Mjerenje uspješnosti investicijskih fondova

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Učitavanje podataka i pomoćnih biblioteka

Prilikom proučavanja podataka primjetili smo da vrijednost fonda ErsteAdriaticEquity za 24.1.2016. poprilično odskače od okolnih datuma. Pretragom na stranici Erste grupe ustvrdili smo pogrešku u unosu podataka te smo ručno ispravili vrijednost.

```
library(reshape2)
library(dplyr)
library(magrittr)
library(ggplot2)
library(stringr)
library(xts)
require(quantmod)
require(PerformanceAnalytics)
require(reshape2)
source('data_extraction.r')
xs <- read_normalize('./investicijski_fondovi_data.csv')
```

Priprema i analiza podataka

Podjela prema tipovima fondova

```
investment_funds <- c("ERSTEAdriaticEquity", "OTPMeridian20", "ZBAktiv")
pension_funds <- c("RaiffeisenDMF", "ERSTEPlaviEXPERT", "ERSTEPlaviPROTECT")
market_portfolio <- c("CROBEX")

all_funds <- c(investment_funds, pension_funds)
data_columns <- c(pension_funds, investment_funds, market_portfolio)
```

Povrati

Računanje dnevnih povrata prema formuli: R(t) = log(S(t)/S(t-1))

```
diff_function_log <- function(St, St_minus_one) log(St) - log(St_minus_one)
xs.returns <- to_time_series_diff_df(xs, data_columns, diff_function_log)
xs.returns.summary <- summary(xs.returns[data_columns] * 365)
data.frame(unclass(xs.returns.summary), check.names = FALSE, stringsAsFactors = FALSE)
```

```
##        RaiffeisenDMF   ERSTEPlaviEXPERT  ERSTEPlaviPROTECT
## 1 Min.   :-5.79209   Min.   :-5.73634   Min.   :-2.06945
## 2 1st Qu.:-0.18757   1st Qu.:-0.19025   1st Qu.:-0.05239
## 3 Median : 0.02441   Median : 0.02279   Median : 0.04456
## 4 Mean   : 0.06451   Mean   : 0.07278   Mean   : 0.06709
## 5 3rd Qu.: 0.31443   3rd Qu.: 0.39346   3rd Qu.: 0.20759
## 6 Max.   : 8.91872   Max.   : 4.58776   Max.   : 3.22798
```

```
##   ERSTEAdriaticEquity       OTPMeridian20             ZBAktiv
## 1 Min.   :-18.08756   Min.   :-23.51025   Min.   :-13.47776
## 2 1st Qu.: -0.48492   1st Qu.: -0.35673   1st Qu.: -0.41271
## 3 Median :  0.00000   Median :  0.00000   Median :  0.00000
## 4 Mean   :  0.01423   Mean   :  0.01395   Mean   :  0.03645
## 5 3rd Qu.:  0.50246   3rd Qu.:  0.63048   3rd Qu.:  0.61738
## 6 Max.   : 21.67018   Max.   : 13.60614   Max.   : 34.35281
##                CROBEX
## 1 Min.   :-17.43339
## 2 1st Qu.: -0.58382
## 3 Median :  0.00000
## 4 Mean   : -0.00203
## 5 3rd Qu.:  0.67653
## 6 Max.   : 31.25453
```

Mjere raspršenosti

Prikaz vrijednosti standardne devijacije i varijance za svaki fond

```
variances <- apply(xs.returns[all_funds] * 365, 2, var, na.rm = T)
std.devs <- apply(xs.returns[all_funds] * sqrt(365), 2, sd, na.rm = T)

data.frame(std.devs, variances)
```

