_{\mathrm{o}}(\mathbf{x}^{t+1},\mathbf{y}^{t+1},\mathbf{x}^{t},\mathbf{y}^{t}) = \text{TECHCH} \times \text{PEFFCH} \times \text{SCH}$$

in which TECHCH represents technical change, PEFFCH represents pure efficiency change, and SCH represents scale change. The scale-change and pure-efficiency-change components are decompositions of efficiency change calculated relative to constant returns to scale: EFFCH = PEFFCH × SCH. The EFFCH term refers to efficiency change calculated under constant returns to scale, and PEFFCH is efficiency change calculated under variable returns to scale.

[17]To derive the full decomposition, including the scale-change component, requires calculation of an additional two programming problems: these are $D_{\mathrm{o}}^{t}(\mathbf{x}^{t}, \mathbf{y}^{t})$ and $D_{\mathrm{o}}^{t+1}(\mathbf{x}^{t+1}, \mathbf{y}^{t+1})$ relative to the variable-returns-to-scale technology.

TABLE 1—AVERAGE ANNUAL GROWTH RATES: GROSS DOMESTIC PRODUCT, CAPITAL AND LABOR, 1979–1988

Country	Gross domestic product	Capital	Labor
Australia	0.02728	0.02962	0.01594
Austria	0.01845	0.02018	0.00598
Belgium	0.01387	0.01213	0.00534
Canada	0.02744	0.04154	0.01325
Denmark	0.01745	0.01496	0.00566
Finland	0.03143	0.02863	0.00674
France	0.01464	0.02242	0.00722
Germany	0.01468	0.01961	0.00404
Greece	0.01269	0.01612	0.00486
Ireland	0.00856	0.02505	0.01425
Italy	0.02347	0.01674	0.00568
Japan	0.03380	0.05179	0.00770
Norway	0.03008	0.03210	0.00847
Spain	0.01853	0.02778	0.01016
Sweden	0.01972	0.01792	0.00403
United Kingdom	0.02094	0.01967	0.00368
United States	0.02615	0.02503	0.01111
Sample:	0.02113	0.02478	0.00789

III. Data and Results

We calculate productivity growth and its components for a sample of 17 OECD countries over the period 1979–1988 using data from the Penn World Tables (Mark 5). These data are built up from the benchmark studies of the International Comparison Program of the United Nations and national-account data. The procedures used to create the data set are discussed in some detail in Robert Summers and Alan Heston (1991). The resulting adjustments imply that "*real* international quantity comparison can be made both between countries and over time" (Summers and Heston, 1991 p. 1). The international prices are average world prices of final goods, rather than prices of a specific benchmark country.

Our measure of aggregate output is gross domestic product (GDP); capital stock and employment are our aggregate input proxies. GDP and capital stock are measured in 1985 international prices. Employment is retrieved from real GDP per worker, and capital is retrieved from capital stock per worker. (Capital stock does not include residential construction but does include gross domestic investment in producers' durables, as well as nonresidential construction. These are the cumulated and depreciated sums of past investment.)

Our method constructs a best-practice frontier from the data in the sample (i.e., we are constructing a world frontier and comparing individual countries to that frontier). Technology in any given period is represented as an output distance function. In the context here, where we have only one aggregate output, the output distance function becomes equivalent to a frontier production function in the sense that the frontier gives maximum feasible output given inputs (see footnote 5).

Our final sample consists of the 17 OECD countries for which capital-stock data were available over the 1979–1988 period: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Japan, Norway, Spain, Sweden, the United Kingdom, and the United States. We begin with a summary table of average annual growth rates of output, capital, and labor for each country in our sample. As seen in Table 1, growth in GDP averaged 2.1 percent per year over the entire 1979–1988 period for our sample. Japan had the highest average annual growth in

TABLE 2—CAPITAL–LABOR RATIOS (÷1000): SELECTED YEARS

Country	Year 1979	1983	1988
Australia	27.1535	28.6079	31.0267
Austria	25.1911	26.8039	28.9676
Belgium	40.9993	42.0072	43.8347
Canada	33.2425	37.9180	43.7224
Denmark	28.7653	28.4569	31.5111
Finland	38.8871	42.5726	48.1981
France	33.8774	36.4611	39.3318
Germany	33.6583	35.9226	39.2456
Greece	13.8322	14.9700	15.4544
Ireland	20.6891	23.5429	22.9544
Italy	30.2798	31.8558	33.7694
Japan	34.2422	41.6261	52.4700
Norway	41.9271	45.8818	52.8168
Spain	23.3333	24.8568	27.7107
Sweden	22.5562	23.6967	25.8723
United Kingdom	20.1372	20.7440	23.5700
United States	28.9230	29.7380	33.1471

