

Table of contents

Table of contents	1
Computation time in total	1
Tolerance of physicality constraint violation	1
Experimental condition	1
True object	2
Tester objects	2
Cases	2
Computation time of estimators	3
Table	3
Histogram	3
MSE of empirical distributions	7
Consistency test	8
MSE of estimators	9
Comparison of analytical results	9
Comparison of parametrization	10
Comparison of estimators	11
Physicality violation test	12
Test of equality constraint violation	12
Test of inequality constraint violation	15
Eigenvalue	15
Sum of unphysical eigenvalues	23

Computation time in total

Total	0.5573378841082255min.
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Tolerance of physicality constraint violation

		True	False
Tolerance at physicality violation test	equality constraint	1.00e-13	1.00e-05
	inequality constraint	1.00e-05	1.00e-05

Experimental condition

Type of tomography	StandardQst
Nrep	10
N	[10, 100]
RNG seed	777

True object

Type	State
Dim	2
Vec	[0.70710678 0.0426784 0.05018503 0.63291038]

Tester objects

0	Type	Povm
	Dim	2
	Number of outcomes	2
	Vecs	[[0.70384373 0.6307768 -0.07036698 -0.03852733] [0.71036983 -0.6307768 0.07036698 0.03852733]]
1	Type	Povm
	Dim	2
	Number of outcomes	2
	Vecs	[[0.72395843 0.01968471 0.63888099 -0.00407311] [0.69025513 -0.01968471 -0.63888099 0.00407311]]
2	Type	Povm
	Dim	2
	Number of outcomes	2
	Vecs	[[0.6964465 0.00830651 -0.02153699 0.63860616] [0.71776707 -0.00830651 0.02153699 -0.63860616]]

Cases

	Name	Param	Tomography	Estimator
0	Linear (True)	True	StandardQst	Linear
1	Linear (False)	False	StandardQst	Linear
2	ProjectedLinear (True)	True	StandardQst	ProjectedLinear
3	ProjectedLinear (False)	False	StandardQst	ProjectedLinear
4	Maximum-Likelihood (True)	True	StandardQst	LossMinimization
5	Maximum-Likelihood (False)	False	StandardQst	LossMinimization

6	Least Squares (True)	True	StandardQst	LossMinimization
7	Least Squares (False)	False	StandardQst	LossMinimization

Computation time of estimators

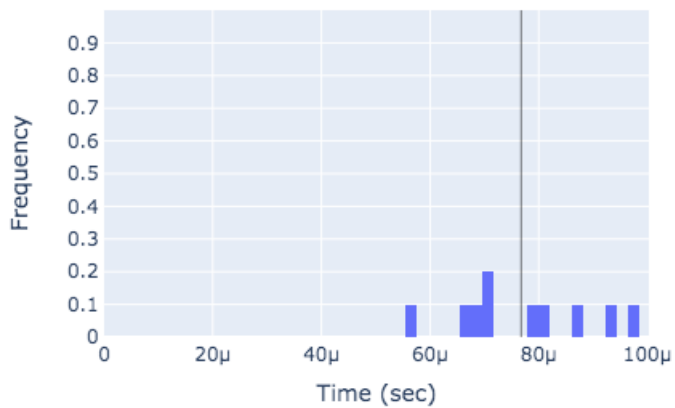
Table

	Name	N	Nrep	Mean (sec)	Std (sec)
0	Linear (True)	10	10	0.000077	0.000012
1	"	100	10	0.000077	0.000030
2	Linear (False)	10	10	0.000074	0.000011
3	"	100	10	0.000061	0.000010
4	ProjectedLinear (True)	10	10	0.008937	0.006642
5	"	100	10	0.002958	0.004545
6	ProjectedLinear (False)	10	10	0.006772	0.005016
7	"	100	10	0.002530	0.004047
8	Maximum-Likelihood (True)	10	10	0.488324	0.099040
9	"	100	10	0.465219	0.128858
10	Maximum-Likelihood (False)	10	10	0.685621	0.209755
11	"	100	10	1.034957	0.712243
12	Least Squares (True)	10	10	0.239782	0.175603
13	"	100	10	0.079726	0.038838
14	Least Squares (False)	10	10	0.233295	0.123365
15	"	100	10	0.095616	0.067979

Histogram

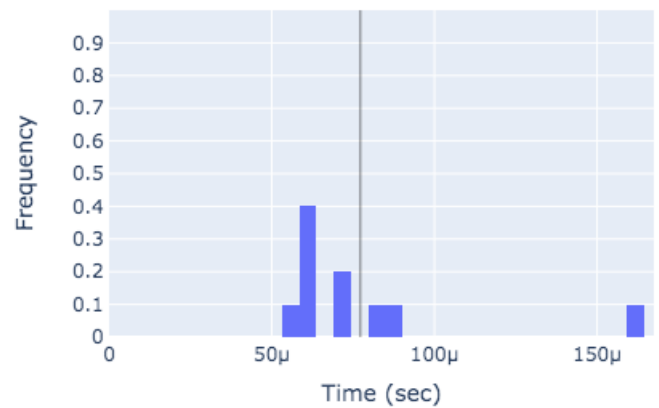
Case 0: Linear (True)

N=10, Nrep=10
Mean = 7.677078247070312e-05(sec)



Min: 5.7220458984375e-05
Max: 9.822845458984375e-05
|Max-Min|: 4.100799560546875e-05

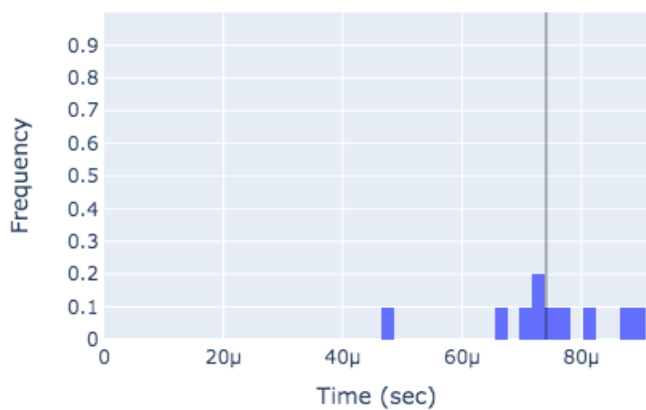
N=100, Nrep=10
Mean = 7.722377777099609e-05(sec)



Min: 5.602836608886719e-05
Max: 0.0001621246337890625
|Max-Min|: 0.00010609626770019531

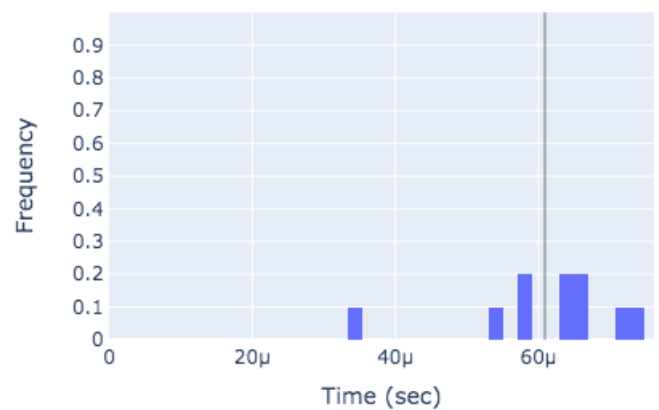
Case 1: Linear (False)

N=10, Nrep=10
Mean = 7.407665252685547e-05(sec)



Min: 4.696846008300781e-05
Max: 8.916854858398438e-05
|Max-Min|: 4.220008850097656e-05

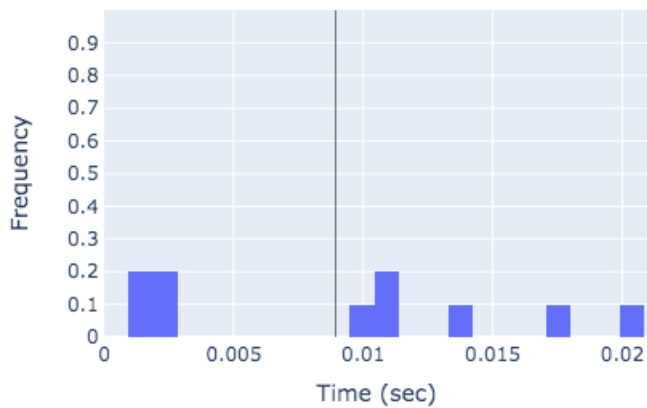
N=100, Nrep=10
Mean = 6.0892105102539064e-05(sec)



Min: 3.4809112548828125e-05
Max: 7.414817810058594e-05
|Max-Min|: 3.933906555175781e-05

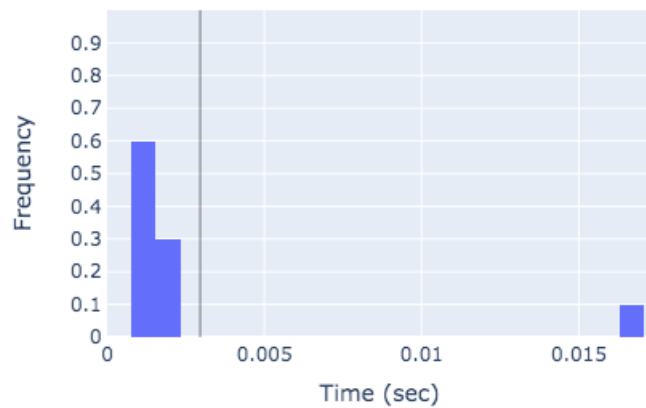
Case 2: ProjectedLinear (True)

N=10, Nrep=10
Mean = 0.008936595916748048(sec)



Min: 0.0011203289031982422
Max: 0.020079851150512695
|Max-Min|: 0.018959522247314453

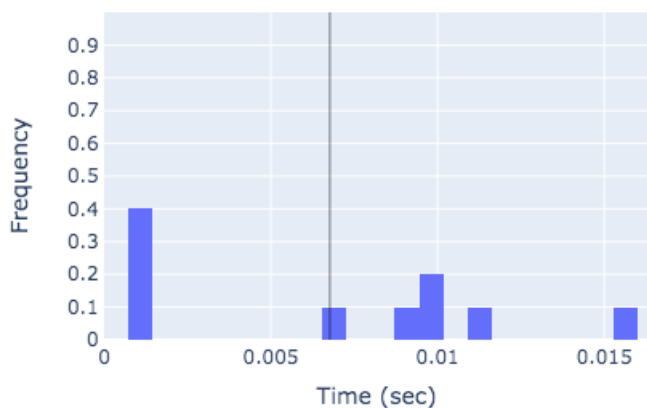
N=100, Nrep=10
Mean = 0.0029584407806396485(sec)



Min: 0.001035928726196289
Max: 0.016547441482543945
|Max-Min|: 0.015511512756347656

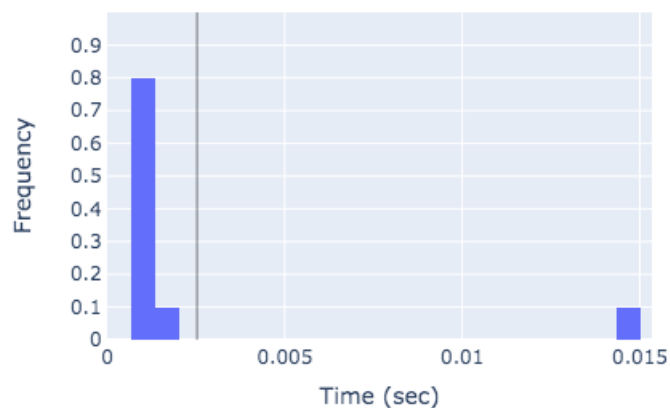
Case 3: ProjectedLinear (False)

N=10, Nrep=10
Mean = 0.006771588325500488(sec)



Min: 0.0010786056518554688
Max: 0.01561117172241211
|Max-Min|: 0.01453256607055664

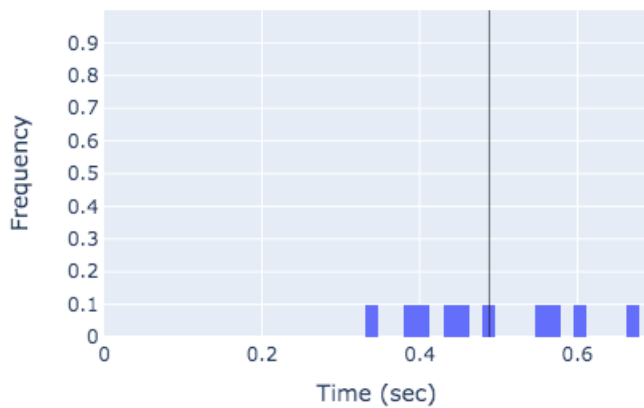
N=100, Nrep=10
Mean = 0.0025304555892944336(sec)



Min: 0.0009958744049072266
Max: 0.014654874801635742
|Max-Min|: 0.013659000396728516

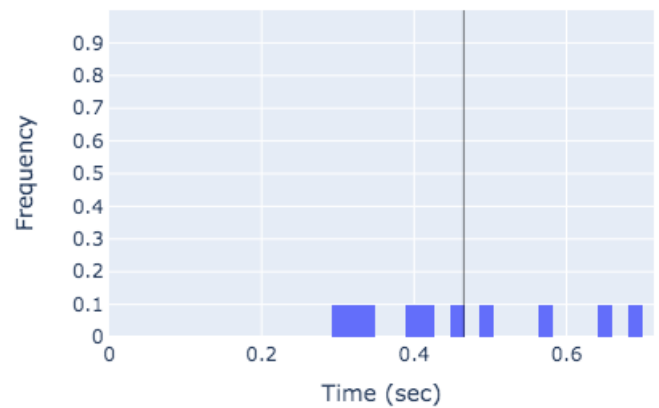
Case 4: Maximum-Likelihood (True)

N=10, Nrep=10
Mean = 0.488324499130249(sec)



Min: 0.3434326648712158
Max: 0.6739966869354248
|Max-Min|: 0.330564022064209

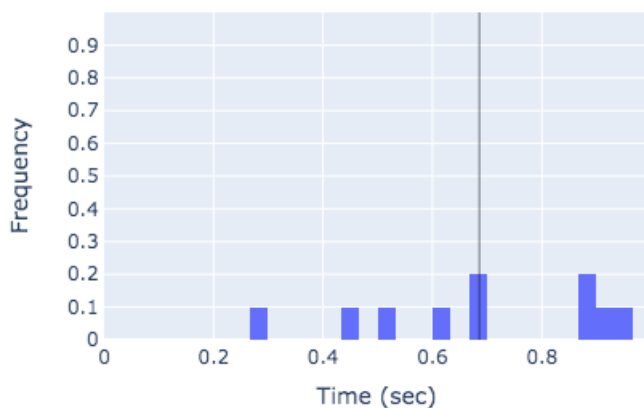
N=100, Nrep=10
Mean = 0.46521930694580077(sec)



Min: 0.30645155906677246
Max: 0.6950726509094238
|Max-Min|: 0.38862109184265137

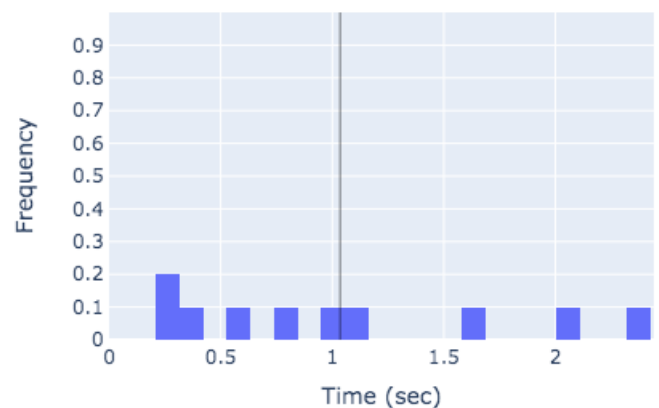
Case 5: Maximum-Likelihood (False)

N=10, Nrep=10
Mean = 0.6856214046478272(sec)



