

Table of contents

Table of contents	1
Computation time in total	1
Tolerance of physicality constraint violation	1
Experimental condition	1
True object	2
Tester objects	2
Cases	2
Computation time of estimators	3
Table	3
Histogram	3
MSE of empirical distributions	7
Consistency test	8
MSE of estimators	9
Comparison of analytical results	9
Comparison of parametrization	10
Comparison of estimators	11
Physicality violation test	12
Test of equality constraint violation	12
Test of inequality constraint violation	15
Eigenvalue	15
Sum of unphysical eigenvalues	23

Computation time in total

Total	0.7788017392158508min.
-------	------------------------

Tolerance of physicality constraint violation

		True	False
Tolerance at physicality violation test	equality constraint	1.00e-13	1.00e-05
	inequality constraint	1.00e-05	1.00e-05

Experimental condition

Type of tomography	StandardQst
Nrep	10
N	[10, 100]
RNG seed	777

True object

Type	State
Dim	2
Vec	[0.70710678 0. 0. 0.6363961]

Tester objects

0	Type	Povm
	Dim	2
	Number of outcomes	2
	Vecs	[[0.70710678 0.6363961 0. 0.] [0.70710678 -0.6363961 0. 0.]]
1	Type	Povm
	Dim	2
	Number of outcomes	2
	Vecs	[[0.70710678 0. 0.6363961 0.] [0.70710678 0. -0.6363961 0.]]
2	Type	Povm
	Dim	2
	Number of outcomes	2
	Vecs	[[0.70710678 0. 0. 0.6363961] [0.70710678 0. 0. -0.6363961]]

Cases

	Name	Param	Tomography	Estimator
0	Linear (True)	True	StandardQst	Linear
1	Linear (False)	False	StandardQst	Linear
2	ProjectedLinear (True)	True	StandardQst	ProjectedLinear
3	ProjectedLinear (False)	False	StandardQst	ProjectedLinear
4	Maximum-Likelihood (True)	True	StandardQst	LossMinimization
5	Maximum-Likelihood (False)	False	StandardQst	LossMinimization

6	Least Squares (True)	True	StandardQst	LossMinimization
7	Least Squares (False)	False	StandardQst	LossMinimization

Computation time of estimators

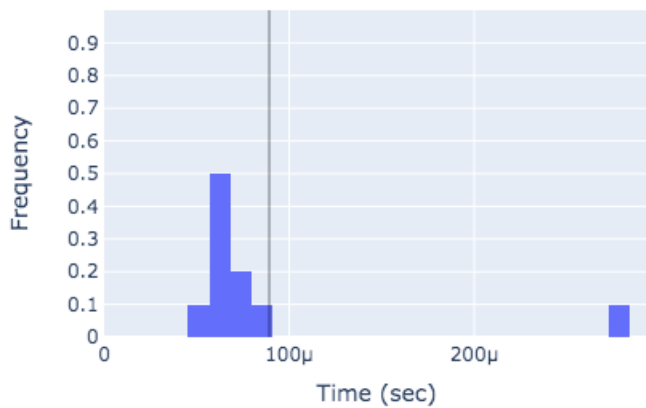
Table

	Name	N	Nrep	Mean (sec)	Std (sec)
0	Linear (True)	10	10	0.000089	0.000065
1	"	100	10	0.000063	0.000018
2	Linear (False)	10	10	0.000074	0.000022
3	"	100	10	0.000060	0.000022
4	ProjectedLinear (True)	10	10	0.008301	0.005186
5	"	100	10	0.003116	0.005154
6	ProjectedLinear (False)	10	10	0.009795	0.008066
7	"	100	10	0.002710	0.002614
8	Maximum-Likelihood (True)	10	10	0.444052	0.065212
9	"	100	10	0.392163	0.078838
10	Maximum-Likelihood (False)	10	10	1.704152	2.582865
11	"	100	10	1.854094	2.315696
12	Least Squares (True)	10	10	0.057638	0.019307
13	"	100	10	0.062231	0.025751
14	Least Squares (False)	10	10	0.068573	0.030691
15	"	100	10	0.065700	0.016822

Histogram

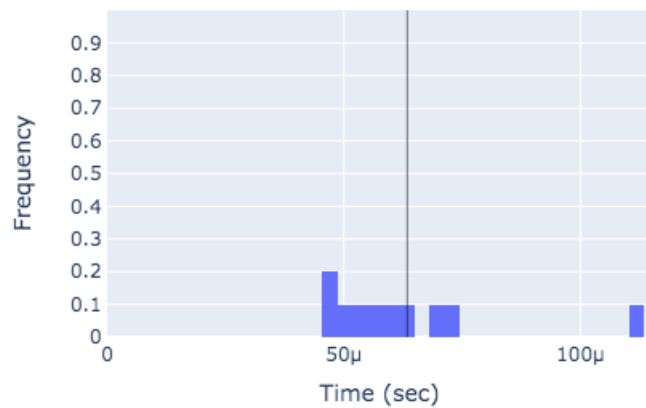
Case 0: Linear (True)

N=10, Nrep=10
Mean = 8.921623229980468e-05(sec)



Min: 5.602836608886719e-05
Max: 0.0002830028533935547
|Max-Min|: 0.0002269744873046875

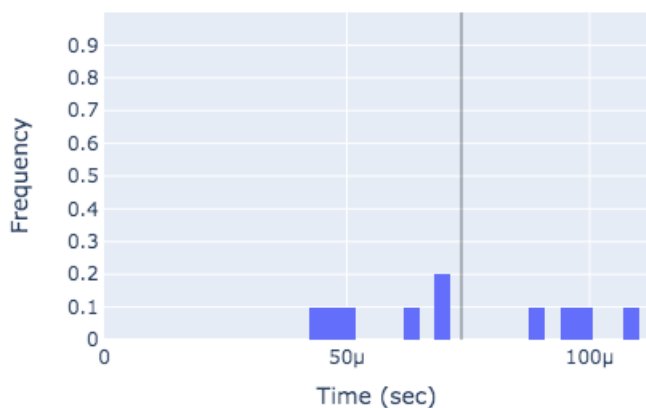
N=100, Nrep=10
Mean = 6.339550018310546e-05(sec)



Min: 4.696846008300781e-05
Max: 0.00011181831359863281
|Max-Min|: 6.4849853515625e-05

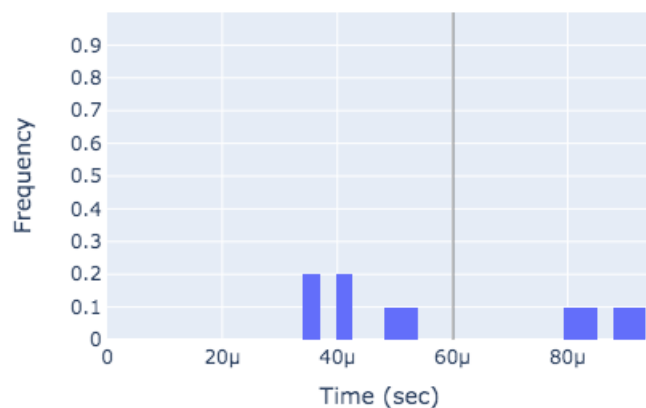
Case 1: Linear (False)

N=10, Nrep=10
Mean = 7.359981536865234e-05(sec)



Min: 4.410743713378906e-05
Max: 0.00010895729064941406
|Max-Min|: 6.4849853515625e-05

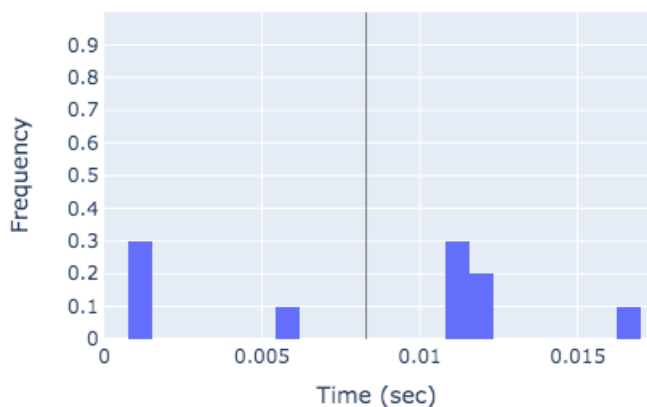
N=100, Nrep=10
Mean = 6.012916564941406e-05(sec)



Min: 3.504753112792969e-05
Max: 9.179115295410156e-05
|Max-Min|: 5.6743621826171875e-05

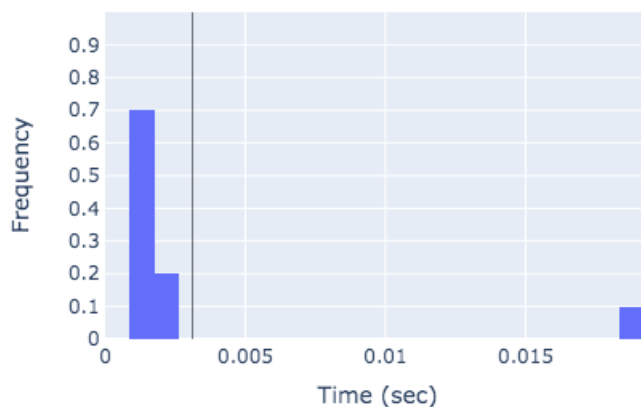
Case 2: ProjectedLinear (True)

N=10, Nrep=10
Mean = 0.00830092430114746(sec)



Min: 0.0010371208190917969
Max: 0.01648235321044922
|Max-Min|: 0.015445232391357422

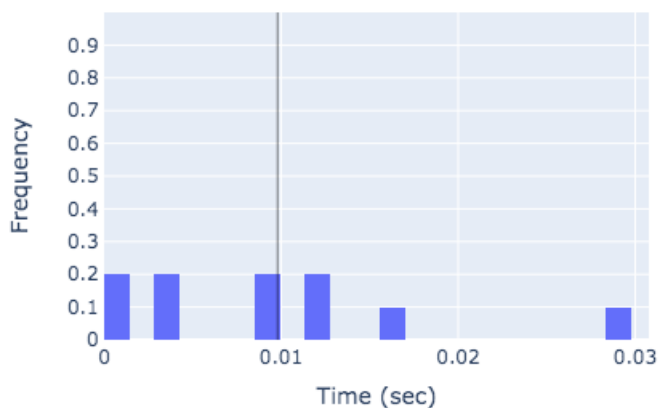
N=100, Nrep=10
Mean = 0.003116297721862793(sec)



Min: 0.0011067390441894531
Max: 0.018559694290161133
|Max-Min|: 0.01745295524597168

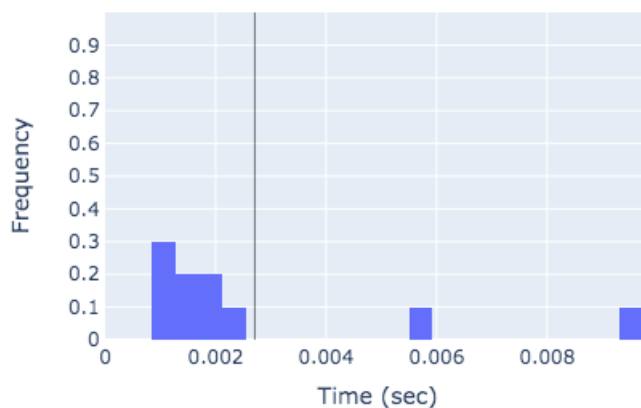
Case 3: ProjectedLinear (False)

N=10, Nrep=10
Mean = 0.009795427322387695(sec)



Min: 0.0010139942169189453
Max: 0.02935194969177246
|Max-Min|: 0.028337955474853516

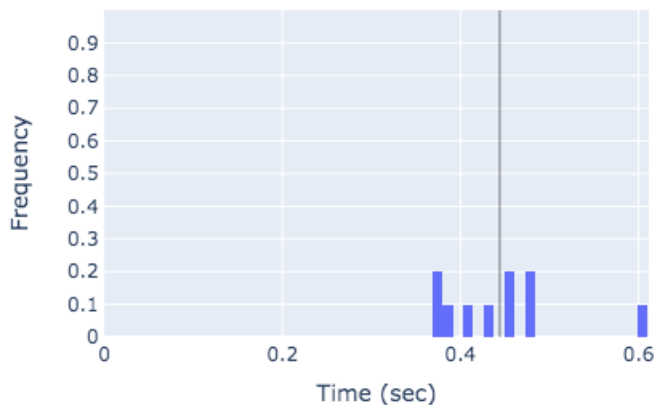
N=100, Nrep=10
Mean = 0.0027097225189208984(sec)



Min: 0.0009739398956298828
Max: 0.009436607360839844
|Max-Min|: 0.008462667465209961

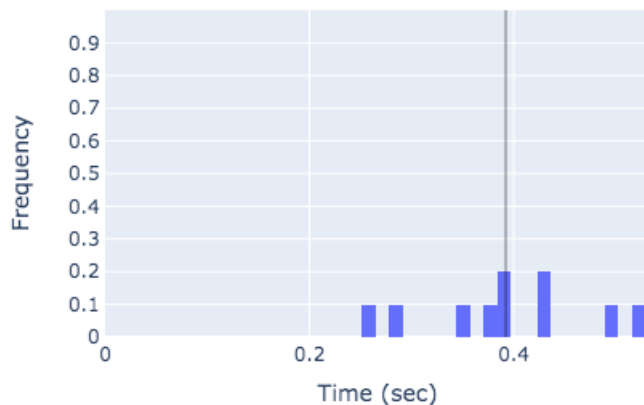
Case 4: Maximum-Likelihood (True)

N=10, Nrep=10
Mean = 0.44405169486999513(sec)



Min: 0.369657039642334
Max: 0.5999460220336914
|Max-Min|: 0.23028898239135742

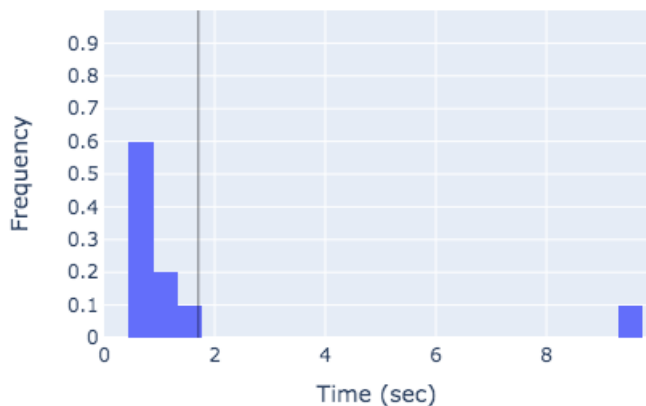
N=100, Nrep=10
Mean = 0.3921628475189209(sec)



Min: 0.2555124759674072
Max: 0.5200240612030029
|Max-Min|: 0.2645115852355957

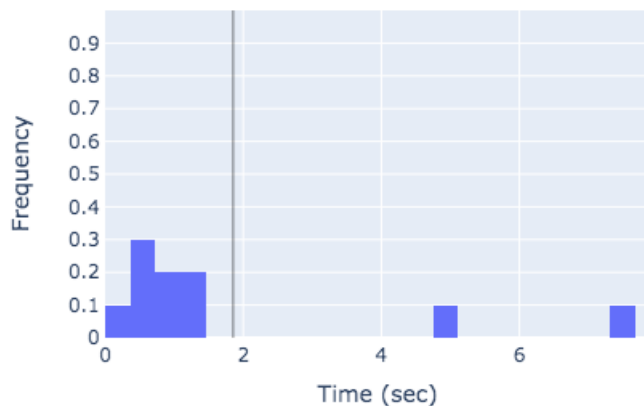
Case 5: Maximum-Likelihood (False)

N=10, Nrep=10
Mean = 1.7041515827178955(sec)



Min: 0.5558240413665771
Max: 9.409453630447388
|Max-Min|: 8.85362958908081

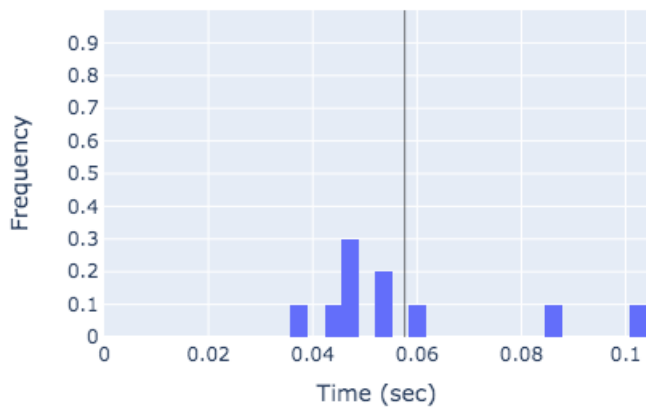
N=100, Nrep=10
Mean = 1.8540937662124635(sec)



