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Computation time in total

Total 0.5503757317860921min.

Tolerance of physicality constraint violation

		True	False
Tolerance at physicality	equality constraint	1.00e-13	1.00e-05
violation test	inequality constraint	1.00e-05	1.00e-05

Experimental condition

Type of tomography	StandardQst
Nrep	10
N	[10, 100]
RNG seed	777

True object

Туре	State
Dim	2
Vec	[0.70710678 0.01900582 -0.10294554 0.65537864]

Tester objects

0	Туре	Povm
	Dim	2
	Number of outcomes	2
	Vecs	[[0.72718114 0.63323685 -0.06643122 -0.05867981] [0.68703242 -0.63323685 0.06643122 0.05867981]]
1	Туре	Povm
	Dim	2
	Number of outcomes	2
	Vecs	[[0.69794903 -0.04457267 0.63539095 0.0786084] [0.71626453 0.04457267 -0.63539095 -0.0786084]]
2	Туре	Povm
	Dim	2
	Number of outcomes	2
	Vecs	[[0.72807441 -0.09343513 -0.04036465 0.63004429] [0.68613915 0.09343513 0.04036465 -0.63004429]]

Cases

	Name	Param	Tomography	Estimator
0	Linear (True)	True	StandardQst	Linear
1	Linear (False)	False	StandardQst	Linear
2	ProjectedLinear (True)	True	StandardQst	ProjectedLinear
3	ProjectedLinear (False)	False	StandardQst	ProjectedLinear
4	Maximum-Likelihood (True)	True	StandardQst	LossMinimization
5	Maximum-Likelihood (False)	False	StandardQst	LossMinimization

6	Least Squares (True)	True	StandardQst	LossMinimization
7	Least Squares (False)	False	StandardQst	LossMinimization

Computation time of estimators

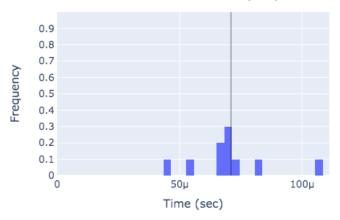
Table

	Name	N	Nrep	Mean (sec)	Std (sec)
0	Linear (True)	10	10	0.000071	0.000016
1	П	100	10	0.000061	0.000010
2	Linear (False)	10	10	0.000064	0.000013
3	П	100	10	0.000054	0.000014
4	ProjectedLinear (True)	10	10	0.009921	0.006278
5	П	100	10	0.004590	0.004766
6	ProjectedLinear (False)	10	10	0.009209	0.005524
7	П	100	10	0.003897	0.004519
8	Maximum-Likelihood (True)	10	10	0.481922	0.138619
9	П	100	10	0.511826	0.119586
10	Maximum-Likelihood (False)	10	10	0.689375	0.270307
11	П	100	10	0.701542	0.419390
12	Least Squares (True)	10	10	0.234638	0.103964
13	н	100	10	0.144703	0.117185
14	Least Squares (False)	10	10	0.282629	0.098945
15	П	100	10	0.227753	0.143212

Histogram

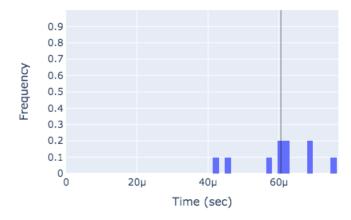
Case 0: Linear (True)

N=10, Nrep=10 Mean = 7.092952728271484e-05(sec)



Min: 4.601478576660156e-05 Max: 0.00010800361633300781 |Max-Min|: 6.198883056640625e-05

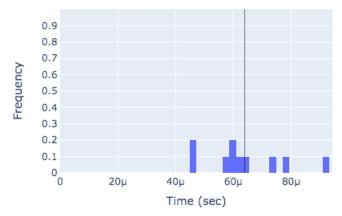
N=100, Nrep=10 Mean = 6.053447723388672e-05(sec)



Min: 4.1961669921875e-05 Max: 7.510185241699219e-05 |Max-Min|: 3.314018249511719e-05

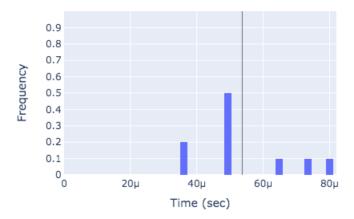
Case 1: Linear (False)

N=10, Nrep=10 Mean = 6.394386291503906e-05(sec)



Min: 4.601478576660156e-05 Max: 9.202957153320312e-05 |Max-Min|: 4.601478576660156e-05

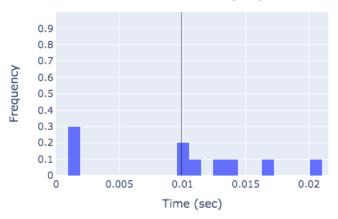
N=100, Nrep=10 Mean = 5.369186401367187e-05(sec)



Min: 3.600120544433594e-05 Max: 7.987022399902344e-05 |Max-Min|: 4.38690185546875e-05

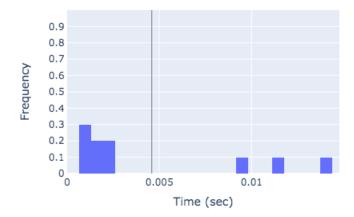
Case 2: ProjectedLinear (True)

N=10, Nrep=10 Mean = 0.00992147922515869(sec)



Min: 0.0014493465423583984 Max: 0.020581960678100586 |Max-Min|: 0.019132614135742188

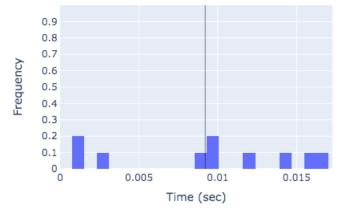
N=100, Nrep=10 Mean = 0.004590249061584473(sec)



Min: 0.0010421276092529297 Max: 0.014111042022705078 |Max-Min|: 0.013068914413452148

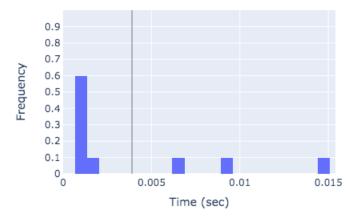
Case 3: ProjectedLinear (False)

N=10, Nrep=10 Mean = 0.009208750724792481(sec)



Min: 0.0010271072387695312 Max: 0.016499996185302734 |Max-Min|: 0.015472888946533203