Min: 0.2960331439971924
Max: 0.9620890617370605
|Max-Min|: 0.6660559177398682

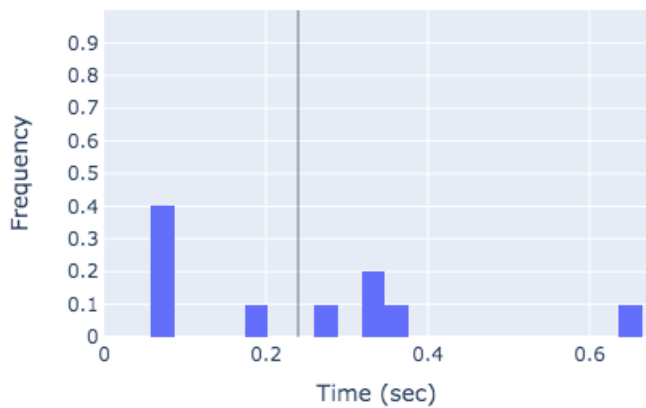
N=100, Nrep=10
Mean = 1.0349571466445924(sec)



Min: 0.22514605522155762
Max: 2.3354783058166504
|Max-Min|: 2.1103322505950928

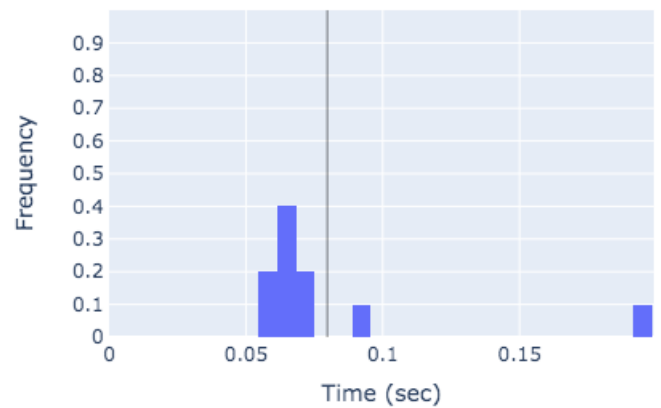
Case 6: Least Squares (True)

N=10, Nrep=10
Mean = 0.23978190422058104(sec)



Min: 0.06609606742858887
Max: 0.6443595886230469
|Max-Min|: 0.578263521194458

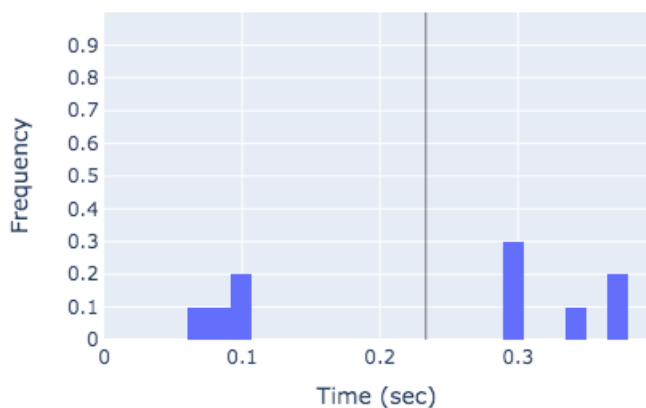
N=100, Nrep=10
Mean = 0.07972593307495117(sec)



Min: 0.055510759353637695
Max: 0.19224190711975098
|Max-Min|: 0.13673114776611328

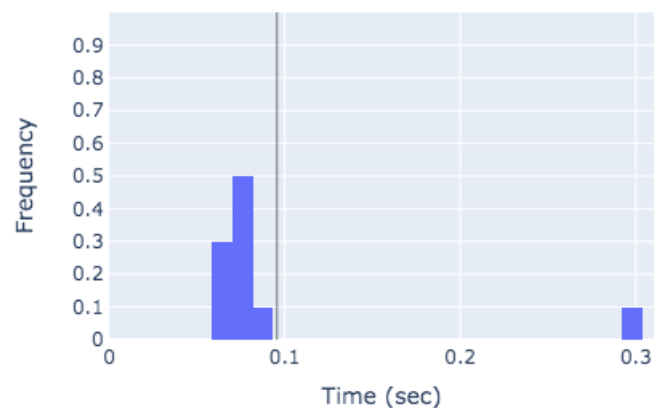
Case 7: Least Squares (False)

N=10, Nrep=10
Mean = 0.23329498767852783(sec)



Min: 0.07552123069763184
Max: 0.3797900676727295
|Max-Min|: 0.30426883697509766

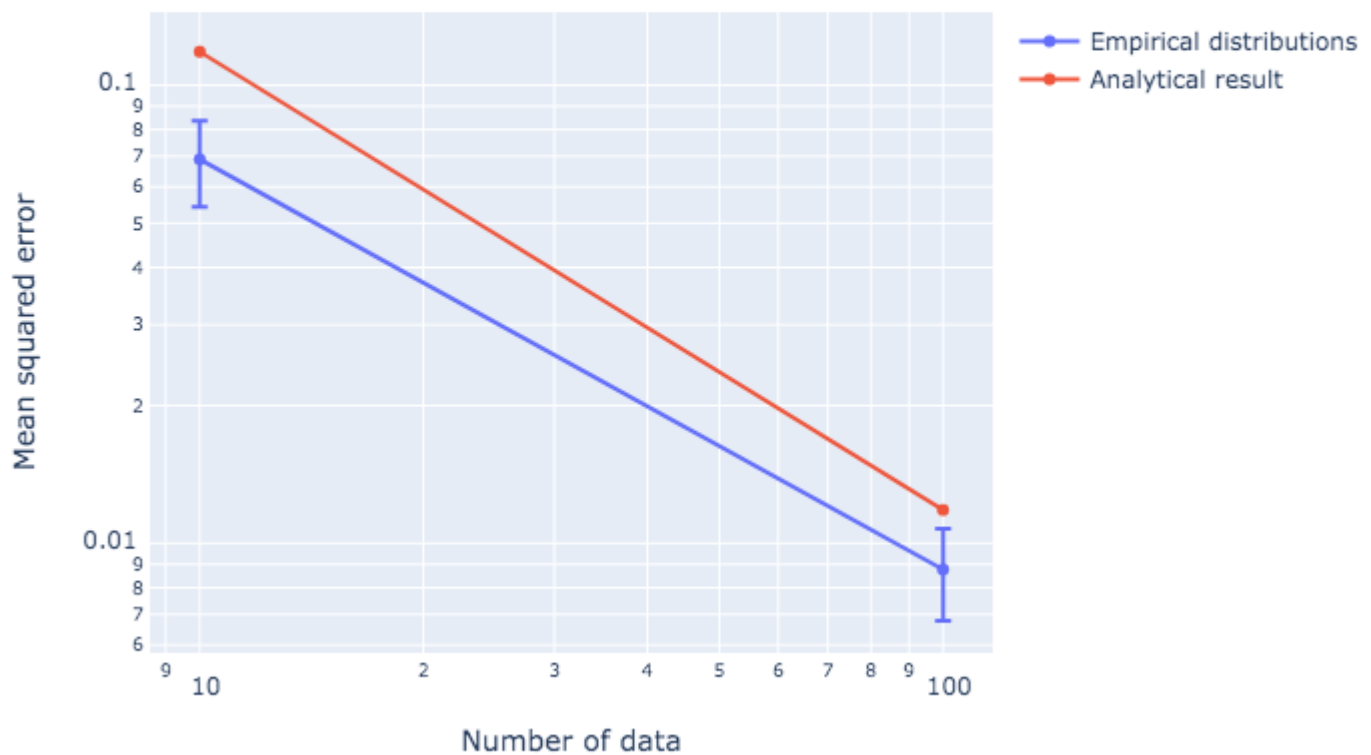
N=100, Nrep=10
Mean = 0.09561607837677003(sec)



Min: 0.06488800048828125
Max: 0.298581600189209
|Max-Min|: 0.23369359970092773

MSE of empirical distributions

Mean squared error



Consistency test

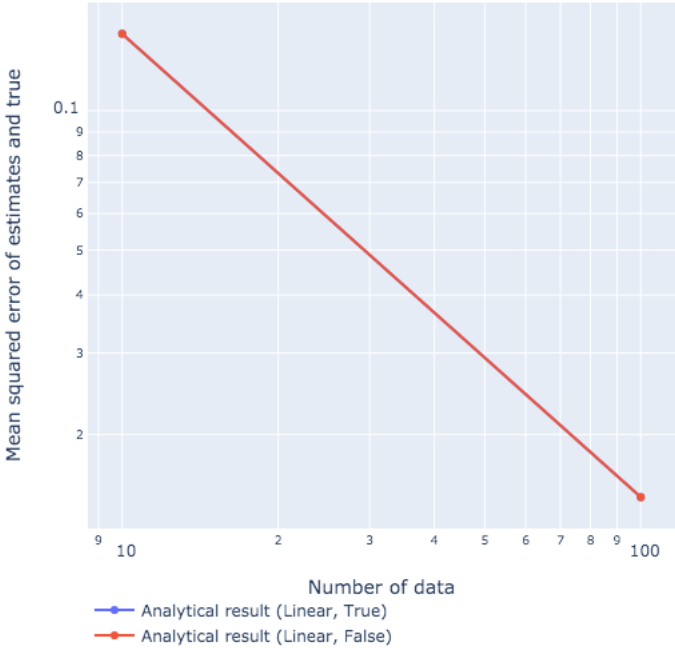
	Name	Type of tomography	Param	Estimator	Loss	Algo	Squared Error to True	Possibly OK	To be checked
0	Linear (True)	Standard Qst	True	Linear	None	None	9.29e-33	OK	not need debug
1	Linear (False)	Standard Qst	False	Linear	None	None	4.05e-32	OK	not need debug
2	ProjectedLinear (True)	Standard Qst	True	Projected Linear	None	None	6.40e-32	OK	not need debug
3	ProjectedLinear (False)	Standard Qst	False	Projected Linear	None	None	7.75e-32	OK	not need debug
4	Maximum-Likelihood (True)	Standard Qst	True	Loss Minimization	Weighted Relative Entropy	Projected Gradient Descent Backtracking	4.80e-18	OK	not need debug
5	Maximum-Likelihood (False)	Standard Qst	False	Loss Minimization	Weighted Relative Entropy	Projected Gradient Descent Backtracking	8.19e-09	NG	not need debug
6	Least Squares (True)	Standard Qst	True	Loss Minimization	Weighted Probability Based Squared Error	Projected Gradient Descent Backtracking	9.28e-31	OK	not need debug

7	Least Squares (False)	Standard Qst	False	Loss Minimization	Weighted Probability Based Squared Error	Projected Gradient Descent Backtracking	1.02e-30	OK	not need debug
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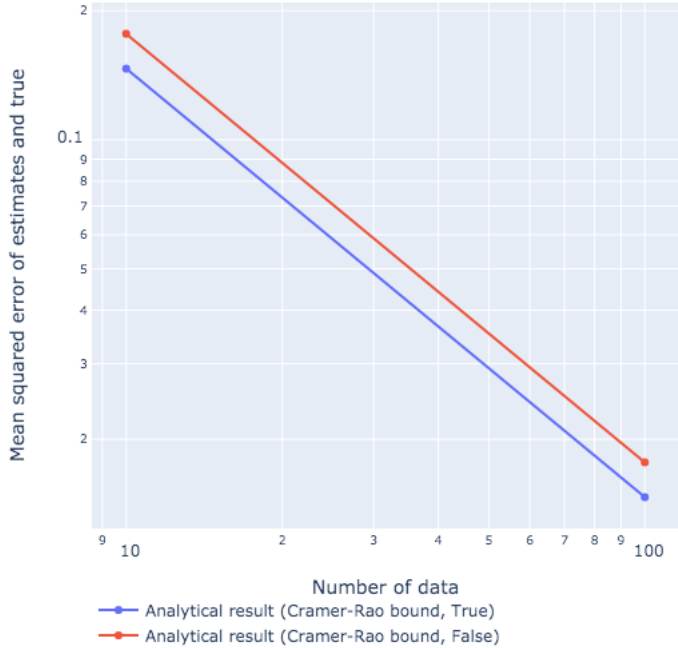
MSE of estimators

Comparison of analytical results

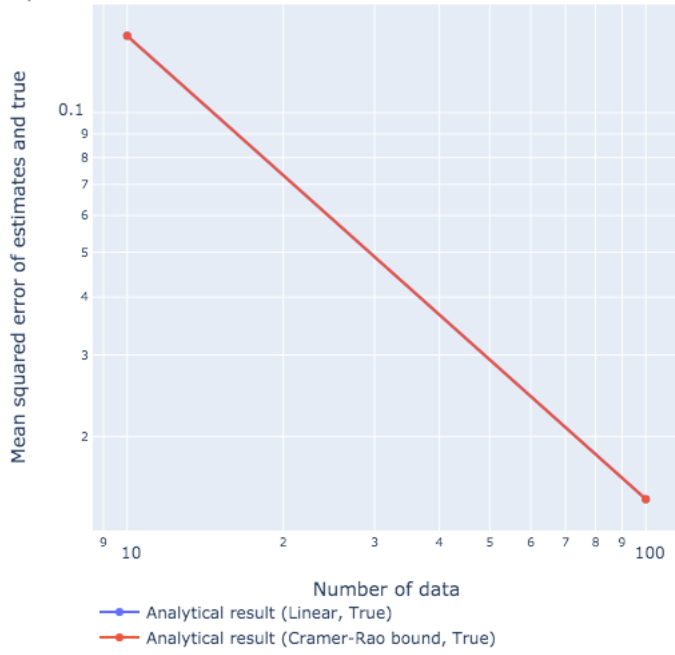
Mean squared error
Analytical result
estimator=Linear



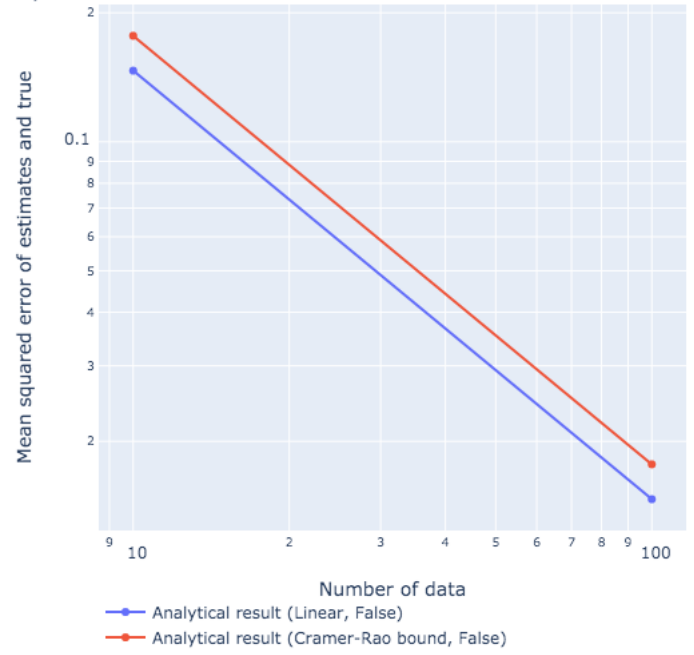
Mean squared error
Analytical result
Cramer-Rao bound



