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## Computation time in total

Total	0.6709725181261699min.
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## Tolerance of physicality constraint violation

		True	False
Tolerance at physicality violation test	equality constraint	1.00e-13	1.00e-05
	inequality constraint	1.00e-05	1.00e-05

## Experimental condition

Type of tomography	StandardQst
Nrep	10
N	[10, 100]
RNG seed	777

### True object

Type	State
Dim	2
Vec	[0.70710678 0. 0. 0.6363961 ]

### Tester objects

0	Type	Povm
	Dim	2
	Number of outcomes	2
	Vecs	[[ 0.70710678 0.6363961 0. 0. ] [ 0.70710678 -0.6363961 0. 0. ]]
1	Type	Povm
	Dim	2
	Number of outcomes	2
	Vecs	[[ 0.70710678 0. 0.6363961 0. ] [ 0.70710678 0. -0.6363961 0. ]]
2	Type	Povm
	Dim	2
	Number of outcomes	2
	Vecs	[[ 0.70710678 0. 0. 0.6363961 ] [ 0.70710678 0. 0. -0.6363961 ]]

### Cases

	Name	Param	Tomography	Estimator
0	Linear (True)	True	StandardQst	Linear
1	Linear (False)	False	StandardQst	Linear
2	ProjectedLinear (True)	True	StandardQst	ProjectedLinear
3	ProjectedLinear (False)	False	StandardQst	ProjectedLinear
4	Maximum-Likelihood (True)	True	StandardQst	LossMinimization
5	Maximum-Likelihood (False)	False	StandardQst	LossMinimization

6	Least Squares (True)	True	StandardQst	LossMinimization
7	Least Squares (False)	False	StandardQst	LossMinimization

## Computation time of estimators

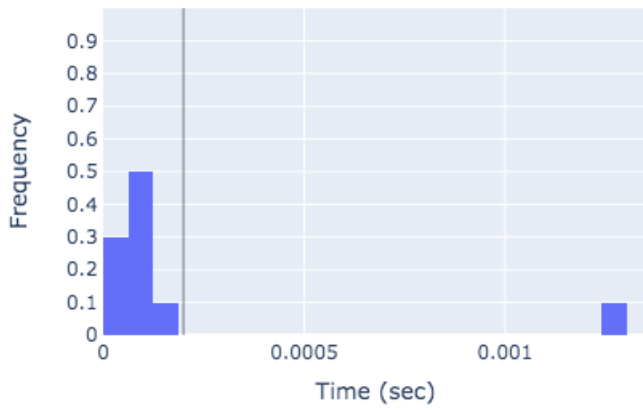
Table

	Name	N	Nrep	Mean (sec)	Std (sec)
0	Linear (True)	10	10	0.000200	0.000365
1	"	100	10	0.000074	0.000023
2	Linear (False)	10	10	0.000076	0.000048
3	"	100	10	0.000105	0.000145
4	ProjectedLinear (True)	10	10	0.006886	0.004584
5	"	100	10	0.002246	0.002883
6	ProjectedLinear (False)	10	10	0.006153	0.004075
7	"	100	10	0.001939	0.002474
8	Maximum-Likelihood (True)	10	10	0.550861	0.131363
9	"	100	10	0.487633	0.133638
10	Maximum-Likelihood (False)	10	10	1.286001	1.753712
11	"	100	10	1.420074	1.519164
12	Least Squares (True)	10	10	0.057098	0.019072
13	"	100	10	0.047338	0.006008
14	Least Squares (False)	10	10	0.076766	0.025825
15	"	100	10	0.082384	0.027363

## Histogram

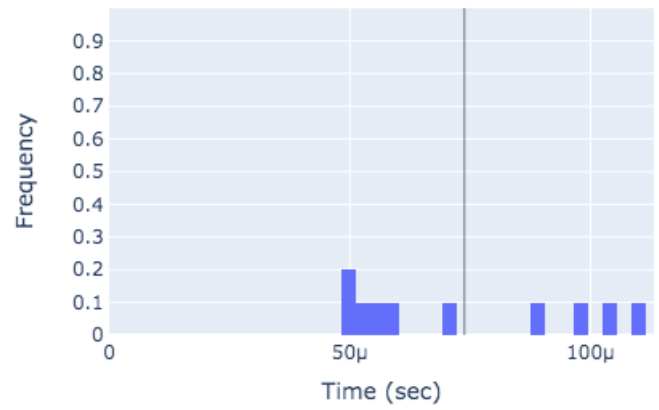
Case 0: Linear (True)

N=10, Nrep=10  
Mean = 0.0001997709274291992(sec)



Min: 5.316734313964844e-05  
Max: 0.0012929439544677734  
|Max-Min|: 0.001239776611328125

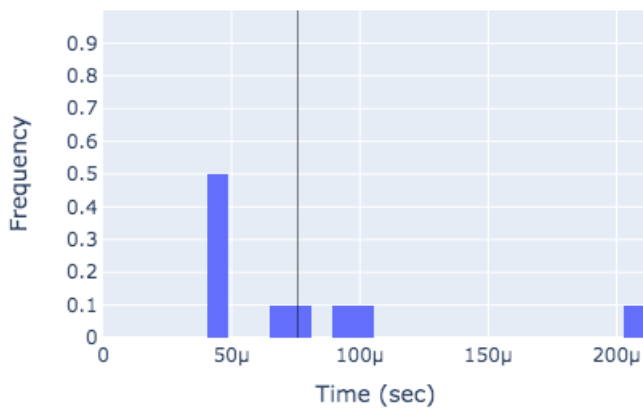
N=100, Nrep=10  
Mean = 7.381439208984376e-05(sec)



Min: 4.982948303222656e-05  
Max: 0.00011014938354492188  
|Max-Min|: 6.031990051269531e-05

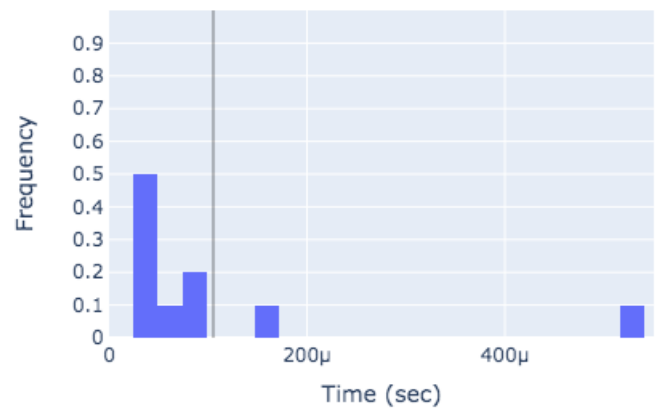
### Case 1: Linear (False)

N=10, Nrep=10  
Mean = 7.584095001220704e-05(sec)



Min: 4.1961669921875e-05  
Max: 0.0002040863037109375  
|Max-Min|: 0.0001621246337890625

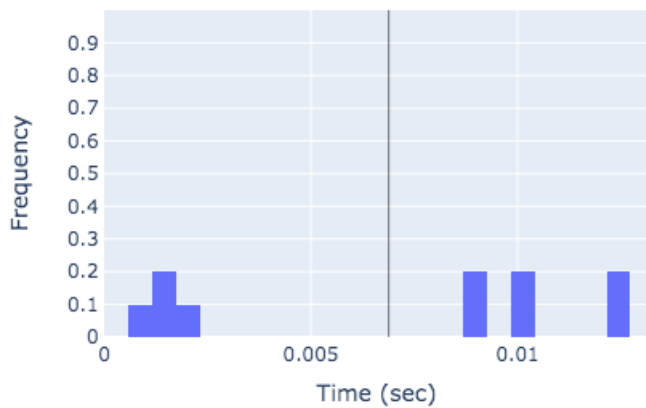
N=100, Nrep=10  
Mean = 0.00010528564453125(sec)



Min: 3.3855438232421875e-05  
Max: 0.0005259513854980469  
|Max-Min|: 0.000492095947265625

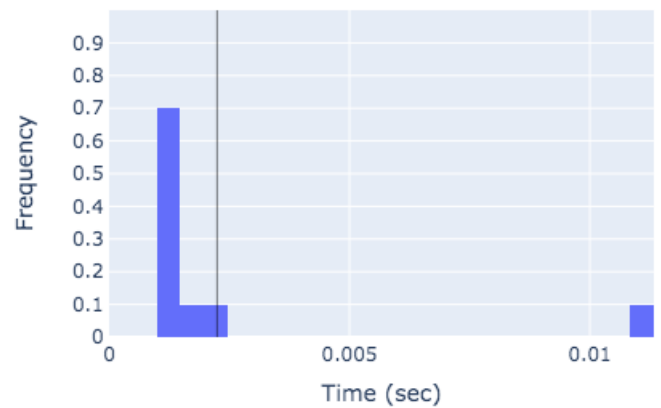
### Case 2: ProjectedLinear (True)

N=10, Nrep=10  
Mean = 0.006885838508605957(sec)



Min: 0.0010259151458740234  
Max: 0.012598752975463867  
|Max-Min|: 0.011572837829589844

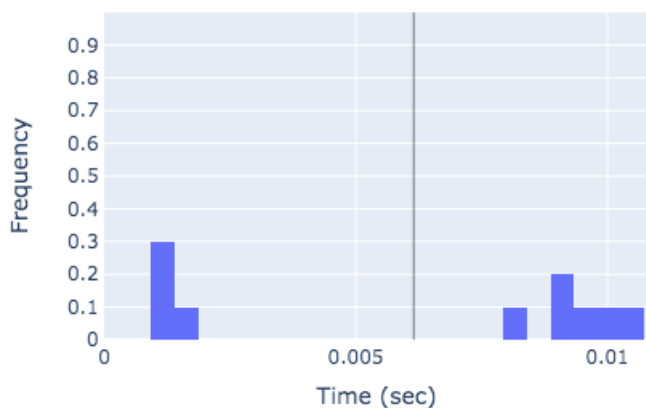
N=100, Nrep=10  
Mean = 0.0022456884384155274(sec)



Min: 0.0009970664978027344  
Max: 0.010838985443115234  
|Max-Min|: 0.0098419189453125

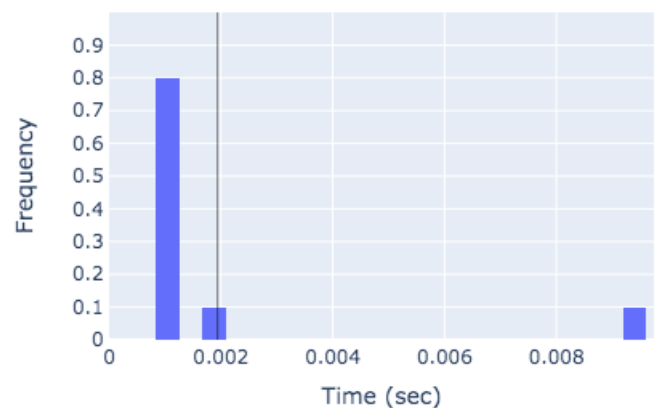
### Case 3: ProjectedLinear (False)

N=10, Nrep=10  
Mean = 0.006153392791748047(sec)



Min: 0.0010230541229248047  
Max: 0.010358572006225586  
|Max-Min|: 0.009335517883300781

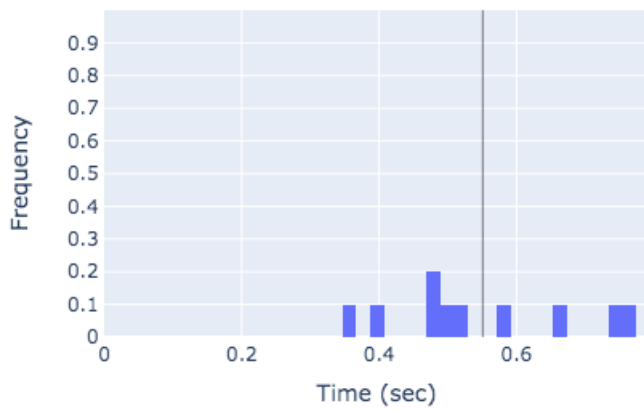
N=100, Nrep=10  
Mean = 0.001939249038696289(sec)



Min: 0.0009720325469970703  
Max: 0.009325981140136719  
|Max-Min|: 0.008353948593139648

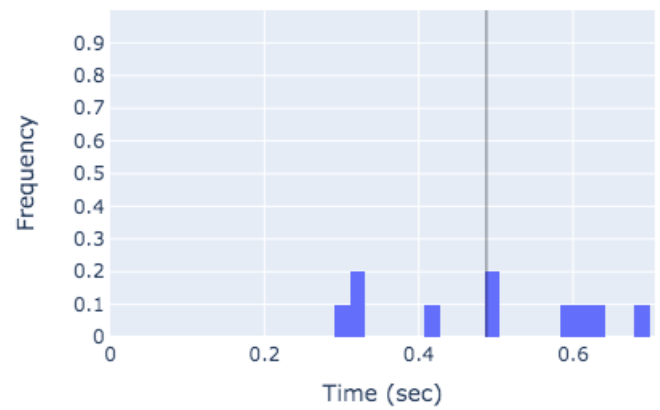
### Case 4: Maximum-Likelihood (True)

N=10, Nrep=10  
Mean = 0.5508614540100097(sec)



Min: 0.3642098903656006  
Max: 0.772050142288208  
|Max-Min|: 0.4078402519226074

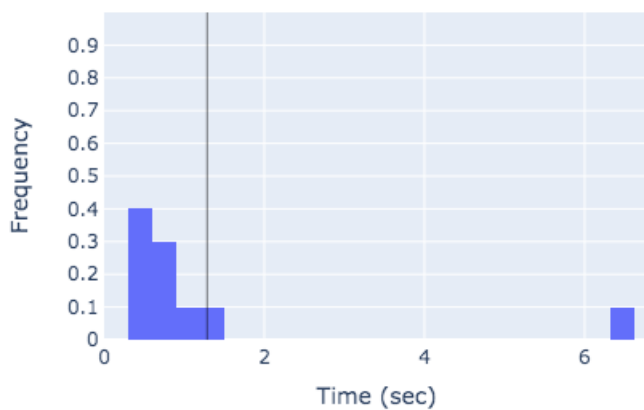
N=100, Nrep=10  
Mean = 0.4876333475112915(sec)



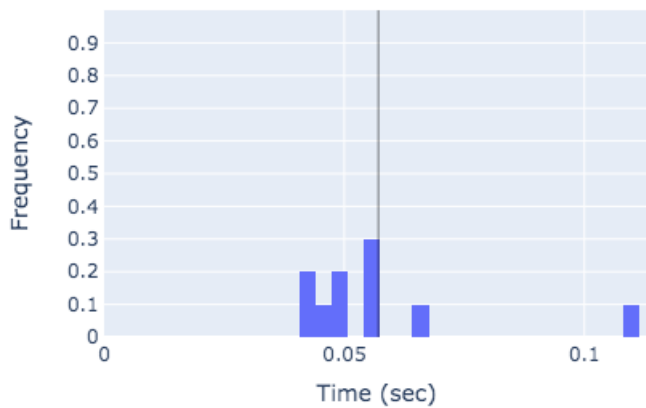
Min: 0.29795384407043457  
Max: 0.6864383220672607  
|Max-Min|: 0.38848447799682617

### Case 5: Maximum-Likelihood (False)

N=10, Nrep=10  
Mean = 1.286001205444336(sec)

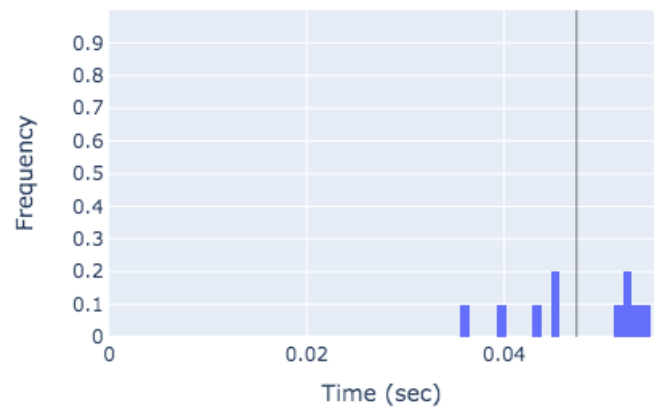


N=10, Nrep=10  
Mean = 0.05709784030914307(sec)