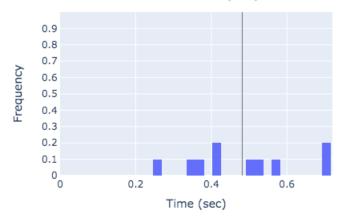
N=100, Nrep=10 Mean = 0.003896641731262207(sec)



Min: 0.0009860992431640625 Max: 0.014715909957885742 |Max-Min|: 0.01372981071472168

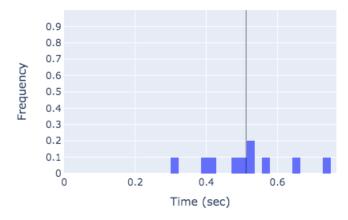
Case 4: Maximum-Likelihood (True)

N=10, Nrep=10 Mean = 0.4819220542907715(sec)



Min: 0.25034189224243164 Max: 0.698004961013794 |Max-Min|: 0.4476630687713623

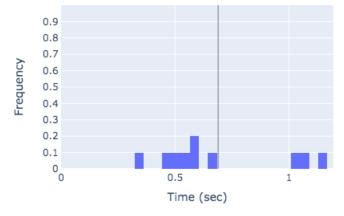
N=100, Nrep=10 Mean = 0.5118257522583007(sec)



Min: 0.3158562183380127 Max: 0.7444441318511963 |Max-Min|: 0.4285879135131836

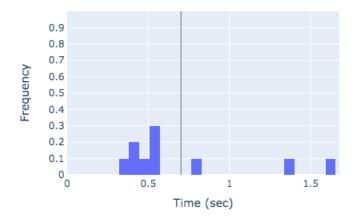
Case 5: Maximum-Likelihood (False)

N=10, Nrep=10 Mean = 0.6893749475479126(sec)



Min: 0.3487892150878906 Max: 1.1547038555145264 |Max-Min|: 0.8059146404266357

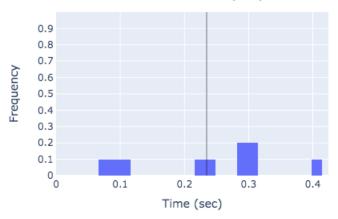
N=100, Nrep=10 Mean = 0.7015421390533447(sec)



Min: 0.34050464630126953 Max: 1.613386869430542 |Max-Min|: 1.2728822231292725

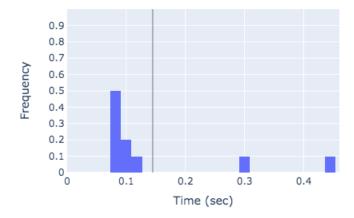
Case 6: Least Squares (True)

N=10, Nrep=10 Mean = 0.23463819026947022(sec)



Min: 0.07610726356506348 Max: 0.40777134895324707 |Max-Min|: 0.3316640853881836

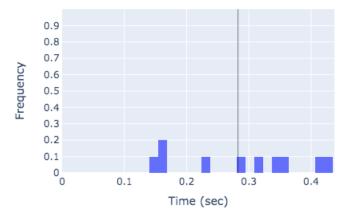
N=100, Nrep=10 Mean = 0.14470269680023193(sec)



Min: 0.07891273498535156 Max: 0.44204115867614746 |Max-Min|: 0.3631284236907959

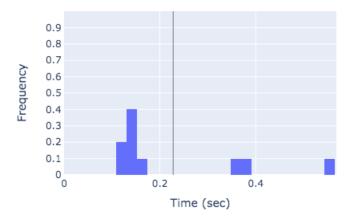
Case 7: Least Squares (False)

N=10, Nrep=10 Mean = 0.28262927532196047(sec)



Min: 0.14296388626098633 Max: 0.42344021797180176 |Max-Min|: 0.28047633171081543

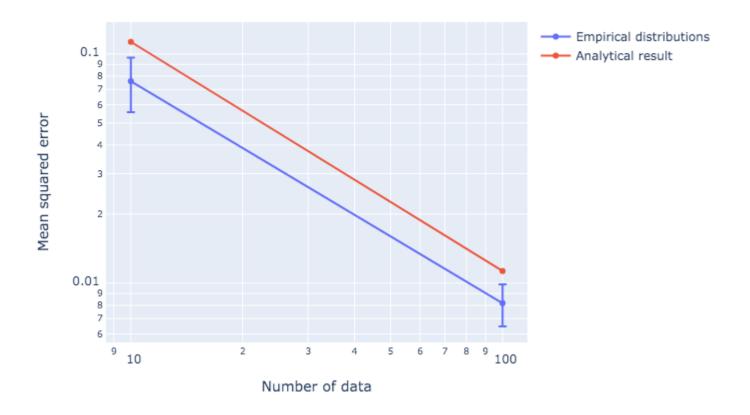
N=100, Nrep=10 Mean = 0.2277531147003174(sec)



Min: 0.11190080642700195 Max: 0.5467355251312256 |Max-Min|: 0.43483471870422363

MSE of empirical distributions

Mean squared error

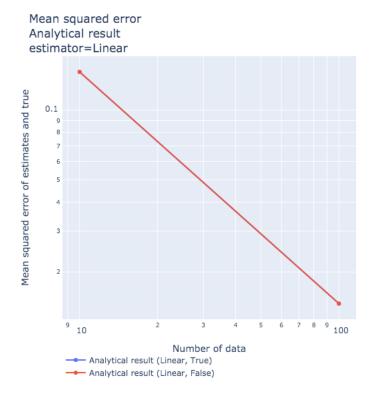


Consistency test

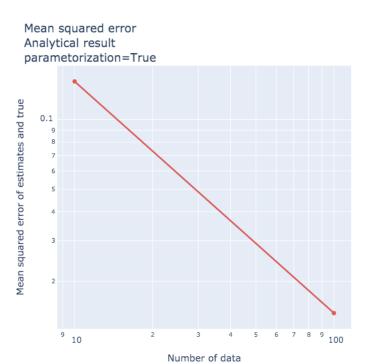
	Name	Type of tomography	Param	Estimator	Loss	Algo	Squared Error to True	Possibly OK	To be checked
0	Linear (True)	Standard Qst	True	Linear	None	None	1.00e-32	ОК	not need debug
1	Linear (False)	Standard Qst	False	Linear	None	None	2.59e-32	ОК	not need debug
2	ProjectedLinear (True)	Standard Qst	True	Projected Linear	None	None	1.66e-32	ОК	not need debug
3	ProjectedLinear (False)	Standard Qst	False	Projected Linear	None	None	3.06e-32	OK	not need debug
4	Maximum-Likelihood (True)	Standard Qst	True	Loss Minimization	Weighted Relative Entropy	Projected Gradient Descent Backtracking	4.61e-18	OK	not need debug
5	Maximum-Likelihood (False)	Standard Qst	False	Loss Minimization	Weighted Relative Entropy	Projected Gradient Descent Backtracking	1.08e-08	NG	not need debug
6	Least Squares (True)	Standard Qst	True	Loss Minimization	Weighted Probability Based Squared Error	Projected Gradient Descent Backtracking	1.30e-29	OK	not need debug

