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Computation time in total

Total	0.6929421305656434min.

Tolerance of physicality constraint violation

		True	False
Tolerance at physicality violation test	**equality constraint**	1.00e-13	1.00e-05
	inequality constraint	1.00e-05	1.00e-05

Experimental condition

Type of tomography	StandardQst
Nrep	10
N	[10, 100]
RNG seed	777

True object

Type	State
Dim	2
Vec	[0.70710678 0. 0. 0.6363961]

Tester objects

0	**Type**	Povm
	Dim	2
	Number of outcomes	2
	Vecs	[[0.70710678 0.6363961 0. 0.] [0.70710678 -0.6363961 0. 0.]]
1	**Type**	Povm
	Dim	2
	Number of outcomes	2
	Vecs	[[0.70710678 0. 0.6363961 0.] [0.70710678 0. -0.6363961 0.]]
2	**Type**	Povm
	Dim	2
	Number of outcomes	2
	Vecs	[[0.70710678 0. 0. 0.6363961] [0.70710678 0. 0. -0.6363961]]

Cases

	Name	Param	Tomography	Estimator
0	Linear (True)	True	StandardQst	Linear
1	Linear (False)	False	StandardQst	Linear
2	ProjectedLinear (True)	True	StandardQst	ProjectedLinear
3	ProjectedLinear (False)	False	StandardQst	ProjectedLinear
4	Maximum-Likelihood (True)	True	StandardQst	LossMinimization
5	Maximum-Likelihood (False)	False	StandardQst	LossMinimization

| 6 | Least Squares (True) | True | StandardQst | LossMinimization |
| 7 | Least Squares (False) | False | StandardQst | LossMinimization |

Computation time of estimators

Table

	Name	N	Nrep	Mean (sec)	Std (sec)
0	Linear (True)	10	10	0.000109	0.000152
1	"	100	10	0.000050	0.000011
2	Linear (False)	10	10	0.000276	0.000513
3	"	100	10	0.000227	0.000419
4	ProjectedLinear (True)	10	10	0.008760	0.006971
5	"	100	10	0.002760	0.004113
6	ProjectedLinear (False)	10	10	0.006854	0.004681
7	"	100	10	0.002220	0.003300
8	Maximum-Likelihood (True)	10	10	0.429890	0.059878
9	"	100	10	0.384906	0.079138
10	Maximum-Likelihood (False)	10	10	1.474548	1.759708
11	"	100	10	1.606115	1.784696
12	Least Squares (True)	10	10	0.050073	0.006656
13	"	100	10	0.048755	0.009736
14	Least Squares (False)	10	10	0.074239	0.033793
15	"	100	10	0.067871	0.014087

Histogram

Case 0: Linear (True)

N=10, Nrep=10
Mean = 0.00010921955108642579(sec)
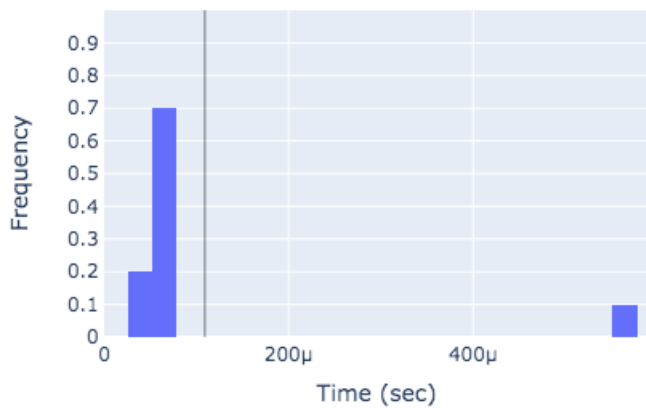
Min: 3.910064697265625e-05
Max: 0.0005643367767333984
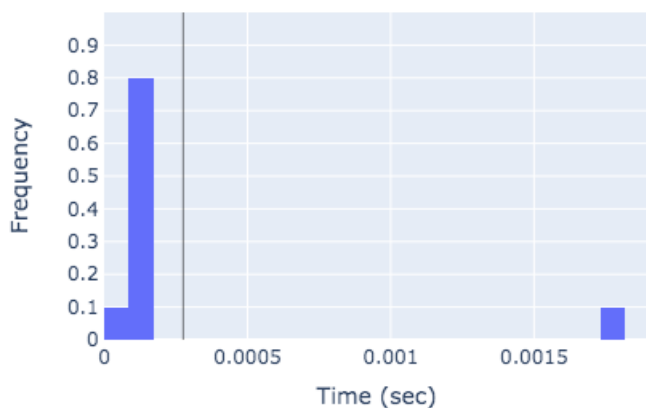|Max-Min|: 0.0005252361297607422

N=100, Nrep=10
Mean = 5.018711090087906e-05(sec)
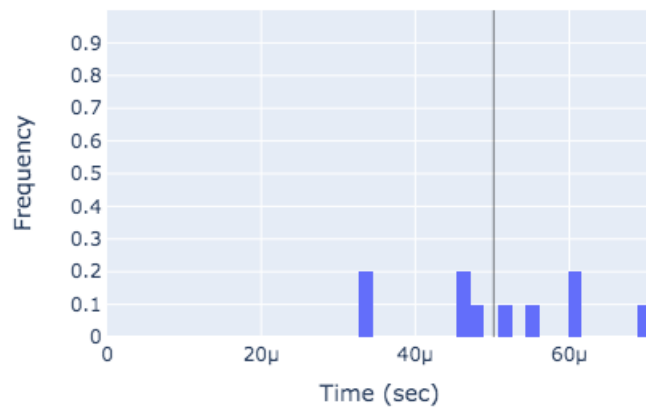
Min: 3.266334533691406e-05
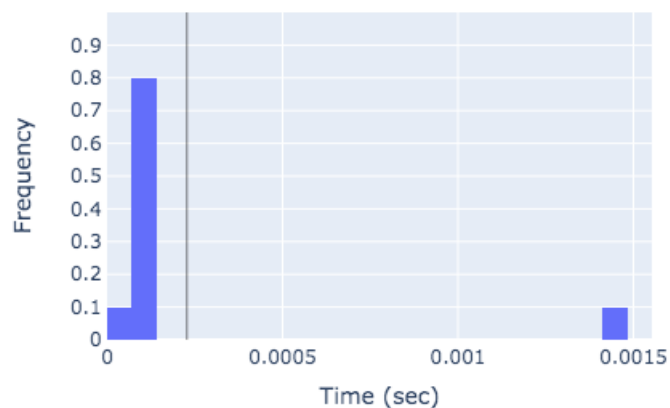Max: 6.890296936035156e-05
|Max-Min|: 3.62396240234375e-05

Case 1: Linear (False)

N=10, Nrep=10
Mean = 0.00027575492858886717(sec)

Min: 8.177757263183594e-05
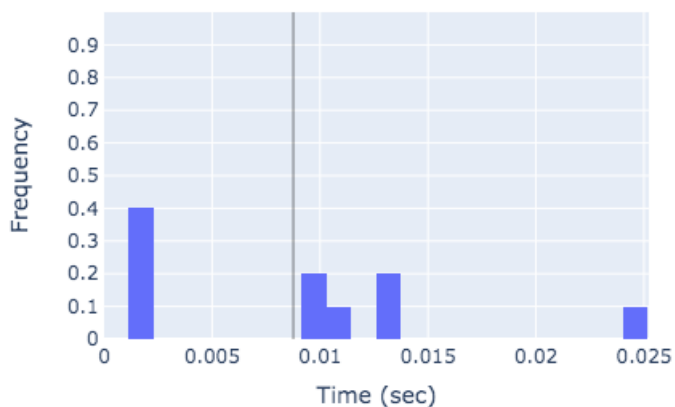Max: 0.0018138885498046875
|Max-Min|: 0.0017321109771728516

N=100, Nrep=10
Mean = 0.0002274751663208008(sec)

Min: 6.914138793945312e-05
Max: 0.0014820098876953125
|Max-Min|: 0.0014128684997558594

Case 2: ProjectedLinear (True)

N=10, Nrep=10
Mean = 0.00875999927520752(sec)

Min: 0.0012171268463134766
Max: 0.02408289909362793
|Max-Min|: 0.022865772247314453

N=100, Nrep=10
Mean = 0.002759575843811035(sec)
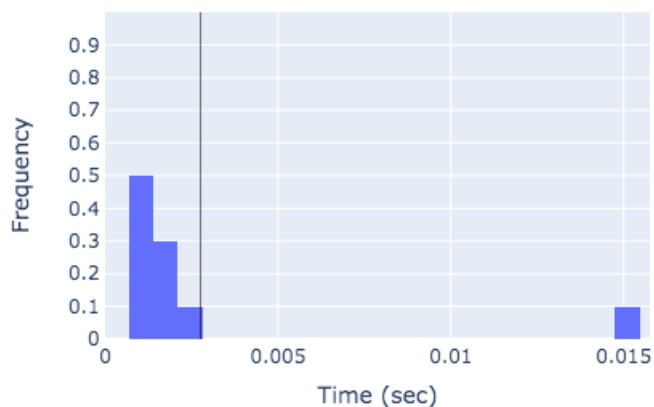
Min: 0.0010013580322265625
Max: 0.015055179595947266
|Max-Min|: 0.014053821563720703

Case 3: ProjectedLinear (False)

N=10, Nrep=10
Mean = 0.006853723526000976(sec)
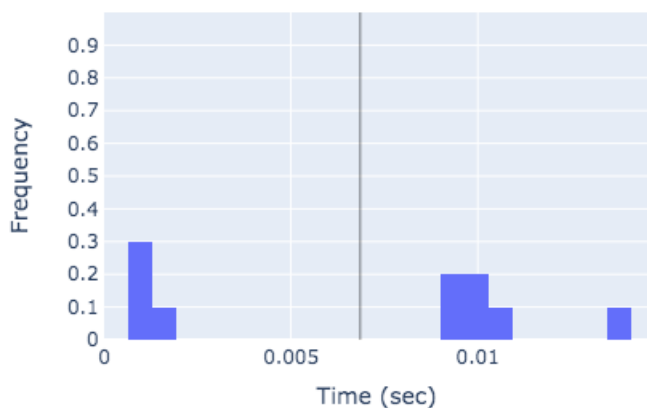
Min: 0.0010979175567626953
Max: 0.013937234878540039
|Max-Min|: 0.012839317321777344

N=100, Nrep=10
Mean = 0.0022202491760253905(sec)
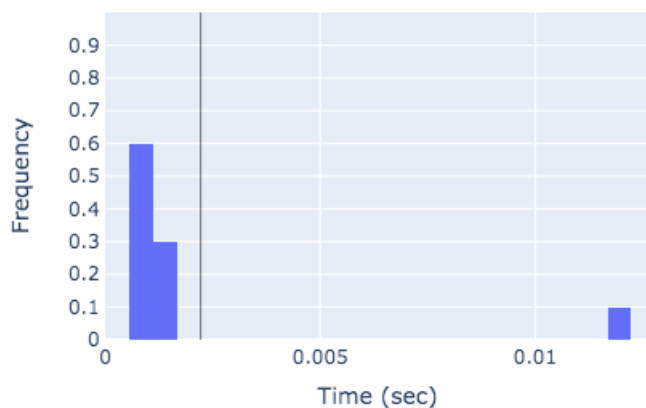
Min: 0.0009851455688476562
Max: 0.01210784912109375
|Max-Min|: 0.011122703552246094

Case 4: Maximum-Likelihood (True)

N=10, Nrep=10
Mean = 0.42988977432250974(sec)
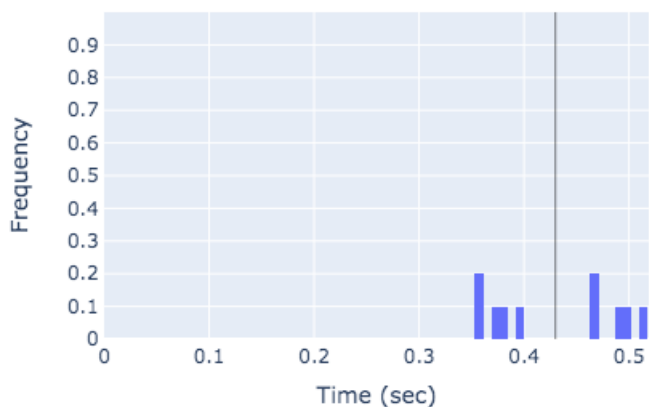
Min: 0.3540523052215576
Max: 0.5110030174255371
|Max-Min|: 0.1569507122039795

N=100, Nrep=10
Mean = 0.38490607738494875(sec)
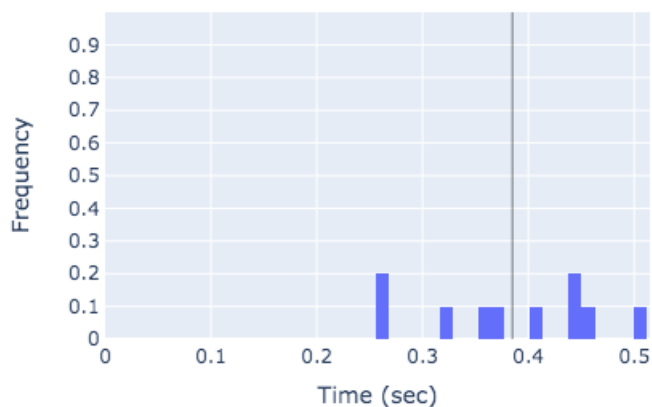
Min: 0.25916314125061035
Max: 0.5026500225067139
|Max-Min|: 0.24348688125610352

Case 5: Maximum-Likelihood (False)

N=10, Nrep=10
Mean = 1.4745483875274659(sec)
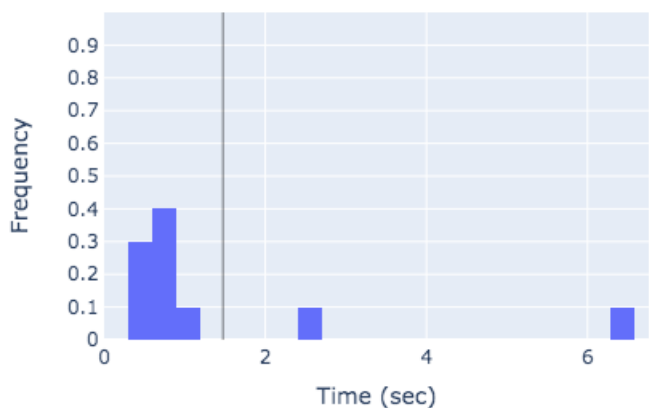
Min: 0.46915769577026367
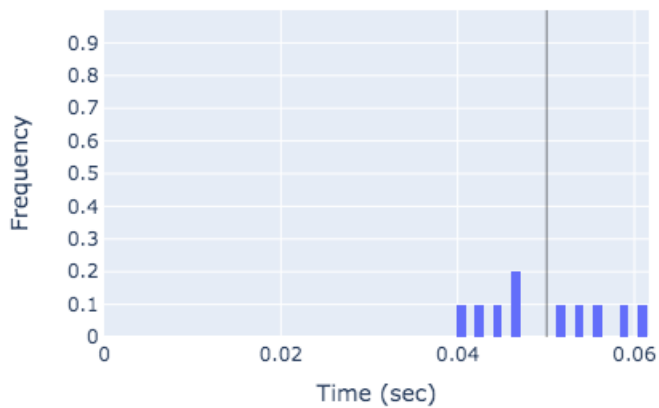Max: 6.460079908370972
|Max-Min|: 5.990922212600708

N=100, Nrep=10
Mean = 1.606115198135376(sec)
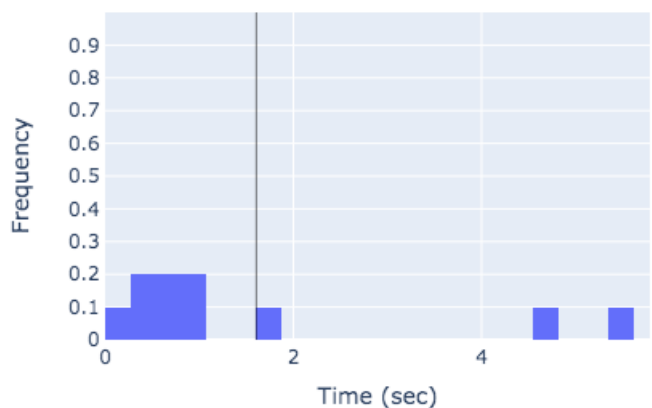
Min: 0.17540216445922852
Max: 5.5205419063568115
|Max-Min|: 5.345139741897583