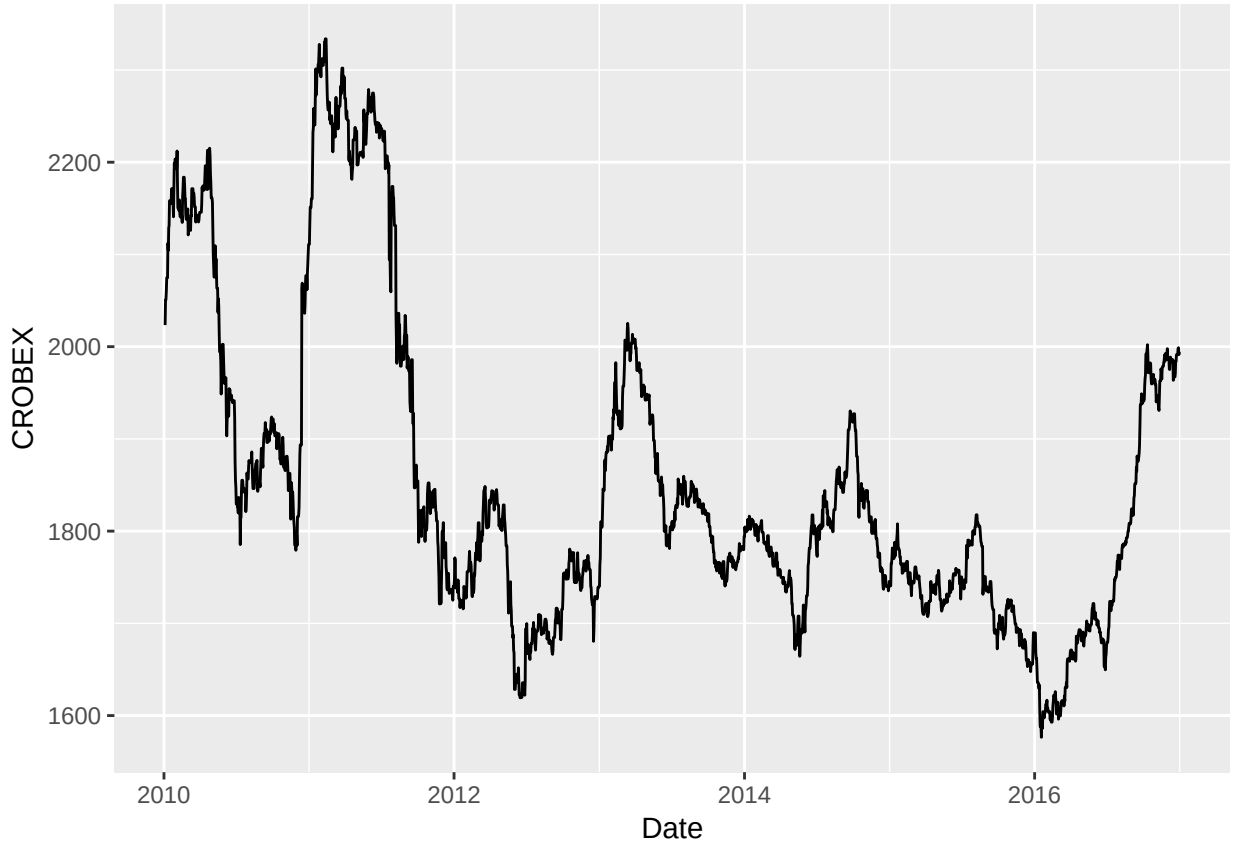
```
##                       std.devs variances
## ERSTEAdriaticEquity 0.08446841 2.6042430
## OTPMeridian20       0.09034363 2.9791195
## ZBAktiv             0.08987277 2.9481469
## RaiffeisenDMF       0.03555447 0.4614040
## ERSTEPlaviEXPERT    0.04020710 0.5900631
## ERSTEPlaviPROTECT   0.01835943 0.1230300
```

Grafički prikaz podataka

Prikaz vrijednosti CROBEX-a po danima

Kretanje vrijednosti burzovnog indeksa od početka 2010. godine do kraja 2016. godine.

