Table of contents

Computation time in total

Total	0.5864518443743388min.

Tolerance of physicality constraint violation

		True	False
Tolerance at physicality violation test	**equality constraint**	1.00e-13	1.00e-05
	inequality constraint	1.00e-05	1.00e-05

Experimental condition

Type of tomography	StandardQst
Nrep	10
N	[10, 100]
RNG seed	777

True object

Type	State
Dim	2
Vec	[0.70710678 0.04314638 -0.03754928 0.64700626]

Tester objects

0	**Type**	Povm
	Dim	2
	Number of outcomes	2
	Vecs	[[0.67045551 0.63151808 -0.00840615 -0.03564561] [0.74375805 -0.63151808 0.00840615 0.03564561]]
1	**Type**	Povm
	Dim	2
	Number of outcomes	2
	Vecs	[[0.72596364 -0.04732708 0.64096095 -0.08116688] [0.68824993 0.04732708 -0.64096095 0.08116688]]
2	**Type**	Povm
	Dim	2
	Number of outcomes	2
	Vecs	[[0.71790027 0.02984105 -0.02029088 0.66221865] [0.6963133 -0.02984105 0.02029088 -0.66221865]]

Cases

	Name	Param	Tomography	Estimator
0	Linear (True)	True	StandardQst	Linear
1	Linear (False)	False	StandardQst	Linear
2	ProjectedLinear (True)	True	StandardQst	ProjectedLinear
3	ProjectedLinear (False)	False	StandardQst	ProjectedLinear
4	Maximum-Likelihood (True)	True	StandardQst	LossMinimization
5	Maximum-Likelihood (False)	False	StandardQst	LossMinimization

|	Name			
6	Least Squares (True)	True	StandardQst	LossMinimization
7	Least Squares (False)	False	StandardQst	LossMinimization

Computation time of estimators

Table

	Name	N	Nrep	Mean (sec)	Std (sec)
0	Linear (True)	10	10	0.000101	0.000040
1	"	100	10	0.000063	0.000003
2	Linear (False)	10	10	0.000061	0.000015
3	"	100	10	0.000058	0.000017
4	ProjectedLinear (True)	10	10	0.012303	0.010736
5	"	100	10	0.005264	0.006800
6	ProjectedLinear (False)	10	10	0.010316	0.007869
7	"	100	10	0.005542	0.008532
8	Maximum-Likelihood (True)	10	10	0.565193	0.140795
9	"	100	10	0.597748	0.201294
10	Maximum-Likelihood (False)	10	10	0.661443	0.126227
11	"	100	10	0.876040	0.326420
12	Least Squares (True)	10	10	0.232593	0.146467
13	"	100	10	0.116983	0.088654
14	Least Squares (False)	10	10	0.283130	0.136286
15	"	100	10	0.151872	0.062631

Histogram

Case 0: Linear (True)

N=10, Nrep=10
Mean = 0.00010132789611816406(sec)
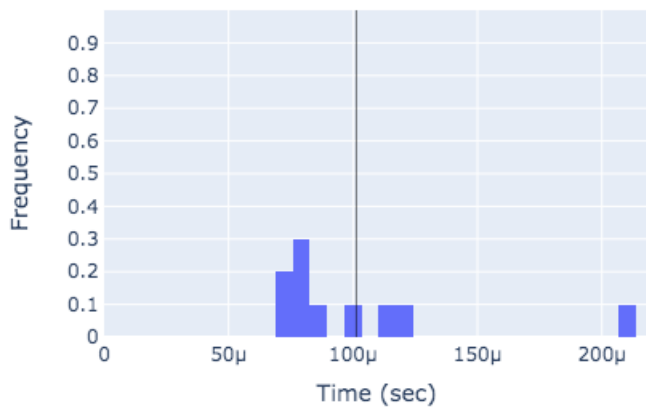
Min: 7.414817810058594e-05
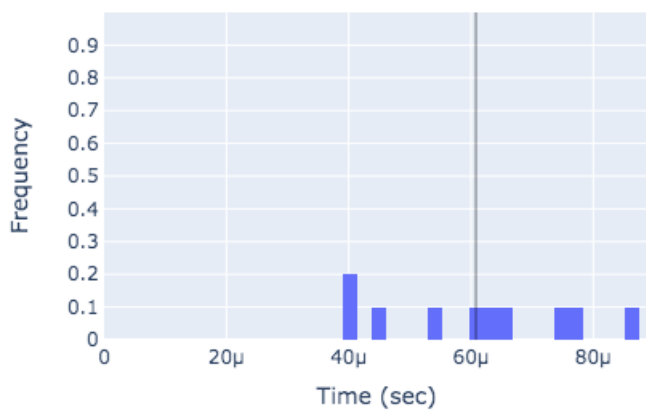Max: 0.00021195411682128906
|Max-Min|: 0.00013780593872070312

N=100, Nrep=10
Mean = 6.346702575683593e-05(sec)
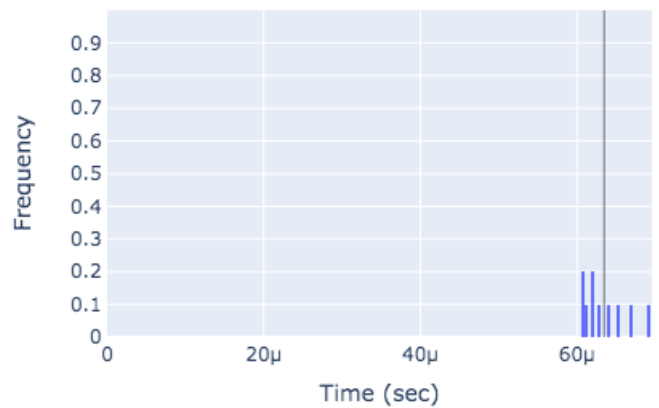
Min: 6.079673767089844e-05
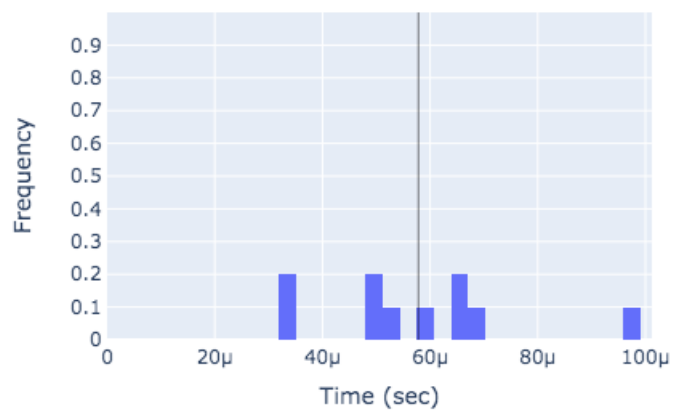Max: 6.914138793945312e-05
|Max-Min|: 8.344650268554688e-06

Case 1: Linear (False)

N=10, Nrep=10
Mean = 6.079673767089844e-05(sec)

Min: 4.076957702636719e-05
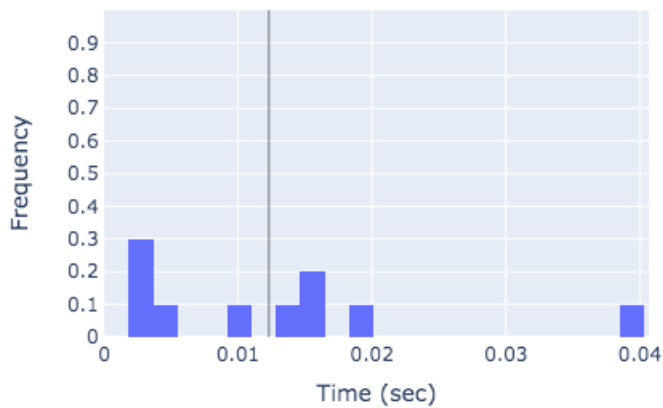Max: 8.678436279296875e-05
|Max-Min|: 4.601478576660156e-05

N=100, Nrep=10
Mean = 5.7816505432128906e-05(sec)

Min: 3.409385681152344e-05
Max: 9.799003601074219e-05
|Max-Min|: 6.389617919921875e-05

Case 2: ProjectedLinear (True)

N=10, Nrep=10
Mean = 0.012302899360656738(sec)

Frequency

0   0.01   0.02   0.03   0.04
Time (sec)

Min: 0.002210855484008789
Max: 0.03883981704711914
|Max-Min|: 0.03662896156311035

N=100, Nrep=10
Mean = 0.0052642345428466795(sec)

Frequency

0   0.005   0.01   0.015   0.02
Time (sec)
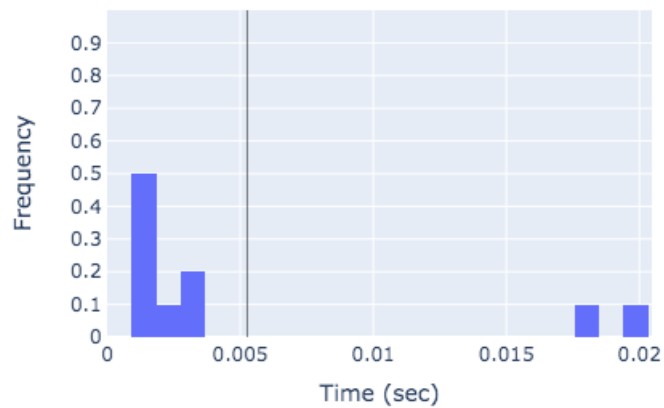
Min: 0.0010547637939453125
Max: 0.019542932510375977
|Max-Min|: 0.018488168716430664

Case 3: ProjectedLinear (False)

N=10, Nrep=10
Mean = 0.010316109657287598(sec)

Frequency

0   0.01   0.02
Time (sec)
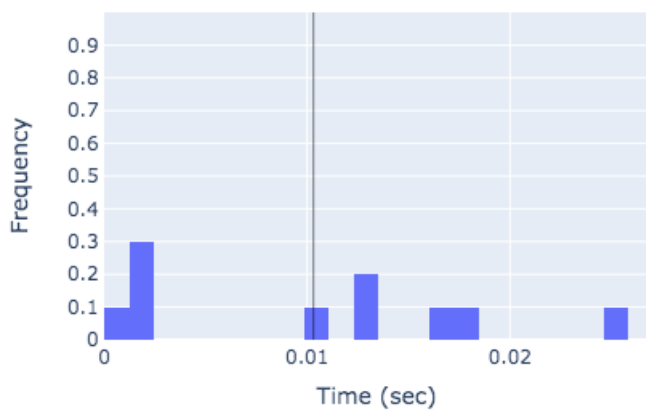
Min: 0.00099706649780227344
Max: 0.025609731674194336
|Max-Min|: 0.0246126651763916

N=100, Nrep=10
Mean = 0.0055419921875(sec)

Frequency

0   0.01   0.02
Time (sec)

Min: 0.000965118408203125
Max: 0.0272030830383300078
|Max-Min|: 0.026237964630126953

Case 4: Maximum-Likelihood (True)
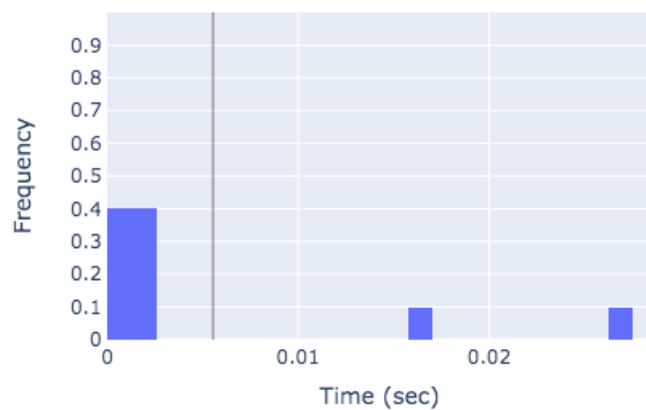
N=10, Nrep=10
Mean = 0.5651931047439576(sec)

N=100, Nrep=10
Mean = 0.5977484464645386(sec)
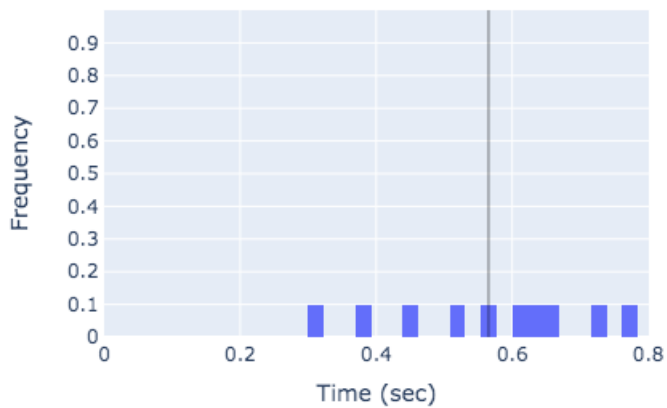
Min: 0.316011905670166
Max: 0.7777693271636963
|Max-Min|: 0.4617574214935303
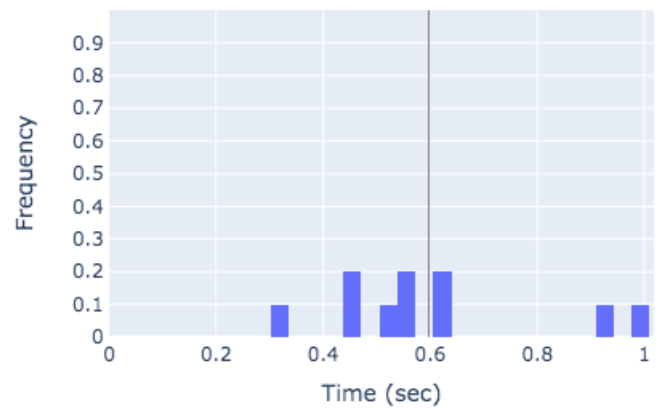
Min: 0.31116509437561035
Max: 0.9842596054077148
|Max-Min|: 0.6730945110321045

Case 5: Maximum-Likelihood (False)

N=10, Nrep=10
Mean = 0.6614428758621216(sec)

N=100, Nrep=10
Mean = 0.8760395765304565(sec)
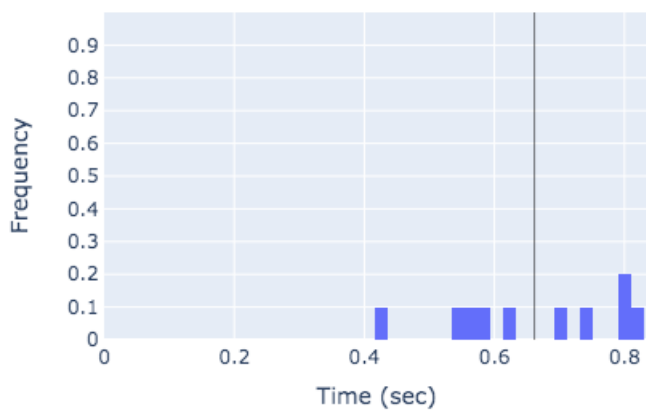
Min: 0.42223668098449707
Max: 0.81766676902771
|Max-Min|: 0.3954300880432129
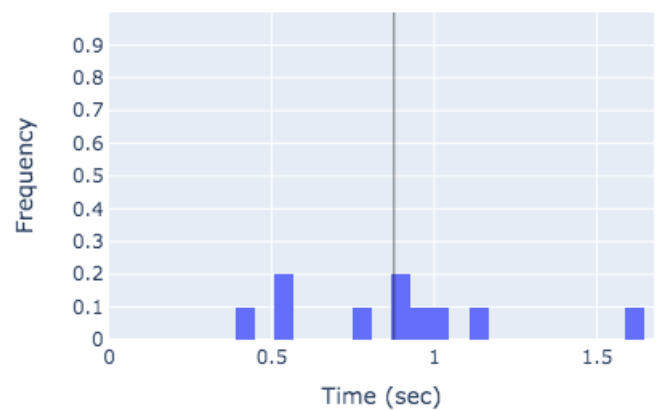
Min: 0.41898107528686523
Max: 1.6150541305541992
|Max-Min|: 1.196073055267334

