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## Computation time in total

Total	0.6161385258038838min.
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## Tolerance of physicality constraint violation

		True	False
Tolerance at physicality violation test	equality constraint	1.00e-13	1.00e-05
	inequality constraint	1.00e-05	1.00e-05

## Experimental condition

Type of tomography	StandardQst
Nrep	10
N	[10, 100]
RNG seed	777

### True object

Type	State
Dim	2
Vec	[ 0.70710678 0.04314638 -0.03754928 0.64700626]

### Tester objects

0	Type	Povm
	Dim	2
	Number of outcomes	2
	Vecs	[[ 0.67045551 0.63151808 -0.00840615 -0.03564561] [ 0.74375805 -0.63151808 0.00840615 0.03564561]]
1	Type	Povm
	Dim	2
	Number of outcomes	2
	Vecs	[[ 0.72596364 -0.04732708 0.64096095 -0.08116688] [ 0.68824993 0.04732708 -0.64096095 0.08116688]]
2	Type	Povm
	Dim	2
	Number of outcomes	2
	Vecs	[[ 0.71790027 0.02984105 -0.02029088 0.66221865] [ 0.6963133 -0.02984105 0.02029088 -0.66221865]]

### Cases

	Name	Param	Tomography	Estimator
0	Linear (True)	True	StandardQst	Linear
1	Linear (False)	False	StandardQst	Linear
2	ProjectedLinear (True)	True	StandardQst	ProjectedLinear
3	ProjectedLinear (False)	False	StandardQst	ProjectedLinear
4	Maximum-Likelihood (True)	True	StandardQst	LossMinimization
5	Maximum-Likelihood (False)	False	StandardQst	LossMinimization

6	Least Squares (True)	True	StandardQst	LossMinimization
7	Least Squares (False)	False	StandardQst	LossMinimization

## Computation time of estimators

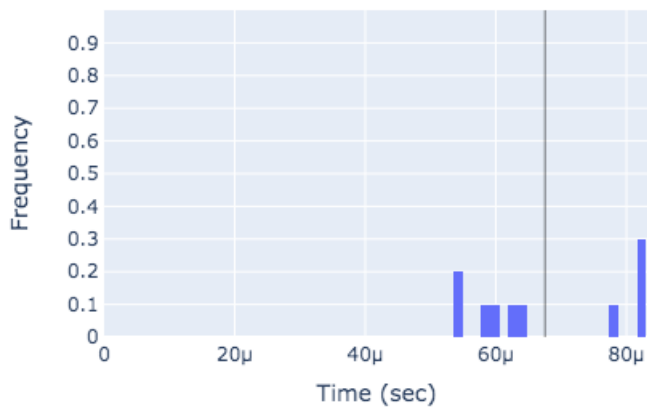
Table

	Name	N	Nrep	Mean (sec)	Std (sec)
0	Linear (True)	10	10	0.000068	0.000011
1	"	100	10	0.000116	0.000179
2	Linear (False)	10	10	0.000197	0.000374
3	"	100	10	0.000070	0.000023
4	ProjectedLinear (True)	10	10	0.007736	0.005522
5	"	100	10	0.002862	0.003518
6	ProjectedLinear (False)	10	10	0.006411	0.004334
7	"	100	10	0.002896	0.003439
8	Maximum-Likelihood (True)	10	10	0.525048	0.115335
9	"	100	10	0.635163	0.266584
10	Maximum-Likelihood (False)	10	10	0.716170	0.262806
11	"	100	10	0.923316	0.570008
12	Least Squares (True)	10	10	0.301161	0.232186
13	"	100	10	0.135413	0.091682
14	Least Squares (False)	10	10	0.280553	0.107080
15	"	100	10	0.159651	0.059279

## Histogram

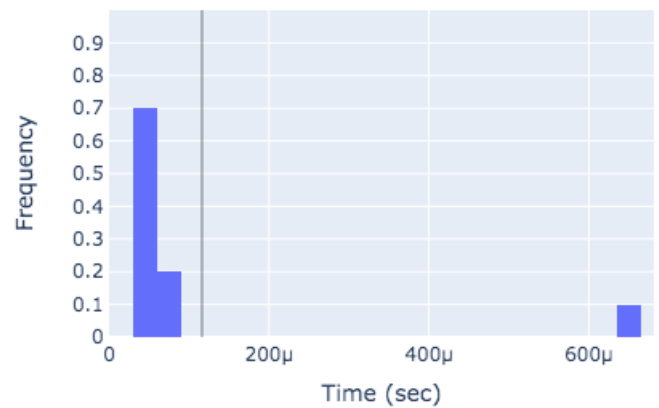
Case 0: Linear (True)

N=10, Nrep=10  
Mean = 6.75201416015625e-05(sec)



Min: 5.3882598876953125e-05  
Max: 8.20159912109375e-05  
|Max-Min|: 2.8133392333984375e-05

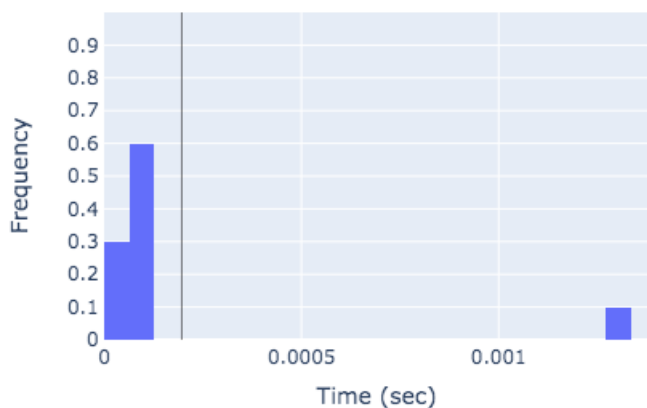
N=100, Nrep=10  
Mean = 0.0001161336898803711(sec)



Min: 4.601478576660156e-05  
Max: 0.0006508827209472656  
|Max-Min|: 0.0006048679351806641

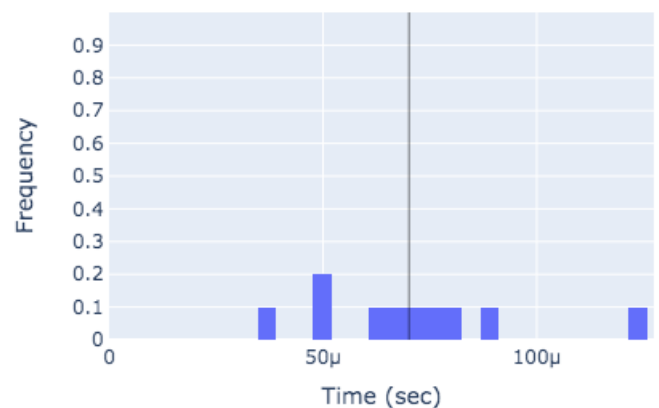
### Case 1: Linear (False)

N=10, Nrep=10  
Mean = 0.00019681453704833984(sec)



Min: 4.57763671875e-05  
Max: 0.0013170242309570312  
|Max-Min|: 0.0012712478637695312

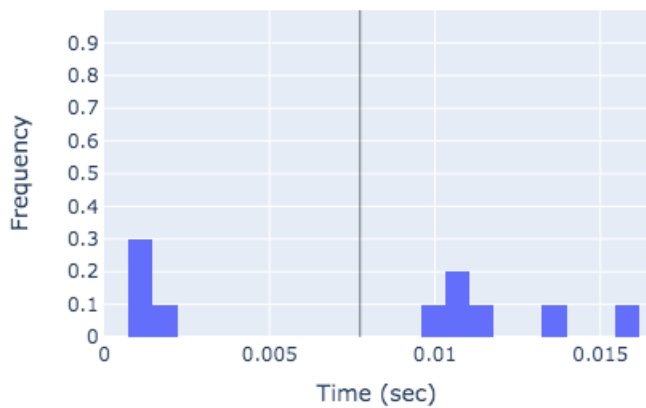
N=100, Nrep=10  
Mean = 7.011890411376954e-05(sec)



Min: 3.62396240234375e-05  
Max: 0.00012302398681640625  
|Max-Min|: 8.678436279296875e-05

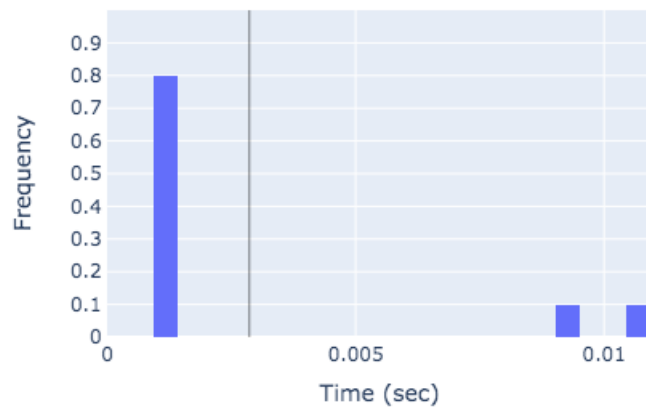
### Case 2: ProjectedLinear (True)

N=10, Nrep=10  
Mean = 0.007736325263977051(sec)



Min: 0.0010139942169189453  
Max: 0.01573014259338379  
|Max-Min|: 0.014716148376464844

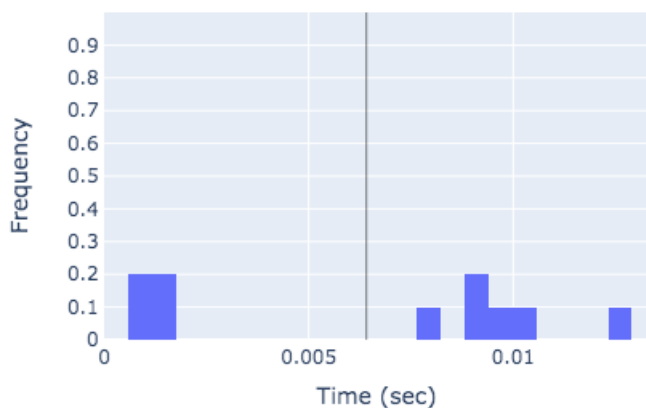
N=100, Nrep=10  
Mean = 0.002862071990966797(sec)



Min: 0.0009877681732177734  
Max: 0.010485649108886719  
|Max-Min|: 0.009497880935668945

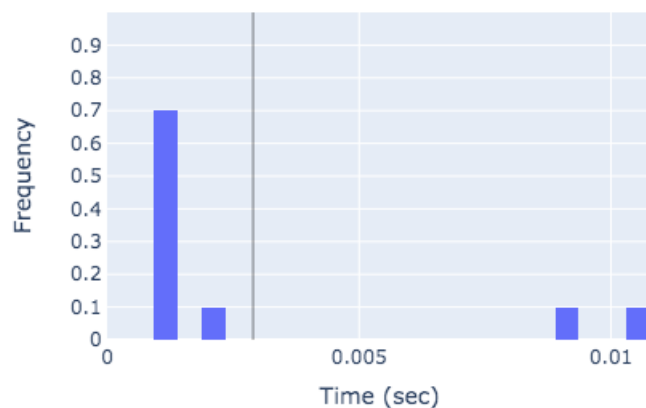
### Case 3: ProjectedLinear (False)

N=10, Nrep=10  
Mean = 0.006410980224609375(sec)



Min: 0.0009987354278564453  
Max: 0.012728691101074219  
|Max-Min|: 0.011729955673217773

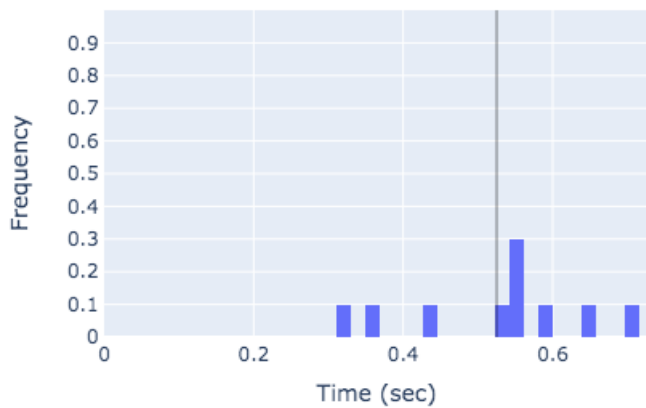
N=100, Nrep=10  
Mean = 0.002895545959472656(sec)



Min: 0.0009806156158447266  
Max: 0.010341882705688477  
|Max-Min|: 0.00936126708984375

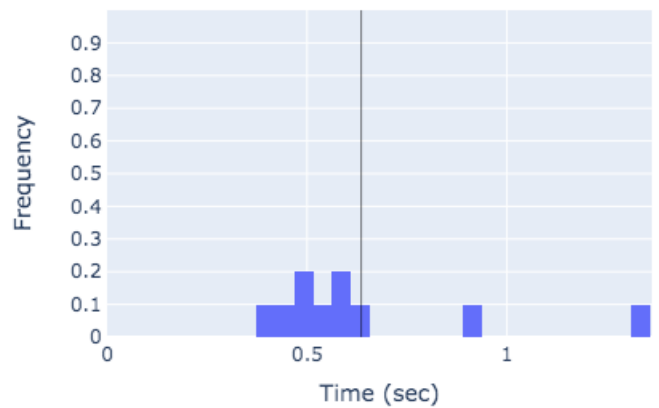
### Case 4: Maximum-Likelihood (True)

N=10, Nrep=10  
Mean = 0.5250484228134156(sec)



Min: 0.3221898078918457  
Max: 0.7092688083648682  
|Max-Min|: 0.38707900047302246

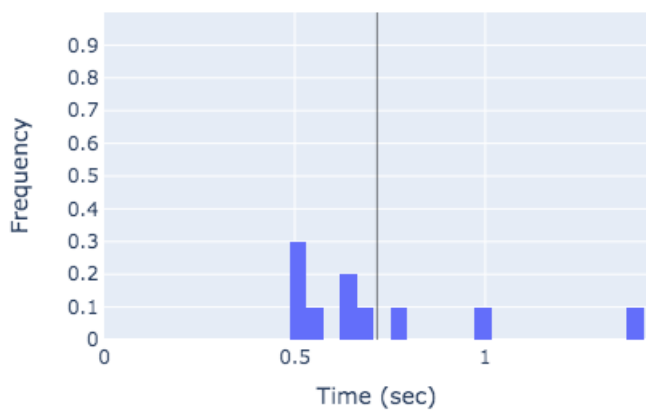
N=100, Nrep=10  
Mean = 0.6351628303527832(sec)



Min: 0.3791530132293701  
Max: 1.315096139907837  
|Max-Min|: 0.9359431266784668

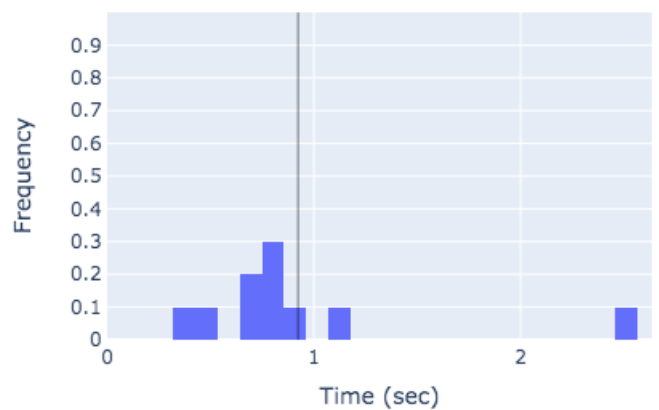
### Case 5: Maximum-Likelihood (False)

N=10, Nrep=10  
Mean = 0.7161700963973999(sec)



Min: 0.5007107257843018  
Max: 1.3848459720611572  
|Max-Min|: 0.8841352462768555

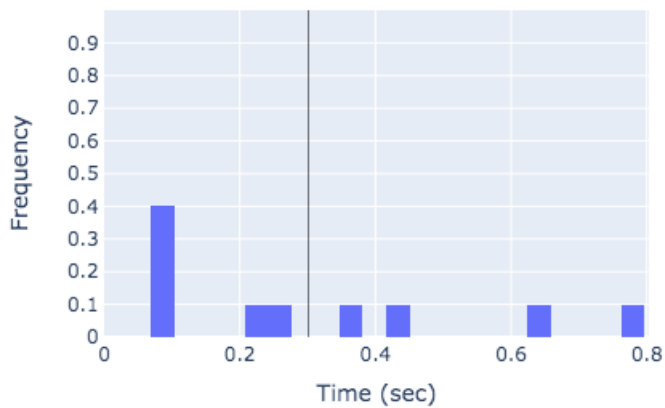
N=100, Nrep=10  
Mean = 0.9233158111572266(sec)



