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Computation time in total

Total	0.5482404192288717min.
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Tolerance of physicality constraint violation

		True	False
Tolerance at physicality violation test	equality constraint	1.00e-13	1.00e-05
	inequality constraint	1.00e-05	1.00e-05

Experimental condition

Type of tomography	StandardQst
Nrep	10
N	[10, 100]
RNG seed	777

True object

Type	State
Dim	2
Vec	[0.70710678 0.04314638 -0.03754928 0.64700626]

Tester objects

0	Type	Povm
	Dim	2
	Number of outcomes	2
	Vecs	[[0.67045551 0.63151808 -0.00840615 -0.03564561] [0.74375805 -0.63151808 0.00840615 0.03564561]]
1	Type	Povm
	Dim	2
	Number of outcomes	2
	Vecs	[[0.72596364 -0.04732708 0.64096095 -0.08116688] [0.68824993 0.04732708 -0.64096095 0.08116688]]
2	Type	Povm
	Dim	2
	Number of outcomes	2
	Vecs	[[0.71790027 0.02984105 -0.02029088 0.66221865] [0.6963133 -0.02984105 0.02029088 -0.66221865]]

Cases

	Name	Param	Tomography	Estimator
0	Linear (True)	True	StandardQst	Linear
1	Linear (False)	False	StandardQst	Linear
2	ProjectedLinear (True)	True	StandardQst	ProjectedLinear
3	ProjectedLinear (False)	False	StandardQst	ProjectedLinear
4	Maximum-Likelihood (True)	True	StandardQst	LossMinimization
5	Maximum-Likelihood (False)	False	StandardQst	LossMinimization

6	Least Squares (True)	True	StandardQst	LossMinimization
7	Least Squares (False)	False	StandardQst	LossMinimization

Computation time of estimators

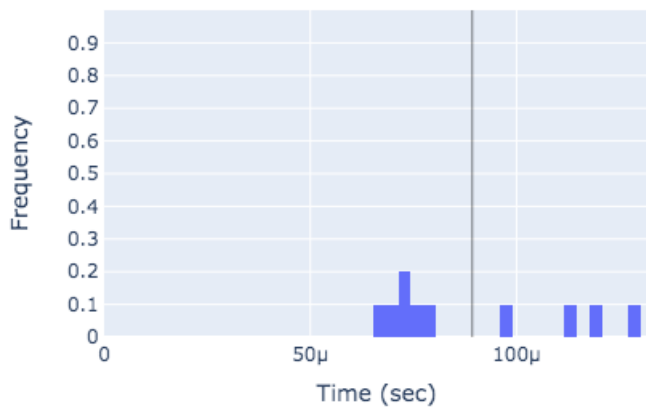
Table

	Name	N	Nrep	Mean (sec)	Std (sec)
0	Linear (True)	10	10	0.000089	0.000022
1	"	100	10	0.000083	0.000043
2	Linear (False)	10	10	0.000092	0.000030
3	"	100	10	0.000154	0.000203
4	ProjectedLinear (True)	10	10	0.009561	0.007971
5	"	100	10	0.005138	0.007851
6	ProjectedLinear (False)	10	10	0.006580	0.004468
7	"	100	10	0.003032	0.003636
8	Maximum-Likelihood (True)	10	10	0.523598	0.132858
9	"	100	10	0.574862	0.205882
10	Maximum-Likelihood (False)	10	10	0.617612	0.116209
11	"	100	10	0.775926	0.332860
12	Least Squares (True)	10	10	0.248239	0.167760
13	"	100	10	0.116347	0.084048
14	Least Squares (False)	10	10	0.257915	0.107904
15	"	100	10	0.150216	0.072488

Histogram

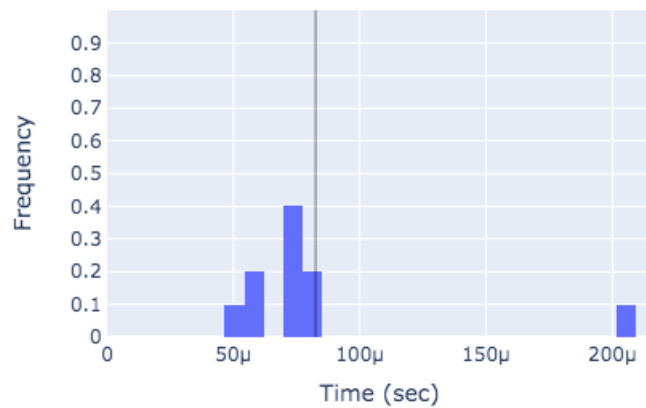
Case 0: Linear (True)

N=10, Nrep=10
Mean = 8.9263916015625e-05(sec)



Min: 6.699562072753906e-05
Max: 0.0001289844512939453
|Max-Min|: 6.198883056640625e-05

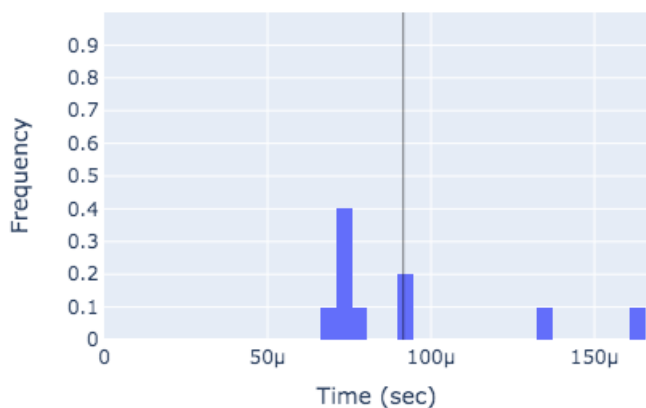
N=100, Nrep=10
Mean = 8.251667022705079e-05(sec)



Min: 5.2928924560546875e-05
Max: 0.0002079010009765625
|Max-Min|: 0.00015497207641601562

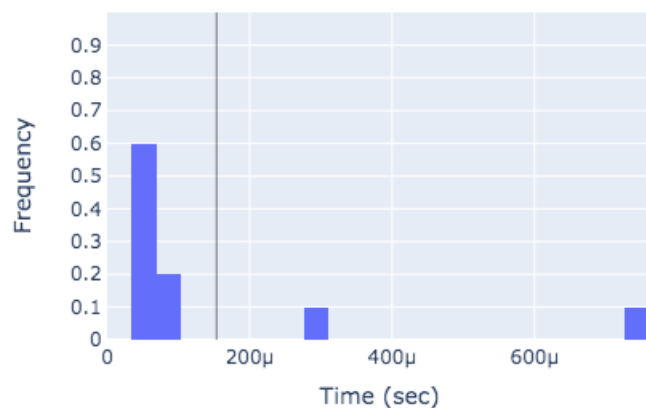
Case 1: Linear (False)

N=10, Nrep=10
Mean = 9.150505065917969e-05(sec)



Min: 6.723403930664062e-05
Max: 0.00016188621520996094
|Max-Min|: 9.465217590332031e-05

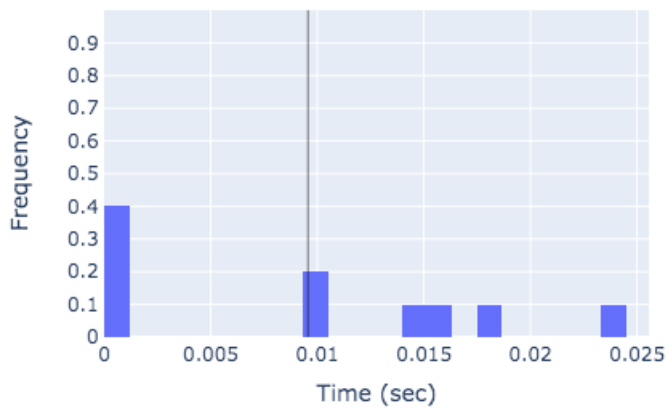
N=100, Nrep=10
Mean = 0.0001535654067993164(sec)



Min: 3.790855407714844e-05
Max: 0.0007300376892089844
|Max-Min|: 0.0006921291351318359

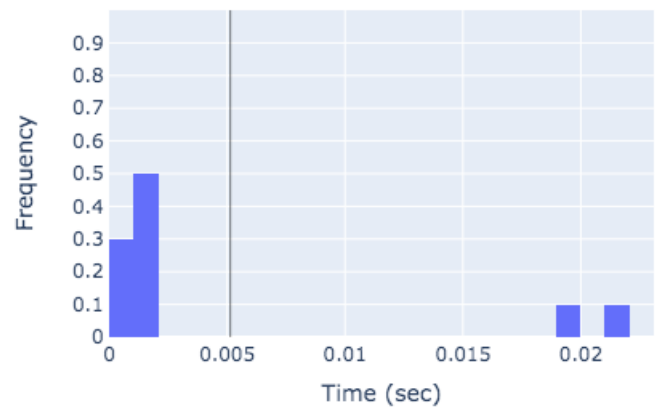
Case 2: ProjectedLinear (True)

N=10, Nrep=10
Mean = 0.009561467170715331(sec)



Min: 0.0010640621185302734
Max: 0.024386882781982422
|Max-Min|: 0.02332282066345215

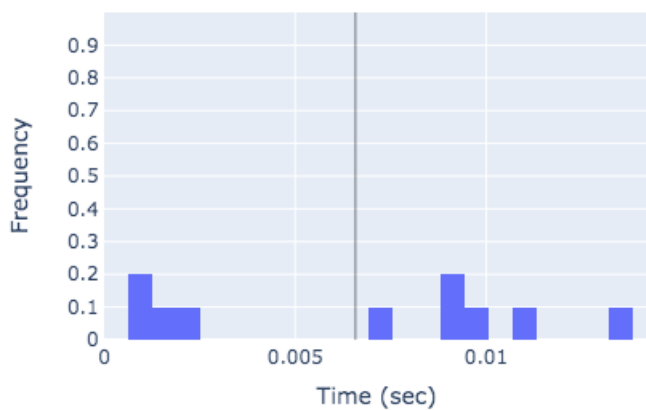
N=100, Nrep=10
Mean = 0.005137920379638672(sec)



Min: 0.0010213851928710938
Max: 0.022044658660888672
|Max-Min|: 0.021023273468017578

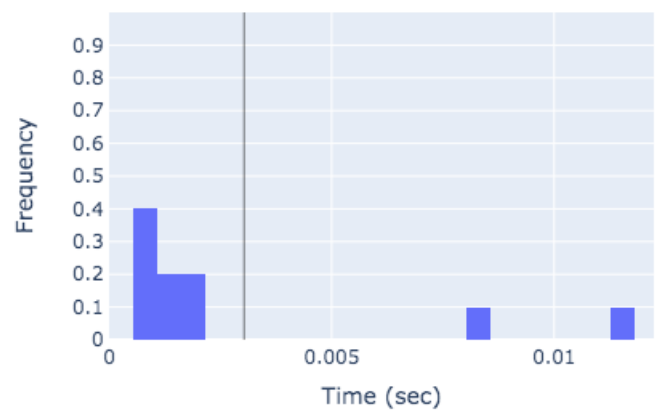
Case 3: ProjectedLinear (False)

N=10, Nrep=10
Mean = 0.006579542160034179(sec)



Min: 0.001055002212524414
Max: 0.013631343841552734
|Max-Min|: 0.01257634162902832

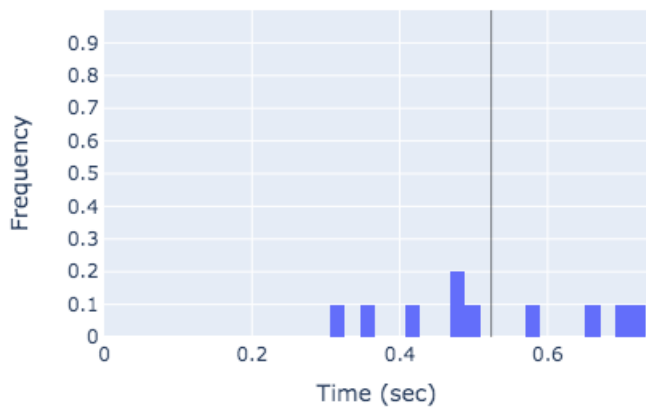
N=100, Nrep=10
Mean = 0.0030318737030029298(sec)



Min: 0.0009768009185791016
Max: 0.011703252792358398
|Max-Min|: 0.010726451873779297

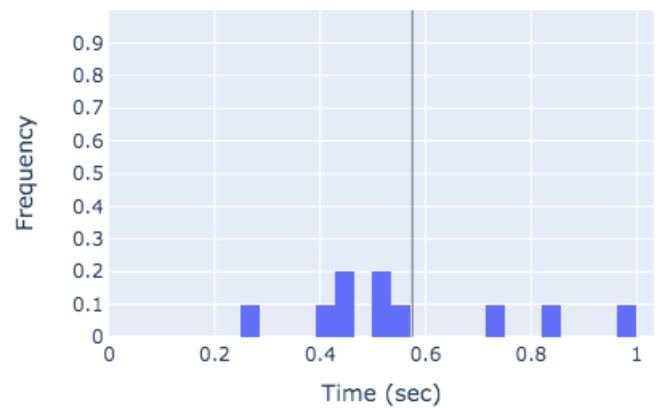
Case 4: Maximum-Likelihood (True)

N=10, Nrep=10
Mean = 0.5235982656478881(sec)



Min: 0.3096938133239746
Max: 0.7164947986602783
|Max-Min|: 0.4068009853363037

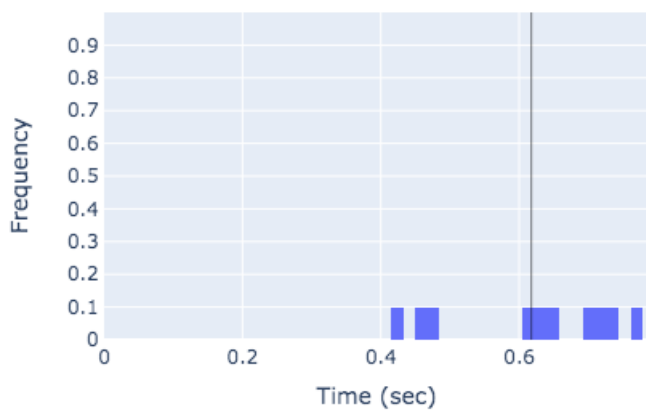
N=100, Nrep=10
Mean = 0.5748616456985474(sec)



Min: 0.2837862968444824
Max: 0.9969861507415771
|Max-Min|: 0.7131998538970947

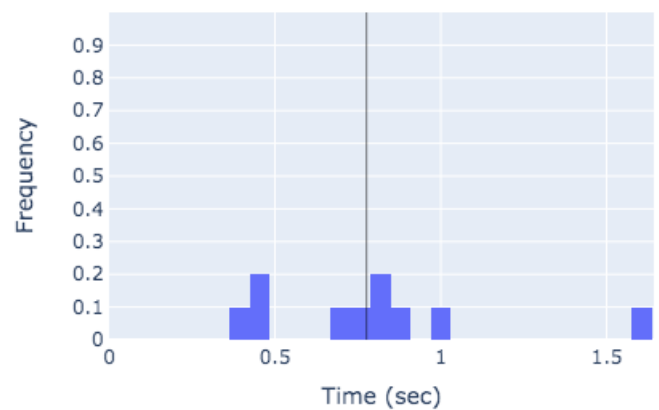
Case 5: Maximum-Likelihood (False)

N=10, Nrep=10
Mean = 0.6176115274429321(sec)



Min: 0.4242079257965088
Max: 0.7701539993286133
|Max-Min|: 0.3459460735321045

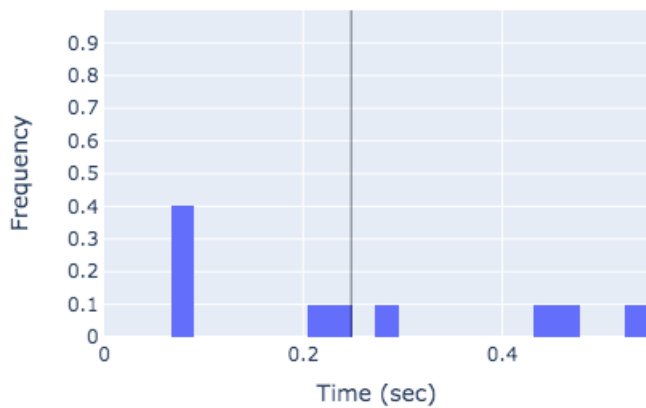
N=100, Nrep=10
Mean = 0.7759257793426514(sec)