Mean squared error
Analytical result
parameterization=True

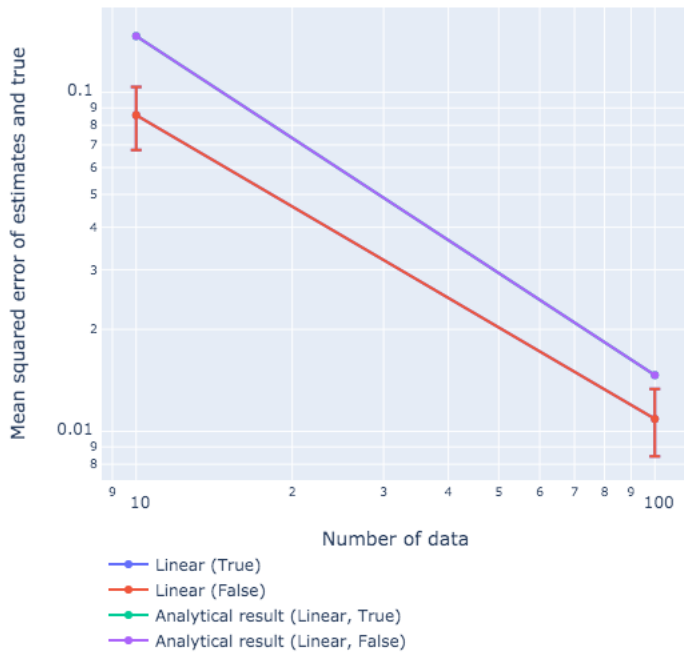


Mean squared error
Analytical result
parameterization=False

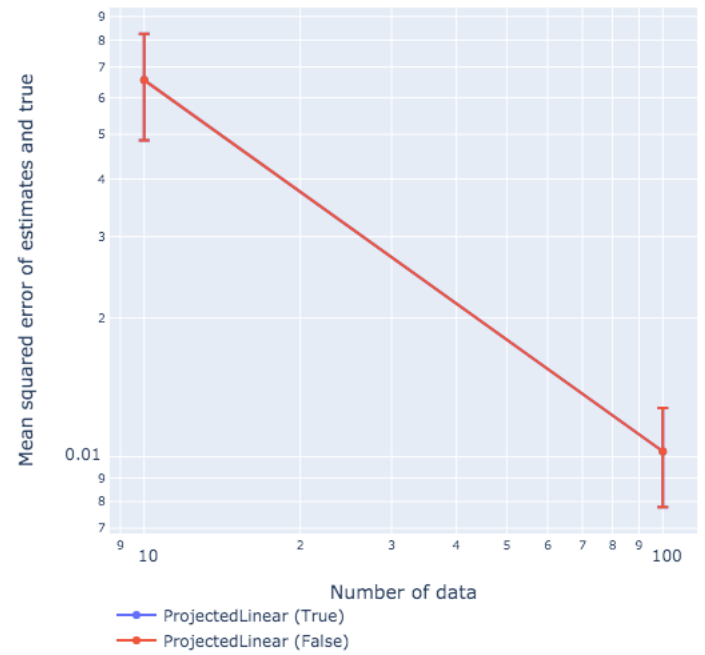


Comparison of parametrization

Mean squared error
Estimator=Linear



Mean squared error
Estimator=ProjectedLinear

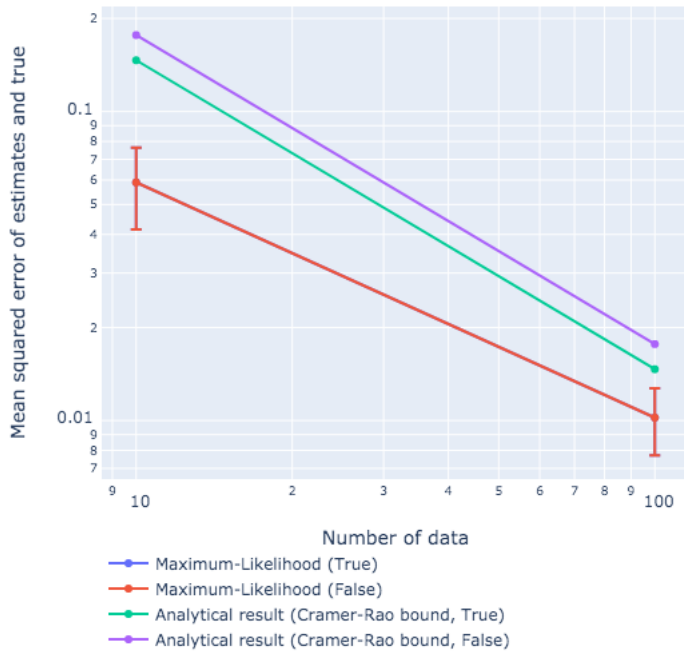


Mean squared error

Estimator=LossMinimization

Loss=WeightedRelativeEntropy

Algo=ProjectedGradientDescentBacktracking

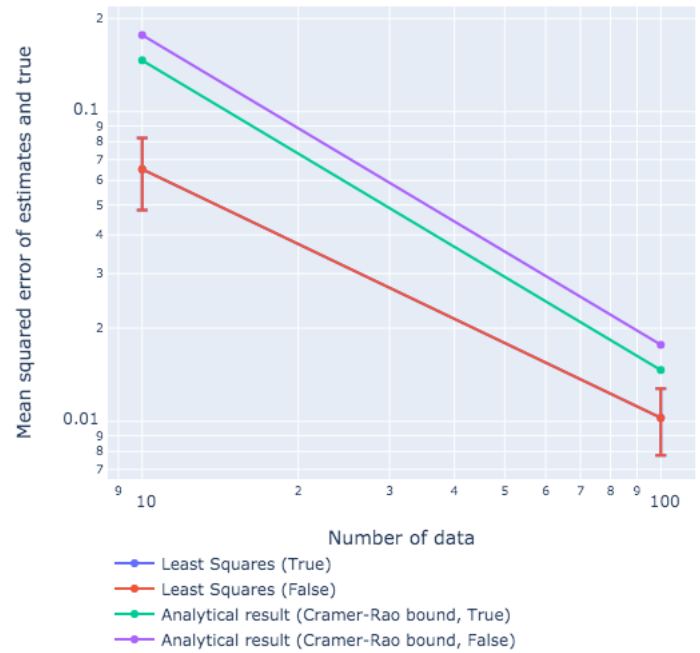


Mean squared error

Estimator=LossMinimization

Loss=WeightedProbabilityBasedSquaredError

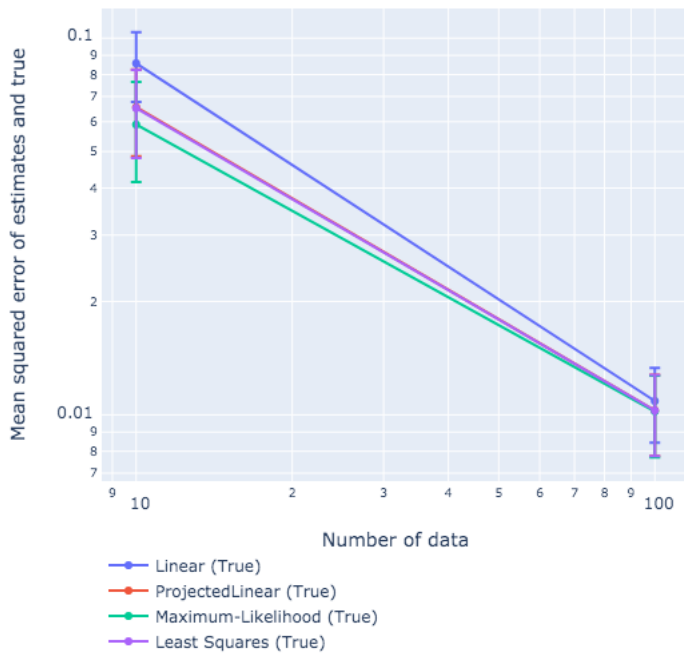
Algo=ProjectedGradientDescentBacktracking



Comparison of estimators

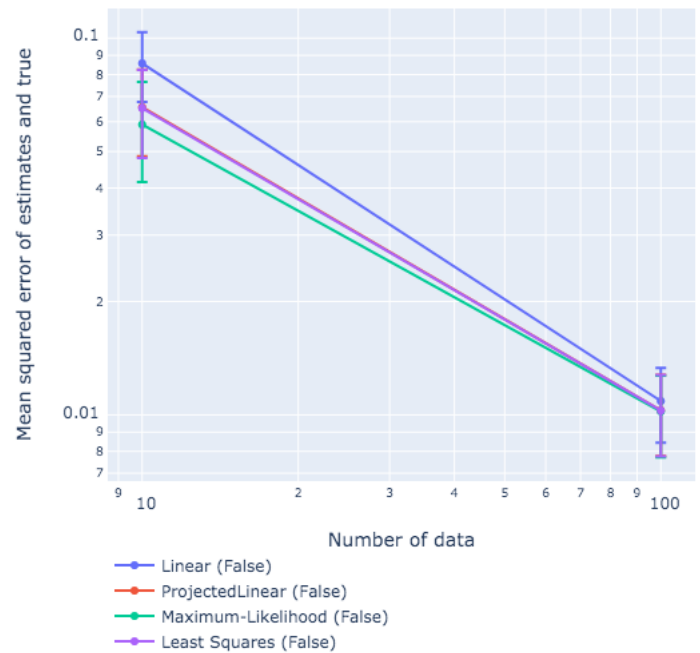
Mean squared error

parametrization=True



Mean squared error

parametrization=False

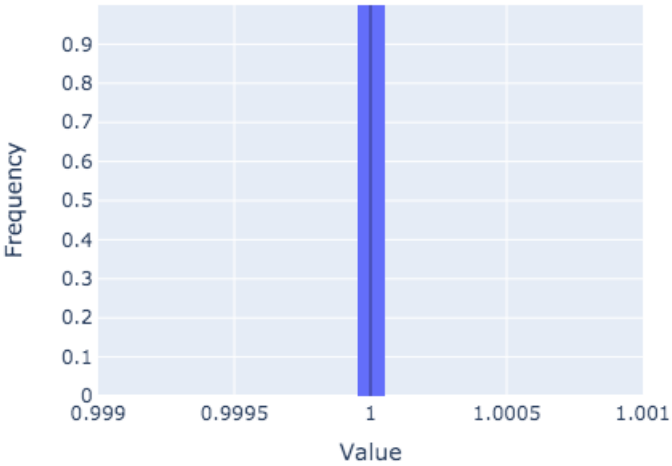


Physicality violation test

Test of equality constraint violation

Case 0: Linear (True)

N=10, Nrep=10



Min: 0.9999999999999997
Max: 0.9999999999999999
|Max-Min|: 2.220446049250313e-16

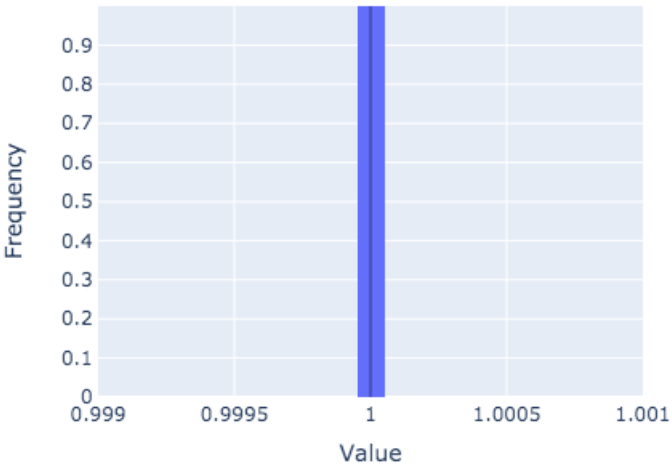
N=100, Nrep=10



Min: 0.9999999999999998
Max: 0.9999999999999998
|Max-Min|: 0.0

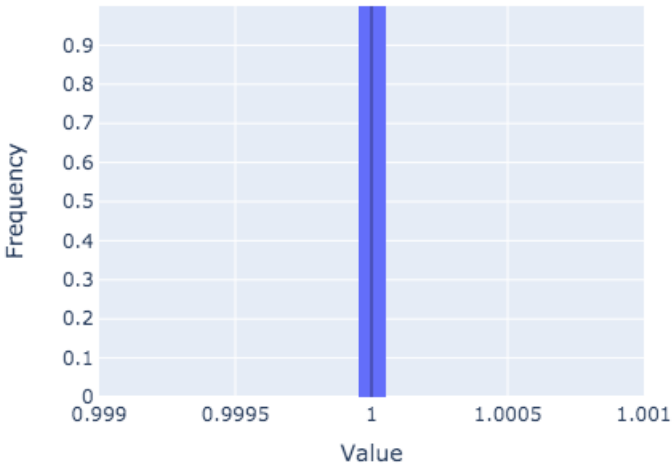
Case 1: Linear (False)

N=10, Nrep=10



Min: 1.0
Max: 1.0000000000000004
|Max-Min|: 4.440892098500626e-16

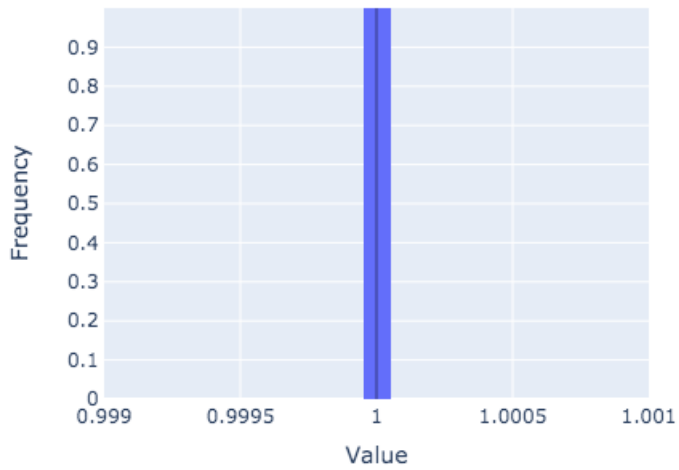
N=100, Nrep=10



Min: 1.0000000000000002
Max: 1.0000000000000004
|Max-Min|: 2.220446049250313e-16

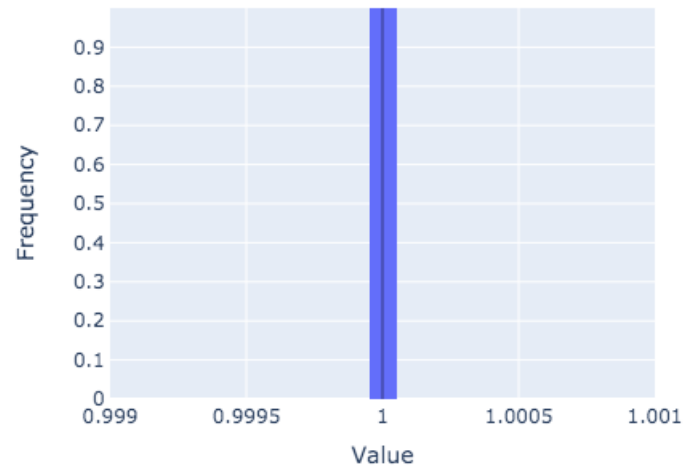
Case 2: ProjectedLinear (True)

N=10, Nrep=10



Min: 0.9999999999999998
Max: 0.9999999999999998
|Max-Min|: 0.0

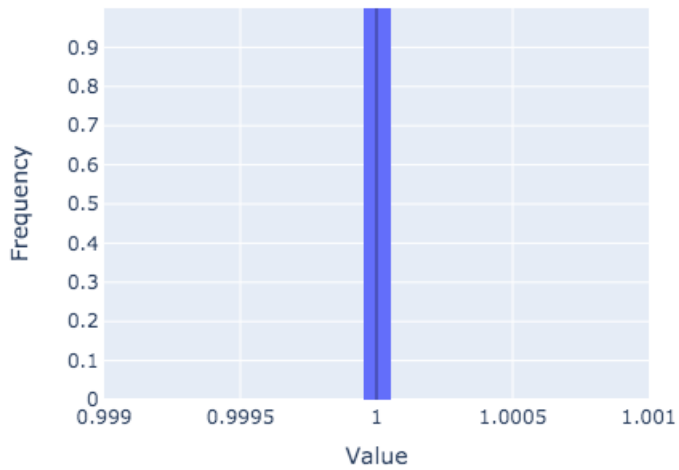
N=100, Nrep=10



Min: 0.9999999999999998
Max: 0.9999999999999998
|Max-Min|: 0.0

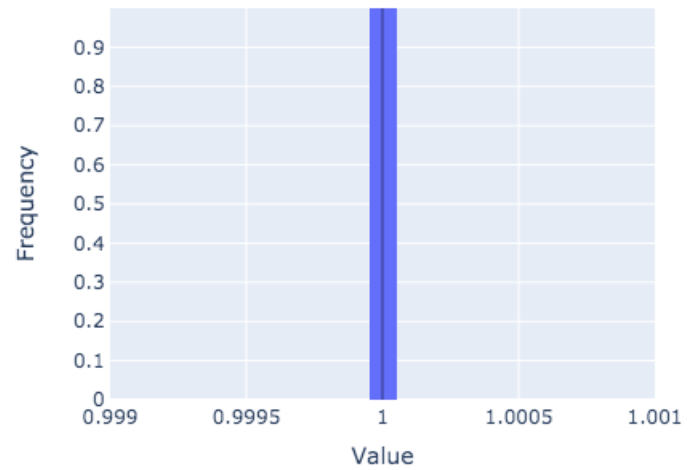
Case 3: ProjectedLinear (False)

N=10, Nrep=10



Min: 0.9999999999999998
Max: 1.0000001778133014
|Max-Min|: 1.7781330163835207e-07

