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## Computation time in total

Total	0.6531767805417379min.
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## Tolerance of physicality constraint violation

		True	False
Tolerance at physicality violation test	equality constraint	1.00e-13	1.00e-05
	inequality constraint	1.00e-05	1.00e-05

## Experimental condition

Type of tomography	StandardQst
Nrep	10
N	[10, 100]
RNG seed	777

### True object

Type	State
Dim	2
Vec	[0.70710678 0. 0. 0.6363961 ]

### Tester objects

0	Type	Povm
	Dim	2
	Number of outcomes	2
	Vecs	[[ 0.70710678 0.6363961 0. 0. ] [ 0.70710678 -0.6363961 0. 0. ]]
1	Type	Povm
	Dim	2
	Number of outcomes	2
	Vecs	[[ 0.70710678 0. 0.6363961 0. ] [ 0.70710678 0. -0.6363961 0. ]]
2	Type	Povm
	Dim	2
	Number of outcomes	2
	Vecs	[[ 0.70710678 0. 0. 0.6363961 ] [ 0.70710678 0. 0. -0.6363961 ]]

### Cases

	Name	Param	Tomography	Estimator
0	Linear (True)	True	StandardQst	Linear
1	Linear (False)	False	StandardQst	Linear
2	ProjectedLinear (True)	True	StandardQst	ProjectedLinear
3	ProjectedLinear (False)	False	StandardQst	ProjectedLinear
4	Maximum-Likelihood (True)	True	StandardQst	LossMinimization
5	Maximum-Likelihood (False)	False	StandardQst	LossMinimization

6	Least Squares (True)	True	StandardQst	LossMinimization
7	Least Squares (False)	False	StandardQst	LossMinimization

## Computation time of estimators

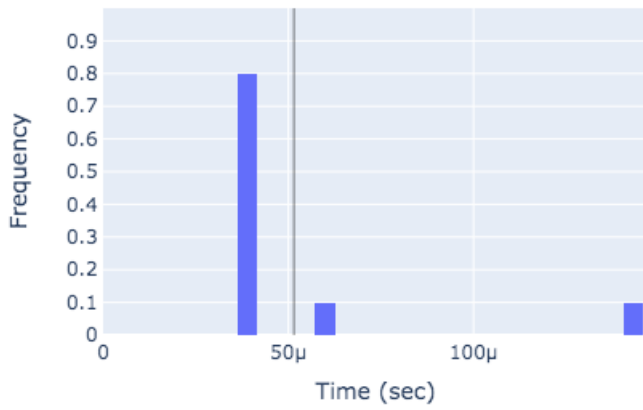
Table

	Name	N	Nrep	Mean (sec)	Std (sec)
0	Linear (True)	10	10	0.000052	0.000031
1	"	100	10	0.000042	0.000018
2	Linear (False)	10	10	0.000070	0.000025
3	"	100	10	0.000065	0.000027
4	ProjectedLinear (True)	10	10	0.006673	0.004439
5	"	100	10	0.002071	0.002860
6	ProjectedLinear (False)	10	10	0.006963	0.004960
7	"	100	10	0.002004	0.002694
8	Maximum-Likelihood (True)	10	10	0.482074	0.183801
9	"	100	10	0.422318	0.145371
10	Maximum-Likelihood (False)	10	10	1.311640	1.708695
11	"	100	10	1.466943	1.676517
12	Least Squares (True)	10	10	0.046890	0.004389
13	"	100	10	0.048040	0.009702
14	Least Squares (False)	10	10	0.057579	0.011405
15	"	100	10	0.065638	0.010987

## Histogram

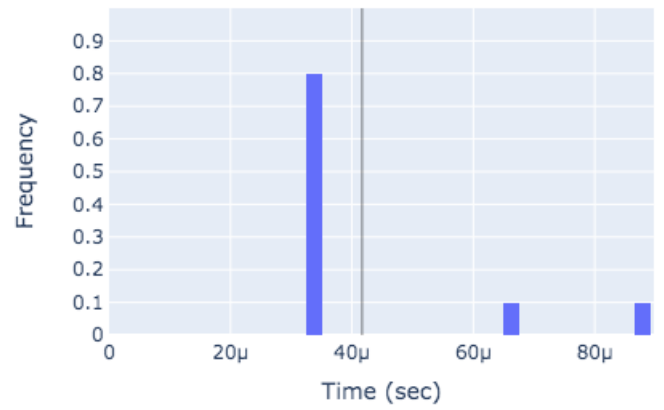
Case 0: Linear (True)

N=10, Nrep=10  
Mean = 5.1522254943847655e-05(sec)



Min: 3.7670135498046875e-05  
Max: 0.0001418590545654297  
|Max-Min|: 0.00010418891906738281

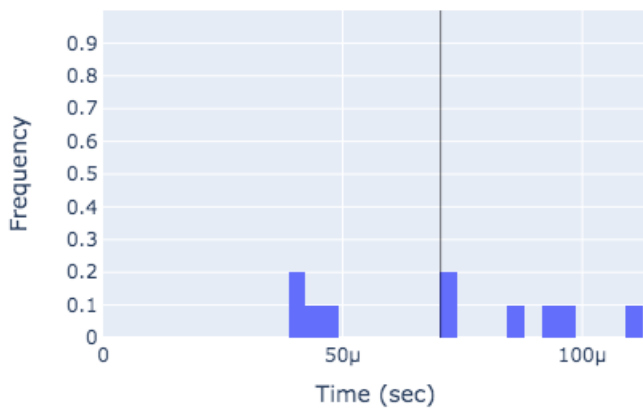
N=100, Nrep=10  
Mean = 4.167556762695313e-05(sec)



Min: 3.2901763916015625e-05  
Max: 8.702278137207031e-05  
|Max-Min|: 5.412101745605469e-05

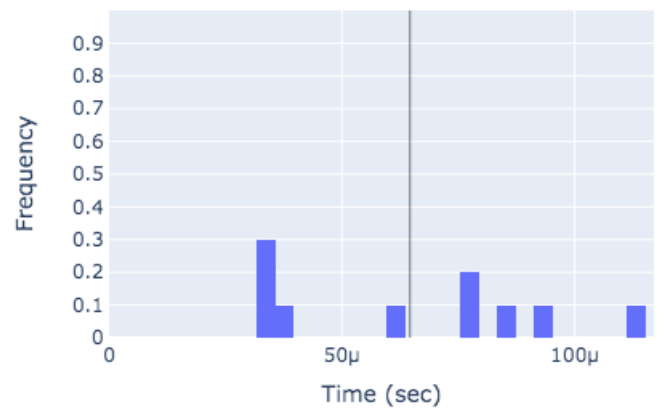
### Case 1: Linear (False)

N=10, Nrep=10  
Mean = 7.040500640869141e-05(sec)



Min: 3.981590270996094e-05  
Max: 0.00011014938354492188  
|Max-Min|: 7.033348083496094e-05

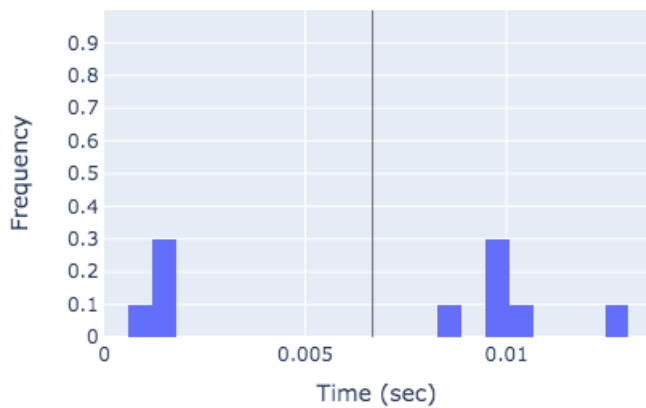
N=100, Nrep=10  
Mean = 6.461143493652344e-05(sec)



Min: 3.361701965332031e-05  
Max: 0.00011301040649414062  
|Max-Min|: 7.939338684082031e-05

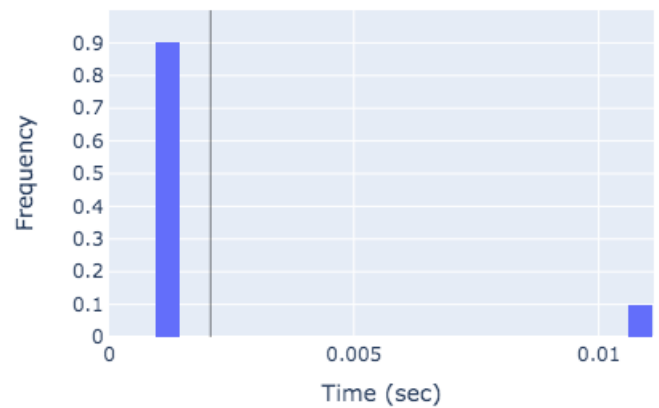
### Case 2: ProjectedLinear (True)

N=10, Nrep=10  
Mean = 0.006672716140747071(sec)



Min: 0.0010929107666015625  
Max: 0.012946367263793945  
|Max-Min|: 0.011853456497192383

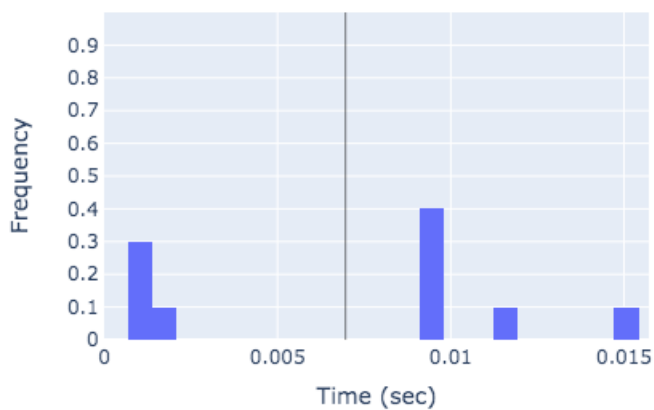
N=100, Nrep=10  
Mean = 0.002070760726928711(sec)



Min: 0.001009225845336914  
Max: 0.01064300537109375  
|Max-Min|: 0.009633779525756836

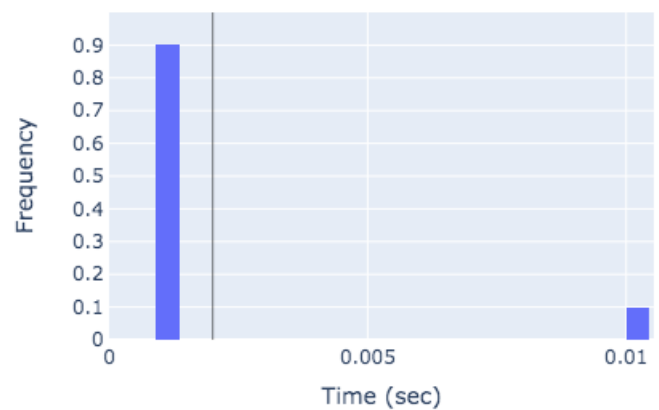
### Case 3: ProjectedLinear (False)

N=10, Nrep=10  
Mean = 0.006963181495666504(sec)



Min: 0.0009999275207519531  
Max: 0.01501321792602539  
|Max-Min|: 0.014013290405273438

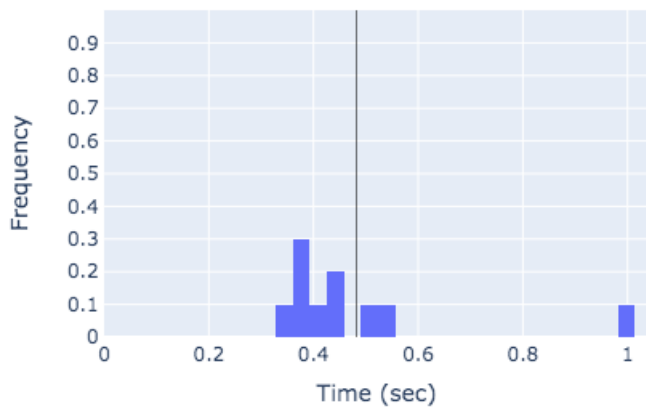
N=100, Nrep=10  
Mean = 0.002004194259643555(sec)



Min: 0.000990152359008789  
Max: 0.01007986068725586  
|Max-Min|: 0.00908970832824707

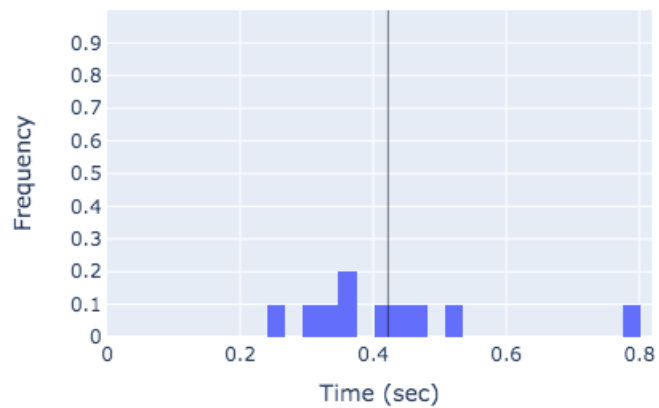
### Case 4: Maximum-Likelihood (True)

N=10, Nrep=10  
Mean = 0.48207385540008546(sec)



Min: 0.35343003273010254  
Max: 1.0079481601715088  
|Max-Min|: 0.6545181274414062

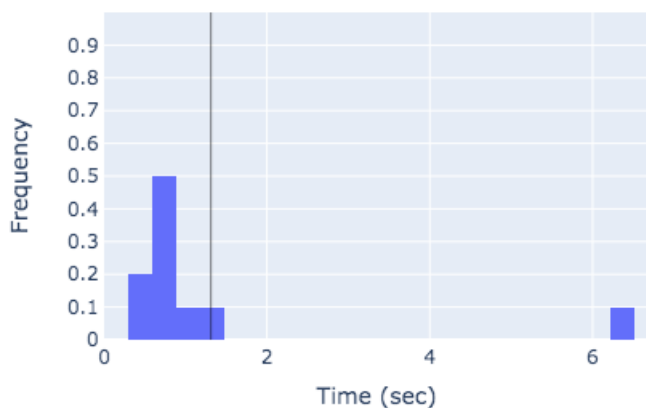
N=100, Nrep=10  
Mean = 0.42231764793396(sec)



Min: 0.25699472427368164  
Max: 0.7920389175415039  
|Max-Min|: 0.5350441932678223

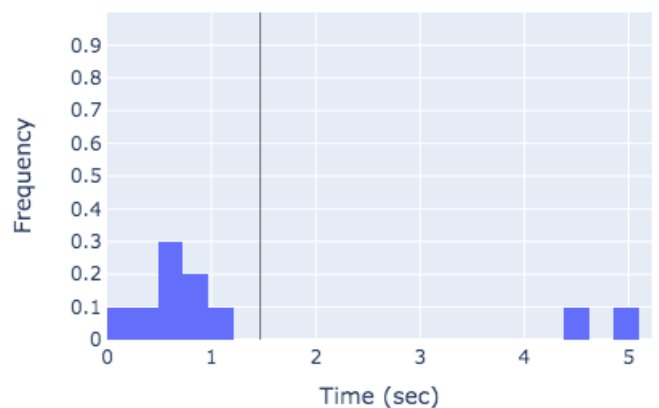
### Case 5: Maximum-Likelihood (False)

N=10, Nrep=10  
Mean = 1.3116395950317383(sec)



Min: 0.477755069732666  
Max: 6.394351959228516  
|Max-Min|: 5.91659688949585

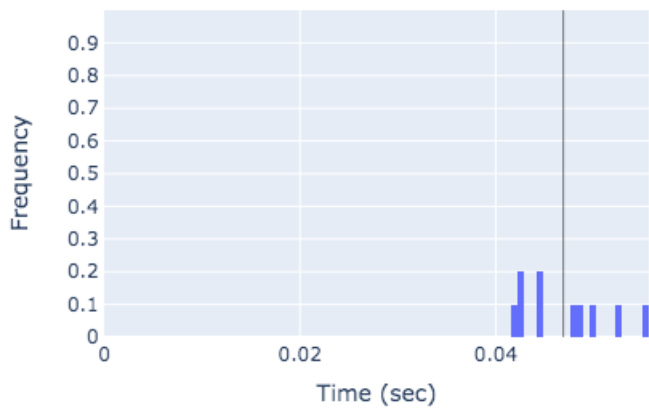
N=100, Nrep=10  
Mean = 1.4669431686401366(sec)



