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Computation time in total

Total	0.6180076162020366min.

Tolerance of physicality constraint violation

		True	False
Tolerance at physicality violation test	**equality constraint**	1.00e-13	1.00e-05
	inequality constraint	1.00e-05	1.00e-05

Experimental condition

Type of tomography	StandardQst
Nrep	10
N	[10, 100]
RNG seed	777

True object

Type	State
Dim	2
Vec	[0.70710678 0. 0. 0.6363961]

Tester objects

0	Type	Povm
	Dim	2
	Number of outcomes	2
	Vecs	[[0.70710678 0.6363961 0. 0.] [0.70710678 -0.6363961 0. 0.]]
1	Type	Povm
	Dim	2
	Number of outcomes	2
	Vecs	[[0.70710678 0. 0.6363961 0.] [0.70710678 0. -0.6363961 0.]]
2	Type	Povm
	Dim	2
	Number of outcomes	2
	Vecs	[[0.70710678 0. 0. 0.6363961] [0.70710678 0. 0. -0.6363961]]

Cases

	Name	Param	Tomography	Estimator
0	Linear (True)	True	StandardQst	Linear
1	Linear (False)	False	StandardQst	Linear
2	ProjectedLinear (True)	True	StandardQst	ProjectedLinear
3	ProjectedLinear (False)	False	StandardQst	ProjectedLinear
4	Maximum-Likelihood (True)	True	StandardQst	LossMinimization
5	Maximum-Likelihood (False)	False	StandardQst	LossMinimization

|	Name	N	Nrep	Mean (sec)	Std (sec)
6	Least Squares (True)	True	StandardQst		LossMinimization
7	Least Squares (False)	False	StandardQst		LossMinimization

Computation time of estimators

Table

|	Name	N	Nrep	Mean (sec)	Std (sec)
0	Linear (True)	10	10	0.000065	0.000027
1	"	100	10	0.000050	0.000016
2	Linear (False)	10	10	0.000056	0.000019
3	"	100	10	0.000055	0.000043
4	ProjectedLinear (True)	10	10	0.007711	0.005286
5	"	100	10	0.002479	0.003781
6	ProjectedLinear (False)	10	10	0.008206	0.005727
7	"	100	10	0.003273	0.005021
8	Maximum-Likelihood (True)	10	10	0.488195	0.081861
9	"	100	10	0.428425	0.080186
10	Maximum-Likelihood (False)	10	10	1.221275	1.652704
11	"	100	10	1.335999	1.436334
12	Least Squares (True)	10	10	0.047353	0.007416
13	"	100	10	0.044077	0.006829
14	Least Squares (False)	10	10	0.058900	0.012729
15	"	100	10	0.061927	0.012648

Histogram

Case 0: Linear (True)

N=10, Nrep=10
Mean = 6.53982162475586e-05(sec)
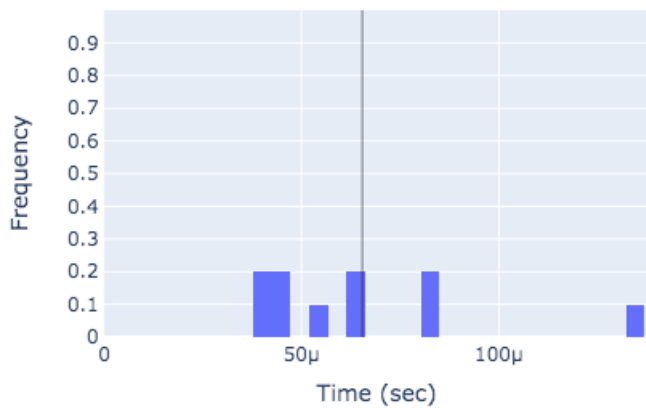
Min: 3.886222839355469e-05
Max: 0.00013327598571777344
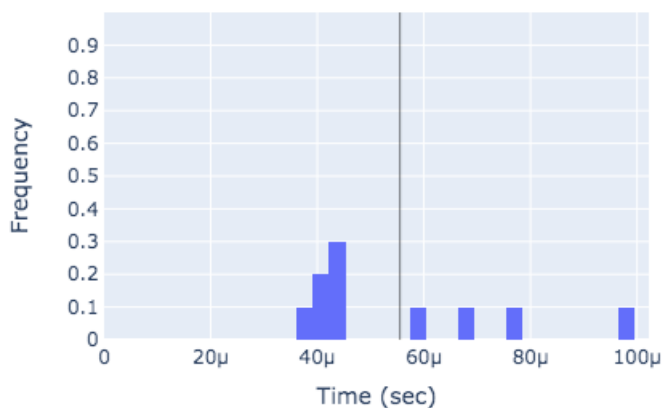|Max-Min|: 9.441375732421875e-05

N=100, Nrep=10
Mean = 5.033016204833984e-05(sec)
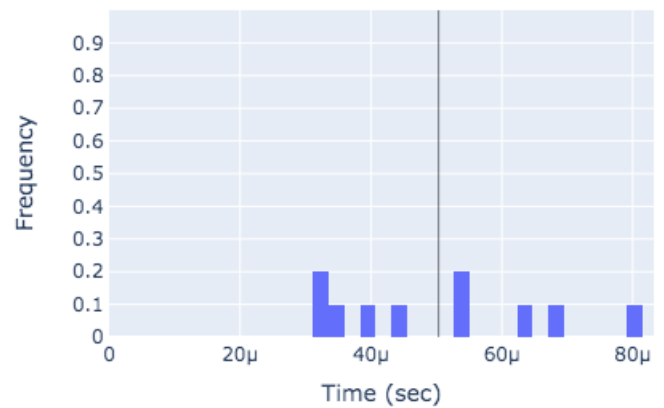
Min: 3.2901763916015625e-05
Max: 8.082389831542969e-05
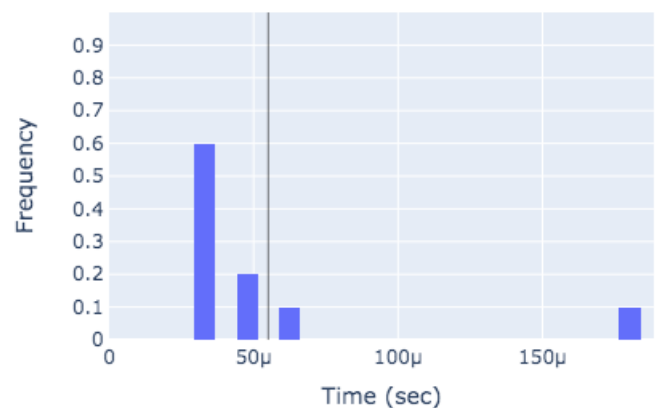|Max-Min|: 4.792213439941406e-05

Case 1: Linear (False)

N=10, Nrep=10
Mean = 5.550384521484375e-05(sec)

Min: 3.886222839355469e-05
Max: 9.918212890625e-05
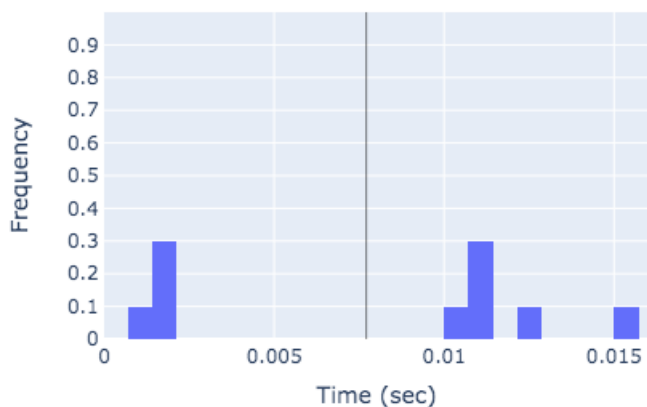|Max-Min|: 6.031990051269531e-05

N=100, Nrep=10
Mean = 5.507469177246094e-05(sec)

Min: 3.409385681152344e-05
Max: 0.0001811981201171875
|Max-Min|: 0.00014710426330566406

Case 2: ProjectedLinear (True)

N=10, Nrep=10
Mean = 0.007711195945739746(sec)

Min: 0.001016855239868164
Max: 0.015308856964111328
|Max-Min|: 0.014292001724243164

N=100, Nrep=10
Mean = 0.0024785518646240233(sec)
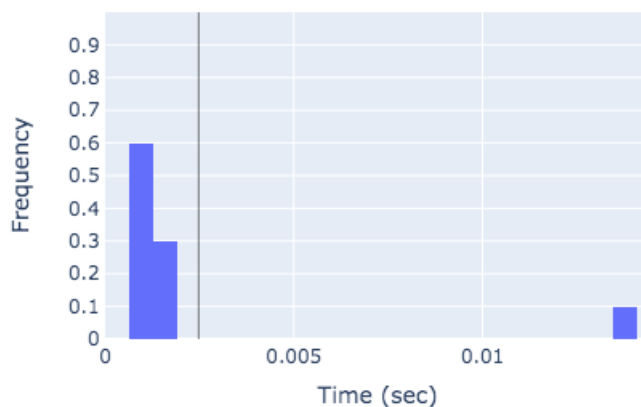
Min: 0.0009851455688476562
Max: 0.013794660568237305
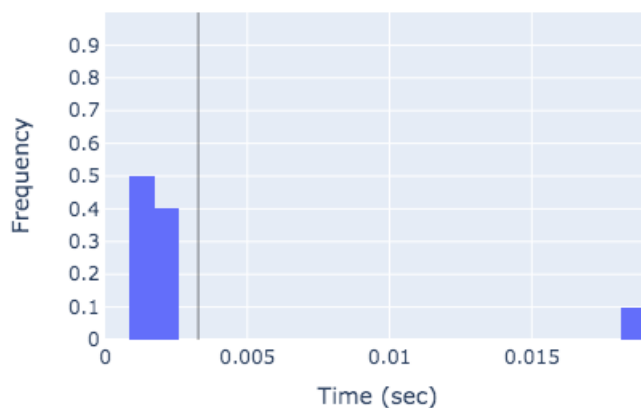|Max-Min|: 0.012809514999389648

Case 3: ProjectedLinear (False)

N=10, Nrep=10
Mean = 0.00820624828338623(sec)
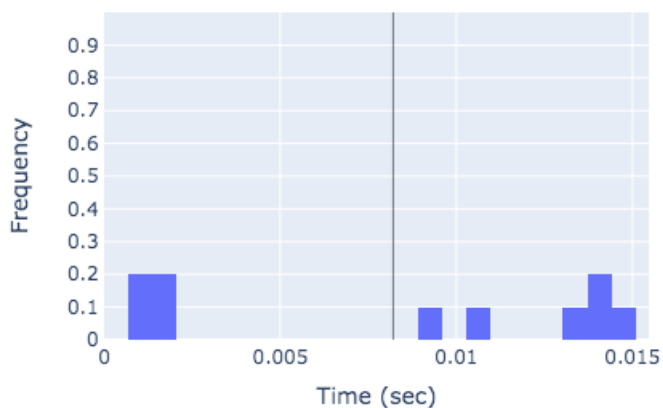
Min: 0.0010631084442138672
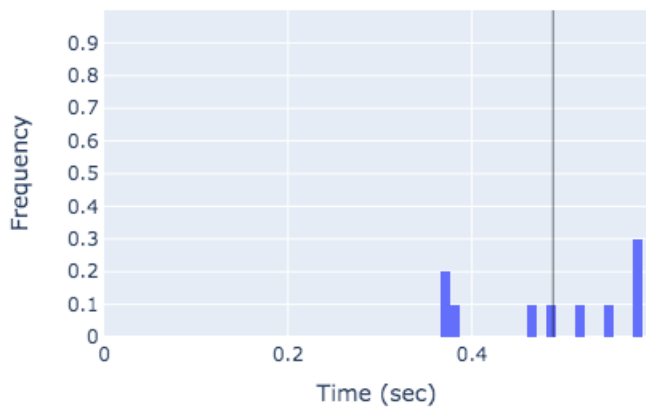Max: 0.014777898788452148
|Max-Min|: 0.013714790344238281

N=100, Nrep=10
Mean = 0.0032728910446166992(sec)

Min: 0.0010020732879638672
Max: 0.018285512924194336
|Max-Min|: 0.017283439963623047

Case 4: Maximum-Likelihood (True)

N=10, Nrep=10
Mean = 0.48819494247436523(sec)

Min: 0.372650146484375
Max: 0.5818569660186768
|Max-Min|: 0.20920681953430176

N=100, Nrep=10
Mean = 0.4284247159957886(sec)
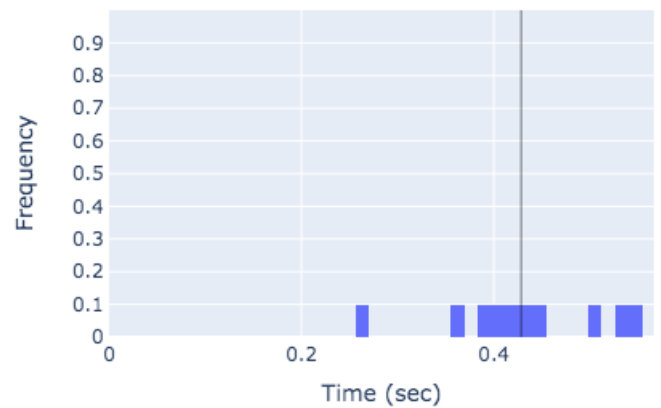
Min: 0.2678561210632324
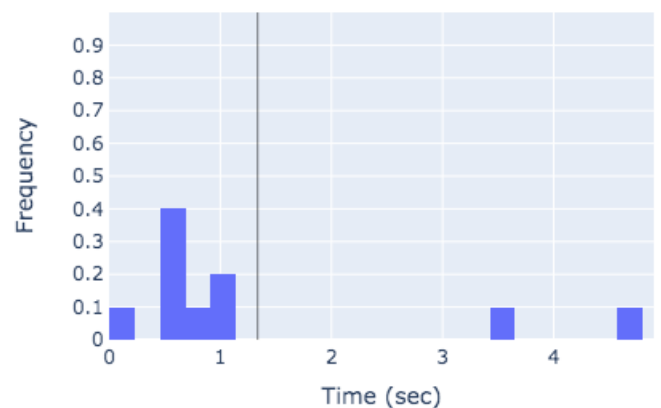Max: 0.5522170066833496
|Max-Min|: 0.2843608856201172

Case 5: Maximum-Likelihood (False)