MSE of estimators

Comparison of analytical results







Mean squared error Analytical result

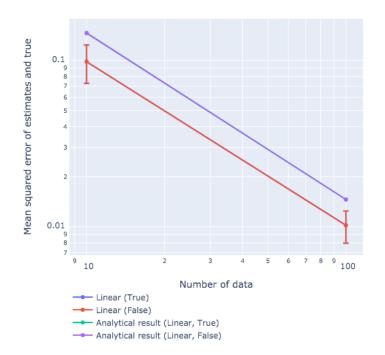


Comparison of parametrization

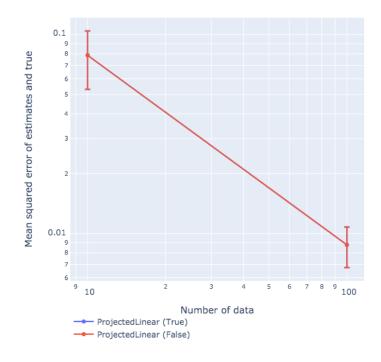
Analytical result (Cramer-Rao bound, True)

Analytical result (Linear, True)

Mean squared error Estimator=Linear



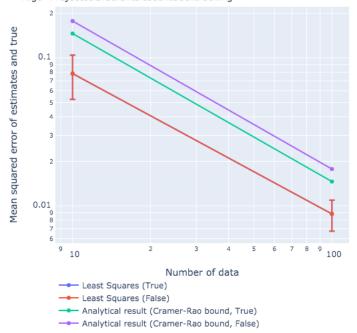
Mean squared error Estimator=ProjectedLinear



Mean squared error Estimator=LossMinimization Loss=WeightedRelativeEntropy Algo=ProjectedGradientDescentBacktracking 0.1 9 8 7 6 5 9 10 Number of data Maximum-Likelihood (True) Maximum-Likelihood (False) Analytical result (Cramer-Rao bound, True)

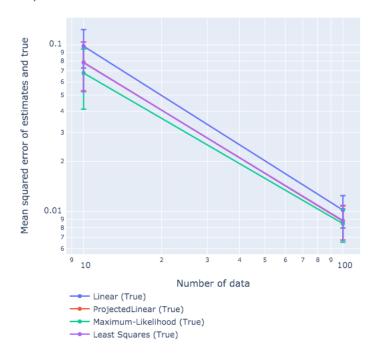
- Analytical result (Cramer-Rao bound, False)

Mean squared error Estimator=LossMinimization Loss=WeightedProbabilityBasedSquaredError Algo=ProjectedGradientDescentBacktracking

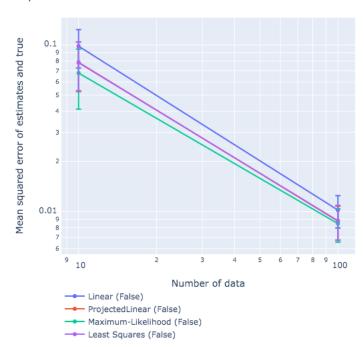


Comparison of estimators

Mean squared error parametrization=True



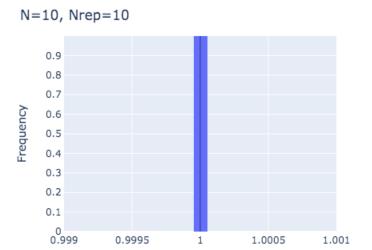
Mean squared error parametrization=False



Physicality violation test

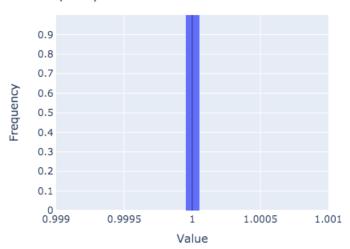
Test of equality constraint violation

Case 0: Linear (True)



Value

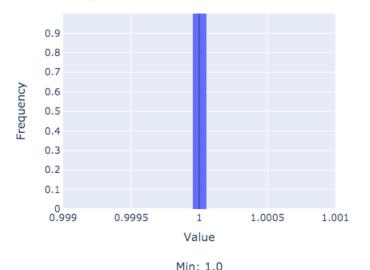
N=100, Nrep=10



Min: 0.99999999999998 Max: 0.99999999999998 |Max-Min|: 0.0

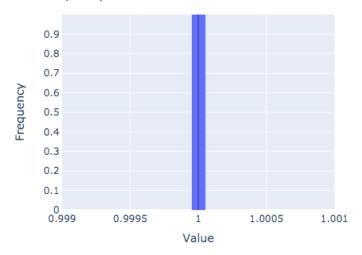
Case 1: Linear (False)

N=10, Nrep=10

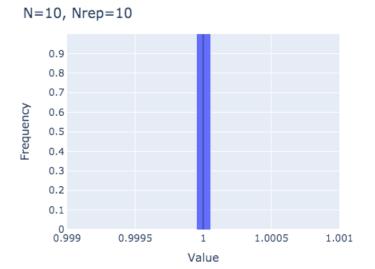


Max: 1.0000000000000004 |Max-Min|: 4.440892098500626e-16

N=100, Nrep=10

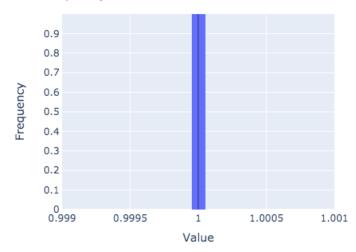


Case 2: ProjectedLinear (True)



Min: 0.99999999999998 Max: 0.99999999999998 |Max-Min|: 0.0

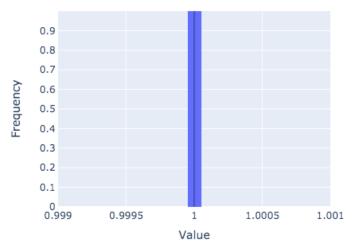
N=100, Nrep=10



Min: 0.99999999999998 Max: 0.99999999999998 |Max-Min|: 0.0

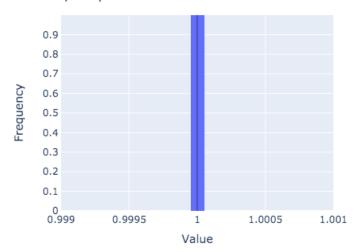
Case 3: ProjectedLinear (False)