Min: 0.390399694442749  
Max: 2.5277698040008545  
|Max-Min|: 2.1373701095581055

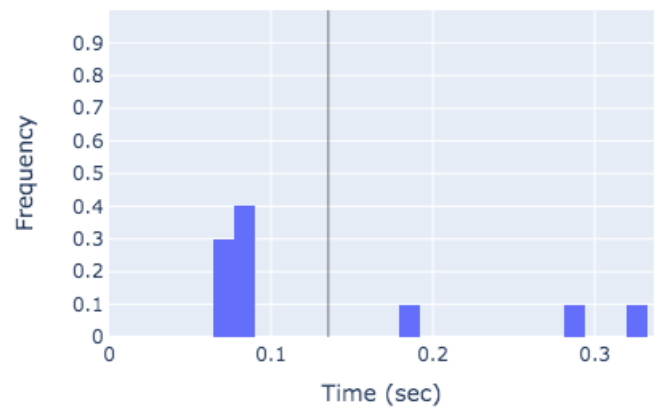
### Case 6: Least Squares (True)

N=10, Nrep=10  
Mean = 0.3011608600616455(sec)



Min: 0.07534456253051758  
Max: 0.7683579921722412  
|Max-Min|: 0.6930134296417236

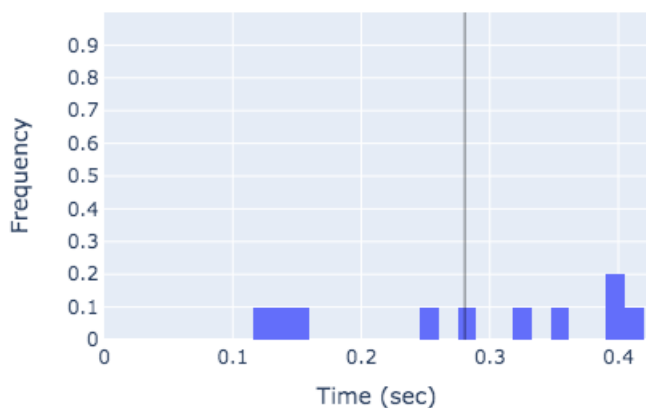
N=100, Nrep=10  
Mean = 0.1354134798049927(sec)



Min: 0.06780385971069336  
Max: 0.3237619400024414  
|Max-Min|: 0.25595808029174805

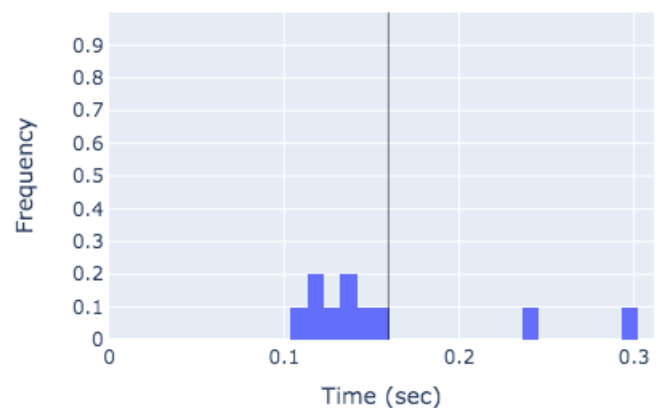
## Case 7: Least Squares (False)

N=10, Nrep=10  
Mean = 0.2805533170700073(sec)



Min: 0.11992025375366211  
Max: 0.4092109203338623  
|Max-Min|: 0.2892906665802002

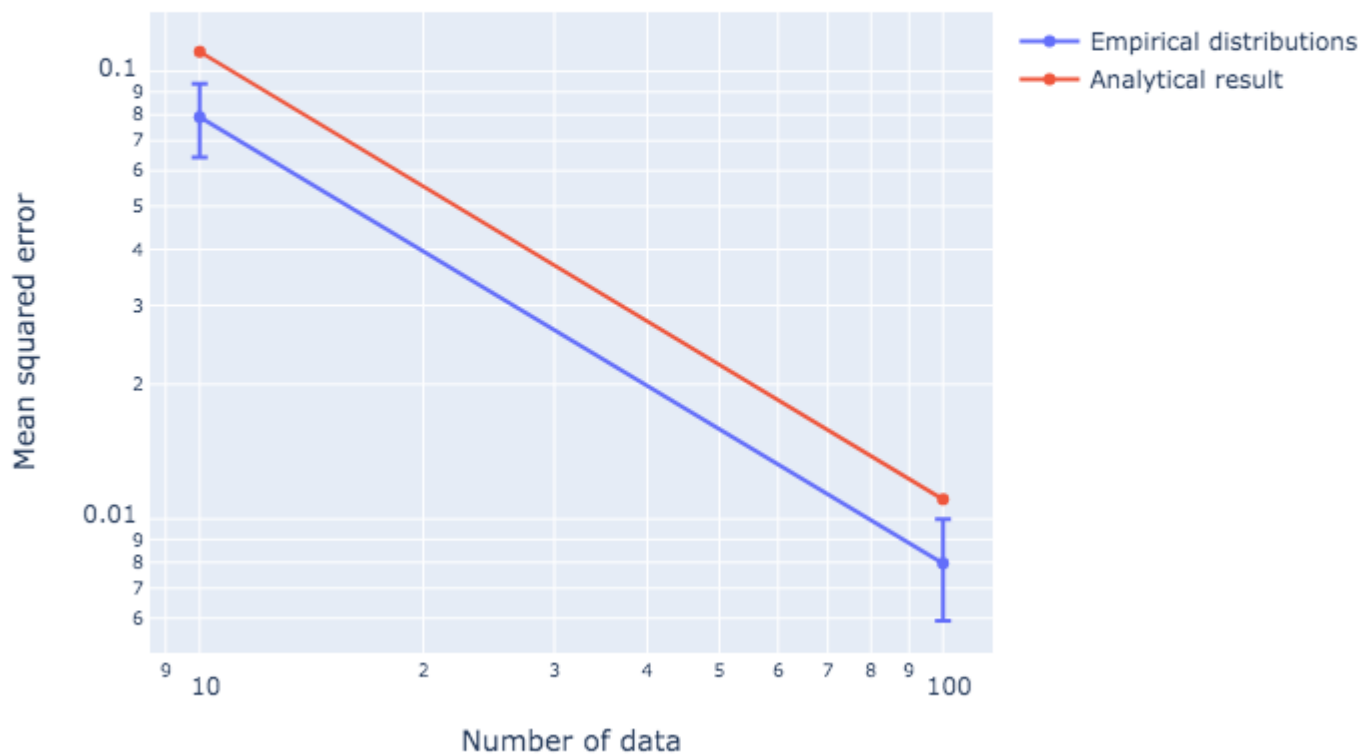
N=100, Nrep=10  
Mean = 0.1596508264541626(sec)



Min: 0.11302876472473145  
Max: 0.3018457889556885  
|Max-Min|: 0.18881702423095703

## MSE of empirical distributions

## Mean squared error



## Consistency test

	Name	Type of tomography	Param	Estimator	Loss	Algo	Squared Error to True	Possibly OK	To be checked
0	Linear (True)	Standard Qst	True	Linear	None	None	3.84e-32	OK	not need debug
1	Linear (False)	Standard Qst	False	Linear	None	None	1.06e-31	OK	not need debug
2	ProjectedLinear (True)	Standard Qst	True	Projected Linear	None	None	2.14e-32	OK	not need debug
3	ProjectedLinear (False)	Standard Qst	False	Projected Linear	None	None	1.04e-31	OK	not need debug
4	Maximum-Likelihood (True)	Standard Qst	True	Loss Minimization	Weighted Relative Entropy	Projected Gradient Descent Backtracking	8.42e-18	OK	not need debug
5	Maximum-Likelihood (False)	Standard Qst	False	Loss Minimization	Weighted Relative Entropy	Projected Gradient Descent Backtracking	1.39e-08	NG	not need debug
6	Least Squares (True)	Standard Qst	True	Loss Minimization	Weighted Probability Based Squared Error	Projected Gradient Descent Backtracking	2.21e-31	OK	not need debug

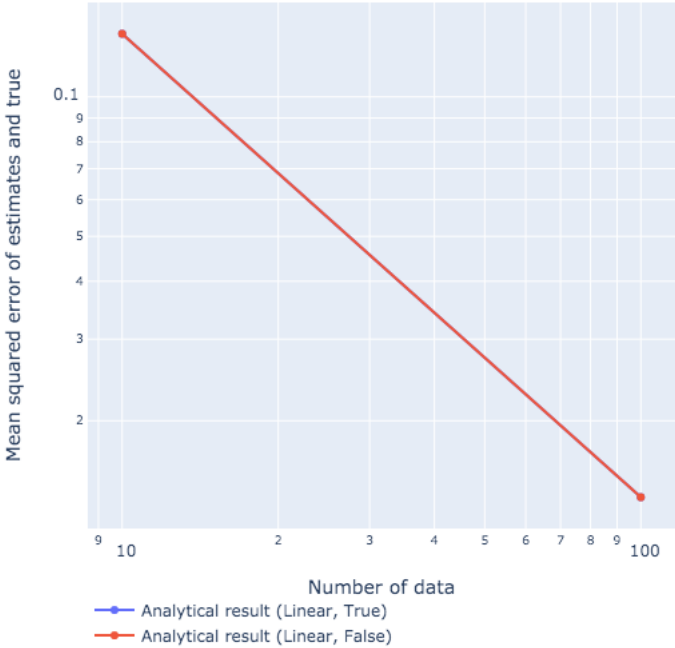


7	Least Squares (False)	Standard Qst	False	Loss Minimization	Weighted Probability Based Squared Error	Projected Gradient Descent Backtracking	4.52e-30	OK	not need debug
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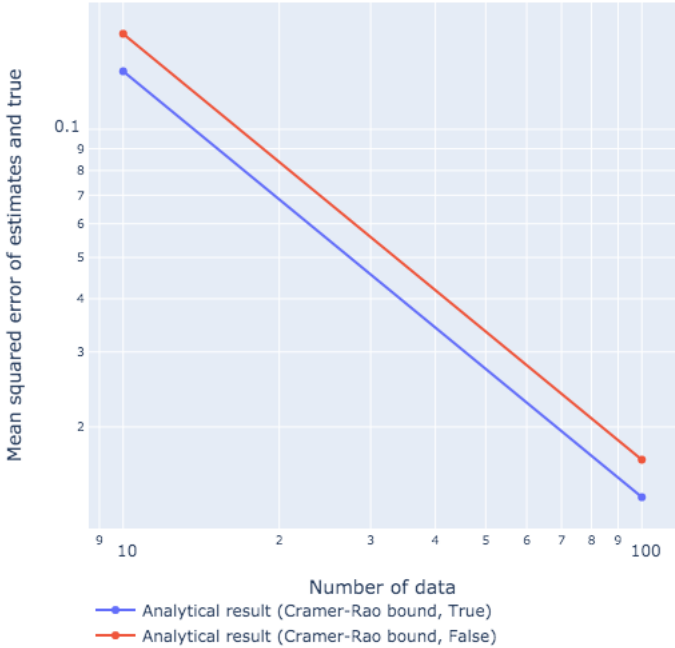
# MSE of estimators

## Comparison of analytical results

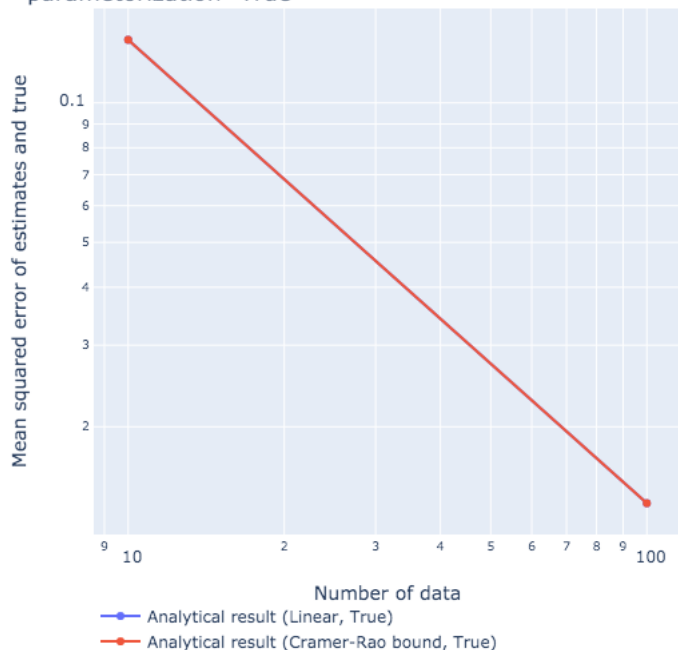
Mean squared error  
Analytical result  
estimator=Linear



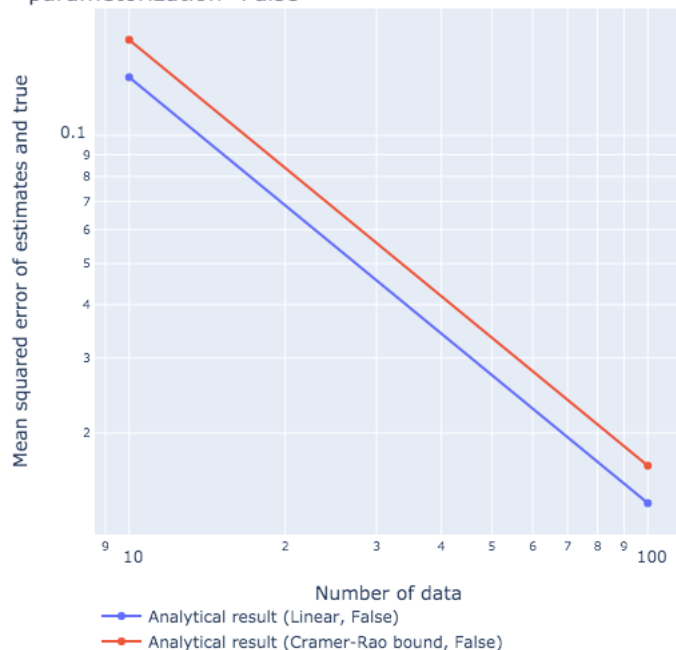
Mean squared error  
Analytical result  
Cramer-Rao bound