Case 6: Least Squares (True)

N=10, Nrep=10
Mean = 0.23259320259094238(sec)
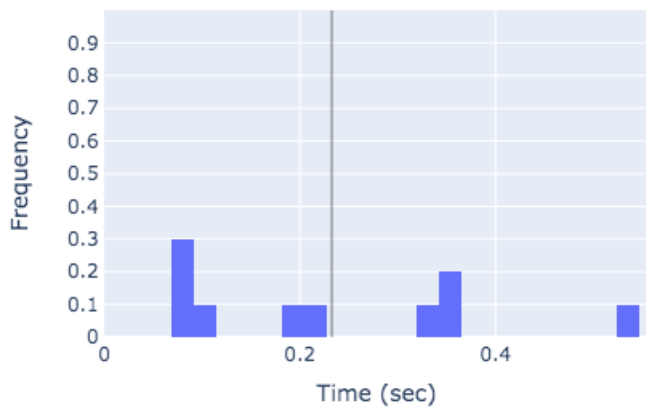
Min: 0.07787084579467773
Max: 0.5339179039001465
|Max-Min|: 0.45604705810546875

N=100, Nrep=10
Mean = 0.11698303222265625(sec)
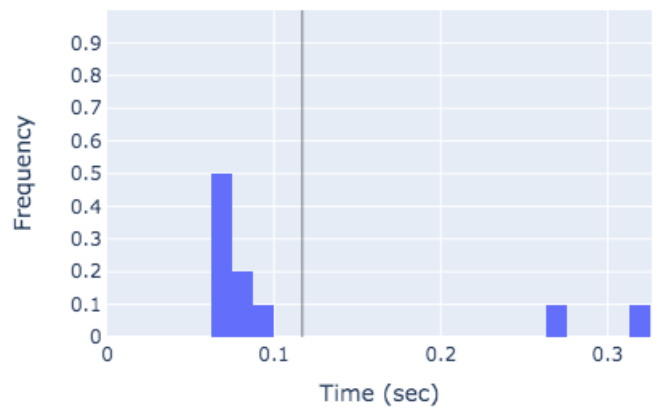
Min: 0.0635530948638916
Max: 0.31397128105163574
|Max-Min|: 0.25041818618774414

Case 7: Least Squares (False)

N=10, Nrep=10
Mean = 0.283130407333374(sec)
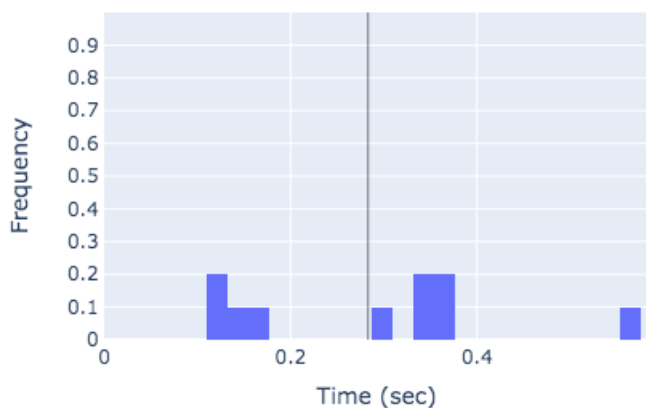
Min: 0.11997532844543457
Max: 0.5622208118438721
|Max-Min|: 0.4422454833984375

N=100, Nrep=10
Mean = 0.15187177658081055(sec)
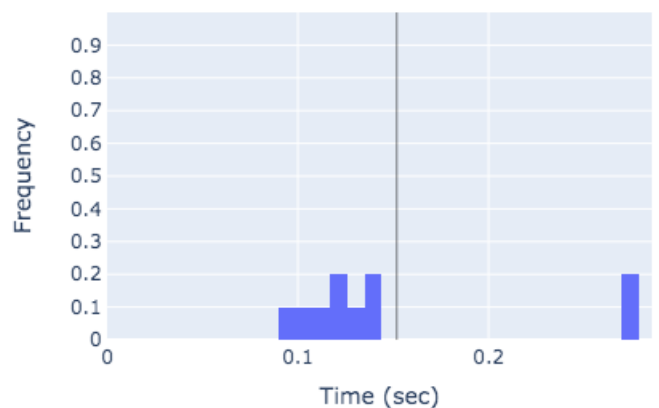
Min: 0.09665679931640625
Max: 0.27661728858947754
|Max-Min|: 0.1799604892730713

MSE of empirical distributions

Mean squared error
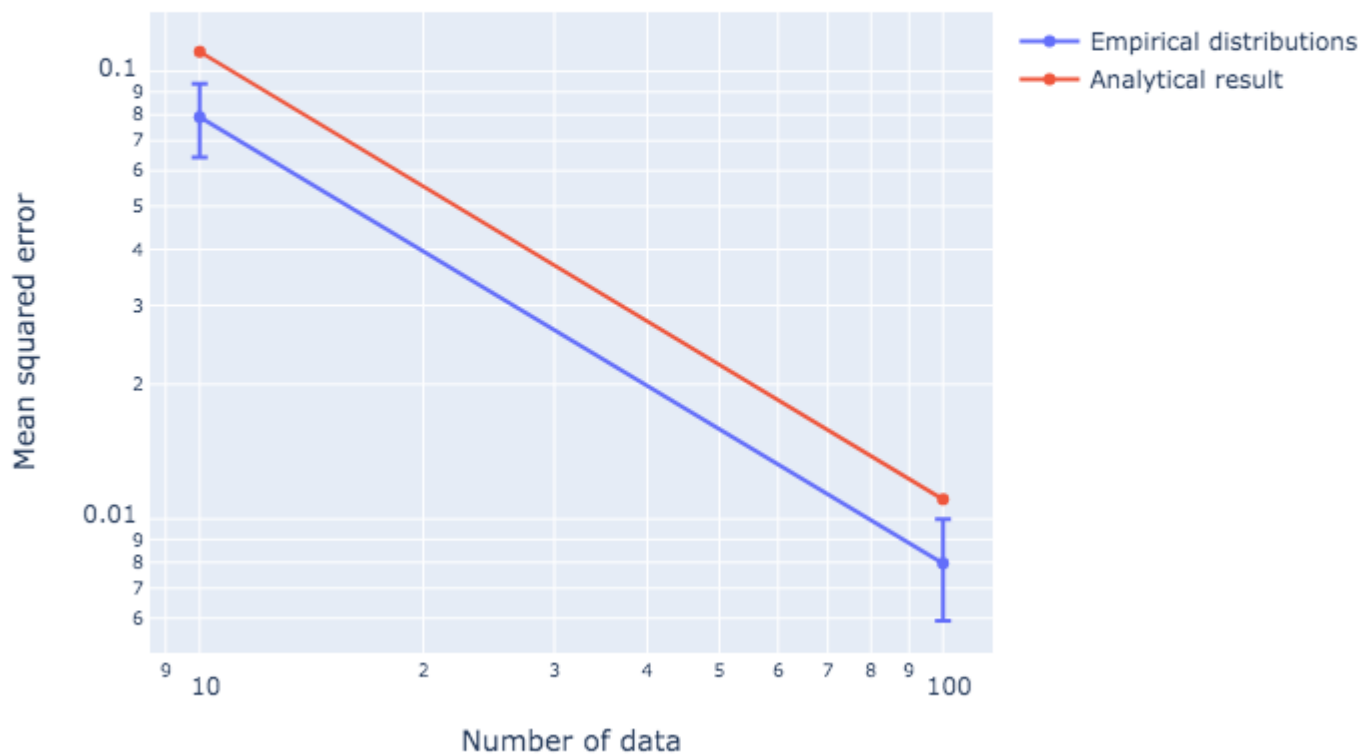
Consistency test

	Name	Type of tomography	Param	Estimator	Loss	Algo	Squared Error to True	Possibly OK	To be checked
0	Linear (True)	Standard Qst	True	Linear	None	None	3.84e-32	OK	not need debug
1	Linear (False)	Standard Qst	False	Linear	None	None	1.06e-31	OK	not need debug
2	ProjectedLinear (True)	Standard Qst	True	Projected Linear	None	None	2.14e-32	OK	not need debug
3	ProjectedLinear (False)	Standard Qst	False	Projected Linear	None	None	1.04e-31	OK	not need debug
4	Maximum-Likelihood (True)	Standard Qst	True	Loss Minimization	Weighted Relative Entropy	Projected Gradient Descent Backtracking	8.42e-18	OK	not need debug
5	Maximum-Likelihood (False)	Standard Qst	False	Loss Minimization	Weighted Relative Entropy	Projected Gradient Descent Backtracking	1.39e-08	NG	not need debug
6	Least Squares (True)	Standard Qst	True	Loss Minimization	Weighted Probability Based Squared Error	Projected Gradient Descent Backtracking	2.21e-31	OK	not need debug

7	Least Squares (False)	Standard Qst	False	Loss Minimization	Weighted Probability Based Squared Error	Projected Gradient Descent Backtracking	4.52e-30	OK	not need debug

MSE of estimators

Comparison of analytical results

Mean squared error
Analytical result
estimator=Linear
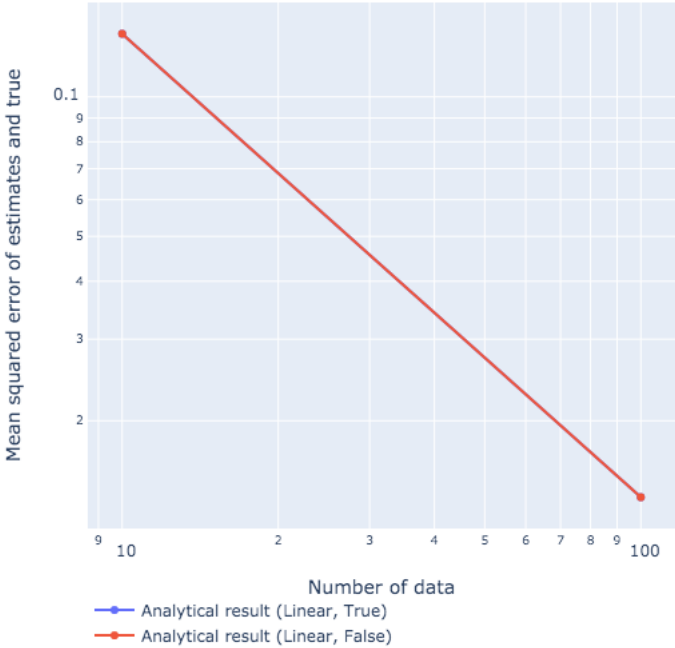
Mean squared error
Analytical result
Cramer-Rao bound
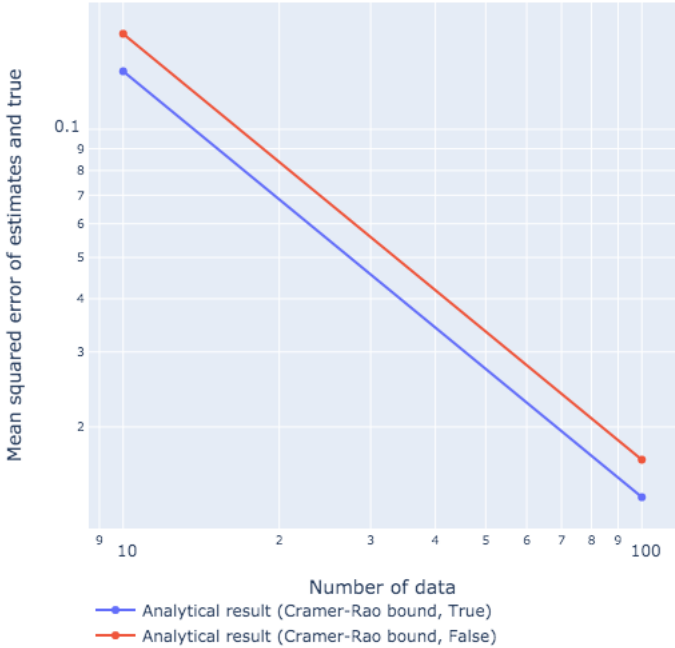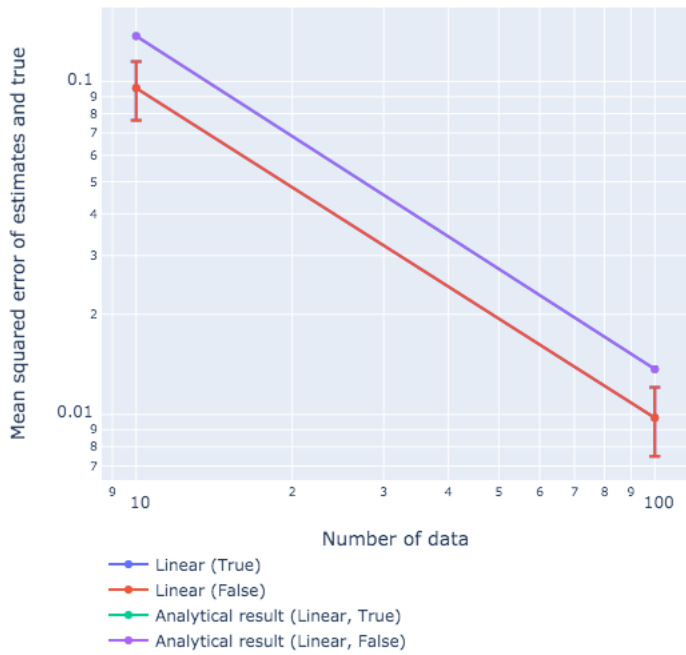
Mean squared error
Analytical result
parametorization=True

Analytical result (Linear, True)
Analytical result (Cramer-Rao bound, True)