Min: 0.12041711807250977  
Max: 4.979774236679077  
|Max-Min|: 4.859357118606567

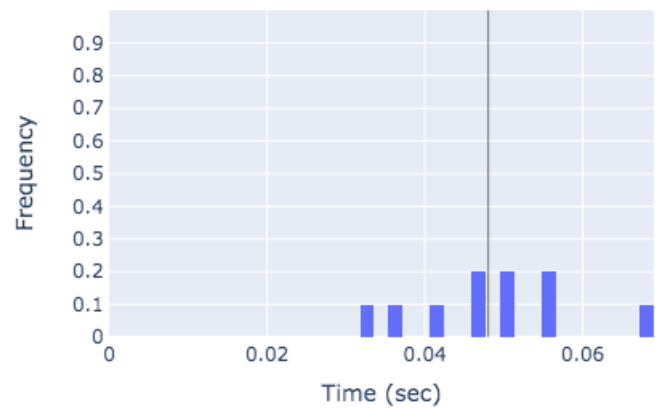
### Case 6: Least Squares (True)

N=10, Nrep=10  
Mean = 0.04689021110534668(sec)



Min: 0.04156041145324707  
Max: 0.054956912994384766  
|Max-Min|: 0.013396501541137695

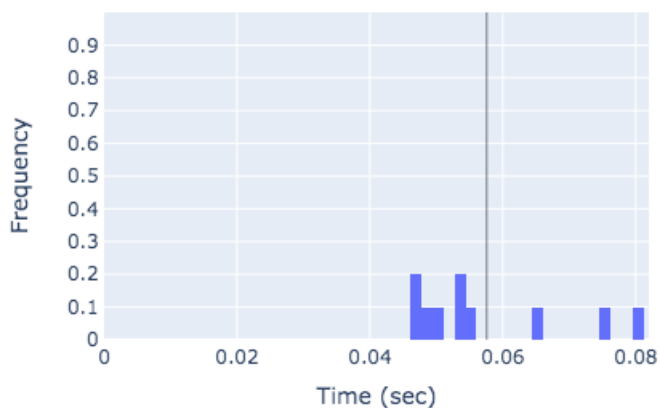
N=100, Nrep=10  
Mean = 0.048040223121643064(sec)



Min: 0.03189873695373535  
Max: 0.06725907325744629  
|Max-Min|: 0.03536033630371094

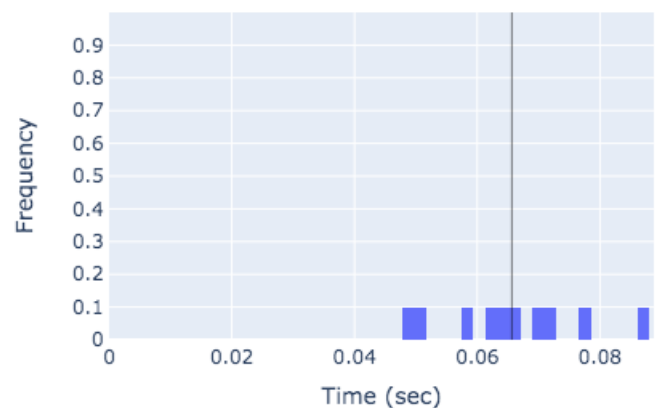
## Case 7: Least Squares (False)

N=10, Nrep=10  
Mean = 0.05757873058319092(sec)



Min: 0.04683804512023926  
Max: 0.08030462265014648  
|Max-Min|: 0.03346657752990723

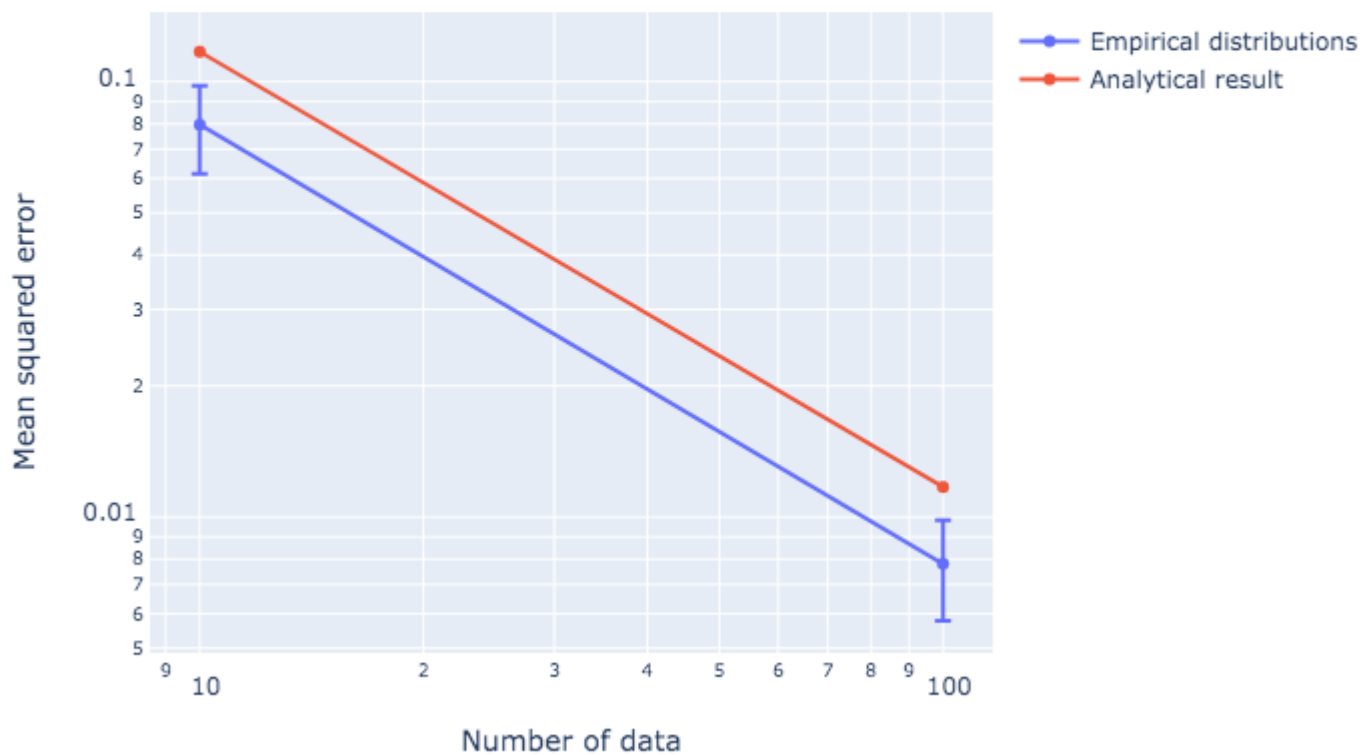
N=100, Nrep=10  
Mean = 0.06563818454742432(sec)



Min: 0.04857301712036133  
Max: 0.08686590194702148  
|Max-Min|: 0.038292884826660156

## MSE of empirical distributions

## Mean squared error



## Consistency test

	Name	Type of tomography	Param	Estimator	Loss	Algo	Squared Error to True	Possibly OK	To be checked
0	Linear (True)	Standard Qst	True	Linear	None	None	0.00e+00	OK	not need debug
1	Linear (False)	Standard Qst	False	Linear	None	None	2.47e-32	OK	not need debug
2	ProjectedLinear (True)	Standard Qst	True	Projected Linear	None	None	1.23e-32	OK	not need debug
3	ProjectedLinear (False)	Standard Qst	False	Projected Linear	None	None	1.11e-31	OK	not need debug
4	Maximum-Likelihood (True)	Standard Qst	True	Loss Minimization	Weighted Relative Entropy	Projected Gradient Descent Backtracking	3.21e-18	OK	not need debug
5	Maximum-Likelihood (False)	Standard Qst	False	Loss Minimization	Weighted Relative Entropy	Projected Gradient Descent Backtracking	5.47e-09	NG	not need debug
6	Least Squares (True)	Standard Qst	True	Loss Minimization	Weighted Probability Based Squared Error	Projected Gradient Descent Backtracking	4.93e-32	OK	not need debug

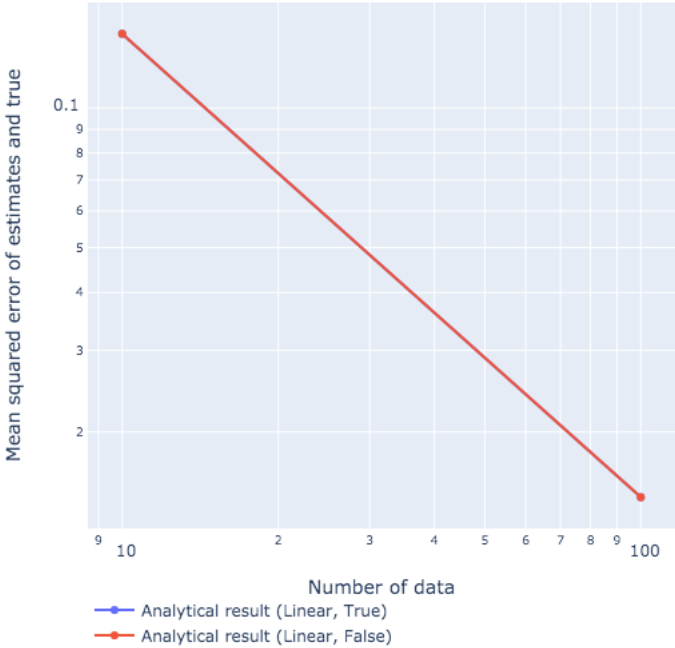


7	Least Squares (False)	Standard Qst	False	Loss Minimization	Weighted Probability Based Squared Error	Projected Gradient Descent Backtracking	1.60e-31	OK	not need debug
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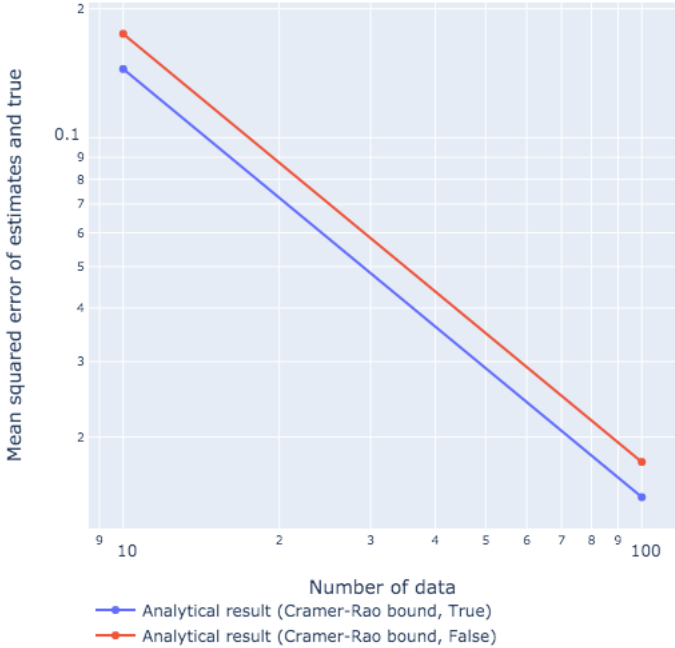
# MSE of estimators

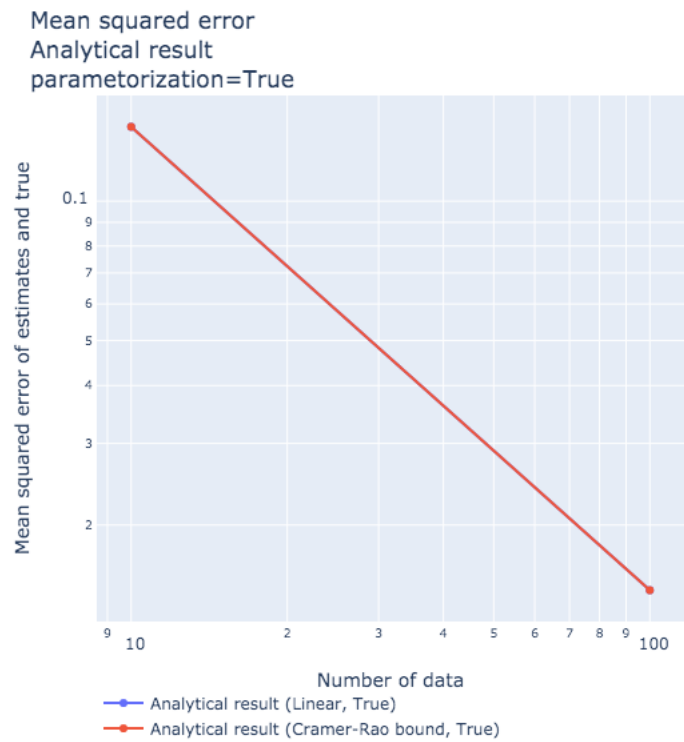
## Comparison of analytical results

Mean squared error  
Analytical result  
estimator=Linear

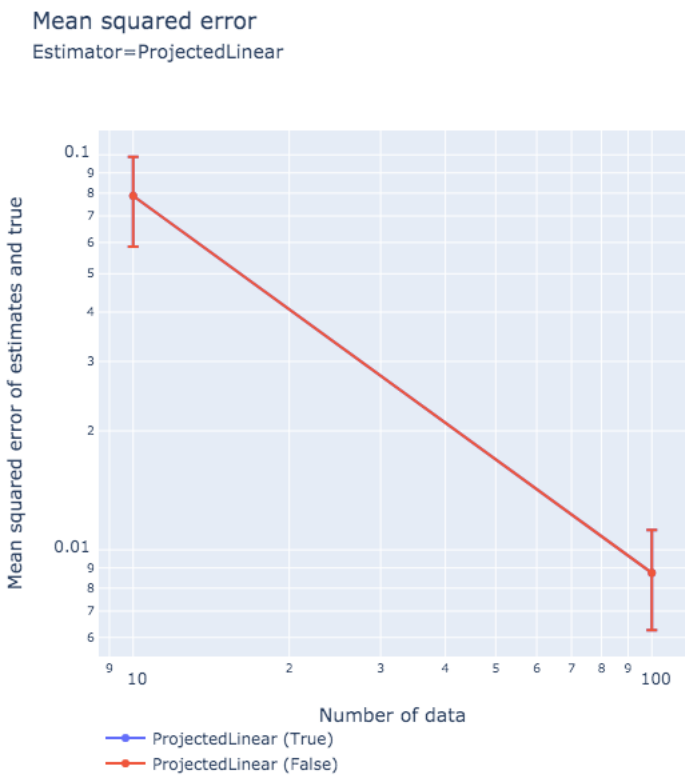
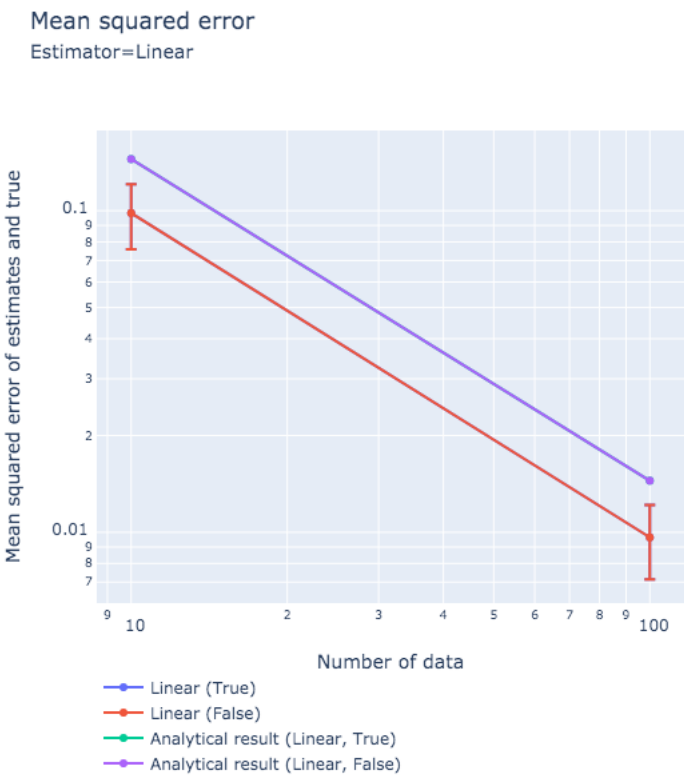