Min: 0.36858296394348145
Max: 1.581549882888794
|Max-Min|: 1.2129669189453125

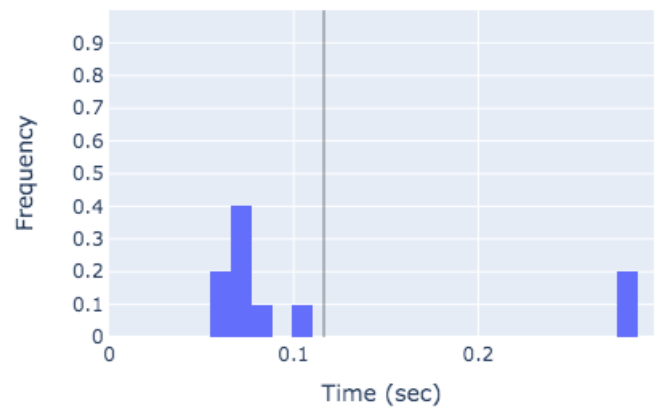
Case 6: Least Squares (True)

N=10, Nrep=10
Mean = 0.24823906421661376(sec)

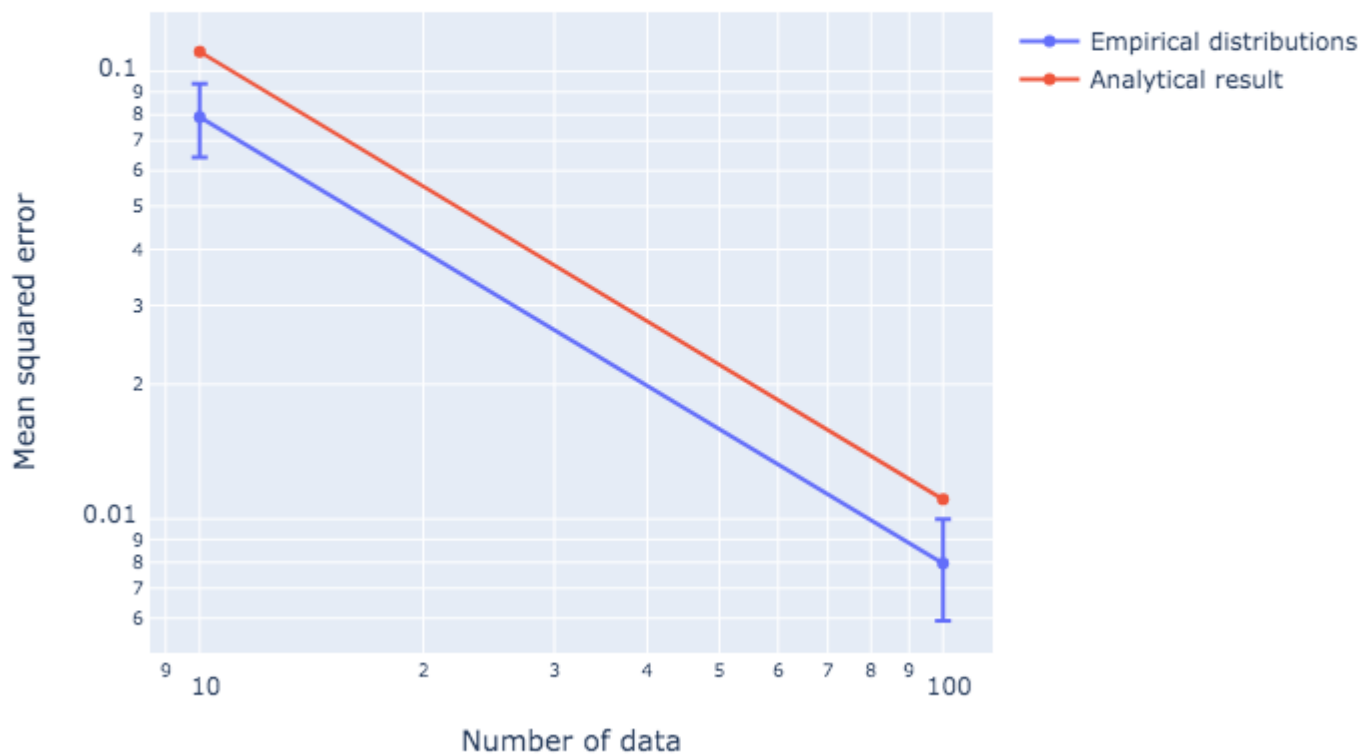


Min: 0.06995296478271484
Max: 0.5246129035949707
|Max-Min|: 0.45465993881225586

N=100, Nrep=10
Mean = 0.1163468599319458(sec)



Mean squared error



Consistency test

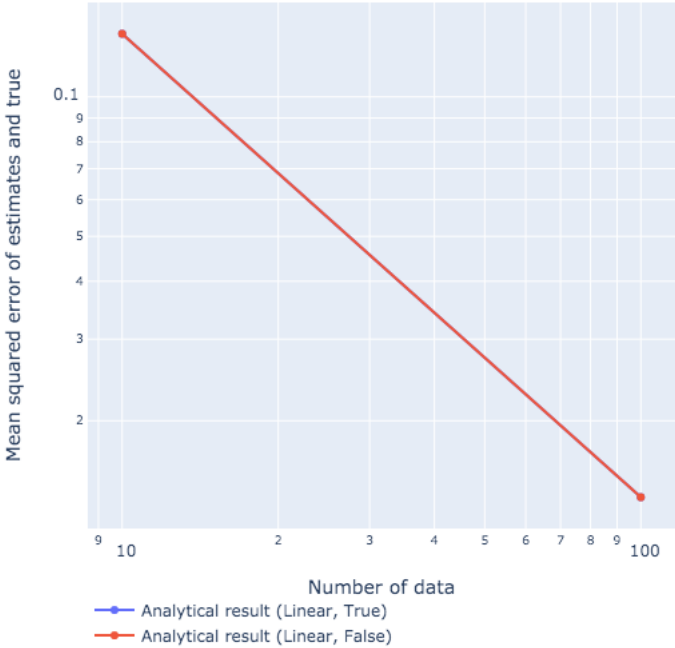
	Name	Type of tomography	Param	Estimator	Loss	Algo	Squared Error to True	Possibly OK	To be checked
0	Linear (True)	Standard Qst	True	Linear	None	None	3.84e-32	OK	not need debug
1	Linear (False)	Standard Qst	False	Linear	None	None	1.06e-31	OK	not need debug
2	ProjectedLinear (True)	Standard Qst	True	Projected Linear	None	None	2.14e-32	OK	not need debug
3	ProjectedLinear (False)	Standard Qst	False	Projected Linear	None	None	1.04e-31	OK	not need debug
4	Maximum-Likelihood (True)	Standard Qst	True	Loss Minimization	Weighted Relative Entropy	Projected Gradient Descent Backtracking	8.42e-18	OK	not need debug
5	Maximum-Likelihood (False)	Standard Qst	False	Loss Minimization	Weighted Relative Entropy	Projected Gradient Descent Backtracking	1.39e-08	NG	not need debug
6	Least Squares (True)	Standard Qst	True	Loss Minimization	Weighted Probability Based Squared Error	Projected Gradient Descent Backtracking	2.21e-31	OK	not need debug

7	Least Squares (False)	Standard Qst	False	Loss Minimization	Weighted Probability Based Squared Error	Projected Gradient Descent Backtracking	4.52e-30	OK	not need debug
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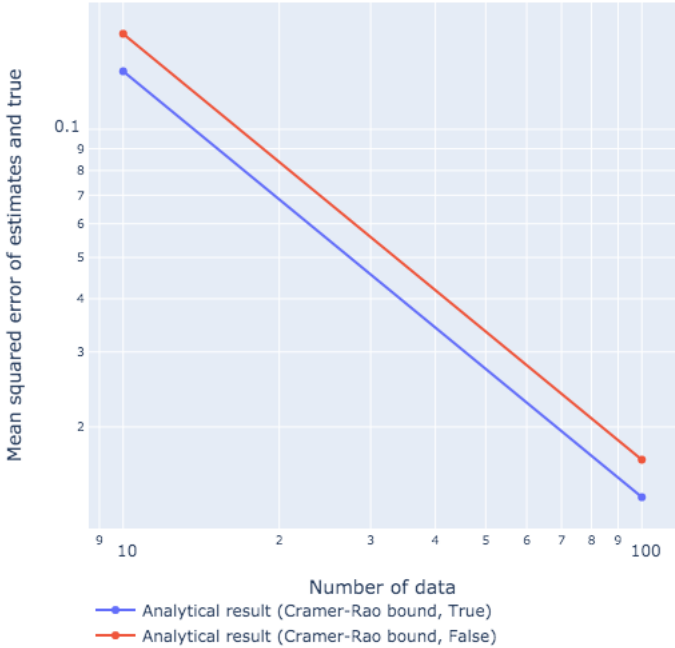
MSE of estimators

Comparison of analytical results

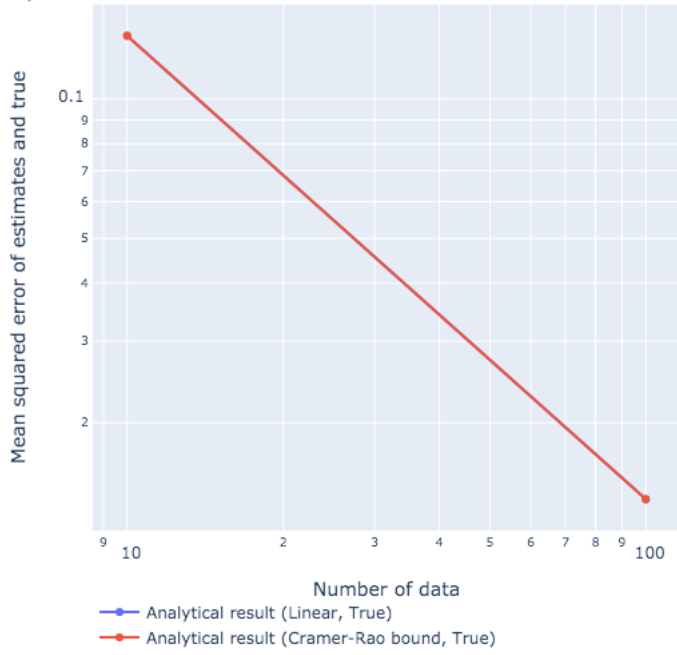
Mean squared error
Analytical result
estimator=Linear



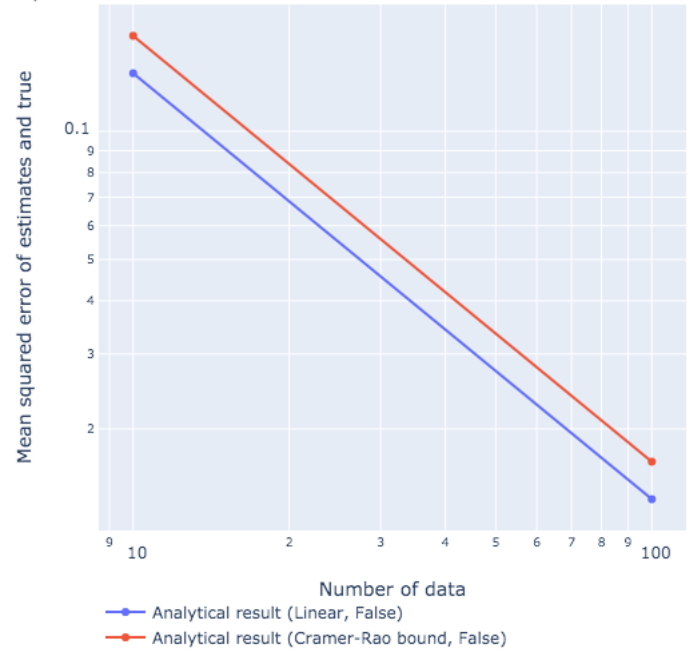
Mean squared error
Analytical result
Cramer-Rao bound