Min: 0.04247689247131348  
Max: 0.11004400253295898  
|Max-Min|: 0.06756711006164551

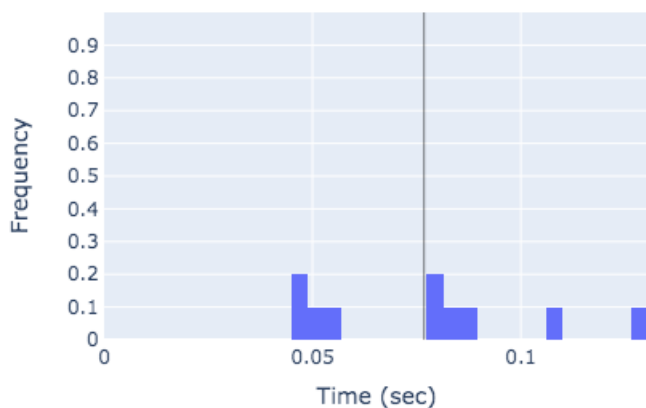
N=100, Nrep=10  
Mean = 0.047337913513183595(sec)



Min: 0.0359959602355957  
Max: 0.05425405502319336  
|Max-Min|: 0.018258094787597656

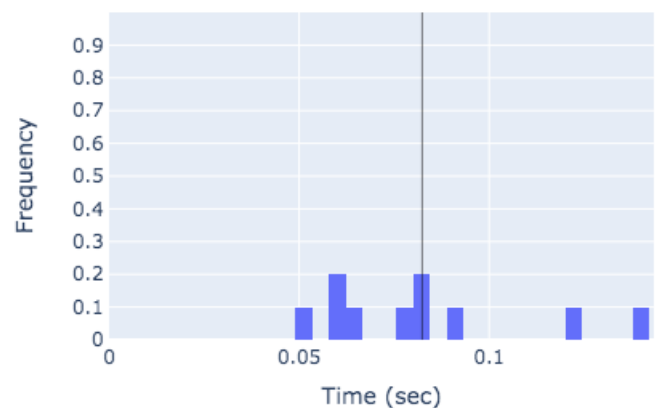
### Case 7: Least Squares (False)

N=10, Nrep=10  
Mean = 0.0767662525177002(sec)



Min: 0.045151472091674805  
Max: 0.1266322135925293  
|Max-Min|: 0.08148074150085449

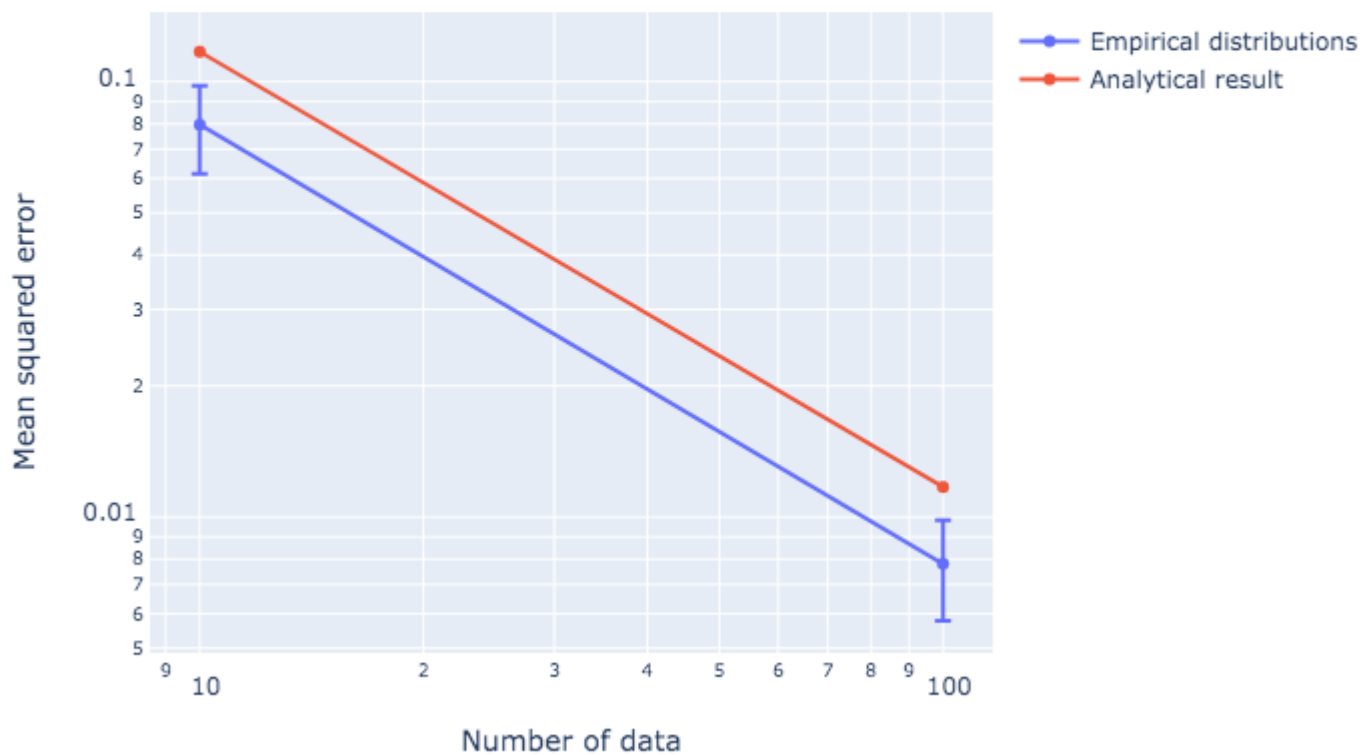
N=100, Nrep=10  
Mean = 0.08238401412963867(sec)



Min: 0.05000448226928711  
Max: 0.138901948928833  
|Max-Min|: 0.0888974666595459

## MSE of empirical distributions

## Mean squared error



## Consistency test

	Name	Type of tomography	Param	Estimator	Loss	Algo	Squared Error to True	Possibly OK	To be checked
0	Linear (True)	Standard Qst	True	Linear	None	None	0.00e+00	OK	not need debug
1	Linear (False)	Standard Qst	False	Linear	None	None	2.47e-32	OK	not need debug
2	ProjectedLinear (True)	Standard Qst	True	Projected Linear	None	None	1.23e-32	OK	not need debug
3	ProjectedLinear (False)	Standard Qst	False	Projected Linear	None	None	1.11e-31	OK	not need debug
4	Maximum-Likelihood (True)	Standard Qst	True	Loss Minimization	Weighted Relative Entropy	Projected Gradient Descent Backtracking	3.21e-18	OK	not need debug
5	Maximum-Likelihood (False)	Standard Qst	False	Loss Minimization	Weighted Relative Entropy	Projected Gradient Descent Backtracking	5.47e-09	NG	not need debug
6	Least Squares (True)	Standard Qst	True	Loss Minimization	Weighted Probability Based Squared Error	Projected Gradient Descent Backtracking	4.93e-32	OK	not need debug

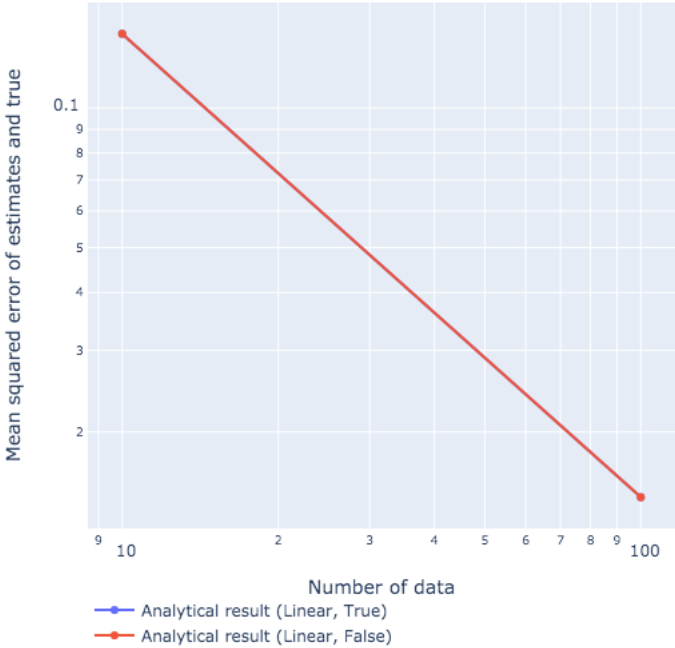


7	Least Squares (False)	Standard Qst	False	Loss Minimization	Weighted Probability Based Squared Error	Projected Gradient Descent Backtracking	1.60e-31	OK	not need debug
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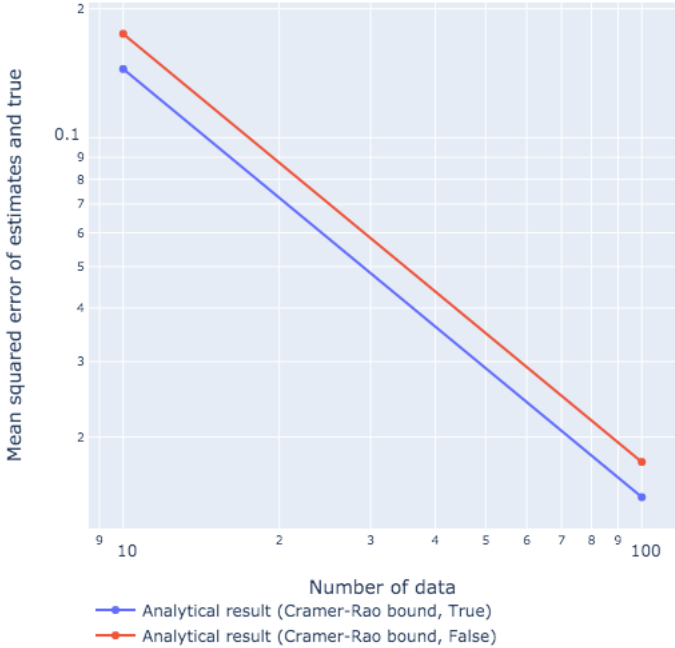
# MSE of estimators

## Comparison of analytical results

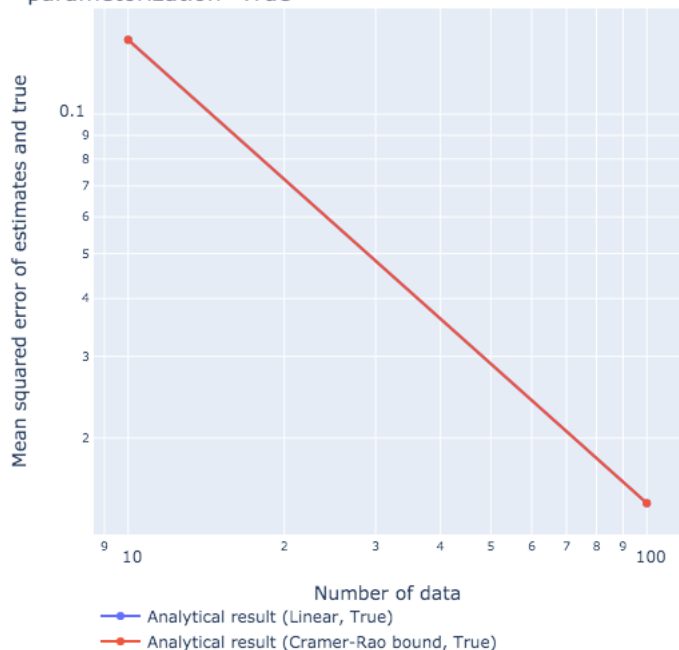
Mean squared error  
Analytical result  
estimator=Linear



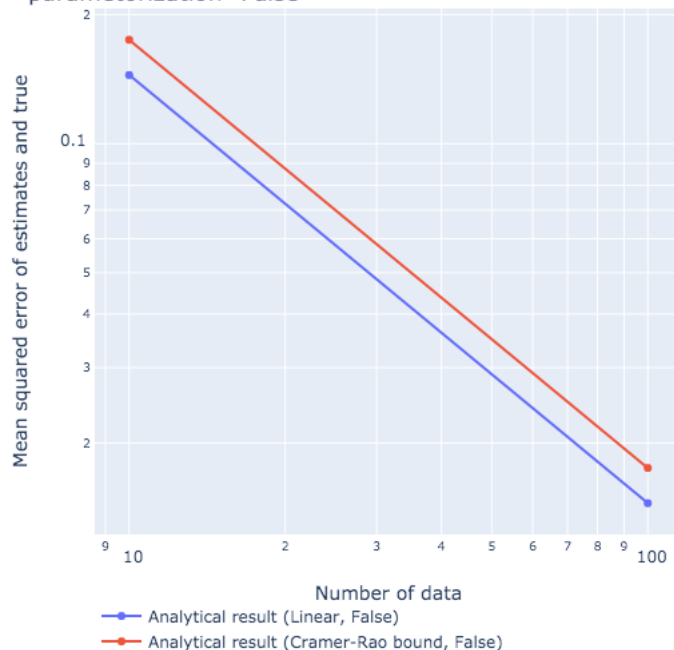
Mean squared error  
Analytical result  
Cramer-Rao bound