Min: 0.2196671962738037
Max: 7.523585081100464
|Max-Min|: 7.30391788482666

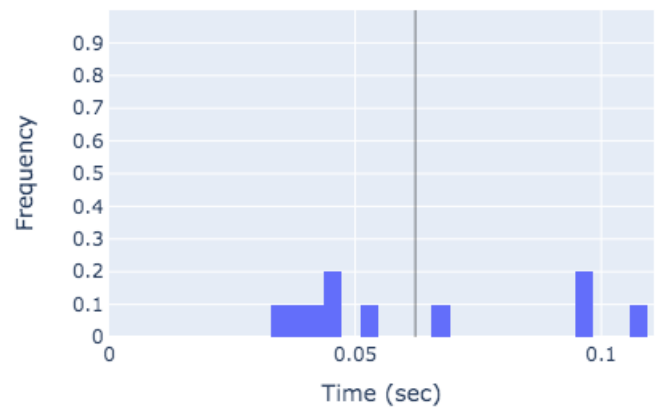
Case 6: Least Squares (True)

N=10, Nrep=10
Mean = 0.05763804912567139(sec)



Min: 0.0361940860748291
Max: 0.10118341445922852
|Max-Min|: 0.06498932838439941

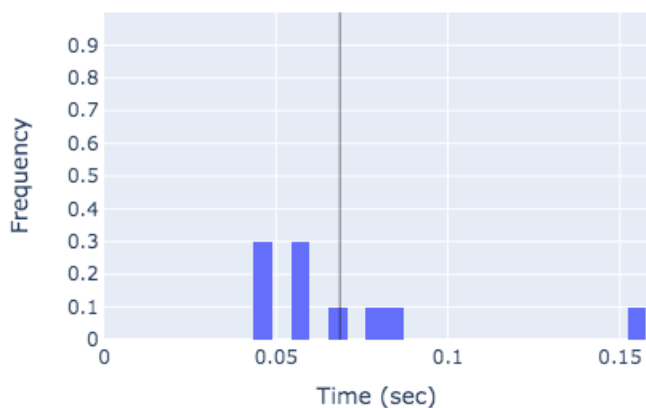
N=100, Nrep=10
Mean = 0.0622307300567627(sec)



Min: 0.03418588638305664
Max: 0.10704994201660156
|Max-Min|: 0.07286405563354492

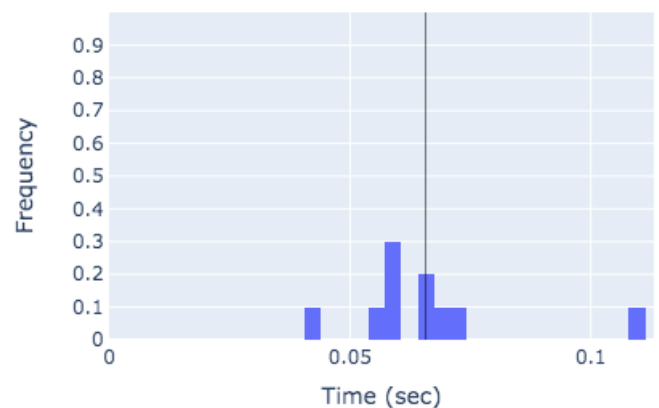
Case 7: Least Squares (False)

N=10, Nrep=10
Mean = 0.06857328414916992(sec)



Min: 0.044283151626586914
Max: 0.153001070022583
|Max-Min|: 0.1087179183959961

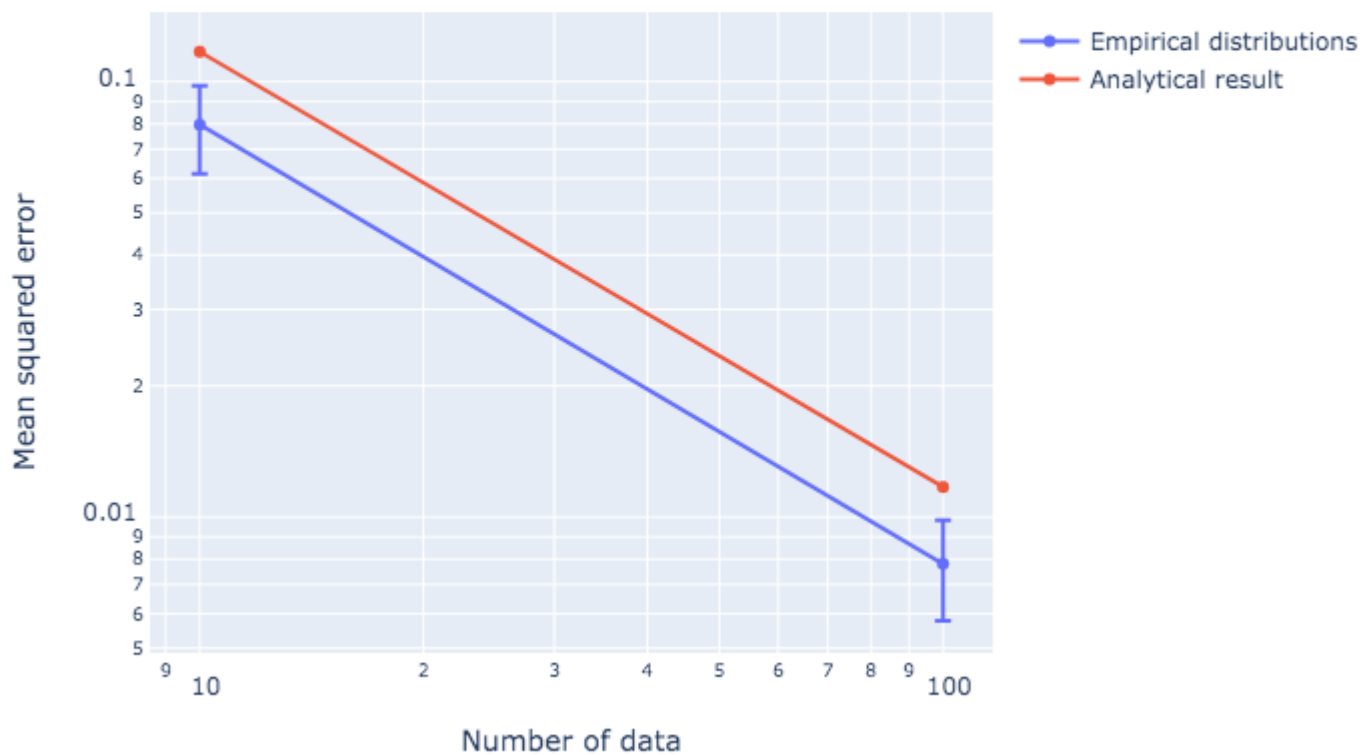
N=100, Nrep=10
Mean = 0.06569976806640625(sec)



Min: 0.04227423667907715
Max: 0.10970211029052734
|Max-Min|: 0.0674278736114502

MSE of empirical distributions

Mean squared error



Consistency test

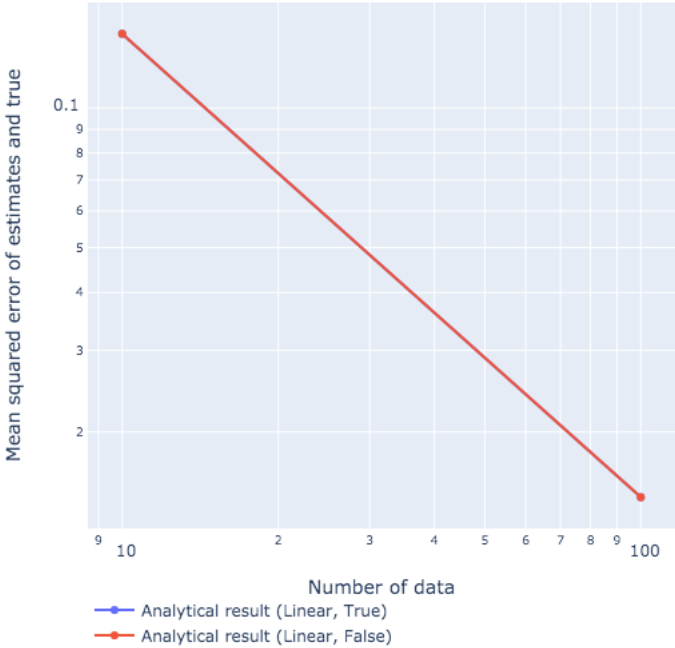
	Name	Type of tomography	Param	Estimator	Loss	Algo	Squared Error to True	Possibly OK	To be checked
0	Linear (True)	Standard Qst	True	Linear	None	None	0.00e+00	OK	not need debug
1	Linear (False)	Standard Qst	False	Linear	None	None	2.47e-32	OK	not need debug
2	ProjectedLinear (True)	Standard Qst	True	Projected Linear	None	None	1.23e-32	OK	not need debug
3	ProjectedLinear (False)	Standard Qst	False	Projected Linear	None	None	1.11e-31	OK	not need debug
4	Maximum-Likelihood (True)	Standard Qst	True	Loss Minimization	Weighted Relative Entropy	Projected Gradient Descent Backtracking	3.21e-18	OK	not need debug
5	Maximum-Likelihood (False)	Standard Qst	False	Loss Minimization	Weighted Relative Entropy	Projected Gradient Descent Backtracking	5.47e-09	NG	not need debug
6	Least Squares (True)	Standard Qst	True	Loss Minimization	Weighted Probability Based Squared Error	Projected Gradient Descent Backtracking	4.93e-32	OK	not need debug

7	Least Squares (False)	Standard Qst	False	Loss Minimization	Weighted Probability Based Squared Error	Projected Gradient Descent Backtracking	1.60e-31	OK	not need debug
---	-----------------------	--------------	-------	-------------------	--	---	----------	----	----------------

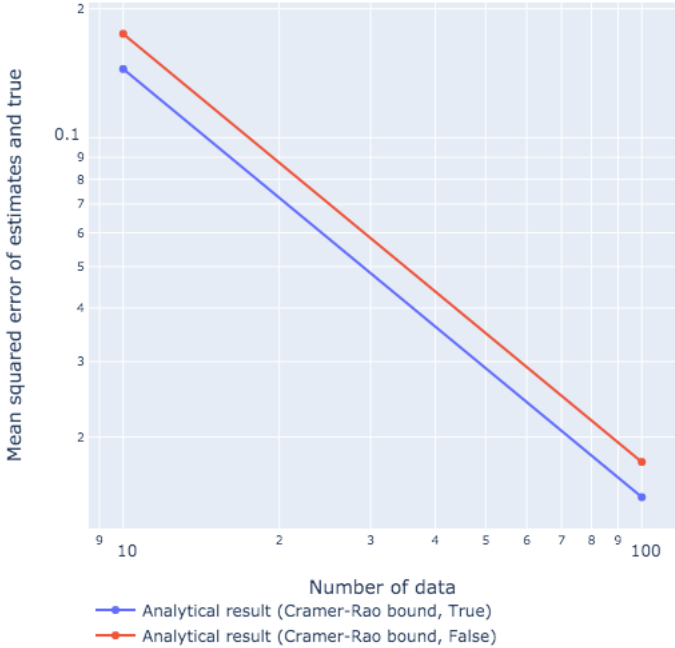
MSE of estimators

Comparison of analytical results

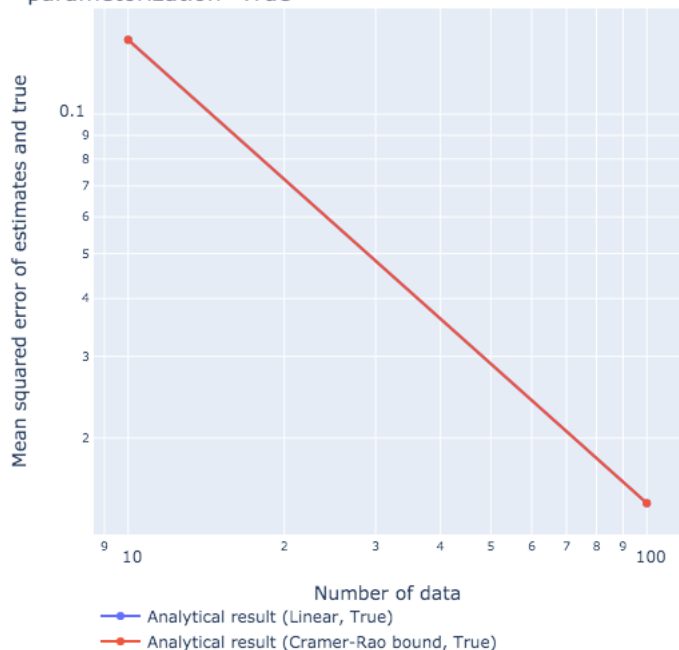
Mean squared error
Analytical result
estimator=Linear



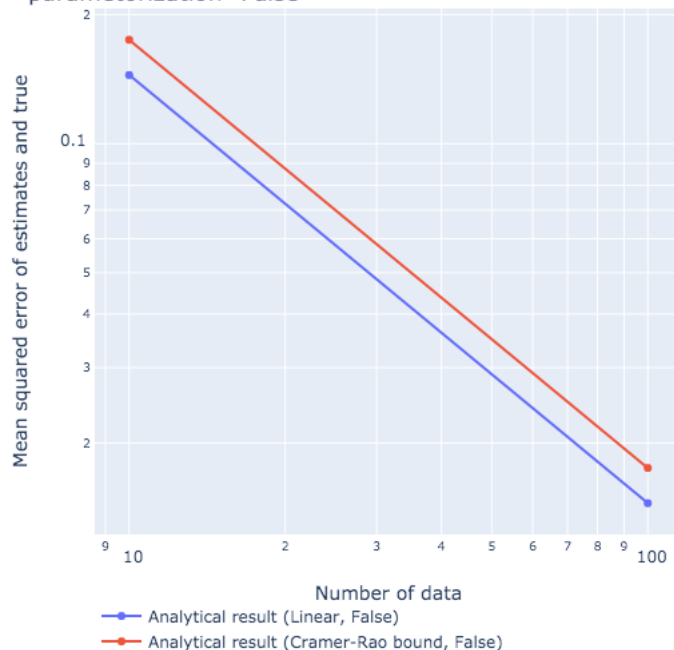
Mean squared error
Analytical result
Cramer-Rao bound



