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Computation time in total

Total	3.5925628026326497min.

Tolerance of physicality constraint violation

Tolerance	1.000000e-13

Experimental condition

Type of tomography	StandardQst
Nrep	100
N	[100, 1000]

RNG seed	777

True object

Type	State
Dim	2
Vec	[0.70710678 0.40824829 0.40824829 0.40824829]

Tester objects

0	**Type**	Povm
	Dim	2
	Number of outcomes	2
	Vecs	[[0.70710678 0.70710678 0. 0.] [0.70710678 -0.70710678 0. 0.]]
1	**Type**	Povm
	Dim	2
	Number of outcomes	2
	Vecs	[[0.70710678 0. 0.70710678 0.] [0.70710678 0. -0.70710678 0.]]
2	**Type**	Povm
	Dim	2
	Number of outcomes	2
	Vecs	[[0.70710678 0. 0. 0.70710678] [0.70710678 0. 0. -0.70710678]]

Cases

	Name	Param	Tomography	Estimator
0	Linear(True)	True	StandardQst	Linear
1	Linear(False)	False	StandardQst	Linear
2	ProjectedLinear(True)	True	StandardQst	ProjectedLinear
3	ProjectedLinear(False)	False	StandardQst	ProjectedLinear
4	Maximum-Likelihood(True)	True	StandardQst	LossMinimization
5	Maximum-Likelihood(False)	False	StandardQst	LossMinimization
6	Least Squares(True)	True	StandardQst	LossMinimization
7	Least Squares(False)	False	StandardQst	LossMinimization

Computation time of estimators

Table

	Name	N	Mean (sec)	Std (sec)
0	Linear(True)	100	0.000096	0.000077
1	"	1000	0.000071	0.000044
2	Linear(False)	100	0.000148	0.000374
3	"	1000	0.000111	0.000212
4	ProjectedLinear(True)	100	0.022970	0.020456
5	"	1000	0.018429	0.018391
6	ProjectedLinear(False)	100	0.014940	0.014440
7	"	1000	0.013898	0.014615
8	Maximum-Likelihood(True)	100	0.518814	0.340321
9	"	1000	0.366112	0.229931
10	Maximum-Likelihood(False)	100	0.533254	0.374313
11	"	1000	0.357877	0.286086
12	Least Squares(True)	100	0.070367	0.017472
13	"	1000	0.066537	0.019404
14	Least Squares(False)	100	0.089244	0.024315
15	"	1000	0.082669	0.032323

Histogram

Case 0: Linear(True)

N=100, Nrep=100
Mean = 9.591102600097656e-05(sec)
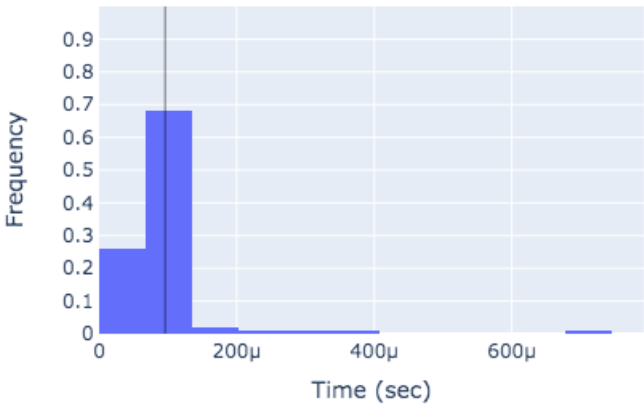
Min: 4.601478576660156e-05
Max: 0.0007240772247314453
|Max-Min|: 0.0006780624389648438

N=1000, Nrep=100
Mean = 7.099628448486327e-05(sec)
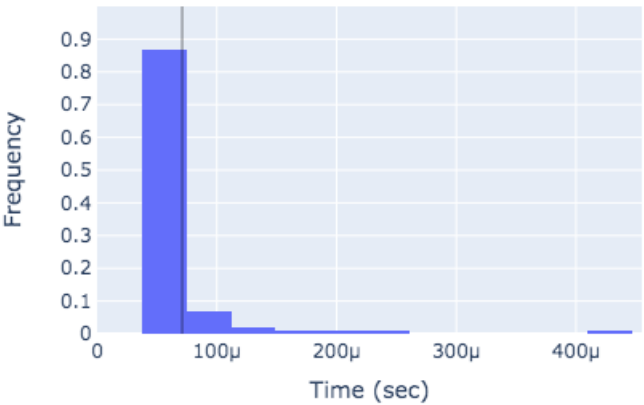
Min: 4.506111145019531e-05
Max: 0.0004177093505859375
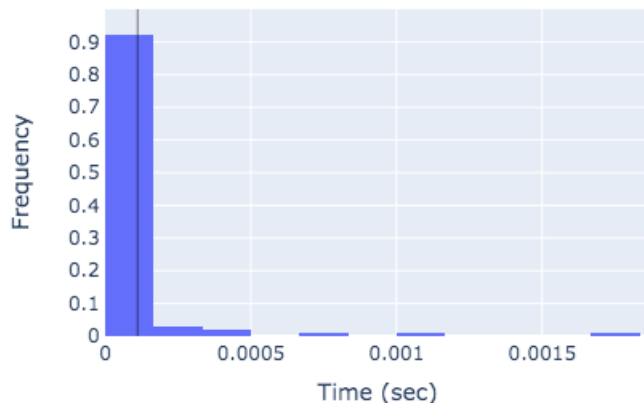|Max-Min|: 0.0003726482391357422

Case 1: Linear(False)

N=100, Nrep=100
Mean = 0.00014840841293334962(sec)
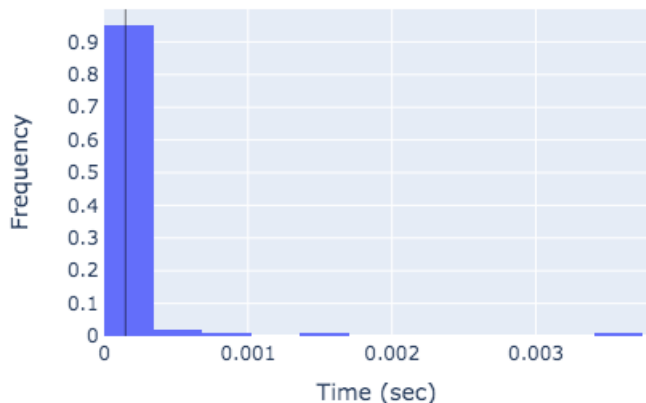
Min: 4.291534423828125e-05
Max: 0.003444194793701172
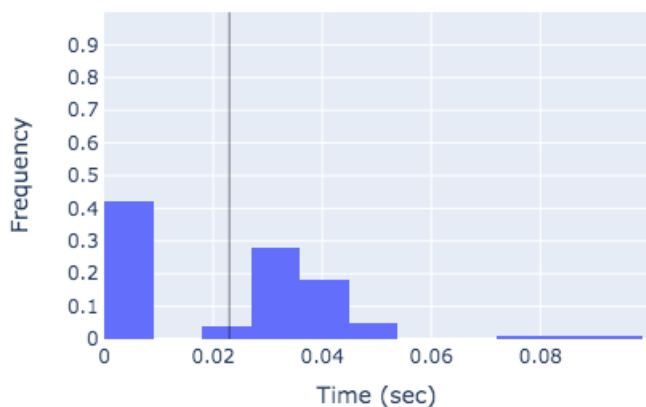|Max-Min|: 0.0034012794494628906

N=1000, Nrep=100
Mean = 0.00011134147644042969(sec)

Min: 3.3855438232421875e-05
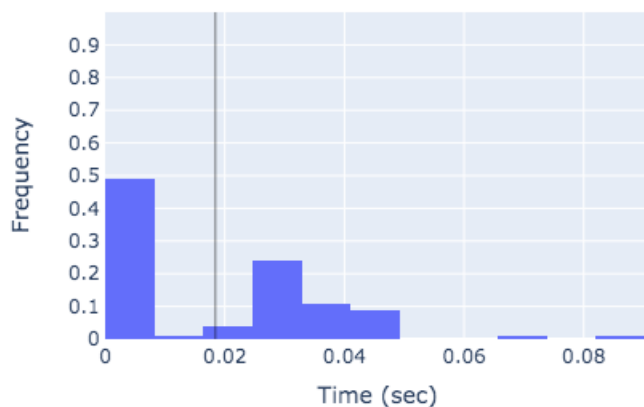Max: 0.0017027854919433594
|Max-Min|: 0.0016689300537109375

Case 2: ProjectedLinear(True)

N=100, Nrep=100
Mean = 0.02296987771987915(sec)
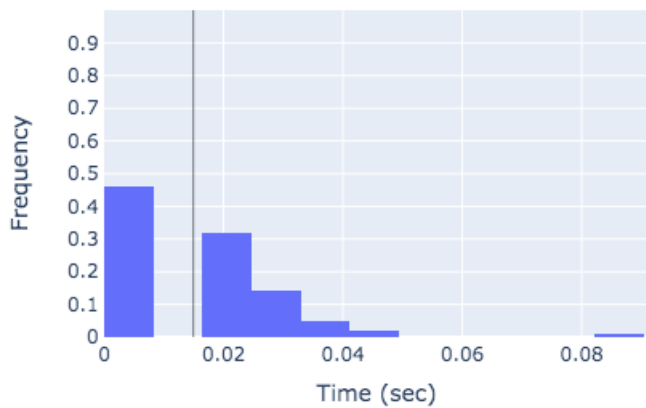

Min: 0.0010612010955810547
Max: 0.09090590476989746
|Max-Min|: 0.0898447036743164

N=1000, Nrep=100
Mean = 0.01842876672744751(sec)

Min: 0.0008339881896972656
Max: 0.08284616470336914
|Max-Min|: 0.08201217651367188

Case 3: ProjectedLinear(False)

N=100, Nrep=100
Mean = 0.014939839839935303(sec)

N=1000, Nrep=100
Mean = 0.013897964954376221(sec)

Min: 0.0008397102355957031
Max: 0.08303689956665039
|Max-Min|: 0.08219718933105469
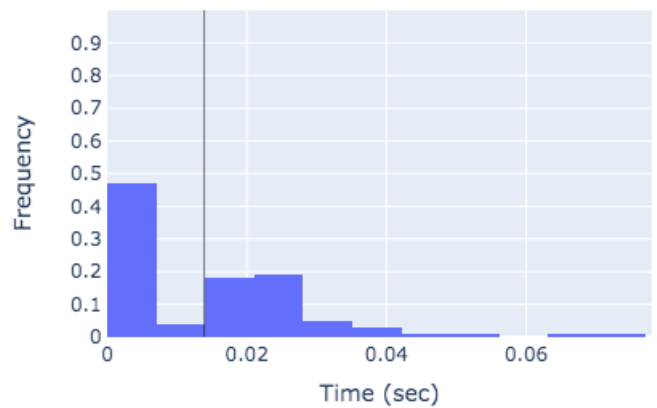
Min: 0.0008177757263183594
Max: 0.07099103927612305
|Max-Min|: 0.07017326354980469

Case 4: Maximum-Likelihood(True)

N=100, Nrep=100
Mean = 0.5188143253326416(sec)

N=1000, Nrep=100
Mean = 0.3661117362976074(sec)
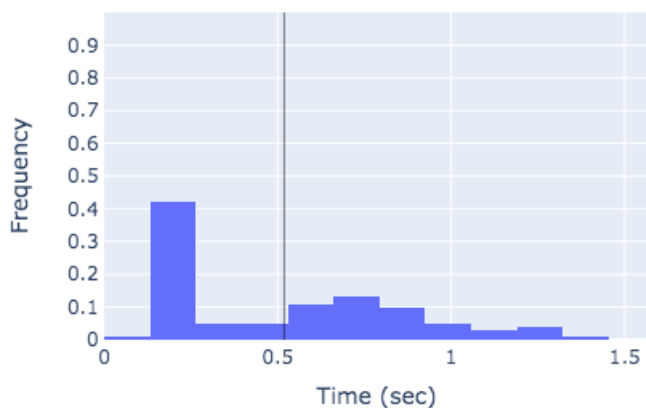
Min: 0.11529326438903809
Max: 1.4374771118164062
|Max-Min|: 1.3221838474273682
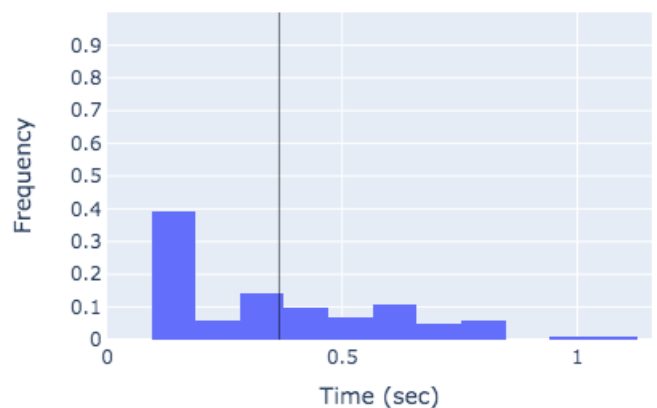
Min: 0.12348723411560059
Max: 1.0645689964294434
|Max-Min|: 0.9410817623138428

Case 5: Maximum-Likelihood(False)

N=100, Nrep=100
Mean = 0.5332543158531189(sec)
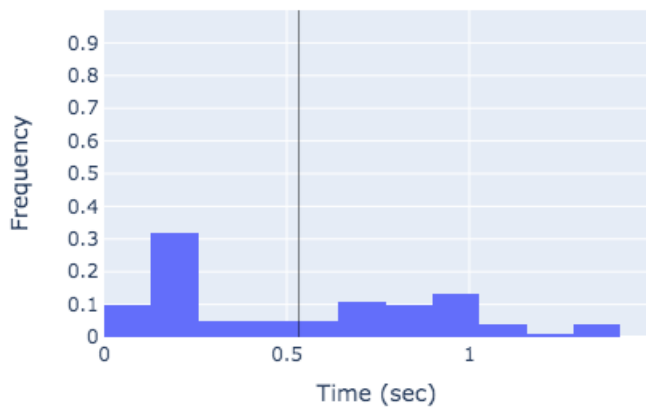
Min: 0.07770395278930664
Max: 1.362936019897461
|Max-Min|: 1.2852320671081543

N=1000, Nrep=100
Mean = 0.3578765678405762(sec)
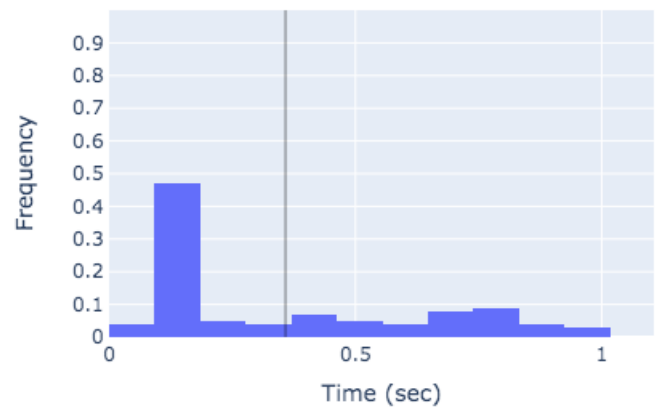
Min: 0.08855915069580078
Max: 1.0130281448364258
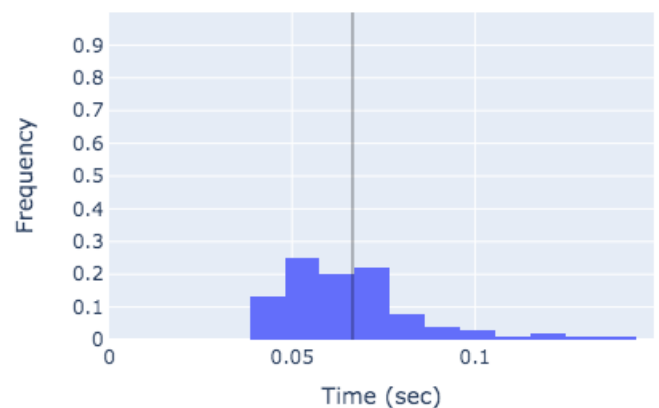|Max-Min|: 0.924468994140625