Mean squared error
Analytical result
parameterization=True

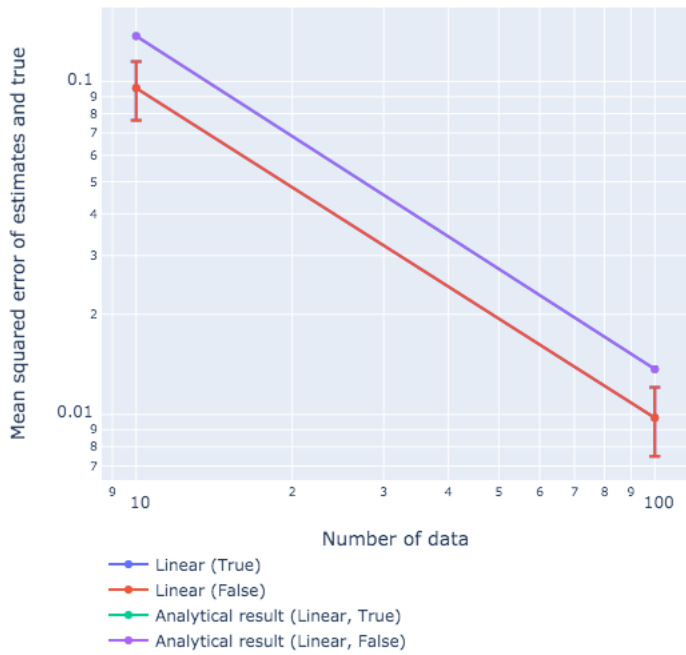


Mean squared error
Analytical result
parameterization=False

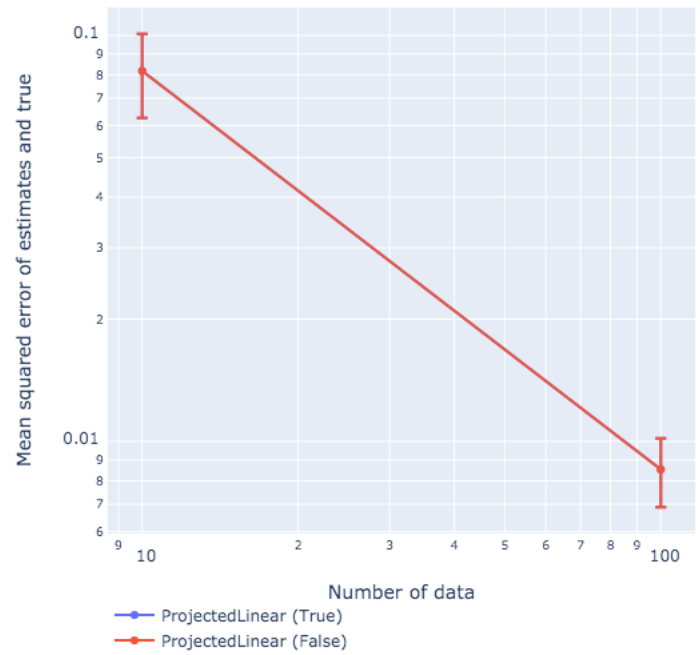


Comparison of parametrization

Mean squared error
Estimator=Linear



Mean squared error
Estimator=ProjectedLinear

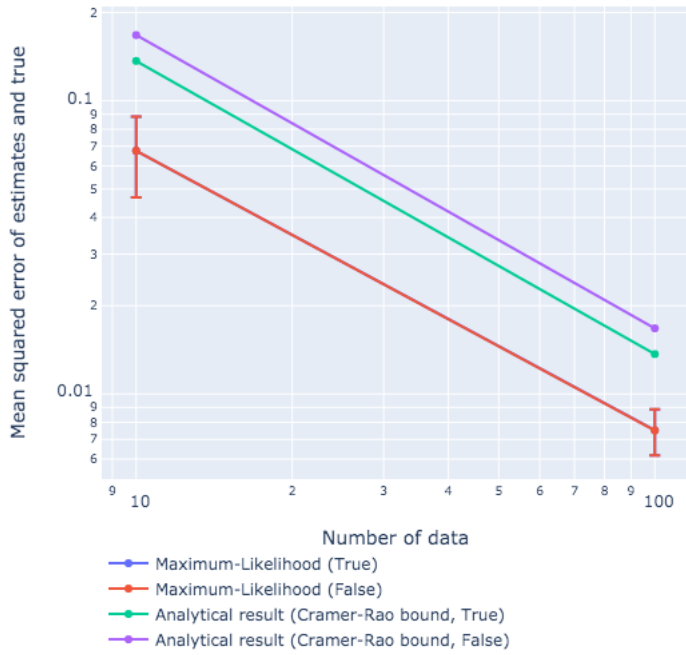


Mean squared error

Estimator=LossMinimization

Loss=WeightedRelativeEntropy

Algo=ProjectedGradientDescentBacktracking

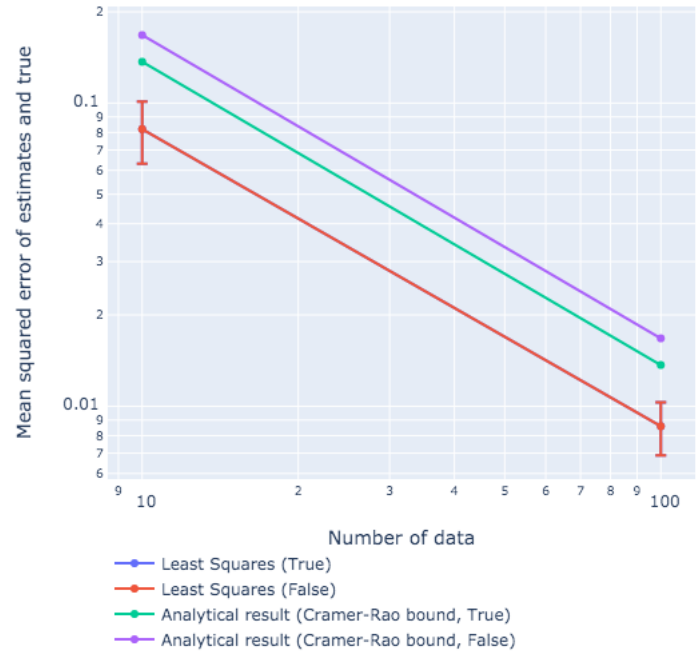


Mean squared error

Estimator=LossMinimization

Loss=WeightedProbabilityBasedSquaredError

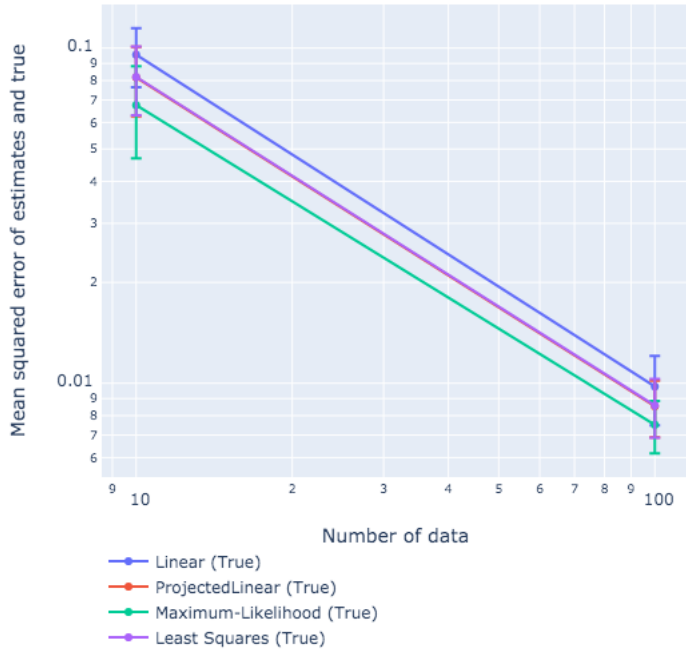
Algo=ProjectedGradientDescentBacktracking



Comparison of estimators

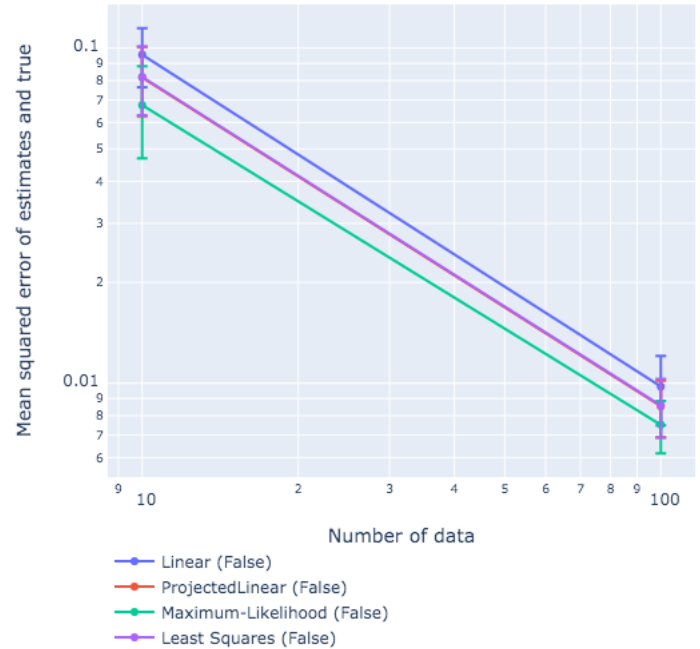
Mean squared error

parametrization=True



Mean squared error

parametrization=False

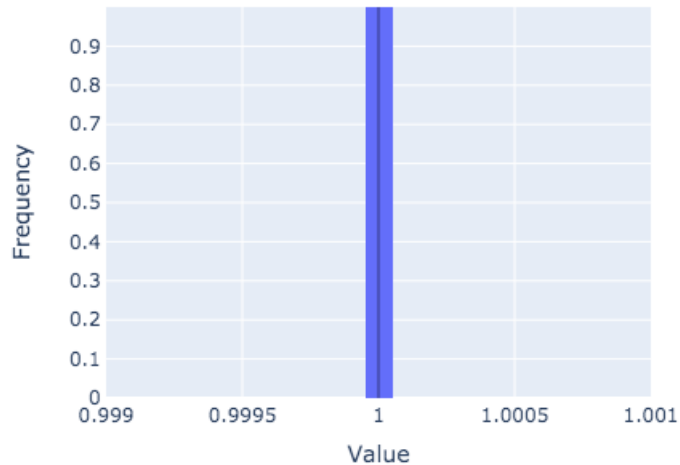


Physicality violation test

Test of equality constraint violation

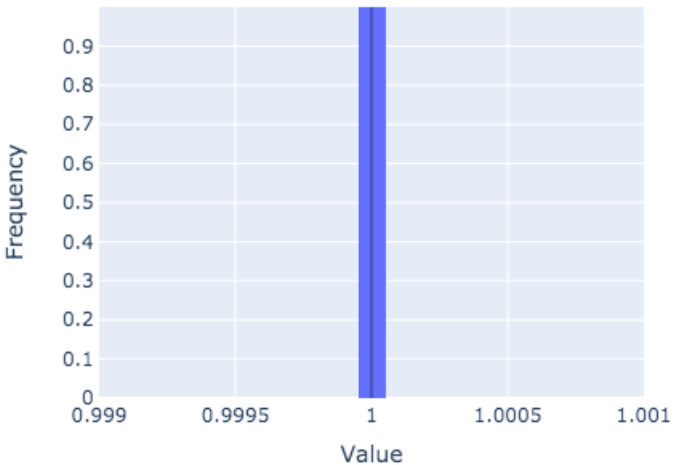
Case 0: Linear (True)

N=10, Nrep=10



Min: 0.9999999999999997
Max: 0.9999999999999999
|Max-Min|: 2.220446049250313e-16

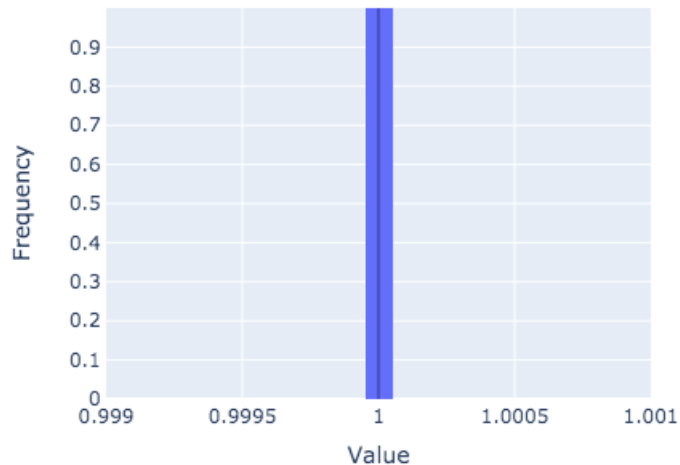
N=100, Nrep=10



Min: 0.9999999999999998
Max: 0.9999999999999999
|Max-Min|: 1.1102230246251565e-16

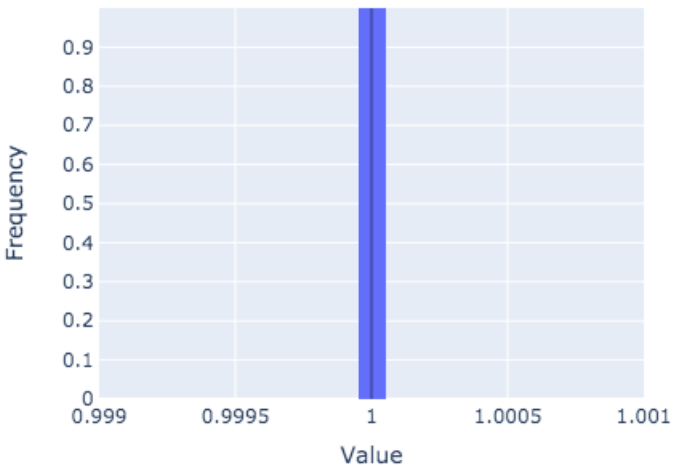
Case 1: Linear (False)

N=10, Nrep=10



Min: 0.9999999999999998
Max: 1.0
|Max-Min|: 2.220446049250313e-16

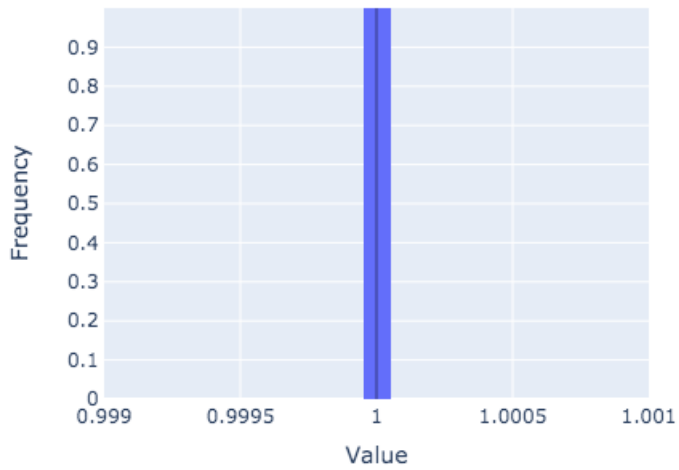
N=100, Nrep=10



Min: 0.9999999999999998
Max: 1.0
|Max-Min|: 2.220446049250313e-16

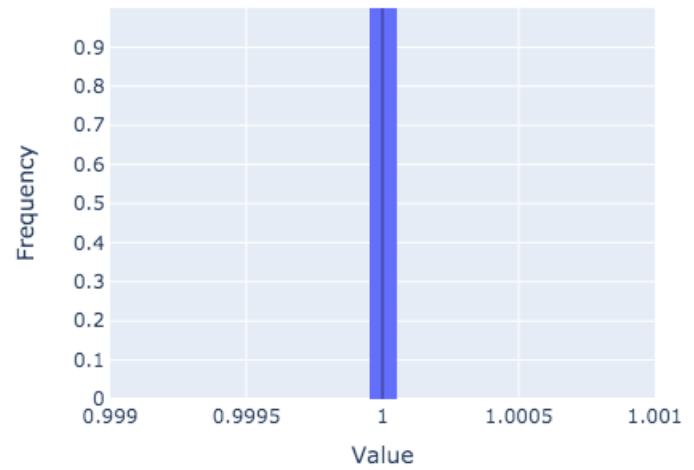
Case 2: ProjectedLinear (True)

N=10, Nrep=10



Min: 0.9999999999999998
Max: 0.9999999999999998
|Max-Min|: 0.0

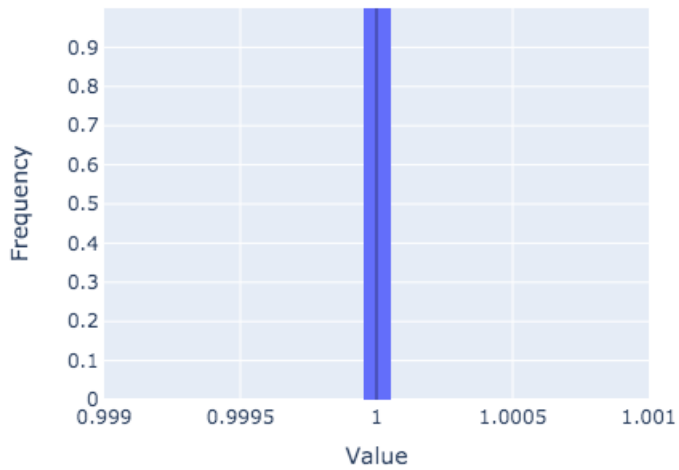
N=100, Nrep=10



Min: 0.9999999999999998
Max: 0.9999999999999998
|Max-Min|: 0.0

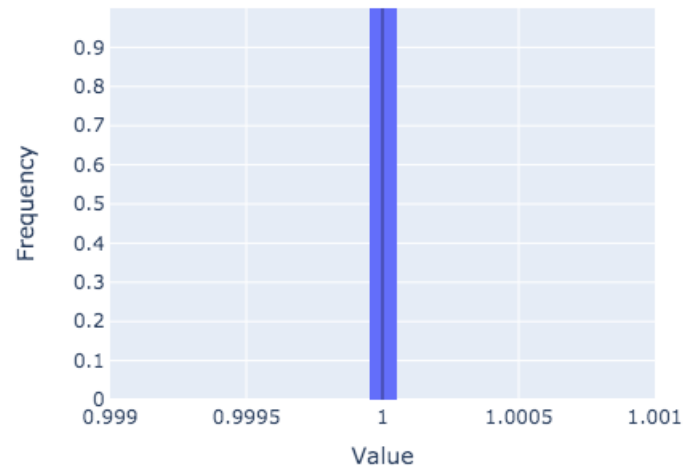
Case 3: ProjectedLinear (False)

N=10, Nrep=10



Min: 0.9999999999999998
Max: 1.0000001786892174
|Max-Min|: 1.786892176447452e-07

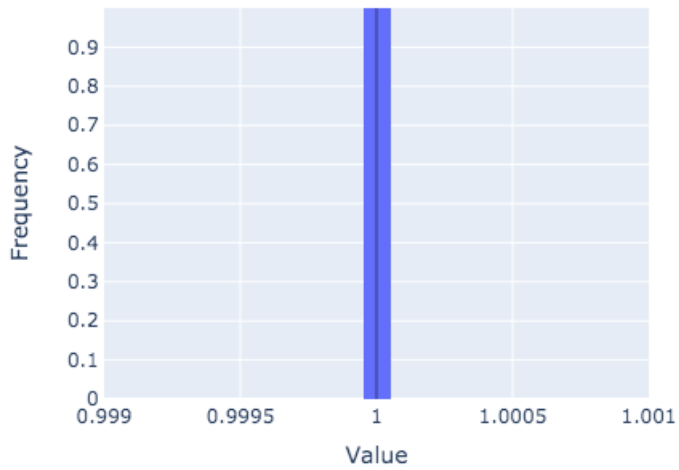
N=100, Nrep=10



Min: 0.9999999999999997
Max: 1.000000130591659
|Max-Min|: 1.3059165937523431e-07

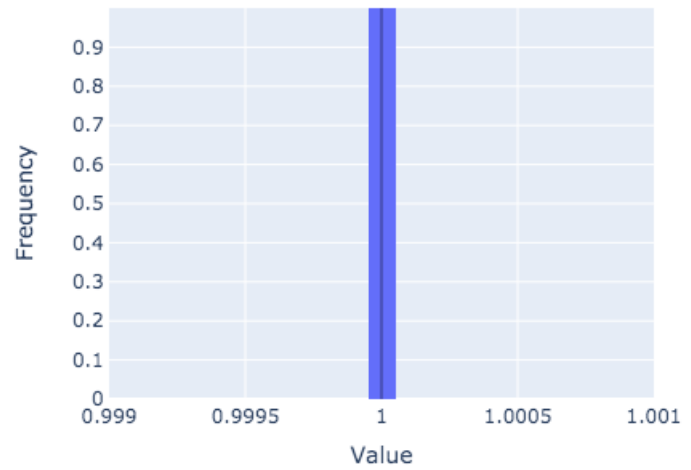
Case 4: Maximum-Likelihood (True)

