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Computation time in total

Total	0.8562086423238119min.

Tolerance of physicality constraint violation

		True	False
Tolerance at physicality violation test	equality constraint	1.00e-13	1.00e-05
	inequality constraint	1.00e-05	1.00e-05

Experimental condition

Type of tomography	StandardQst
Nrep	10
N	[10, 100]
RNG seed	777

True object

Type	State
Dim	2
Vec	[0.70710678 0. 0. 0.6363961]

Tester objects

0	**Type**	Povm
	Dim	2
	Number of outcomes	2
	Vecs	[[0.70710678 0.6363961 0. 0.] [0.70710678 -0.6363961 0. 0.]]
1	**Type**	Povm
	Dim	2
	Number of outcomes	2
	Vecs	[[0.70710678 0. 0.6363961 0.] [0.70710678 0. -0.6363961 0.]]
2	**Type**	Povm
	Dim	2
	Number of outcomes	2
	Vecs	[[0.70710678 0. 0. 0.6363961] [0.70710678 0. 0. -0.6363961]]

Cases

	Name	Param	Tomography	Estimator
0	Linear (True)	True	StandardQst	Linear
1	Linear (False)	False	StandardQst	Linear
2	ProjectedLinear (True)	True	StandardQst	ProjectedLinear
3	ProjectedLinear (False)	False	StandardQst	ProjectedLinear
4	Maximum-Likelihood (True)	True	StandardQst	LossMinimization
5	Maximum-Likelihood (False)	False	StandardQst	LossMinimization

	Name	N	Nrep		
6	Least Squares (True)	True	StandardQst		LossMinimization
7	Least Squares (False)	False	StandardQst		LossMinimization

Computation time of estimators

Table

	Name	N	Nrep	Mean (sec)	Std (sec)
0	Linear (True)	10	10	0.000079	0.000025
1	"	100	10	0.000060	0.000014
2	Linear (False)	10	10	0.000079	0.000015
3	"	100	10	0.000067	0.000010
4	ProjectedLinear (True)	10	10	0.008221	0.006059
5	"	100	10	0.002968	0.004499
6	ProjectedLinear (False)	10	10	0.019703	0.027288
7	"	100	10	0.003165	0.004019
8	Maximum-Likelihood (True)	10	10	0.559123	0.094661
9	"	100	10	0.482868	0.156924
10	Maximum-Likelihood (False)	10	10	1.903050	2.655925
11	"	100	10	1.724811	1.408398
12	Least Squares (True)	10	10	0.078670	0.018103
13	"	100	10	0.097506	0.085795
14	Least Squares (False)	10	10	0.128143	0.058307
15	"	100	10	0.128737	0.046979

Histogram

Case 0: Linear (True)

N=10, Nrep=10
Mean = 7.922649383544922e-05(sec)
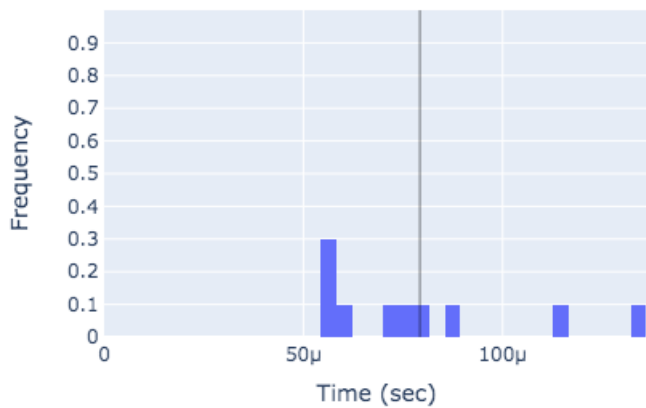
Min: 5.507469177246094e-05
Max: 0.0001327991485595703
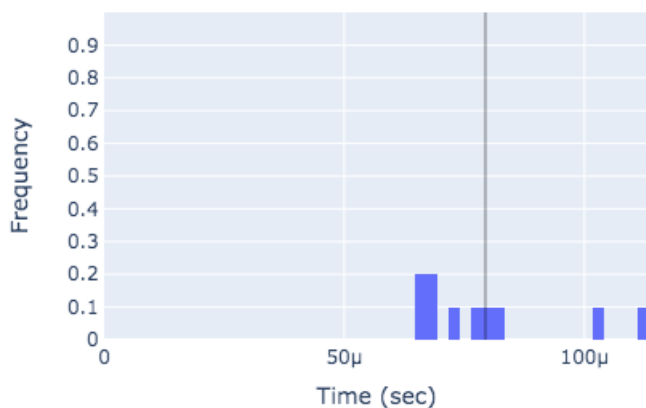|Max-Min|: 7.772445678710938e-05

N=100, Nrep=10
Mean = 5.9556961059570315e-05(sec)
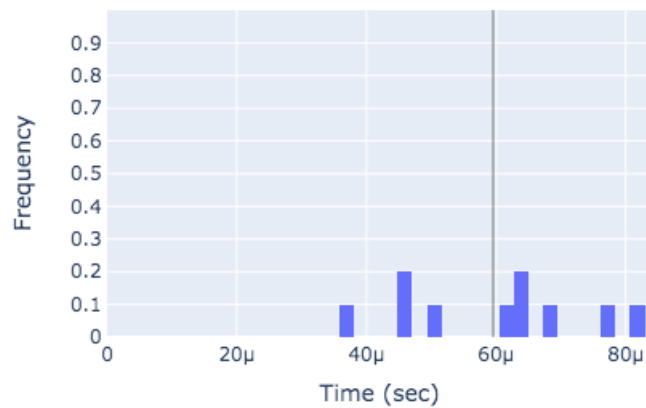
Min: 3.695487976074219e-05
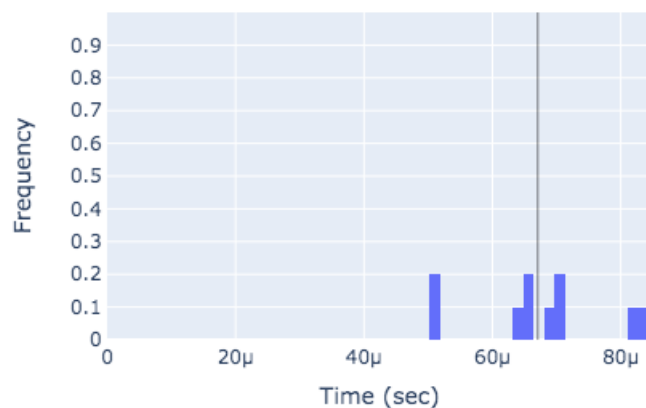Max: 8.177757263183594e-05
|Max-Min|: 4.482269287109375e-05

Case 1: Linear (False)

N=10, Nrep=10
Mean = 7.939338684082031e-05(sec)

Min: 6.4849853515625e-05
Max: 0.00011110305786132812
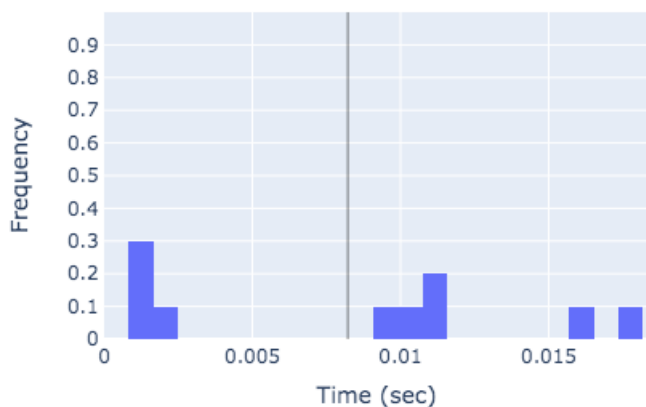|Max-Min|: 4.6253204345703125e-05

N=100, Nrep=10
Mean = 6.706714630126953e-05(sec)

Min: 5.078315734863281e-05
Max: 8.320808410644531e-05
|Max-Min|: 3.24249267578125e-05

Case 2: ProjectedLinear (True)

N=10, Nrep=10
Mean = 0.008220767974853516(sec)

Min: 0.0010318756103515625
Max: 0.01755213737487793
|Max-Min|: 0.016520261764526367

N=100, Nrep=10
Mean = 0.0029681921005249023(sec)
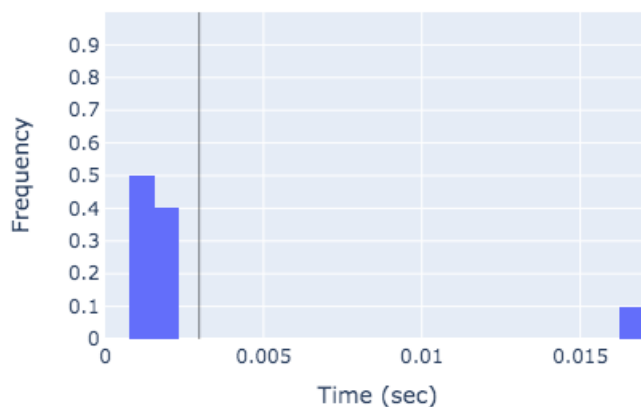
Min: 0.0009784698486328125
Max: 0.01642298698425293
|Max-Min|: 0.015444517135620117

Case 3: ProjectedLinear (False)

N=10, Nrep=10
Mean = 0.019703173637390138(sec)
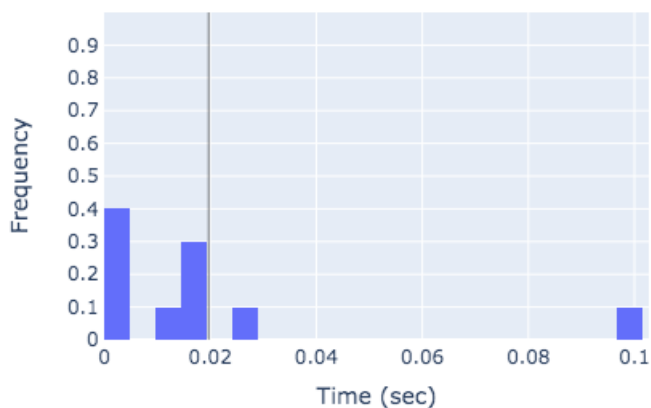
Min: 0.0011601448059082031
Max: 0.09787678718566895
|Max-Min|: 0.09671664237976074

N=100, Nrep=10
Mean = 0.003165459632873535(sec)
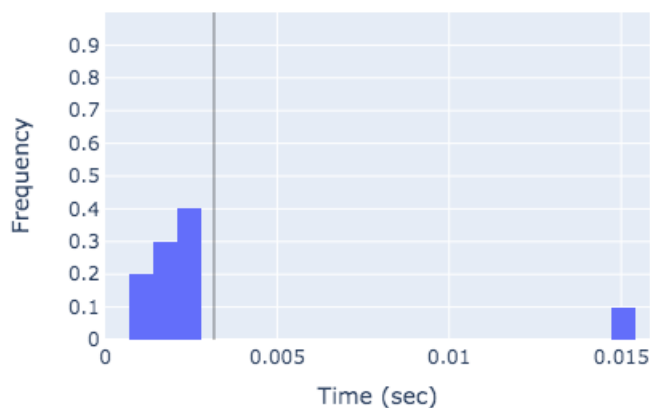
Min: 0.0011301040649414062
Max: 0.015132904052734375
|Max-Min|: 0.014002799987792969

Case 4: Maximum-Likelihood (True)

N=10, Nrep=10
Mean = 0.5591234922409057(sec)
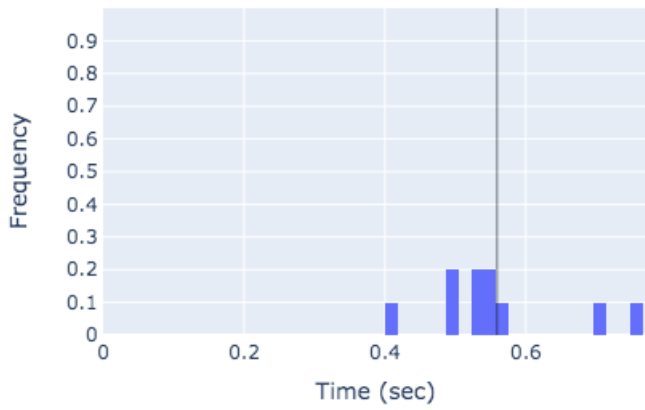
Min: 0.40752243995666504
Max: 0.7557954788208008
|Max-Min|: 0.34827303886413574

N=100, Nrep=10
Mean = 0.48286848068237304(sec)
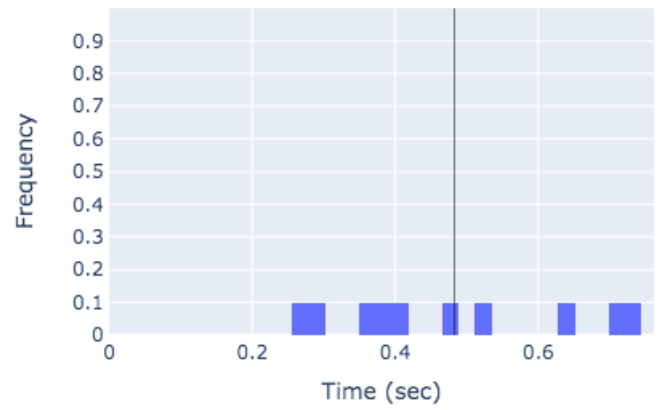
Min: 0.2731318473815918
Max: 0.7384762763977051
|Max-Min|: 0.4653444290161133

Case 5: Maximum-Likelihood (False)

N=10, Nrep=10
Mean = 1.9030502557754516(sec)
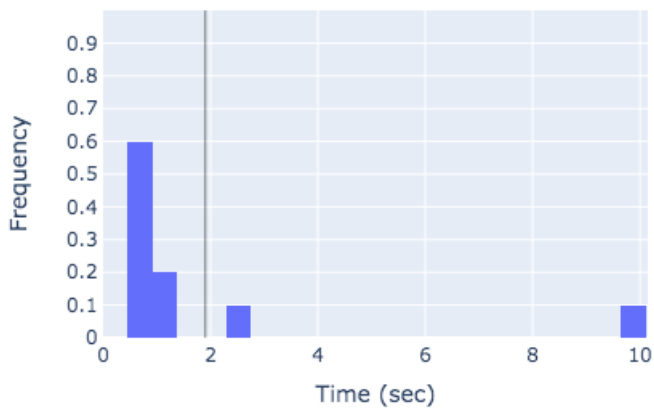
Min: 0.5006980895996094
Max: 9.686901807785034
|Max-Min|: 9.186203718185425

N=100, Nrep=10
Mean = 1.724810743331909(sec)
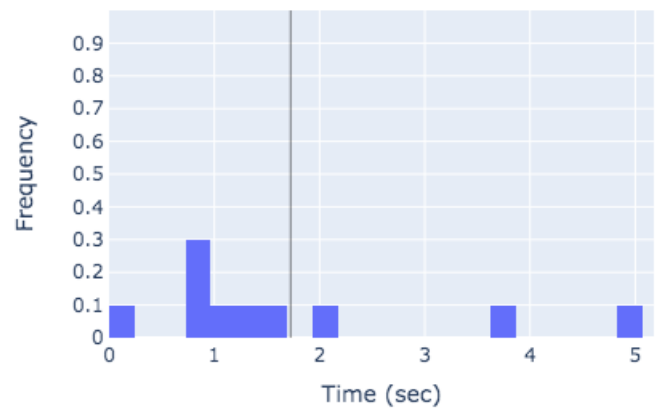
Min: 0.10749983787536621
Max: 4.93526029586792
|Max-Min|: 4.827760457992554

