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Computation time in total

Total	0.576649804910024min.
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Tolerance of physicality constraint violation

		True	False
Tolerance at physicality violation test	equality constraint	1.00e-13	1.00e-05
	inequality constraint	1.00e-05	1.00e-05

Experimental condition

Type of tomography	StandardQst
Nrep	10
N	[10, 100]
RNG seed	777

True object

Type	State
Dim	2
Vec	[0.70710678 0. 0. 0.6363961]

Tester objects

0	Type	Povm
	Dim	2
	Number of outcomes	2
	Vecs	[[0.70710678 0.6363961 0. 0.] [0.70710678 -0.6363961 0. 0.]]
1	Type	Povm
	Dim	2
	Number of outcomes	2
	Vecs	[[0.70710678 0. 0.6363961 0.] [0.70710678 0. -0.6363961 0.]]
2	Type	Povm
	Dim	2
	Number of outcomes	2
	Vecs	[[0.70710678 0. 0. 0.6363961] [0.70710678 0. 0. -0.6363961]]

Cases

	Name	Param	Tomography	Estimator
0	Linear (True)	True	StandardQst	Linear
1	Linear (False)	False	StandardQst	Linear
2	ProjectedLinear (True)	True	StandardQst	ProjectedLinear
3	ProjectedLinear (False)	False	StandardQst	ProjectedLinear
4	Maximum-Likelihood (True)	True	StandardQst	LossMinimization
5	Maximum-Likelihood (False)	False	StandardQst	LossMinimization

6	Least Squares (True)	True	StandardQst	LossMinimization
7	Least Squares (False)	False	StandardQst	LossMinimization

Computation time of estimators

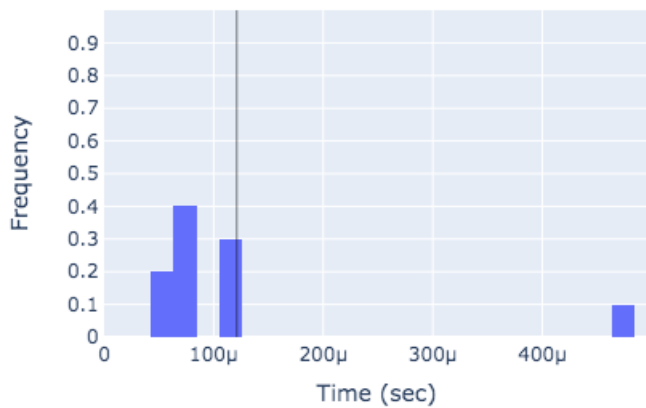
Table

	Name	N	Nrep	Mean (sec)	Std (sec)
0	Linear (True)	10	10	0.000121	0.000121
1	"	100	10	0.000110	0.000136
2	Linear (False)	10	10	0.000064	0.000023
3	"	100	10	0.000055	0.000022
4	ProjectedLinear (True)	10	10	0.007803	0.005413
5	"	100	10	0.002321	0.002860
6	ProjectedLinear (False)	10	10	0.008179	0.005788
7	"	100	10	0.002255	0.002801
8	Maximum-Likelihood (True)	10	10	0.405405	0.052859
9	"	100	10	0.358899	0.066465
10	Maximum-Likelihood (False)	10	10	1.191799	1.577707
11	"	100	10	1.250740	1.348042
12	Least Squares (True)	10	10	0.051348	0.008311
13	"	100	10	0.048920	0.005830
14	Least Squares (False)	10	10	0.071058	0.031740
15	"	100	10	0.060824	0.007197

Histogram

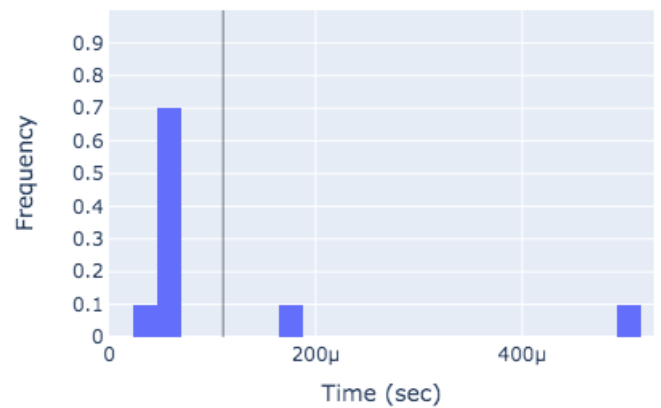
Case 0: Linear (True)

N=10, Nrep=10
Mean = 0.00012106895446777344(sec)



Min: 5.4836273193359375e-05
Max: 0.0004761219024658203
|Max-Min|: 0.00042128562927246094

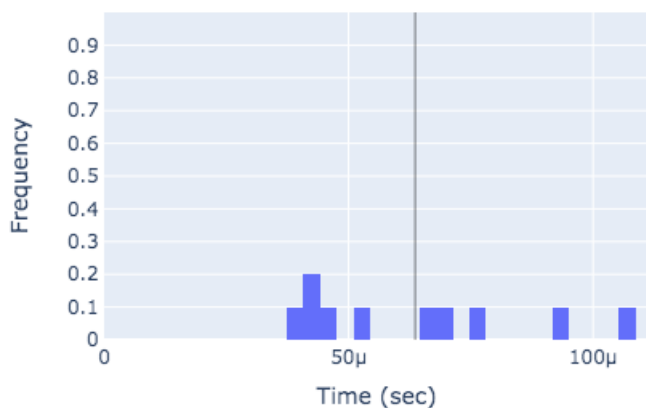
N=100, Nrep=10
Mean = 0.00011036396026611328(sec)



Min: 3.62396240234375e-05
Max: 0.0005040168762207031
|Max-Min|: 0.000467772521972656

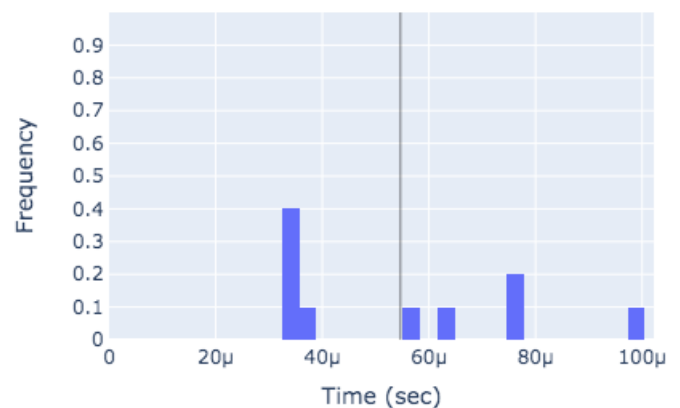
Case 1: Linear (False)

N=10, Nrep=10
Mean = 6.361007690429688e-05(sec)



Min: 4.00543212890625e-05
Max: 0.00010800361633300781
|Max-Min|: 6.794929504394531e-05

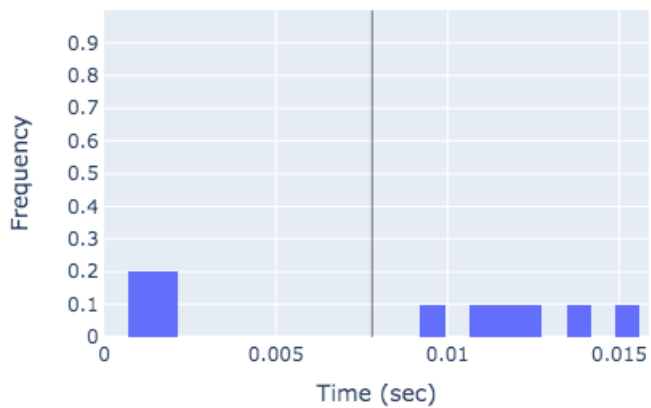
N=100, Nrep=10
Mean = 5.464553833007812e-05(sec)



Min: 3.409385681152344e-05
Max: 9.894371032714844e-05
|Max-Min|: 6.4849853515625e-05

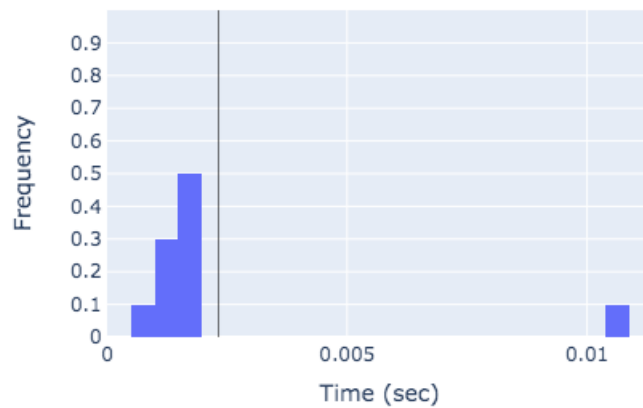
Case 2: ProjectedLinear (True)

N=10, Nrep=10
Mean = 0.007803201675415039(sec)



Min: 0.000985860824584961
Max: 0.015152931213378906
|Max-Min|: 0.014167070388793945

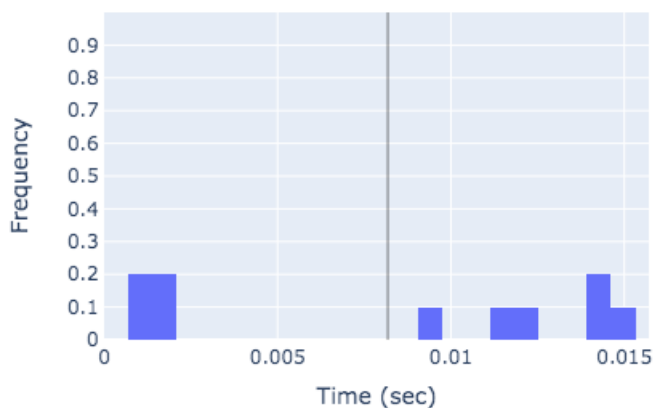
N=100, Nrep=10
Mean = 0.0023205757141113283(sec)



Min: 0.0009660720825195312
Max: 0.010853290557861328
|Max-Min|: 0.009887218475341797

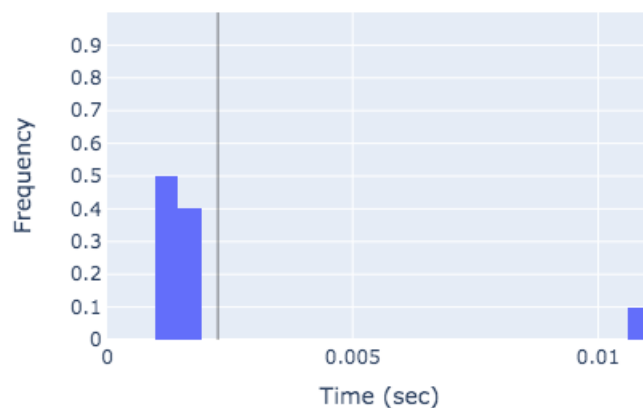
Case 3: ProjectedLinear (False)

N=10, Nrep=10
Mean = 0.008178973197937011(sec)



Min: 0.001088857650756836
Max: 0.015011072158813477
|Max-Min|: 0.01392221450805664

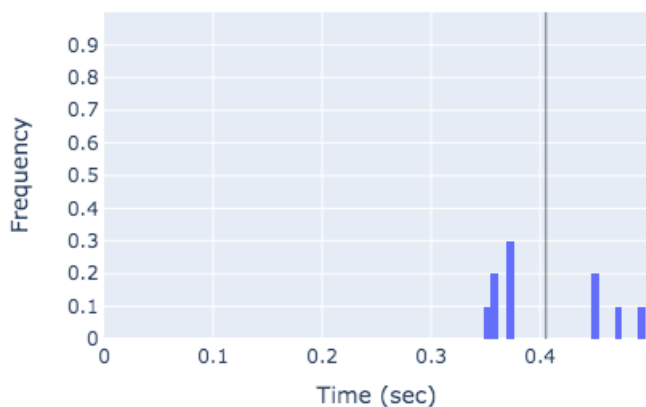
N=100, Nrep=10
Mean = 0.0022548675537109376(sec)



Min: 0.0009756088256835938
Max: 0.010612726211547852
|Max-Min|: 0.009637117385864258

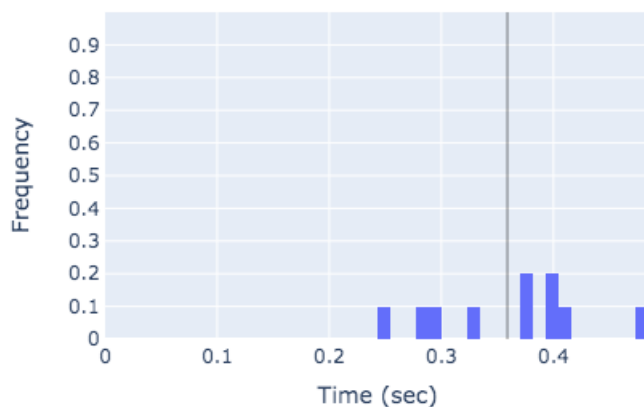
Case 4: Maximum-Likelihood (True)

N=10, Nrep=10
Mean = 0.40540480613708496(sec)



Min: 0.35073304176330566
Max: 0.49265289306640625
|Max-Min|: 0.14191985130310059

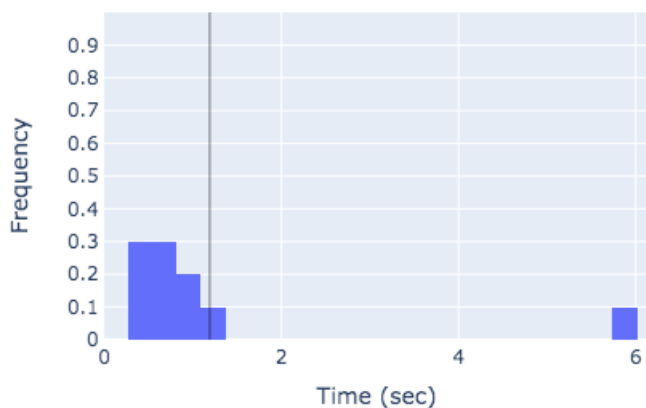
N=100, Nrep=10
Mean = 0.35889856815338134(sec)



Min: 0.24348211288452148
Max: 0.47447705268859863
|Max-Min|: 0.23099493980407715

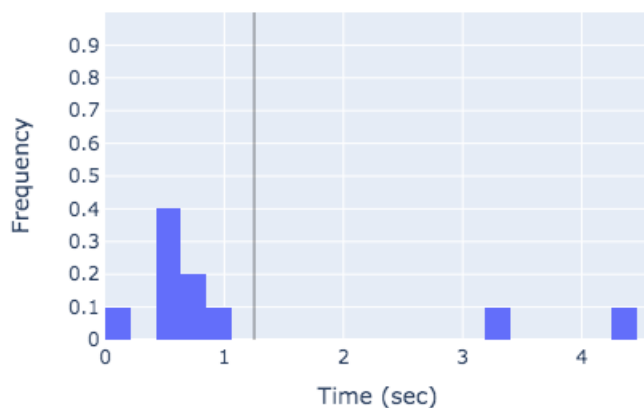
Case 5: Maximum-Likelihood (False)

N=10, Nrep=10
Mean = 1.191798996925354(sec)



Min: 0.40854597091674805
Max: 5.874047517776489
|Max-Min|: 5.465501546859741

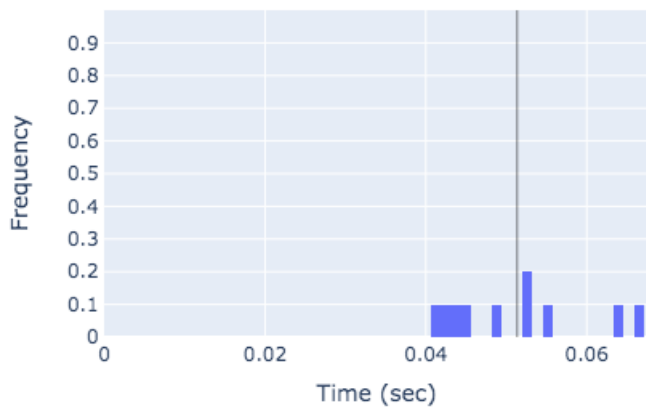
N=100, Nrep=10
Mean = 1.2507399559020995(sec)