Mean squared error  
Analytical result  
Cramer-Rao bound





# Comparison of parametrization

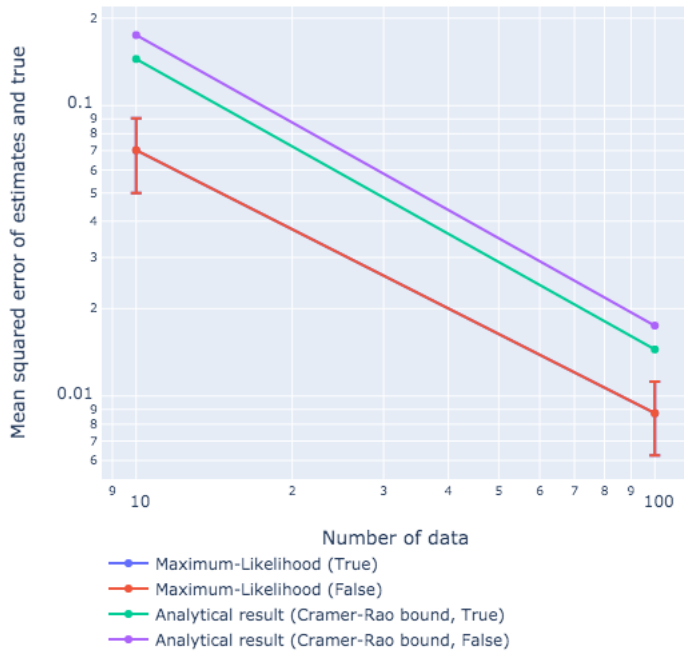


### Mean squared error

Estimator=LossMinimization

Loss=WeightedRelativeEntropy

Algo=ProjectedGradientDescentBacktracking

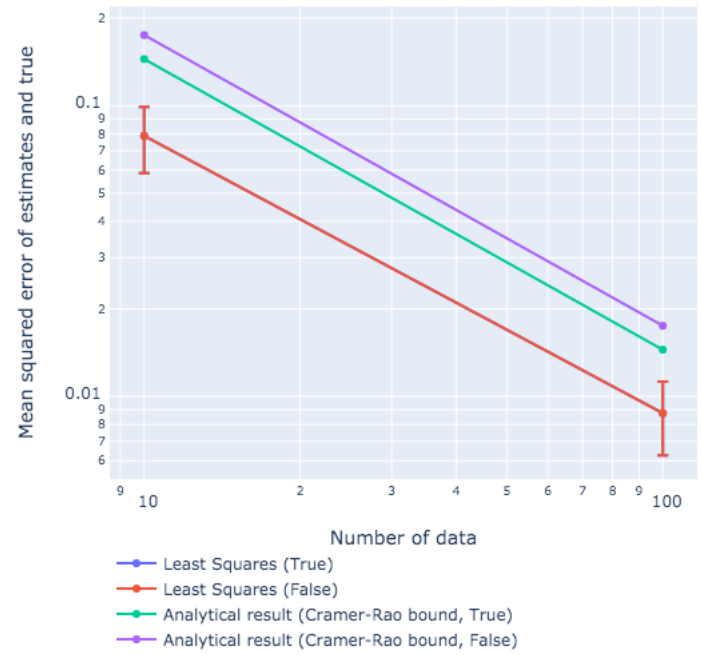


### Mean squared error

Estimator=LossMinimization

Loss=WeightedProbabilityBasedSquaredError

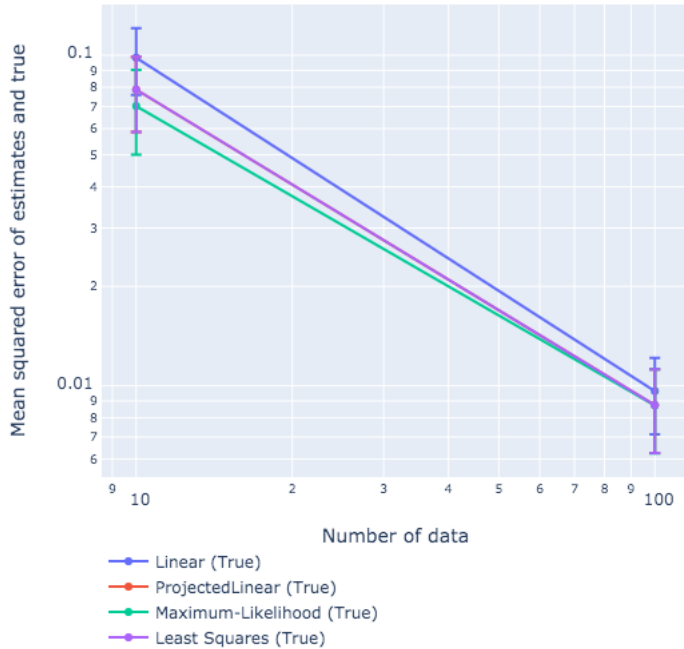
Algo=ProjectedGradientDescentBacktracking



## Comparison of estimators

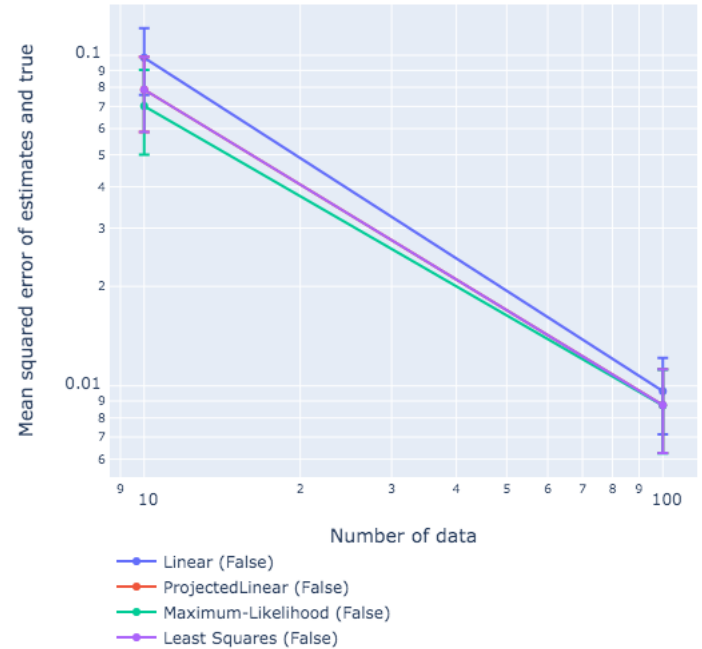
### Mean squared error

parametrization=True



### Mean squared error

parametrization=False

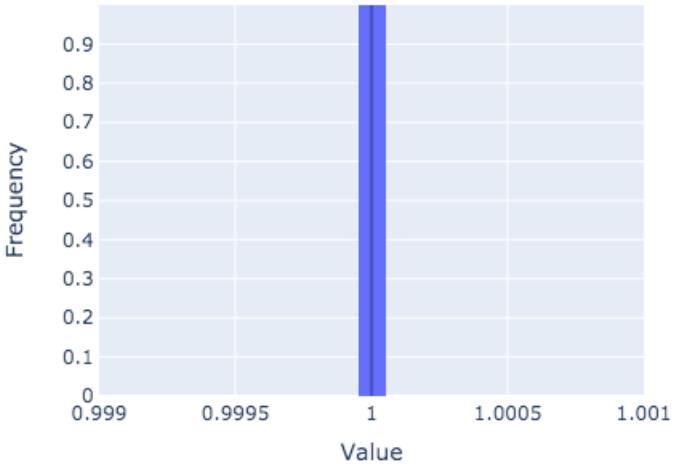


# Physicality violation test

## Test of equality constraint violation

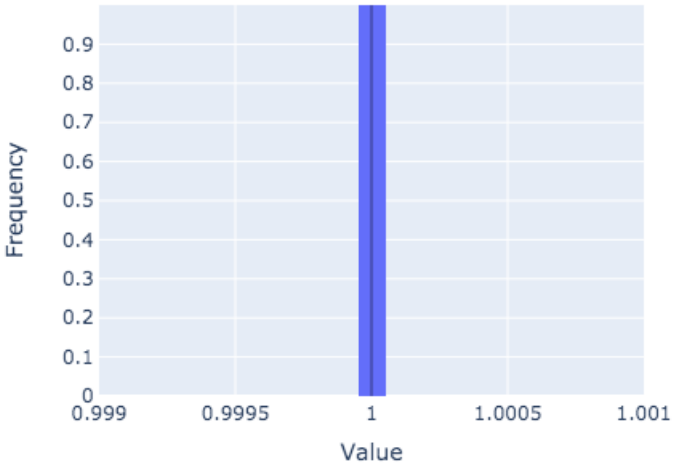
### Case 0: Linear (True)

N=10, Nrep=10



Min: 0.9999999999999997  
Max: 0.9999999999999998  
|Max-Min|: 1.1102230246251565e-16

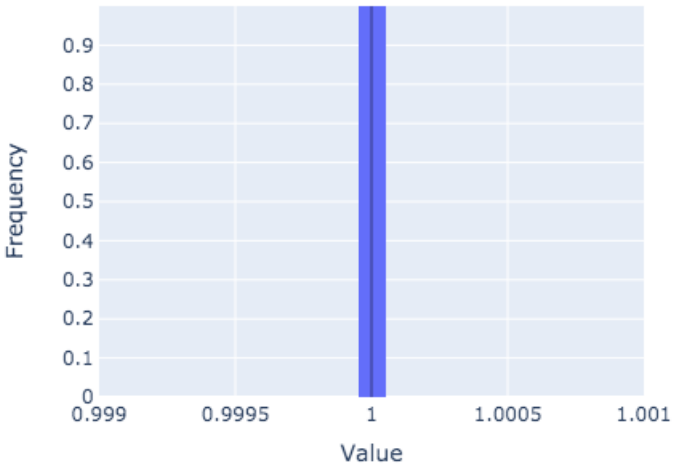
N=100, Nrep=10



Min: 0.9999999999999998  
Max: 0.9999999999999998  
|Max-Min|: 0.0

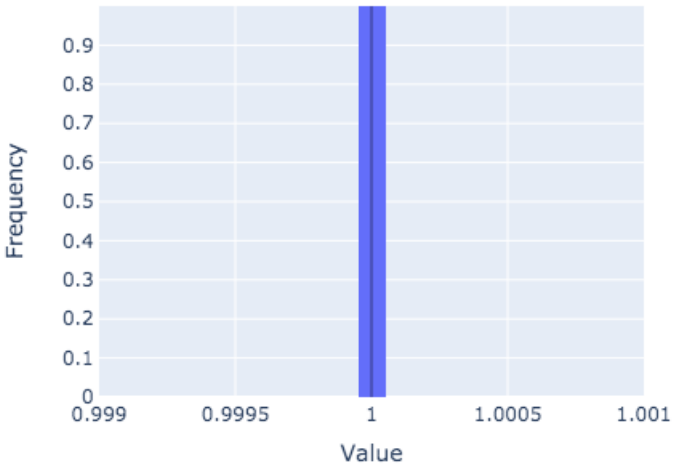
### Case 1: Linear (False)

N=10, Nrep=10



Min: 0.9999999999999998  
Max: 1.0000000000000002  
|Max-Min|: 4.440892098500626e-16

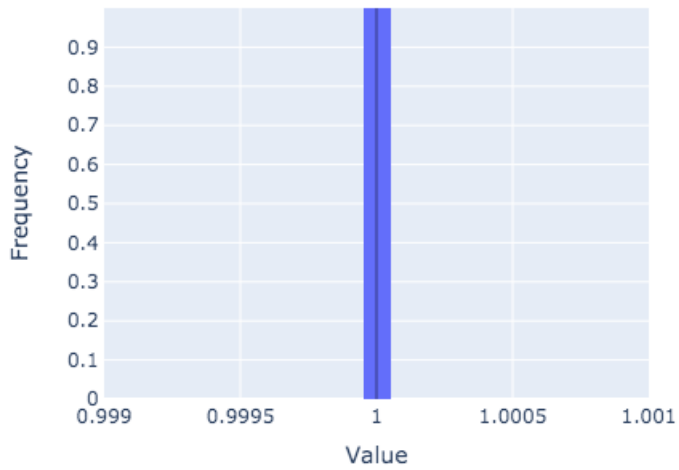
N=100, Nrep=10



Min: 0.9999999999999999  
Max: 1.0000000000000002  
|Max-Min|: 3.3306690738754696e-16

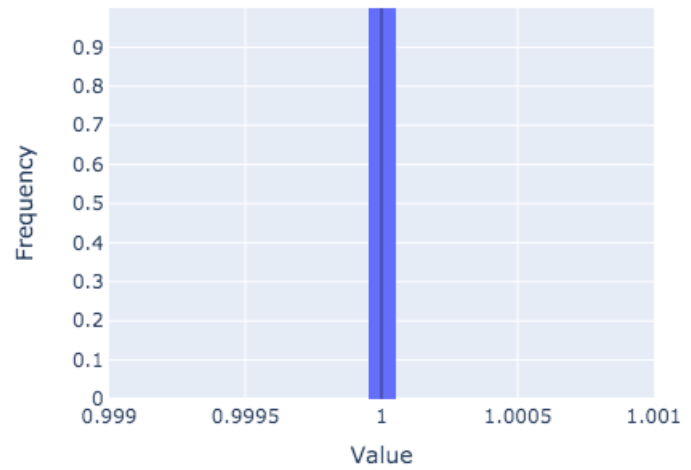
### Case 2: ProjectedLinear (True)

N=10, Nrep=10



Min: 0.9999999999999998  
Max: 0.9999999999999998  
|Max-Min|: 0.0

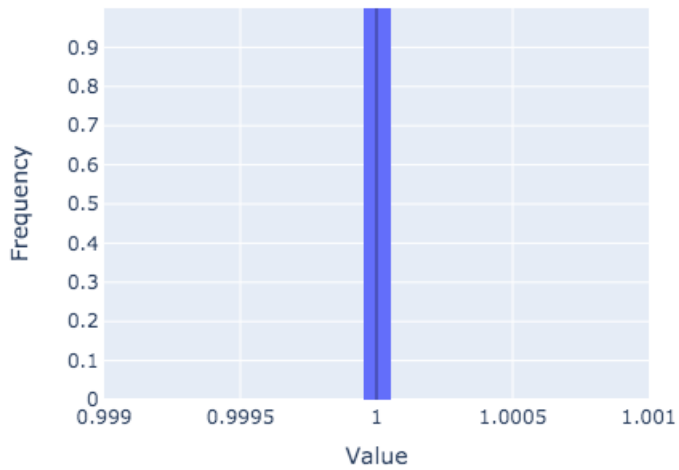
N=100, Nrep=10



Min: 0.9999999999999998  
Max: 0.9999999999999998  
|Max-Min|: 0.0

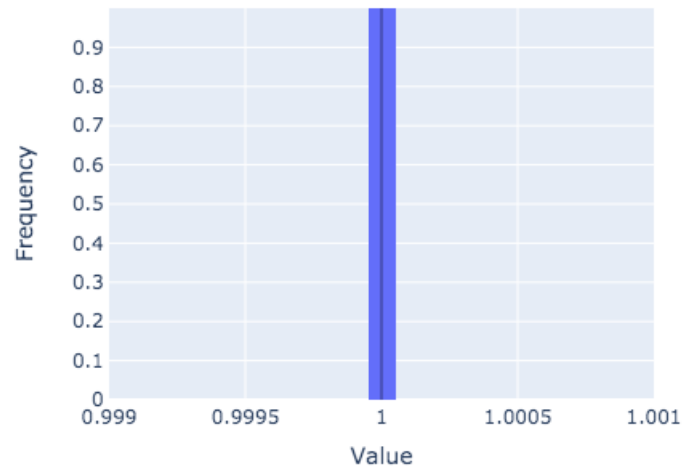
### Case 3: ProjectedLinear (False)

N=10, Nrep=10



Min: 0.9999999999999996  
Max: 1.00000014753393  
|Max-Min|: 1.4753393040578544e-07