Mean squared error
Analytical result
parametorization=False
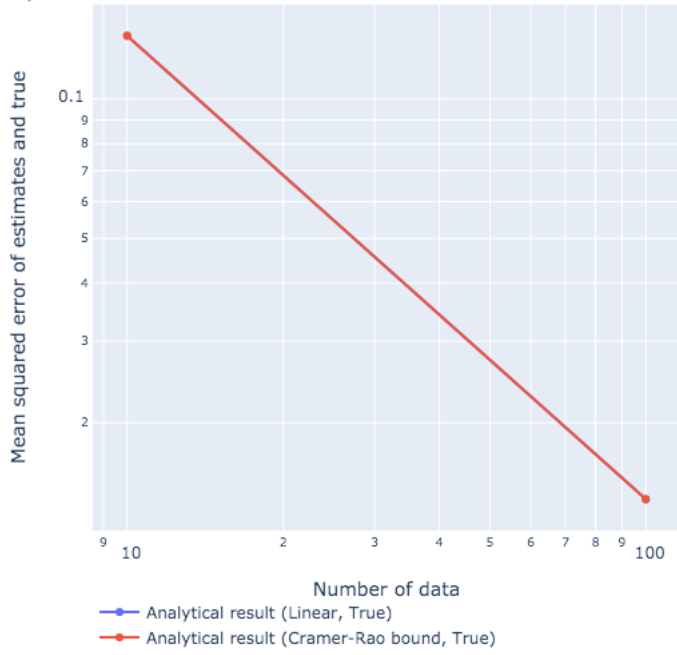
Analytical result (Linear, False)
Analytical result (Cramer-Rao bound, False)
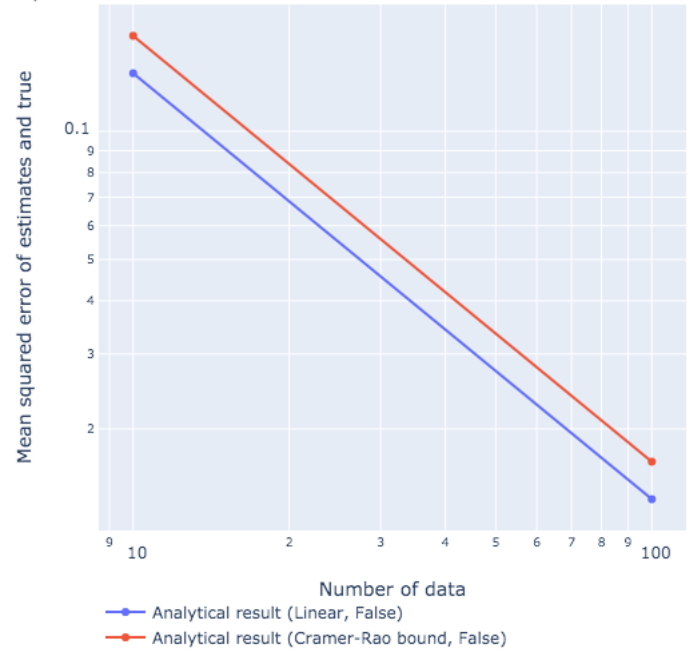
Comparison of parametrization

Mean squared error
Estimator=Linear

Linear (True)
Linear (False)
Analytical result (Linear, True)
Analytical result (Linear, False)

Mean squared error
Estimator=ProjectedLinear

ProjectedLinear (True)
ProjectedLinear (False)

10
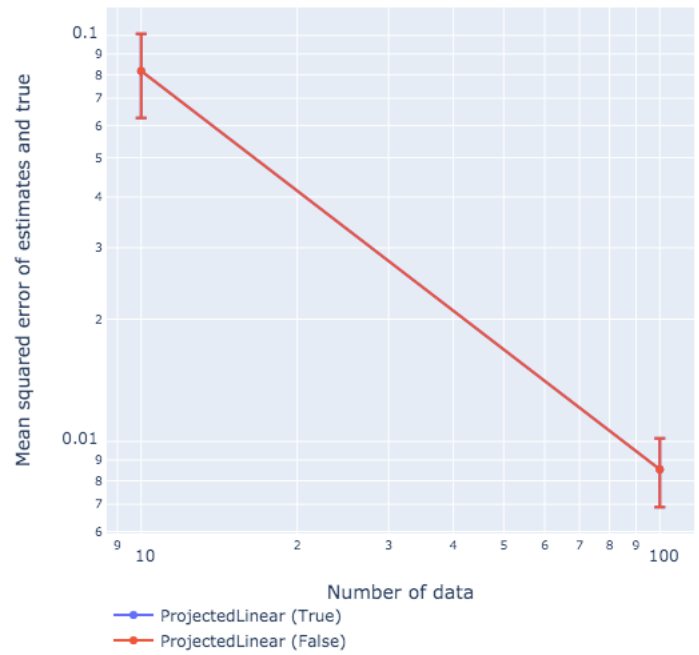
Mean squared error
Estimator=LossMinimization
Loss=WeightedRelativeEntropy
Algo=ProjectedGradientDescentBacktracking

Legend:
- Maximum-Likelihood (True)
- Maximum-Likelihood (False)
- Analytical result (Cramer-Rao bound, True)
- Analytical result (Cramer-Rao bound, False)
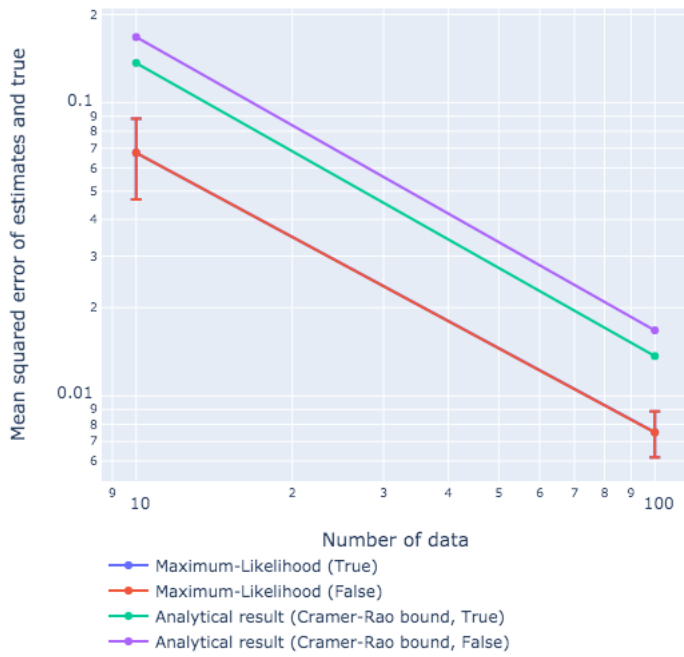
Mean squared error
Estimator=LossMinimization
Loss=WeightedProbabilityBasedSquaredError
Algo=ProjectedGradientDescentBacktracking

Legend:
- Least Squares (True)
- Least Squares (False)
- Analytical result (Cramer-Rao bound, True)
- Analytical result (Cramer-Rao bound, False)
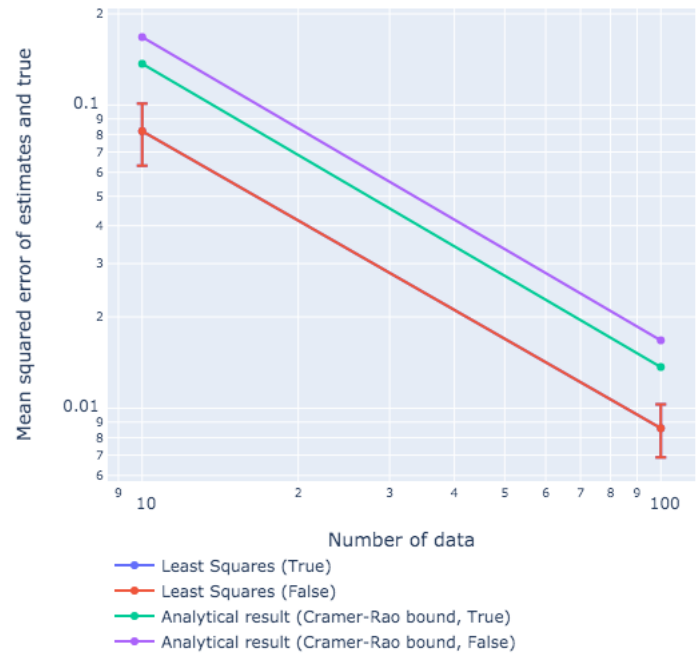
Comparison of estimators

Mean squared error
parametrization=True

Legend:
- Linear (True)
- ProjectedLinear (True)
- Maximum-Likelihood (True)
- Least Squares (True)

Mean squared error
parametrization=False

Legend:
- Linear (False)
- ProjectedLinear (False)
- Maximum-Likelihood (False)
- Least Squares (False)

11
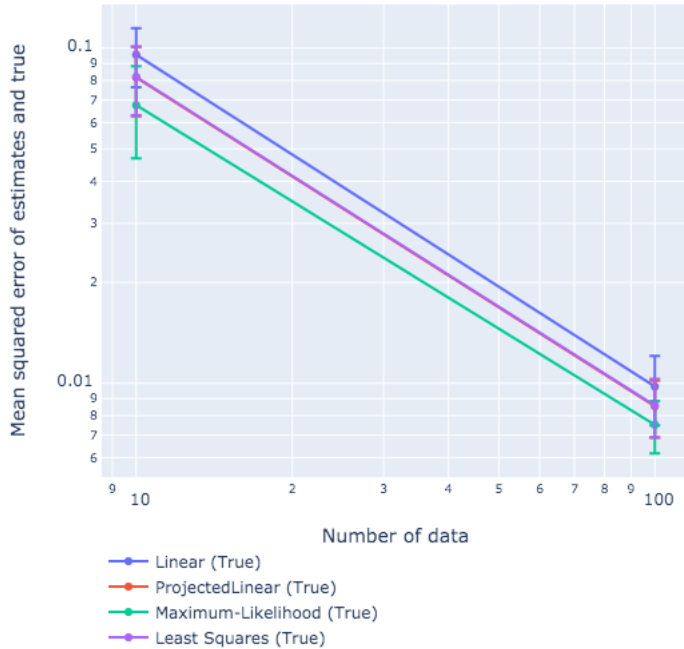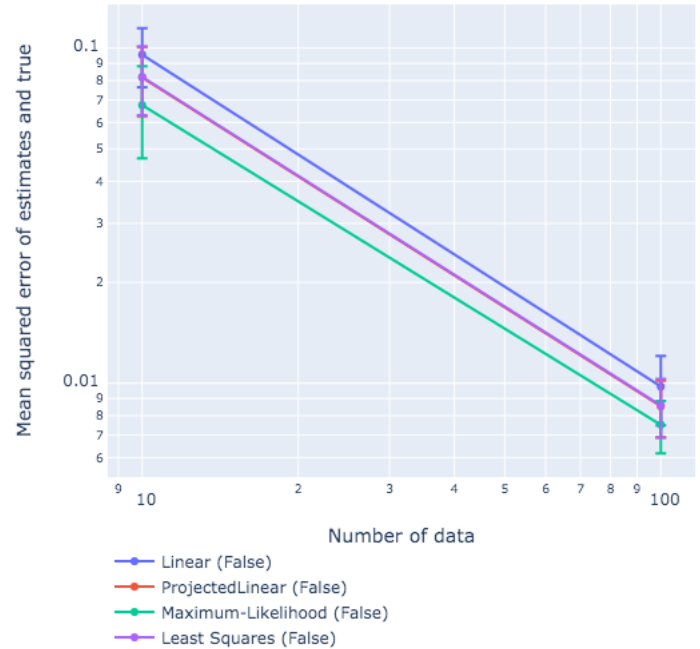
Physicality violation test

Test of equality constraint violation

Case 0: Linear (True)
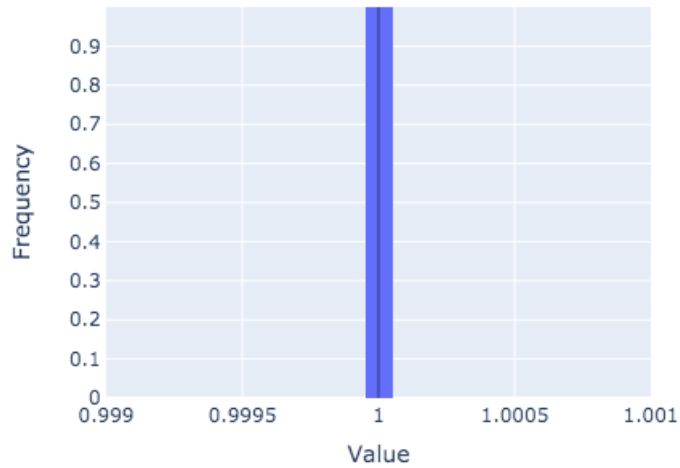
N=10, Nrep=10

Min: 0.9999999999999997
Max: 0.9999999999999999
|Max-Min|: 2.220446049250313e-16
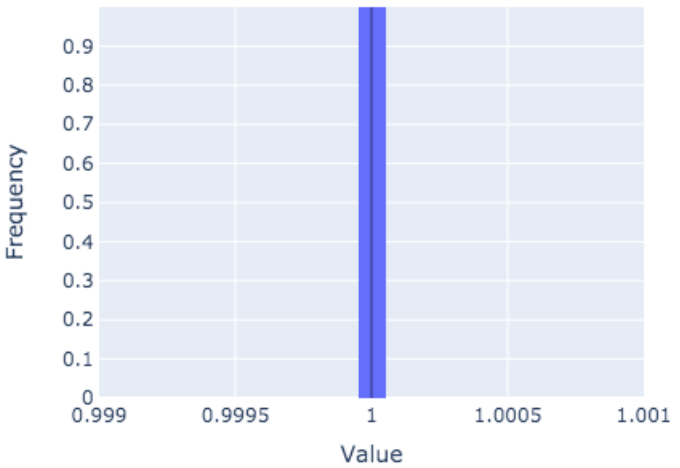
N=100, Nrep=10

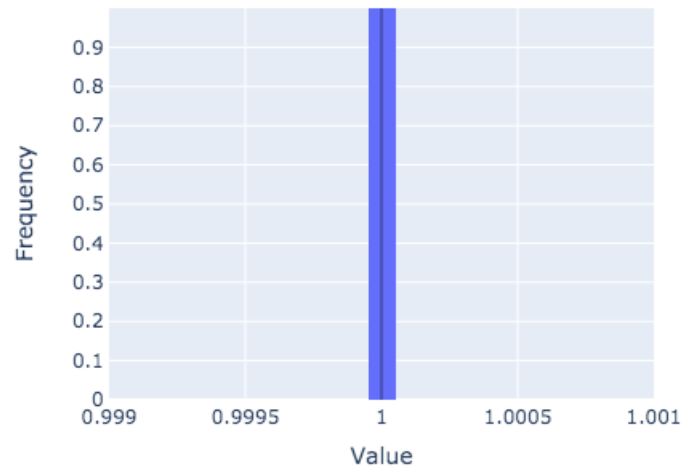Min: 0.9999999999999998
Max: 0.9999999999999999
|Max-Min|: 1.1102230246251565e-16