Case 6: Least Squares (True)

N=10, Nrep=10
Mean = 0.05007250308990478(sec)

Min: 0.04013967514038086
Max: 0.0606081485748291
|Max-Min|: 0.020468473434448242

N=100, Nrep=10
Mean = 0.0487548828125(sec)
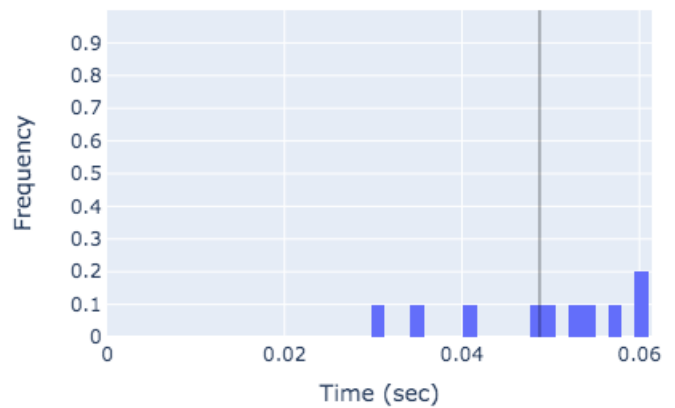
Min: 0.030167341232299805
Max: 0.05991816520690918
|Max-Min|: 0.029750823974609375

Case 7: Least Squares (False)

N=10, Nrep=10
Mean = 0.07423882484436035(sec)
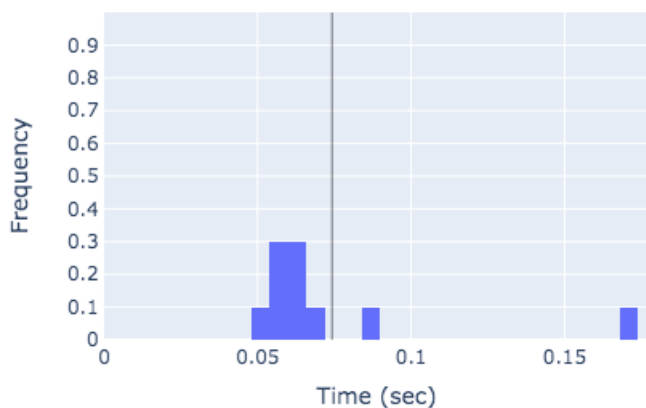
Min: 0.05149698257446289
Max: 0.17146754264831543
|Max-Min|: 0.11997056007385254

N=100, Nrep=10
Mean = 0.06787095069885254(sec)

Min: 0.04977774620056152
Max: 0.10312175750732422
|Max-Min|: 0.053344011306762695

MSE of empirical distributions
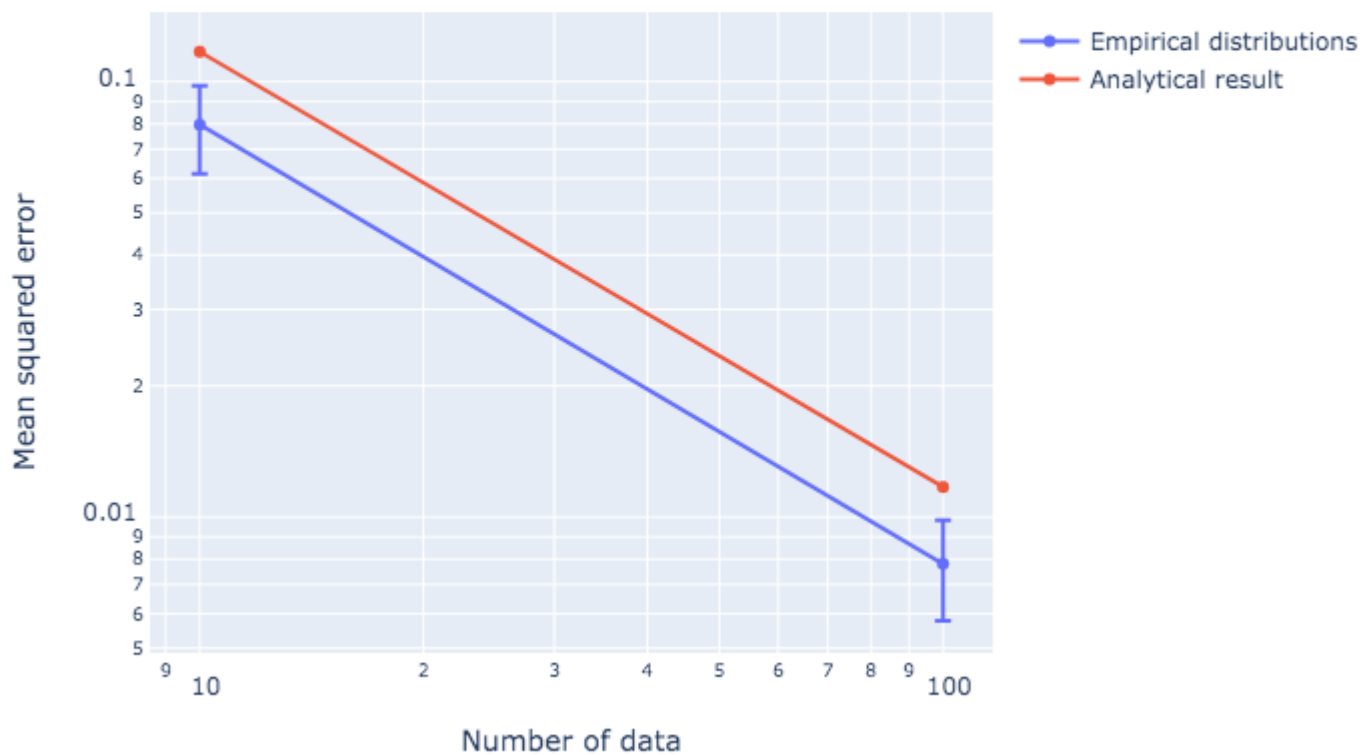
Mean squared error

Consistency test

	Name	Type of tomography	Param	Estimator	Loss	Algo	Squared Error to True	Possibly OK	To be checked
0	Linear (True)	Standard Qst	True	Linear	None	None	0.00e+00	OK	not need debug
1	Linear (False)	Standard Qst	False	Linear	None	None	2.47e-32	OK	not need debug
2	ProjectedLinear (True)	Standard Qst	True	Projected Linear	None	None	1.23e-32	OK	not need debug
3	ProjectedLinear (False)	Standard Qst	False	Projected Linear	None	None	1.11e-31	OK	not need debug
4	Maximum-Likelihood (True)	Standard Qst	True	Loss Minimization	Weighted Relative Entropy	Projected Gradient Descent Backtracking	3.21e-18	OK	not need debug
5	Maximum-Likelihood (False)	Standard Qst	False	Loss Minimization	Weighted Relative Entropy	Projected Gradient Descent Backtracking	5.47e-09	NG	not need debug
6	Least Squares (True)	Standard Qst	True	Loss Minimization	Weighted Probability Based Squared Error	Projected Gradient Descent Backtracking	4.93e-32	OK	not need debug

7	Least Squares (False)	Standard Qst	False	Loss Minimization	Weighted Probability Based Squared Error	Projected Gradient Descent Backtracking	1.60e-31	OK	not need debug

MSE of estimators

Comparison of analytical results

Mean squared error
Analytical result
estimator=Linear
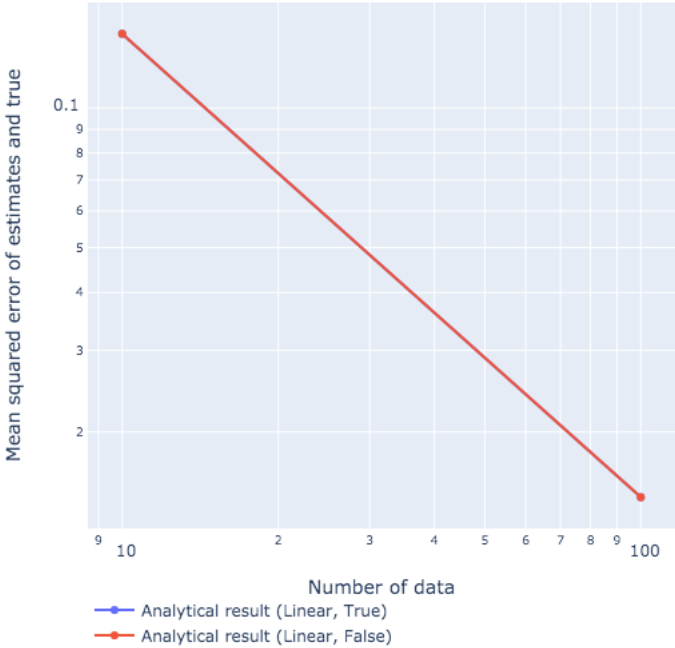
Mean squared error
Analytical result
Cramer-Rao bound
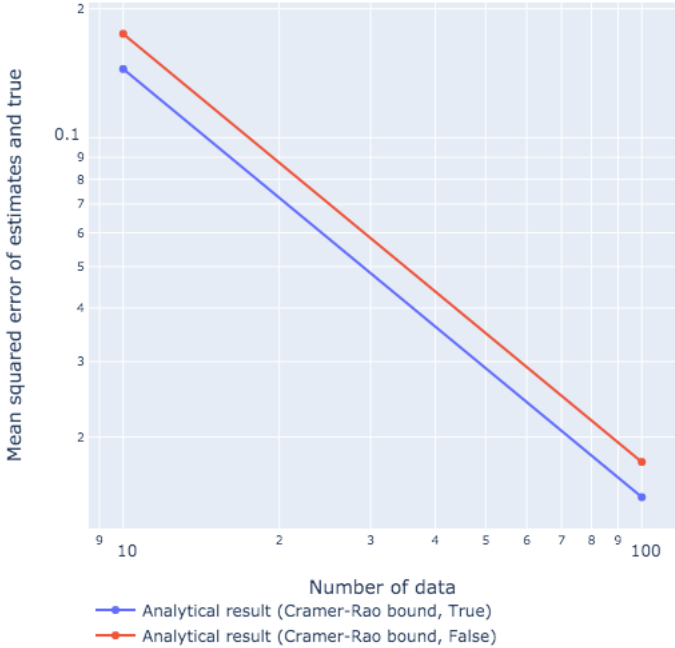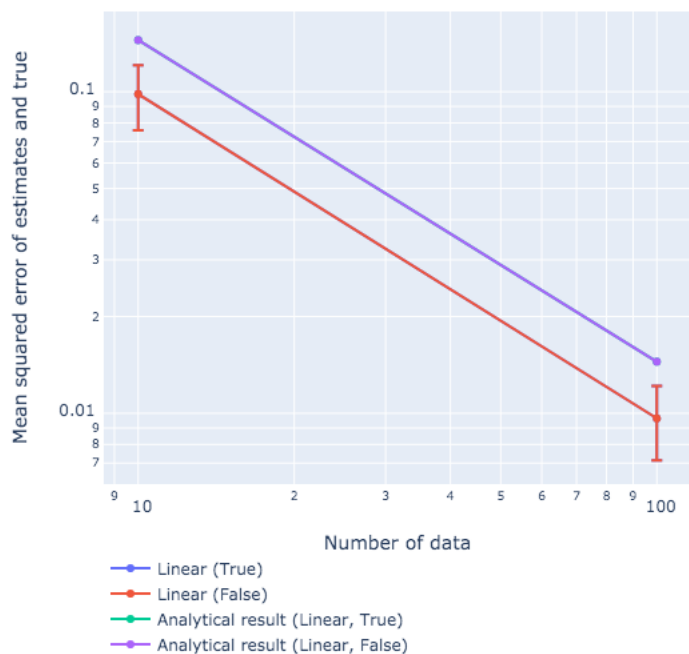
Mean squared error
Analytical result
parametorization=True

Mean squared error of estimates and true

Number of data

- Analytical result (Linear, True)
- Analytical result (Cramer-Rao bound, True)

Mean squared error
Analytical result
parametorization=False

Mean squared error of estimates and true

Number of data
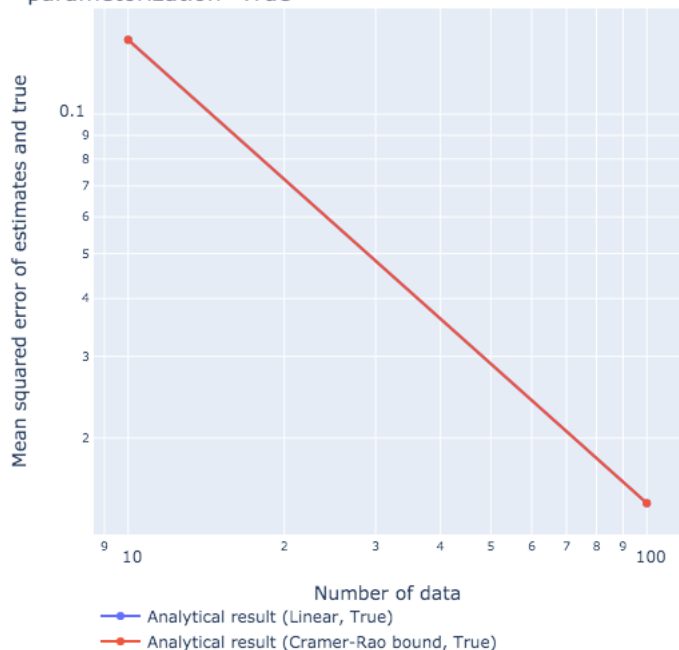
- Analytical result (Linear, False)
- Analytical result (Cramer-Rao bound, False)
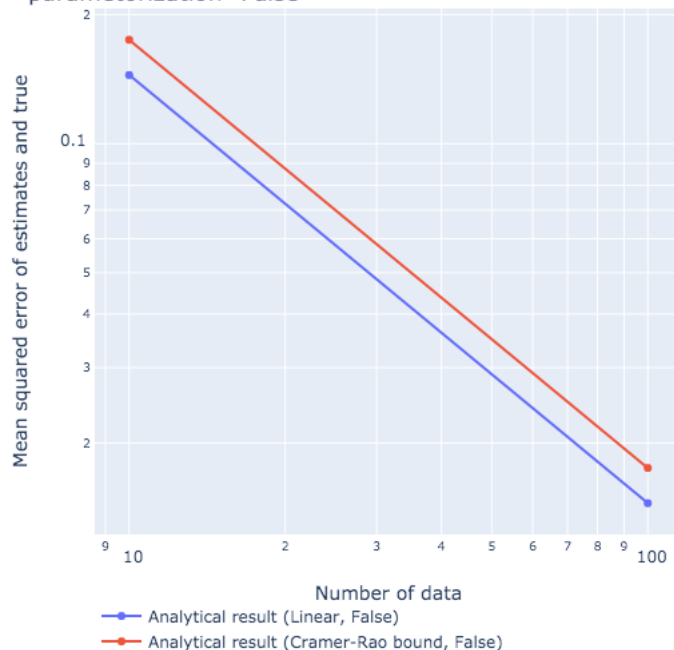
Comparison of parametrization

Mean squared error
Estimator=Linear

Mean squared error of estimates and true

Number of data

- Linear (True)
- Linear (False)
- Analytical result (Linear, True)
- Analytical result (Linear, False)

