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Computation time in total

Total	0.6753138264020284min.
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Tolerance of physicality constraint violation

		True	False
Tolerance at physicality violation test	equality constraint	1.00e-13	1.00e-05
	inequality constraint	1.00e-05	1.00e-05

Experimental condition

Type of tomography	StandardQst
Nrep	10
N	[10, 100]
RNG seed	777

True object

Type	State
Dim	2
Vec	[0.70710678 0.11208398 -0.03085986 0.64541004]

Tester objects

0	Type	Povm
	Dim	2
	Number of outcomes	2
	Vecs	[[0.70621296 0.63082314 0.03954112 0.00696134] [0.7080006 -0.63082314 -0.03954112 -0.00696134]]
1	Type	Povm
	Dim	2
	Number of outcomes	2
	Vecs	[[0.70953915 0.00994222 0.64613632 0.08130017] [0.70467441 -0.00994222 -0.64613632 -0.08130017]]
2	Type	Povm
	Dim	2
	Number of outcomes	2
	Vecs	[[0.75226367 -0.08640619 0.00622688 0.63087349] [0.66194989 0.08640619 -0.00622688 -0.63087349]]

Cases

	Name	Param	Tomography	Estimator
0	Linear (True)	True	StandardQst	Linear
1	Linear (False)	False	StandardQst	Linear
2	ProjectedLinear (True)	True	StandardQst	ProjectedLinear
3	ProjectedLinear (False)	False	StandardQst	ProjectedLinear
4	Maximum-Likelihood (True)	True	StandardQst	LossMinimization
5	Maximum-Likelihood (False)	False	StandardQst	LossMinimization

6	Least Squares (True)	True	StandardQst	LossMinimization
7	Least Squares (False)	False	StandardQst	LossMinimization

Computation time of estimators

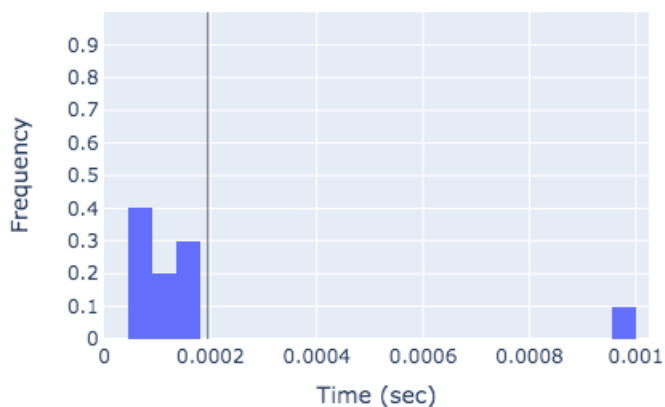
Table

	Name	N	Nrep	Mean (sec)	Std (sec)
0	Linear (True)	10	10	0.000195	0.000263
1	"	100	10	0.000456	0.001113
2	Linear (False)	10	10	0.000096	0.000052
3	"	100	10	0.000080	0.000034
4	ProjectedLinear (True)	10	10	0.011068	0.009616
5	"	100	10	0.008957	0.014032
6	ProjectedLinear (False)	10	10	0.014005	0.015282
7	"	100	10	0.016040	0.036869
8	Maximum-Likelihood (True)	10	10	0.696642	0.277371
9	"	100	10	0.644034	0.177177
10	Maximum-Likelihood (False)	10	10	0.817931	0.399357
11	"	100	10	1.094831	0.433017
12	Least Squares (True)	10	10	0.236932	0.172469
13	"	100	10	0.139705	0.107072
14	Least Squares (False)	10	10	0.233487	0.132767
15	"	100	10	0.137425	0.087755

Histogram

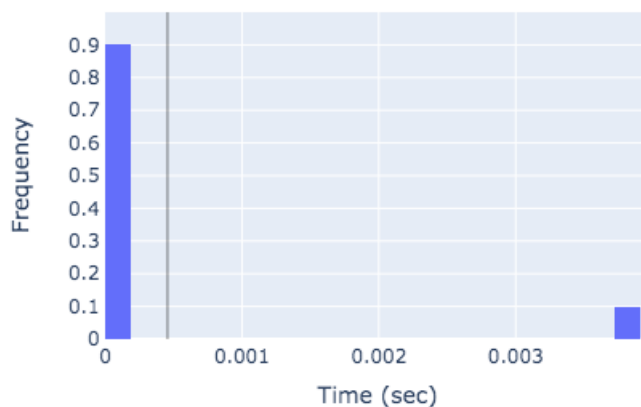
Case 0: Linear (True)

N=10, Nrep=10
Mean = 0.0001950502395629883(sec)



Min: 7.009506225585938e-05
Max: 0.0009791851043701172
|Max-Min|: 0.0009090900421142578

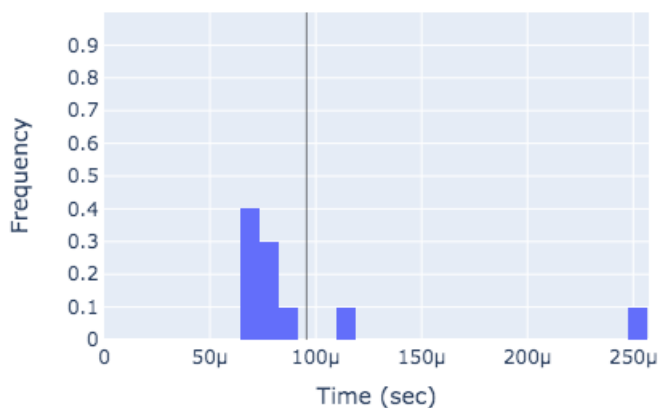
N=100, Nrep=10
Mean = 0.0004558086395263672(sec)



Min: 6.890296936035156e-05
Max: 0.0037941932678222656
|Max-Min|: 0.003725290298461914

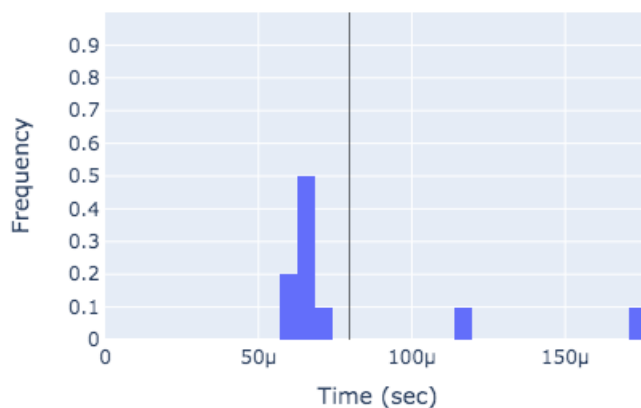
Case 1: Linear (False)

N=10, Nrep=10
Mean = 9.565353393554687e-05(sec)



Min: 6.4849853515625e-05
Max: 0.00024819374084472656
|Max-Min|: 0.00018334388732910156

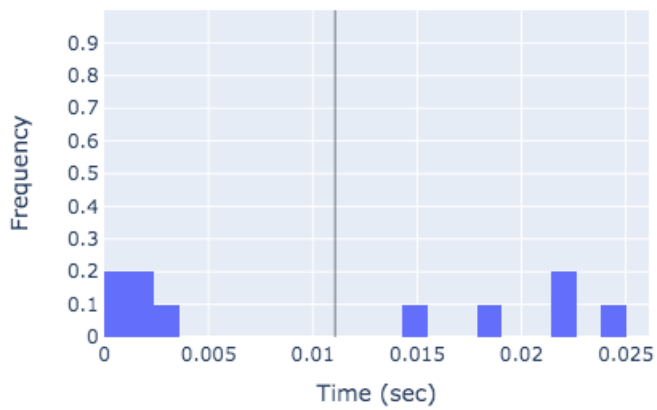
N=100, Nrep=10
Mean = 7.965564727783204e-05(sec)



Min: 5.793571472167969e-05
Max: 0.00017189979553222656
|Max-Min|: 0.00011396408081054688

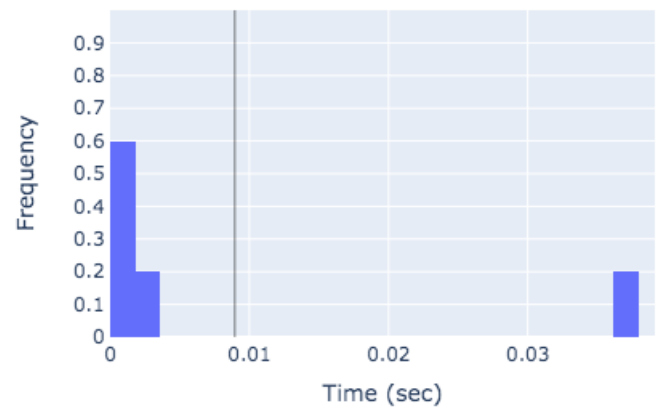
Case 2: ProjectedLinear (True)

N=10, Nrep=10
Mean = 0.011067652702331543(sec)