Case 6: Least Squares (True)

N=10, Nrep=10
Mean = 0.07867021560668945(sec)
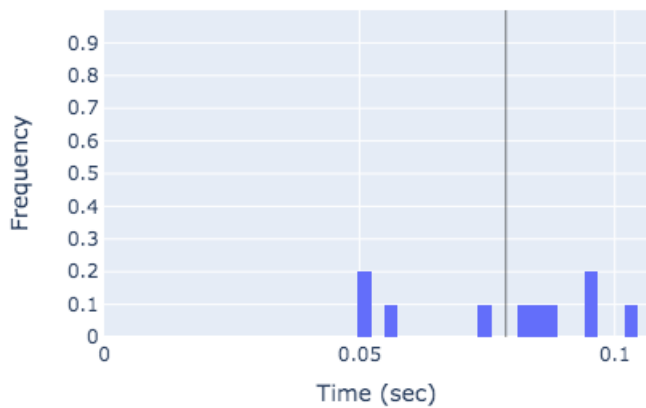
Min: 0.05174994468688965
Max: 0.1040487289428711
|Max-Min|: 0.052298784255981445

N=100, Nrep=10
Mean = 0.09750628471374512(sec)
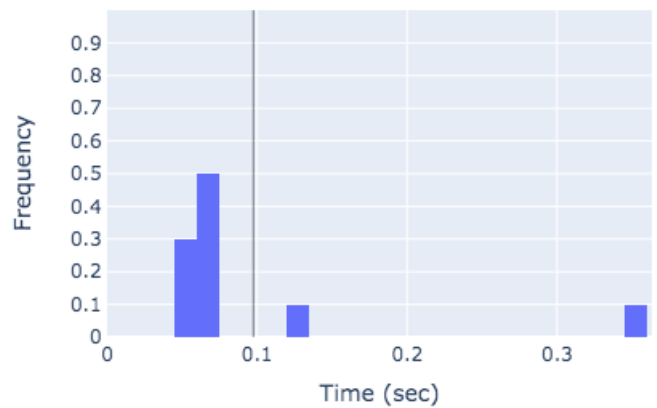
Min: 0.04793810844421387
Max: 0.3481769561767578
|Max-Min|: 0.30023884773254395

Case 7: Least Squares (False)

N=10, Nrep=10
Mean = 0.1281428098678589(sec)
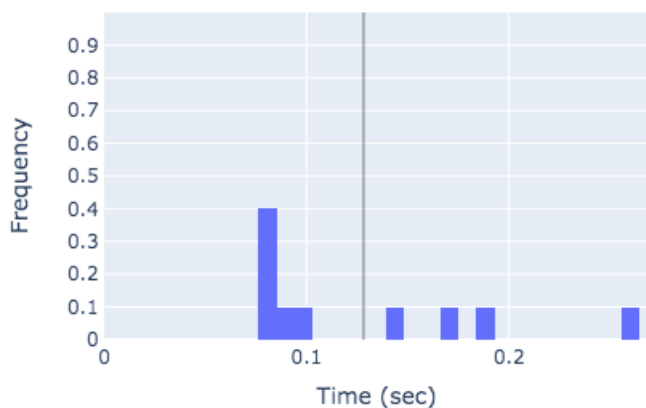
Min: 0.08069396018981934
Max: 0.26009202003479004
|Max-Min|: 0.1793980598449707

N=100, Nrep=10
Mean = 0.1287367343902588(sec)
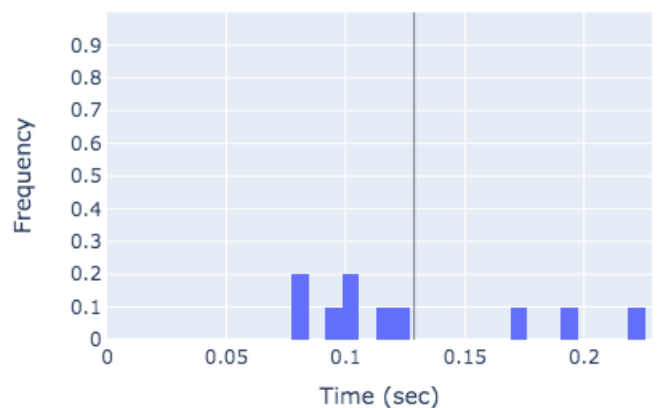
Min: 0.08029675483703613
Max: 0.22133827209472656
|Max-Min|: 0.14104151725769043

MSE of empirical distributions

Mean squared error
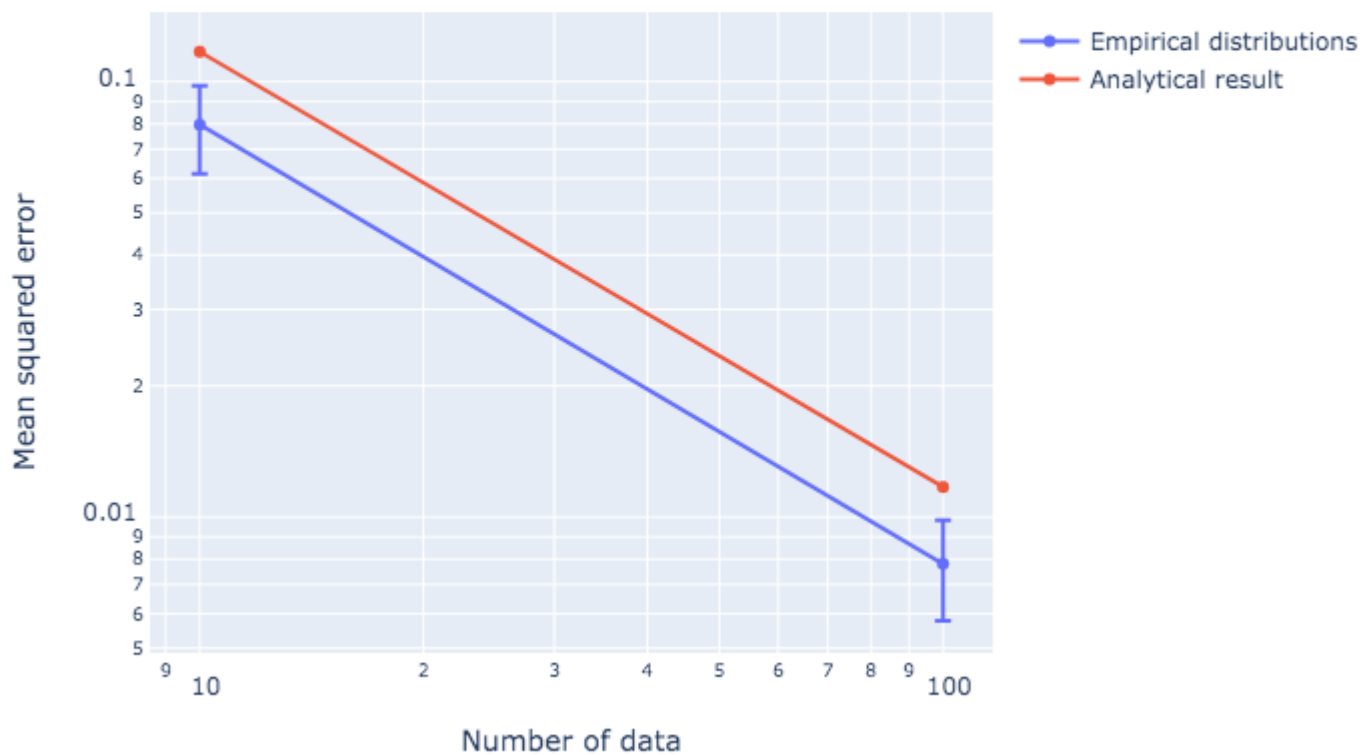
Consistency test

	Name	Type of tomography	Param	Estimator	Loss	Algo	Squared Error to True	Possibly OK	To be checked
0	Linear (True)	Standard Qst	True	Linear	None	None	0.00e+00	OK	not need debug
1	Linear (False)	Standard Qst	False	Linear	None	None	2.47e-32	OK	not need debug
2	ProjectedLinear (True)	Standard Qst	True	Projected Linear	None	None	1.23e-32	OK	not need debug
3	ProjectedLinear (False)	Standard Qst	False	Projected Linear	None	None	1.11e-31	OK	not need debug
4	Maximum-Likelihood (True)	Standard Qst	True	Loss Minimization	Weighted Relative Entropy	Projected Gradient Descent Backtracking	3.21e-18	OK	not need debug
5	Maximum-Likelihood (False)	Standard Qst	False	Loss Minimization	Weighted Relative Entropy	Projected Gradient Descent Backtracking	5.47e-09	NG	not need debug
6	Least Squares (True)	Standard Qst	True	Loss Minimization	Weighted Probability Based Squared Error	Projected Gradient Descent Backtracking	4.93e-32	OK	not need debug

7	Least Squares (False)	Standard Qst	False	Loss Minimization	Weighted Probability Based Squared Error	Projected Gradient Descent Backtracking	1.60e-31	OK	not need debug

MSE of estimators

Comparison of analytical results

Mean squared error
Analytical result
estimator=Linear
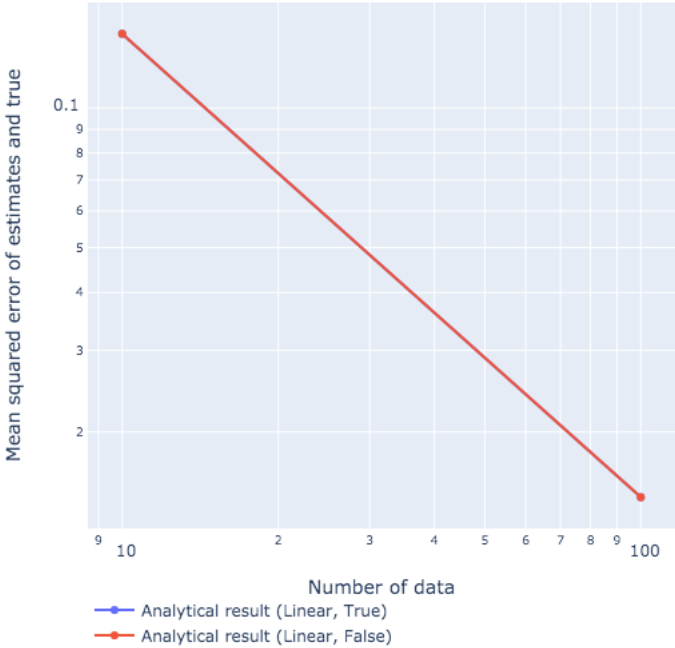
Mean squared error
Analytical result
Cramer-Rao bound
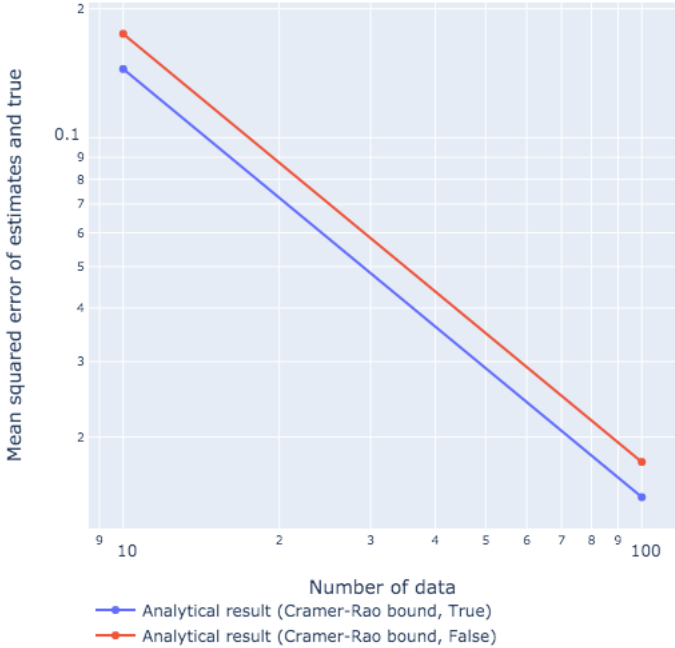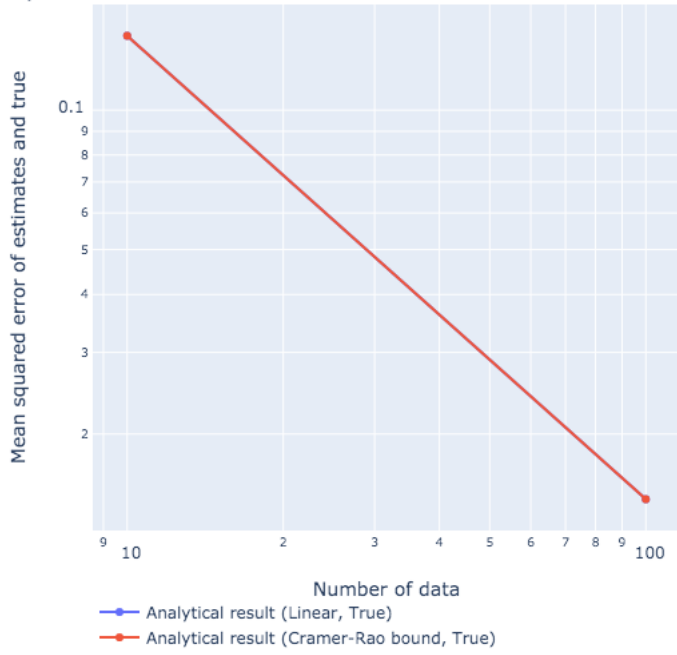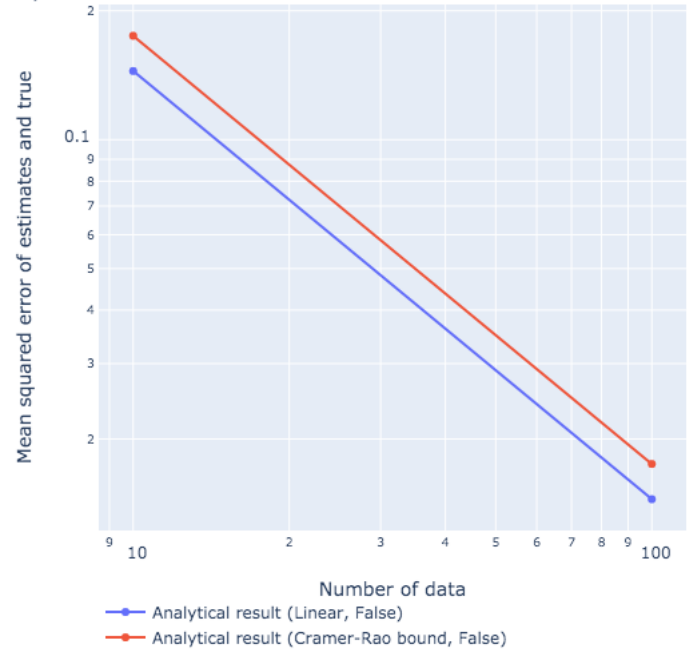
Comparison of parametrization
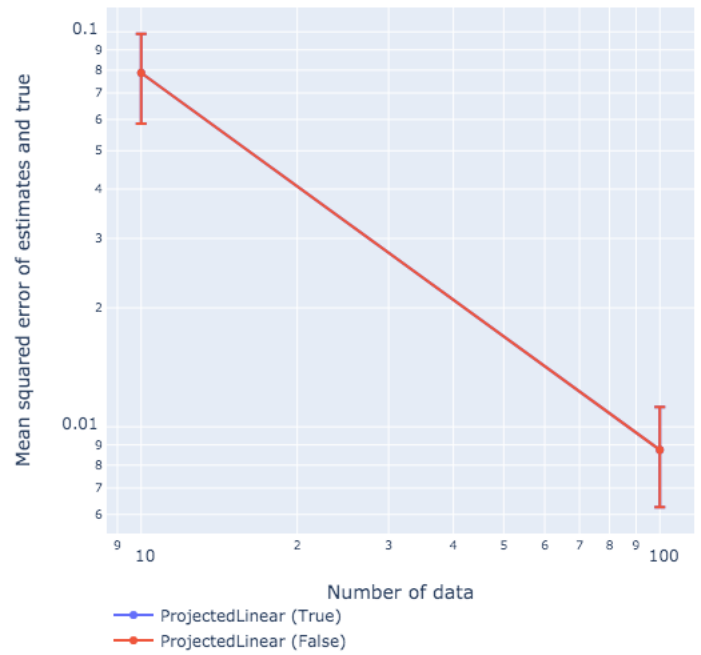
Mean squared error
Estimator=LossMinimization
Loss=WeightedRelativeEntropy
Algo=ProjectedGradientDescentBacktracking

