CMPT 360 Assign 2

1.4 questions 1,3,5,7,9

1.5 questions 1,7,11

2.3 questions 1, 2, 5, 6, 8, 9, 10, 11, 15, 21, 22, 24, 29

Tyler Kuipers

September 8, 2015

1

## Question 1

- 1.  $x = \{1, -1\}$
- 2.  $x = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$
- 3.  $x = \{0, 1, 4, 9, 16, 25, 36, 49, 64, 81\}$
- 4.  $x = \{\}$

## Question 3

- 1. They are equal
- 2. Not Equal
- 3. Not Equal

## Question 5

Let p and q be propositions:

- p: It is below freezing.
- q: It is snowing
- 1.  $p \wedge q$
- 2.  $p \land \neg q$
- 3.  $\neg p \land \neg q$
- 4.  $p \lor q$
- 5.  $p \rightarrow q$
- 6.  $(p \lor q) \land (p \to \neg q)$
- 7.  $p \leftrightarrow q$

# Question 6

Let p, q, and r be propositions:

- p: You have the flu.
- q: You miss the final exam.
- r: You pass the course.
- 1. You have the flu so you miss the final.
- 2. You don't miss the final exam and because of this, you can't fail the course.
- 3. You miss the final so you fail the course.
- 4. You either have the flu, miss the final, or fail the course. (or any combination of those three)

- 5. You either have the flu and fail or miss the final and fail. (or both)
- 6. You either have the flu and miss the final, or don't have the flu and pass. (or both)

# Question 7

Let p and q be propositions:

- p: You drive over 65 mph.
- q: You get a speeding ticket.
- $1. \neg p$
- 2.  $p \land \neg q$
- 3.  $p \rightarrow q$
- 4.  $\neg p \rightarrow \neg q$
- 5.  $p \rightarrow q$
- 6.  $\neg p \land q$
- 7.  $q \rightarrow p$

## Question 13

Each inhabitant of a remote village always tells the truth or always lies. A villager will only give a 'Yes' or a 'No' response to a question a tourist asks. Suppose you are a tourist visiting this area and come to a fork in the road. One branch leads to the ruins you want to visit; the other branch leads deep into the jungle. A villager is standing at the fork in the road. What one question can you ask the villager to determine which branch to take?

I would ask 'If I were to ask you whether the path to the right leads to the ruins, would you say yes?'

#### Let:

p represent whether or not the villager is a liar.

q represent whether or not the branch to the right leads to the ruins. (hidden information).

r represent the villager's answer to the question 'Does the path to the right lead to the ruins?'.

s represent the villager's answer to the question 'What would you say if I asked whether the path to the right leads to the ruins?'

p	$\mid q \mid$	$r = (p \leftrightarrow \neg q) \oplus (\neg p \leftrightarrow q)$	$s = (p \leftrightarrow \neg r) \oplus (\neg p \leftrightarrow r)$
False	False	False	False
False	True	True	True
True	False	True	False
True	True	False	True

In the above table, we show that if the villager is a liar, and will always lie, we can use that to determine the path. Since we expect him to lie, we ask whether his answer would be yes to taking the path on the right. This lets us know the truth because

if he were lying about whether the path on the right leads to the ruins, he will lie again and they will cancel out. If the villager is not a liar, then we have nothing to worry about. Because of this, if we ask 'If I were to ask you whether the path to the right leads to the ruins, would you say yes?', and the villager says yes, we can be confident that the path to the right is correct.

## Question 19

State the converse and contrapositive of each of the following implications.

- 1. Converse: If I ski tomorrow, it will snow today.

  Contrapositive: If it doesn't snow today, I won't ski tomorrow.
- 2. Converse: Whenever I come to class, there will be a quiz. Contrapositive: If there is not a quiz, I won't come to class.
- 3. Converse: If a positive integer has no divisors other than one and itself, it is a prime number.

  Contrapositive: If a positive integer has divisors other than one and itself, it is not prime.

### Question 21

Construct a truth table for each of the following compound prepositions.

1.	$p \land \neg p$	
	p	$p \land \neg p$
	0	0
	1	0

2.  $p \lor \neg p$ 

<u> </u>	
p	$p \vee \neg p$
0	1
1	1

3.  $(p \lor \neg q) \to q$ 

p	q	$\neg q$	$p \vee \neg q$	$(p \vee \neg q) \to q$
0	0	1	1	0
0	1	0	0	1
1	0	1	1	0
1	1	0	1	1

 $4. \ (p \vee \neg q) \to q$ 

p	q	$p \rightarrow q$	$p \wedge q$	$(p \lor q) \to (p \land q)$
0	0	1	1	1
0	1	1	1	0
1	0	0	0	0
1	1	1	1	1

5.  $(p \to q) \leftrightarrow (\neg q \to \neg p)$ 

p	q	$p \rightarrow q$	$\neg q \to \neg p$	$ \begin{array}{ccc} (p & \rightarrow & q) & \leftrightarrow \\ (\neg q \rightarrow \neg p) & & \end{array} $
0	0	1	1	1
0	1	1	1	1
1	0	0	0	1
1	1	1	1	1

6.  $(p \to q) \leftrightarrow (q \to p)$ 

p	q	$p \rightarrow q$	$q \rightarrow p$	$(p \to q) \leftrightarrow (q \to$
				p)
0	0	1	1	1
0	1	1	0	0
1	0	0	1	1
1	1	1	1	1