TABLE 3—TECHNICAL EFFICIENCY: SELECTED YEARS (CONSTANT RETURNS TO SCALE)

Country	Year 1979	1983	1988
Australia	1.1962	1.2284	1.2357
Austria	1.3406	1.3159	1.3746
Belgium	1.2865	1.2567	1.3684
Canada	1.1493	1.0935	1.1600
Denmark	1.6534	1.5082	1.6276
Finland	1.7131	1.5006	1.5546
France	1.2903	1.2487	1.3857
Germany	1.3786	1.3481	1.4344
Greece	1.0936	1.1224	1.1411
Ireland	1.4130	1.5984	1.6749
Italy	1.3269	1.2392	1.2879
Japan	1.7202	1.5249	1.5402
Norway	1.3340	1.2530	1.2493
Spain	1.3950	1.4548	1.5414
Sweden	1.1694	1.1204	1.1589
United Kingdom	1.0846	1.0593	1.0816
United States	1.0000	1.0000	1.0000

GDP (3.4 percent), and Ireland had the lowest (less than 1 percent). Evidence concerning the capital–labor mix in these countries is summarized in Table 2.

Since the basic component of the Malmquist index is related to measures of technical efficiency, we also report technical efficiency for the countries in our sample for selected years in Table 3. Values of unity imply that the country is on the world frontier in the associated year. Values exceeding unity imply that the country is below the frontier or technically inefficient. For the years reported in Table 3, as well as the intermediate years, the United States is consistently technically efficient, under all three types of returns to scale. In fact, the United States is the only country determining the frontier in the constant-returns-to-scale version of technology. Perhaps surprisingly, Japan is one of the least technically efficient countries in the sample.

Figure 3 gives a visual summary of the data and formation of the frontier for the constant-returns-to-scale case. Here we have plotted output per unit of labor against the capital–labor ratio for France (F), Germany (G), Japan (J), the United Kingdom (UK), and the United States (US) for 1979 and 1988.[18] The figure shows why we find the United States to be technically efficient relative to the world frontier: it has the highest ratio of output to capital in the sample.

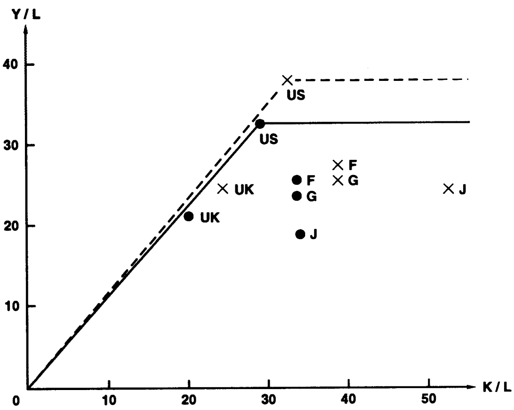

FIGURE 3. OUTPUT–LABOR (Y/L) AND CAPITAL–LABOR (K/L) RATIOS FOR SELECTED COUNTRIES, 1979 (●) AND 1988 (×)

Next we calculate Malmquist productivity indexes as well as the efficiency-change, technical-change, and scale-change components for each country in our sample. Since

[18]We thank an anonymous referee for suggesting this diagram.

TABLE 4—DECOMPOSITION WITH SCALE EFFECTS

Country	Average annual changes				
	Malmquist index (MALM)	Technical change (TECHCH)	Efficiency change (EFFCH)	Pure efficiency change (PEFFCH)	Scale change (SCH)
Australia	0.9973	1.0009	0.9964	0.9978	0.9986
Austria	0.9981	1.0009	0.9972	1.0023	0.9950
Belgium	1.0092	1.0161	0.9932	0.9905	1.0027
Canada	1.0151	1.0161	0.9990	0.9979	1.0011
Denmark	1.0026	1.0009	1.0017	1.0047	0.9971
Finland	1.0272	1.0161	1.0108	1.0065	1.0043
France	1.0081	1.0161	0.9921	0.9918	1.0003
Germany	1.0117	1.0161	0.9956	0.9954	1.0002
Greece	0.9962	1.0009	0.9953	1.0000	0.9953
Ireland	0.9821	1.0009	0.9813	1.0000	0.9813
Italy	1.0195	1.0161	1.0033	1.0037	0.9996
Japan	1.0287	1.0161	1.0124	1.0123	1.0001
Norway	1.0236	1.0161	1.0073	1.0000	1.0073
Spain	0.9898	1.0009	0.9890	0.9894	0.9960
Sweden	1.0019	1.0009	1.0010	1.0051	0.9960
United Kingdom	1.0012	1.0009	1.0003	1.0006	0.9997
United States	1.0085	1.0085	1.0000	1.0000	1.0000
Mean:	1.0070	1.0085	0.9986	0.9999	0.9987