Maximum-Likelihood (True)
Maximum-Likelihood (False)
Analytical result (Cramer-Rao bound, True)
Analytical result (Cramer-Rao bound, False)
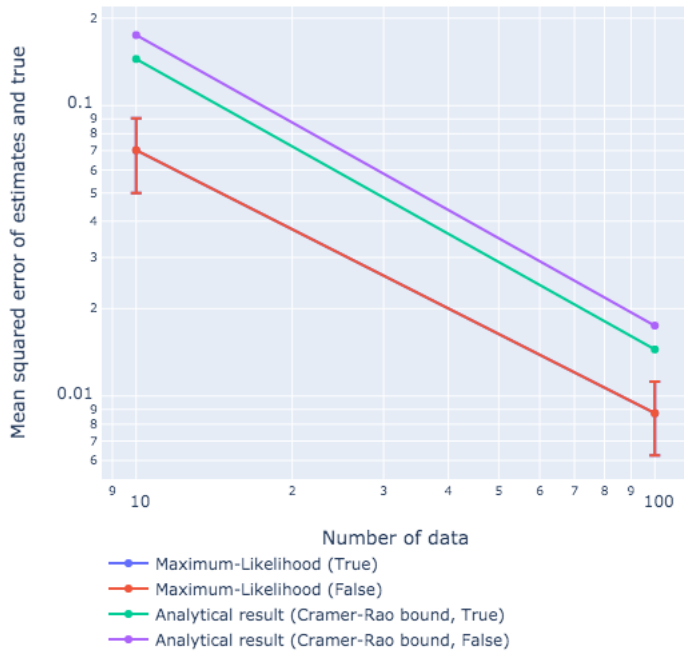
Mean squared error
Estimator=LossMinimization
Loss=WeightedProbabilityBasedSquaredError
Algo=ProjectedGradientDescentBacktracking

Least Squares (True)
Least Squares (False)
Analytical result (Cramer-Rao bound, True)
Analytical result (Cramer-Rao bound, False)
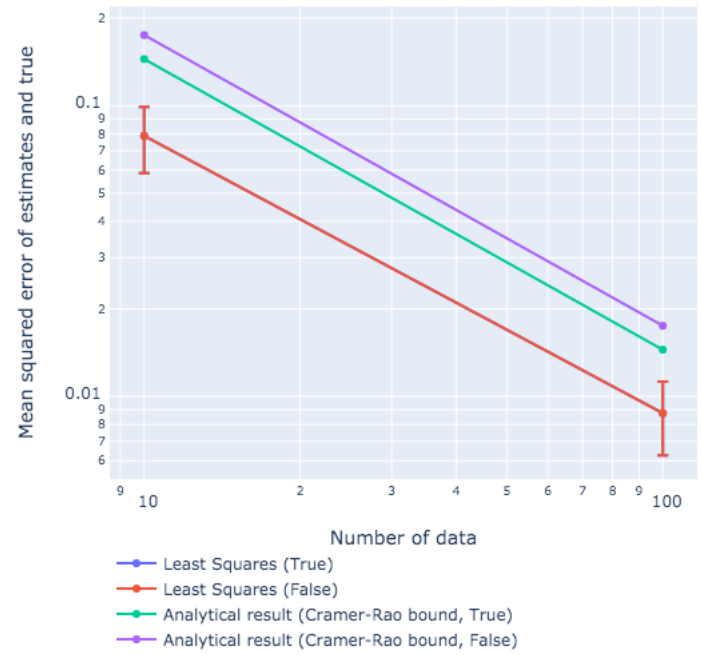
Comparison of estimators

Mean squared error
parametrization=True

Linear (True)
ProjectedLinear (True)
Maximum-Likelihood (True)
Least Squares (True)

Mean squared error
parametrization=False
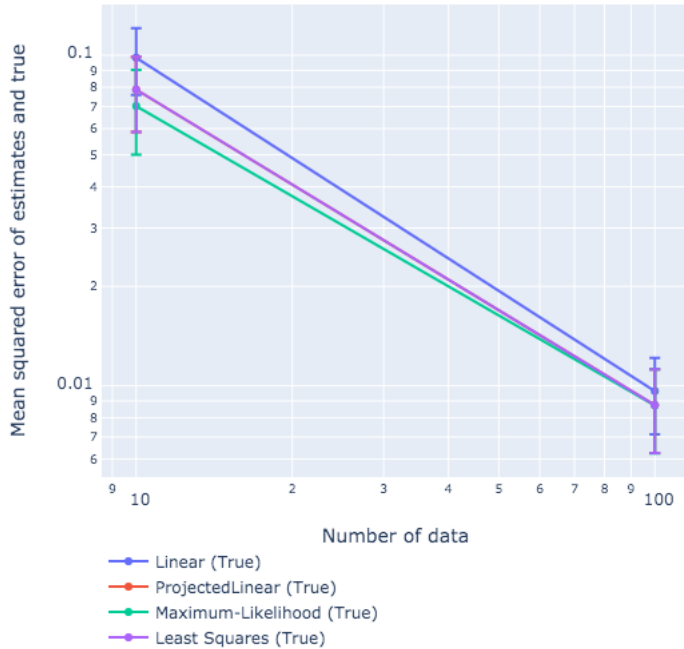
Linear (False)
ProjectedLinear (False)
Maximum-Likelihood (False)
Least Squares (False)
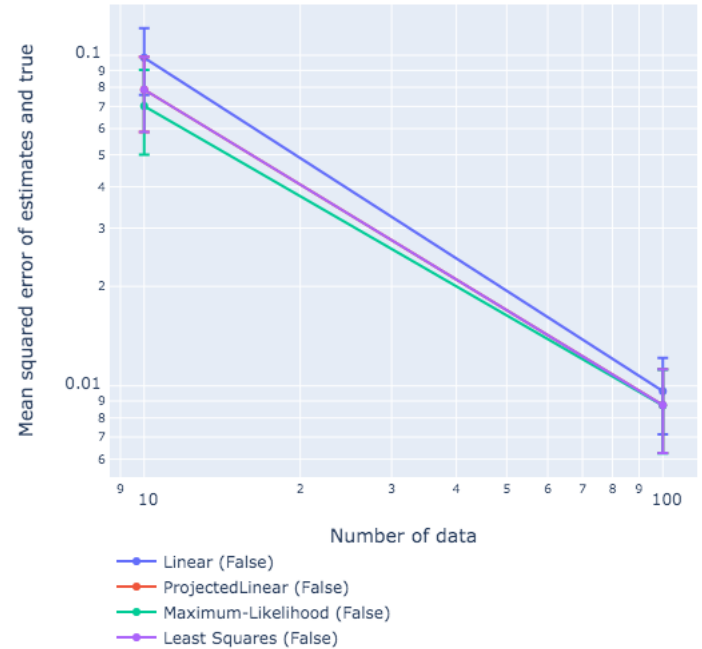
Physicality violation test

Test of equality constraint violation

Case 0: Linear (True)
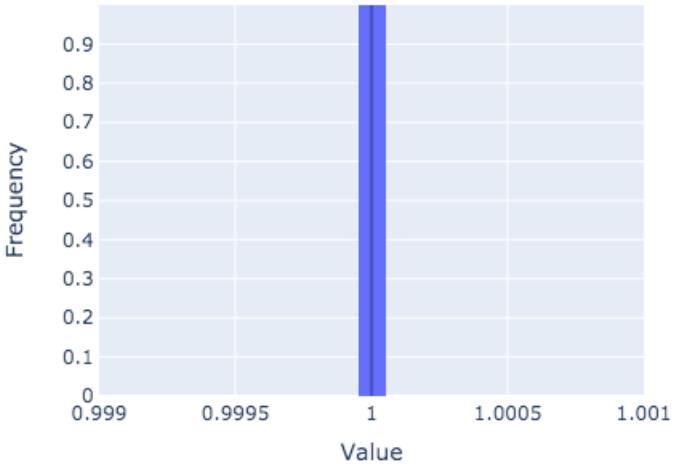
N=10, Nrep=10

Min: 0.9999999999999997
Max: 0.9999999999999998
|Max-Min|: 1.1102230246251565e-16

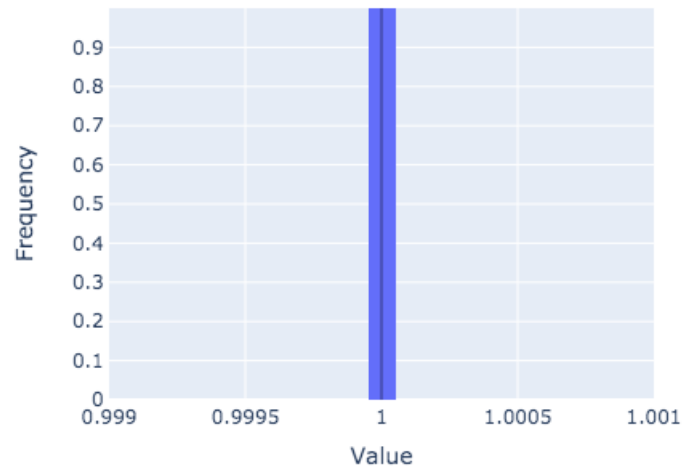N=100, Nrep=10
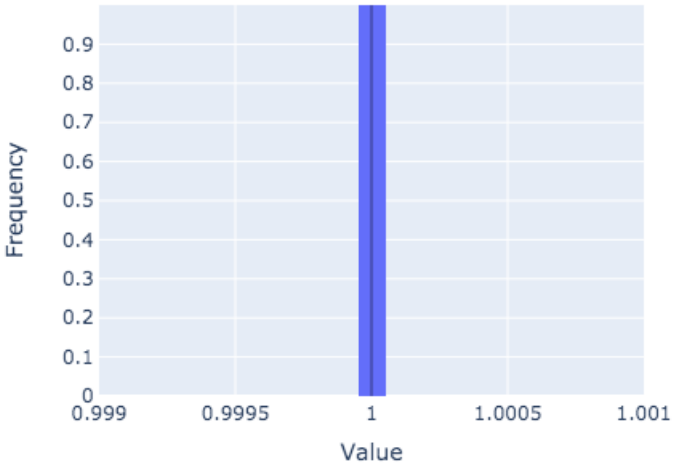
Min: 0.9999999999999998
Max: 0.9999999999999998
|Max-Min|: 0.0

Case 1: Linear (False)
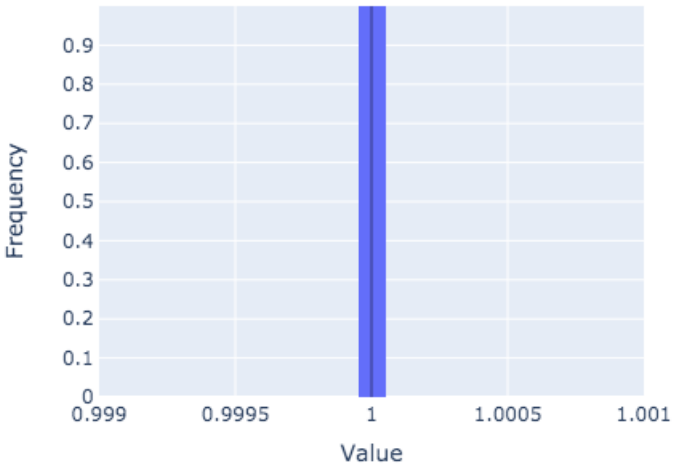
N=10, Nrep=10

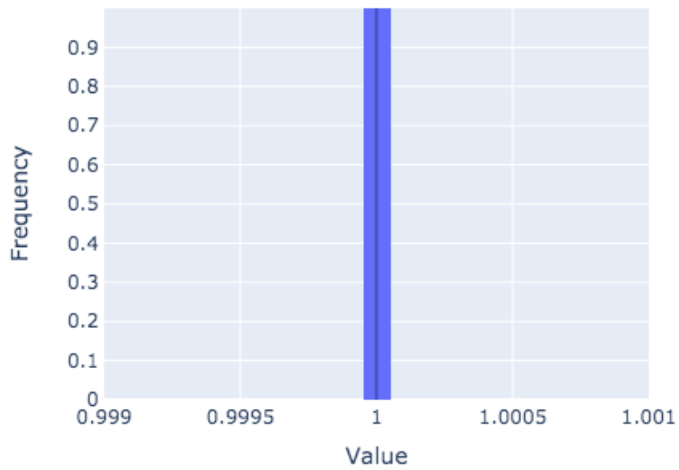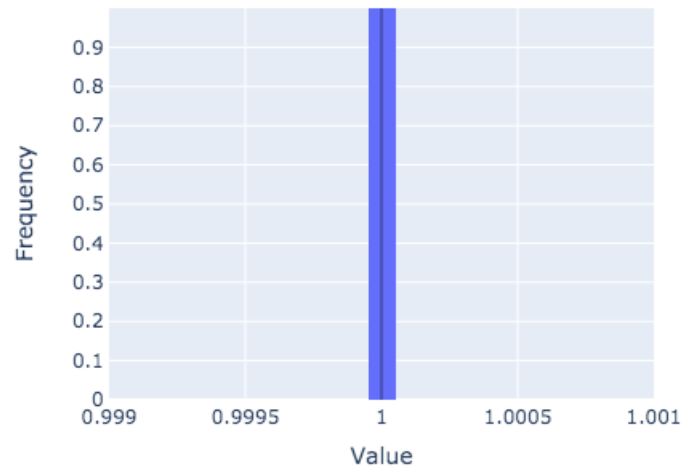Min: 0.9999999999999998
Max: 1.0000000000000002
|Max-Min|: 4.440892098500626e-16