N=10, Nrep=10
Mean = 1.2212751865386964(sec)
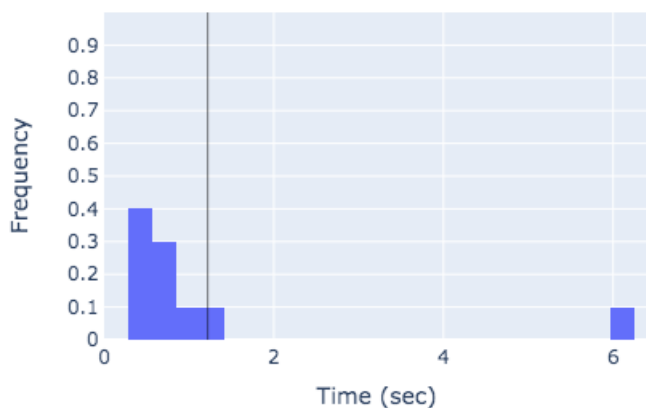
Min: 0.44734716415405273
Max: 6.136350870132446
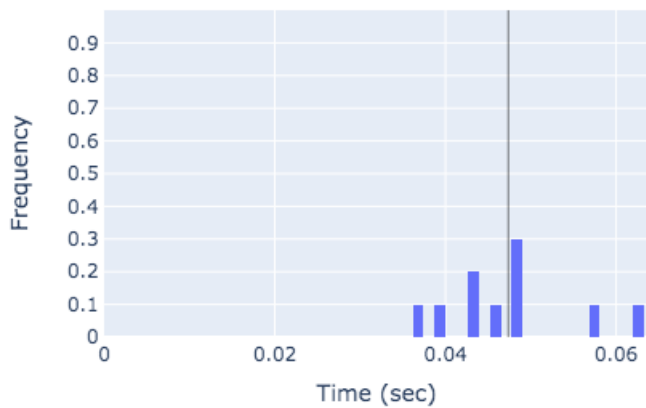|Max-Min|: 5.6890037059783936

N=100, Nrep=10
Mean = 1.3359986066818237(sec)

Min: 0.10594916343688965
Max: 4.672011137008667
|Max-Min|: 4.566061973571777

Case 6: Least Squares (True)

N=10, Nrep=10
Mean = 0.04735319614410401(sec)

Min: 0.036714792251586914
Max: 0.06253504753112793
|Max-Min|: 0.025820255279541016

N=100, Nrep=10
Mean = 0.044076728820800784(sec)
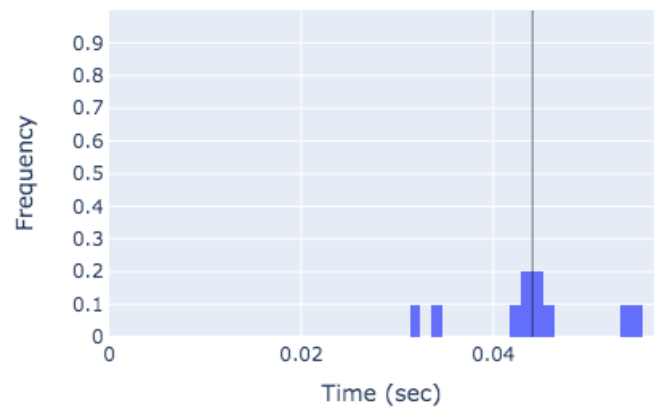
Min: 0.032384634017944336
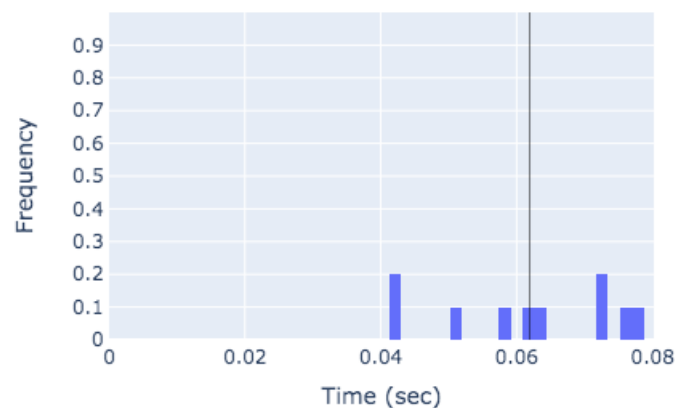Max: 0.055548906326293945
|Max-Min|: 0.02316427230834961

Case 7: Least Squares (False)

N=10, Nrep=10
Mean = 0.05890040397644043(sec)
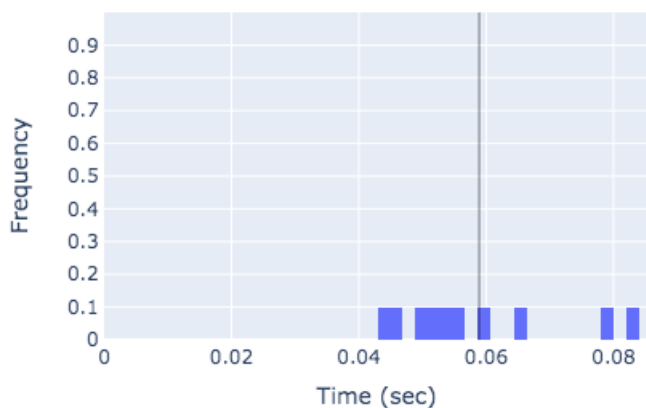
Min: 0.04453301429748535
Max: 0.08359503746032715
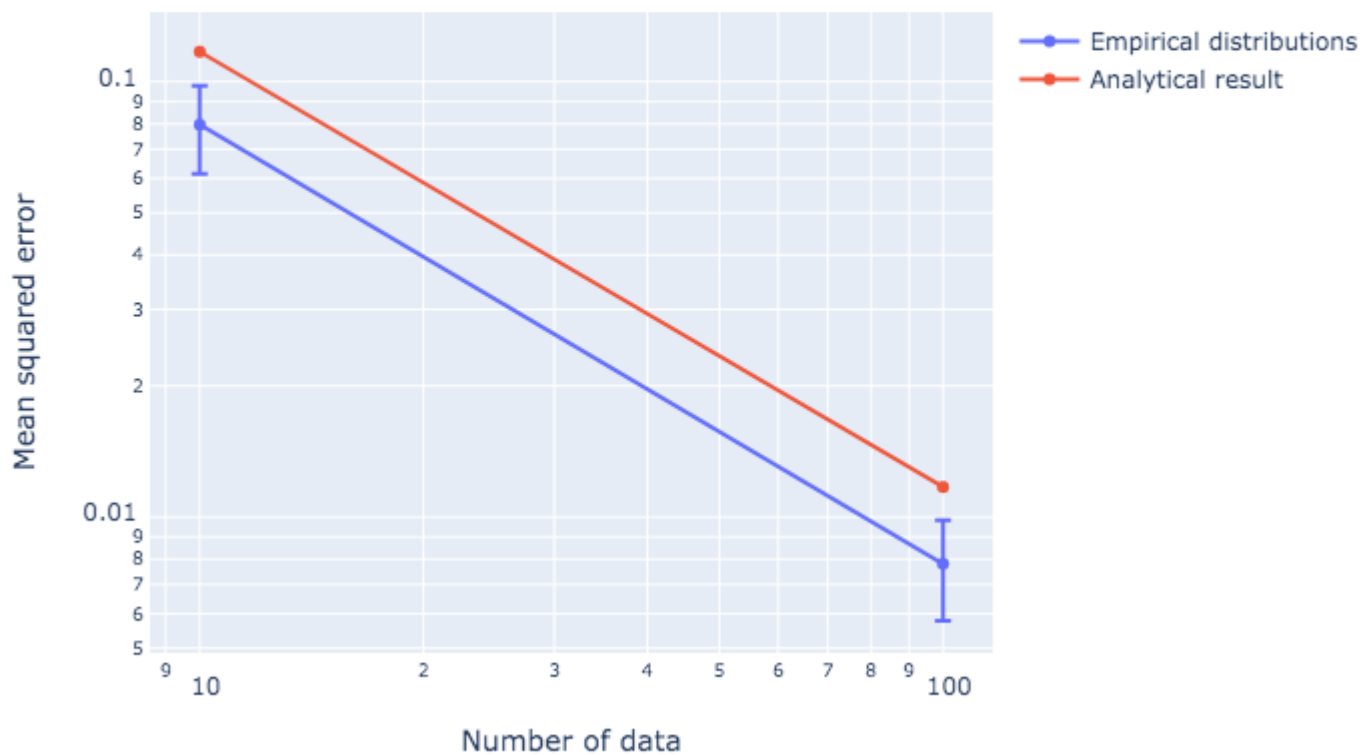|Max-Min|: 0.0390620231628418

N=100, Nrep=10
Mean = 0.06192672252655029(sec)

Min: 0.04259800910949707
Max: 0.0784111 0229492188
|Max-Min|: 0.035813093185424805

MSE of empirical distributions

Mean squared error

Consistency test

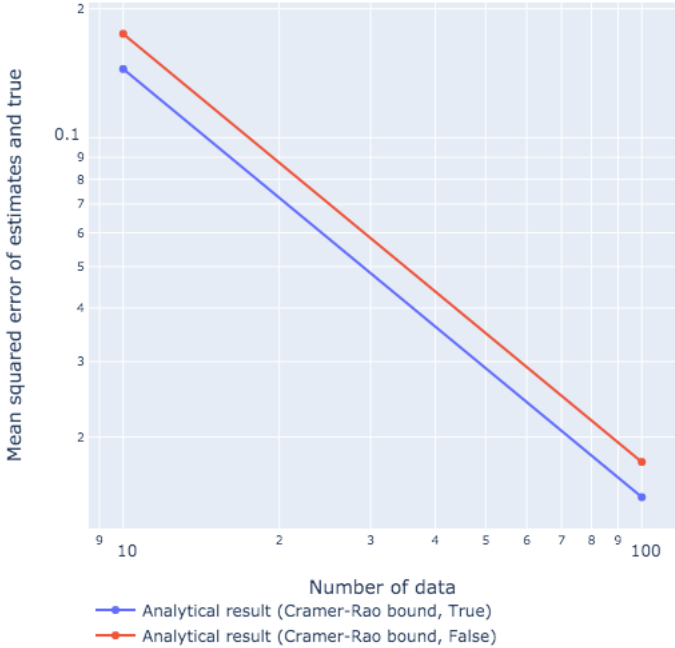|	Name	Type of tomography	Param	Estimator	Loss	Algo	Squared Error to True	Possibly OK	To be checked
0	Linear (True)	Standard Qst	True	Linear	None	None	0.00e+00	OK	not need debug
1	Linear (False)	Standard Qst	False	Linear	None	None	2.47e-32	OK	not need debug
2	ProjectedLinear (True)	Standard Qst	True	Projected Linear	None	None	1.23e-32	OK	not need debug
3	ProjectedLinear (False)	Standard Qst	False	Projected Linear	None	None	1.11e-31	OK	not need debug
4	Maximum-Likelihood (True)	Standard Qst	True	Loss Minimization	Weighted Relative Entropy	Projected Gradient Descent Backtracking	3.21e-18	OK	not need debug
5	Maximum-Likelihood (False)	Standard Qst	False	Loss Minimization	Weighted Relative Entropy	Projected Gradient Descent Backtracking	5.47e-09	NG	not need debug
6	Least Squares (True)	Standard Qst	True	Loss Minimization	Weighted Probability Based Squared Error	Projected Gradient Descent Backtracking	4.93e-32	OK	not need debug

MSE of estimators

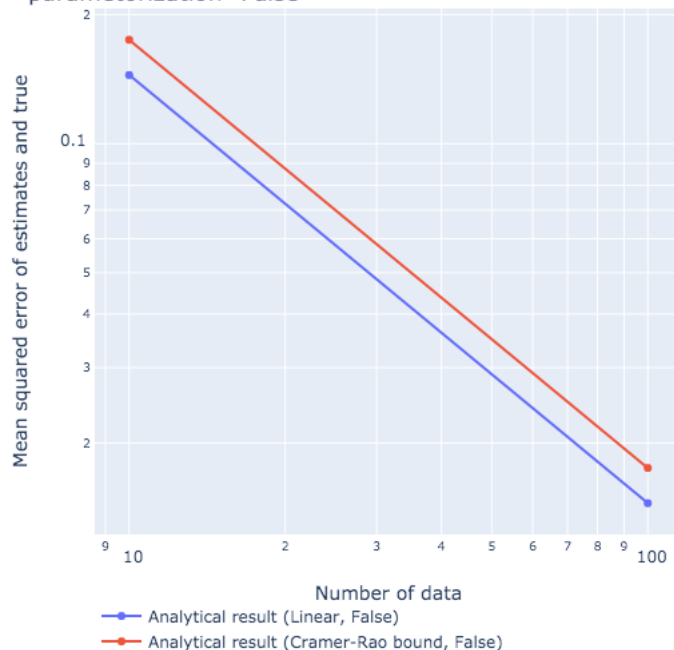## Comparison of analytical results
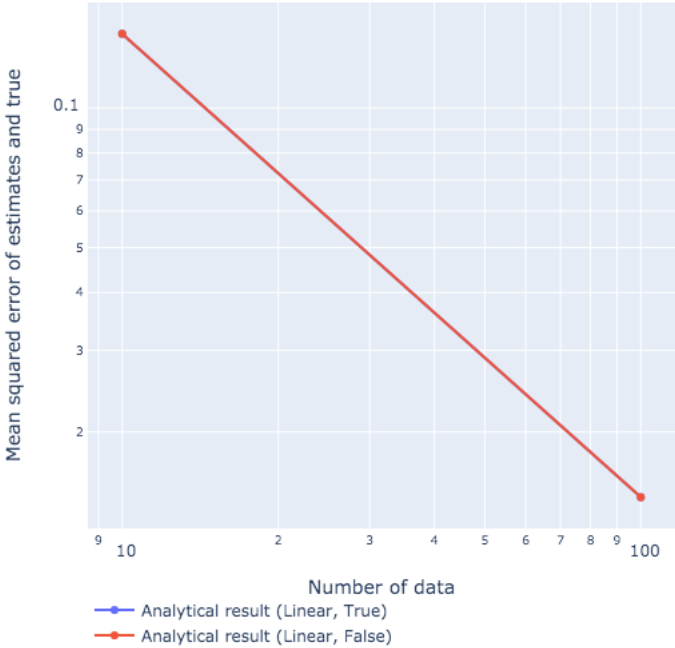
Mean squared error
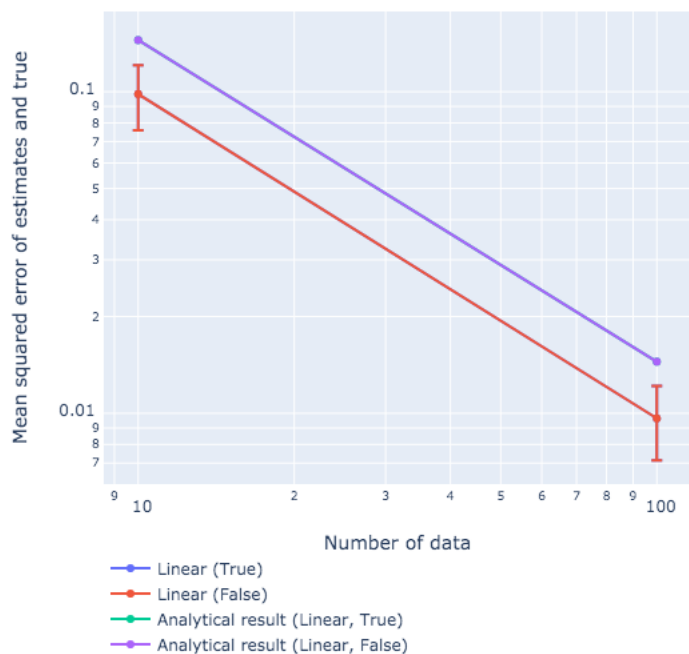Analytical result
parametorization=True