Min: 0.10896134376525879
Max: 4.35776424407959
|Max-Min|: 4.248802900314331

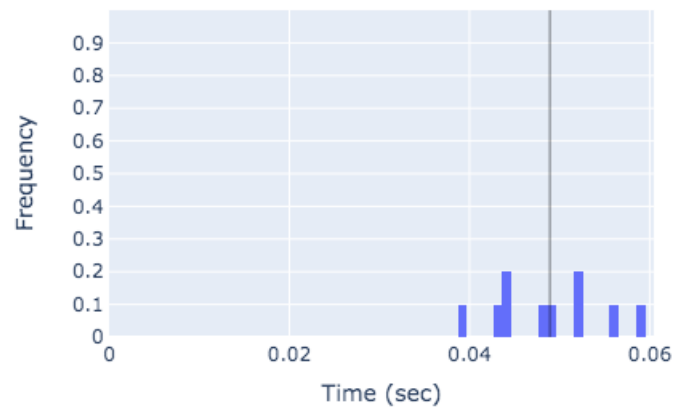
Case 6: Least Squares (True)

N=10, Nrep=10
Mean = 0.05134806632995605(sec)



Min: 0.04111504554748535
Max: 0.06645369529724121
|Max-Min|: 0.02533864974975586

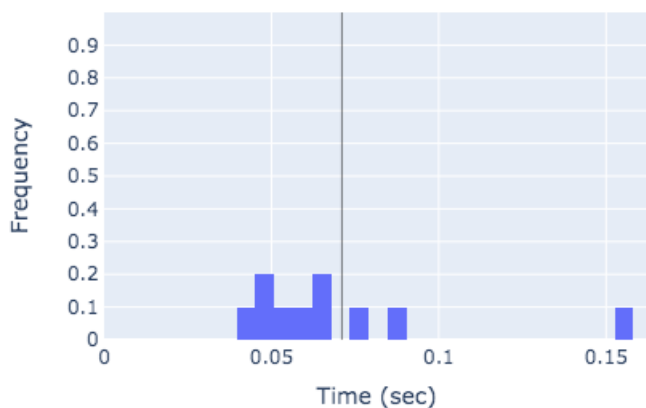
N=100, Nrep=10
Mean = 0.04891963005065918(sec)



Min: 0.039629220962524414
Max: 0.05947113037109375
|Max-Min|: 0.019841909408569336

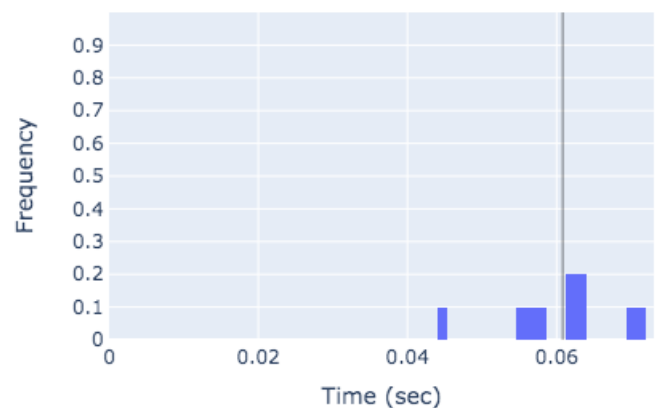
Case 7: Least Squares (False)

N=10, Nrep=10
Mean = 0.07105758190155029(sec)



Min: 0.044137001037597656
Max: 0.157135009765625
|Max-Min|: 0.11299800872802734

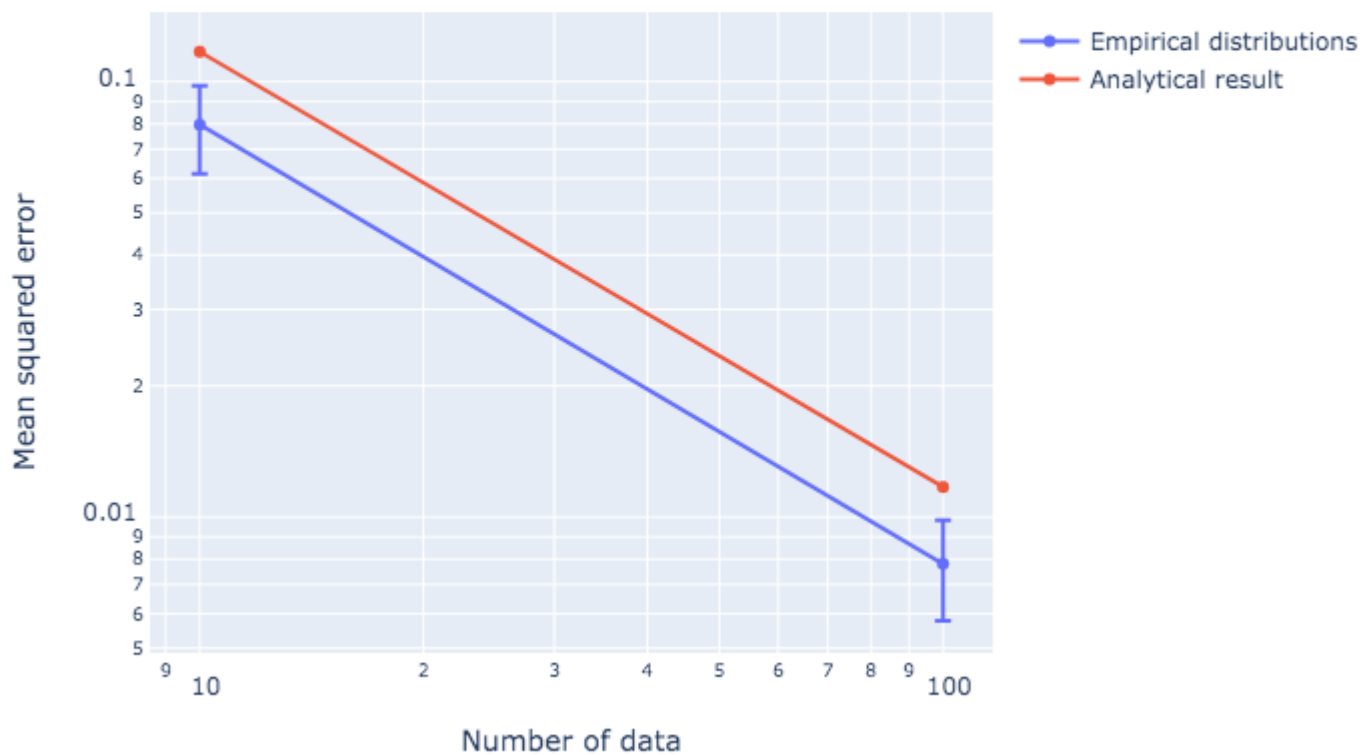
N=100, Nrep=10
Mean = 0.060823917388916016(sec)



Min: 0.04505324363708496
Max: 0.07170581817626953
|Max-Min|: 0.02665257453918457

MSE of empirical distributions

Mean squared error



Consistency test

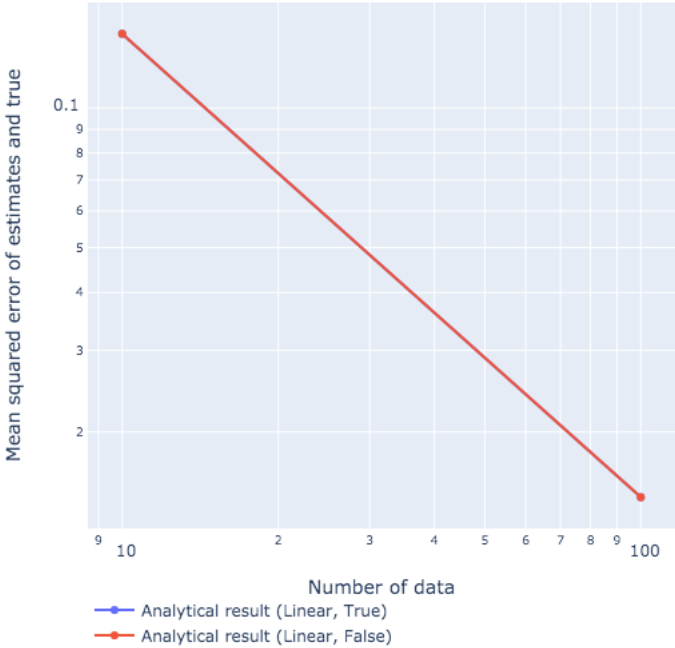
	Name	Type of tomography	Param	Estimator	Loss	Algo	Squared Error to True	Possibly OK	To be checked
0	Linear (True)	Standard Qst	True	Linear	None	None	0.00e+00	OK	not need debug
1	Linear (False)	Standard Qst	False	Linear	None	None	2.47e-32	OK	not need debug
2	ProjectedLinear (True)	Standard Qst	True	Projected Linear	None	None	1.23e-32	OK	not need debug
3	ProjectedLinear (False)	Standard Qst	False	Projected Linear	None	None	1.11e-31	OK	not need debug
4	Maximum-Likelihood (True)	Standard Qst	True	Loss Minimization	Weighted Relative Entropy	Projected Gradient Descent Backtracking	3.21e-18	OK	not need debug
5	Maximum-Likelihood (False)	Standard Qst	False	Loss Minimization	Weighted Relative Entropy	Projected Gradient Descent Backtracking	5.47e-09	NG	not need debug
6	Least Squares (True)	Standard Qst	True	Loss Minimization	Weighted Probability Based Squared Error	Projected Gradient Descent Backtracking	4.93e-32	OK	not need debug

7	Least Squares (False)	Standard Qst	False	Loss Minimization	Weighted Probability Based Squared Error	Projected Gradient Descent Backtracking	1.60e-31	OK	not need debug
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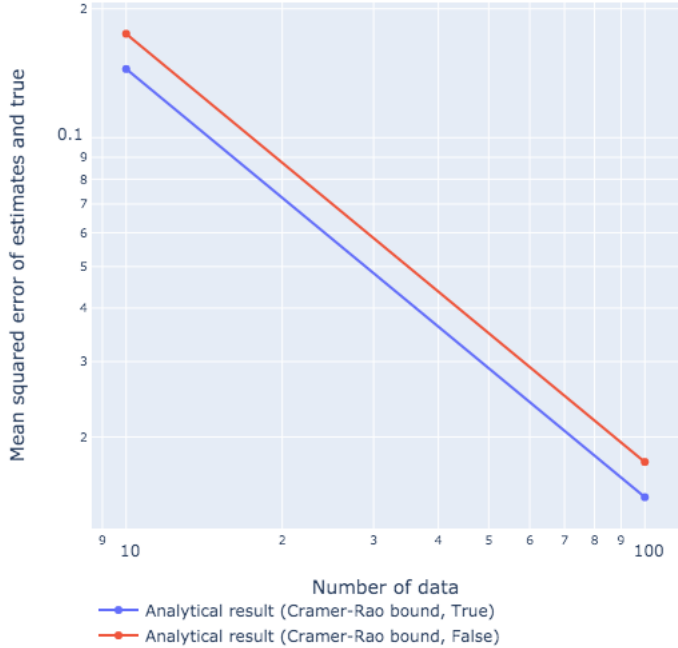
MSE of estimators

Comparison of analytical results

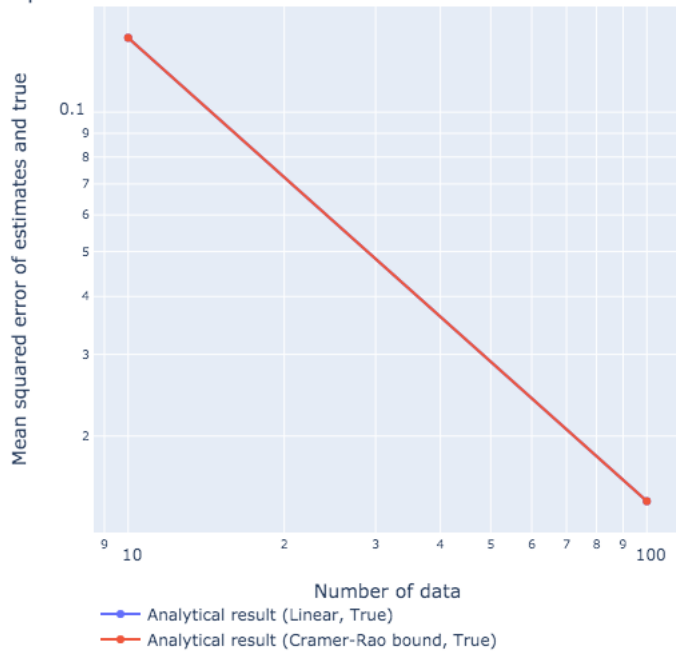
Mean squared error
Analytical result
estimator=Linear



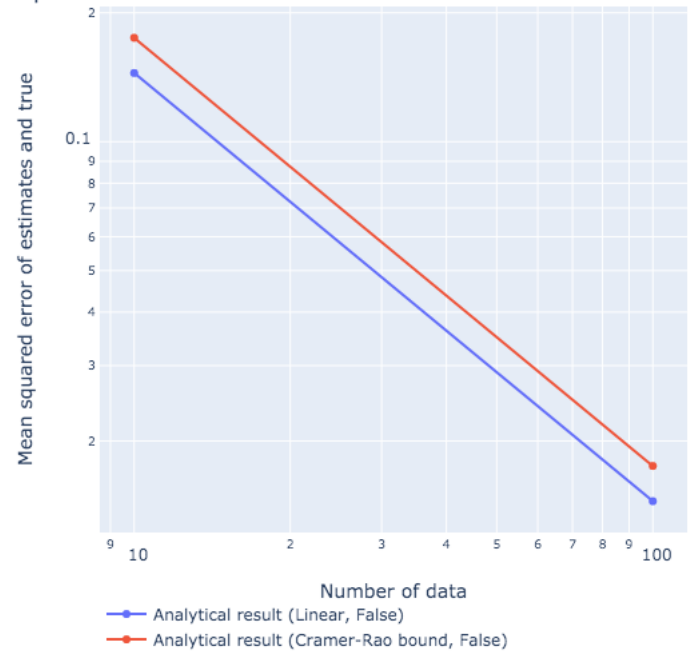
Mean squared error
Analytical result
Cramer-Rao bound