N=100, Nrep=10
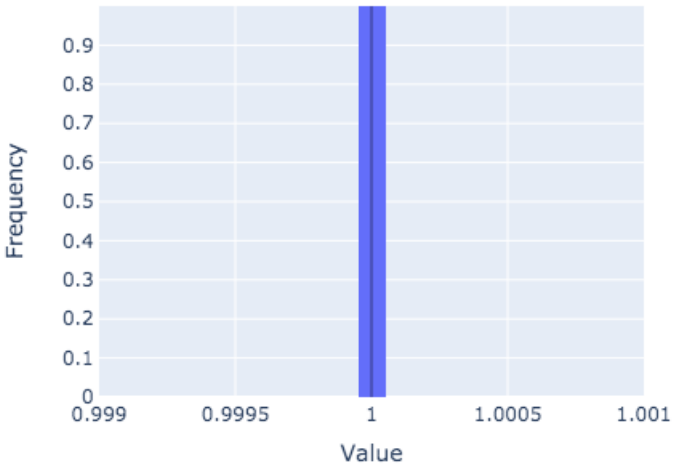
Min: 0.9999999999999999
Max: 1.0000000000000002
|Max-Min|: 3.3306690738754696e-16

Case 2: ProjectedLinear (True)
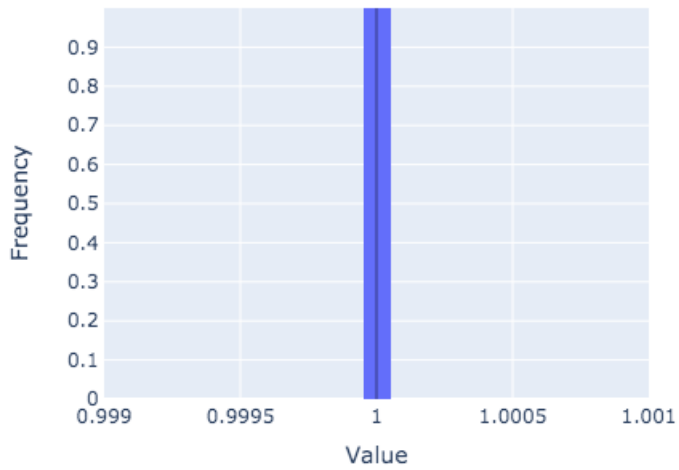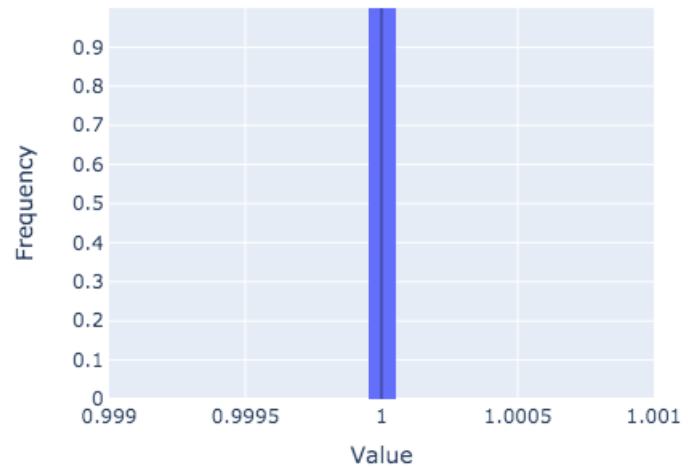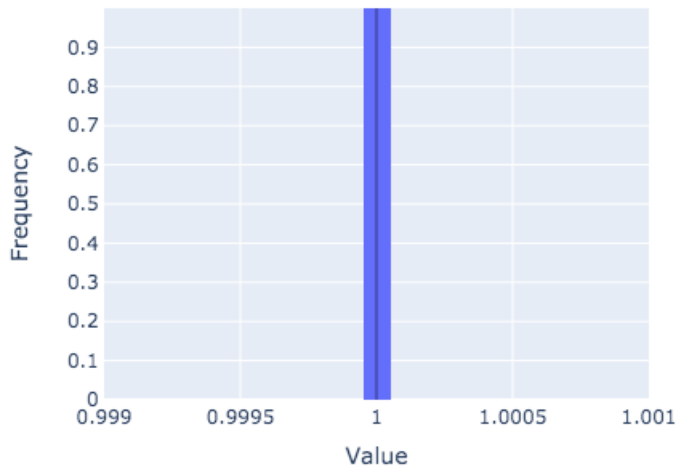
N=10, Nrep=10

Min: 0.9999999999999998
Max: 0.9999999999999998
|Max-Min|: 0.0
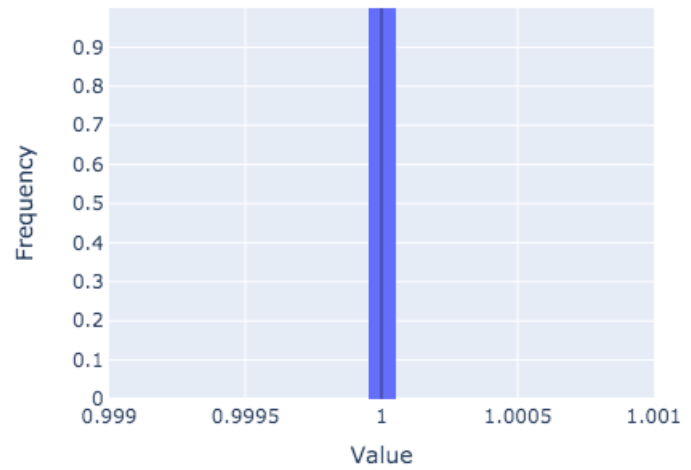
N=100, Nrep=10

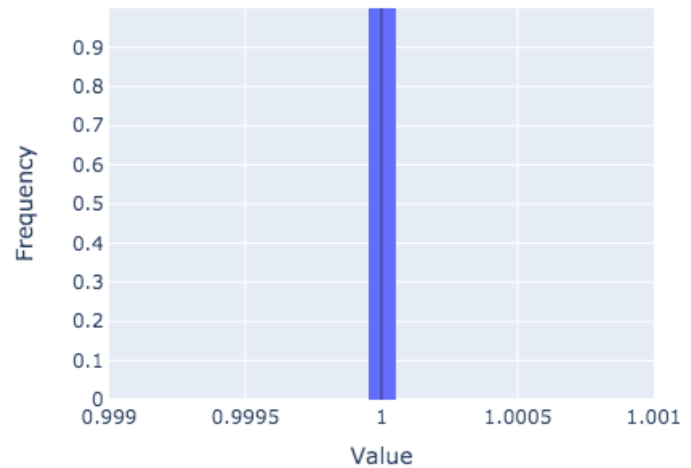Min: 0.9999999999999998
Max: 0.9999999999999998
|Max-Min|: 0.0

Case 3: ProjectedLinear (False)
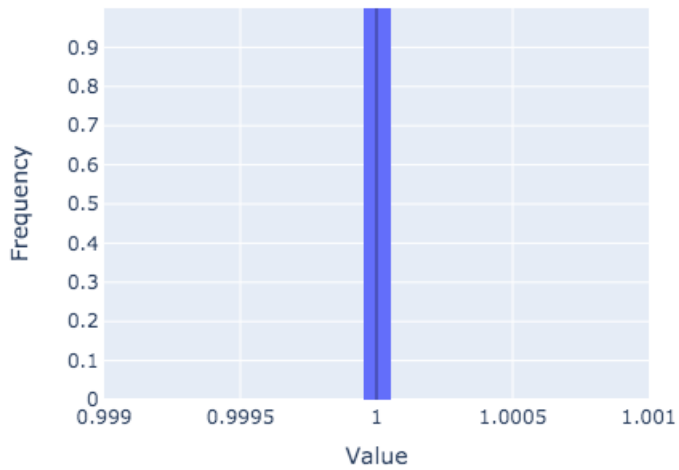
N=10, Nrep=10

Min: 0.9999999999999996
Max: 1.00000014753393
|Max-Min|: 1.4753393040578544e-07

N=100, Nrep=10

Min: 0.9999999999999997
Max: 1.0000001280394155
|Max-Min|: 1.280394158476028e-07

Case 4: Maximum-Likelihood (True)
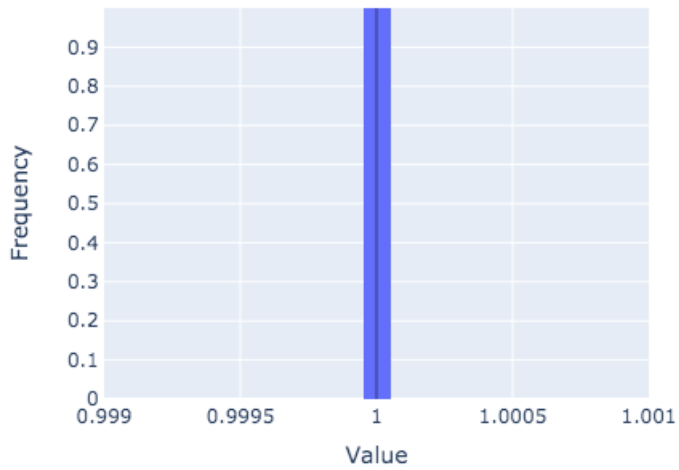
N=10, Nrep=10

Min: 0.9999999999999998
Max: 0.9999999999999998
|Max-Min|: 0.0
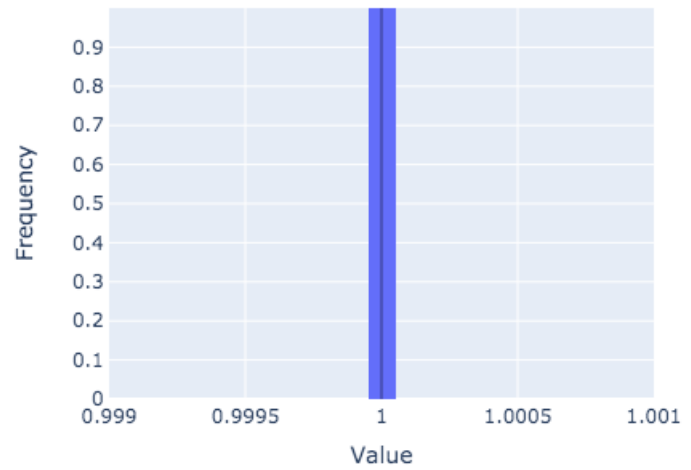
N=100, Nrep=10

Min: 0.9999999999999998
Max: 0.9999999999999998
|Max-Min|: 0.0

Case 5: Maximum-Likelihood (False)

N=10, Nrep=10

Min: 1.000000006567707
Max: 1.0000001622723005
|Max-Min|: 1.5570459344083076e-07

N=100, Nrep=10

Min: 1.0000000121352572
Max: 1.00000001810607821
|Max-Min|: 1.689255249015531e-07

Case 6: Least Squares (True)
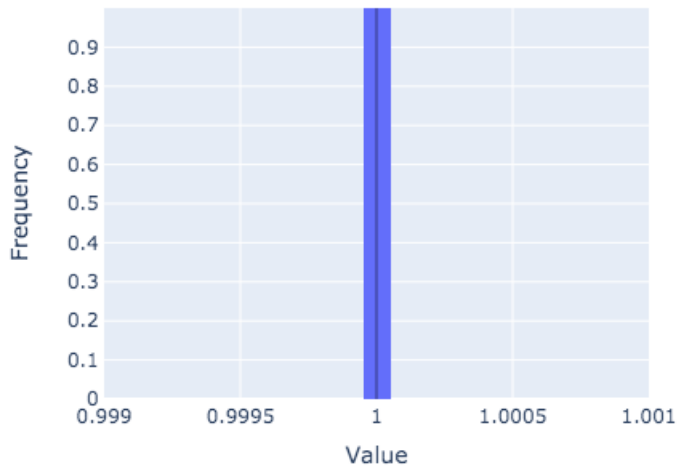
N=10, Nrep=10

Min: 0.9999999999999998
Max: 0.9999999999999998
|Max-Min|: 0.0

N=100, Nrep=10

Min: 0.9999999999999998
Max: 0.9999999999999998
|Max-Min|: 0.0

Case 7: Least Squares (False)

N=10, Nrep=10

Min: 0.9999999999999998
Max: 1.0000001781169368
|Max-Min|: 1.7811693697744602e-07

N=100, Nrep=10

Min: 0.9999999999999998
Max: 1.0000001555680968
|Max-Min|: 1.5556809707106822e-07

Test of inequality constraint violation

Eigenvalue
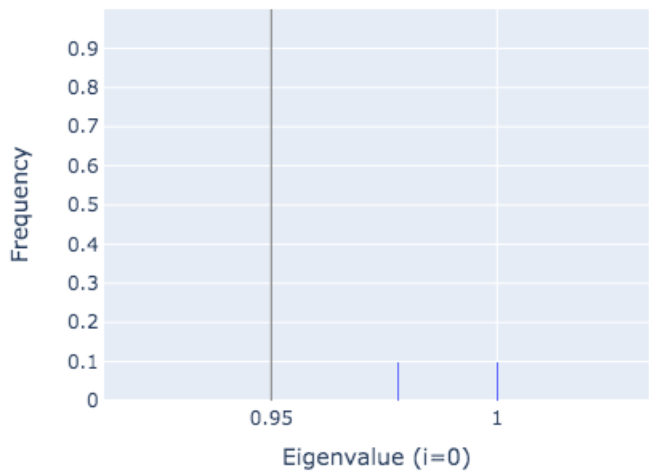
Case 0: Linear (True)
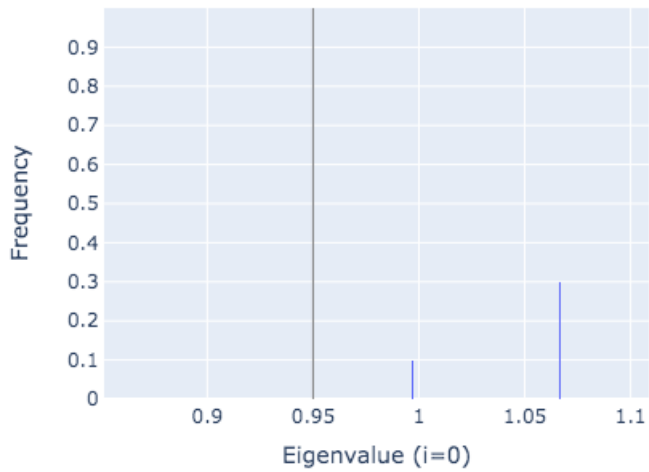
N=10, Nrep=10