Mean squared error
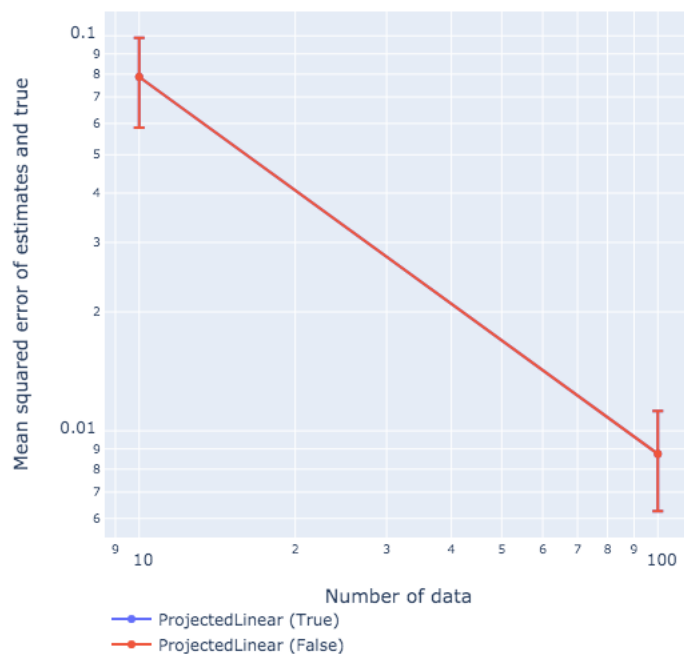Analytical result
parametorization=False

Comparison of parametrization

Mean squared error
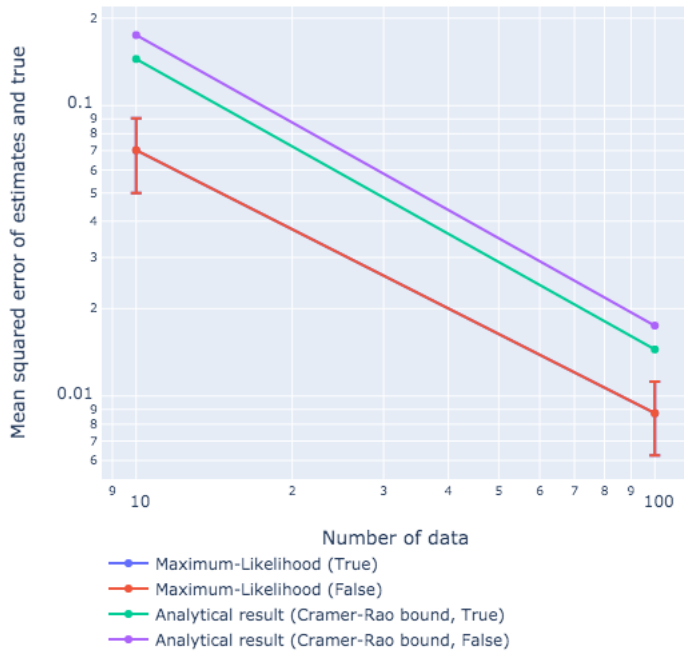Estimator=Linear

Mean squared error
Estimator=ProjectedLinear

Mean squared error
Estimator=LossMinimization
Loss=WeightedRelativeEntropy
Algo=ProjectedGradientDescentBacktracking

- Maximum-Likelihood (True)
- Maximum-Likelihood (False)
- Analytical result (Cramer-Rao bound, True)
- Analytical result (Cramer-Rao bound, False)

Mean squared error
Estimator=LossMinimization
Loss=WeightedProbabilityBasedSquaredError
Algo=ProjectedGradientDescentBacktracking

- Least Squares (True)
- Least Squares (False)
- Analytical result (Cramer-Rao bound, True)
- Analytical result (Cramer-Rao bound, False)
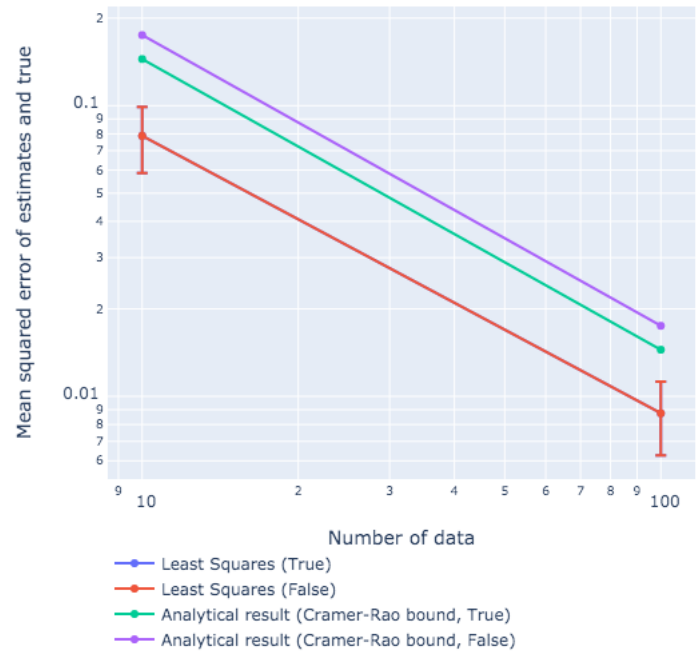
Comparison of estimators

Mean squared error
parametrization=True

- Linear (True)
- ProjectedLinear (True)
- Maximum-Likelihood (True)
- Least Squares (True)

Mean squared error
parametrization=False

- Linear (False)
- ProjectedLinear (False)
- Maximum-Likelihood (False)
- Least Squares (False)

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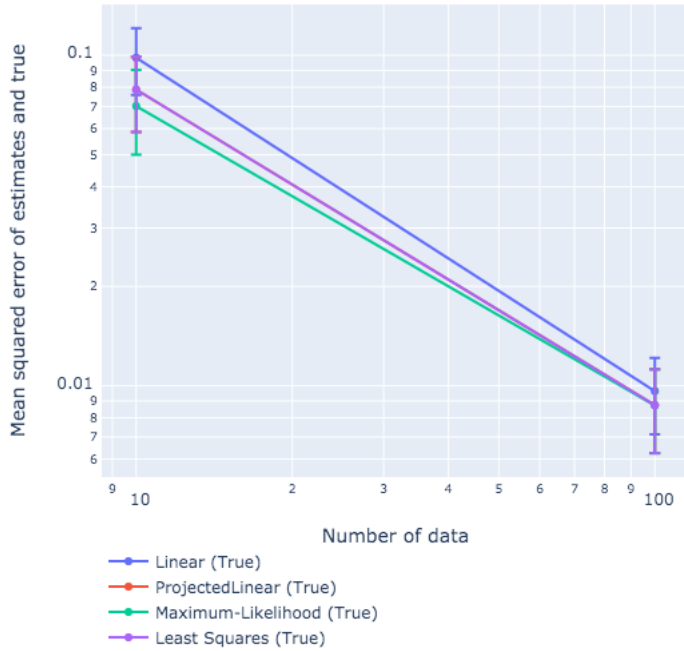
Physicality violation test

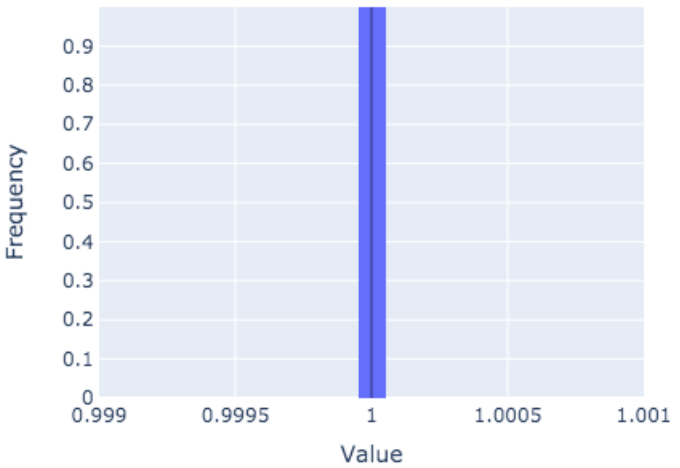## Test of equality constraint violation

Case 0: Linear (True)
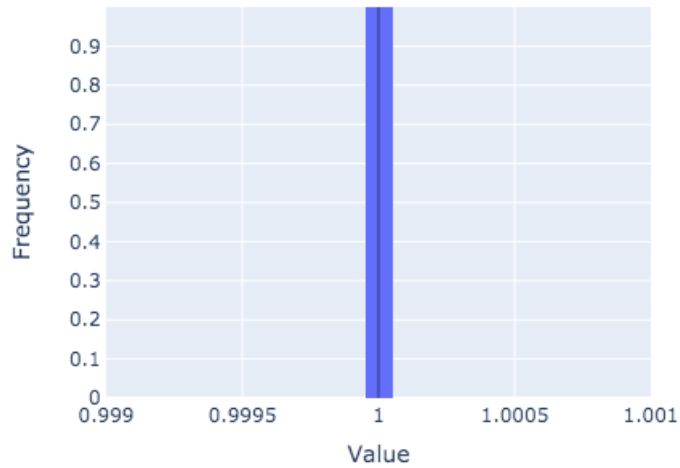
N=10, Nrep=10

Min: 0.9999999999999997
Max: 0.9999999999999998
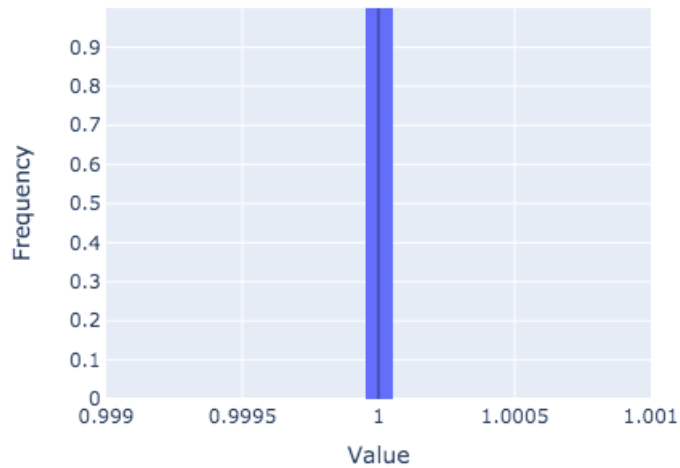|Max-Min|: 1.1102230246251565e-16

N=100, Nrep=10

Min: 0.9999999999999998
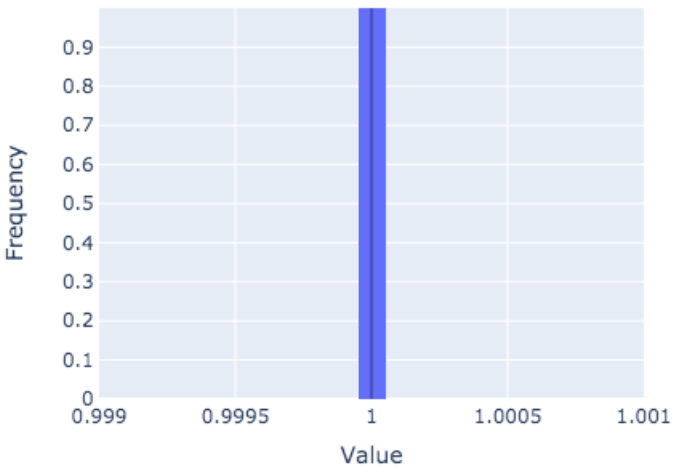Max: 0.9999999999999998
|Max-Min|: 0.0

Case 1: Linear (False)