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Min: 0.001100778579711914
Max: 0.02491307258605957
|Max-Min|: 0.023812294006347656
```

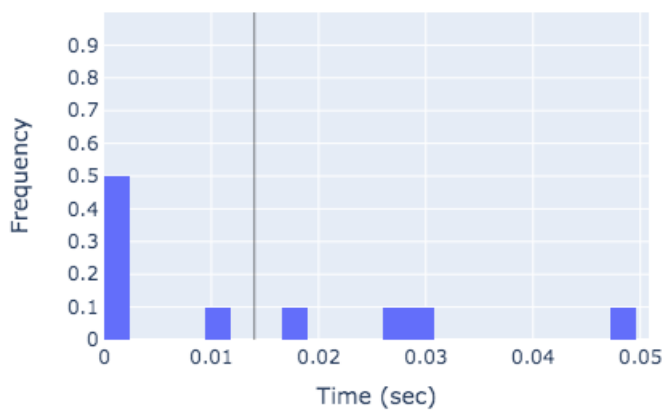
N=100, Nrep=10
Mean = 0.00895678997039795(sec)



Min: 0.0011699199676513672
Max: 0.03731894493103027
|Max-Min|: 0.036149024963378906

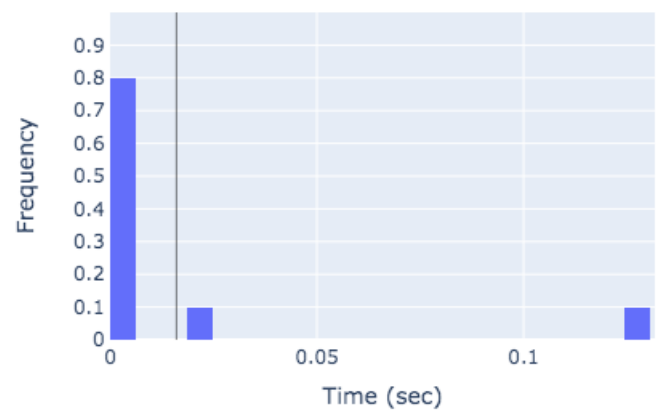
Case 3: ProjectedLinear (False)

N=10, Nrep=10
Mean = 0.014005422592163086(sec)



Min: 0.0011587142944335938
Max: 0.04842996597290039
|Max-Min|: 0.0472712516784668

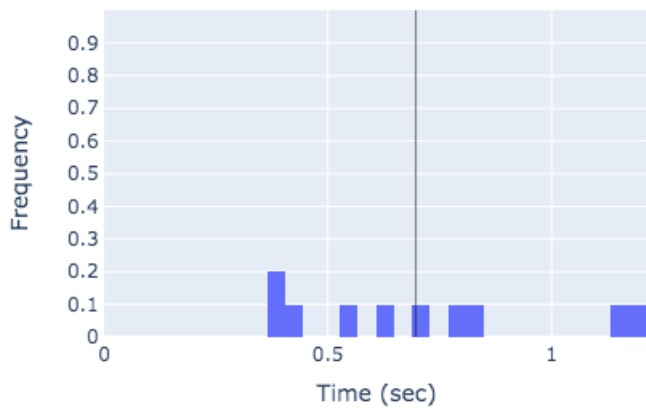
N=100, Nrep=10
Mean = 0.01603987216949463(sec)



Min: 0.0011868476867675781
Max: 0.12553000450134277
|Max-Min|: 0.1243431568145752

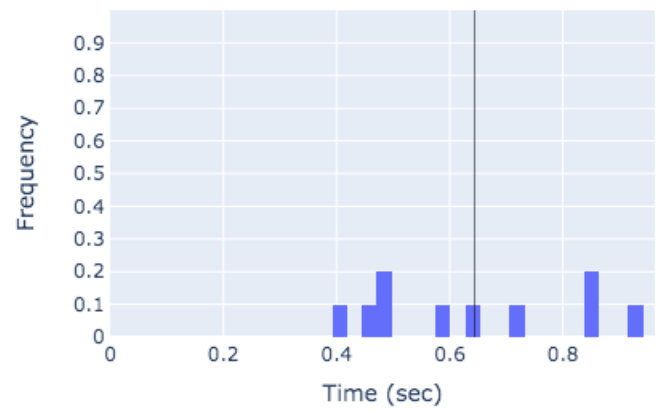
Case 4: Maximum-Likelihood (True)

N=10, Nrep=10
Mean = 0.6966415882110596(sec)



Min: 0.36759114265441895
Max: 1.176896572113037
|Max-Min|: 0.8093054294586182

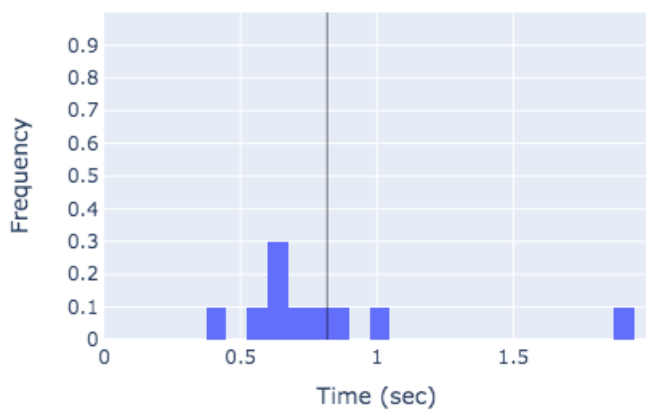
N=100, Nrep=10
Mean = 0.6440342187881469(sec)



Min: 0.4132828712463379
Max: 0.9362344741821289
|Max-Min|: 0.522951602935791

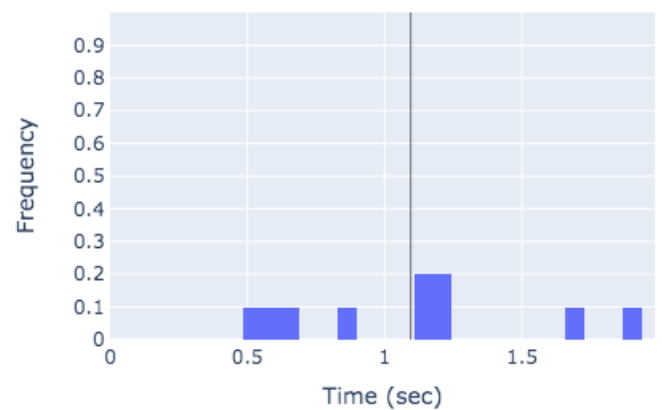
Case 5: Maximum-Likelihood (False)

N=10, Nrep=10
Mean = 0.8179309368133545(sec)



Min: 0.42667341232299805
Max: 1.922178030014038
|Max-Min|: 1.49550461769104

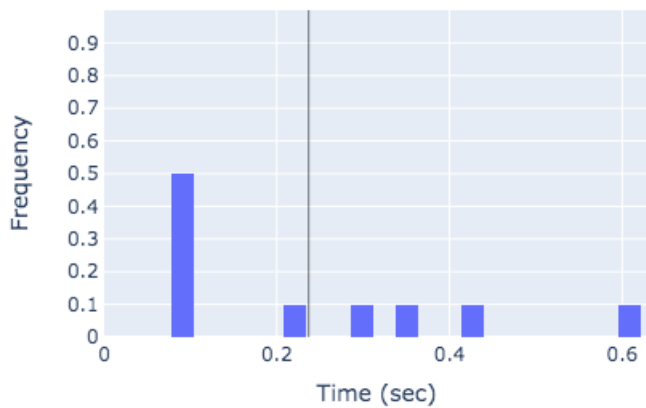
N=100, Nrep=10
Mean = 1.0948306560516357(sec)



Min: 0.532585859298706
Max: 1.915055274963379
|Max-Min|: 1.3824694156646729

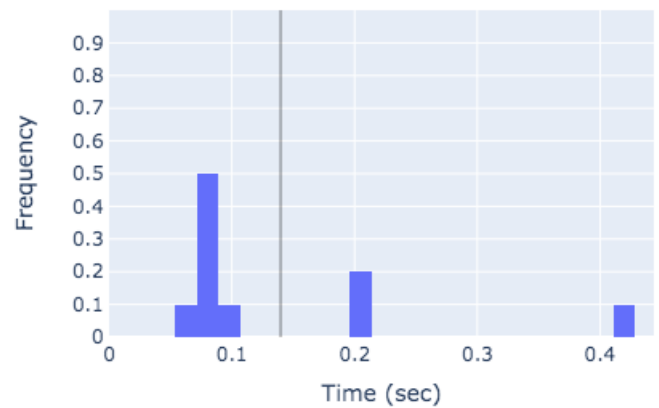
Case 6: Least Squares (True)

N=10, Nrep=10
Mean = 0.23693227767944336(sec)



Min: 0.08672404289245605
Max: 0.6049962043762207
|Max-Min|: 0.5182721614837646

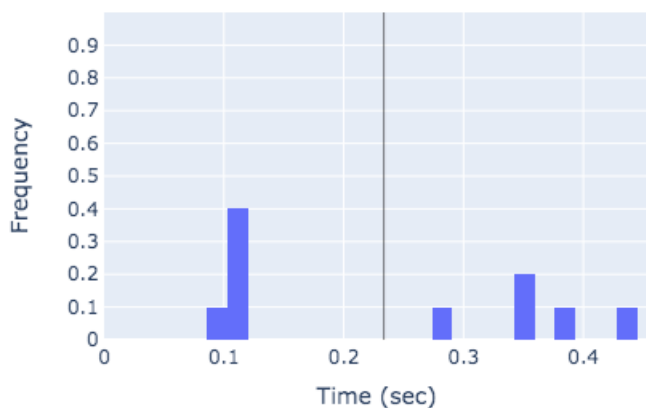
N=100, Nrep=10
Mean = 0.1397045612335205(sec)



Min: 0.06882095336914062
Max: 0.42597389221191406
|Max-Min|: 0.35715293884277344

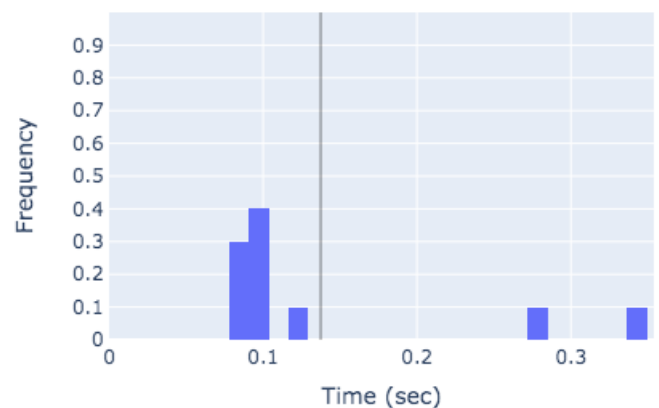
Case 7: Least Squares (False)

N=10, Nrep=10
Mean = 0.23348743915557862(sec)



Min: 0.09544968605041504
Max: 0.4375619888305664
|Max-Min|: 0.34211230278015137

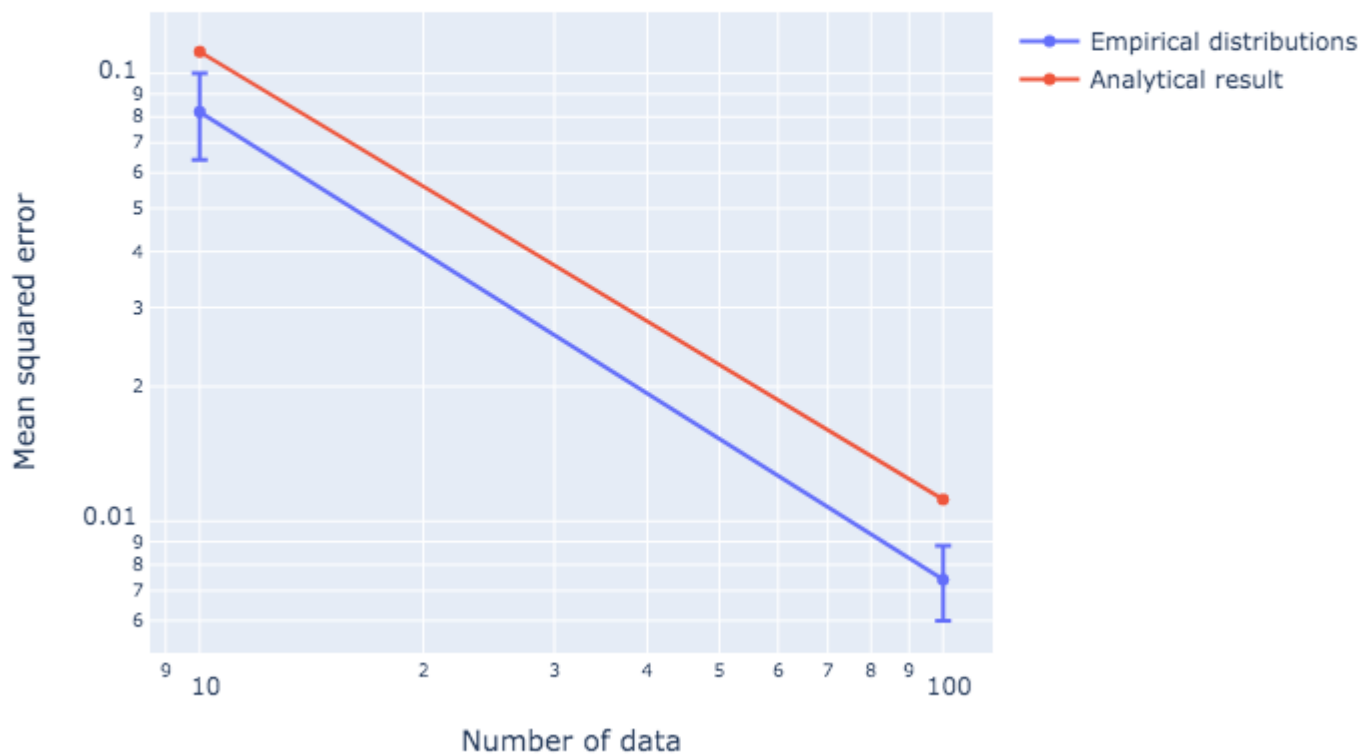
N=100, Nrep=10
Mean = 0.13742537498474122(sec)



Min: 0.08202290534973145
Max: 0.3409557342529297
|Max-Min|: 0.25893282890319824

MSE of empirical distributions

Mean squared error



Consistency test

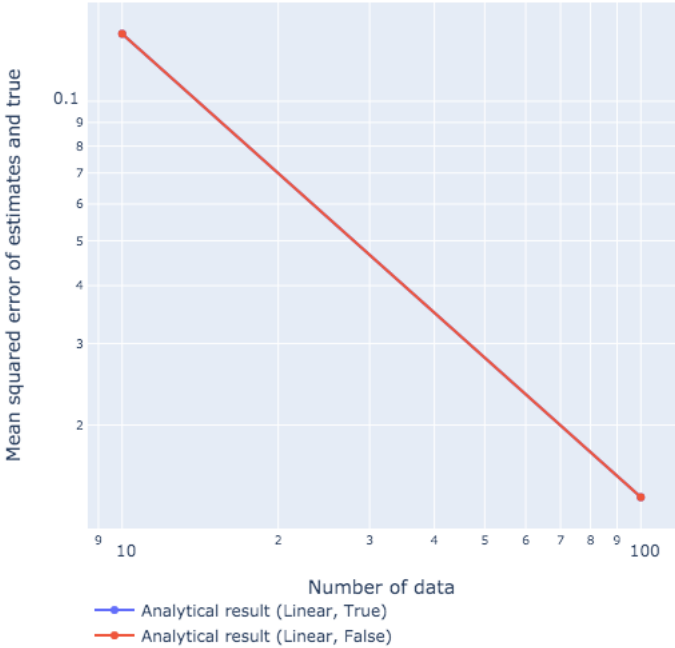