N=10, Nrep=10



Min: 0.9999999999999998
Max: 0.9999999999999998
|Max-Min|: 0.0

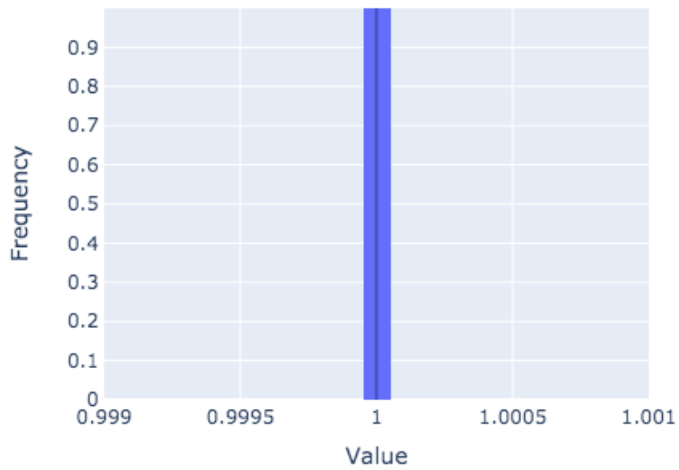
N=100, Nrep=10



Min: 0.9999999999999998
Max: 0.9999999999999998
|Max-Min|: 0.0

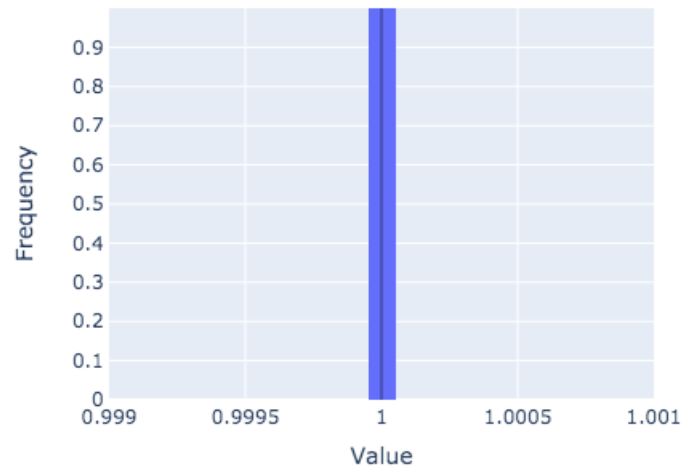
Case 5: Maximum-Likelihood (False)

N=10, Nrep=10



Min: 1.0000000061404672
Max: 1.0000001784357189
|Max-Min|: 1.7229525162143489e-07

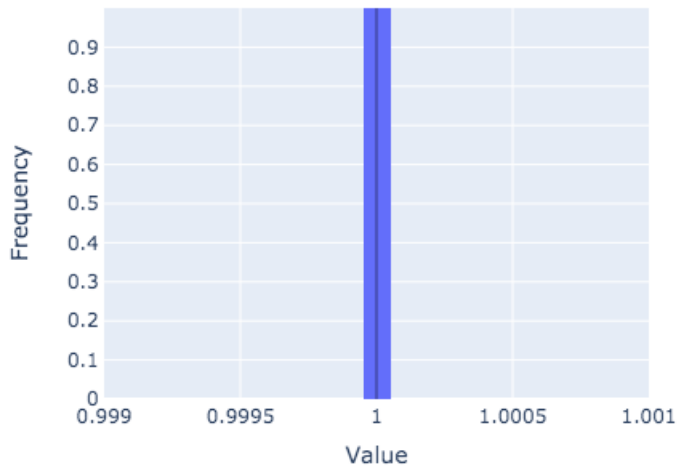
N=100, Nrep=10



Min: 1.000000007962313
Max: 1.000000181521831
|Max-Min|: 1.7355951786868218e-07

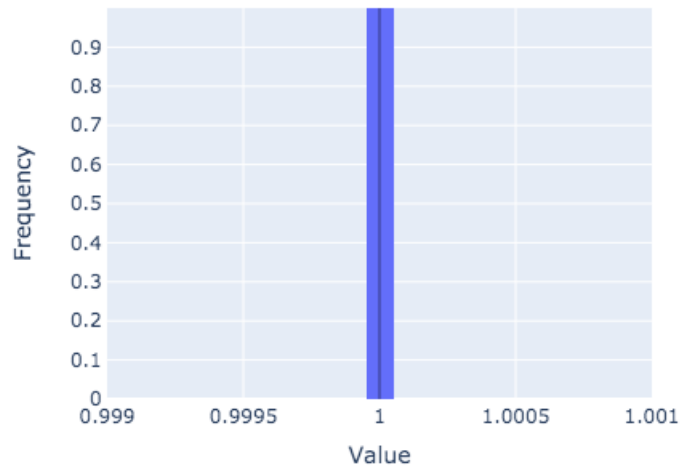
Case 6: Least Squares (True)

N=10, Nrep=10



Min: 0.9999999999999998
Max: 0.9999999999999998
|Max-Min|: 0.0

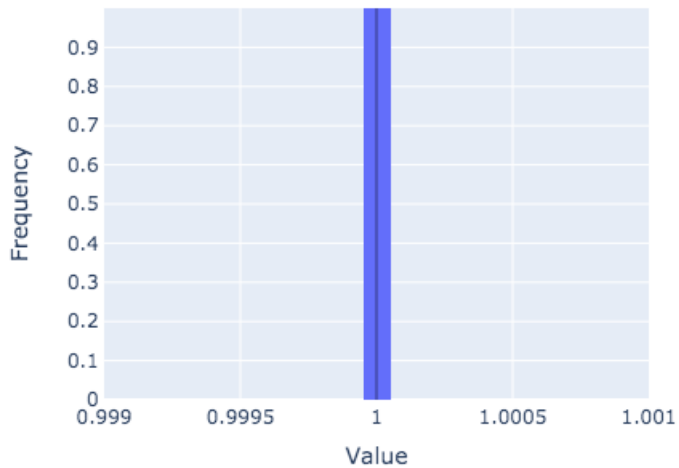
N=100, Nrep=10



Min: 0.9999999999999998
Max: 0.9999999999999998
|Max-Min|: 0.0

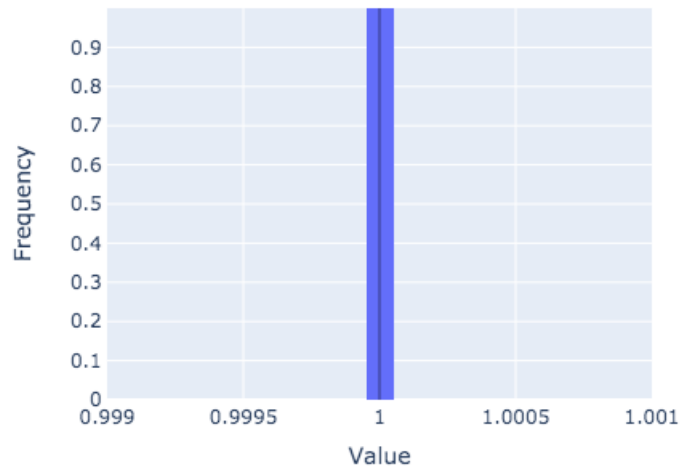
Case 7: Least Squares (False)

N=10, Nrep=10



Min: 1.0
Max: 1.00000015436117
|Max-Min|: 1.5436117006828454e-07

N=100, Nrep=10



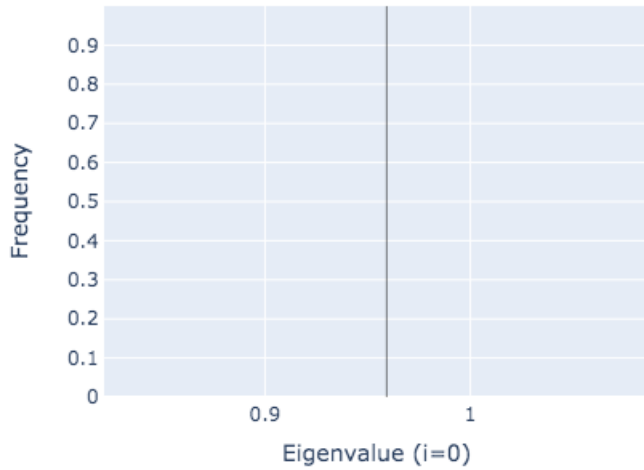
Min: 0.9999999999999998
Max: 1.0000001612437763
|Max-Min|: 1.612437765530217e-07

Test of inequality constraint violation

Eigenvalue

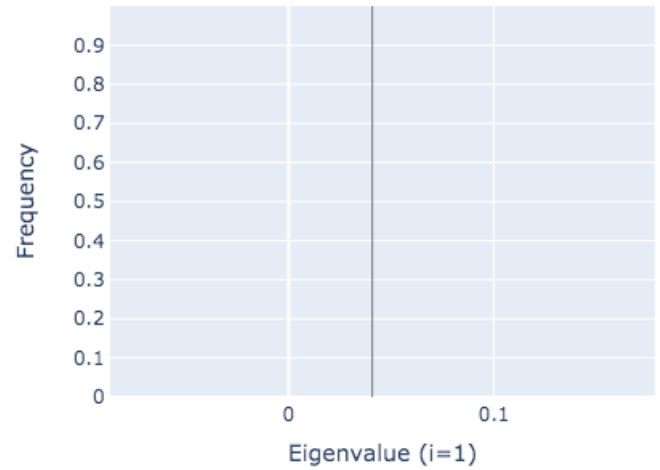
Case 0: Linear (True)

N=10, Nrep=10



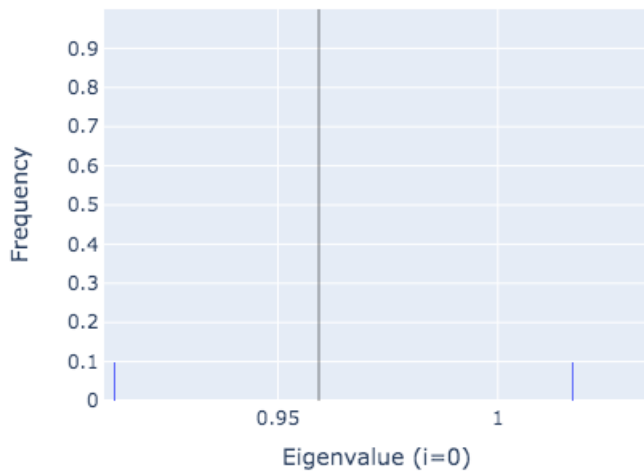
Min: 0.8217439063611547
Max: 1.0868807432325331
|Max-Min|: 0.26513683687137846

N=10, Nrep=10



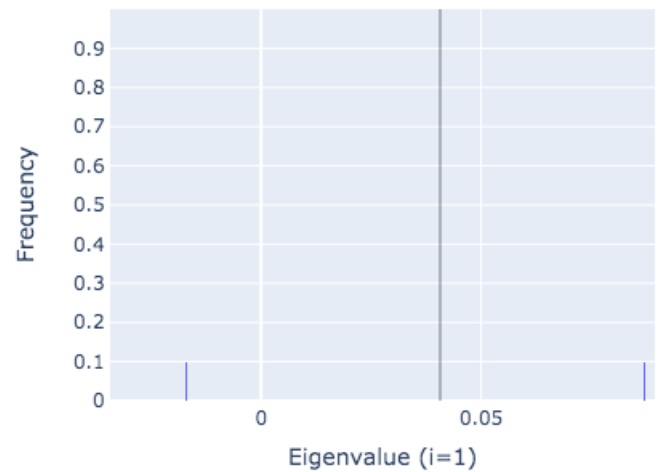
Min: -0.08688074323253331
Max: 0.1782560936388451
|Max-Min|: 0.2651368368713784

N=100, Nrep=10



Min: 0.9105490618058564
Max: 1.0342338199919683
|Max-Min|: 0.12368475818611191

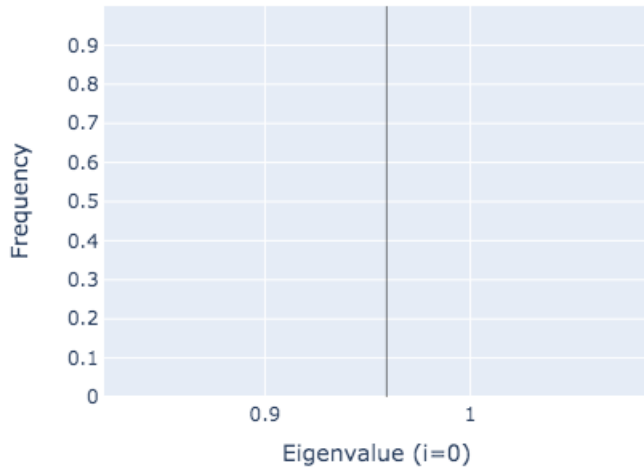
N=100, Nrep=10



Min: -0.03423381999196835
Max: 0.08945093819414349
|Max-Min|: 0.12368475818611184

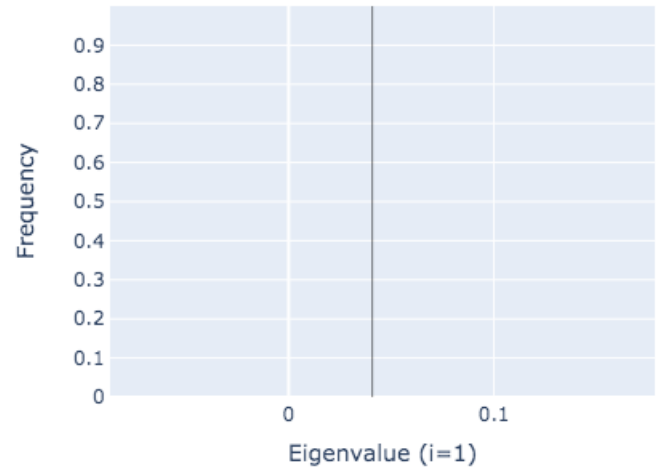
Case 1: Linear (False)

N=10, Nrep=10



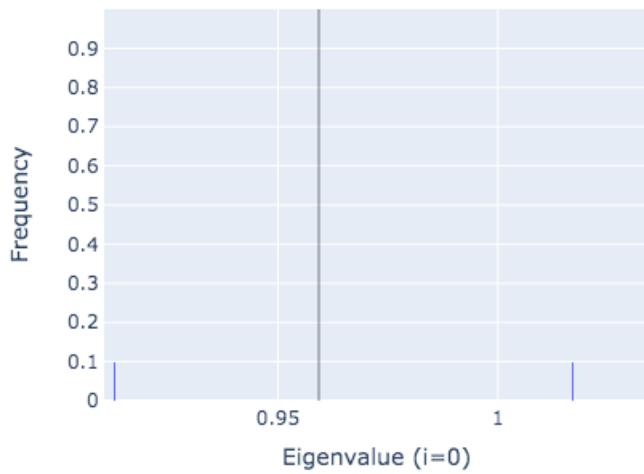
Min: 0.8217439063611547
Max: 1.0868807432325331
|Max-Min|: 0.26513683687137846

N=10, Nrep=10



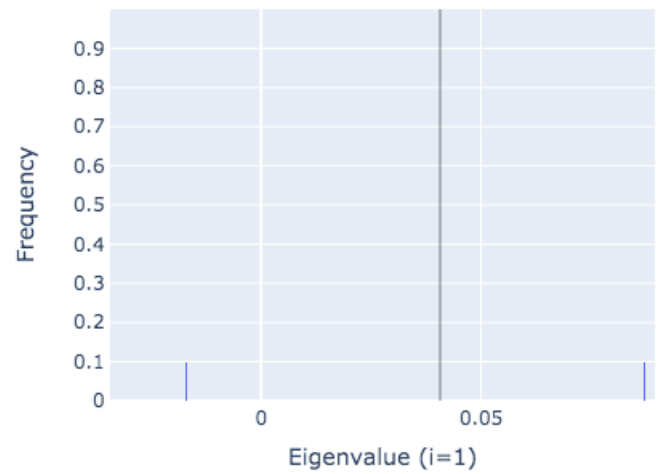
Min: -0.08688074323253318
Max: 0.17825609363884512
|Max-Min|: 0.2651368368713783