Min: 0.8513641844631532
Max: 1.1085806194501846
|Max-Min|: 0.2572164349870314
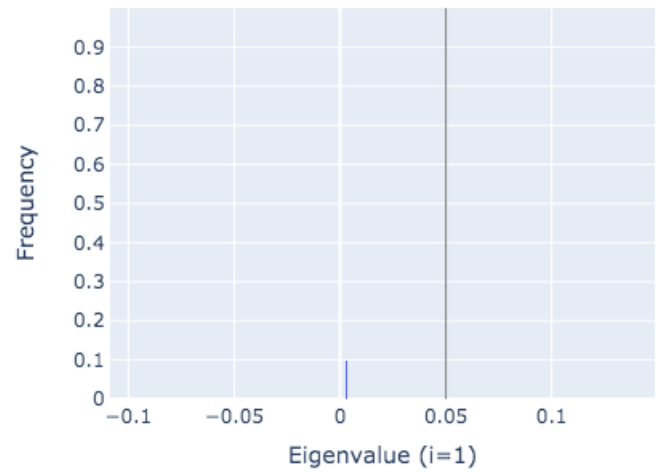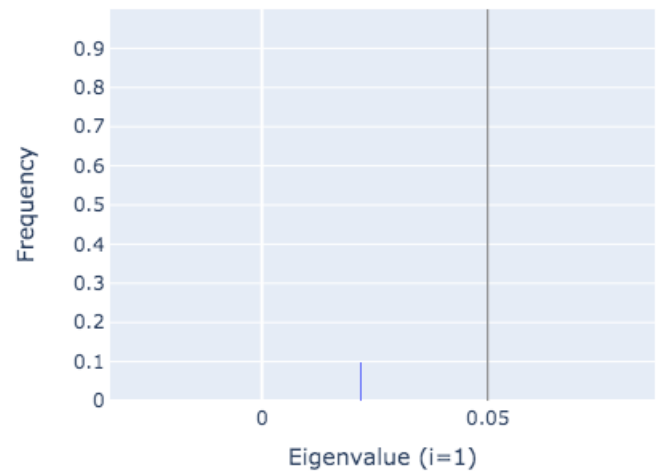
N=10, Nrep=10

Min: -0.10858061945018471
Max: 0.14863581553684663
|Max-Min|: 0.25721643498703134
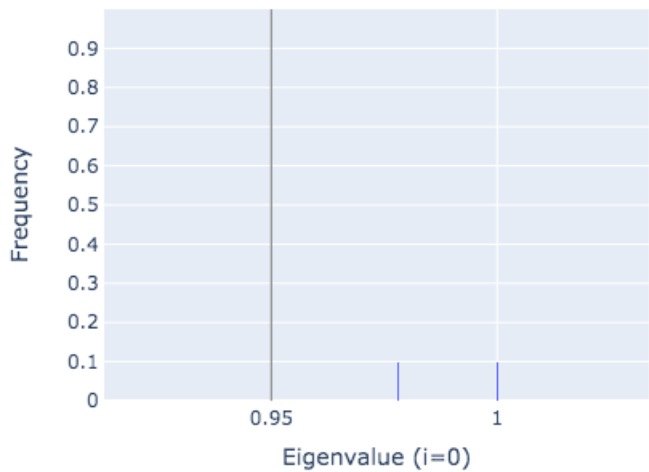
N=100, Nrep=10

Min: 0.9130584510098023
Max: 1.0335647646019095
|Max-Min|: 0.12050631359210717

N=100, Nrep=10
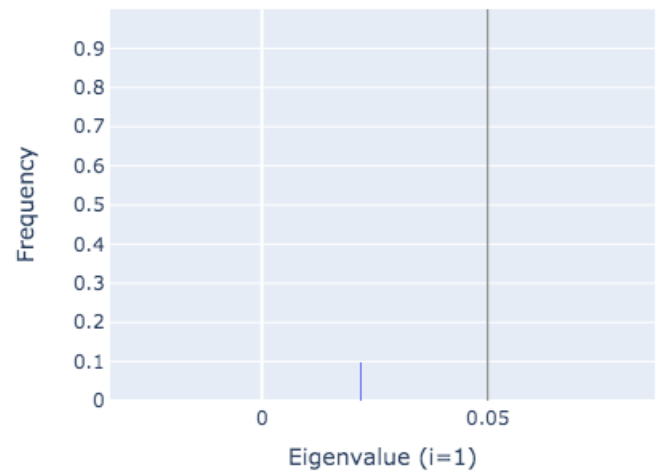
Min: -0.03356476460190986
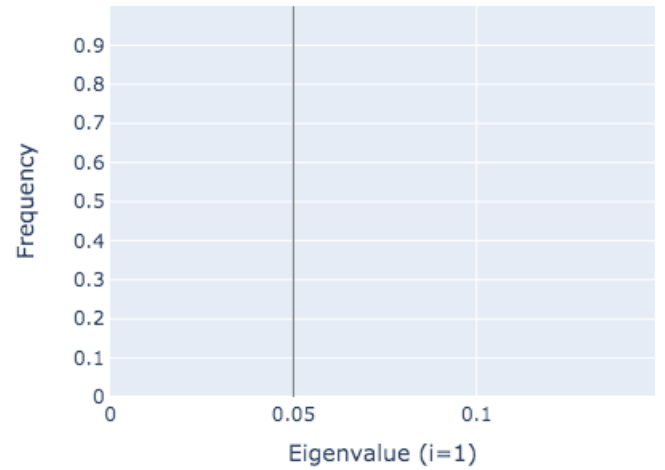Max: 0.0869415489901976
|Max-Min|: 0.12050631359210745

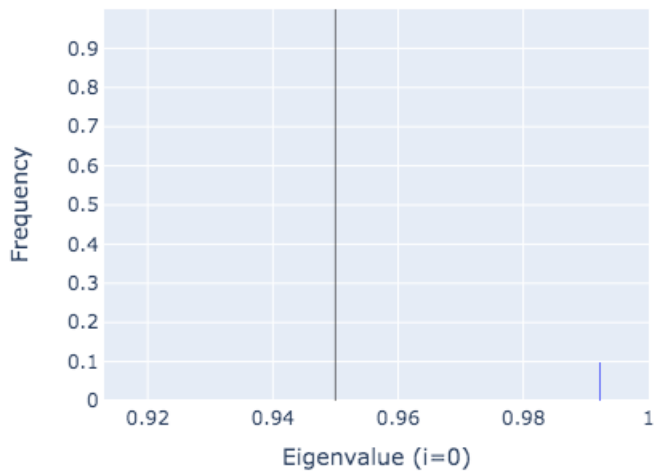**Case 1: Linear (False)**

N=10, Nrep=10

Eigenvalue (i=0)

Min: 0.8513641844631532
Max: 1.1085806194501846
|Max-Min|: 0.2572164349870314

N=10, Nrep=10

Eigenvalue (i=1)
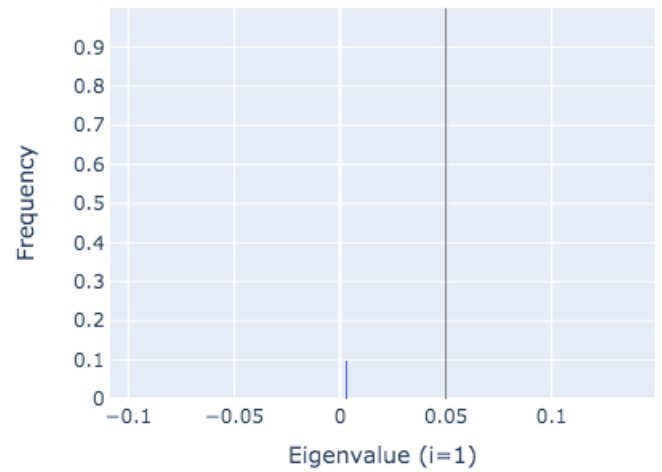
Min: -0.10858061945018459
Max: 0.14863581553684663
|Max-Min|: 0.25721643498703123

N=100, Nrep=10

Eigenvalue (i=0)

Min: 0.9130584510098023
Max: 1.0335647646019095
|Max-Min|: 0.12050631359210717

N=100, Nrep=10

Eigenvalue (i=1)

Min: -0.03356476460190975
Max: 0.08694154899019772
|Max-Min|: 0.12050631359210748

Case 2: ProjectedLinear (True)
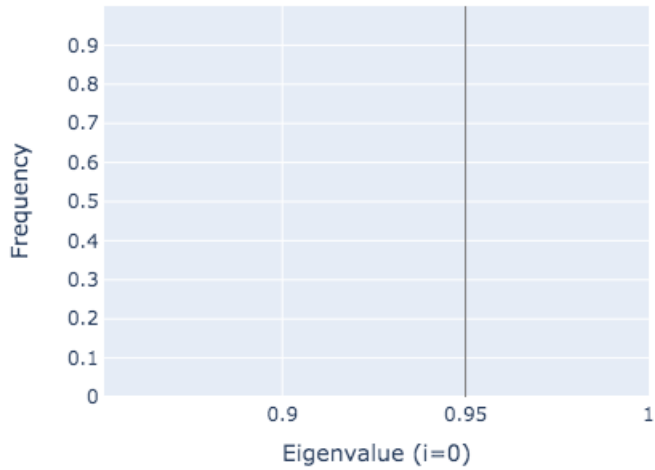
N=10, Nrep=10

Min: 0.8513641844631531
Max: 1.0000000737669648
|Max-Min|: 0.14863588930381166

N=10, Nrep=10

Min: -7.376696509533986e-08
Max: 0.14863581553684668
|Max-Min|: 0.14863588930381177

N=100, Nrep=10

Min: 0.9130584510098023
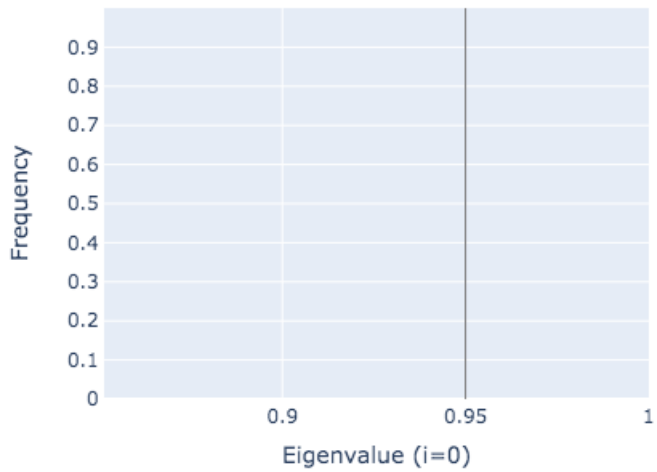Max: 1.0000000640197078
|Max-Min|: 0.08694161300990544

N=100, Nrep=10
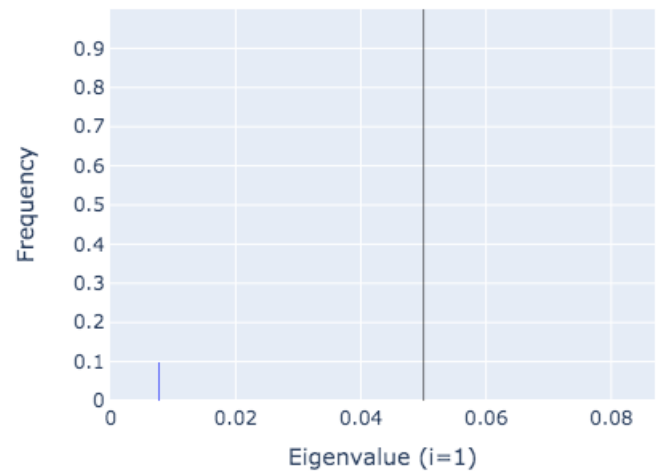
Min: -6.401970792556323e-08
Max: 0.0869415489901976
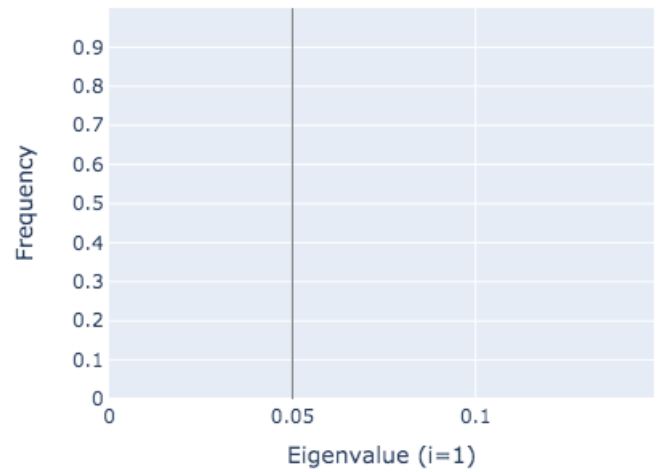|Max-Min|: 0.08694161300990552
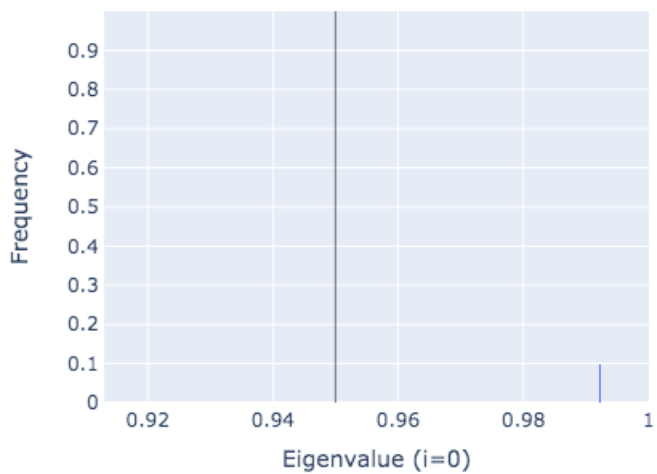
Case 3: ProjectedLinear (False)

N=10, Nrep=10

Min: 0.8513641844631532
Max: 1.00000014753393
|Max-Min|: 0.14863596307077676

N=10, Nrep=10

Min: -2.7755575615628914e-17
Max: 0.14863581553684696
|Max-Min|: 0.148635815536847

N=100, Nrep=10

Min: 0.9130584510098023
Max: 1.0000001280394155
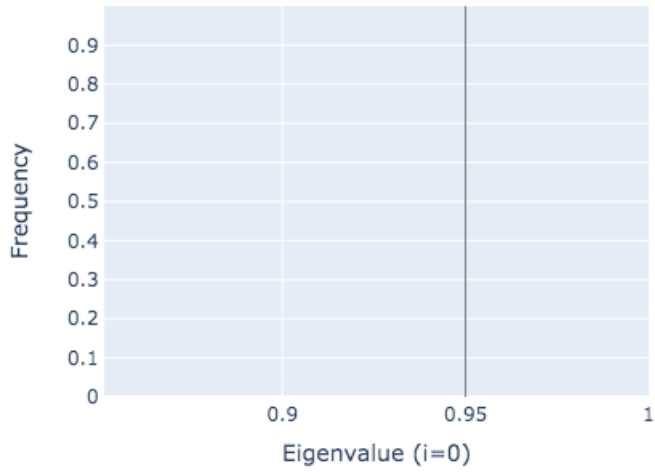|Max-Min|: 0.0869416770296132