N=10, Nrep=10



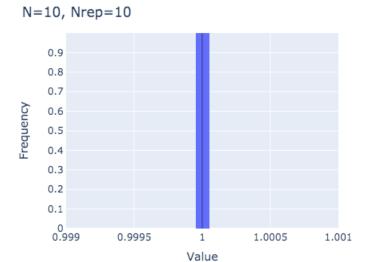
Min: 0.999999999999998 Max: 1.0000001807827894 |Max-Min|: 1.807827896271874e-07

N=100, Nrep=10



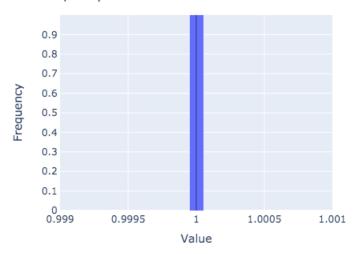
Min: 0.999999999999998 Max: 1.0000001551111417 |Max-Min|: 1.55111141930675e-07

Case 4: Maximum-Likelihood (True)



Min: 0.99999999999998 Max: 0.99999999999998 |Max-Min|: 0.0

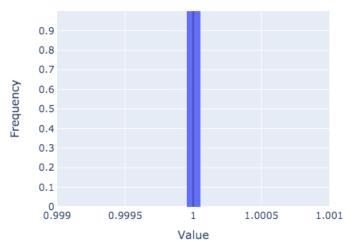
N=100, Nrep=10



Min: 0.99999999999998 Max: 0.99999999999998 |Max-Min|: 0.0

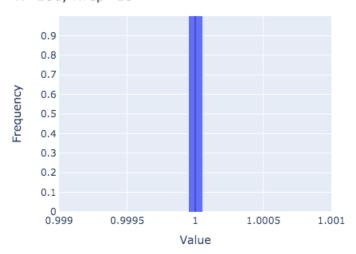
Case 5: Maximum-Likelihood (False)

N=10, Nrep=10



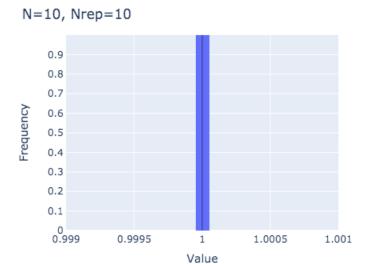
Min: 1.00000002701252 Max: 1.00000015572761 |Max-Min|: 1.530263578075619e-07

N=100, Nrep=10

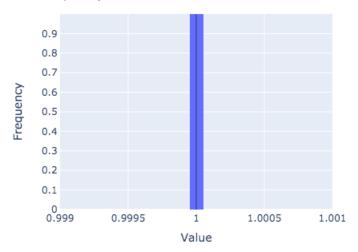


Min: 1.00000007540976 Max: 1.0000001541710055 |Max-Min|: 1.4663002945169978e-07

Case 6: Least Squares (True)



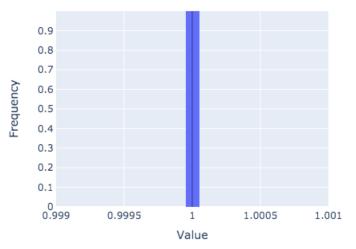
Min: 0.99999999999998 Max: 0.99999999999998 |Max-Min|: 0.0 N=100, Nrep=10



Min: 0.99999999999998 Max: 0.99999999999998 |Max-Min|: 0.0

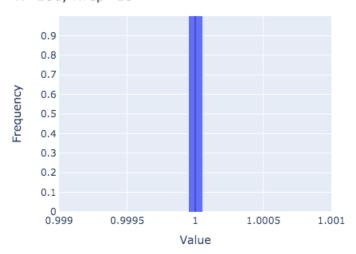
Case 7: Least Squares (False)

N=10, Nrep=10



Min: 1.000000000000013 Max: 1.0000001660502706 |Max-Min|: 1.6605026931237887e-07

N=100, Nrep=10

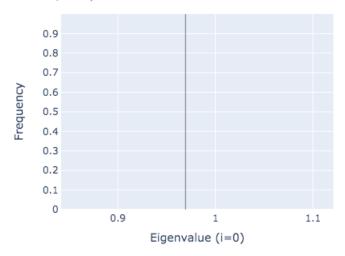


Min: 1.000000000000013 Max: 1.0000001531930411 |Max-Min|: 1.5319303980909638e-07

Test of inequality constraint violation

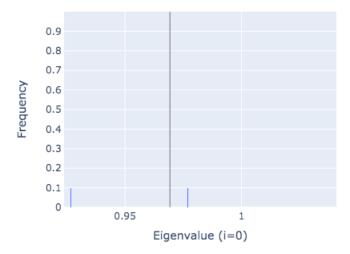
Eigenvalue

Case 0: Linear (True)



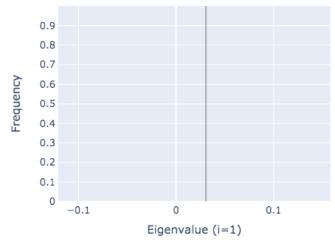
Min: 0.8418102069718651 Max: 1.1208478114533518 |Max-Min|: 0.2790376044814866

N=100, Nrep=10



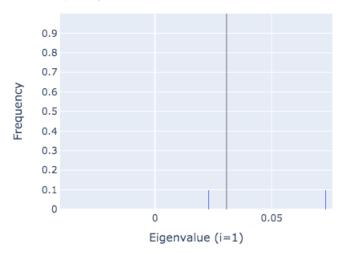
Min: 0.9239932464088522 Max: 1.0406614551236344 |Max-Min|: 0.11666820871478223

N=10, Nrep=10



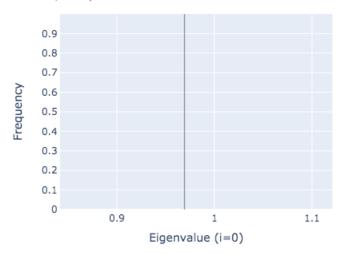
Min: -0.12084781145335197 Max: 0.15818979302813466 |Max-Min|: 0.2790376044814866

N=100, Nrep=10



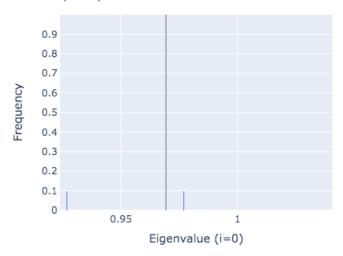
Min: -0.04066145512363449 Max: 0.07600675359114764 |Max-Min|: 0.11666820871478213