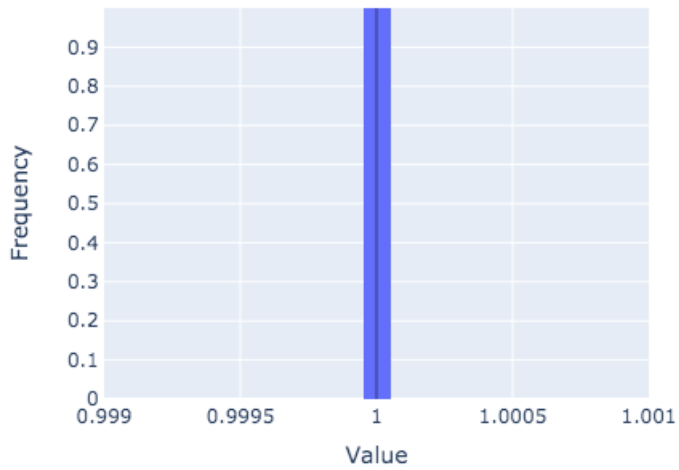
N=100, Nrep=10



Min: 0.9999999999999998
Max: 1.0000000927630224
|Max-Min|: 9.276302259664249e-08

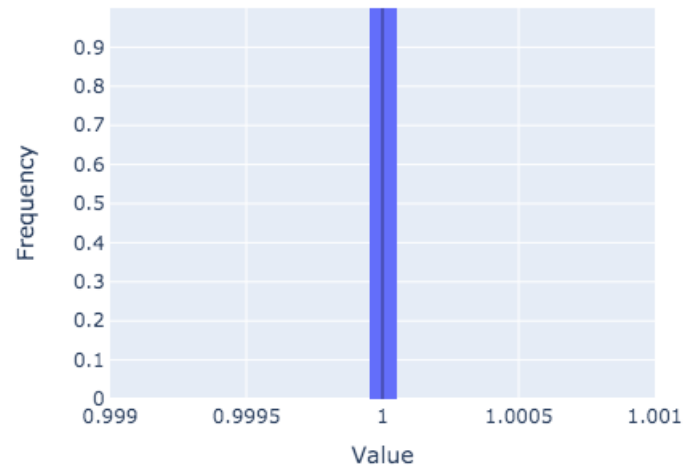
Case 4: Maximum-Likelihood (True)

N=10, Nrep=10



Min: 0.9999999999999998
Max: 0.9999999999999998
|Max-Min|: 0.0

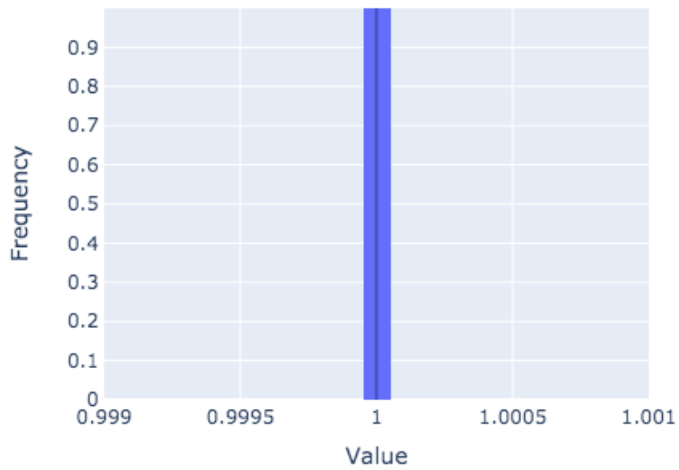
N=100, Nrep=10



Min: 0.9999999999999998
Max: 0.9999999999999998
|Max-Min|: 0.0

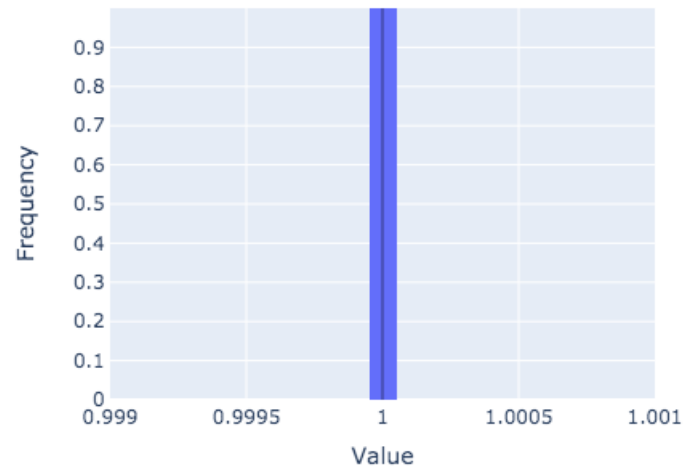
Case 5: Maximum-Likelihood (False)

N=10, Nrep=10



Min: 1.000000010541924
Max: 1.0000001624469406
|Max-Min|: 1.5190501656547895e-07

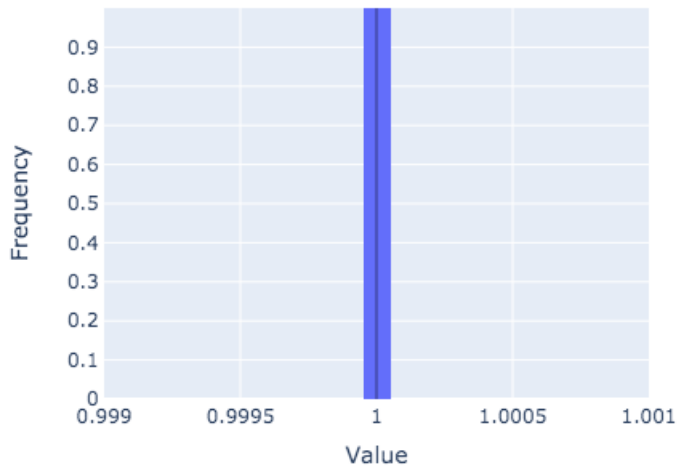
N=100, Nrep=10



Min: 1.000000008836664
Max: 1.0000001021955032
|Max-Min|: 9.335883910388532e-08

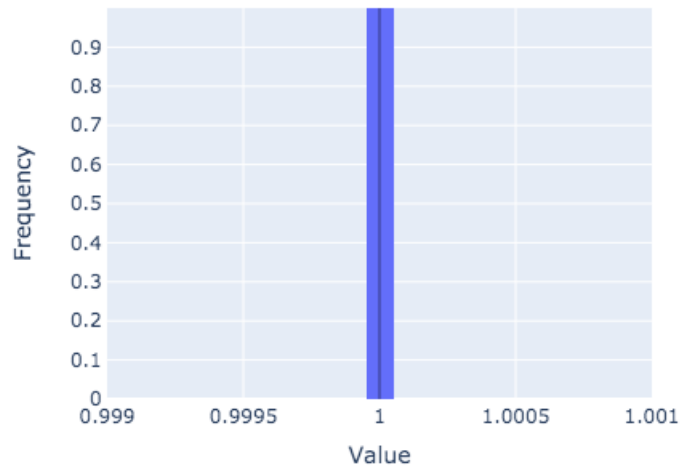
Case 6: Least Squares (True)

N=10, Nrep=10



Min: 0.9999999999999998
Max: 0.9999999999999998
|Max-Min|: 0.0

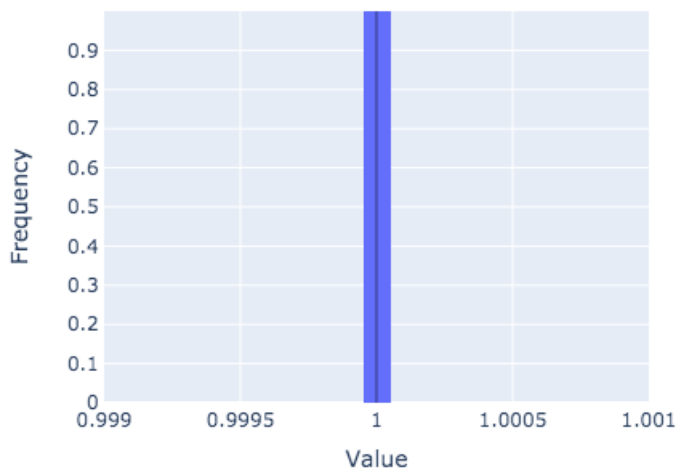
N=100, Nrep=10



Min: 0.9999999999999998
Max: 0.9999999999999998
|Max-Min|: 0.0

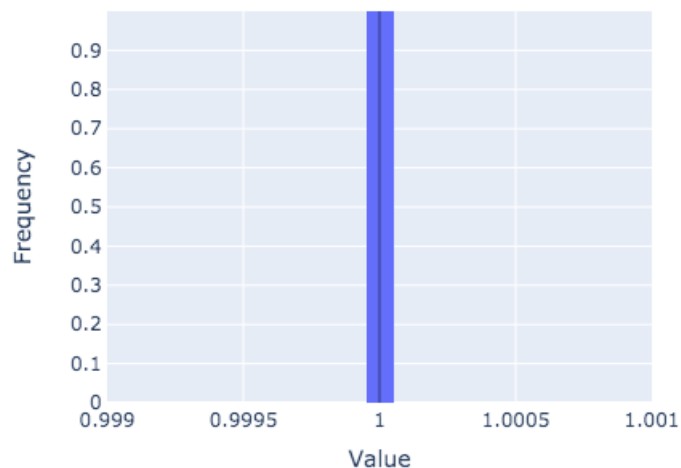
Case 7: Least Squares (False)

N=10, Nrep=10



Min: 0.9999999999999998
Max: 1.000000149383655
|Max-Min|: 1.4938365522354502e-07

N=100, Nrep=10



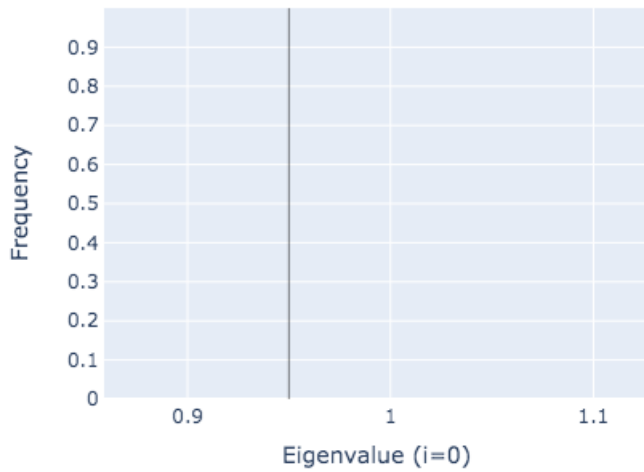
Min: 0.9999999999999998
Max: 1.0000000989943127
|Max-Min|: 9.899431296922501e-08

Test of inequality constraint violation

Eigenvalue

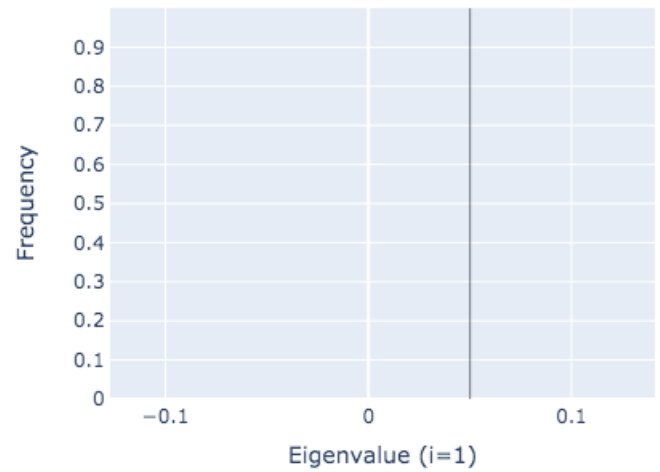
Case 0: Linear (True)

N=10, Nrep=10



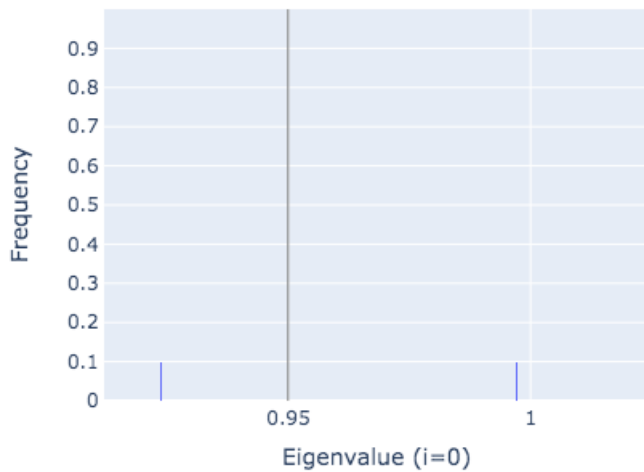
Min: 0.8589495035535653
Max: 1.1272800413447277
|Max-Min|: 0.26833053779116245

N=10, Nrep=10



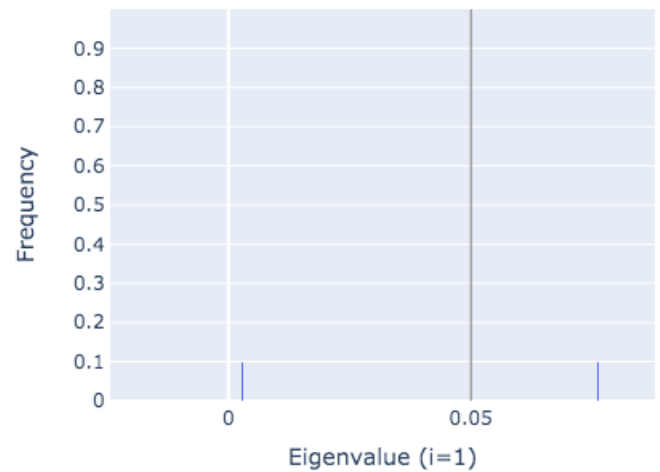
Min: -0.12728004134472798
Max: 0.14105049644643455
|Max-Min|: 0.26833053779116256

N=100, Nrep=10



Min: 0.9120872001442579
Max: 1.024317269762404
|Max-Min|: 0.11223006961814597

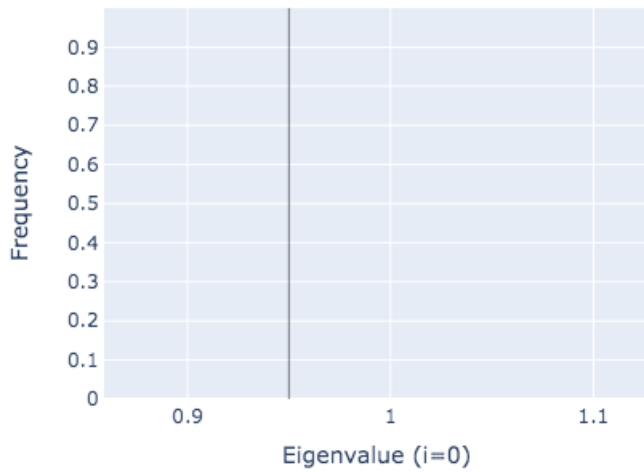
N=100, Nrep=10



Min: -0.024317269762403972
Max: 0.08791279985574182
|Max-Min|: 0.1122300696181458

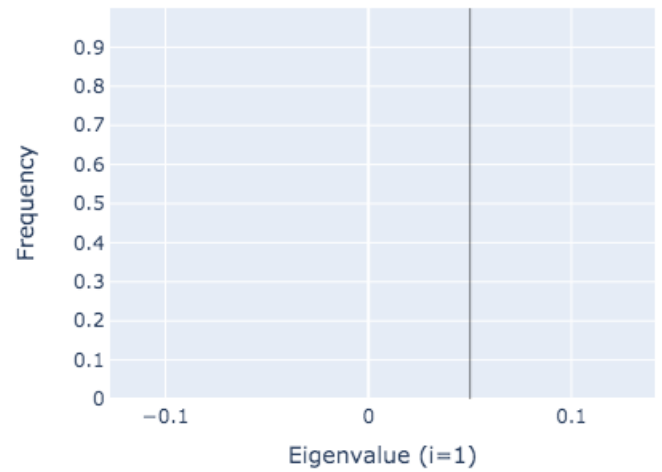
Case 1: Linear (False)

N=10, Nrep=10



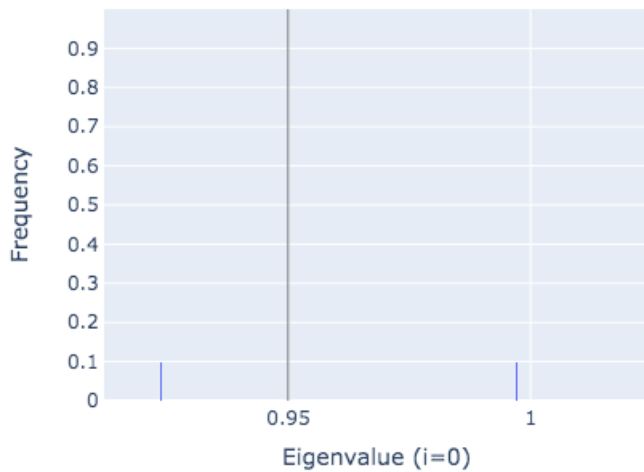
Min: 0.8589495035535657
Max: 1.127280041344728
|Max-Min|: 0.2683305377911622