N=10, Nrep=10

Min: 0.9999999999999998
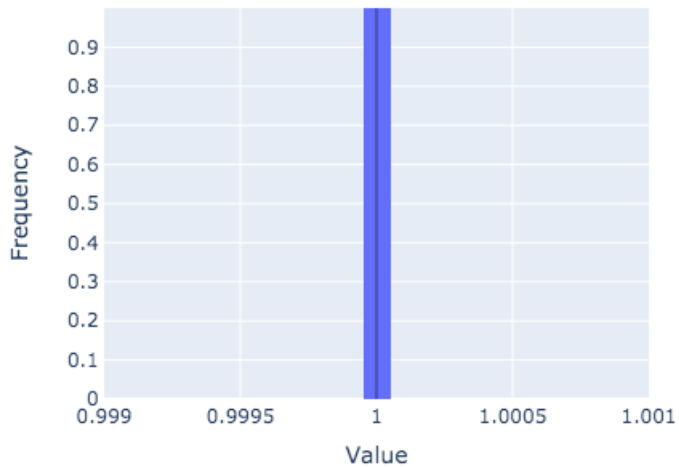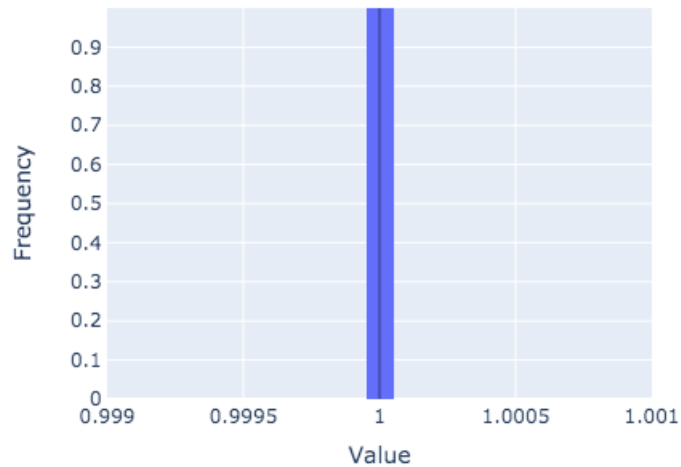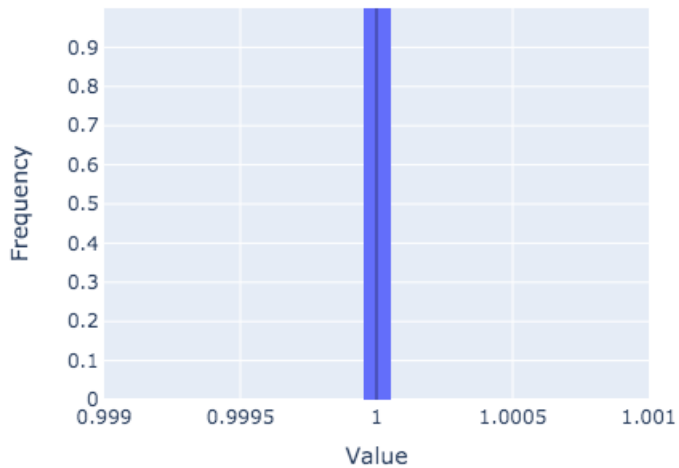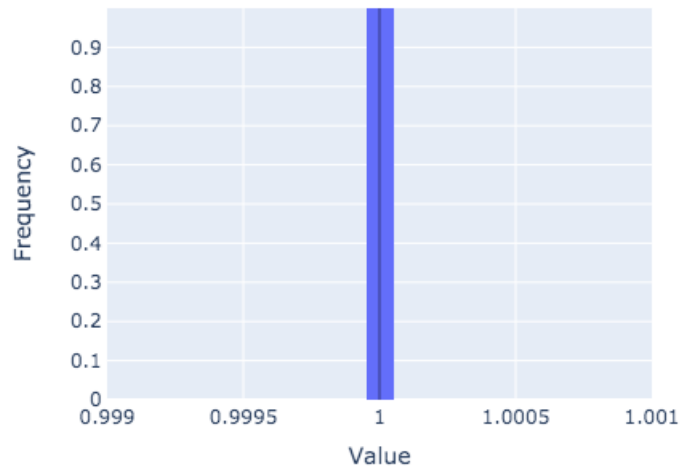Max: 1.0000000000000002
|Max-Min|: 4.440892098500626e-16

N=100, Nrep=10

Min: 0.9999999999999999
Max: 1.0000000000000002
|Max-Min|: 3.3306690738754696e-16

Case 2: ProjectedLinear (True)

N=10, Nrep=10

Min: 0.9999999999999998
Max: 0.9999999999999998
|Max-Min|: 0.0

N=100, Nrep=10

Min: 0.9999999999999998
Max: 0.9999999999999998
|Max-Min|: 0.0

Case 3: ProjectedLinear (False)

N=10, Nrep=10

Min: 0.9999999999999996
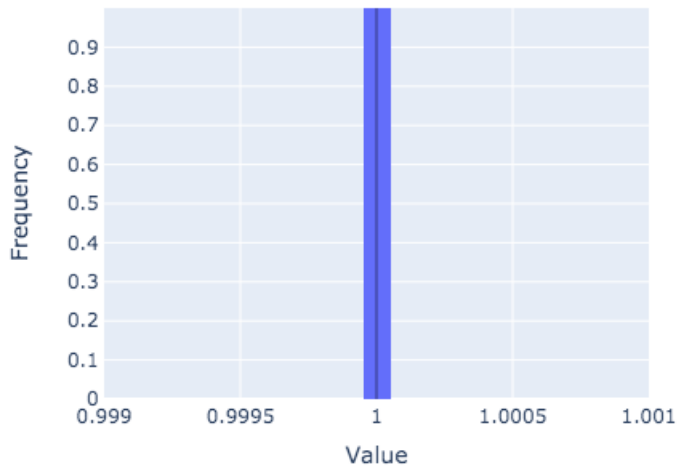Max: 1.00000014753393
|Max-Min|: 1.4753393040578544e-07

N=100, Nrep=10

Min: 0.9999999999999997
Max: 1.0000001280394155
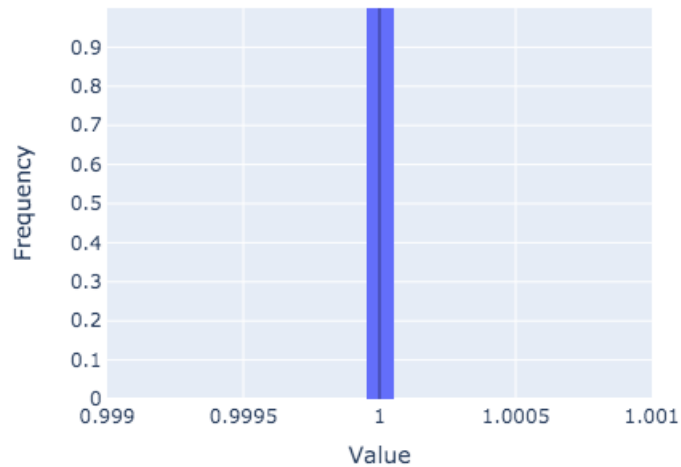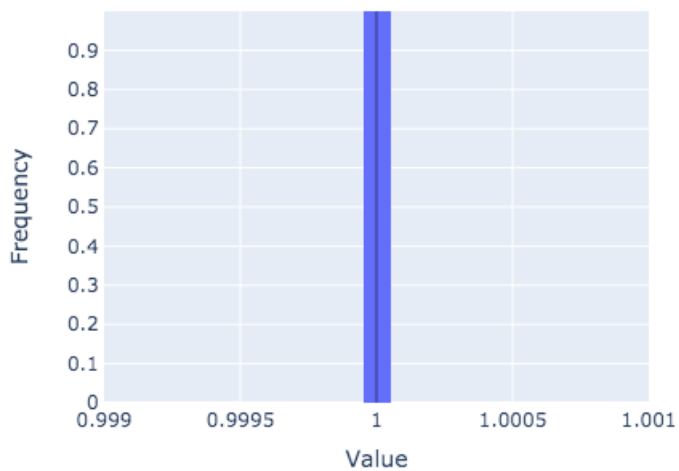|Max-Min|: 1.280394158476028e-07

Case 4: Maximum-Likelihood (True)

N=10, Nrep=10

Min: 0.9999999999999998
Max: 0.9999999999999998
|Max-Min|: 0.0

N=100, Nrep=10

Min: 0.9999999999999998
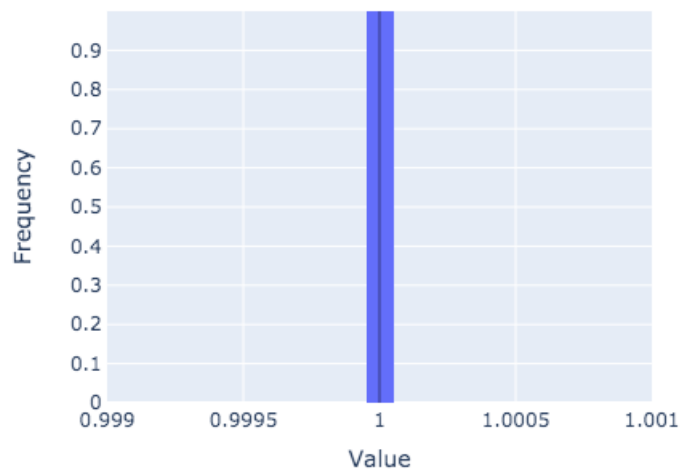Max: 0.9999999999999998
|Max-Min|: 0.0

Case 5: Maximum-Likelihood (False)

N=10, Nrep=10

Min: 1.000000006567707
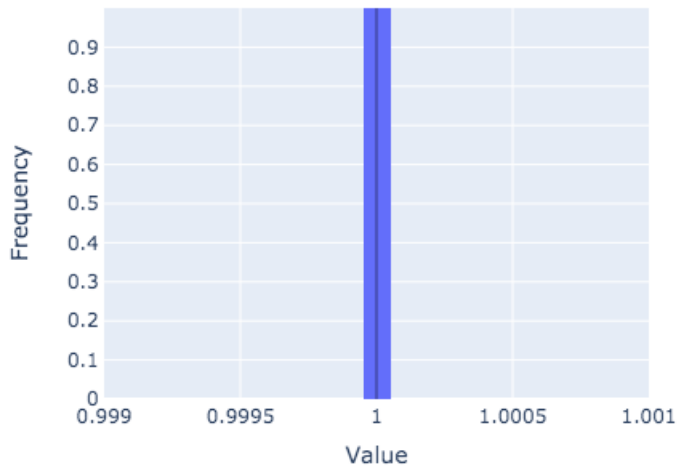Max: 1.0000001622723005
|Max-Min|: 1.5570459344083076e-07

N=100, Nrep=10

Min: 1.0000000121352572
Max: 1.00000001810607821
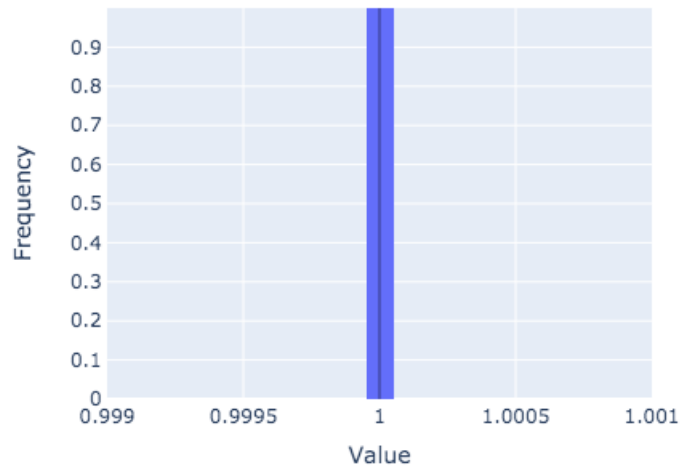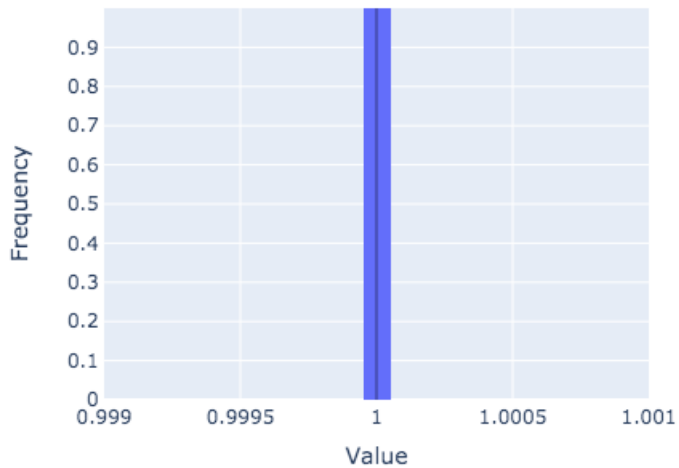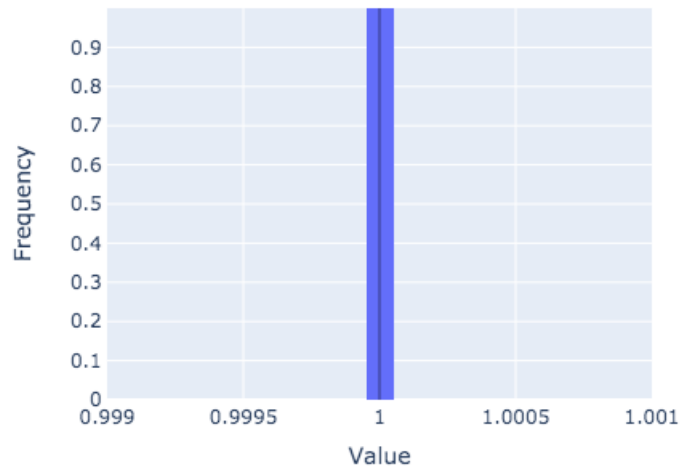|Max-Min|: 1.689255249015531e-07

Case 6: Least Squares (True)

N=10, Nrep=10

Min: 0.9999999999999998
Max: 0.9999999999999998
|Max-Min|: 0.0

N=100, Nrep=10

Min: 0.9999999999999998
Max: 0.9999999999999998
|Max-Min|: 0.0

Case 7: Least Squares (False)

N=10, Nrep=10

Min: 0.9999999999999998
Max: 1.0000001781169368
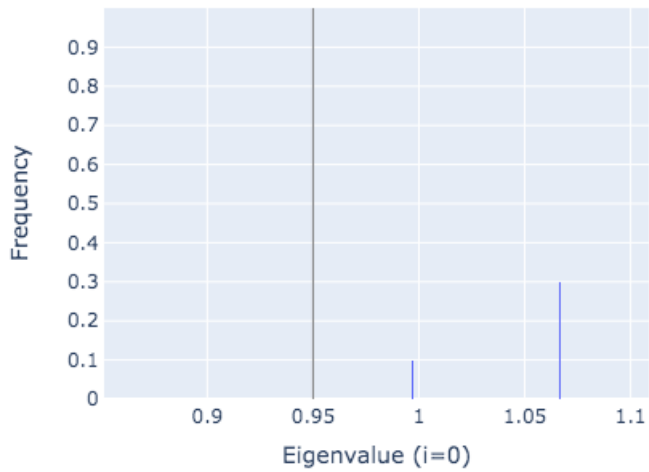|Max-Min|: 1.7811693697744602e-07