Mean squared error
Analytical result
parameterization=True

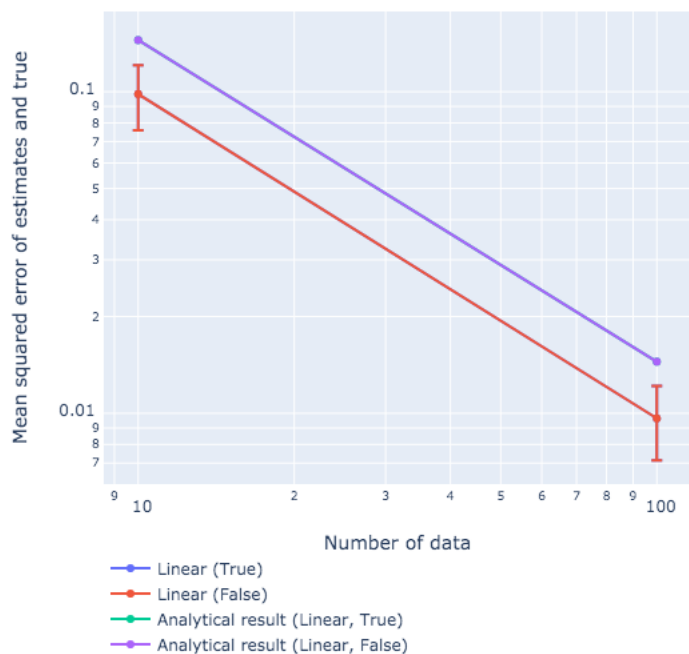


Mean squared error
Analytical result
parameterization=False

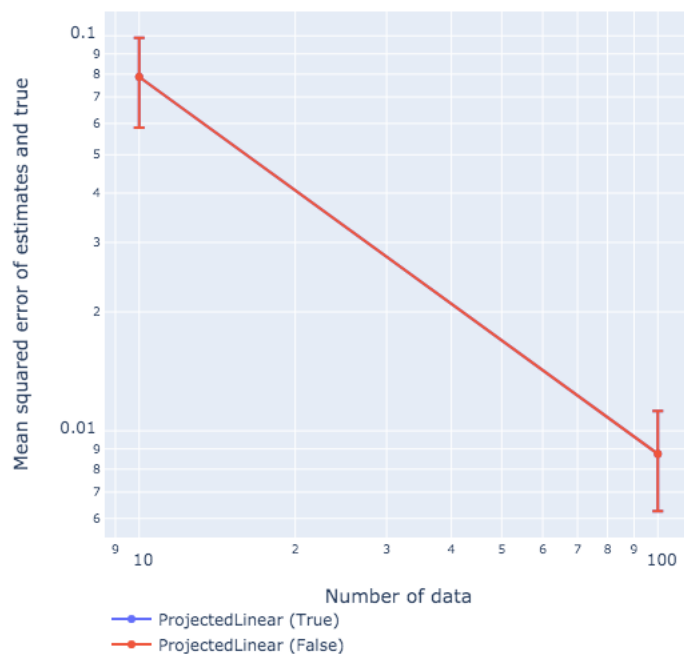


Comparison of parametrization

Mean squared error
Estimator=Linear



Mean squared error
Estimator=ProjectedLinear

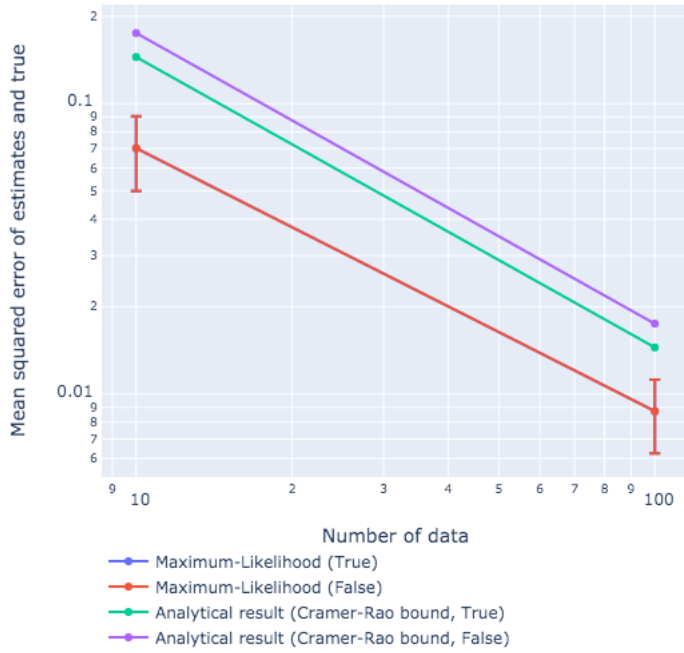


Mean squared error

Estimator=LossMinimization

Loss=WeightedRelativeEntropy

Algo=ProjectedGradientDescentBacktracking

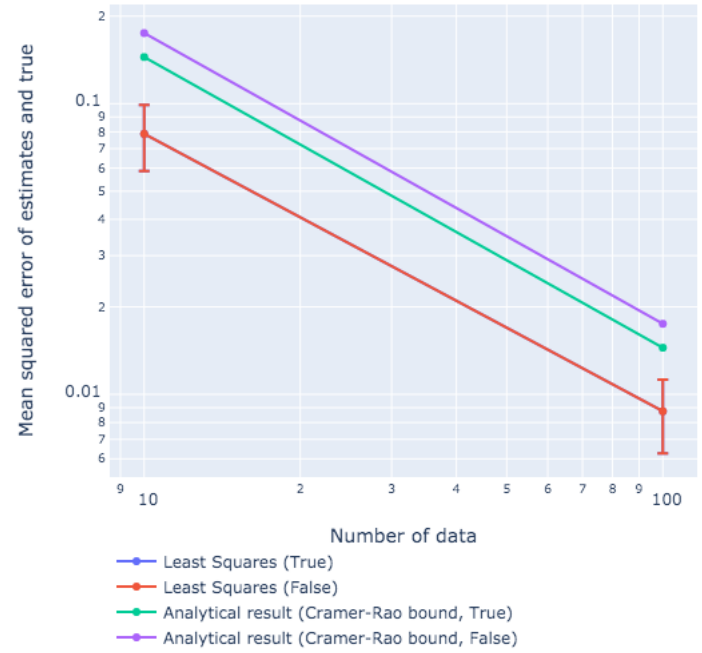


Mean squared error

Estimator=LossMinimization

Loss=WeightedProbabilityBasedSquaredError

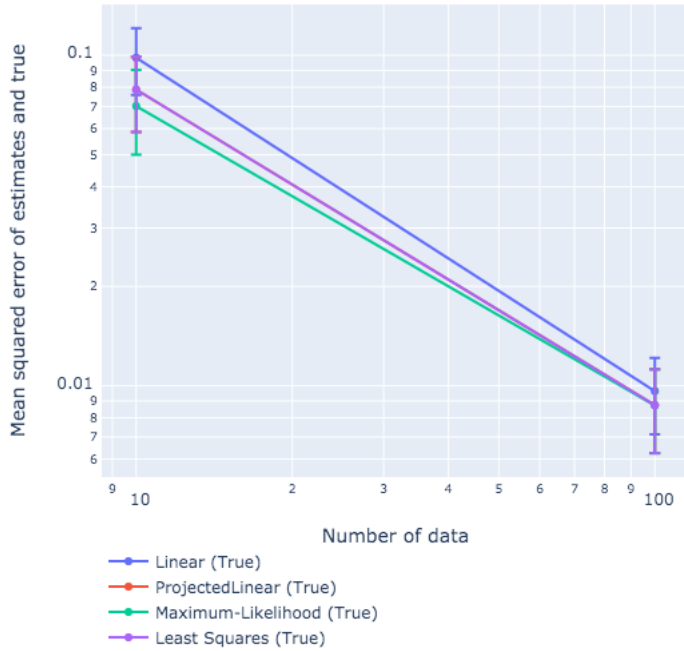
Algo=ProjectedGradientDescentBacktracking



Comparison of estimators

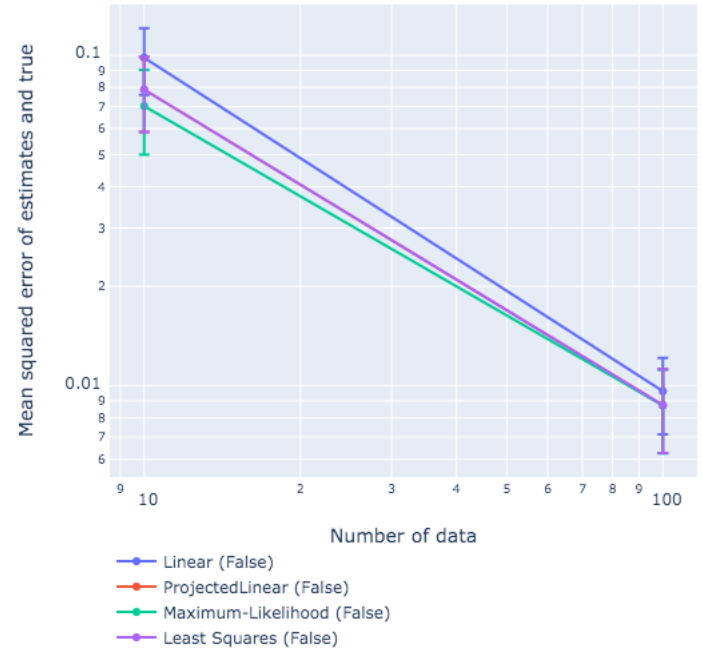
Mean squared error

parametrization=True



Mean squared error

parametrization=False

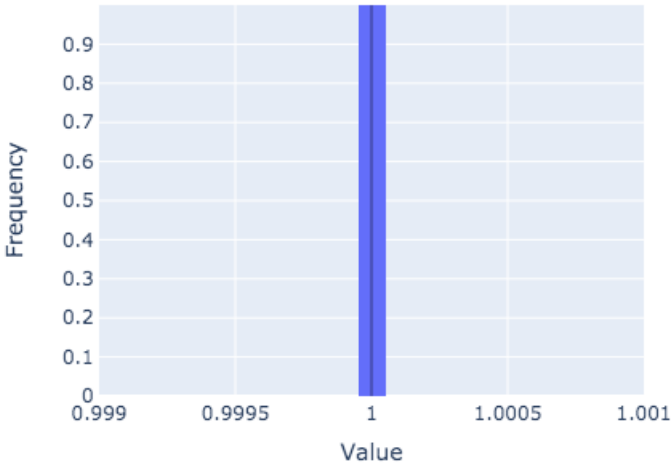


Physicality violation test

Test of equality constraint violation

Case 0: Linear (True)

N=10, Nrep=10



Min: 0.9999999999999997
Max: 0.9999999999999998
|Max-Min|: 1.1102230246251565e-16

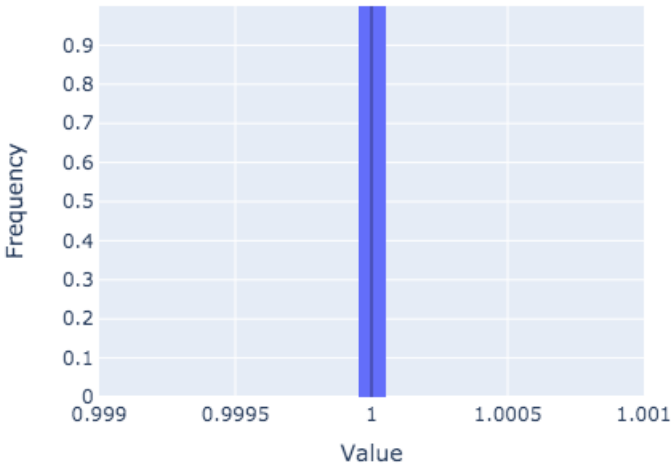
N=100, Nrep=10



Min: 0.9999999999999998
Max: 0.9999999999999998
|Max-Min|: 0.0

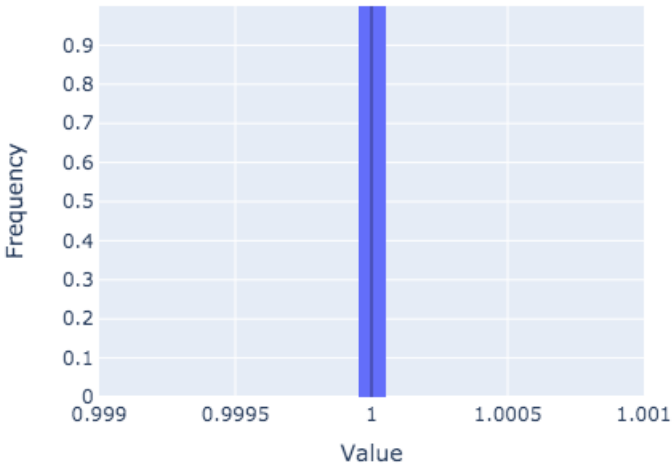
Case 1: Linear (False)

N=10, Nrep=10



Min: 0.9999999999999998
Max: 1.0000000000000002
|Max-Min|: 4.440892098500626e-16

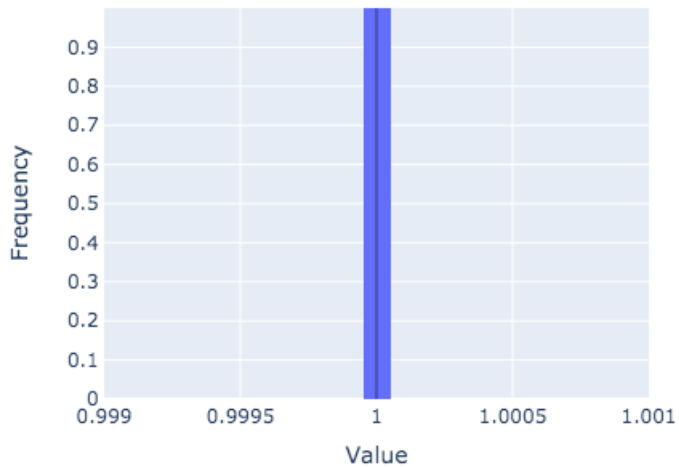
N=100, Nrep=10



Min: 0.9999999999999999
Max: 1.0000000000000002
|Max-Min|: 3.3306690738754696e-16

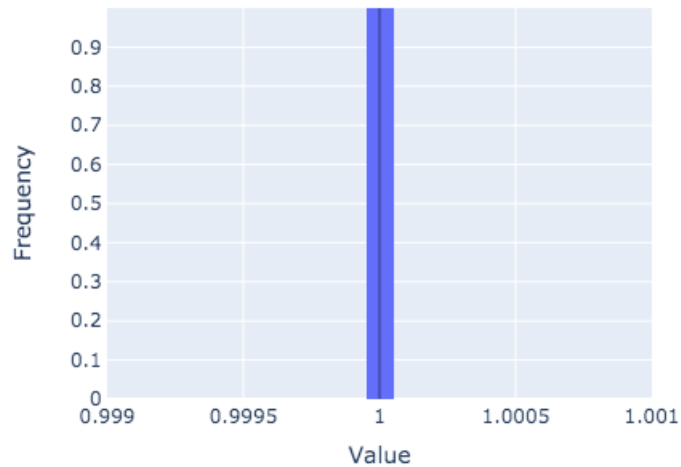
Case 2: ProjectedLinear (True)

N=10, Nrep=10



Min: 0.9999999999999998
Max: 0.9999999999999998
|Max-Min|: 0.0

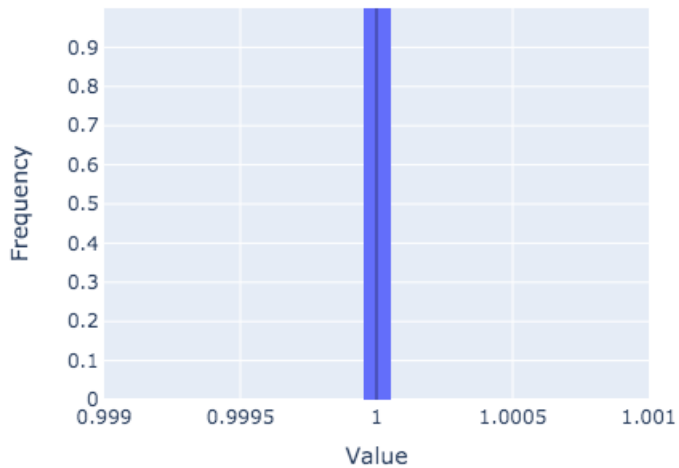
N=100, Nrep=10



Min: 0.9999999999999998
Max: 0.9999999999999998
|Max-Min|: 0.0

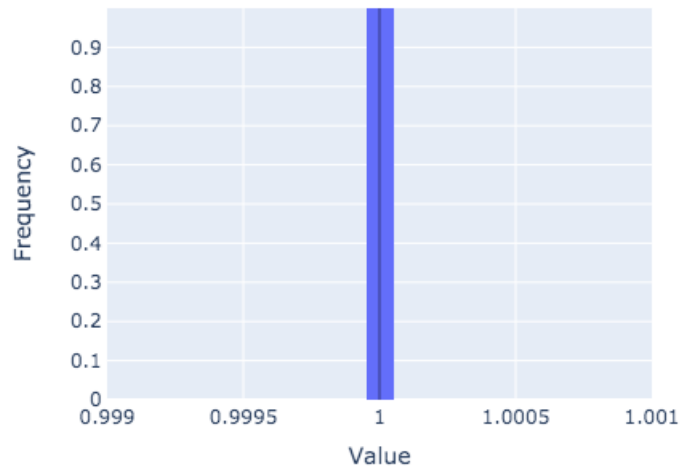
Case 3: ProjectedLinear (False)