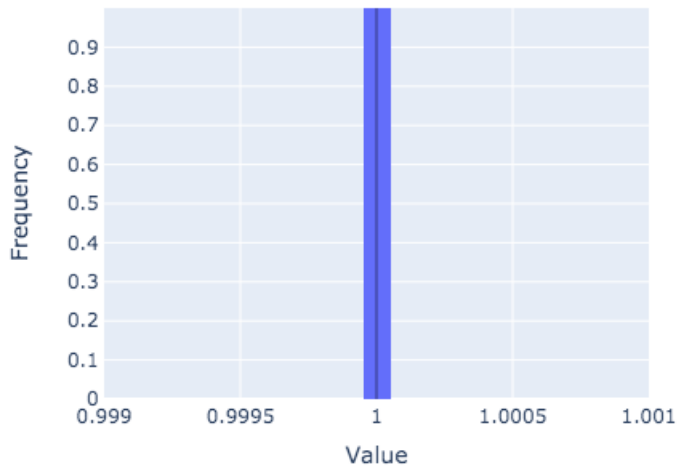
N=100, Nrep=10



Min: 0.9999999999999997  
Max: 1.0000001280394155  
|Max-Min|: 1.280394158476028e-07

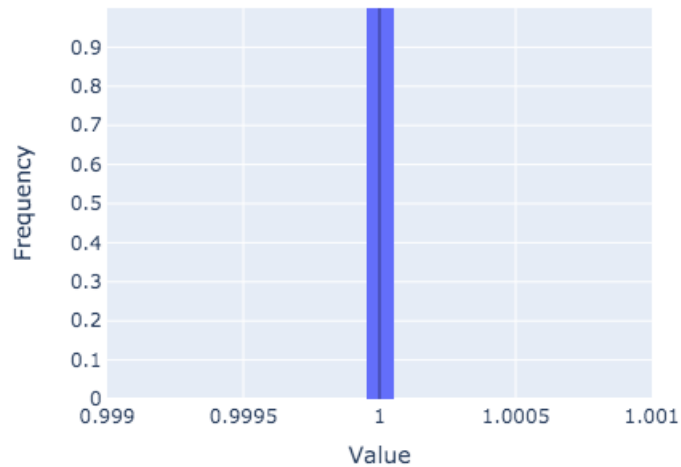
### Case 4: Maximum-Likelihood (True)

N=10, Nrep=10



Min: 0.9999999999999998  
Max: 0.9999999999999998  
|Max-Min|: 0.0

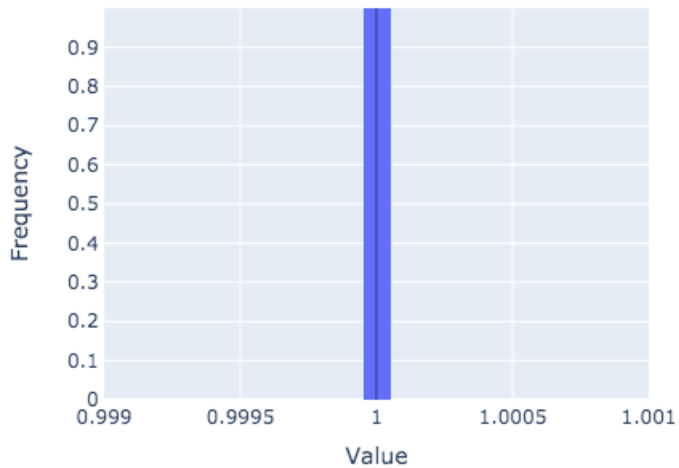
N=100, Nrep=10



Min: 0.9999999999999998  
Max: 0.9999999999999998  
|Max-Min|: 0.0

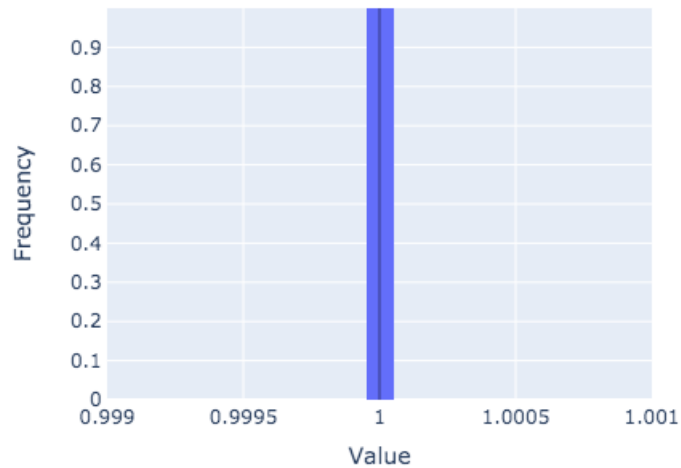
### Case 5: Maximum-Likelihood (False)

N=10, Nrep=10



Min: 1.0000000006567707  
Max: 1.0000001622723005  
|Max-Min|: 1.5570459344083076e-07

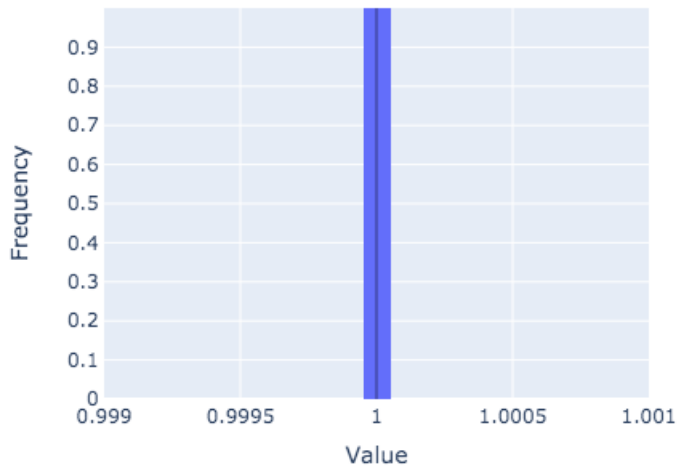
N=100, Nrep=10



Min: 1.0000000121352572  
Max: 1.0000001810607821  
|Max-Min|: 1.689255249015531e-07

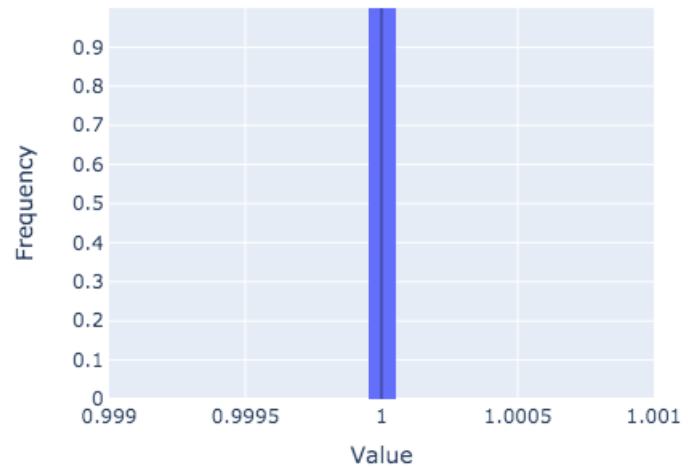
### Case 6: Least Squares (True)

N=10, Nrep=10



Min: 0.9999999999999998  
Max: 0.9999999999999998  
|Max-Min|: 0.0

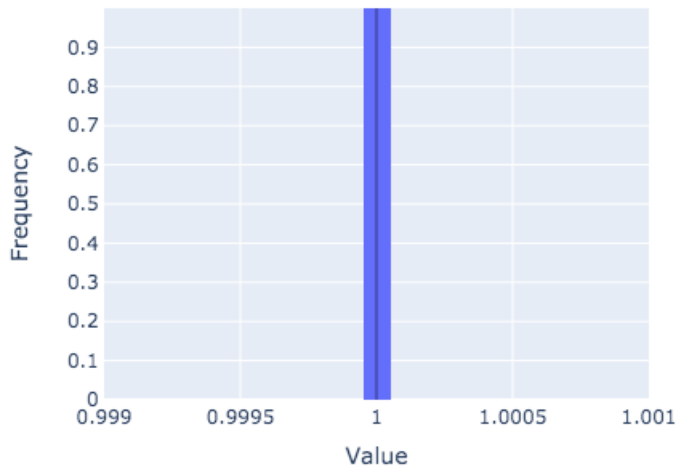
N=100, Nrep=10



Min: 0.9999999999999998  
Max: 0.9999999999999998  
|Max-Min|: 0.0

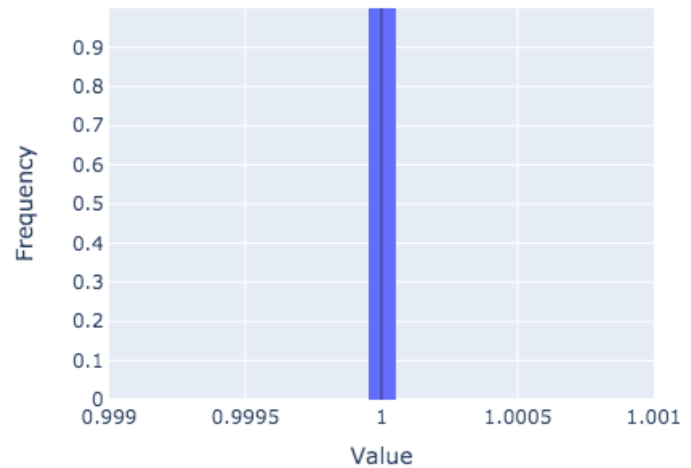
### Case 7: Least Squares (False)

N=10, Nrep=10



Min: 0.9999999999999998  
Max: 1.0000001781169368  
|Max-Min|: 1.7811693697744602e-07

N=100, Nrep=10



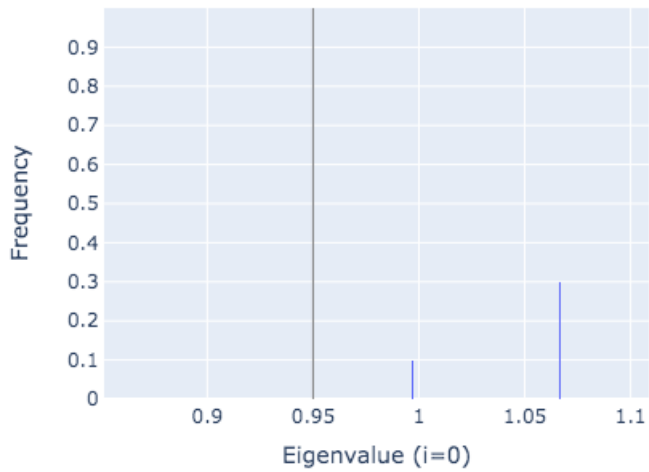
Min: 0.9999999999999998  
Max: 1.0000001555680968  
|Max-Min|: 1.5556809707106822e-07

## Test of inequality constraint violation

### Eigenvalue

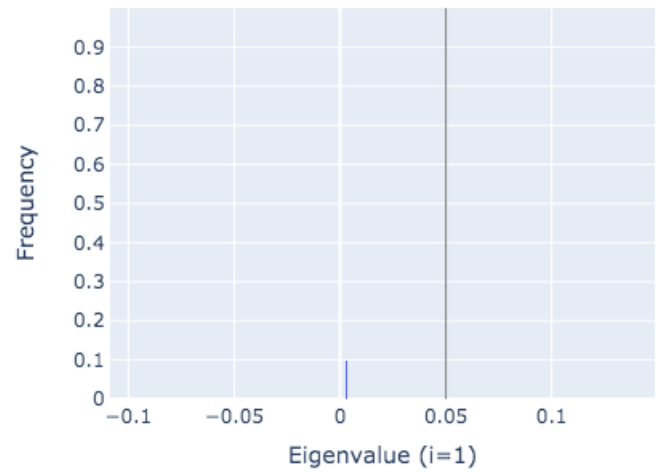
#### Case 0: Linear (True)

N=10, Nrep=10



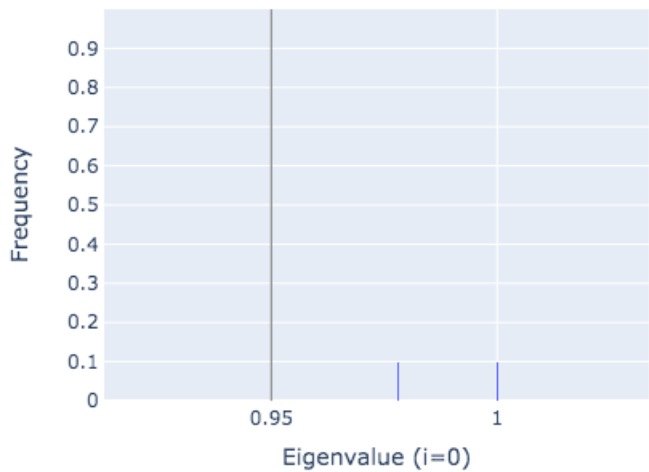
Min: 0.8513641844631532  
 Max: 1.1085806194501846  
 |Max-Min|: 0.2572164349870314

N=10, Nrep=10



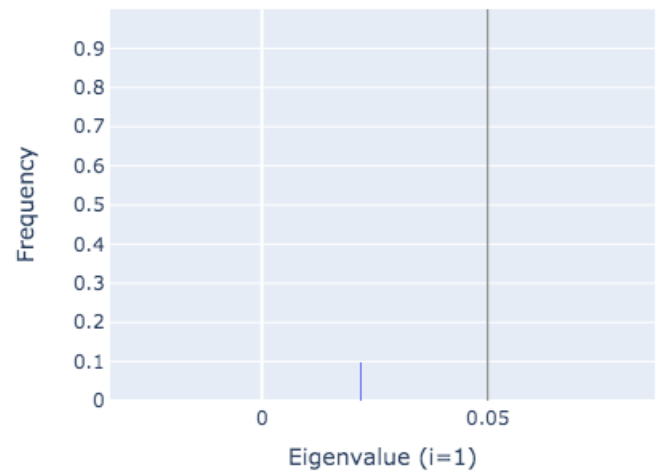
Min: -0.10858061945018471  
 Max: 0.14863581553684663  
 |Max-Min|: 0.2572164349870314

N=100, Nrep=10



Min: 0.9130584510098023  
 Max: 1.0335647646019095  
 |Max-Min|: 0.12050631359210717

N=100, Nrep=10

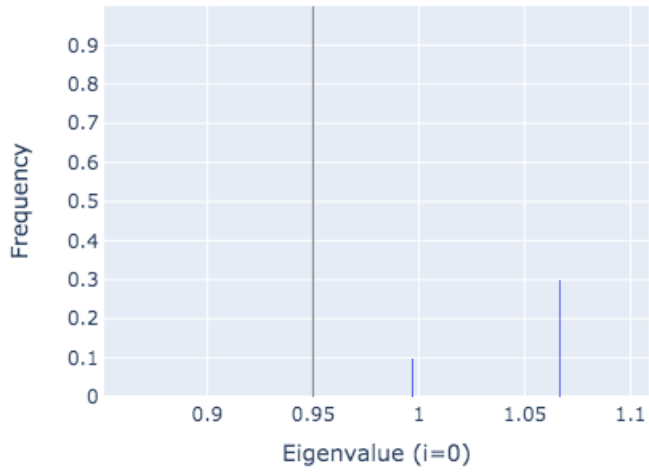


Min: -0.03356476460190986  
 Max: 0.0869415489901976  
 |Max-Min|: 0.12050631359210745

Case 1: Linear (False)

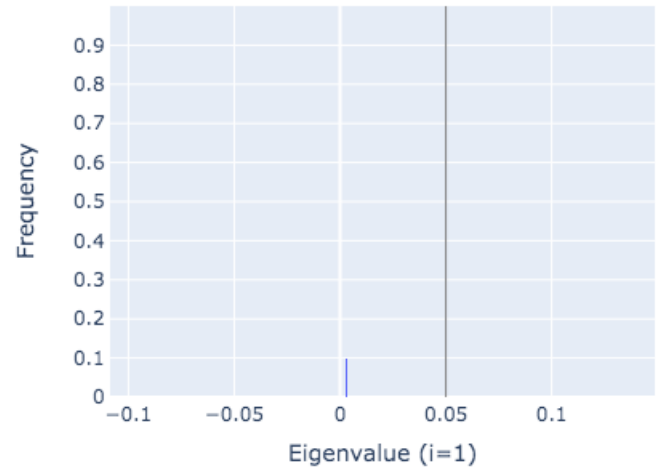


N=10, Nrep=10



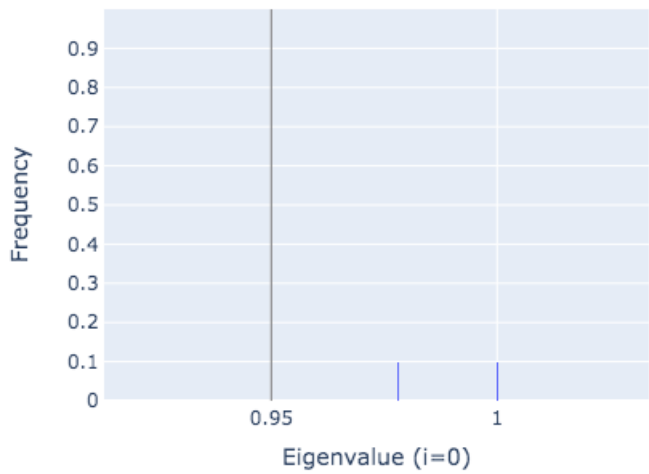
Min: 0.8513641844631532  
 Max: 1.1085806194501846  
 |Max-Min|: 0.2572164349870314