Case 6: Least Squares(True)

N=100, Nrep=100
Mean = 0.07036726713180542(sec)
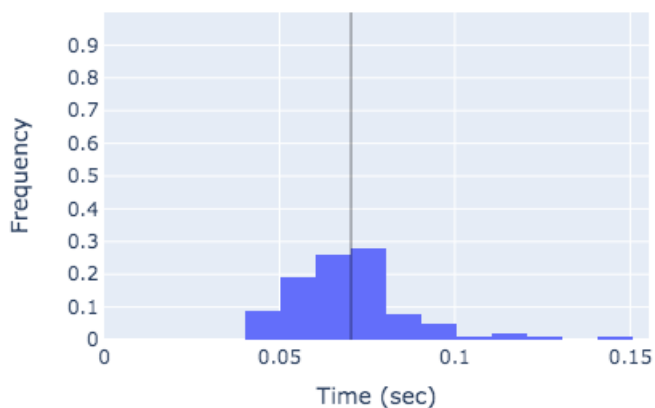
Min: 0.04486989974975586
Max: 0.14526796340942383
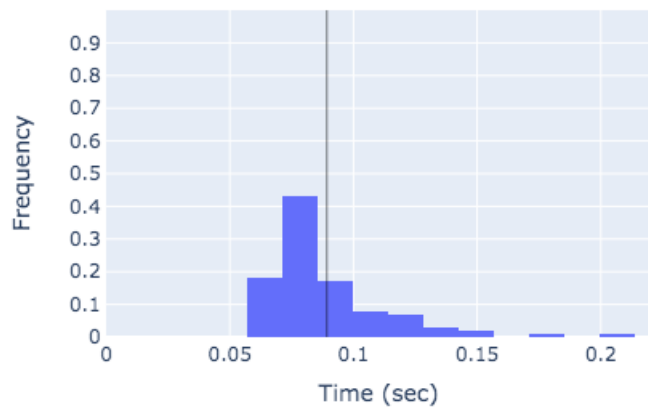|Max-Min|: 0.10039806365966797

N=1000, Nrep=100
Mean = 0.06653718709945679(sec)

Min: 0.04337573051452637
Max: 0.13929033279418945
|Max-Min|: 0.09591460227966309

Case 7: Least Squares(False)

N=100, Nrep=100
Mean = 0.08924393892288209(sec)

Min: 0.063446044921875
Max: 0.20595502853393555
|Max-Min|: 0.14250898361206055

N=1000, Nrep=100
Mean = 0.08266923666000366(sec)
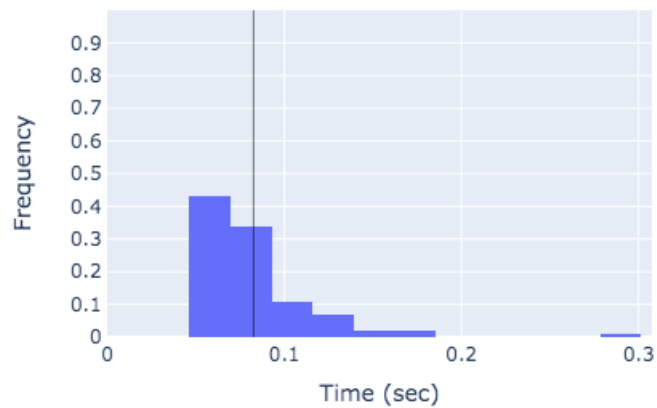
Min: 0.0524296760559082
Max: 0.2842676639556885
|Max-Min|: 0.23183798789978027

# MSE of empirical distributions

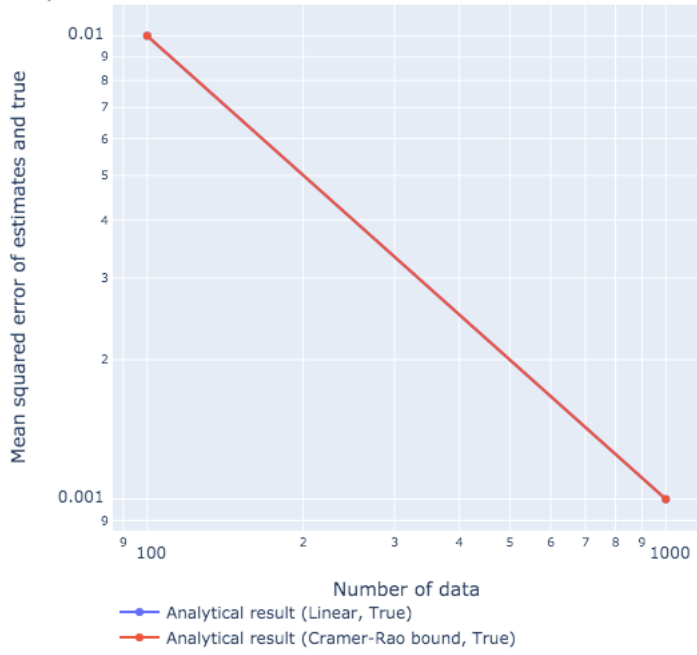## Mean squared error

Consistency test

	Name	Type of tomography	Param	Estimator	Loss	Algo	Result
0	Linear(True)	Standard Qst	True	Linear	None	None	9.24e-33
1	Linear(False)	Standard Qst	False	Linear	None	None	5.85e-32
2	ProjectedLinear(True)	Standard Qst	True	Projected Linear	None	None	2.10e-31
3	ProjectedLinear(False)	Standard Qst	False	Projected Linear	None	None	3.05e-31
4	Maximum-Likelihood(True)	Standard Qst	True	Loss Minimization	Weighted Relative Entropy	Projected Gradient Descent Backtracking	4.26e-27
5	Maximum-Likelihood(False)	Standard Qst	False	Loss Minimization	Weighted Relative Entropy	Projected Gradient Descent Backtracking	6.37e-28
6	Least Squares(True)	Standard Qst	True	Loss Minimization	Weighted Probability Based Squared Error	Projected Gradient Descent Backtracking	1.66e-29
7	Least Squares(False)	Standard Qst	False	Loss Minimization	Weighted Probability Based Squared Error	Projected Gradient Descent Backtracking	2.97e-29

MSE of estimators

Comparison of analytical results
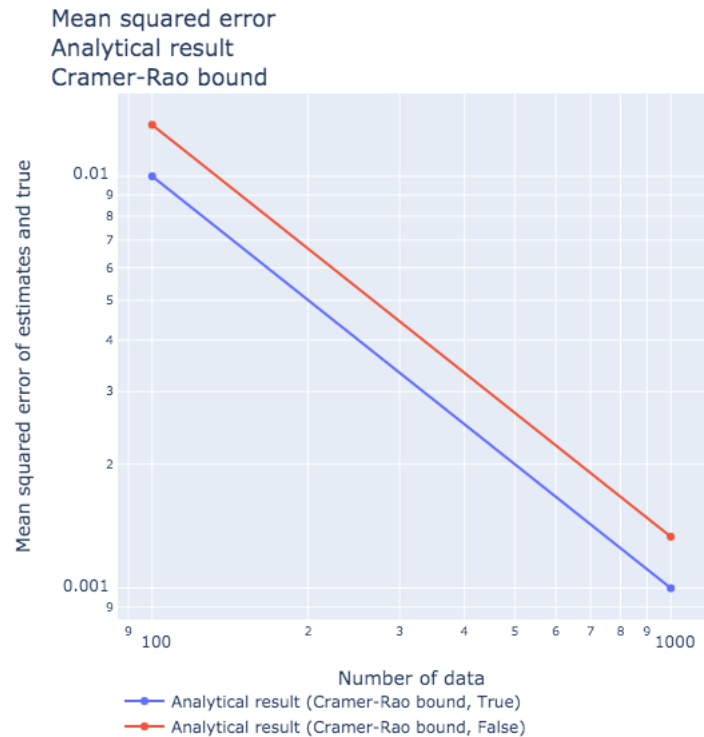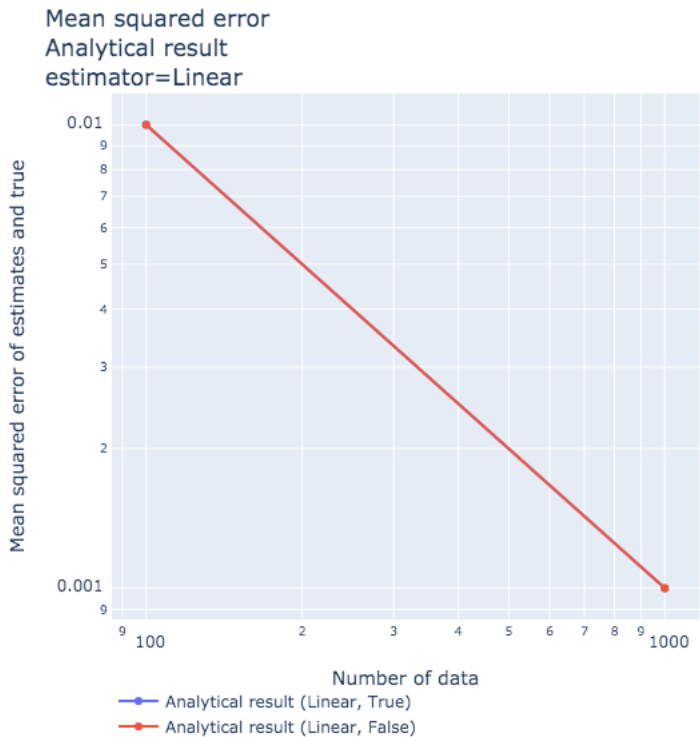
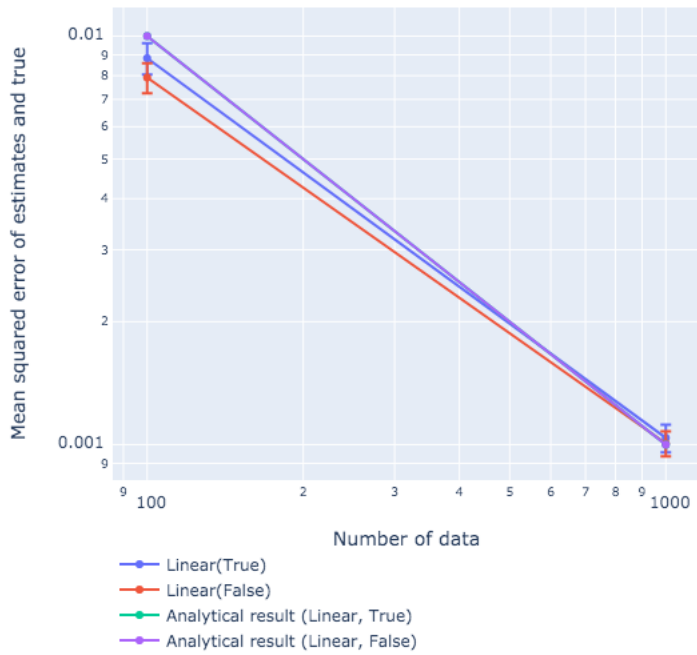## Mean squared error
Analytical result
parametorization=True

Analytical result (Linear, True)
Analytical result (Cramer-Rao bound, True)

Mean squared error
Analytical result
parametorization=False

Analytical result (Linear, False)
Analytical result (Cramer-Rao bound, False)
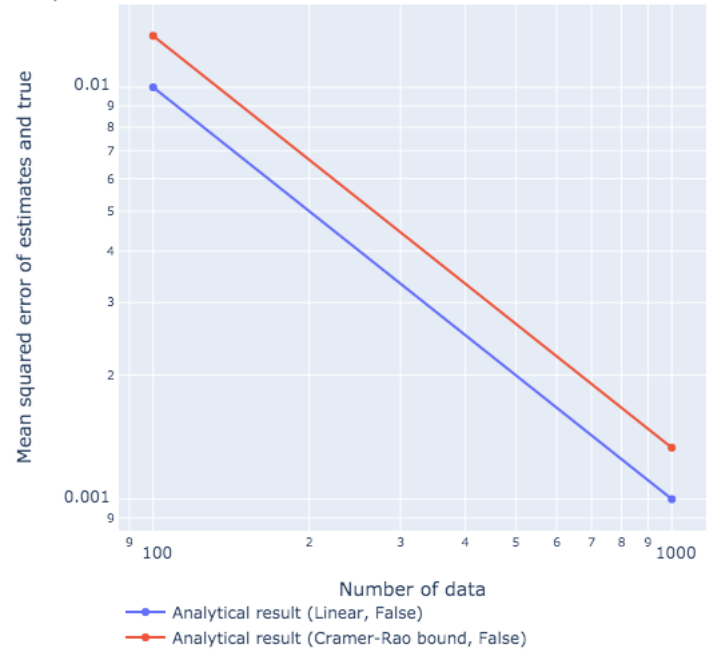
Comparison of parametrization

Mean squared error
Estimator=Linear

Linear(True)
Linear(False)
Analytical result (Linear, True)
Analytical result (Linear, False)

Mean squared error
Estimator=ProjectedLinear

ProjectedLinear(True)
ProjectedLinear(False)
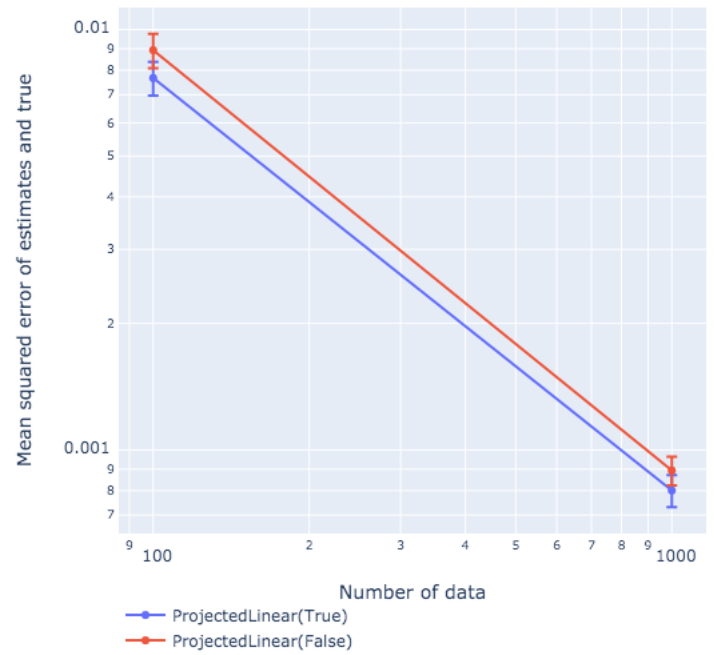
Mean squared error

Estimator=LossMinimization
Loss=WeightedRelativeEntropy
Algo=ProjectedGradientDescentBacktracking

- Maximum-Likelihood(True)
- Maximum-Likelihood(False)
- Analytical result (Cramer-Rao bound, True)
- Analytical result (Cramer-Rao bound, False)
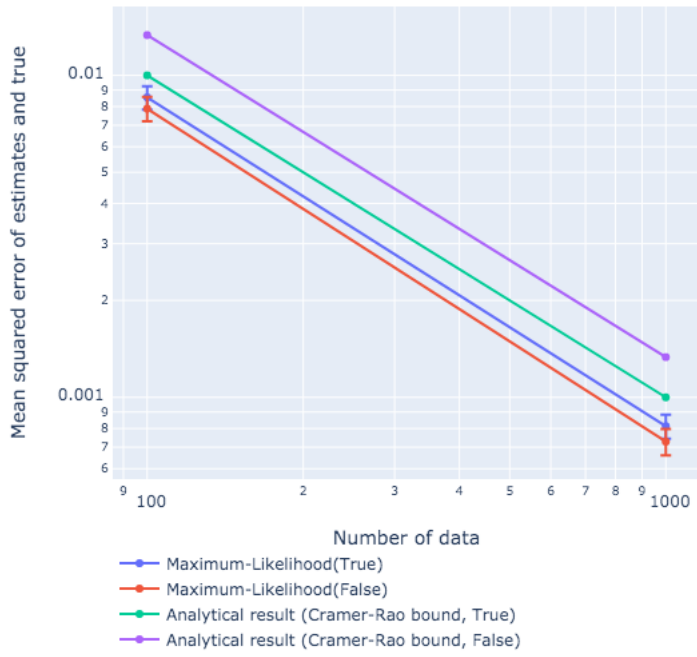
Mean squared error