N=100, Nrep=10



Min: 0.9105490618058564
Max: 1.0342338199919683
|Max-Min|: 0.12368475818611191

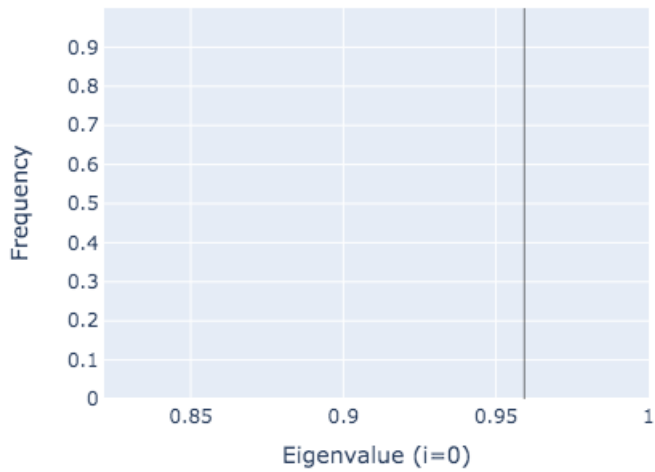
N=100, Nrep=10



Min: -0.03423381999196848
Max: 0.08945093819414354
|Max-Min|: 0.12368475818611202

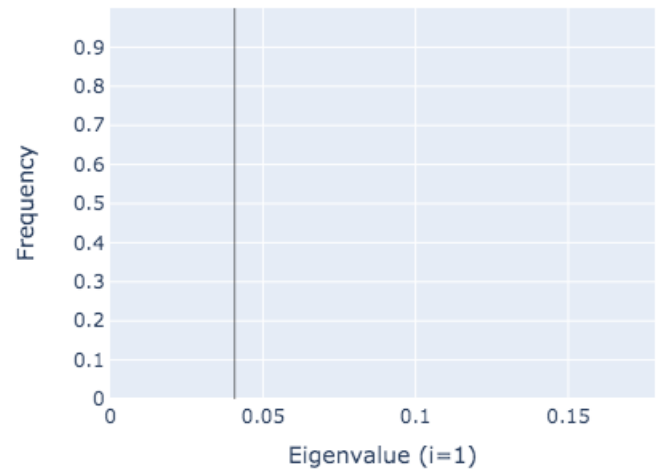
Case 2: ProjectedLinear (True)

N=10, Nrep=10



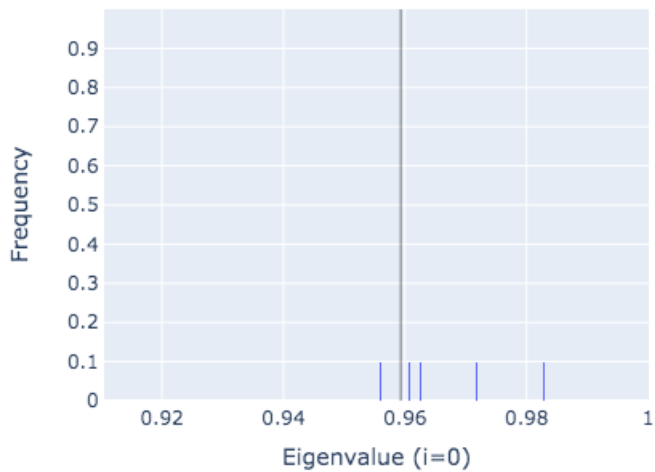
Min: 0.8217439063611546
Max: 1.0000000893446086
|Max-Min|: 0.17825618298345403

N=10, Nrep=10



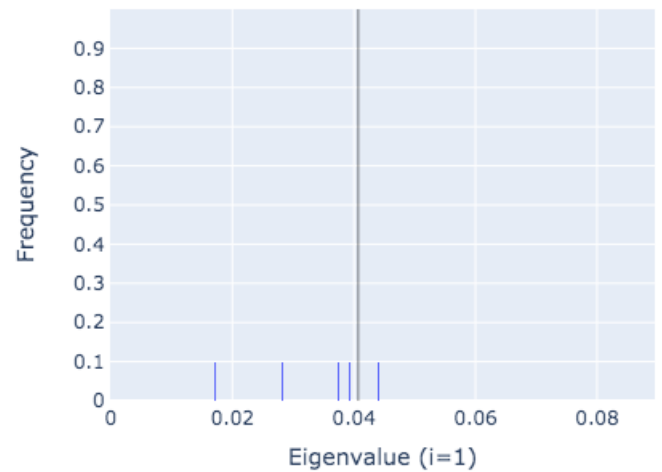
Min: -8.934460889523099e-08
Max: 0.17825609363884518
|Max-Min|: 0.17825618298345408

N=100, Nrep=10



Min: 0.9105490618058562
Max: 1.0000000652958296
|Max-Min|: 0.08945100348997348

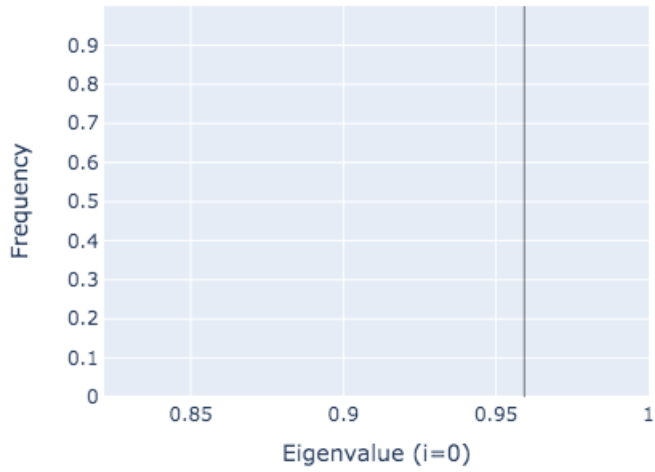
N=100, Nrep=10



Min: -6.529582974659776e-08
Max: 0.08945093819414354
|Max-Min|: 0.08945100348997329

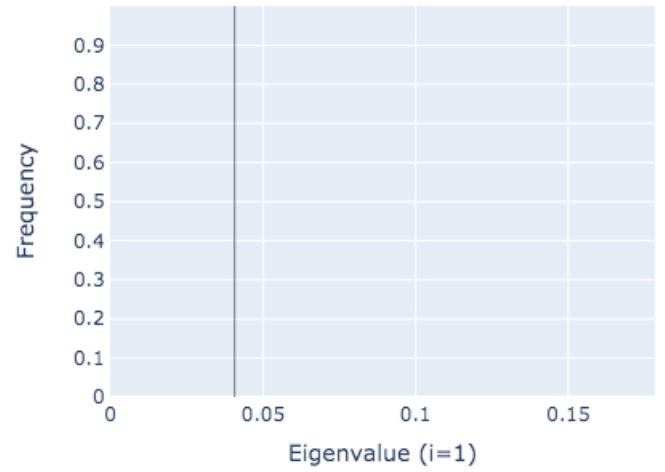
Case 3: ProjectedLinear (False)

N=10, Nrep=10



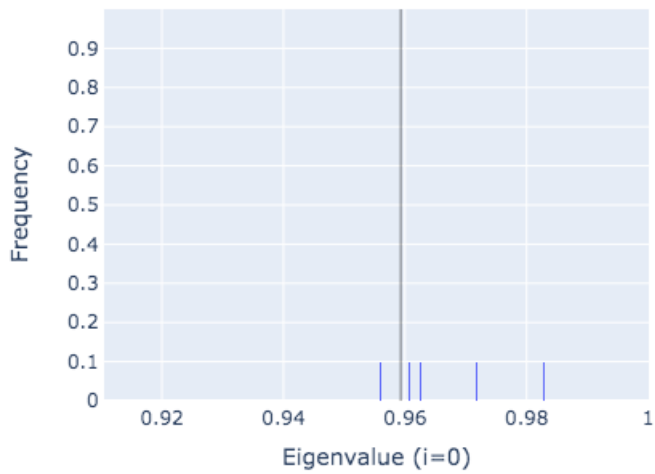
Min: 0.8217439063611548
 Max: 1.0000001786892174
 |Max-Min|: 0.17825627232806263

N=10, Nrep=10



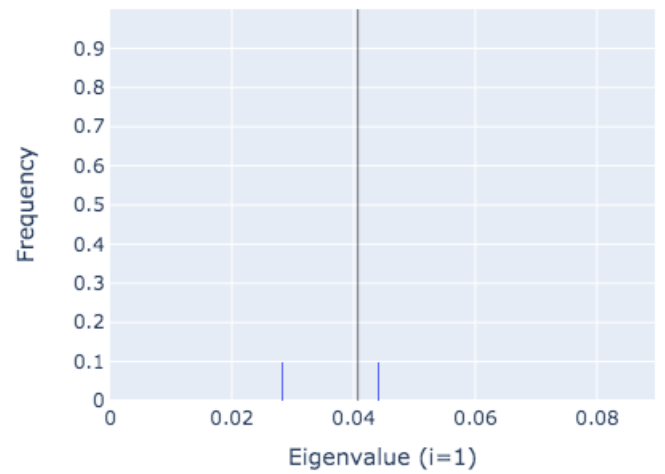
Min: -8.326672684688674e-17
 Max: 0.1782560936388453
 |Max-Min|: 0.17825609363884537

N=100, Nrep=10



Min: 0.9105490618058562
 Max: 1.0000001305916593
 |Max-Min|: 0.08945106878580311

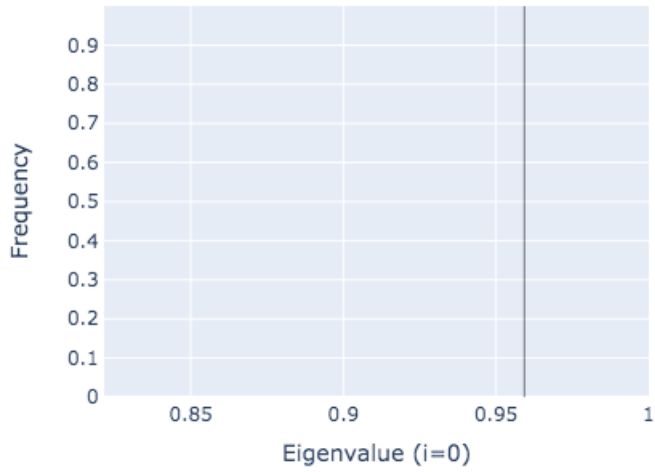
N=100, Nrep=10



Min: -1.3183898417423734e-16
 Max: 0.08945093819414349
 |Max-Min|: 0.08945093819414363

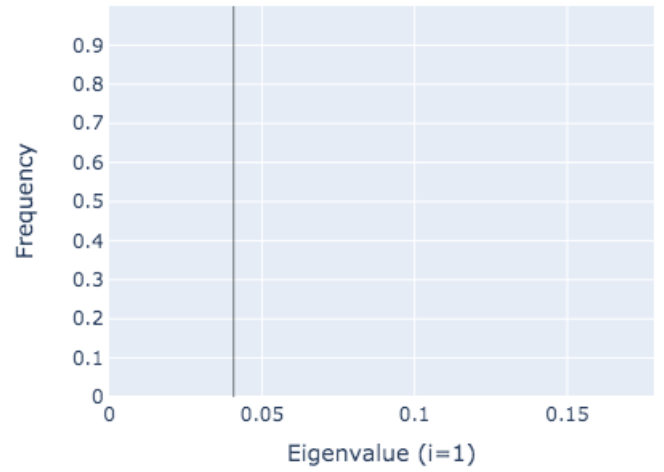
Case 4: Maximum-Likelihood (True)

N=10, Nrep=10



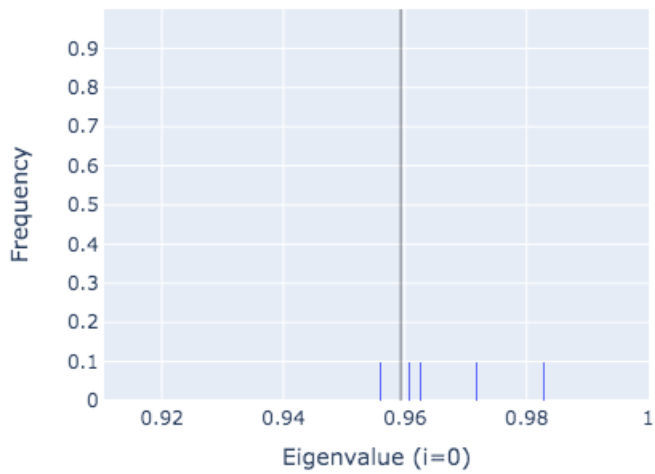
Min: 0.8217439099529941
 Max: 1.000000080374627
 |Max-Min|: 0.1782561704216329

N=10, Nrep=10



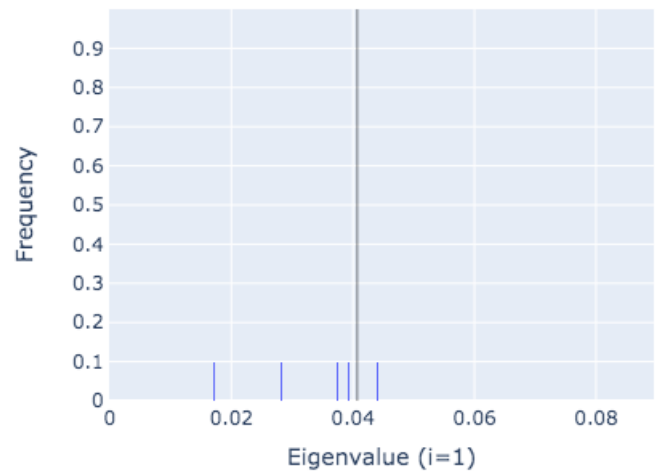
Min: -8.037462703724962e-08
 Max: 0.17825609004700563
 |Max-Min|: 0.17825617042163266

N=100, Nrep=10



Min: 0.9105490612563245
 Max: 1.0000000786009928
 |Max-Min|: 0.08945101734466832

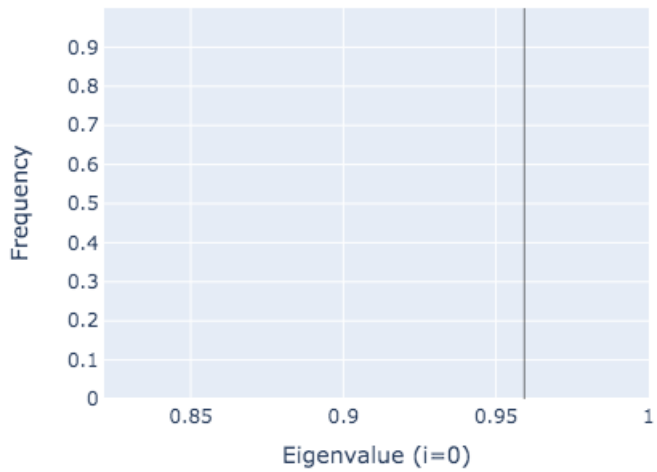
N=100, Nrep=10



Min: -7.860099304800652e-08
 Max: 0.08945093874367524
 |Max-Min|: 0.08945101734466829

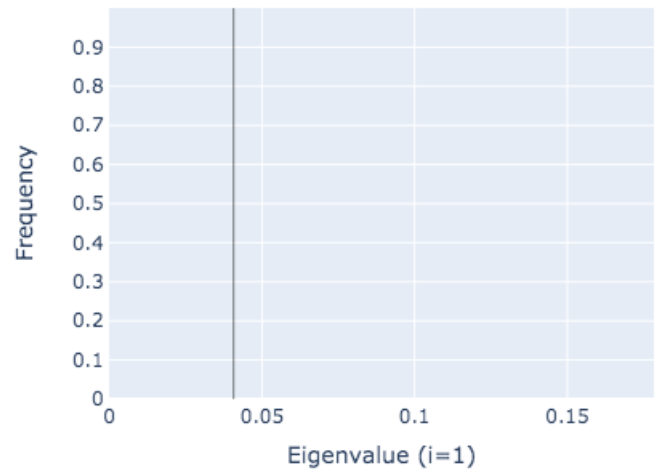
Case 5: Maximum-Likelihood (False)