Case 1: Linear (False)



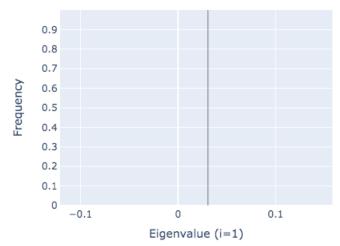
Min: 0.8418102069718654 Max: 1.120847811453352 |Max-Min|: 0.2790376044814866

N=100, Nrep=10



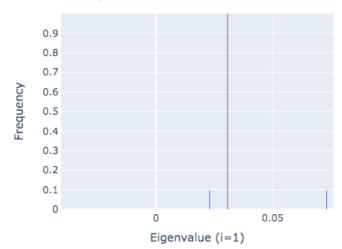
Min: 0.9239932464088524 Max: 1.0406614551236344 |Max-Min|: 0.116668208714782

N=10, Nrep=10



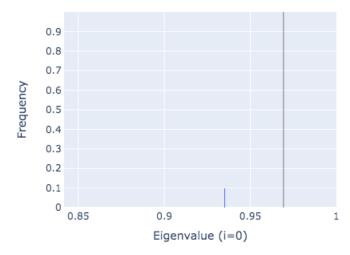
Min: -0.12084781145335197 Max: 0.15818979302813474 |Max-Min|: 0.27903760448148673

N=100, Nrep=10



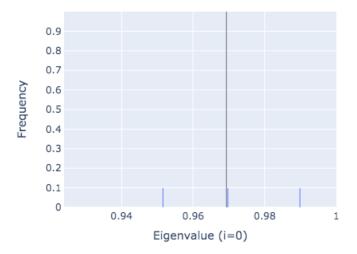
Min: -0.040661455123634296 Max: 0.07600675359114792 |Max-Min|: 0.11666820871478221

Case 2: ProjectedLinear (True)



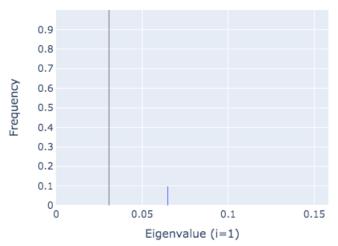
Min: 0.8418102069718649 Max: 1.0000000903913944 |Max-Min|: 0.15818988341952944

N=100, Nrep=10



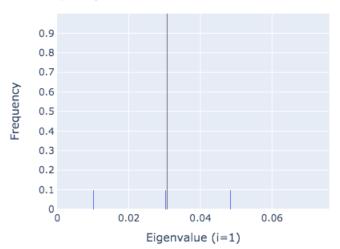
Min: 0.923993246408852 Max: 1.0000000775555709 |Max-Min|: 0.0760068311467188

N=10, Nrep=10



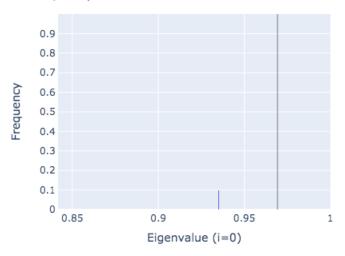
Min: -9.039139473032698e-08 Max: 0.15818979302813482 |Max-Min|: 0.15818988341952955

N=100, Nrep=10



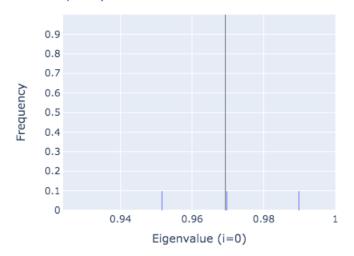
Min: -7.755557092283677e-08 Max: 0.07600675359114771 |Max-Min|: 0.07600683114671863

Case 3: ProjectedLinear (False)



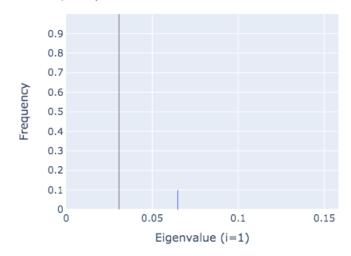
Min: 0.8418102069718654 Max: 1.0000001807827896 |Max-Min|: 0.15818997381092426

N=100, Nrep=10



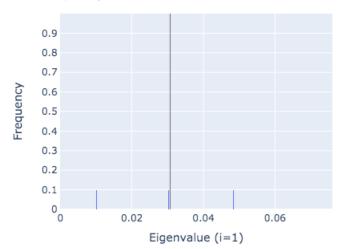
Min: 0.9239932464088519 Max: 1.0000001551111417 |Max-Min|: 0.07600690870228977

N=10, Nrep=10



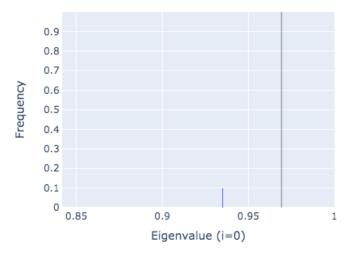
Min: -8.326672684688674e-17 Max: 0.15818979302813488 |Max-Min|: 0.15818979302813496

N=100, Nrep=10



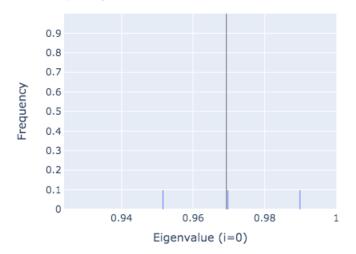
Min: 2.935477382004503e-17 Max: 0.07600675359114775 |Max-Min|: 0.07600675359114772

Case 4: Maximum-Likelihood (True)



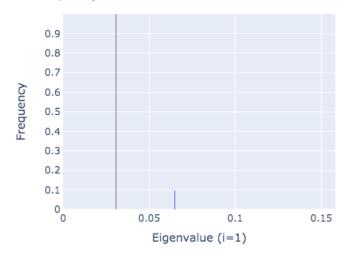
Min: 0.841810205970211 Max: 1.0000000674320337 |Max-Min|: 0.1581898614618228

N=100, Nrep=10



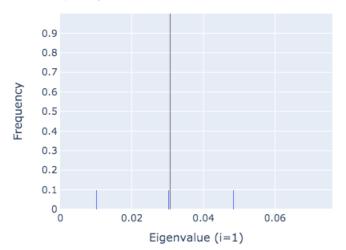
Min: 0.9239932443919643 Max: 1.0000000667579059 |Max-Min|: 0.07600682236594158