N=100, Nrep=10

Min: 0.9999999999999998
Max: 1.0000001555680968
|Max-Min|: 1.5556809707106822e-07
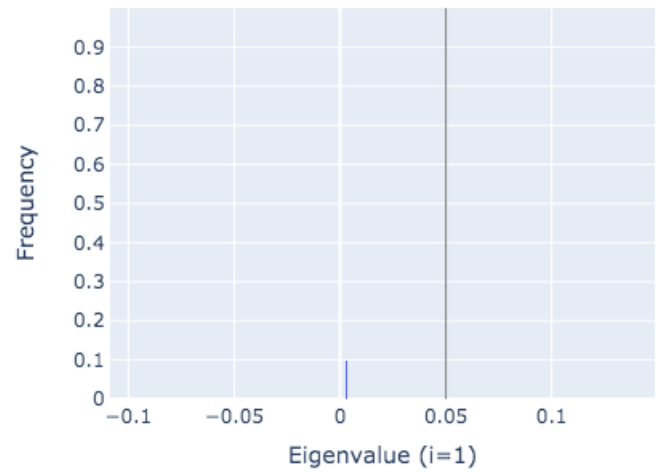
Test of inequality constraint violation

Eigenvalue
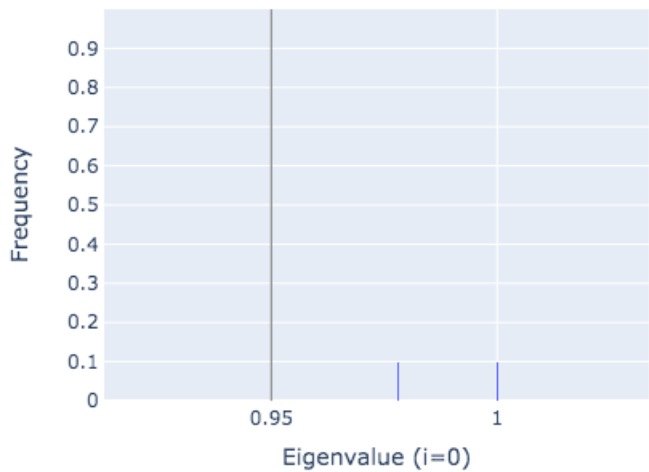
Case 0: Linear (True)

N=10, Nrep=10

Min: 0.8513641844631532
Max: 1.1085806194501846
|Max-Min|: 0.2572164349870314
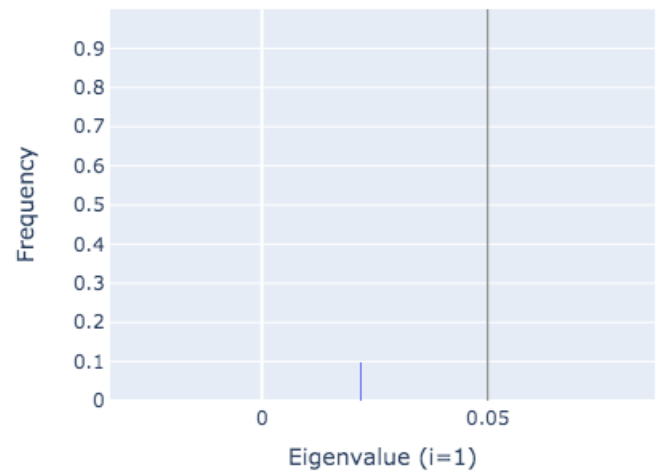
N=10, Nrep=10

Min: -0.10858061945018471
Max: 0.14863581553684663
|Max-Min|: 0.25721643498703134

N=100, Nrep=10

Min: 0.9130584510098023
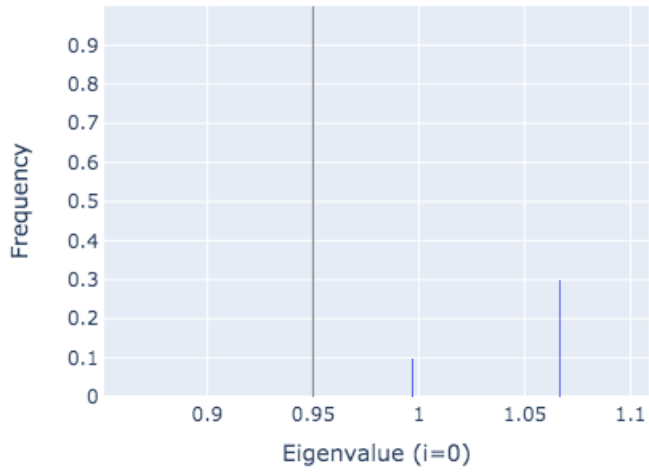Max: 1.0335647646019095
|Max-Min|: 0.12050631359210717

N=100, Nrep=10

Min: -0.03356476460190986
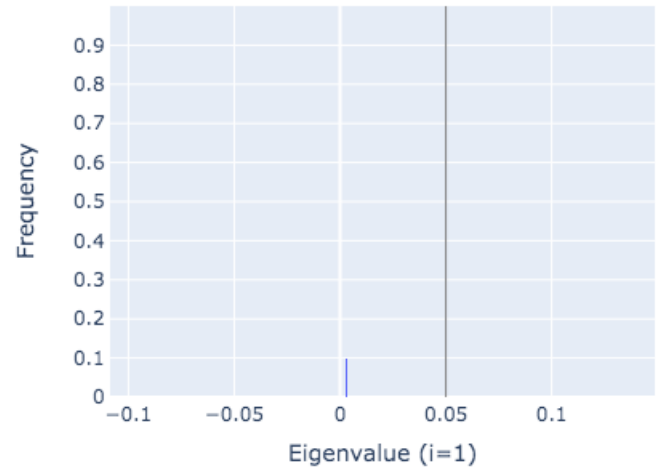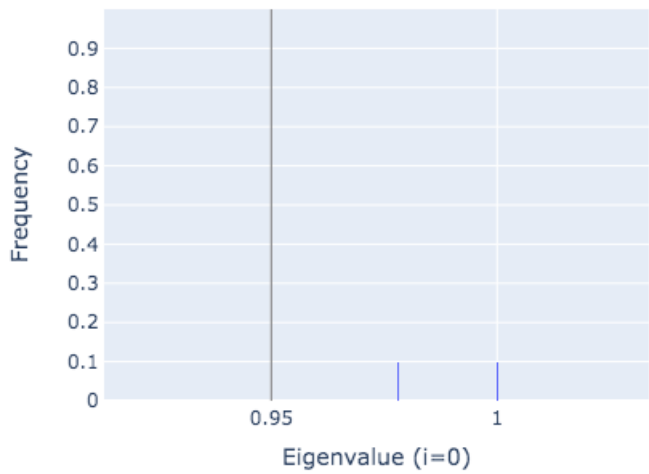Max: 0.0869415489901976
|Max-Min|: 0.12050631359210745

Case 1: Linear (False)

N=10, Nrep=10

Min: 0.8513641844631532
Max: 1.1085806194501846
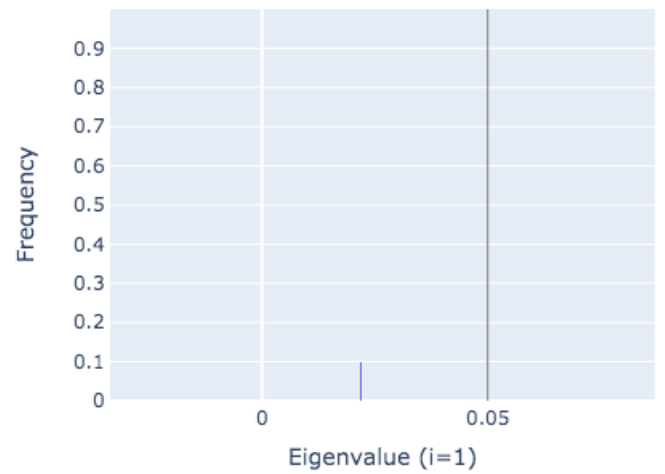|Max-Min|: 0.2572164349870314

N=10, Nrep=10

Min: -0.10858061945018459
Max: 0.14863581553684663
|Max-Min|: 0.25721643498703123

N=100, Nrep=10

Min: 0.9130584510098023
Max: 1.0335647646019095
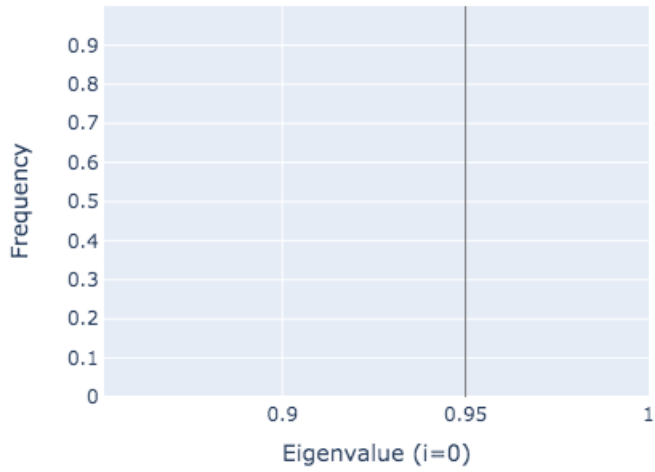|Max-Min|: 0.12050631359210717

N=100, Nrep=10

Min: -0.03356476460190975
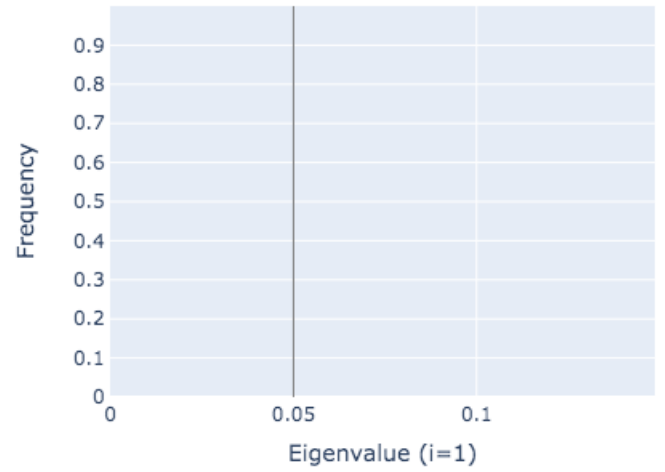Max: 0.08694154899019772
|Max-Min|: 0.12050631359210748

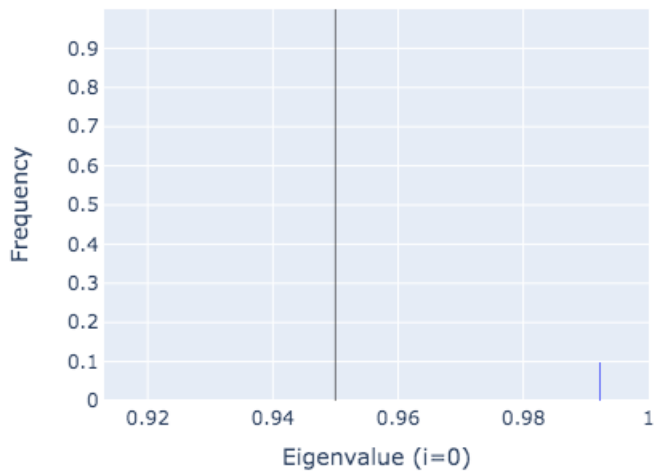**Case 2: ProjectedLinear (True)**

N=10, Nrep=10

Min: 0.8513641844631531
Max: 1.0000000737669648
|Max-Min|: 0.14863588930381166
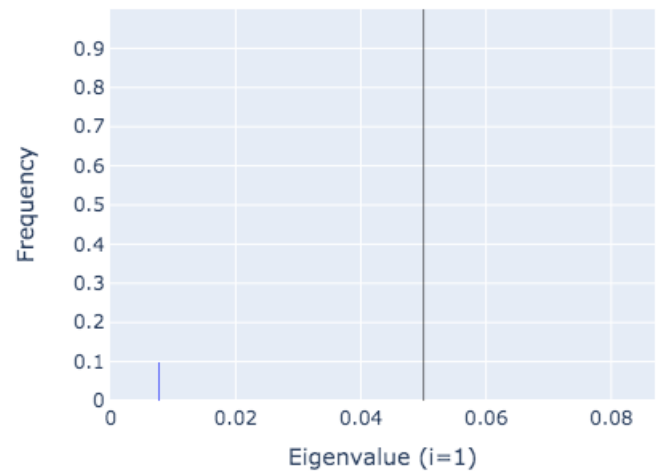
N=10, Nrep=10

Min: -7.376696509533986e-08
Max: 0.14863581553684668
|Max-Min|: 0.14863588930381177

N=100, Nrep=10

Min: 0.9130584510098023
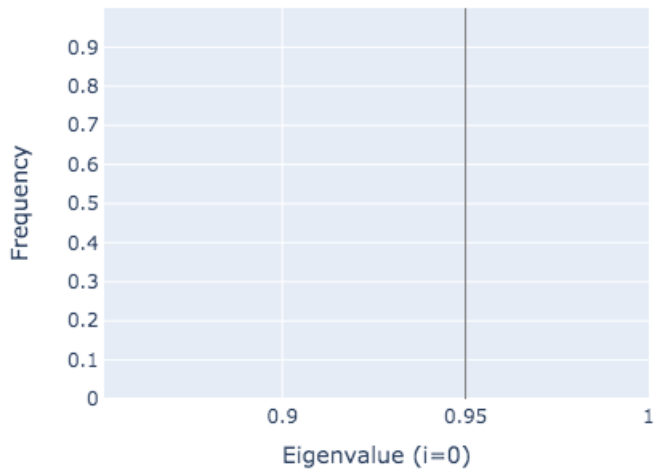Max: 1.0000000640197078
|Max-Min|: 0.08694161300990544

N=100, Nrep=10

Min: -6.401970792556323e-08
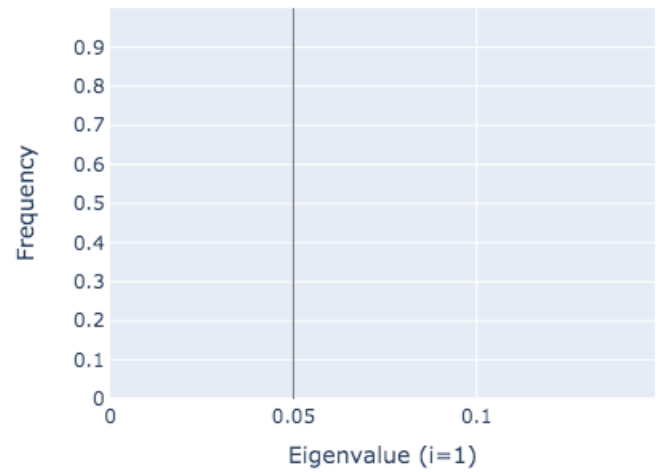Max: 0.0869415489901976
|Max-Min|: 0.08694161300990552