Mean squared error
Estimator=ProjectedLinear

Mean squared error of estimates and true

Number of data

- ProjectedLinear (True)
- ProjectedLinear (False)
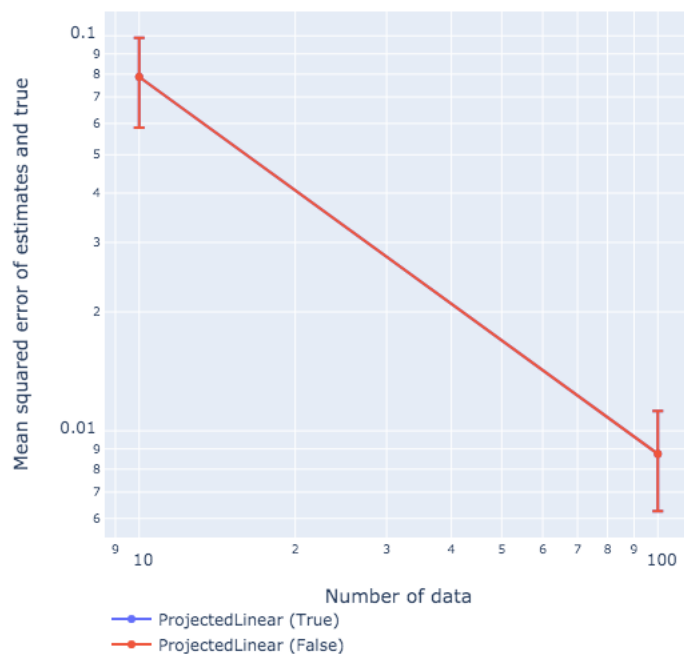
Mean squared error
Estimator=LossMinimization
Loss=WeightedRelativeEntropy
Algo=ProjectedGradientDescentBacktracking

Legend:
- Maximum-Likelihood (True)
- Maximum-Likelihood (False)
- Analytical result (Cramer-Rao bound, True)
- Analytical result (Cramer-Rao bound, False)
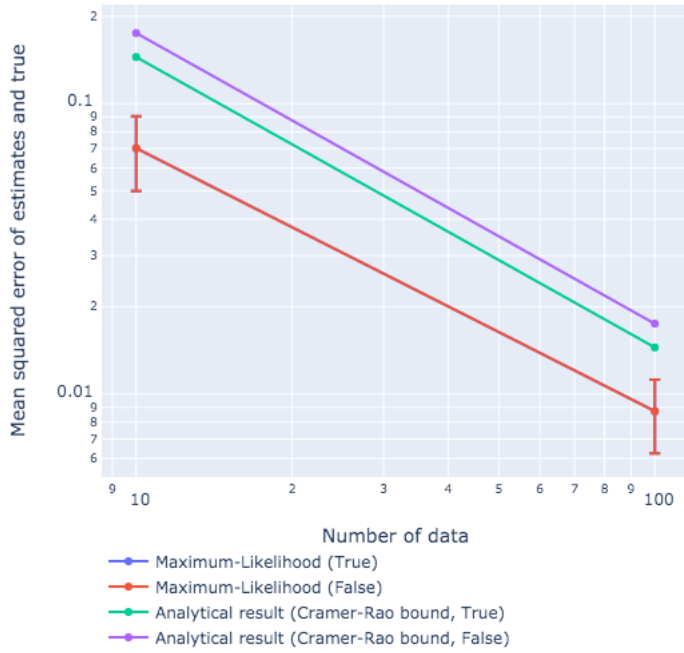
Mean squared error
Estimator=LossMinimization
Loss=WeightedProbabilityBasedSquaredError
Algo=ProjectedGradientDescentBacktracking

Legend:
- Least Squares (True)
- Least Squares (False)
- Analytical result (Cramer-Rao bound, True)
- Analytical result (Cramer-Rao bound, False)
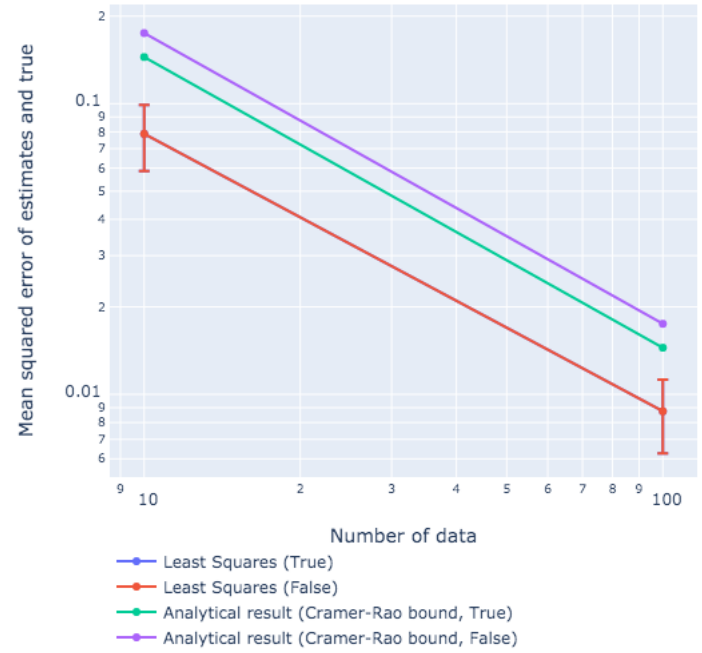
Comparison of estimators

Mean squared error
parametrization=True

Legend:
- Linear (True)
- ProjectedLinear (True)
- Maximum-Likelihood (True)
- Least Squares (True)

Mean squared error
parametrization=False
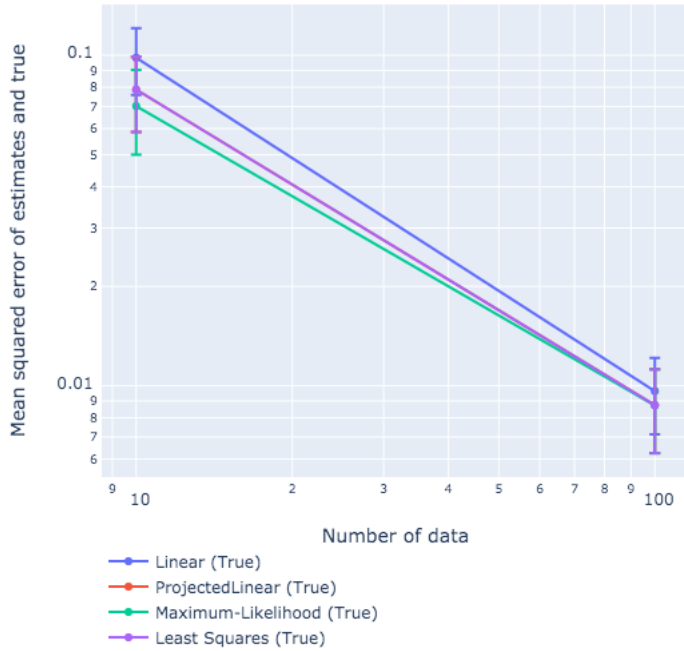
Legend:
- Linear (False)
- ProjectedLinear (False)
- Maximum-Likelihood (False)
- Least Squares (False)

Physicality violation test

Test of equality constraint violation

Case 0: Linear (True)
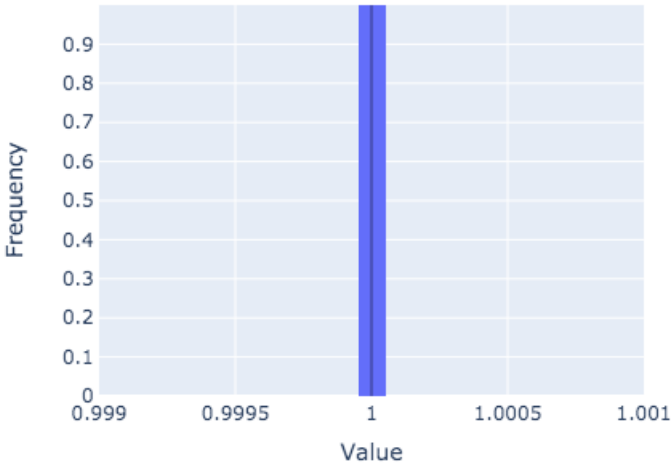
N=10, Nrep=10

Min: 0.9999999999999997
Max: 0.9999999999999998
|Max-Min|: 1.1102230246251565e-16

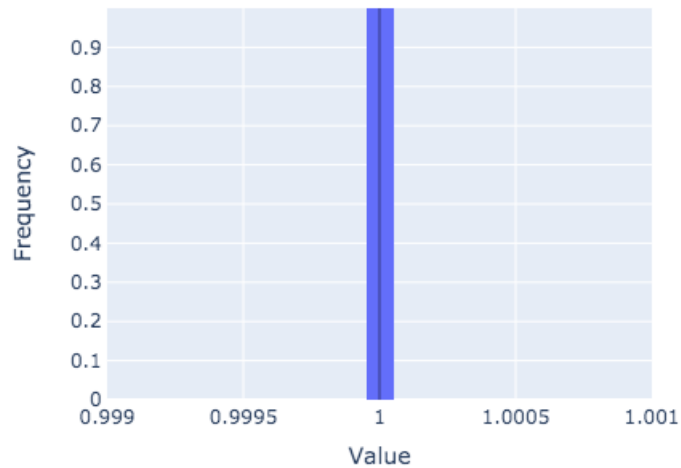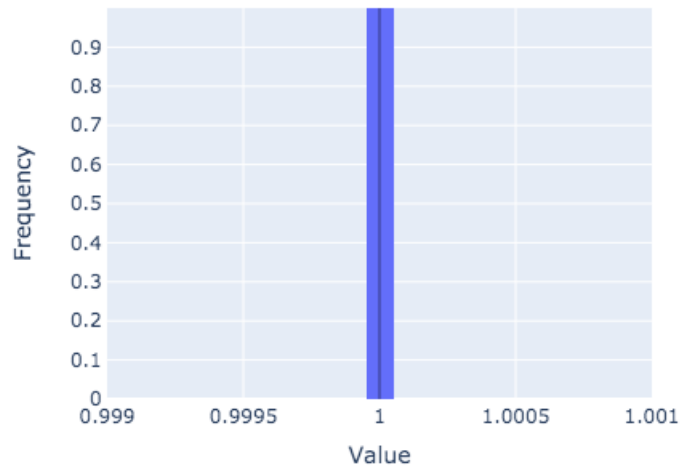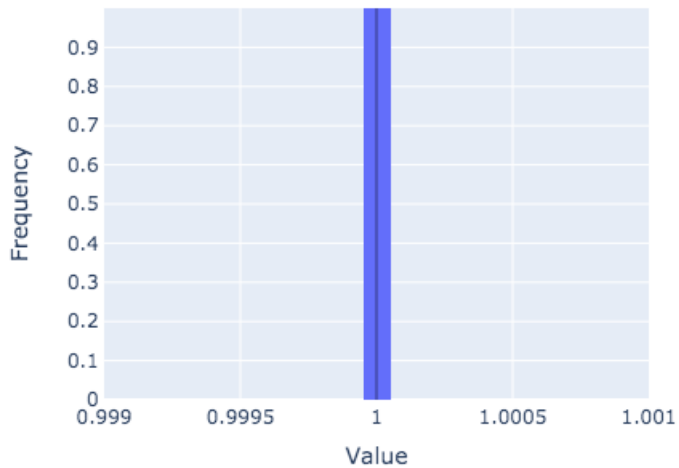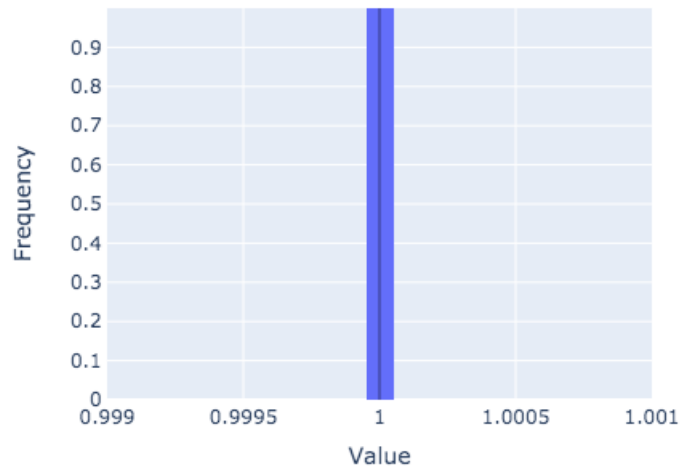N=100, Nrep=10

Min: 0.9999999999999998
Max: 0.9999999999999998
|Max-Min|: 0.0

Case 1: Linear (False)
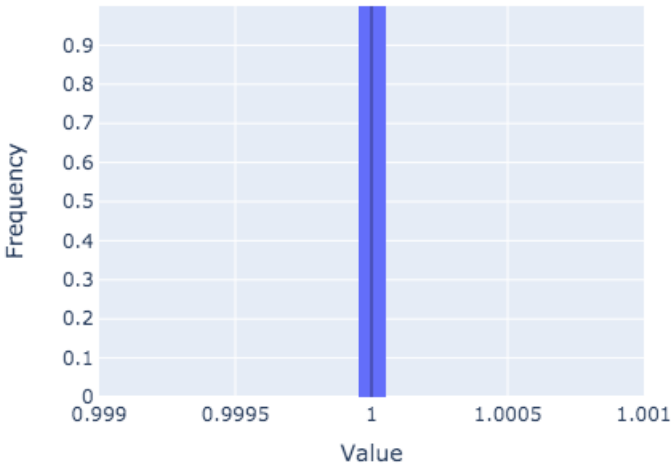
N=10, Nrep=10

Min: 0.9999999999999998
Max: 1.0000000000000002
|Max-Min|: 4.440892098500626e-16

N=100, Nrep=10
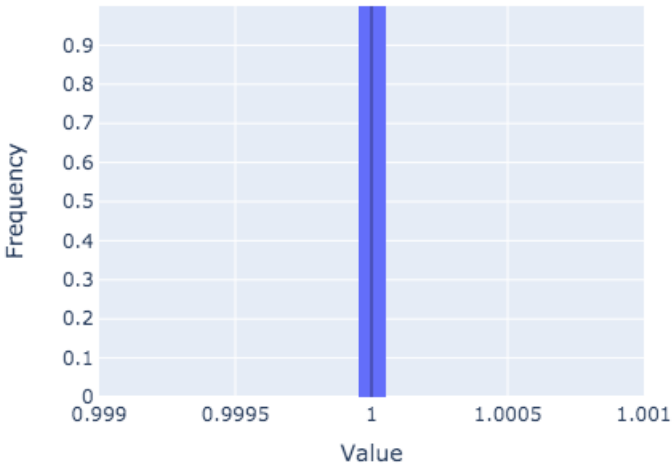
Min: 0.9999999999999999
Max: 1.0000000000000002
|Max-Min|: 3.3306690738754696e-16

Case 2: ProjectedLinear (True)
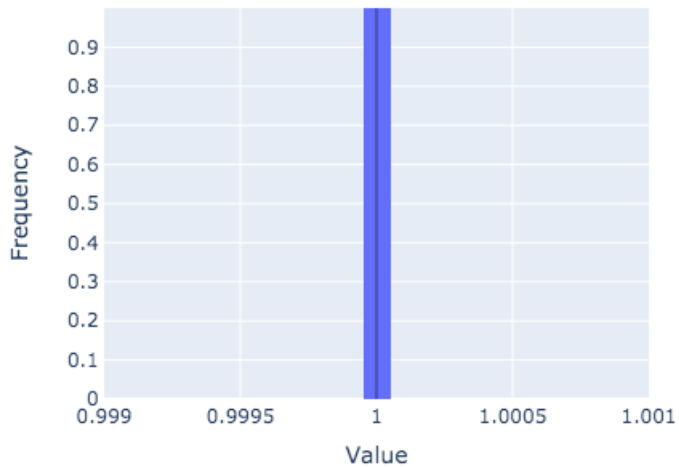
N=10, Nrep=10

Min: 0.9999999999999998
Max: 0.9999999999999998
|Max-Min|: 0.0
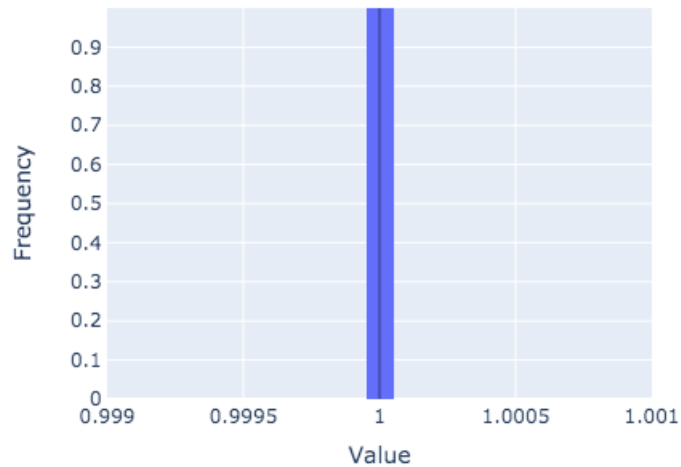
N=100, Nrep=10