Mean squared error  
Analytical result  
parameterization=True

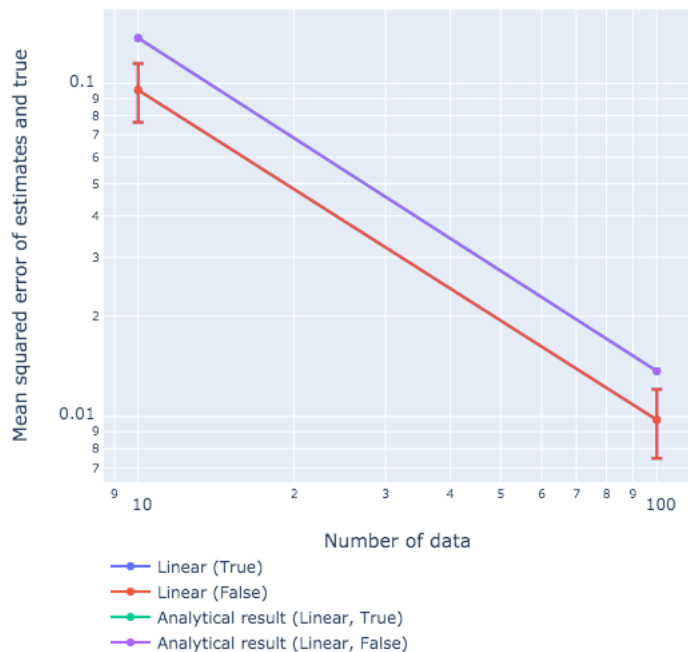


Mean squared error  
Analytical result  
parameterization=False

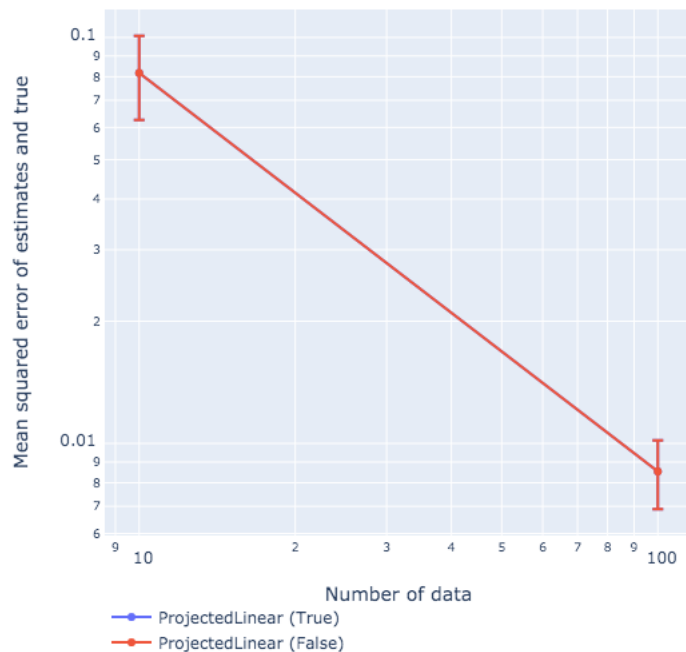


## Comparison of parametrization

Mean squared error  
Estimator=Linear



Mean squared error  
Estimator=ProjectedLinear

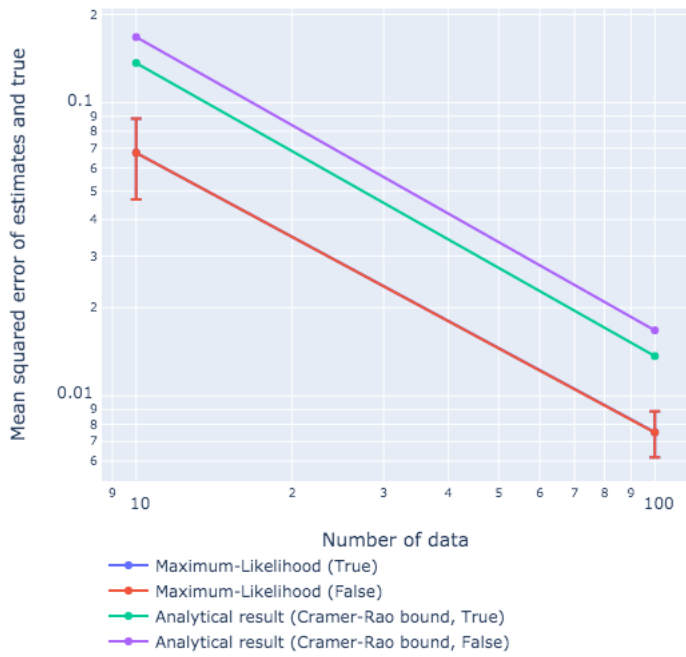


### Mean squared error

Estimator=LossMinimization

Loss=WeightedRelativeEntropy

Algo=ProjectedGradientDescentBacktracking

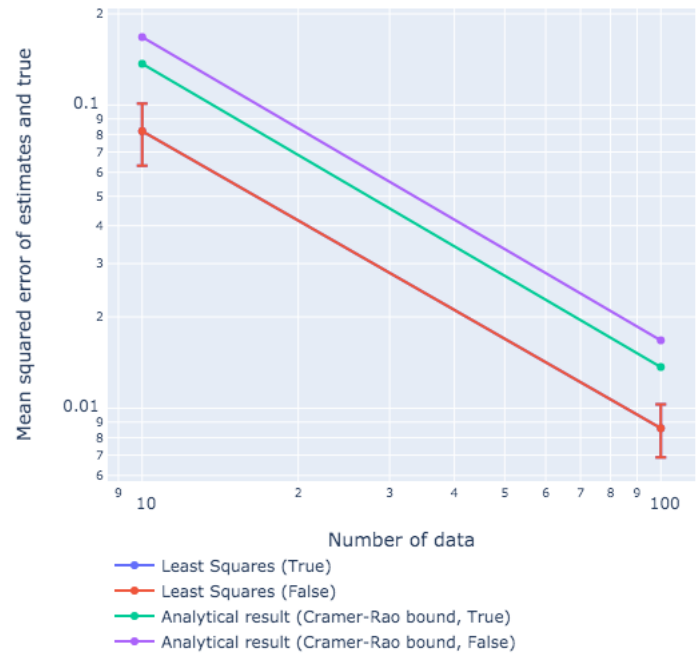


### Mean squared error

Estimator=LossMinimization

Loss=WeightedProbabilityBasedSquaredError

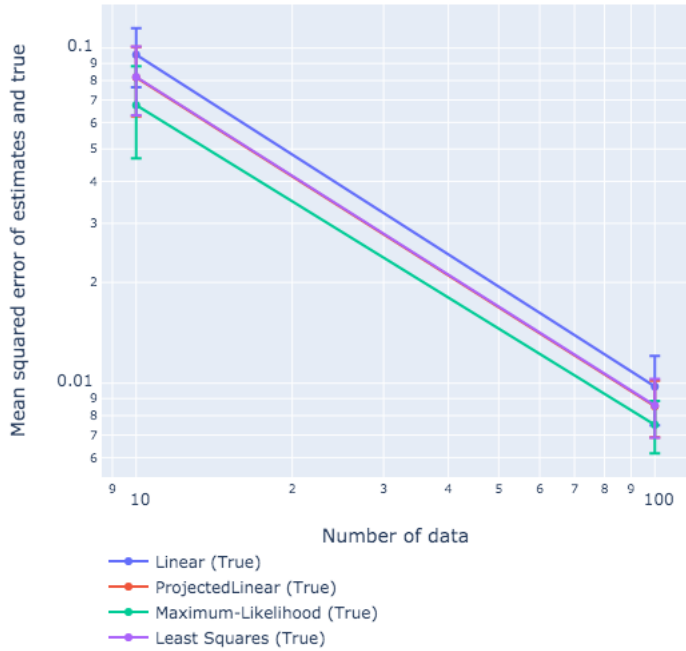
Algo=ProjectedGradientDescentBacktracking



## Comparison of estimators

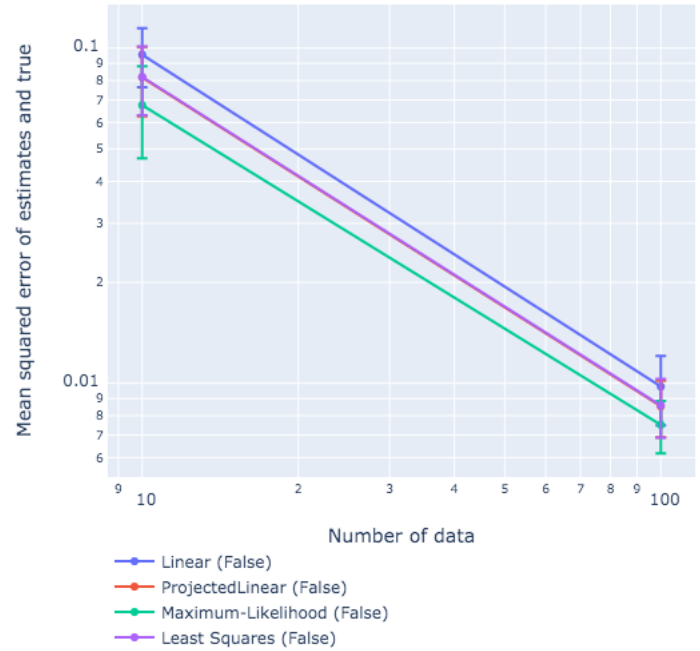
### Mean squared error

parametrization=True



### Mean squared error

parametrization=False

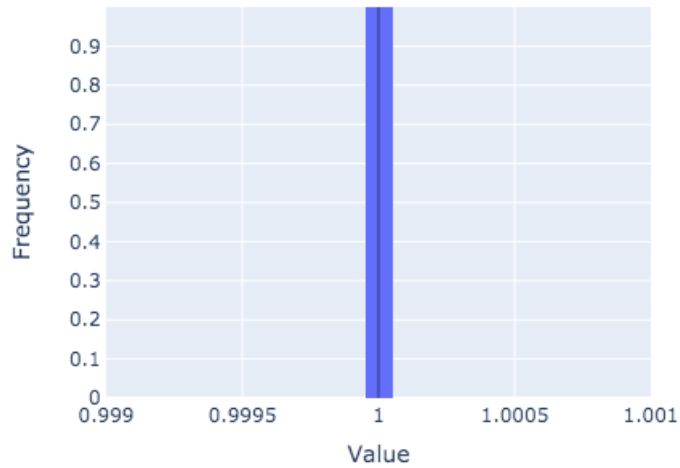


# Physicality violation test

## Test of equality constraint violation

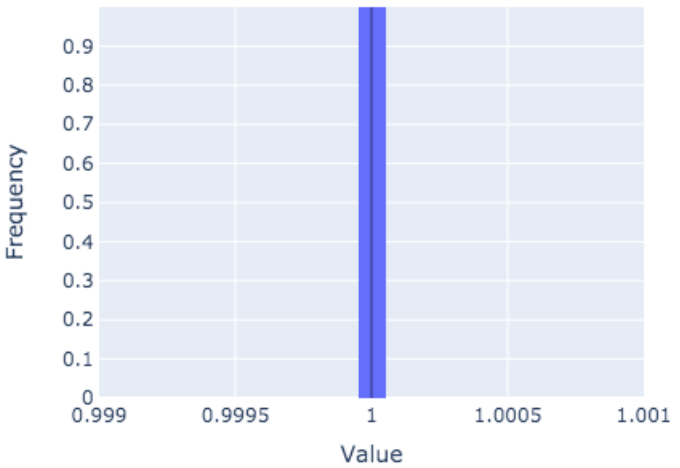
### Case 0: Linear (True)

N=10, Nrep=10



Min: 0.9999999999999997  
Max: 0.9999999999999999  
|Max-Min|: 2.220446049250313e-16

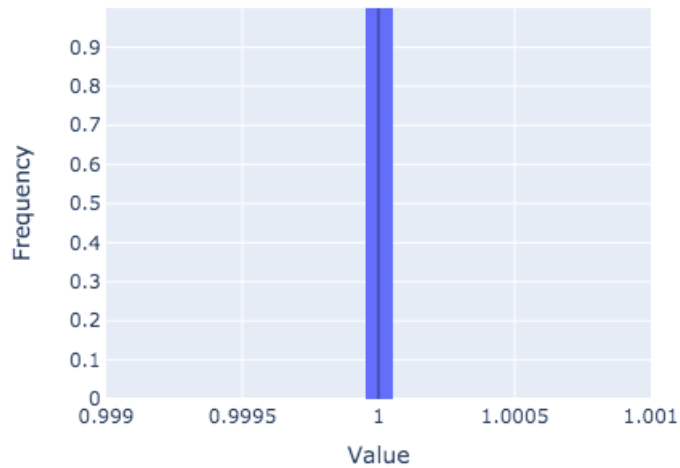
N=100, Nrep=10



Min: 0.9999999999999998  
Max: 0.9999999999999999  
|Max-Min|: 1.1102230246251565e-16

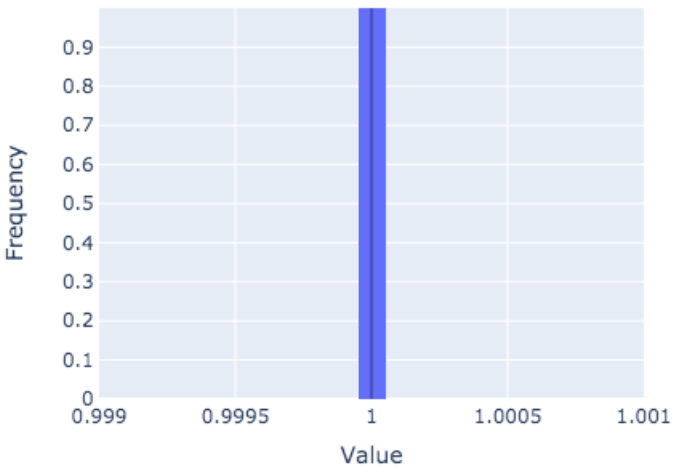
### Case 1: Linear (False)

N=10, Nrep=10



Min: 0.9999999999999998  
Max: 1.0  
|Max-Min|: 2.220446049250313e-16

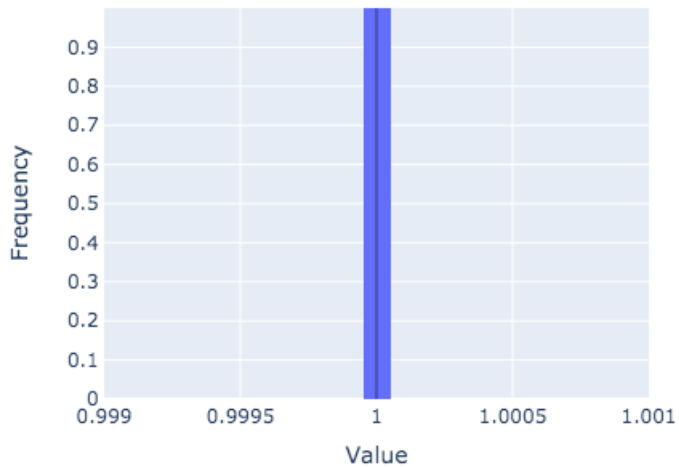
N=100, Nrep=10



Min: 0.9999999999999998  
Max: 1.0  
|Max-Min|: 2.220446049250313e-16

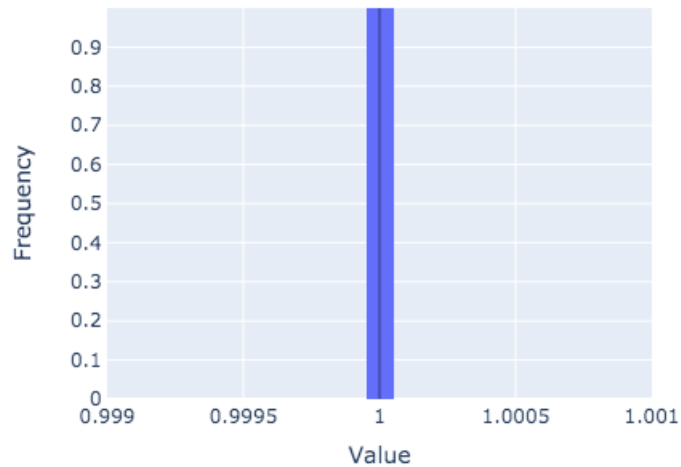
### Case 2: ProjectedLinear (True)

N=10, Nrep=10



Min: 0.9999999999999998  
Max: 0.9999999999999998  
|Max-Min|: 0.0

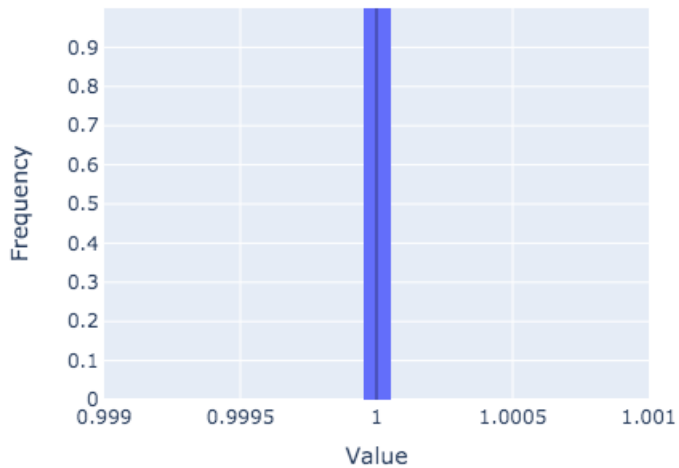
N=100, Nrep=10



Min: 0.9999999999999998  
Max: 0.9999999999999998  
|Max-Min|: 0.0

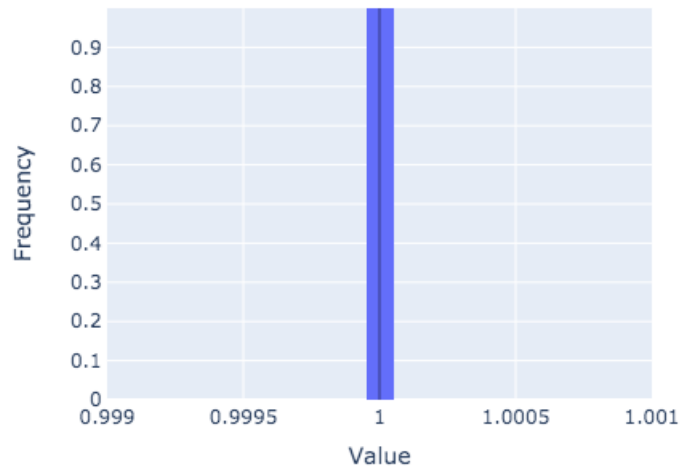
### Case 3: ProjectedLinear (False)

N=10, Nrep=10



Min: 0.9999999999999998  
Max: 1.0000001786892174  
|Max-Min|: 1.786892176447452e-07