Estimator=LossMinimization
Loss=WeightedProbabilityBasedSquaredError
Algo=ProjectedGradientDescentBacktracking

- Least Squares(True)
- Least Squares(False)
- Analytical result (Cramer-Rao bound, True)
- Analytical result (Cramer-Rao bound, False)
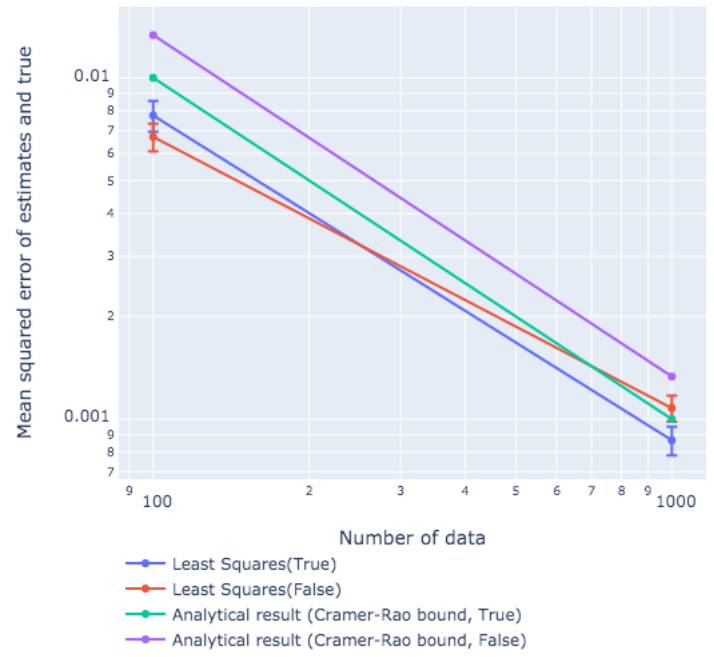
Comparison of estimators

Mean squared error
parametrization=True

- Linear(True)
- ProjectedLinear(True)
- Maximum-Likelihood(True)
- Least Squares(True)

Mean squared error
parametrization=False

- Linear(False)
- ProjectedLinear(False)
- Maximum-Likelihood(False)
- Least Squares(False)

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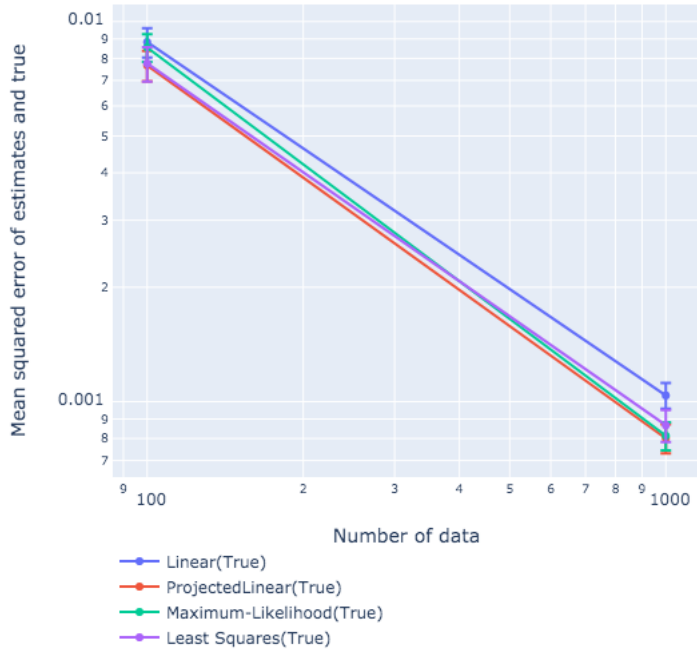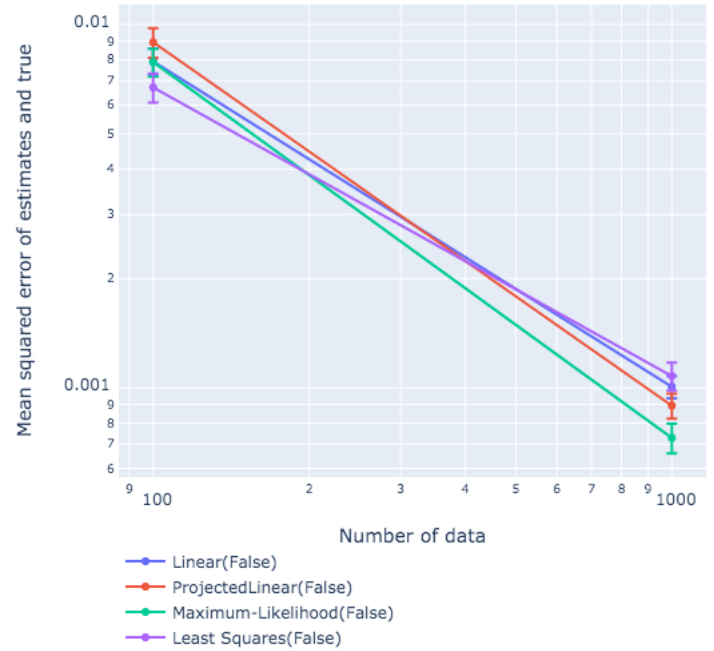
Physicality violation test

Test of equality constraint violation

Case 0: Linear(True)
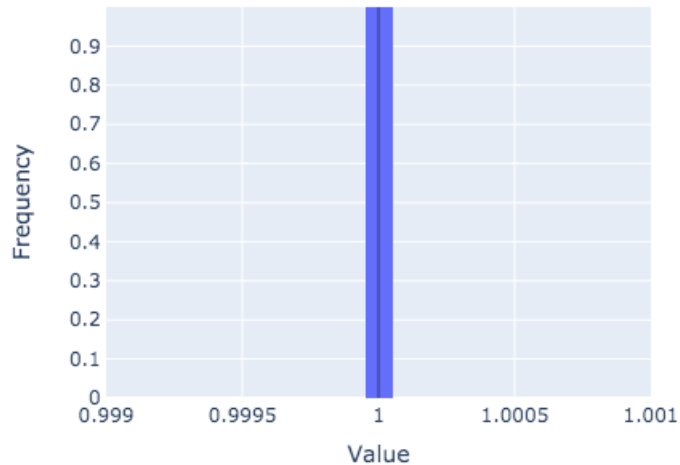
N=100, Nrep=100

Min: 0.9999999999999998
Max: 0.9999999999999998
|Max-Min|: 0.0
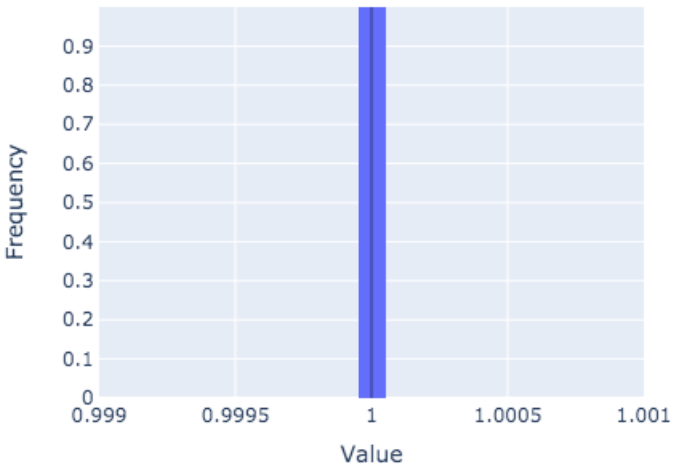
N=1000, Nrep=100

Min: 0.9999999999999998
Max: 0.9999999999999998
|Max-Min|: 0.0

Case 1: Linear(False)
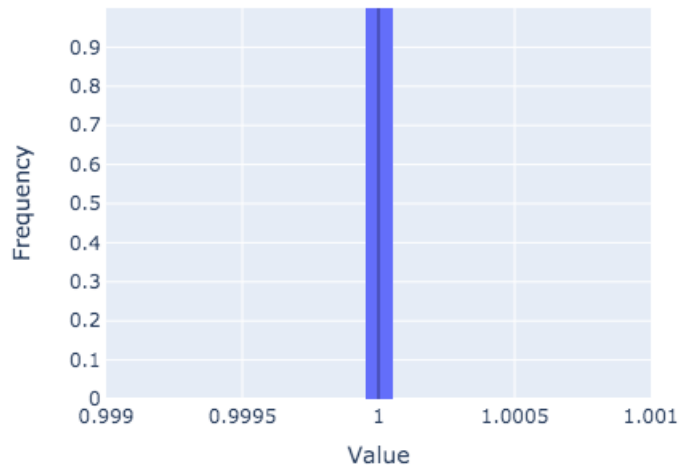
N=100, Nrep=100

Min: 0.9999999999999998
Max: 1.0000000000000002
|Max-Min|: 4.440892098500626e-16

N=1000, Nrep=100
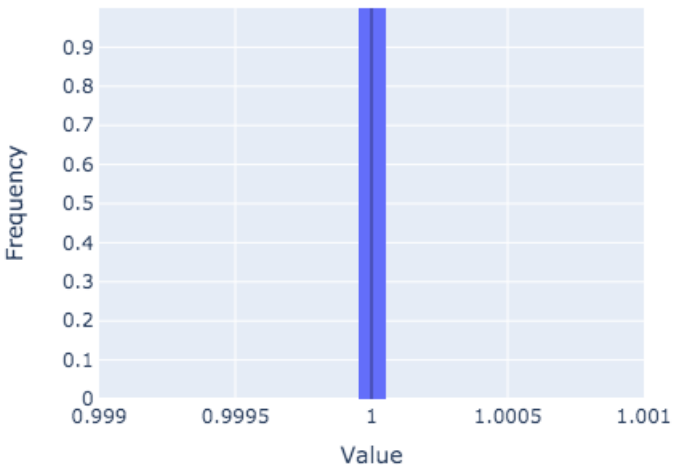
Min: 0.9999999999999998
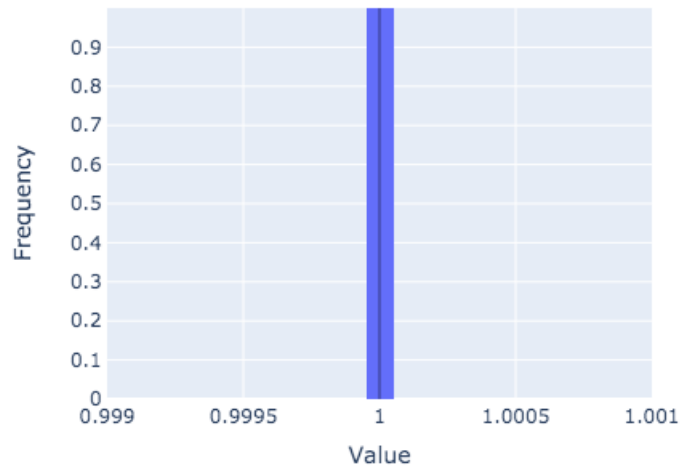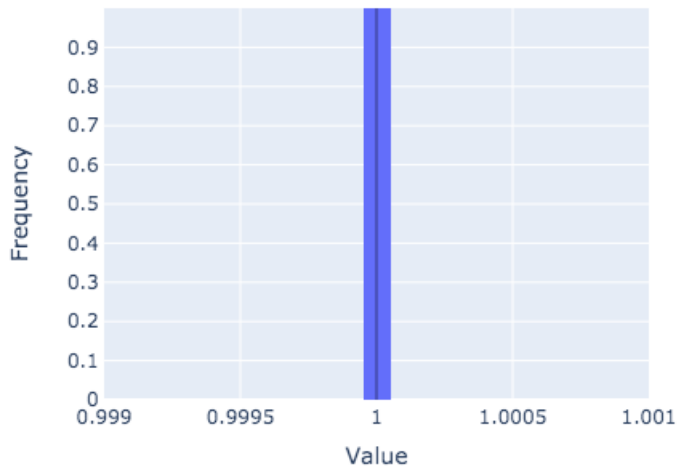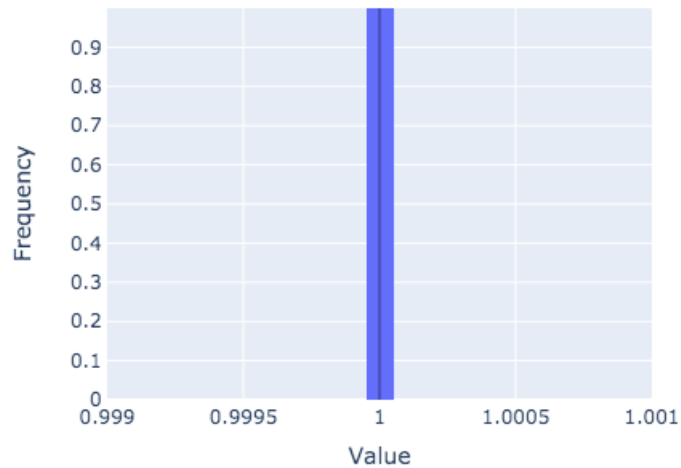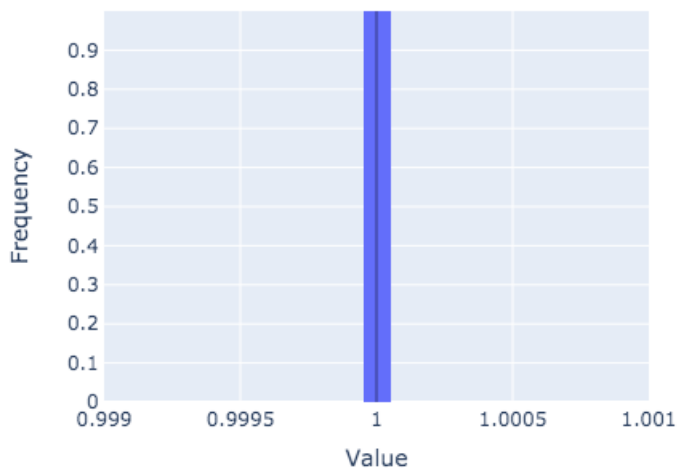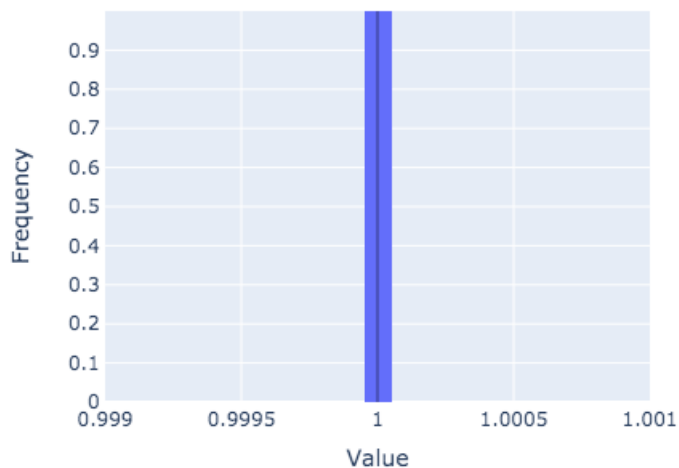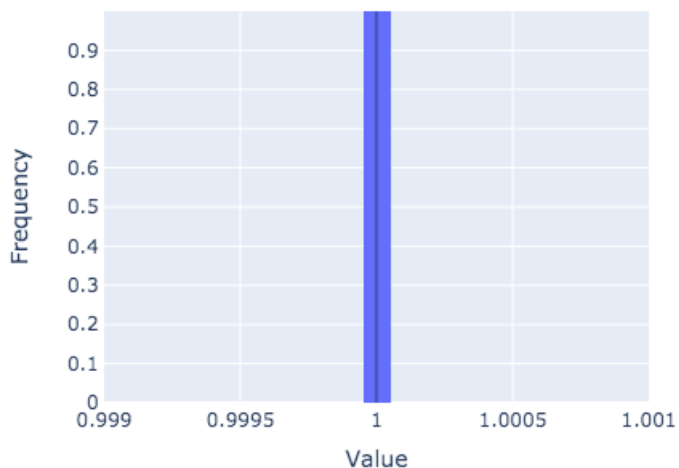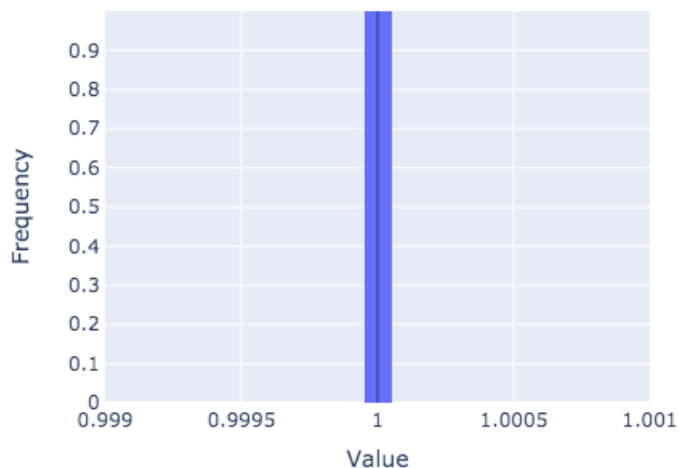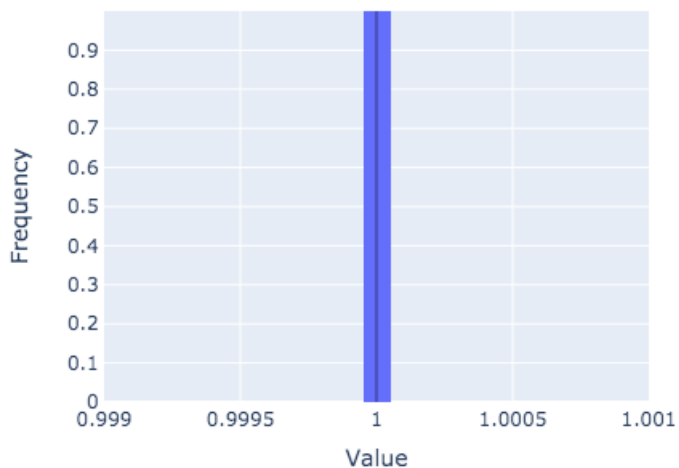Max: 1.0000000000000002
|Max-Min|: 4.440892098500626e-16