Mean squared error  
Analytical result  
parameterization=True

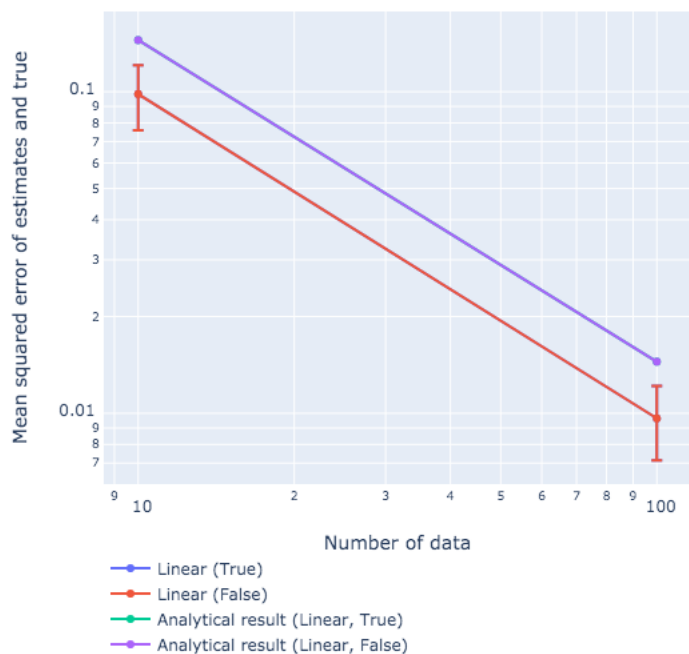


Mean squared error  
Analytical result  
parameterization=False

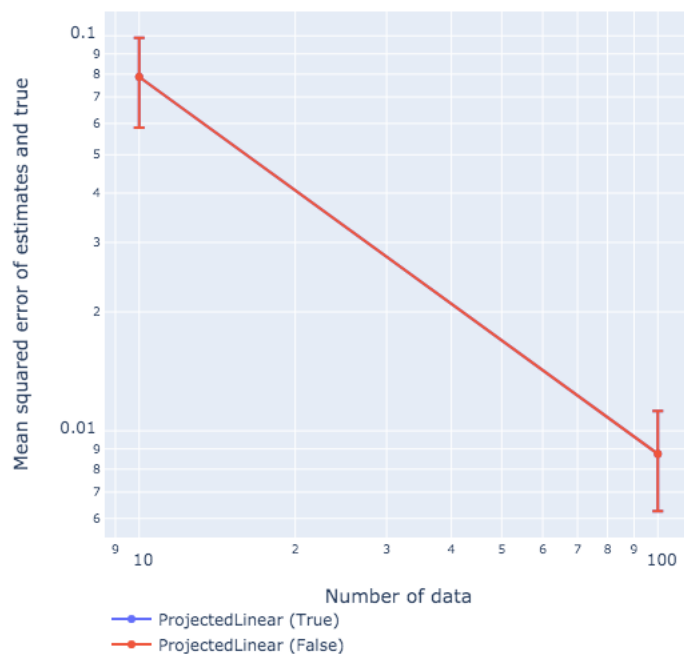


## Comparison of parametrization

Mean squared error  
Estimator=Linear



Mean squared error  
Estimator=ProjectedLinear

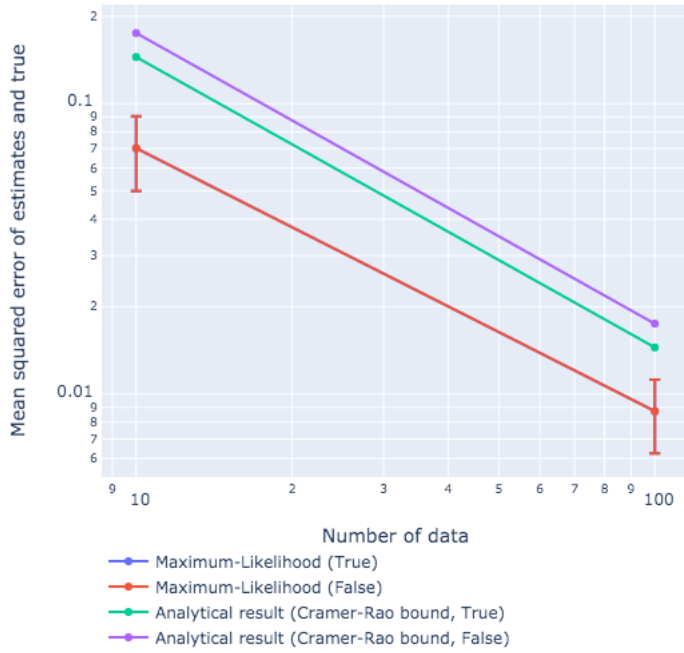


### Mean squared error

Estimator=LossMinimization

Loss=WeightedRelativeEntropy

Algo=ProjectedGradientDescentBacktracking

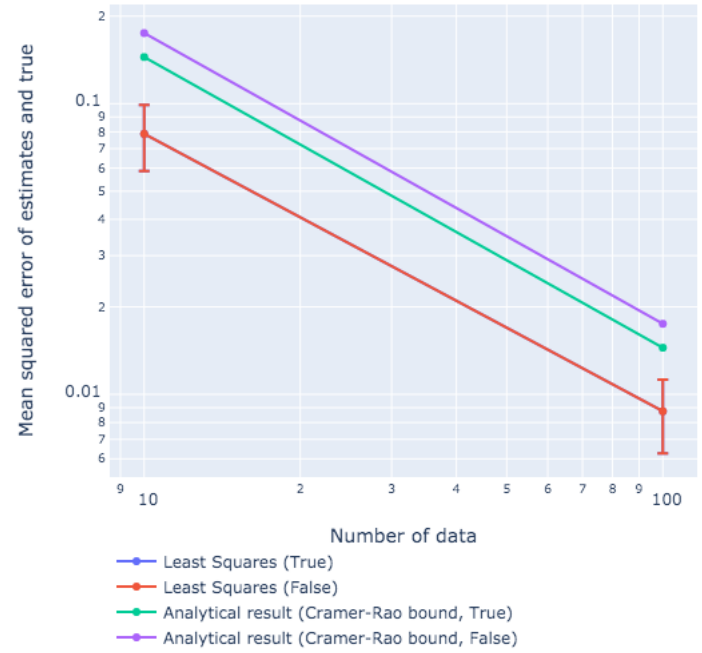


### Mean squared error

Estimator=LossMinimization

Loss=WeightedProbabilityBasedSquaredError

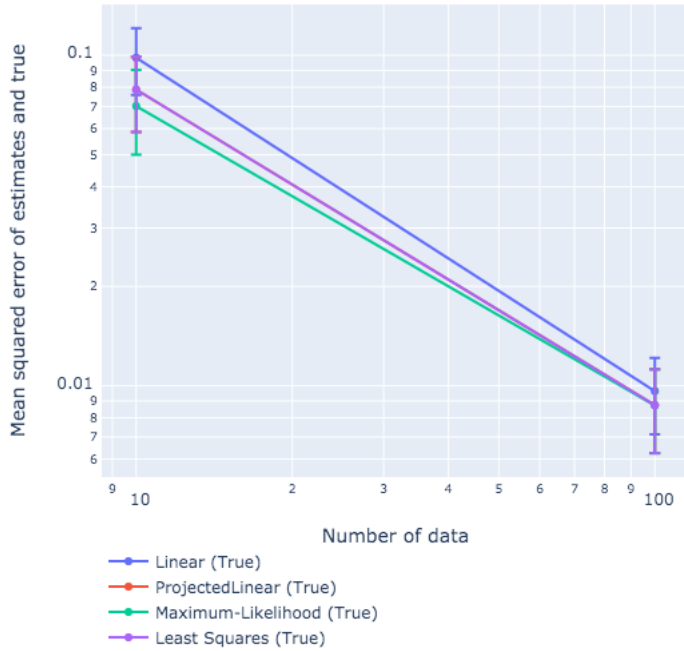
Algo=ProjectedGradientDescentBacktracking



## Comparison of estimators

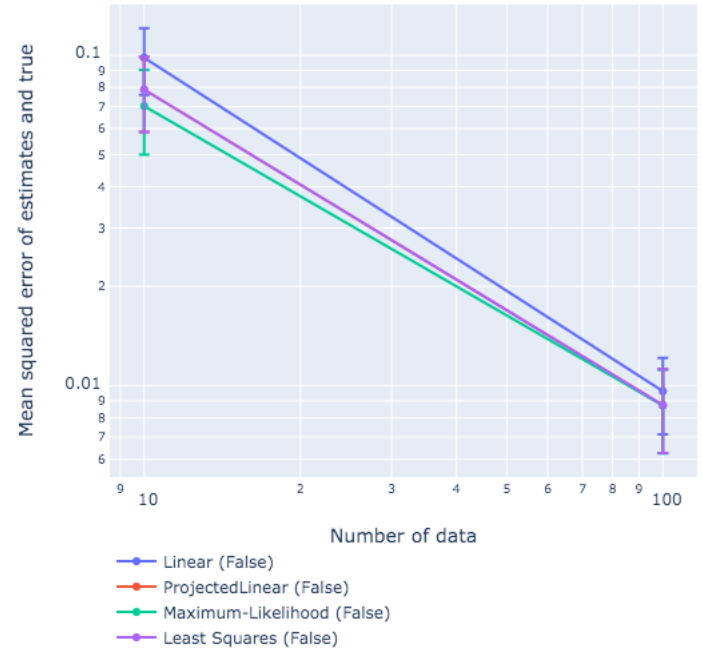
### Mean squared error

parametrization=True



### Mean squared error

parametrization=False

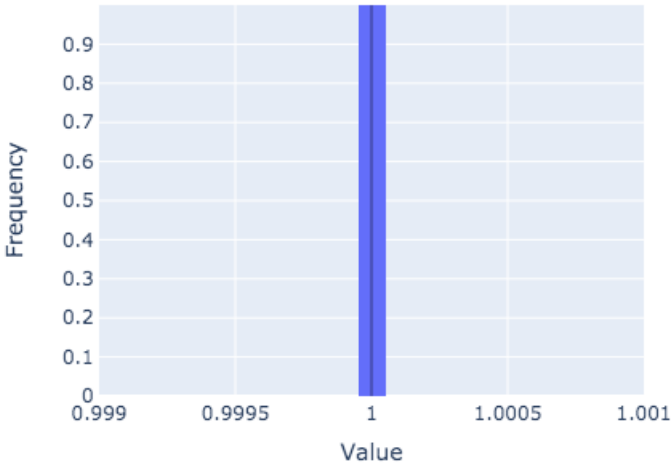


# Physicality violation test

## Test of equality constraint violation

### Case 0: Linear (True)

N=10, Nrep=10



Min: 0.9999999999999997  
Max: 0.9999999999999998  
|Max-Min|: 1.1102230246251565e-16

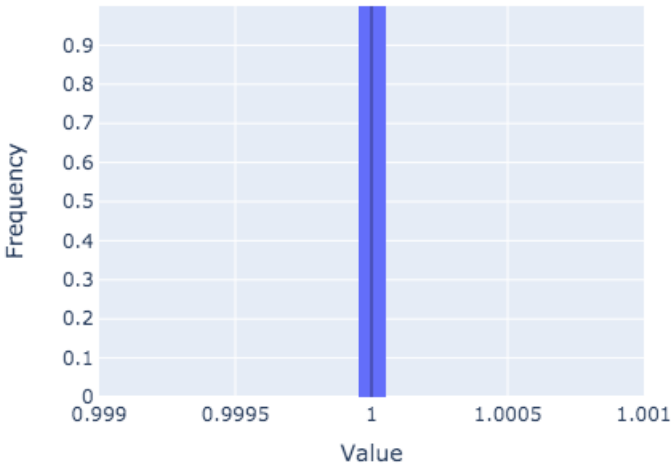
N=100, Nrep=10



Min: 0.9999999999999998  
Max: 0.9999999999999998  
|Max-Min|: 0.0

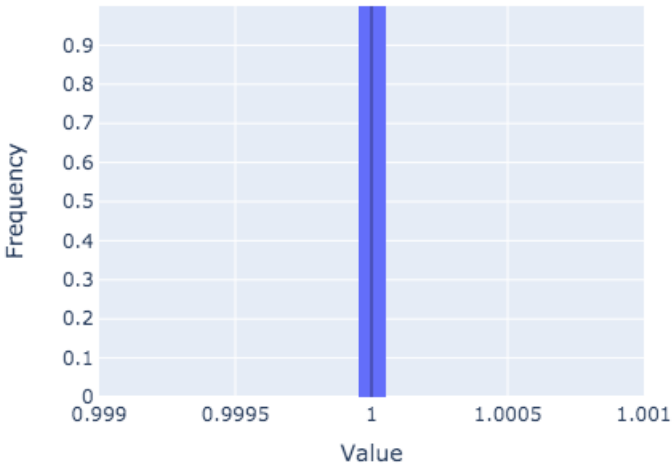
### Case 1: Linear (False)

N=10, Nrep=10



Min: 0.9999999999999998  
Max: 1.0000000000000002  
|Max-Min|: 4.440892098500626e-16

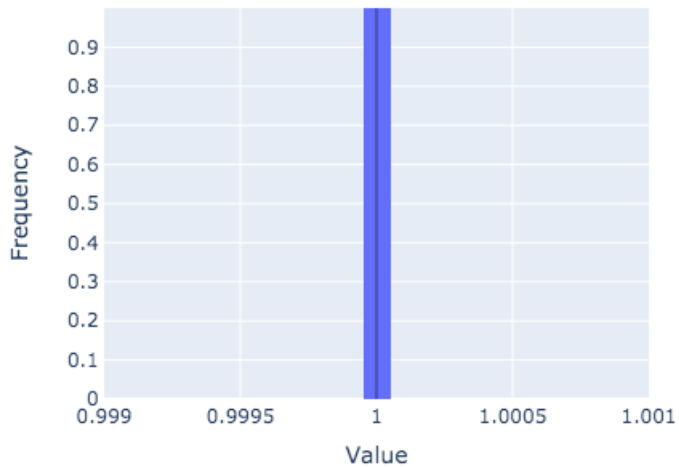
N=100, Nrep=10



Min: 0.9999999999999999  
Max: 1.0000000000000002  
|Max-Min|: 3.3306690738754696e-16

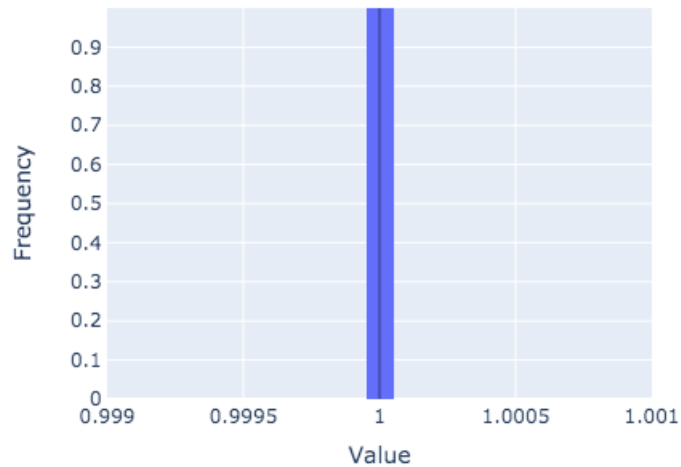
### Case 2: ProjectedLinear (True)

N=10, Nrep=10



Min: 0.9999999999999998  
Max: 0.9999999999999998  
|Max-Min|: 0.0

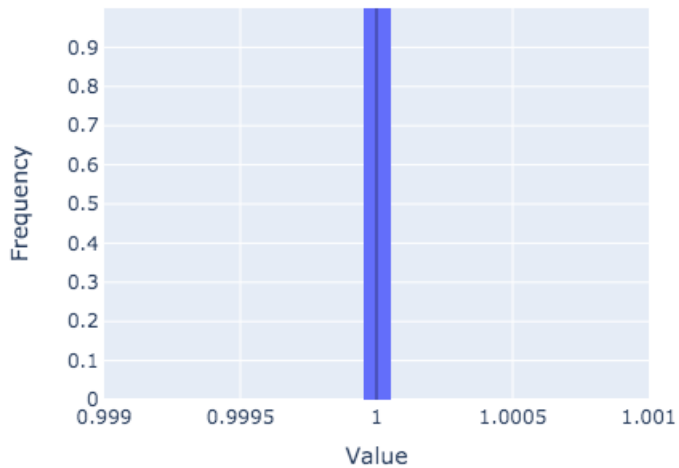
N=100, Nrep=10



Min: 0.9999999999999998  
Max: 0.9999999999999998  
|Max-Min|: 0.0

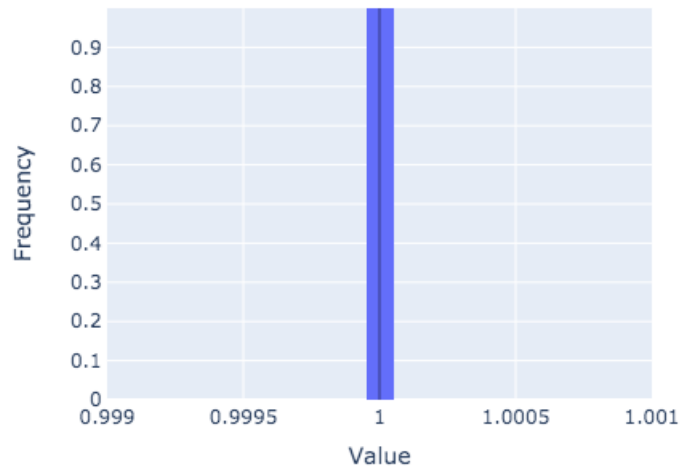
### Case 3: ProjectedLinear (False)

N=10, Nrep=10



Min: 0.9999999999999996  
Max: 1.00000014753393  
|Max-Min|: 1.4753393040578544e-07

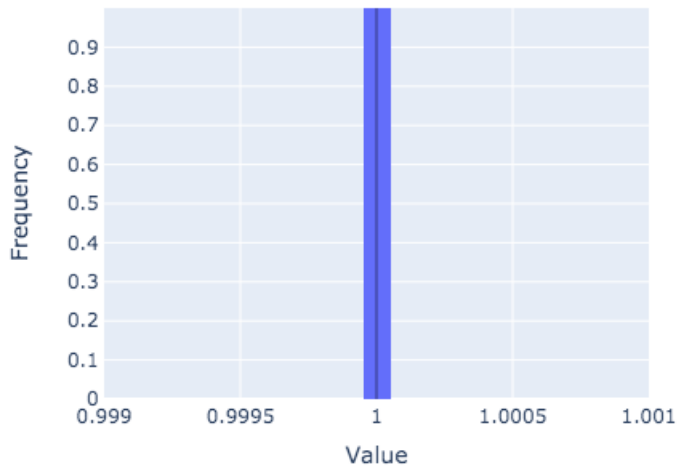
N=100, Nrep=10



Min: 0.9999999999999997  
Max: 1.0000001280394155  
|Max-Min|: 1.280394158476028e-07