N=10, Nrep=10



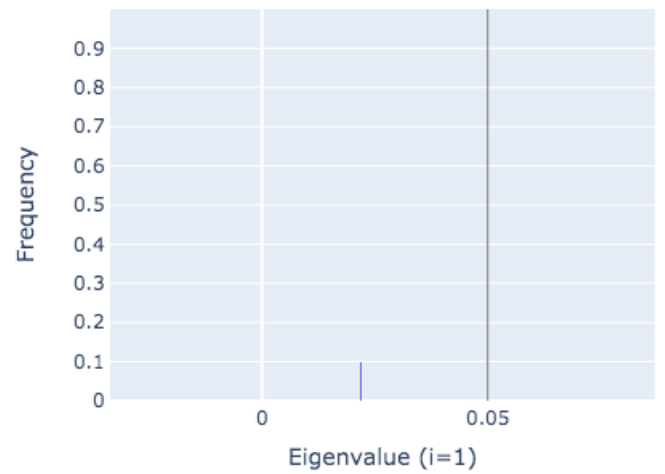
Min: -0.10858061945018459  
 Max: 0.14863581553684663  
 |Max-Min|: 0.25721643498703123

N=100, Nrep=10



Min: 0.9130584510098023  
 Max: 1.0335647646019095  
 |Max-Min|: 0.12050631359210717

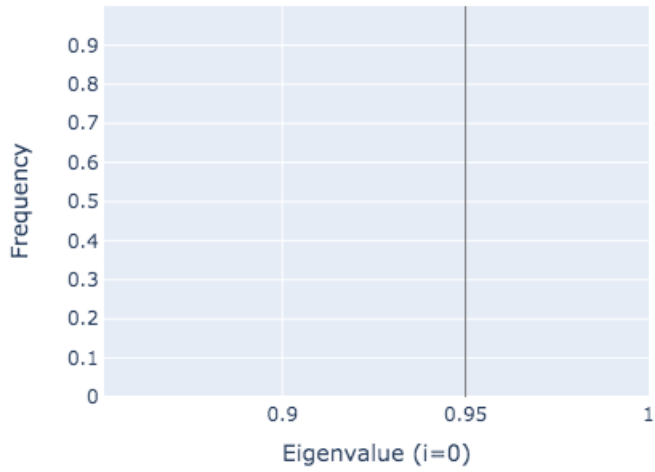
N=100, Nrep=10



Min: -0.03356476460190975  
 Max: 0.08694154899019772  
 |Max-Min|: 0.12050631359210748

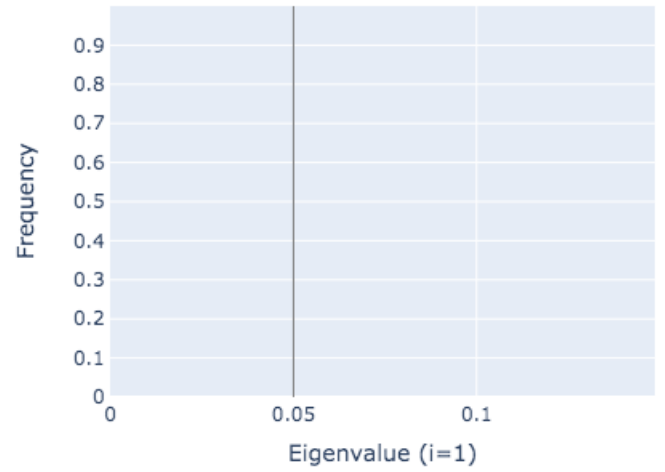
Case 2: ProjectedLinear (True)

N=10, Nrep=10



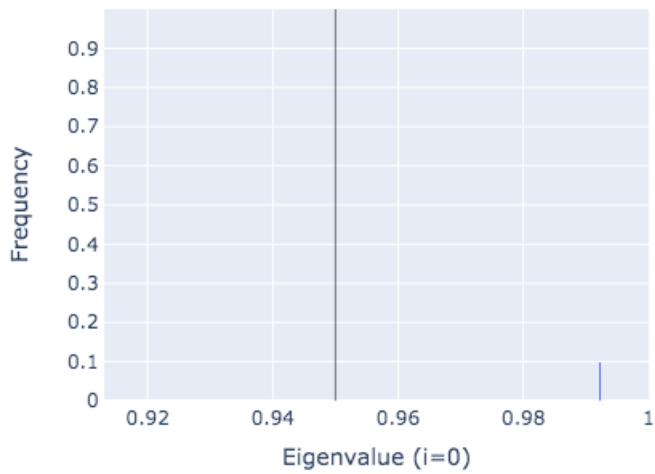
Min: 0.8513641844631531  
 Max: 1.0000000737669648  
 |Max-Min|: 0.14863588930381166

N=10, Nrep=10



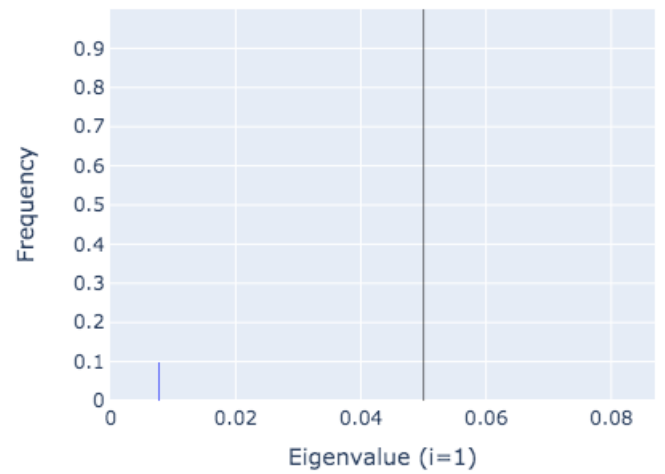
Min: -7.376696509533986e-08  
 Max: 0.14863581553684668  
 |Max-Min|: 0.14863588930381177

N=100, Nrep=10



Min: 0.9130584510098023  
 Max: 1.0000000640197078  
 |Max-Min|: 0.08694161300990544

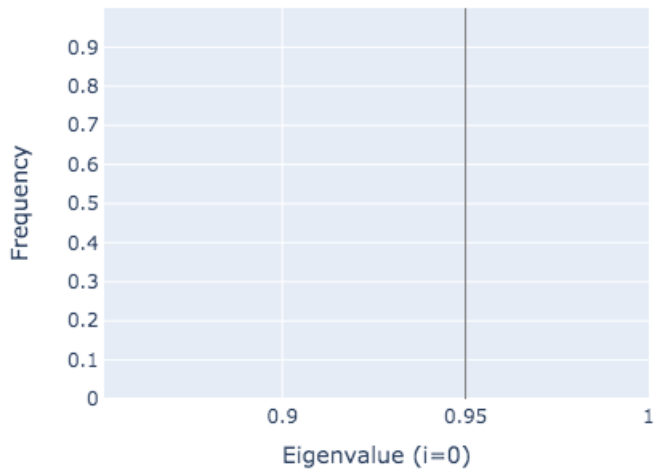
N=100, Nrep=10



Min: -6.401970792556323e-08  
 Max: 0.0869415489901976  
 |Max-Min|: 0.08694161300990552

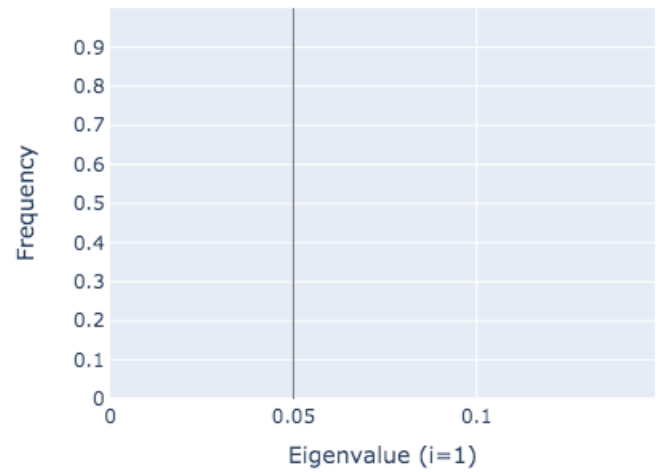
Case 3: ProjectedLinear (False)

N=10, Nrep=10



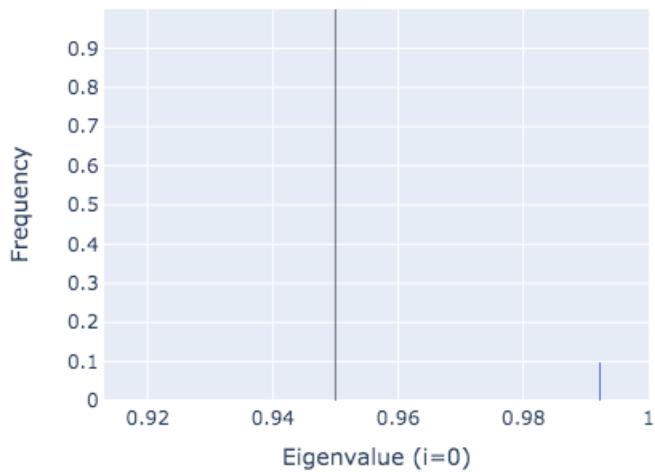
Min: 0.8513641844631532  
 Max: 1.00000014753393  
 |Max-Min|: 0.14863596307077676

N=10, Nrep=10



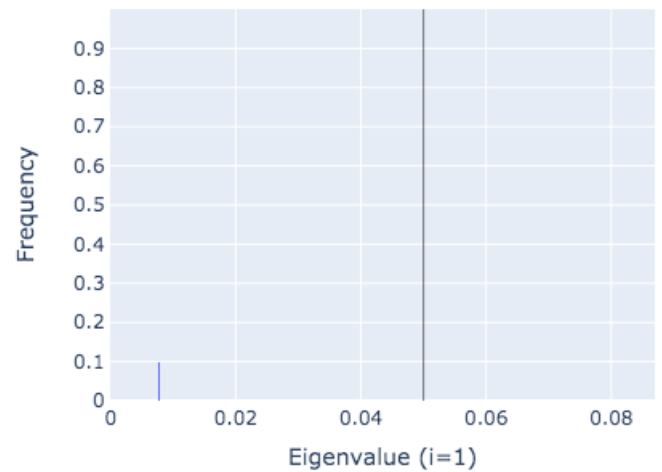
Min: -2.7755575615628914e-17  
 Max: 0.14863581553684696  
 |Max-Min|: 0.148635815536847

N=100, Nrep=10



Min: 0.9130584510098023  
 Max: 1.0000001280394155  
 |Max-Min|: 0.0869416770296132

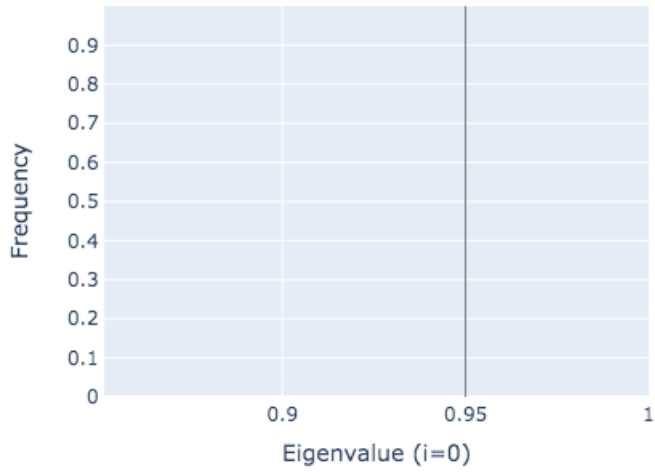
N=100, Nrep=10



Min: -5.730008481585891e-17  
 Max: 0.0869415489901976  
 |Max-Min|: 0.08694154899019765

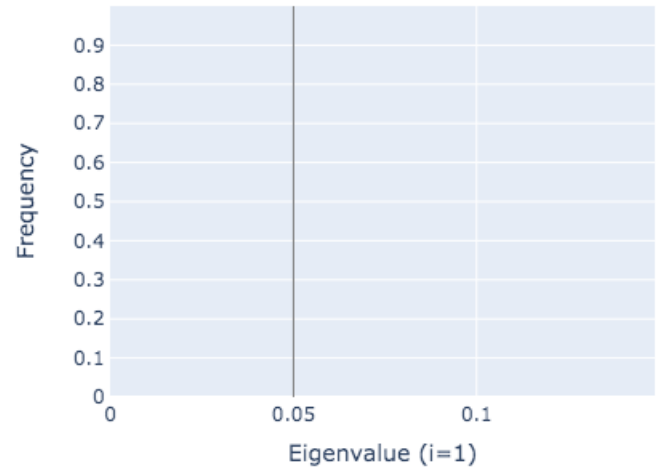
#### Case 4: Maximum-Likelihood (True)

N=10, Nrep=10



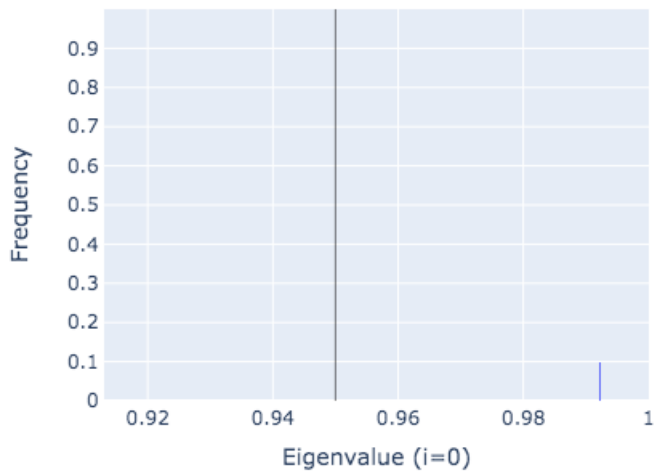
Min: 0.8513641812250503  
 Max: 1.0000000702659644  
 |Max-Min|: 0.14863588904091407

N=10, Nrep=10



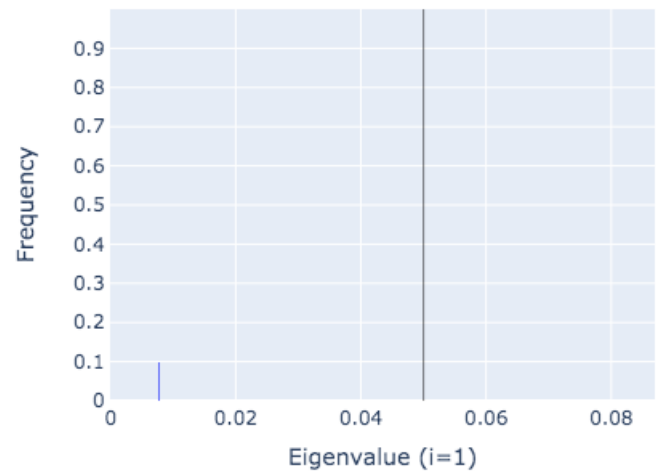
Min: -7.026596454684508e-08  
 Max: 0.14863581877494952  
 |Max-Min|: 0.14863588904091407

N=100, Nrep=10



Min: 0.9130584509383943  
 Max: 1.000000078401553  
 |Max-Min|: 0.0869416274631587