Case 1: Linear (False)
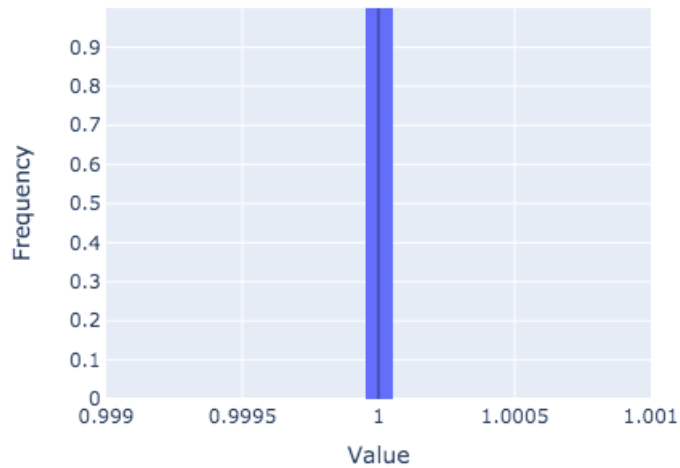
N=10, Nrep=10

Min: 0.9999999999999998
Max: 1.0
|Max-Min|: 2.220446049250313e-16
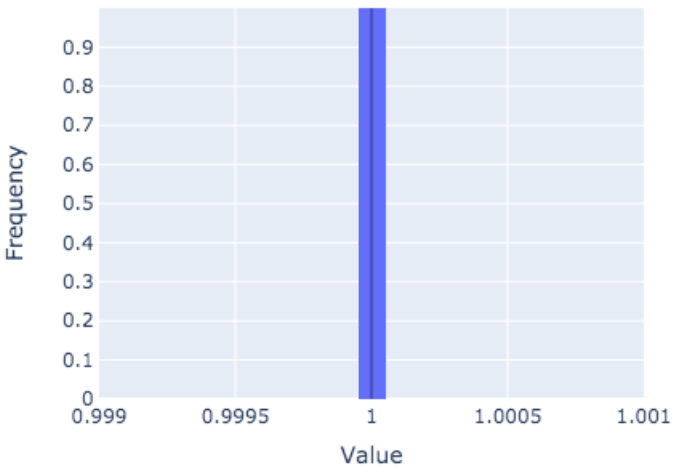
N=100, Nrep=10

Min: 0.9999999999999998
Max: 1.0
|Max-Min|: 2.220446049250313e-16

Case 2: ProjectedLinear (True)
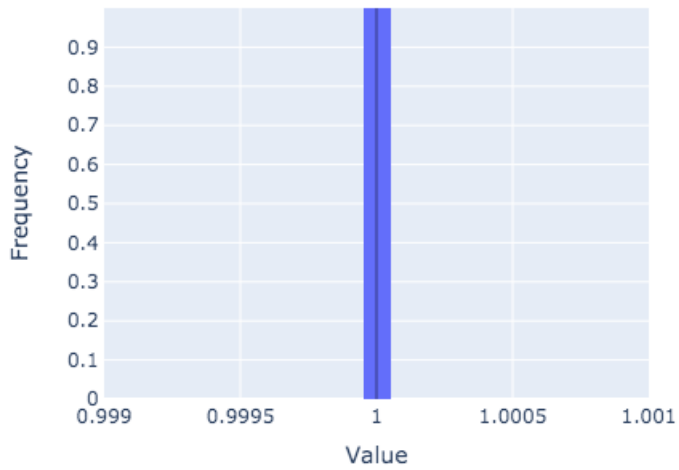
N=10, Nrep=10

Min: 0.9999999999999998
Max: 0.9999999999999998
|Max-Min|: 0.0
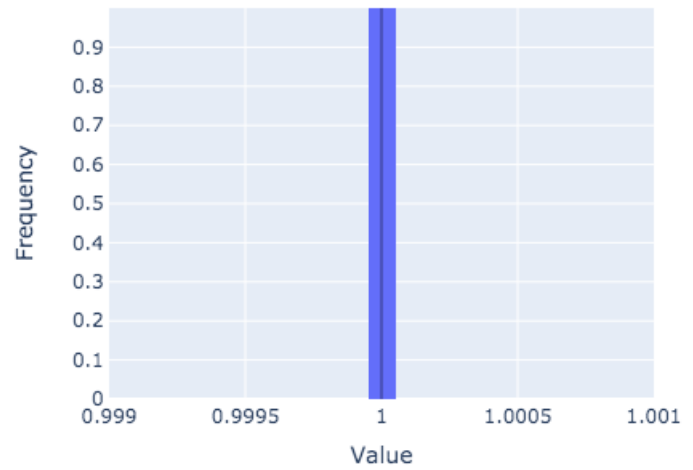
N=100, Nrep=10

Min: 0.9999999999999998
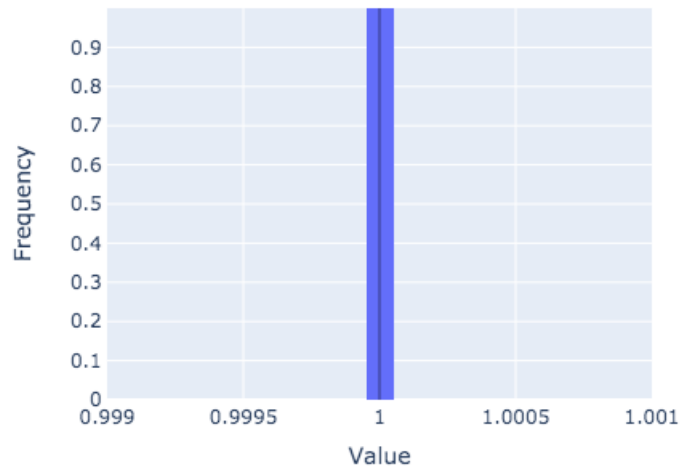Max: 0.9999999999999998
|Max-Min|: 0.0

Case 3: ProjectedLinear (False)
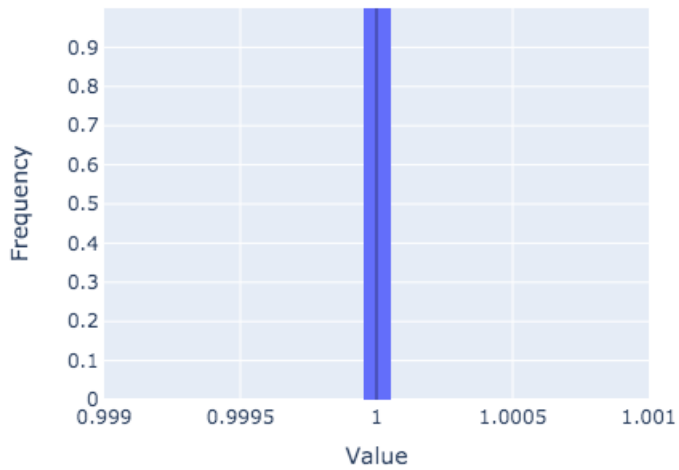
N=10, Nrep=10

Min: 0.9999999999999998
Max: 1.0000001786892174
|Max-Min|: 1.786892176447452e-07

N=100, Nrep=10
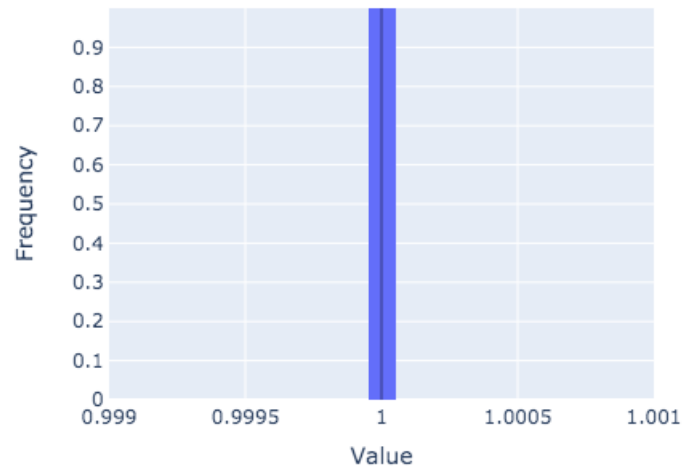
Min: 0.9999999999999997
Max: 1.000000130591659
|Max-Min|: 1.3059165937523431e-07

Case 4: Maximum-Likelihood (True)
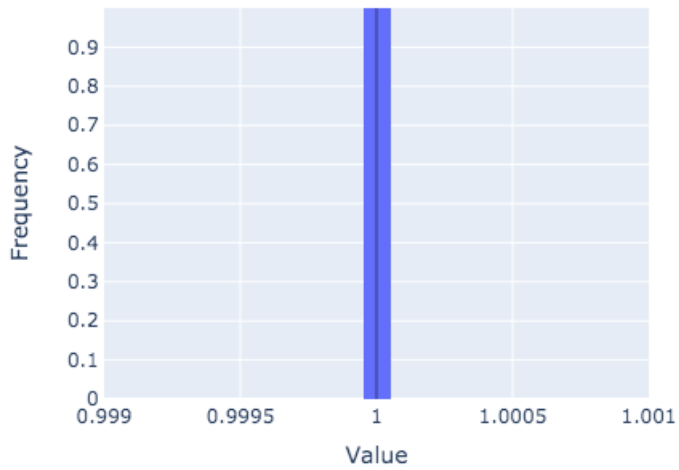
N=10, Nrep=10

Min: 0.9999999999999998
Max: 0.9999999999999998
|Max-Min|: 0.0
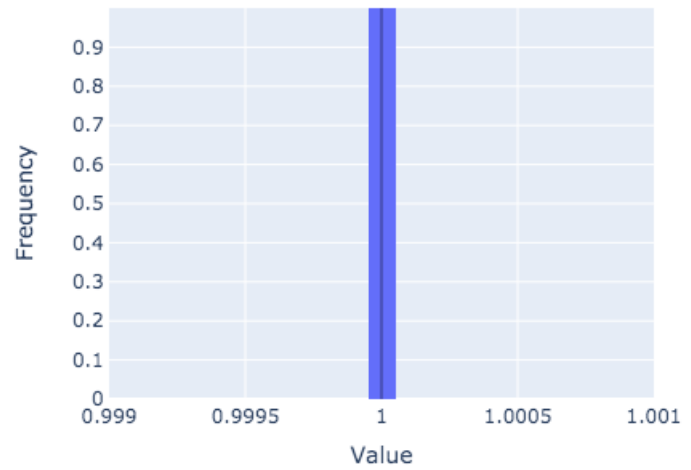
N=100, Nrep=10

Min: 0.9999999999999998
Max: 0.9999999999999998
|Max-Min|: 0.0

Case 5: Maximum-Likelihood (False)
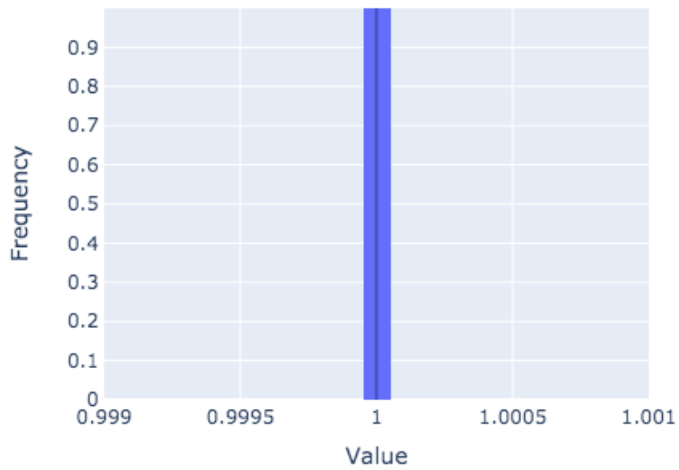
N=10, Nrep=10

Min: 1.000000061404672
Max: 1.0000001784357189
|Max-Min|: 1.7229525162143489e-07

N=100, Nrep=10

Min: 1.000000007962313
Max: 1.000000181521831
|Max-Min|: 1.7355951786868218e-07

Case 6: Least Squares (True)
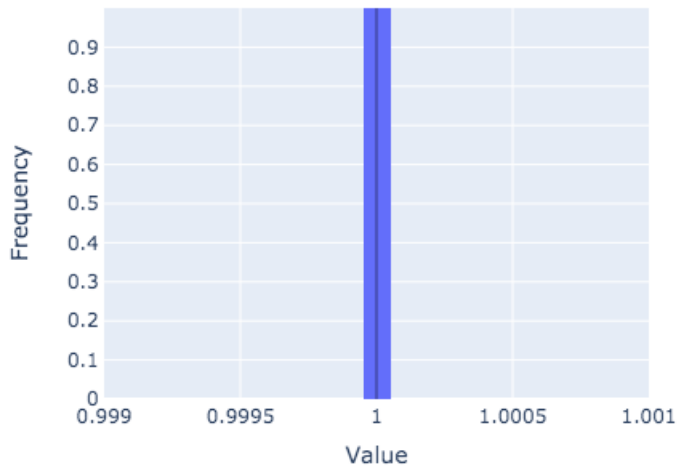
N=10, Nrep=10

Min: 0.9999999999999998
Max: 0.9999999999999998
|Max-Min|: 0.0
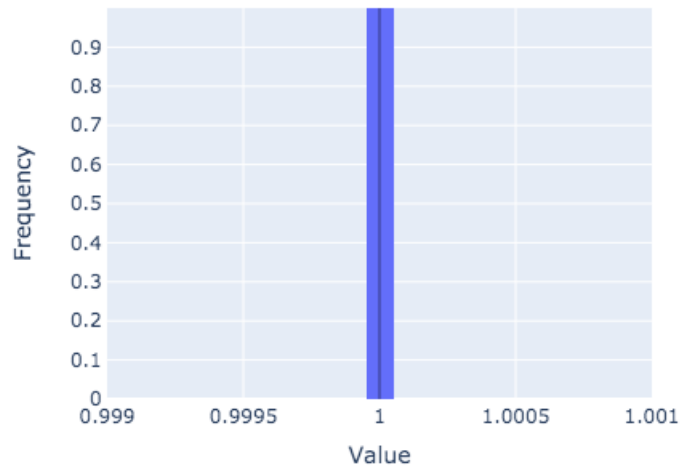
N=100, Nrep=10

Min: 0.9999999999999998
Max: 0.9999999999999998
|Max-Min|: 0.0

Case 7: Least Squares (False)
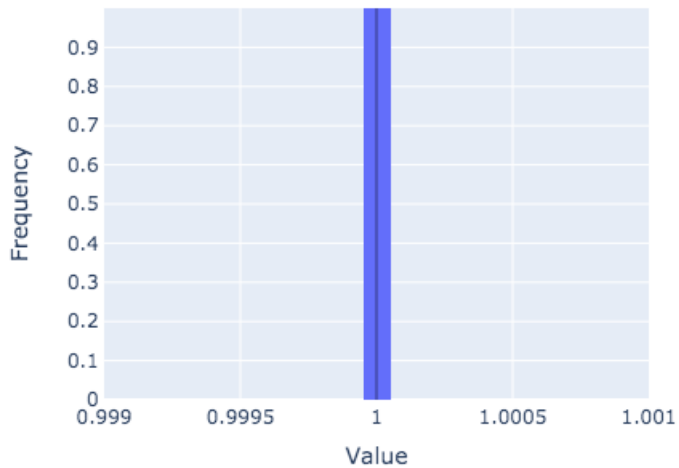
N=10, Nrep=10

Min: 1.0
Max: 1.00000015436117
|Max-Min|: 1.5436117006828454e-07

N=100, Nrep=10

Min: 0.9999999999999998
Max: 1.00000001612437763
|Max-Min|: 1.612437765530217e-07