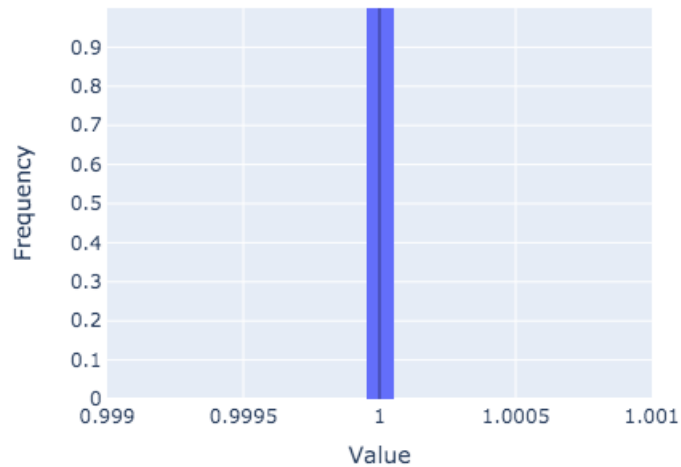
### Case 4: Maximum-Likelihood (True)

N=10, Nrep=10



Min: 0.9999999999999998  
Max: 0.9999999999999998  
|Max-Min|: 0.0

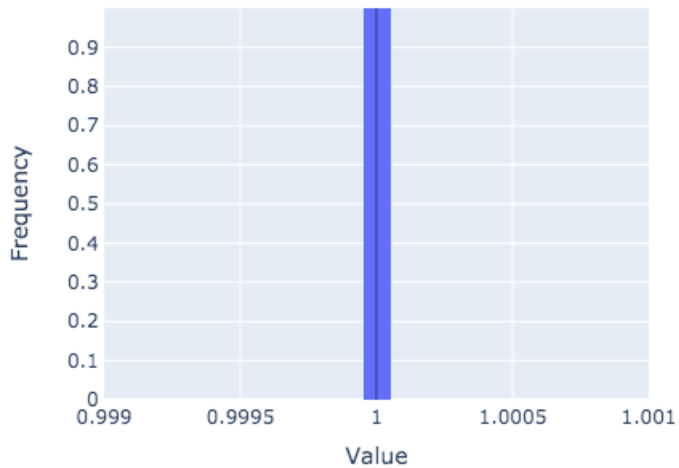
N=100, Nrep=10



Min: 0.9999999999999998  
Max: 0.9999999999999998  
|Max-Min|: 0.0

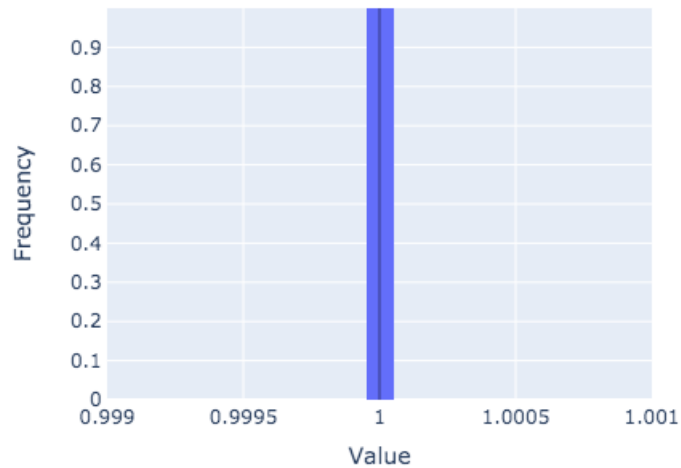
### Case 5: Maximum-Likelihood (False)

N=10, Nrep=10



Min: 1.0000000006567707  
Max: 1.0000001622723005  
|Max-Min|: 1.5570459344083076e-07

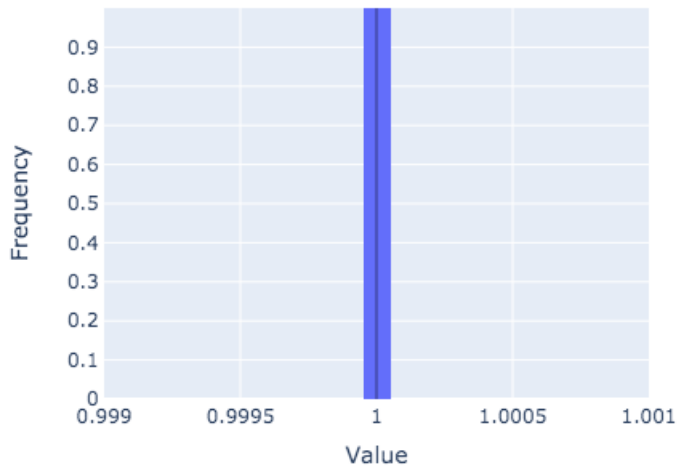
N=100, Nrep=10



Min: 1.0000000121352572  
Max: 1.0000001810607821  
|Max-Min|: 1.689255249015531e-07

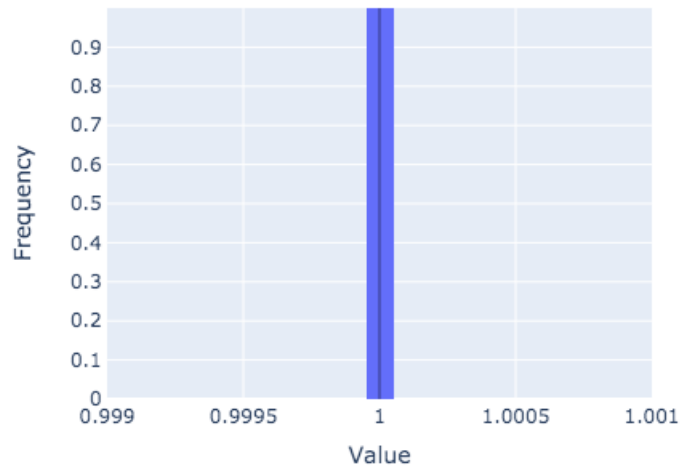
### Case 6: Least Squares (True)

N=10, Nrep=10



Min: 0.9999999999999998  
Max: 0.9999999999999998  
|Max-Min|: 0.0

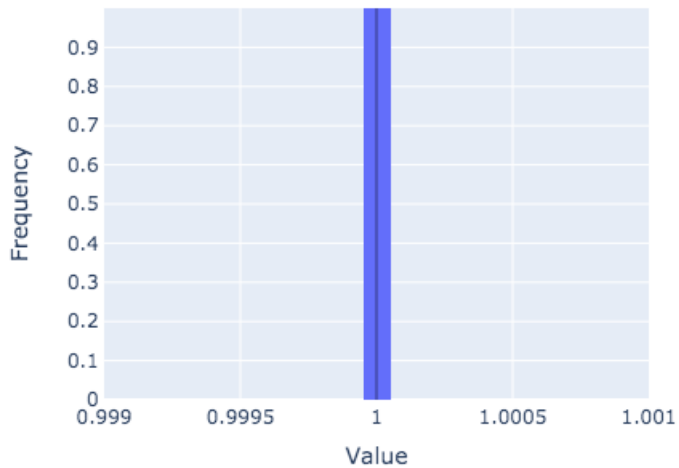
N=100, Nrep=10



Min: 0.9999999999999998  
Max: 0.9999999999999998  
|Max-Min|: 0.0

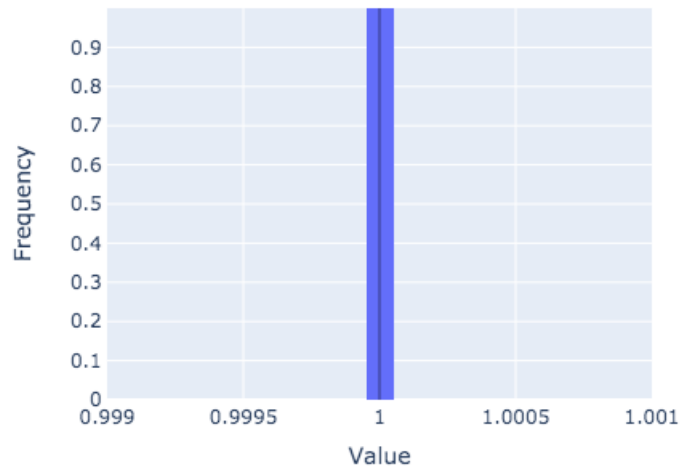
### Case 7: Least Squares (False)

N=10, Nrep=10



Min: 0.9999999999999998  
Max: 1.0000001781169368  
|Max-Min|: 1.7811693697744602e-07

N=100, Nrep=10



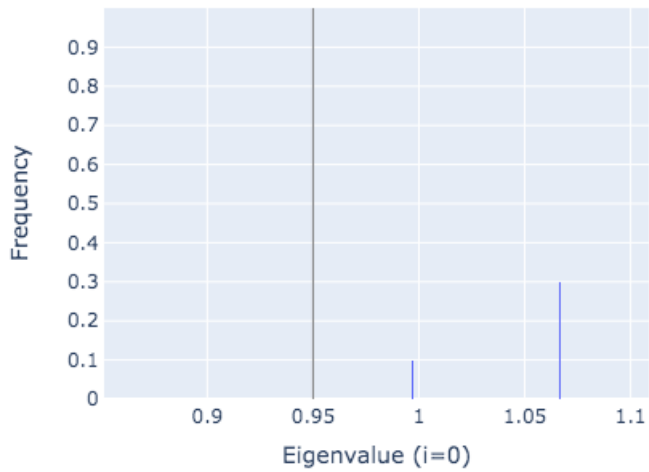
Min: 0.9999999999999998  
Max: 1.0000001555680968  
|Max-Min|: 1.5556809707106822e-07

## Test of inequality constraint violation

### Eigenvalue

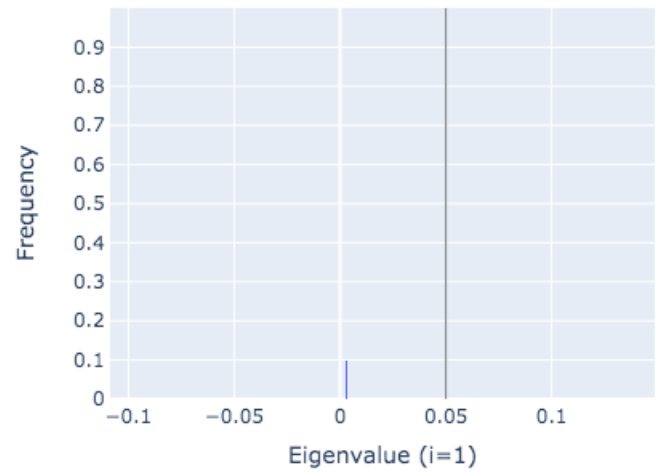
#### Case 0: Linear (True)

N=10, Nrep=10



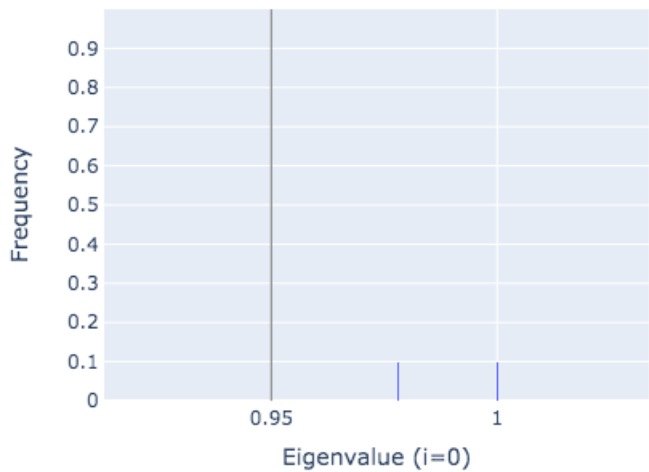
Min: 0.8513641844631532  
 Max: 1.1085806194501846  
 |Max-Min|: 0.2572164349870314

N=10, Nrep=10



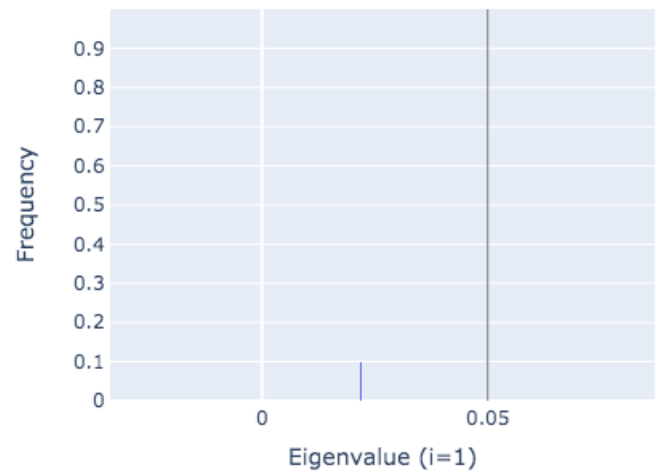
Min: -0.10858061945018471  
 Max: 0.14863581553684663  
 |Max-Min|: 0.2572164349870314

N=100, Nrep=10



Min: 0.9130584510098023  
 Max: 1.0335647646019095  
 |Max-Min|: 0.12050631359210717

N=100, Nrep=10

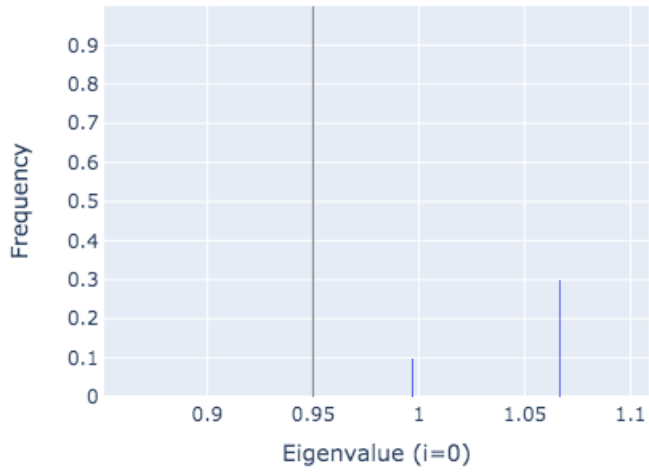


Min: -0.03356476460190986  
 Max: 0.0869415489901976  
 |Max-Min|: 0.12050631359210745

Case 1: Linear (False)

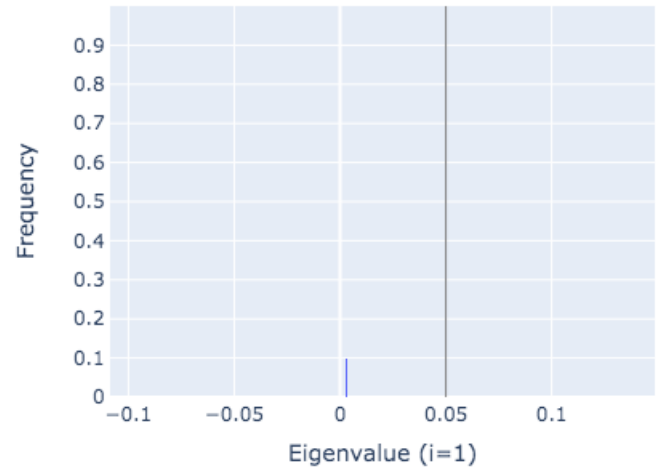


N=10, Nrep=10



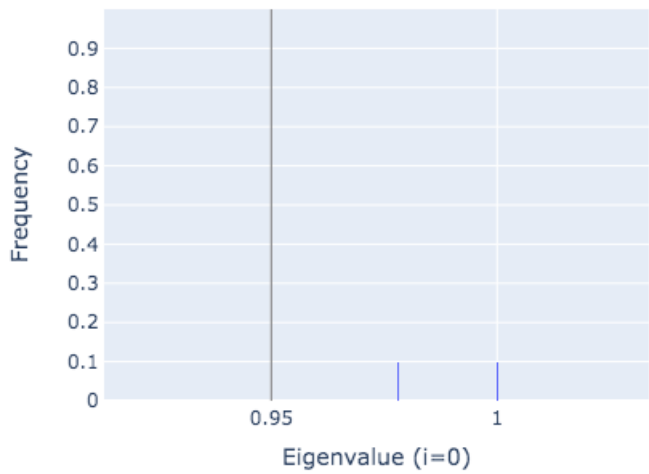
Min: 0.8513641844631532  
Max: 1.1085806194501846  
|Max-Min|: 0.2572164349870314