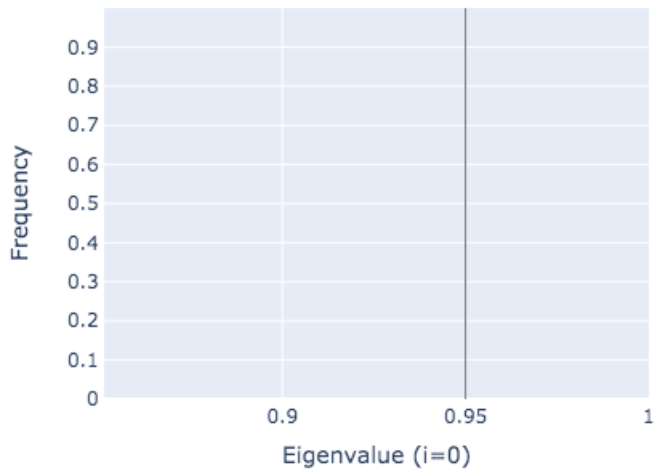
N=100, Nrep=10



Min: -7.840155328241432e-08  
 Max: 0.08694154906160545  
 |Max-Min|: 0.08694162746315873

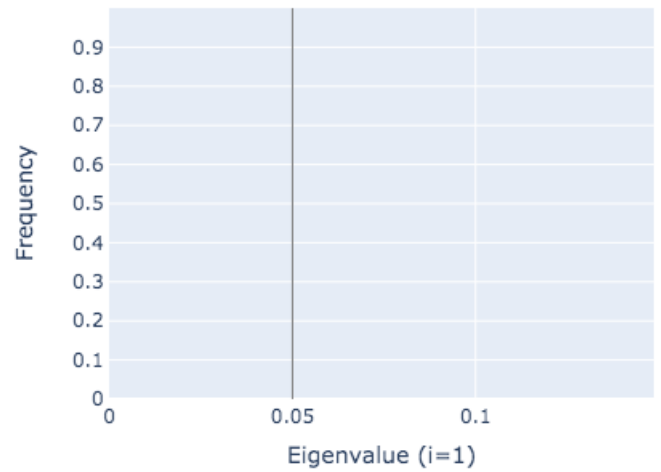
## Case 5: Maximum-Likelihood (False)

N=10, Nrep=10



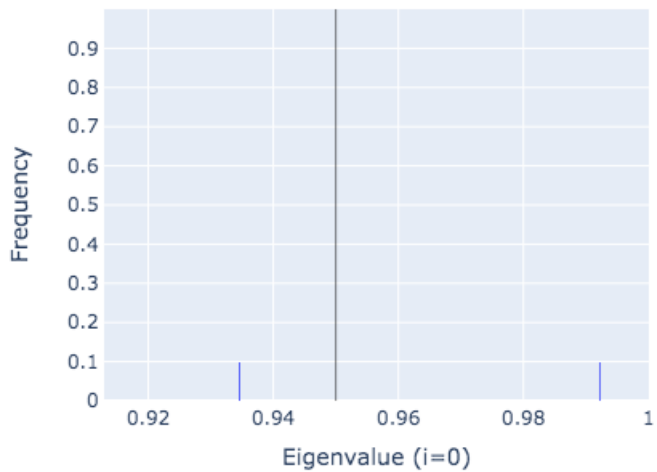
Min: 0.851427240055816  
 Max: 1.000000162272261  
 |Max-Min|: 0.1485729222164449

N=10, Nrep=10



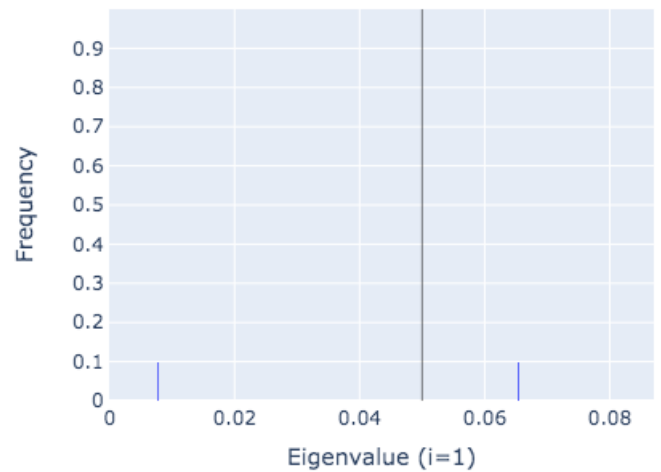
Min: -1.3877787807814457e-17  
 Max: 0.14857279250213073  
 |Max-Min|: 0.14857279250213073

N=100, Nrep=10



Min: 0.9130380513737355  
 Max: 1.0000001810607806  
 |Max-Min|: 0.08696212968704509

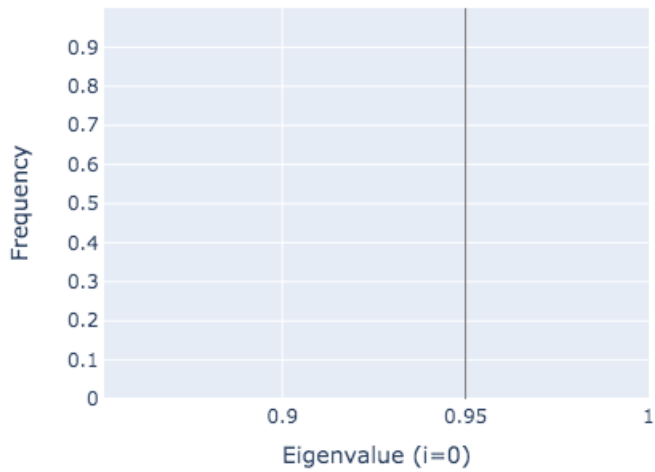
N=100, Nrep=10



Min: -1.0061396160665481e-16  
 Max: 0.08696196076152173  
 |Max-Min|: 0.08696196076152182

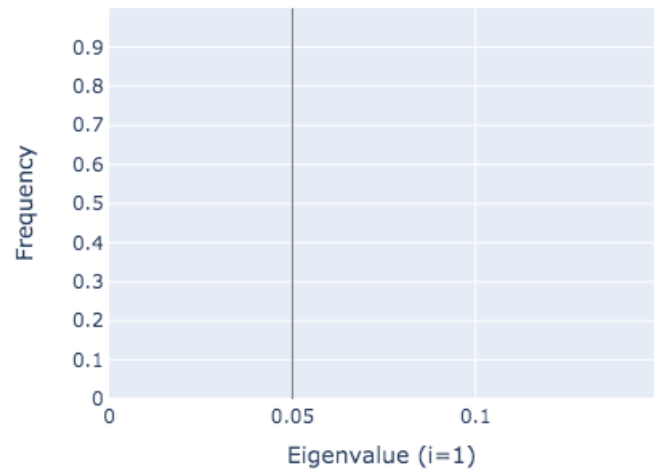
## Case 6: Least Squares (True)

N=10, Nrep=10



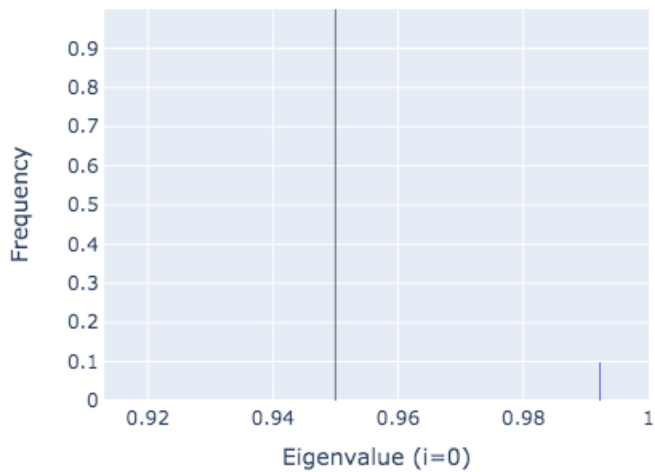
Min: 0.851364184463153  
 Max: 1.0000000761055894  
 |Max-Min|: 0.1486358916424364

N=10, Nrep=10



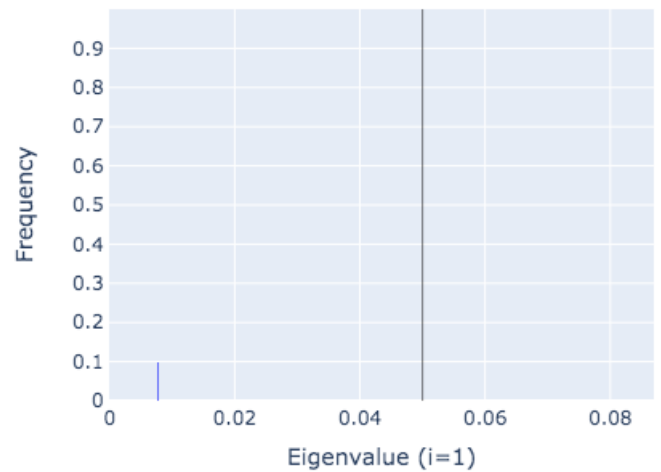
Min: -7.610558962201086e-08  
 Max: 0.14863581553684674  
 |Max-Min|: 0.14863589164243635

N=100, Nrep=10



Min: 0.9130584510098028  
 Max: 1.0000000598780594  
 |Max-Min|: 0.08694160886825664

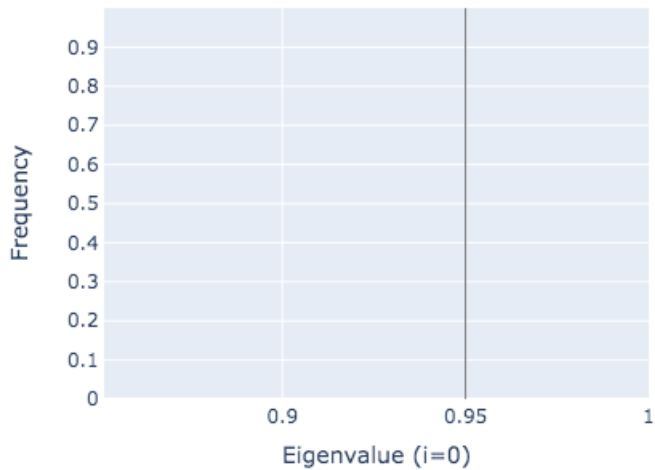
N=100, Nrep=10



Min: -5.987805948691623e-08  
 Max: 0.0869415489901971  
 |Max-Min|: 0.08694160886825658

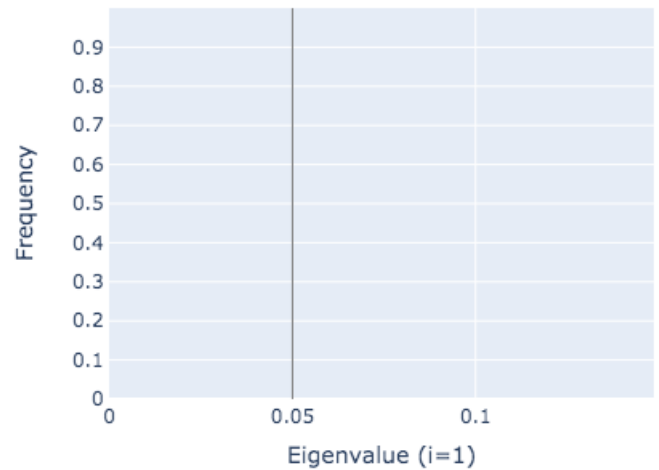
## Case 7: Least Squares (False)

N=10, Nrep=10



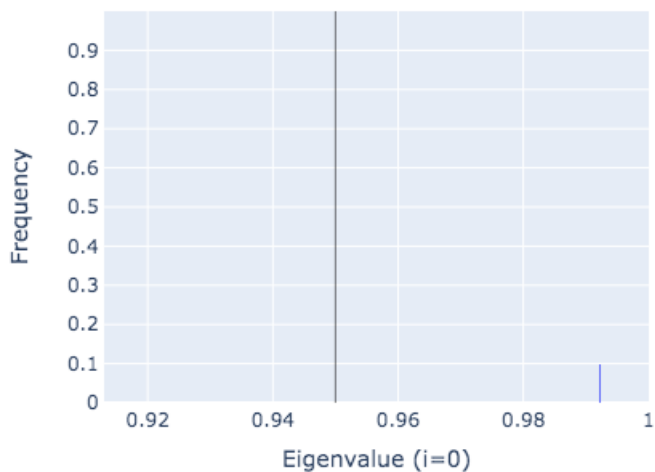
Min: 0.8513641844631532  
Max: 1.0000001781169368  
|Max-Min|: 0.14863599365378355

N=10, Nrep=10



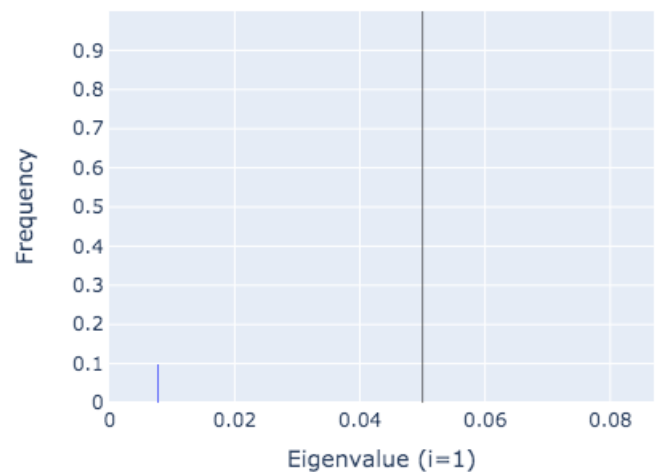
Min: -7.632783294297951e-17  
Max: 0.14863581553684663  
|Max-Min|: 0.1486358155368467

N=100, Nrep=10



Min: 0.9130584510098023  
Max: 1.0000001555680968  
|Max-Min|: 0.08694170455829453

N=100, Nrep=10



Min: 3.06287113727155e-18  
Max: 0.08694154899019754  
|Max-Min|: 0.08694154899019754

## Sum of unphysical eigenvalues

### Case 0: Linear (True)

N=10

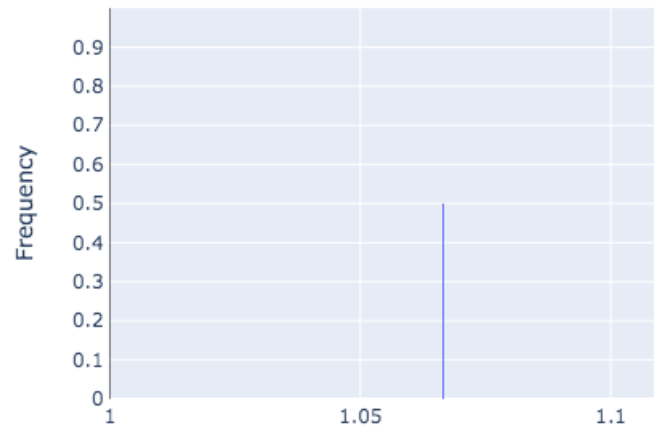
Number of unphysical estimates=6

N=10, Nrep=10



Min: -0.10858061945018471  
 Max: -0.009175077217315732  
 |Max-Min|: 0.09940554223286899

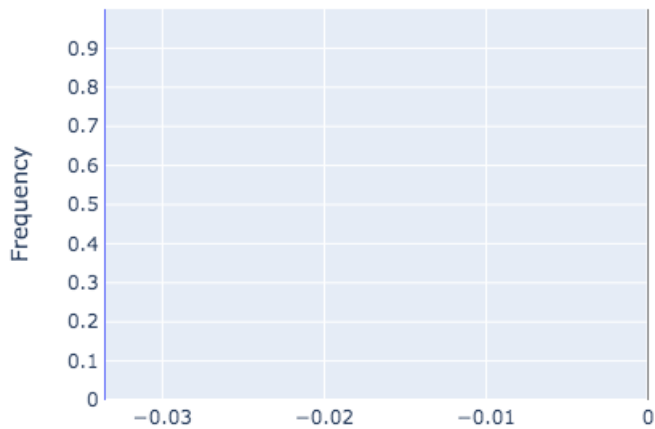
N=10, Nrep=10



Min: 1.0091750772173156  
 Max: 1.1085806194501846  
 |Max-Min|: 0.09940554223286902

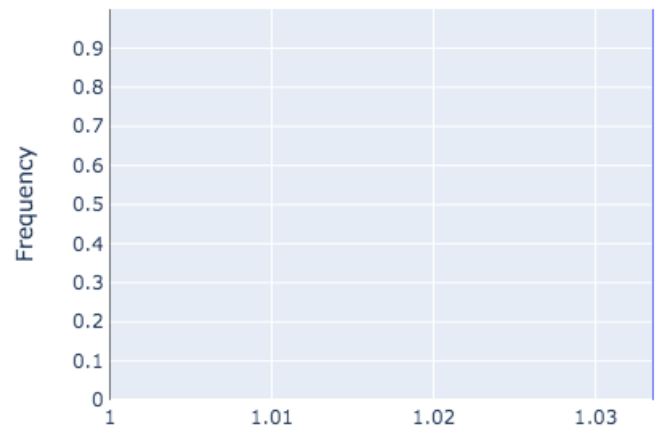
N=100  
 Number of unphysical estimates=1