Mean squared error
Analytical result
parameterization=True

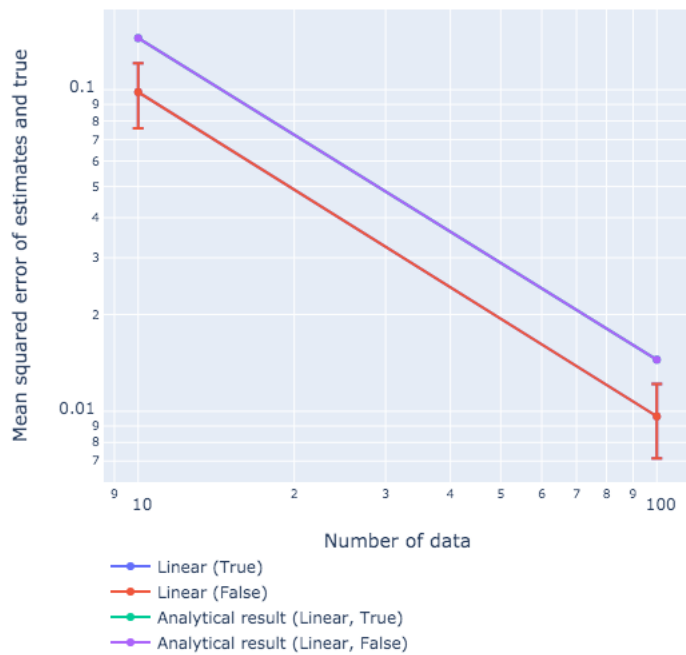


Mean squared error
Analytical result
parameterization=False

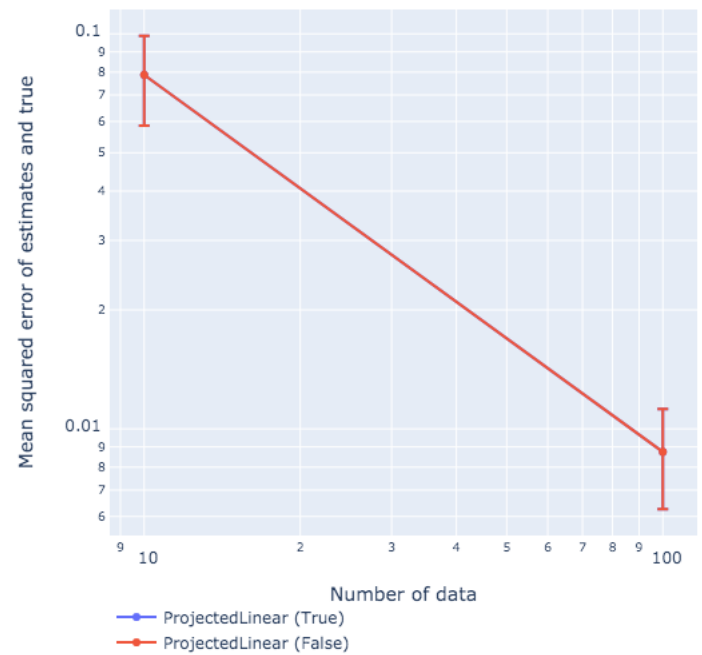


Comparison of parametrization

Mean squared error
Estimator=Linear



Mean squared error
Estimator=ProjectedLinear

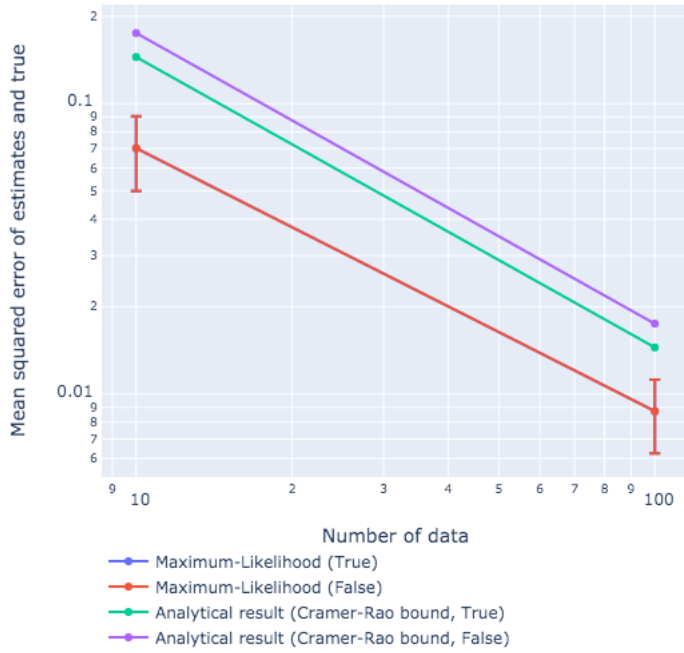


Mean squared error

Estimator=LossMinimization

Loss=WeightedRelativeEntropy

Algo=ProjectedGradientDescentBacktracking

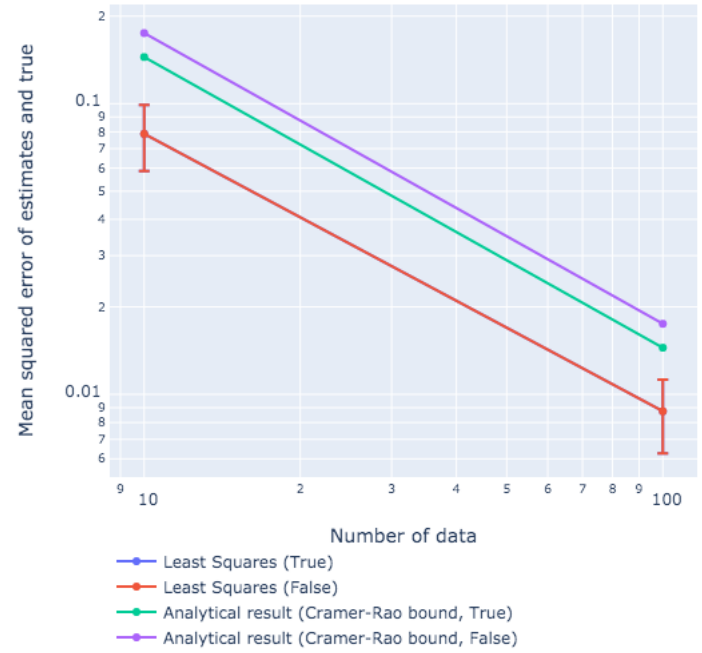


Mean squared error

Estimator=LossMinimization

Loss=WeightedProbabilityBasedSquaredError

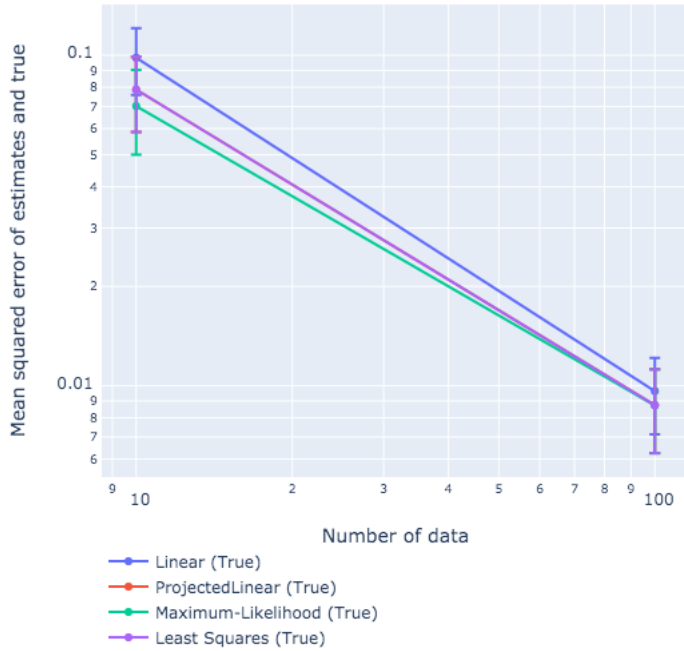
Algo=ProjectedGradientDescentBacktracking



Comparison of estimators

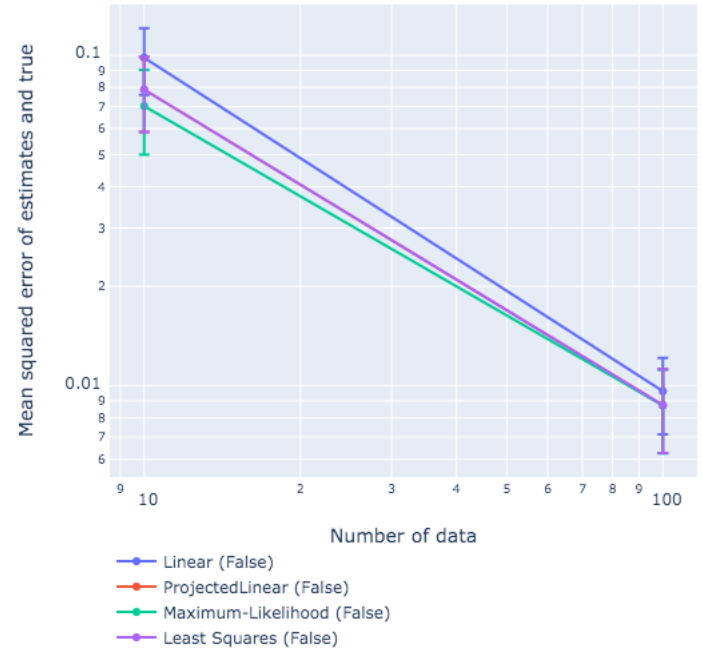
Mean squared error

parametrization=True



Mean squared error

parametrization=False

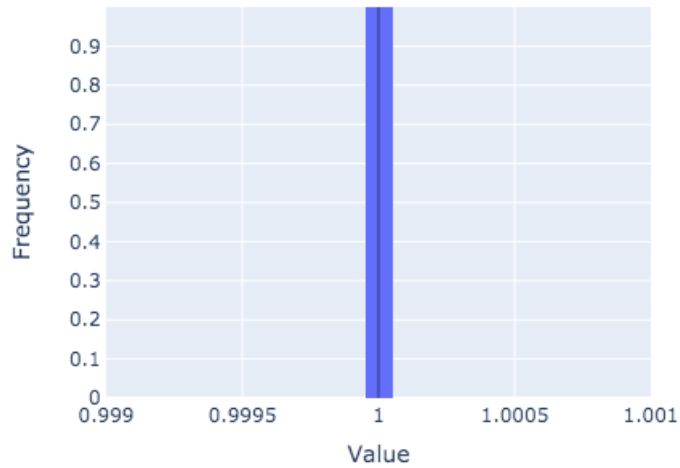


Physicality violation test

Test of equality constraint violation

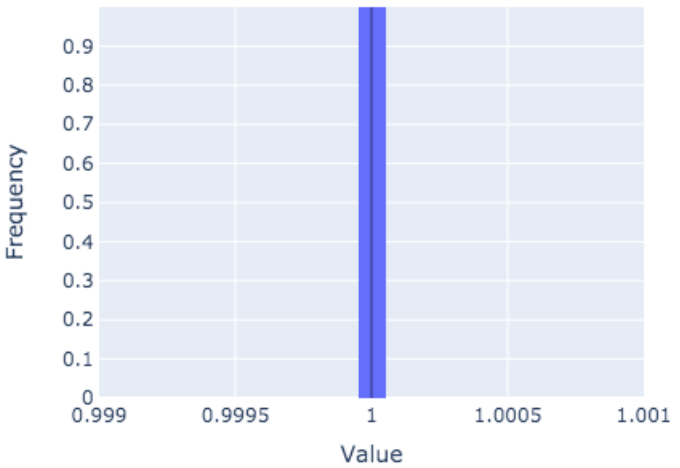
Case 0: Linear (True)

N=10, Nrep=10



Min: 0.9999999999999997
Max: 0.9999999999999998
|Max-Min|: 1.1102230246251565e-16

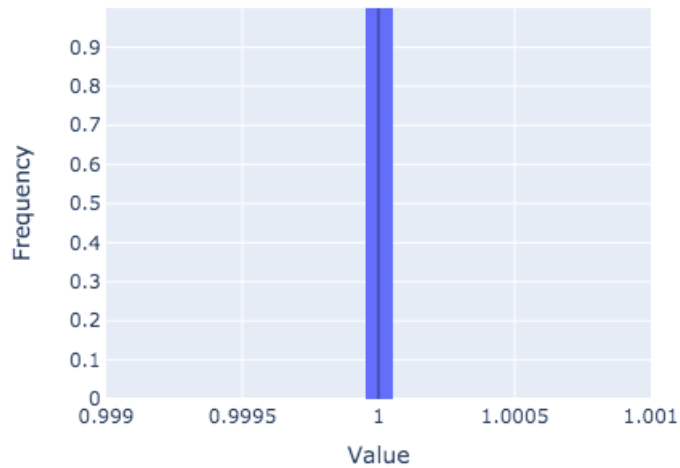
N=100, Nrep=10



Min: 0.9999999999999998
Max: 0.9999999999999998
|Max-Min|: 0.0

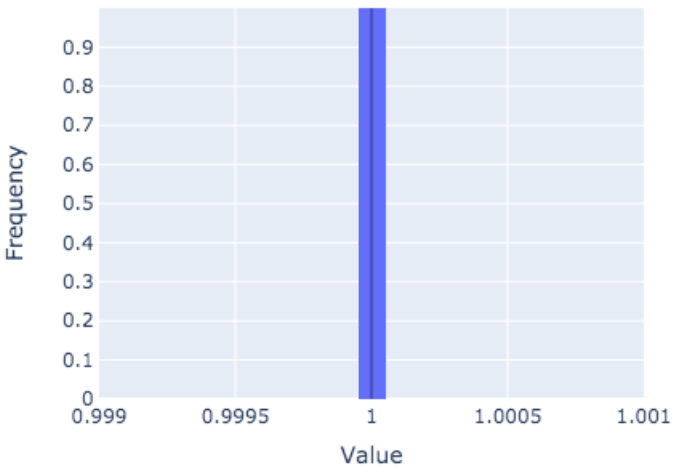
Case 1: Linear (False)

N=10, Nrep=10



Min: 0.9999999999999998
Max: 1.0000000000000002
|Max-Min|: 4.440892098500626e-16

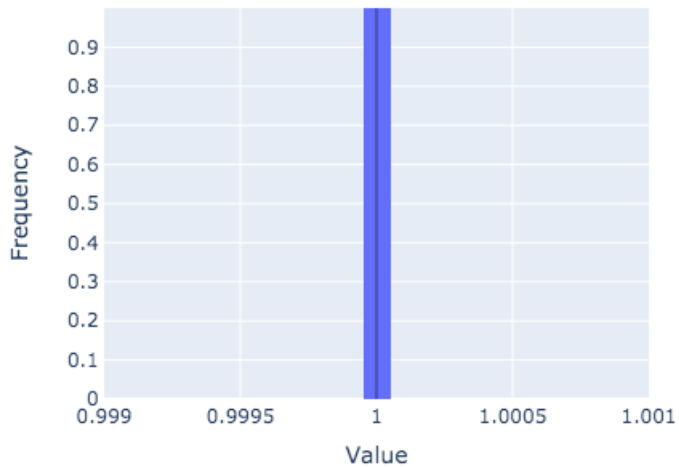
N=100, Nrep=10



Min: 0.9999999999999999
Max: 1.0000000000000002
|Max-Min|: 3.3306690738754696e-16

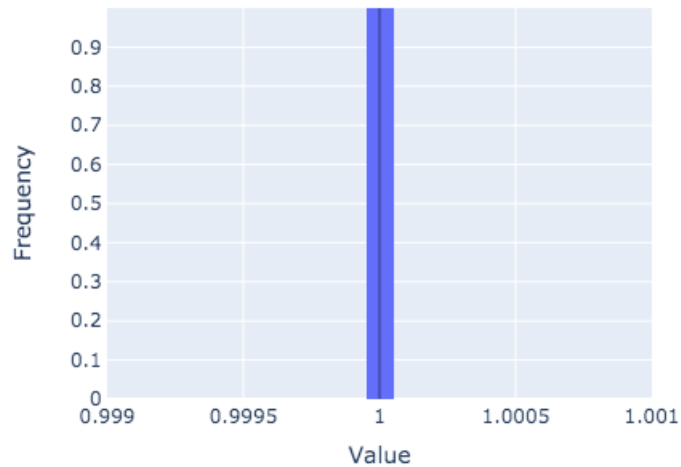
Case 2: ProjectedLinear (True)

N=10, Nrep=10



Min: 0.9999999999999998
Max: 0.9999999999999998
|Max-Min|: 0.0

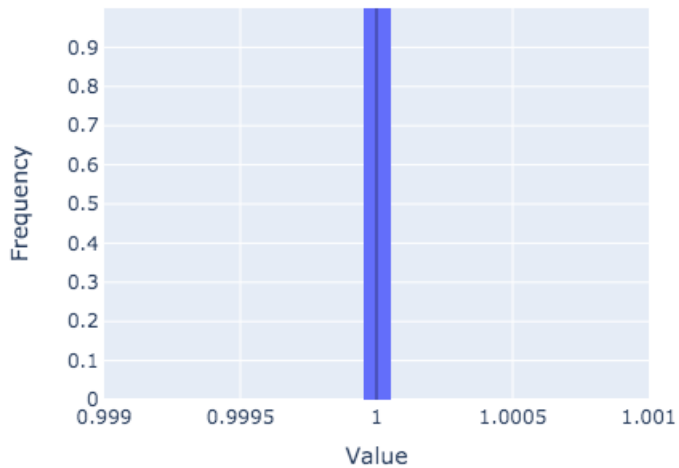
N=100, Nrep=10



Min: 0.9999999999999998
Max: 0.9999999999999998
|Max-Min|: 0.0

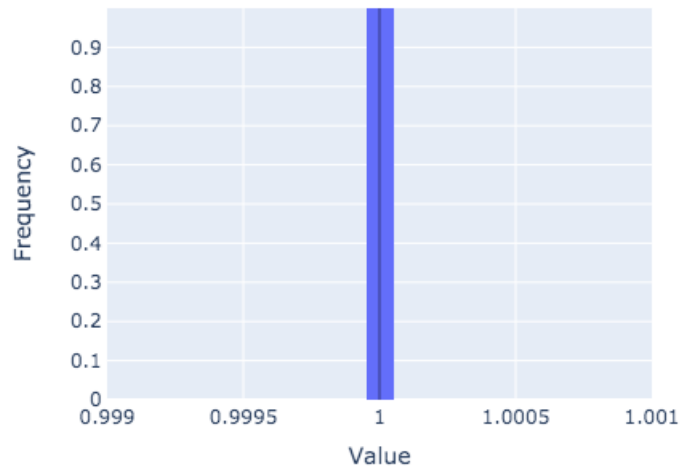
Case 3: ProjectedLinear (False)