Min: 0.9999999999999998
Max: 0.9999999999999998
|Max-Min|: 0.0

Case 3: ProjectedLinear (False)
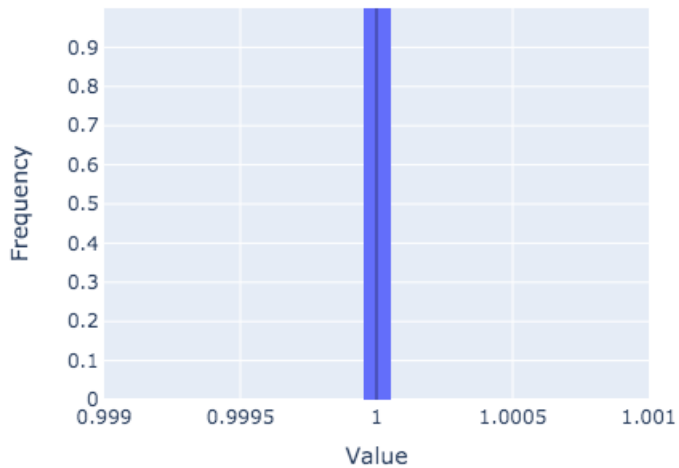
N=10, Nrep=10

Min: 0.9999999999999996
Max: 1.00000014753393
|Max-Min|: 1.4753393040578544e-07

N=100, Nrep=10
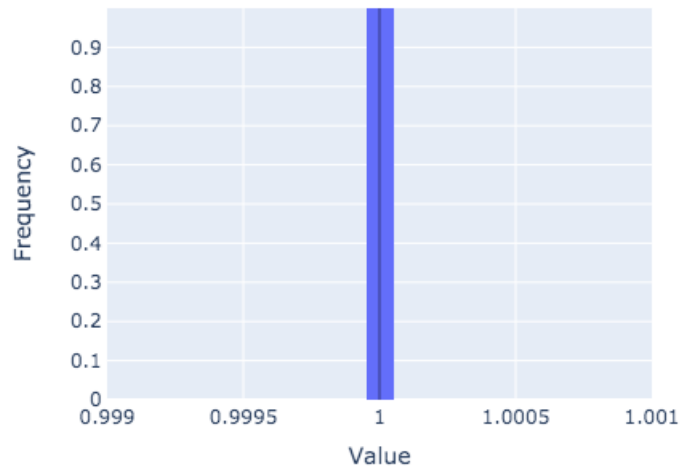
Min: 0.9999999999999997
Max: 1.0000001280394155
|Max-Min|: 1.280394158476028e-07

Case 4: Maximum-Likelihood (True)
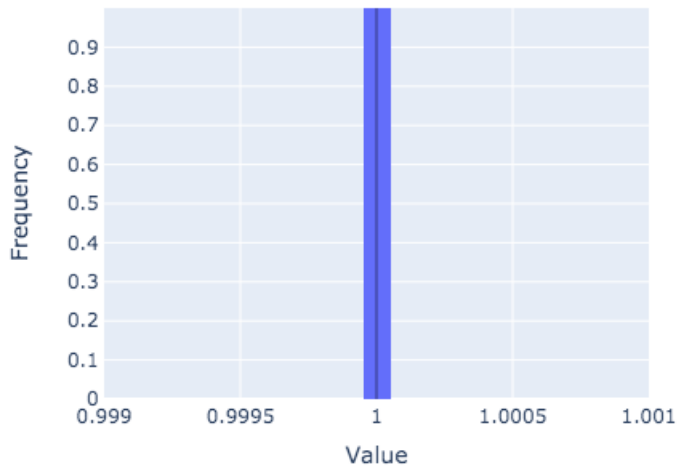
N=10, Nrep=10

Min: 0.9999999999999998
Max: 0.9999999999999998
|Max-Min|: 0.0
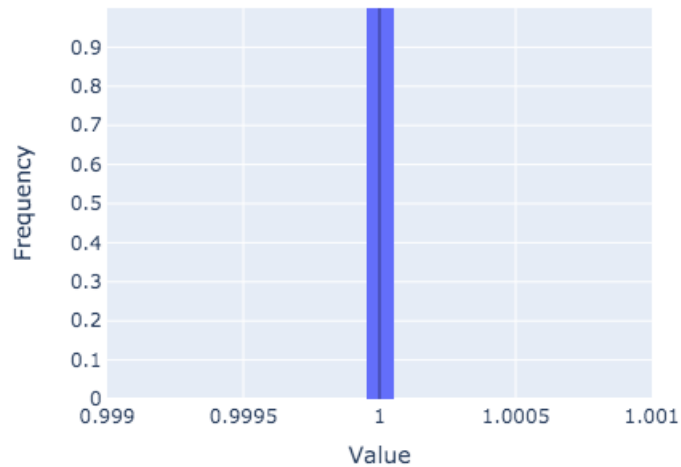
N=100, Nrep=10

Min: 0.9999999999999998
Max: 0.9999999999999998
|Max-Min|: 0.0

Case 5: Maximum-Likelihood (False)
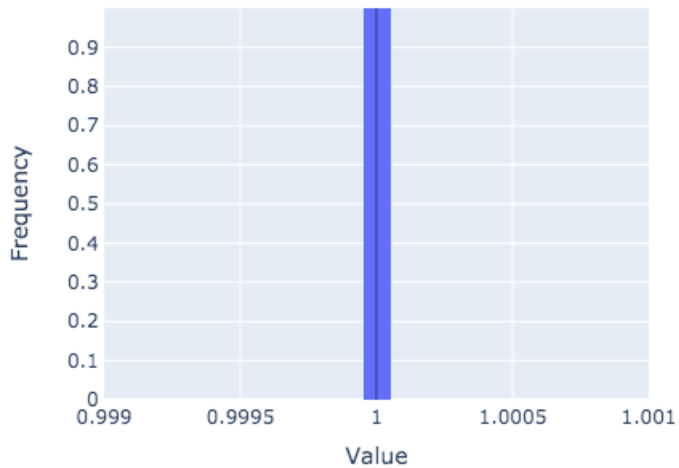
N=10, Nrep=10

Min: 1.000000006567707
Max: 1.0000001622723005
|Max-Min|: 1.5570459344083076e-07

N=100, Nrep=10

Min: 1.0000000121352572
Max: 1.00000001810607821
|Max-Min|: 1.689255249015531e-07

Case 6: Least Squares (True)
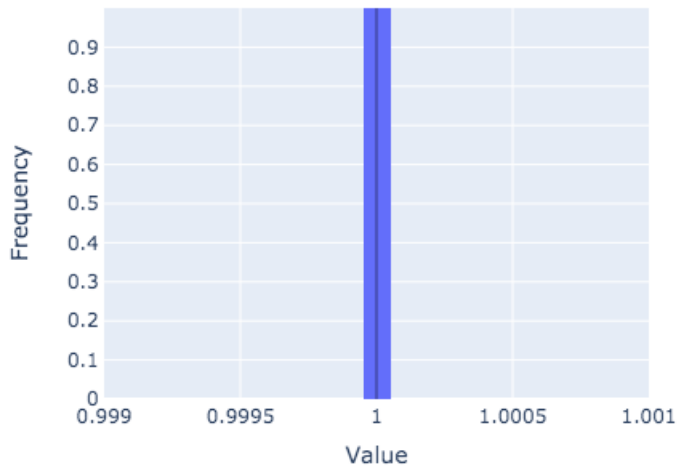
N=10, Nrep=10

Min: 0.9999999999999998
Max: 0.9999999999999998
|Max-Min|: 0.0

N=100, Nrep=10

Min: 0.9999999999999998
Max: 0.9999999999999998
|Max-Min|: 0.0

Case 7: Least Squares (False)

N=10, Nrep=10

Min: 0.9999999999999998
Max: 1.0000001781169368
|Max-Min|: 1.7811693697744602e-07

N=100, Nrep=10

Min: 0.9999999999999998
Max: 1.0000001555680968
|Max-Min|: 1.5556809707106822e-07

Test of inequality constraint violation

Eigenvalue

Case 0: Linear (True)
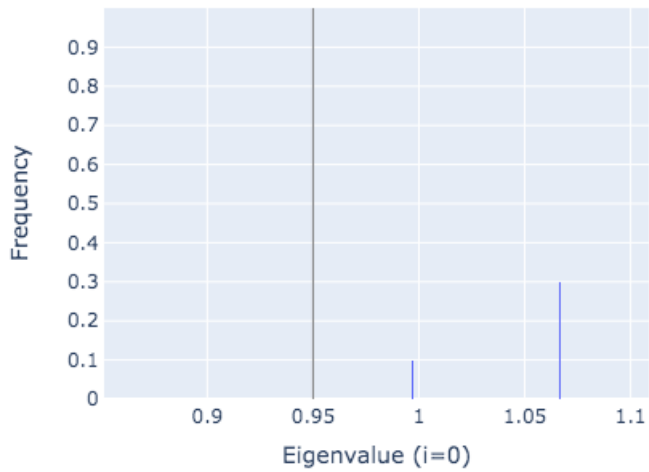
N=10, Nrep=10

Min: 0.8513641844631532
Max: 1.1085806194501846
|Max-Min|: 0.2572164349870314
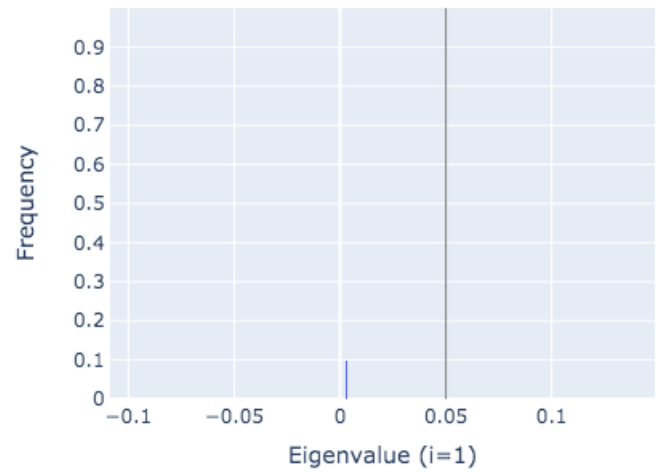
N=10, Nrep=10

Min: -0.10858061945018471
Max: 0.14863581553684663
|Max-Min|: 0.25721643498703134
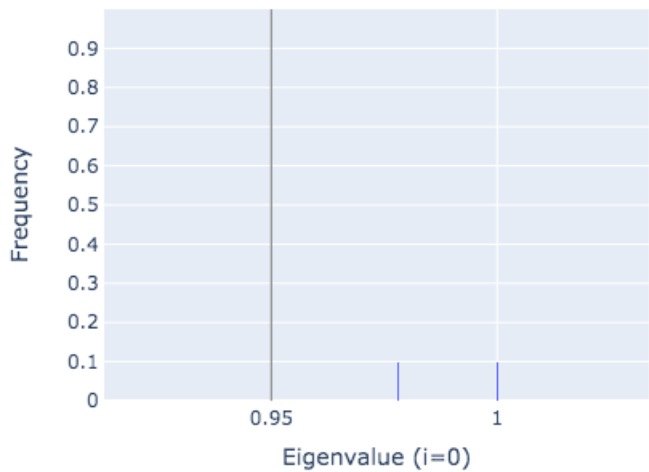
N=100, Nrep=10

Min: 0.9130584510098023
Max: 1.0335647646019095
|Max-Min|: 0.12050631359210717

N=100, Nrep=10
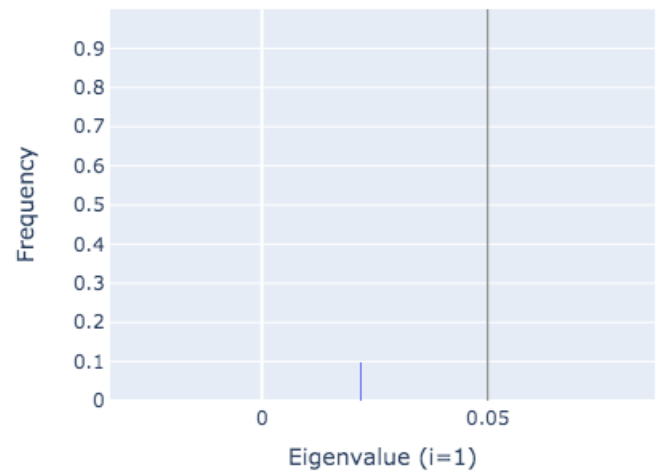
Min: -0.03356476460190986
Max: 0.0869415489901976
|Max-Min|: 0.12050631359210745
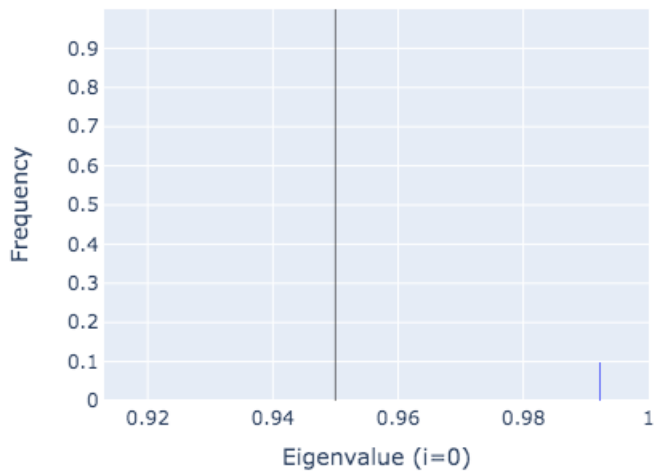
Case 1: Linear (False)

N=10, Nrep=10

Eigenvalue (i=0)

Min: 0.8513641844631532
Max: 1.1085806194501846
|Max-Min|: 0.2572164349870314

N=10, Nrep=10

Eigenvalue (i=1)
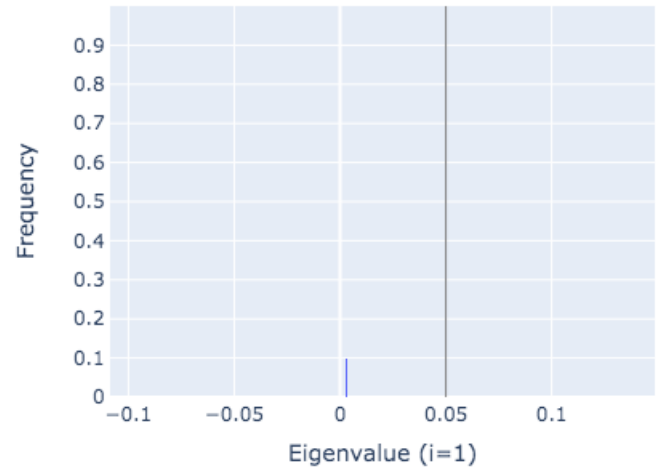
Min: -0.10858061945018459
Max: 0.14863581553684663
|Max-Min|: 0.25721643498703123

N=100, Nrep=10
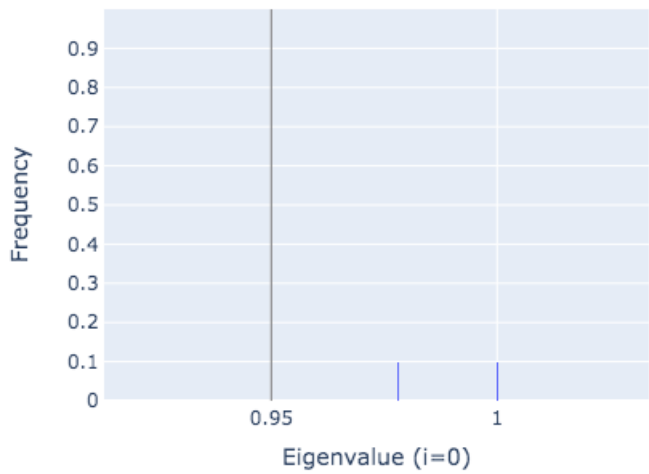
Eigenvalue (i=0)