N=10, Nrep=10



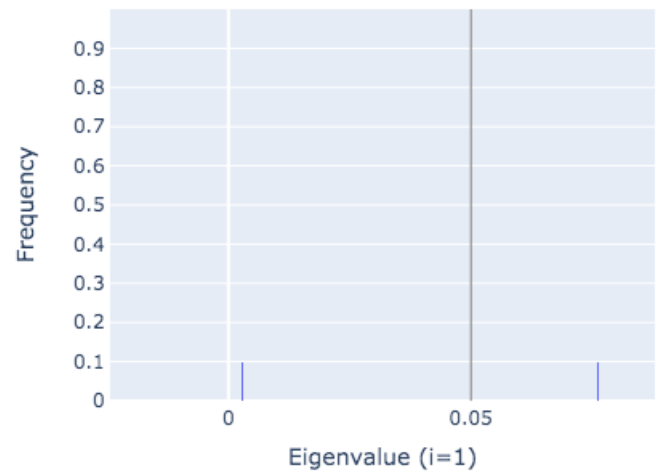
Min: -0.12728004134472776
Max: 0.14105049644643464
|Max-Min|: 0.2683305377911624

N=100, Nrep=10



Min: 0.9120872001442583
Max: 1.0243172697624043
|Max-Min|: 0.11223006961814608

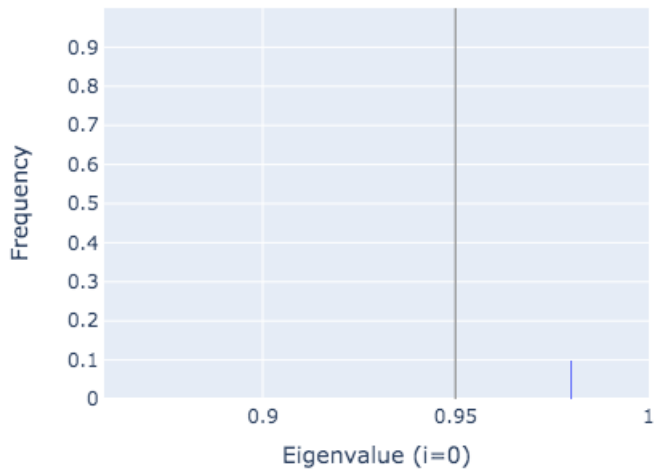
N=100, Nrep=10



Min: -0.024317269762403865
Max: 0.08791279985574188
|Max-Min|: 0.11223006961814574

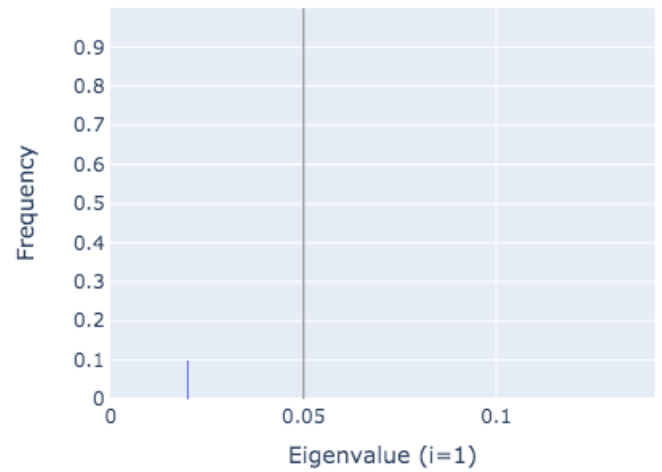
Case 2: ProjectedLinear (True)

N=10, Nrep=10



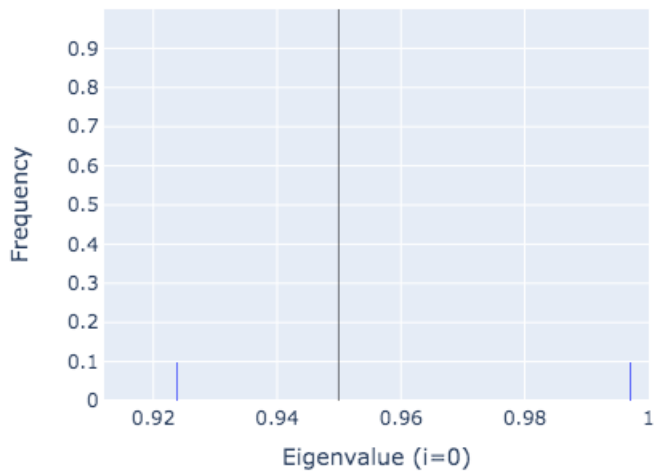
Min: 0.858949503553565
Max: 1.0000000889066505
|Max-Min|: 0.14105058535308546

N=10, Nrep=10



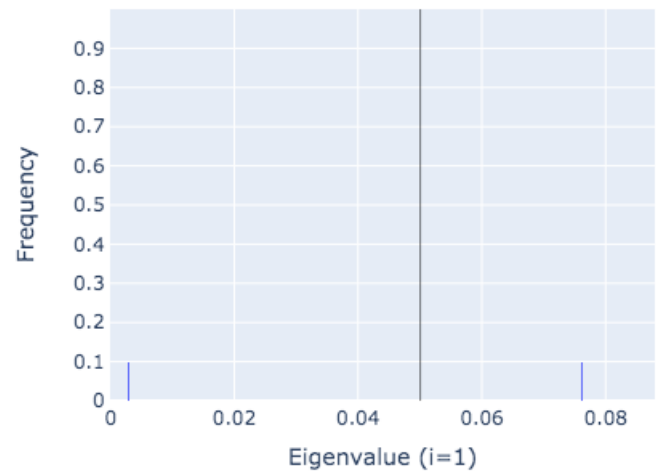
Min: -8.890665082958438e-08
Max: 0.14105049644643472
|Max-Min|: 0.14105058535308554

N=100, Nrep=10



Min: 0.9120872001442577
Max: 1.000000046381511
|Max-Min|: 0.08791284623725337

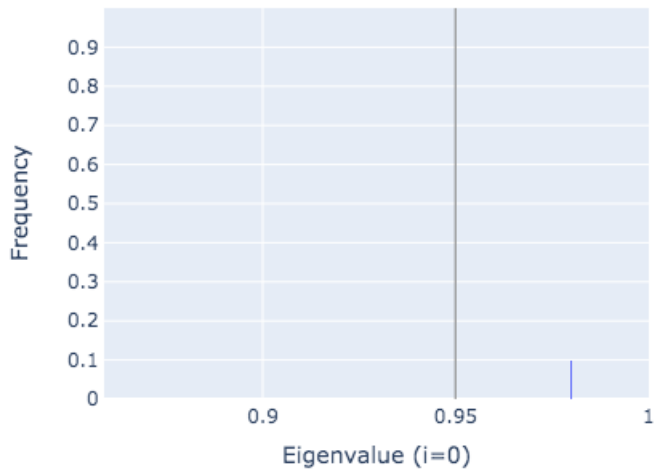
N=100, Nrep=10



Min: -4.638151134168933e-08
Max: 0.08791279985574205
|Max-Min|: 0.08791284623725339

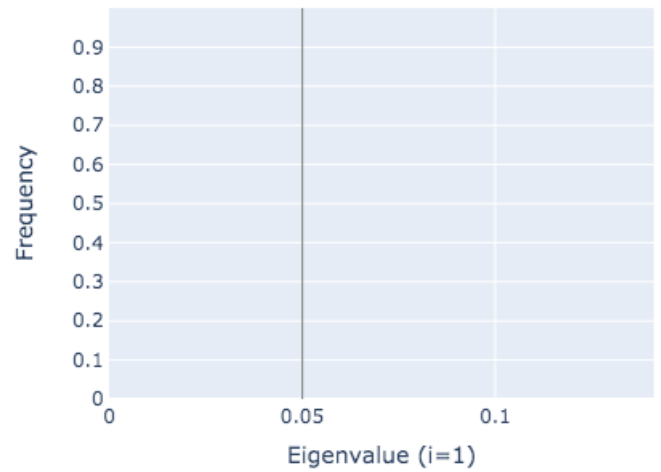
Case 3: ProjectedLinear (False)

N=10, Nrep=10



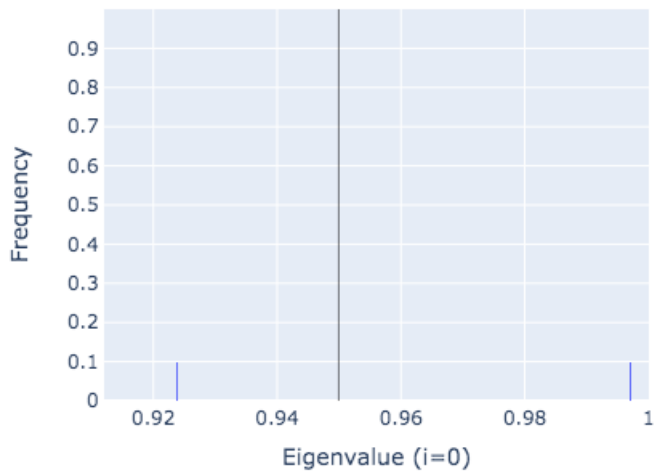
Min: 0.8589495035535654
 Max: 1.0000001778133014
 |Max-Min|: 0.14105067425973605

N=10, Nrep=10



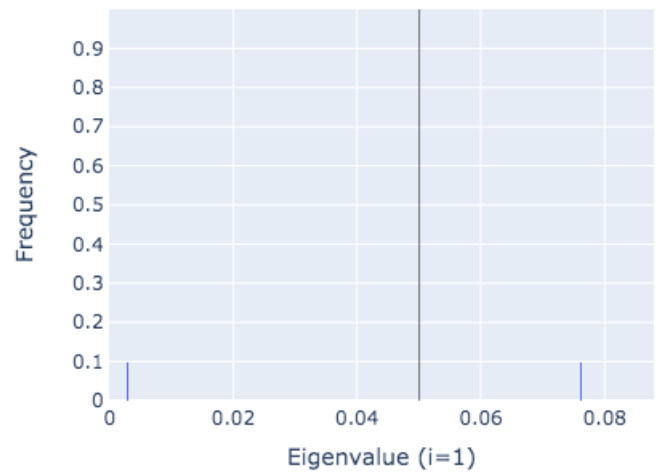
Min: -5.4643789493269423e-17
 Max: 0.14105049644643436
 |Max-Min|: 0.14105049644643441

N=100, Nrep=10



Min: 0.9120872001442579
 Max: 1.0000000927630224
 |Max-Min|: 0.08791289261876445

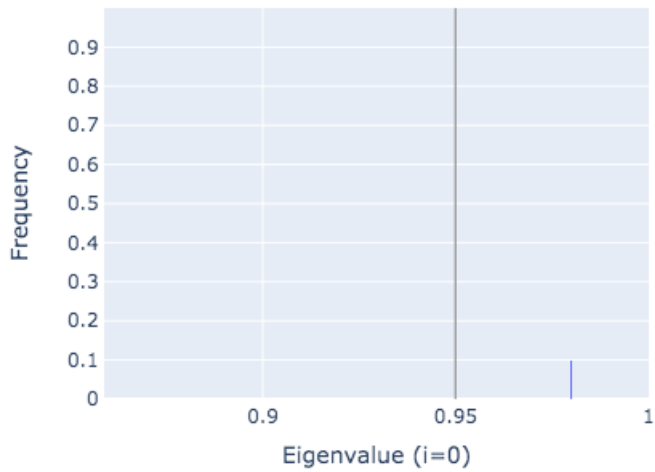
N=100, Nrep=10



Min: -4.7704895589362195e-17
 Max: 0.08791279985574182
 |Max-Min|: 0.08791279985574187

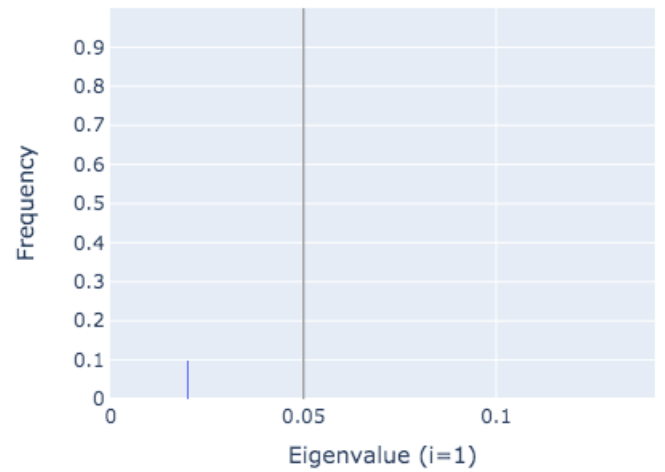
Case 4: Maximum-Likelihood (True)

N=10, Nrep=10



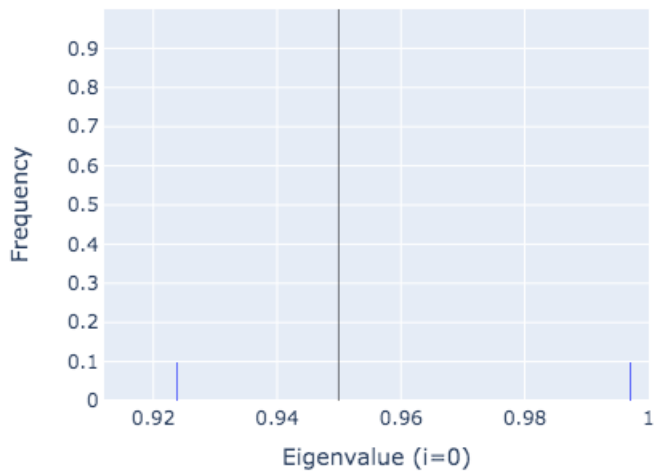
Min: 0.8589495013167568
 Max: 1.0000000703415943
 |Max-Min|: 0.14105056902483748

N=10, Nrep=10



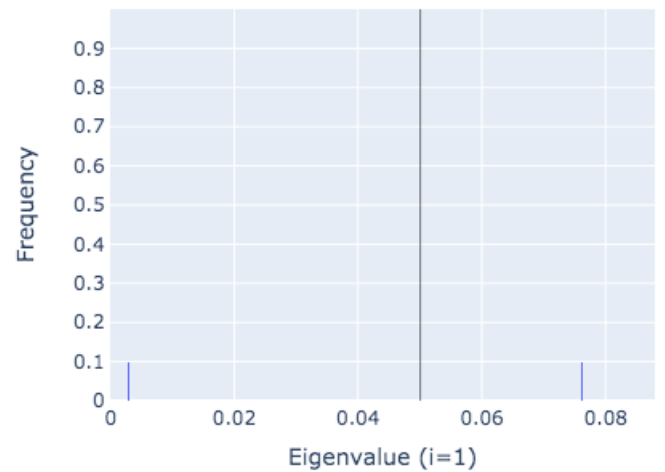
Min: -7.034159459767819e-08
 Max: 0.141050498683243
 |Max-Min|: 0.1410505690248376

N=100, Nrep=10



Min: 0.9120872001948318
 Max: 1.0000000885036828
 |Max-Min|: 0.08791288830885102

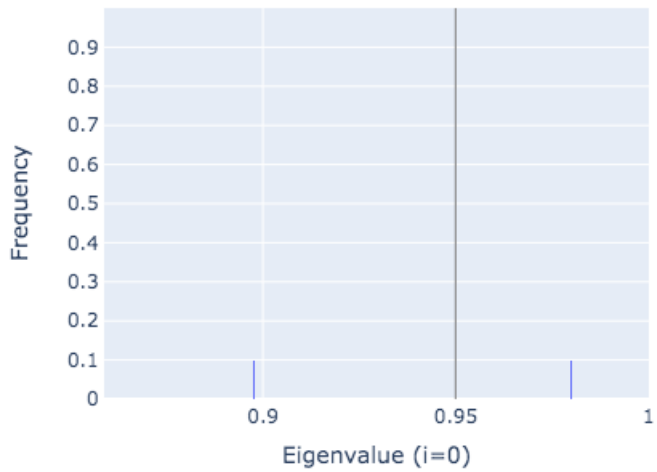
N=100, Nrep=10



Min: -8.850368309748502e-08
 Max: 0.08791279980516796
 |Max-Min|: 0.08791288830885105