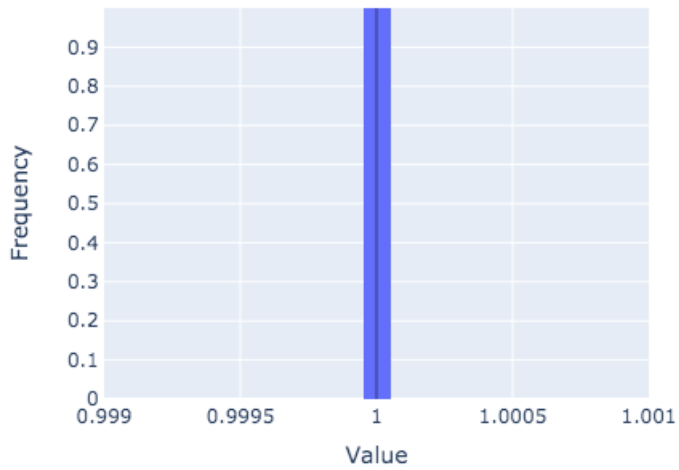
N=100, Nrep=10



Min: 0.9999999999999997  
Max: 1.000000130591659  
|Max-Min|: 1.3059165937523431e-07

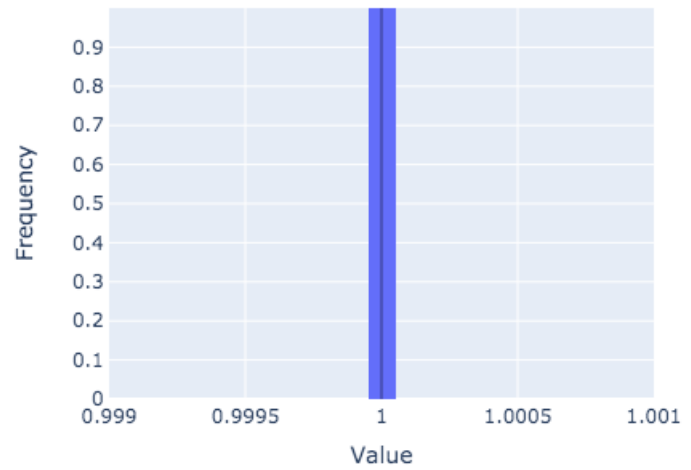
### Case 4: Maximum-Likelihood (True)

N=10, Nrep=10



Min: 0.9999999999999998  
Max: 0.9999999999999998  
|Max-Min|: 0.0

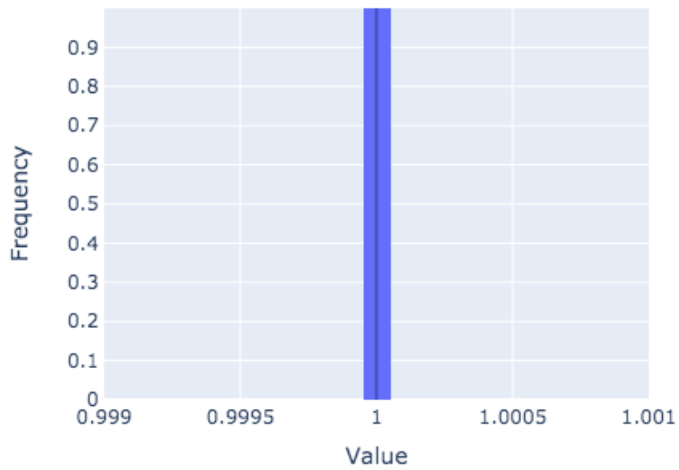
N=100, Nrep=10



Min: 0.9999999999999998  
Max: 0.9999999999999998  
|Max-Min|: 0.0

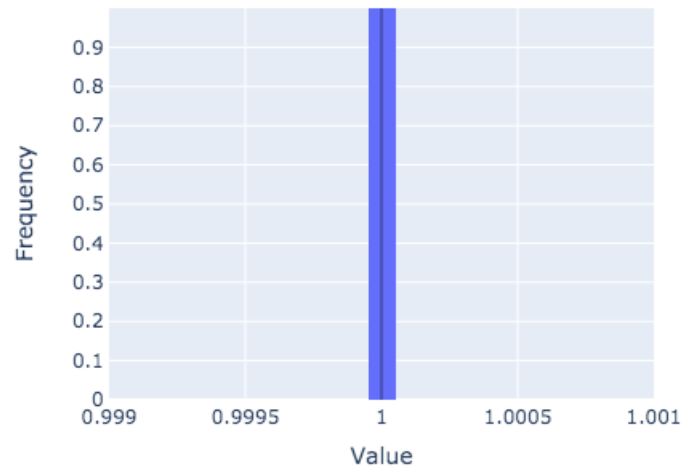
### Case 5: Maximum-Likelihood (False)

N=10, Nrep=10



Min: 1.0000000061404672  
Max: 1.0000001784357189  
|Max-Min|: 1.7229525162143489e-07

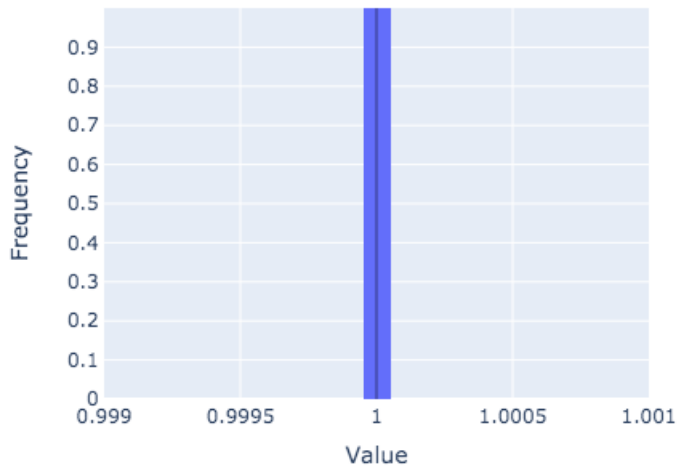
N=100, Nrep=10



Min: 1.000000007962313  
Max: 1.000000181521831  
|Max-Min|: 1.7355951786868218e-07

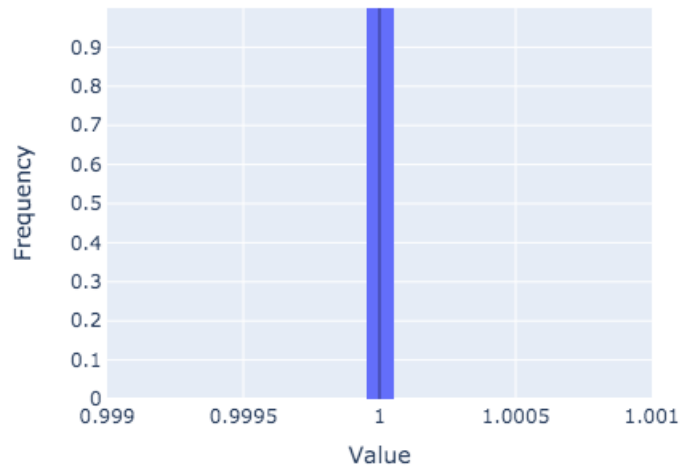
### Case 6: Least Squares (True)

N=10, Nrep=10



Min: 0.9999999999999998  
Max: 0.9999999999999998  
|Max-Min|: 0.0

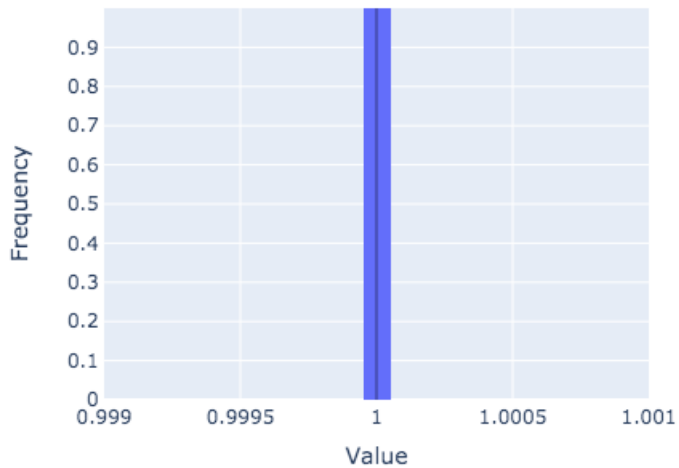
N=100, Nrep=10



Min: 0.9999999999999998  
Max: 0.9999999999999998  
|Max-Min|: 0.0

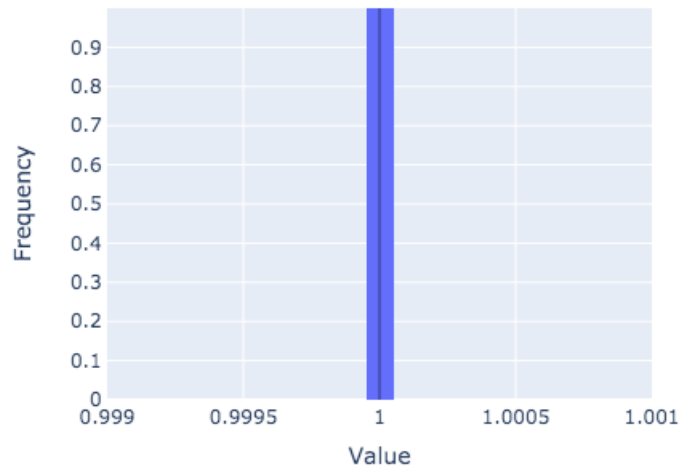
### Case 7: Least Squares (False)

N=10, Nrep=10



Min: 1.0  
Max: 1.00000015436117  
|Max-Min|: 1.5436117006828454e-07

N=100, Nrep=10



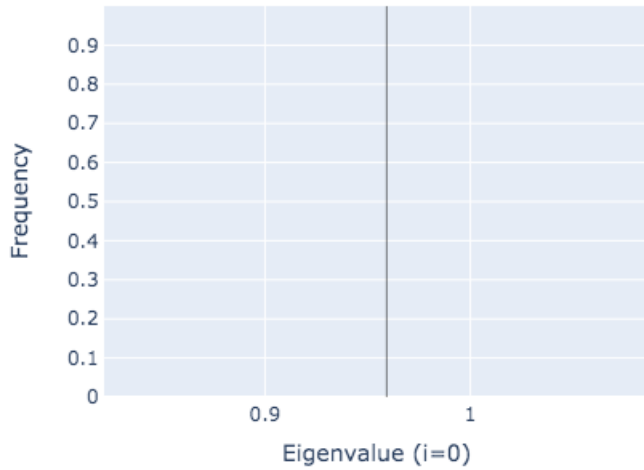
Min: 0.9999999999999998  
Max: 1.0000001612437763  
|Max-Min|: 1.612437765530217e-07

## Test of inequality constraint violation

### Eigenvalue

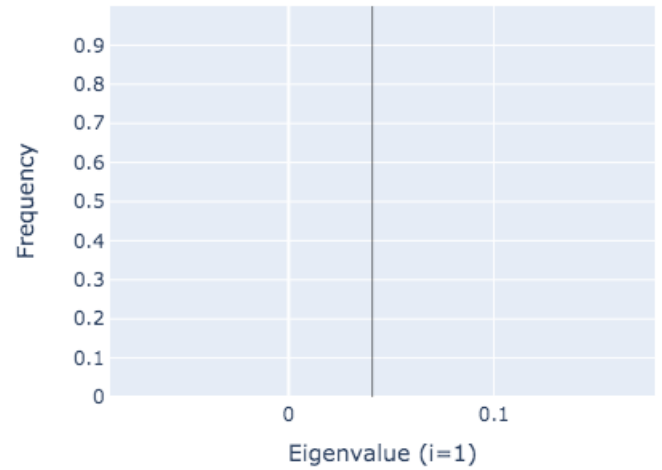
#### Case 0: Linear (True)

N=10, Nrep=10



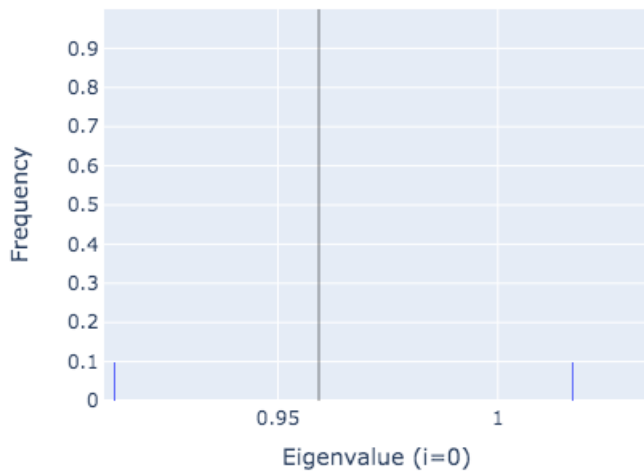
Min: 0.8217439063611547  
Max: 1.0868807432325331  
|Max-Min|: 0.26513683687137846

N=10, Nrep=10



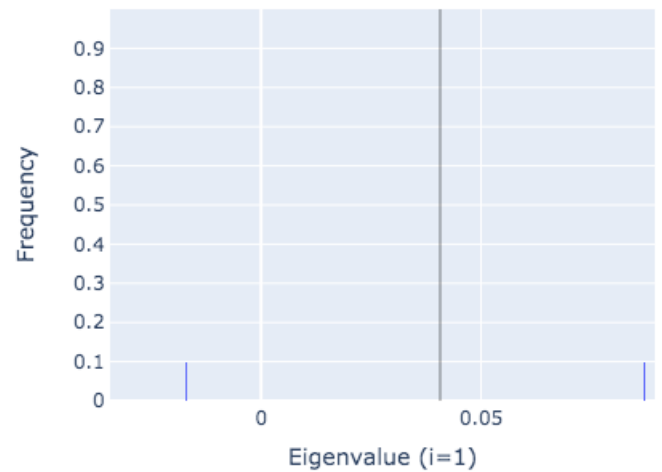
Min: -0.08688074323253331  
Max: 0.1782560936388451  
|Max-Min|: 0.2651368368713784

N=100, Nrep=10



Min: 0.9105490618058564  
Max: 1.0342338199919683  
|Max-Min|: 0.12368475818611191

N=100, Nrep=10

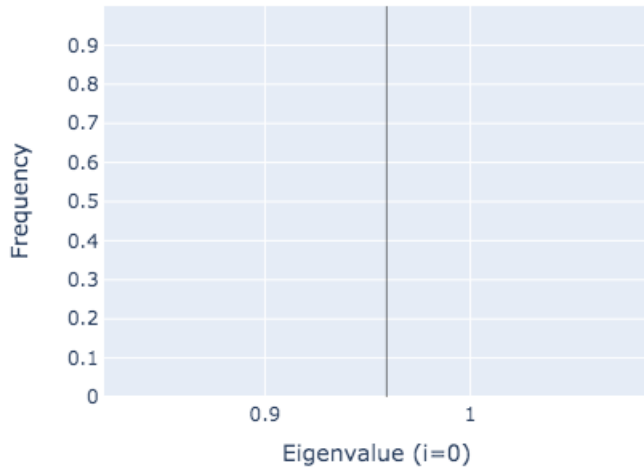


Min: -0.03423381999196835  
Max: 0.08945093819414349  
|Max-Min|: 0.12368475818611184

Case 1: Linear (False)

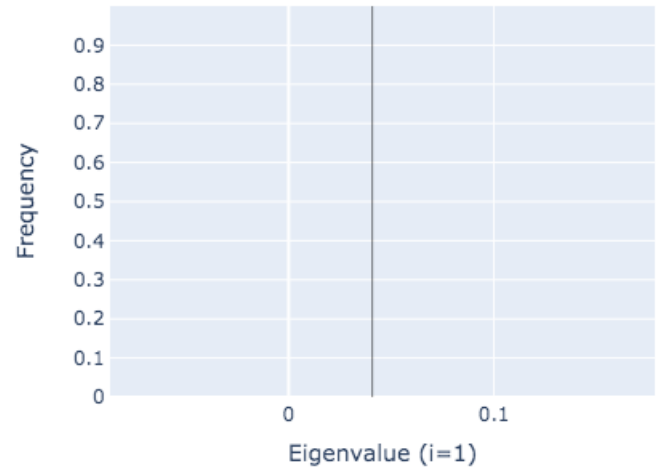


N=10, Nrep=10



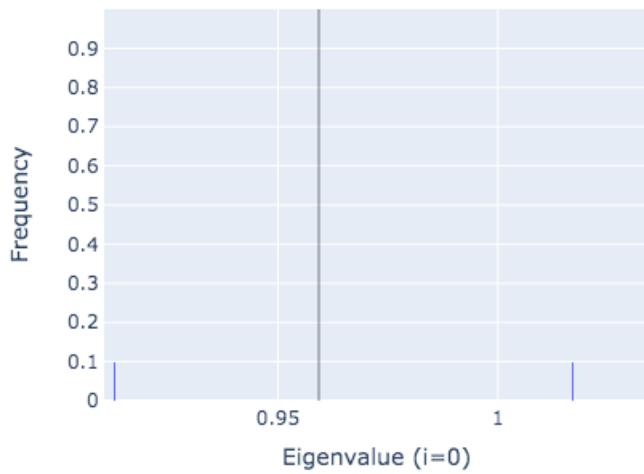
Min: 0.8217439063611547  
Max: 1.0868807432325331  
|Max-Min|: 0.26513683687137846