Case 2: ProjectedLinear(True)
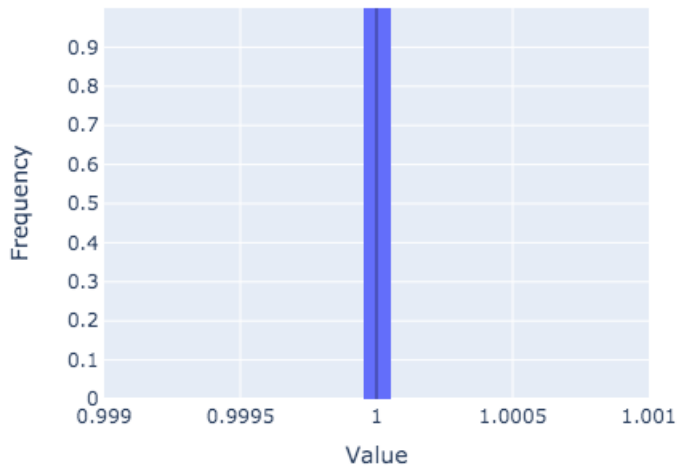
N=100, Nrep=100

Min: 0.9999999999999998
Max: 0.9999999999999998
|Max-Min|: 0.0

N=1000, Nrep=100

Min: 0.9999999999999998
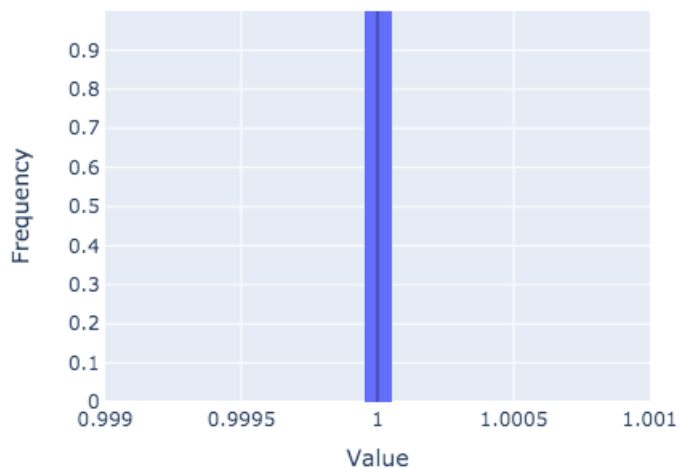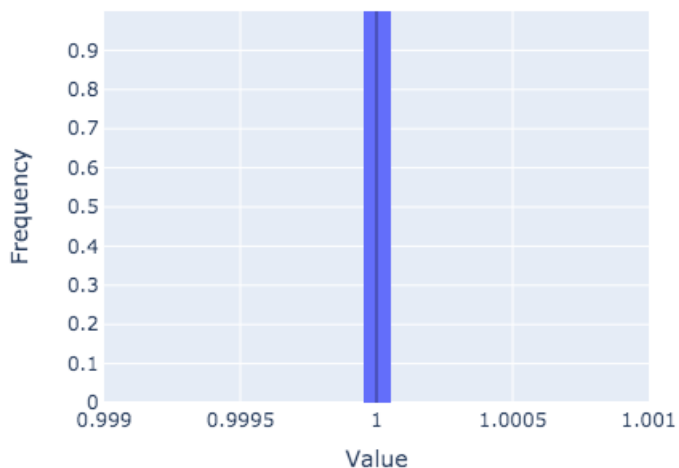Max: 0.9999999999999998
|Max-Min|: 0.0

Case 3: ProjectedLinear(False)

N=100, Nrep=100

Min: 0.9999999999999998
Max: 0.9999999999999998
|Max-Min|: 0.0

N=1000, Nrep=100

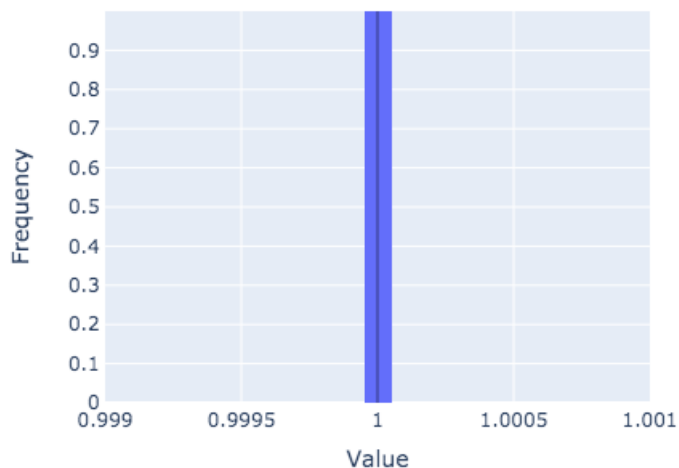Min: 0.9999999999999998
Max: 0.9999999999999998
|Max-Min|: 0.0

Case 4: Maximum-Likelihood(True)

N=100, Nrep=100

Min: 0.9999999999999998
Max: 0.9999999999999998
|Max-Min|: 0.0

N=1000, Nrep=100

Min: 0.9999999999999998
Max: 0.9999999999999998
|Max-Min|: 0.0

Case 5: Maximum-Likelihood(False)

N=100, Nrep=100

Min: 0.9999999999999998
Max: 0.9999999999999998
|Max-Min|: 0.0

N=1000, Nrep=100

Min: 0.9999999999999998
Max: 0.9999999999999998
|Max-Min|: 0.0

Case 6: Least Squares(True)

N=100, Nrep=100

Min: 0.9999999999999998
Max: 0.9999999999999998
|Max-Min|: 0.0

N=1000, Nrep=100

Min: 0.9999999999999998
Max: 0.9999999999999998
|Max-Min|: 0.0

Case 7: Least Squares(False)

N=100, Nrep=100

Min: 0.9999999999999998
Max: 0.9999999999999998
|Max-Min|: 0.0

N=1000, Nrep=100

Min: 0.9999999999999998
Max: 0.9999999999999998
|Max-Min|: 0.0

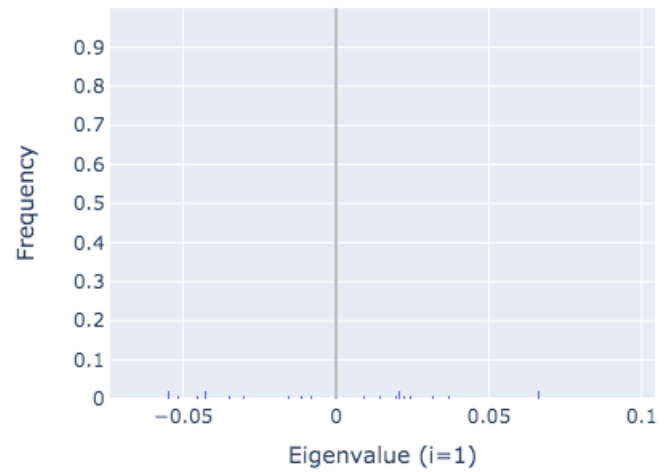# Test of inequality constraint violation
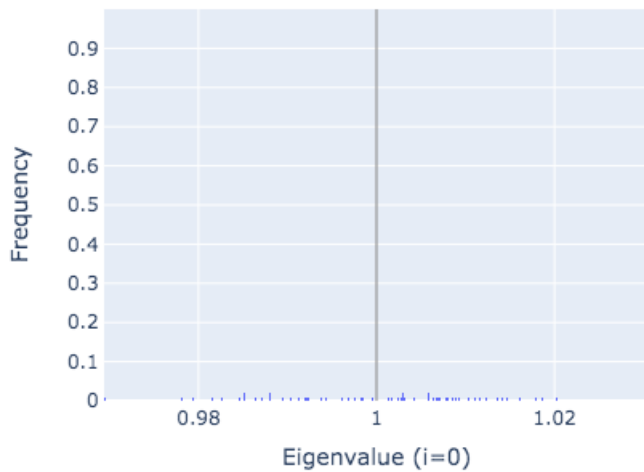
Eigenvalue

Case 0: Linear(True)
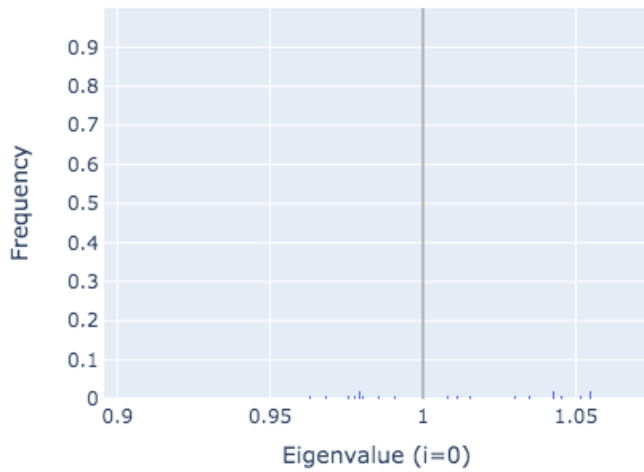
N=100, Nrep=100

Min: 0.8957271787481875
Max: 1.0738466694161428
|Max-Min|: 0.1781194906679553

N=100, Nrep=100

Min: -0.07384666941614303
Max: 0.10427282125181224
|Max-Min|: 0.17811949066795527

N=1000, Nrep=100

Min: 0.9694773689966321
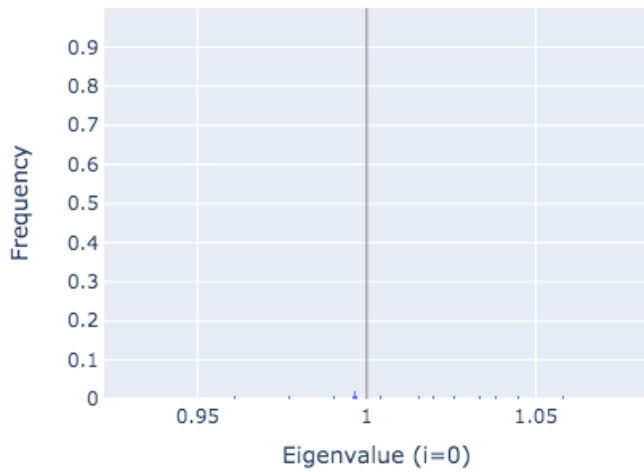Max: 1.0305223840706443
|Max-Min|: 0.06104501507401228

N=1000, Nrep=100
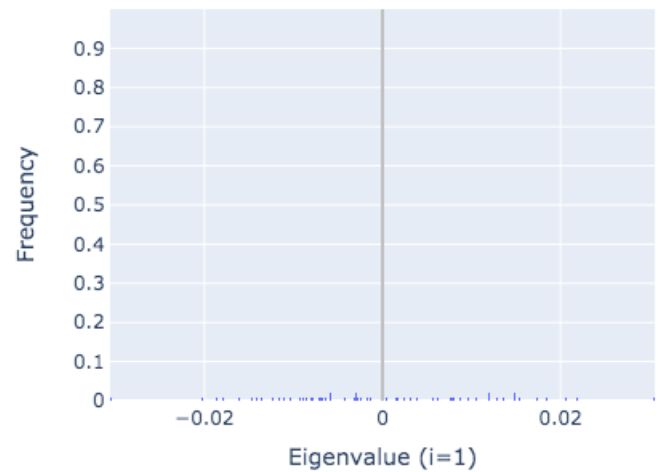
Min: -0.030522384070644254
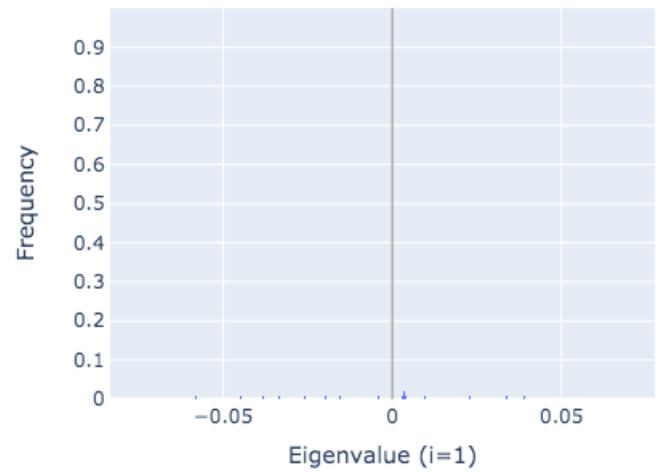Max: 0.030522631003367565
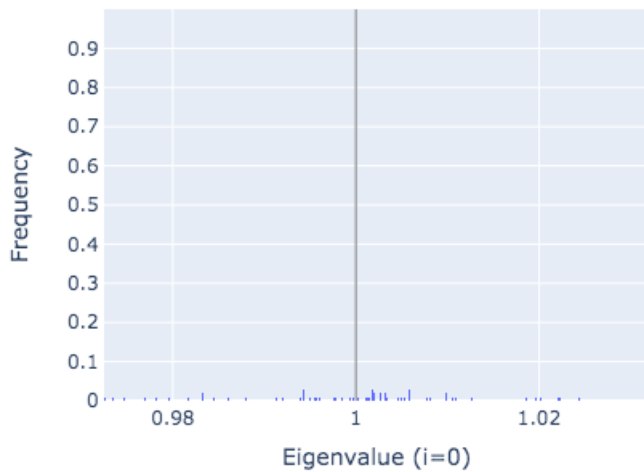|Max-Min|: 0.06104501507401182

Case 1: Linear(False)