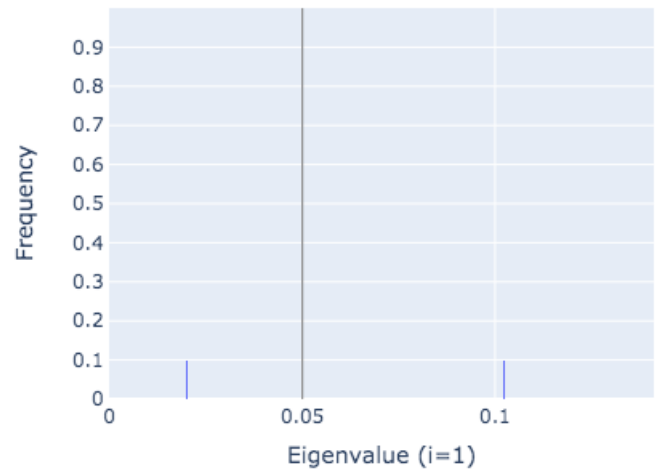
Case 5: Maximum-Likelihood (False)

N=10, Nrep=10



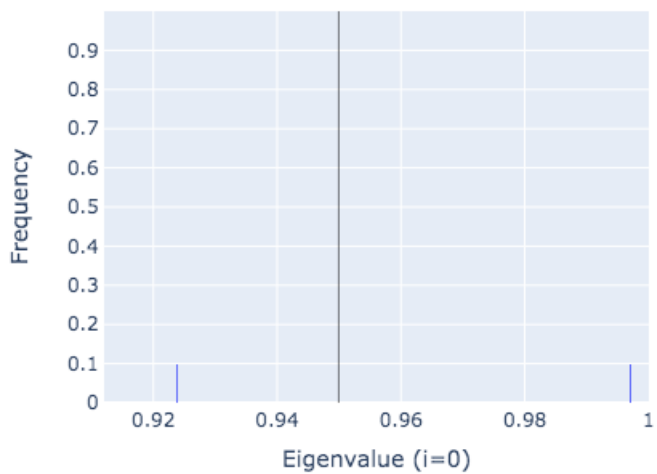
Min: 0.8589249966926165
 Max: 1.0000001624469381
 |Max-Min|: 0.14107516575432166

N=10, Nrep=10



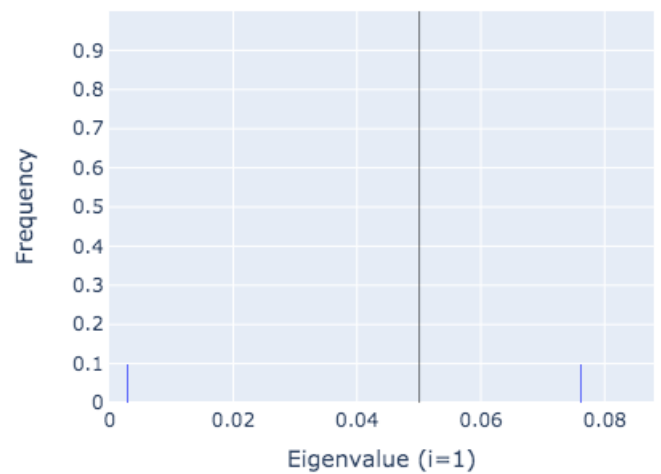
Min: -1.1611805267319752e-16
 Max: 0.14107502853532802
 |Max-Min|: 0.14107502853532813

N=100, Nrep=10



Min: 0.9120699929271241
 Max: 1.0000001021955032
 |Max-Min|: 0.08793010926837908

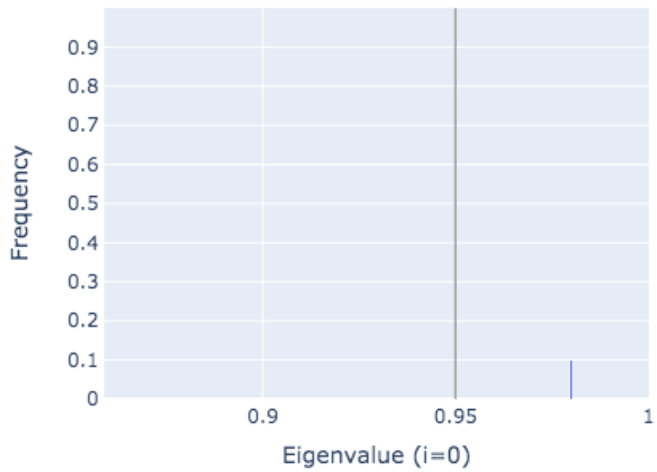
N=100, Nrep=10



Min: 1.5395670849294163e-16
 Max: 0.0879300206789945
 |Max-Min|: 0.08793002067899434

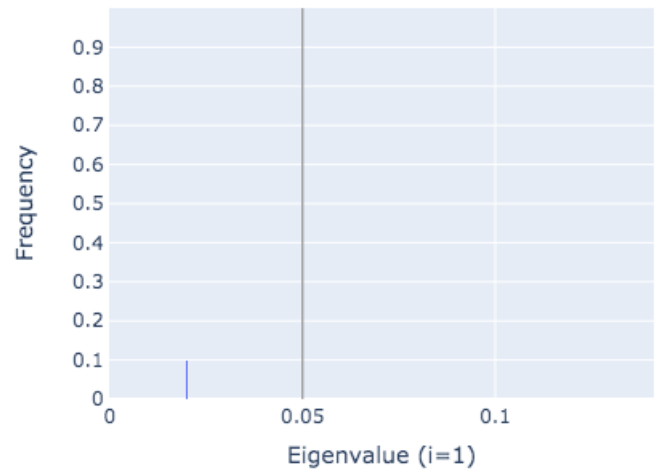
Case 6: Least Squares (True)

N=10, Nrep=10



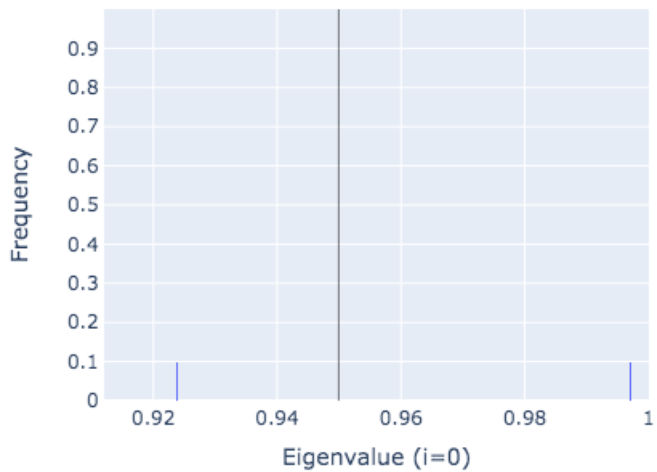
Min: 0.8589495035535648
 Max: 1.000000083720352
 |Max-Min|: 0.14105058016678718

N=10, Nrep=10



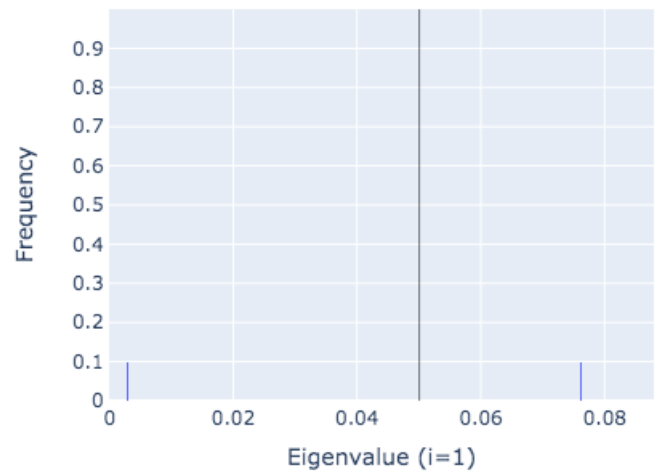
Min: -8.372035236597908e-08
 Max: 0.14105049644643503
 |Max-Min|: 0.1410505801667874

N=100, Nrep=10



Min: 0.9120872001442585
 Max: 1.0000000857315419
 |Max-Min|: 0.08791288558728338

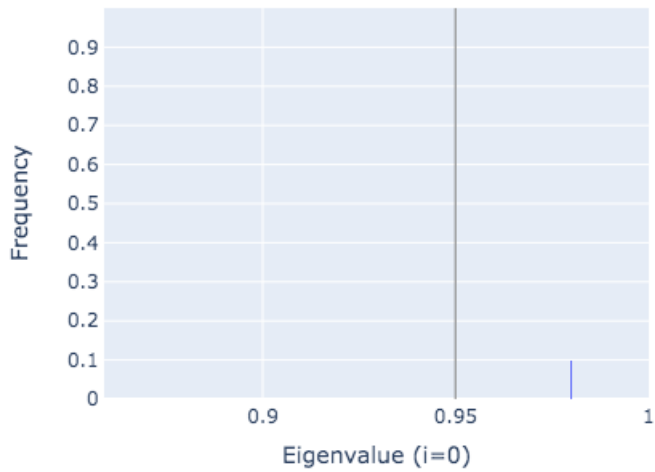
N=100, Nrep=10



Min: -8.573154199921496e-08
 Max: 0.08791279985574135
 |Max-Min|: 0.08791288558728336

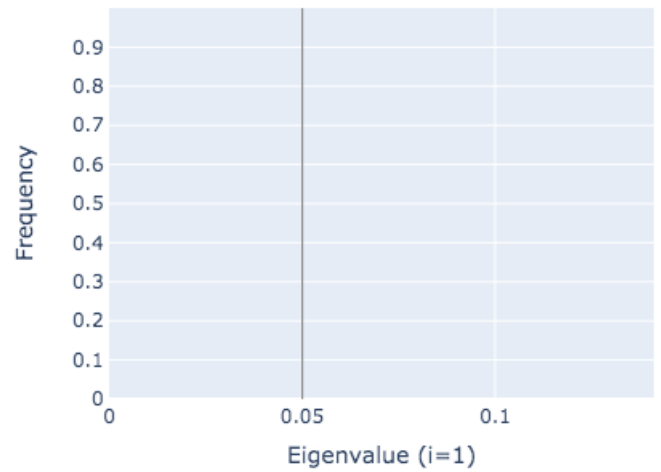
Case 7: Least Squares (False)

N=10, Nrep=10



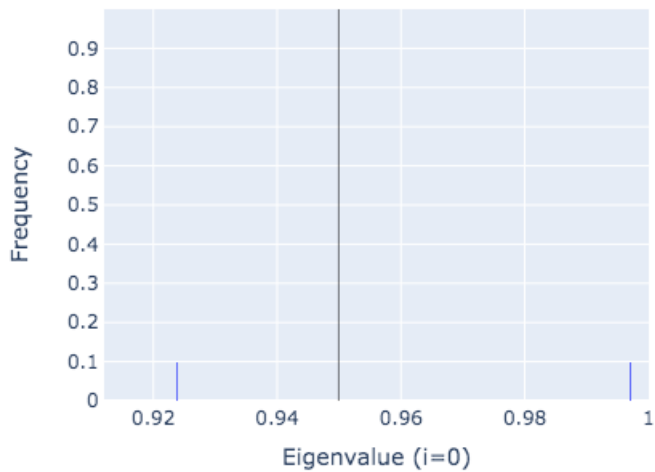
Min: 0.8589495035535653
 Max: 1.000000149383655
 |Max-Min|: 0.14105064583008975

N=10, Nrep=10



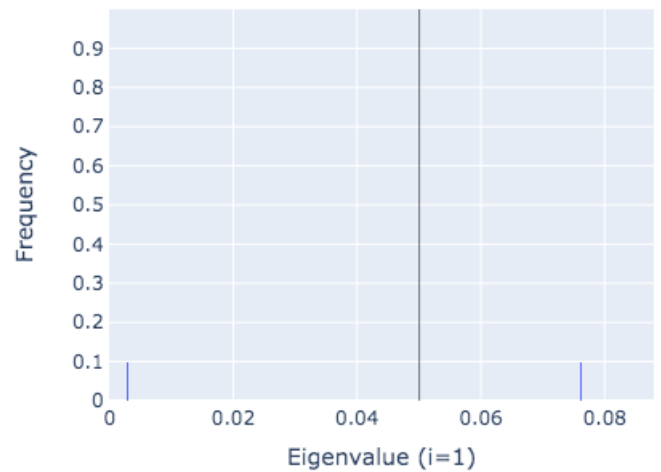
Min: -1.3183898417423734e-16
 Max: 0.1410504964464345
 |Max-Min|: 0.14105049644643464

N=100, Nrep=10



Min: 0.9120872001442578
 Max: 1.0000000989943127
 |Max-Min|: 0.08791289885005493

N=100, Nrep=10



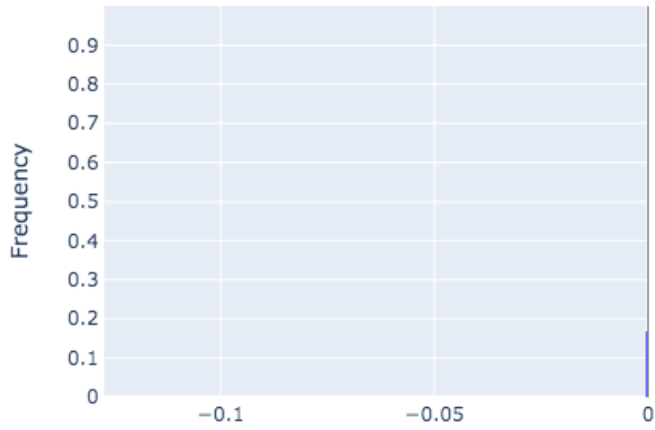
Min: 1.474514954580286e-17
 Max: 0.0879127998557419
 |Max-Min|: 0.08791279985574188

Sum of unphysical eigenvalues

Case 0: Linear (True)

N=10
 Number of unphysical estimates=6

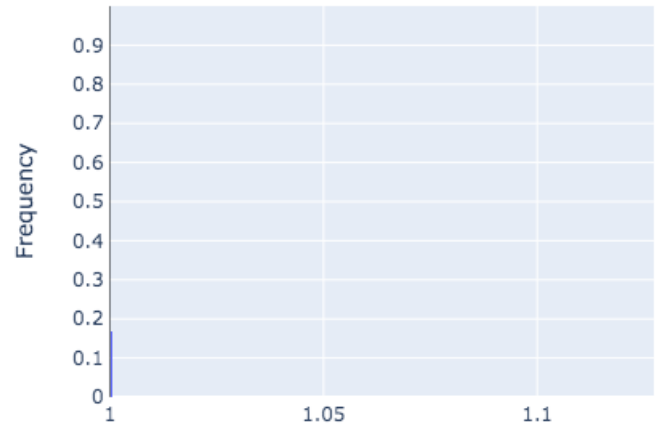
N=10, Nrep=10



Sum of negative eigenvalues (<0)

Min: -0.12728004134472798
Max: -0.0003526250257380667
|Max-Min|: 0.1269274163189899

N=10, Nrep=10

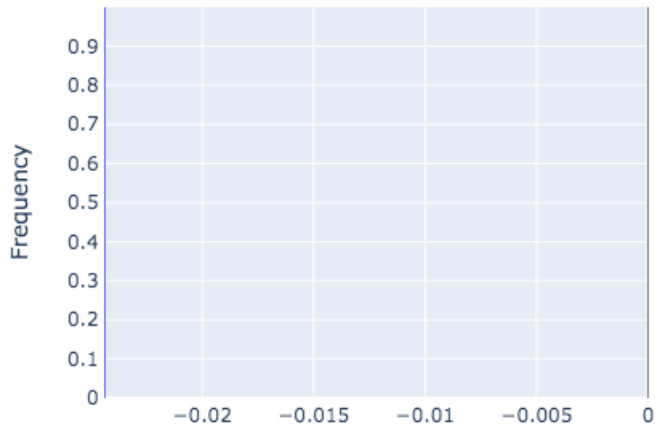


Sum of non-negative eigenvalues (>1)