Case 3: ProjectedLinear (False)
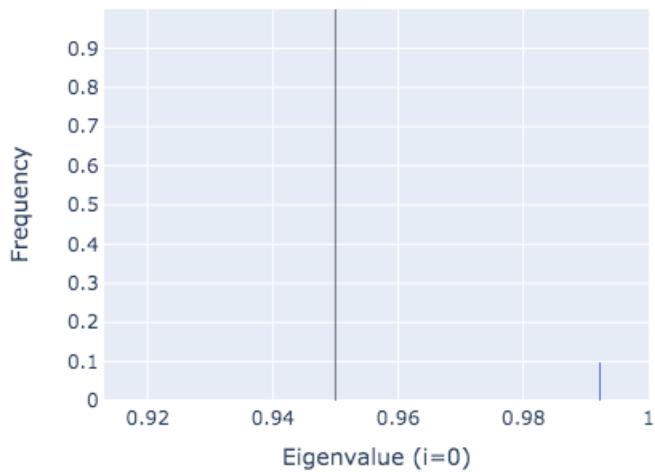
N=10, Nrep=10

Min: 0.8513641844631532
Max: 1.00000014753393
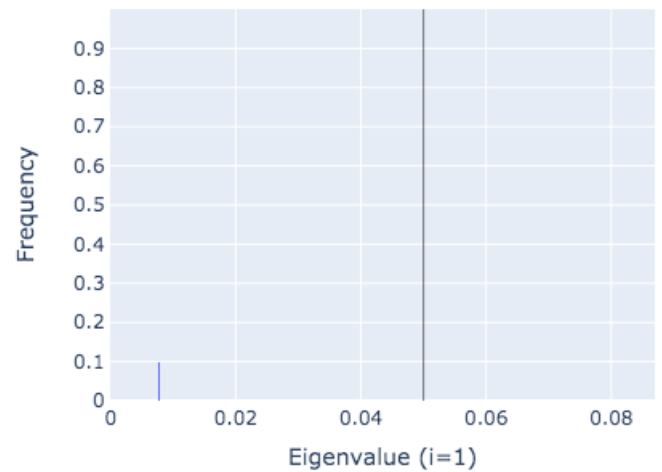|Max-Min|: 0.14863596307077676

N=10, Nrep=10

Min: -2.7755575615628914e-17
Max: 0.14863581553684696
|Max-Min|: 0.148635815536847

N=100, Nrep=10

Min: 0.9130584510098023
Max: 1.0000001280394155
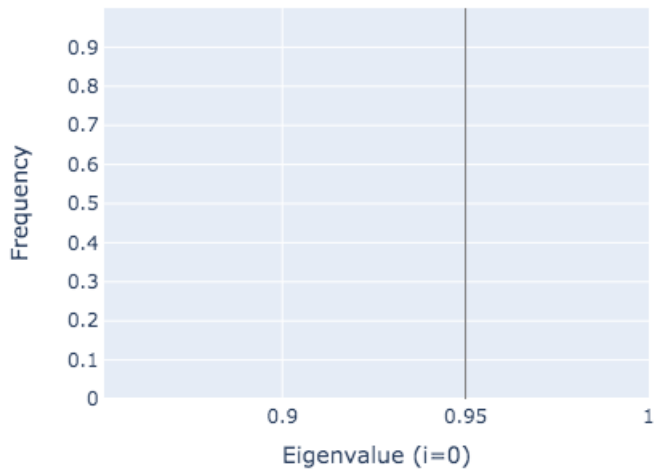|Max-Min|: 0.0869416770296132

N=100, Nrep=10

Min: -5.730008481585891e-17
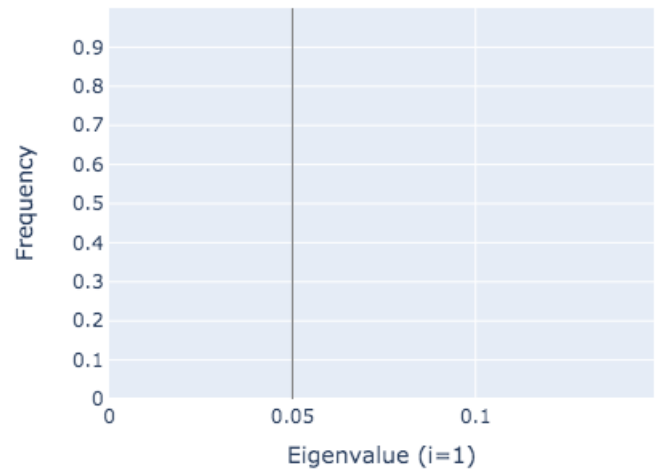Max: 0.0869415489901976
|Max-Min|: 0.08694154899019765

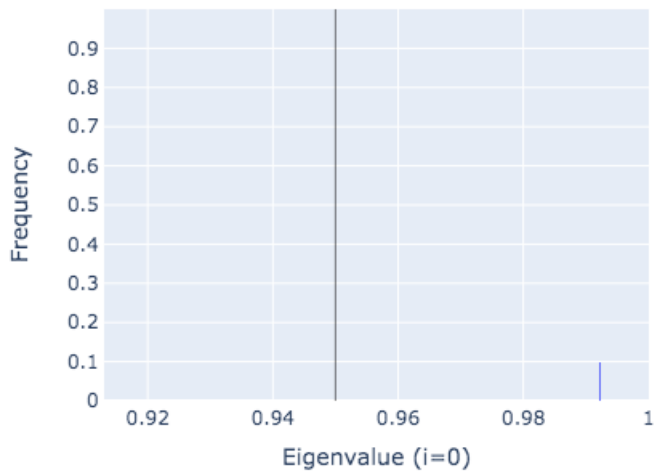**Case 4: Maximum-Likelihood (True)**

N=10, Nrep=10

Frequency vs Eigenvalue (i=0)

Min: 0.8513641812250503
Max: 1.0000000702659644
|Max-Min|: 0.14863588904091407

N=10, Nrep=10

Frequency vs Eigenvalue (i=1)

Min: -7.026596454684508e-08
Max: 0.14863581877494952
|Max-Min|: 0.14863588904091407

N=100, Nrep=10

Frequency vs Eigenvalue (i=0)

Min: 0.9130584509383943
Max: 1.000000078401553
|Max-Min|: 0.0869416274631587

N=100, Nrep=10

Frequency vs Eigenvalue (i=1)
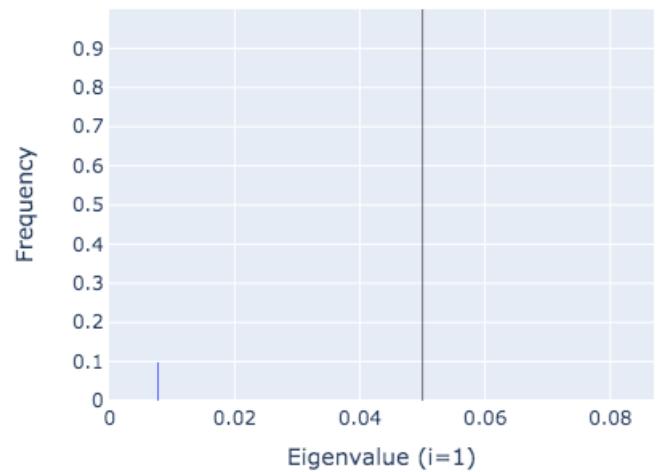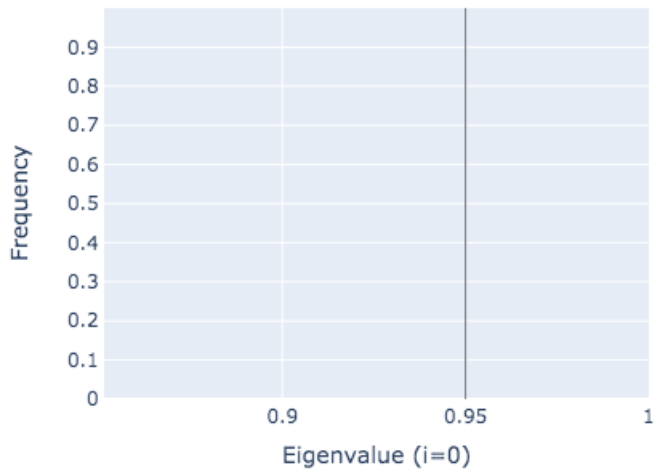
Min: -7.840155328241432e-08
Max: 0.08694154906160545
|Max-Min|: 0.08694162746315873

Case 5: Maximum-Likelihood (False)

N=10, Nrep=10

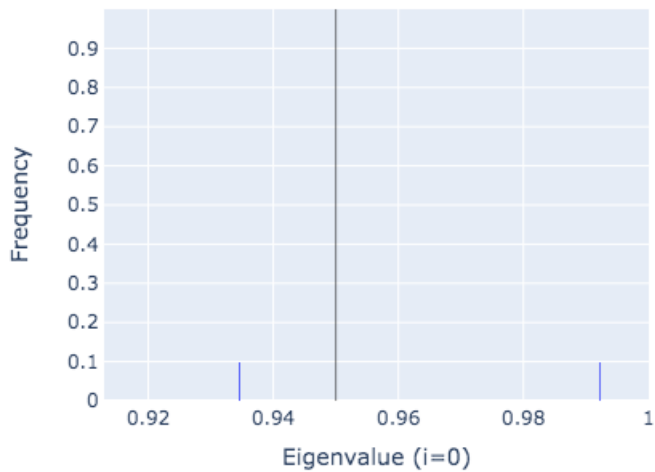Frequency vs Eigenvalue (i=0)

Min: 0.851427240055816
Max: 1.000000162272261
|Max-Min|: 0.1485729222164449

N=10, Nrep=10

Frequency vs Eigenvalue (i=1)
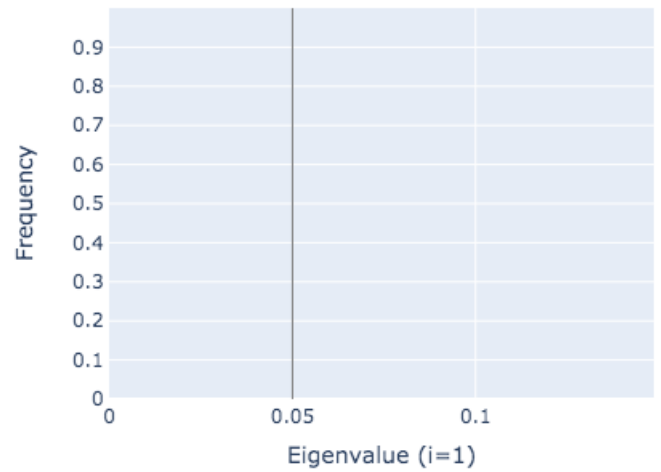
Min: -1.3877787807814457e-17
Max: 0.14857279250213073
|Max-Min|: 0.14857279250213073

N=100, Nrep=10

Frequency vs Eigenvalue (i=0)

Min: 0.9130380513737355
Max: 1.0000001810607806
|Max-Min|: 0.08696212968704509

N=100, Nrep=10

Frequency vs Eigenvalue (i=1)
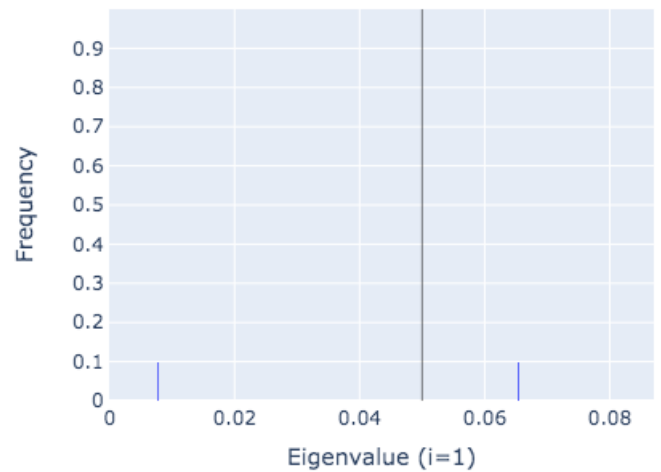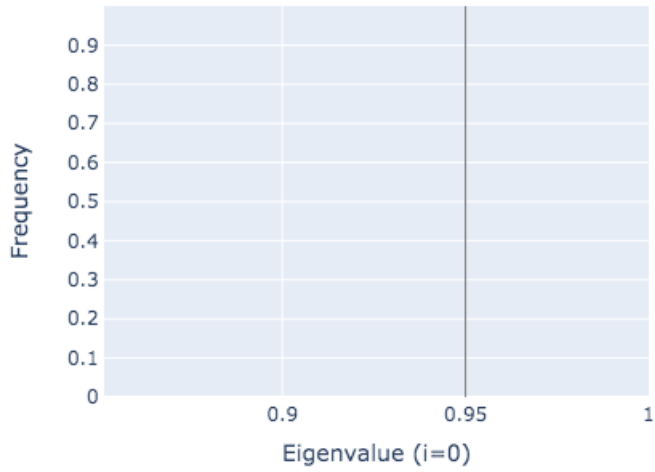
Min: -1.0061396160665481e-16
Max: 0.08696196076152173
|Max-Min|: 0.08696196076152182
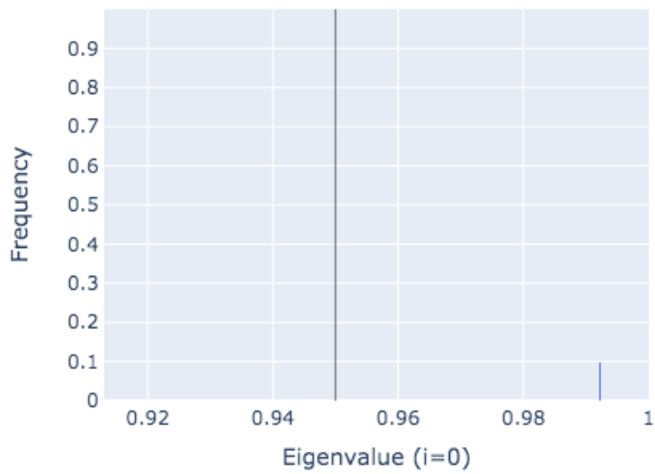
Case 6: Least Squares (True)