N=10, Nrep=10



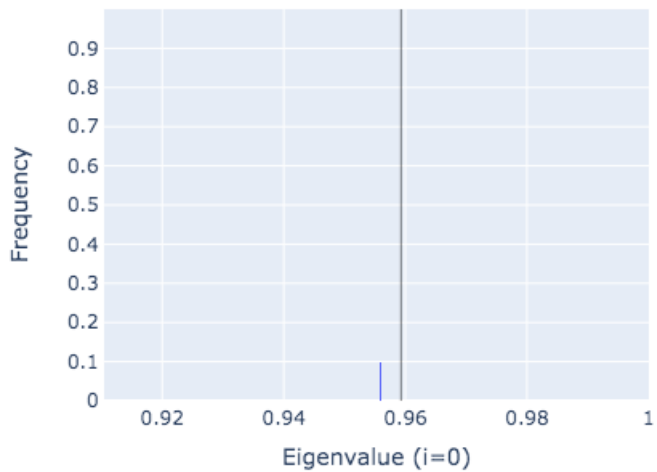
Min: 0.8217419203339902
Max: 1.0000001784357186
|Max-Min|: 0.17825825810172846

N=10, Nrep=10



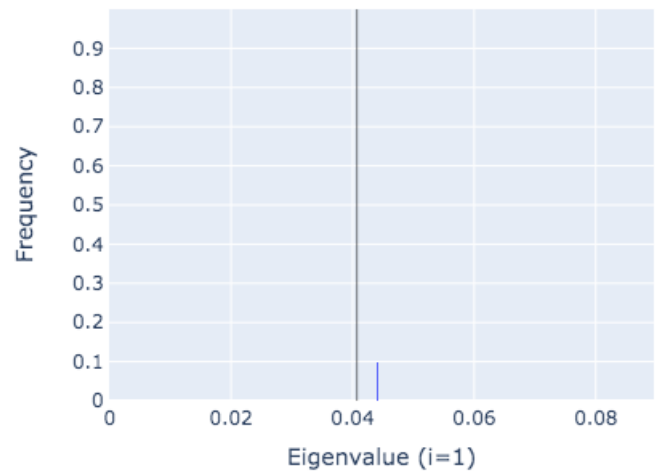
Min: 1.1102230246251565e-16
Max: 0.17825808580647715
|Max-Min|: 0.17825808580647703

N=100, Nrep=10



Min: 0.9105361357403978
Max: 1.0000001815218298
|Max-Min|: 0.08946404578143197

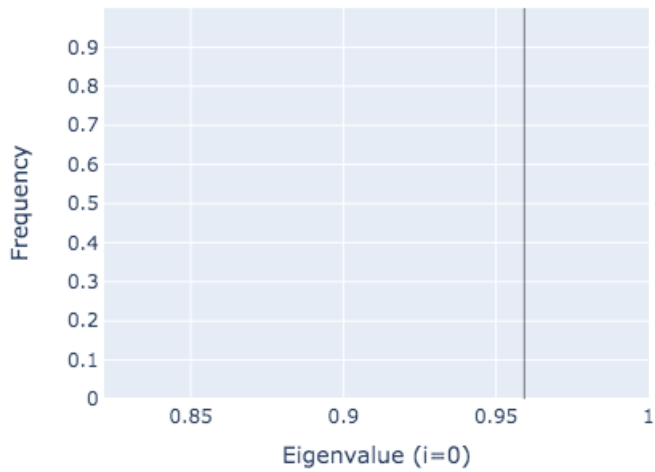
N=100, Nrep=10



Min: 1.175708835843281e-15
Max: 0.08946387235627798
|Max-Min|: 0.0894638723562768

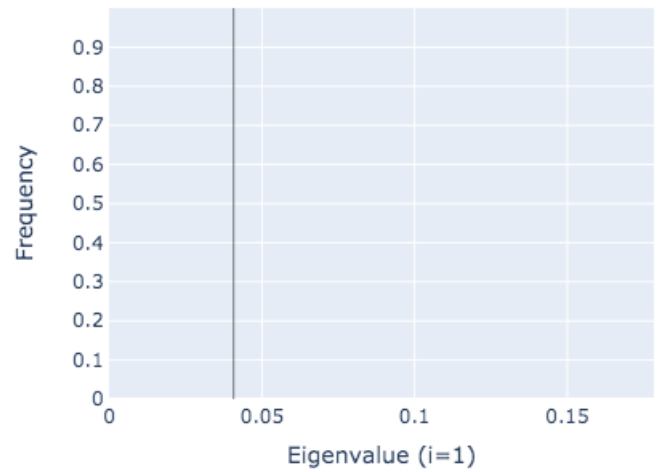
Case 6: Least Squares (True)

N=10, Nrep=10



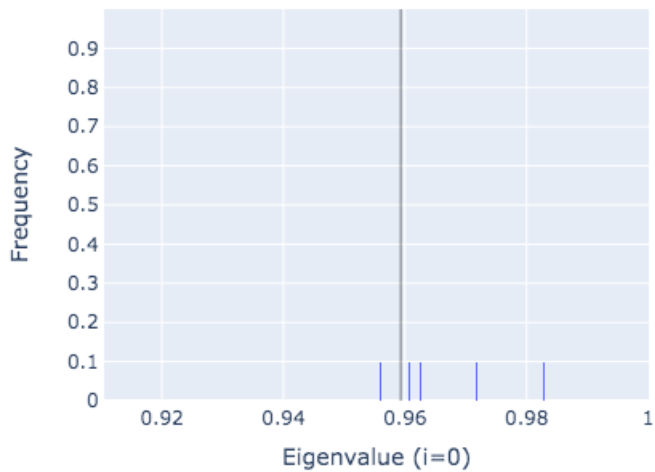
Min: 0.8217439063611545
 Max: 1.000000080061129
 |Max-Min|: 0.1782561736999746

N=10, Nrep=10



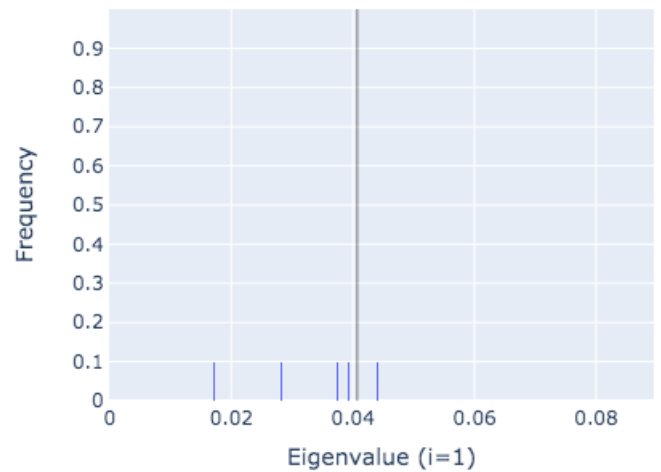
Min: -8.006112933722775e-08
 Max: 0.17825609363884526
 |Max-Min|: 0.1782561736999746

N=100, Nrep=10



Min: 0.9105490618058559
 Max: 1.000000069820626
 |Max-Min|: 0.08945100801477002

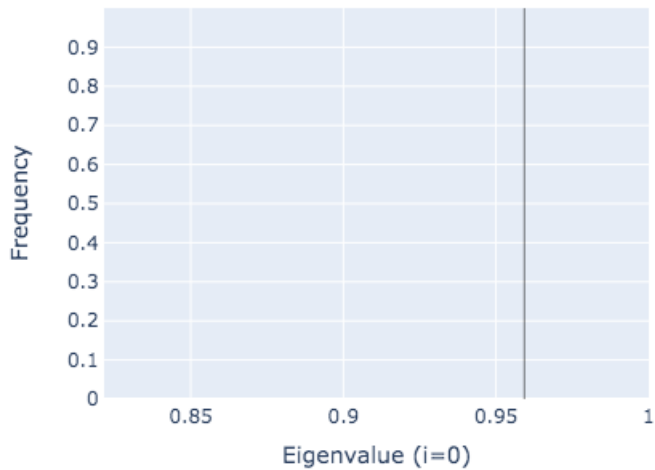
N=100, Nrep=10



Min: -6.982062614006856e-08
 Max: 0.0894509381941439
 |Max-Min|: 0.08945100801477004

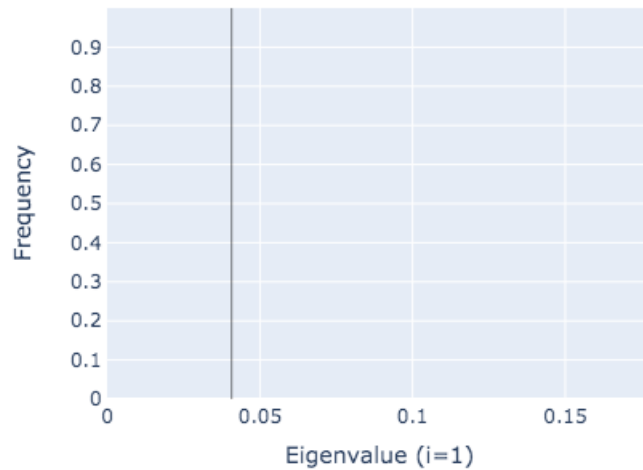
Case 7: Least Squares (False)

N=10, Nrep=10



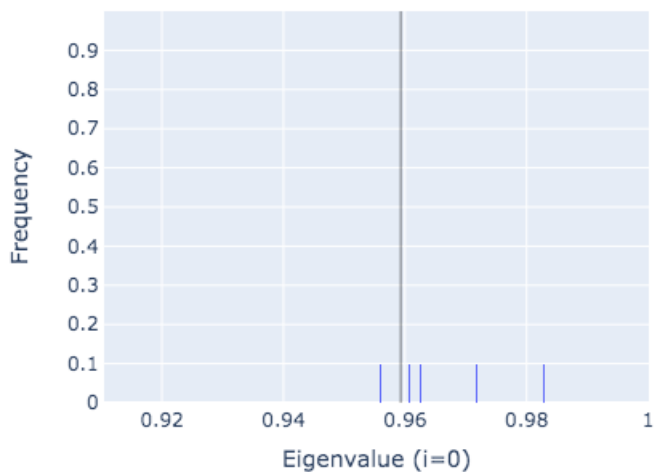
Min: 0.821743906361156
 Max: 1.00000015436117
 |Max-Min|: 0.17825624800001405

N=10, Nrep=10



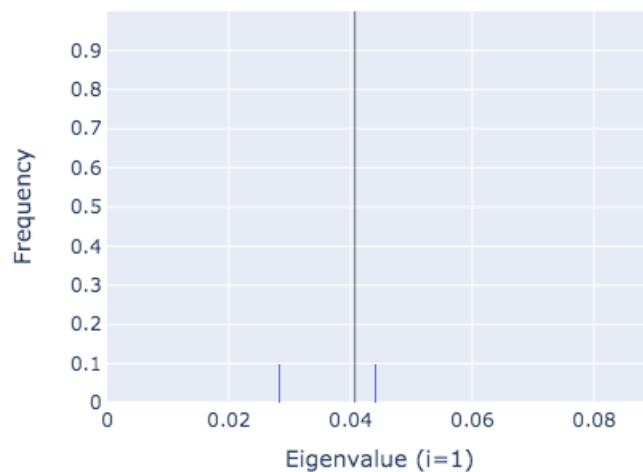
Min: -1.3877787807814457e-16
 Max: 0.1782560936388442
 |Max-Min|: 0.17825609363884434

N=100, Nrep=10



Min: 0.910549061805857
 Max: 1.000000161243776
 |Max-Min|: 0.08945109943791907

N=100, Nrep=10



Min: 4.7704895589362195e-17
 Max: 0.0894509381941427
 |Max-Min|: 0.08945093819414265

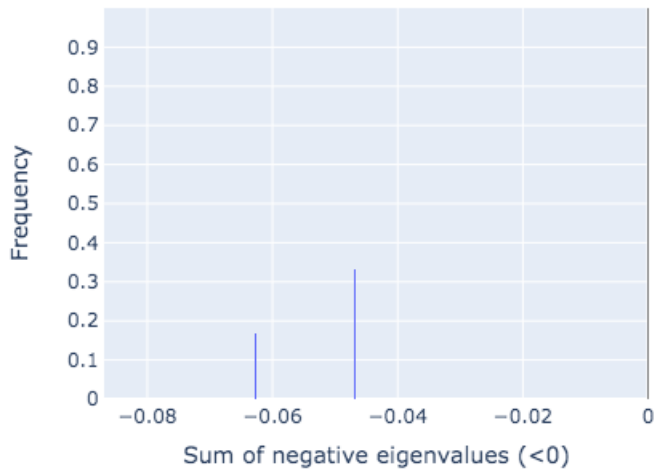
Sum of unphysical eigenvalues

Case 0: Linear (True)

N=10

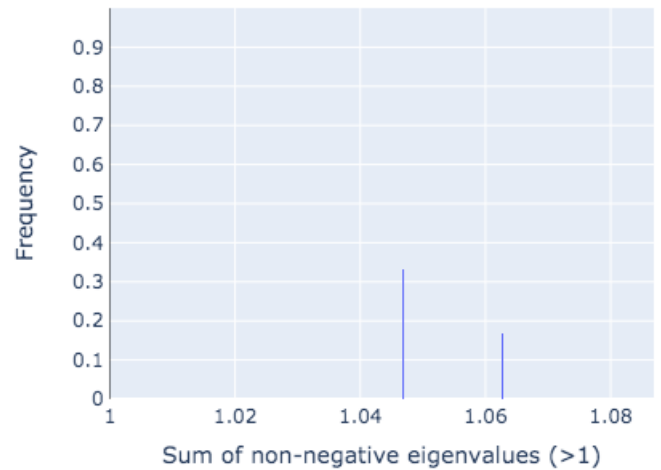
Number of unphysical estimates=6

N=10, Nrep=10



Min: -0.08688074323253331
 Max: -0.002998403797154006
 |Max-Min|: 0.0838823394353793

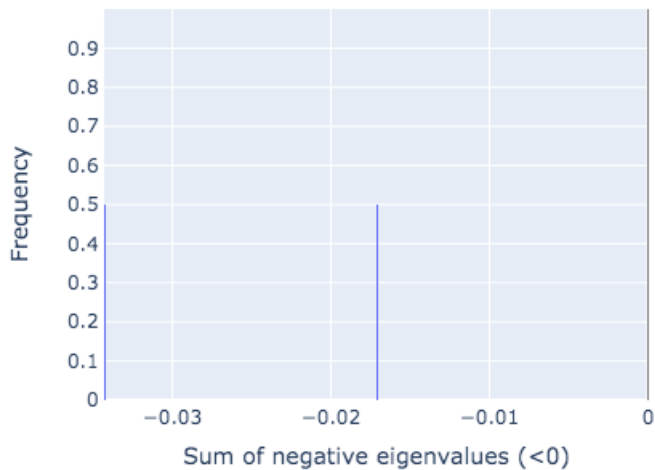
N=10, Nrep=10



Min: 1.0029984037971538
 Max: 1.0868807432325331
 |Max-Min|: 0.0838823394353793

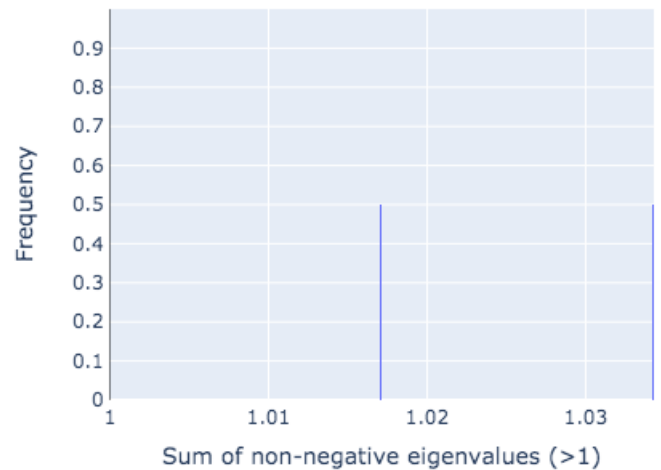
N=100
 Number of unphysical estimates=2

N=100, Nrep=10



Min: -0.03423381999196835
 Max: -0.017068185480011048
 |Max-Min|: 0.017165634511957305