Test of inequality constraint violation

Eigenvalue

Case 0: Linear (True)

N=10, Nrep=10

Eigenvalue (i=0)

Min: 0.8217439063611547
Max: 1.0868807432325331
|Max-Min|: 0.26513683687137846

N=10, Nrep=10

Eigenvalue (i=1)
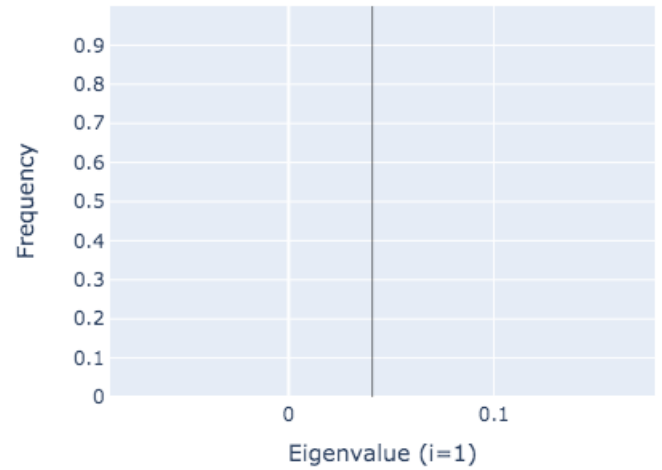
Min: -0.08688074323253331
Max: 0.1782560936388451
|Max-Min|: 0.2651368368713784

N=100, Nrep=10

Eigenvalue (i=0)

Min: 0.9105490618058564
Max: 1.0342338199919683
|Max-Min|: 0.12368475818611191

N=100, Nrep=10

Eigenvalue (i=1)
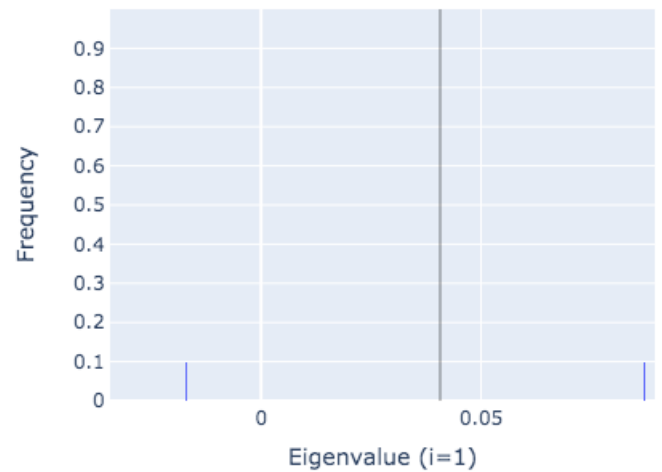
Min: -0.03423381999196835
Max: 0.08945093819414349
|Max-Min|: 0.12368475818611184

Case 1: Linear (False)
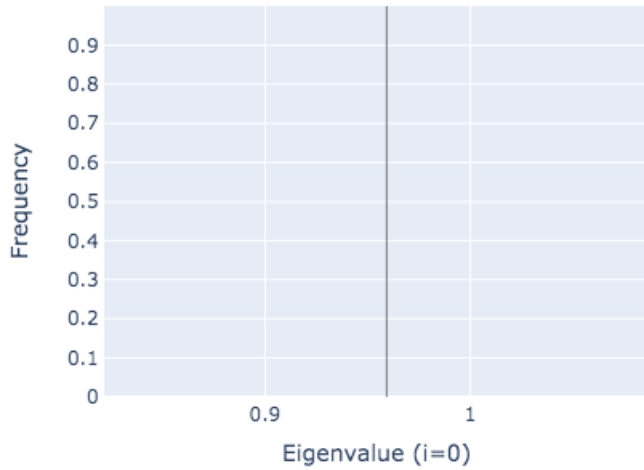
N=10, Nrep=10

Min: 0.8217439063611547
Max: 1.0868807432325331
|Max-Min|: 0.26513683687137846
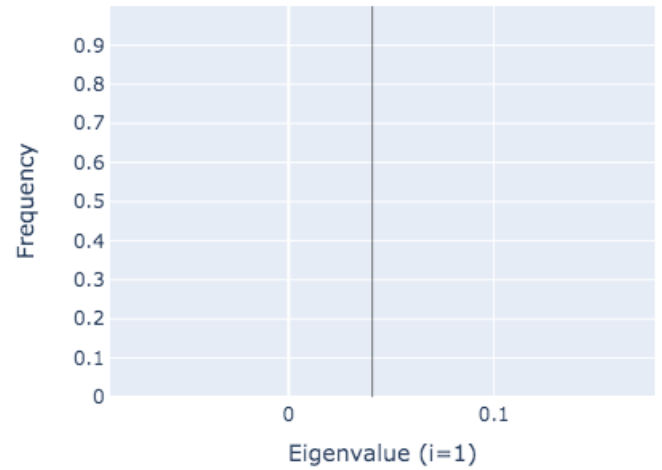
N=10, Nrep=10

Min: -0.08688074323253318
Max: 0.17825609363884512
|Max-Min|: 0.2651368368713783

N=100, Nrep=10
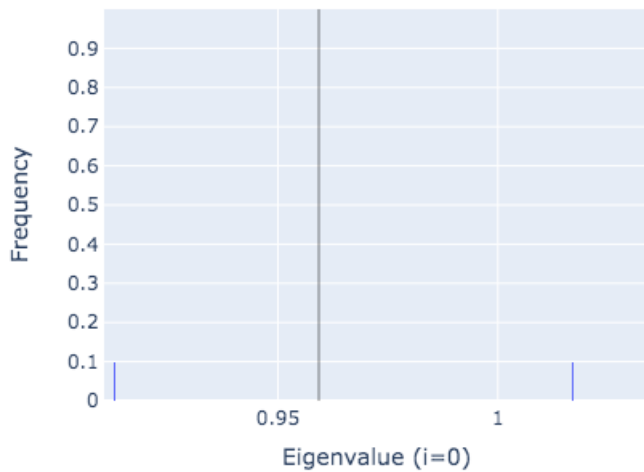
Min: 0.9105490618058564
Max: 1.0342338199919683
|Max-Min|: 0.12368475818611191

N=100, Nrep=10
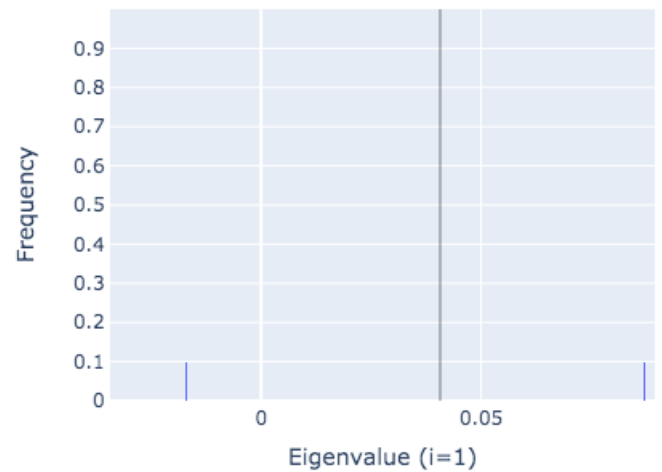
Min: -0.03423381999196848
Max: 0.08945093819414354
|Max-Min|: 0.12368475818611202

Case 2: ProjectedLinear (True)
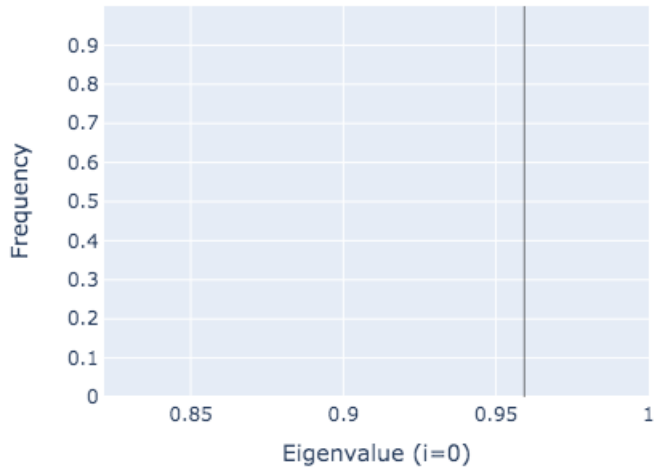
N=10, Nrep=10

Min: 0.8217439063611546
Max: 1.0000000893446086
|Max-Min|: 0.17825618298345403

N=10, Nrep=10
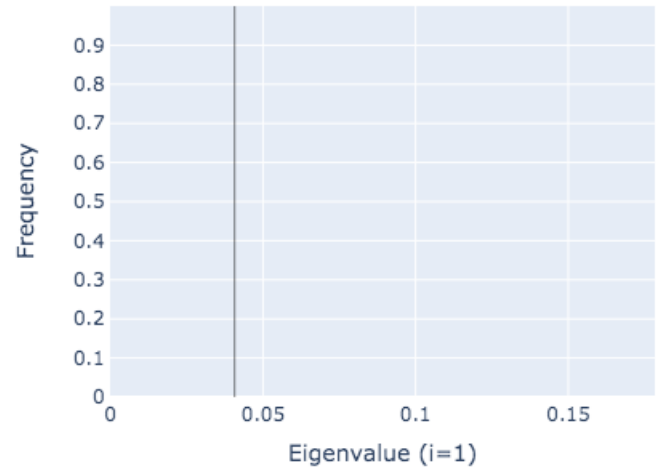
Min: -8.934460889523099e-08
Max: 0.17825609363884518
|Max-Min|: 0.17825618298345408

N=100, Nrep=10
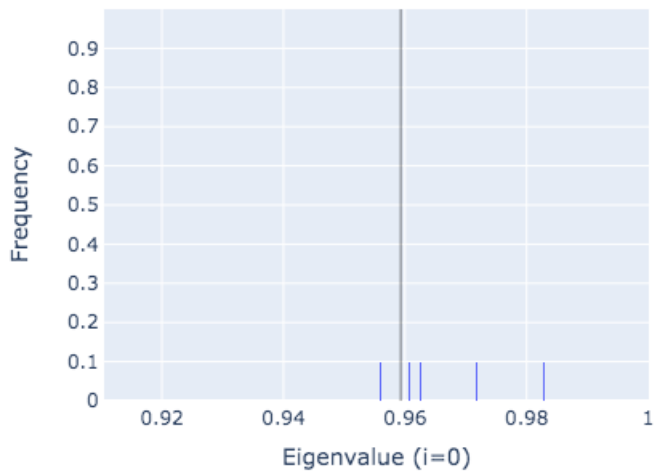
Min: 0.9105490618058562
Max: 1.0000000652958296
|Max-Min|: 0.08945100348997348

N=100, Nrep=10
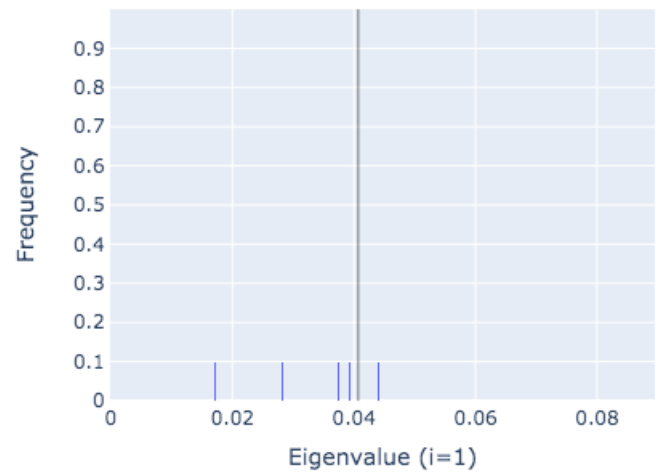
Min: -6.529582974659776e-08
Max: 0.08945093819414354
|Max-Min|: 0.08945100348997329
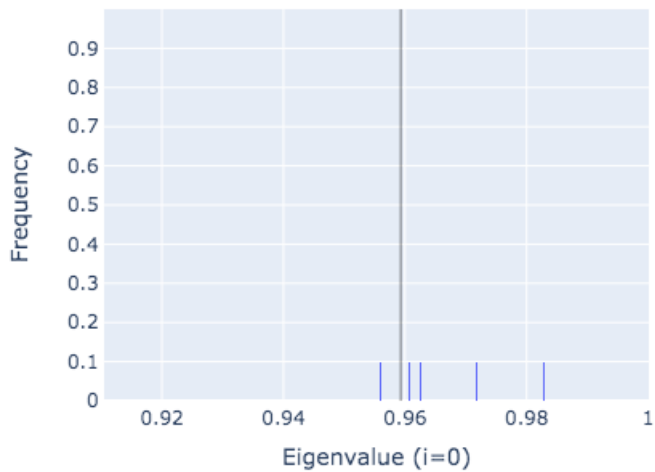
Case 3: ProjectedLinear (False)
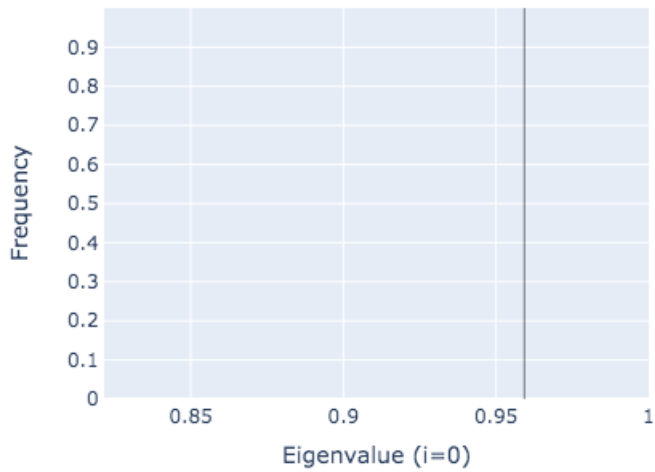
N=10, Nrep=10

Min: 0.8217439063611548
Max: 1.0000001786892174
|Max-Min|: 0.17825627232806263

N=10, Nrep=10
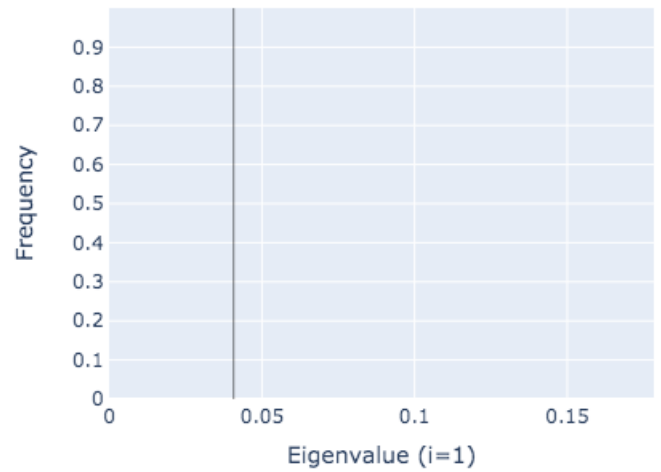
Min: -8.326672684688674e-17
Max: 0.1782560936388453
|Max-Min|: 0.17825609363884537