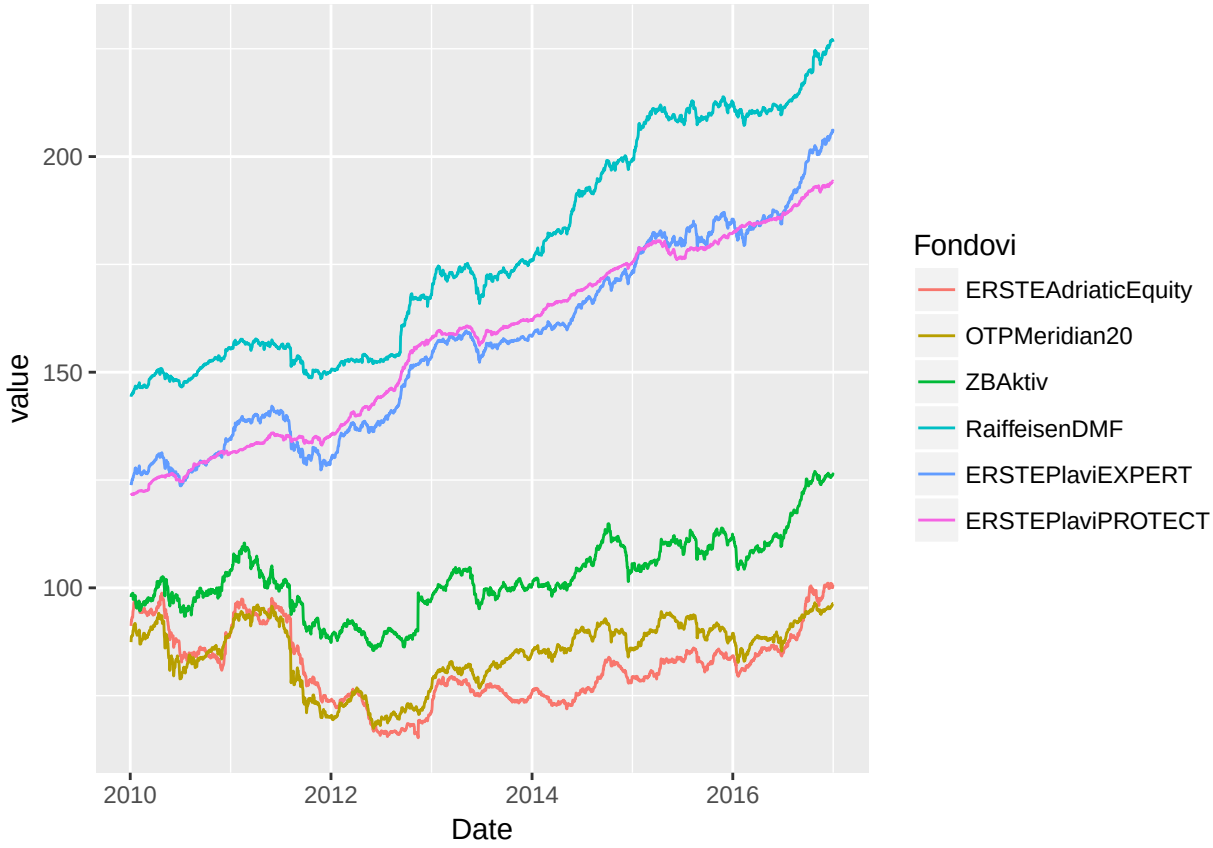
```
ggplot(xs, aes(Date, CROBEX)) + geom_line()
```



Prikaz vrijednosti investicijskih i mirovinskih fondova po danima

Iz grafa se čini kako investicijski fondovi imaju veći apsolutan rast, uz veću prosječnu vrijednost.