	Name	Type of tomography	Param	Estimator	Loss	Algo	Squared Error to True	Possibly OK	To be checked
0	Linear (True)	Standard Qst	True	Linear	None	None	1.71e-32	OK	not need debug
1	Linear (False)	Standard Qst	False	Linear	None	None	2.18e-32	OK	not need debug
2	ProjectedLinear (True)	Standard Qst	True	Projected Linear	None	None	1.58e-32	OK	not need debug
3	ProjectedLinear (False)	Standard Qst	False	Projected Linear	None	None	2.48e-32	OK	not need debug
4	Maximum-Likelihood (True)	Standard Qst	True	Loss Minimization	Weighted Relative Entropy	Projected Gradient Descent Backtracking	9.52e-17	OK	not need debug
5	Maximum-Likelihood (False)	Standard Qst	False	Loss Minimization	Weighted Relative Entropy	Projected Gradient Descent Backtracking	1.28e-08	NG	not need debug
6	Least Squares (True)	Standard Qst	True	Loss Minimization	Weighted Probability Based Squared Error	Projected Gradient Descent Backtracking	1.51e-30	OK	not need debug

7	Least Squares (False)	Standard Qst	False	Loss Minimization	Weighted Probability Based Squared Error	Projected Gradient Descent Backtracking	1.77e-30	OK	not need debug
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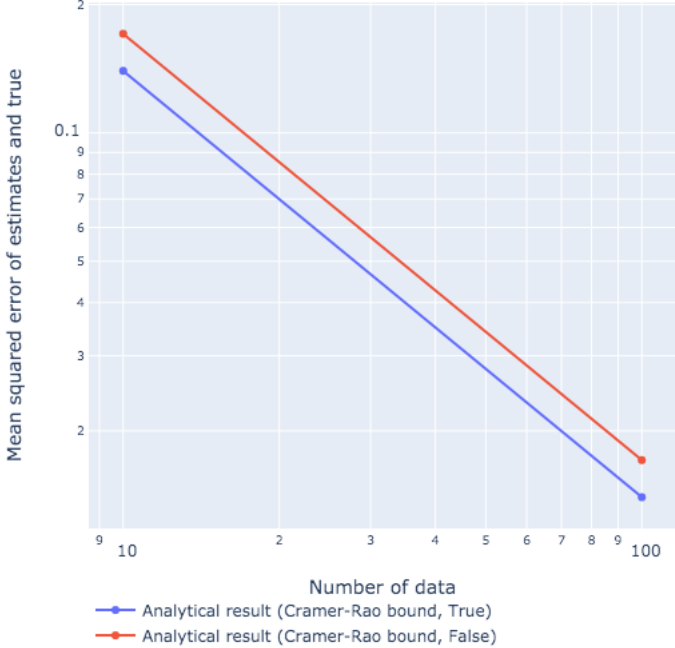
MSE of estimators

Comparison of analytical results

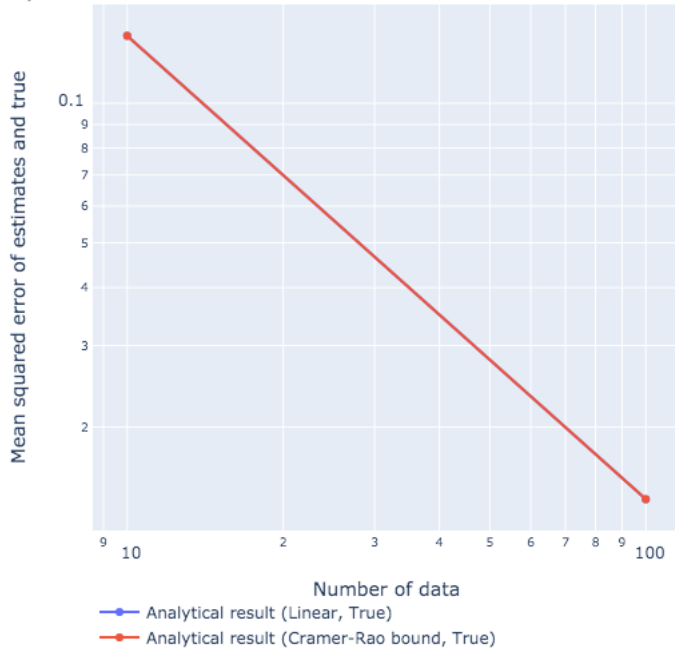
Mean squared error
Analytical result
estimator=Linear



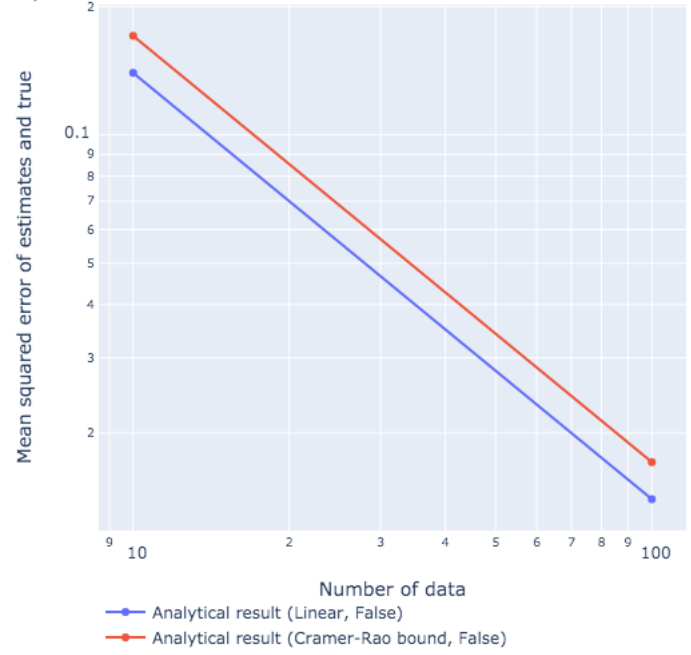
Mean squared error
Analytical result
Cramer-Rao bound



Mean squared error
Analytical result
parameterization=True

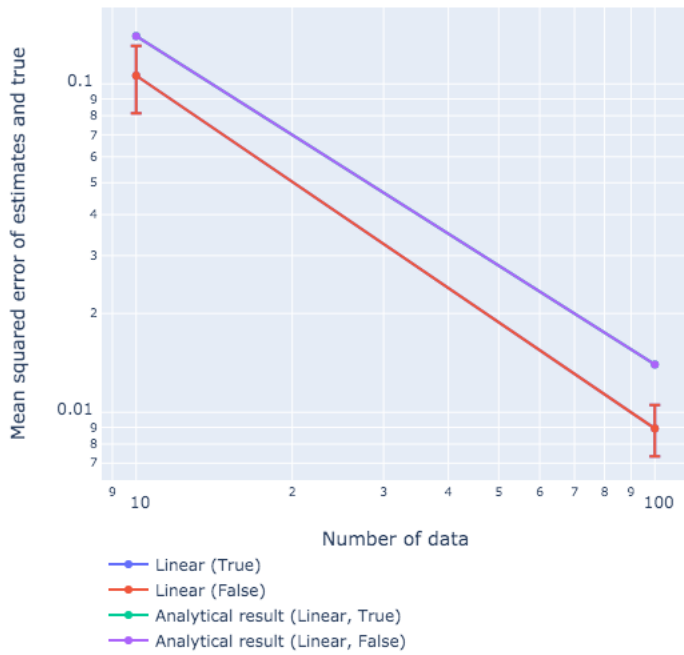


Mean squared error
Analytical result
parameterization=False

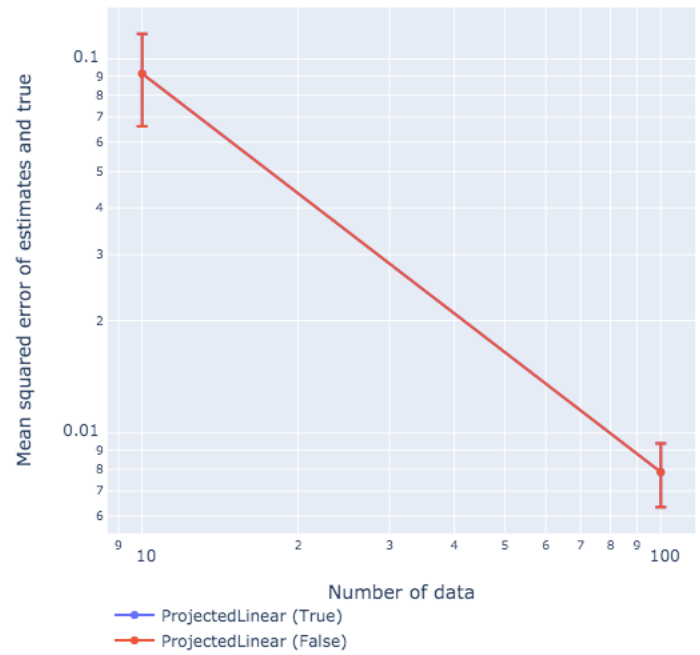


Comparison of parametrization

Mean squared error
Estimator=Linear



Mean squared error
Estimator=ProjectedLinear

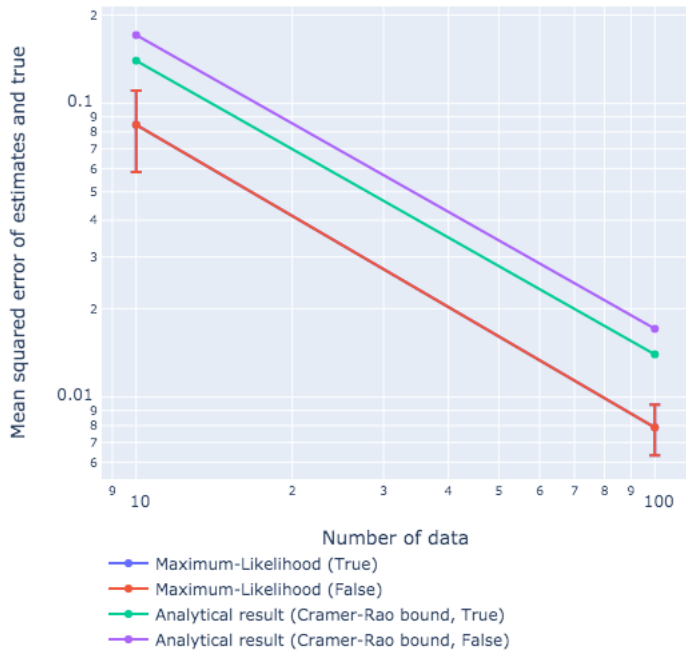


Mean squared error

Estimator=LossMinimization

Loss=WeightedRelativeEntropy

Algo=ProjectedGradientDescentBacktracking

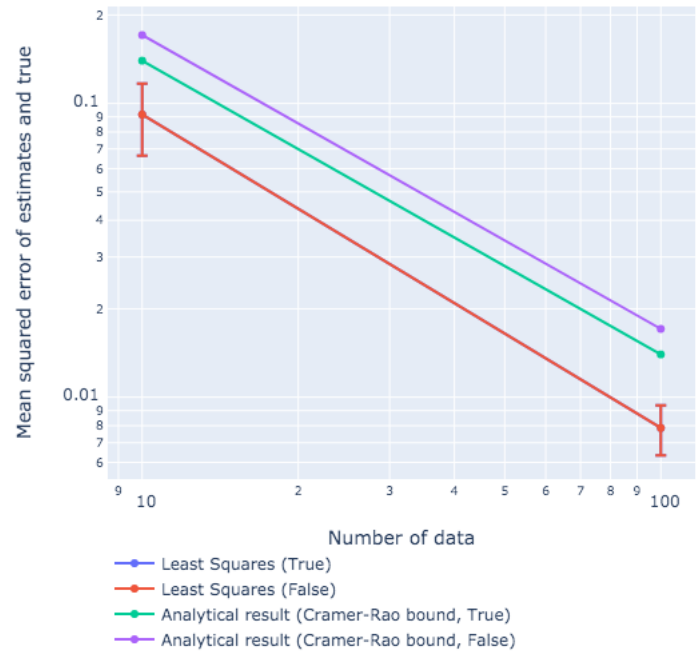


Mean squared error

Estimator=LossMinimization

Loss=WeightedProbabilityBasedSquaredError

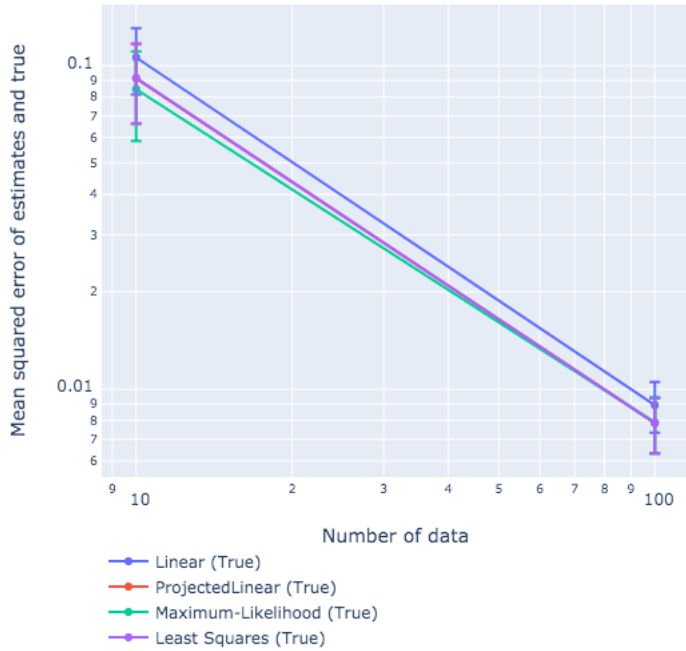
Algo=ProjectedGradientDescentBacktracking



Comparison of estimators

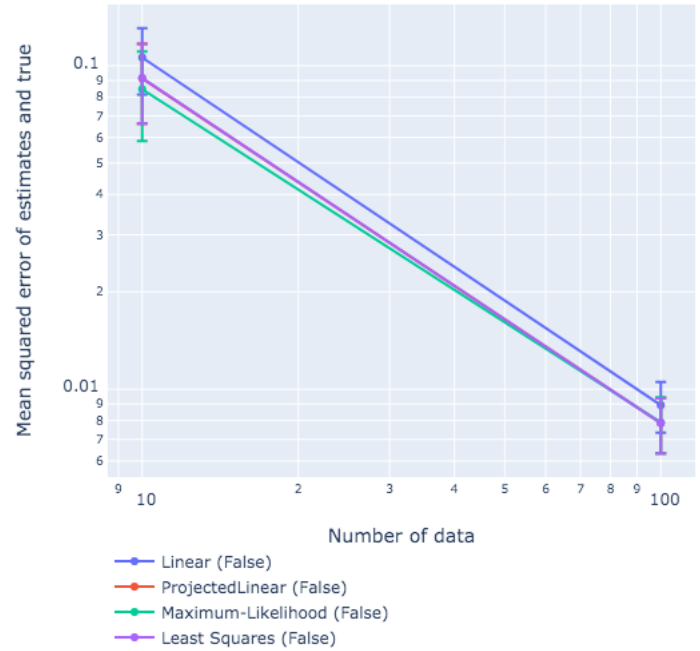
Mean squared error

parametrization=True