Min: 0.9130584510098023
Max: 1.0335647646019095
|Max-Min|: 0.12050631359210717

N=100, Nrep=10

Eigenvalue (i=1)
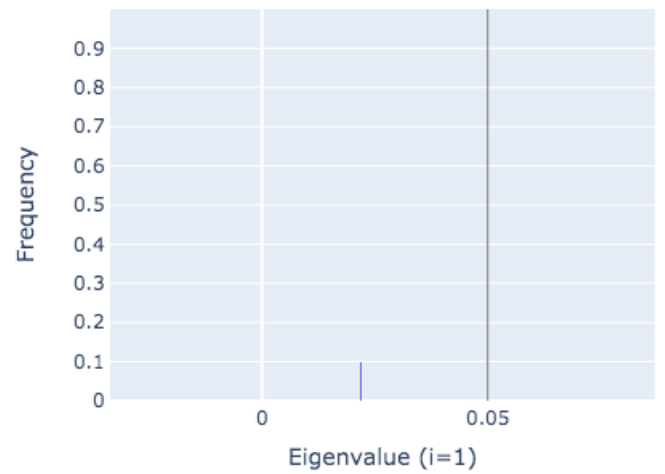
Min: -0.03356476460190975
Max: 0.08694154899019772
|Max-Min|: 0.12050631359210748

Case 2: ProjectedLinear (True)

N=10, Nrep=10

Eigenvalue (i=0)
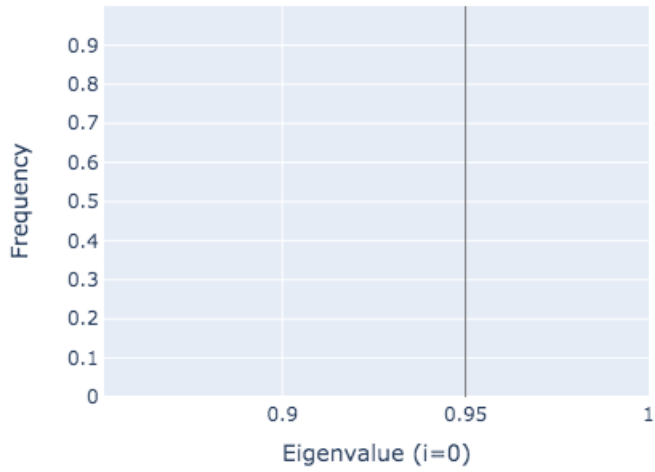
Min: 0.8513641844631531
Max: 1.0000000737669648
|Max-Min|: 0.14863588930381166

N=10, Nrep=10

Eigenvalue (i=1)
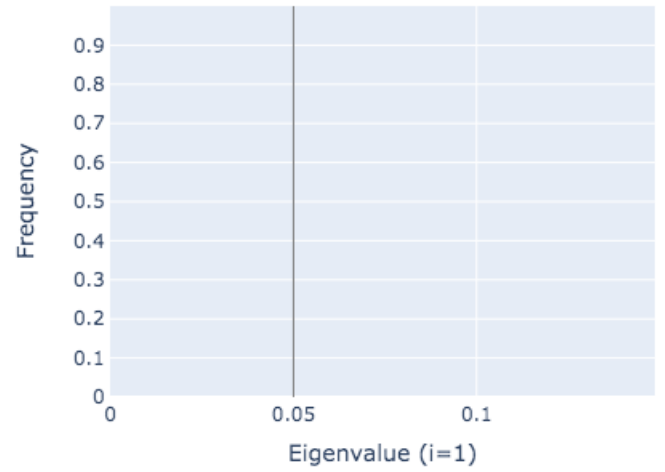
Min: -7.376696509533986e-08
Max: 0.14863581553684668
|Max-Min|: 0.14863588930381177

N=100, Nrep=10

Eigenvalue (i=0)

Min: 0.9130584510098023
Max: 1.0000000640197078
|Max-Min|: 0.08694161300990544

N=100, Nrep=10

Eigenvalue (i=1)
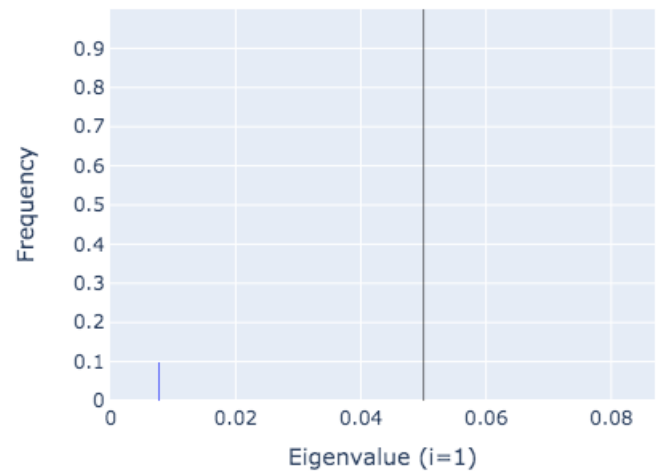
Min: -6.401970792556323e-08
Max: 0.0869415489901976
|Max-Min|: 0.08694161300990552

Case 3: ProjectedLinear (False)

N=10, Nrep=10

Eigenvalue (i=0)
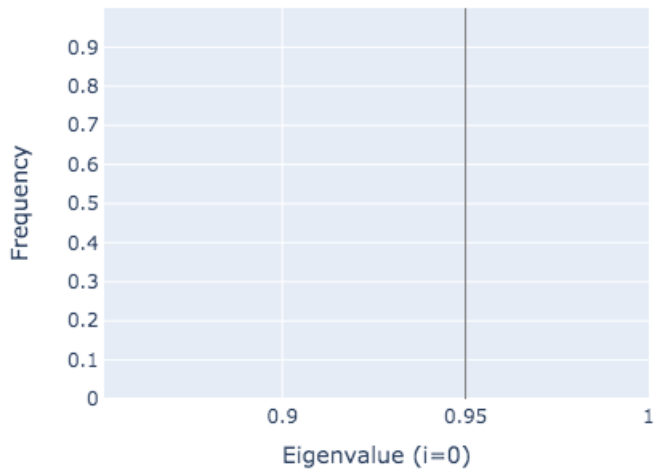
Min: 0.8513641844631532
Max: 1.00000014753393
|Max-Min|: 0.14863596307077676

N=10, Nrep=10

Eigenvalue (i=1)
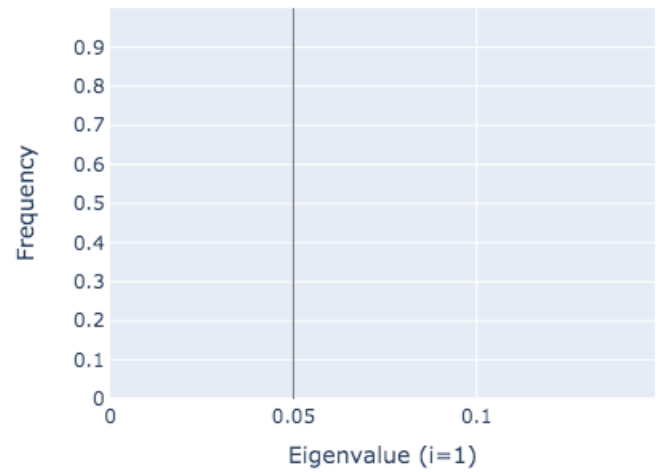
Min: -2.7755575615628914e-17
Max: 0.14863581553684696
|Max-Min|: 0.148635815536847

N=100, Nrep=10

Eigenvalue (i=0)
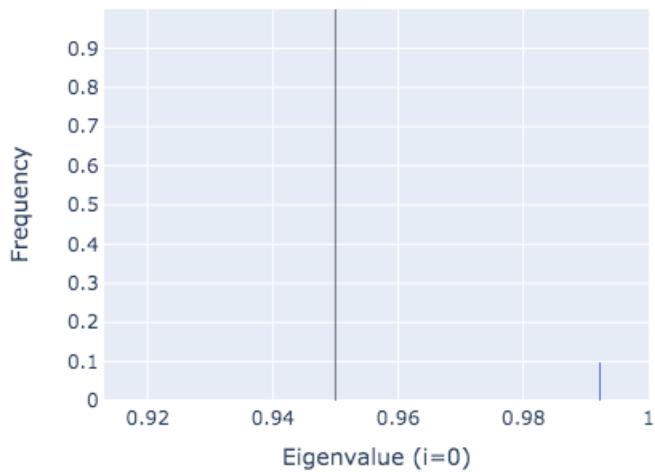
Min: 0.9130584510098023
Max: 1.0000001280394155
|Max-Min|: 0.0869416770296132

N=100, Nrep=10

Eigenvalue (i=1)
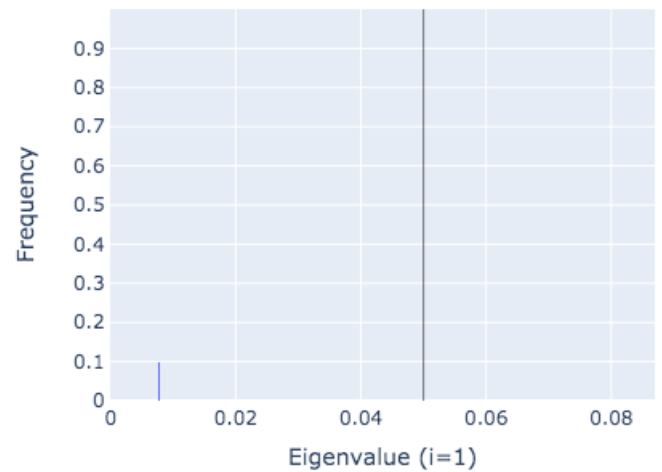
Min: -5.730008481585891e-17
Max: 0.0869415489901976
|Max-Min|: 0.08694154899019765

Case 4: Maximum-Likelihood (True)

N=10, Nrep=10

Eigenvalue (i=0)
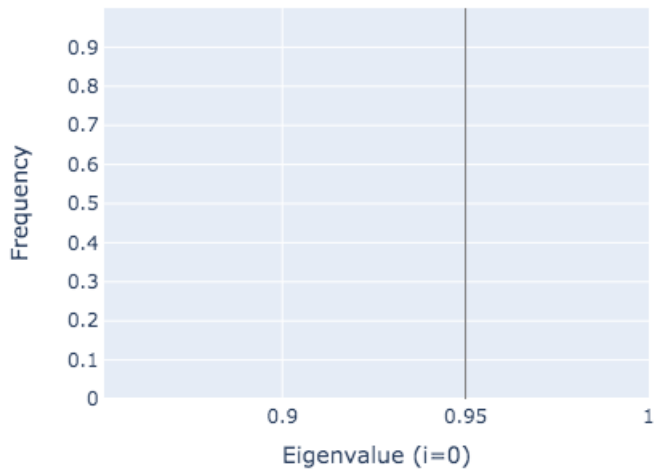
Min: 0.8513641812250503
Max: 1.0000000702659644
|Max-Min|: 0.14863588904091407

N=10, Nrep=10

Eigenvalue (i=1)
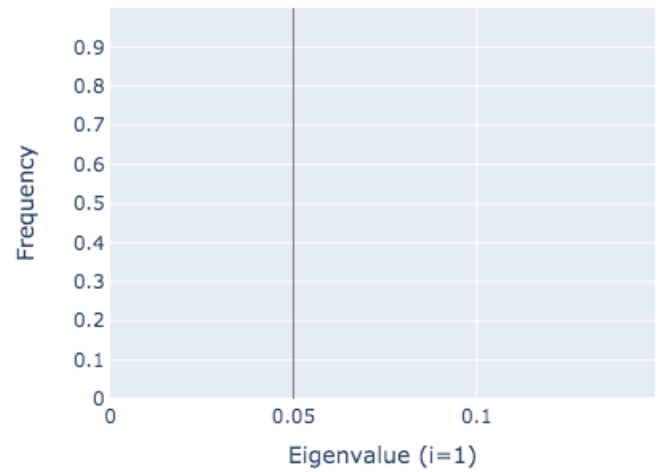
Min: -7.026596454684508e-08
Max: 0.14863581877494952
|Max-Min|: 0.14863588904091407

N=100, Nrep=10

Eigenvalue (i=0)
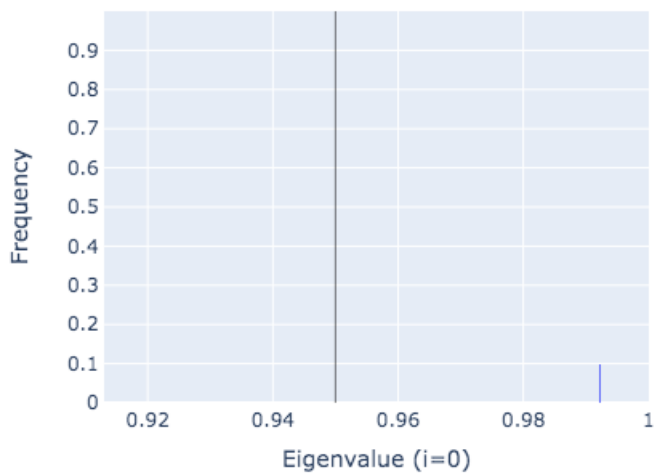
Min: 0.9130584509383943
Max: 1.000000078401553
|Max-Min|: 0.0869416274631587

N=100, Nrep=10

Eigenvalue (i=1)
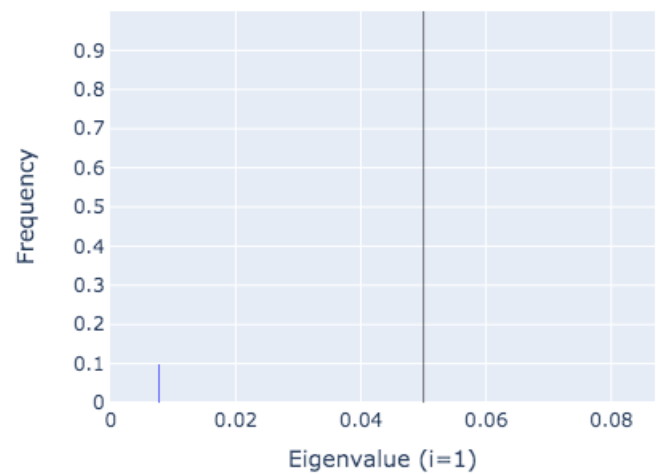
Min: -7.840155328241432e-08
Max: 0.08694154906160545
|Max-Min|: 0.08694162746315873

Case 5: Maximum-Likelihood (False)

N=10, Nrep=10

Eigenvalue (i=0)
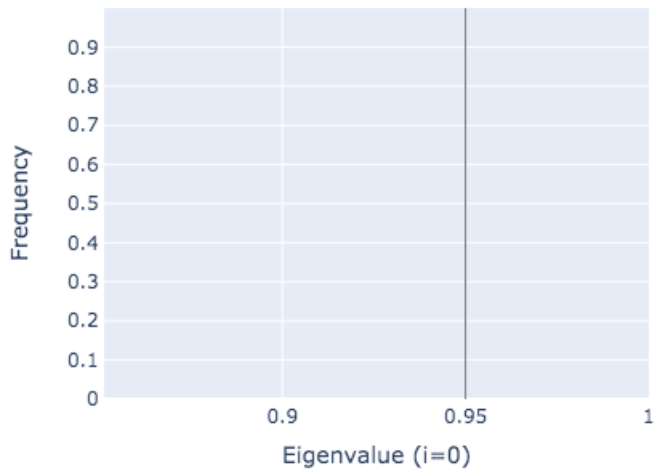
Min: 0.851427240055816
Max: 1.000000162272261
|Max-Min|: 0.1485729222164449

N=10, Nrep=10

Eigenvalue (i=1)
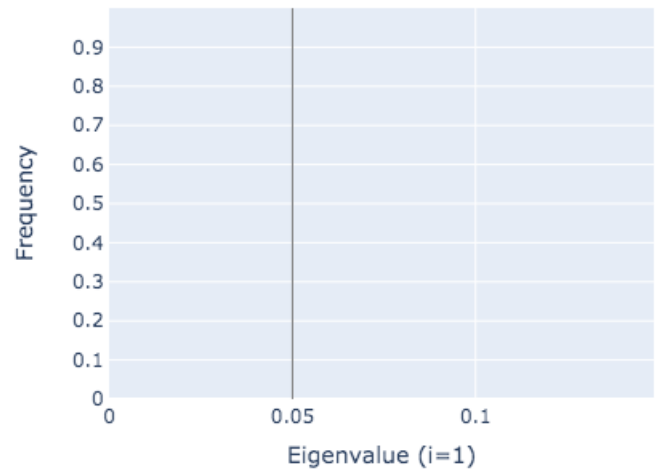
Min: -1.3877787807814457e-17
Max: 0.14857279250213073
|Max-Min|: 0.14857279250213073