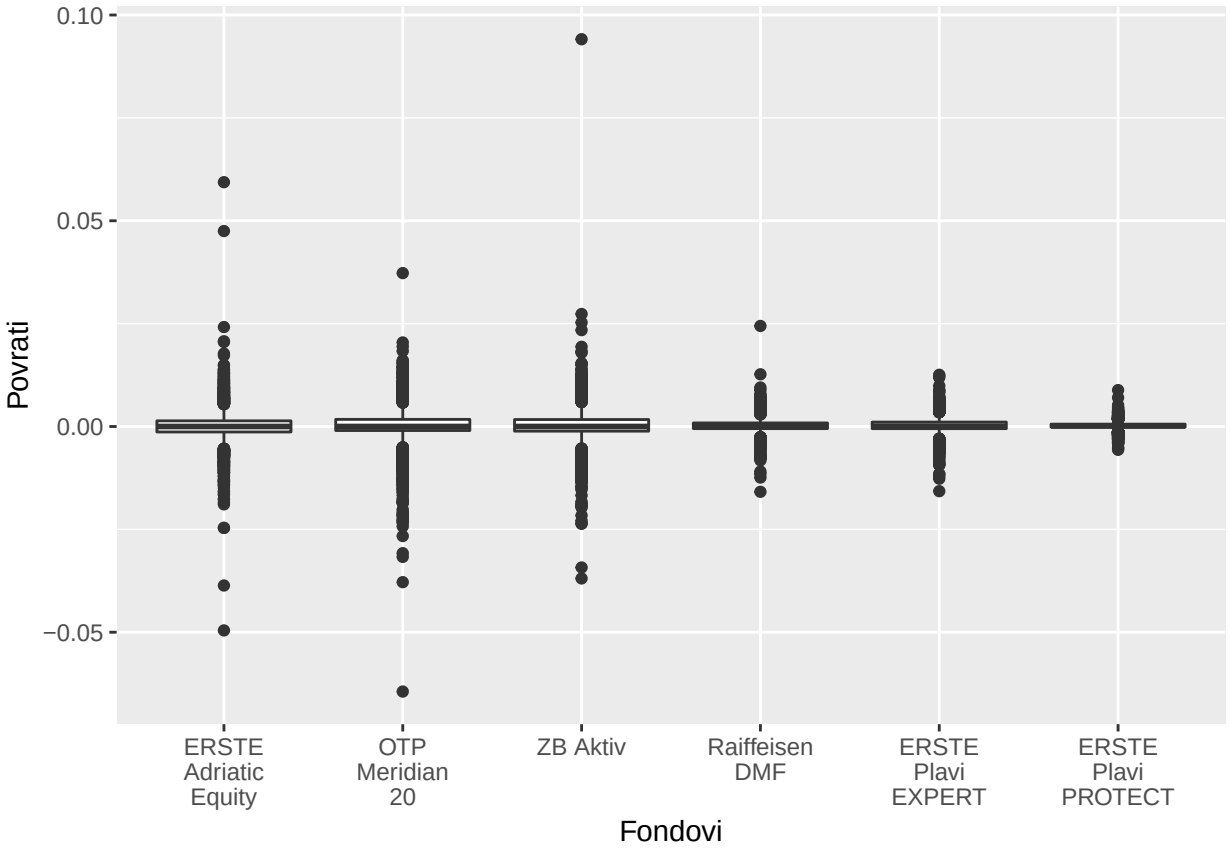
```
df <- melt(xs[c("Date", investment_funds, pension_funds)],
           id.vars = 'Date',
           variable.name = 'Fondovi')
ggplot(df, aes(Date, value)) + geom_line(aes(colour = Fondovi))
```