N=100, Nrep=100

Min: 0.9224926034855522
Max: 1.0833523806414103
|Max-Min|: 0.16085977715585809
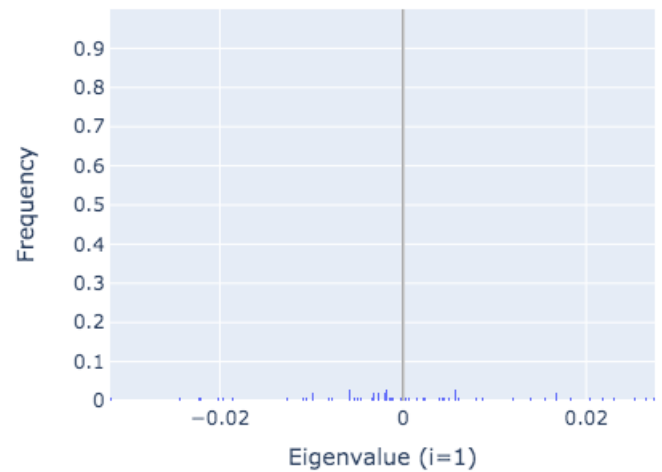
N=100, Nrep=100

Min: -0.08335238064140982
Max: 0.07750739651444777
|Max-Min|: 0.1608597771558576

N=1000, Nrep=100

Min: 0.972573803759793
Max: 1.0319107067920326
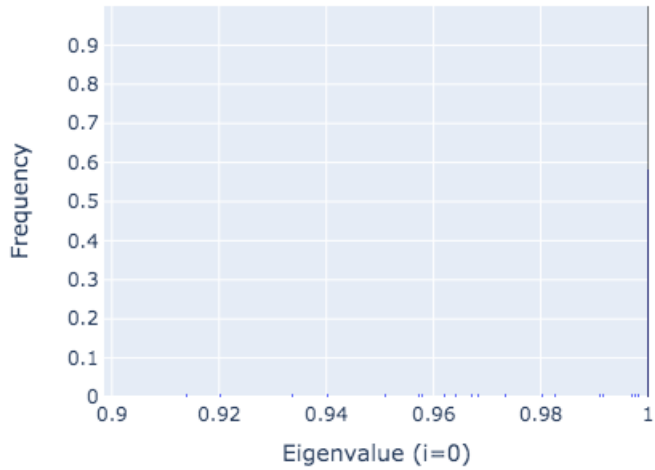|Max-Min|: 0.059336903032239596

N=1000, Nrep=100

Min: -0.031910706792032915
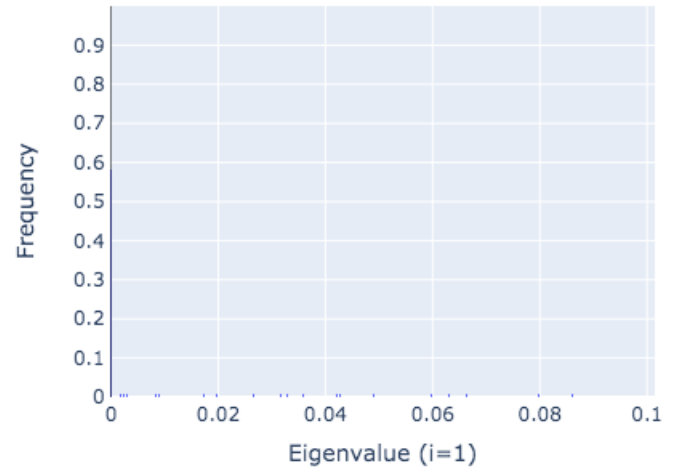Max: 0.02742619624020604
|Max-Min|: 0.05933690303223896

Case 2: ProjectedLinear(True)
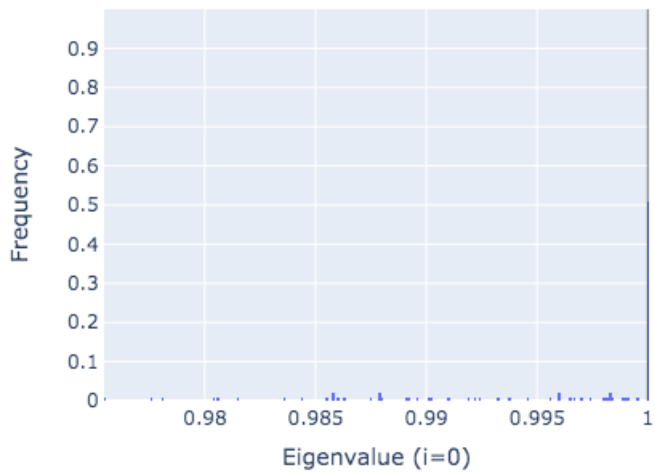
N=100, Nrep=100

Min: 0.8986226285598948
Max: 1.0000000000006035
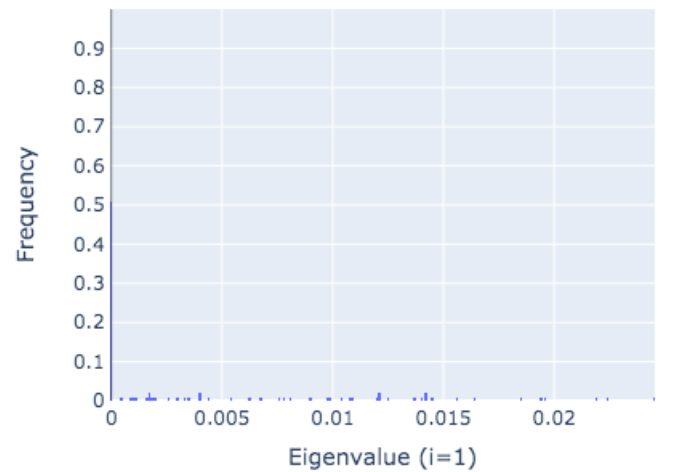|Max-Min|: 0.10137737144070869

N=100, Nrep=100

Min: -6.039008229280467e-13
Max: 0.10137737144010506
|Max-Min|: 0.10137737144070896

N=1000, Nrep=100

Min: 0.975538641962985
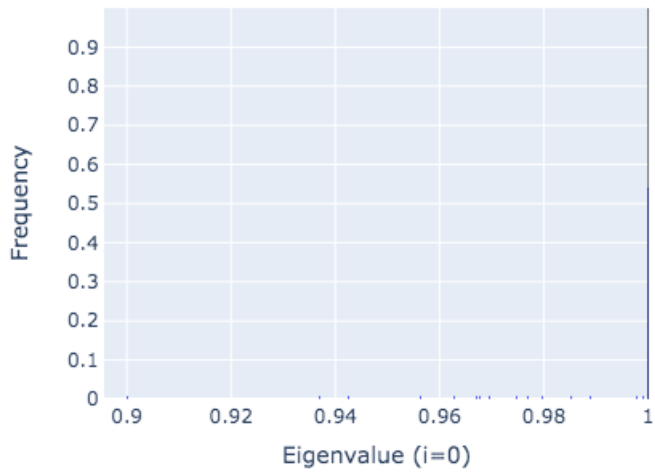Max: 1.000000000005184
|Max-Min|: 0.02446135804219906

N=1000, Nrep=100

Min: -5.184662910032525e-12
Max: 0.024461358037014748
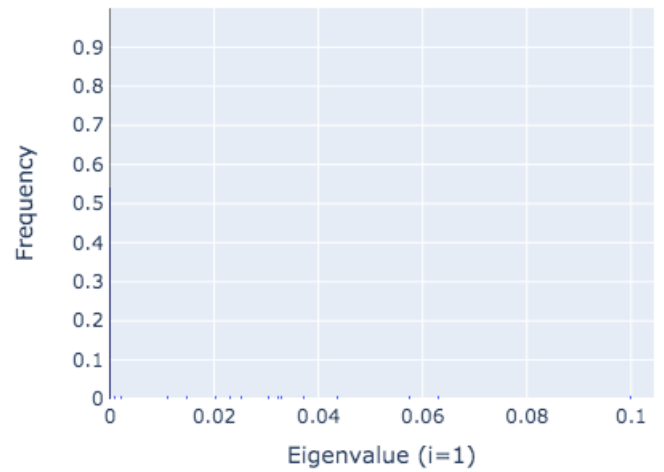|Max-Min|: 0.02446135804219941

Case 3: ProjectedLinear(False)
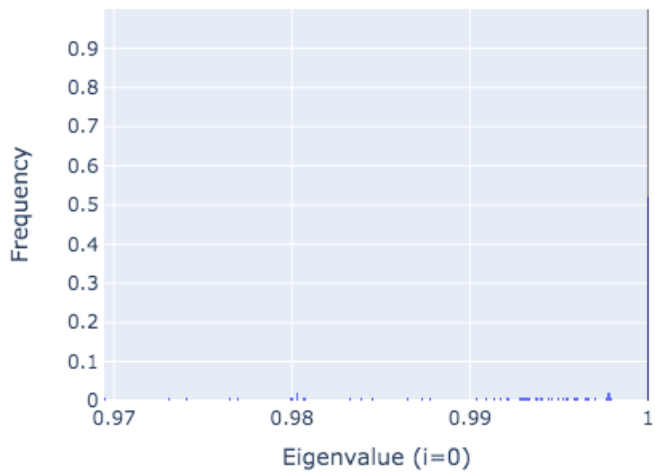
N=100, Nrep=100

Min: 0.8957271787481873
Max: 1.0000000000009297
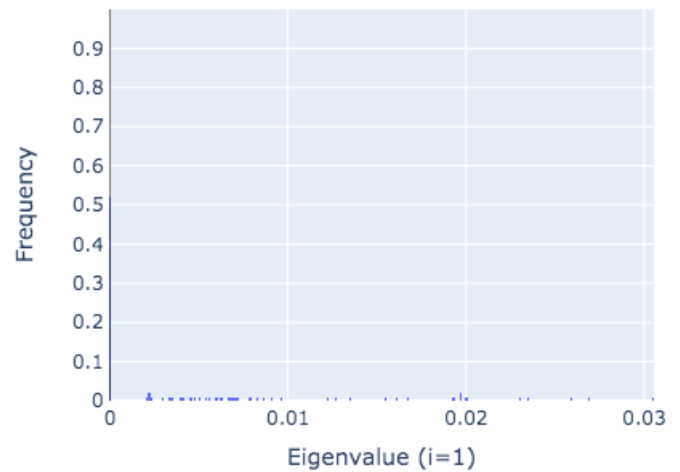|Max-Min|: 0.1042728212527424

N=100, Nrep=100

Min: -9.303964064993807e-13
Max: 0.10427282125181248
|Max-Min|: 0.10427282125274287

N=1000, Nrep=100

Min: 0.9695157079374443
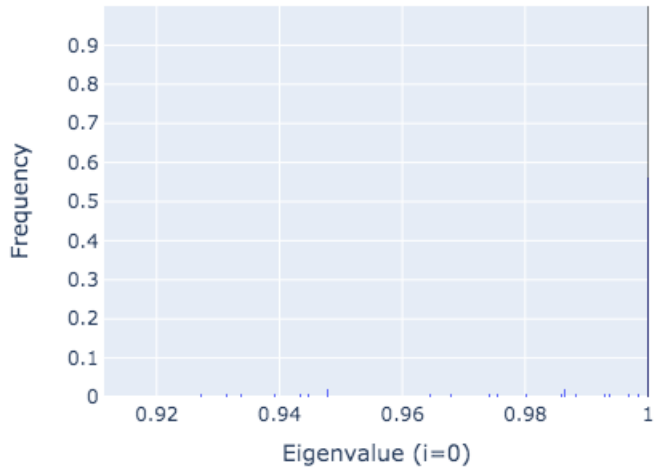Max: 1.000000000976561
|Max-Min|: 0.030484293039116706

N=1000, Nrep=100

Min: -9.765616615728533e-10
Max: 0.03048429206255536
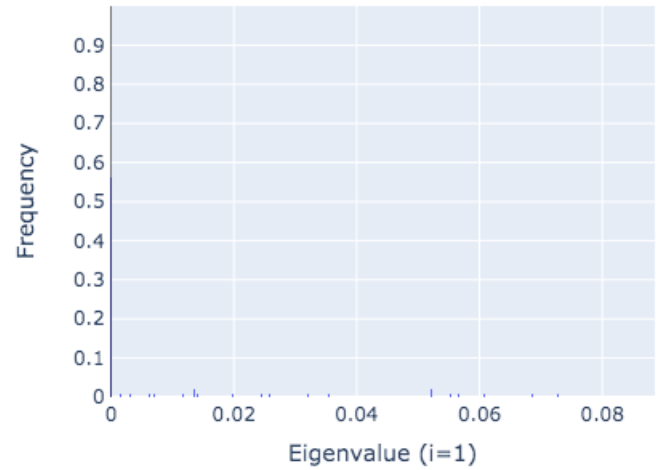|Max-Min|: 0.03048429303911702

Case 4: Maximum-Likelihood(True)

N=100, Nrep=100

Eigenvalue (i=0)

Min: 0.9114608211527193
Max: 1.0000000000053535
|Max-Min|: 0.08853917885263418
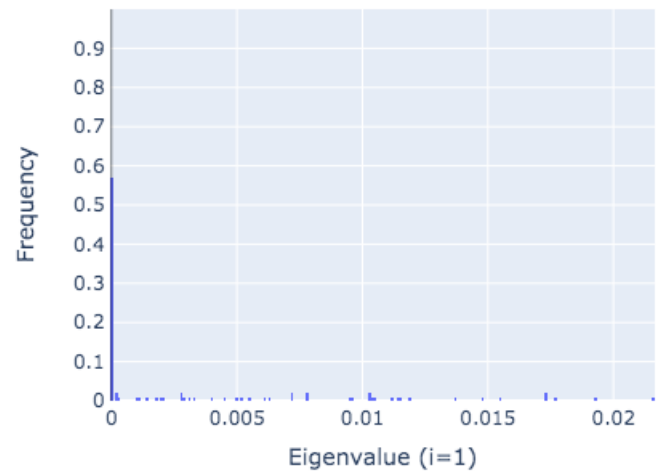
N=100, Nrep=100

Eigenvalue (i=1)

Min: -5.354217988982624e-12
Max: 0.08853917884728026
|Max-Min|: 0.08853917885263447

N=1000, Nrep=100

Eigenvalue (i=0)

Min: 0.9783555200918326
Max: 1.0000000001123612
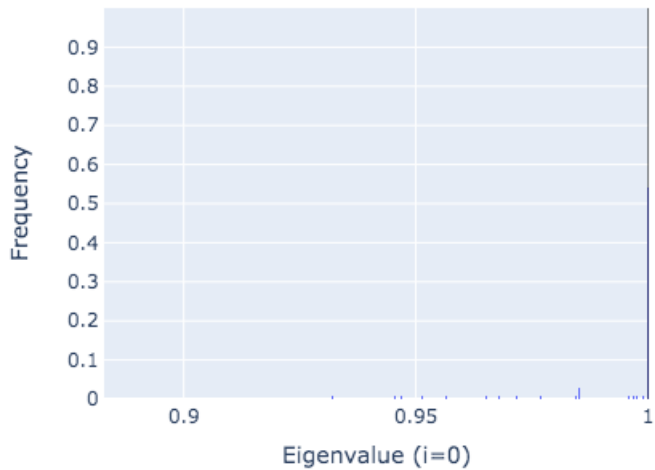|Max-Min|: 0.021644480020528656

N=1000, Nrep=100

Eigenvalue (i=1)

Min: -1.1236145599302914e-10
Max: 0.02164447990816662
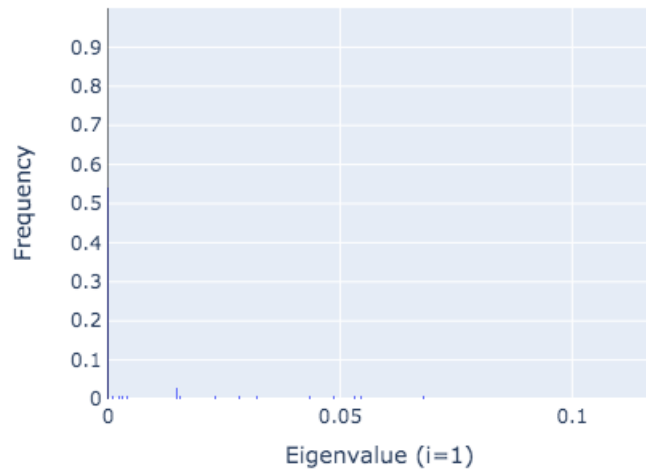|Max-Min|: 0.021644480020528076