N=100, Nrep=10

Eigenvalue (i=0)
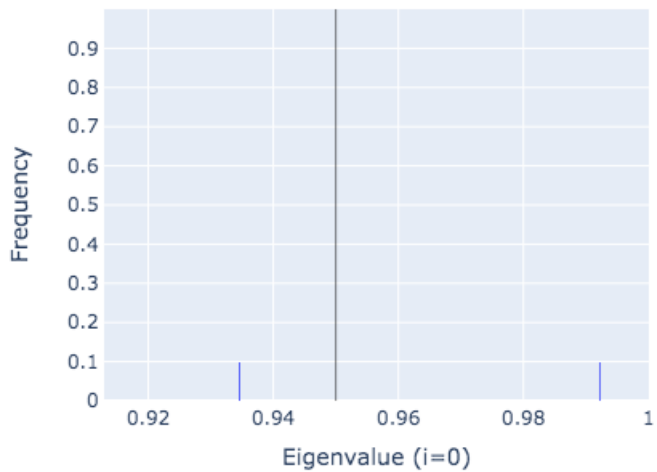
Min: 0.9130380513737355
Max: 1.0000001810607806
|Max-Min|: 0.08696212968704509

N=100, Nrep=10

Eigenvalue (i=1)
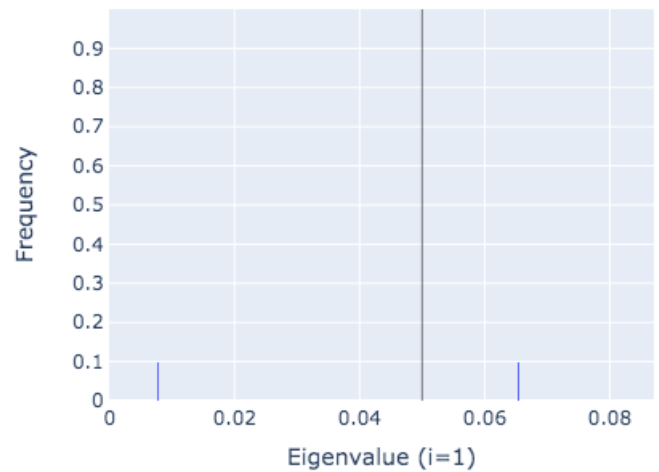
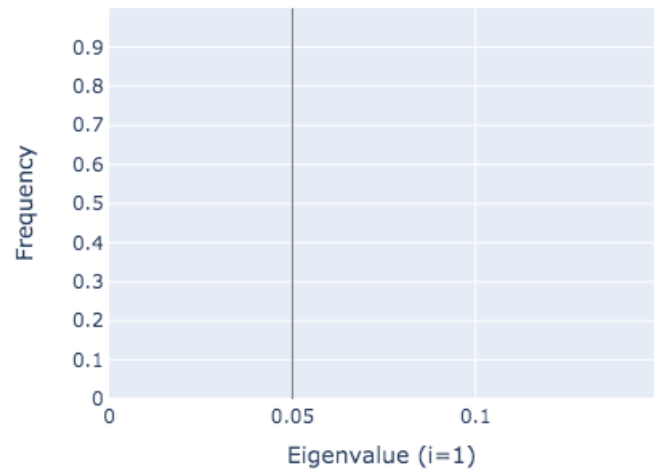Min: -1.0061396160665481e-16
Max: 0.08696196076152173
|Max-Min|: 0.08696196076152182

Case 6: Least Squares (True)
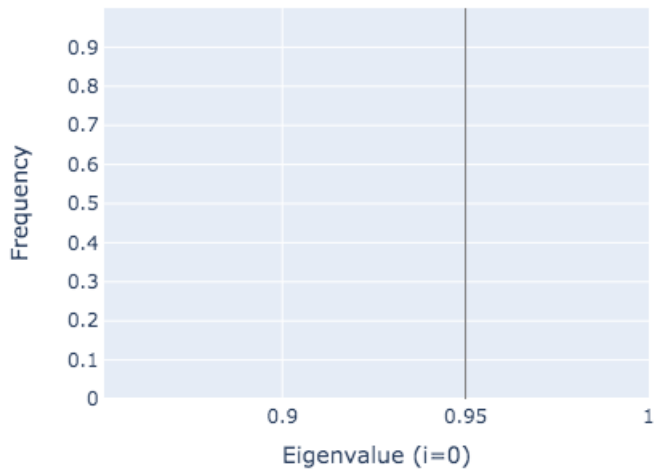
N=10, Nrep=10

Min: 0.851364184463153
Max: 1.0000000761055894
|Max-Min|: 0.1486358916424364

N=10, Nrep=10

Min: -7.610558962201086e-08
Max: 0.14863581553684674
|Max-Min|: 0.14863589164243635

N=100, Nrep=10
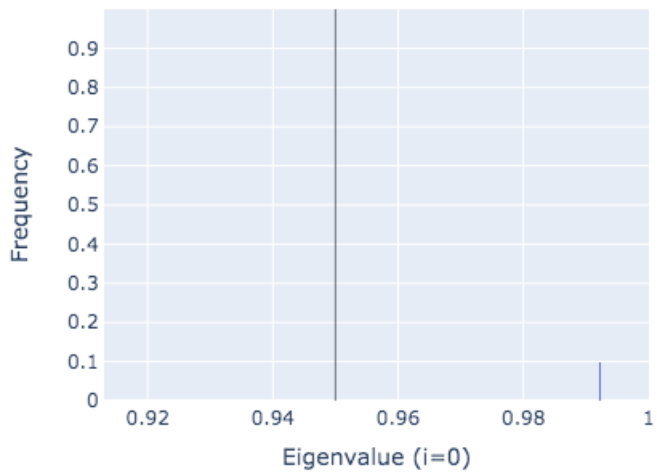
Min: 0.9130584510098028
Max: 1.0000000598780594
|Max-Min|: 0.08694160886825664

N=100, Nrep=10
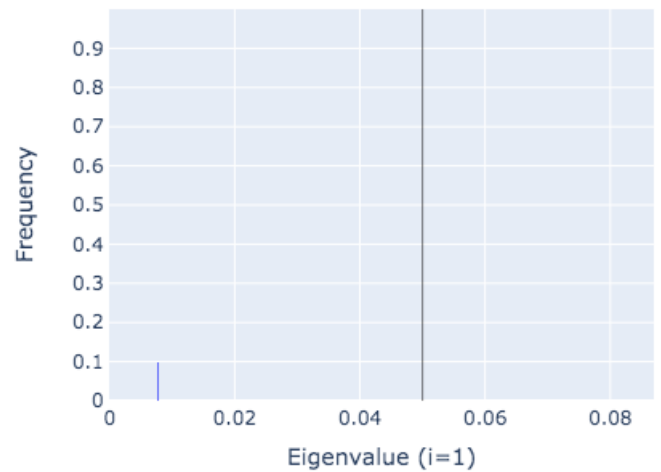
Min: -5.987805948691623e-08
Max: 0.0869415489901971
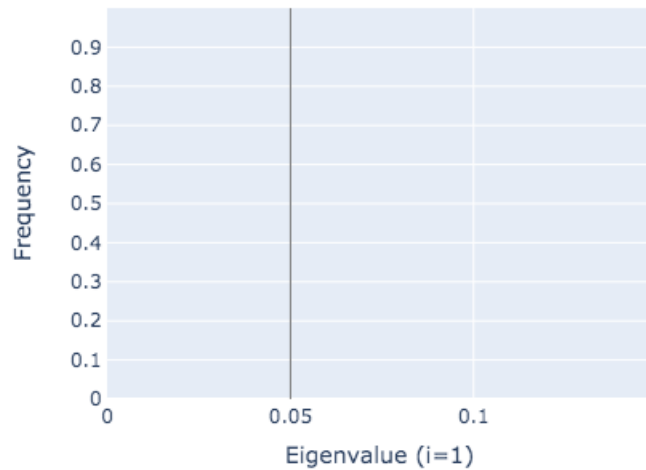|Max-Min|: 0.08694160886825658
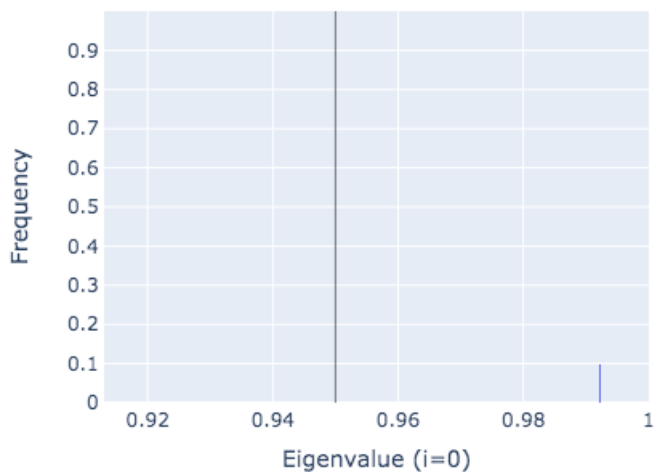
Case 7: Least Squares (False)
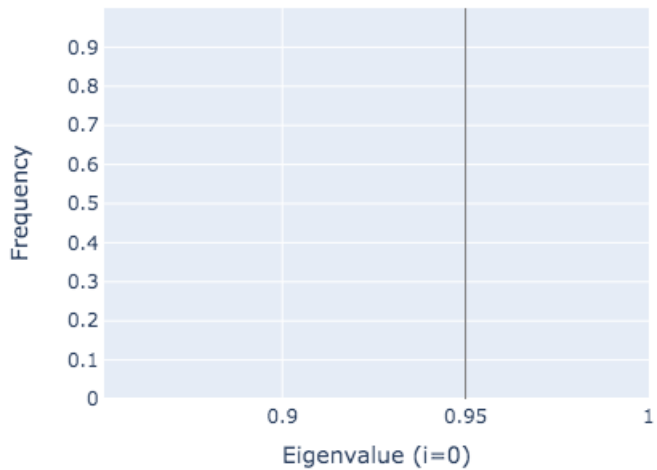
N=10, Nrep=10

Min: 0.8513641844631532
Max: 1.0000001781169368
|Max-Min|: 0.14863599365378355

N=10, Nrep=10

Min: -7.632783294297951e-17
Max: 0.14863581553684663
|Max-Min|: 0.1486358155368467

N=100, Nrep=10

Min: 0.9130584510098023
Max: 1.0000001555680968
|Max-Min|: 0.08694170455829453

N=100, Nrep=10
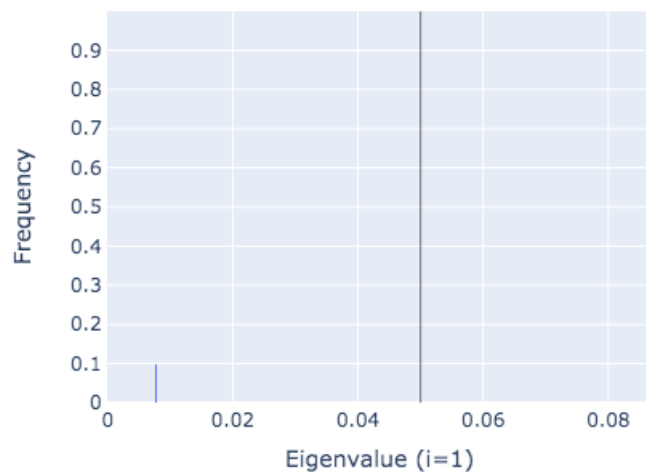
Min: 3.06287113727155e-18
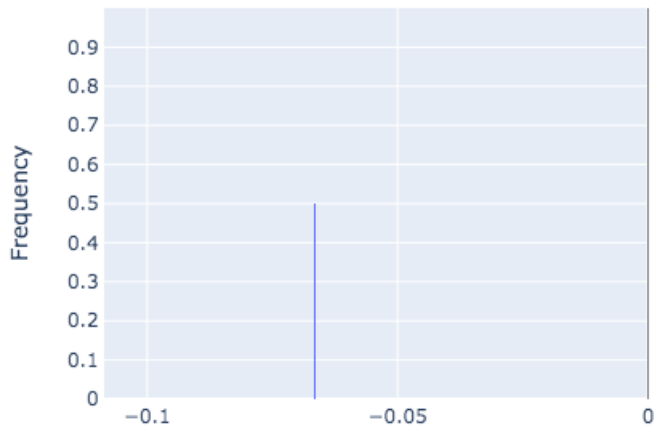Max: 0.08694154899019754
|Max-Min|: 0.08694154899019754
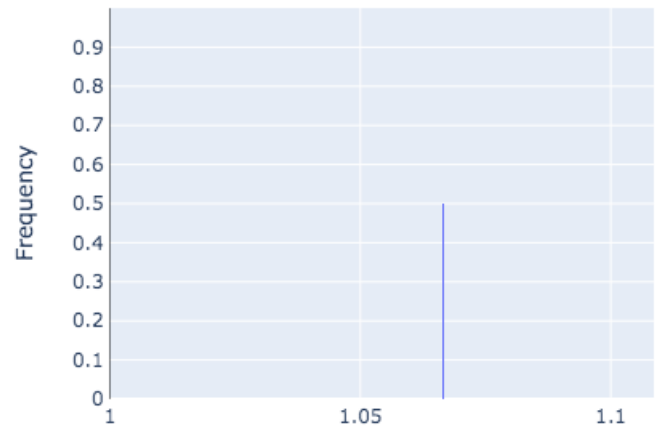
Sum of unphysical eigenvalues

Case 0: Linear (True)

N=10
Number of unphysical estimates=6

Frequency

Sum of negative eigenvalues (<0)

Min: -0.10858061945018471
Max: -0.009175077217315732
|Max-Min|: 0.09940554223286899

Frequency

Sum of non-negative eigenvalues (>1)

Min: 1.0091750772173156
Max: 1.1085806194501846
|Max-Min|: 0.09940554223286902

N=100
Number of unphysical estimates=1

N=100, Nrep=10

Frequency

Sum of negative eigenvalues (<0)

Min: -0.03356476460190986
Max: -0.03356476460190986
|Max-Min|: 0.0

N=100, Nrep=10

Frequency

Sum of non-negative eigenvalues (>1)
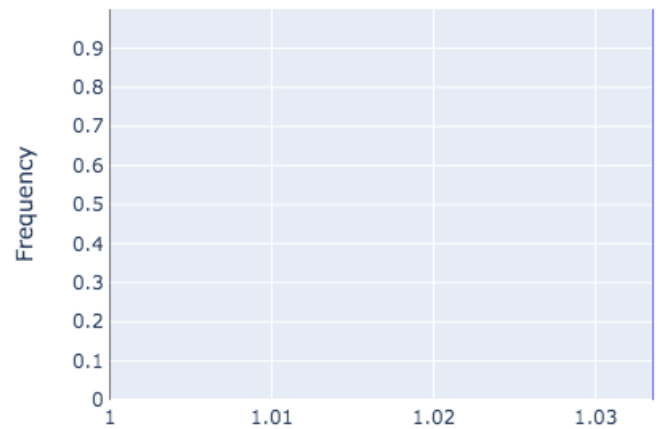
Min: 1.0335647646019095
Max: 1.0335647646019095
|Max-Min|: 0.0

Case 1: Linear (False)

N=10
Number of unphysical estimates=6

N=10, Nrep=10
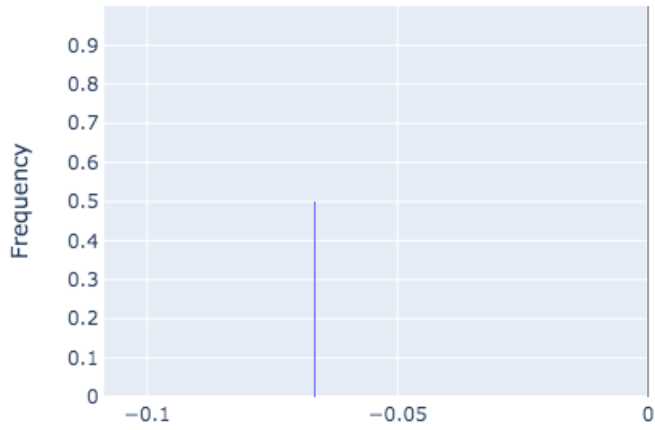
Min: -0.10858061945018459
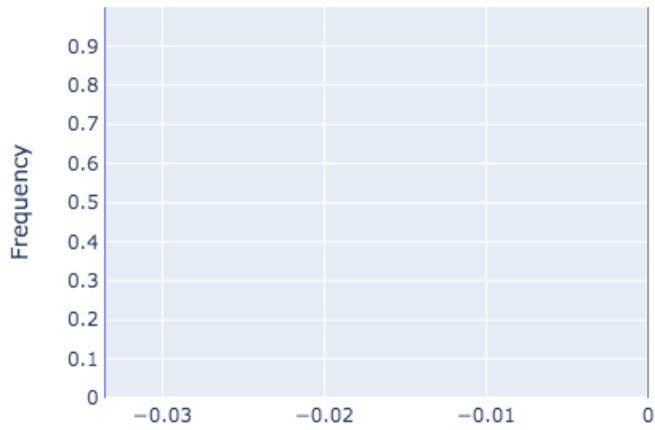Max: -0.009175077217315586
|Max-Min|: 0.099405542232869