N=10, Nrep=10



Min: 0.9999999999999996
Max: 1.00000014753393
|Max-Min|: 1.4753393040578544e-07

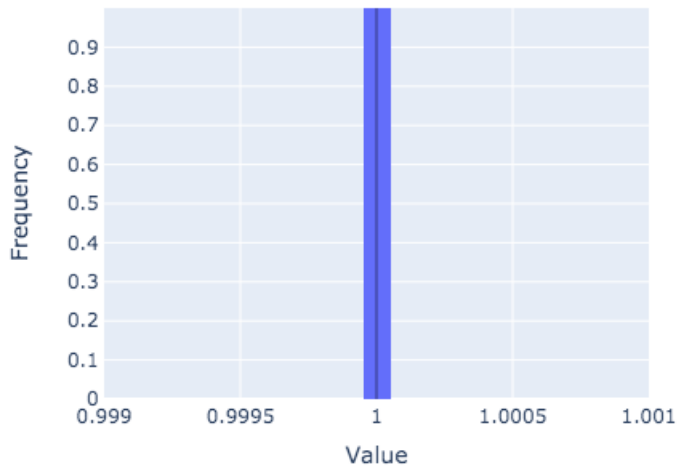
N=100, Nrep=10



Min: 0.9999999999999997
Max: 1.0000001280394155
|Max-Min|: 1.280394158476028e-07

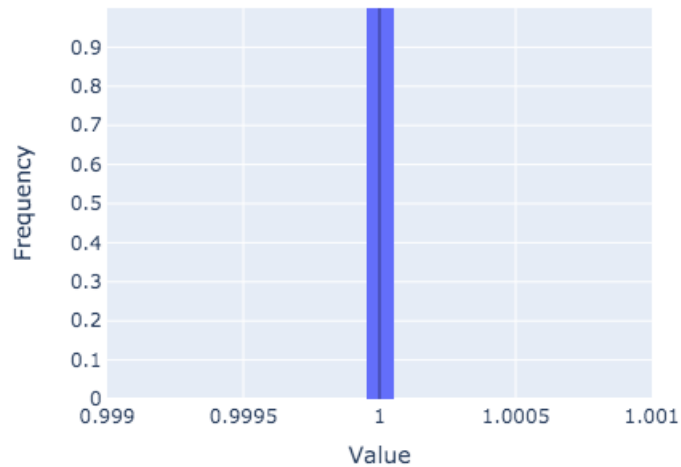
Case 4: Maximum-Likelihood (True)

N=10, Nrep=10



Min: 0.9999999999999998
Max: 0.9999999999999998
|Max-Min|: 0.0

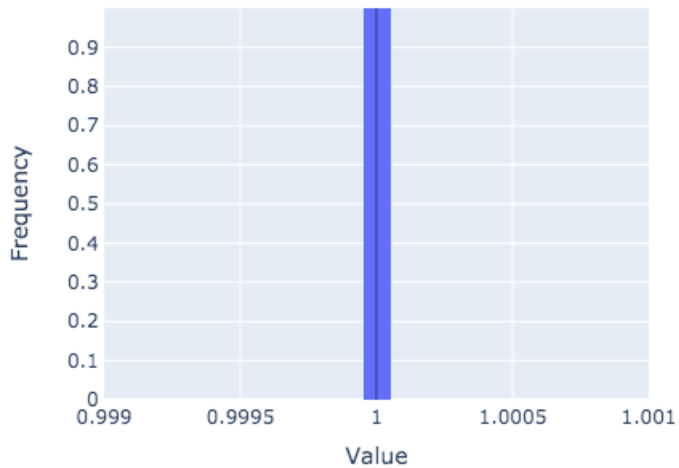
N=100, Nrep=10



Min: 0.9999999999999998
Max: 0.9999999999999998
|Max-Min|: 0.0

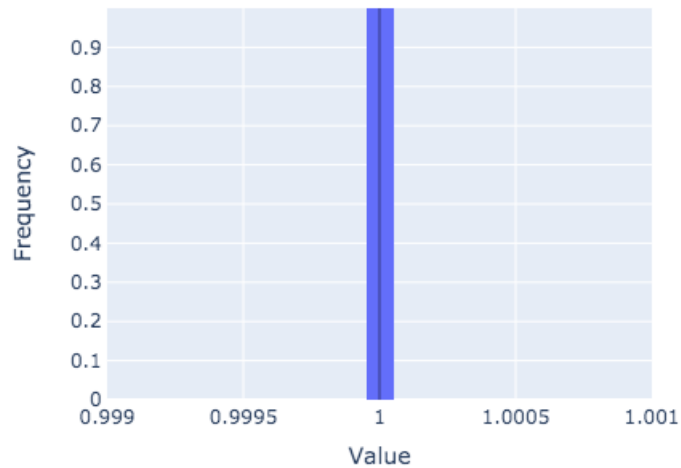
Case 5: Maximum-Likelihood (False)

N=10, Nrep=10



Min: 1.0000000006567707
Max: 1.0000001622723005
|Max-Min|: 1.5570459344083076e-07

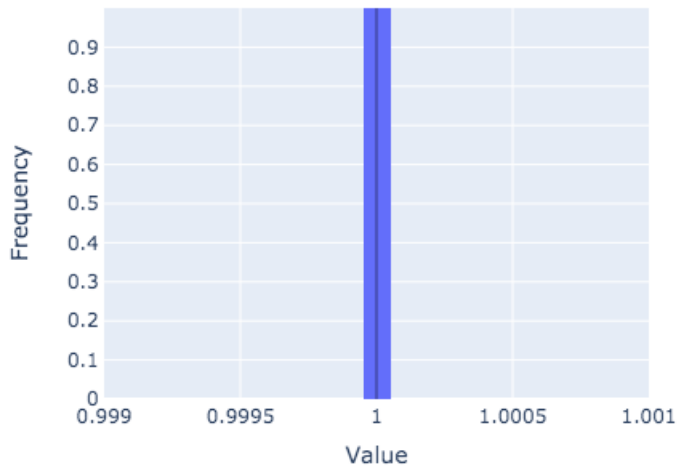
N=100, Nrep=10



Min: 1.0000000121352572
Max: 1.0000001810607821
|Max-Min|: 1.689255249015531e-07

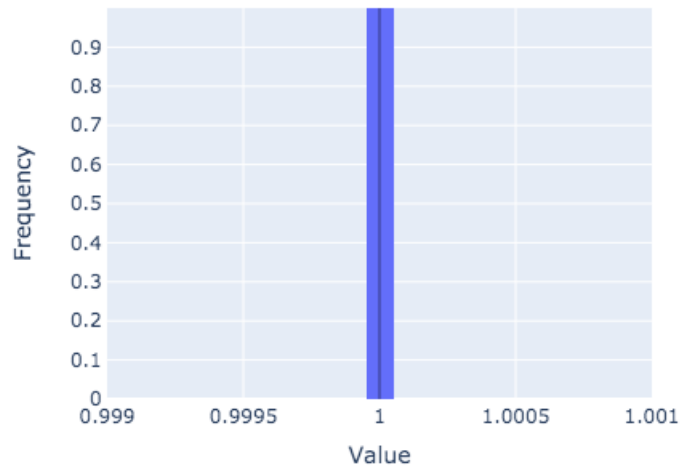
Case 6: Least Squares (True)

N=10, Nrep=10



Min: 0.9999999999999998
Max: 0.9999999999999998
|Max-Min|: 0.0

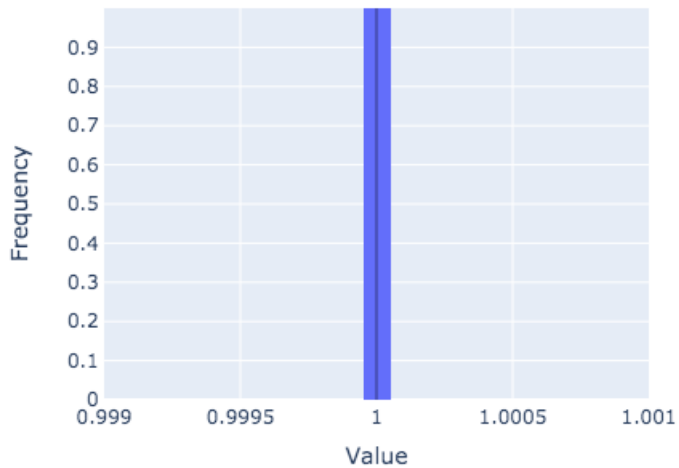
N=100, Nrep=10



Min: 0.9999999999999998
Max: 0.9999999999999998
|Max-Min|: 0.0

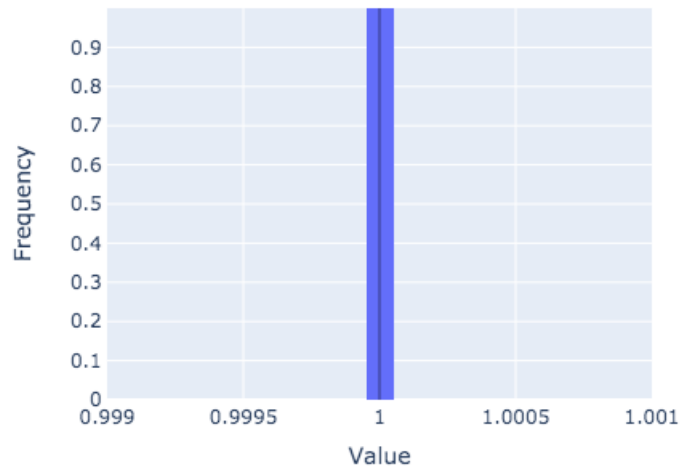
Case 7: Least Squares (False)

N=10, Nrep=10



Min: 0.9999999999999998
Max: 1.0000001781169368
|Max-Min|: 1.7811693697744602e-07

N=100, Nrep=10



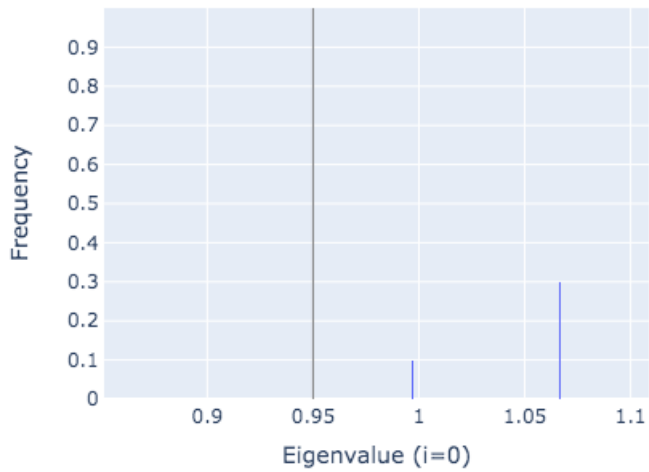
Min: 0.9999999999999998
Max: 1.0000001555680968
|Max-Min|: 1.5556809707106822e-07

Test of inequality constraint violation

Eigenvalue

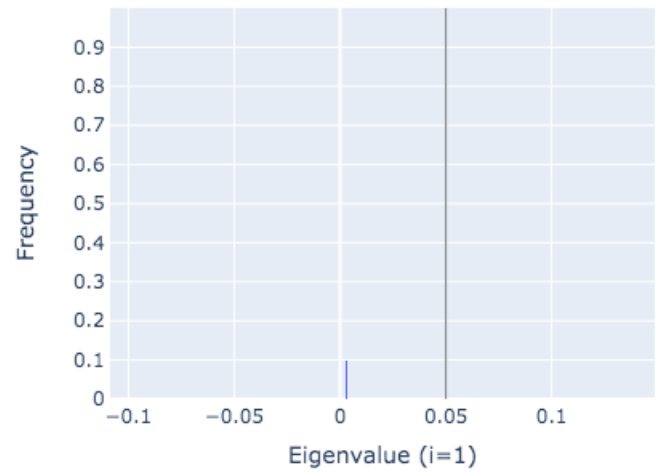
Case 0: Linear (True)

N=10, Nrep=10



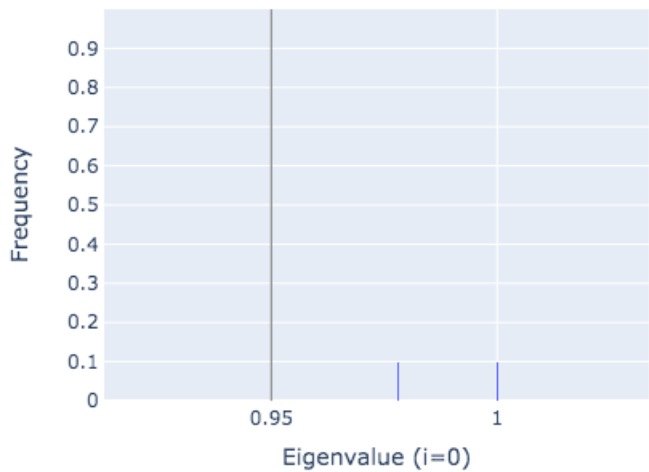
Min: 0.8513641844631532
 Max: 1.1085806194501846
 |Max-Min|: 0.2572164349870314

N=10, Nrep=10



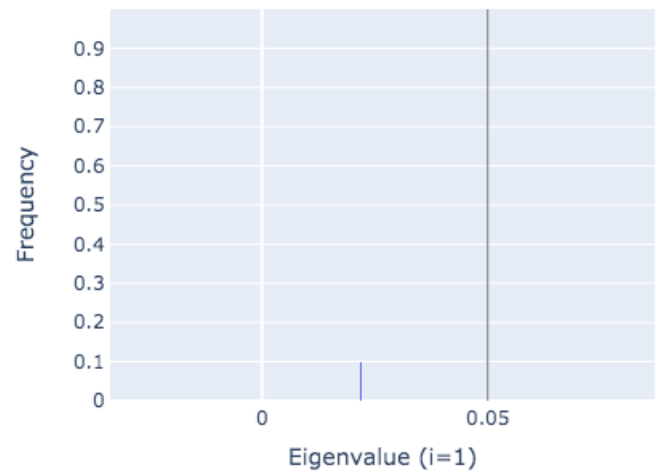
Min: -0.10858061945018471
 Max: 0.14863581553684663
 |Max-Min|: 0.2572164349870314

N=100, Nrep=10



Min: 0.9130584510098023
 Max: 1.0335647646019095
 |Max-Min|: 0.12050631359210717

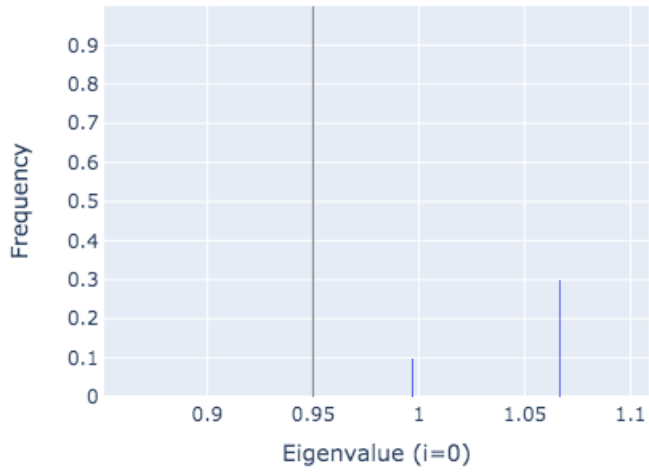
N=100, Nrep=10



Min: -0.03356476460190986
 Max: 0.0869415489901976
 |Max-Min|: 0.12050631359210745

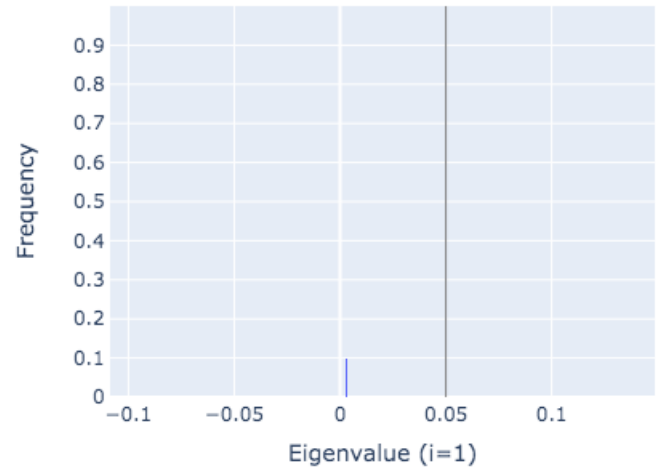
Case 1: Linear (False)

