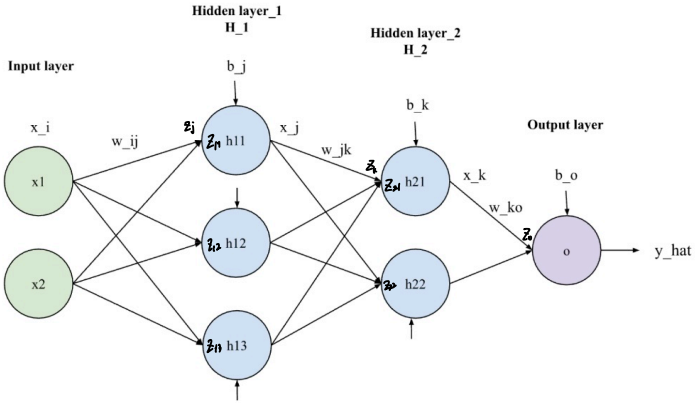
Exercise 2



x_i = input data

x_j = input to H_2 from H_1

x_k = input to O from H_2

b_j = bias at H_1

b_k = bias at H_2

b_o = bias at O

$\hat{y}$ = prediction

activation function at H_1: linear function

activation function at H_2: sigmoid

activation function at O: sigmoid

$$\frac{\partial L}{\partial w_{ko}} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial z_o} \cdot \frac{\partial z_o}{\partial w_{ko}}$$

$$= (\hat{y} - y) \cdot \hat{y} \cdot (1 - \hat{y}) \cdot \sigma(z_k)$$

$$= (\hat{y} - y) \cdot \hat{y} \cdot (1 - \hat{y}) \cdot x_k$$

$$\Rightarrow w_{ko\_new} = w_{ko} - \lambda \cdot (\hat{y} - y) \cdot \hat{y} \cdot (1 - \hat{y}) \cdot x_k$$

$$\frac{\partial L}{\partial w_{jk}} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial z_o} \cdot \frac{\partial z_o}{\partial x_k} \cdot \frac{\partial x_k}{\partial z_k} \cdot \frac{\partial z_k}{\partial w_{jk}}$$

$$= (\hat{y} - y) \cdot \hat{y} \cdot (1 - \hat{y}) \cdot w_{ko} \cdot \sigma'(z_k) \cdot \sigma(z_k)$$

$$= (\hat{y} - y) \cdot \hat{y} \cdot (1 - \hat{y}) \cdot w_{ko} \cdot x_k \cdot (1 - x_k) \cdot x_j$$

$$\Rightarrow w_{jk\_new} = w_{jk} - \lambda \cdot (\hat{y} - y) \cdot \hat{y} \cdot (1 - \hat{y}) \cdot w_{ko} \cdot x_k \cdot (1 - x_k) \cdot x_j$$

$$\frac{\partial L}{\partial w_{ij}} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial z_o} \cdot \frac{\partial z_o}{\partial x_k} \cdot \frac{\partial x_k}{\partial z_k} \cdot \frac{\partial z_k}{\partial x_j} \cdot \frac{\partial x_j}{\partial z_j} \cdot \frac{\partial z_j}{\partial w_{ij}}$$

$$= (\hat{y} - y) \cdot \hat{y} \cdot (1 - \hat{y}) \cdot w_{ko} \cdot \sigma'(z_k) \cdot w_{jk} \cdot 1 \cdot x_i$$

$$= (\hat{y} - y) \cdot \hat{y} \cdot (1 - \hat{y}) \cdot w_{ko} \cdot x_k \cdot (1 - x_k) \cdot w_{jk} \cdot x_i$$

$$\Rightarrow w_{ij\_new} = w_{ij} - \lambda (\hat{y} - y) \hat{y} (1 - \hat{y}) w_{ko} \cdot x_k \cdot (1 - x_k) \cdot w_{jk} \cdot x_i$$