N=100, Nrep=10

Min: 0.9105490618058562
Max: 1.0000001305916593
|Max-Min|: 0.08945106878580311

N=100, Nrep=10
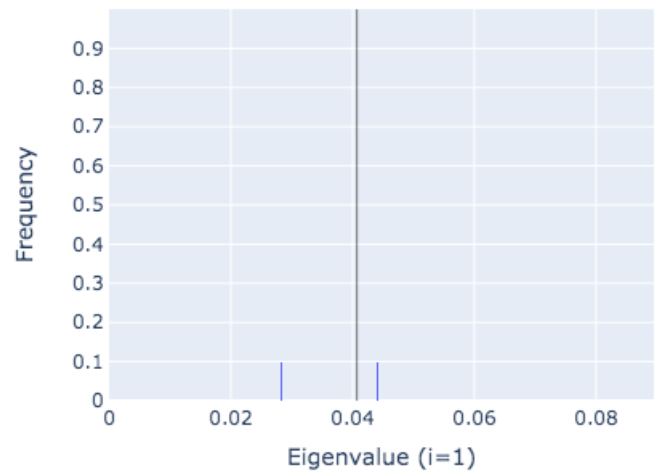
Min: -1.3183898417423734e-16
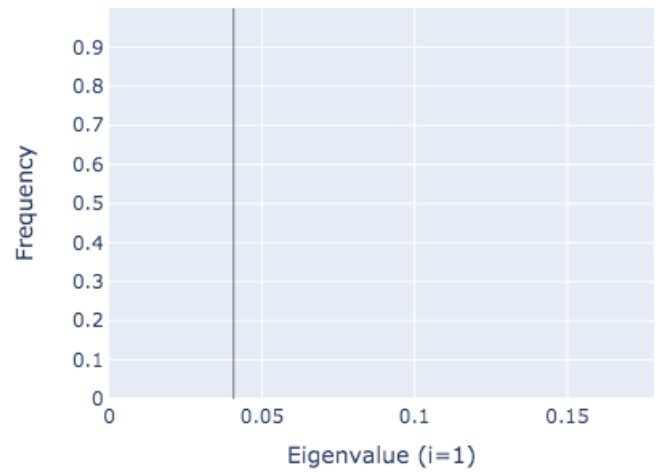Max: 0.08945093819414349
|Max-Min|: 0.08945093819414363
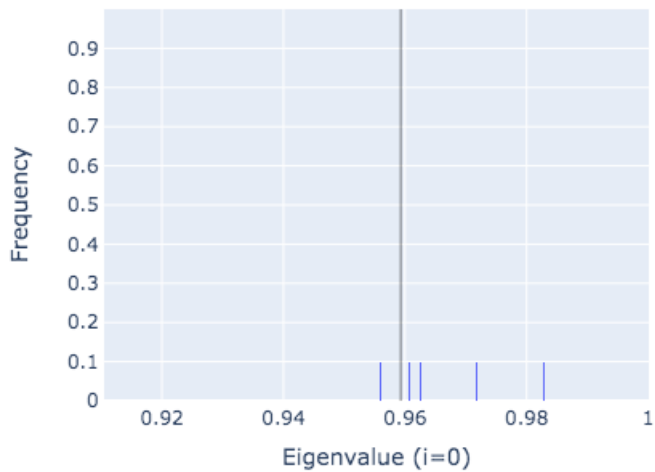
Case 4: Maximum-Likelihood (True)

N=10, Nrep=10

Eigenvalue (i=0)
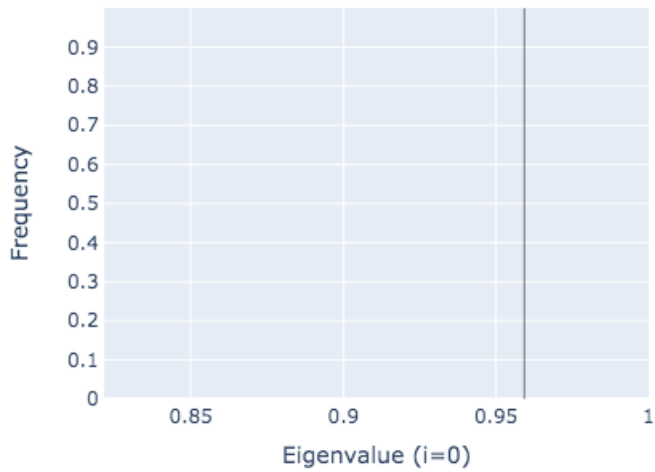
Min: 0.8217439099529941
Max: 1.000000080374627
|Max-Min|: 0.1782561704216329

N=10, Nrep=10

Eigenvalue (i=1)

Min: -8.037462703724962e-08
Max: 0.17825609004700563
|Max-Min|: 0.17825617042163266

N=100, Nrep=10

Eigenvalue (i=0)

Min: 0.9105490612563245
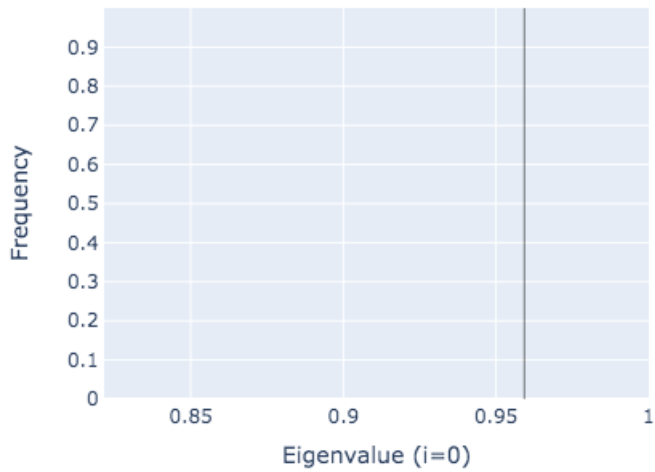Max: 1.0000000786009928
|Max-Min|: 0.08945101734466832

N=100, Nrep=10

Eigenvalue (i=1)
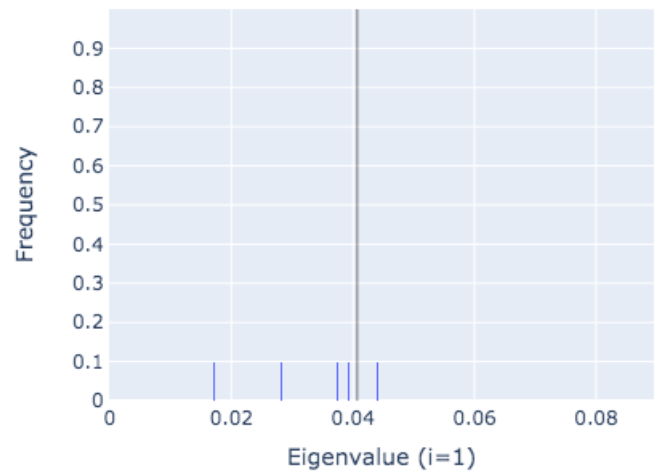
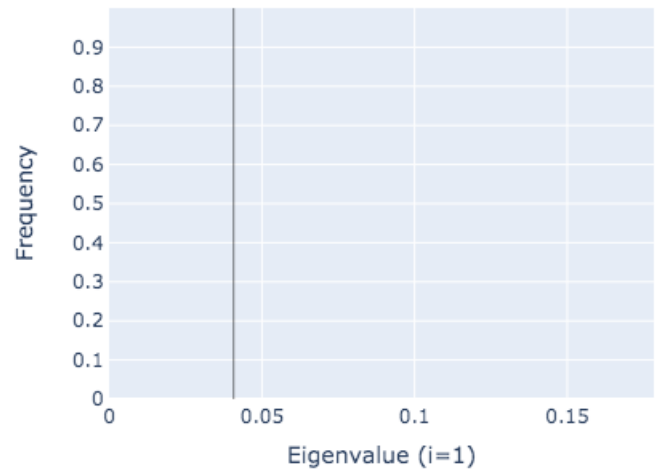Min: -7.860099304800652e-08
Max: 0.08945093874367524
|Max-Min|: 0.08945101734466829
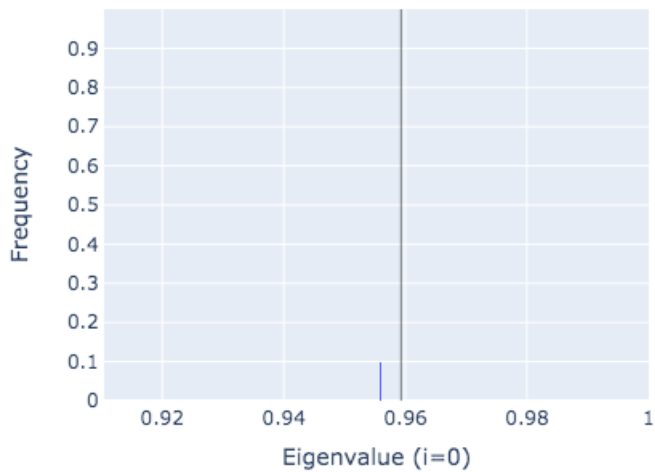
Case 5: Maximum-Likelihood (False)

N=10, Nrep=10

Eigenvalue (i=0)

Min: 0.8217419203339902
Max: 1.0000001784357186
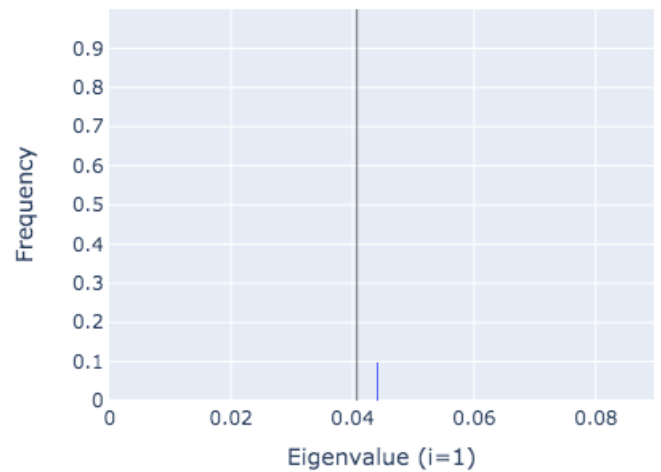|Max-Min|: 0.17825825810172846

N=10, Nrep=10

Eigenvalue (i=1)

Min: 1.1102230246251565e-16
Max: 0.17825808580647715
|Max-Min|: 0.17825808580647703

N=100, Nrep=10

Eigenvalue (i=0)

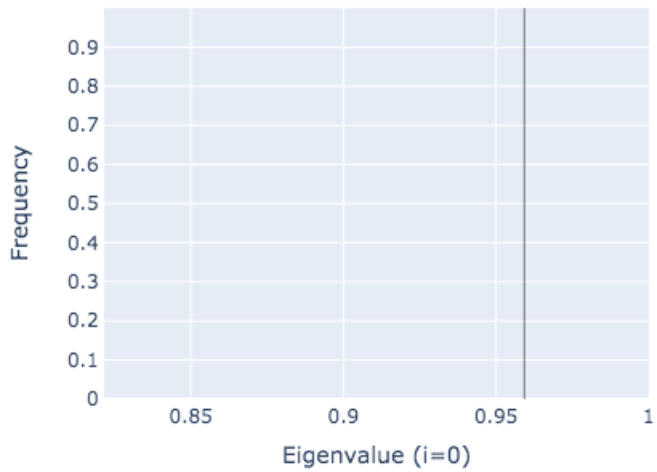Min: 0.9105361357403978
Max: 1.0000001815218298
|Max-Min|: 0.08946404578143197

N=100, Nrep=10

Eigenvalue (i=1)

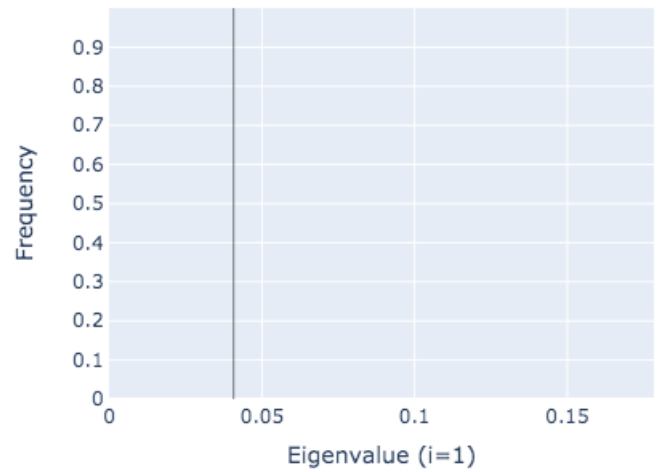Min: 1.175708835843281e-15
Max: 0.08946387235627798
|Max-Min|: 0.0894638723562768

Case 6: Least Squares (True)
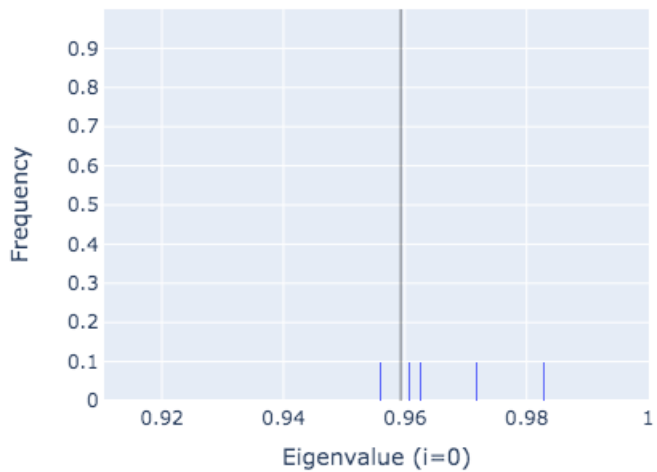
N=10, Nrep=10

Min: 0.8217439063611545
Max: 1.000000080061129
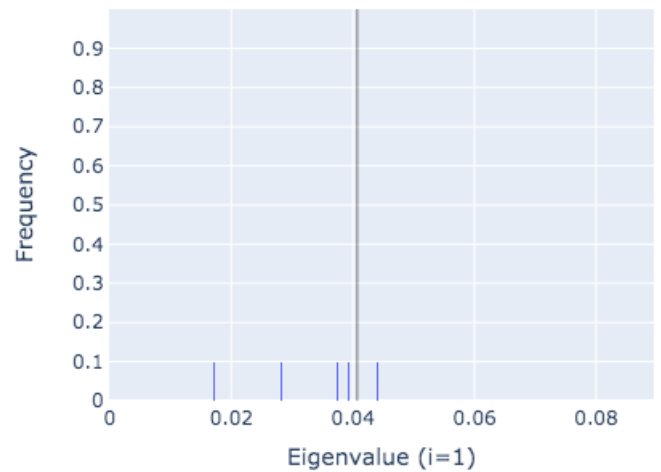|Max-Min|: 0.1782561736999746