Mean squared error

parametrization=False

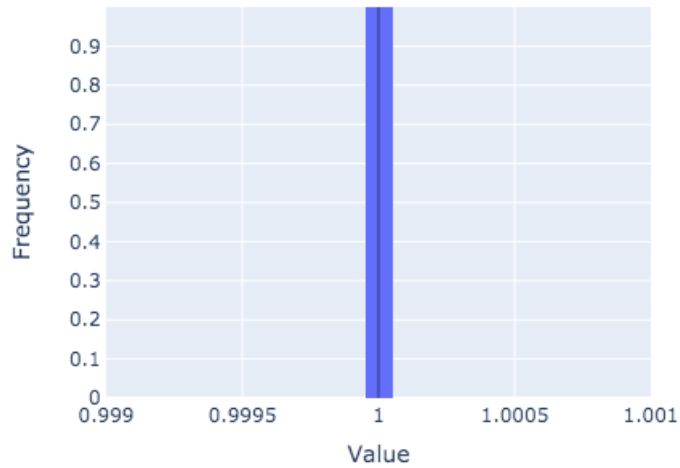


Physicality violation test

Test of equality constraint violation

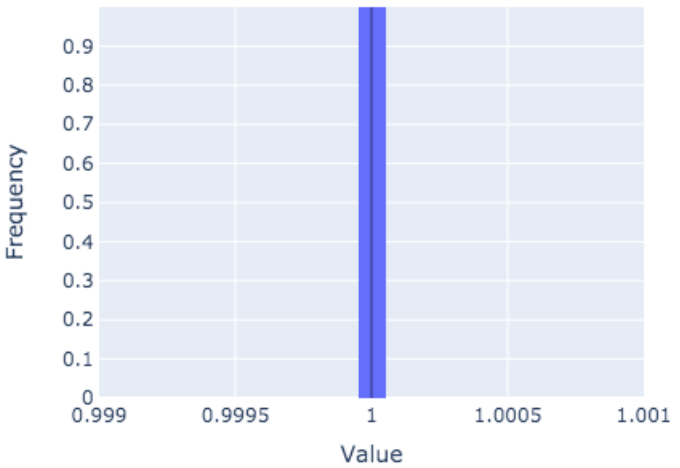
Case 0: Linear (True)

N=10, Nrep=10



Min: 0.9999999999999997
Max: 0.9999999999999999
|Max-Min|: 2.220446049250313e-16

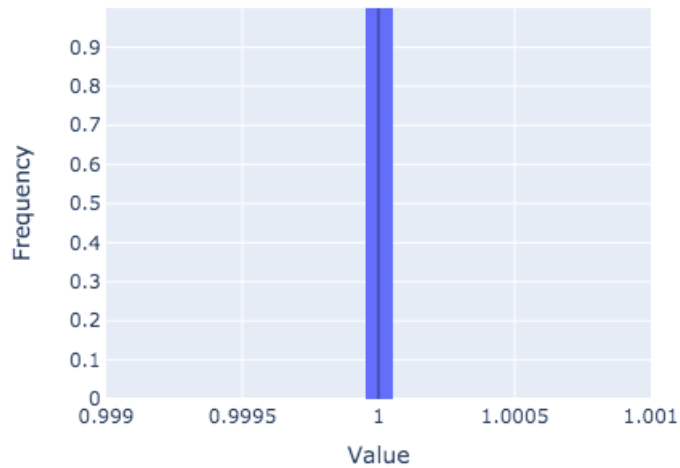
N=100, Nrep=10



Min: 0.9999999999999998
Max: 0.9999999999999998
|Max-Min|: 0.0

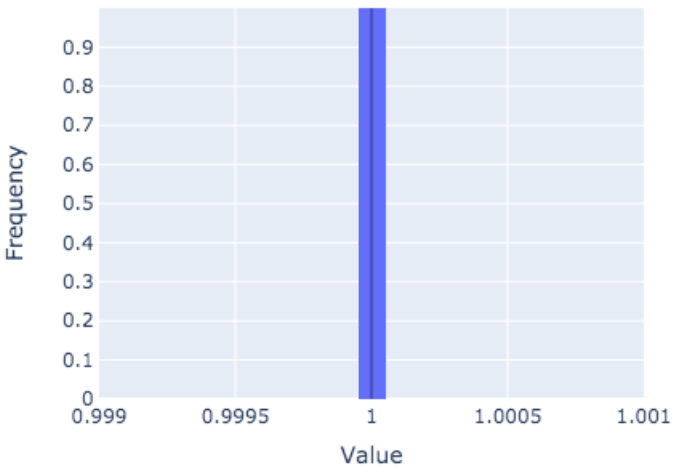
Case 1: Linear (False)

N=10, Nrep=10



Min: 1.0
Max: 1.0000000000000002
|Max-Min|: 2.220446049250313e-16

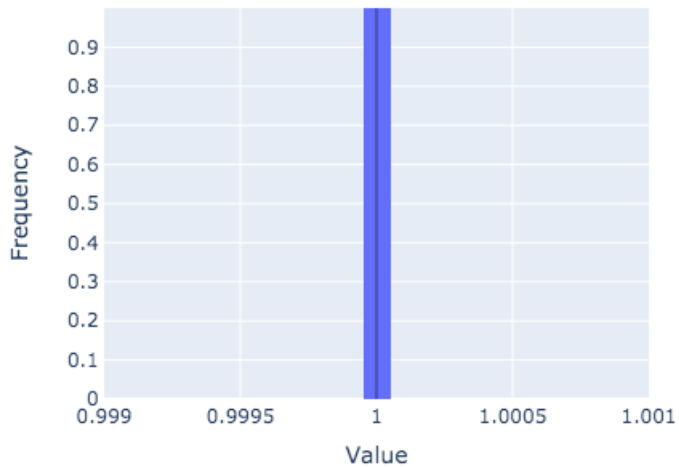
N=100, Nrep=10



Min: 0.9999999999999999
Max: 1.0000000000000002
|Max-Min|: 3.3306690738754696e-16

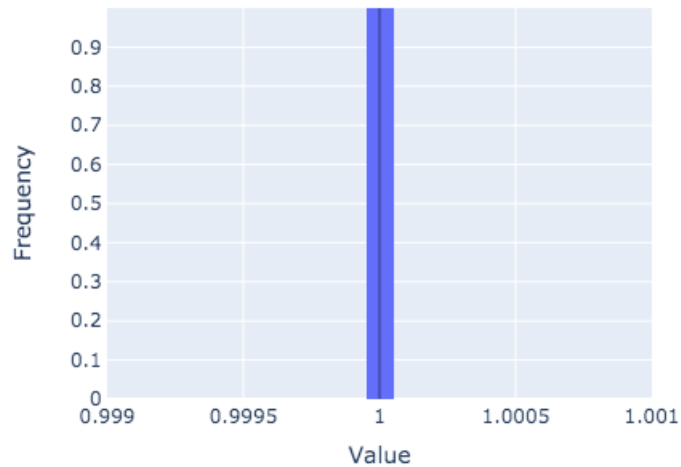
Case 2: ProjectedLinear (True)

N=10, Nrep=10



Min: 0.9999999999999998
Max: 0.9999999999999998
|Max-Min|: 0.0

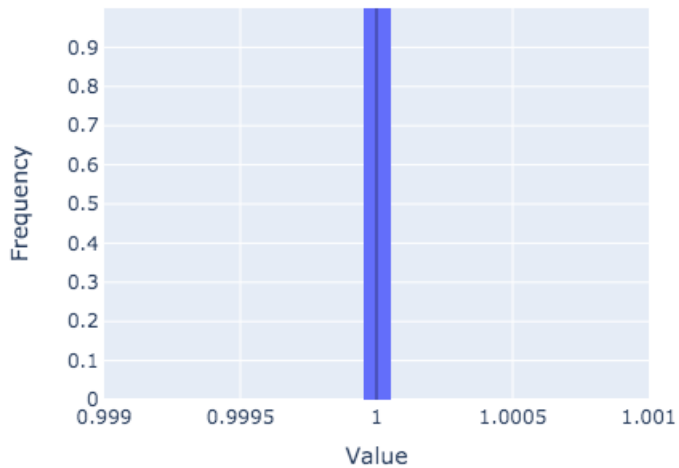
N=100, Nrep=10



Min: 0.9999999999999998
Max: 0.9999999999999998
|Max-Min|: 0.0

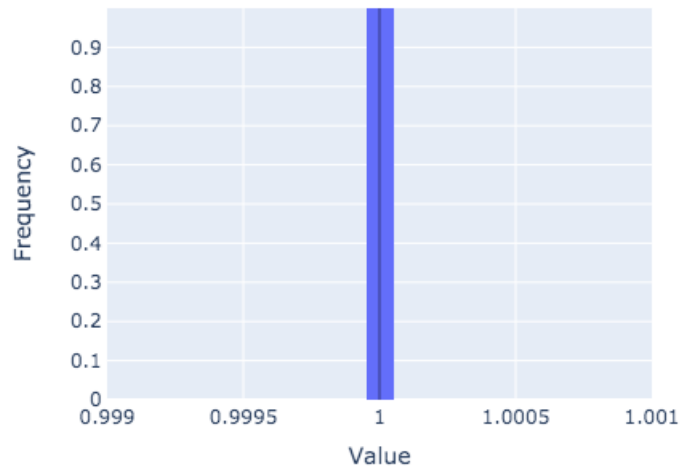
Case 3: ProjectedLinear (False)

N=10, Nrep=10



Min: 0.9999999999999998
Max: 1.0000001516482666
|Max-Min|: 1.5164826683289334e-07

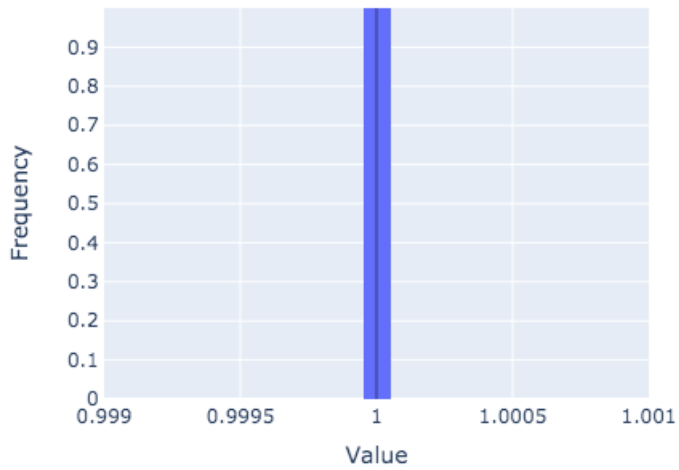
N=100, Nrep=10



Min: 0.9999999999999997
Max: 1.0000001494878719
|Max-Min|: 1.494878721919335e-07

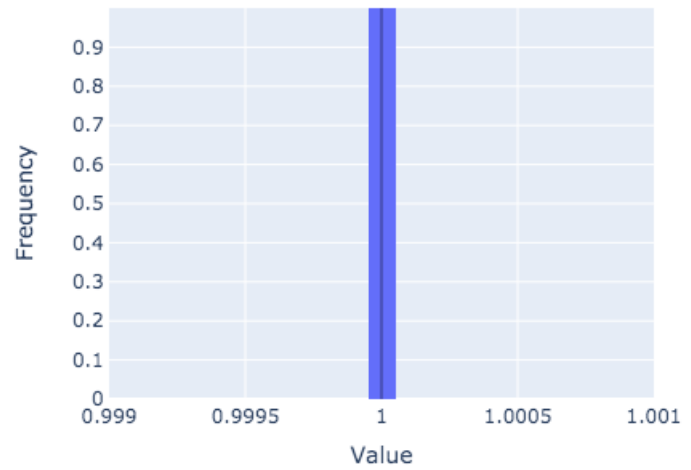
Case 4: Maximum-Likelihood (True)

N=10, Nrep=10



Min: 0.9999999999999998
Max: 0.9999999999999998
|Max-Min|: 0.0

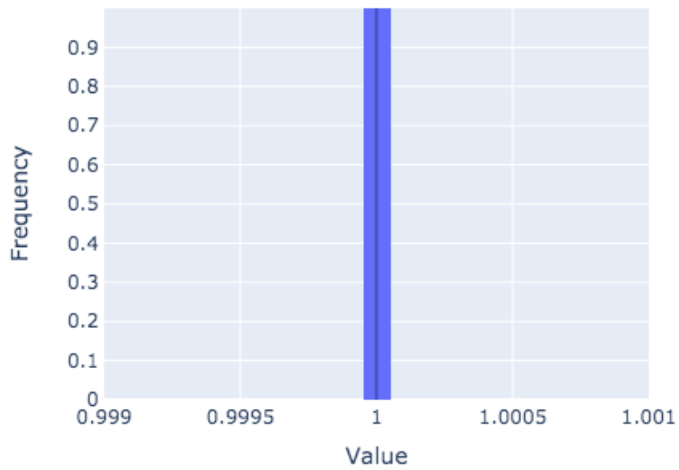
N=100, Nrep=10



Min: 0.9999999999999998
Max: 0.9999999999999998
|Max-Min|: 0.0

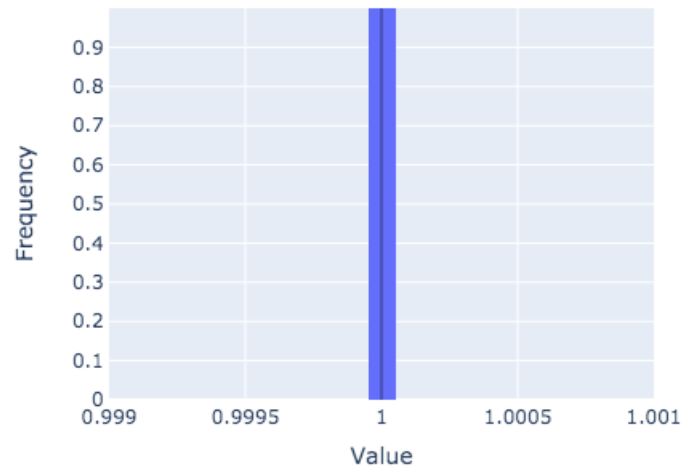
Case 5: Maximum-Likelihood (False)

N=10, Nrep=10



Min: 1.0000000041294725
Max: 1.000000155262015
|Max-Min|: 1.5113254248078078e-07

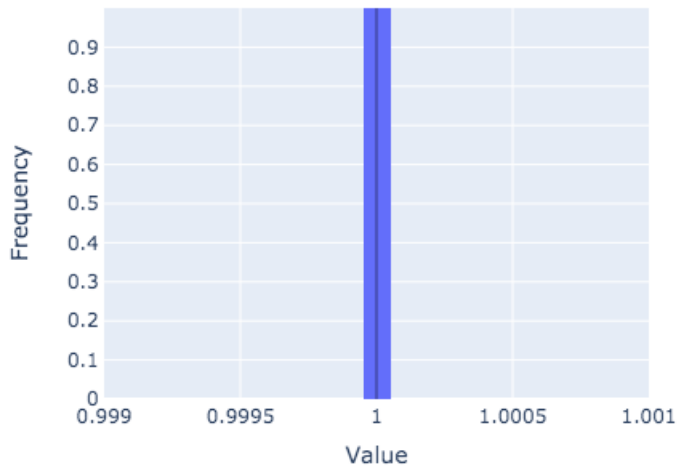
N=100, Nrep=10



Min: 1.0000000143103447
Max: 1.000000138231927
|Max-Min|: 1.2392158232898964e-07

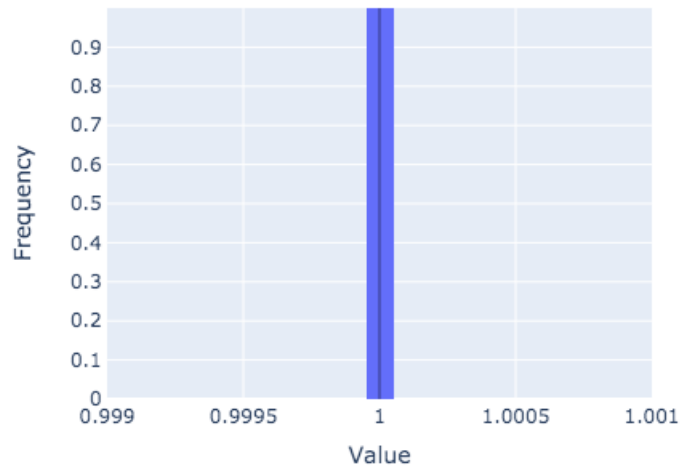
Case 6: Least Squares (True)

N=10, Nrep=10



Min: 0.9999999999999998
Max: 0.9999999999999998
|Max-Min|: 0.0

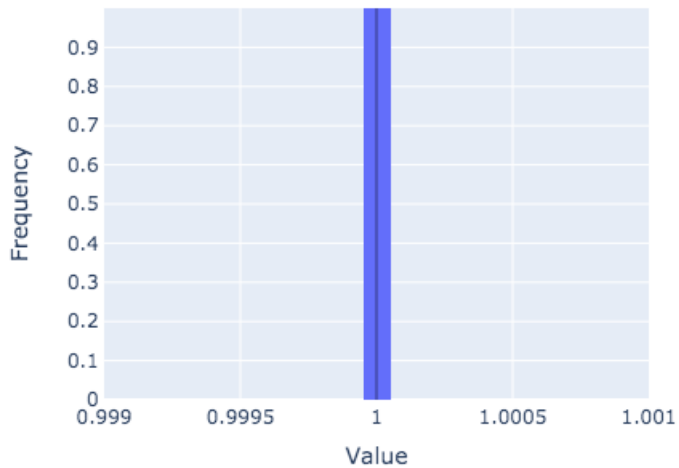
N=100, Nrep=10



Min: 0.9999999999999998
Max: 0.9999999999999998
|Max-Min|: 0.0

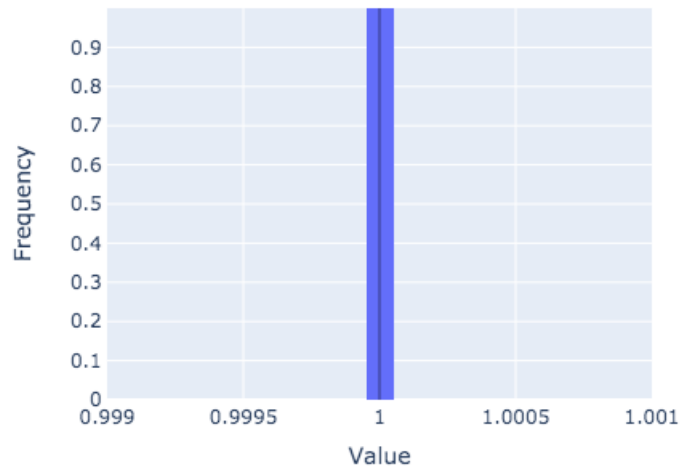
Case 7: Least Squares (False)

N=10, Nrep=10