N=10, Nrep=10



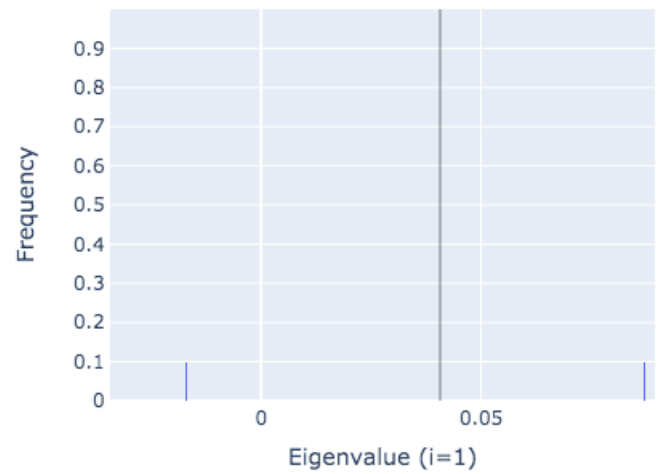
Min: -0.08688074323253318  
Max: 0.17825609363884512  
|Max-Min|: 0.2651368368713783

N=100, Nrep=10



Min: 0.9105490618058564  
Max: 1.0342338199919683  
|Max-Min|: 0.12368475818611191

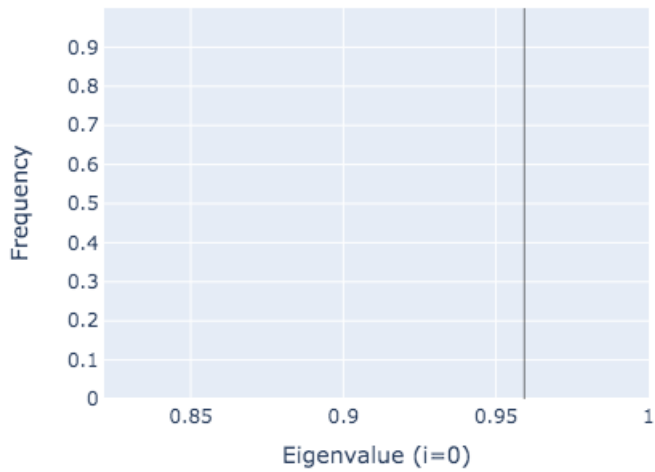
N=100, Nrep=10



Min: -0.03423381999196848  
Max: 0.08945093819414354  
|Max-Min|: 0.12368475818611202

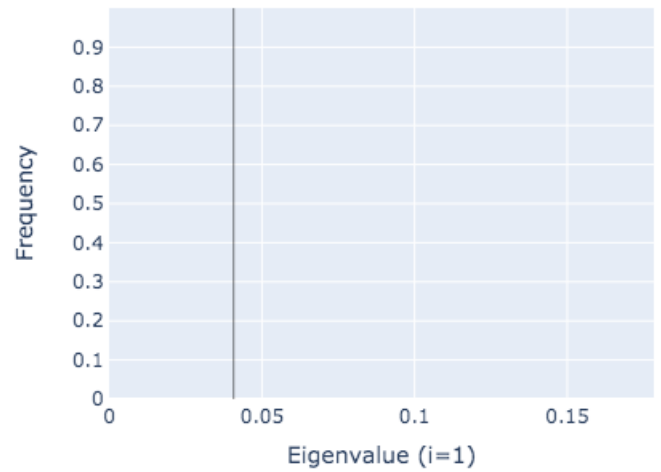
Case 2: ProjectedLinear (True)

N=10, Nrep=10



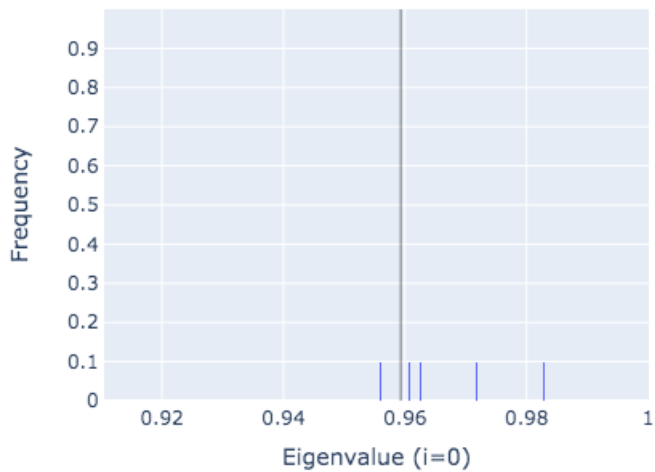
Min: 0.8217439063611546  
 Max: 1.0000000893446086  
 |Max-Min|: 0.17825618298345403

N=10, Nrep=10



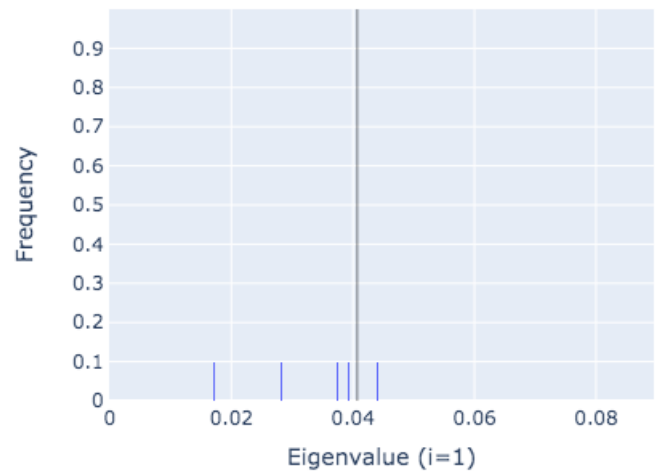
Min: -8.934460889523099e-08  
 Max: 0.17825609363884518  
 |Max-Min|: 0.17825618298345408

N=100, Nrep=10



Min: 0.9105490618058562  
 Max: 1.0000000652958296  
 |Max-Min|: 0.08945100348997348

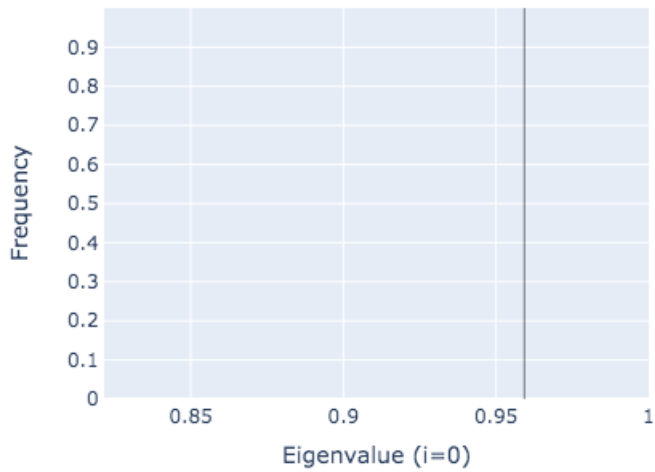
N=100, Nrep=10



Min: -6.529582974659776e-08  
 Max: 0.08945093819414354  
 |Max-Min|: 0.08945100348997329

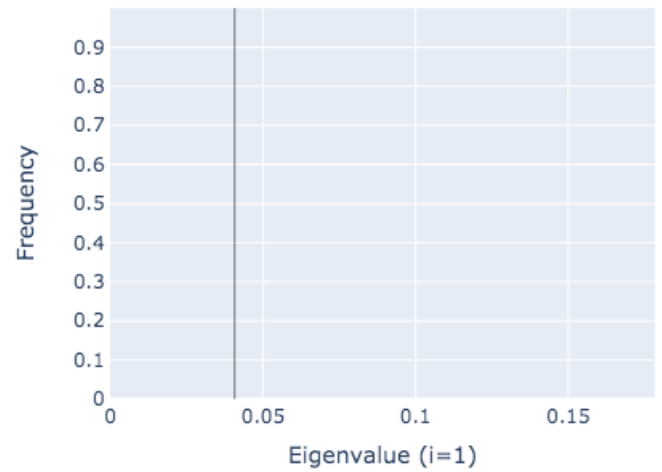
Case 3: ProjectedLinear (False)

N=10, Nrep=10



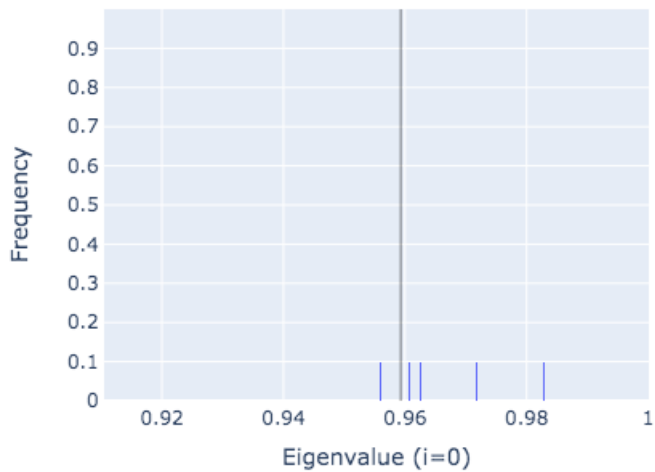
Min: 0.8217439063611548  
 Max: 1.0000001786892174  
 |Max-Min|: 0.17825627232806263

N=10, Nrep=10



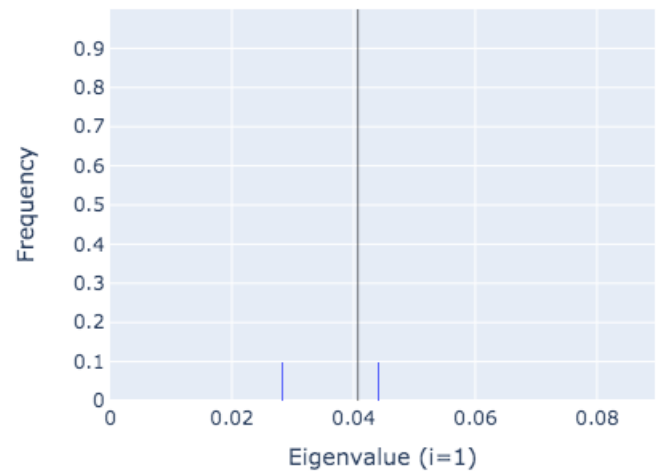
Min: -8.326672684688674e-17  
 Max: 0.1782560936388453  
 |Max-Min|: 0.17825609363884537

N=100, Nrep=10



Min: 0.9105490618058562  
 Max: 1.0000001305916593  
 |Max-Min|: 0.08945106878580311

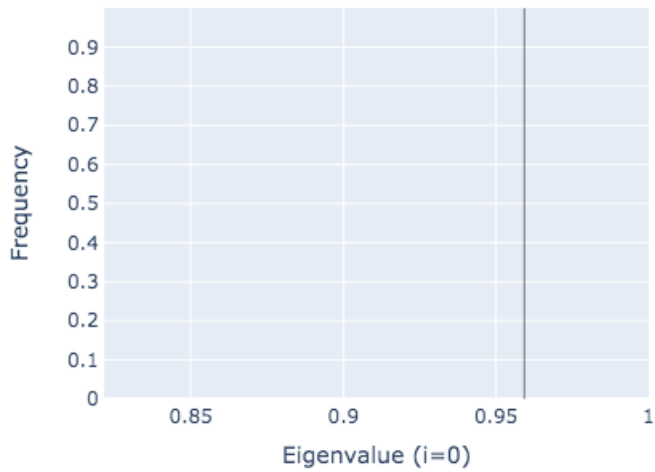
N=100, Nrep=10



Min: -1.3183898417423734e-16  
 Max: 0.08945093819414349  
 |Max-Min|: 0.08945093819414363

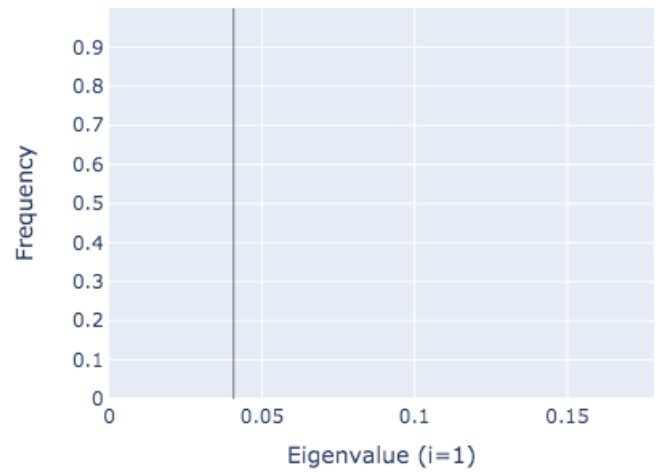
#### Case 4: Maximum-Likelihood (True)

N=10, Nrep=10



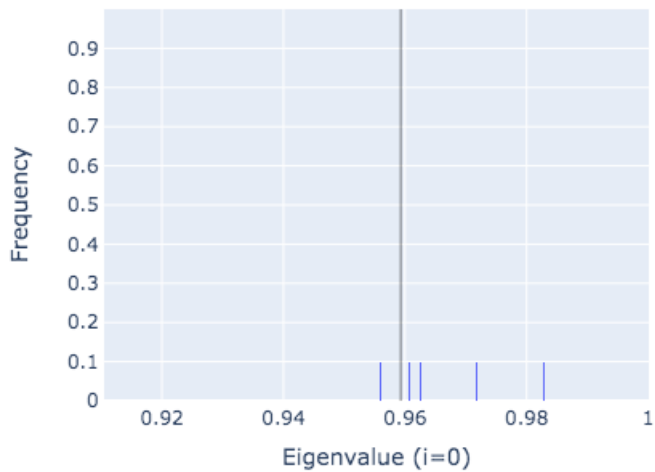
Min: 0.8217439099529941  
 Max: 1.000000080374627  
 |Max-Min|: 0.1782561704216329

N=10, Nrep=10



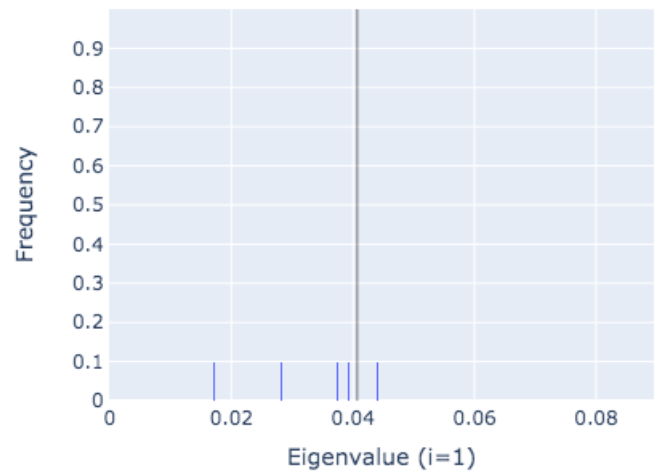
Min: -8.037462703724962e-08  
 Max: 0.17825609004700563  
 |Max-Min|: 0.17825617042163266

N=100, Nrep=10



Min: 0.9105490612563245  
 Max: 1.0000000786009928  
 |Max-Min|: 0.08945101734466832

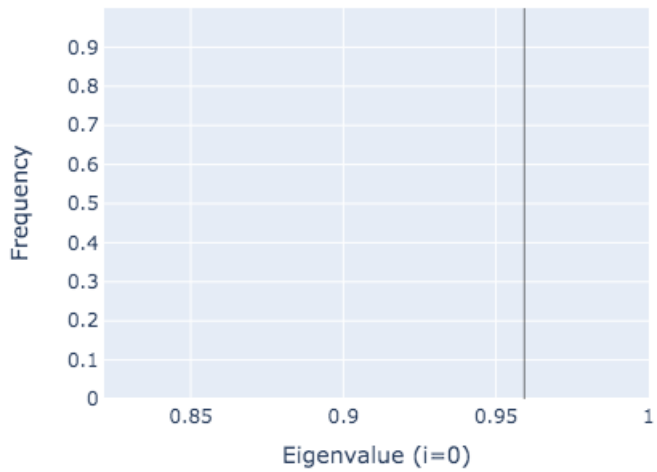
N=100, Nrep=10



Min: -7.860099304800652e-08  
 Max: 0.08945093874367524  
 |Max-Min|: 0.08945101734466829