N=100, Nrep=10
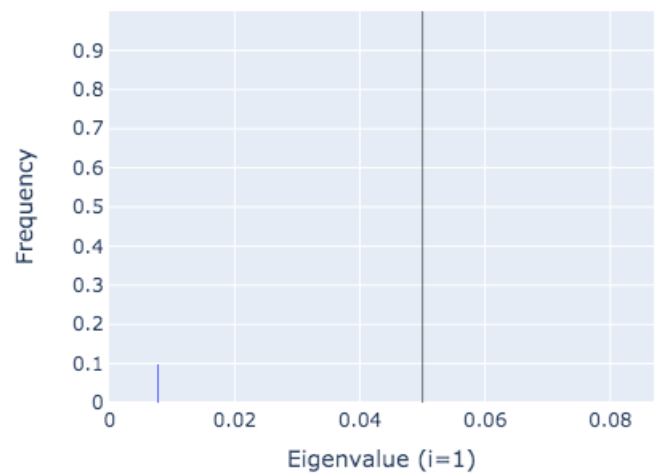
Min: -5.730008481585891e-17
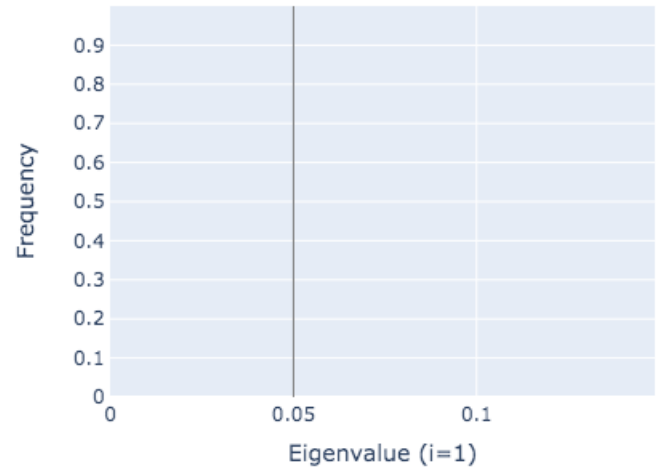Max: 0.0869415489901976
|Max-Min|: 0.08694154899019765
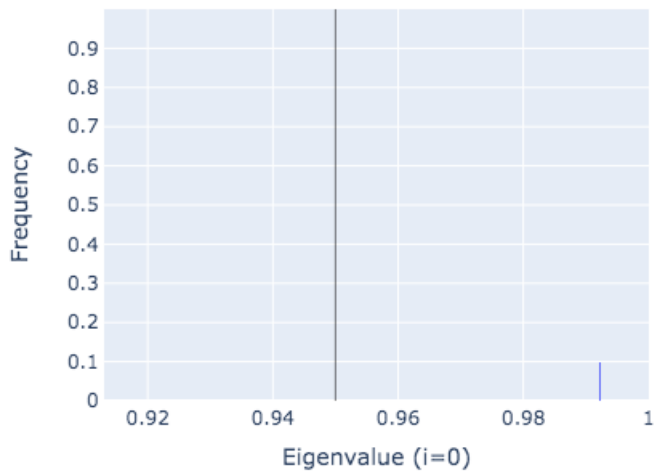
Case 4: Maximum-Likelihood (True)

N=10, Nrep=10

Eigenvalue (i=0)

Min: 0.8513641812250503
Max: 1.0000000702659644
|Max-Min|: 0.14863588904091407
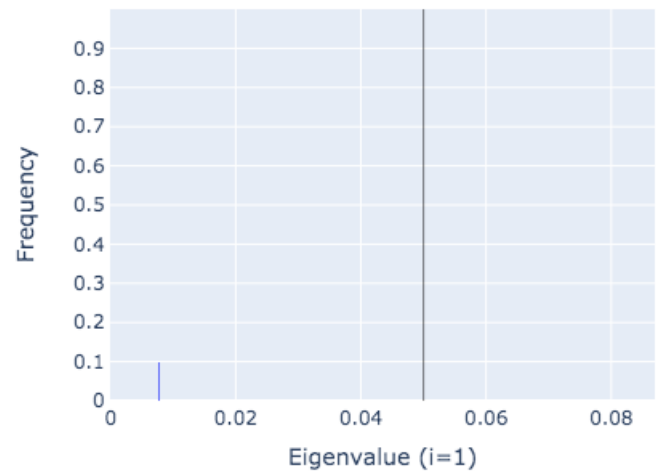
N=10, Nrep=10

Eigenvalue (i=1)

Min: -7.026596454684508e-08
Max: 0.14863581877494952
|Max-Min|: 0.14863588904091407

N=100, Nrep=10

Eigenvalue (i=0)

Min: 0.9130584509383943
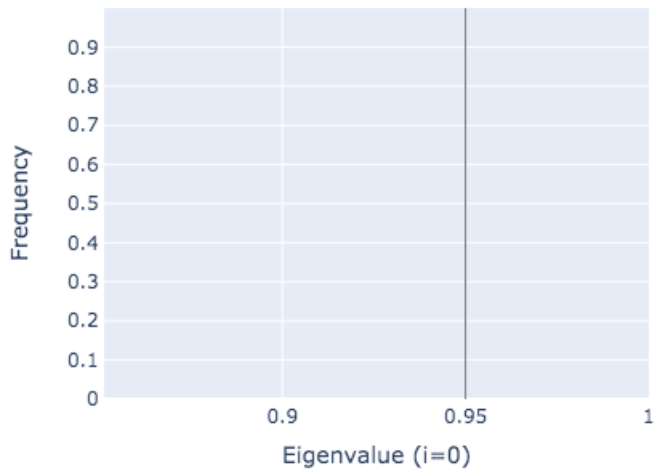Max: 1.000000078401553
|Max-Min|: 0.0869416274631587

N=100, Nrep=10

Eigenvalue (i=1)

Min: -7.840155328241432e-08
Max: 0.08694154906160545
|Max-Min|: 0.08694162746315873
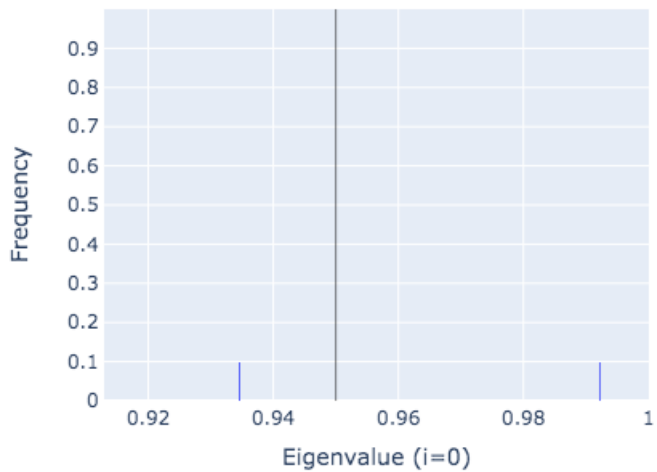
Case 5: Maximum-Likelihood (False)

N=10, Nrep=10

Eigenvalue (i=0)

Min: 0.851427240055816
Max: 1.000000162272261
|Max-Min|: 0.1485729222164449
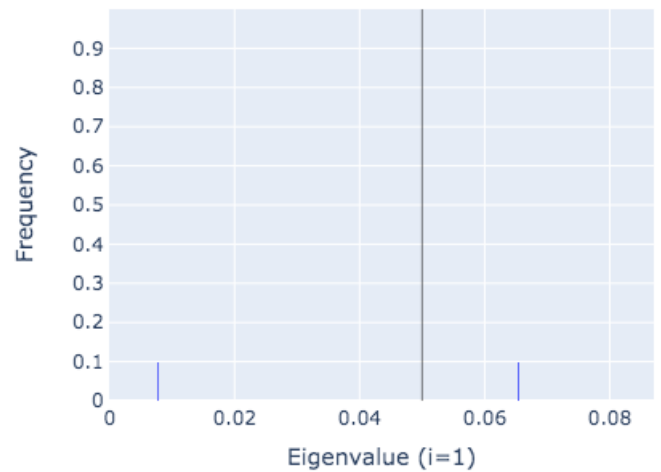
N=10, Nrep=10

Eigenvalue (i=1)
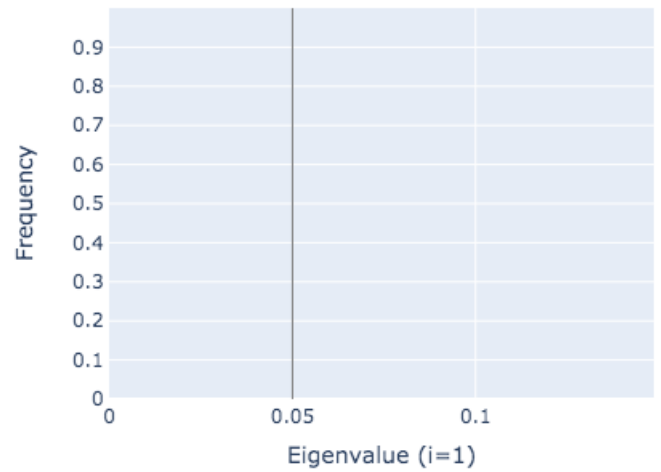
Min: -1.3877787807814457e-17
Max: 0.14857279250213073
|Max-Min|: 0.14857279250213073

N=100, Nrep=10

Eigenvalue (i=0)

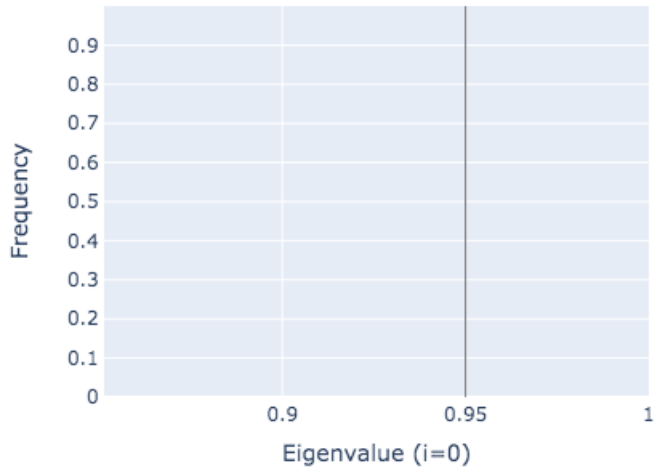Min: 0.9130380513737355
Max: 1.0000001810607806
|Max-Min|: 0.08696212968704509

N=100, Nrep=10

Eigenvalue (i=1)

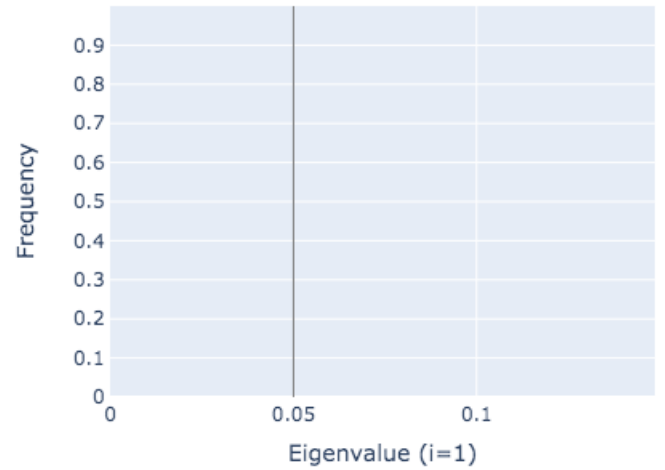Min: -1.0061396160665481e-16
Max: 0.08696196076152173
|Max-Min|: 0.08696196076152182
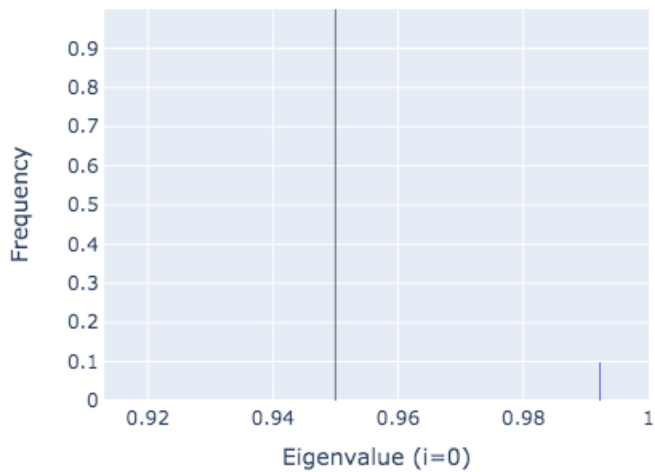
Case 6: Least Squares (True)

N=10, Nrep=10

Eigenvalue (i=0)

Min: 0.851364184463153
Max: 1.0000000761055894
|Max-Min|: 0.1486358916424364
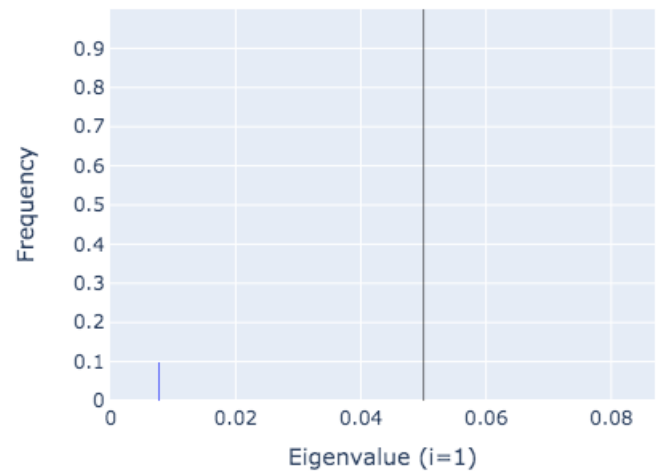
N=10, Nrep=10

Eigenvalue (i=1)

Min: -7.610558962201086e-08
Max: 0.14863581553684674
|Max-Min|: 0.14863589164243635

N=100, Nrep=10

Eigenvalue (i=0)

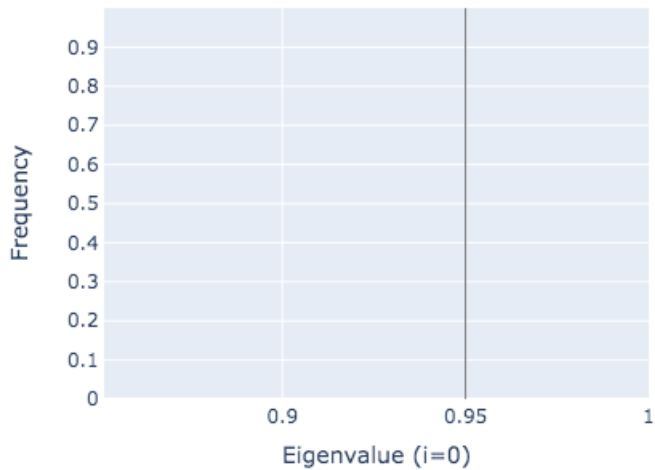Min: 0.9130584510098028
Max: 1.0000000598780594
|Max-Min|: 0.08694160886825664

N=100, Nrep=10

Eigenvalue (i=1)