Min: 0.9999999999999998
Max: 1.0000001793816624
|Max-Min|: 1.793816626349809e-07

N=100, Nrep=10



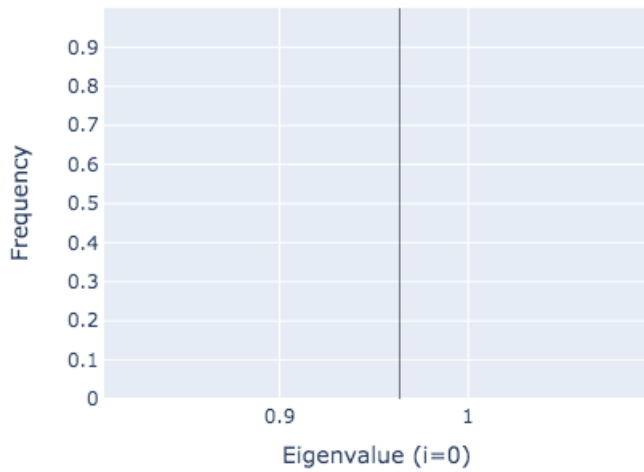
Min: 0.9999999999999998
Max: 1.0000001451628096
|Max-Min|: 1.451628097726143e-07

Test of inequality constraint violation

Eigenvalue

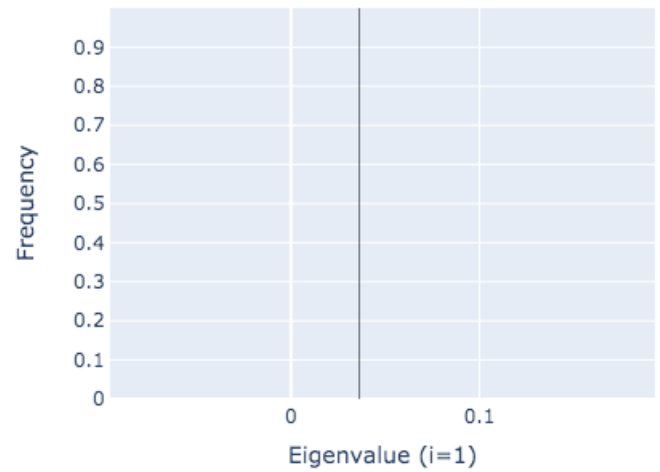
Case 0: Linear (True)

N=10, Nrep=10



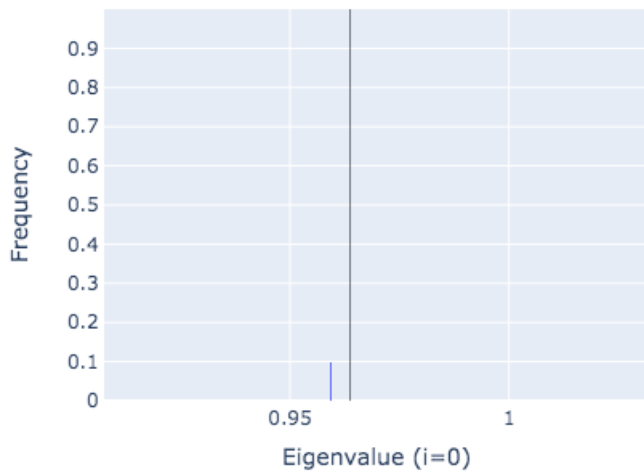
Min: 0.8069977960718091
Max: 1.0958047895355603
|Max-Min|: 0.28880699346375116

N=10, Nrep=10



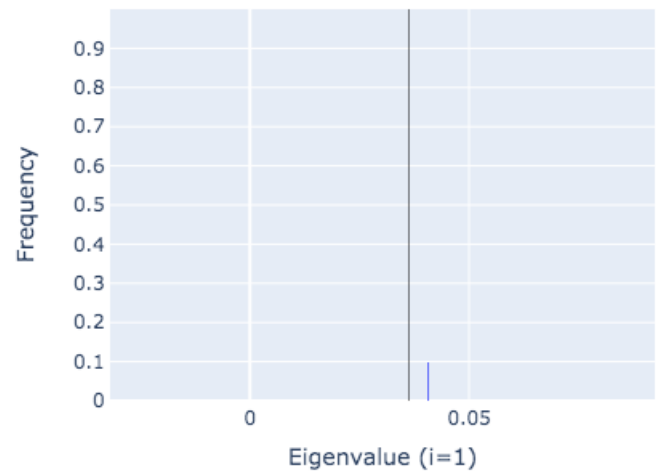
Min: -0.0958047895355604
Max: 0.19300220392819065
|Max-Min|: 0.28880699346375105

N=100, Nrep=10



Min: 0.9076879594874542
Max: 1.0318530568570035
|Max-Min|: 0.12416509736954928

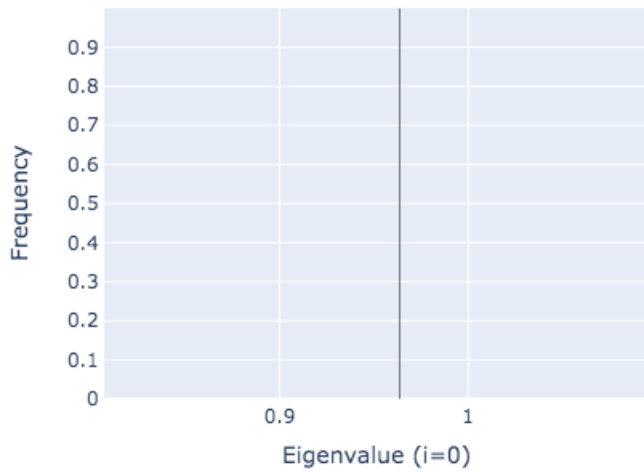
N=100, Nrep=10



Min: -0.03185305685700378
Max: 0.09231204051254559
|Max-Min|: 0.12416509736954937

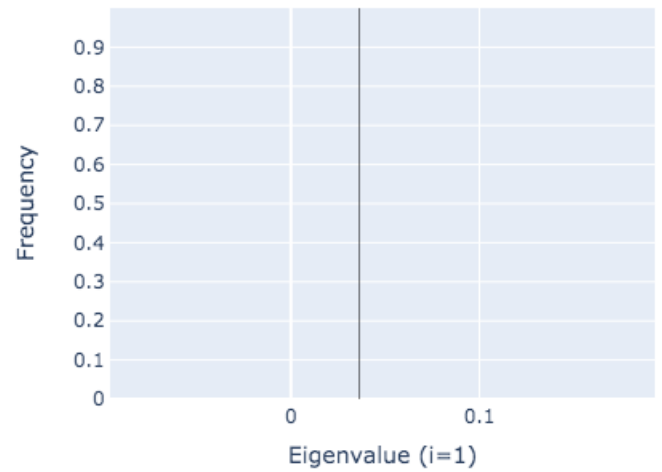
Case 1: Linear (False)

N=10, Nrep=10



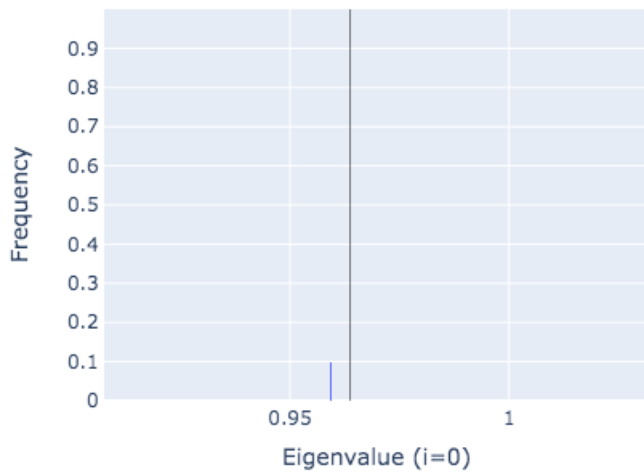
Min: 0.8069977960718093
Max: 1.0958047895355603
|Max-Min|: 0.28880699346375094

N=10, Nrep=10



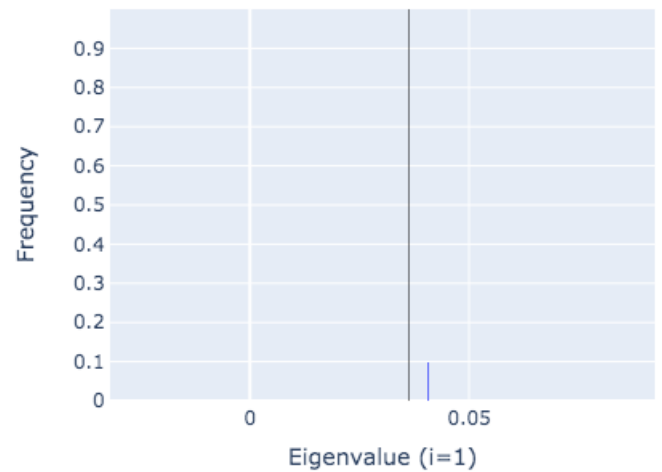
Min: -0.09580478953556018
Max: 0.19300220392819067
|Max-Min|: 0.2888069934637508

N=100, Nrep=10



Min: 0.9076879594874543
Max: 1.0318530568570035
|Max-Min|: 0.12416509736954917

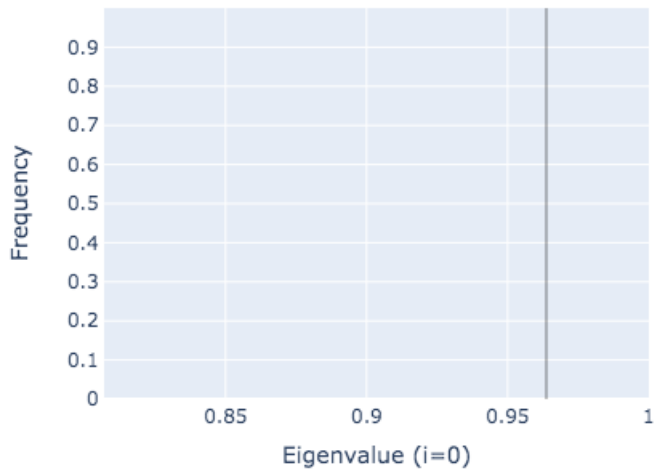
N=100, Nrep=10



Min: -0.03185305685700368
Max: 0.09231204051254595
|Max-Min|: 0.12416509736954963

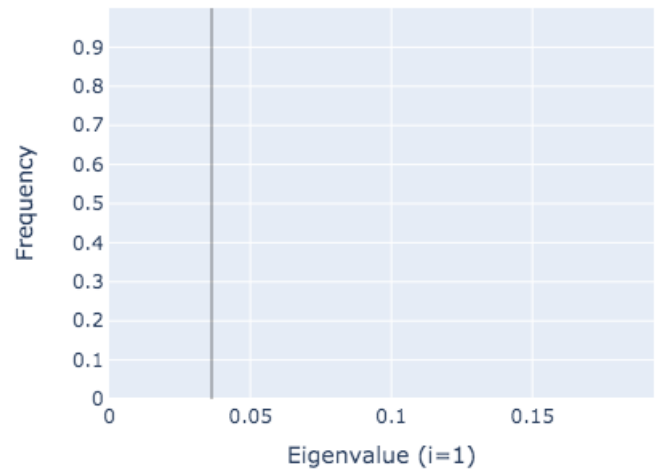
Case 2: ProjectedLinear (True)

N=10, Nrep=10



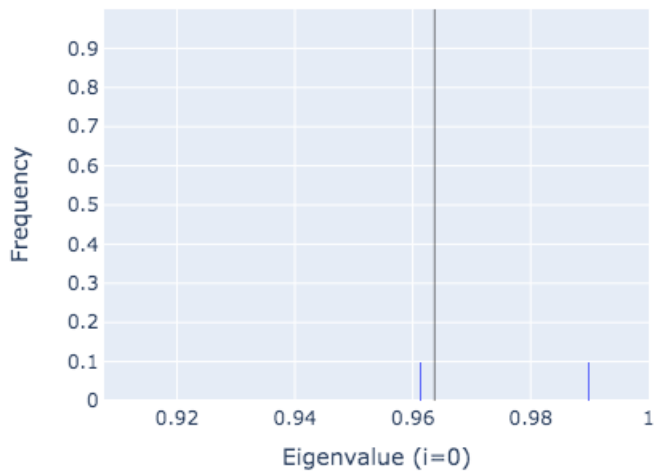
Min: 0.8069977960718093
 Max: 1.0000000758241332
 |Max-Min|: 0.19300227975232387

N=10, Nrep=10



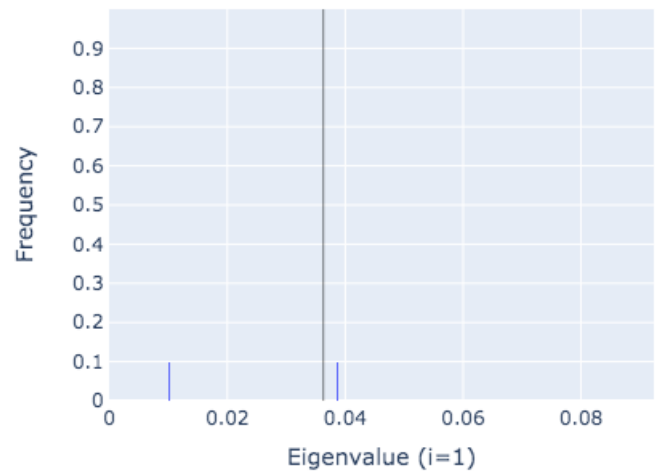
Min: -7.582413344420225e-08
 Max: 0.19300220392819056
 |Max-Min|: 0.193002279752324

N=100, Nrep=10



Min: 0.9076879594874541
 Max: 1.000000074743936
 |Max-Min|: 0.09231211525648186

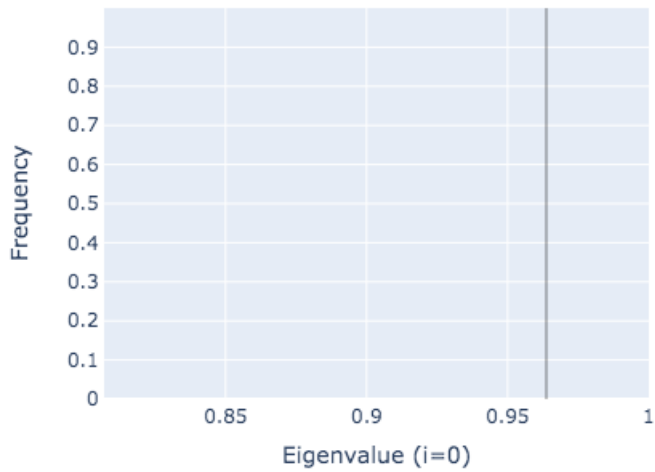
N=100, Nrep=10



Min: -7.474393607861951e-08
 Max: 0.0923120405125457
 |Max-Min|: 0.09231211525648178

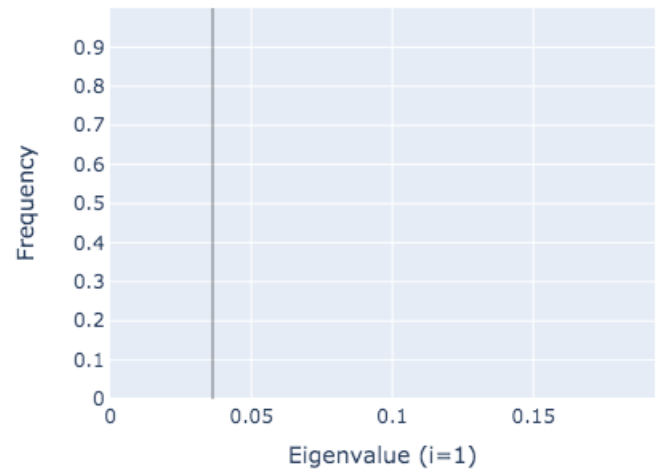
Case 3: ProjectedLinear (False)

N=10, Nrep=10



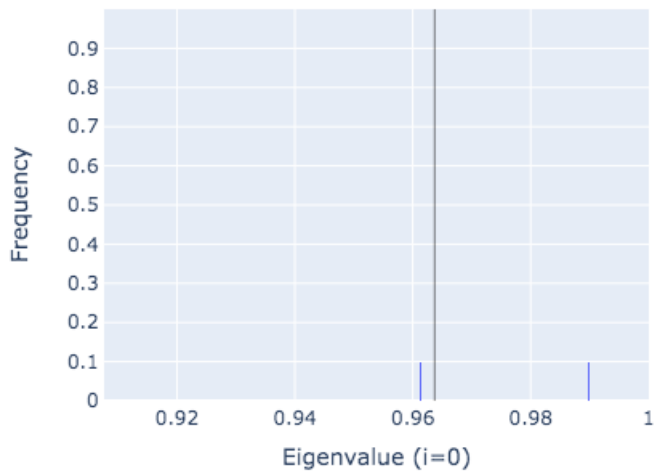
Min: 0.8069977960718095
Max: 1.0000001516482668
|Max-Min|: 0.19300235557645729

N=10, Nrep=10