N=100, Nrep=10



Min: -0.03356476460190986  
 Max: -0.03356476460190986  
 |Max-Min|: 0.0

N=100, Nrep=10



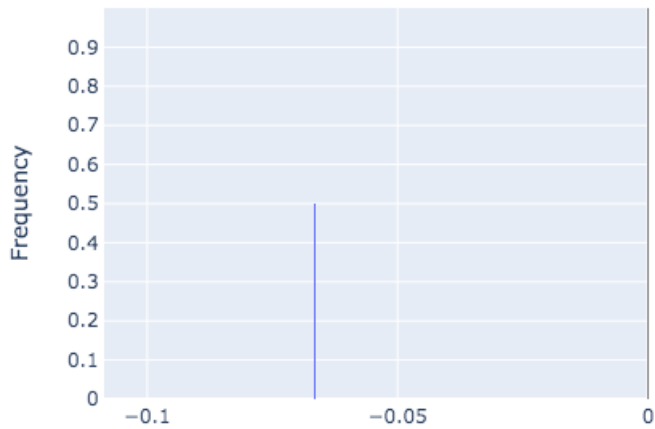
Min: 1.0335647646019095  
 Max: 1.0335647646019095  
 |Max-Min|: 0.0

### Case 1: Linear (False)

N=10  
 Number of unphysical estimates=6

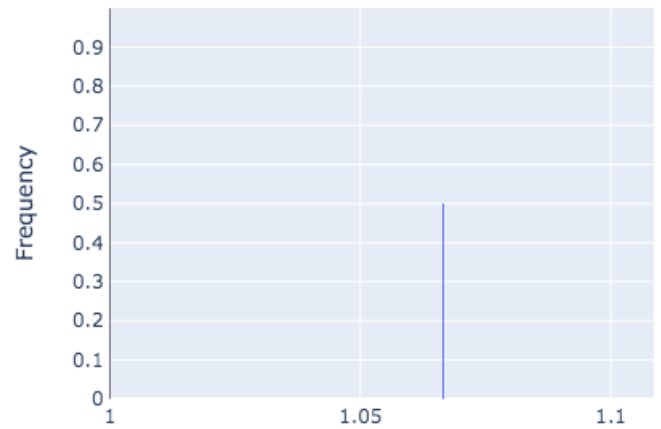


N=10, Nrep=10



Min: -0.10858061945018459  
 Max: -0.009175077217315586  
 |Max-Min|: 0.099405542232869

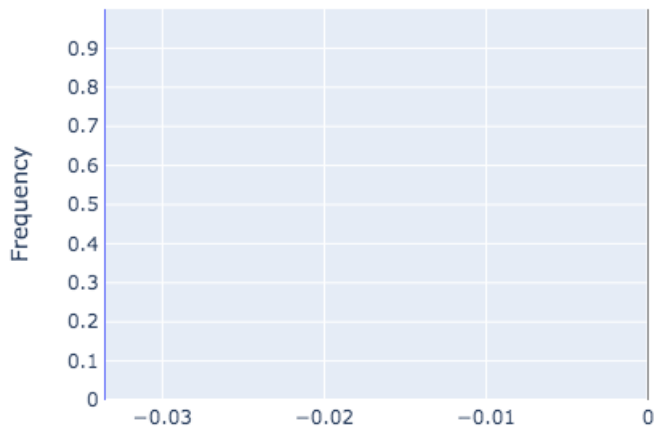
N=10, Nrep=10



Min: 1.0091750772173156  
 Max: 1.1085806194501846  
 |Max-Min|: 0.09940554223286902

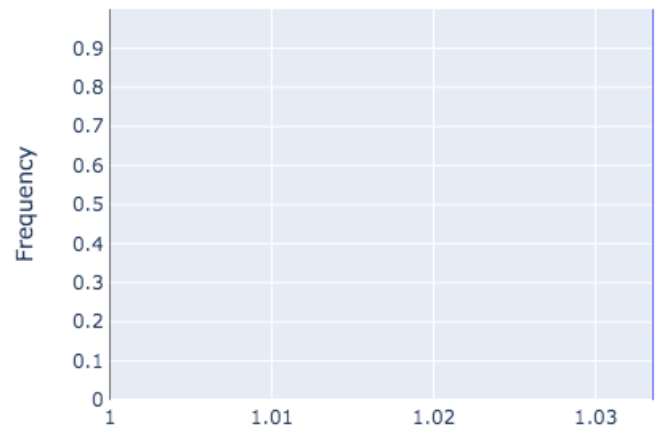
N=100  
 Number of unphysical estimates=1

N=100, Nrep=10



Min: -0.03356476460190975  
 Max: -0.03356476460190975  
 |Max-Min|: 0.0

N=100, Nrep=10

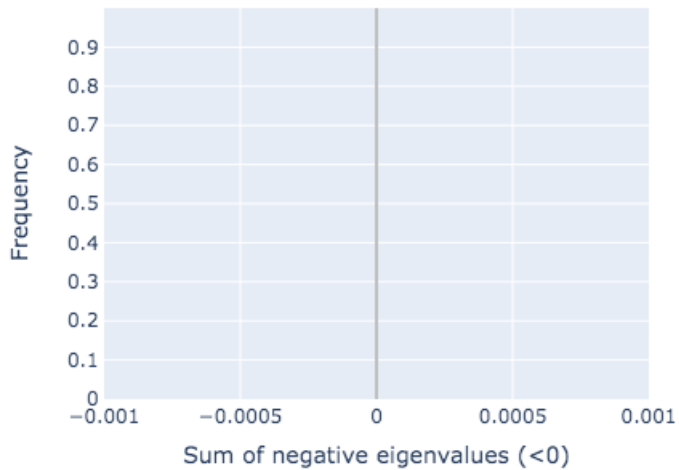


Min: 1.0335647646019095  
 Max: 1.0335647646019095  
 |Max-Min|: 0.0

## Case 2: ProjectedLinear (True)

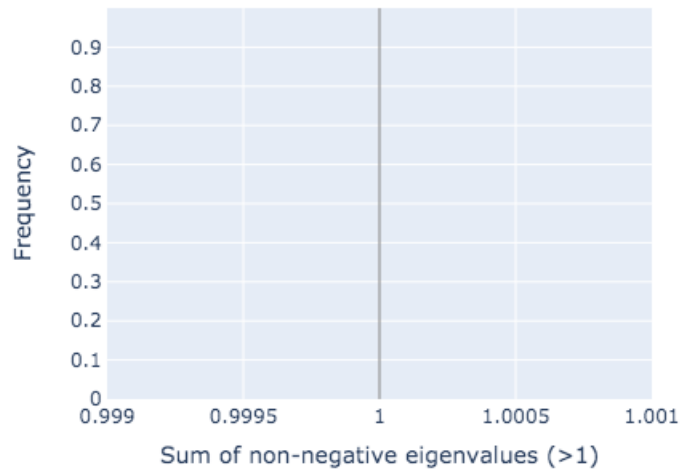
N=10  
 Number of unphysical estimates=0

N=10, Nrep=10



Min: -  
Max: -  
|Max-Min|: -

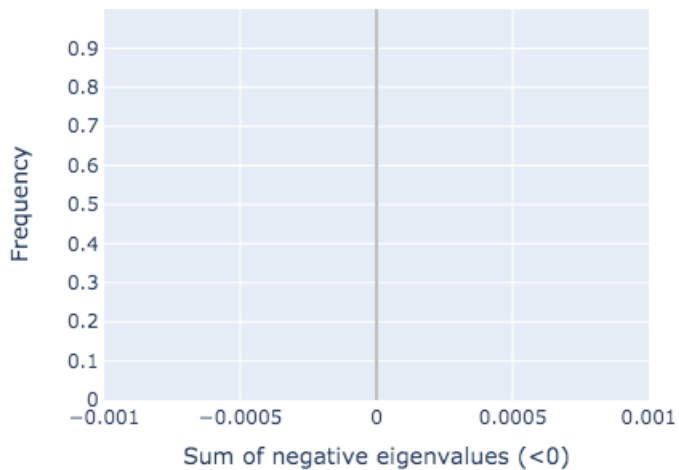
N=10, Nrep=10



Min: -  
Max: -  
|Max-Min|: -

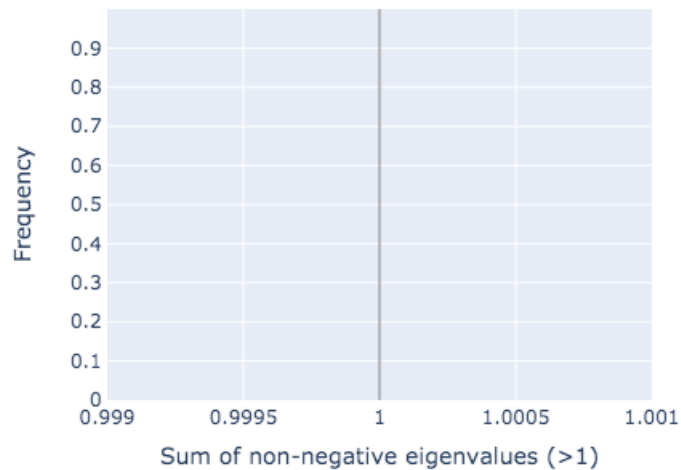
N=100  
Number of unphysical estimates=0

N=100, Nrep=10



Min: -  
Max: -  
|Max-Min|: -

N=100, Nrep=10

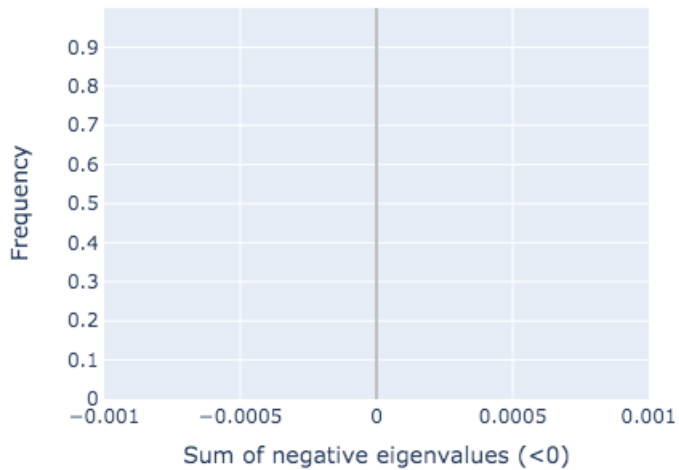


Min: -  
Max: -  
|Max-Min|: -

### Case 3: ProjectedLinear (False)

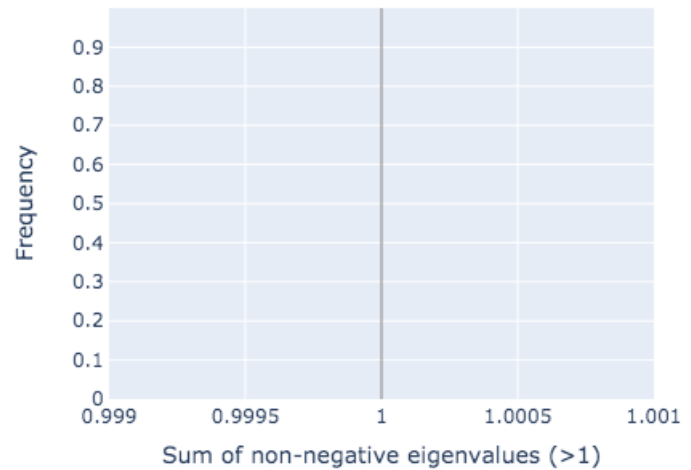
N=10  
Number of unphysical estimates=0

N=10, Nrep=10



Min: -  
Max: -  
|Max-Min|: -

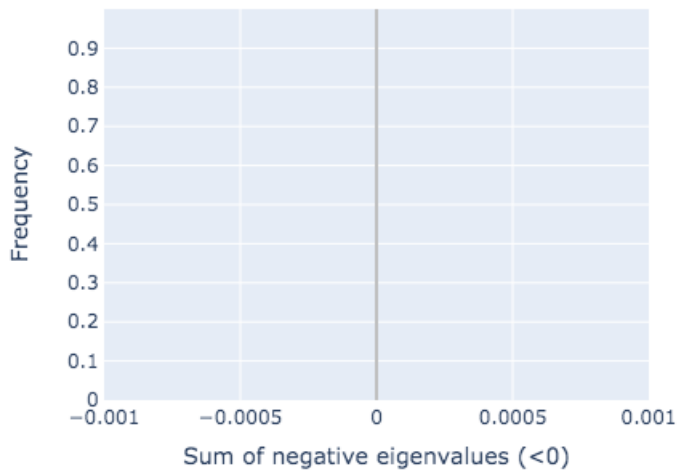
N=10, Nrep=10



Min: -  
Max: -  
|Max-Min|: -

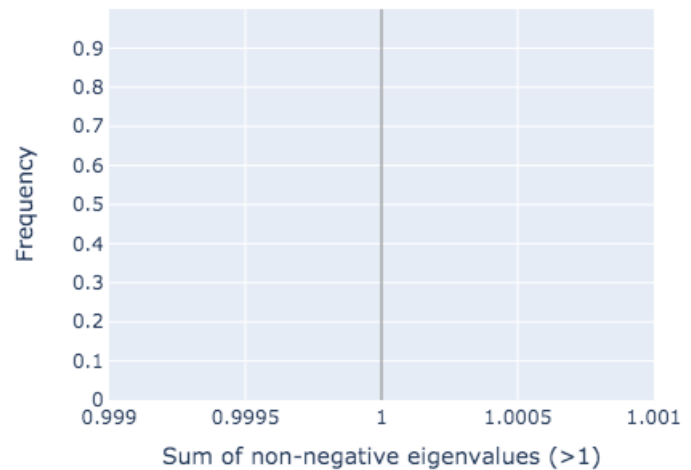
N=100  
Number of unphysical estimates=0