N=10, Nrep=10



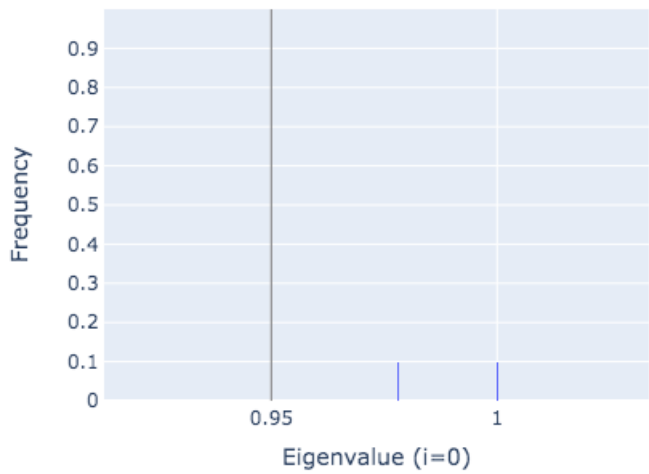
Min: 0.8513641844631532
 Max: 1.1085806194501846
 |Max-Min|: 0.2572164349870314

N=10, Nrep=10



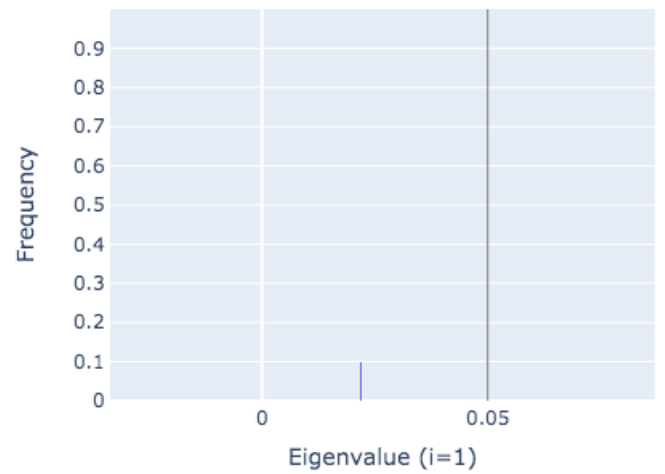
Min: -0.10858061945018459
 Max: 0.14863581553684663
 |Max-Min|: 0.25721643498703123

N=100, Nrep=10



Min: 0.9130584510098023
 Max: 1.0335647646019095
 |Max-Min|: 0.12050631359210717

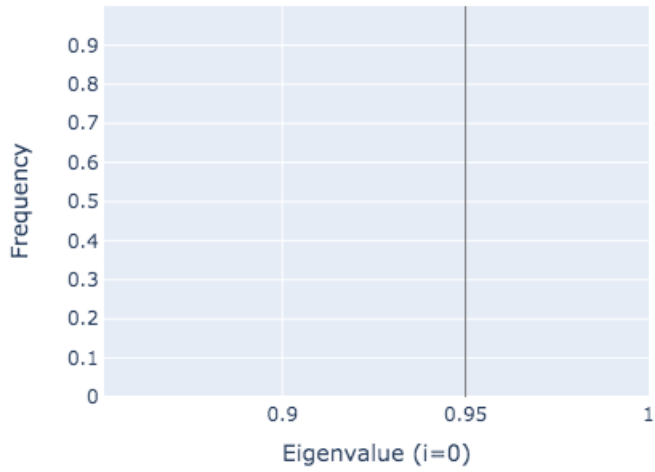
N=100, Nrep=10



Min: -0.03356476460190975
 Max: 0.08694154899019772
 |Max-Min|: 0.12050631359210748

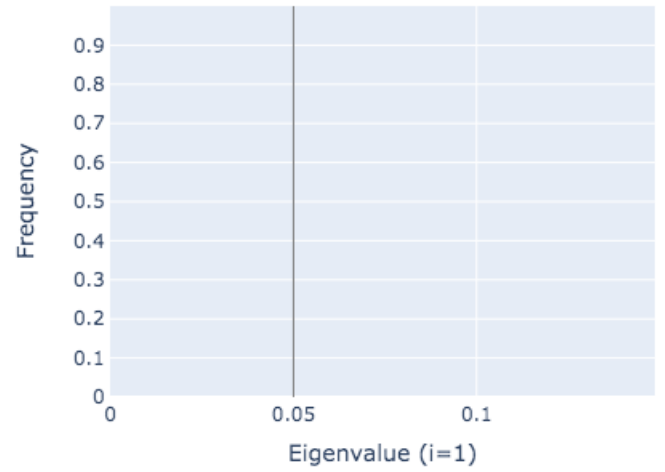
Case 2: ProjectedLinear (True)

N=10, Nrep=10



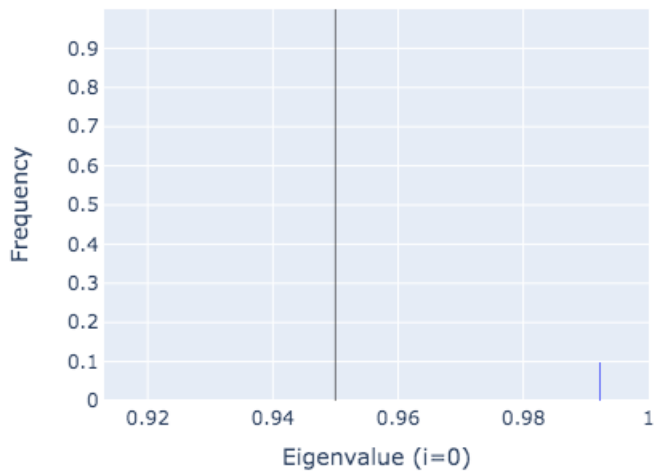
Min: 0.8513641844631531
Max: 1.0000000737669648
|Max-Min|: 0.14863588930381166

N=10, Nrep=10



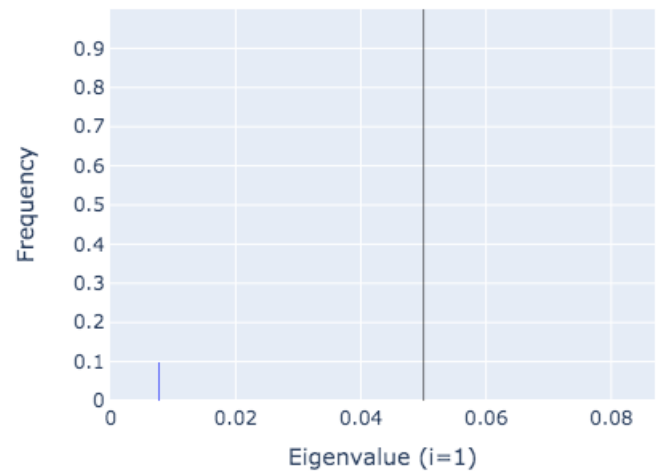
Min: -7.376696509533986e-08
Max: 0.14863581553684668
|Max-Min|: 0.14863588930381177

N=100, Nrep=10



Min: 0.9130584510098023
Max: 1.0000000640197078
|Max-Min|: 0.08694161300990544

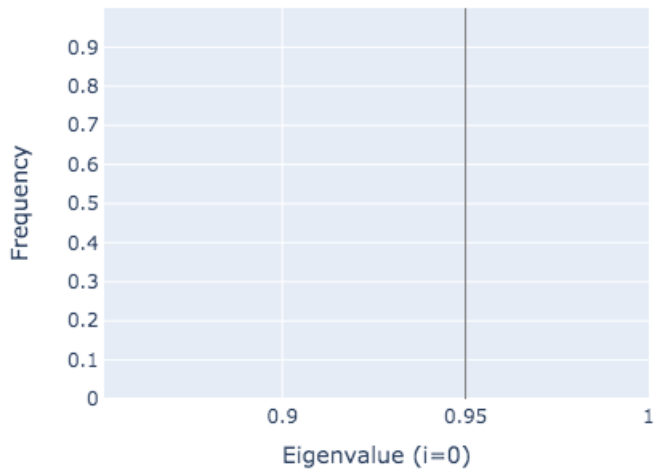
N=100, Nrep=10



Min: -6.401970792556323e-08
Max: 0.0869415489901976
|Max-Min|: 0.08694161300990552

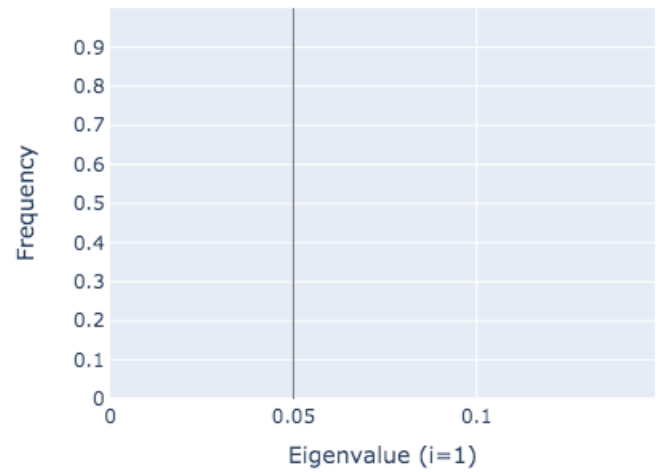
Case 3: ProjectedLinear (False)

N=10, Nrep=10



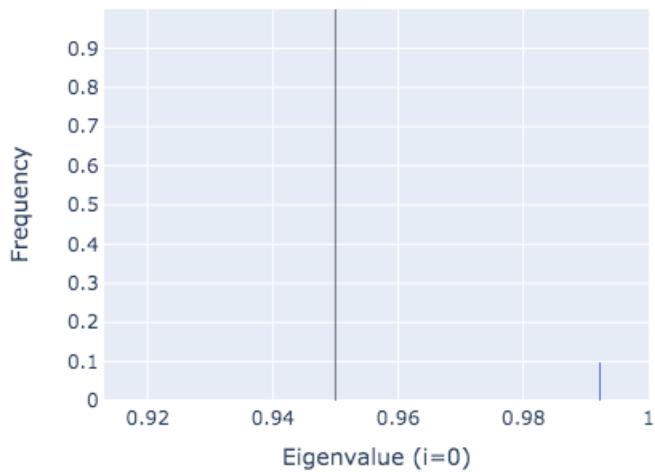
Min: 0.8513641844631532
 Max: 1.00000014753393
 |Max-Min|: 0.14863596307077676

N=10, Nrep=10



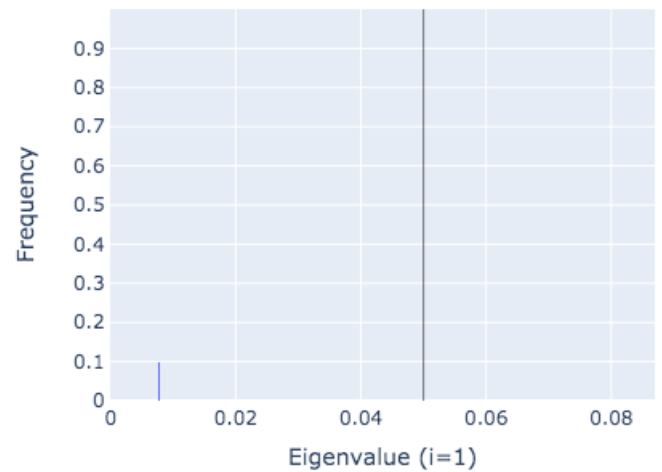
Min: -2.7755575615628914e-17
 Max: 0.14863581553684696
 |Max-Min|: 0.148635815536847

N=100, Nrep=10



Min: 0.9130584510098023
 Max: 1.0000001280394155
 |Max-Min|: 0.0869416770296132

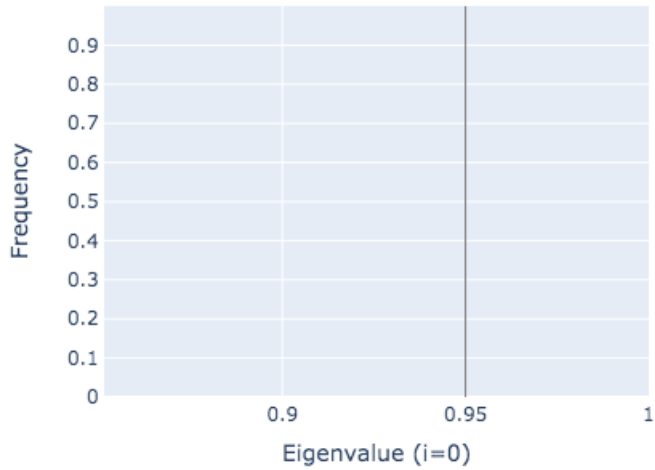
N=100, Nrep=10



Min: -5.730008481585891e-17
 Max: 0.0869415489901976
 |Max-Min|: 0.08694154899019765

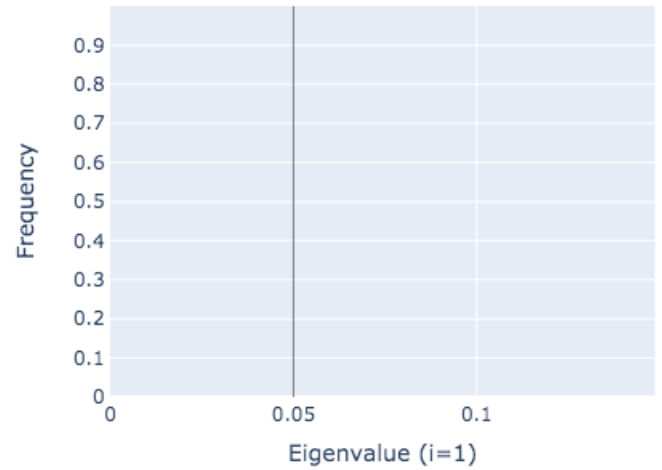
Case 4: Maximum-Likelihood (True)

N=10, Nrep=10



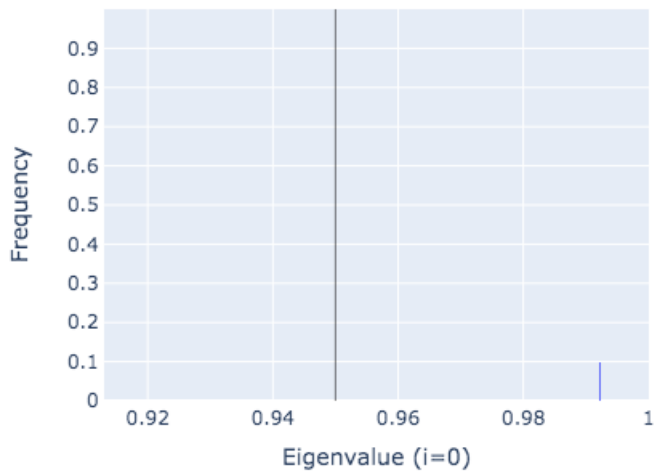
Min: 0.8513641812250503
Max: 1.0000000702659644
|Max-Min|: 0.14863588904091407

N=10, Nrep=10



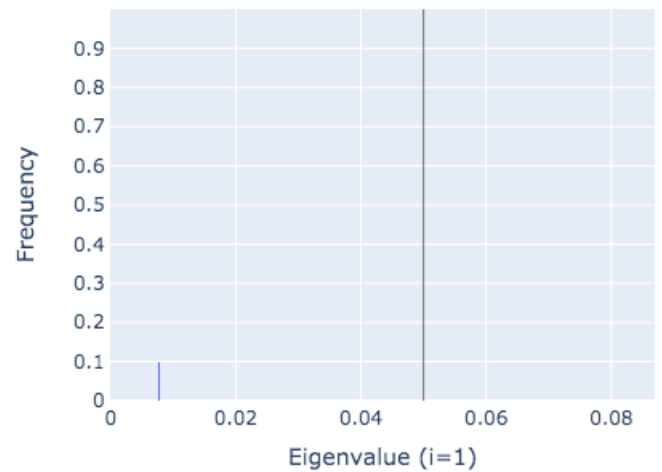
Min: -7.026596454684508e-08
Max: 0.14863581877494952
|Max-Min|: 0.14863588904091407