this is an index based on discrete time, each country will have an index for every pair of years. This entails calculating the component distance functions using linear-programming methods as described in the previous section. We calculated 918 linear-programming problems.

Instead of presenting the disaggregated results for each country and year, we turn to a summary description of the average performance of each country over the entire 1979–1988 time period.[19] Recall that if the value of the Malmquist index or any of its components is less than 1, that denotes regress or deterioration in performance, whereas values greater than 1 denote improvements in the relevant performance. Also recall that these measures capture performance relative to the best practice in the sample, where best practice represents a "world frontier," and the world is defined as the countries in our sample. Looking first at the bottom of Table 4, we see that, on average, productivity increased slightly over the 1979–1988 period for the countries in our sample: the average change in the Malmquist productivity index was less than 1 percent per year for our sample as a whole.[20] On average, that growth was due to innovation (TECHCH) rather than improvements in efficiency (EFFCH).

Turning to the country-by-country results, we note that Japan has the highest total factor productivity change in the sample at 2.9 percent per year on average, almost half of which is due to improvements in efficiency. In fact, Japan's rate of efficiency change was the highest in the sample (i.e., Japan was especially good at moving toward the frontier or "catching up"). Based on the constant-returns-to-scale technology, U.S. total factor productivity change was slightly higher than the sample average (0.85 percent compared to 0.70 percent), all of which was due to innovation or technical change.

Although the average results with respect to technical change are suggestive, they do

[19]Since the Malmquist index is multiplicative, these averages are also multiplicative (i.e., they are geometric means).

[20]Subtracting 1 from the number reported in the table gives average increase or decrease per annum for the relevant time period and relevant performance measure.

TABLE 5—COUNTRIES SHIFTING THE FRONTIER

Year	Country
1979–1980	—
1980–1981	United States
1981–1982	—
1982–1983	United States
1983–1984	United States
1984–1985	United States
1985–1986	United States
1986–1987	United States
1987–1988	United States

not allow us to identify which countries are shifting the frontier over time. The technical-change component of the Malmquist index tells us what happened to the frontier at the input level and mix of each country, but not whether that country actually caused the frontier to shift. In order to provide evidence as to which countries were the "innovators," we can look at the component distance functions in the technical-change index. Specifically, if

$$TC^k > 1$$

$$D_o^t(\mathbf{x}^{k,t+1}, \mathbf{y}^{k,t+1}) > 1$$

and

$$D_o^{k,t+1}(\mathbf{x}^{k,t+1}, \mathbf{y}^{k,t+1}) = 1$$

then that country has contributed to a shift in the frontier between period t and $t+1$. Since the United States alone determined the frontier in each year under constant returns to scale and nonincreasing returns to scale, it is classified as the sole innovator given those technologies (see Table 5).[21]

Disaggregated results for each country are available from the authors upon request. To give some idea of the pattern of productivity *growth* and its components, we include an illustration for Japan and the United States (see Figs. 4 and 5, respectively). We include the cumulated Malmquist index as well as the cumulated efficiency-change and technical-change components. These are

[21]In 1979 and 1981, the frontier shifted backward slightly.