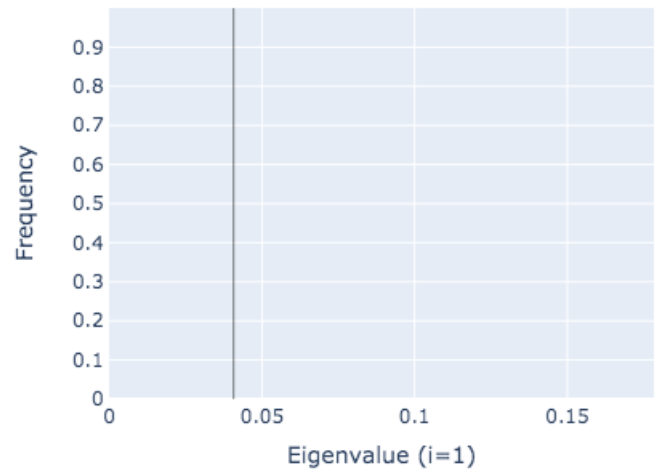
## Case 5: Maximum-Likelihood (False)

N=10, Nrep=10



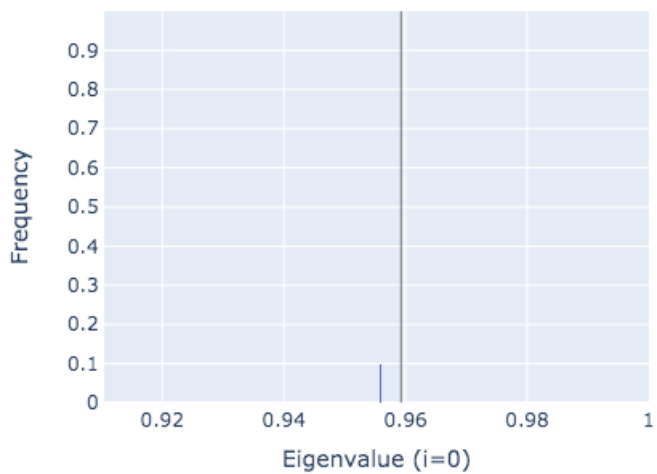
Min: 0.8217419203339902  
Max: 1.0000001784357186  
|Max-Min|: 0.17825825810172846

N=10, Nrep=10



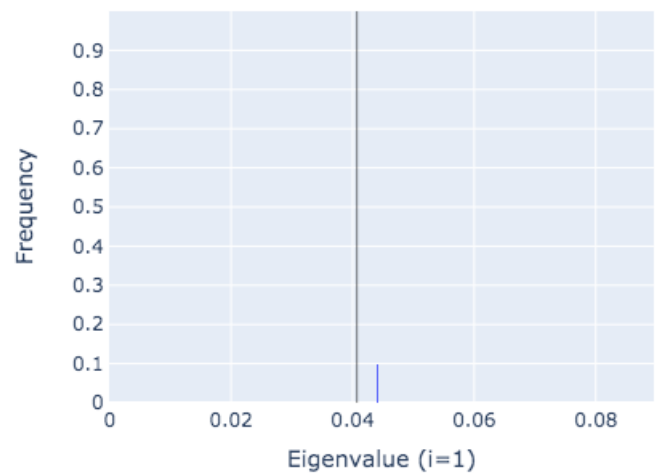
Min: 1.1102230246251565e-16  
Max: 0.17825808580647715  
|Max-Min|: 0.17825808580647703

N=100, Nrep=10



Min: 0.9105361357403978  
Max: 1.0000001815218298  
|Max-Min|: 0.08946404578143197

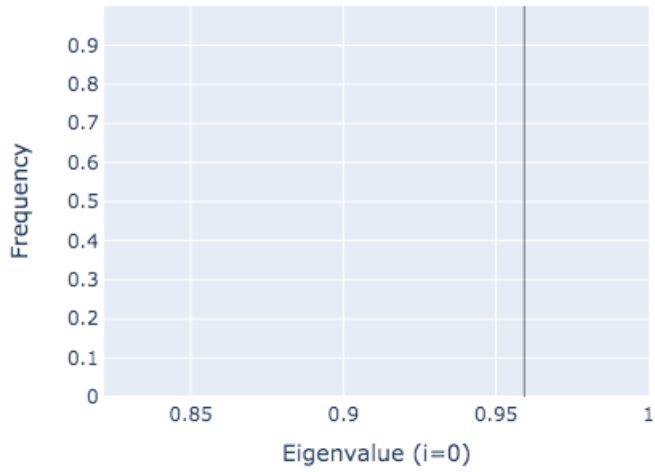
N=100, Nrep=10



Min: 1.175708835843281e-15  
Max: 0.08946387235627798  
|Max-Min|: 0.0894638723562768

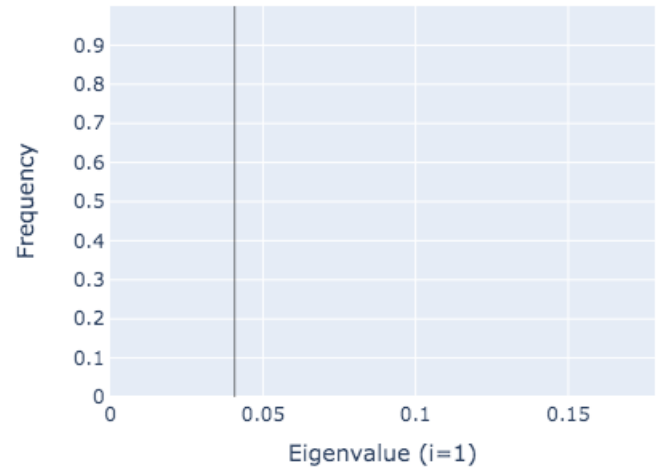
## Case 6: Least Squares (True)

N=10, Nrep=10



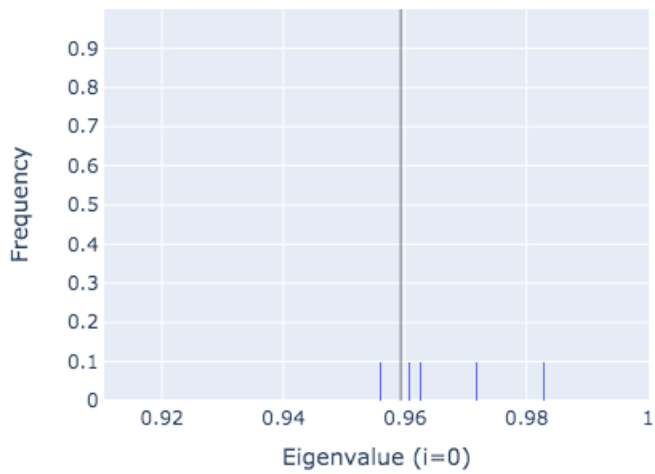
Min: 0.8217439063611545  
 Max: 1.000000080061129  
 |Max-Min|: 0.1782561736999746

N=10, Nrep=10



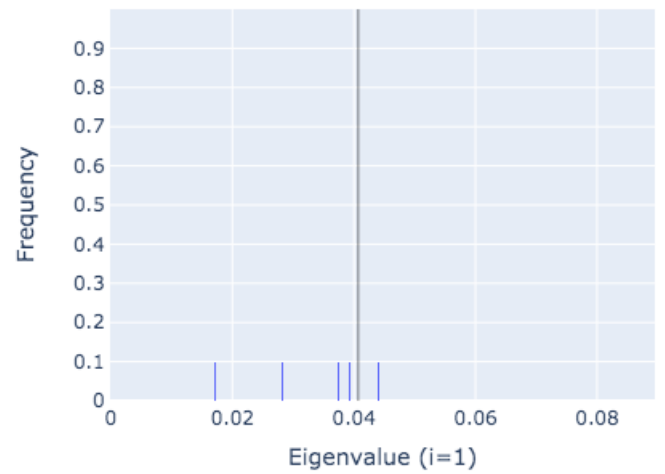
Min: -8.006112933722775e-08  
 Max: 0.17825609363884526  
 |Max-Min|: 0.1782561736999746

N=100, Nrep=10



Min: 0.9105490618058559  
 Max: 1.000000069820626  
 |Max-Min|: 0.08945100801477002

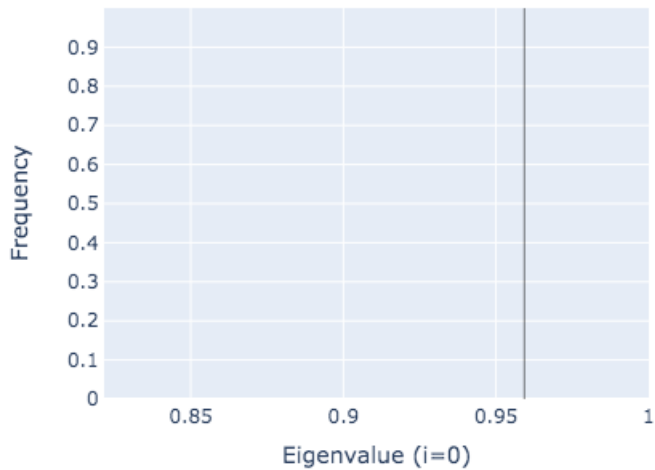
N=100, Nrep=10



Min: -6.982062614006856e-08  
 Max: 0.0894509381941439  
 |Max-Min|: 0.08945100801477004

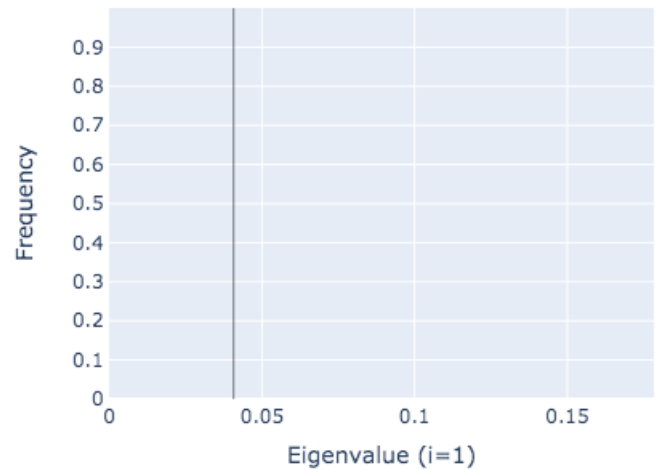
## Case 7: Least Squares (False)

N=10, Nrep=10



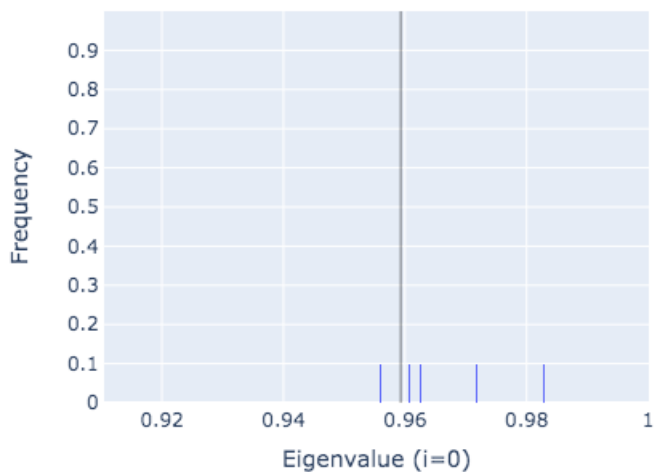
Min: 0.821743906361156  
 Max: 1.00000015436117  
 |Max-Min|: 0.17825624800001405

N=10, Nrep=10



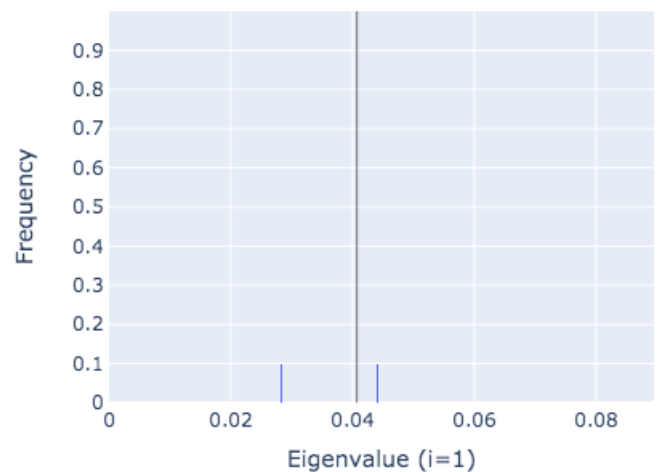
Min: -1.3877787807814457e-16  
 Max: 0.1782560936388442  
 |Max-Min|: 0.17825609363884434

N=100, Nrep=10



Min: 0.910549061805857  
 Max: 1.000000161243776  
 |Max-Min|: 0.08945109943791907

N=100, Nrep=10



Min: 4.7704895589362195e-17  
 Max: 0.0894509381941427  
 |Max-Min|: 0.08945093819414265

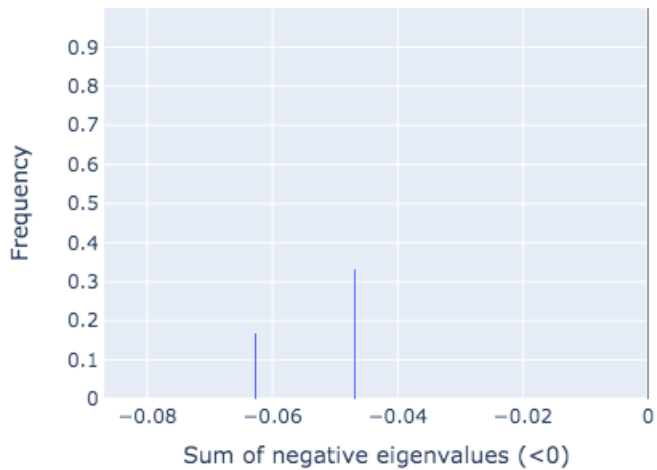
## Sum of unphysical eigenvalues

### Case 0: Linear (True)

N=10

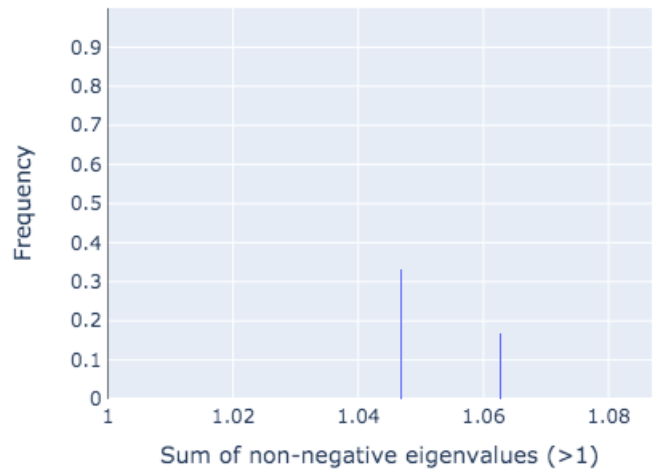
Number of unphysical estimates=6

N=10, Nrep=10



Min: -0.08688074323253331  
 Max: -0.002998403797154006  
 |Max-Min|: 0.0838823394353793

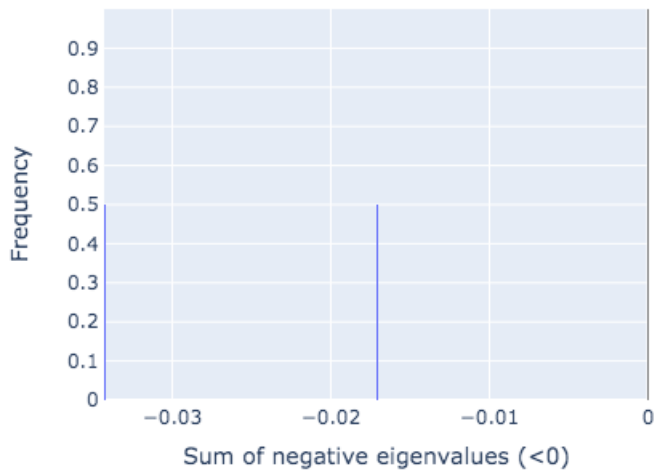
N=10, Nrep=10



Min: 1.0029984037971538  
 Max: 1.0868807432325331  
 |Max-Min|: 0.0838823394353793