N=10, Nrep=10



Min: 0.9999999999999996
Max: 1.00000014753393
|Max-Min|: 1.4753393040578544e-07

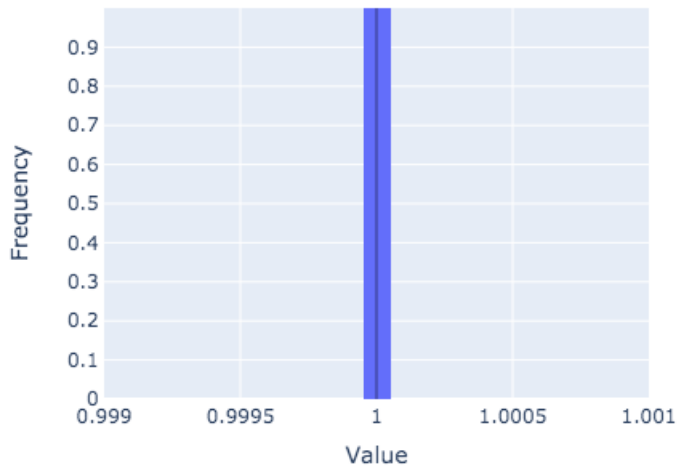
N=100, Nrep=10



Min: 0.9999999999999997
Max: 1.0000001280394155
|Max-Min|: 1.280394158476028e-07

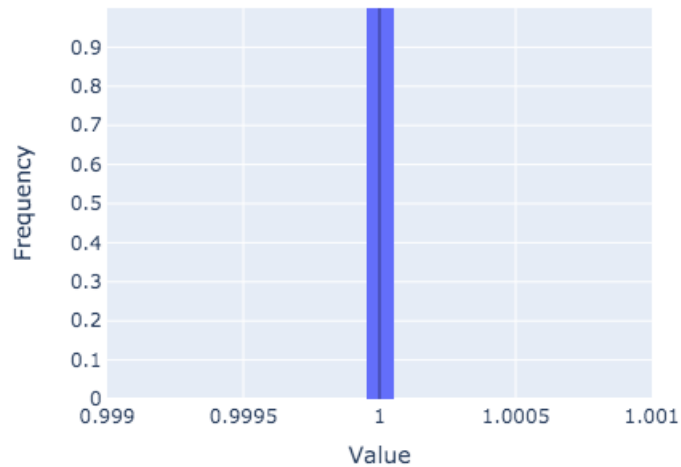
Case 4: Maximum-Likelihood (True)

N=10, Nrep=10



Min: 0.9999999999999998
Max: 0.9999999999999998
|Max-Min|: 0.0

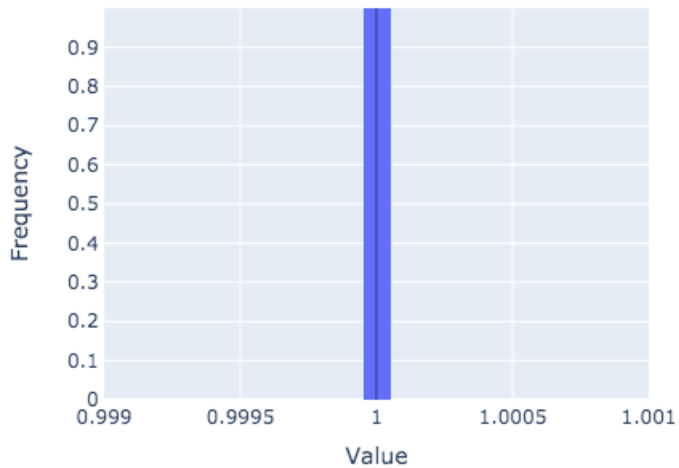
N=100, Nrep=10



Min: 0.9999999999999998
Max: 0.9999999999999998
|Max-Min|: 0.0

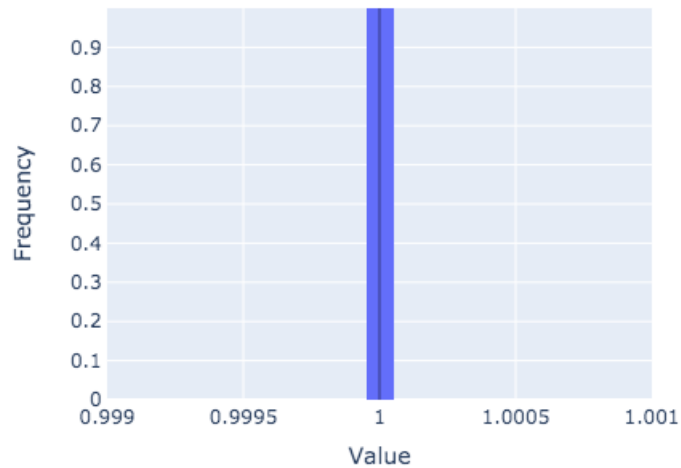
Case 5: Maximum-Likelihood (False)

N=10, Nrep=10



Min: 1.0000000006567707
Max: 1.0000001622723005
|Max-Min|: 1.5570459344083076e-07

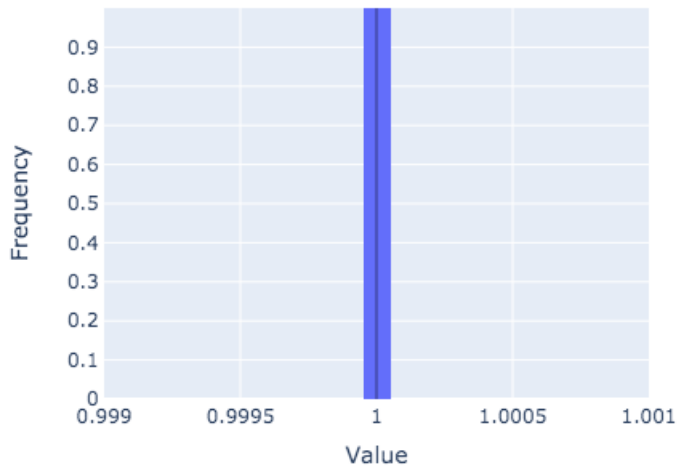
N=100, Nrep=10



Min: 1.0000000121352572
Max: 1.0000001810607821
|Max-Min|: 1.689255249015531e-07

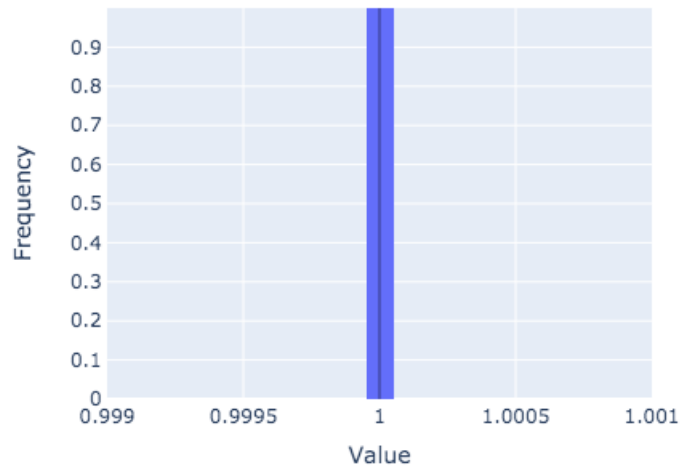
Case 6: Least Squares (True)

N=10, Nrep=10



Min: 0.9999999999999998
Max: 0.9999999999999998
|Max-Min|: 0.0

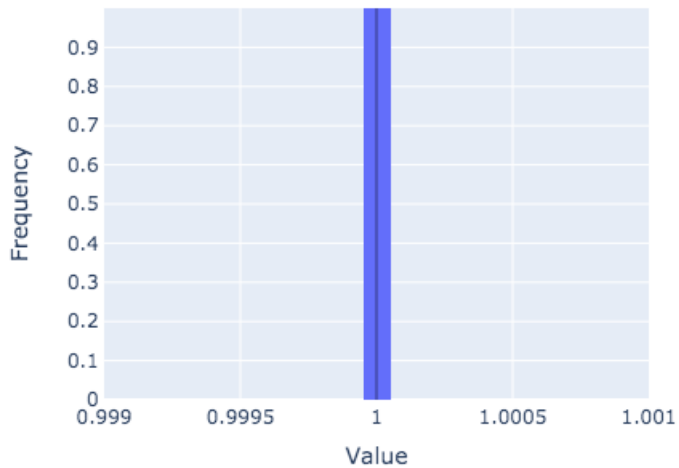
N=100, Nrep=10



Min: 0.9999999999999998
Max: 0.9999999999999998
|Max-Min|: 0.0

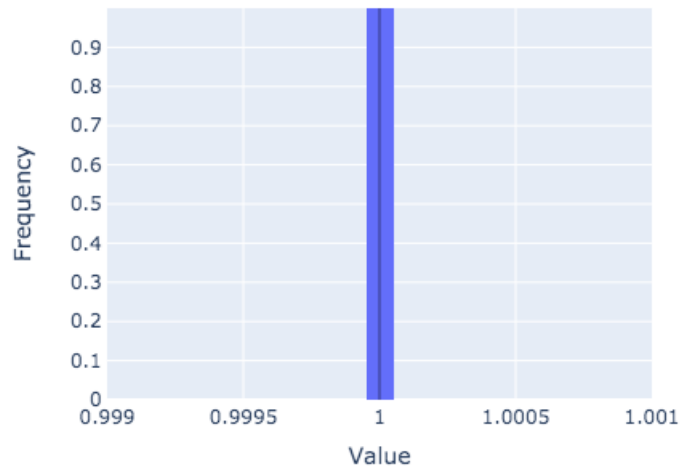
Case 7: Least Squares (False)

N=10, Nrep=10



Min: 0.9999999999999998
Max: 1.0000001781169368
|Max-Min|: 1.7811693697744602e-07

N=100, Nrep=10



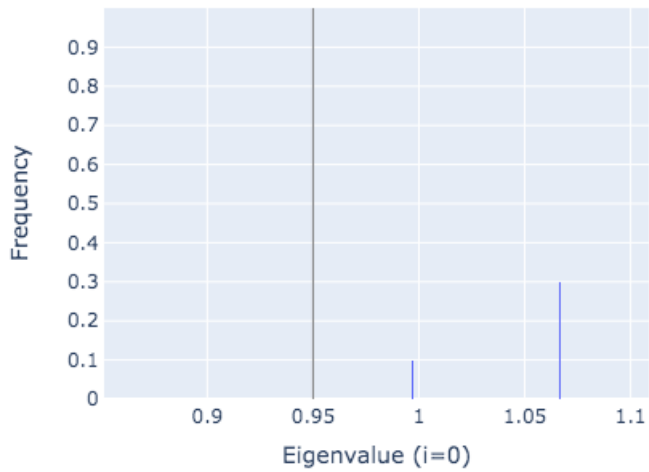
Min: 0.9999999999999998
Max: 1.0000001555680968
|Max-Min|: 1.5556809707106822e-07

Test of inequality constraint violation

Eigenvalue

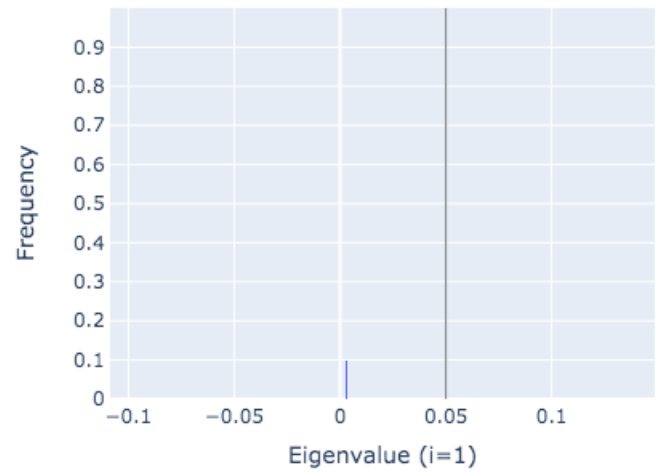
Case 0: Linear (True)

N=10, Nrep=10



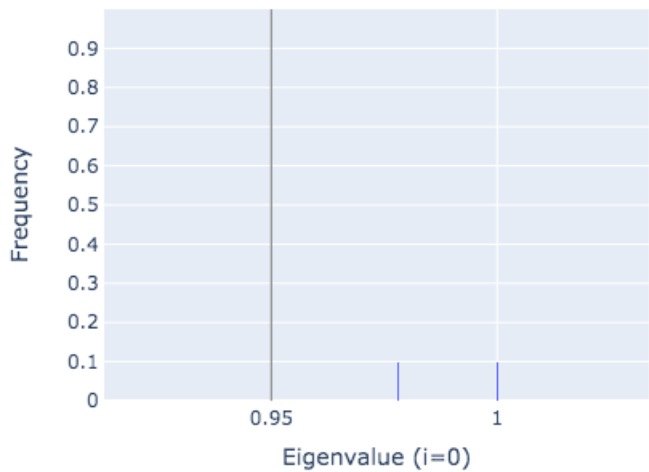
Min: 0.8513641844631532
 Max: 1.1085806194501846
 |Max-Min|: 0.2572164349870314

N=10, Nrep=10



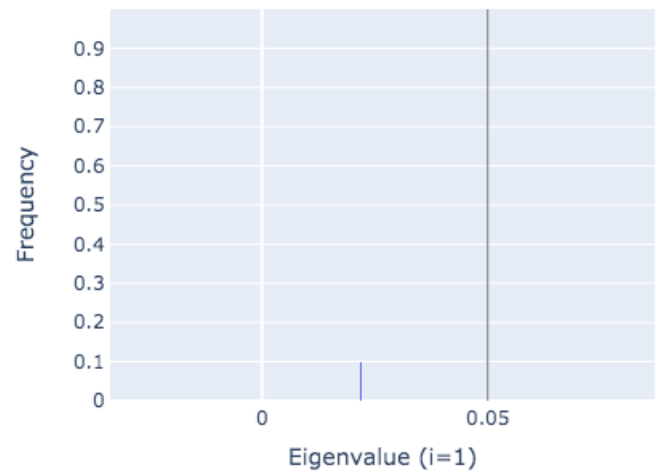
Min: -0.10858061945018471
 Max: 0.14863581553684663
 |Max-Min|: 0.2572164349870314

N=100, Nrep=10



Min: 0.9130584510098023
 Max: 1.0335647646019095
 |Max-Min|: 0.12050631359210717

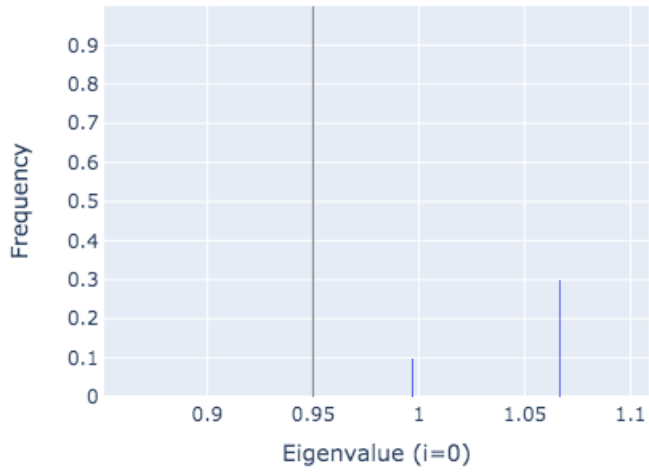
N=100, Nrep=10



Min: -0.03356476460190986
 Max: 0.0869415489901976
 |Max-Min|: 0.12050631359210745

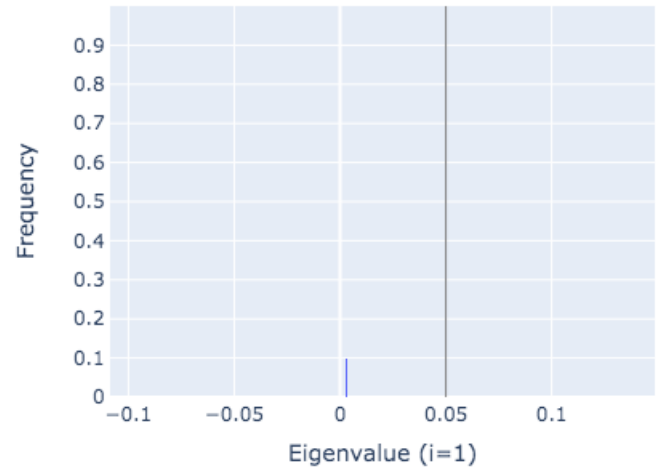
Case 1: Linear (False)