N=10, Nrep=10
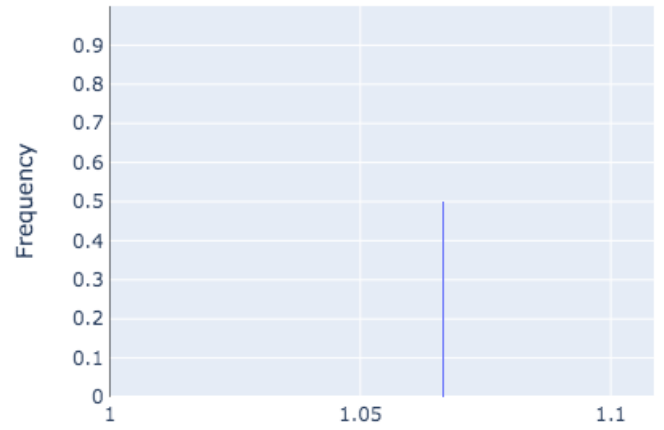
Min: 1.0091750772173156
Max: 1.1085806194501846
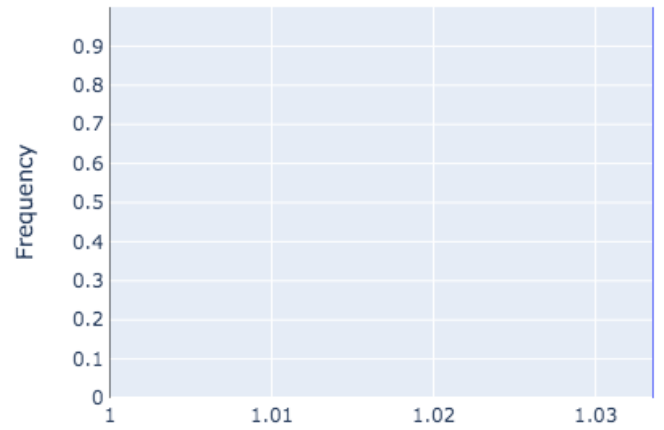|Max-Min|: 0.09940554223286902

N=100
Number of unphysical estimates=1

N=100, Nrep=10

Min: -0.03356476460190975
Max: -0.03356476460190975
|Max-Min|: 0.0
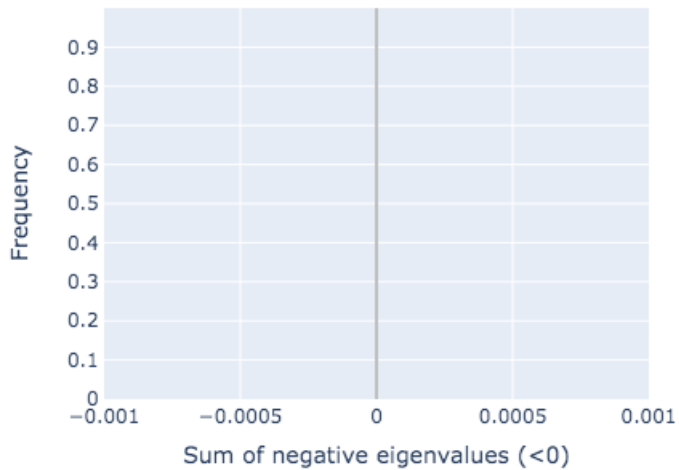
N=100, Nrep=10

Min: 1.0335647646019095
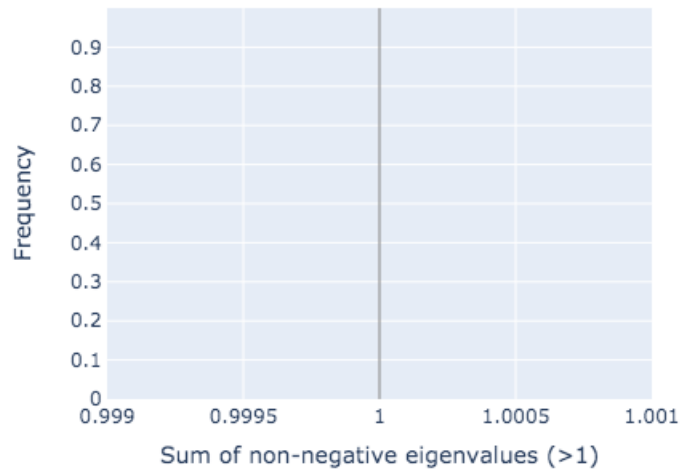Max: 1.0335647646019095
|Max-Min|: 0.0

Case 2: ProjectedLinear (True)

N=10
Number of unphysical estimates=0

N=10, Nrep=10

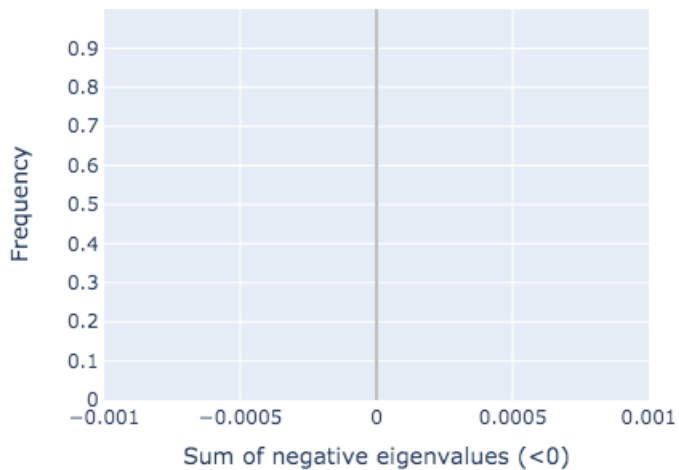Frequency vs Sum of negative eigenvalues (<0)

Min: -
Max: -
|Max-Min|: -

N=10, Nrep=10

Frequency vs Sum of non-negative eigenvalues (>1)

Min: -
Max: -
|Max-Min|: -

N=100
Number of unphysical estimates=0

N=100, Nrep=10

Frequency vs Sum of negative eigenvalues (<0)

Min: -
Max: -
|Max-Min|: -

N=100, Nrep=10

Frequency vs Sum of non-negative eigenvalues (>1)
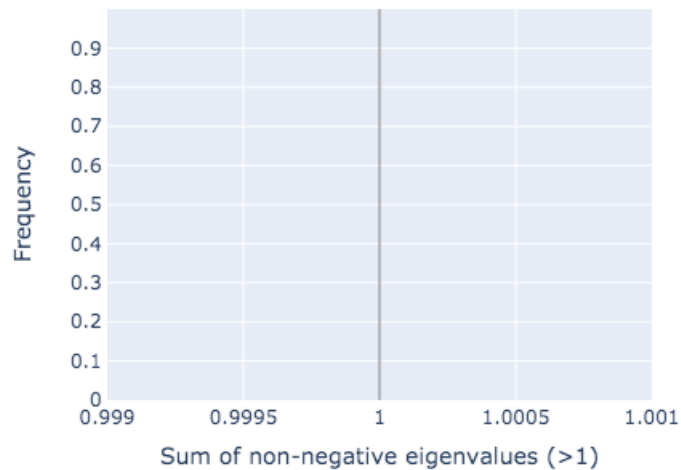
Min: -
Max: -
|Max-Min|: -

Case 3: ProjectedLinear (False)
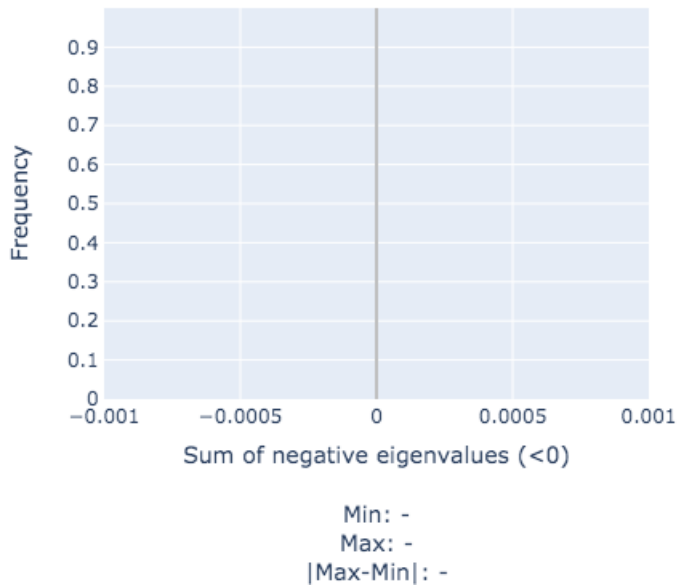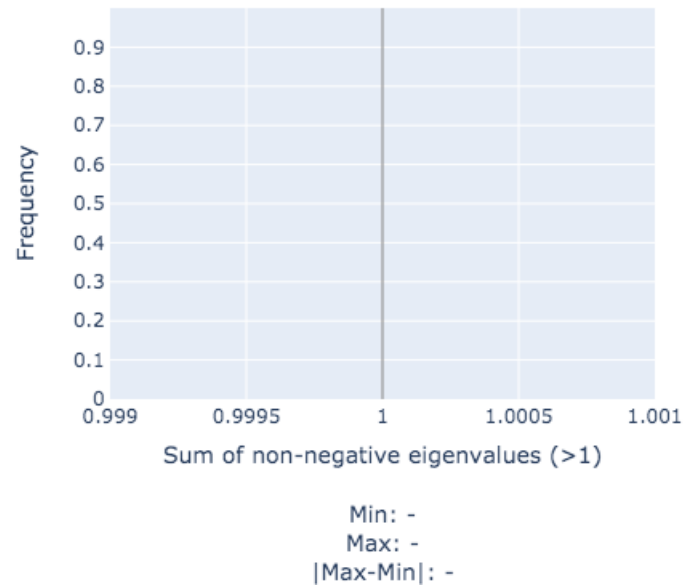
N=10
Number of unphysical estimates=0

N=10, Nrep=10

Sum of negative eigenvalues (<0)

Min: -
Max: -
|Max-Min|: -

N=10, Nrep=10

Sum of non-negative eigenvalues (>1)

Min: -
Max: -
|Max-Min|: -

N=100
Number of unphysical estimates=0

N=100, Nrep=10

Sum of negative eigenvalues (<0)

Min: -
Max: -
|Max-Min|: -

N=100, Nrep=10

Sum of non-negative eigenvalues (>1)

Min: -
Max: -
|Max-Min|: -
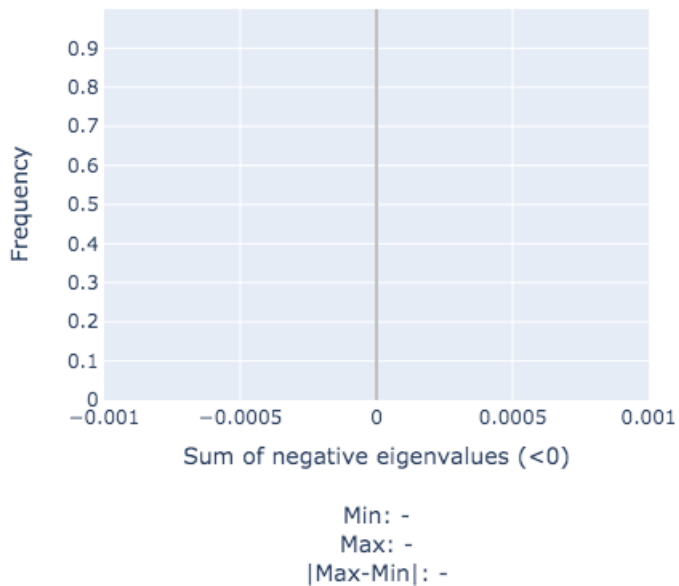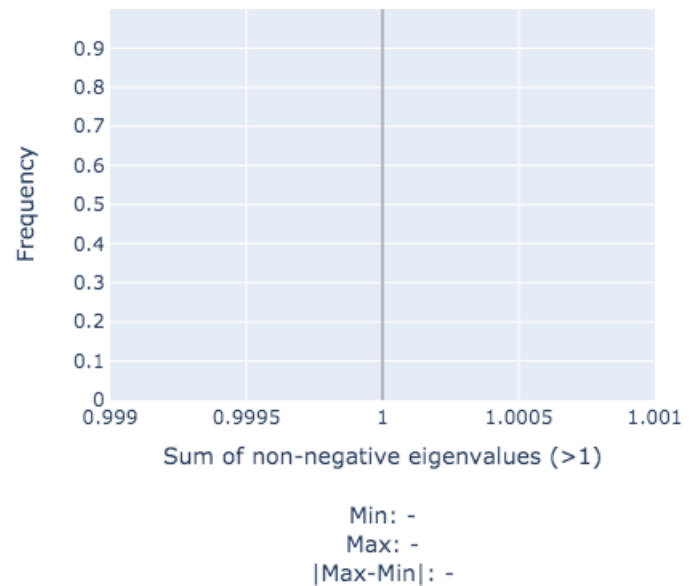
Case 4: Maximum-Likelihood (True)

N=10
Number of unphysical estimates=0
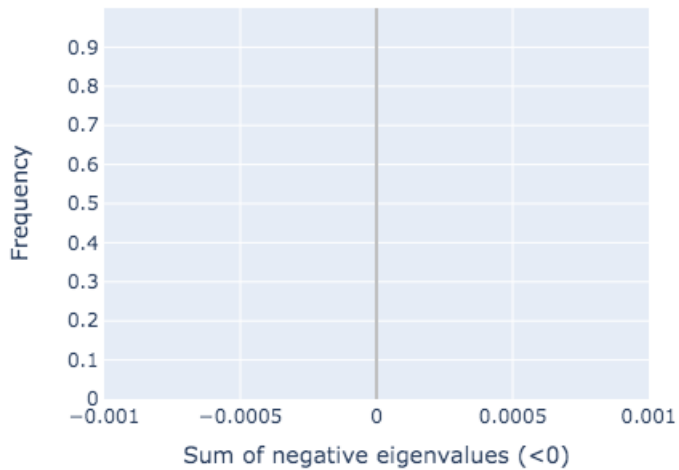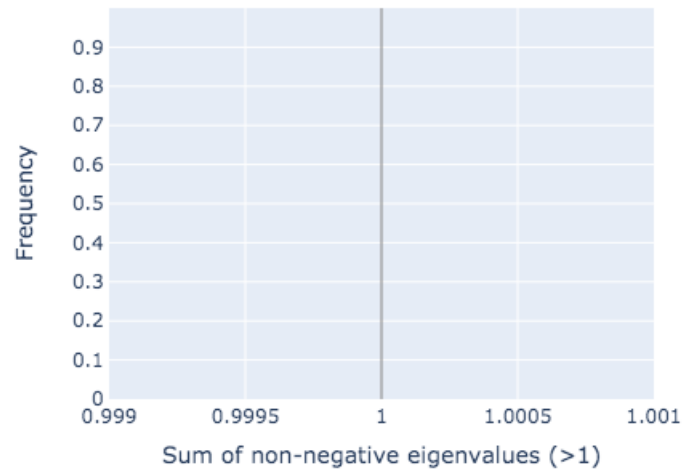
N=10, Nrep=10