N=10, Nrep=10



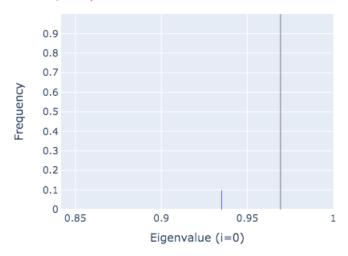
Min: -6.743203388032137e-08 Max: 0.15818979402978878 |Max-Min|: 0.15818986146182265

N=100, Nrep=10



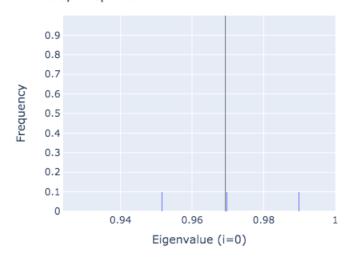
Min: -6.675790605037424e-08 Max: 0.07600675560803535 |Max-Min|: 0.0760068223659414

Case 5: Maximum-Likelihood (False)



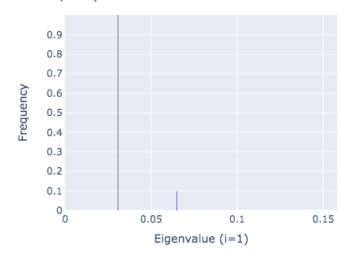
Min: 0.8418193521194106 Max: 1.00000015572761 |Max-Min|: 0.1581808036081993

N=100, Nrep=10



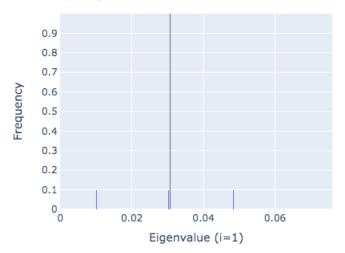
Min: 0.9239674033539598 Max: 1.0000001541709906 |Max-Min|: 0.07603275081703087

N=10, Nrep=10



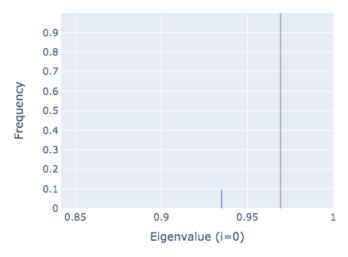
Min: 9.71445146547012e-17 Max: 0.15818065058184155 |Max-Min|: 0.15818065058184144

N=100, Nrep=10



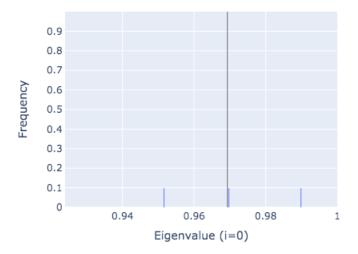
Min: 1.1796119636642288e-16 Max: 0.07603260922147073 |Max-Min|: 0.07603260922147062

Case 6: Least Squares (True)



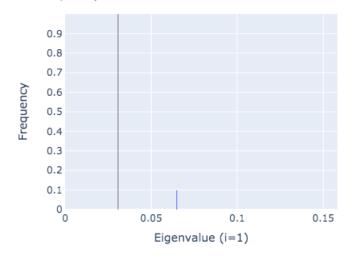
Min: 0.8418102069718657 Max: 1.0000000805599834 |Max-Min|: 0.15818987358811765

N=100, Nrep=10



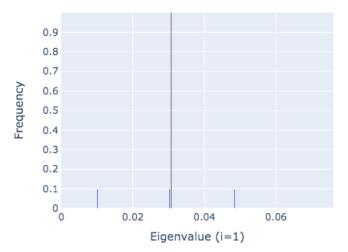
Min: 0.9239932464088507 Max: 1.0000000896591925 |Max-Min|: 0.0760068432503418

N=10, Nrep=10



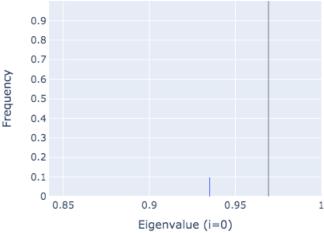
Min: -8.055998357292538e-08 Max: 0.158189793028134 |Max-Min|: 0.15818987358811756

N=100, Nrep=10



Min: -8.965919272847073e-08 Max: 0.0760067535911491 |Max-Min|: 0.07600684325034182

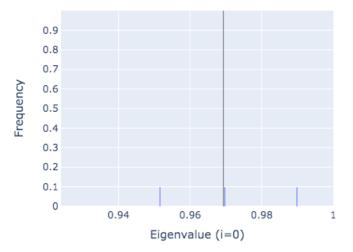
Case 7: Least Squares (False)



Min: 0.8418102069718676 Max: 1.00000016605027

|Max-Min|: 0.1581899590784024

N=100, Nrep=10



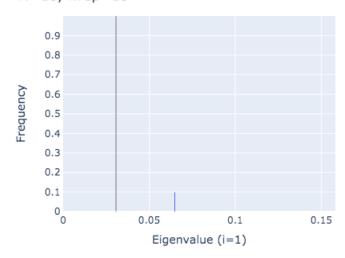
Min: 0.9239932464088527 Max: 1.0000001531930411 |Max-Min|: 0.07600690678418842

Sum of unphysical eigenvalues

Case 0: Linear (True)

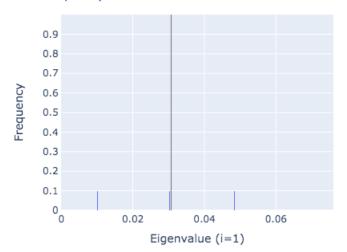
Number of unphysical estimates=7

N=10, Nrep=10

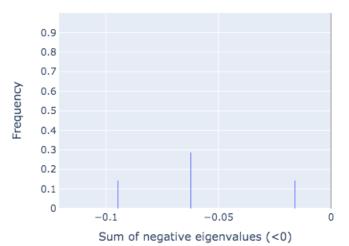


Min: -1.2836953722228372e-16 Max: 0.15818979302813618 |Max-Min|: 0.15818979302813632

N=100, Nrep=10



Min: -6.938893903907228e-18 Max: 0.07600675359114963 |Max-Min|: 0.07600675359114964