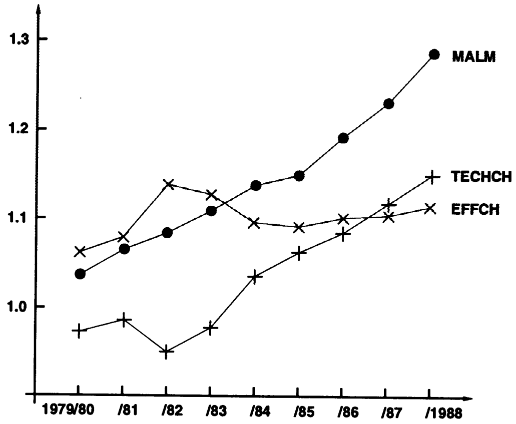

FIGURE 4. CUMULATED RESULTS (CONSTANT RETURNS TO SCALE), JAPAN

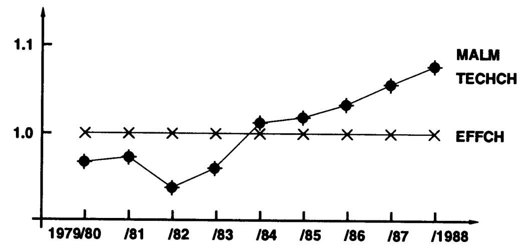

FIGURE 5. CUMULATED RESULTS (CONSTANT RETURNS TO SCALE), UNITED STATES

calculated as the sequential multiplicative sums of the annual indexes, since the index itself is multiplicative. Summaries of the total compounded growth between 1979 and 1988 for all countries are found in Table 6.

We note that the Malmquist productivity index as defined in (6) does not satisfy the circular test. This means that the index is not path-independent (i.e., if we calculated the Malmquist index directly between the endpoint years 1979 and 1988 and solved for the appropriate root, we would not necessarily get the same average changes reported in Table 4). On the other hand, the simpler indexes from (4) and (5) do satisfy the circular test. These are included in Table 7 and are labeled CCD1 and CCD2. Since the Malmquist index is defined as the geometric mean of CCD1 and CCD2, it is

TABLE 6—CUMULATED PRODUCTIVITY, 1979–1988

Country	Malmquist index (MALM)	Technical change (TECHCH)	Efficiency change (EFFCH)	Pure efficiency change (PEFFCH)	Scale change (SCH)
Australia	0.9757	1.0079	0.9681	0.9805	0.9874
Austria	0.9830	1.0079	0.9753	1.0206	0.9556
Belgium	1.0859	1.1551	0.9401	0.9179	1.0242
Canada	1.1444	1.1551	0.9907	0.9813	1.0096
Denmark	1.0238	1.0079	1.0158	1.0430	0.9740
Finland	1.2729	1.1551	1.1020	1.0599	1.0397
France	1.0756	1.1551	0.9311	0.9283	1.0031
Germany	1.1102	1.1551	0.9611	0.9593	1.0019
Greece	0.9660	1.0079	0.9584	1.0000	0.9584
Ireland	0.8503	1.0079	0.8436	1.0000	0.8436
Italy	1.1898	1.1548	1.0303	1.0338	0.9966
Japan	1.2901	1.1551	1.1169	1.1161	1.0007
Norway	1.2334	1.1551	1.0678	1.0000	1.0678
Spain	0.9122	1.0079	0.9050	0.9085	0.9962
Sweden	1.0171	1.0079	1.0091	1.0466	0.9642
United Kingdom	1.0107	1.0079	1.0028	1.0056	0.9972
United States	1.0790	1.0790	1.0000	1.0000	1.0000

TABLE 7—COMPARISON OF MALMQUIST TO CCD INDEX (CONSTANT RETURNS TO SCALE)

	Geometric mean		
Country	Malmquist index	CCD1 index	CCD2 index
Australia	0.9973	0.9973	0.9973
Austria	0.9981	0.9981	0.9981
Belgium	1.0092	1.0092	1.0092
Canada	1.0151	1.0151	1.0151
Denmark	1.0026	1.0026	1.0026
Finland	1.0272	1.0272	1.0272
France	1.0081	1.0081	1.0081
Germany	1.0117	1.0117	1.0117
Greece	0.9962	0.9962	0.9962
Ireland	0.9821	0.9821	0.9821
Italy	1.0195	1.0195	1.0195
Japan	1.0287	1.0287	1.0287
Norway	1.0236	1.0236	1.0236
Spain	0.9898	0.9898	0.9898
Sweden	1.0019	1.0019	1.0019
United Kingdom	1.0012	1.0012	1.0012
United States	1.0085	1.0161	1.0009
Sample	1.0070	1.0075	1.0066

bounded by or equal to indexes (4) and (5).[22]