N=10, Nrep=10



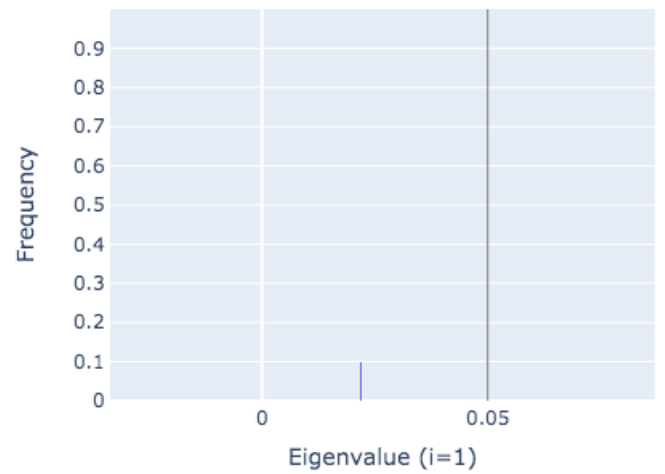
Min: -0.10858061945018459  
Max: 0.14863581553684663  
|Max-Min|: 0.25721643498703123

N=100, Nrep=10



Min: 0.9130584510098023  
Max: 1.0335647646019095  
|Max-Min|: 0.12050631359210717

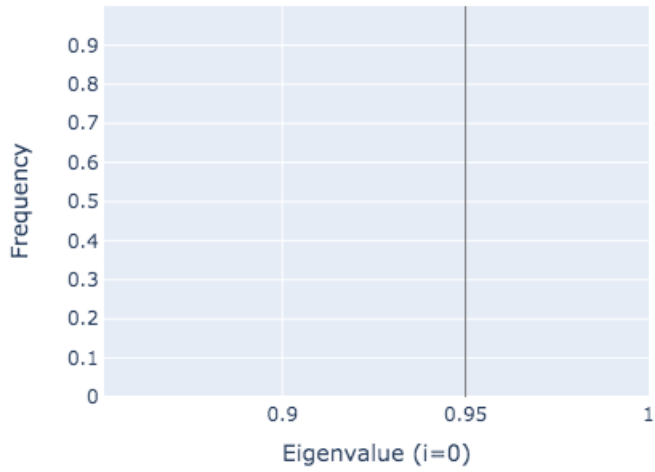
N=100, Nrep=10



Min: -0.03356476460190975  
Max: 0.08694154899019772  
|Max-Min|: 0.12050631359210748

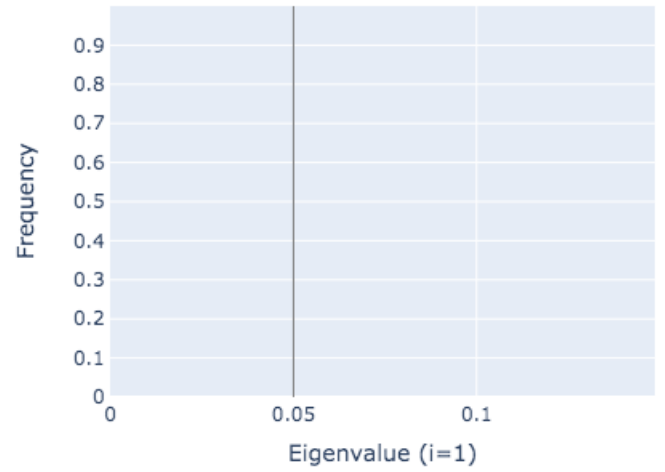
Case 2: ProjectedLinear (True)

N=10, Nrep=10



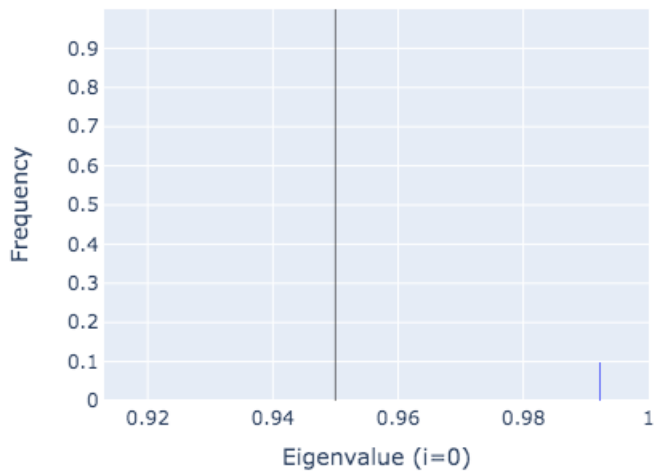
Min: 0.8513641844631531  
 Max: 1.0000000737669648  
 |Max-Min|: 0.14863588930381166

N=10, Nrep=10



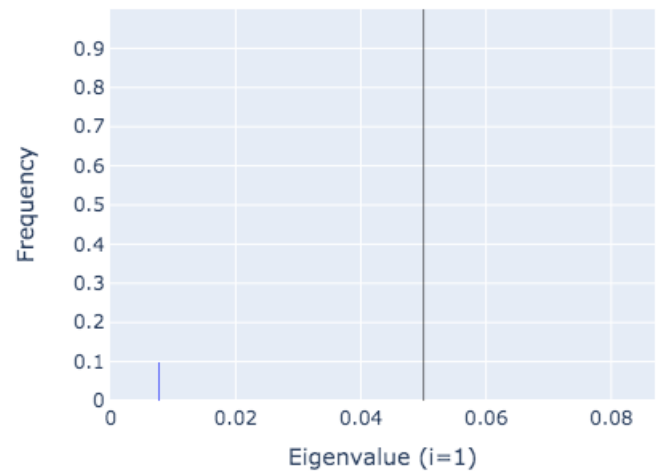
Min: -7.376696509533986e-08  
 Max: 0.14863581553684668  
 |Max-Min|: 0.14863588930381177

N=100, Nrep=10



Min: 0.9130584510098023  
 Max: 1.0000000640197078  
 |Max-Min|: 0.08694161300990544

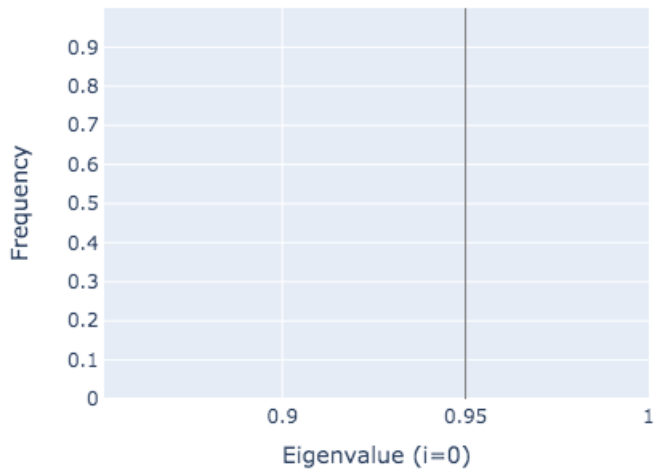
N=100, Nrep=10



Min: -6.401970792556323e-08  
 Max: 0.0869415489901976  
 |Max-Min|: 0.08694161300990552

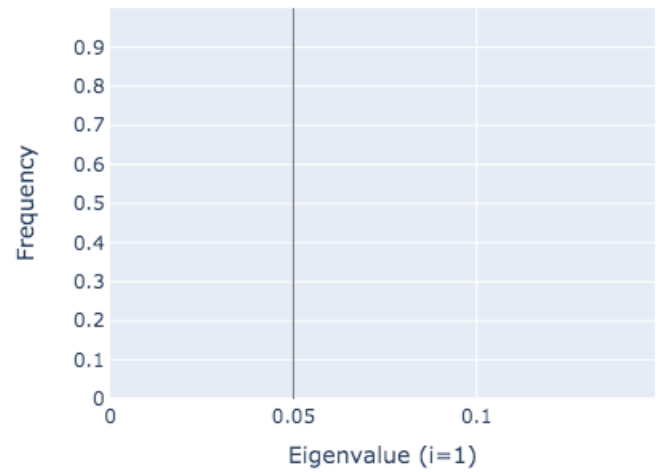
Case 3: ProjectedLinear (False)

N=10, Nrep=10



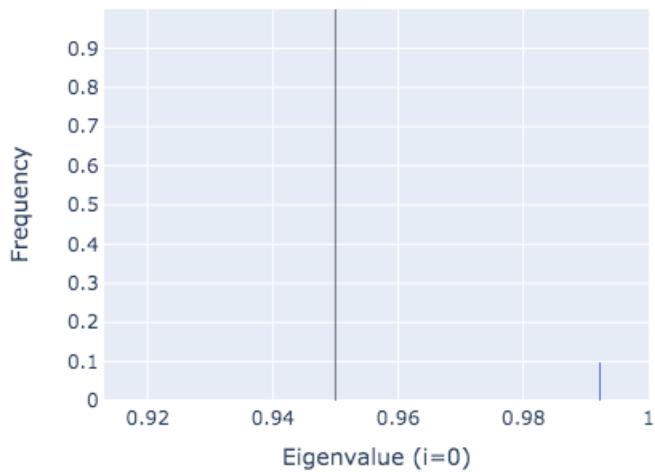
Min: 0.8513641844631532  
Max: 1.00000014753393  
|Max-Min|: 0.14863596307077676

N=10, Nrep=10



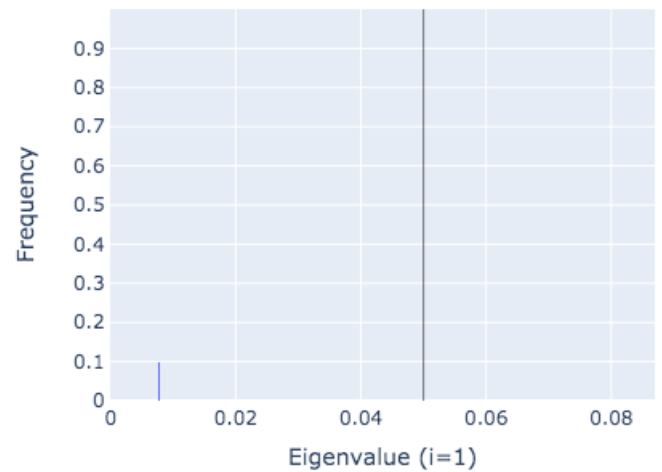
Min: -2.7755575615628914e-17  
Max: 0.14863581553684696  
|Max-Min|: 0.148635815536847

N=100, Nrep=10



Min: 0.9130584510098023  
Max: 1.0000001280394155  
|Max-Min|: 0.0869416770296132

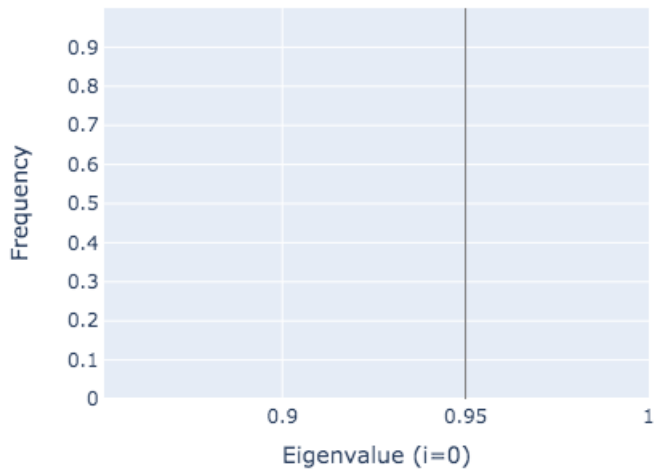
N=100, Nrep=10



Min: -5.730008481585891e-17  
Max: 0.0869415489901976  
|Max-Min|: 0.08694154899019765

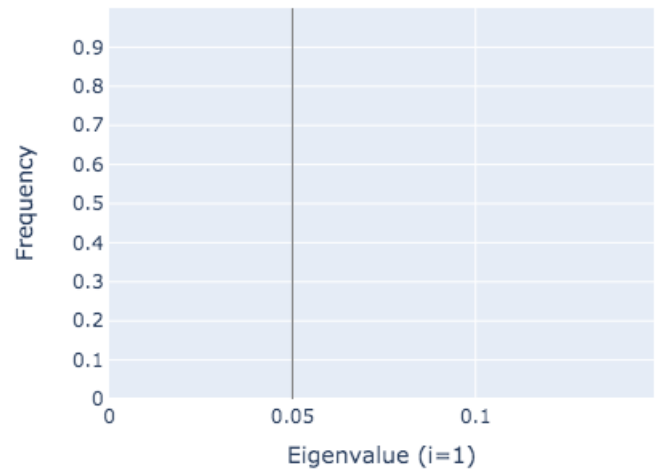
#### Case 4: Maximum-Likelihood (True)

N=10, Nrep=10



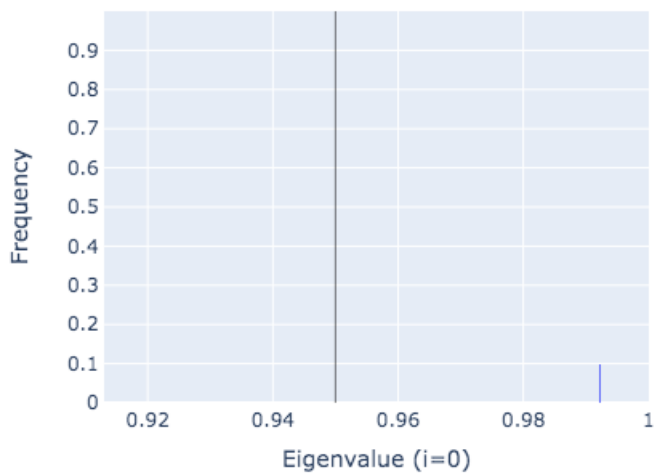
Min: 0.8513641812250503  
Max: 1.0000000702659644  
|Max-Min|: 0.14863588904091407

N=10, Nrep=10



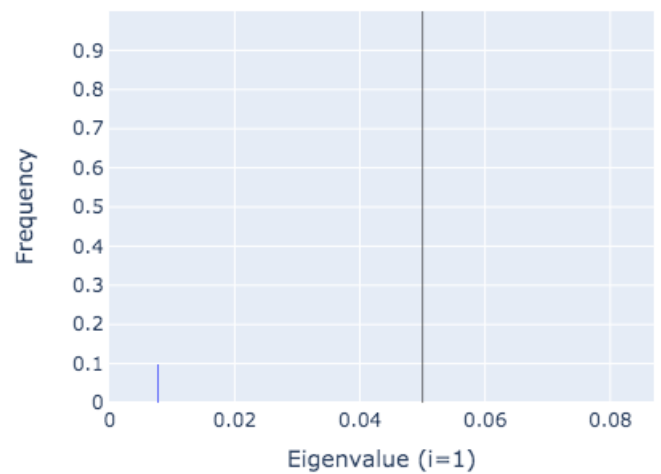
Min: -7.026596454684508e-08  
Max: 0.14863581877494952  
|Max-Min|: 0.14863588904091407

N=100, Nrep=10



Min: 0.9130584509383943  
Max: 1.000000078401553  
|Max-Min|: 0.0869416274631587

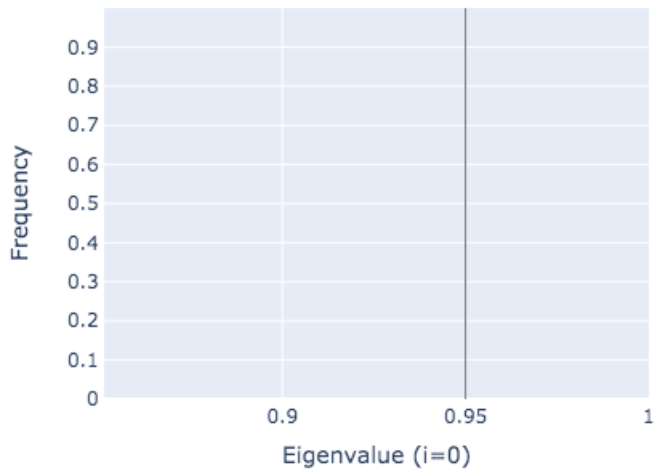
N=100, Nrep=10



Min: -7.840155328241432e-08  
Max: 0.08694154906160545  
|Max-Min|: 0.08694162746315873