Min: 1.0003526250257377
Max: 1.1272800413447277
|Max-Min|: 0.1269274163189896

N=100
Number of unphysical estimates=1

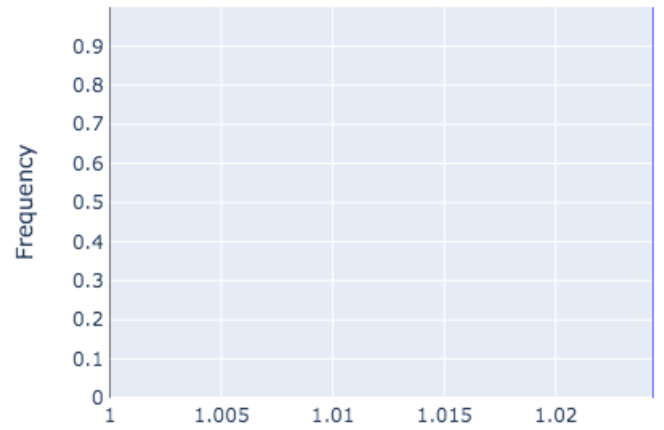
N=100, Nrep=10



Sum of negative eigenvalues (<0)

Min: -0.024317269762403972
Max: -0.024317269762403972
|Max-Min|: 0.0

N=100, Nrep=10



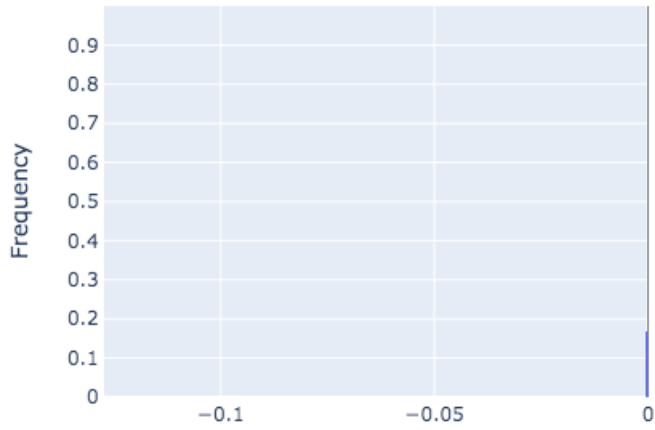
Sum of non-negative eigenvalues (>1)

Min: 1.024317269762404
Max: 1.024317269762404
|Max-Min|: 0.0

Case 1: Linear (False)

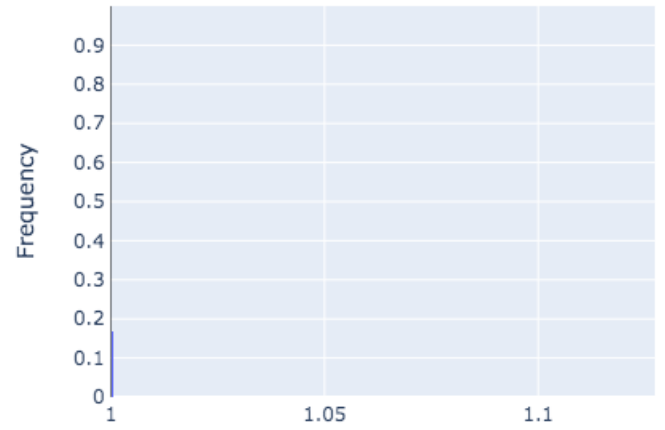
N=10
Number of unphysical estimates=6

N=10, Nrep=10



Min: -0.12728004134472776
Max: -0.00035262502573775445
|Max-Min|: 0.12692741631899002

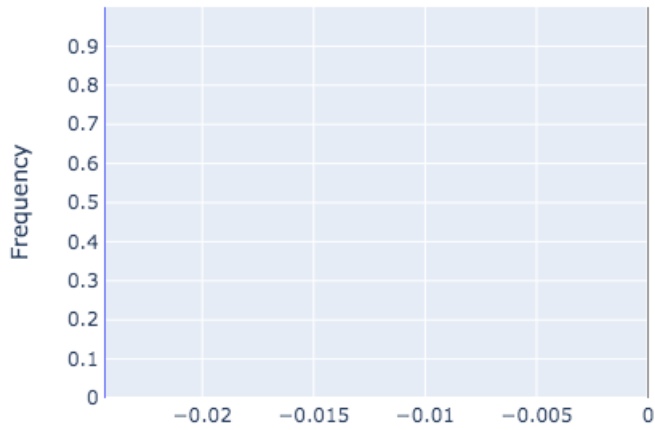
N=10, Nrep=10



Min: 1.000352625025738
Max: 1.127280041344728
|Max-Min|: 0.12692741631898996

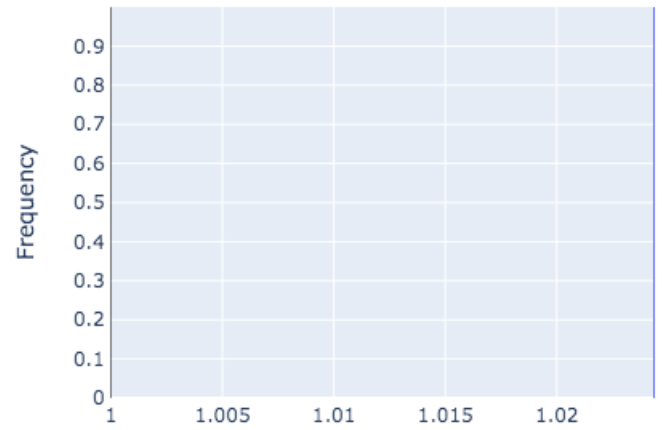
N=100
Number of unphysical estimates=1

N=100, Nrep=10



Min: -0.024317269762403865
Max: -0.024317269762403865
|Max-Min|: 0.0

N=100, Nrep=10

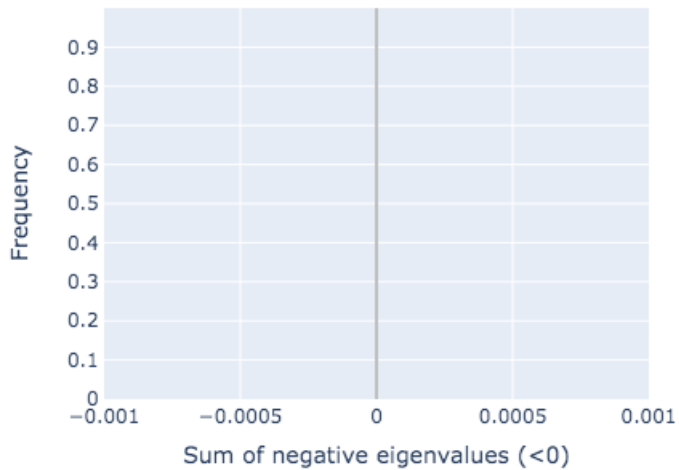


Min: 1.0243172697624043
Max: 1.0243172697624043
|Max-Min|: 0.0

Case 2: ProjectedLinear (True)

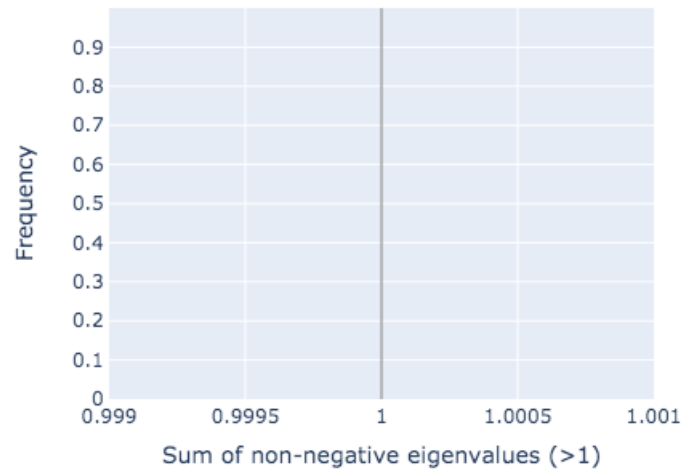
N=10
Number of unphysical estimates=0

N=10, Nrep=10



Min: -
Max: -
|Max-Min|: -

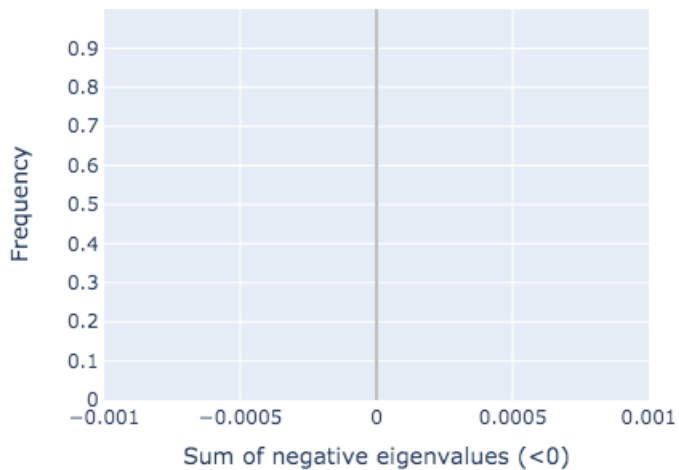
N=10, Nrep=10



Min: -
Max: -
|Max-Min|: -

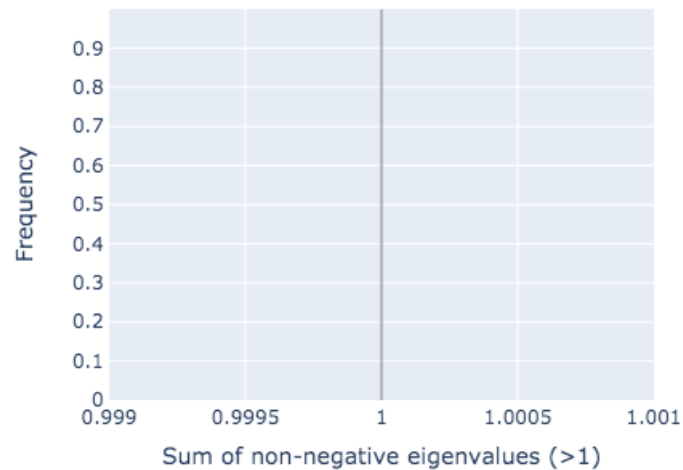
N=100
Number of unphysical estimates=0

N=100, Nrep=10



Min: -
Max: -
|Max-Min|: -

N=100, Nrep=10

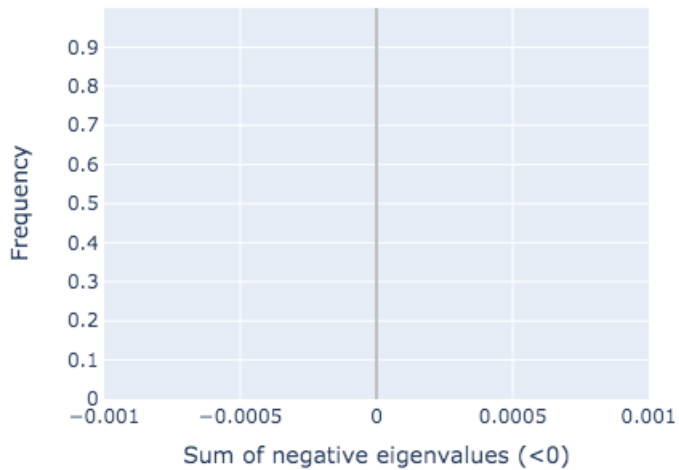


Min: -
Max: -
|Max-Min|: -

Case 3: ProjectedLinear (False)

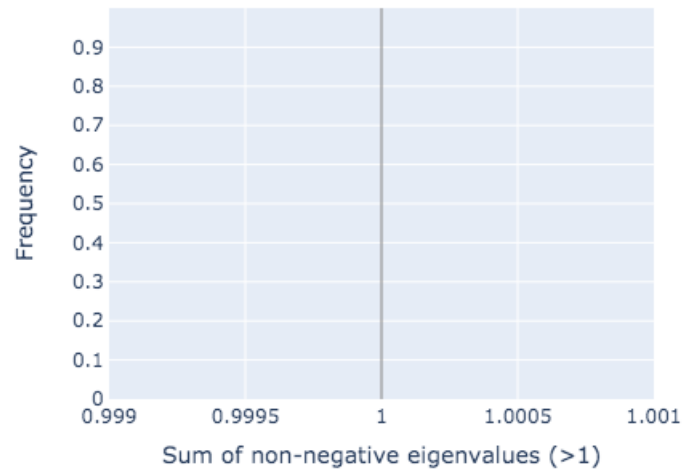
N=10
Number of unphysical estimates=0

N=10, Nrep=10



Min: -
Max: -
|Max-Min|: -

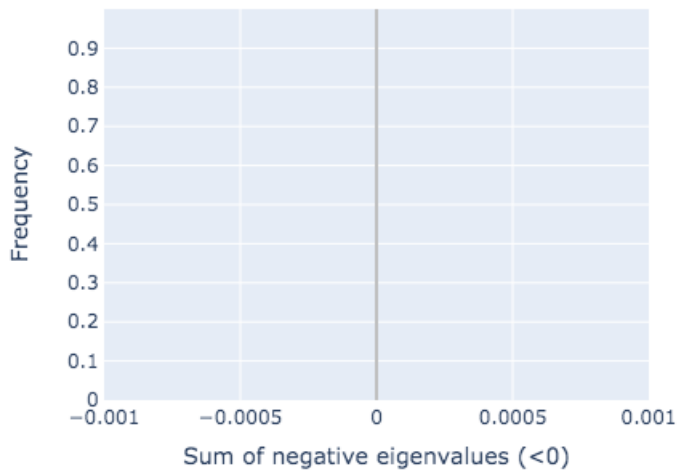
N=10, Nrep=10



Min: -
Max: -
|Max-Min|: -

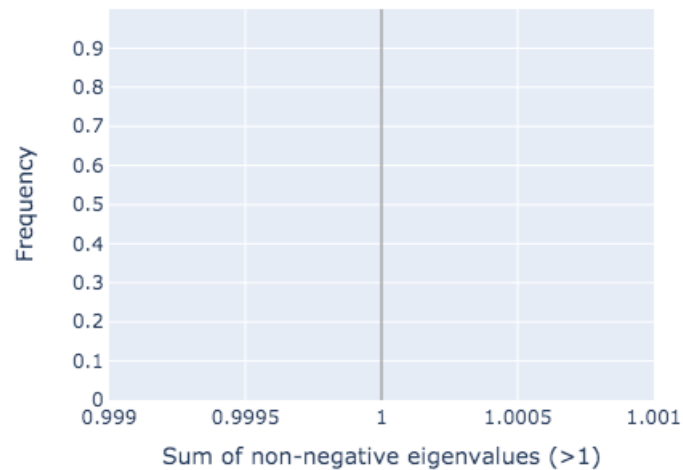
N=100
Number of unphysical estimates=0

N=100, Nrep=10



Min: -
Max: -
|Max-Min|: -

N=100, Nrep=10

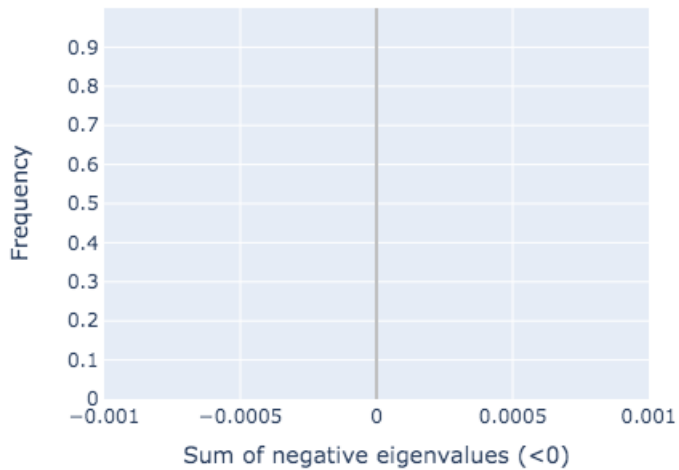


Min: -
Max: -
|Max-Min|: -

Case 4: Maximum-Likelihood (True)

