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18 April 2003

M E M O R A N D U M

To: SAROPS File

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Subject: Method of Weighting Particles

This memorandum discusses a method for weighting particles that accounts for scenario probabilities, failure to detect probabilities, and resampling.

1. Particles Weights Before Resampling.

Suppose there are M scenarios and the mth scenario has probability r_m assigned to it with

$$\sum_{m=1}^{M} r_m = 1$$

For each scenario we generate N particles. For $m = 1,\ldots,M$ and $n = 1,\ldots,N$, we let $q(m,n)$ be the failure to detect probability for the nth particle in the mth scenario. Initially we set

$$q(m,n) = 1 \text{ for } m = 1,\ldots,M; n = 1,\ldots,N$$

Prior to any resampling, the probability that particle (m,n) represents the target and is undetected is

$$p(m,n) = \frac{r_m}{N} q(m,n). \tag{1}$$

This is the posterior probability of this particle "being" the target and not being detected by the search to date. The values of $p(m,n)$ are used to produce the composite probability distribution for target location and the *POS* over all scenarios. The later is computed by

$$POS = 1 - \sum_{m=1}^{M} \sum_{n=1}^{N} \frac{r_m}{N} q(m,n). \tag{2}$$

As Jack Frost has point out, this approach has the virtue that it is easy to reweight the scenario probabilities without regenerating the scenarios. One simply adjusts the r_m values to reflect the new weights. If the user wishes to delete a scenario, he can simple set $r_m = 0$ for that scenario and adjust the other scenario probabilities to add to 1. If the user wishes to add a scenario, this can be done by giving the new scenario a weight $r_{M+1} > 0$ and adjusting the remaining weights so that the sum of the $M+1$ scenario probabilities equals 1. Only the replications for this new scenario need to be generated and run through the search effort to date to computer their failure to detect probabilities.

Probability of success conditioned on scenario *m* being true is also simple to calculate. It is

$$POS(m) = 1 - \frac{1}{N} \sum_{n=1}^{N} q(m,n). \tag{3}$$

The posterior probability map for target location given the *m*th scenario is true is easily generated by mapping only the particles with in scenario *m* and using the weights

$$\hat{p}(n \mid m) = q(m,n) / N \tag{4}$$

for the *n*th particle in scenario *m*.

2. Resampling

In order to allow for resampling we must add an addition component to the scenario weights, namely the cumulative value of $Q(m) = 1 - POS(m)$ prior to the last resampling of scenario *m*. Initially $Q(m) = 1$ for all scenarios. Immediately after scenario *m* has been

resampled there are again N particles and each particle has $q(m,n)=1$. To account for the previous unsuccessful search, equations (1) - (4) must be modified as follows.

$$p(m,n)=\frac{r_m Q(m)}{N}q(m,n) \tag{1'}$$

$$POS=1-\sum_{m=1}^{M}\sum_{n=1}^{N}\frac{r_m Q(m)}{N}q(m,n) \tag{2'}$$

$$POS(m)=1-\frac{Q(m)}{N}\sum_{n=1}^{N}q(m,n) \tag{3'}$$

$$\hat{p}(n\,|\,m)=Q(m)q(m,n)/N\,. \tag{4'}$$

Note that equations (1') - (4') reduce to equations (1) - (4) when $Q(m)=1$.