N=10, Nrep=10

Eigenvalue (i=0)

Min: 0.851364184463153
Max: 1.0000000761055894
|Max-Min|: 0.1486358916424364
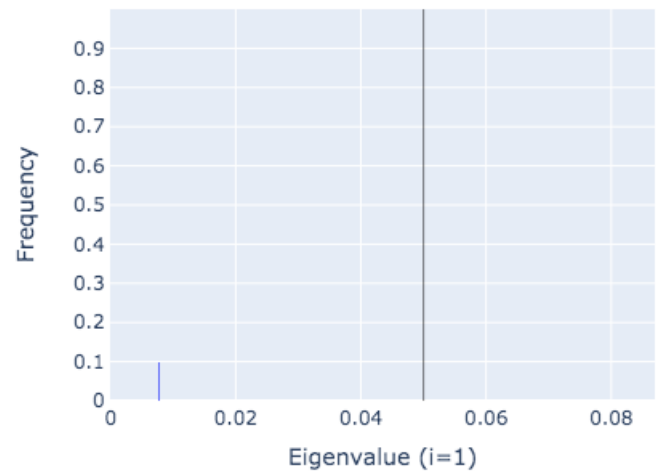
N=10, Nrep=10

Eigenvalue (i=1)
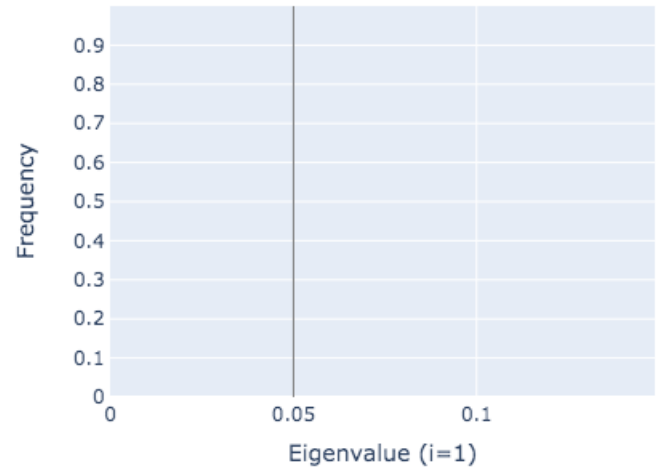
Min: -7.610558962201086e-08
Max: 0.14863581553684674
|Max-Min|: 0.14863589164243635

N=100, Nrep=10

Eigenvalue (i=0)

Min: 0.9130584510098028
Max: 1.0000000598780594
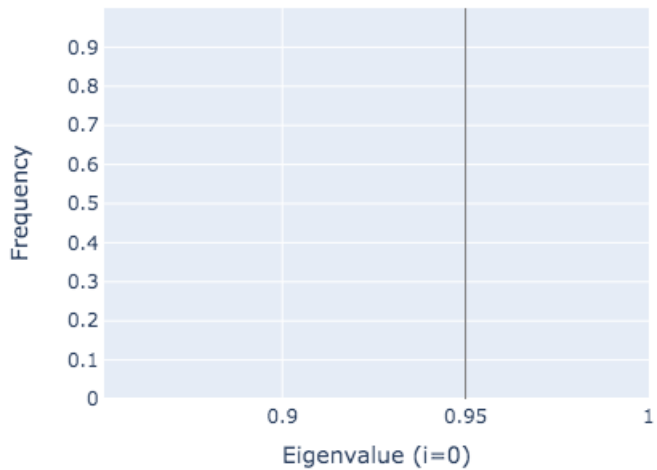|Max-Min|: 0.08694160886825664

N=100, Nrep=10

Eigenvalue (i=1)

Min: -5.987805948691623e-08
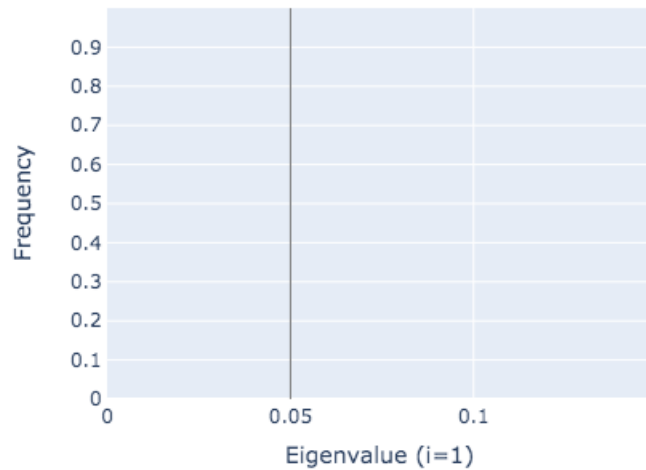Max: 0.0869415489901971
|Max-Min|: 0.08694160886825658

Case 7: Least Squares (False)

N=10, Nrep=10 — Eigenvalue (i=0)
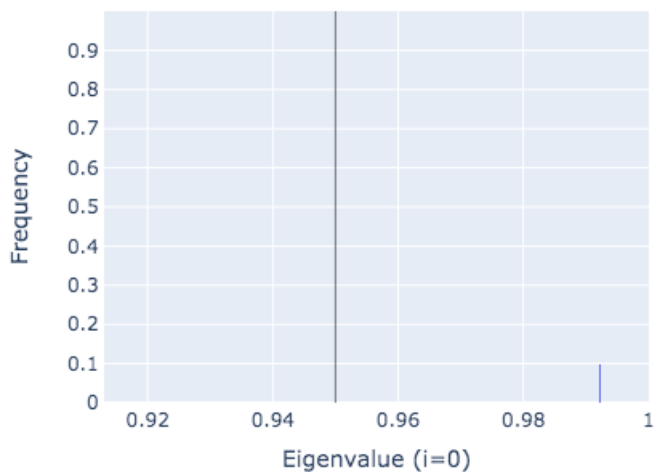
Min: 0.8513641844631532
Max: 1.0000001781169368
|Max-Min|: 0.14863599365378355

N=10, Nrep=10 — Eigenvalue (i=1)

Min: -7.632783294297951e-17
Max: 0.14863581553684663
|Max-Min|: 0.1486358155368467

N=100, Nrep=10 — Eigenvalue (i=0)

Min: 0.9130584510098023
Max: 1.0000001555680968
|Max-Min|: 0.08694170455829453

N=100, Nrep=10 — Eigenvalue (i=1)
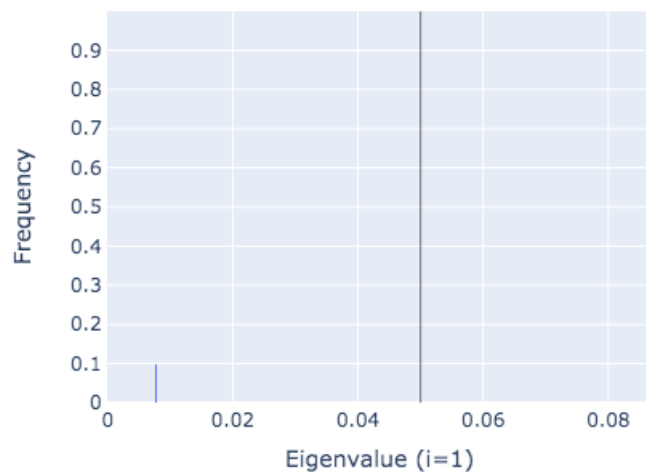
Min: 3.06287113727155e-18
Max: 0.08694154899019754
|Max-Min|: 0.08694154899019754

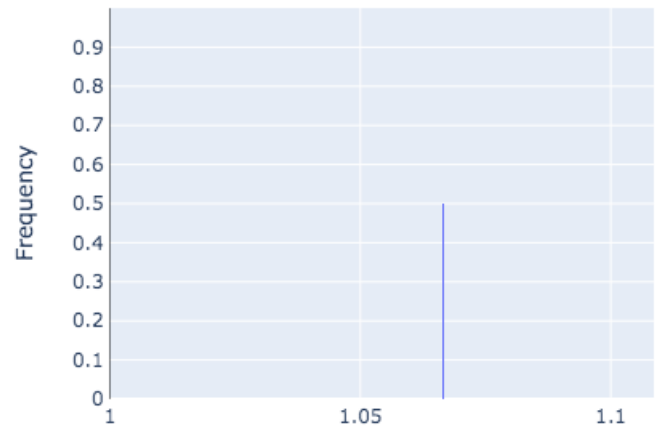## Sum of unphysical eigenvalues

Case 0: Linear (True)

N=10
Number of unphysical estimates=6

N=10, Nrep=10

Sum of negative eigenvalues (<0)
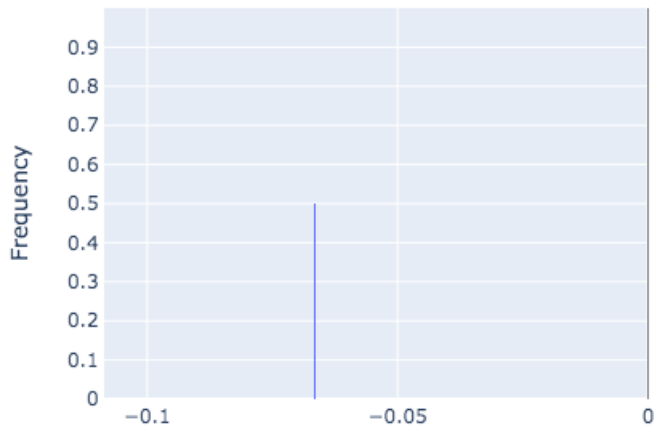
Min: -0.10858061945018471
Max: -0.009175077217315732
|Max-Min|: 0.09940554223286899

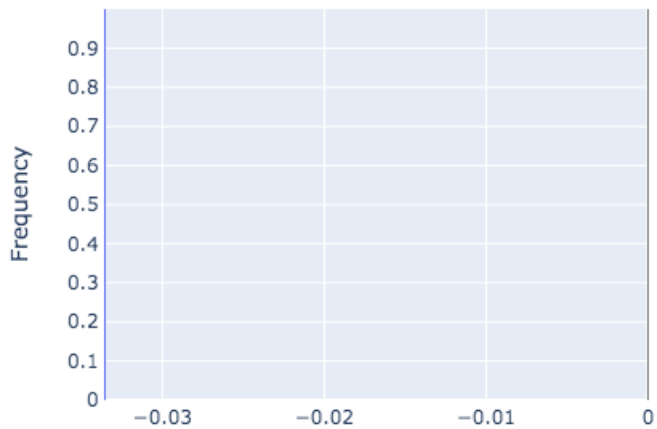## N=10, Nrep=10

Sum of non-negative eigenvalues (>1)

Min: 1.0091750772173156
Max: 1.1085806194501846
|Max-Min|: 0.09940554223286902

N=100
Number of unphysical estimates=1

N=100, Nrep=10

Sum of negative eigenvalues (<0)

Min: -0.03356476460190986
Max: -0.03356476460190986
|Max-Min|: 0.0

N=100, Nrep=10
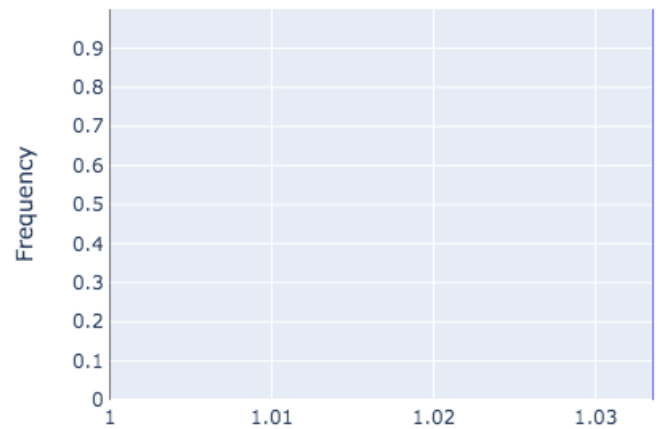
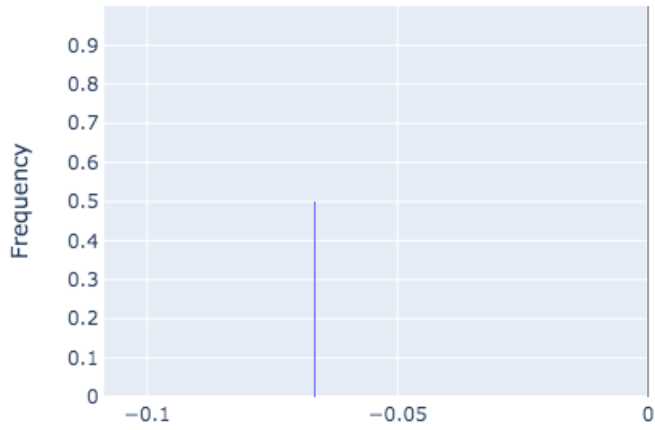Sum of non-negative eigenvalues (>1)

Min: 1.0335647646019095
Max: 1.0335647646019095
|Max-Min|: 0.0

Case 1: Linear (False)

N=10
Number of unphysical estimates=6

N=10, Nrep=10

Sum of negative eigenvalues (<0)

Min: -0.10858061945018459
Max: -0.009175077217315586
|Max-Min|: 0.099405542232869
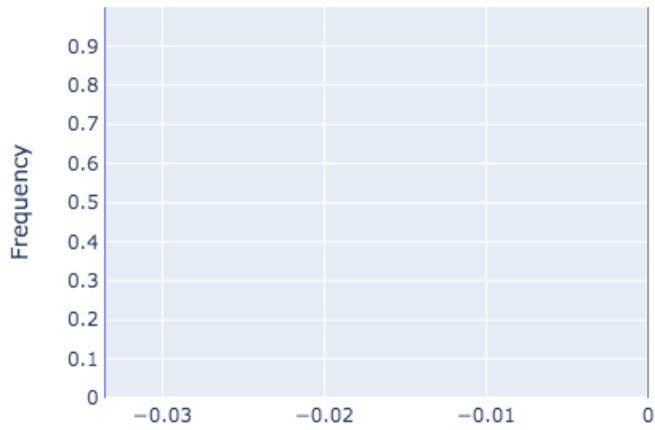
N=10, Nrep=10
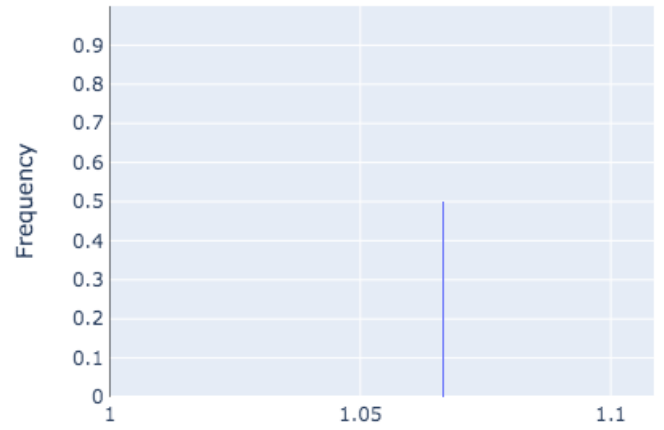
Sum of non-negative eigenvalues (>1)