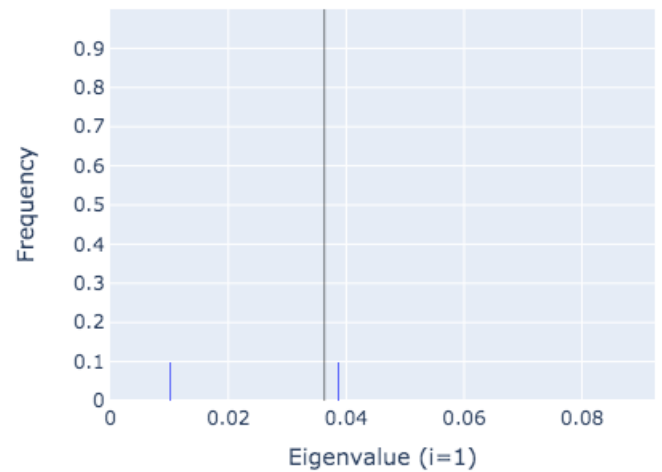
Min: -5.551115123125783e-17
Max: 0.19300220392819056
|Max-Min|: 0.19300220392819062

N=100, Nrep=10



Min: 0.9076879594874538
Max: 1.0000001494878719
|Max-Min|: 0.09231219000041802

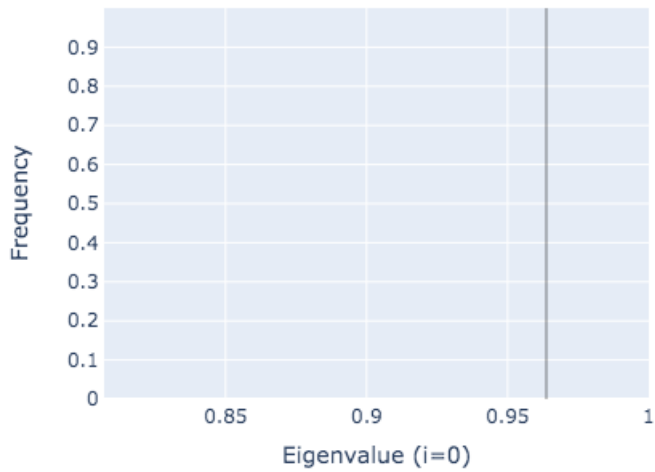
N=100, Nrep=10



Min: -4.683753385137379e-17
Max: 0.09231204051254595
|Max-Min|: 0.09231204051254599

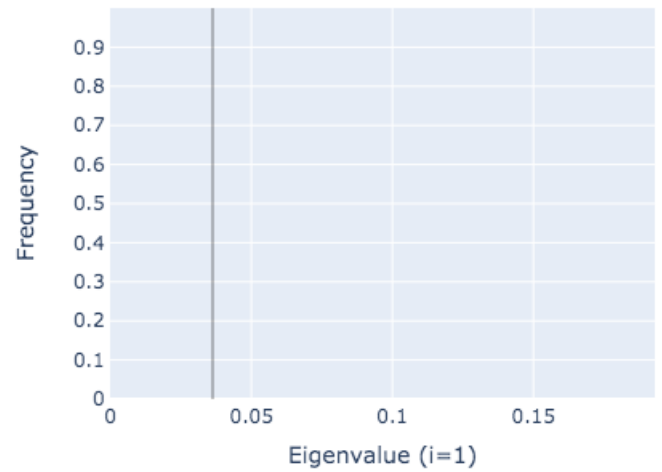
Case 4: Maximum-Likelihood (True)

N=10, Nrep=10



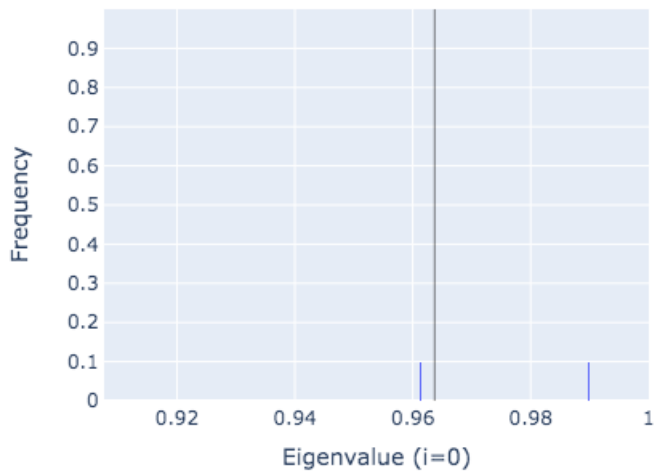
Min: 0.8069977967475155
Max: 1.0000000873548198
|Max-Min|: 0.19300229060730434

N=10, Nrep=10



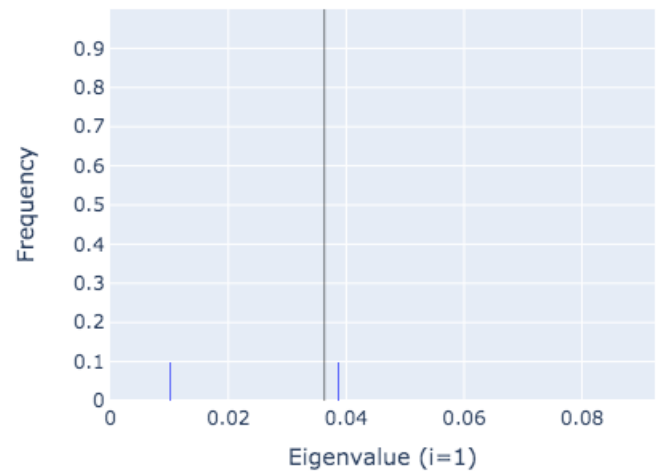
Min: -8.735481991928307e-08
Max: 0.1930022032524843
|Max-Min|: 0.19300229060730423

N=100, Nrep=10



Min: 0.9076879586550686
Max: 1.0000000897838994
|Max-Min|: 0.09231213112883085

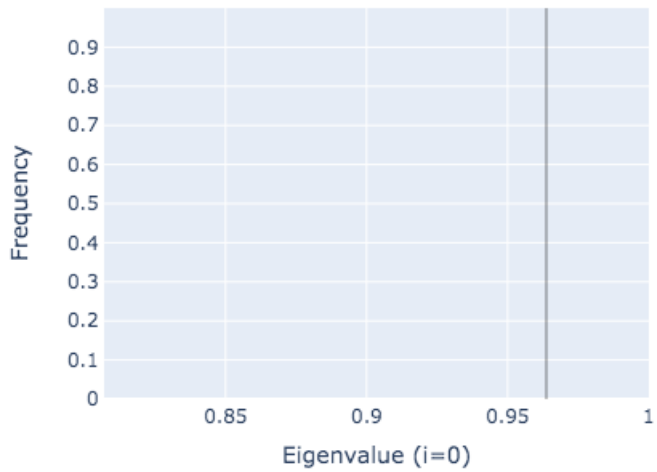
N=100, Nrep=10



Min: -8.97838996266262e-08
Max: 0.09231204134493125
|Max-Min|: 0.09231213112883087

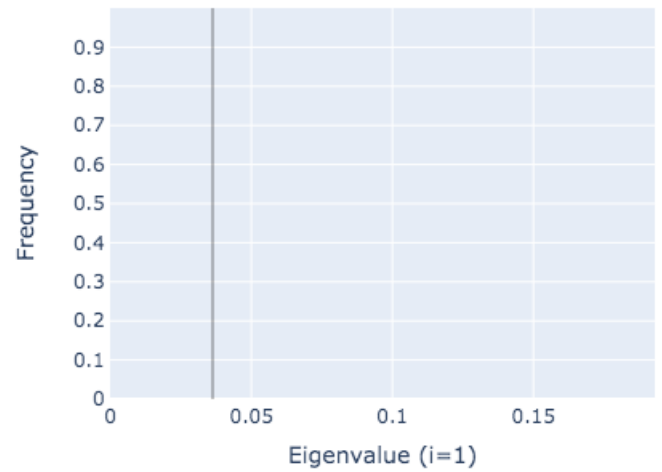
Case 5: Maximum-Likelihood (False)

N=10, Nrep=10



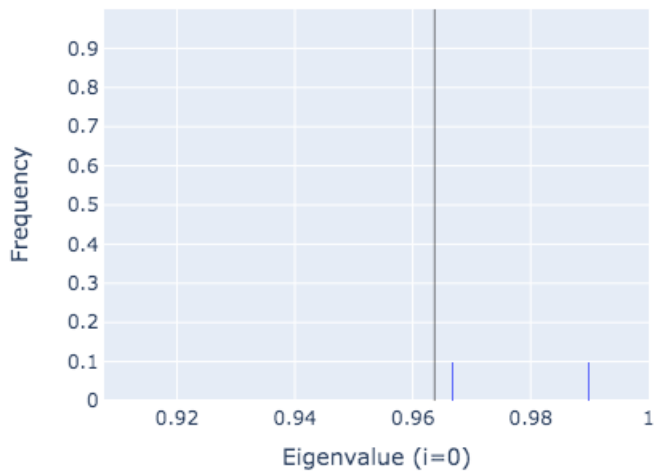
Min: 0.806988949568848
 Max: 1.0000001552619584
 |Max-Min|: 0.19301120569311037

N=10, Nrep=10



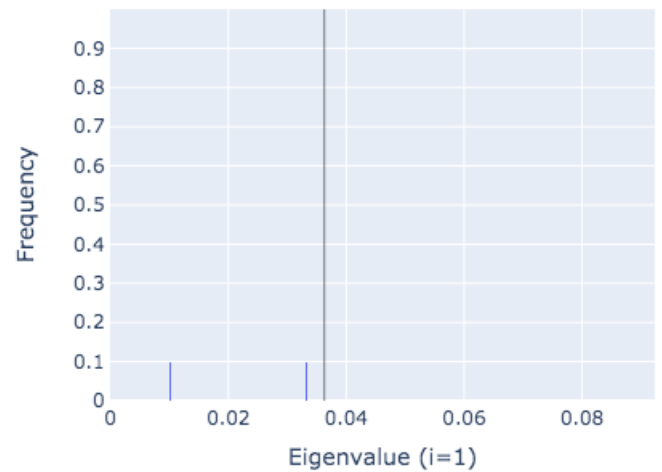
Min: 2.0122792321330962e-16
 Max: 0.1930110545606244
 |Max-Min|: 0.1930110545606242

N=100, Nrep=10



Min: 0.9076784030287195
 Max: 1.0000001382305568
 |Max-Min|: 0.09232173520183728

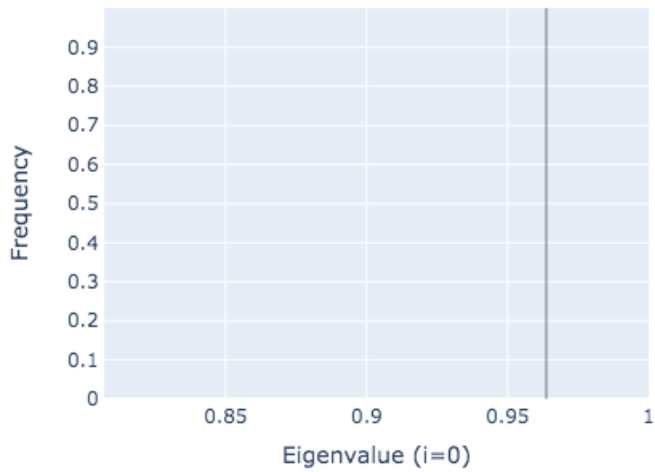
N=100, Nrep=10



Min: 4.597017211338539e-17
 Max: 0.09232161304743344
 |Max-Min|: 0.0923216130474334

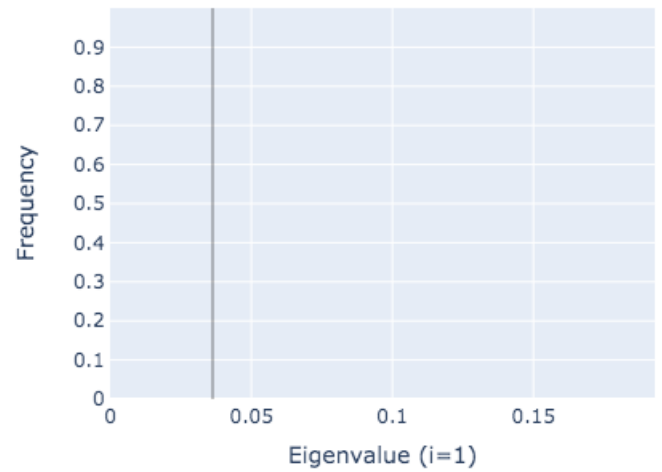
Case 6: Least Squares (True)

N=10, Nrep=10



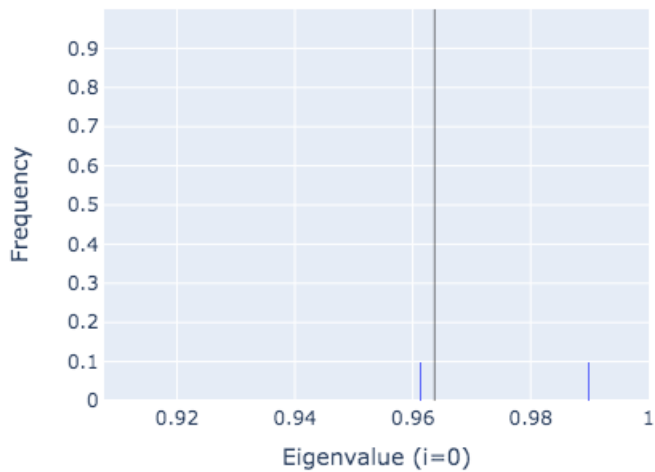
Min: 0.8069977960718084
 Max: 1.0000000895255958
 |Max-Min|: 0.1930022934537874

N=10, Nrep=10



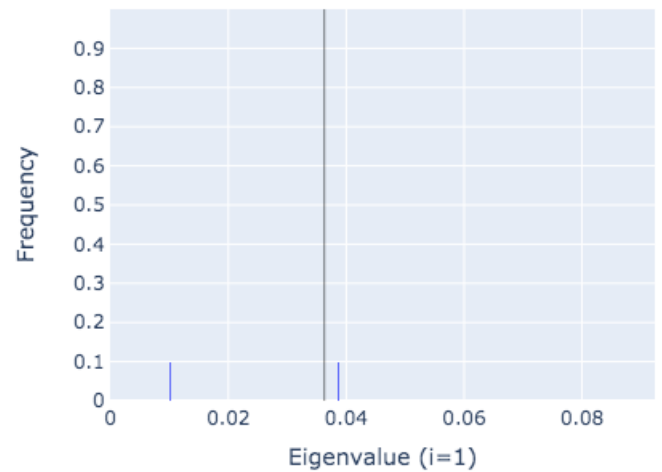
Min: -8.952559623146006e-08
 Max: 0.19300220392819128
 |Max-Min|: 0.19300229345378753

N=100, Nrep=10



Min: 0.9076879594874527
 Max: 1.0000000628573955
 |Max-Min|: 0.09231210336994278

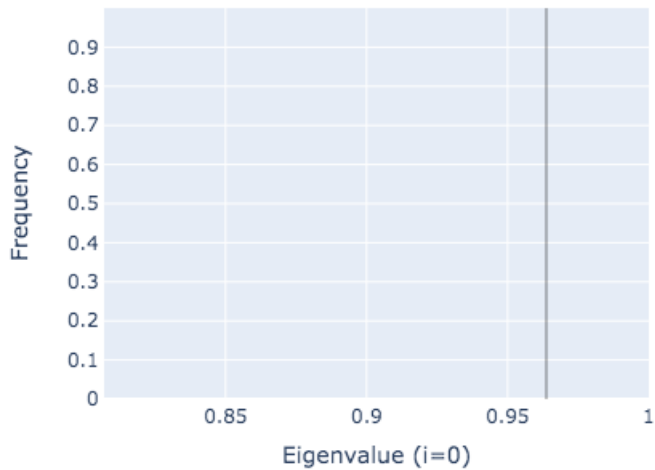
N=100, Nrep=10



Min: -6.285739573980154e-08
 Max: 0.09231204051254706
 |Max-Min|: 0.0923121033699428

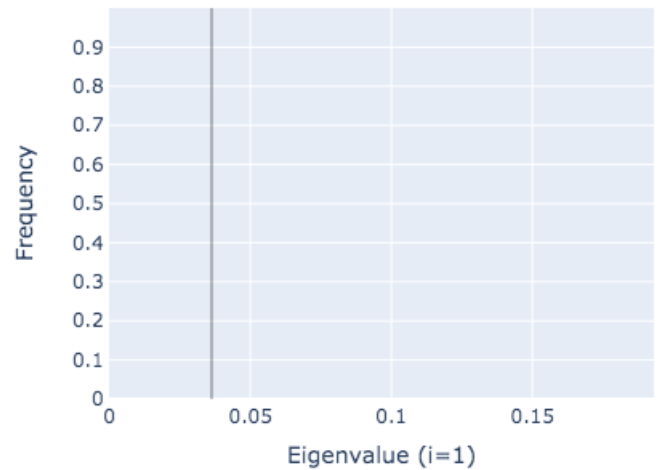
Case 7: Least Squares (False)

N=10, Nrep=10



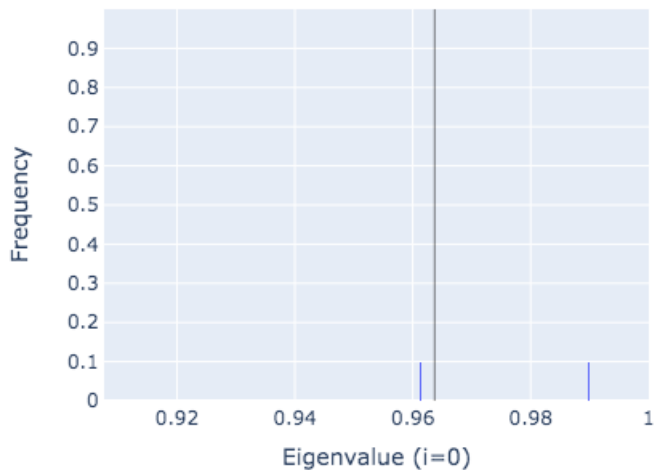