N=100, Nrep=10



Min: 0.9130584509383943
Max: 1.000000078401553
|Max-Min|: 0.0869416274631587

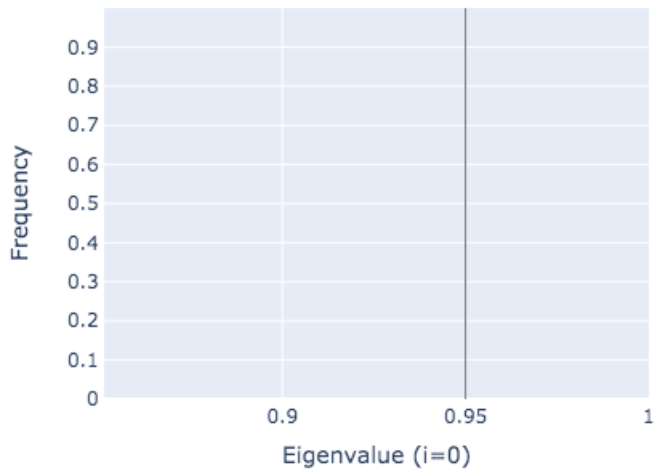
N=100, Nrep=10



Min: -7.840155328241432e-08
Max: 0.08694154906160545
|Max-Min|: 0.08694162746315873

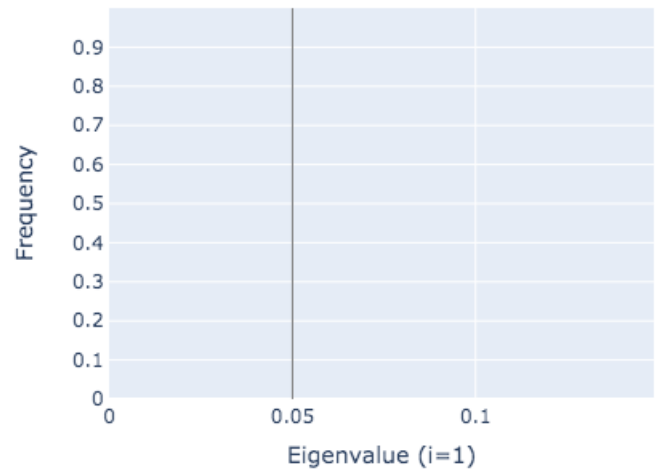
Case 5: Maximum-Likelihood (False)

N=10, Nrep=10



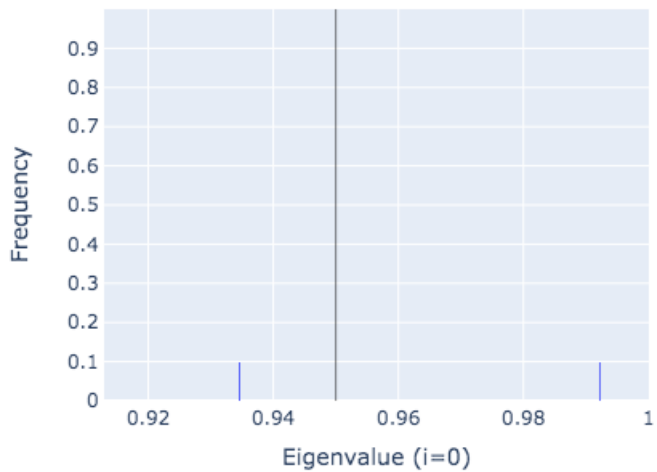
Min: 0.851427240055816
 Max: 1.000000162272261
 |Max-Min|: 0.1485729222164449

N=10, Nrep=10



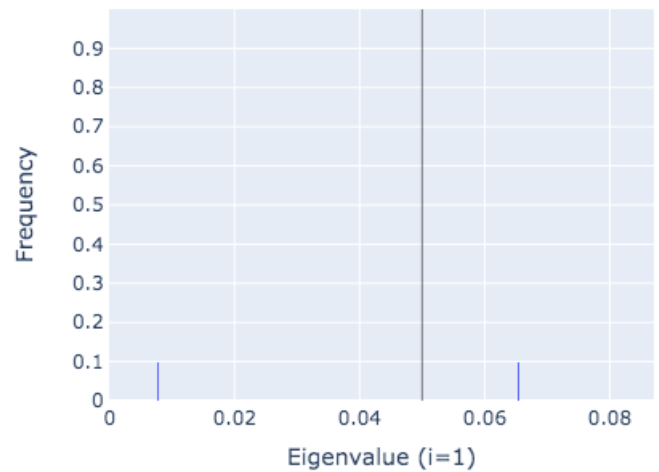
Min: -1.3877787807814457e-17
 Max: 0.14857279250213073
 |Max-Min|: 0.14857279250213073

N=100, Nrep=10



Min: 0.9130380513737355
 Max: 1.0000001810607806
 |Max-Min|: 0.08696212968704509

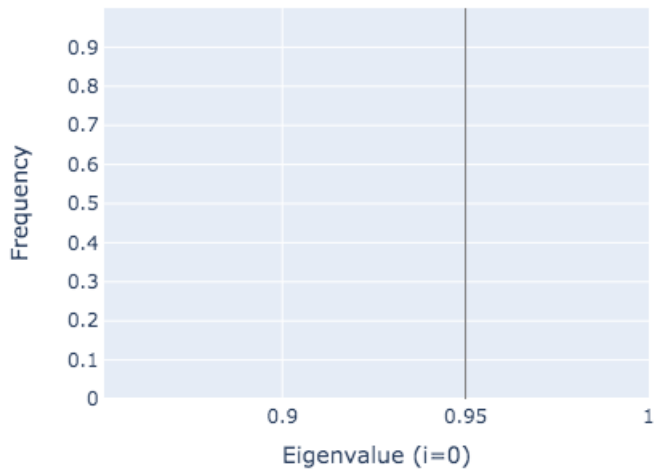
N=100, Nrep=10



Min: -1.0061396160665481e-16
 Max: 0.08696196076152173
 |Max-Min|: 0.08696196076152182

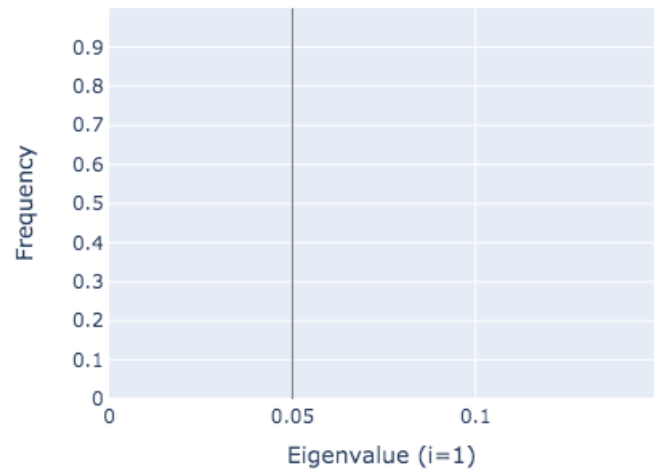
Case 6: Least Squares (True)

N=10, Nrep=10



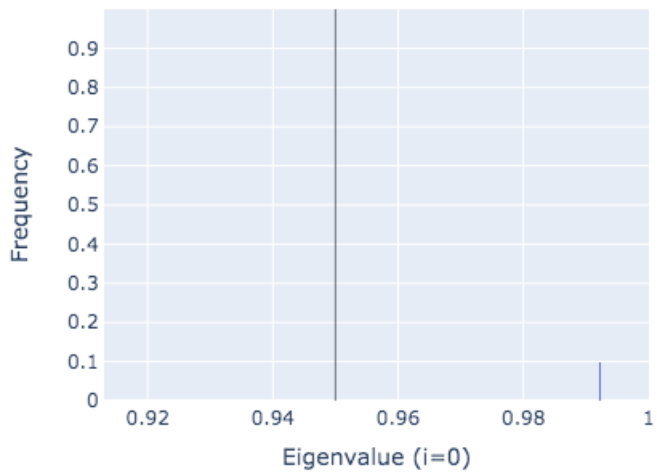
Min: 0.851364184463153
Max: 1.0000000761055894
|Max-Min|: 0.1486358916424364

N=10, Nrep=10



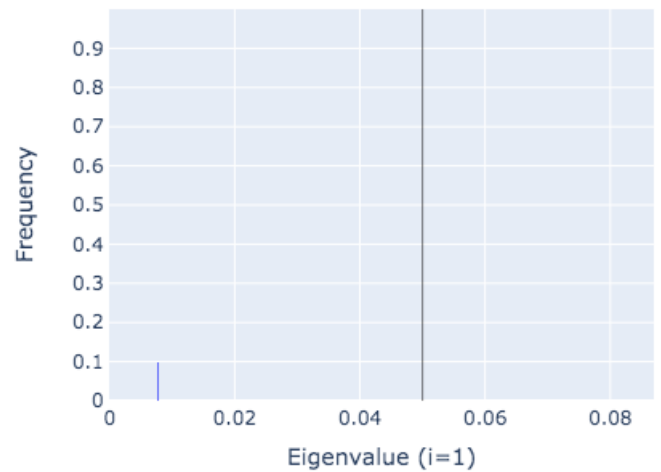
Min: -7.610558962201086e-08
Max: 0.14863581553684674
|Max-Min|: 0.14863589164243635

N=100, Nrep=10



Min: 0.9130584510098028
Max: 1.0000000598780594
|Max-Min|: 0.08694160886825664

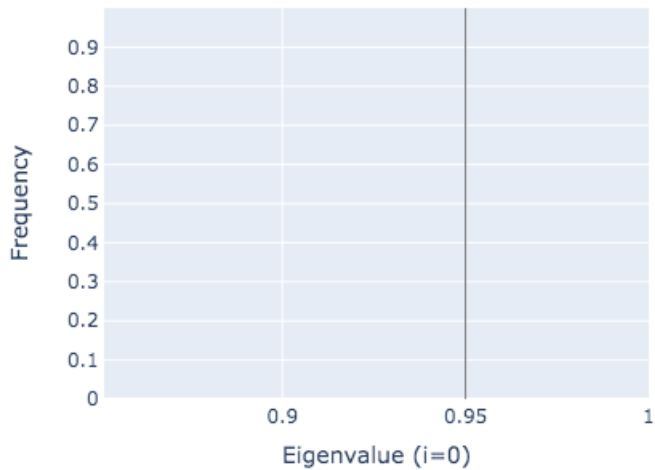
N=100, Nrep=10



Min: -5.987805948691623e-08
Max: 0.0869415489901971
|Max-Min|: 0.08694160886825658

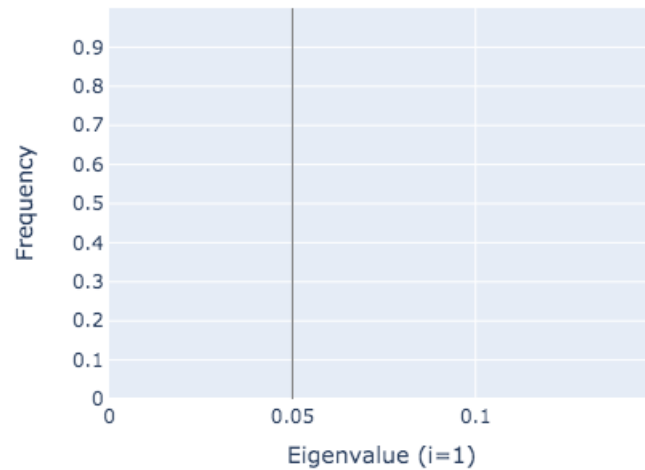
Case 7: Least Squares (False)

N=10, Nrep=10



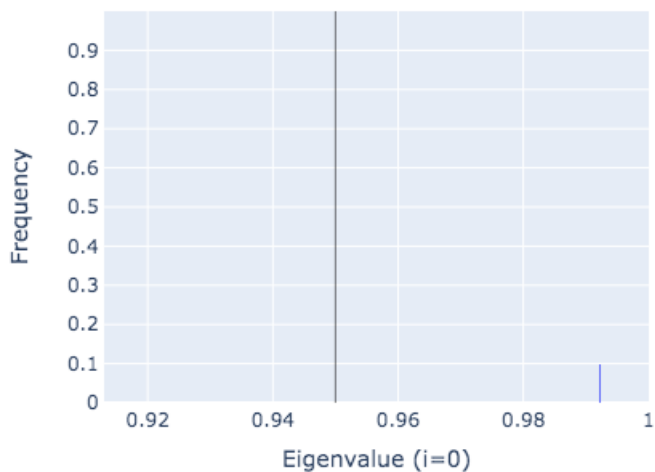
Min: 0.8513641844631532
 Max: 1.0000001781169368
 |Max-Min|: 0.14863599365378355

N=10, Nrep=10



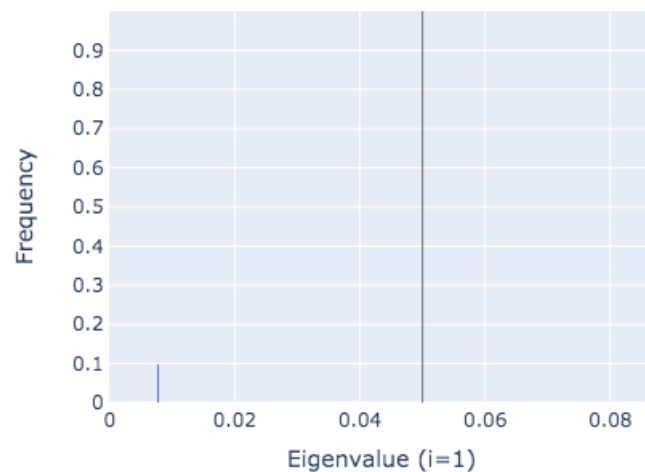
Min: -7.632783294297951e-17
 Max: 0.14863581553684663
 |Max-Min|: 0.1486358155368467

N=100, Nrep=10



Min: 0.9130584510098023
 Max: 1.0000001555680968
 |Max-Min|: 0.08694170455829453

N=100, Nrep=10



Min: 3.06287113727155e-18
 Max: 0.08694154899019754
 |Max-Min|: 0.08694154899019754

Sum of unphysical eigenvalues

Case 0: Linear (True)

N=10

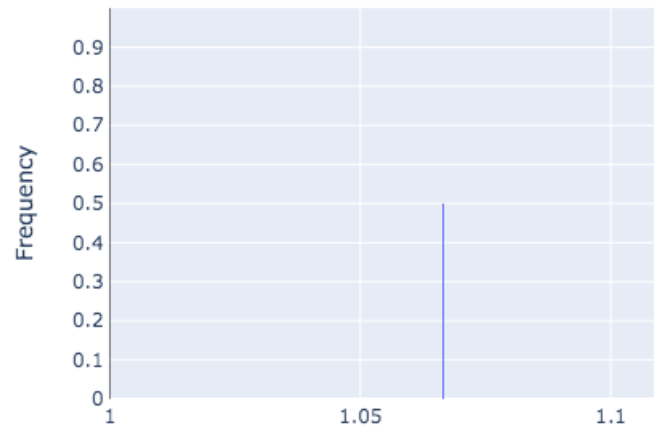
Number of unphysical estimates=6

N=10, Nrep=10



Min: -0.10858061945018471
 Max: -0.009175077217315732
 |Max-Min|: 0.09940554223286899