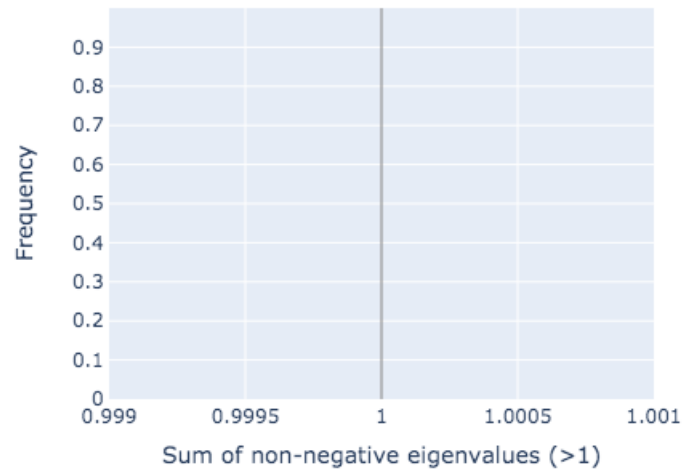
N=10
Number of unphysical estimates=0

N=10, Nrep=10



Min: -
Max: -
|Max-Min|: -

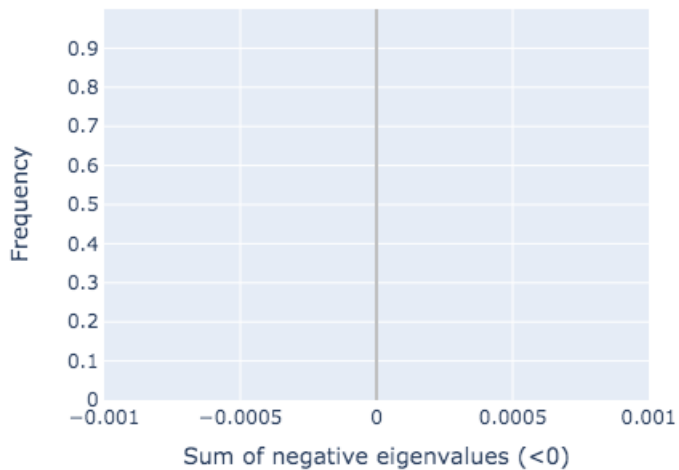
N=10, Nrep=10



Min: -
Max: -
|Max-Min|: -

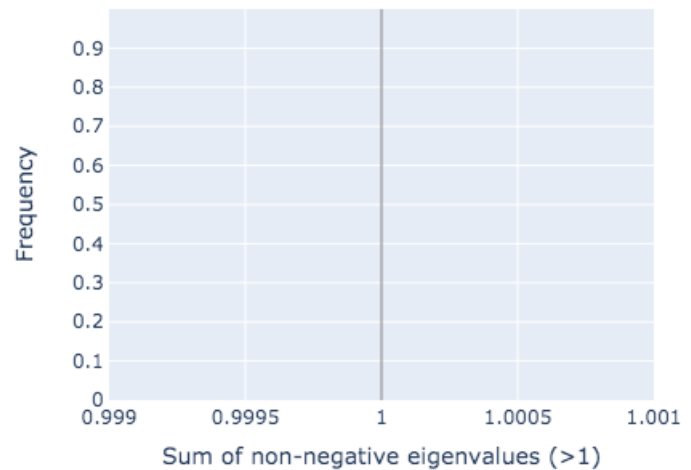
N=100
Number of unphysical estimates=0

N=100, Nrep=10



Min: -
Max: -
|Max-Min|: -

N=100, Nrep=10

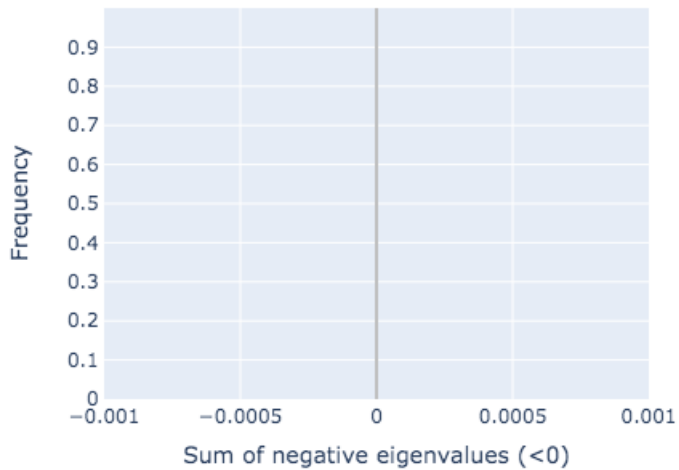


Min: -
Max: -
|Max-Min|: -

Case 5: Maximum-Likelihood (False)

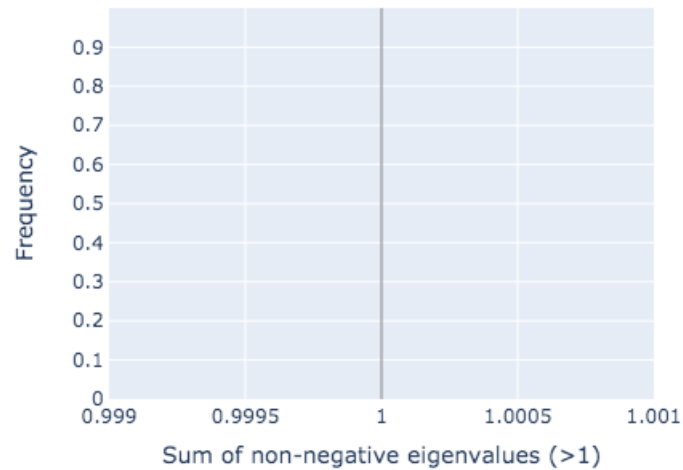
N=10
Number of unphysical estimates=0

N=10, Nrep=10



Min: -
Max: -
|Max-Min|: -

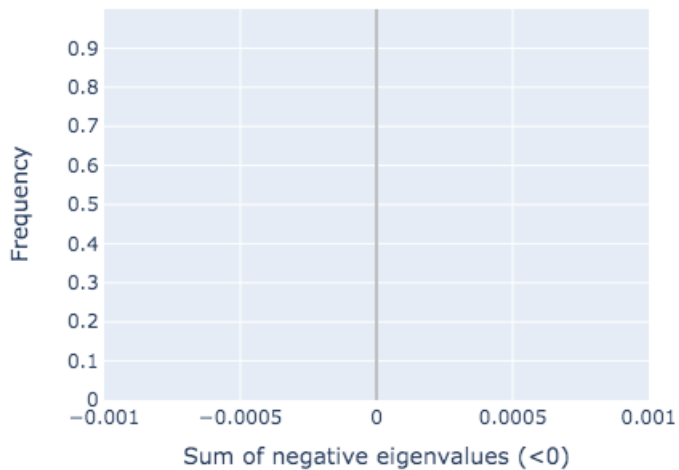
N=10, Nrep=10



Min: -
Max: -
|Max-Min|: -

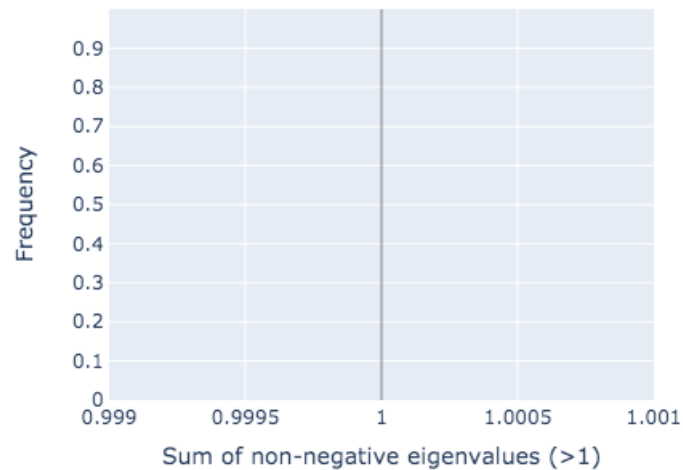
N=100
Number of unphysical estimates=0

N=100, Nrep=10



Min: -
Max: -
|Max-Min|: -

N=100, Nrep=10

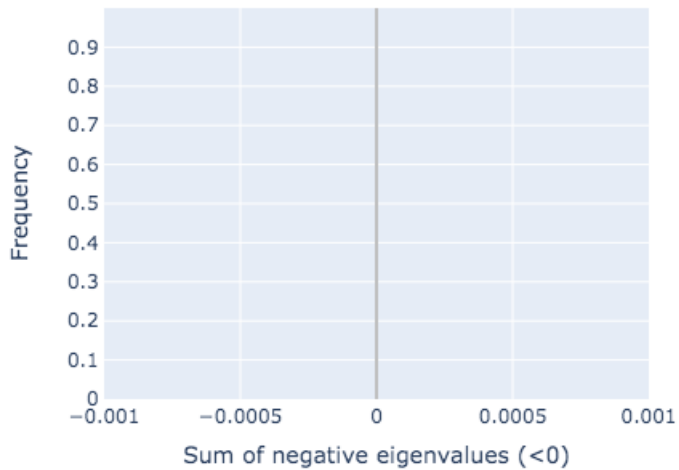


Min: -
Max: -
|Max-Min|: -

Case 6: Least Squares (True)

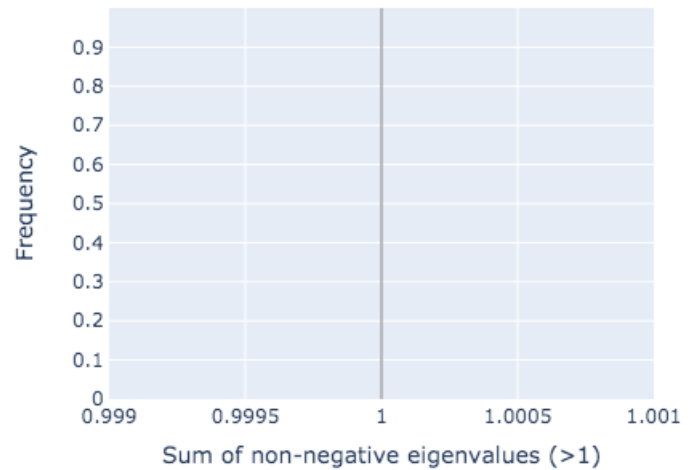
N=10
Number of unphysical estimates=0

N=10, Nrep=10



Min: -
Max: -
|Max-Min|: -

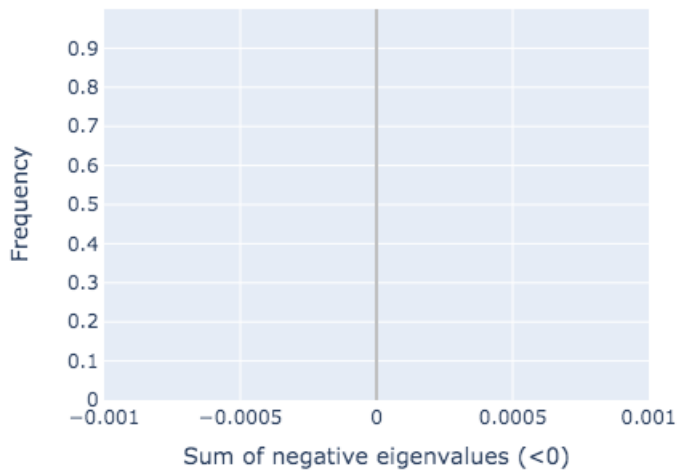
N=10, Nrep=10



Min: -
Max: -
|Max-Min|: -

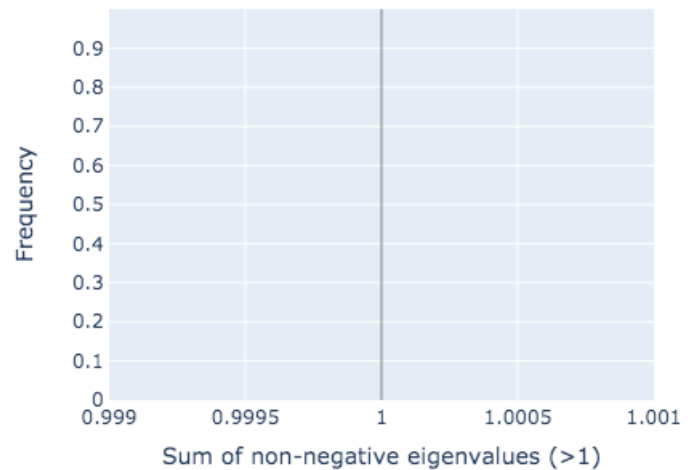
N=100
Number of unphysical estimates=0

N=100, Nrep=10



Min: -
Max: -
|Max-Min|: -

N=100, Nrep=10

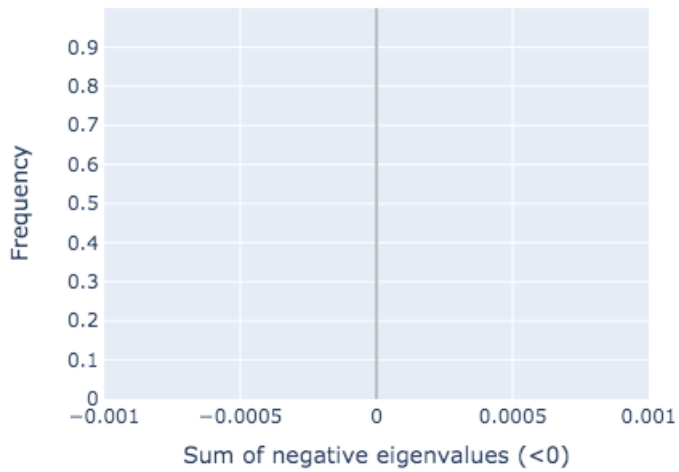


Min: -
Max: -
|Max-Min|: -

Case 7: Least Squares (False)

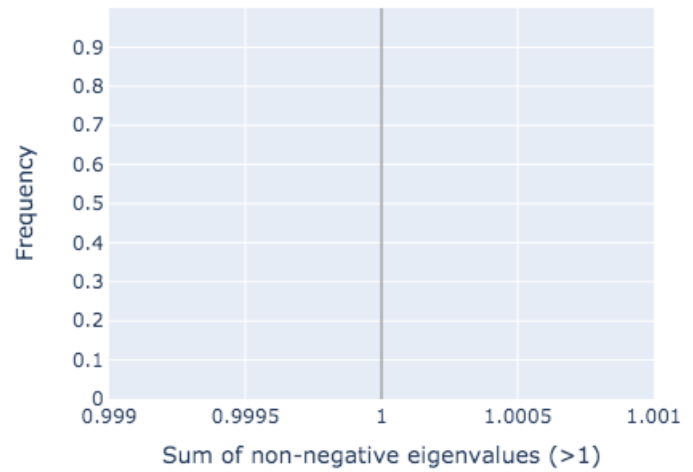
N=10
Number of unphysical estimates=0

N=10, Nrep=10



Min: -
Max: -
|Max-Min|: -

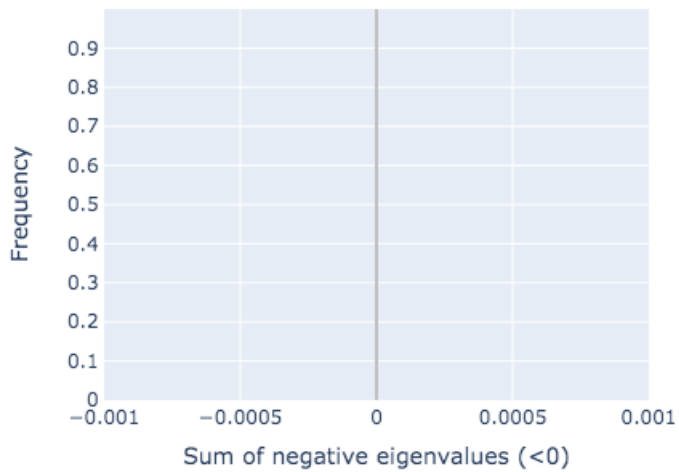
N=10, Nrep=10



Min: -
Max: -
|Max-Min|: -

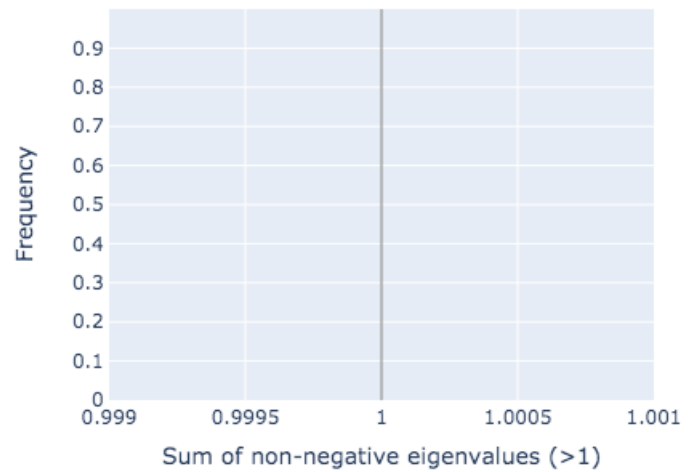
N=100
Number of unphysical estimates=0

N=100, Nrep=10



Min: -
Max: -
|Max-Min|: -

N=100, Nrep=10



Min: -
Max: -
|Max-Min|: -