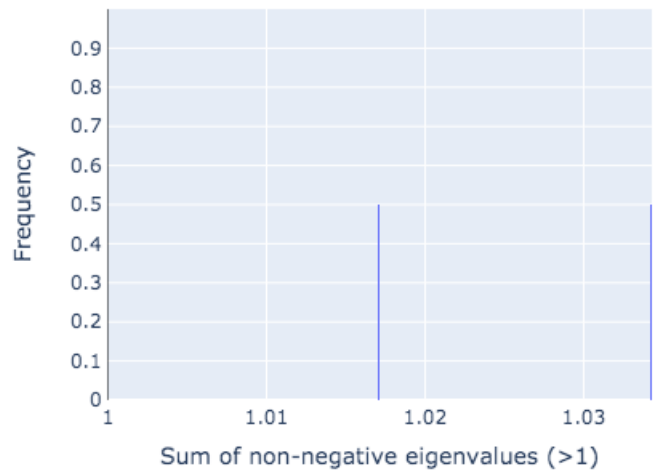
N=100  
 Number of unphysical estimates=2

N=100, Nrep=10



Min: -0.03423381999196835  
 Max: -0.017068185480011048  
 |Max-Min|: 0.017165634511957305

N=100, Nrep=10



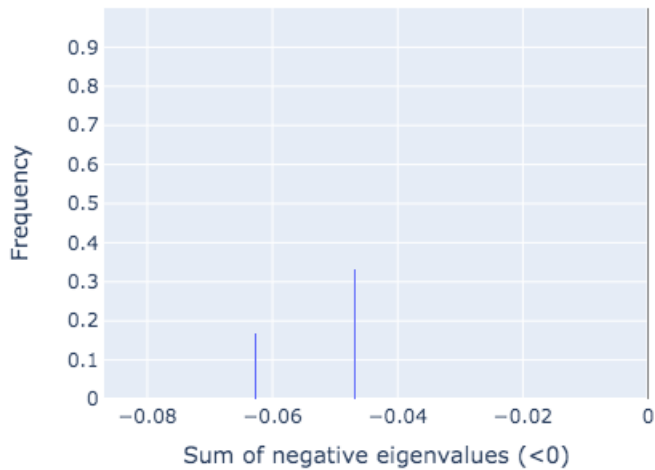
Min: 1.017068185480011  
 Max: 1.0342338199919683  
 |Max-Min|: 0.017165634511957295

### Case 1: Linear (False)

N=10  
 Number of unphysical estimates=6

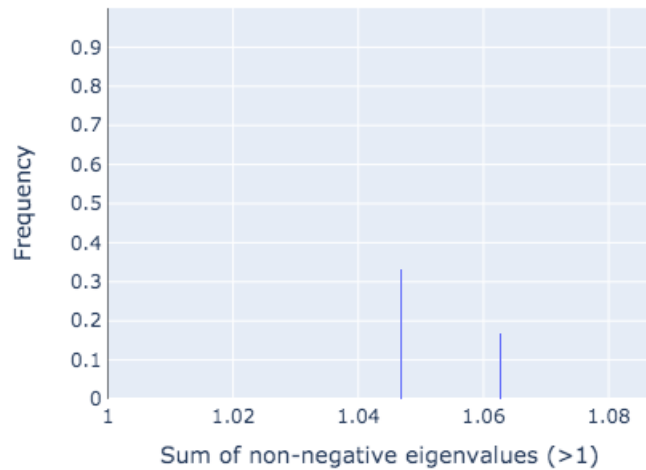


N=10, Nrep=10



Min: -0.08688074323253318  
 Max: -0.0029984037971540756  
 |Max-Min|: 0.08388233943537911

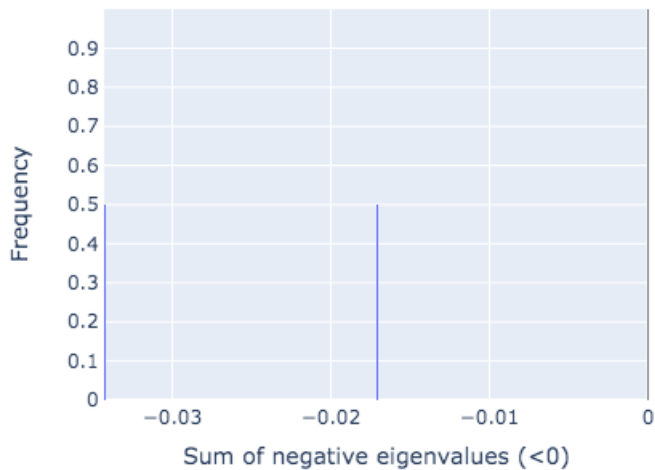
N=10, Nrep=10



Min: 1.0029984037971538  
 Max: 1.0868807432325331  
 |Max-Min|: 0.08388233943537937

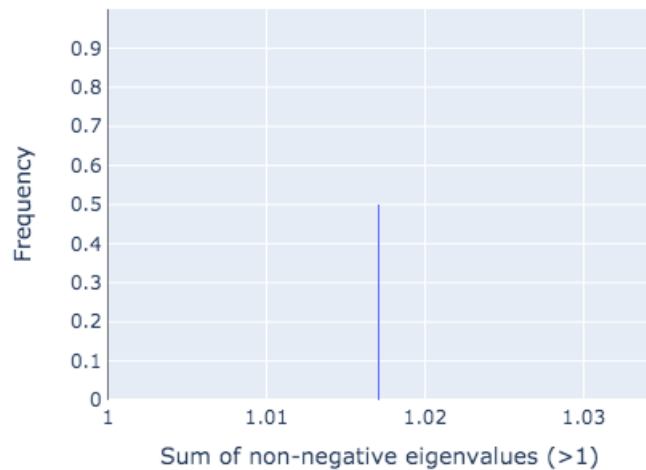
N=100  
 Number of unphysical estimates=2

N=100, Nrep=10



Min: -0.03423381999196848  
 Max: -0.017068185480010926  
 |Max-Min|: 0.017165634511957552

N=100, Nrep=10

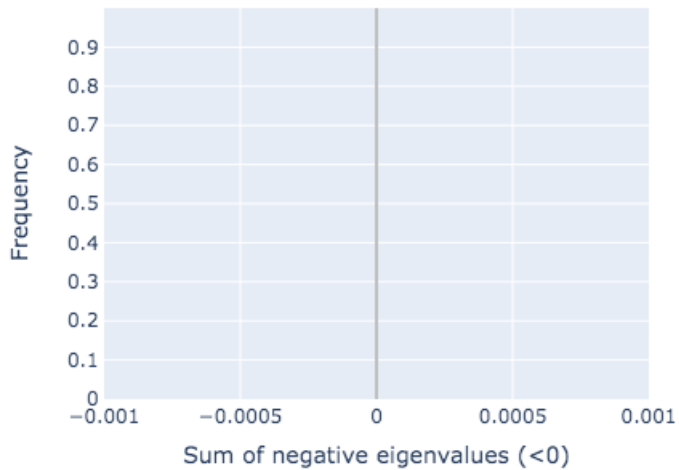


Min: 1.017068185480011  
 Max: 1.0342338199919683  
 |Max-Min|: 0.017165634511957295

## Case 2: ProjectedLinear (True)

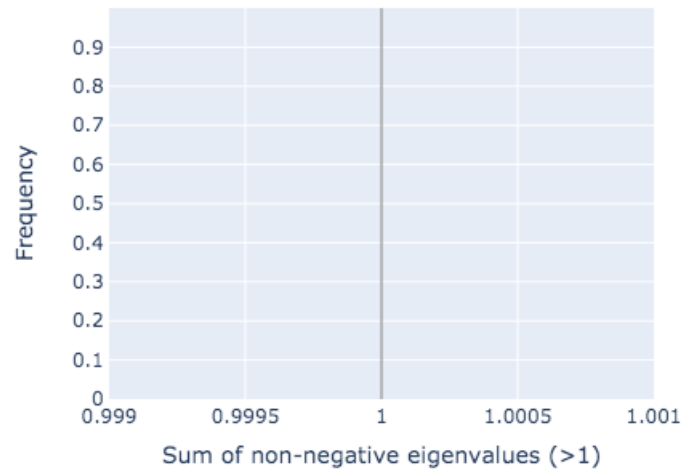
N=10  
 Number of unphysical estimates=0

N=10, Nrep=10



Min: -  
Max: -  
|Max-Min|: -

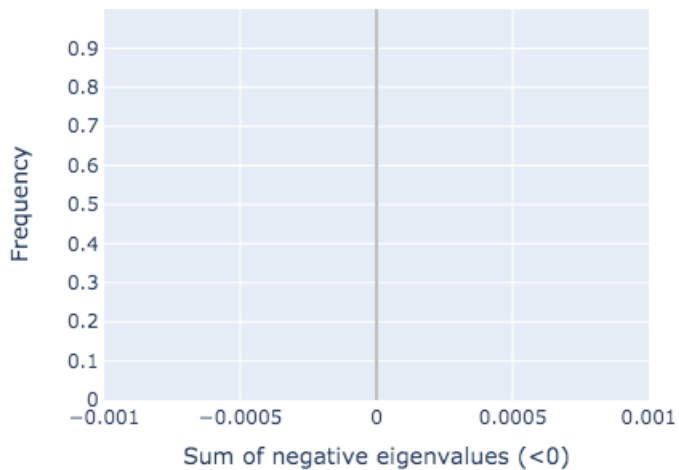
N=10, Nrep=10



Min: -  
Max: -  
|Max-Min|: -

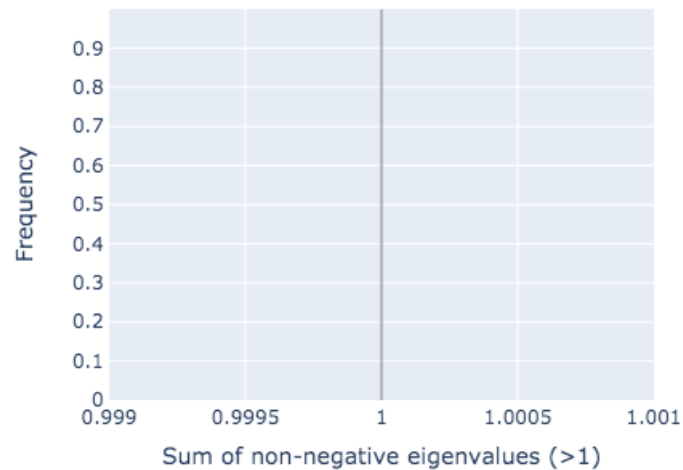
N=100  
Number of unphysical estimates=0

N=100, Nrep=10



Min: -  
Max: -  
|Max-Min|: -

N=100, Nrep=10

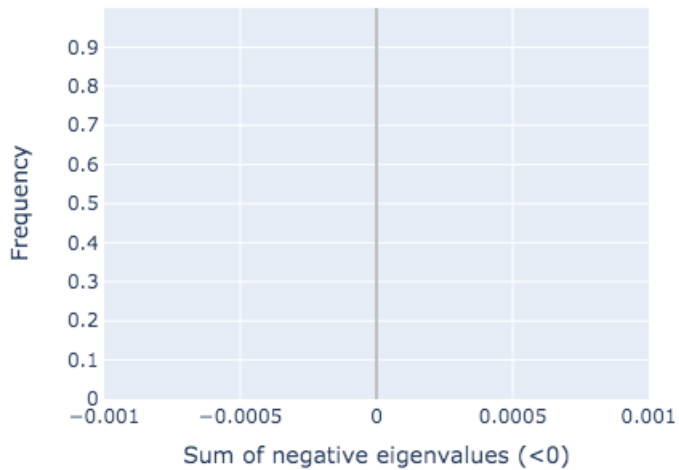


Min: -  
Max: -  
|Max-Min|: -

### Case 3: ProjectedLinear (False)

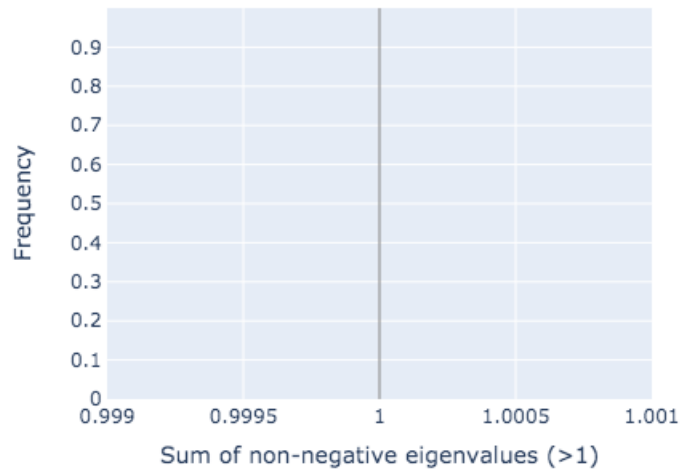
N=10  
Number of unphysical estimates=0

N=10, Nrep=10



Min: -  
Max: -  
|Max-Min|: -

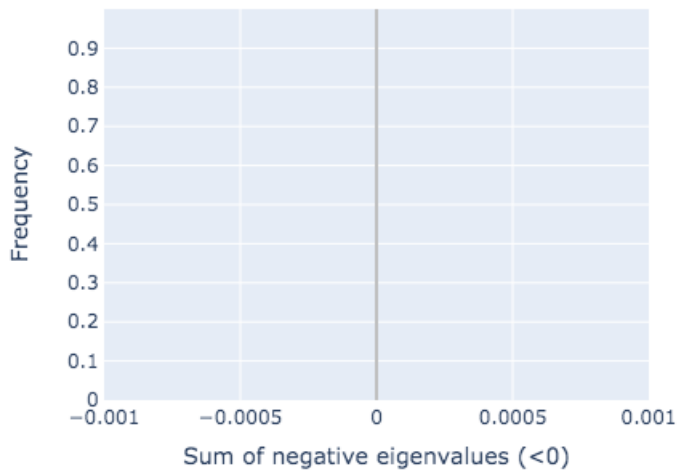
N=10, Nrep=10



Min: -  
Max: -  
|Max-Min|: -

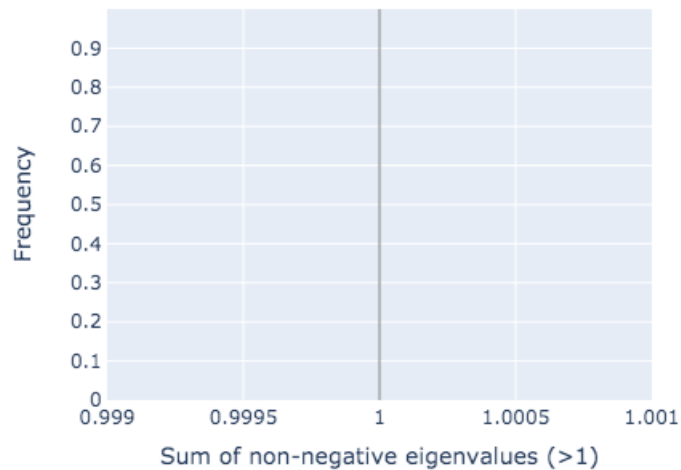
N=100  
Number of unphysical estimates=0