N=10, Nrep=10



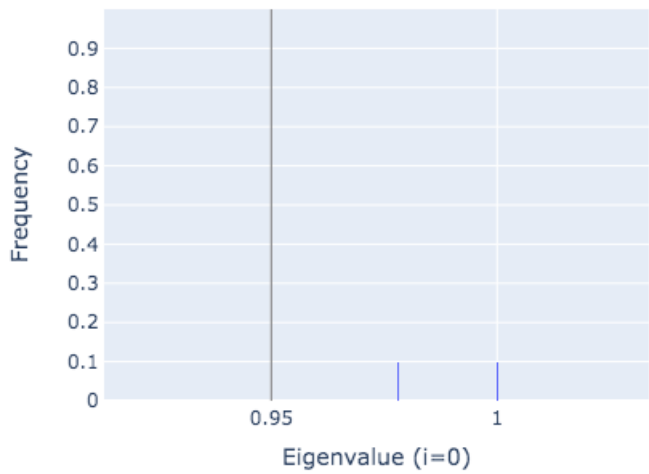
Min: 0.8513641844631532
 Max: 1.1085806194501846
 |Max-Min|: 0.2572164349870314

N=10, Nrep=10



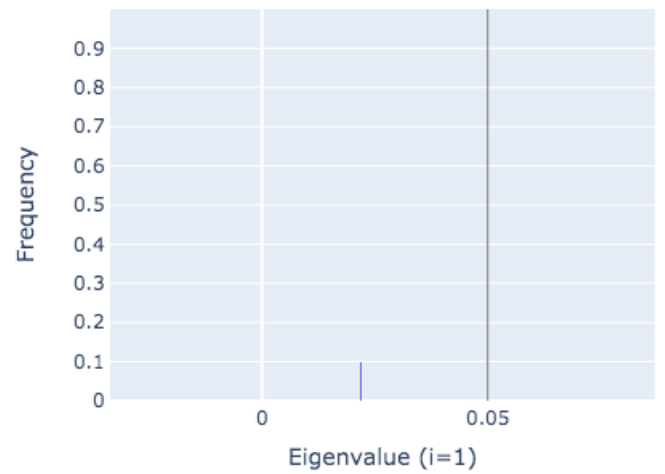
Min: -0.10858061945018459
 Max: 0.14863581553684663
 |Max-Min|: 0.25721643498703123

N=100, Nrep=10



Min: 0.9130584510098023
 Max: 1.0335647646019095
 |Max-Min|: 0.12050631359210717

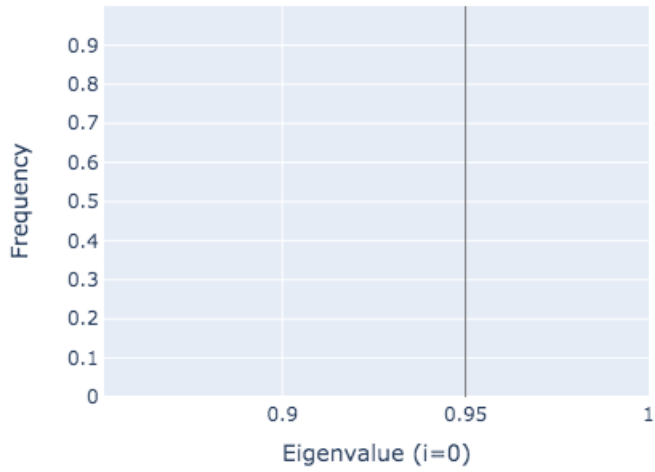
N=100, Nrep=10



Min: -0.03356476460190975
 Max: 0.08694154899019772
 |Max-Min|: 0.12050631359210748

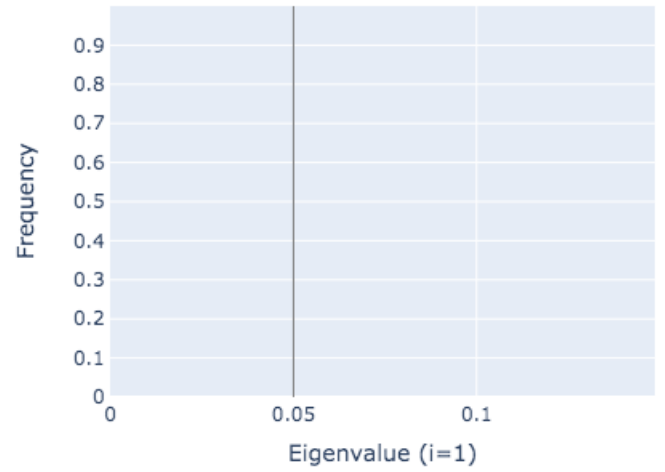
Case 2: ProjectedLinear (True)

N=10, Nrep=10



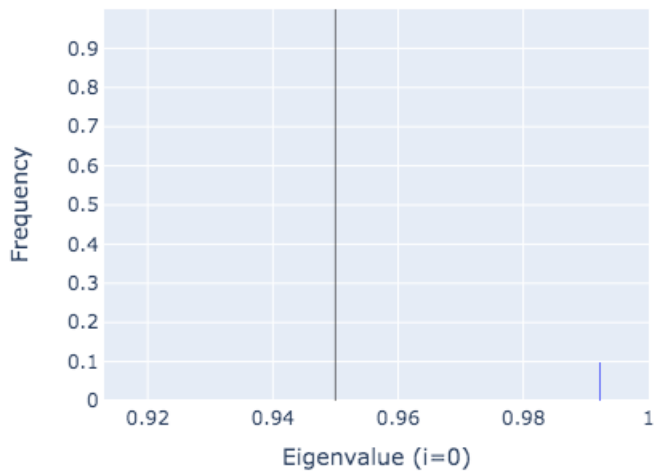
Min: 0.8513641844631531
Max: 1.0000000737669648
|Max-Min|: 0.14863588930381166

N=10, Nrep=10



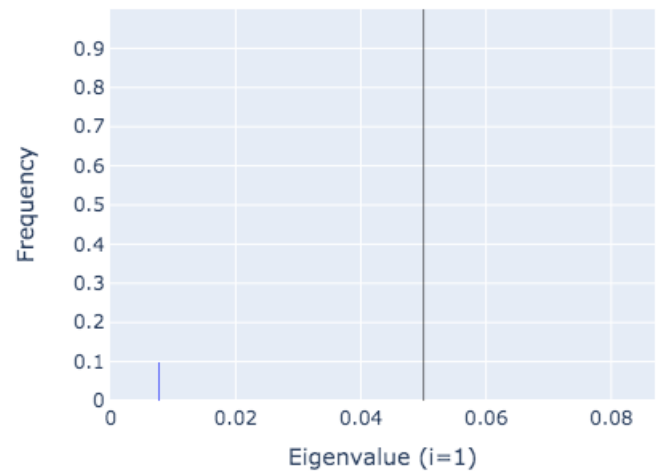
Min: -7.376696509533986e-08
Max: 0.14863581553684668
|Max-Min|: 0.14863588930381177

N=100, Nrep=10



Min: 0.9130584510098023
Max: 1.0000000640197078
|Max-Min|: 0.08694161300990544

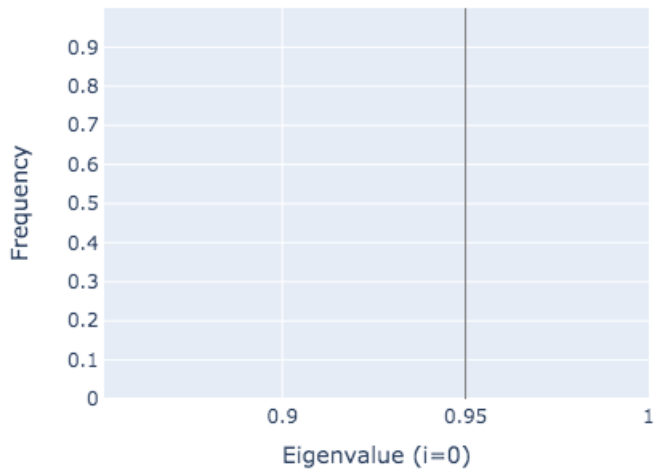
N=100, Nrep=10



Min: -6.401970792556323e-08
Max: 0.0869415489901976
|Max-Min|: 0.08694161300990552

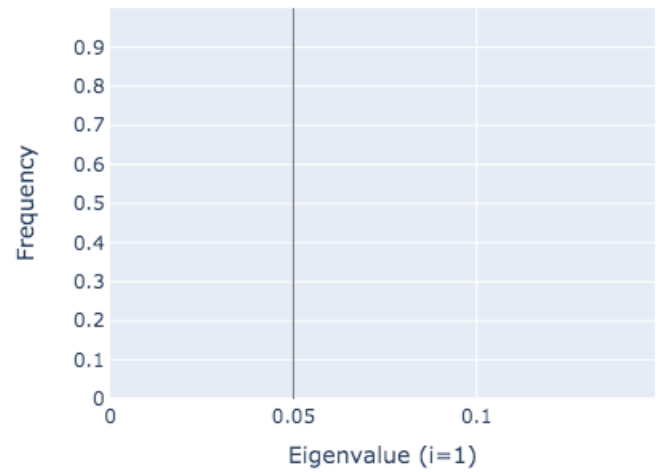
Case 3: ProjectedLinear (False)

N=10, Nrep=10



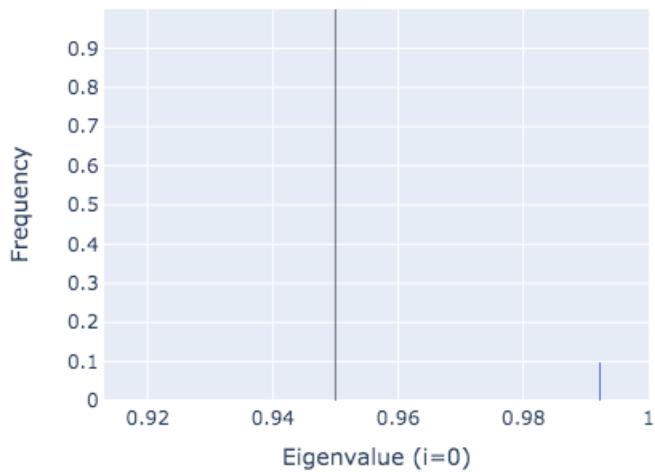
Min: 0.8513641844631532
Max: 1.00000014753393
|Max-Min|: 0.14863596307077676

N=10, Nrep=10



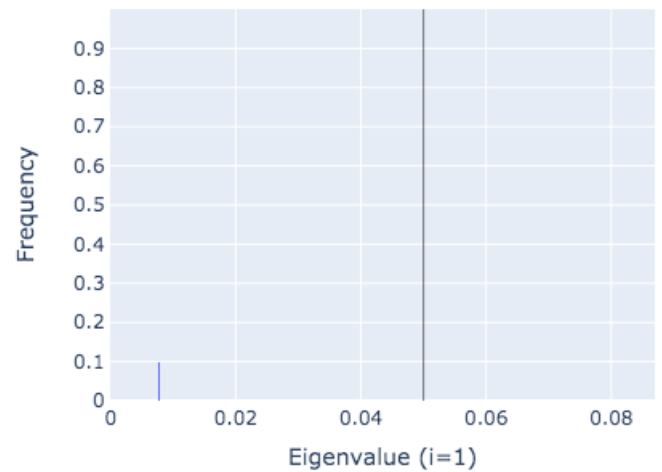
Min: -2.7755575615628914e-17
Max: 0.14863581553684696
|Max-Min|: 0.148635815536847

N=100, Nrep=10



Min: 0.9130584510098023
Max: 1.0000001280394155
|Max-Min|: 0.0869416770296132

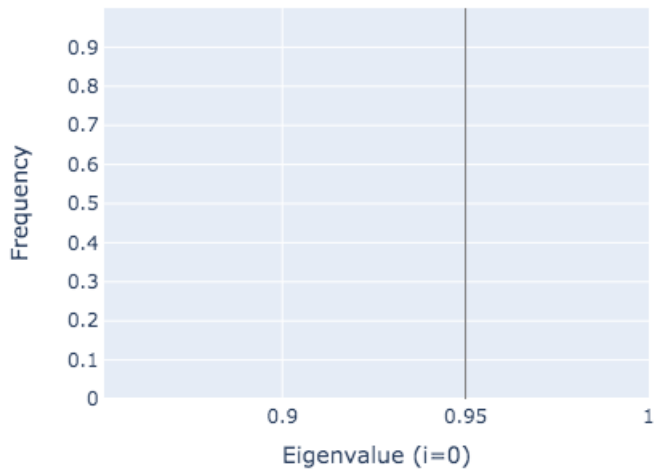
N=100, Nrep=10



Min: -5.730008481585891e-17
Max: 0.0869415489901976
|Max-Min|: 0.08694154899019765

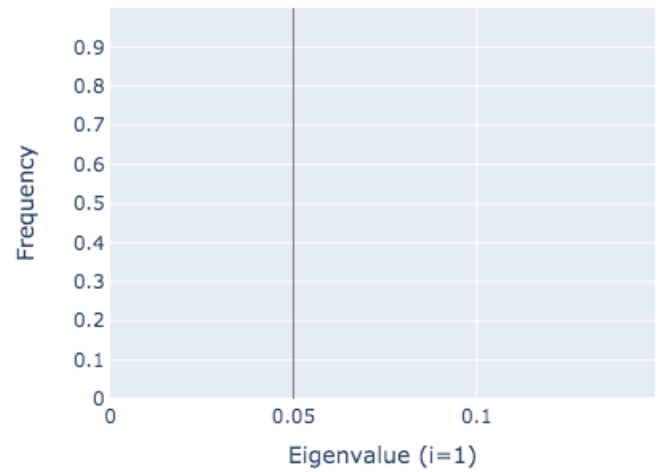
Case 4: Maximum-Likelihood (True)

N=10, Nrep=10



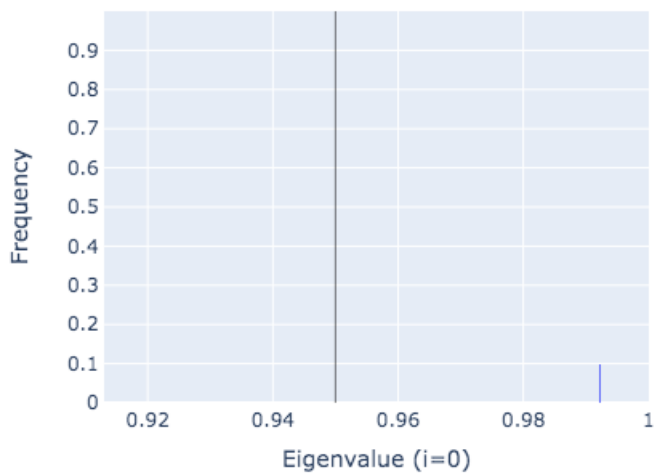
Min: 0.8513641812250503
Max: 1.0000000702659644
|Max-Min|: 0.14863588904091407

N=10, Nrep=10



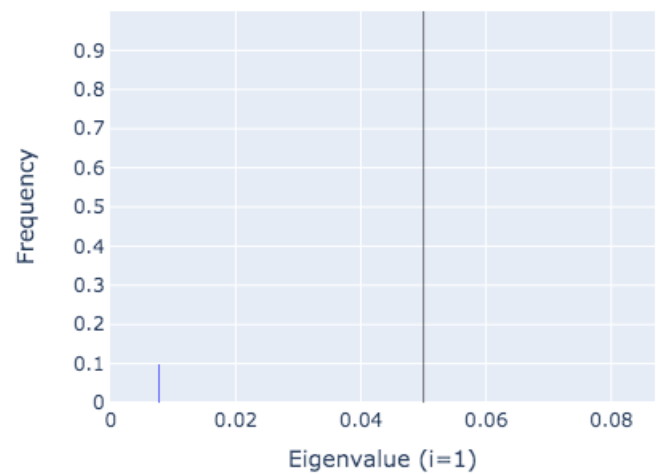
Min: -7.026596454684508e-08
Max: 0.14863581877494952
|Max-Min|: 0.14863588904091407