N=100, Nrep=10

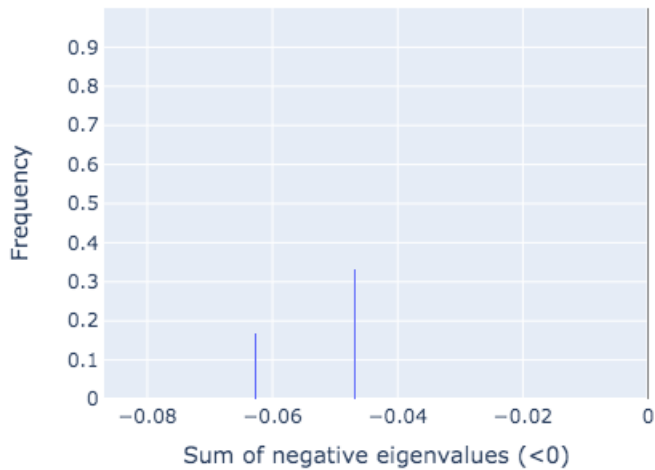


Min: 1.017068185480011
 Max: 1.0342338199919683
 |Max-Min|: 0.017165634511957295

Case 1: Linear (False)

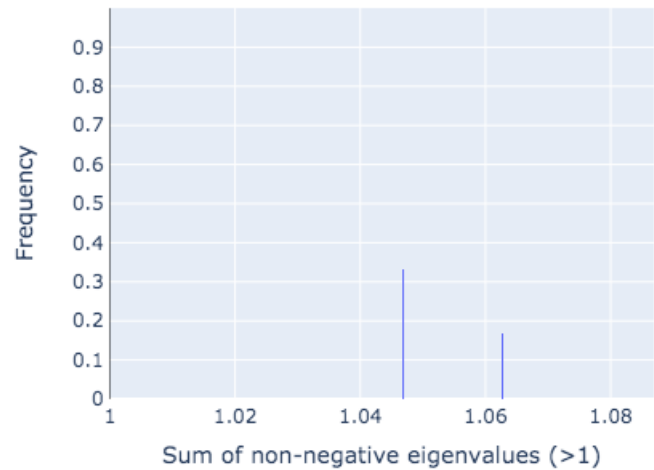
N=10
 Number of unphysical estimates=6

N=10, Nrep=10



Min: -0.08688074323253318
 Max: -0.0029984037971540756
 |Max-Min|: 0.08388233943537911

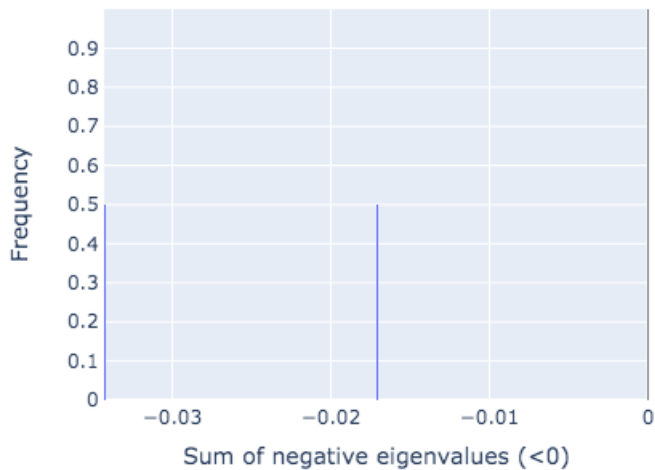
N=10, Nrep=10



Min: 1.0029984037971538
 Max: 1.0868807432325331
 |Max-Min|: 0.08388233943537937

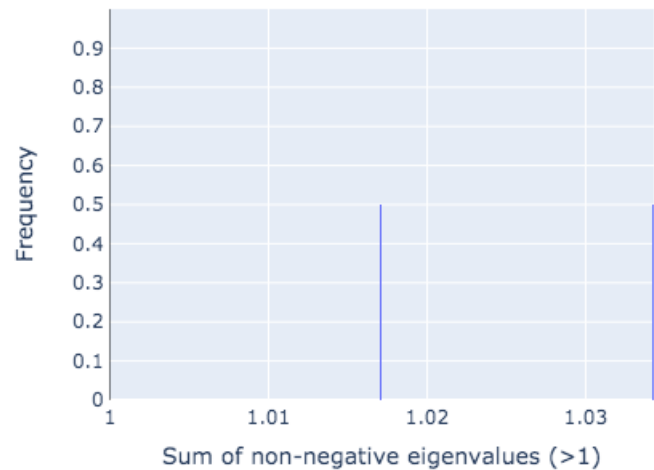
N=100
 Number of unphysical estimates=2

N=100, Nrep=10



Min: -0.03423381999196848
 Max: -0.017068185480010926
 |Max-Min|: 0.017165634511957552

N=100, Nrep=10

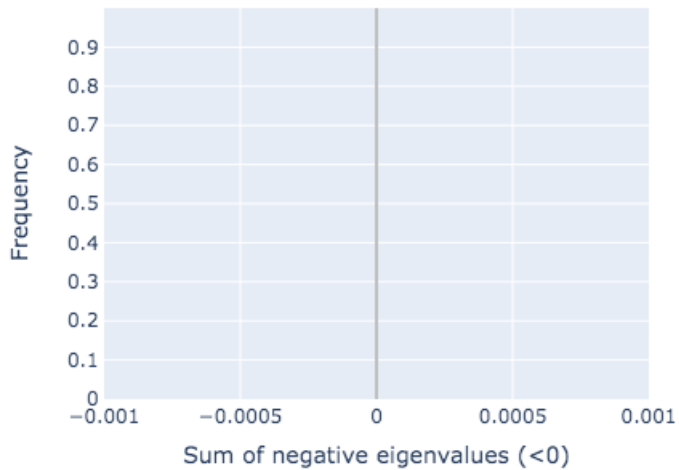


Min: 1.017068185480011
 Max: 1.0342338199919683
 |Max-Min|: 0.017165634511957295

Case 2: ProjectedLinear (True)

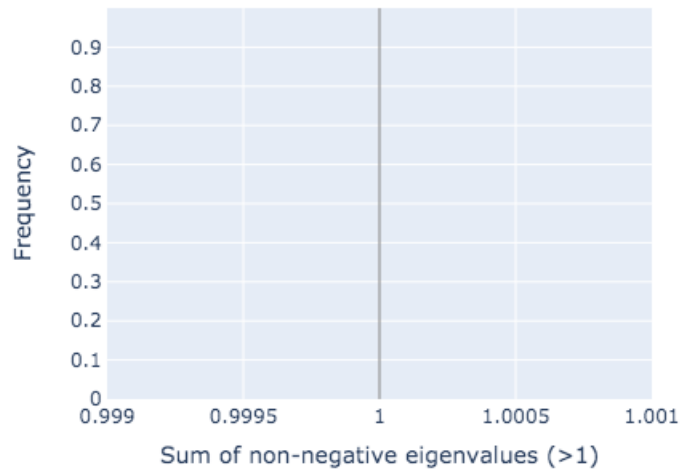
N=10
 Number of unphysical estimates=0

N=10, Nrep=10



Min: -
Max: -
|Max-Min|: -

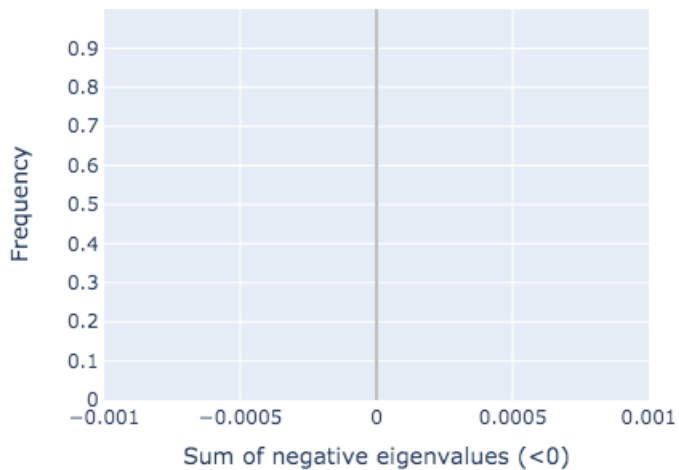
N=10, Nrep=10



Min: -
Max: -
|Max-Min|: -

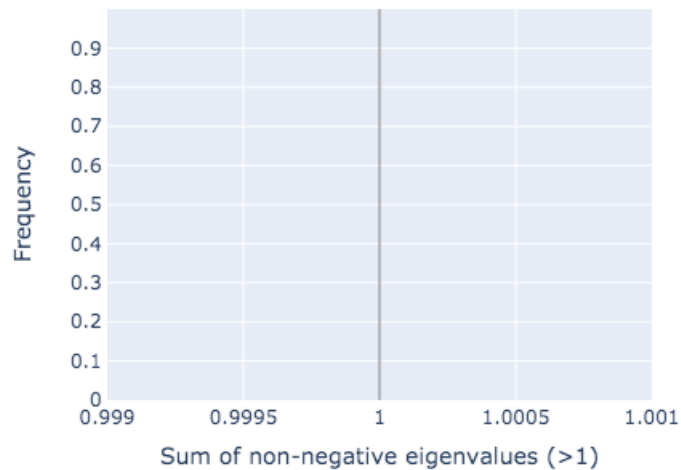
N=100
Number of unphysical estimates=0

N=100, Nrep=10



Min: -
Max: -
|Max-Min|: -

N=100, Nrep=10

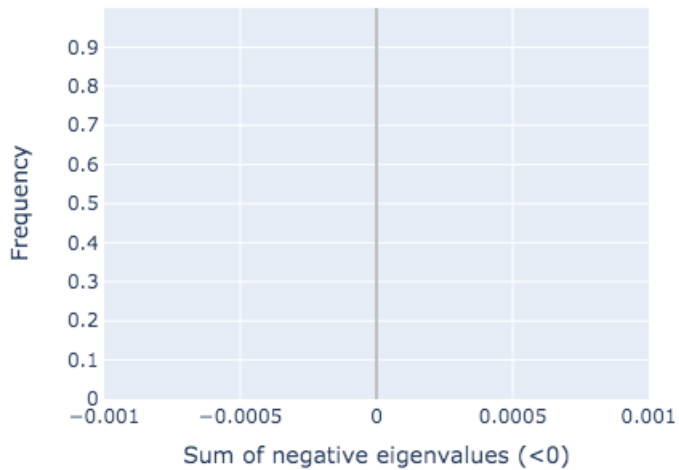


Min: -
Max: -
|Max-Min|: -

Case 3: ProjectedLinear (False)

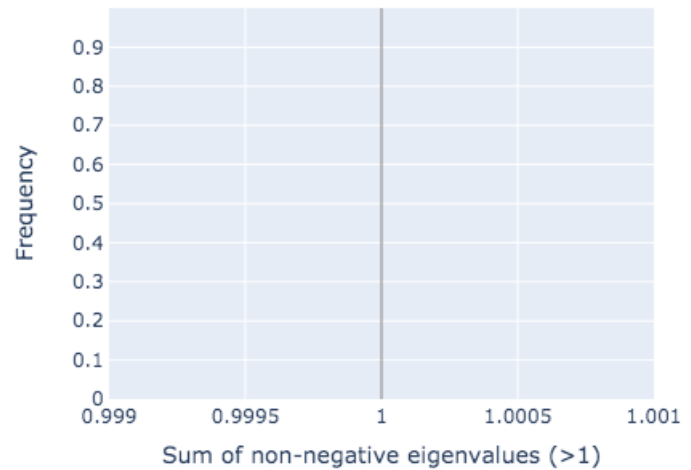
N=10
Number of unphysical estimates=0

N=10, Nrep=10



Min: -
Max: -
|Max-Min|: -

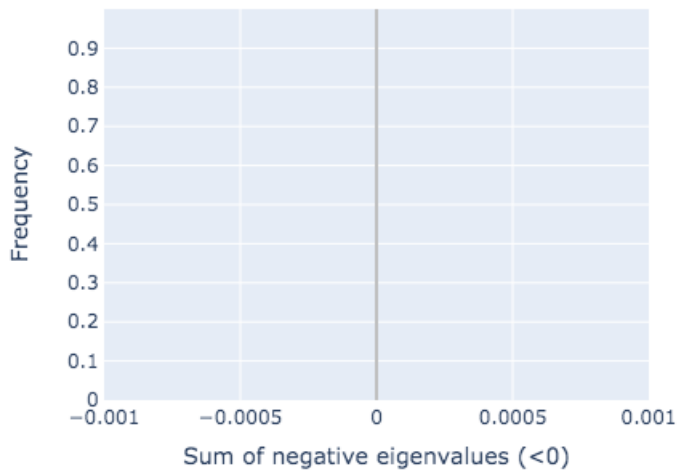
N=10, Nrep=10



Min: -
Max: -
|Max-Min|: -

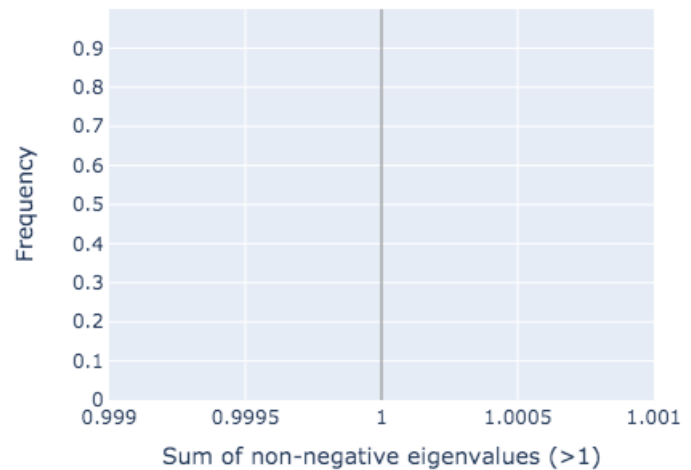
N=100
Number of unphysical estimates=0

N=100, Nrep=10



Min: -
Max: -
|Max-Min|: -

N=100, Nrep=10

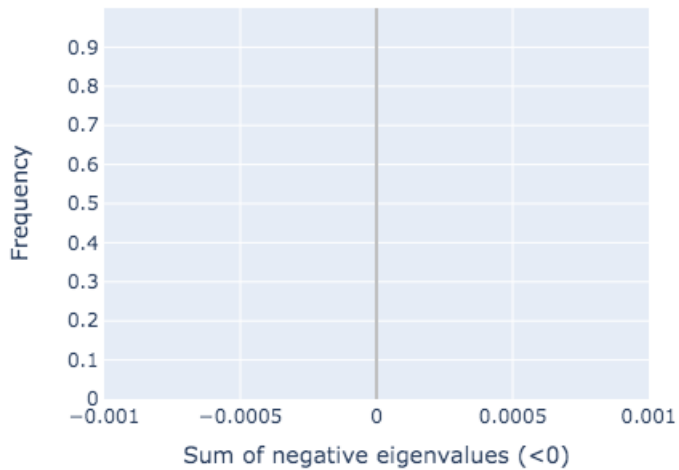


Min: -
Max: -
|Max-Min|: -

Case 4: Maximum-Likelihood (True)