N=10, Nrep=10

Min: -8.006112933722775e-08
Max: 0.17825609363884526
|Max-Min|: 0.1782561736999746

N=100, Nrep=10

Min: 0.9105490618058559
Max: 1.000000069820626
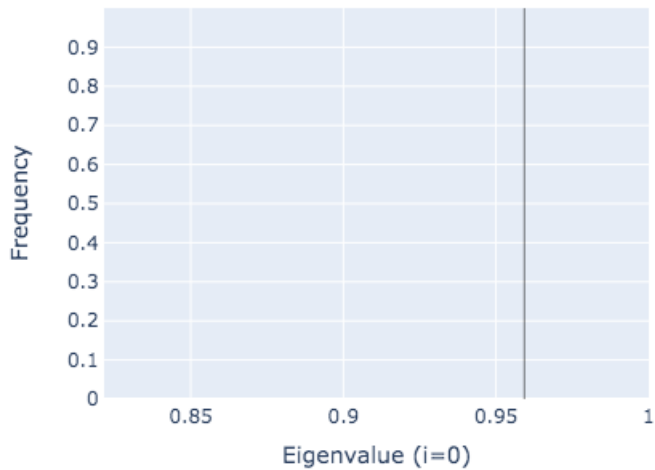|Max-Min|: 0.08945100801477002

N=100, Nrep=10

Min: -6.982062614006856e-08
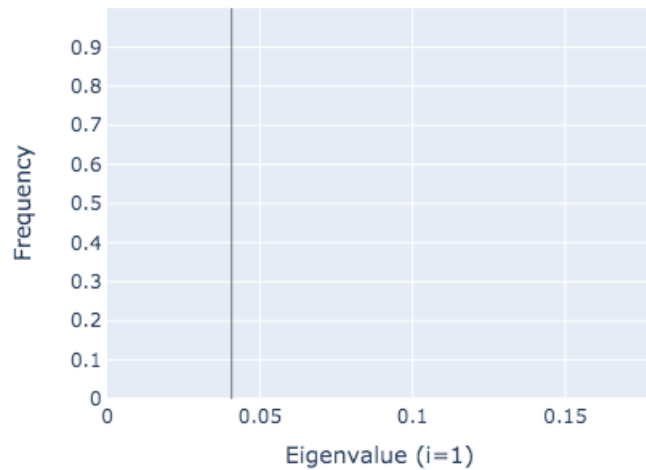Max: 0.0894509381941439
|Max-Min|: 0.08945100801477004

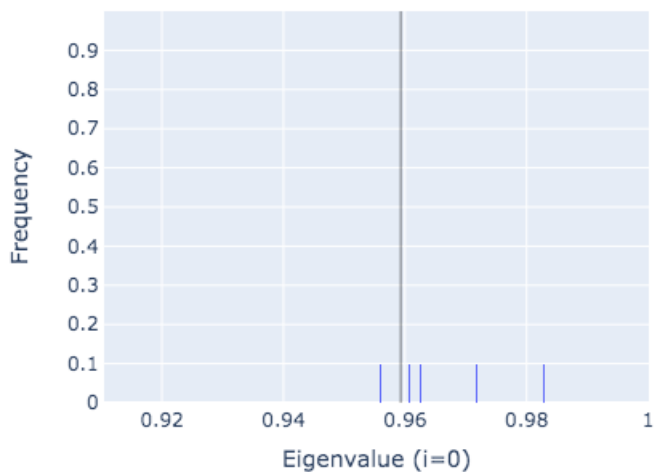**Case 7: Least Squares (False)**

N=10, Nrep=10

Eigenvalue (i=0)

Min: 0.821743906361156
Max: 1.00000015436117
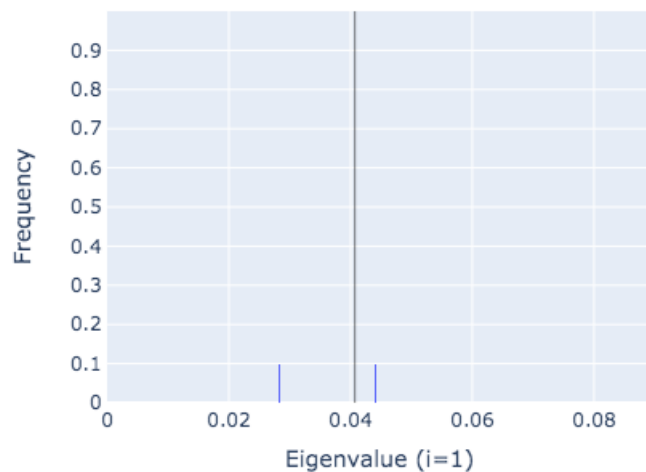|Max-Min|: 0.17825624800001405

N=10, Nrep=10

Eigenvalue (i=1)

Min: -1.3877787807814457e-16
Max: 0.1782560936388442
|Max-Min|: 0.17825609363884434

N=100, Nrep=10

Eigenvalue (i=0)

Min: 0.910549061805857
Max: 1.000000161243776
|Max-Min|: 0.08945109943791907

N=100, Nrep=10

Eigenvalue (i=1)

Min: 4.7704895589362195e-17
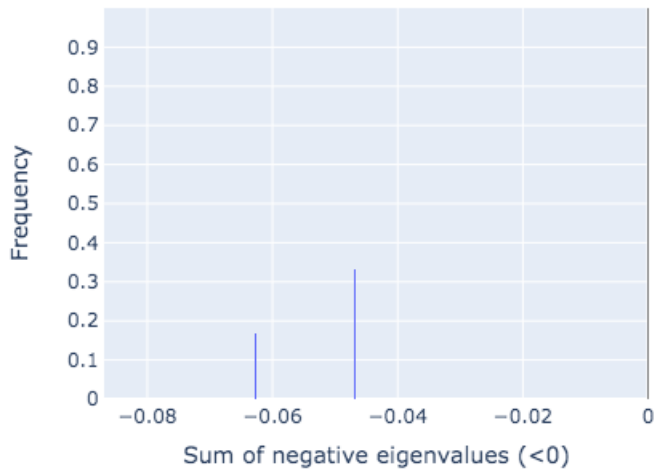Max: 0.0894509381941427
|Max-Min|: 0.08945093819414265

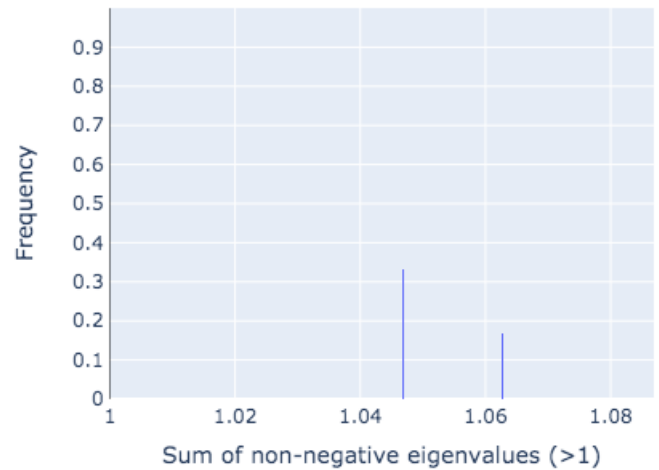## Sum of unphysical eigenvalues

Case 0: Linear (True)

N=10
Number of unphysical estimates=6

N=10, Nrep=10

Sum of negative eigenvalues (<0)

Min: -0.08688074323253331
Max: -0.002998403797154006
|Max-Min|: 0.0838823394353793

N=10, Nrep=10

Sum of non-negative eigenvalues (>1)
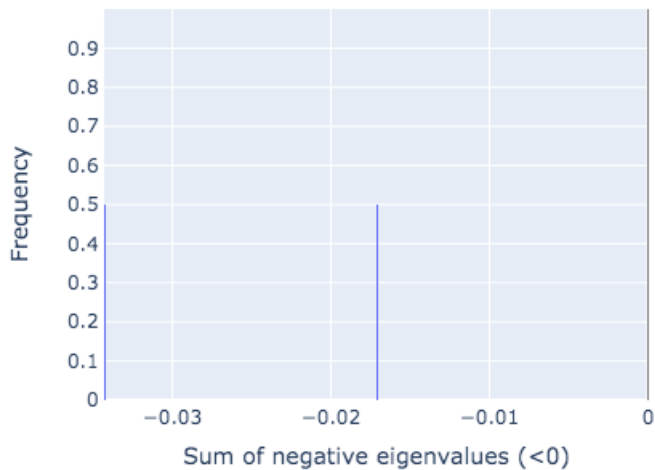
Min: 1.0029984037971538
Max: 1.0868807432325331
|Max-Min|: 0.08388233943537937

N=100
Number of unphysical estimates=2

N=100, Nrep=10

Sum of negative eigenvalues (<0)

Min: -0.03423381999196835
Max: -0.017068185480011048
|Max-Min|: 0.017165634511957305

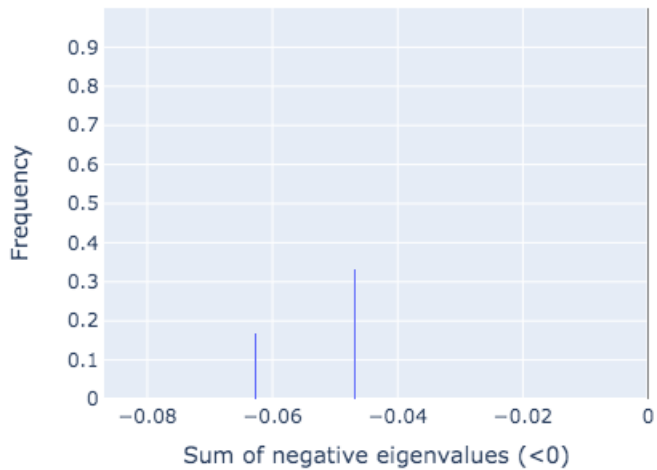## N=100, Nrep=10

Sum of non-negative eigenvalues (>1)
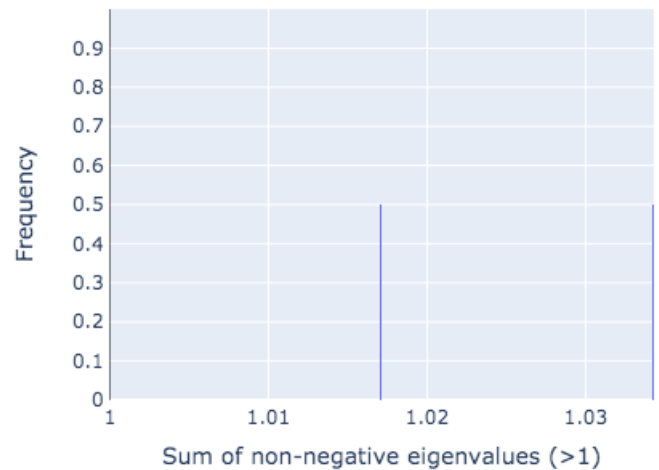
Min: 1.017068185480011
Max: 1.0342338199919683
|Max-Min|: 0.017165634511957295

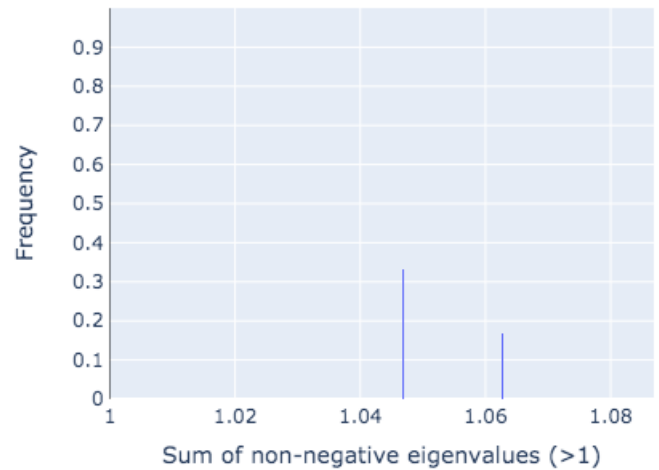## Case 1: Linear (False)

N=10
Number of unphysical estimates=6

N=10, Nrep=10

Sum of negative eigenvalues (<0)

Min: -0.08688074323253318
Max: -0.0029984037971540756
|Max-Min|: 0.08388233943537911

N=10, Nrep=10
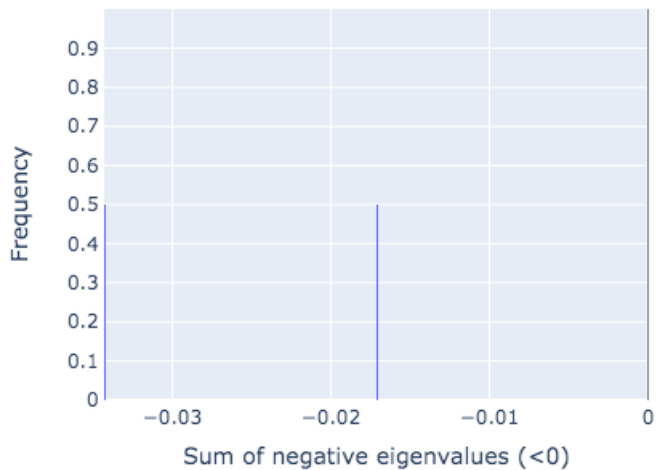

Sum of non-negative eigenvalues (>1)

Min: 1.0029984037971538
Max: 1.0868807432325331
|Max-Min|: 0.08388233943537937

N=100
Number of unphysical estimates=2

N=100, Nrep=10

Sum of negative eigenvalues (<0)

Min: -0.03423381999196848
Max: -0.017068185480010926
|Max-Min|: 0.017165634511957552

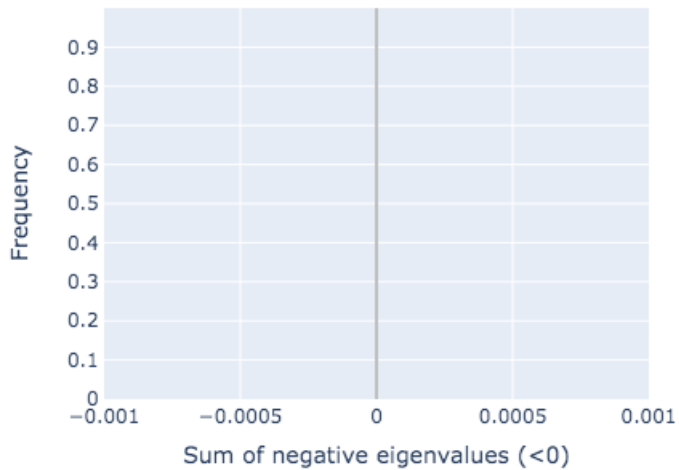## N=100, Nrep=10

Sum of non-negative eigenvalues (>1)

Min: 1.017068185480011
Max: 1.0342338199919683
|Max-Min|: 0.017165634511957295

Case 2: ProjectedLinear (True)