N=100, Nrep=10



Min: 0.9130584509383943
Max: 1.000000078401553
|Max-Min|: 0.0869416274631587

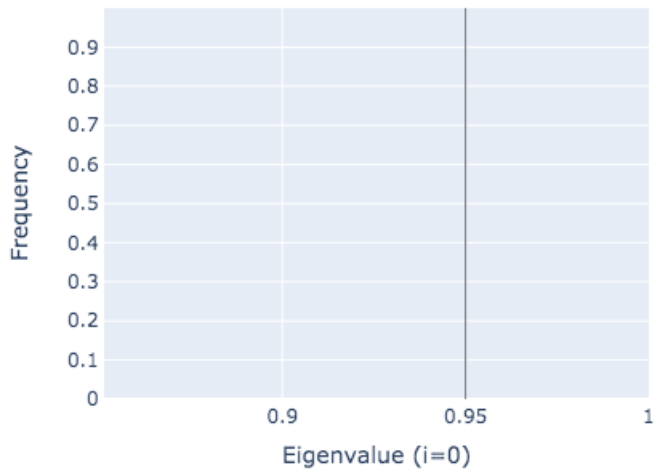
N=100, Nrep=10



Min: -7.840155328241432e-08
Max: 0.08694154906160545
|Max-Min|: 0.08694162746315873

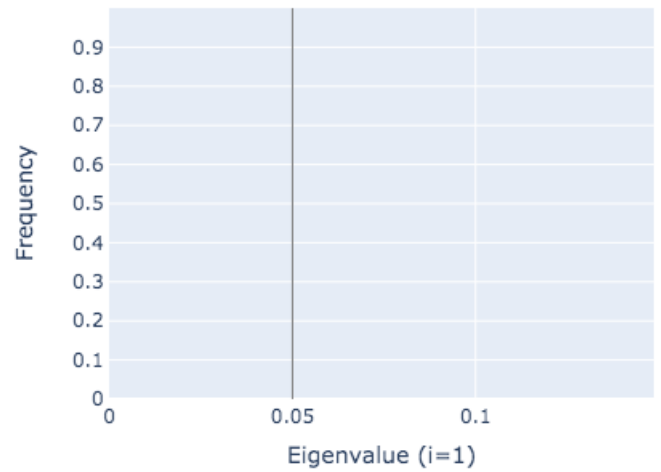
Case 5: Maximum-Likelihood (False)

N=10, Nrep=10



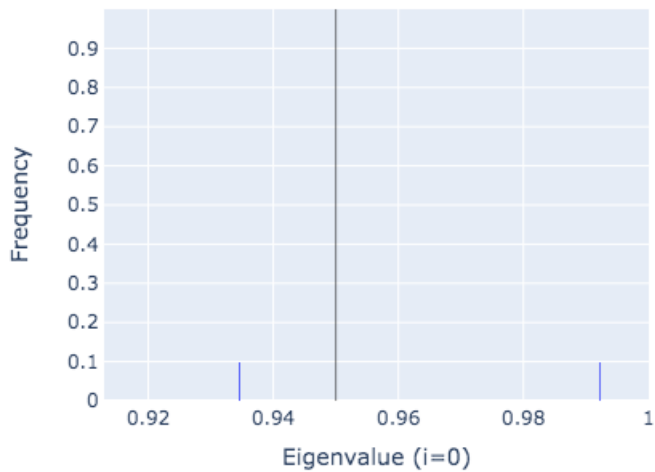
Min: 0.851427240055816
 Max: 1.000000162272261
 |Max-Min|: 0.1485729222164449

N=10, Nrep=10



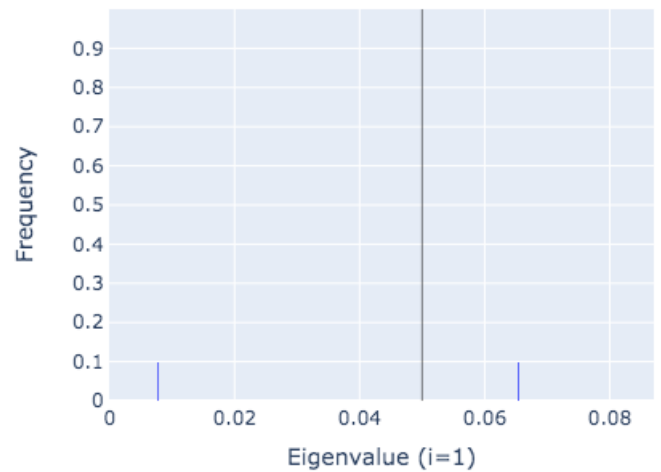
Min: -1.3877787807814457e-17
 Max: 0.14857279250213073
 |Max-Min|: 0.14857279250213073

N=100, Nrep=10



Min: 0.9130380513737355
 Max: 1.0000001810607806
 |Max-Min|: 0.08696212968704509

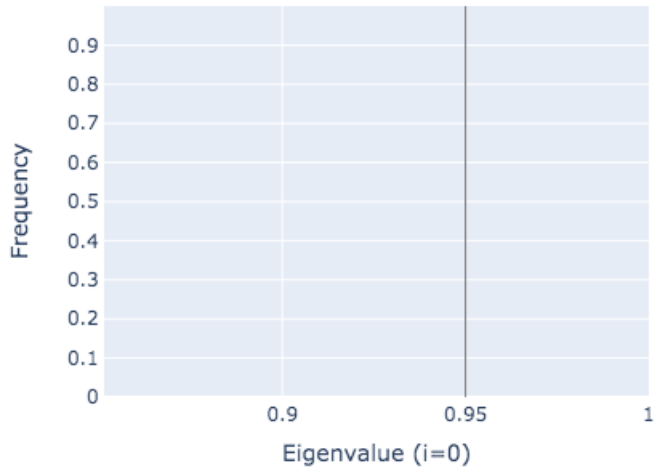
N=100, Nrep=10



Min: -1.0061396160665481e-16
 Max: 0.08696196076152173
 |Max-Min|: 0.08696196076152182

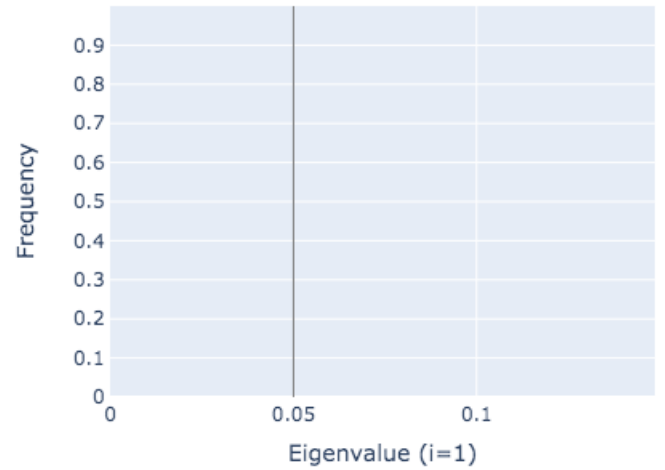
Case 6: Least Squares (True)

N=10, Nrep=10



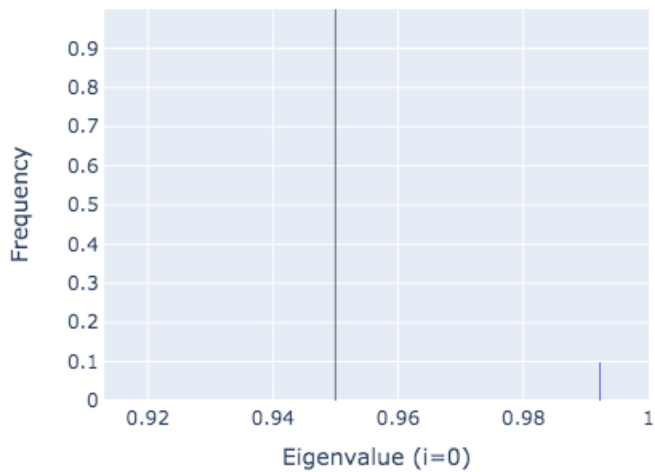
Min: 0.851364184463153
 Max: 1.0000000761055894
 |Max-Min|: 0.1486358916424364

N=10, Nrep=10



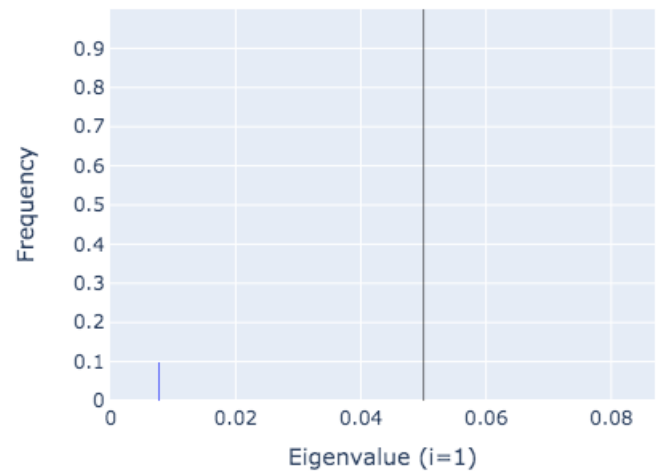
Min: -7.610558962201086e-08
 Max: 0.14863581553684674
 |Max-Min|: 0.14863589164243635

N=100, Nrep=10



Min: 0.9130584510098028
 Max: 1.0000000598780594
 |Max-Min|: 0.08694160886825664

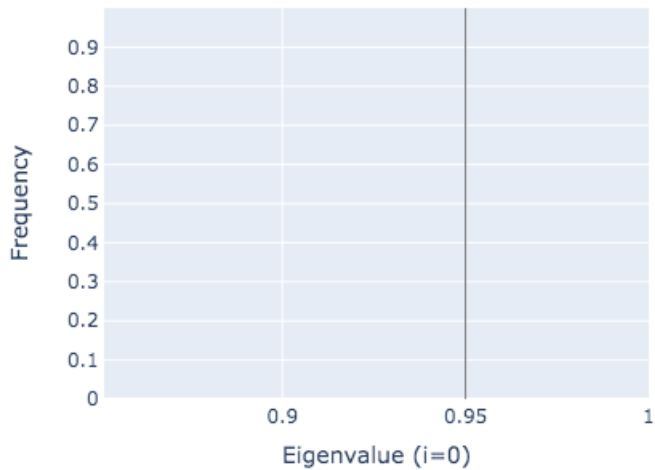
N=100, Nrep=10



Min: -5.987805948691623e-08
 Max: 0.0869415489901971
 |Max-Min|: 0.08694160886825658

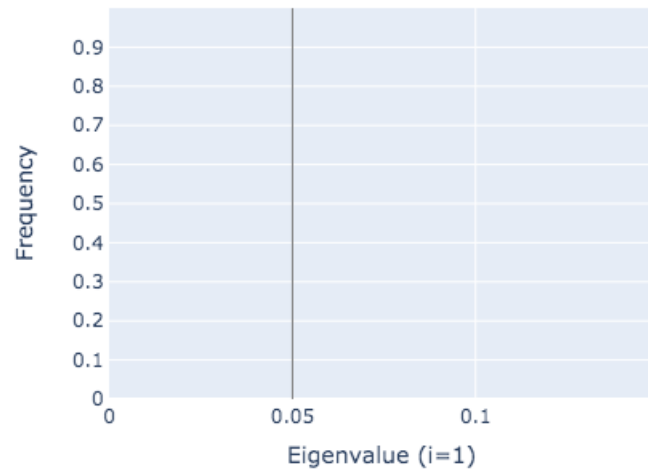
Case 7: Least Squares (False)

N=10, Nrep=10



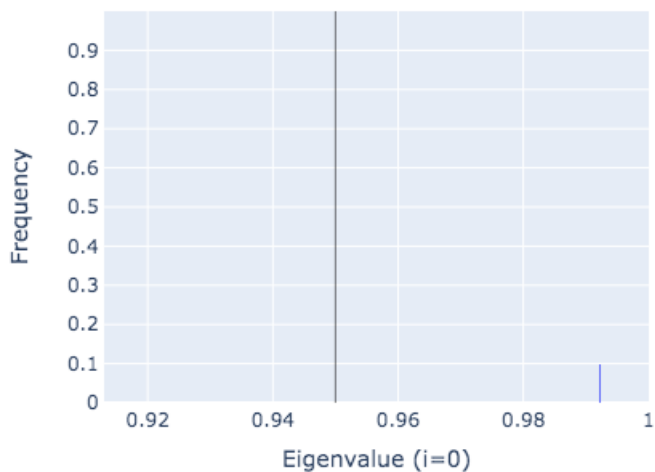
Min: 0.8513641844631532
 Max: 1.0000001781169368
 |Max-Min|: 0.14863599365378355

N=10, Nrep=10



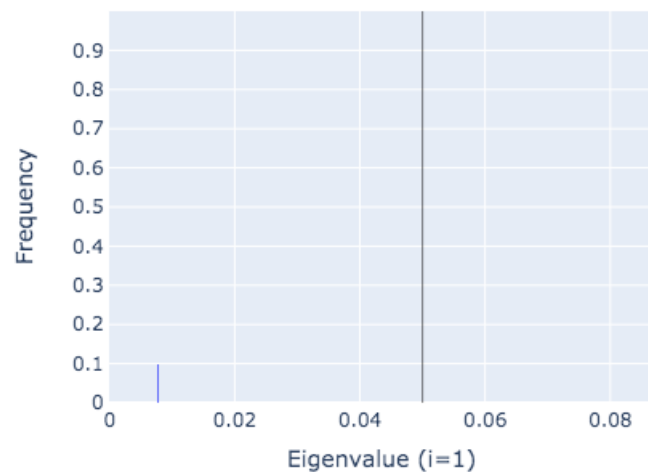
Min: -7.632783294297951e-17
 Max: 0.14863581553684663
 |Max-Min|: 0.1486358155368467

N=100, Nrep=10



Min: 0.9130584510098023
 Max: 1.0000001555680968
 |Max-Min|: 0.08694170455829453

N=100, Nrep=10



Min: 3.06287113727155e-18
 Max: 0.08694154899019754
 |Max-Min|: 0.08694154899019754

Sum of unphysical eigenvalues

Case 0: Linear (True)

N=10

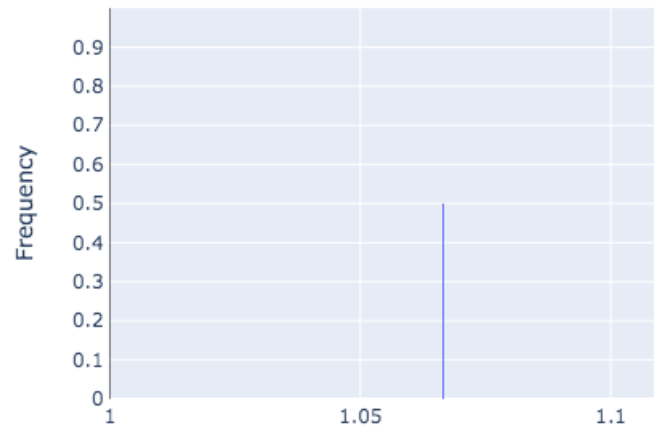
Number of unphysical estimates=6

N=10, Nrep=10



Min: -0.10858061945018471
 Max: -0.009175077217315732
 |Max-Min|: 0.09940554223286899