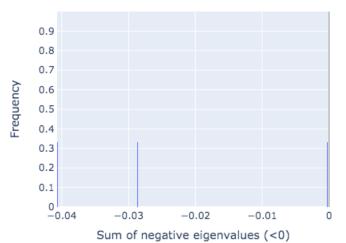


Min: -0.12084781145335197 Max: -0.016008803355067935 |Max-Min|: 0.10483900809828403

N=100

Number of unphysical estimates=3





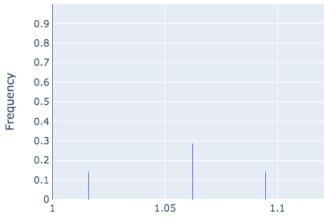
Min: -0.04066145512363449 Max: -0.00020965699256968076 |Max-Min|: 0.04045179813106481

Case 1: Linear (False)

N=10

Number of unphysical estimates=7

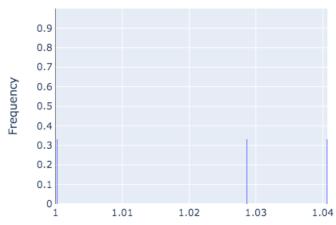




Sum of non-negative eigenvalues (>1)

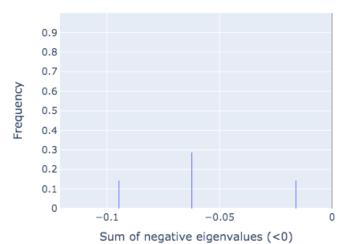
Min: 1.0160088033550676 Max: 1.1208478114533518 |Max-Min|: 0.10483900809828417

N=100, Nrep=10



Sum of non-negative eigenvalues (>1)

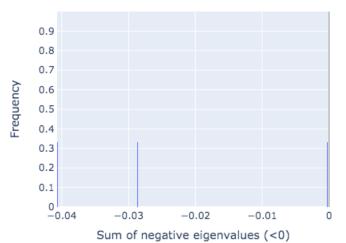
Min: 1.0002096569925696 Max: 1.0406614551236344 |Max-Min|: 0.04045179813106481



Min: -0.12084781145335197 Max: -0.01600880335506777 |Max-Min|: 0.1048390080982842

N=100 Number of unphysical estimates=3





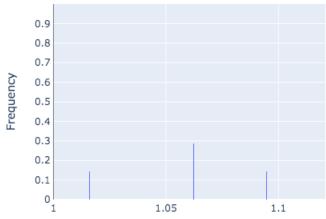
Min: -0.040661455123634296 Max: -0.0002096569925694593 |Max-Min|: 0.04045179813106484

Case 2: ProjectedLinear (True)

N = 10

Number of unphysical estimates=0

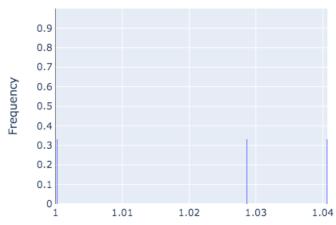




Sum of non-negative eigenvalues (>1)

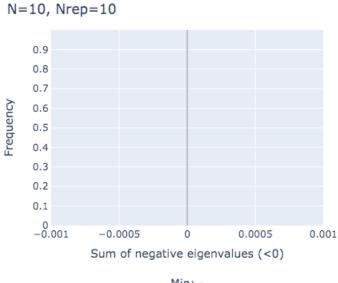
Min: 1.016008803355068 Max: 1.120847811453352 |Max-Min|: 0.10483900809828395

N=100, Nrep=10

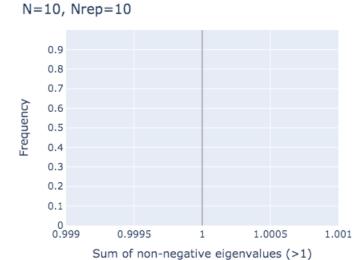


Sum of non-negative eigenvalues (>1)

Min: 1.0002096569925696 Max: 1.0406614551236344 |Max-Min|: 0.04045179813106481



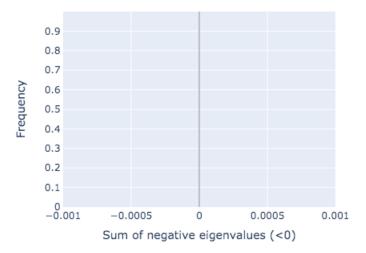
Min: -Max: -|Max-Min|: -



Min: -Max: -|Max-Min|: -

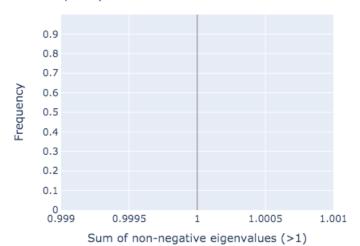
N=100 Number of unphysical estimates=0

N=100, Nrep=10



Min: -Max: -|Max-Min|: -

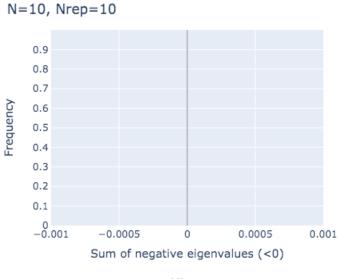
N=100, Nrep=10



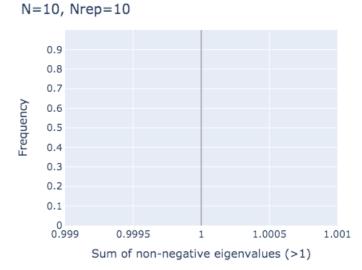
Min: -Max: -|Max-Min|: -

Case 3: ProjectedLinear (False)

N=10 Number of unphysical estimates=0



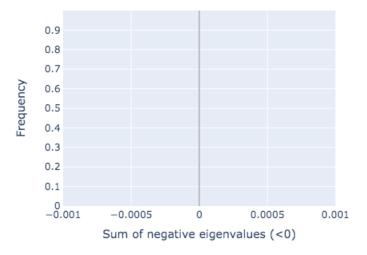
Min: -Max: -|Max-Min|: -



Min: -Max: -|Max-Min|: -

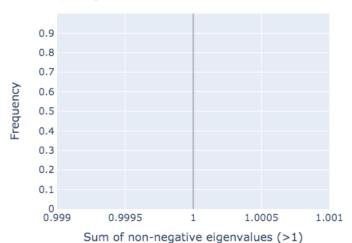
N=100 Number of unphysical estimates=0

N=100, Nrep=10



Min: -Max: -|Max-Min|: -

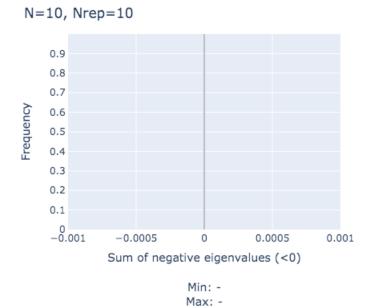
N=100, Nrep=10



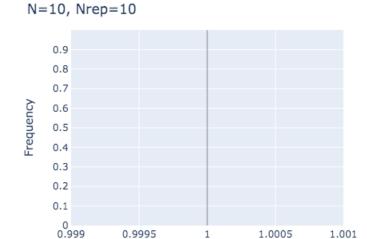
Min: -Max: -|Max-Min|: -

Case 4: Maximum-Likelihood (True)