Min: 0.806997796071808
 Max: 1.0000001793816624
 |Max-Min|: 0.19300238330985442

N=10, Nrep=10



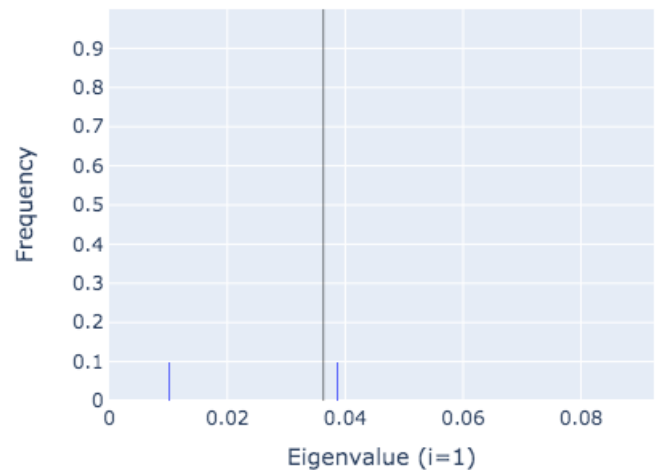
Min: -5.551115123125783e-17
 Max: 0.193002203928192
 |Max-Min|: 0.19300220392819206

N=100, Nrep=10



Min: 0.9076879594874532
 Max: 1.0000001451628093
 |Max-Min|: 0.09231218567535615

N=100, Nrep=10



Min: -1.214306433183765e-17
 Max: 0.09231204051254684
 |Max-Min|: 0.09231204051254685

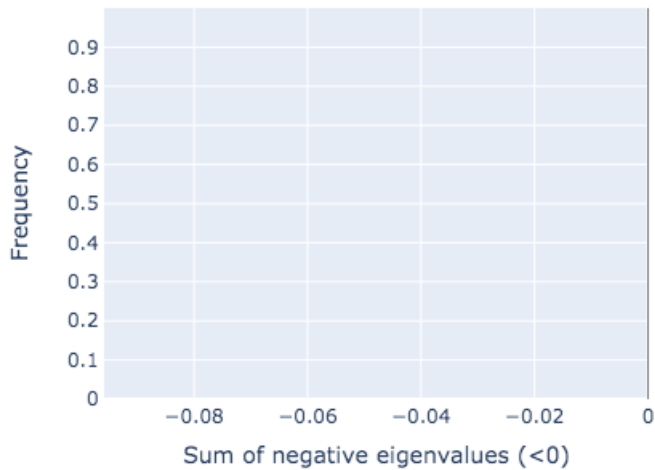
Sum of unphysical eigenvalues

Case 0: Linear (True)

N=10

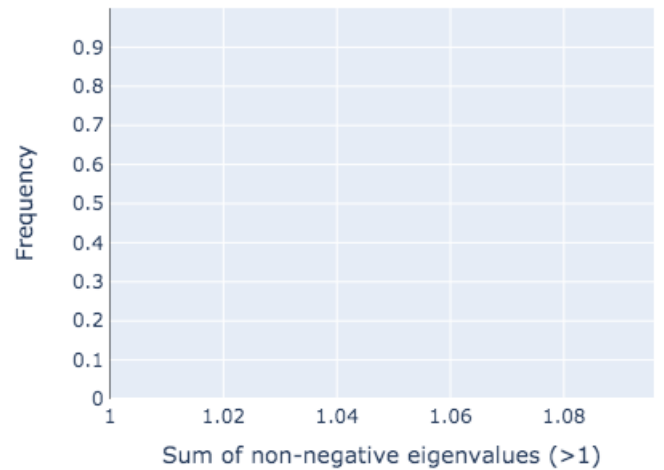
Number of unphysical estimates=5

N=10, Nrep=10



Min: -0.0958047895355604
Max: -0.056400214853479805
|Max-Min|: 0.0394045746820806

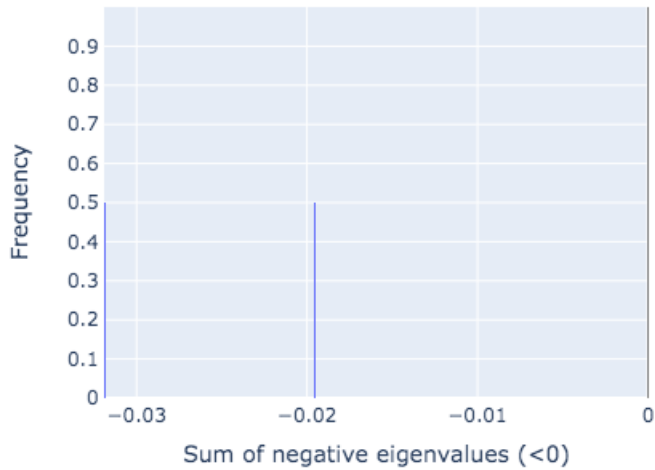
N=10, Nrep=10



Min: 1.0564002148534795
Max: 1.0958047895355603
|Max-Min|: 0.0394045746820806

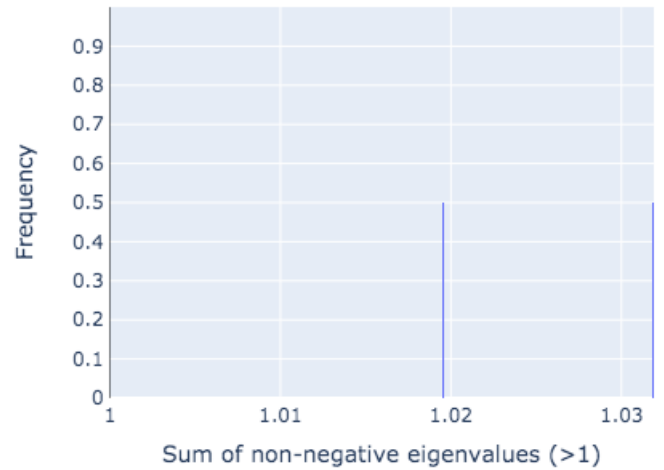
N=100
Number of unphysical estimates=2

N=100, Nrep=10



Min: -0.03185305685700378
Max: -0.01959367436468452
|Max-Min|: 0.012259382492319258

N=100, Nrep=10

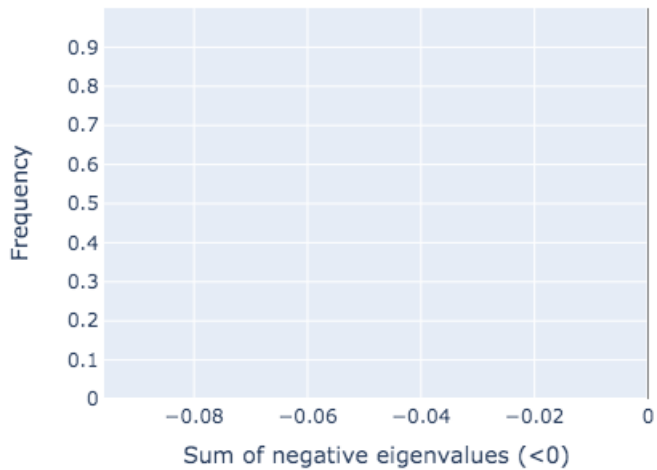


Min: 1.0195936743646843
Max: 1.0318530568570035
|Max-Min|: 0.01225938249231917

Case 1: Linear (False)

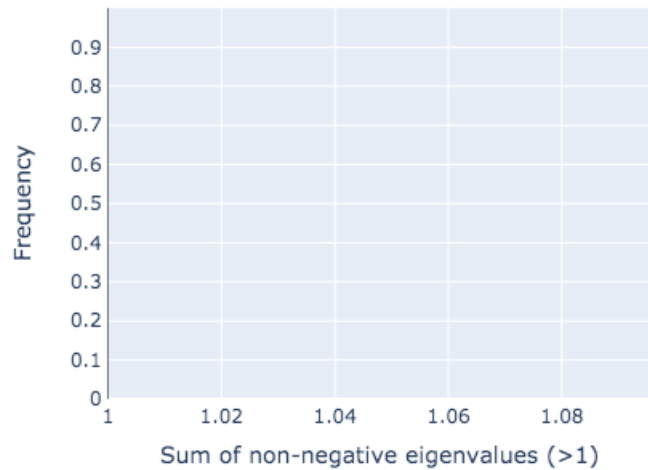
N=10
Number of unphysical estimates=5

N=10, Nrep=10



Min: -0.09580478953556018
 Max: -0.056400214853479506
 |Max-Min|: 0.039404574682080674

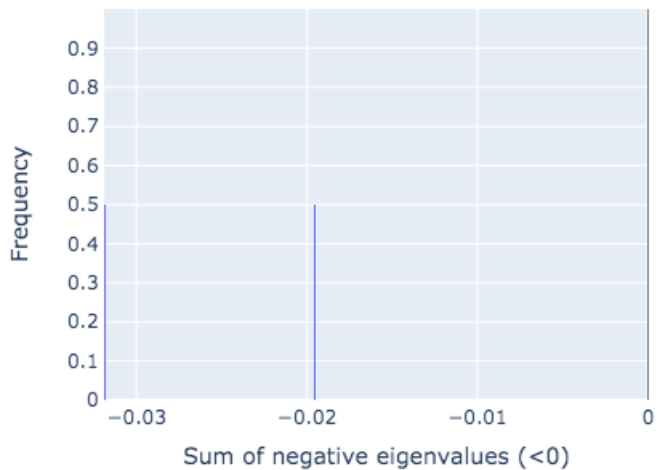
N=10, Nrep=10



Min: 1.0564002148534797
 Max: 1.0958047895355603
 |Max-Min|: 0.039404574682080584

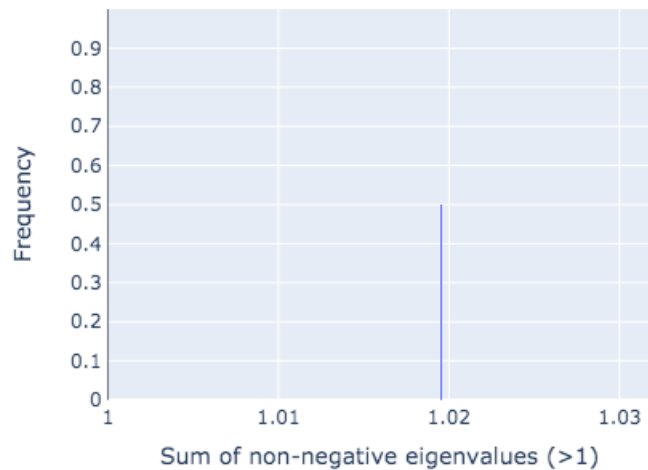
N=100
 Number of unphysical estimates=2

N=100, Nrep=10



Min: -0.03185305685700368
 Max: -0.01959367436468431
 |Max-Min|: 0.012259382492319372

N=100, Nrep=10

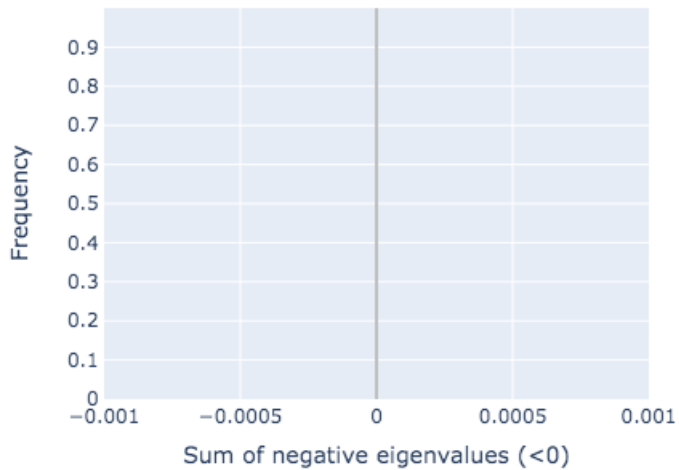


Min: 1.0195936743646845
 Max: 1.0318530568570035
 |Max-Min|: 0.012259382492318949

Case 2: ProjectedLinear (True)

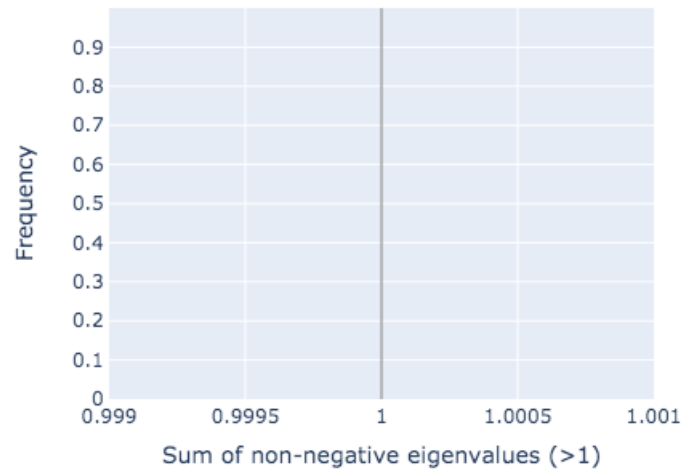
N=10
 Number of unphysical estimates=0

N=10, Nrep=10



Min: -
Max: -
|Max-Min|: -

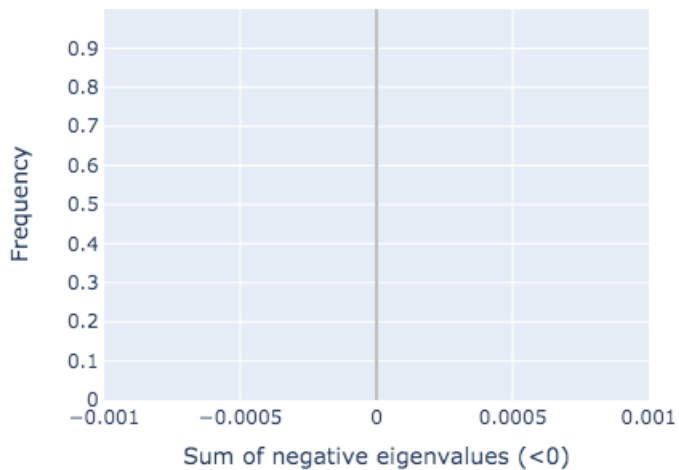
N=10, Nrep=10



Min: -
Max: -
|Max-Min|: -

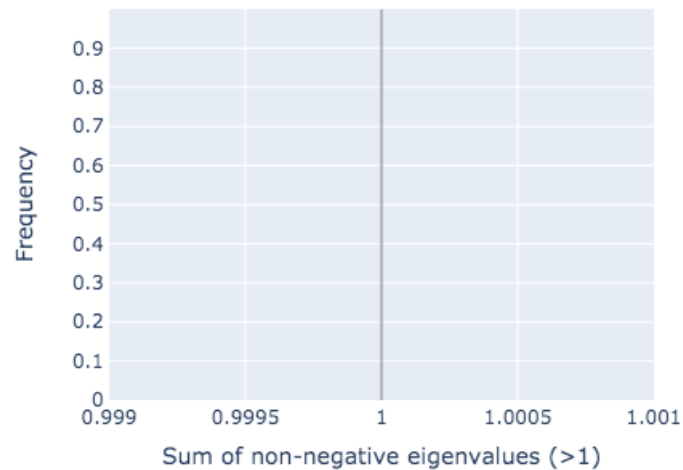
N=100
Number of unphysical estimates=0

N=100, Nrep=10



Min: -
Max: -
|Max-Min|: -

N=100, Nrep=10

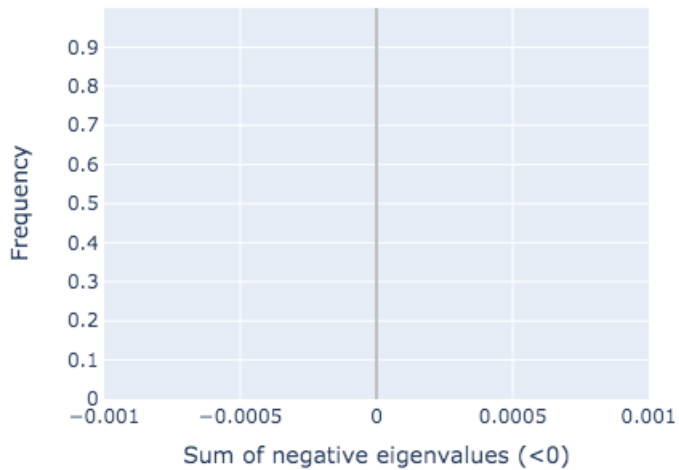