N=100, Nrep=10



Min: -  
Max: -  
|Max-Min|: -

N=100, Nrep=10

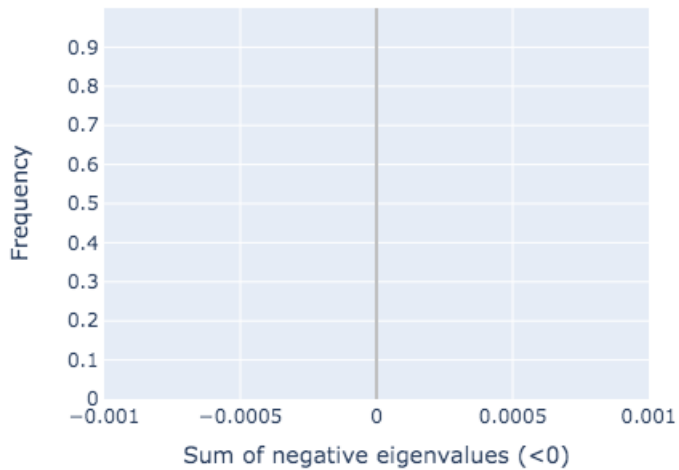


Min: -  
Max: -  
|Max-Min|: -

#### Case 4: Maximum-Likelihood (True)

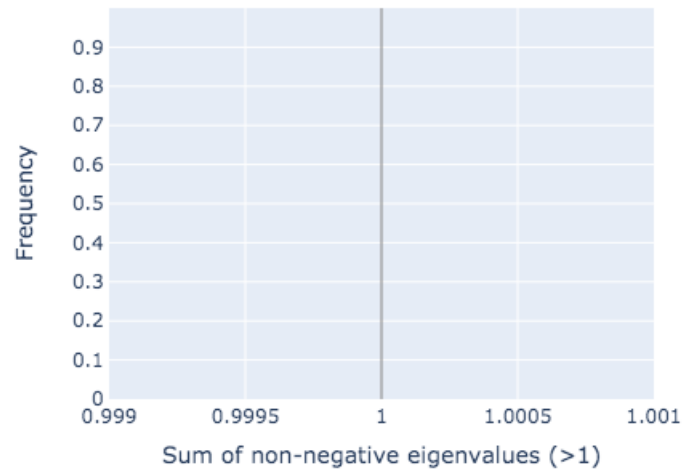
N=10  
Number of unphysical estimates=0

N=10, Nrep=10



Min: -  
Max: -  
|Max-Min|: -

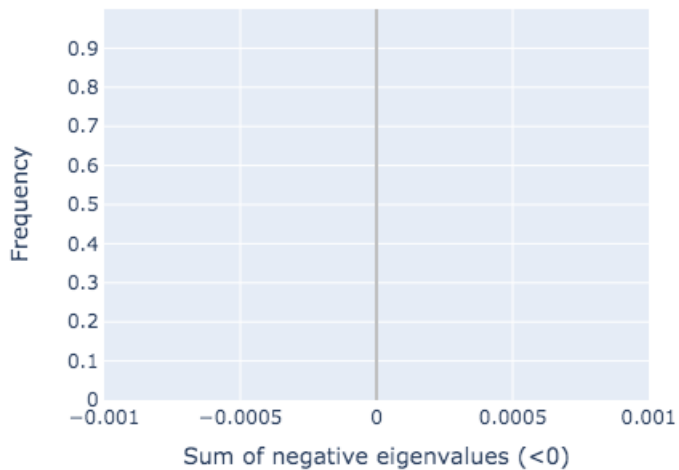
N=10, Nrep=10



Min: -  
Max: -  
|Max-Min|: -

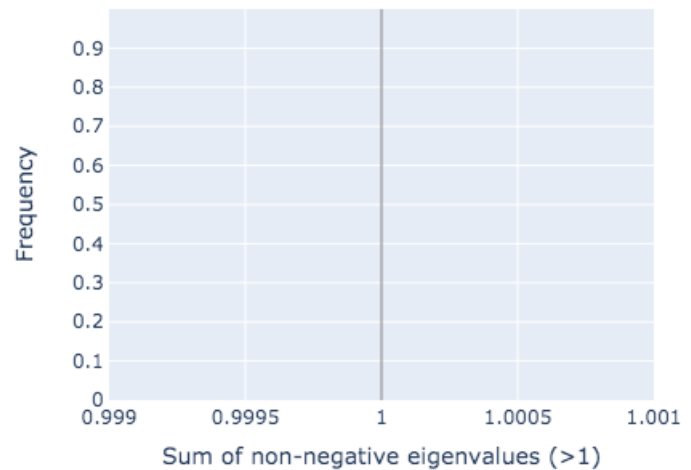
N=100  
Number of unphysical estimates=0

N=100, Nrep=10



Min: -  
Max: -  
|Max-Min|: -

N=100, Nrep=10

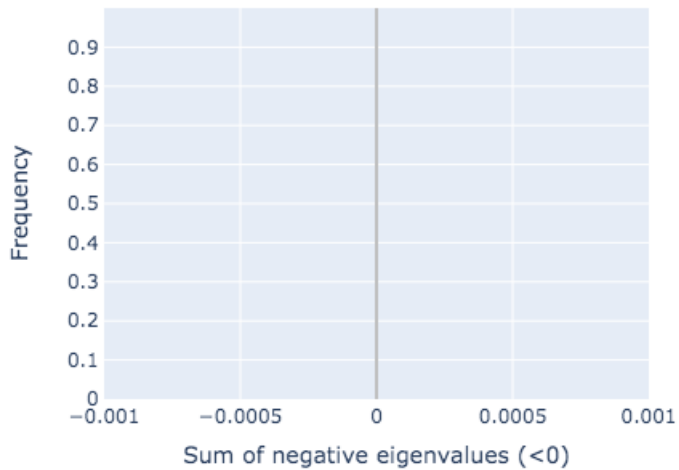


Min: -  
Max: -  
|Max-Min|: -

### Case 5: Maximum-Likelihood (False)

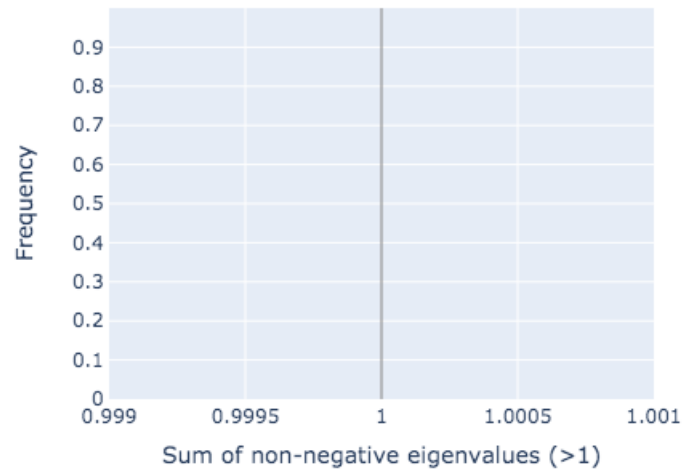
N=10  
Number of unphysical estimates=0

N=10, Nrep=10



Min: -  
Max: -  
|Max-Min|: -

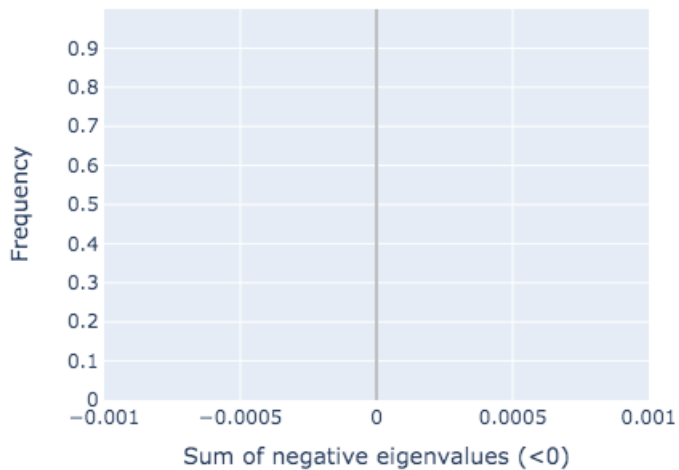
N=10, Nrep=10



Min: -  
Max: -  
|Max-Min|: -

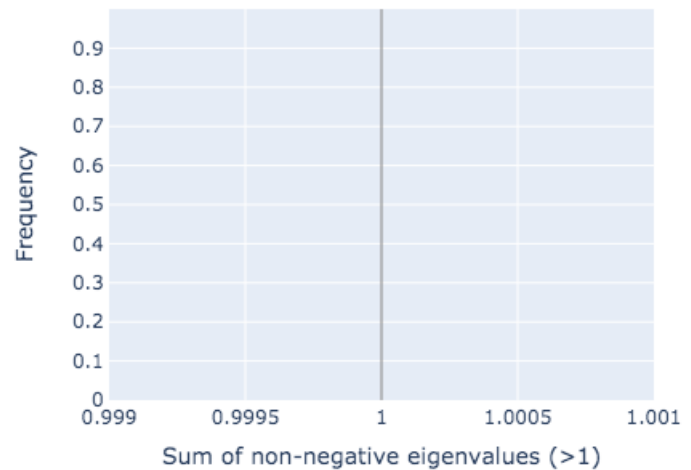
N=100  
Number of unphysical estimates=0

N=100, Nrep=10



Min: -  
Max: -  
|Max-Min|: -

N=100, Nrep=10

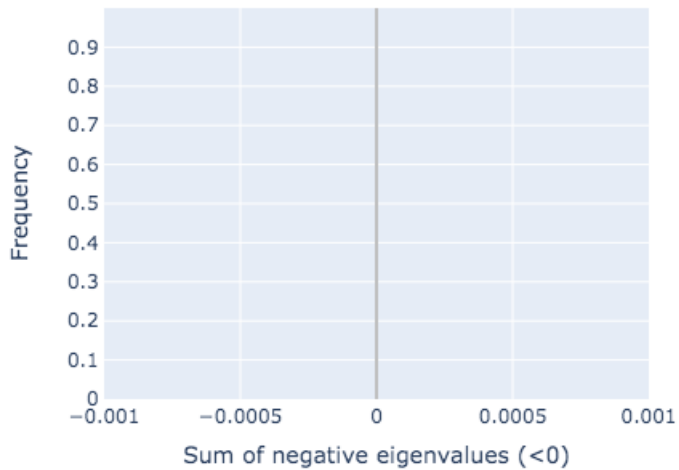


Min: -  
Max: -  
|Max-Min|: -

### Case 6: Least Squares (True)

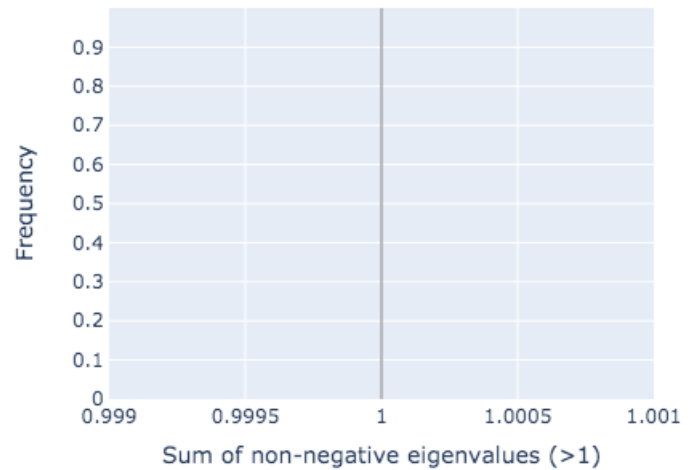
N=10  
Number of unphysical estimates=0

N=10, Nrep=10



Min: -  
Max: -  
|Max-Min|: -

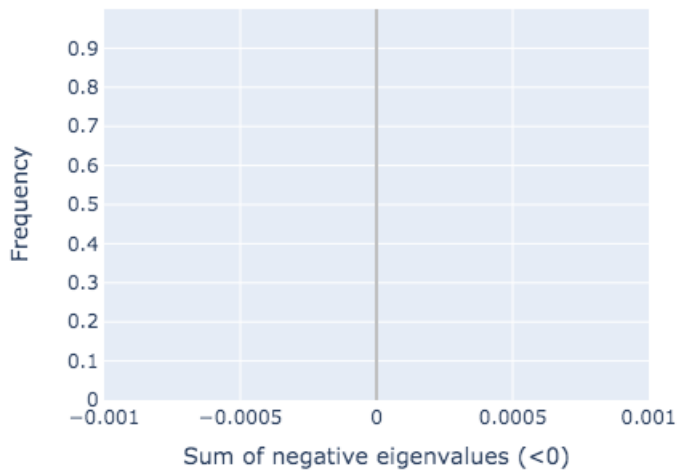
N=10, Nrep=10



Min: -  
Max: -  
|Max-Min|: -

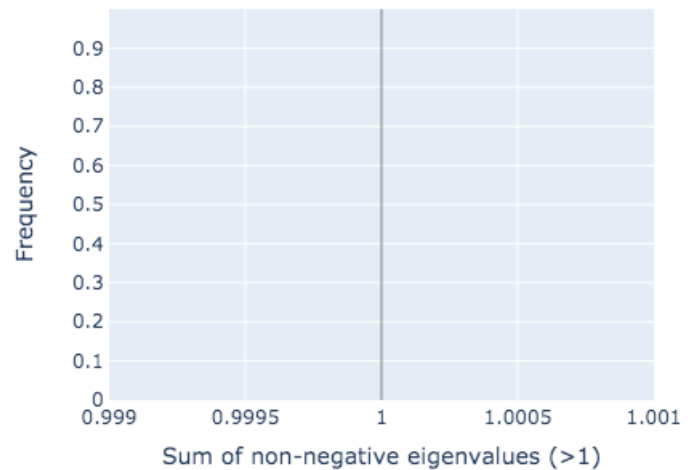
N=100  
Number of unphysical estimates=0

N=100, Nrep=10



Min: -  
Max: -  
|Max-Min|: -

N=100, Nrep=10

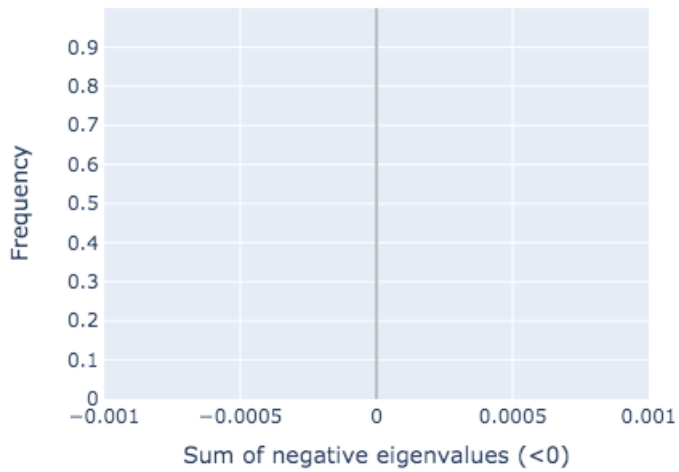


Min: -  
Max: -  
|Max-Min|: -

### Case 7: Least Squares (False)

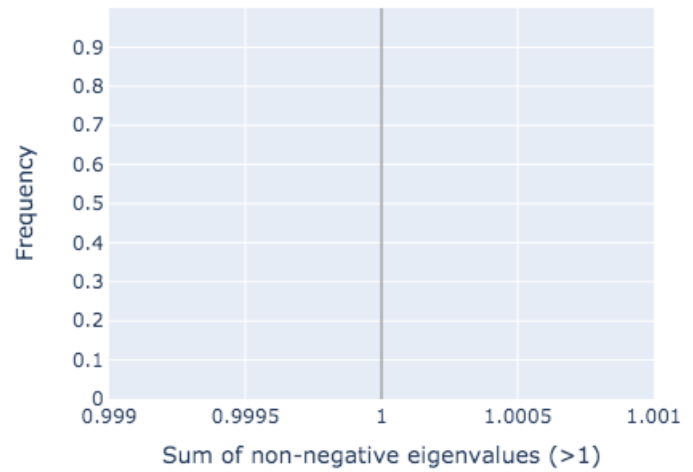
N=10  
Number of unphysical estimates=0

N=10, Nrep=10



Min: -  
Max: -  
|Max-Min|: -

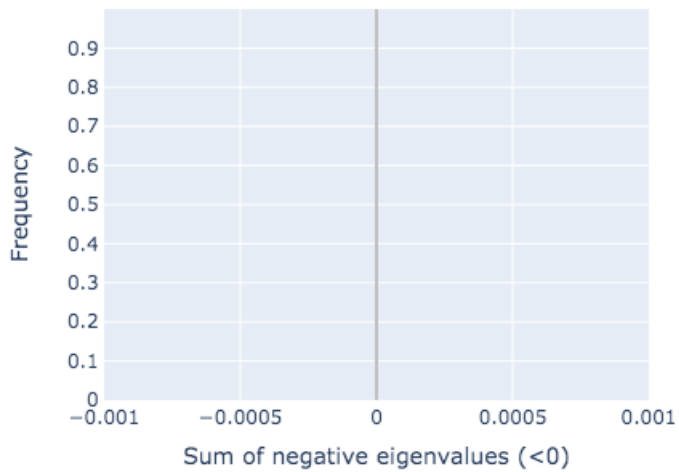
N=10, Nrep=10



Min: -  
Max: -  
|Max-Min|: -

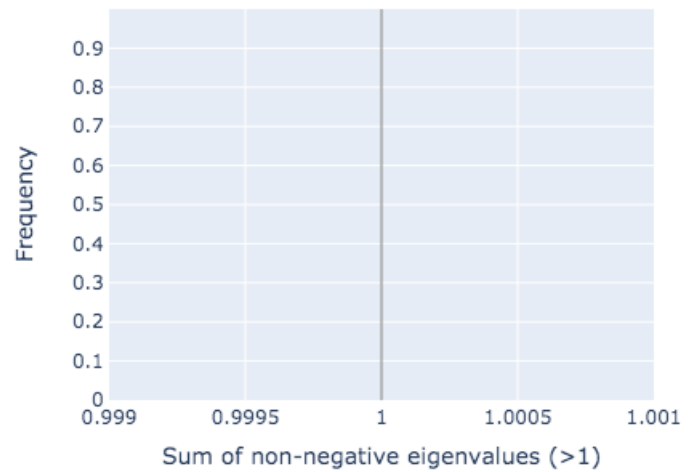
N=100  
Number of unphysical estimates=0

N=100, Nrep=10



Min: -  
Max: -  
|Max-Min|: -

N=100, Nrep=10



Min: -  
Max: -  
|Max-Min|: -