N=10
Number of unphysical estimates=0
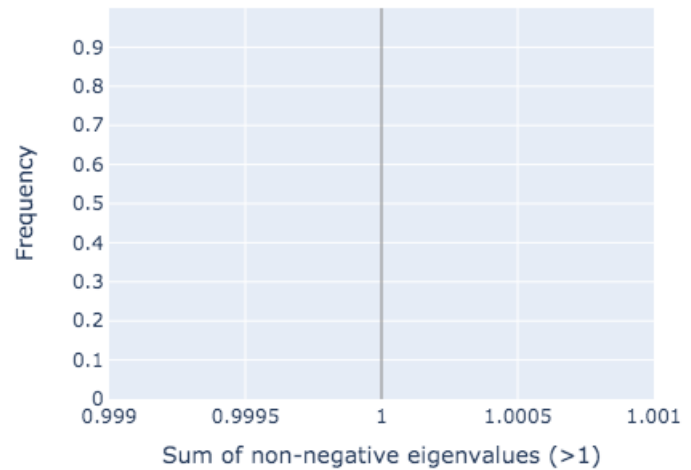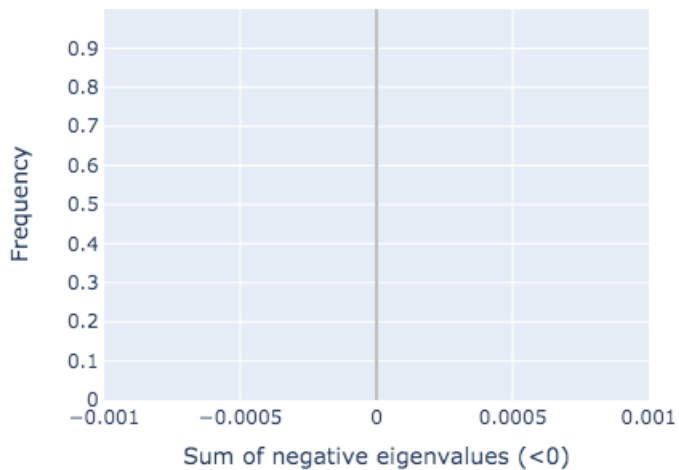
N=10, Nrep=10

Sum of negative eigenvalues (<0)

Min: -
Max: -
|Max-Min|: -

N=10, Nrep=10

Sum of non-negative eigenvalues (>1)

Min: -
Max: -
|Max-Min|: -

N=100
Number of unphysical estimates=0

N=100, Nrep=10

Sum of negative eigenvalues (<0)

Min: -
Max: -
|Max-Min|: -

N=100, Nrep=10

Sum of non-negative eigenvalues (>1)
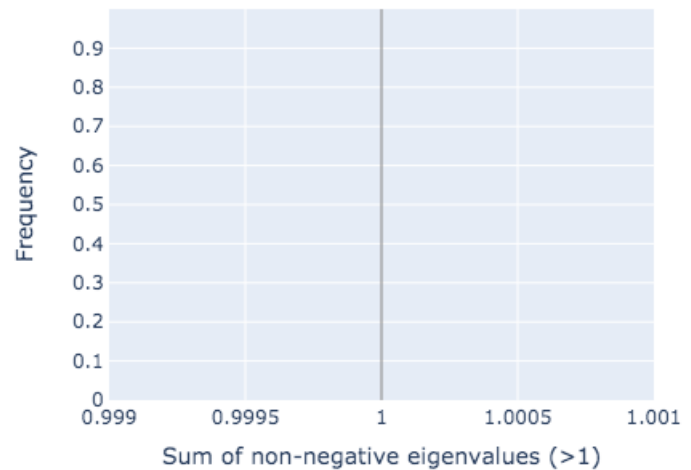
Min: -
Max: -
|Max-Min|: -

Case 3: ProjectedLinear (False)

N=10
Number of unphysical estimates=0

N=10, Nrep=10

Frequency

Sum of negative eigenvalues (<0)

Min: -
Max: -
|Max-Min|: -

N=10, Nrep=10

Frequency

Sum of non-negative eigenvalues (>1)

Min: -
Max: -
|Max-Min|: -

N=100
Number of unphysical estimates=0

N=100, Nrep=10

Frequency

Sum of negative eigenvalues (<0)

Min: -
Max: -
|Max-Min|: -

N=100, Nrep=10

Frequency

Sum of non-negative eigenvalues (>1)

Min: -
Max: -
|Max-Min|: -

Case 4: Maximum-Likelihood (True)

N=10
Number of unphysical estimates=0

N=10, Nrep=10

Frequency (y-axis: 0 to 0.9)
Sum of negative eigenvalues (<0) (x-axis: −0.001 to 0.001)

Min: -
Max: -
|Max-Min|: -

N=10, Nrep=10

Frequency (y-axis: 0 to 0.9)
Sum of non-negative eigenvalues (>1) (x-axis: 0.999 to 1.001)

Min: -
Max: -
|Max-Min|: -

N=100
Number of unphysical estimates=0

N=100, Nrep=10

Frequency (y-axis: 0 to 0.9)
Sum of negative eigenvalues (<0) (x-axis: −0.001 to 0.001)

Min: -
Max: -
|Max-Min|: -

N=100, Nrep=10

Frequency (y-axis: 0 to 0.9)
Sum of non-negative eigenvalues (>1) (x-axis: 0.999 to 1.001)

Min: -
Max: -
|Max-Min|: -

Case 5: Maximum-Likelihood (False)

N=10
Number of unphysical estimates=0

N=10, Nrep=10

Sum of negative eigenvalues (<0)
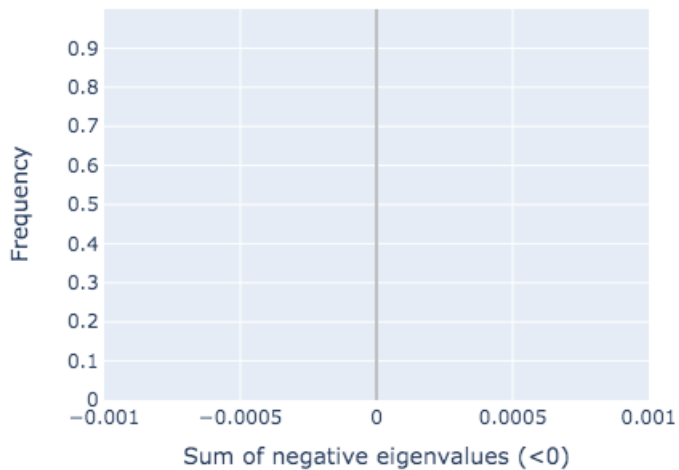
Min: -
Max: -
|Max-Min|: -

N=10, Nrep=10

Sum of non-negative eigenvalues (>1)

Min: -
Max: -
|Max-Min|: -

N=100
Number of unphysical estimates=0

N=100, Nrep=10

Sum of negative eigenvalues (<0)

Min: -
Max: -
|Max-Min|: -

N=100, Nrep=10

Sum of non-negative eigenvalues (>1)

Min: -
Max: -
|Max-Min|: -

Case 6: Least Squares (True)

N=10
Number of unphysical estimates=0

N=10, Nrep=10

Sum of negative eigenvalues (<0)

Min: -
Max: -
|Max-Min|: -

N=10, Nrep=10

Sum of non-negative eigenvalues (>1)

Min: -
Max: -
|Max-Min|: -

N=100
Number of unphysical estimates=0

N=100, Nrep=10

Sum of negative eigenvalues (<0)

Min: -
Max: -
|Max-Min|: -

N=100, Nrep=10

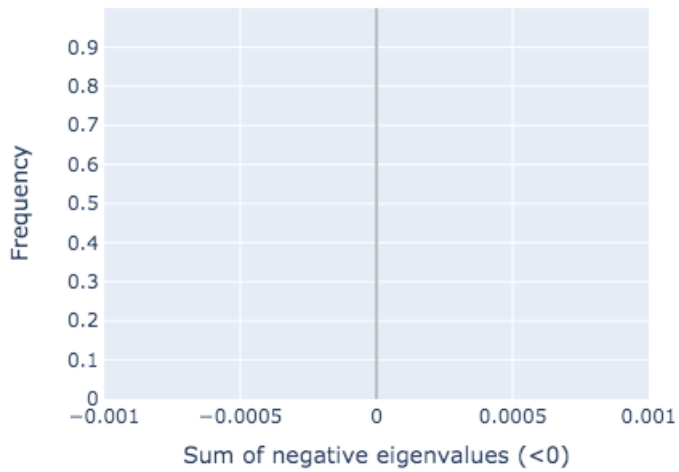Sum of non-negative eigenvalues (>1)

Min: -
Max: -
|Max-Min|: -
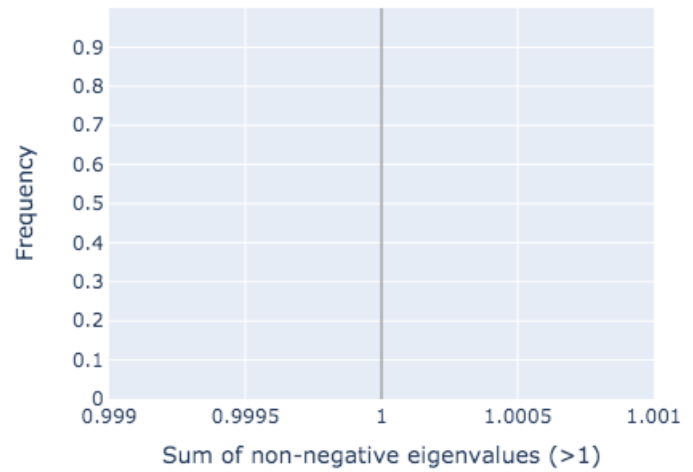
Case 7: Least Squares (False)

N=10
Number of unphysical estimates=0

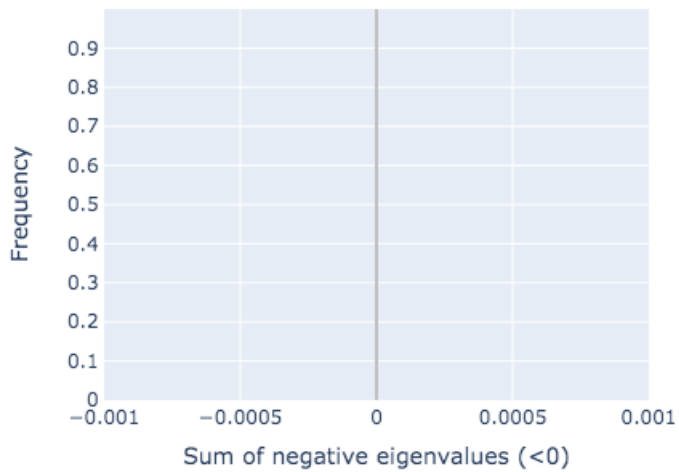## N=10, Nrep=10

Min: -
Max: -
|Max-Min|: -

N=10, Nrep=10

Min: -
Max: -
|Max-Min|: -
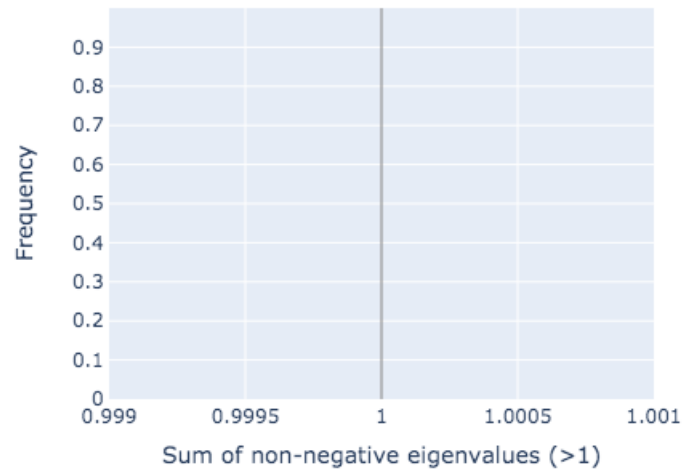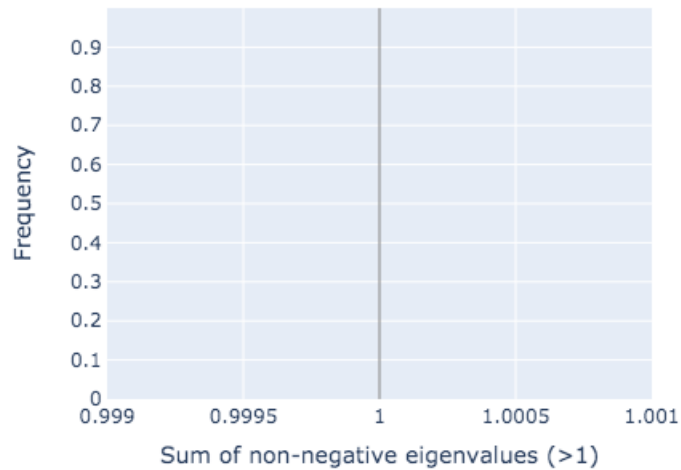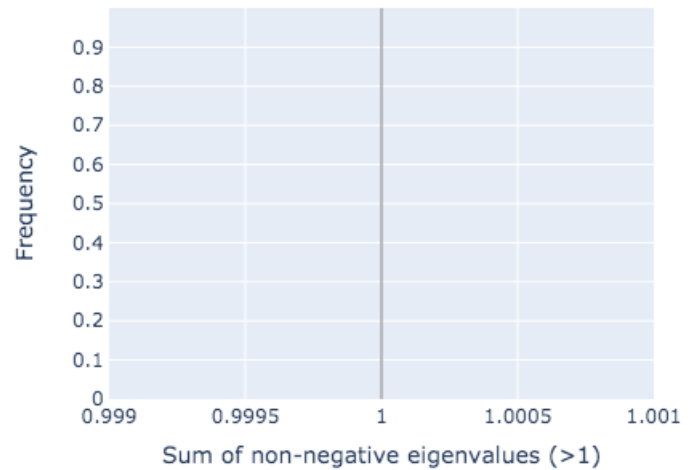
N=100
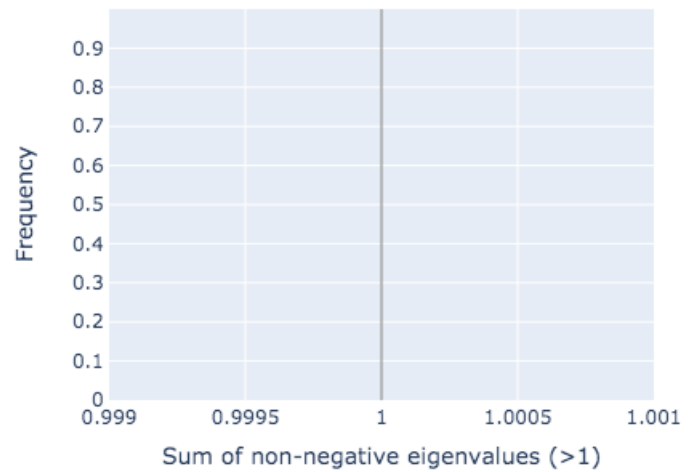**Number of unphysical estimates=0**

N=100, Nrep=10

Min: -
Max: -
|Max-Min|: -

N=100, Nrep=10

Min: -
Max: -
|Max-Min|: -