CPA calculation

Background: We are searching for moving search objects (SOs) from moving search and rescue units (SRUs). The detection probability is a function of the range from the SRU/sensor at the closest point of approach (CPA). When both search objects and SRUs are in motion, CPAs can occur at any relative bearing with respect to the SRU.

Particle (SO) on Leg $L_k^{pID} = \left[(\varphi, \lambda)_{BEGIN}, (\varphi, \lambda)_{END} \right]_k^{pID}$, where pID is a particle ID, k is an index of leg & associated time step T_k^{pID}

Search legs are not infinitely long. Typical search patterns involve multiple legs, often in specific patterns (such as "ladder" searches consisting of a set of straight, parallel, equally spaced search legs connected by shorter cross legs). SRU on leg $L_{nLeg}^{SRU} = \left[(\varphi, \lambda)_{BEGIN}, (\varphi, \lambda)_{END} \right]_{nLeg}^{SRU}$

Because everything is in motion, timing is important. SAROPS uses discrete time steps to simulate continuous motion. The typical time step is 20 minutes.

$$T_k^{pID} = \left(t_{BEGIN}, t_{END} \right)_k^{pID}$$

$$T_{nLeg}^{SRU} = \left(t_{BEGIN}, t_{END} \right)_{nLeq}^{SRU}$$

Position on leg (SO/particle or SRU), linear approximation

$$X(t)\Big|_{BEGIN}^{END} = (\varphi, \lambda)_{BEGIN} \cdot \frac{t_{EN\,D} - t}{t_{END} - t_{BEGIN}} + (\varphi, \lambda)_{END} \cdot \frac{t - t_{BEGIN}}{t_{END} - t_{BEGIN}}, \tag{1}$$
$$t \in \left[t_{BEGIN}, t_{END} \right],$$
$$t_{END} \geq t_{BEGIN}$$

There should be only one CPA per leg. Even with the "one CPA per leg" constraint, using all CPAs from all legs would result in "multiple-counting" of detection opportunities near the waypoints of a search pattern. CPAs are computed only on Set $\left\{ Particle_k^{pID} \right\}$ of SO/particles (legs) with

$$T_{nLeg}^{SRU} \cap T_k^{pID} \neq 0 \tag{2}$$

If $X_{nLeg}^{SRU}(t)$ and $X_k^{pID}(t)$ are the coordinates (Latitude, Longitude)$\equiv (\varphi, \lambda)$ of SRU on nLeg (1) and particle from the Set of particles determined by (2), CPA may be calculate as

$$d = \min_{\varphi, \lambda} \left\| X_{nLeg}^{SRU}(t) - X_k^{pID}(t) \right\|,$$

$$t \in \left(T_{nLeg}^{SRU} \cap T_k^{pID} \right)$$

The CPA distance d is calculated by solving the equation

$$harvesine\left(\frac{d}{R_{Earth}}\right) = harvesine(\varphi_{SRU} - \varphi_{pID}) + \cos\varphi_{SRU}\cos\varphi_{pID}\, harvesine(\lambda_{SRU} - \lambda_{pID}) \quad (3)$$

The $harvesine(x)$ definition is $harvesine(x) = \sin^2\left(\frac{x}{2}\right) = (1 - \cos x)/2$

The origin of the formula (3) is

$$d = 2R_{Earth}\arcsin\left(\sqrt{\sin^2\left(\frac{\varphi_{SRU} - \varphi_{pID}}{2}\right) + \cos\varphi_{SRU}\cos\varphi_{pID}\sin^2\left(\frac{\lambda_{SRU} - \lambda_{pID}}{2}\right)}\right)$$

If multiple CPAs from multiple legs (including cross legs) were calculated, only the one with the smallest CPA is retained and used for search evaluation purposes. This restriction is also supposed to carry across time step boundaries.