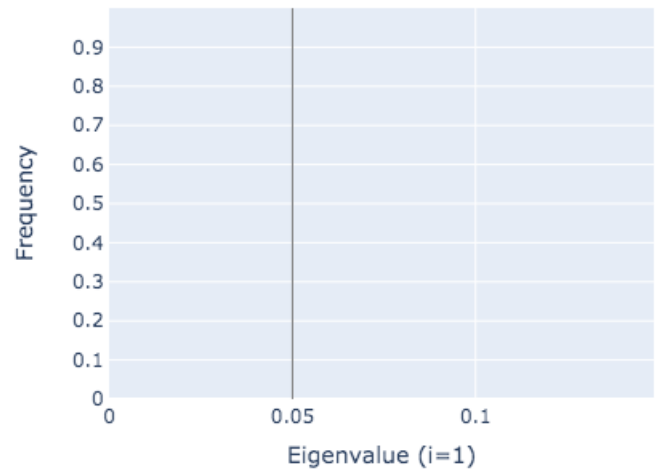
Case 5: Maximum-Likelihood (False)

N=10, Nrep=10



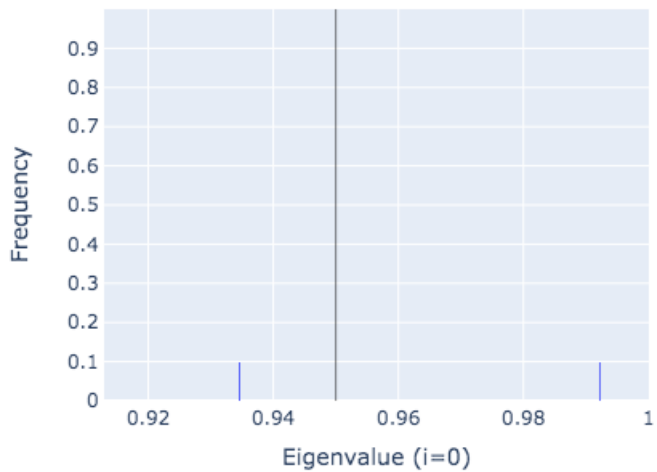
Min: 0.851427240055816  
 Max: 1.000000162272261  
 |Max-Min|: 0.1485729222164449

N=10, Nrep=10



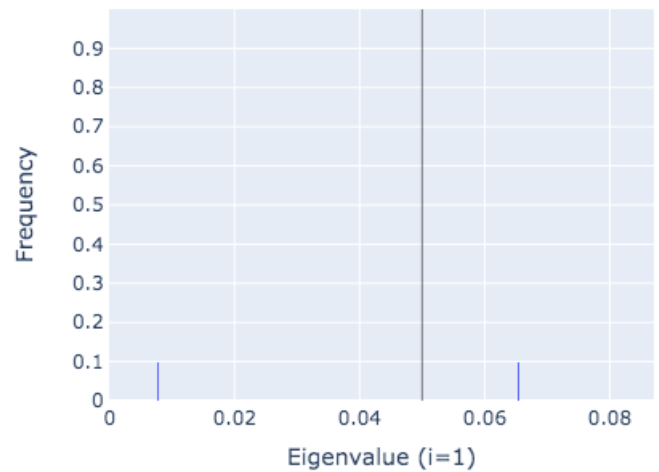
Min: -1.3877787807814457e-17  
 Max: 0.14857279250213073  
 |Max-Min|: 0.14857279250213073

N=100, Nrep=10



Min: 0.9130380513737355  
 Max: 1.0000001810607806  
 |Max-Min|: 0.08696212968704509

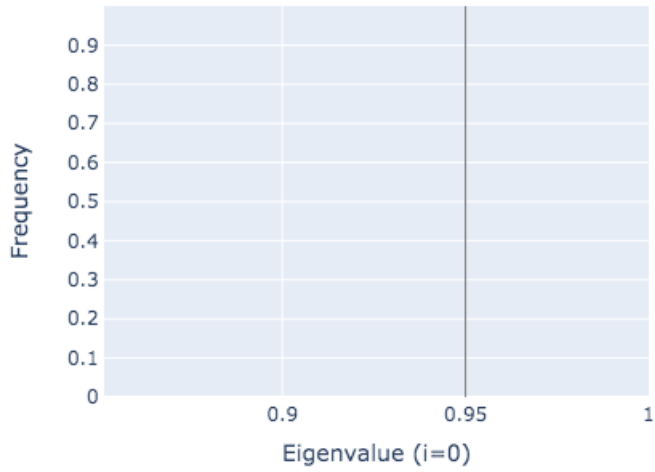
N=100, Nrep=10



Min: -1.0061396160665481e-16  
 Max: 0.08696196076152173  
 |Max-Min|: 0.08696196076152182

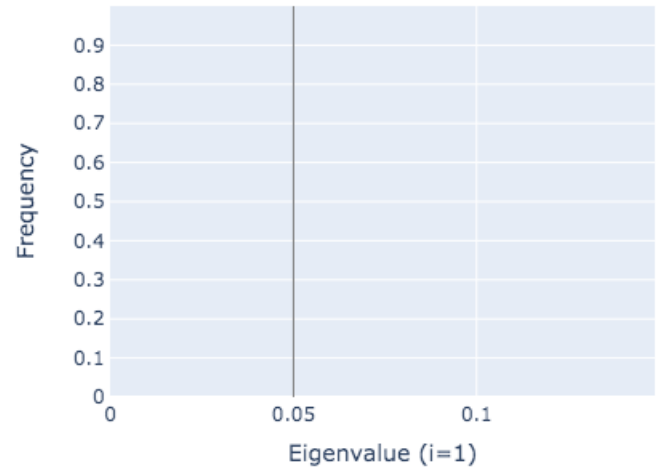
## Case 6: Least Squares (True)

N=10, Nrep=10



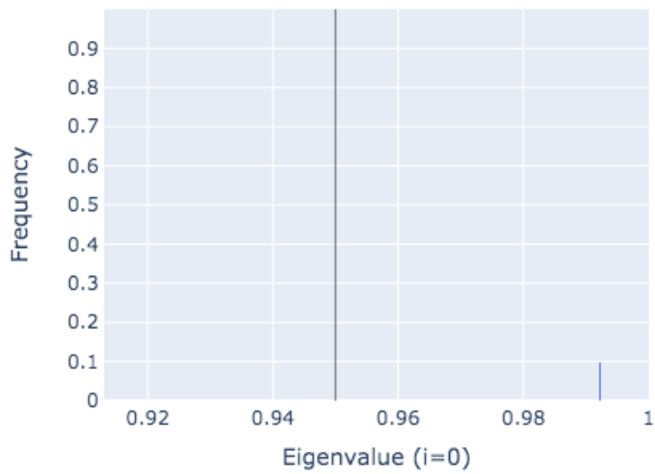
Min: 0.851364184463153  
 Max: 1.0000000761055894  
 |Max-Min|: 0.1486358916424364

N=10, Nrep=10



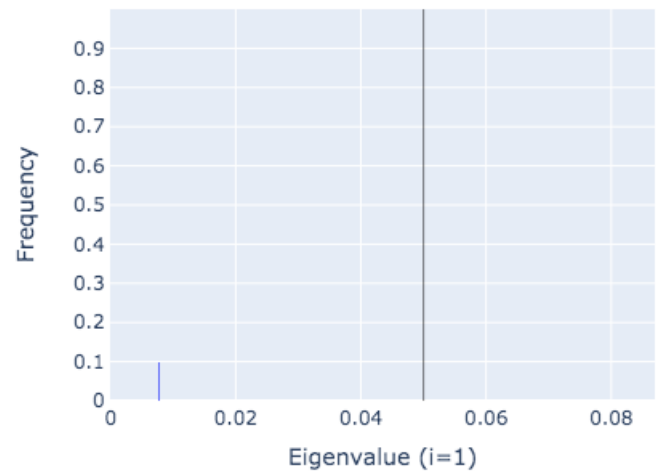
Min: -7.610558962201086e-08  
 Max: 0.14863581553684674  
 |Max-Min|: 0.14863589164243635

N=100, Nrep=10



Min: 0.9130584510098028  
 Max: 1.0000000598780594  
 |Max-Min|: 0.08694160886825664

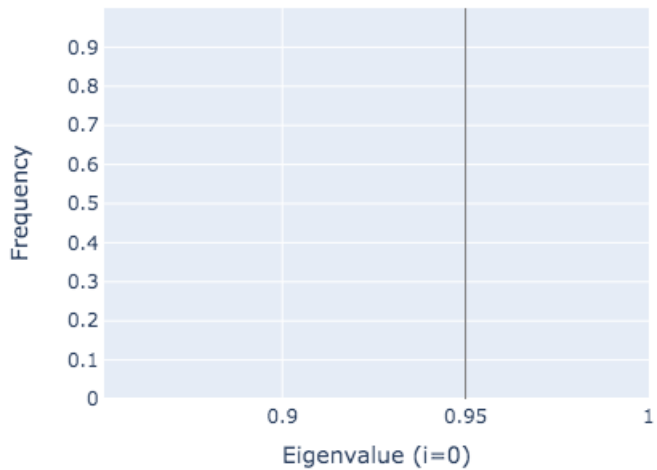
N=100, Nrep=10



Min: -5.987805948691623e-08  
 Max: 0.0869415489901971  
 |Max-Min|: 0.08694160886825658

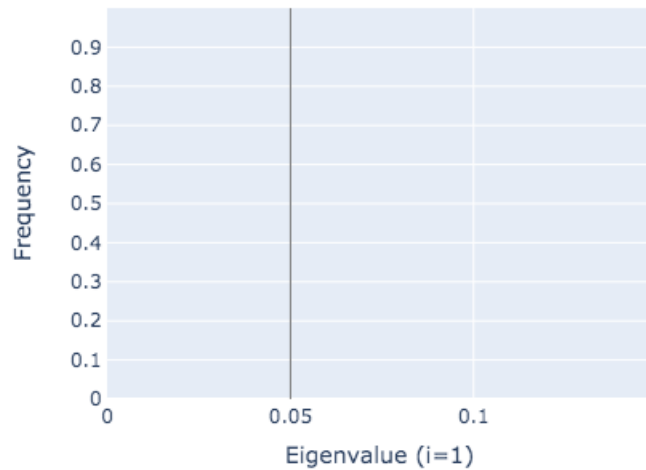
## Case 7: Least Squares (False)

N=10, Nrep=10



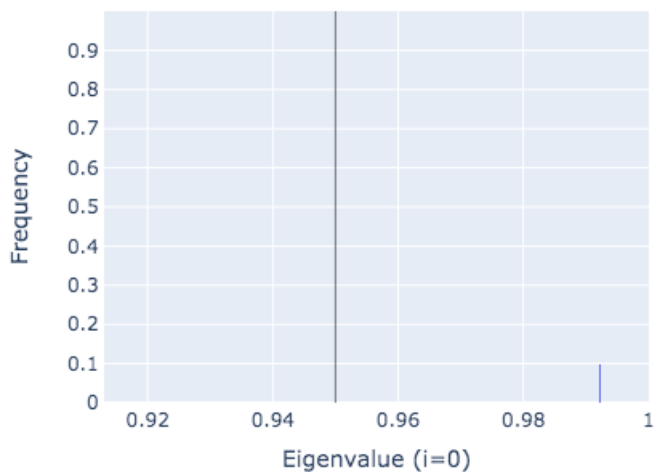
Min: 0.8513641844631532  
 Max: 1.0000001781169368  
 |Max-Min|: 0.14863599365378355

N=10, Nrep=10



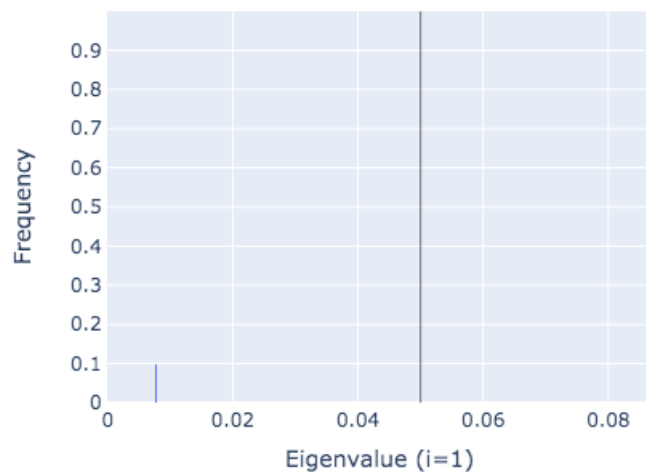
Min: -7.632783294297951e-17  
 Max: 0.14863581553684663  
 |Max-Min|: 0.1486358155368467

N=100, Nrep=10



Min: 0.9130584510098023  
 Max: 1.0000001555680968  
 |Max-Min|: 0.08694170455829453

N=100, Nrep=10



Min: 3.06287113727155e-18  
 Max: 0.08694154899019754  
 |Max-Min|: 0.08694154899019754

## Sum of unphysical eigenvalues

### Case 0: Linear (True)

N=10

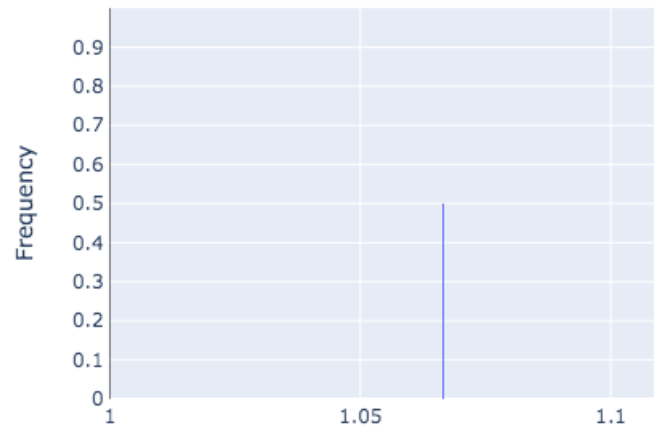
Number of unphysical estimates=6

N=10, Nrep=10



Min: -0.10858061945018471  
 Max: -0.009175077217315732  
 |Max-Min|: 0.09940554223286899

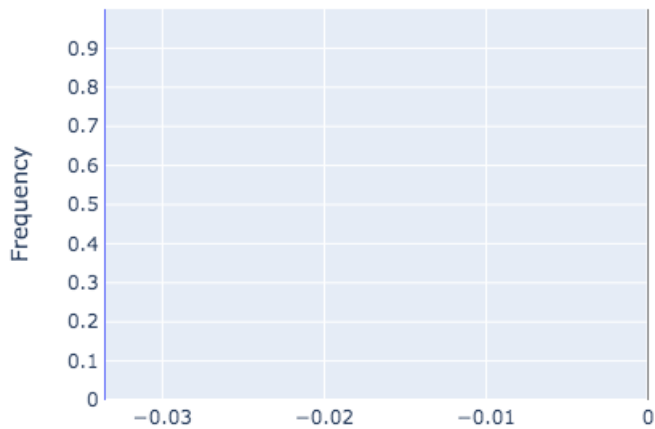
N=10, Nrep=10



Min: 1.0091750772173156  
 Max: 1.1085806194501846  
 |Max-Min|: 0.09940554223286902

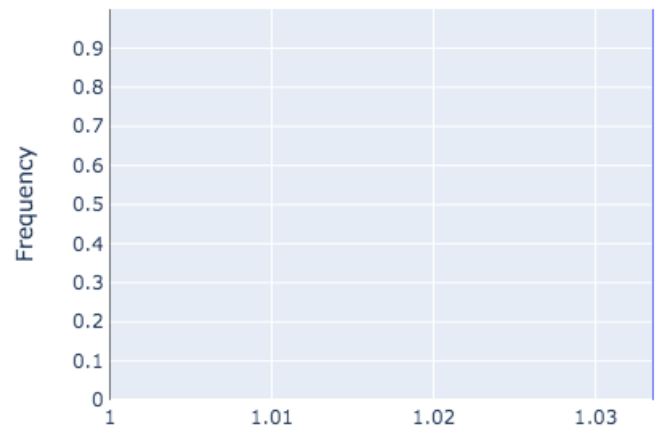
N=100  
 Number of unphysical estimates=1