As a final point of comparison, we present results of calculating total factor productivity growth using the Törnqvist-index formulation of the standard growth-accounting approach. That is, we calculate

$$(18)\quad \dot{TFP}/TFP = \dot{y}/y - \sum_{n=1}^{N} s_n \dot{x}_n / x_n$$

$$= \ln y^{t+1} - \ln y^t - \left[\sum_{N}^{n+1} 1/2(s_n^{t+1} + s_n^t)(\ln x_n^{t+1} - \ln x_n^t) \right]$$

where dots represent time derivatives (proxied here by log differences), and s_n represents input n's share of output. We use shares from John W. Kendrick (1981).[23] These were only available for a subset of our sample: Belgium, Canada, France, Germany, Italy, Japan, Sweden, the United Kingdom, and the United States. The growth rates for GDP, capital, and employment for all 17 countries in our original sample are gathered in Table 1.

[22] For a convincing argument for why the circular test is not important to the construction of a "good" index, see Irving Fisher (1927). Satisfaction of the circular test implies that technical change is neutral.

[23] Since only one share is reported for each country, the Törnqvist index simplifies to $\dot{TFP}/TFP = \ln y^{t+1} - \ln y^t - [\Sigma_N^{n+1} s_n(\ln x_n^{t+1} - \ln x_n^t)]$.

TABLE 8—TOTAL FACTOR PRODUCTIVITY: INCOME-ACCOUNTING APPROACH

Country	Average annual growth rate, 1979–1988
Belgium	0.00611
Canada	0.00084
France	0.00161
Germany	0.00521
Italy	0.01430
Japan	0.01138
Sweden	0.01036
United Kingdom	0.01170
United States	0.00793

Estimates of total factor productivity growth based on equation (18) are displayed in Table 8. As with our results for the Malmquist index, there is slow productivity growth per annum over the 1979–1988 period (the averages over the two slightly different samples are nearly identical; see Table 4). Results for individual countries vary, however. For example, looking at the nine countries for which we could calculate Törnqvist-type indexes, Canada had the third-highest annual average Malmquist productivity growth (at 0.0151), but was ranked last using growth-accounting techniques (at 0.00084). The United Kingdom and Sweden also had fairly dramatic differences on average.

The results of these two methods: the traditional growth-accounting approach and the Malmquist-index approach, are different. Why? We have used the same data (with the exception of the additional input-share data used in our growth-accounting calculations), so the major difference probably lies in the technique applied. One would expect these two approaches to yield comparable results in a world in which there is no inefficiency (see Caves et al., 1982a, b; Färe and Grosskopf, 1990). Note that in using shares to aggregate inputs, the growth-accounting approach introduces another potential source of inefficiency: if observed shares are not cost-minimizing shares (i.e., if factors are not paid their value marginal products as assumed in the growth-accounting approach), the resulting measure of total factor productivity growth will be biased. That is, any technical or allocative inefficiency will appear as deviations in productivity between the two approaches.

There is another reason why the two approaches may yield different estimates of total factor productivity growth. Note that in calculating the results in Table 8 we follow the traditional growth-accounting approach; no attempt was made in our growth-accounting estimates of TFP to make direct multilateral comparisons.[24] Each country is compared only to itself in previous periods, not to a common benchmark. On the other hand, an explicit benchmark is used in the calculation of the Malmquist index of TFP, namely, the world frontier constructed from the data.

Our results here should be interpreted with care. Our sample of countries is arbitrary—they are the countries for which we were able to collect consistent data over this period. The proxies used for capital and labor are not quality- or vintage-adjusted, suggesting that our productivity-growth figures (low as they are), may be upward-biased. The data are also extremely aggregated; disaggregation by sector would be useful, and since the Malmquist index is based on distance functions which are perfect aggregator functions, it is entirely feasible, given appropriate data.

Despite the level of aggregation of our data, we believe that the approach taken here provides important complementary information to traditional approaches to productivity measurement. It also provides a natural way to measure the phenomenon of catching up. Our technical-change component of productivity growth captures shifts in the frontier of technology, providing a natural measure of innovation. This decomposition of total factor productivity growth into catch-up and technical change is therefore useful in distinguishing diffusion of technology and innovation, respectively.

[24]See Caves et al. (1982a) for how such multilateral comparisons may be done.

These techniques could readily be applied at the micro level. One could clearly include resources devoted to research and development as inputs in such a micro model if such data are available. Finally, we note that the Malmquist index as calculated here does not require maintained hypotheses of technical and allocative efficiency implicit in the standard growth-accounting (and Törnqvist-index) approach to total factor productivity growth.

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