Min: -5.987805948691623e-08
Max: 0.0869415489901971
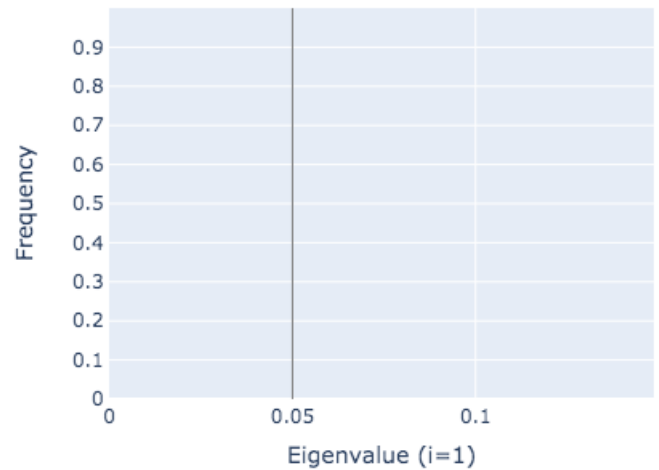|Max-Min|: 0.08694160886825658

Case 7: Least Squares (False)
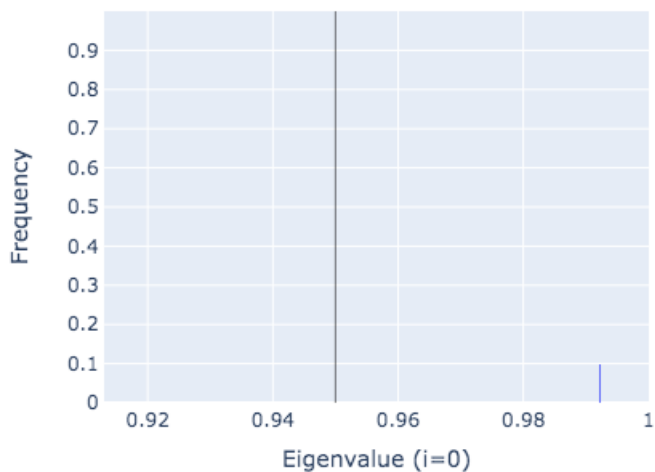
N=10, Nrep=10

Min: 0.8513641844631532
Max: 1.0000001781169368
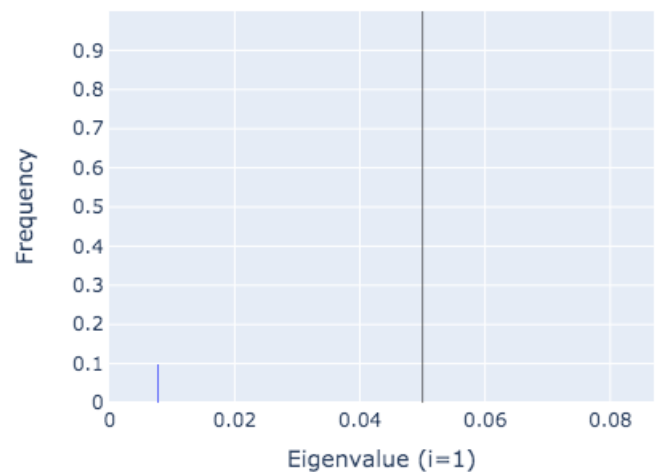|Max-Min|: 0.14863599365378355

N=10, Nrep=10

Min: -7.632783294297951e-17
Max: 0.14863581553684663
|Max-Min|: 0.1486358155368467

N=100, Nrep=10

Min: 0.9130584510098023
Max: 1.0000001555680968
|Max-Min|: 0.08694170455829453

N=100, Nrep=10

Min: 3.06287113727155e-18
Max: 0.08694154899019754
|Max-Min|: 0.08694154899019754

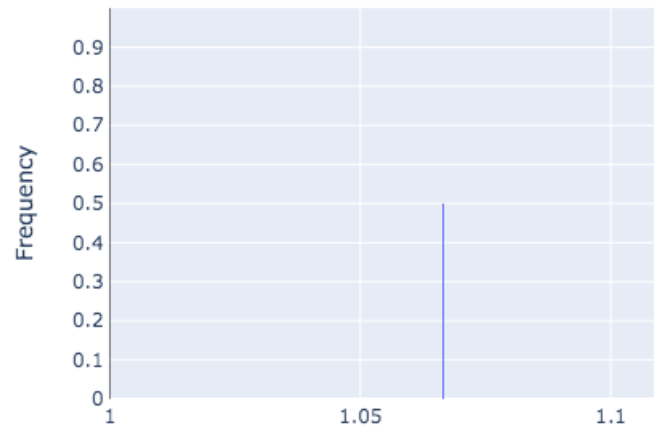## Sum of unphysical eigenvalues

Case 0: Linear (True)

N=10
Number of unphysical estimates=6

N=10, Nrep=10

Sum of negative eigenvalues (<0)

Min: -0.10858061945018471
Max: -0.009175077217315732
|Max-Min|: 0.09940554223286899

N=10, Nrep=10

Sum of non-negative eigenvalues (>1)
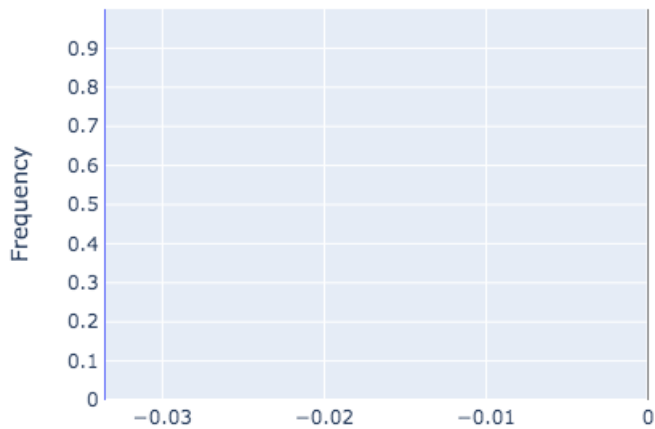
Min: 1.0091750772173156
Max: 1.1085806194501846
|Max-Min|: 0.09940554223286902

N=100
Number of unphysical estimates=1

N=100, Nrep=10

Sum of negative eigenvalues (<0)

Min: -0.03356476460190986
Max: -0.03356476460190986
|Max-Min|: 0.0

N=100, Nrep=10

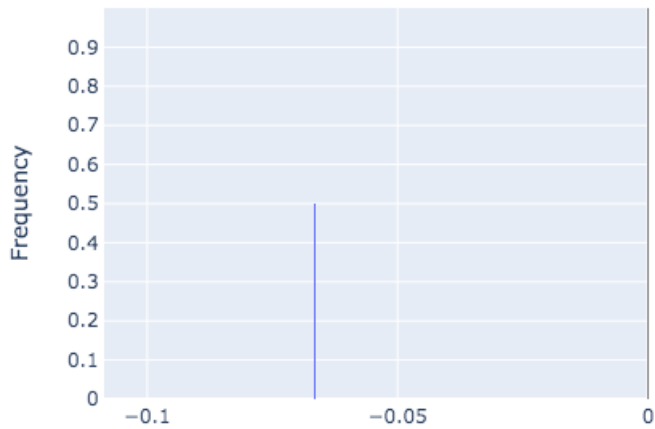Sum of non-negative eigenvalues (>1)
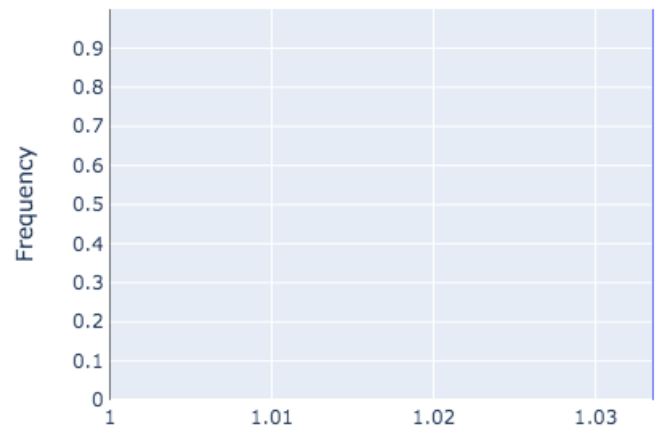
Min: 1.0335647646019095
Max: 1.0335647646019095
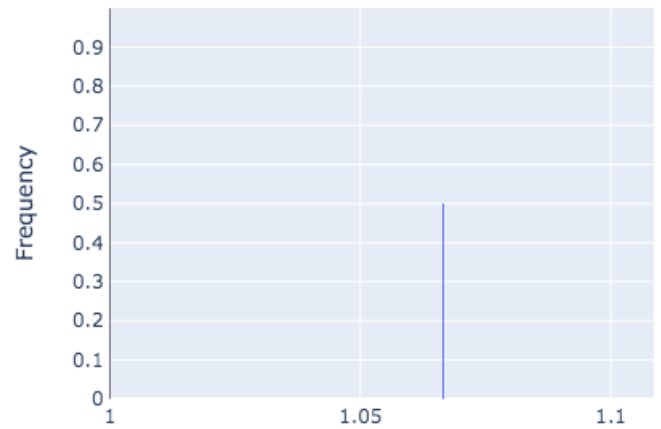|Max-Min|: 0.0

Case 1: Linear (False)

N=10
Number of unphysical estimates=6

N=10, Nrep=10

Min: -0.10858061945018459
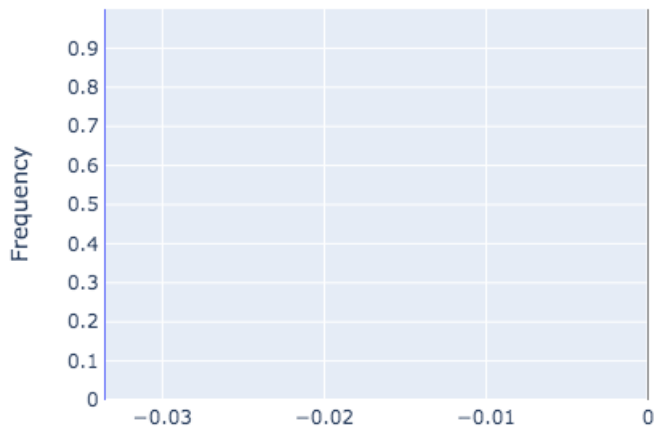Max: -0.009175077217315586
|Max-Min|: 0.099405542232869

N=10, Nrep=10

Min: 1.0091750772173156
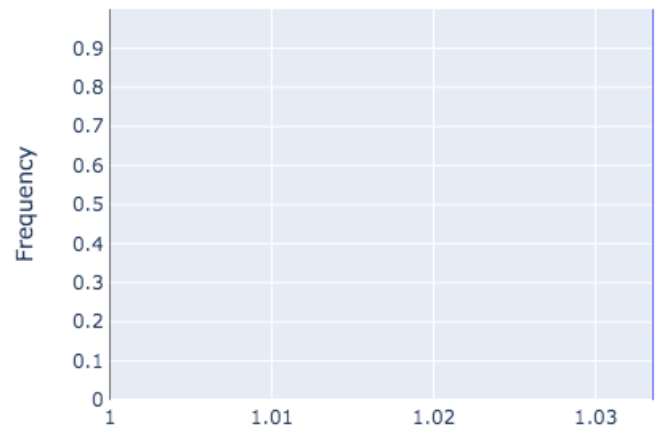Max: 1.1085806194501846
|Max-Min|: 0.09940554223286902

N=100
Number of unphysical estimates=1

N=100, Nrep=10

Min: -0.03356476460190975
Max: -0.03356476460190975
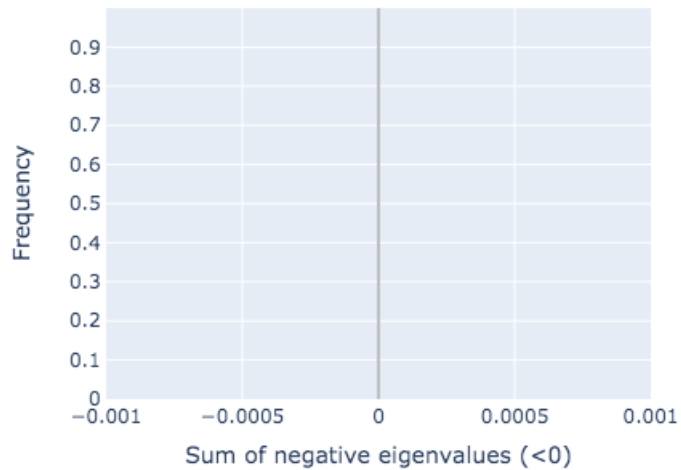|Max-Min|: 0.0

N=100, Nrep=10

Min: 1.0335647646019095
Max: 1.0335647646019095
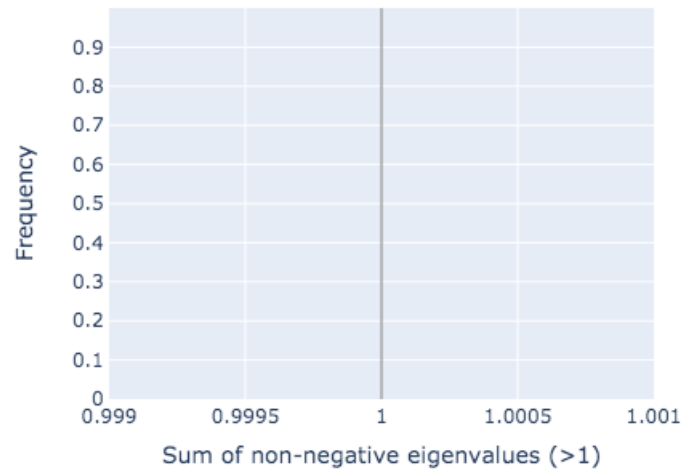|Max-Min|: 0.0

Case 2: ProjectedLinear (True)

N=10
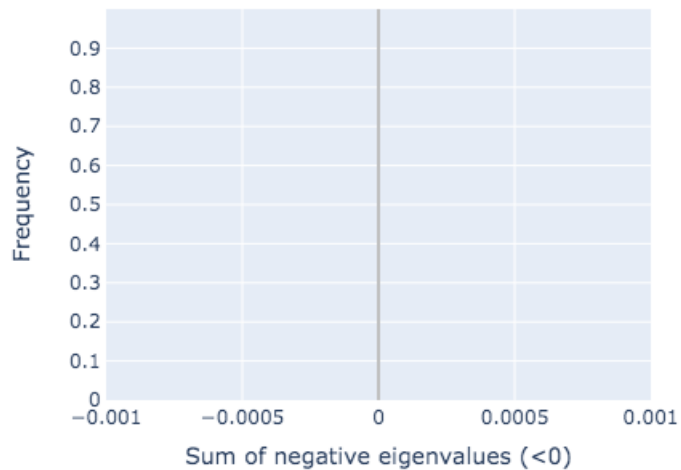**Number of unphysical estimates=0**

N=10, Nrep=10

Sum of negative eigenvalues (<0)

Min: -
Max: -
|Max-Min|: -

N=10, Nrep=10

Sum of non-negative eigenvalues (>1)

Min: -
Max: -
|Max-Min|: -

N=100
Number of unphysical estimates=0

N=100, Nrep=10

Sum of negative eigenvalues (<0)

Min: -
Max: -
|Max-Min|: -

N=100, Nrep=10

Sum of non-negative eigenvalues (>1)

Min: -
Max: -
|Max-Min|: -

Case 3: ProjectedLinear (False)

N=10
Number of unphysical estimates=0

N=10, Nrep=10

Frequency vs Sum of negative eigenvalues (<0)

Min: -
Max: -
|Max-Min|: -

N=10, Nrep=10

Frequency vs Sum of non-negative eigenvalues (>1)