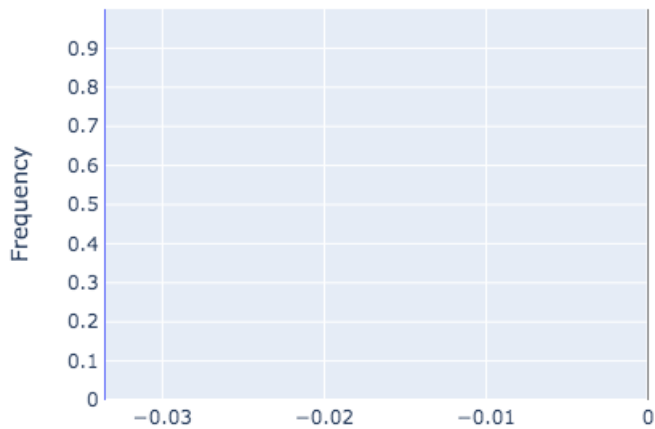
N=10, Nrep=10



Min: 1.0091750772173156
 Max: 1.1085806194501846
 |Max-Min|: 0.09940554223286902

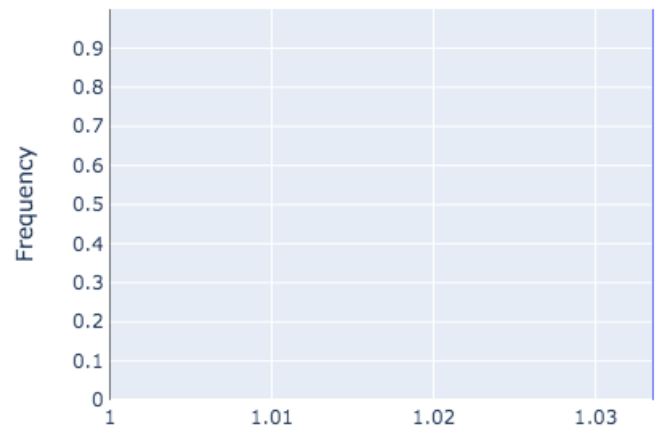
N=100
 Number of unphysical estimates=1

N=100, Nrep=10



Min: -0.03356476460190986
 Max: -0.03356476460190986
 |Max-Min|: 0.0

N=100, Nrep=10

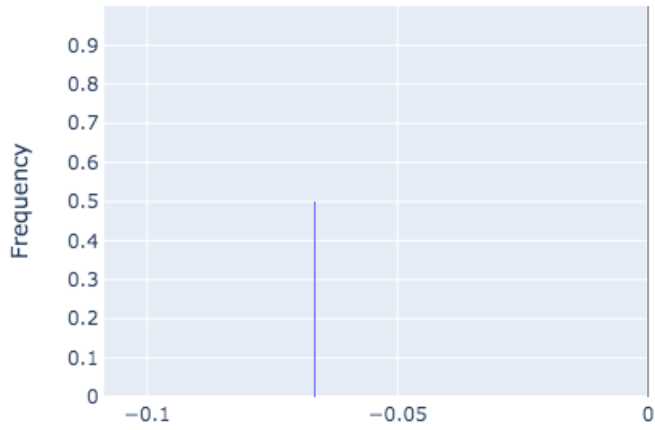


Min: 1.0335647646019095
 Max: 1.0335647646019095
 |Max-Min|: 0.0

Case 1: Linear (False)

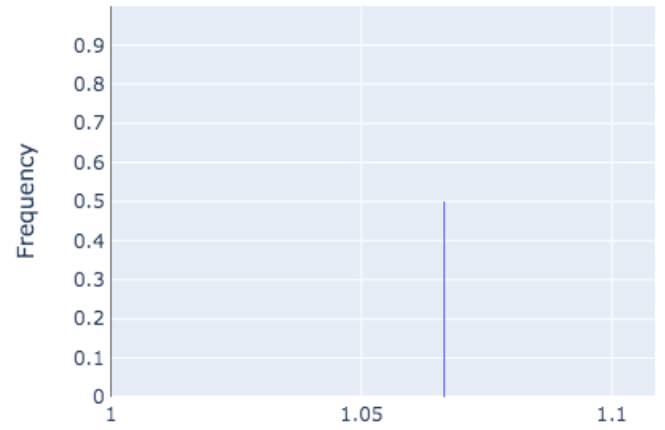
N=10
 Number of unphysical estimates=6

N=10, Nrep=10



Min: -0.10858061945018459
 Max: -0.009175077217315586
 |Max-Min|: 0.099405542232869

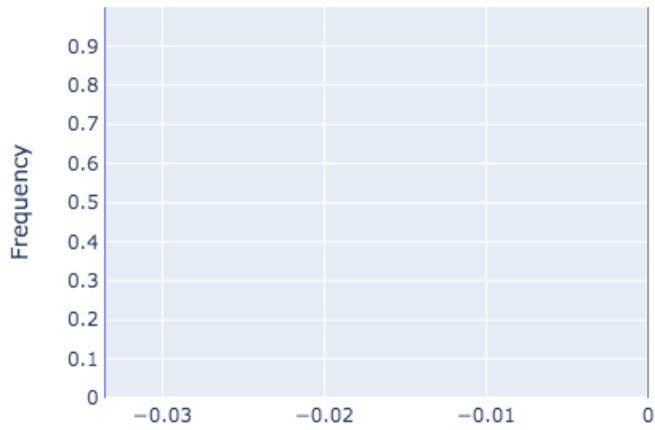
N=10, Nrep=10



Min: 1.0091750772173156
 Max: 1.1085806194501846
 |Max-Min|: 0.09940554223286902

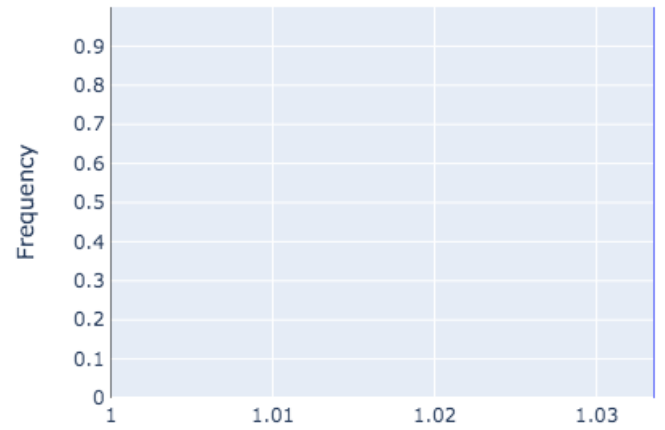
N=100
 Number of unphysical estimates=1

N=100, Nrep=10



Min: -0.03356476460190975
 Max: -0.03356476460190975
 |Max-Min|: 0.0

N=100, Nrep=10

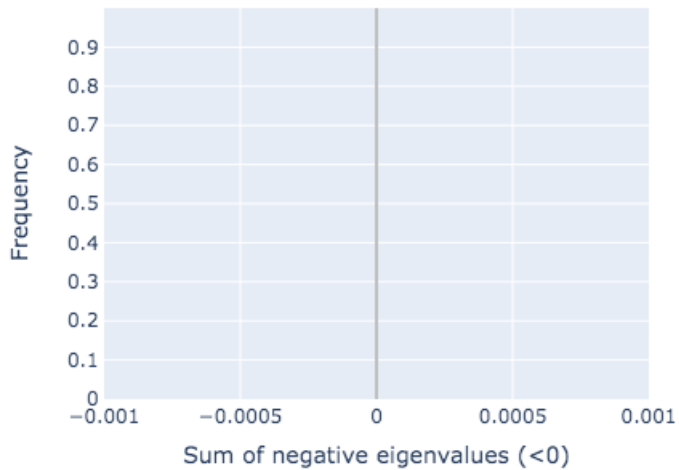


Min: 1.0335647646019095
 Max: 1.0335647646019095
 |Max-Min|: 0.0

Case 2: ProjectedLinear (True)

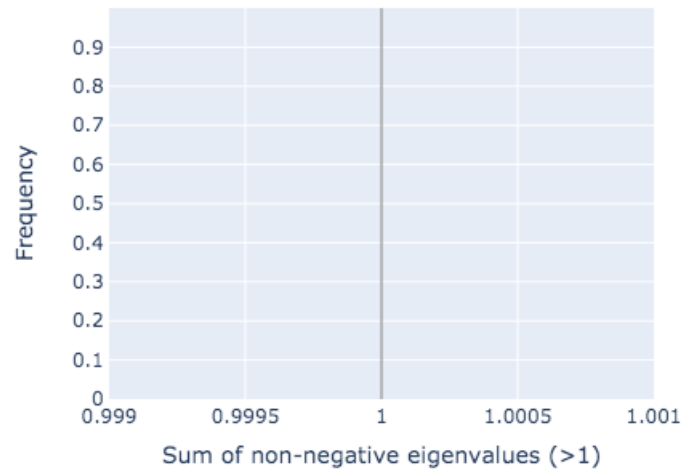
N=10
 Number of unphysical estimates=0

N=10, Nrep=10



Min: -
Max: -
|Max-Min|: -

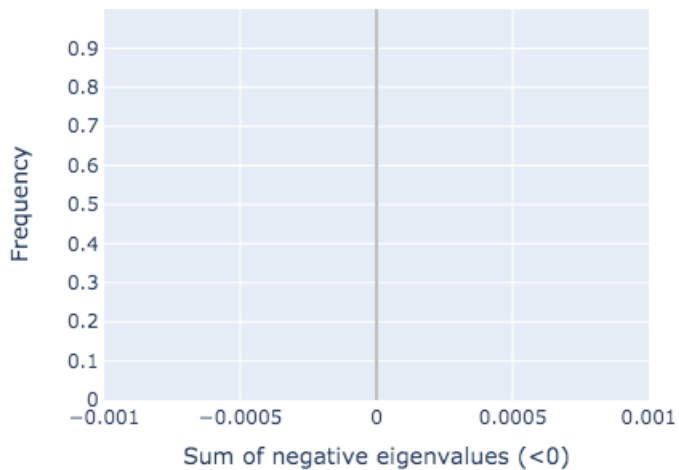
N=10, Nrep=10



Min: -
Max: -
|Max-Min|: -

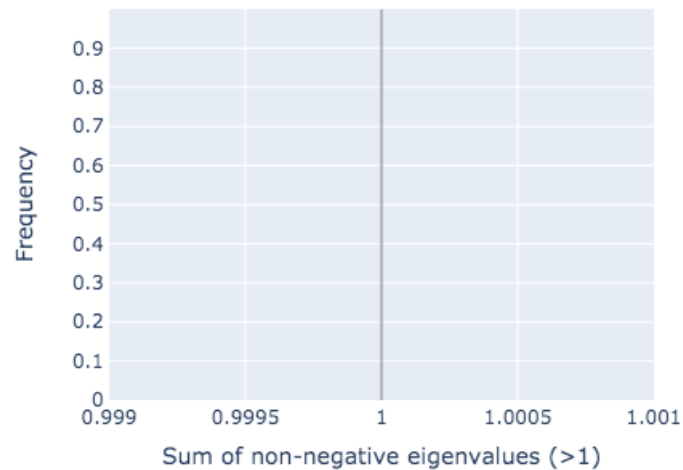
N=100
Number of unphysical estimates=0

N=100, Nrep=10



Min: -
Max: -
|Max-Min|: -

N=100, Nrep=10

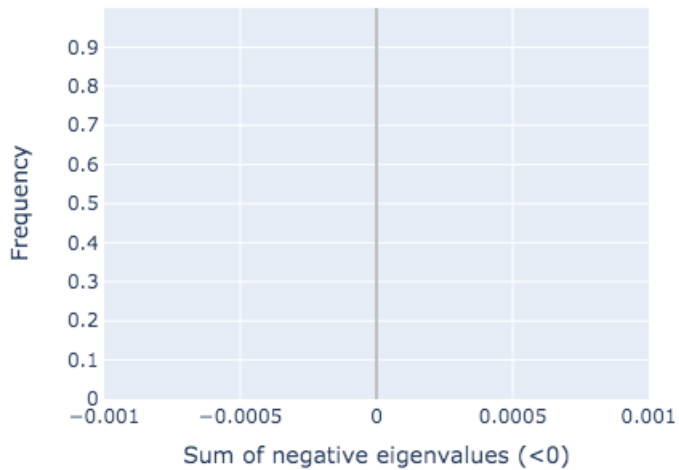


Min: -
Max: -
|Max-Min|: -

Case 3: ProjectedLinear (False)

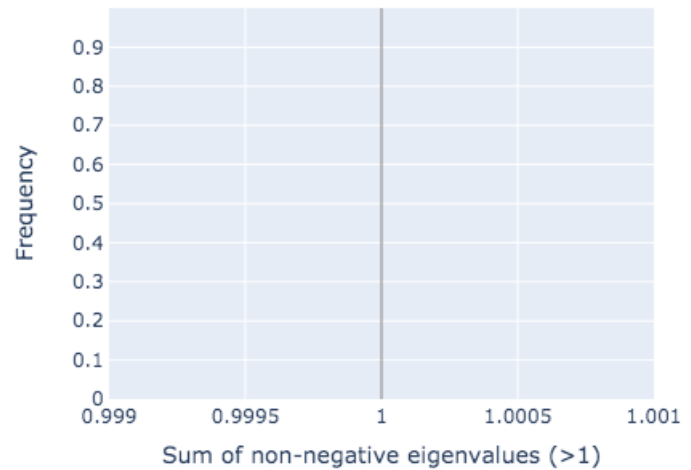
N=10
Number of unphysical estimates=0

N=10, Nrep=10



Min: -
Max: -
|Max-Min|: -

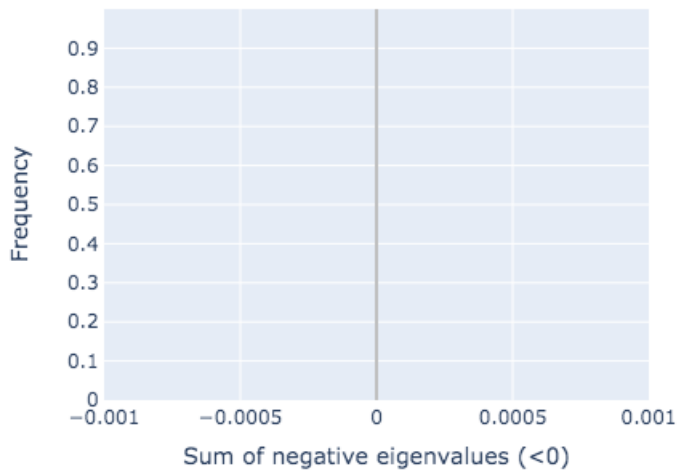
N=10, Nrep=10



Min: -
Max: -
|Max-Min|: -

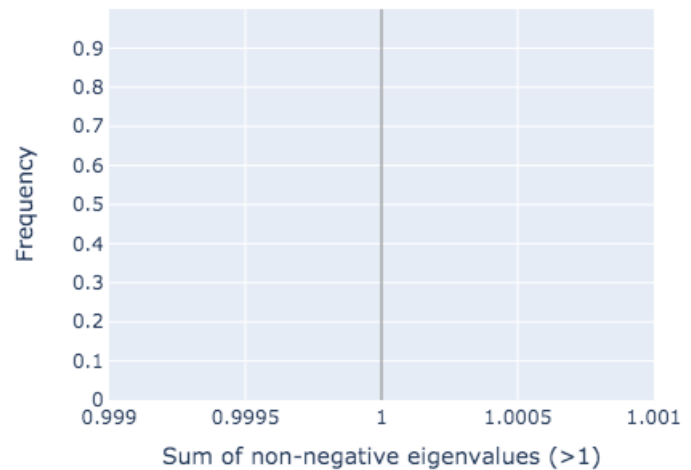
N=100
Number of unphysical estimates=0

N=100, Nrep=10



Min: -
Max: -
|Max-Min|: -

N=100, Nrep=10

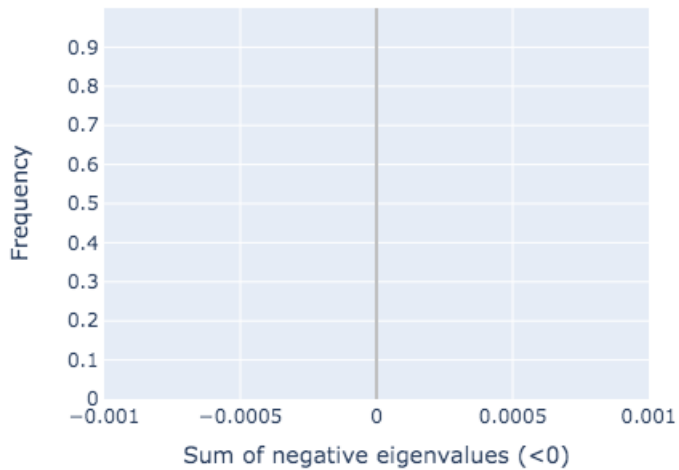


Min: -
Max: -
|Max-Min|: -

Case 4: Maximum-Likelihood (True)