Number of unphysical estimates=0



|Max-Min|: -



Sum of non-negative eigenvalues (>1) Min: -

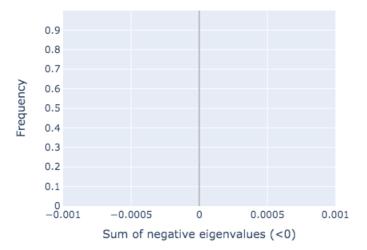
0.9995

Max: -|Max-Min|: - 1.0005

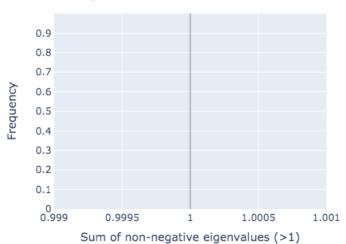
1.001

N=100 Number of unphysical estimates=0

N=100, Nrep=10



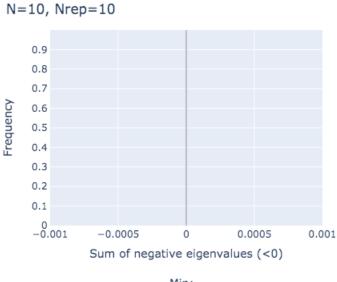
Min: -Max: -|Max-Min|: - N=100, Nrep=10

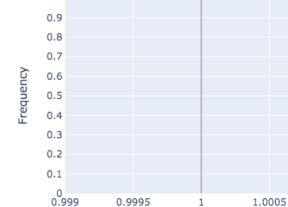


Min: -Max: -|Max-Min|: -

Case 5: Maximum-Likelihood (False)

Number of unphysical estimates=0





Min: -Max: -|Max-Min|: -

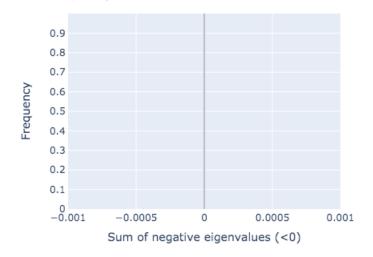
Sum of non-negative eigenvalues (>1)

1.001

Min: -Max: -|Max-Min|: -

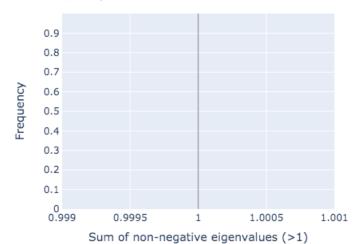
N=100 Number of unphysical estimates=0

N=100, Nrep=10



Min: -Max: -|Max-Min|: - N=100, Nrep=10

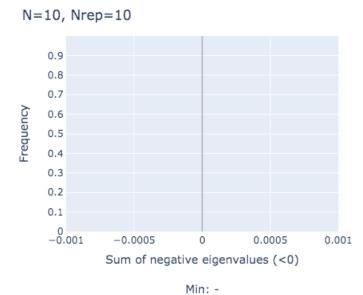
N=10, Nrep=10



Min: -Max: -|Max-Min|: -

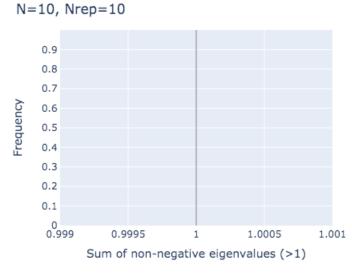
Case 6: Least Squares (True)

N=10 Number of unphysical estimates=0



Max: -

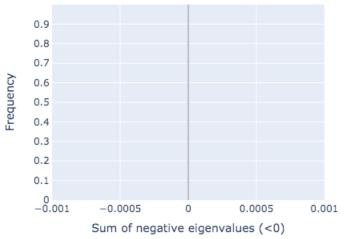
|Max-Min|: -



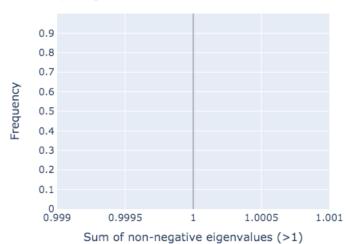
Min: -Max: -|Max-Min|: -

N=100 Number of unphysical estimates=0

N=100, Nrep=10



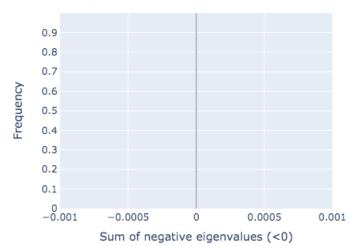
Min: -Max: -|Max-Min|: - N=100, Nrep=10



Min: -Max: -|Max-Min|: -

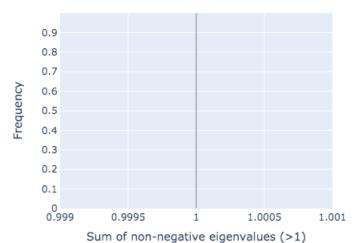
Case 7: Least Squares (False)

Number of unphysical estimates=0



Min: -Max: -|Max-Min|: -

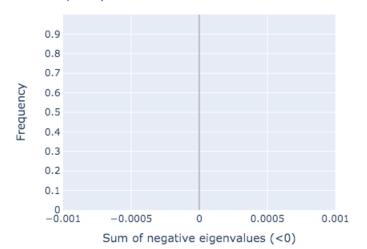
N=10, Nrep=10



Min: -Max: -|Max-Min|: -

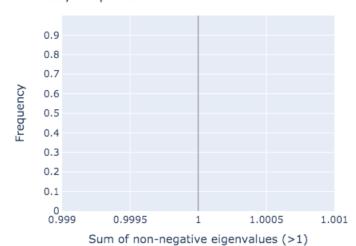
N=100 Number of unphysical estimates=0

N=100, Nrep=10



Min: -Max: -|Max-Min|: -

N=100, Nrep=10



Min: -Max: -|Max-Min|: -