Case 5: Maximum-Likelihood(False)
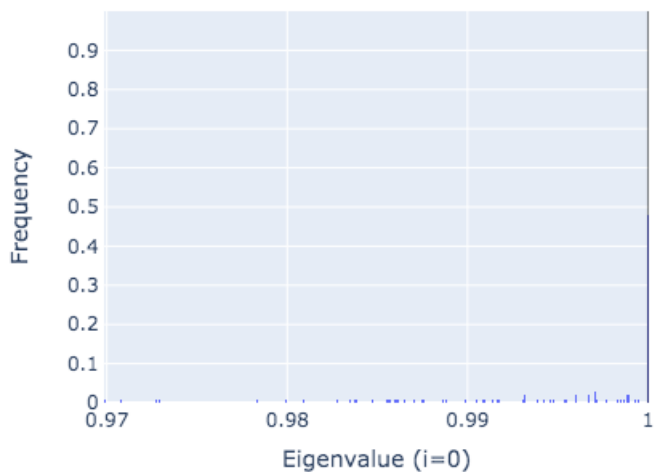
N=100, Nrep=100

Min: 0.8828837965974647
Max: 1.0000000000000102
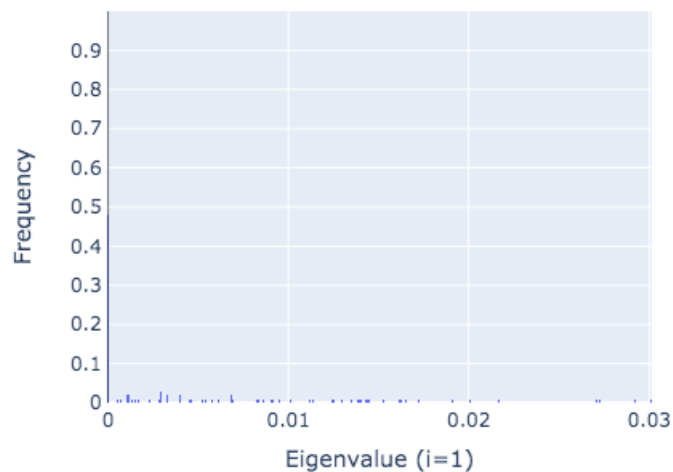|Max-Min|: 0.11711620340254547

N=100, Nrep=100

Min: -1.0664939803058045e-14
Max: 0.11711620340253492
|Max-Min|: 0.11711620340254558

N=1000, Nrep=100

Min: 0.9698872228031326
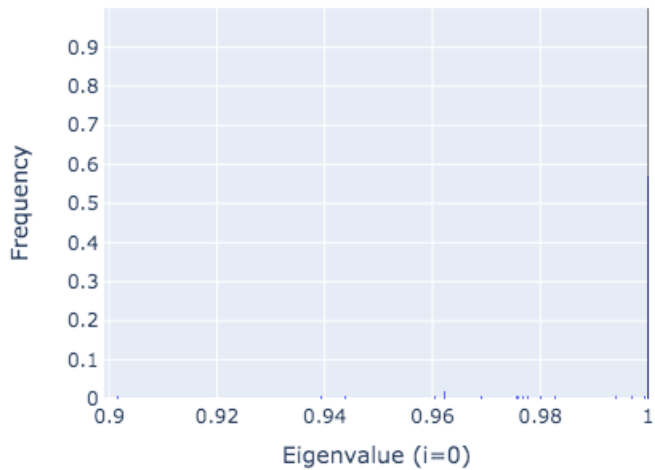Max: 1.0000000000000109
|Max-Min|: 0.030112777196878326

N=1000, Nrep=100

Min: -1.0901152577866491e-14
Max: 0.030112777196866825
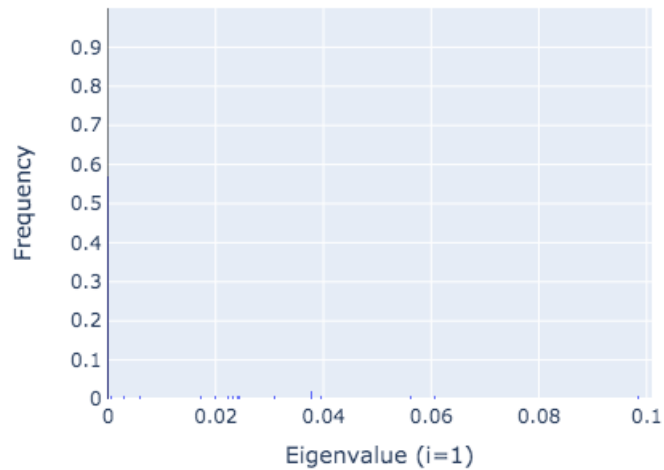|Max-Min|: 0.030112777196877726
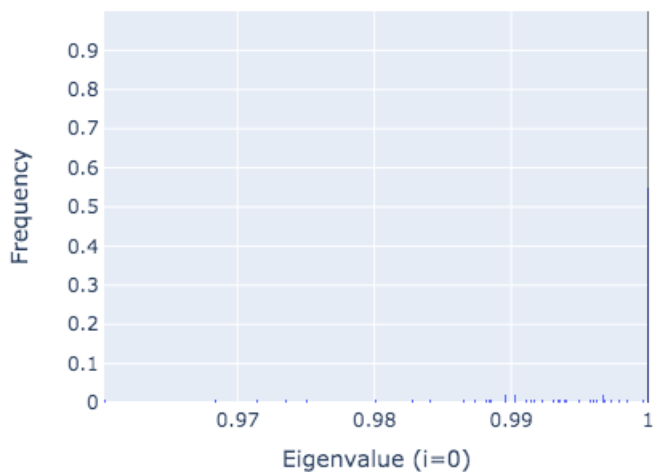
Case 6: Least Squares(True)

N=100, Nrep=100

Eigenvalue (i=0)

Min: 0.8991240357501609
Max: 1.0000000000042972
|Max-Min|: 0.10087596425413636
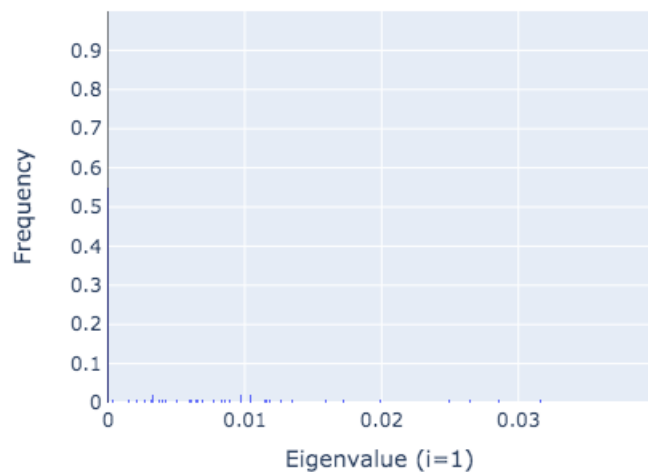
N=100, Nrep=100

Eigenvalue (i=1)

Min: -4.297466172772299e-12
Max: 0.10087596424983913
|Max-Min|: 0.1008759642541366

N=1000, Nrep=100

Eigenvalue (i=0)

Min: 0.9603053314328076
Max: 1.0000000000814833
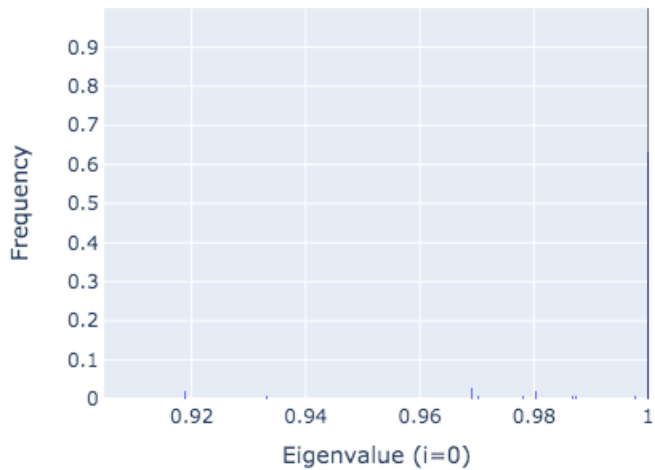|Max-Min|: 0.03969466864867566

N=1000, Nrep=100

Eigenvalue (i=1)

Min: -8.148365220260422e-11
Max: 0.03969466856719204
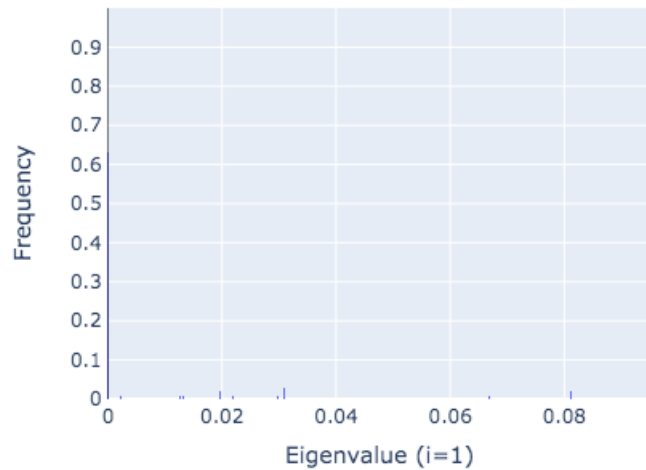|Max-Min|: 0.03969466864867569

Case 7: Least Squares(False)
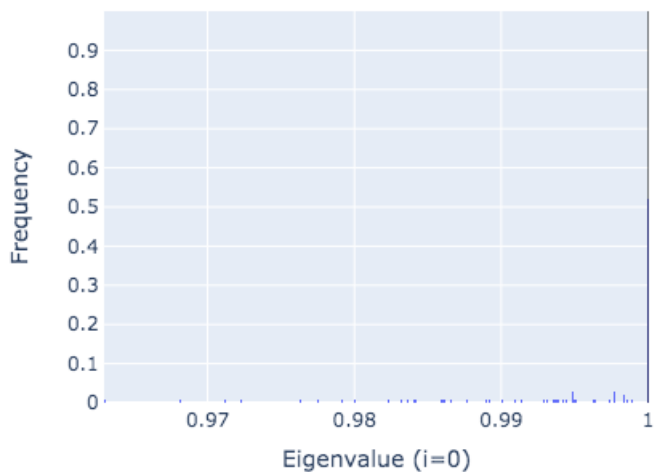
N=100, Nrep=100

Min: 0.9047221468171974
Max: 1.000000000004962
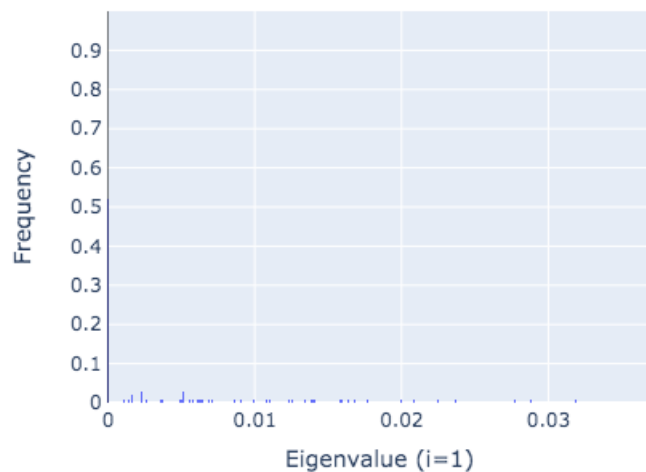|Max-Min|: 0.0952778531877646

N=100, Nrep=100

Min: -4.962265100544321e-12
Max: 0.09527785318280231
|Max-Min|: 0.09527785318776458

N=1000, Nrep=100

Min: 0.9629524581256812
Max: 1.000000000152584
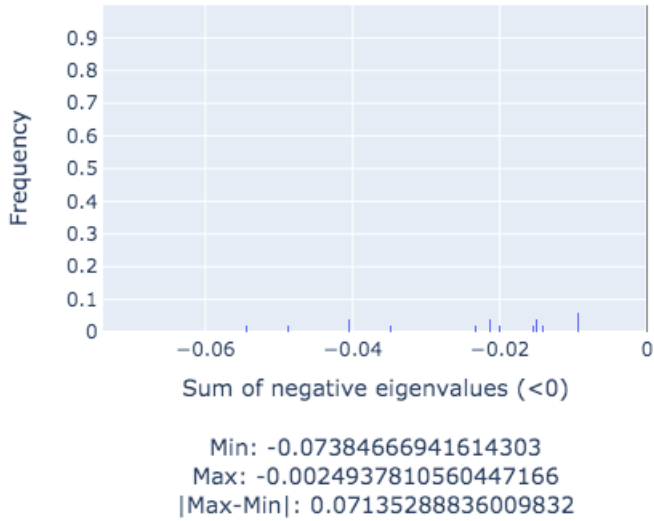|Max-Min|: 0.03704754202690275

N=1000, Nrep=100

Min: -1.525835627677721e-10
Max: 0.03704754187431892
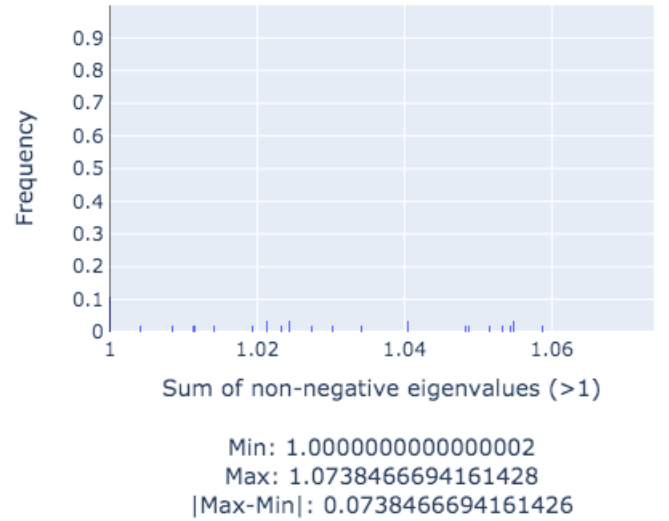|Max-Min|: 0.03704754202690248

Sum of unphysical eigenvalues

Case 0: Linear(True)

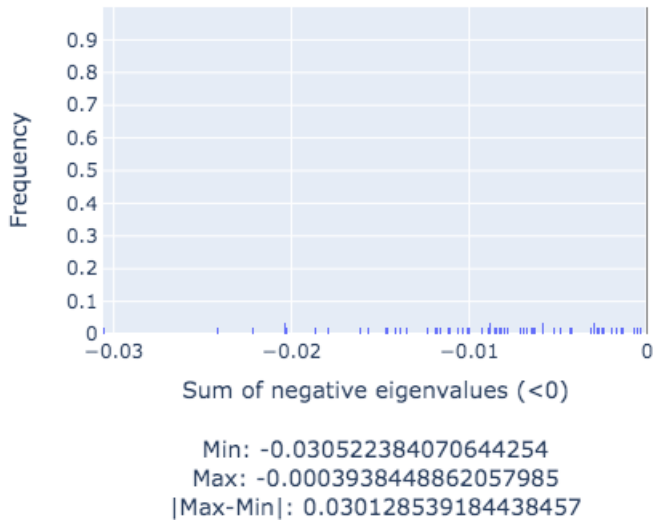**N=100**

N=100, Nrep=100
Number of unphysical estimates=50

Min: -0.07384666941614303
Max: -0.0024937810560447166
|Max-Min|: 0.07135288836009832

N=100, Nrep=100
Number of unphysical estimates=50

Min: 1.0000000000000002
Max: 1.0738466694161428
|Max-Min|: 0.0738466694161426

N=1000

N=1000, Nrep=100
Number of unphysical estimates=55

Min: -0.030522384070644254
Max: -0.0003938448862057985
|Max-Min|: 0.030128539184438457