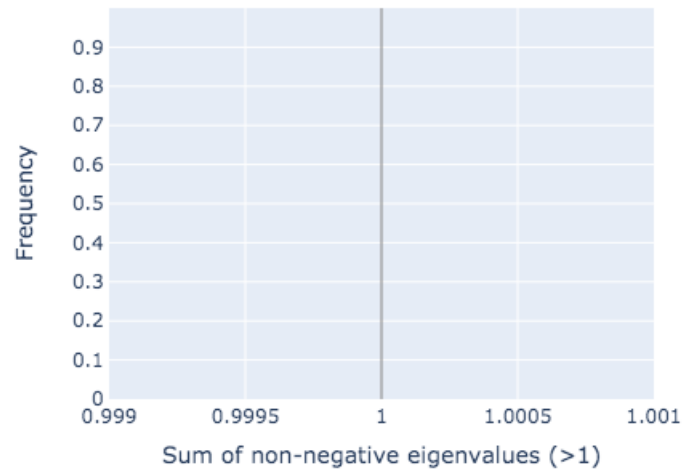
N=10
Number of unphysical estimates=0

N=10, Nrep=10



Min: -
Max: -
|Max-Min|: -

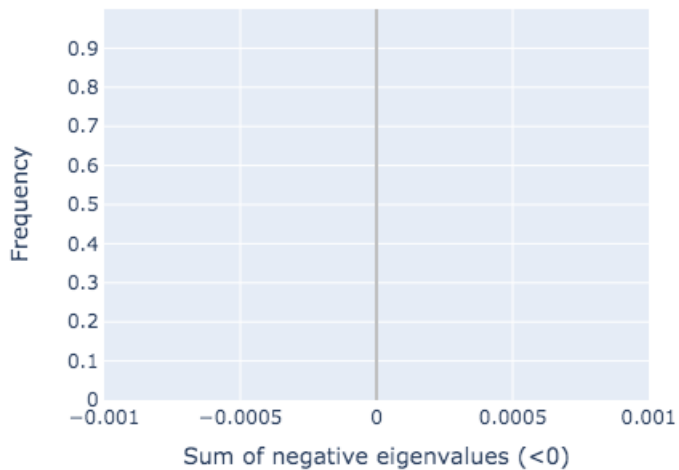
N=10, Nrep=10



Min: -
Max: -
|Max-Min|: -

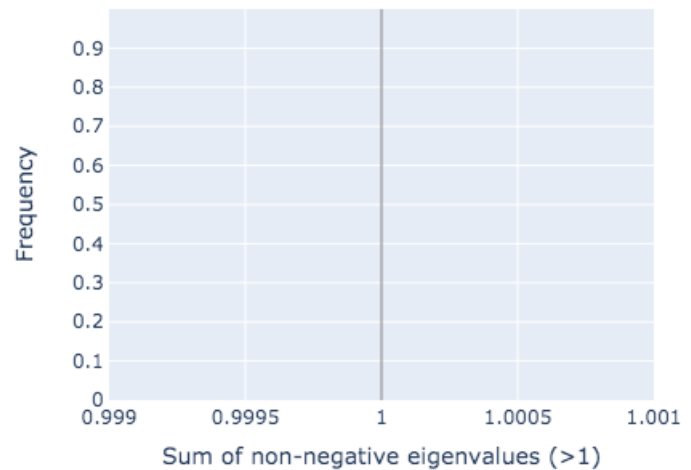
N=100
Number of unphysical estimates=0

N=100, Nrep=10



Min: -
Max: -
|Max-Min|: -

N=100, Nrep=10

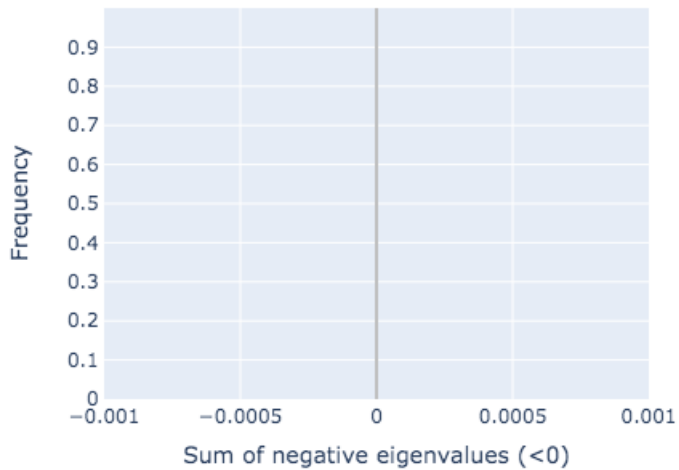


Min: -
Max: -
|Max-Min|: -

Case 5: Maximum-Likelihood (False)

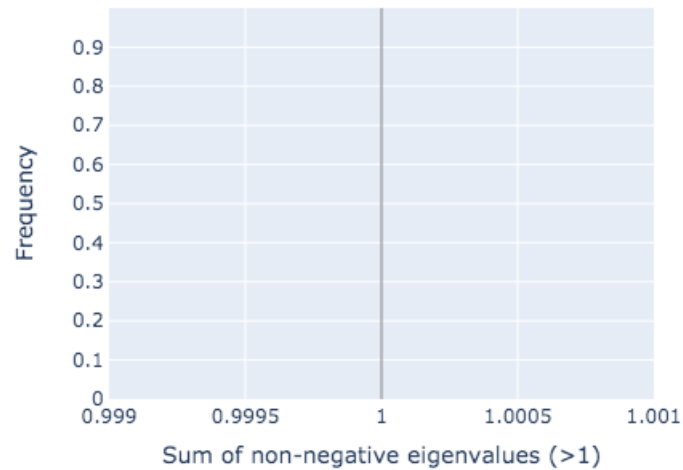
N=10
Number of unphysical estimates=0

N=10, Nrep=10



Min: -
Max: -
|Max-Min|: -

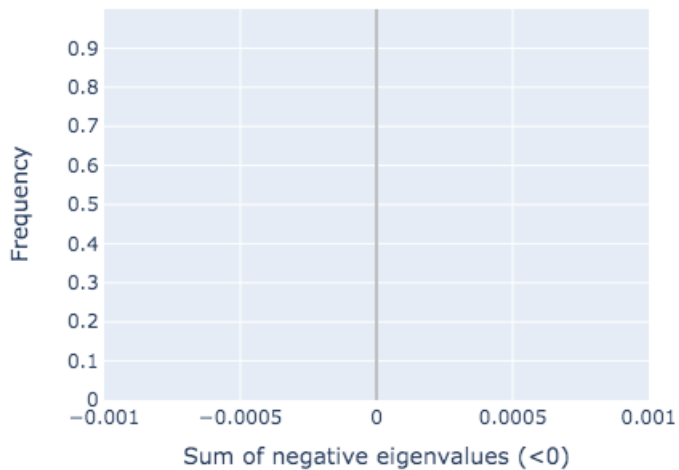
N=10, Nrep=10



Min: -
Max: -
|Max-Min|: -

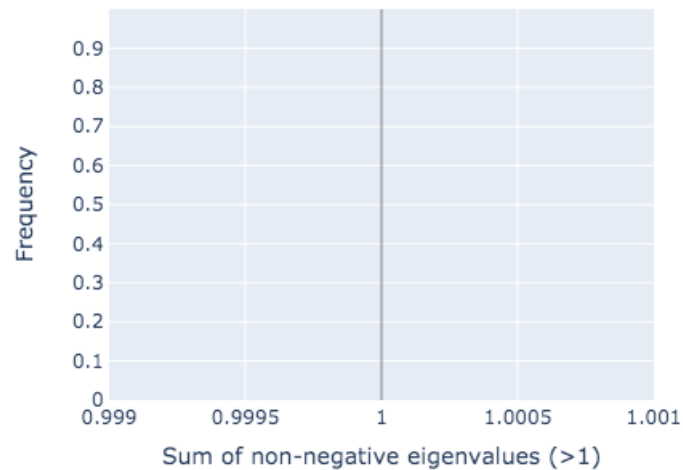
N=100
Number of unphysical estimates=0

N=100, Nrep=10



Min: -
Max: -
|Max-Min|: -

N=100, Nrep=10

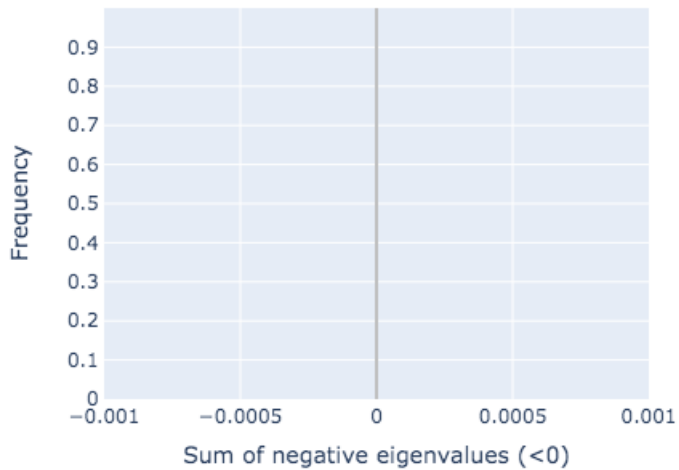


Min: -
Max: -
|Max-Min|: -

Case 6: Least Squares (True)

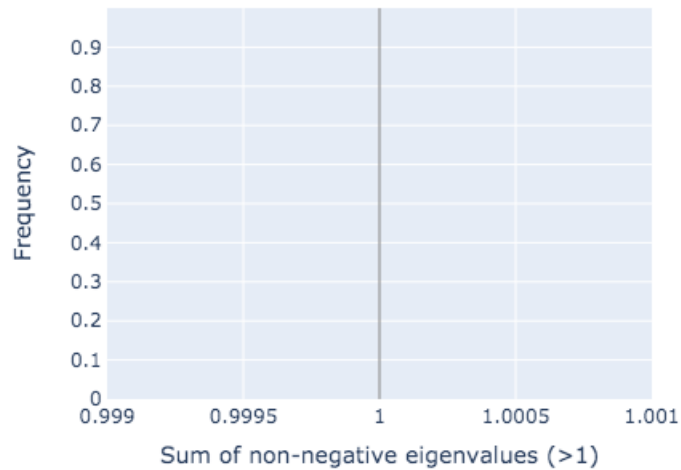
N=10
Number of unphysical estimates=0

N=10, Nrep=10



Min: -
Max: -
|Max-Min|: -

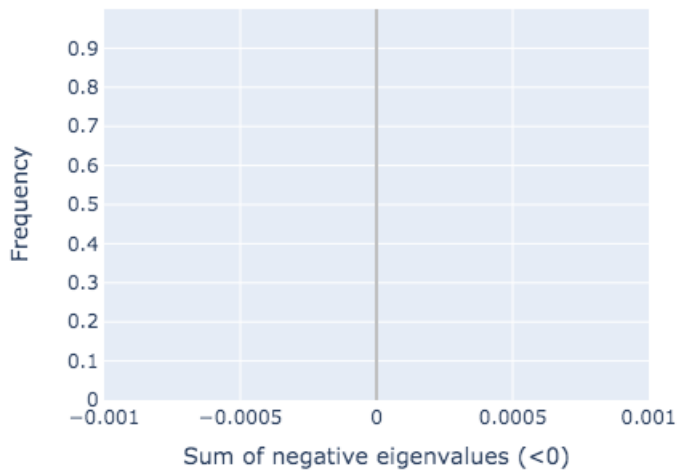
N=10, Nrep=10



Min: -
Max: -
|Max-Min|: -

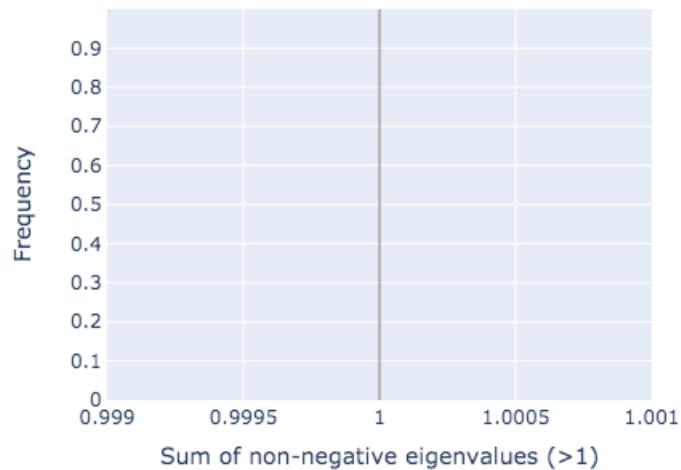
N=100
Number of unphysical estimates=0

N=100, Nrep=10



Min: -
Max: -
|Max-Min|: -

N=100, Nrep=10

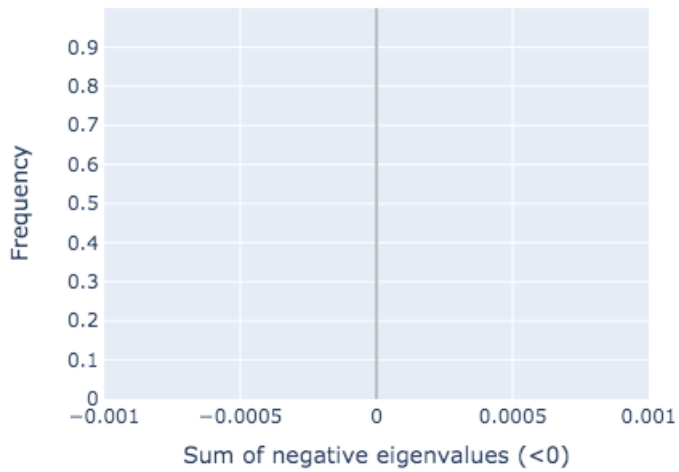


Min: -
Max: -
|Max-Min|: -

Case 7: Least Squares (False)

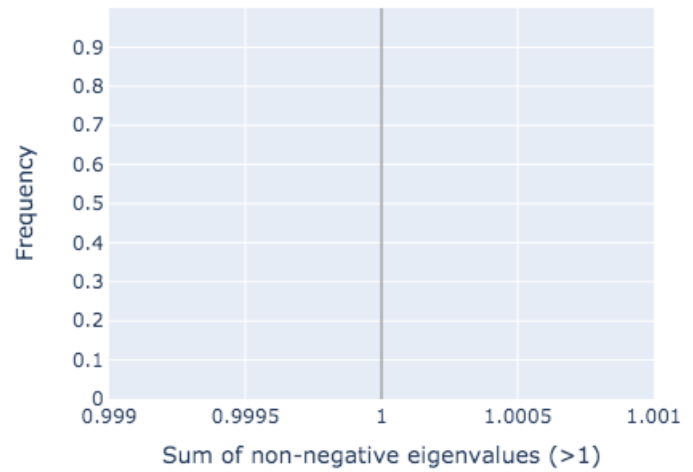
N=10
Number of unphysical estimates=0

N=10, Nrep=10



Min: -
Max: -
|Max-Min|: -

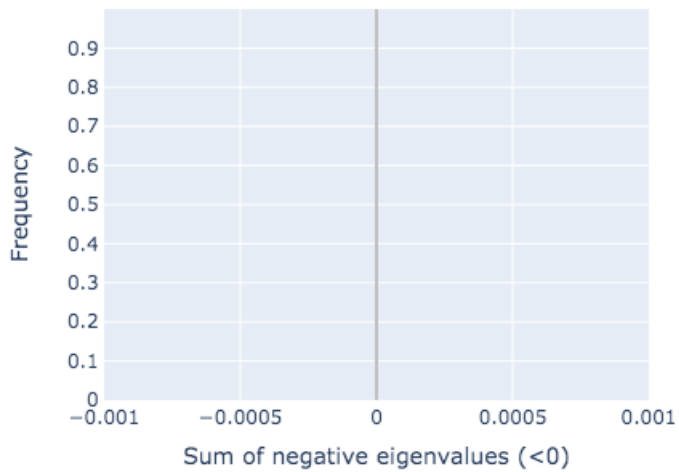
N=10, Nrep=10



Min: -
Max: -
|Max-Min|: -

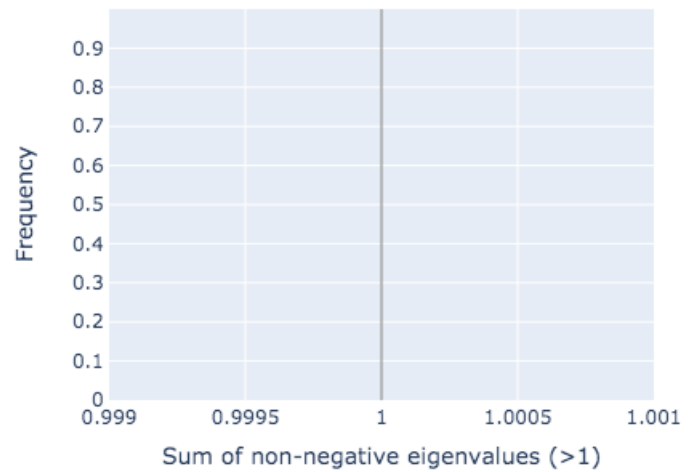
N=100
Number of unphysical estimates=0

N=100, Nrep=10



Min: -
Max: -
|Max-Min|: -

N=100, Nrep=10



Min: -
Max: -
|Max-Min|: -