N=100, Nrep=10



Min: -  
Max: -  
|Max-Min|: -

N=100, Nrep=10

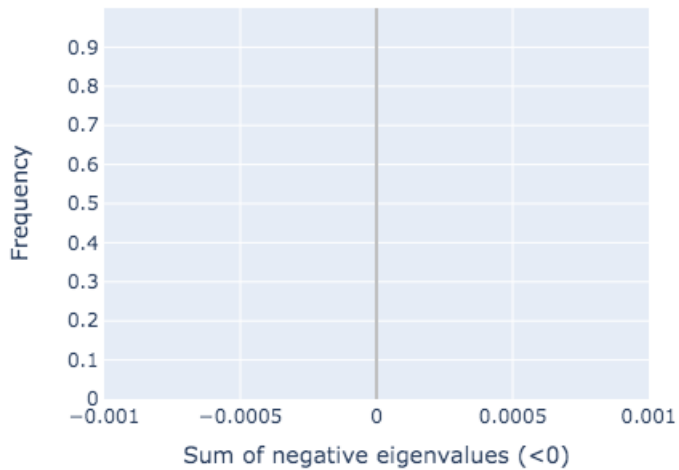


Min: -  
Max: -  
|Max-Min|: -

#### Case 4: Maximum-Likelihood (True)

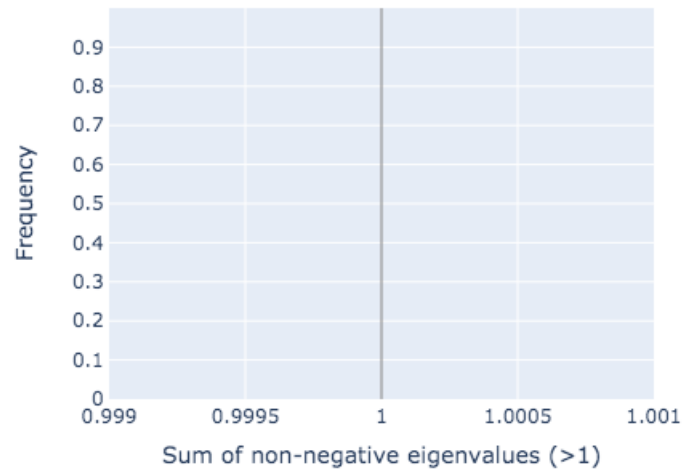
N=10  
Number of unphysical estimates=0

N=10, Nrep=10



Min: -  
Max: -  
|Max-Min|: -

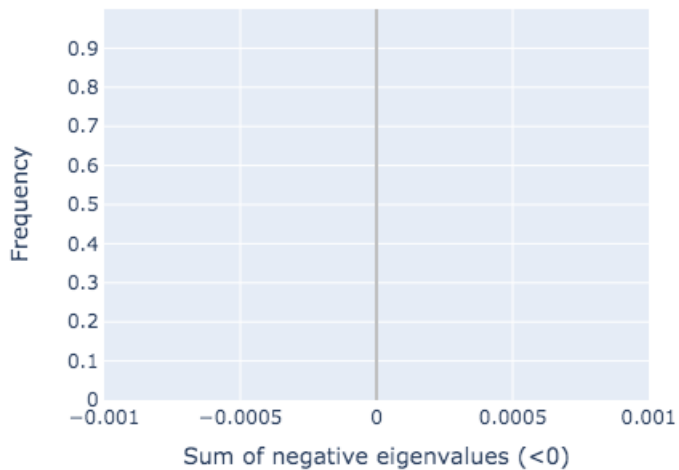
N=10, Nrep=10



Min: -  
Max: -  
|Max-Min|: -

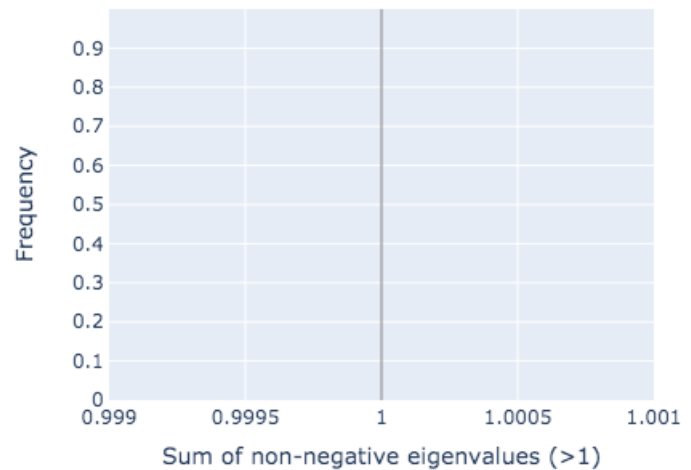
N=100  
Number of unphysical estimates=0

N=100, Nrep=10



Min: -  
Max: -  
|Max-Min|: -

N=100, Nrep=10

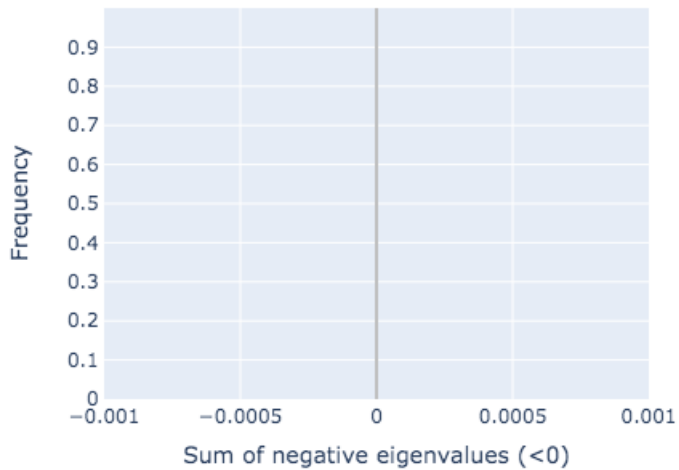


Min: -  
Max: -  
|Max-Min|: -

### Case 5: Maximum-Likelihood (False)

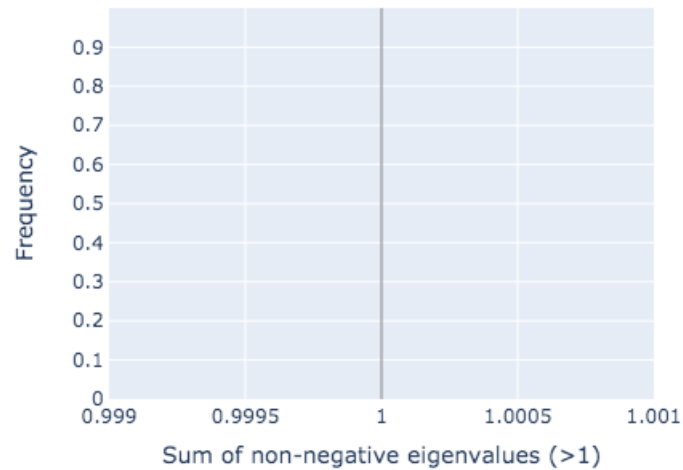
N=10  
Number of unphysical estimates=0

N=10, Nrep=10



Min: -  
Max: -  
|Max-Min|: -

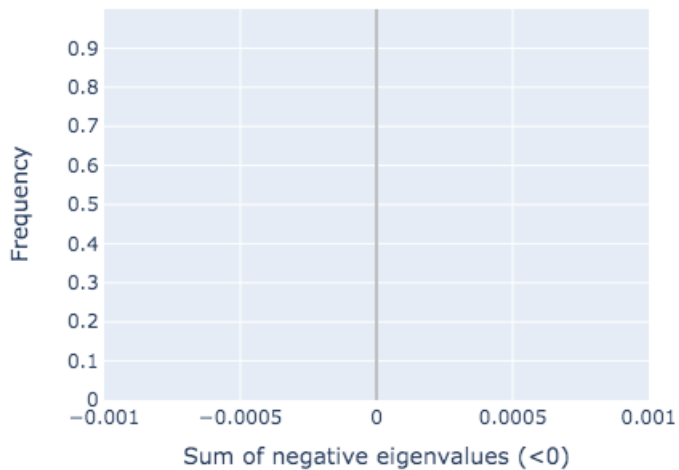
N=10, Nrep=10



Min: -  
Max: -  
|Max-Min|: -

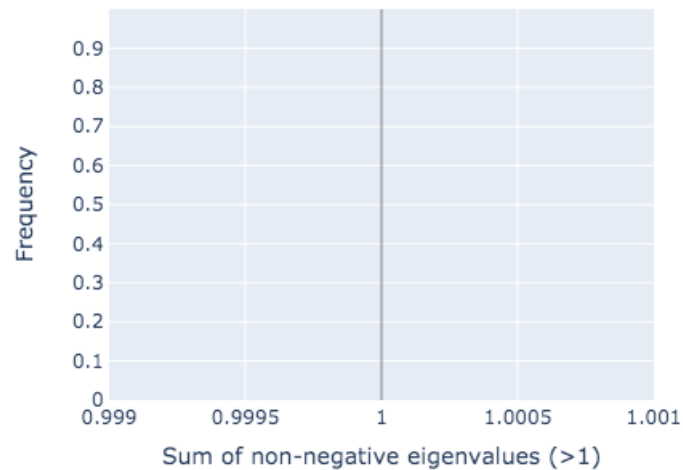
N=100  
Number of unphysical estimates=0

N=100, Nrep=10



Min: -  
Max: -  
|Max-Min|: -

N=100, Nrep=10

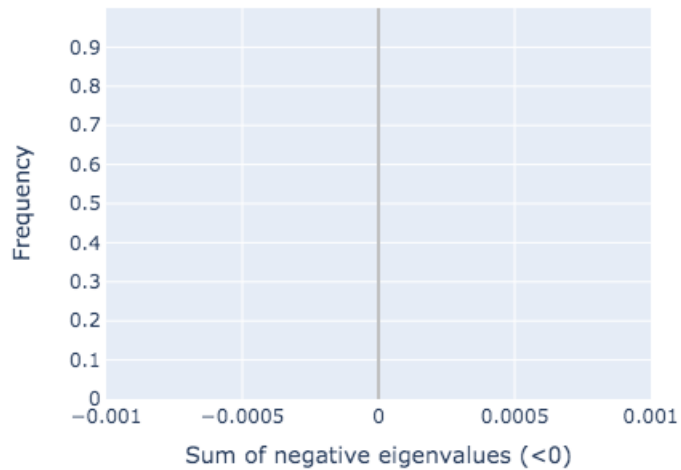


Min: -  
Max: -  
|Max-Min|: -

### Case 6: Least Squares (True)

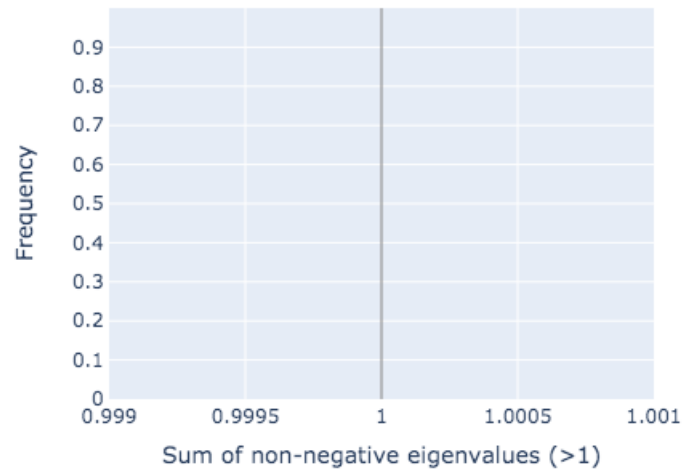
N=10  
Number of unphysical estimates=0

N=10, Nrep=10



Min: -  
Max: -  
|Max-Min|: -

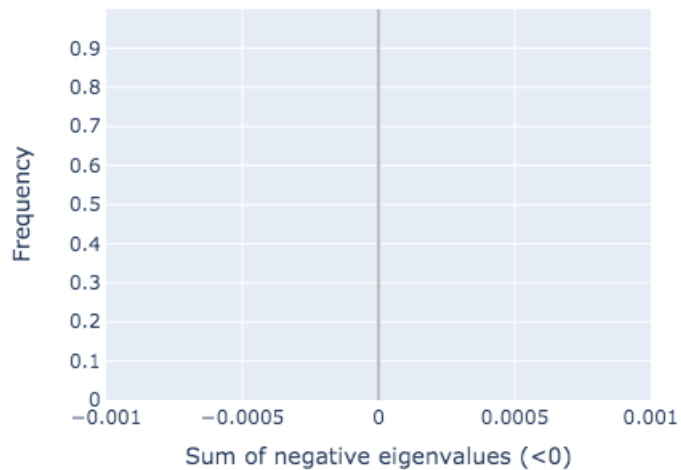
N=10, Nrep=10



Min: -  
Max: -  
|Max-Min|: -

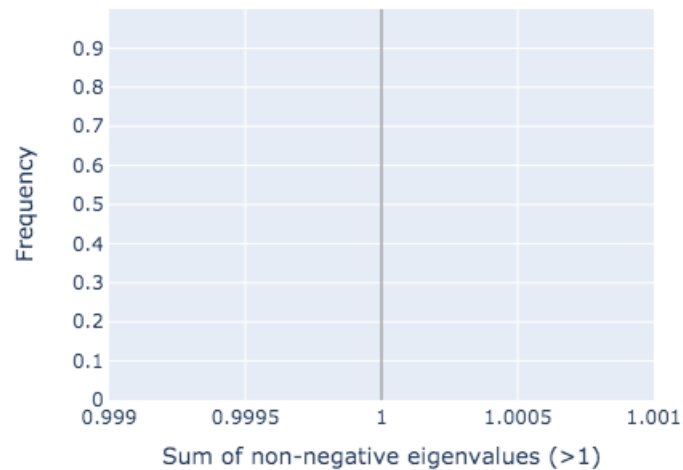
N=100  
Number of unphysical estimates=0

N=100, Nrep=10



Min: -  
Max: -  
|Max-Min|: -

N=100, Nrep=10

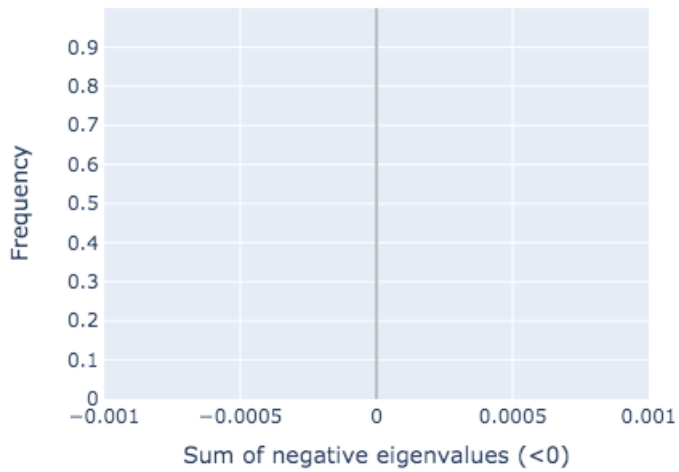


Min: -  
Max: -  
|Max-Min|: -

### Case 7: Least Squares (False)

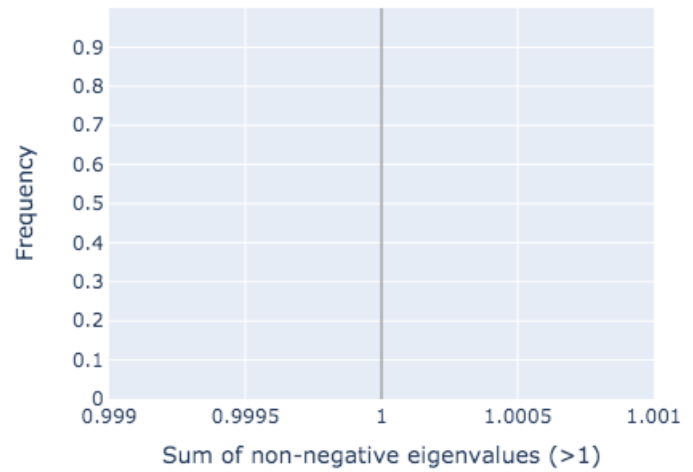
N=10  
Number of unphysical estimates=0

N=10, Nrep=10



Min: -  
Max: -  
|Max-Min|: -

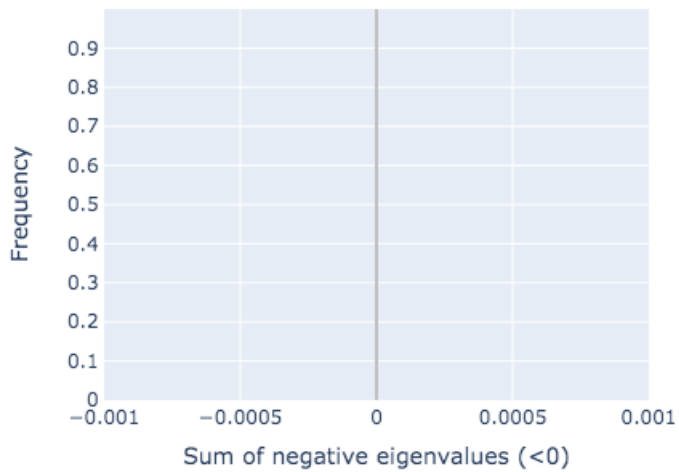
N=10, Nrep=10



Min: -  
Max: -  
|Max-Min|: -

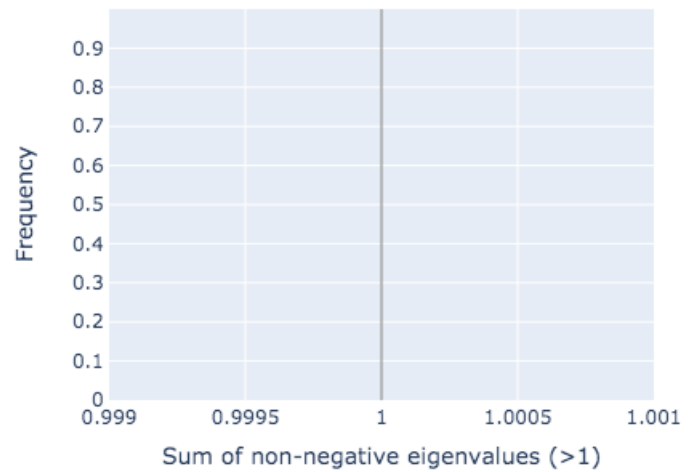
N=100  
Number of unphysical estimates=0

N=100, Nrep=10



Min: -  
Max: -  
|Max-Min|: -

N=100, Nrep=10



Min: -  
Max: -  
|Max-Min|: -