Min: -
Max: -
|Max-Min|: -

N=100
Number of unphysical estimates=0

N=100, Nrep=10

Frequency vs Sum of negative eigenvalues (<0)

Min: -
Max: -
|Max-Min|: -

N=100, Nrep=10

Frequency vs Sum of non-negative eigenvalues (>1)

Min: -
Max: -
|Max-Min|: -

Case 4: Maximum-Likelihood (True)

N=10
Number of unphysical estimates=0

N=10, Nrep=10

Sum of negative eigenvalues (<0)

Min: -
Max: -
|Max-Min|: -

N=10, Nrep=10

Sum of non-negative eigenvalues (>1)

Min: -
Max: -
|Max-Min|: -

N=100
Number of unphysical estimates=0

N=100, Nrep=10

Sum of negative eigenvalues (<0)

Min: -
Max: -
|Max-Min|: -

N=100, Nrep=10

Sum of non-negative eigenvalues (>1)

Min: -
Max: -
|Max-Min|: -

Case 5: Maximum-Likelihood (False)

N=10
Number of unphysical estimates=0

N=10, Nrep=10

Sum of negative eigenvalues (<0)

Min: -
Max: -
|Max-Min|: -

N=10, Nrep=10

Sum of non-negative eigenvalues (>1)

Min: -
Max: -
|Max-Min|: -

N=100
Number of unphysical estimates=0

N=100, Nrep=10

Sum of negative eigenvalues (<0)

Min: -
Max: -
|Max-Min|: -

N=100, Nrep=10

Sum of non-negative eigenvalues (>1)

Min: -
Max: -
|Max-Min|: -

Case 6: Least Squares (True)

N=10
Number of unphysical estimates=0

N=10, Nrep=10

Min: -
Max: -
|Max-Min|: -

N=10, Nrep=10
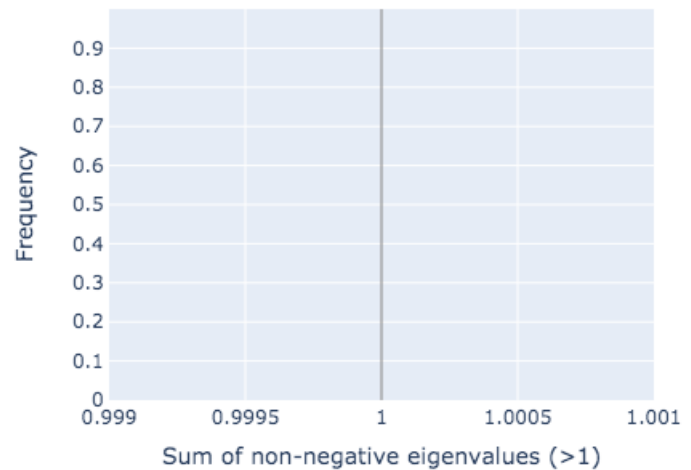

Min: -
Max: -
|Max-Min|: -
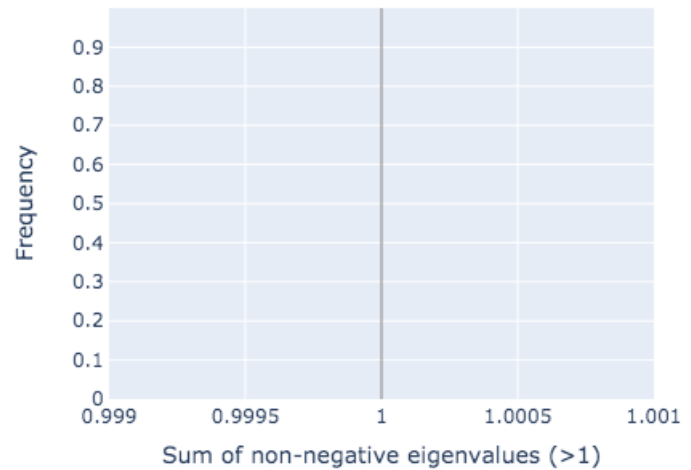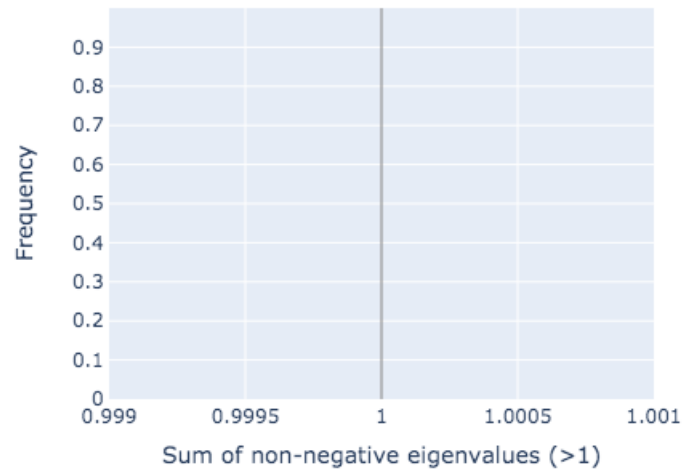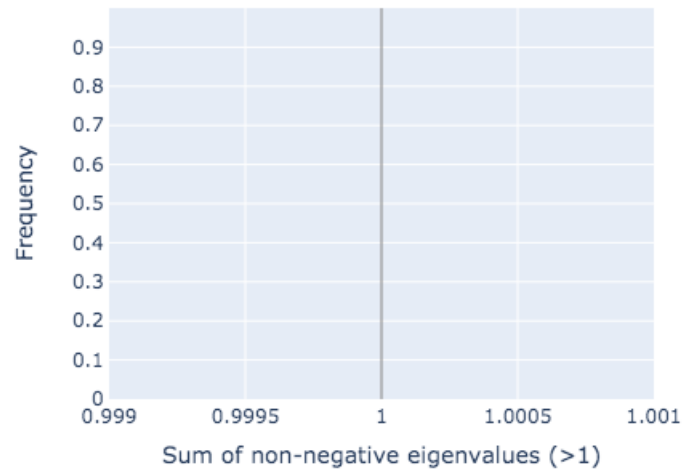
N=100
Number of unphysical estimates=0

N=100, Nrep=10
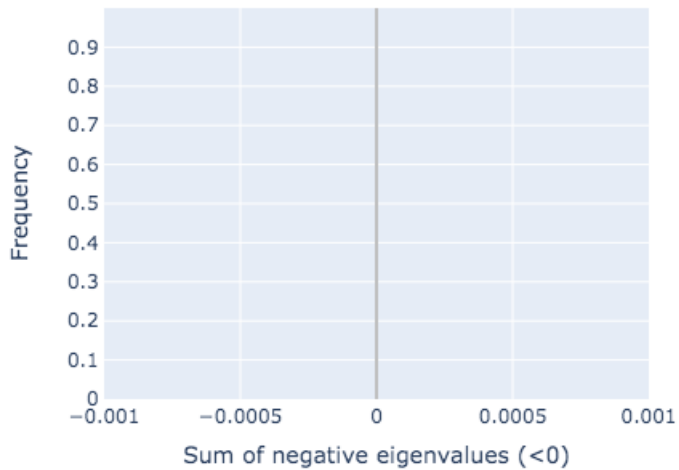

Min: -
Max: -
|Max-Min|: -

N=100, Nrep=10

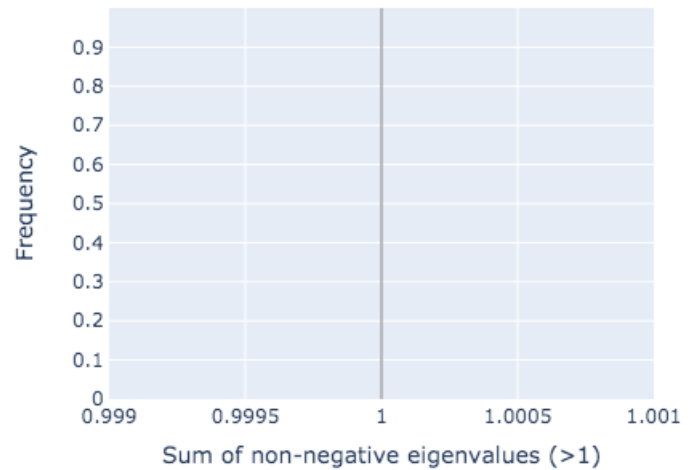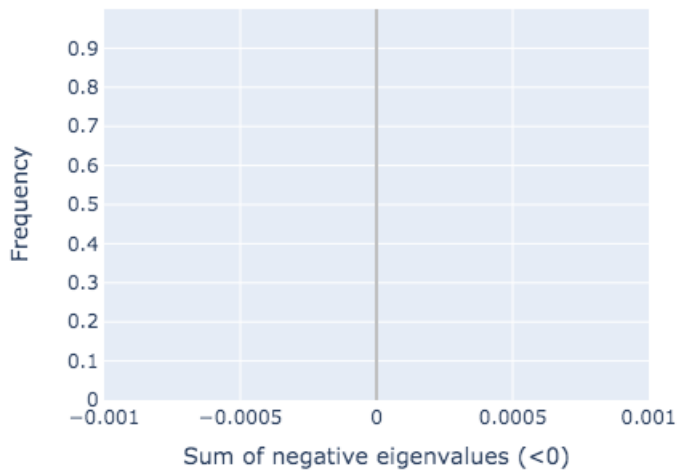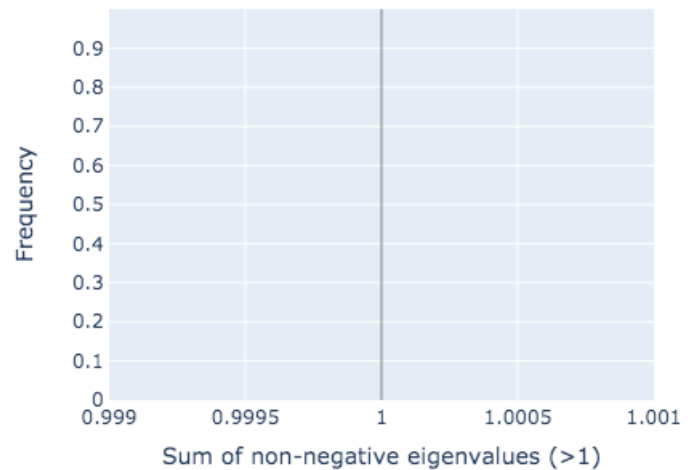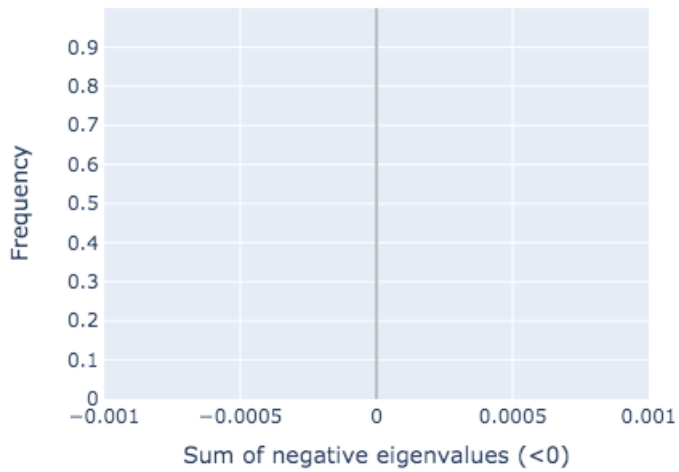Min: -
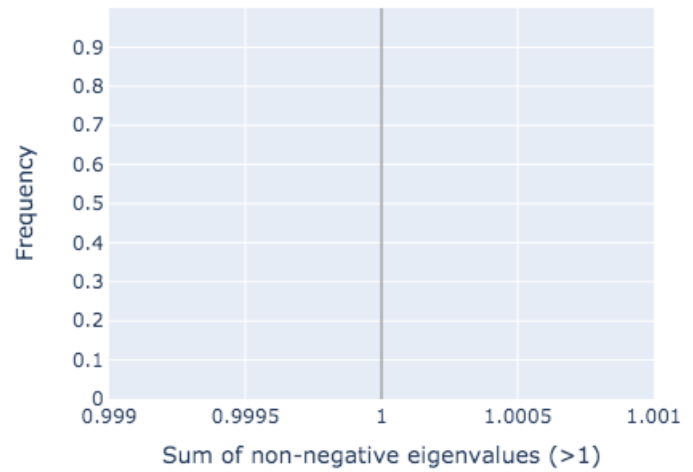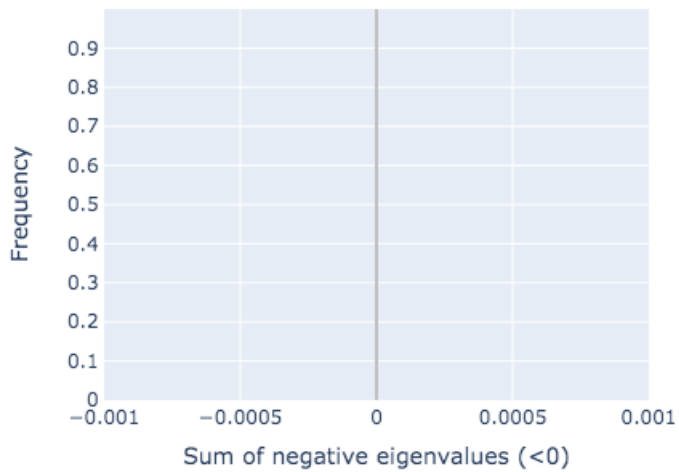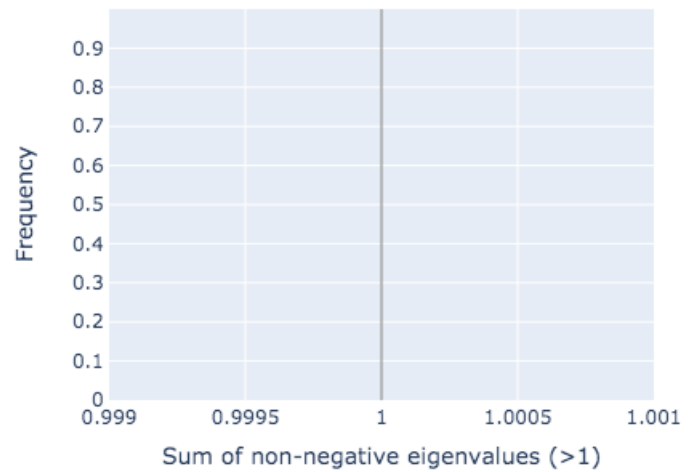Max: -
|Max-Min|: -

Case 7: Least Squares (False)

N=10
Number of unphysical estimates=0

N=10, Nrep=10

Frequency vs. Sum of negative eigenvalues (<0)

Min: -
Max: -
|Max-Min|: -

N=10, Nrep=10

Frequency vs. Sum of non-negative eigenvalues (>1)

Min: -
Max: -
|Max-Min|: -

N=100
Number of unphysical estimates=0

N=100, Nrep=10

Frequency vs. Sum of negative eigenvalues (<0)

Min: -
Max: -
|Max-Min|: -

N=100, Nrep=10

Frequency vs. Sum of non-negative eigenvalues (>1)

Min: -
Max: -
|Max-Min|: -