Min: -
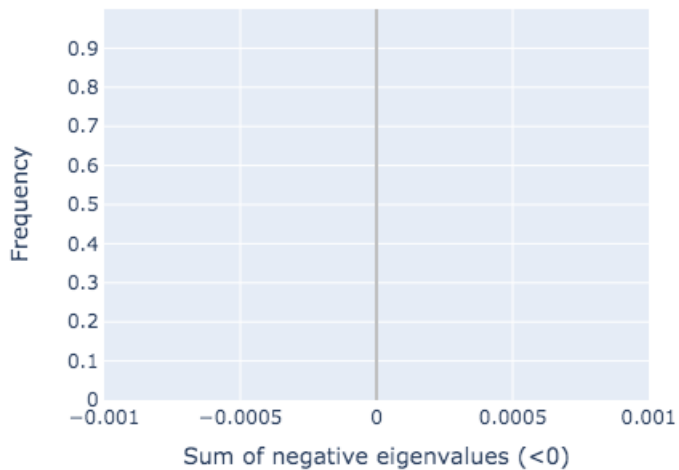Max: -
|Max-Min|: -

N=10, Nrep=10
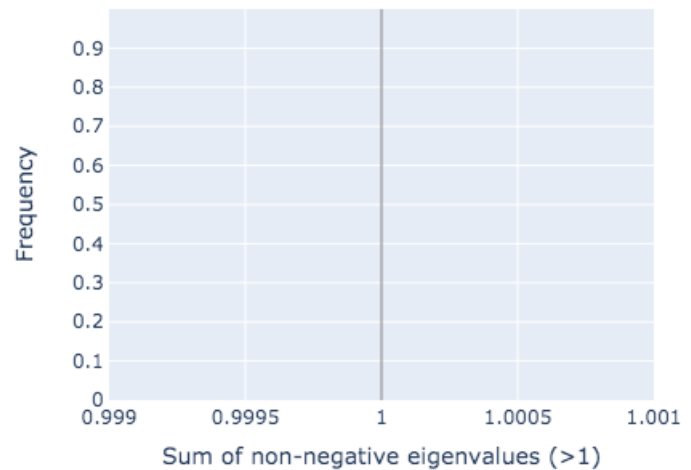

Min: -
Max: -
|Max-Min|: -

N=100
Number of unphysical estimates=0

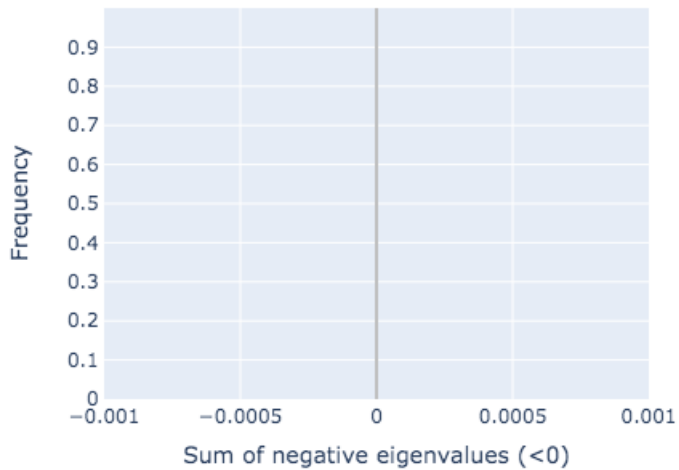## N=100, Nrep=10

Min: -
Max: -
|Max-Min|: -

N=100, Nrep=10

Min: -
Max: -
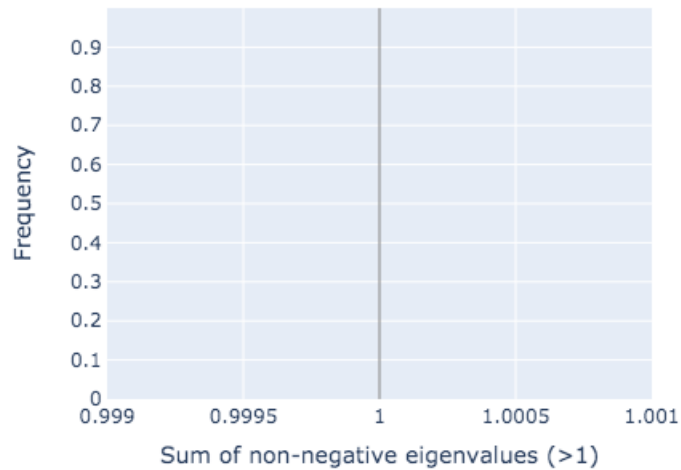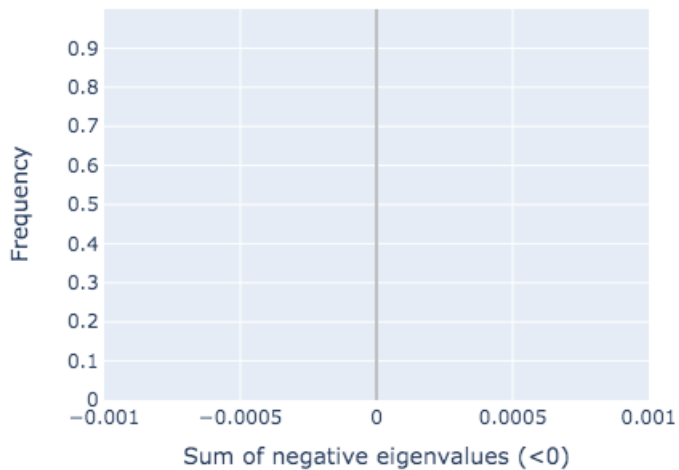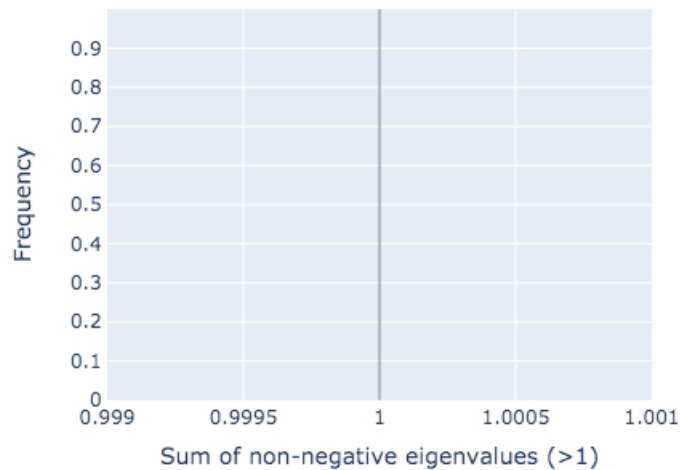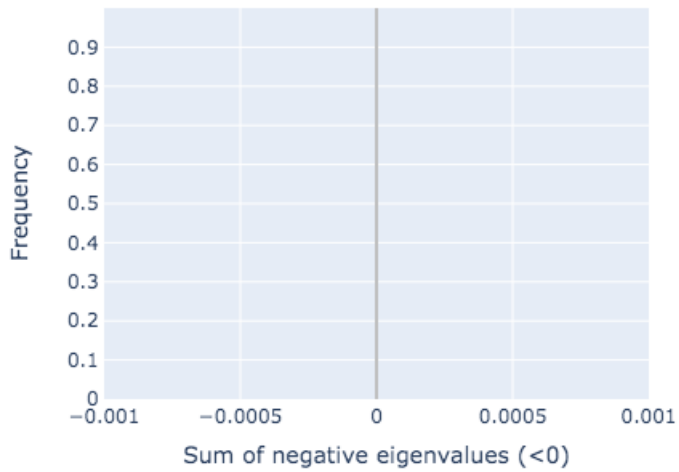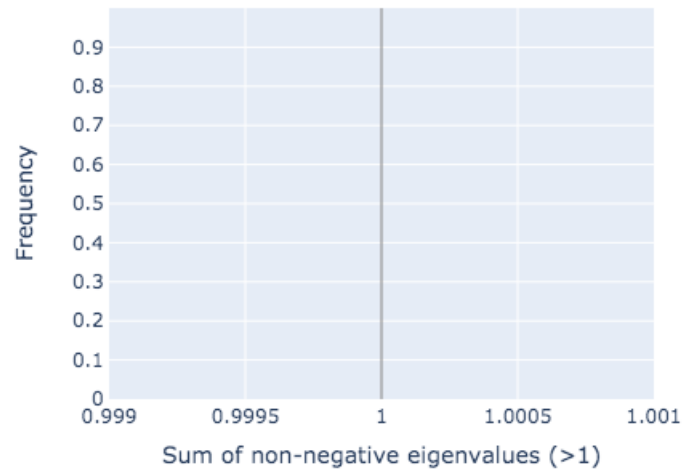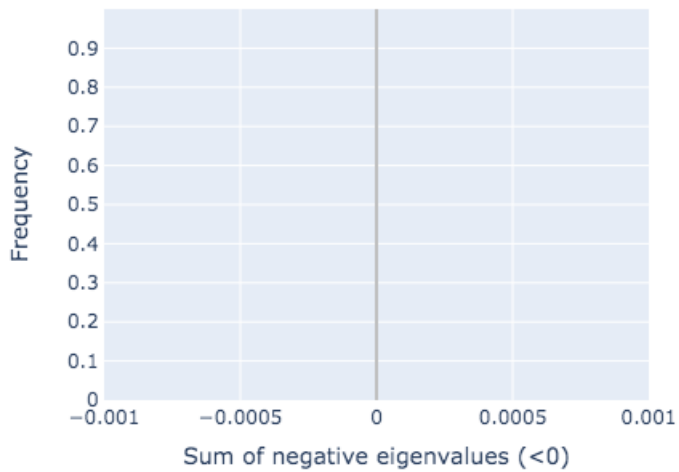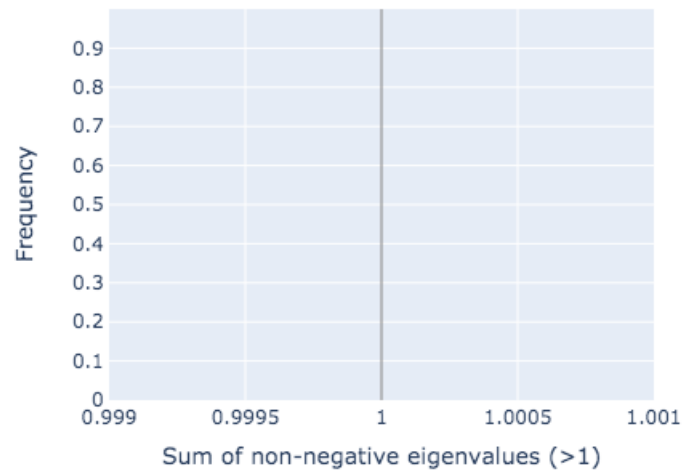|Max-Min|: -

Case 5: Maximum-Likelihood (False)

N=10
Number of unphysical estimates=0

N=10, Nrep=10

Sum of negative eigenvalues (<0)

Min: -
Max: -
|Max-Min|: -

N=10, Nrep=10

Sum of non-negative eigenvalues (>1)

Min: -
Max: -
|Max-Min|: -

N=100
Number of unphysical estimates=0

N=100, Nrep=10

Sum of negative eigenvalues (<0)

Min: -
Max: -
|Max-Min|: -

N=100, Nrep=10

Sum of non-negative eigenvalues (>1)

Min: -
Max: -
|Max-Min|: -

Case 6: Least Squares (True)

N=10
Number of unphysical estimates=0

N=10, Nrep=10

Sum of negative eigenvalues (<0)

Min: -
Max: -
|Max-Min|: -

N=10, Nrep=10

Sum of non-negative eigenvalues (>1)

Min: -
Max: -
|Max-Min|: -

N=100
Number of unphysical estimates=0

N=100, Nrep=10

Sum of negative eigenvalues (<0)

Min: -
Max: -
|Max-Min|: -

N=100, Nrep=10

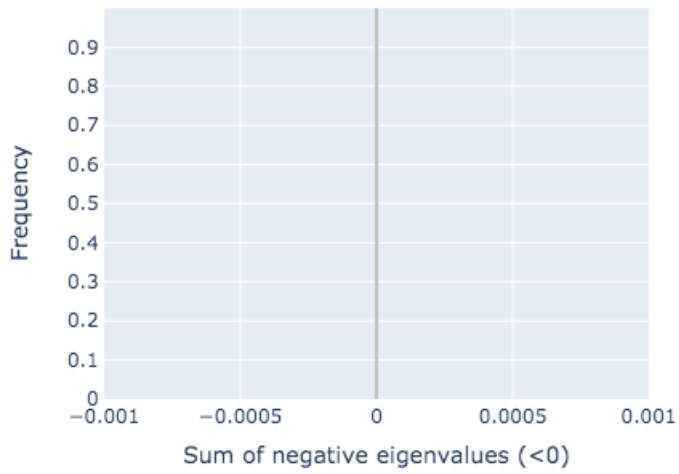Sum of non-negative eigenvalues (>1)

Min: -
Max: -
|Max-Min|: -
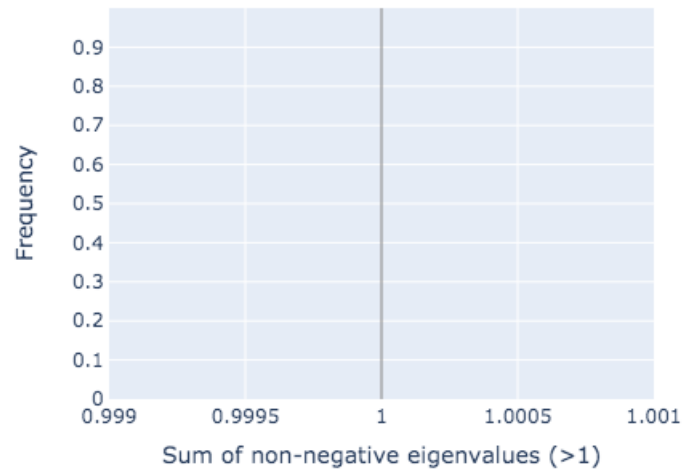
Case 7: Least Squares (False)

N=10
Number of unphysical estimates=0

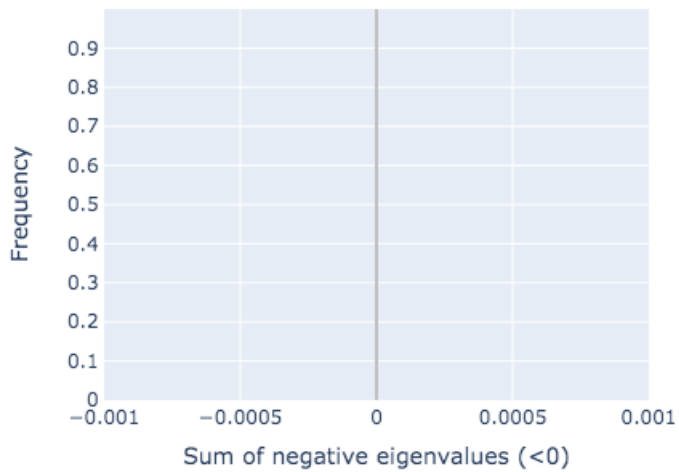## N=10, Nrep=10

Min: -
Max: -
|Max-Min|: -

N=10, Nrep=10

Min: -
Max: -
|Max-Min|: -

N=100
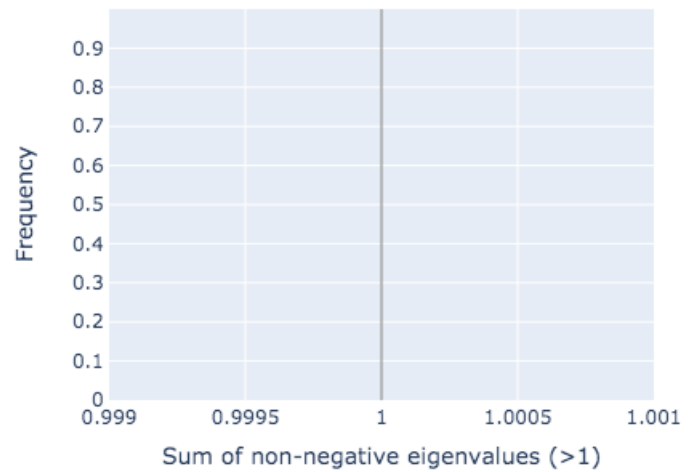**Number of unphysical estimates=0**

N=100, Nrep=10

Min: -
Max: -
|Max-Min|: -

N=100, Nrep=10

Min: -
Max: -
|Max-Min|: -