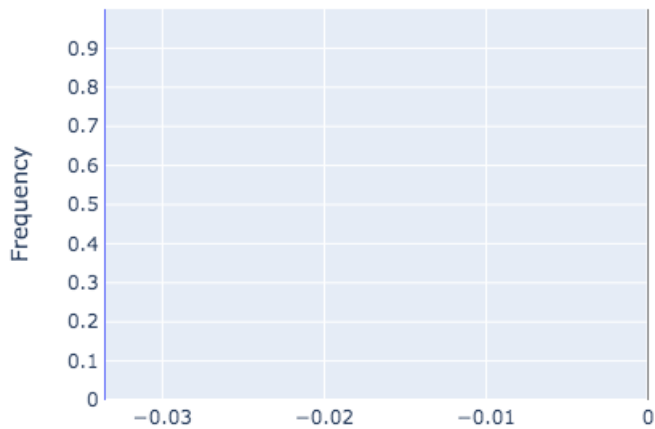
N=10, Nrep=10



Min: 1.0091750772173156
 Max: 1.1085806194501846
 |Max-Min|: 0.09940554223286902

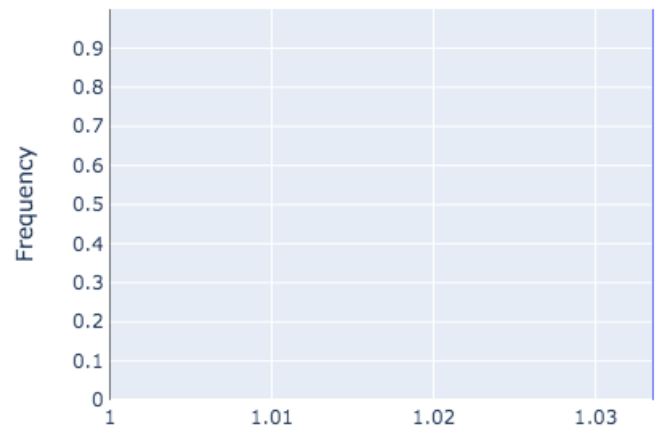
N=100
 Number of unphysical estimates=1

N=100, Nrep=10



Min: -0.03356476460190986
 Max: -0.03356476460190986
 |Max-Min|: 0.0

N=100, Nrep=10

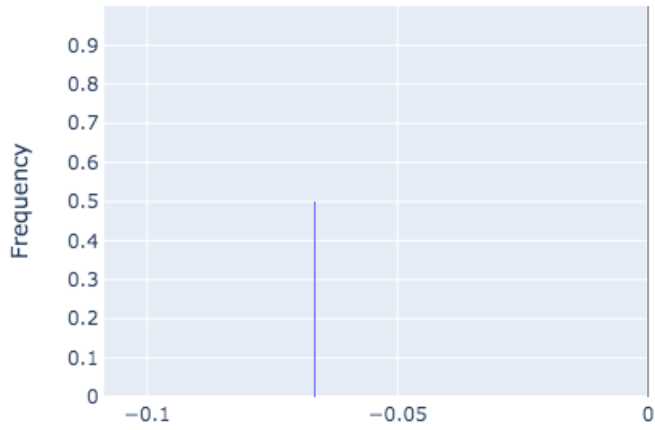


Min: 1.0335647646019095
 Max: 1.0335647646019095
 |Max-Min|: 0.0

Case 1: Linear (False)

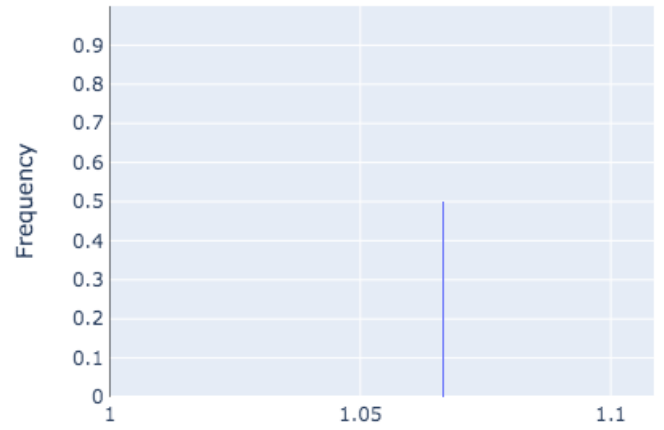
N=10
 Number of unphysical estimates=6

N=10, Nrep=10



Min: -0.10858061945018459
Max: -0.009175077217315586
|Max-Min|: 0.099405542232869

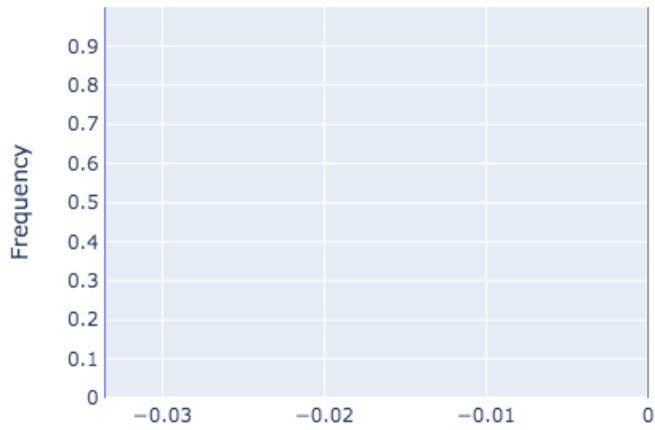
N=10, Nrep=10



Min: 1.0091750772173156
Max: 1.1085806194501846
|Max-Min|: 0.09940554223286902

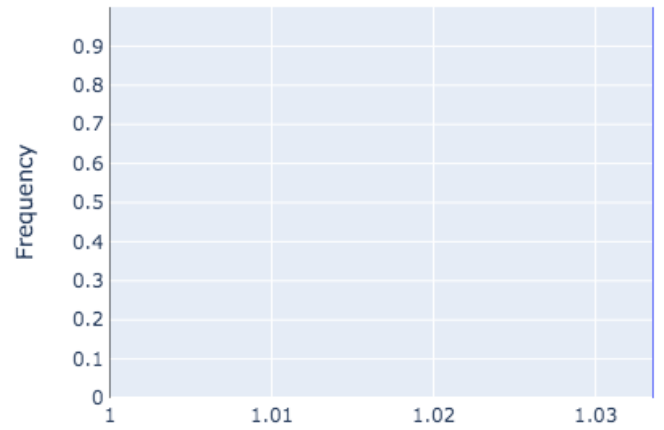
N=100
Number of unphysical estimates=1

N=100, Nrep=10



Min: -0.03356476460190975
Max: -0.03356476460190975
|Max-Min|: 0.0

N=100, Nrep=10

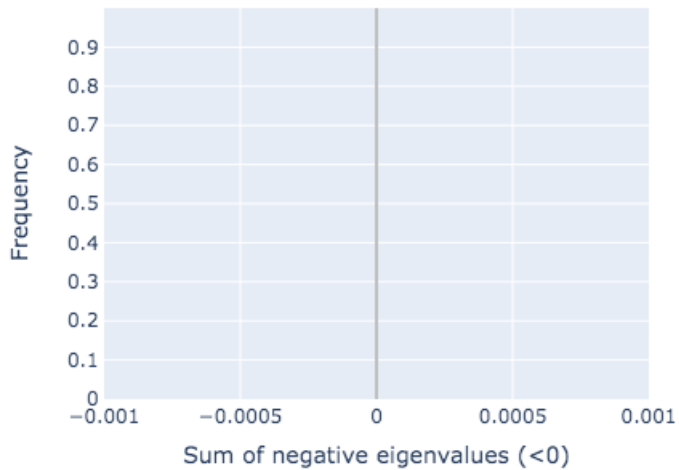


Min: 1.0335647646019095
Max: 1.0335647646019095
|Max-Min|: 0.0

Case 2: ProjectedLinear (True)

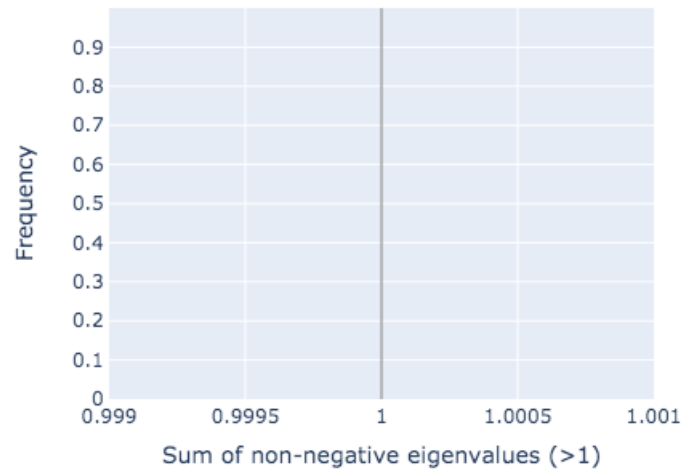
N=10
Number of unphysical estimates=0

N=10, Nrep=10



Min: -
Max: -
|Max-Min|: -

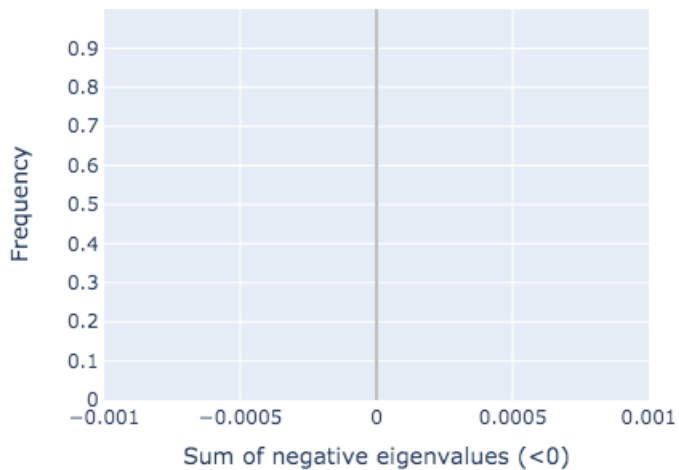
N=10, Nrep=10



Min: -
Max: -
|Max-Min|: -

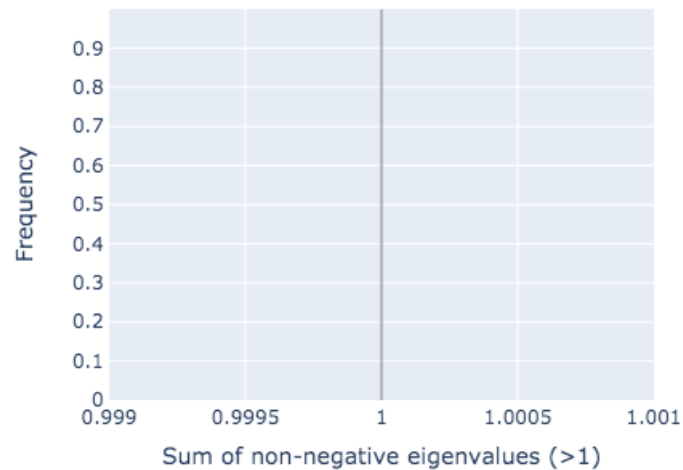
N=100
Number of unphysical estimates=0

N=100, Nrep=10



Min: -
Max: -
|Max-Min|: -

N=100, Nrep=10

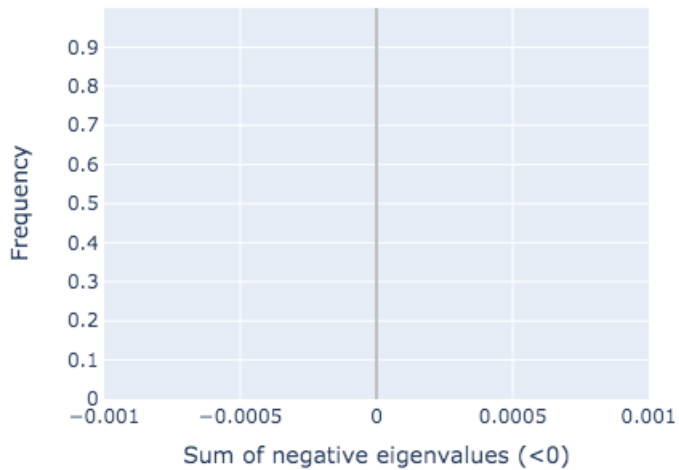


Min: -
Max: -
|Max-Min|: -

Case 3: ProjectedLinear (False)

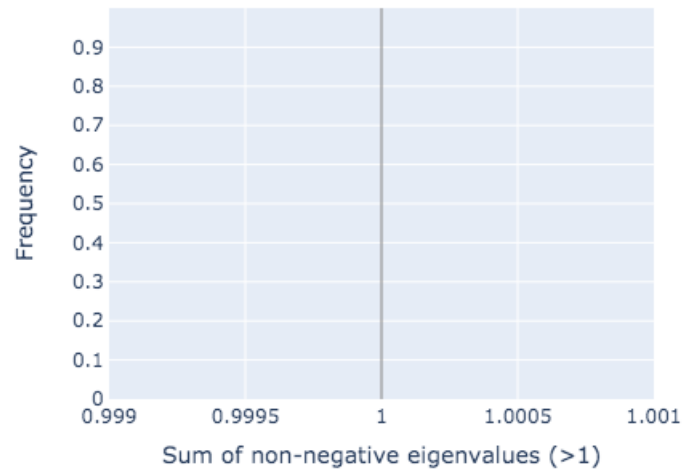
N=10
Number of unphysical estimates=0

N=10, Nrep=10



Min: -
Max: -
|Max-Min|: -

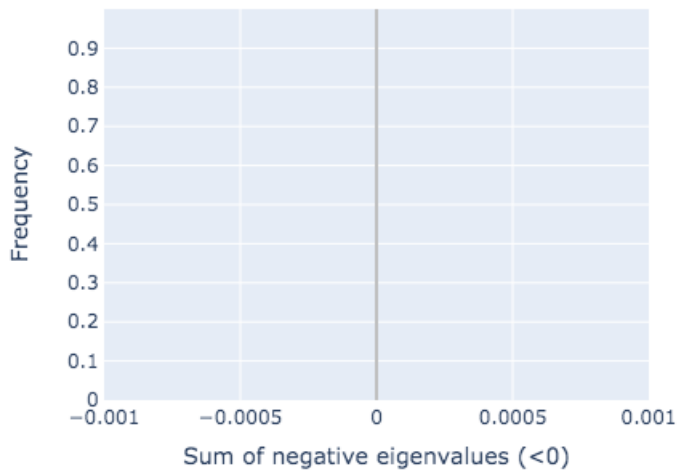
N=10, Nrep=10



Min: -
Max: -
|Max-Min|: -

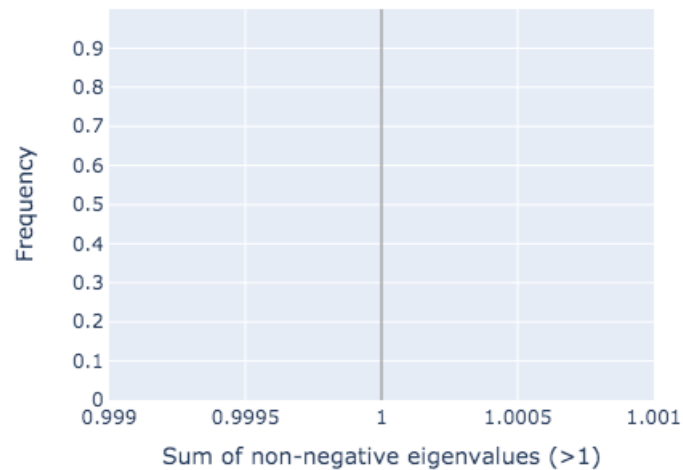
N=100
Number of unphysical estimates=0

N=100, Nrep=10



Min: -
Max: -
|Max-Min|: -

N=100, Nrep=10

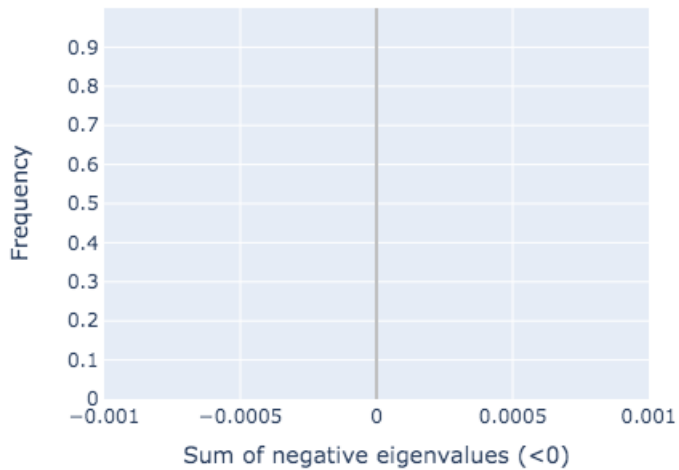


Min: -
Max: -
|Max-Min|: -

Case 4: Maximum-Likelihood (True)