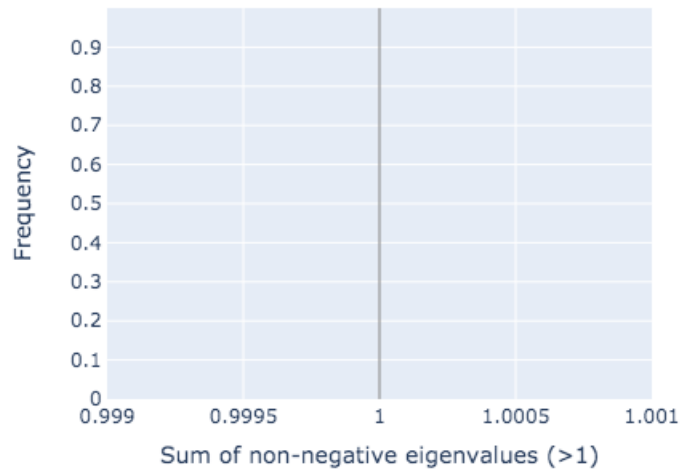
N=10
Number of unphysical estimates=0

N=10, Nrep=10



Min: -
Max: -
|Max-Min|: -

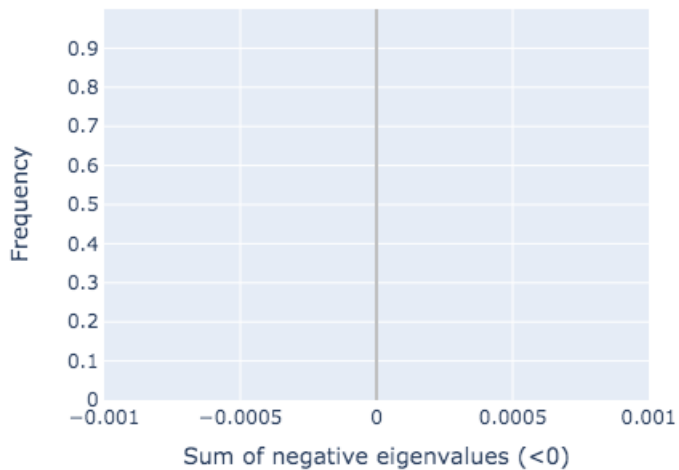
N=10, Nrep=10



Min: -
Max: -
|Max-Min|: -

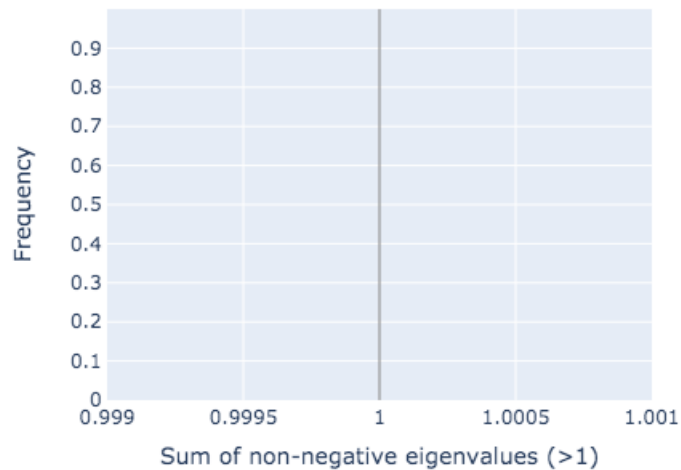
N=100
Number of unphysical estimates=0

N=100, Nrep=10



Min: -
Max: -
|Max-Min|: -

N=100, Nrep=10

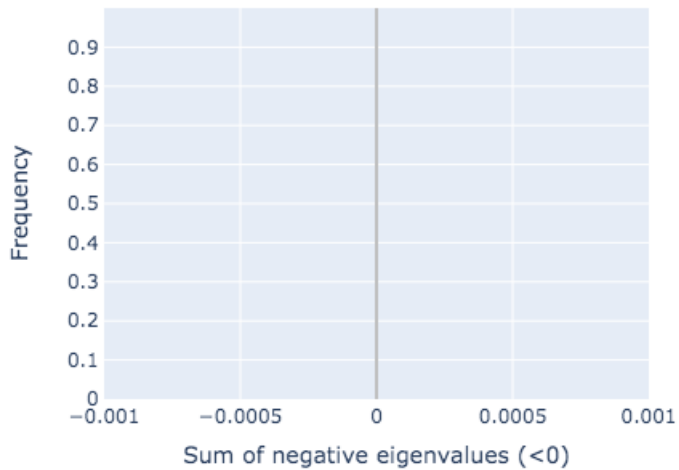


Min: -
Max: -
|Max-Min|: -

Case 5: Maximum-Likelihood (False)

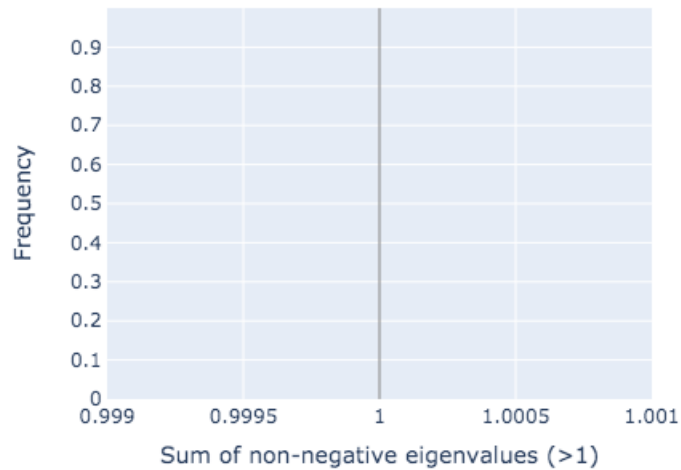
N=10
Number of unphysical estimates=0

N=10, Nrep=10



Min: -
Max: -
|Max-Min|: -

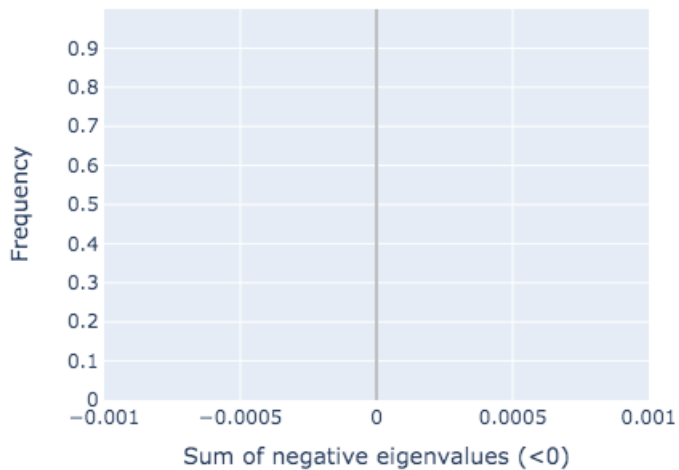
N=10, Nrep=10



Min: -
Max: -
|Max-Min|: -

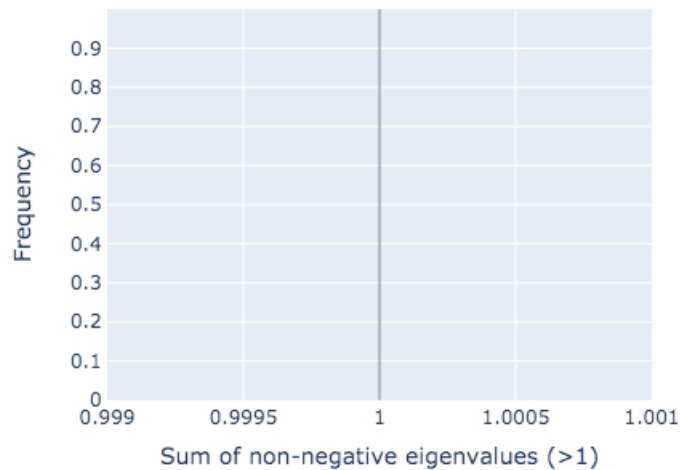
N=100
Number of unphysical estimates=0

N=100, Nrep=10



Min: -
Max: -
|Max-Min|: -

N=100, Nrep=10

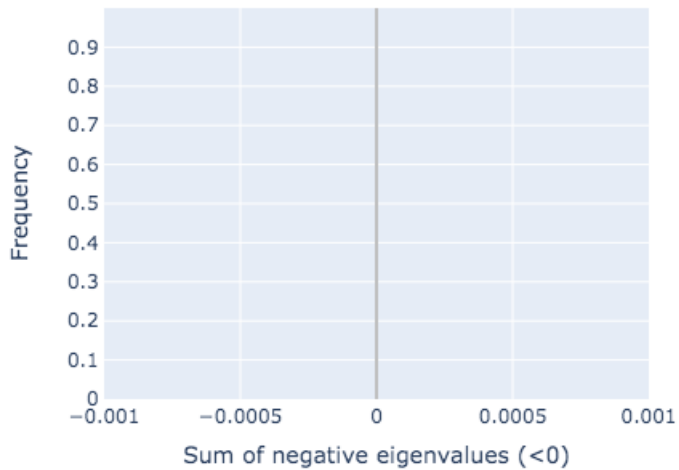


Min: -
Max: -
|Max-Min|: -

Case 6: Least Squares (True)

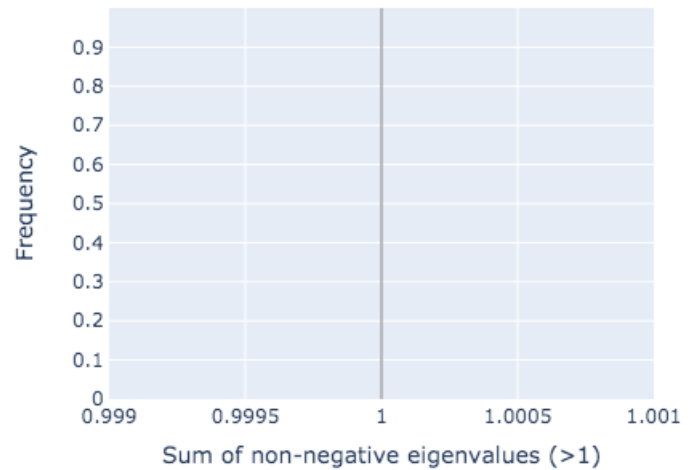
N=10
Number of unphysical estimates=0

N=10, Nrep=10



Min: -
Max: -
|Max-Min|: -

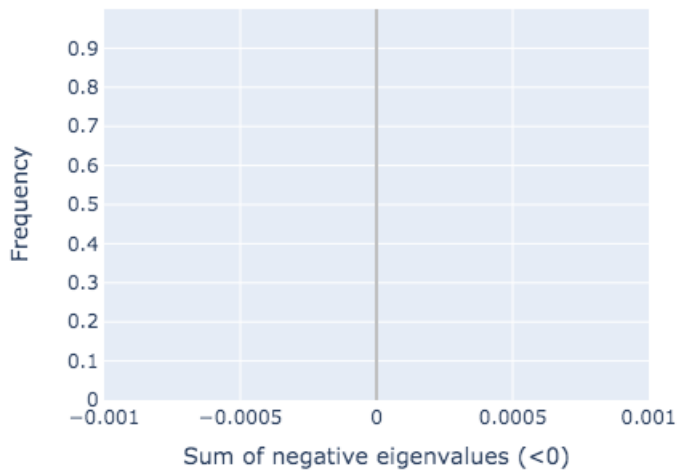
N=10, Nrep=10



Min: -
Max: -
|Max-Min|: -

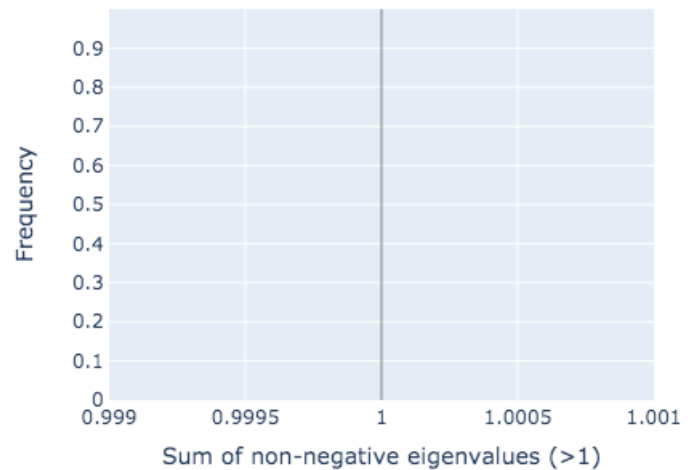
N=100
Number of unphysical estimates=0

N=100, Nrep=10



Min: -
Max: -
|Max-Min|: -

N=100, Nrep=10

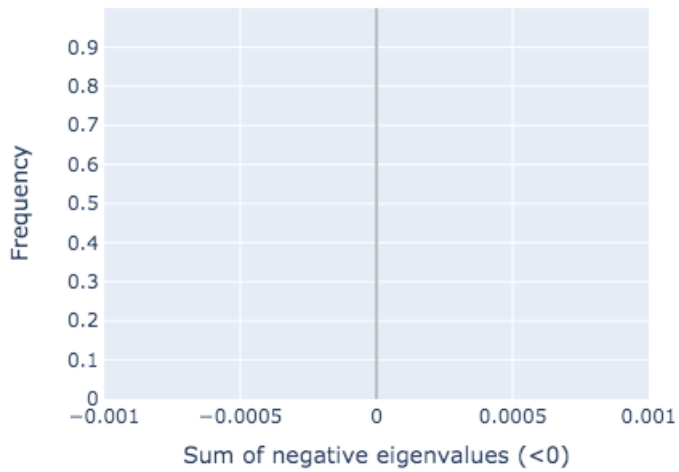


Min: -
Max: -
|Max-Min|: -

Case 7: Least Squares (False)

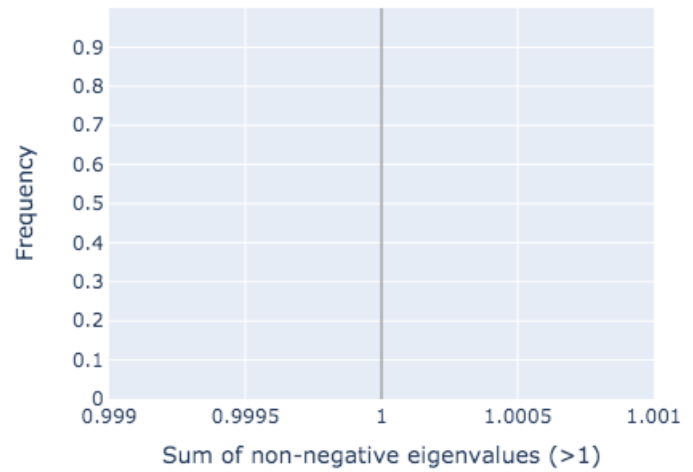
N=10
Number of unphysical estimates=0

N=10, Nrep=10



Min: -
Max: -
|Max-Min|: -

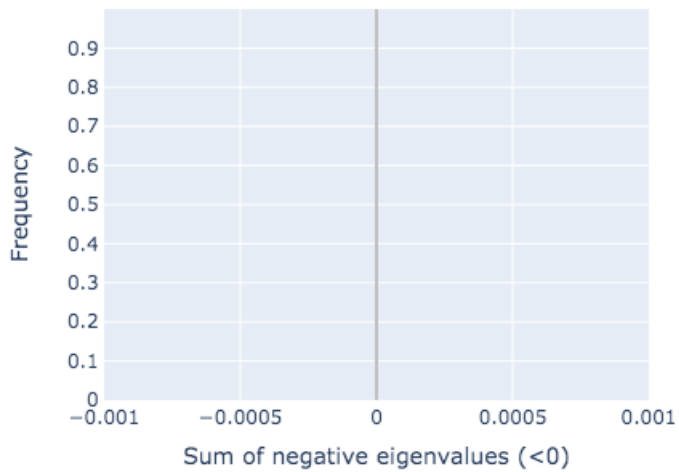
N=10, Nrep=10



Min: -
Max: -
|Max-Min|: -

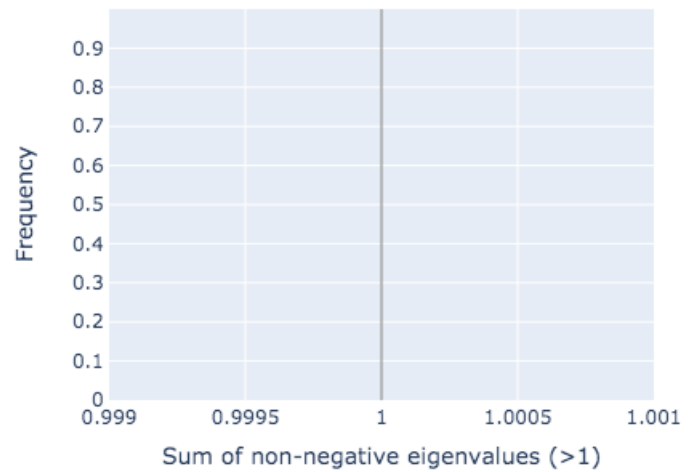
N=100
Number of unphysical estimates=0

N=100, Nrep=10



Min: -
Max: -
|Max-Min|: -

N=100, Nrep=10



Min: -
Max: -
|Max-Min|: -