N=100, Nrep=10



Min: -0.03356476460190986  
 Max: -0.03356476460190986  
 |Max-Min|: 0.0

N=100, Nrep=10



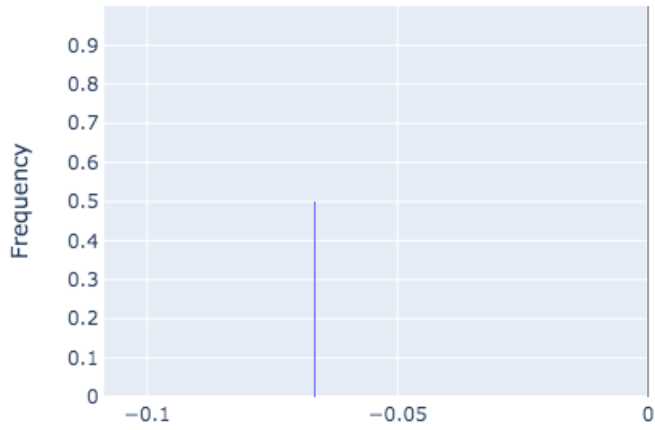
Min: 1.0335647646019095  
 Max: 1.0335647646019095  
 |Max-Min|: 0.0

### Case 1: Linear (False)

N=10  
 Number of unphysical estimates=6

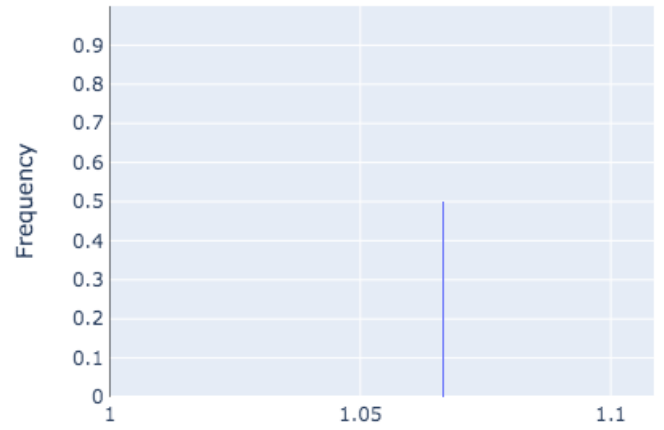


N=10, Nrep=10



Min: -0.10858061945018459  
 Max: -0.009175077217315586  
 |Max-Min|: 0.099405542232869

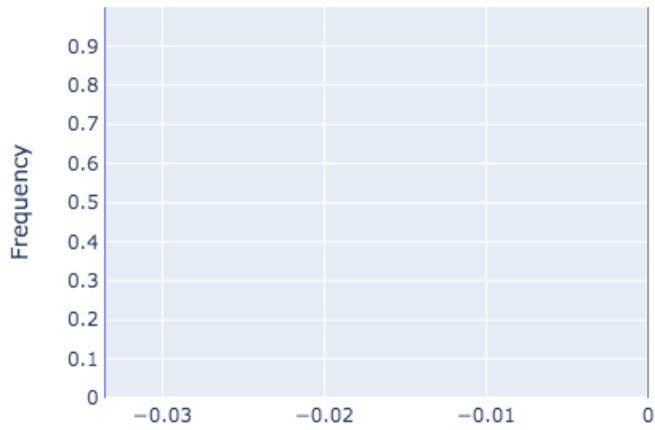
N=10, Nrep=10



Min: 1.0091750772173156  
 Max: 1.1085806194501846  
 |Max-Min|: 0.09940554223286902

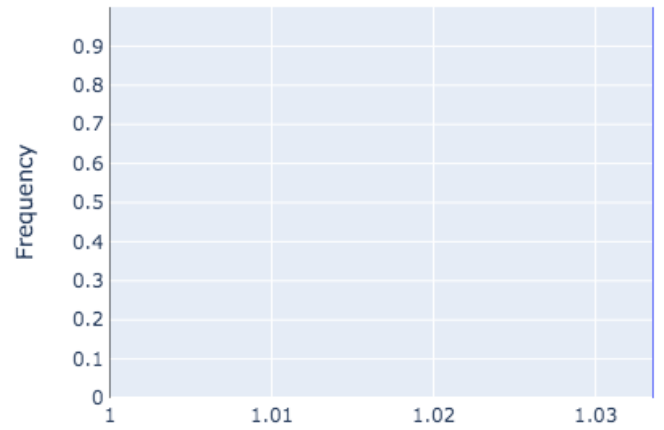
N=100  
 Number of unphysical estimates=1

N=100, Nrep=10



Min: -0.03356476460190975  
 Max: -0.03356476460190975  
 |Max-Min|: 0.0

N=100, Nrep=10

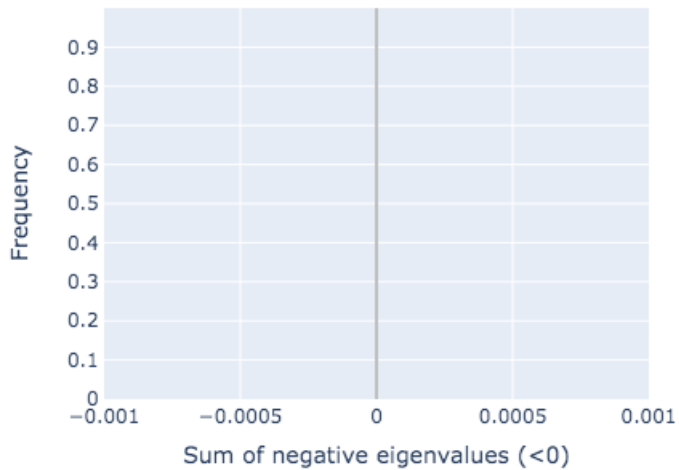


Min: 1.0335647646019095  
 Max: 1.0335647646019095  
 |Max-Min|: 0.0

## Case 2: ProjectedLinear (True)

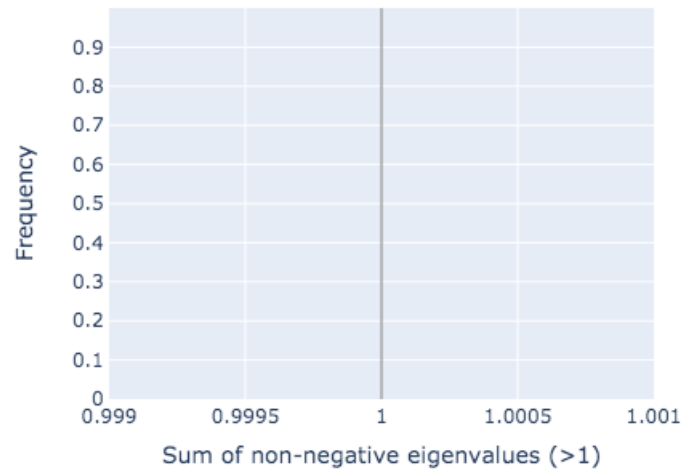
N=10  
 Number of unphysical estimates=0

N=10, Nrep=10



Min: -  
Max: -  
|Max-Min|: -

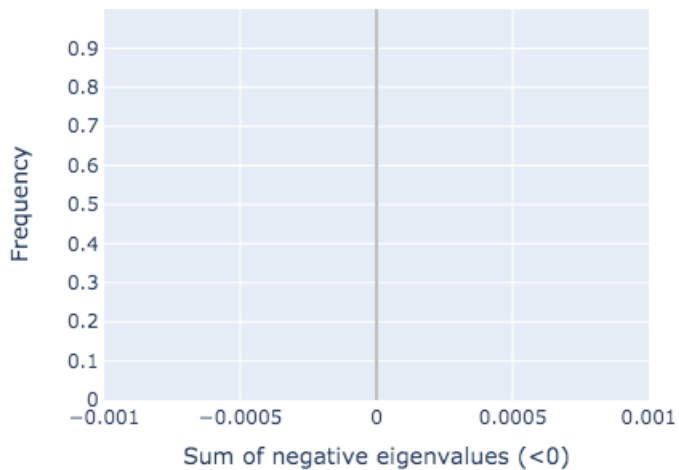
N=10, Nrep=10



Min: -  
Max: -  
|Max-Min|: -

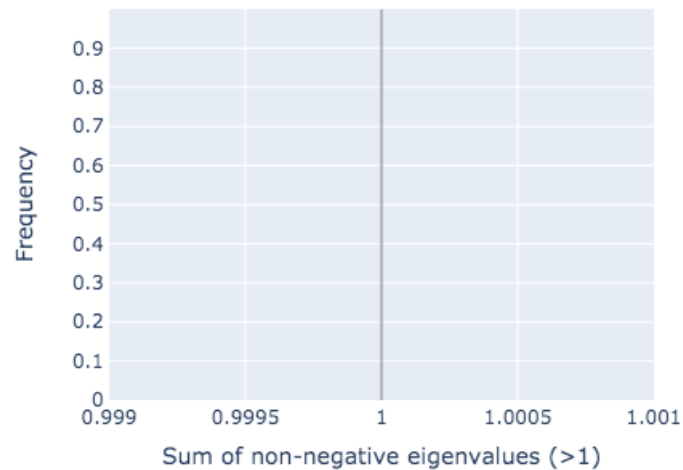
N=100  
Number of unphysical estimates=0

N=100, Nrep=10



Min: -  
Max: -  
|Max-Min|: -

N=100, Nrep=10

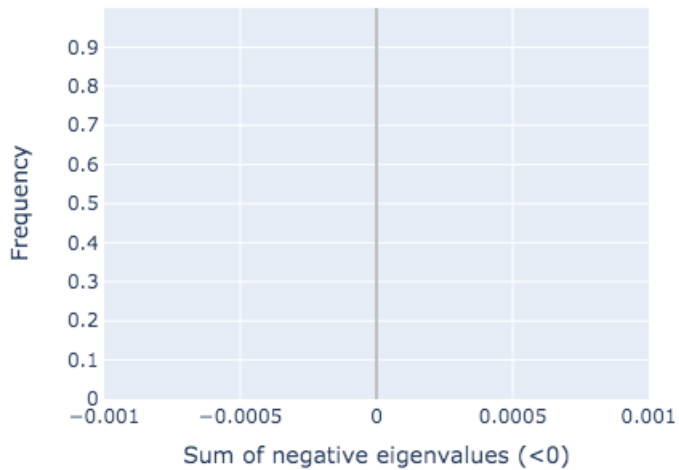


Min: -  
Max: -  
|Max-Min|: -

### Case 3: ProjectedLinear (False)

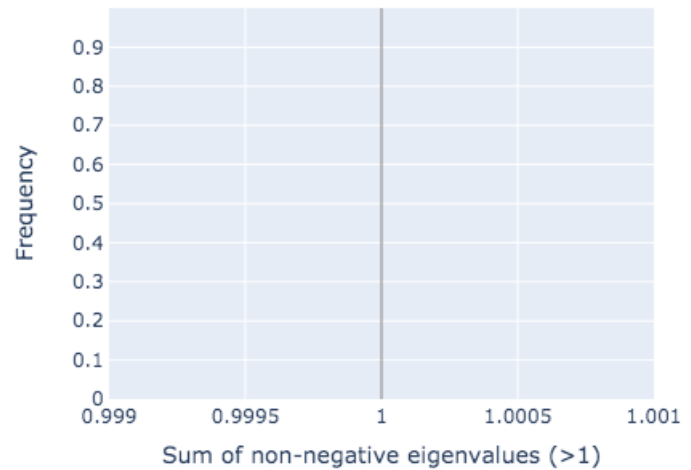
N=10  
Number of unphysical estimates=0

N=10, Nrep=10



Min: -  
Max: -  
|Max-Min|: -

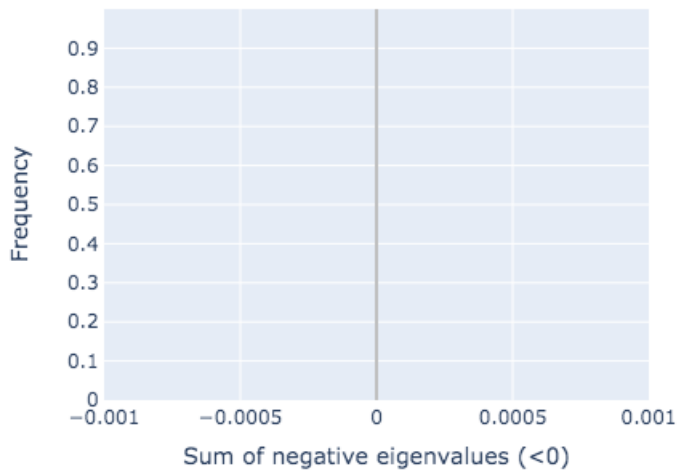
N=10, Nrep=10



Min: -  
Max: -  
|Max-Min|: -

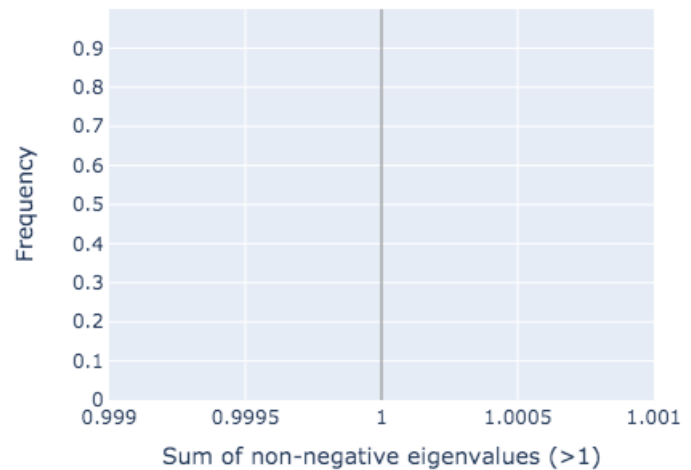
N=100  
Number of unphysical estimates=0

N=100, Nrep=10



Min: -  
Max: -  
|Max-Min|: -

N=100, Nrep=10

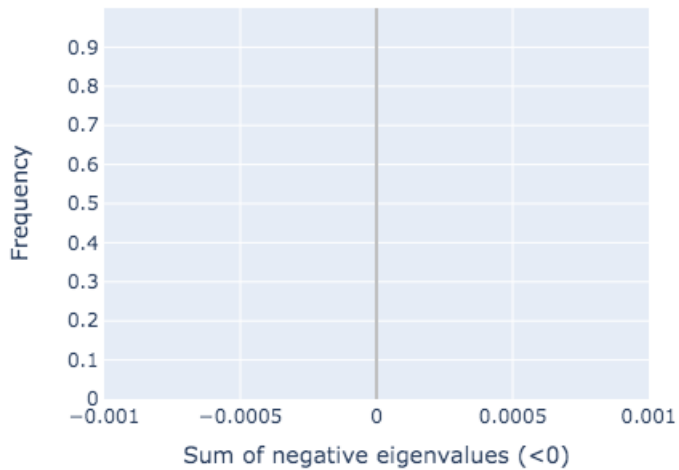


Min: -  
Max: -  
|Max-Min|: -

#### Case 4: Maximum-Likelihood (True)

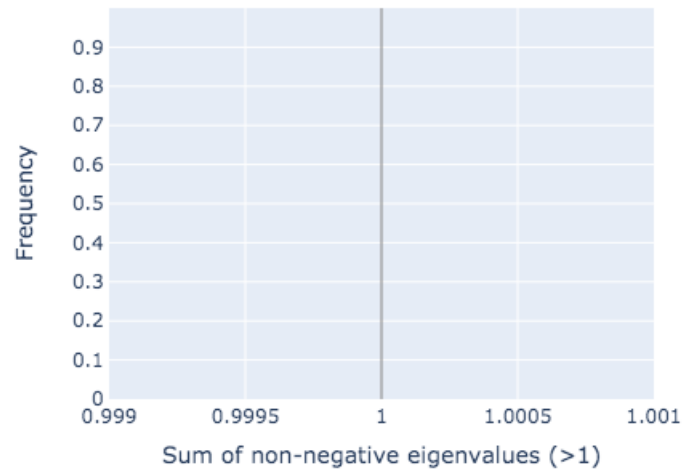
N=10  
Number of unphysical estimates=0

N=10, Nrep=10



Min: -  
Max: -  
|Max-Min|: -

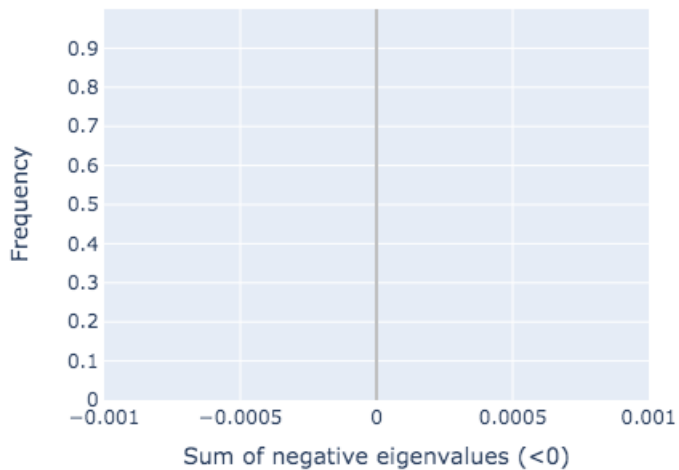
N=10, Nrep=10



Min: -  
Max: -  
|Max-Min|: -

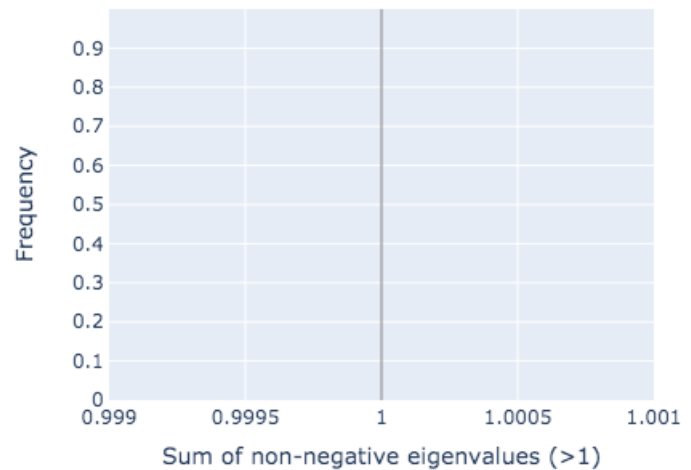
N=100  
Number of unphysical estimates=0