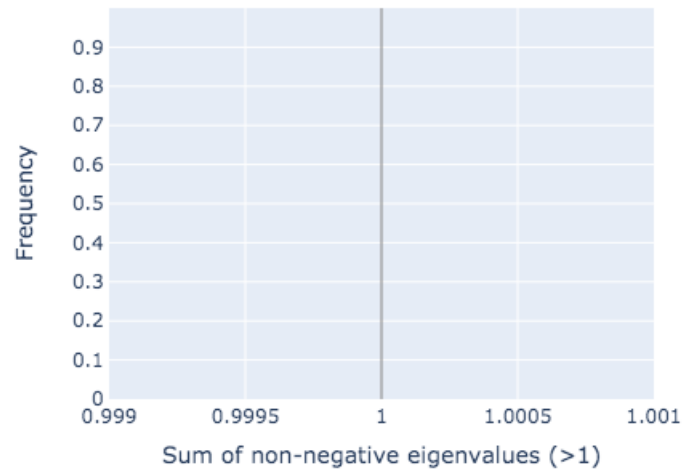
N=10
Number of unphysical estimates=0

N=10, Nrep=10



Min: -
Max: -
|Max-Min|: -

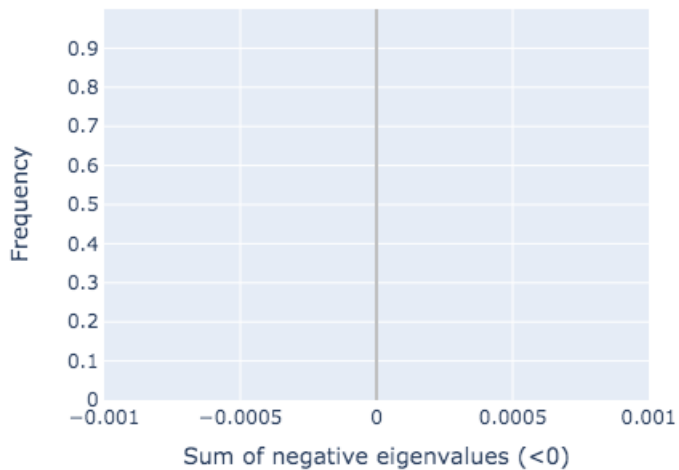
N=10, Nrep=10



Min: -
Max: -
|Max-Min|: -

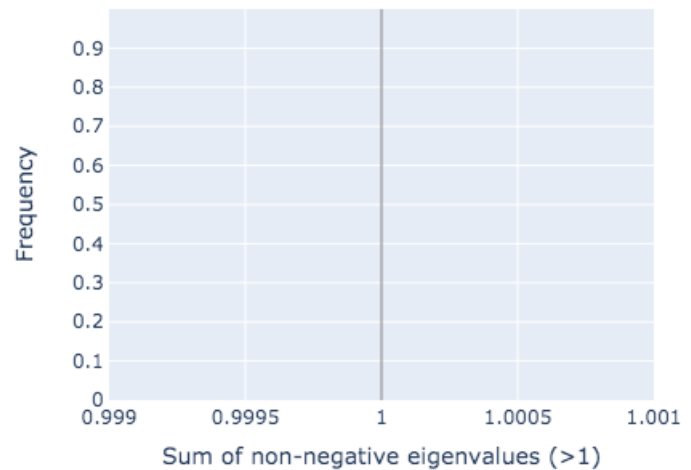
N=100
Number of unphysical estimates=0

N=100, Nrep=10



Min: -
Max: -
|Max-Min|: -

N=100, Nrep=10

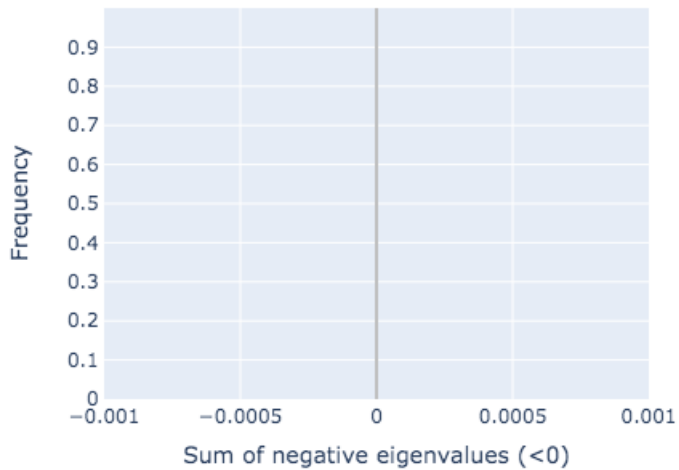


Min: -
Max: -
|Max-Min|: -

Case 5: Maximum-Likelihood (False)

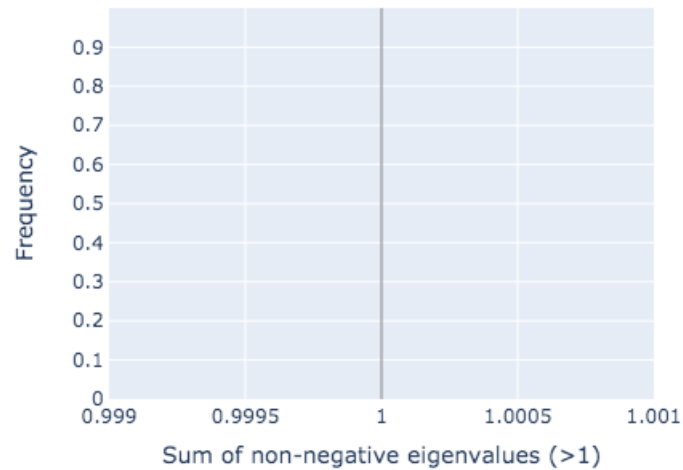
N=10
Number of unphysical estimates=0

N=10, Nrep=10



Min: -
Max: -
|Max-Min|: -

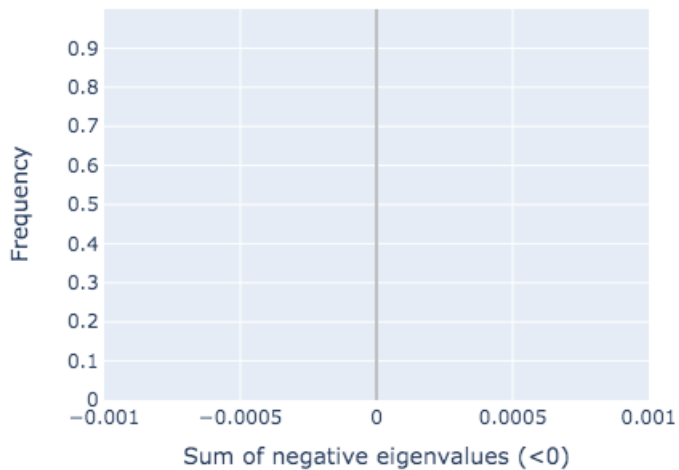
N=10, Nrep=10



Min: -
Max: -
|Max-Min|: -

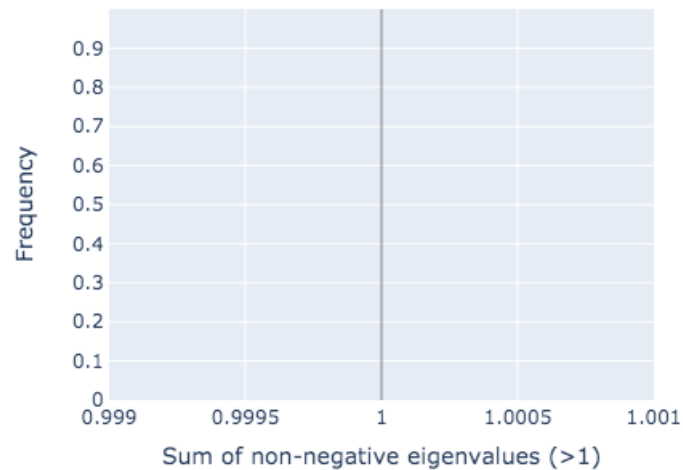
N=100
Number of unphysical estimates=0

N=100, Nrep=10



Min: -
Max: -
|Max-Min|: -

N=100, Nrep=10

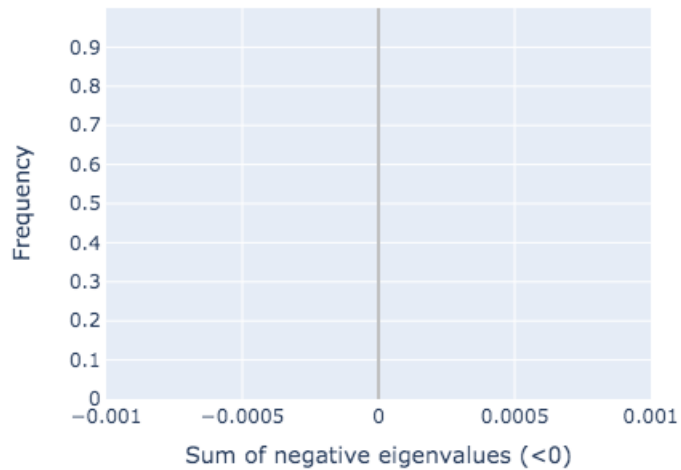


Min: -
Max: -
|Max-Min|: -

Case 6: Least Squares (True)

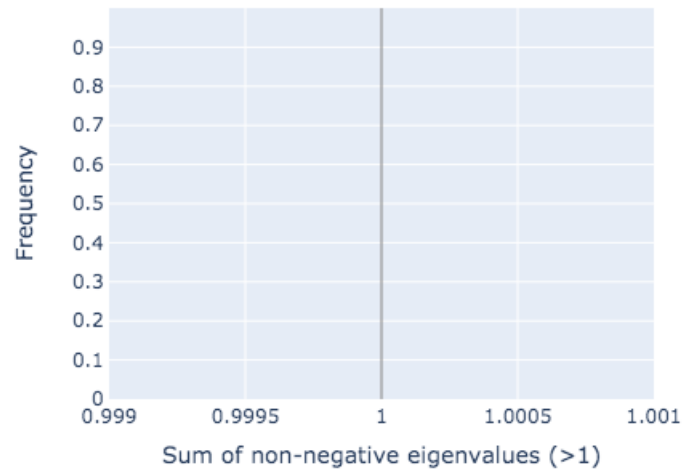
N=10
Number of unphysical estimates=0

N=10, Nrep=10



Min: -
Max: -
|Max-Min|: -

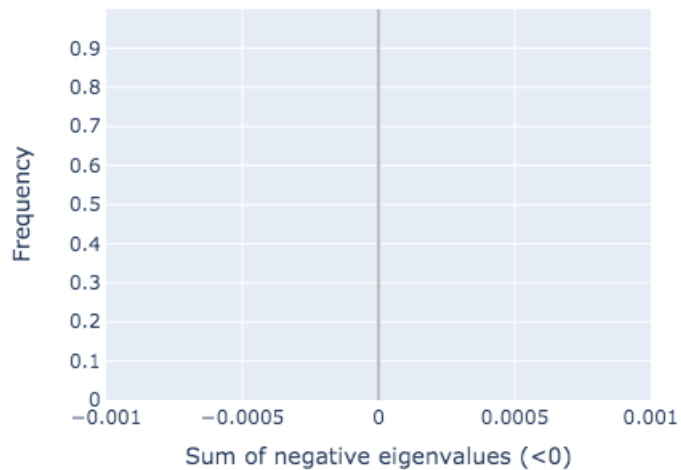
N=10, Nrep=10



Min: -
Max: -
|Max-Min|: -

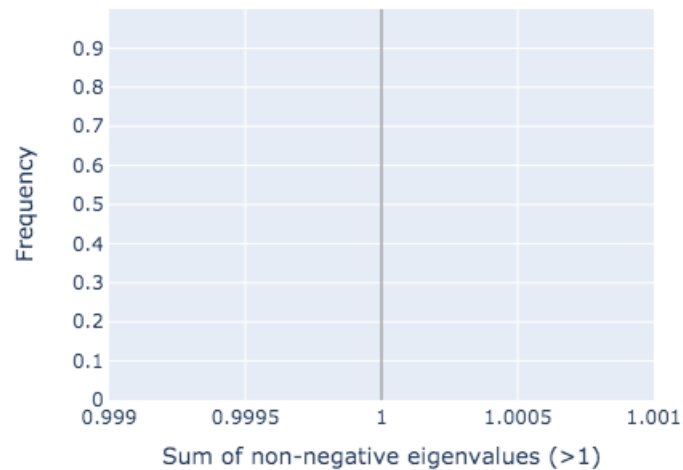
N=100
Number of unphysical estimates=0

N=100, Nrep=10



Min: -
Max: -
|Max-Min|: -

N=100, Nrep=10

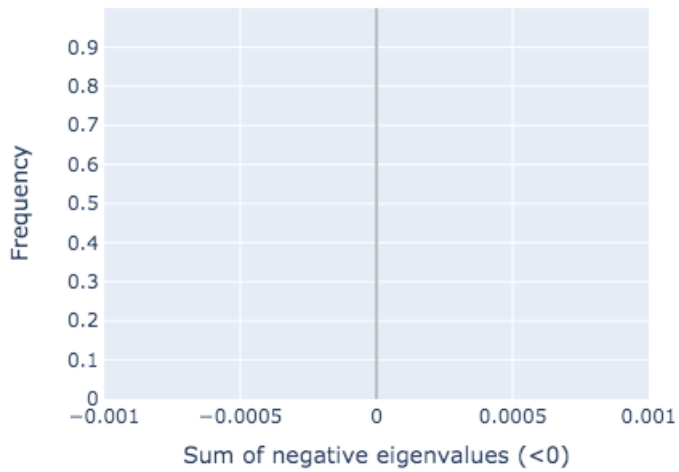


Min: -
Max: -
|Max-Min|: -

Case 7: Least Squares (False)

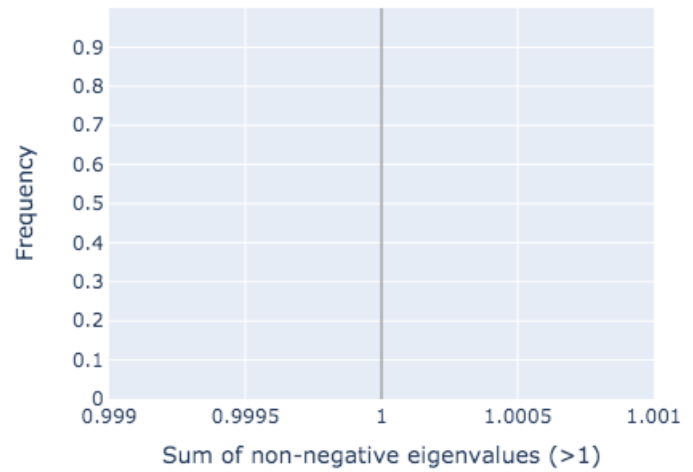
N=10
Number of unphysical estimates=0

N=10, Nrep=10



Min: -
Max: -
|Max-Min|: -

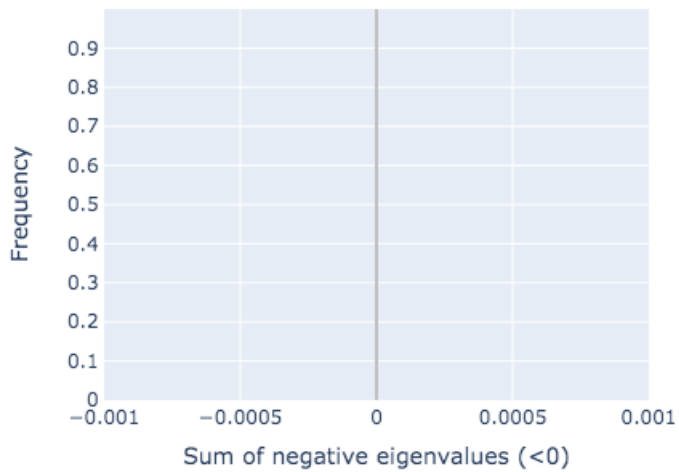
N=10, Nrep=10



Min: -
Max: -
|Max-Min|: -

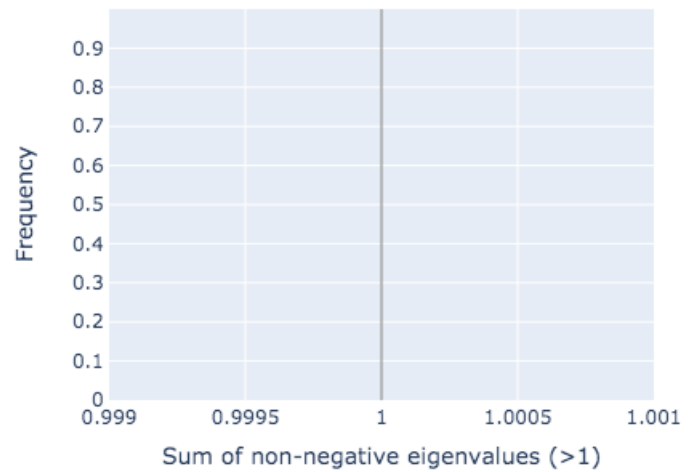
N=100
Number of unphysical estimates=0

N=100, Nrep=10



Min: -
Max: -
|Max-Min|: -

N=100, Nrep=10



Min: -
Max: -
|Max-Min|: -