Min: -
Max: -
|Max-Min|: -

Case 3: ProjectedLinear (False)

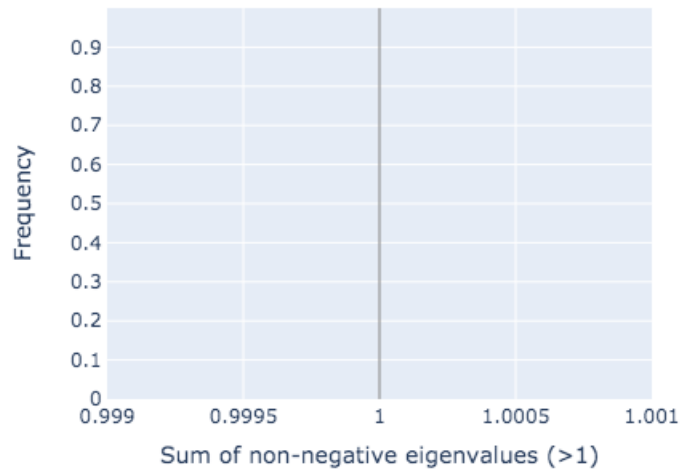
N=10
Number of unphysical estimates=0

N=10, Nrep=10



Min: -
Max: -
|Max-Min|: -

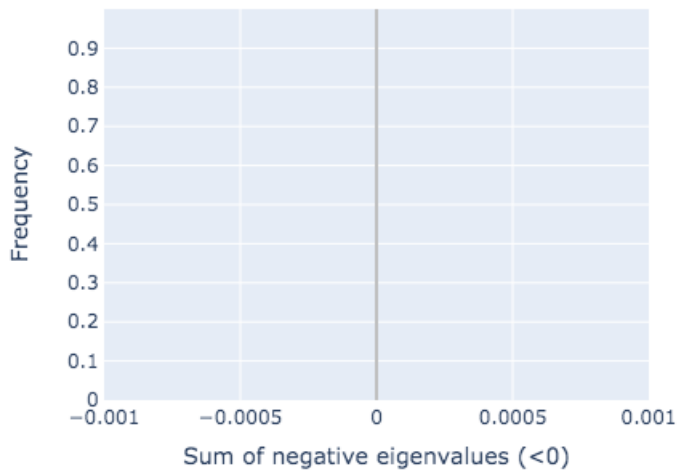
N=10, Nrep=10



Min: -
Max: -
|Max-Min|: -

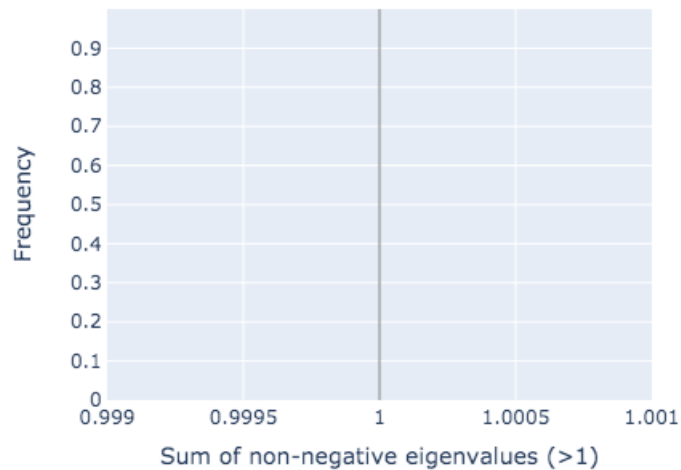
N=100
Number of unphysical estimates=0

N=100, Nrep=10



Min: -
Max: -
|Max-Min|: -

N=100, Nrep=10

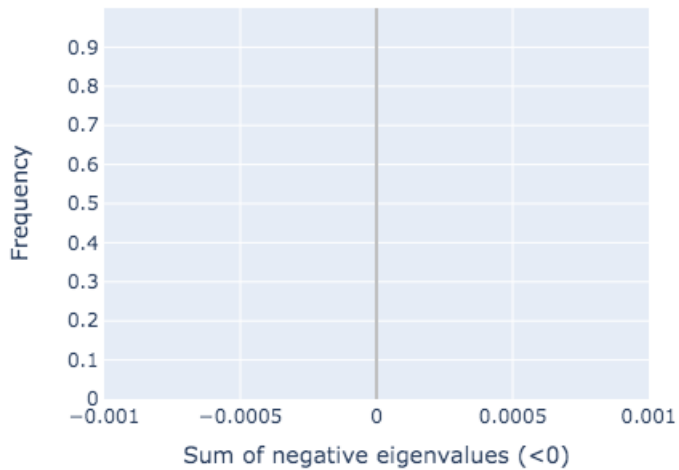


Min: -
Max: -
|Max-Min|: -

Case 4: Maximum-Likelihood (True)

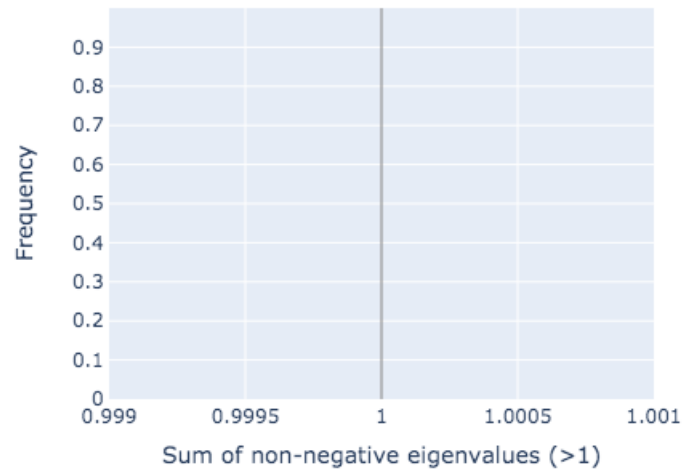
N=10
Number of unphysical estimates=0

N=10, Nrep=10



Min: -
Max: -
|Max-Min|: -

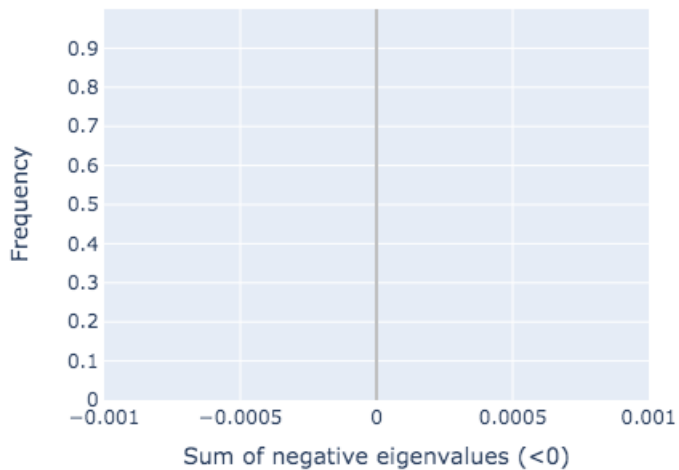
N=10, Nrep=10



Min: -
Max: -
|Max-Min|: -

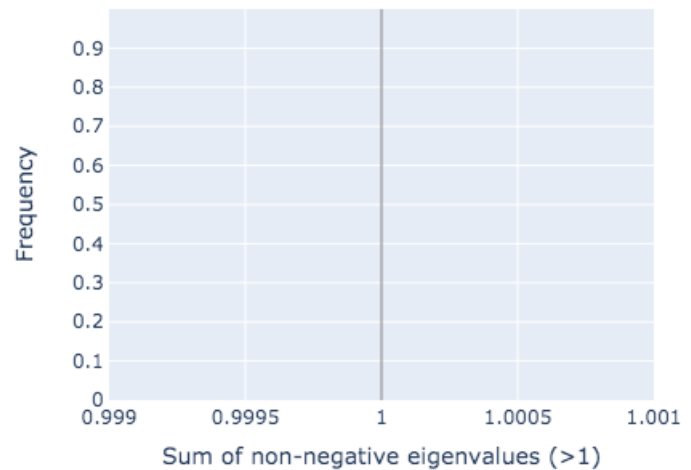
N=100
Number of unphysical estimates=0

N=100, Nrep=10



Min: -
Max: -
|Max-Min|: -

N=100, Nrep=10

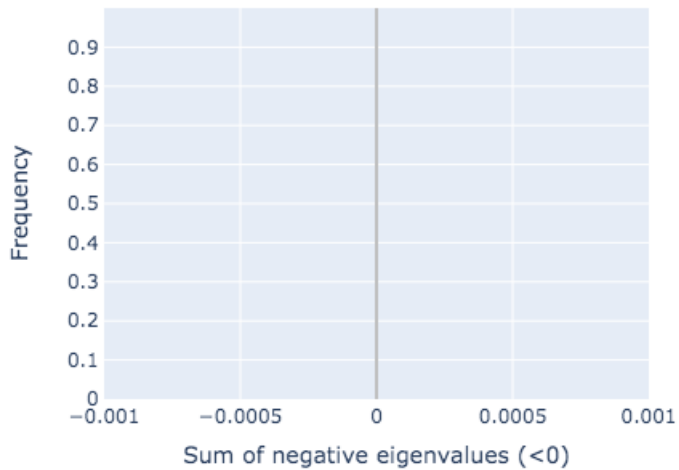


Min: -
Max: -
|Max-Min|: -

Case 5: Maximum-Likelihood (False)

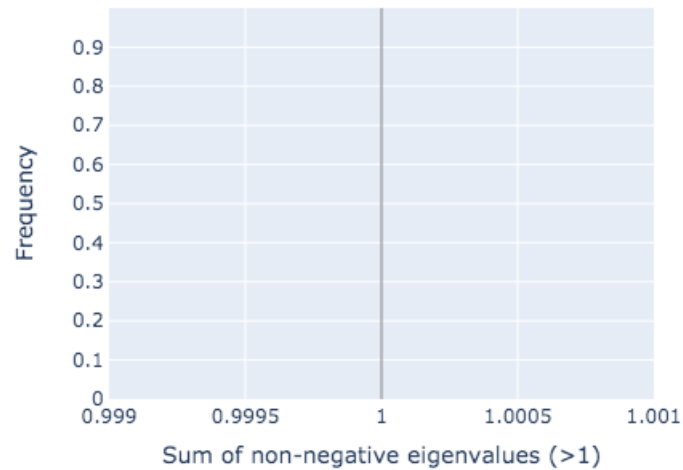
N=10
Number of unphysical estimates=0

N=10, Nrep=10



Min: -
Max: -
|Max-Min|: -

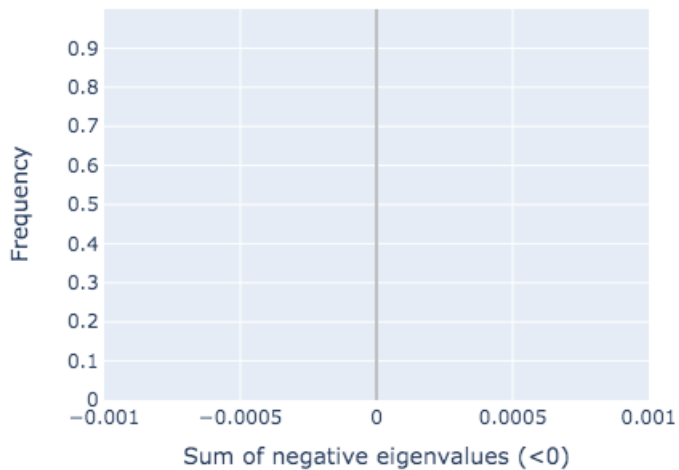
N=10, Nrep=10



Min: -
Max: -
|Max-Min|: -

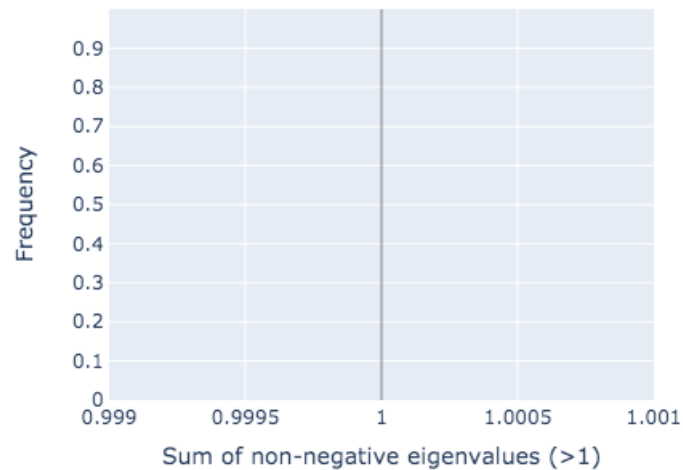
N=100
Number of unphysical estimates=0

N=100, Nrep=10



Min: -
Max: -
|Max-Min|: -

N=100, Nrep=10

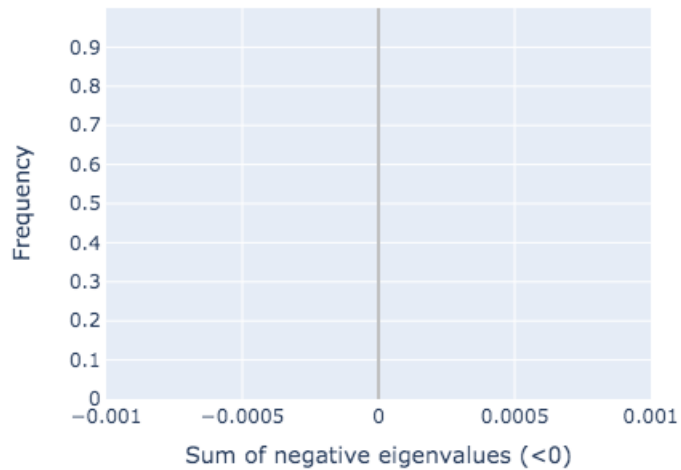


Min: -
Max: -
|Max-Min|: -

Case 6: Least Squares (True)

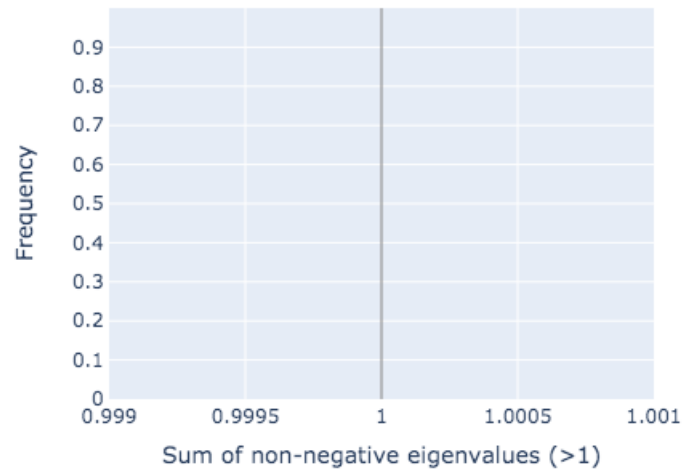
N=10
Number of unphysical estimates=0

N=10, Nrep=10



Min: -
Max: -
|Max-Min|: -

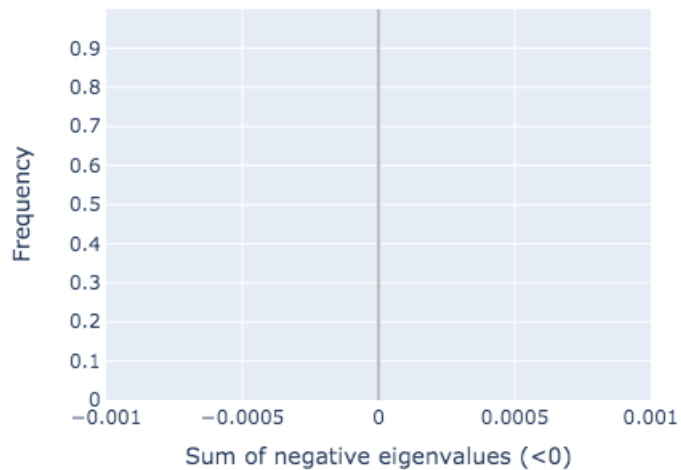
N=10, Nrep=10



Min: -
Max: -
|Max-Min|: -

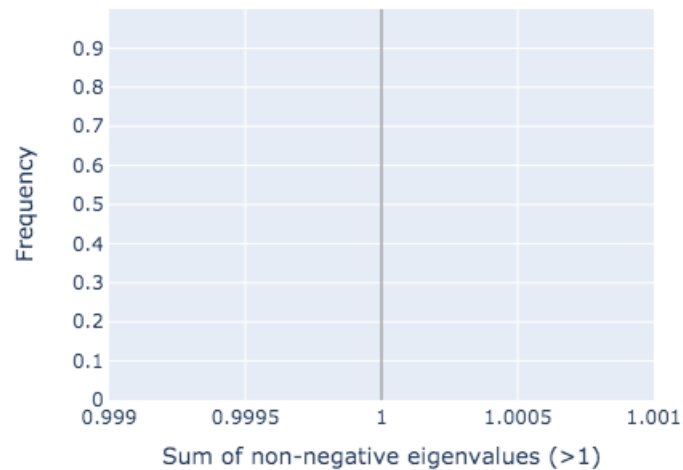
N=100
Number of unphysical estimates=0

N=100, Nrep=10



Min: -
Max: -
|Max-Min|: -

N=100, Nrep=10

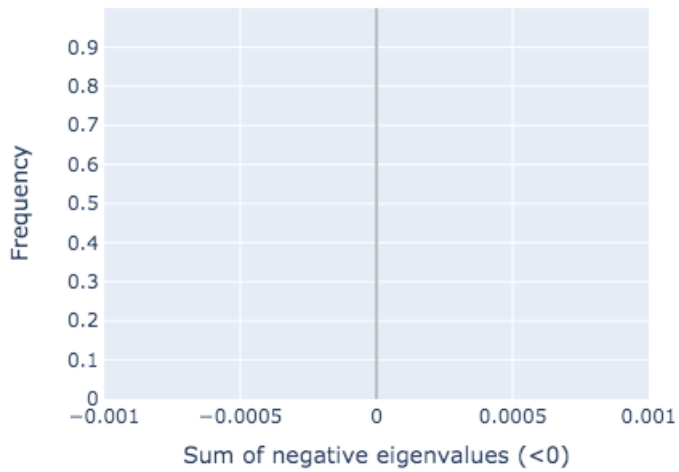


Min: -
Max: -
|Max-Min|: -

Case 7: Least Squares (False)

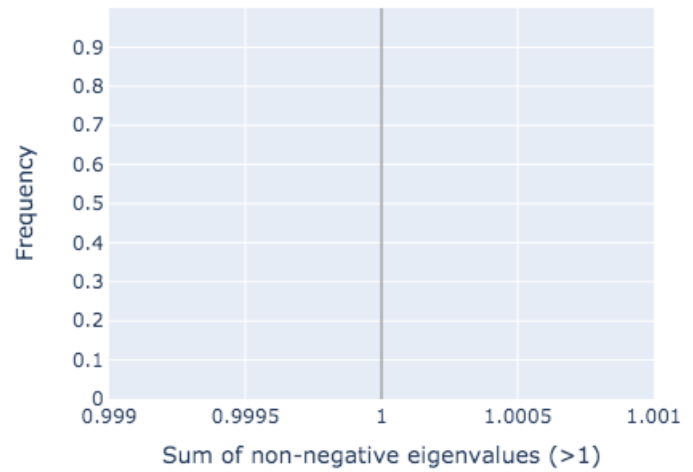
N=10
Number of unphysical estimates=0

N=10, Nrep=10



Min: -
Max: -
|Max-Min|: -

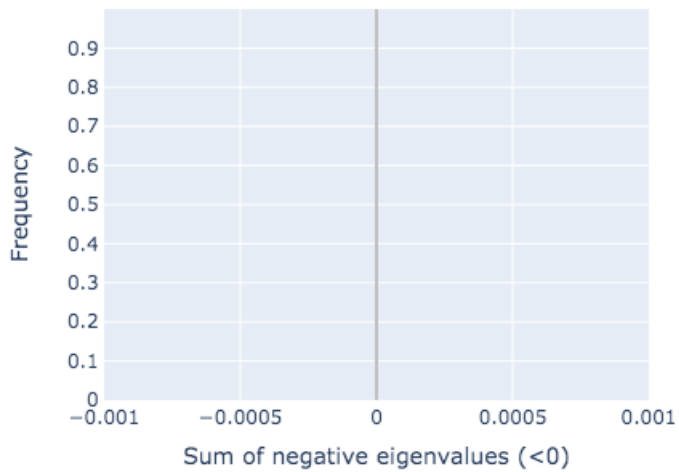
N=10, Nrep=10



Min: -
Max: -
|Max-Min|: -

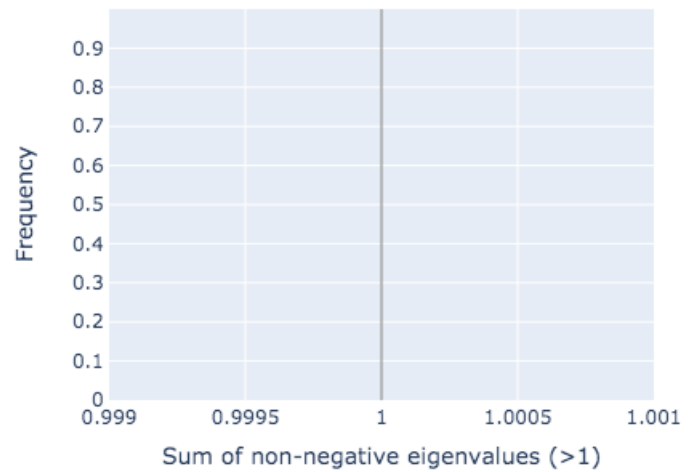
N=100
Number of unphysical estimates=0

N=100, Nrep=10



Min: -
Max: -
|Max-Min|: -

N=100, Nrep=10



Min: -
Max: -
|Max-Min|: -