N=100, Nrep=10



Min: -  
Max: -  
|Max-Min|: -

N=100, Nrep=10

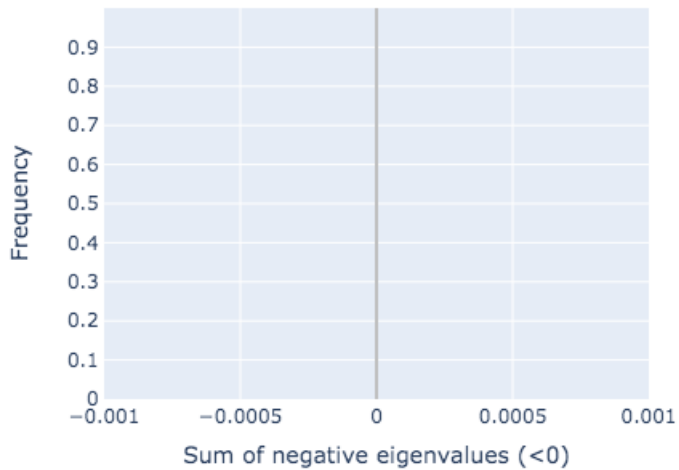


Min: -  
Max: -  
|Max-Min|: -

### Case 5: Maximum-Likelihood (False)

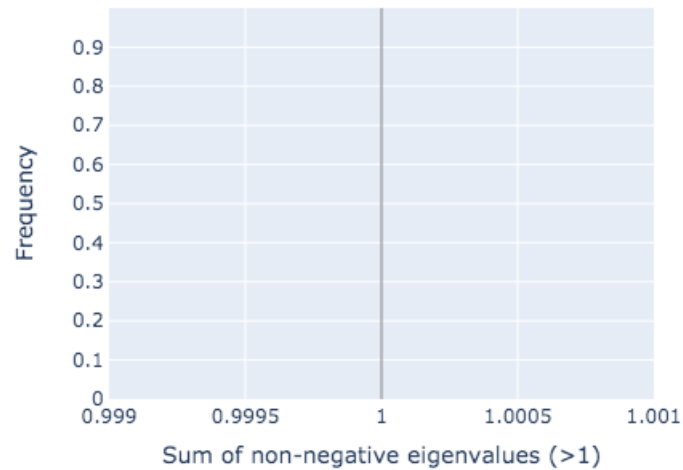
N=10  
Number of unphysical estimates=0

N=10, Nrep=10



Min: -  
Max: -  
|Max-Min|: -

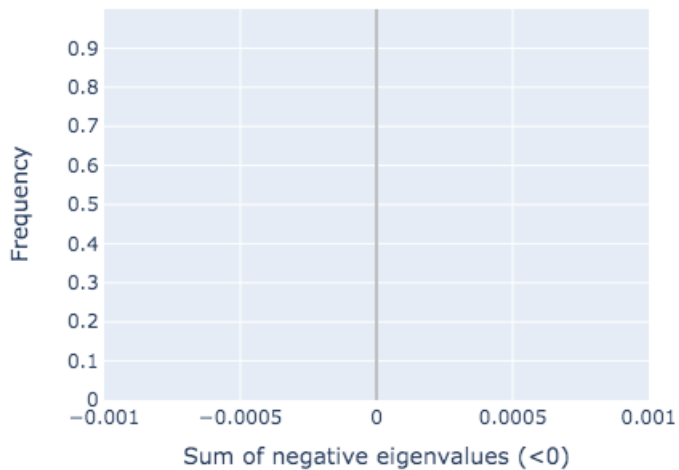
N=10, Nrep=10



Min: -  
Max: -  
|Max-Min|: -

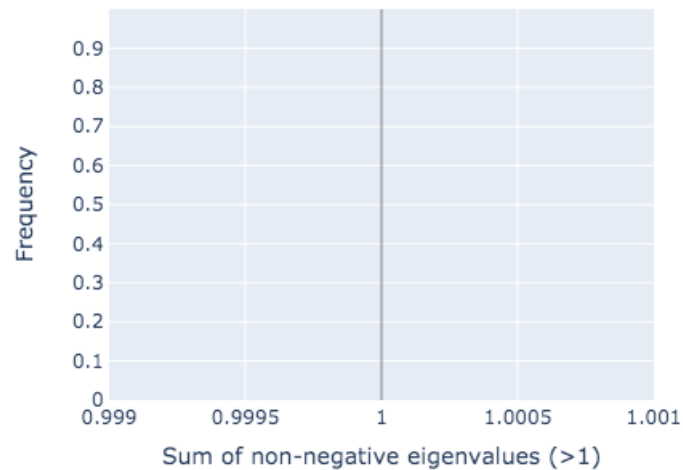
N=100  
Number of unphysical estimates=0

N=100, Nrep=10



Min: -  
Max: -  
|Max-Min|: -

N=100, Nrep=10

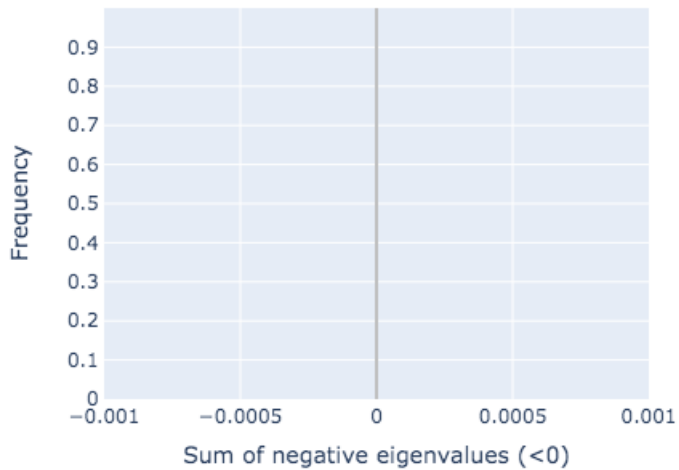


Min: -  
Max: -  
|Max-Min|: -

## Case 6: Least Squares (True)

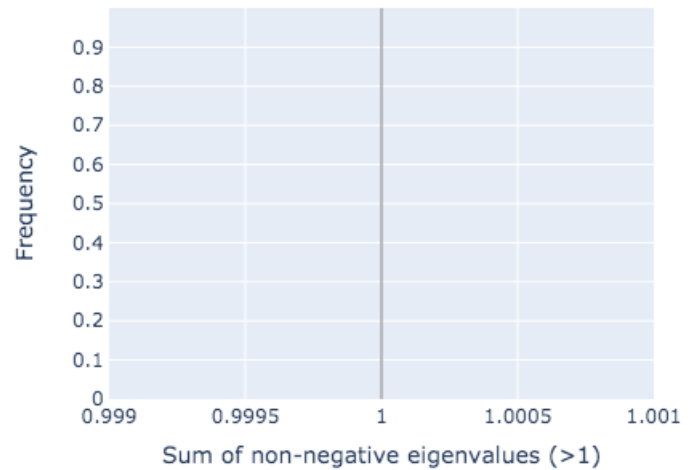
N=10  
Number of unphysical estimates=0

N=10, Nrep=10



Min: -  
Max: -  
|Max-Min|: -

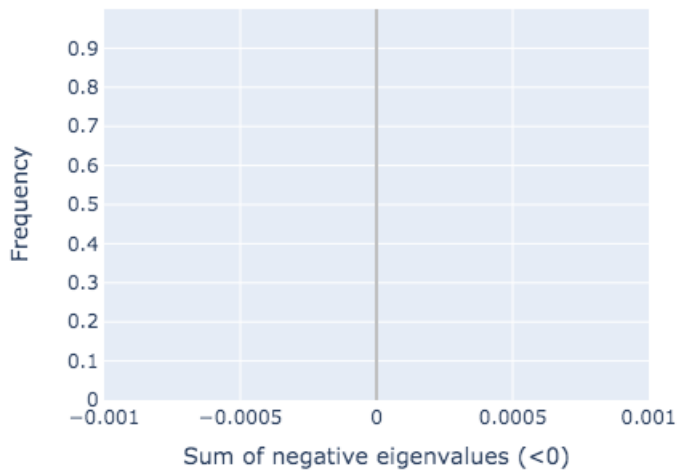
N=10, Nrep=10



Min: -  
Max: -  
|Max-Min|: -

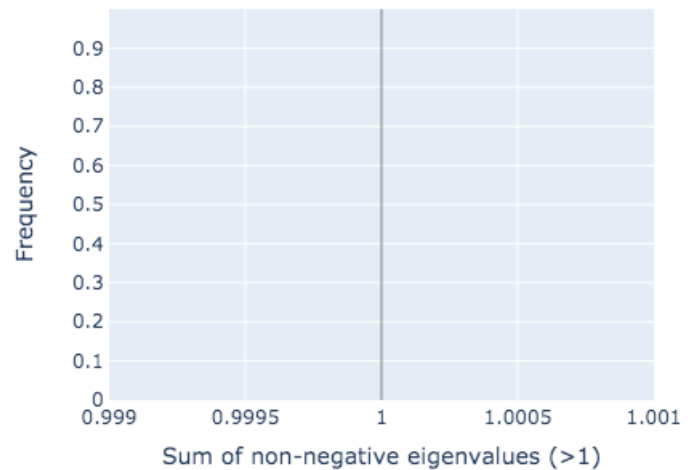
N=100  
Number of unphysical estimates=0

N=100, Nrep=10



Min: -  
Max: -  
|Max-Min|: -

N=100, Nrep=10

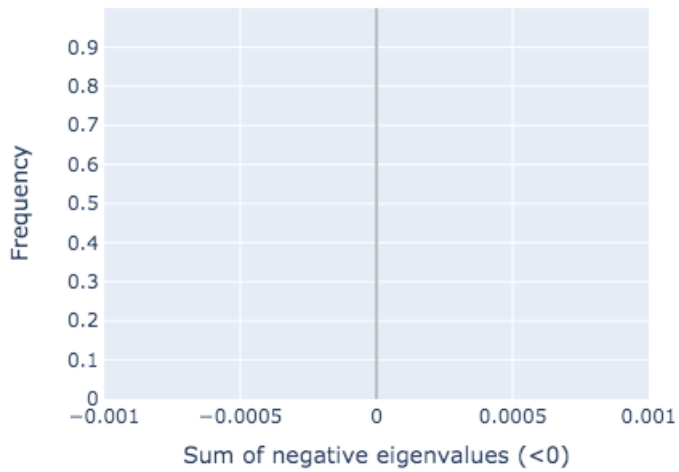


Min: -  
Max: -  
|Max-Min|: -

### Case 7: Least Squares (False)

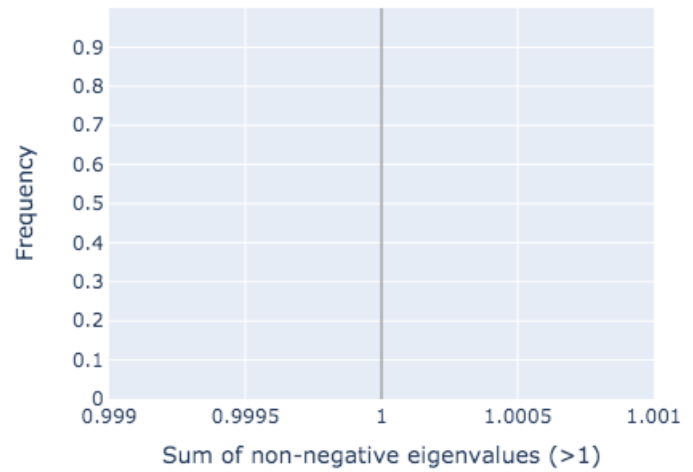
N=10  
Number of unphysical estimates=0

N=10, Nrep=10



Min: -  
Max: -  
|Max-Min|: -

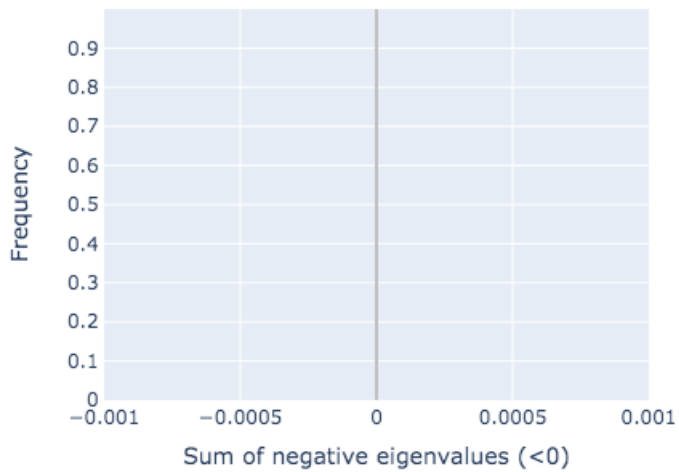
N=10, Nrep=10



Min: -  
Max: -  
|Max-Min|: -

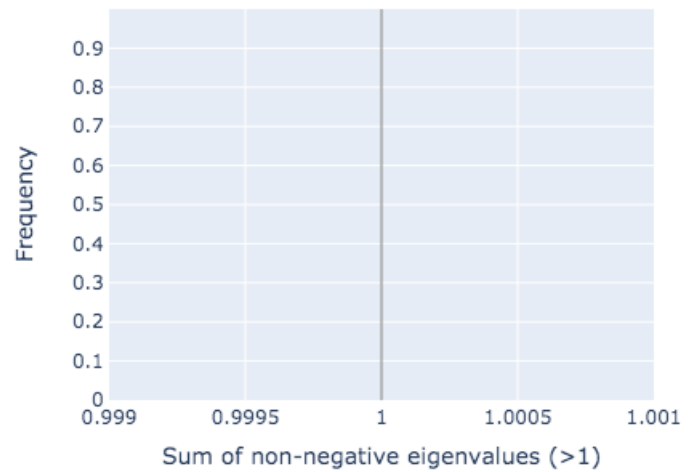
N=100  
Number of unphysical estimates=0

N=100, Nrep=10



Min: -  
Max: -  
|Max-Min|: -

N=100, Nrep=10



Min: -  
Max: -  
|Max-Min|: -