Min: 1.0091750772173156
Max: 1.1085806194501846
|Max-Min|: 0.09940554223286902

N=100
Number of unphysical estimates=1

N=100, Nrep=10

Sum of negative eigenvalues (<0)

Min: -0.03356476460190975
Max: -0.03356476460190975
|Max-Min|: 0.0

N=100, Nrep=10

Sum of non-negative eigenvalues (>1)
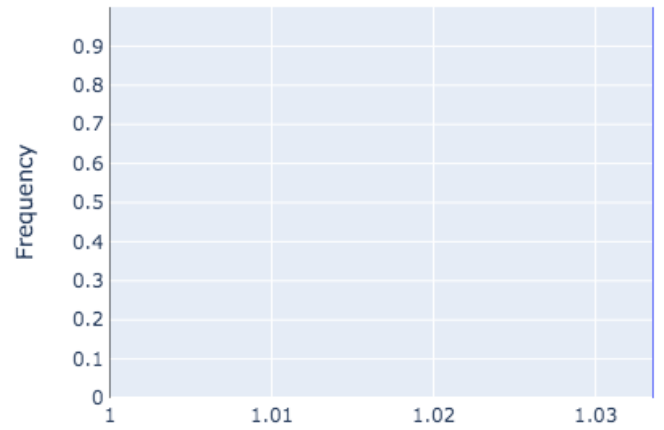
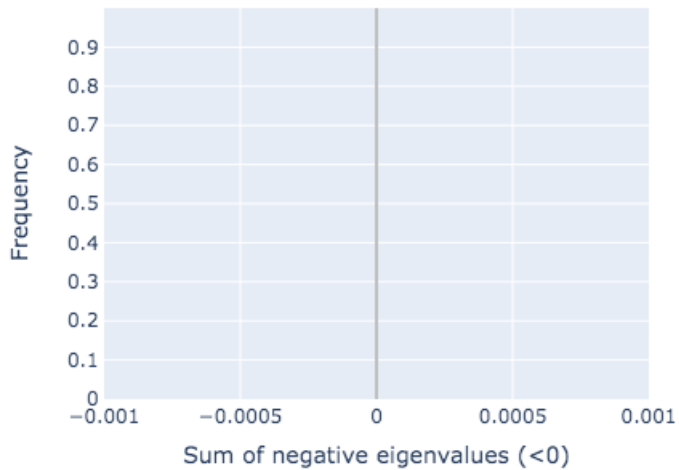Min: 1.0335647646019095
Max: 1.0335647646019095
|Max-Min|: 0.0

Case 2: ProjectedLinear (True)

N=10
Number of unphysical estimates=0

Frequency

Sum of negative eigenvalues (<0)

Min: -
Max: -
|Max-Min|: -

Frequency

Sum of non-negative eigenvalues (>1)

Min: -
Max: -
|Max-Min|: -

N=100
Number of unphysical estimates=0

N=100, Nrep=10

Frequency

Sum of negative eigenvalues (<0)

Min: -
Max: -
|Max-Min|: -

N=100, Nrep=10

Frequency

Sum of non-negative eigenvalues (>1)

Min: -
Max: -
|Max-Min|: -

Case 3: ProjectedLinear (False)

N=10
Number of unphysical estimates=0
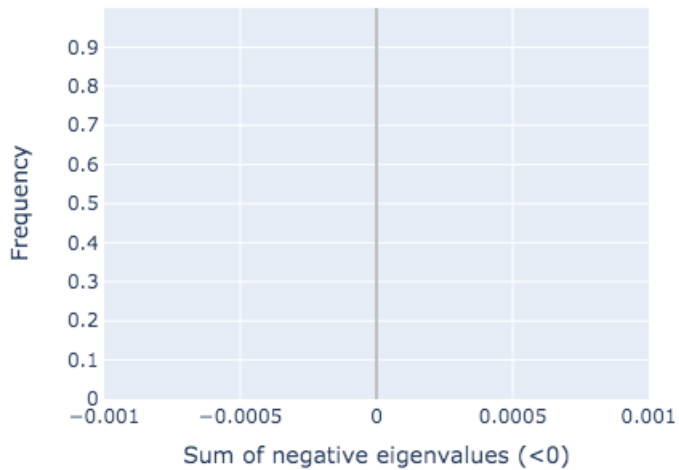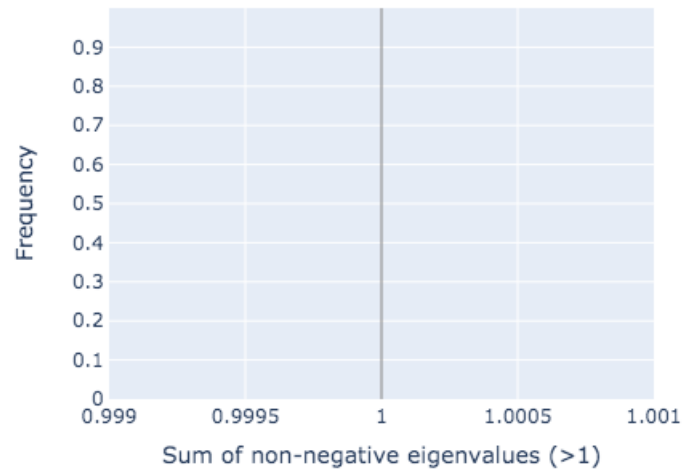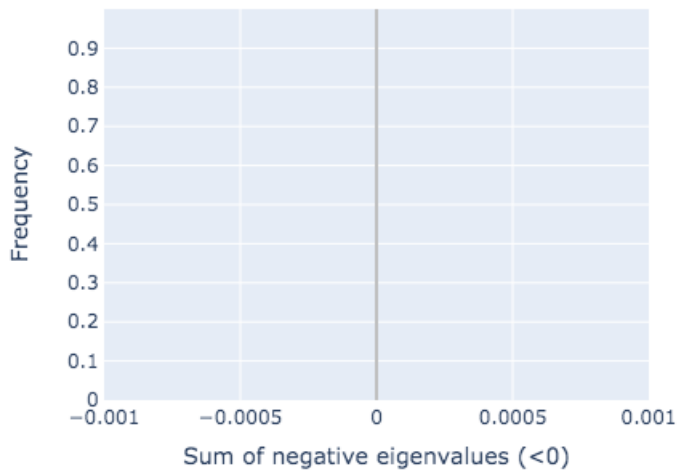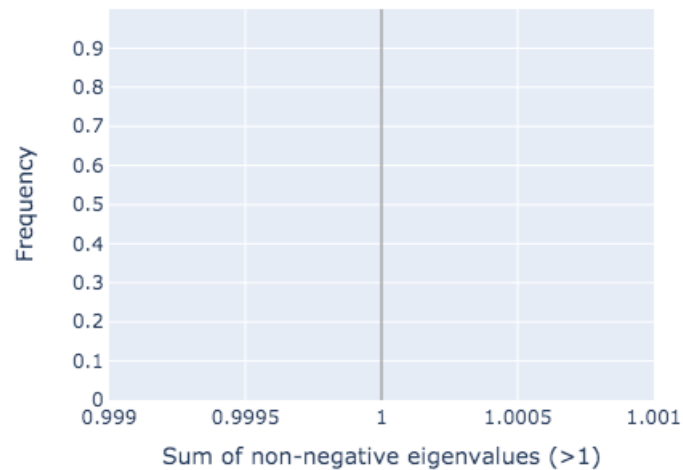
Min: -
Max: -
|Max-Min|: -

Min: -
Max: -
|Max-Min|: -

N=100
Number of unphysical estimates=0

N=100, Nrep=10

N=100, Nrep=10

Min: -
Max: -
|Max-Min|: -

Min: -
Max: -
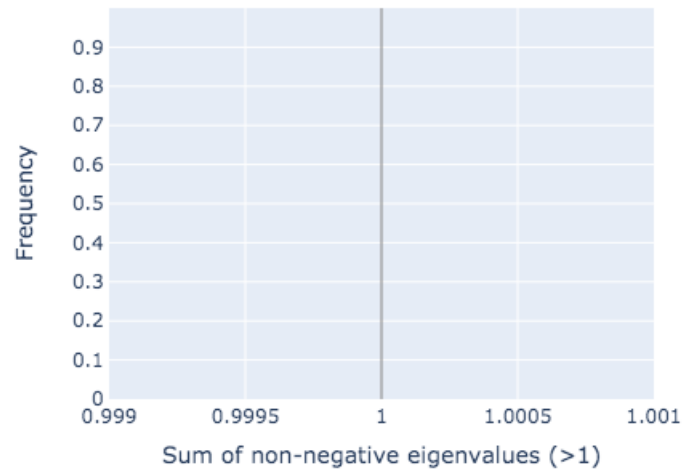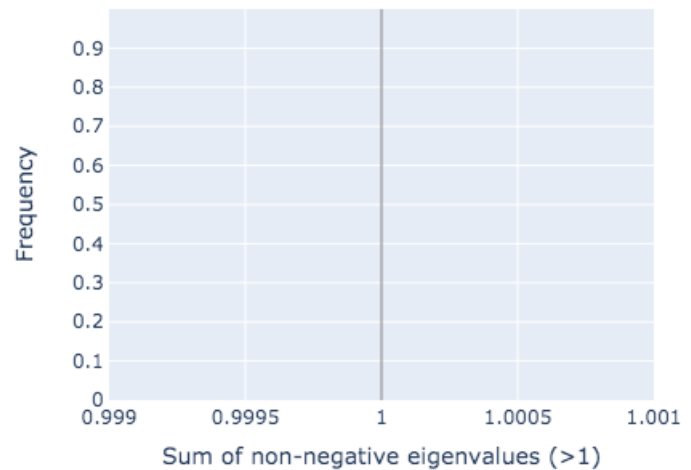|Max-Min|: -

Case 4: Maximum-Likelihood (True)

N=10
Number of unphysical estimates=0

N=10, Nrep=10

Frequency (y-axis)
Sum of negative eigenvalues (<0) (x-axis)

Min: -
Max: -
|Max-Min|: -

N=10, Nrep=10

Frequency (y-axis)
Sum of non-negative eigenvalues (>1) (x-axis)

Min: -
Max: -
|Max-Min|: -

N=100
Number of unphysical estimates=0

N=100, Nrep=10

Frequency (y-axis)
Sum of negative eigenvalues (<0) (x-axis)

Min: -
Max: -
|Max-Min|: -

N=100, Nrep=10

Frequency (y-axis)
Sum of non-negative eigenvalues (>1) (x-axis)
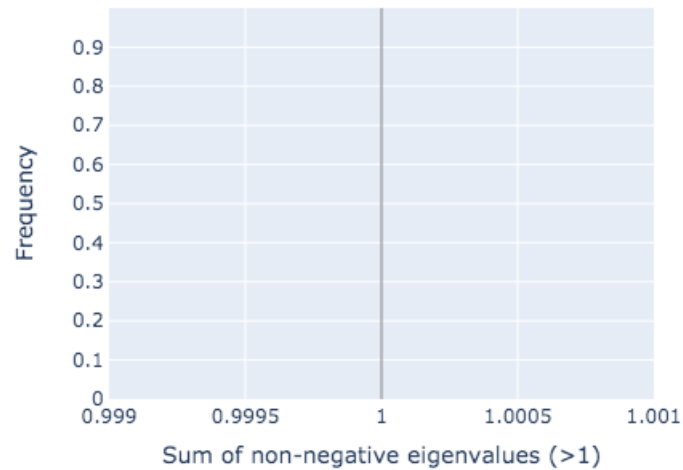
Min: -
Max: -
|Max-Min|: -

Case 5: Maximum-Likelihood (False)

N=10
Number of unphysical estimates=0

N=10, Nrep=10

Sum of negative eigenvalues (<0)

Min: -
Max: -
|Max-Min|: -

N=10, Nrep=10

Sum of non-negative eigenvalues (>1)

Min: -
Max: -
|Max-Min|: -

N=100
Number of unphysical estimates=0

N=100, Nrep=10

Sum of negative eigenvalues (<0)

Min: -
Max: -
|Max-Min|: -

N=100, Nrep=10

Sum of non-negative eigenvalues (>1)

Min: -
Max: -
|Max-Min|: -

Case 6: Least Squares (True)

N=10
Number of unphysical estimates=0

N=10, Nrep=10

Sum of negative eigenvalues (<0)

Min: -
Max: -
|Max-Min|: -

N=10, Nrep=10

Sum of non-negative eigenvalues (>1)

Min: -
Max: -
|Max-Min|: -

N=100
Number of unphysical estimates=0

N=100, Nrep=10

Sum of negative eigenvalues (<0)

Min: -
Max: -
|Max-Min|: -

N=100, Nrep=10

Sum of non-negative eigenvalues (>1)

Min: -
Max: -
|Max-Min|: -

Case 7: Least Squares (False)

N=10
Number of unphysical estimates=0

N=10, Nrep=10

Frequency vs Sum of negative eigenvalues (<0)

Min: -
Max: -
|Max-Min|: -

N=10, Nrep=10

Frequency vs Sum of non-negative eigenvalues (>1)

Min: -
Max: -
|Max-Min|: -

N=100
Number of unphysical estimates=0

N=100, Nrep=10

Frequency vs Sum of negative eigenvalues (<0)

Min: -
Max: -
|Max-Min|: -

N=100, Nrep=10

Frequency vs Sum of non-negative eigenvalues (>1)

Min: -
Max: -
|Max-Min|: -