N=1000, Nrep=100
Number of unphysical estimates=55

Min: 1.0000000000000002
Max: 1.0305223840706443
|Max-Min|: 0.030522384070644115

Case 1: Linear(False)

N=100

N=100, Nrep=100
Number of unphysical estimates=50

Frequency

Sum of negative eigenvalues (<0)

Min: -0.08335238064140982
Max: -0.0024937810560446767
|Max-Min|: 0.08085859958536513

N=100, Nrep=100
Number of unphysical estimates=50

Frequency

Sum of non-negative eigenvalues (>1)

Min: 1.0000000000000002
Max: 1.0833523806414103
|Max-Min|: 0.08335238064141004

N=1000

N=1000, Nrep=100
Number of unphysical estimates=55

Frequency

Sum of negative eigenvalues (<0)

Min: -0.031910706792032915
Max: -0.00013798096125442778
|Max-Min|: 0.03177272583077849

N=1000, Nrep=100
Number of unphysical estimates=55

Frequency

Sum of non-negative eigenvalues (>1)

Min: 1.0000000000000002
Max: 1.0319107067920326
|Max-Min|: 0.03191070679203234

Case 2: ProjectedLinear(True)
N=100

N=100, Nrep=100
Number of unphysical estimates=10

Sum of negative eigenvalues (<0)
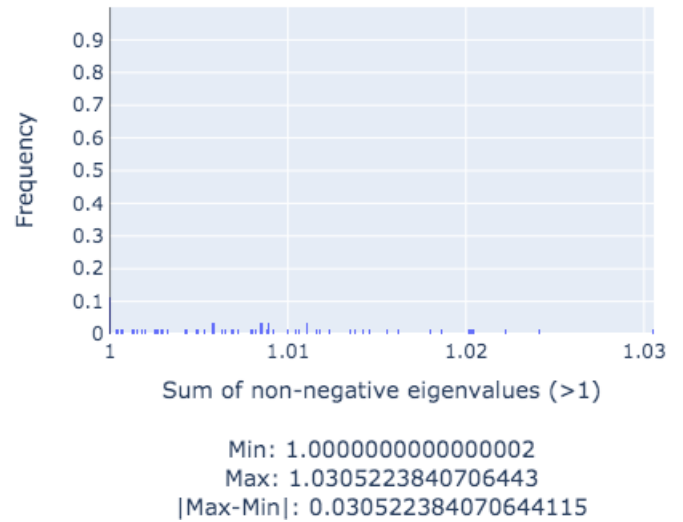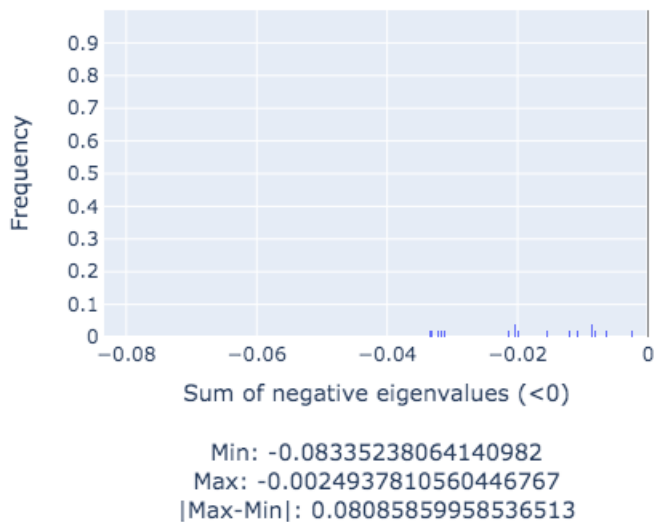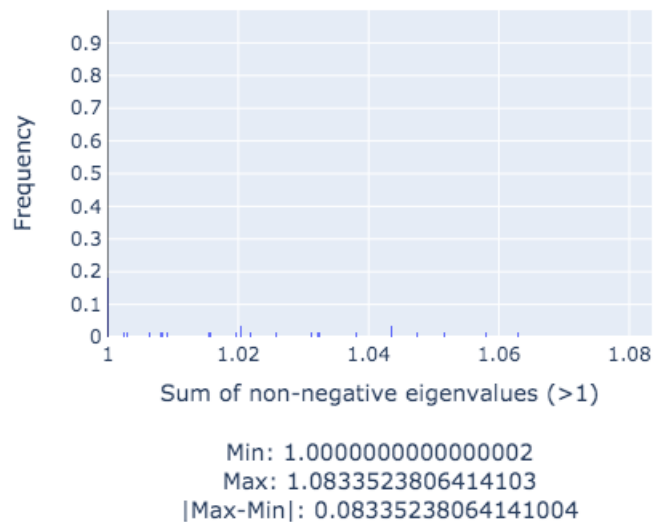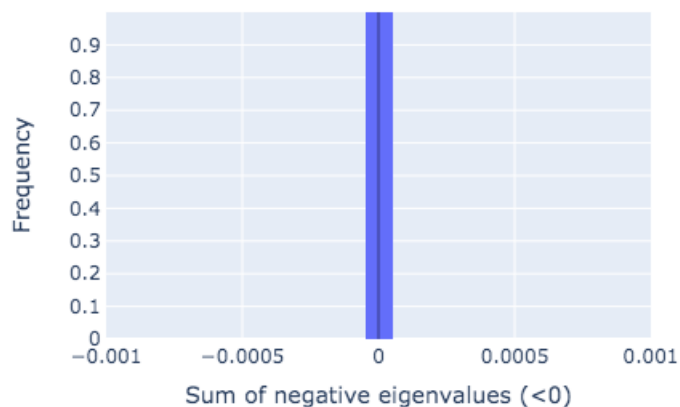
Min: -6.039008229280467e-13
Max: -1.0099746009299816e-14
|Max-Min|: 5.938010769187469e-13

N=100, Nrep=100
Number of unphysical estimates=10

Sum of non-negative eigenvalues (>1)
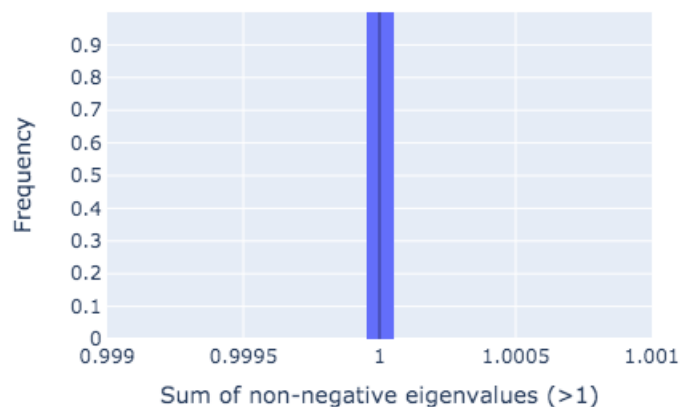
Min: 1.0000000000000002
Max: 1.0000000000006035
|Max-Min|: 6.032951915813101e-13

N=1000

N=1000, Nrep=100
Number of unphysical estimates=32

Sum of negative eigenvalues (<0)

Min: -5.184662910032525e-12
Max: -2.0754279997109768e-14
|Max-Min|: 5.163908630035415e-12

N=1000, Nrep=100
Number of unphysical estimates=32

Sum of non-negative eigenvalues (>1)

Min: 1.0000000000000002
Max: 1.000000000005184
|Max-Min|: 5.183853346579781e-12

Case 3: ProjectedLinear(False)
N=100

N=100, Nrep=100
Number of unphysical estimates=7

Min: -9.303964064993807e-13
Max: -9.670457850491253e-15
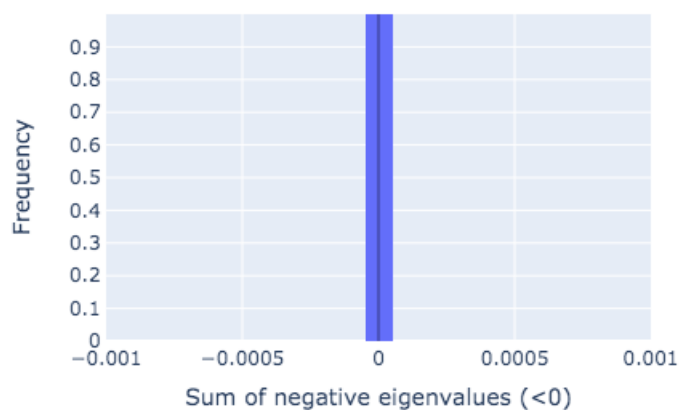|Max-Min|: 9.207259486488895e-13

N=100, Nrep=100
Number of unphysical estimates=7

Min: 1.0000000000000002
Max: 1.0000000000009297
|Max-Min|: 9.29478716216181e-13

N=1000
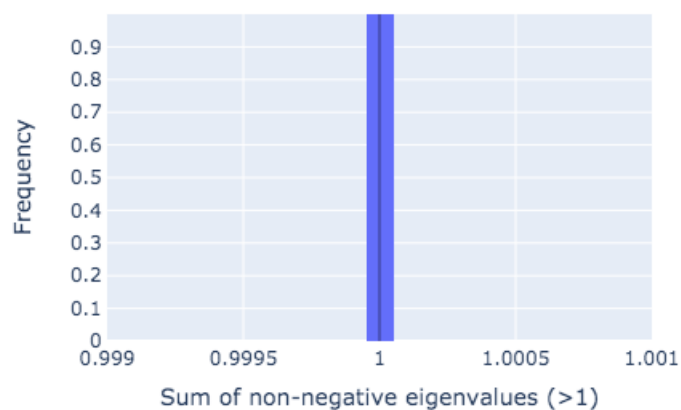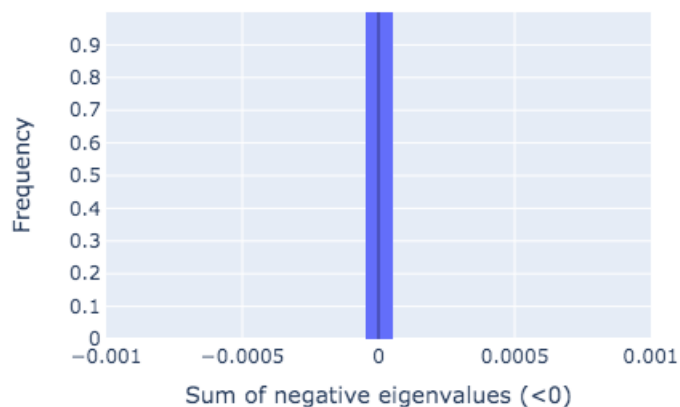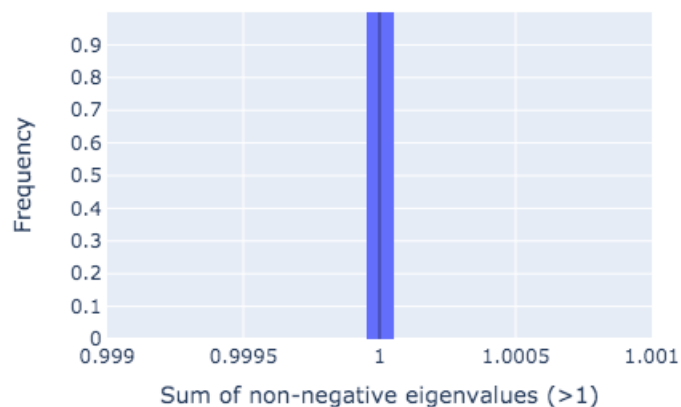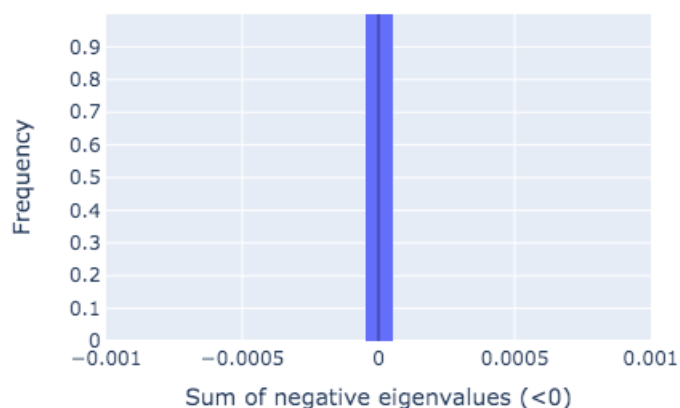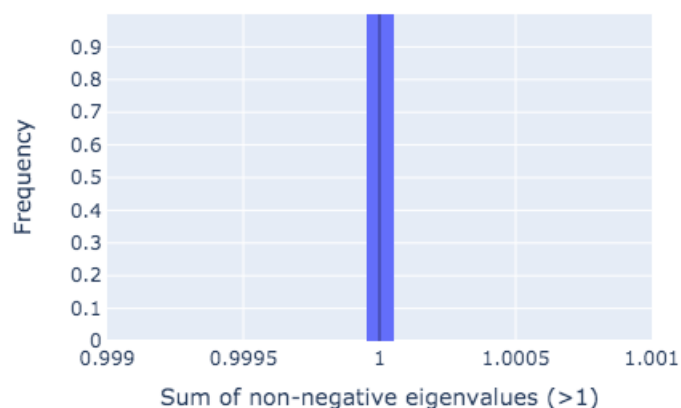
N=1000, Nrep=100
Number of unphysical estimates=33

Min: -9.765616615728533e-10
Max: -2.812716672010479e-14
|Max-Min|: 9.765335344061332e-10

N=1000, Nrep=100
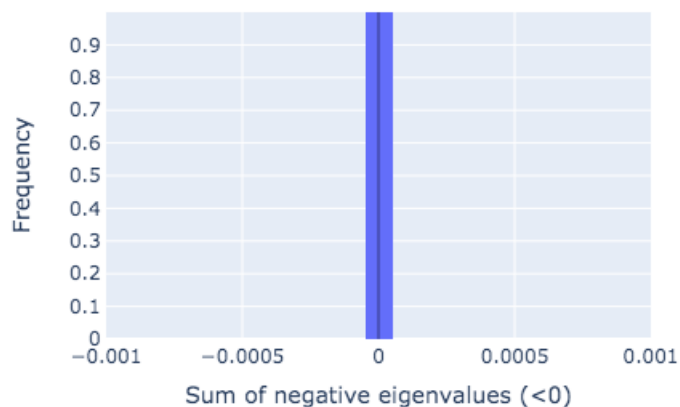Number of unphysical estimates=33

Min: 1.0000000000000002
Max: 1.000000000976561
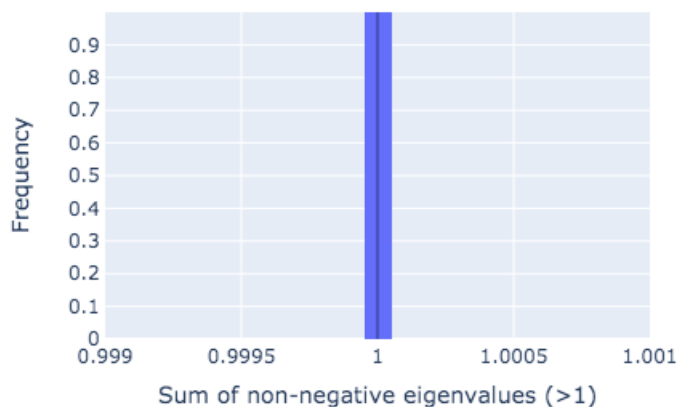|Max-Min|: 9.765608321998798e-10

Case 4: Maximum-Likelihood(True)
N=100

N=100, Nrep=100
Number of unphysical estimates=26

Sum of negative eigenvalues (<0)

Min: -5.354217988982624e-12
Max: -3.1560362675470144e-14
|Max-Min|: 5.322657626307154e-12

N=100, Nrep=100
Number of unphysical estimates=26

Sum of non-negative eigenvalues (>1)

Min: 1.0000000000000002
Max: 1.0000000000053535
|Max-Min|: 5.35327338013758e-12

N=1000

N=1000, Nrep=100
Number of unphysical estimates=56

Sum of negative eigenvalues (<0)
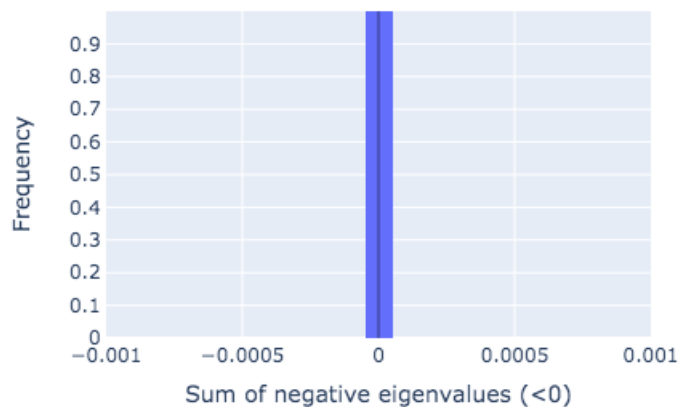
Min: -1.1236145599302914e-10
Max: -9.47597863878011e-14
|Max-Min|: 1.1226669620664135e-10

N=1000, Nrep=100
Number of unphysical estimates=56

Sum of non-negative eigenvalues (>1)
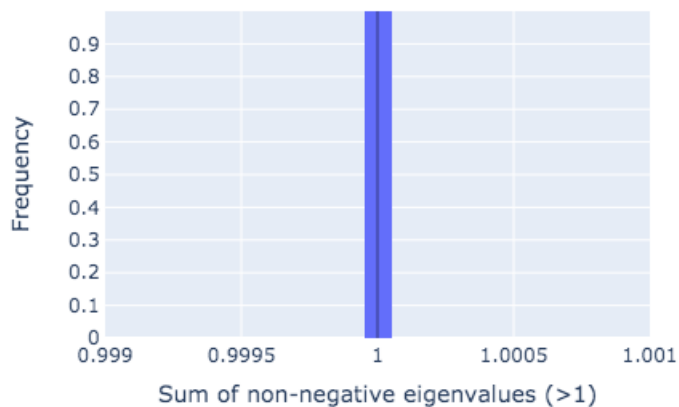
Min: 1.0000000000000002
Max: 1.0000000001123612
|Max-Min|: 1.1236100938560867e-10

Case 5: Maximum-Likelihood(False)
N=100

N=100, Nrep=100
Number of unphysical estimates=0

Min: -1.0664939803058045e-14
Max: -4.722043459913709e-16
|Max-Min|: 1.0192735457066674e-14

N=100, Nrep=100
Number of unphysical estimates=0

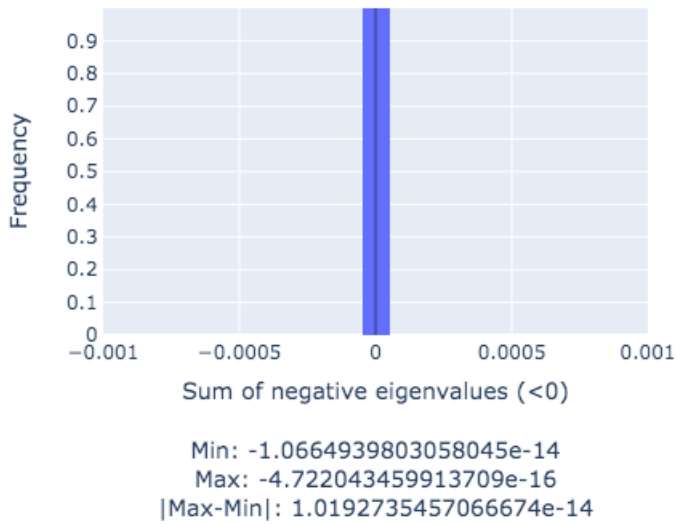Min: 1.0000000000000002
Max: 1.0000000000000102
|Max-Min|: 9.992007221626409e-15

N=1000
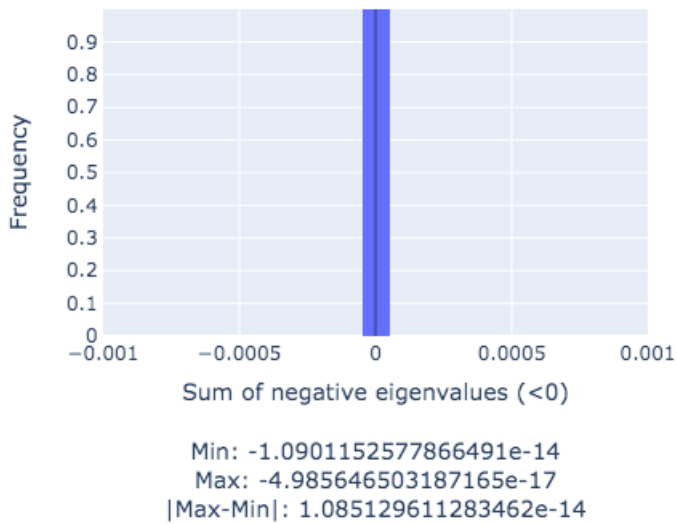
N=1000, Nrep=100
Number of unphysical estimates=0

Min: -1.0901152577866491e-14
Max: -4.985646503187165e-17
|Max-Min|: 1.085129611283462e-14

N=1000, Nrep=100
Number of unphysical estimates=0

Min: 1.0000000000000002
Max: 1.0000000000000109
|Max-Min|: 1.0658141036401503e-14
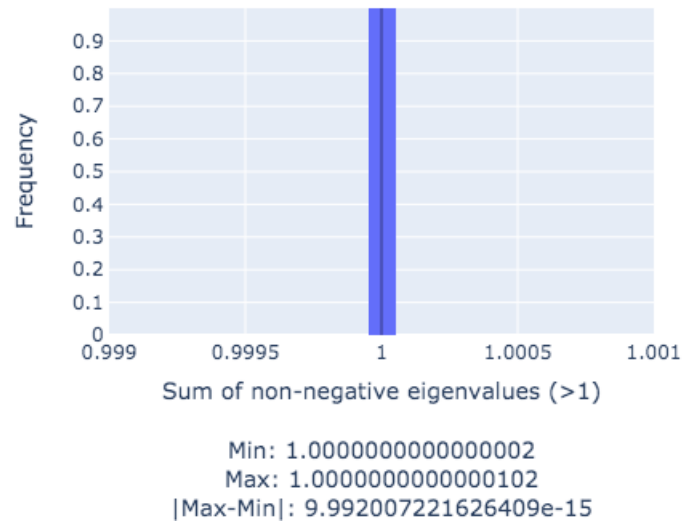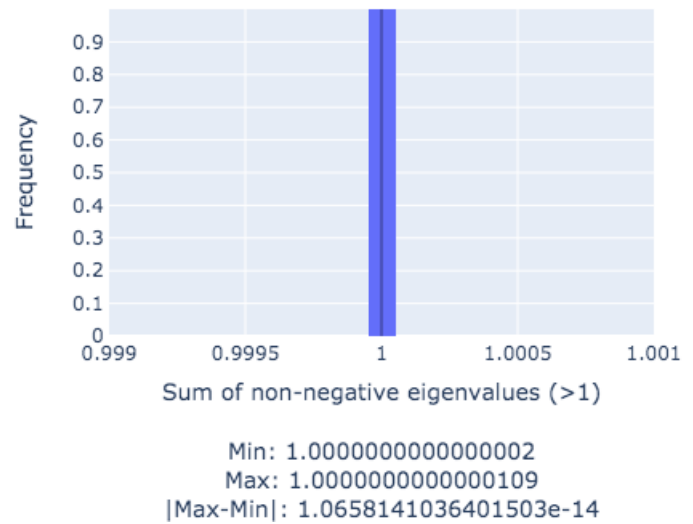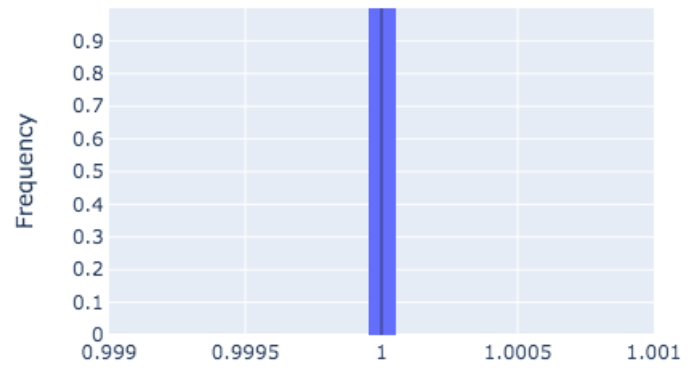
Case 6: Least Squares(True)
N=100

N=100, Nrep=100
Number of unphysical estimates=47
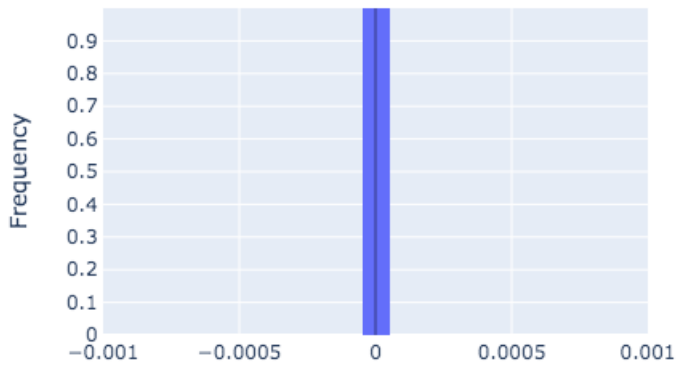
Min: -4.297466172772299e-12
Max: -4.5570814731409415e-14
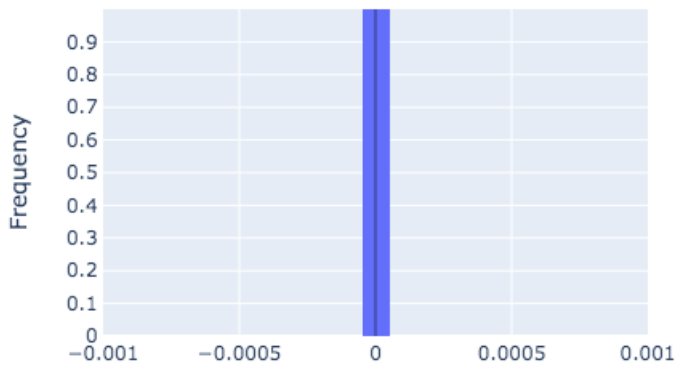|Max-Min|: 4.25189535804089e-12

N=100, Nrep=100
Number of unphysical estimates=47

Min: 1.0000000000000002
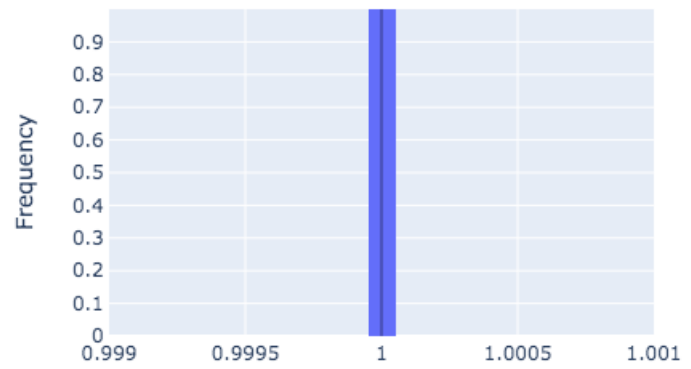Max: 1.0000000000042972
|Max-Min|: 4.297007194509206e-12

N=1000

N=1000, Nrep=100
Number of unphysical estimates=55

Min: -8.148365220260422e-11
Max: -1.466638252898478e-13
|Max-Min|: 8.133698837731437e-11

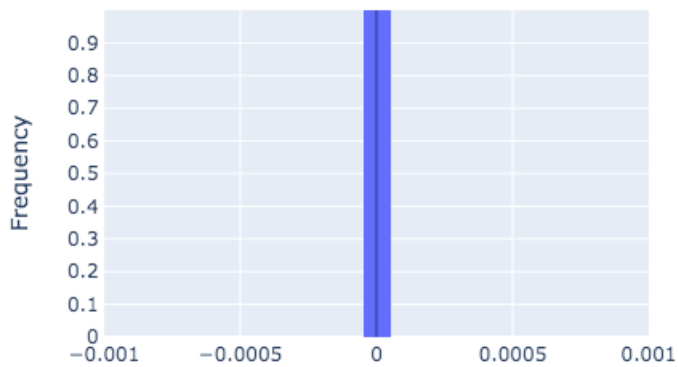N=1000, Nrep=100
Number of unphysical estimates=55

Min: 1.0000000000000002
Max: 1.0000000000814833
|Max-Min|: 8.148304253552396e-11
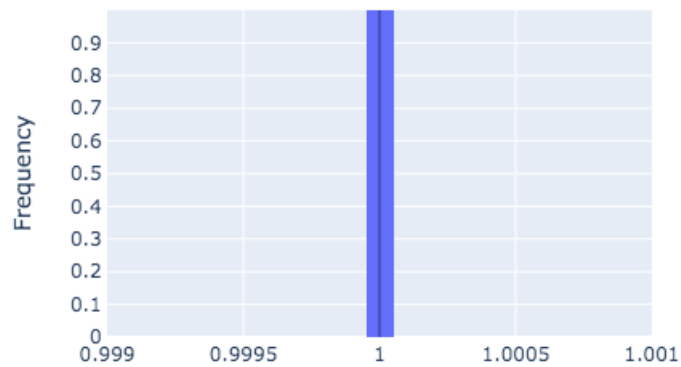
Case 7: Least Squares(False)

N=100

N=100, Nrep=100
Number of unphysical estimates=50

Min: -4.962265100544321e-12
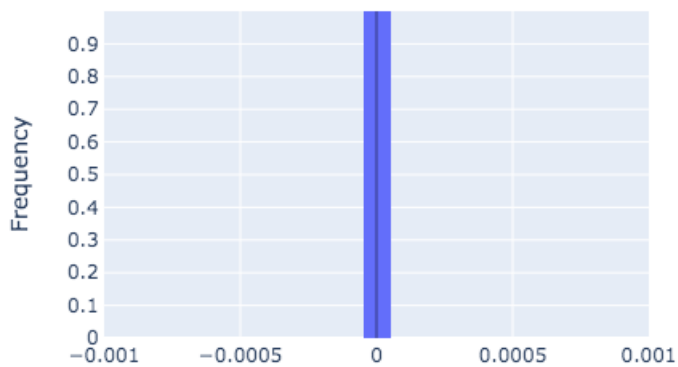Max: -5.0621417462700527e-14
|Max-Min|: 4.91164368308162e-12

N=100, Nrep=100
Number of unphysical estimates=50

Min: 1.0000000000000002
Max: 1.000000000004962
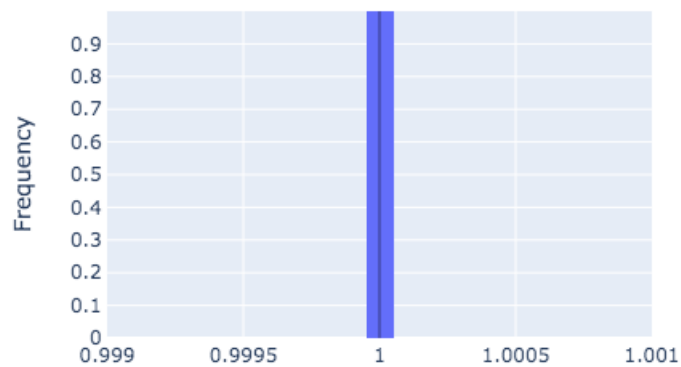|Max-Min|: 4.96180874165475e-12

N=1000

N=1000, Nrep=100
Number of unphysical estimates=52

Min: -1.525835627677721e-10
Max: -1.3819054184148236e-13
|Max-Min|: 1.524453722259306e-10

N=1000, Nrep=100
Number of unphysical estimates=52

Min: 1.0000000000000002
Max: 1.000000000152584
|Max-Min|: 1.5258372343396331e-10