Prikaz boxplotova za sve fondove

Iz ovog se grafa ne može zaključiti mnogo, ali vidi kako su investicijski fondovi (prva tri stupca) na dnevnoj bazi podložniji većim promjenama vrijednosti od mirovinskih, jer ima više stršećih vrijednosti.

```
df.returns <- melt(xs.returns[c("Date", investment_funds, pension_funds)],
                   id.vars = 'Date',
                   variable.name = 'Fondovi')

label_prettify <- function(label) {
  first_matches <- str_match(label, "(^[A-Z]+)([A-Z][a-z]+)(.*)")
  second_matches <- str_match(label, "(^[A-Z][a-z]+)([A-Z]+)")

  first_word <- ifelse(!is.na(first_matches[1, 1]), first_matches[1, 2],
                       second_matches[1, 2])
  second_word <- ifelse(!is.na(first_matches[1, 1]), first_matches[1, 3],
                        second_matches[1, 3])
  second_word <- ifelse(!is.na(first_matches[1, 4]),
                        str_c(second_word, first_matches[1, 4], sep = " "),
                        second_word)

  return(str_c(first_word, second_word,sep = " ") %>% str_wrap(width = 10))
}

ggplot(df.returns, aes(Date, value)) +
  geom_boxplot(aes(Fondovi)) +
  xlab("Fondovi") +
  ylab("Povrati") +
  scale_x_discrete(labels = function(labels) lapply(labels, label_prettify))
```

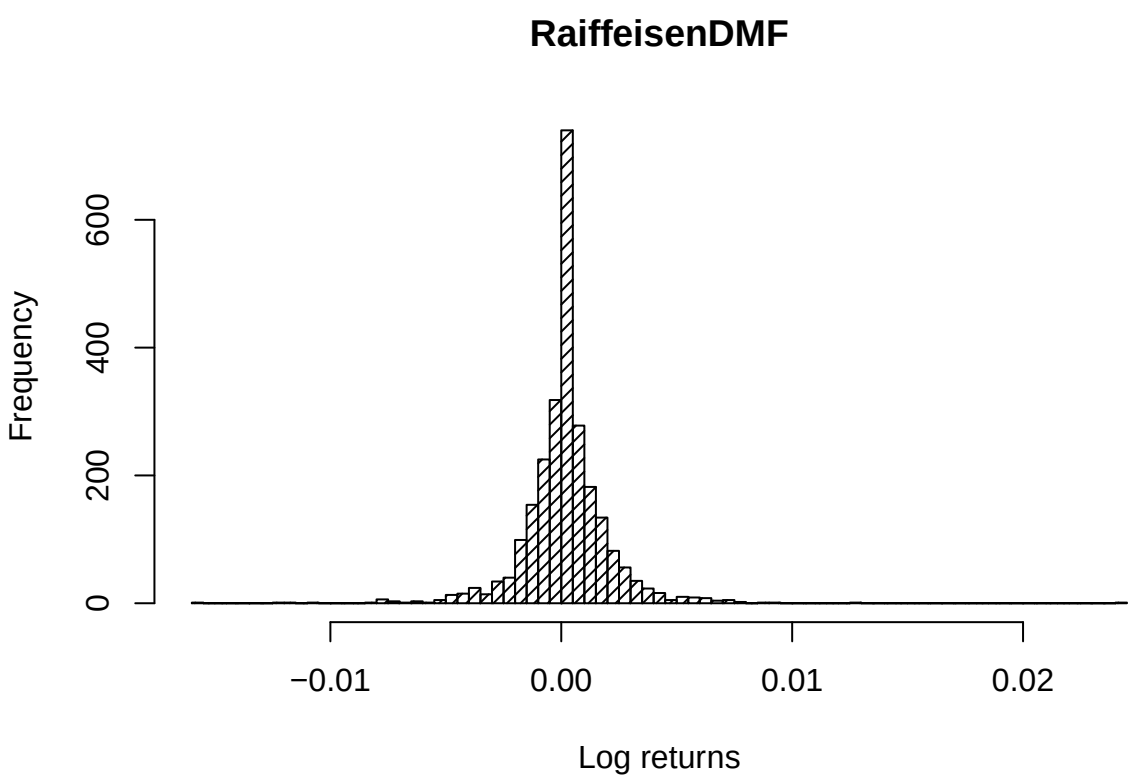


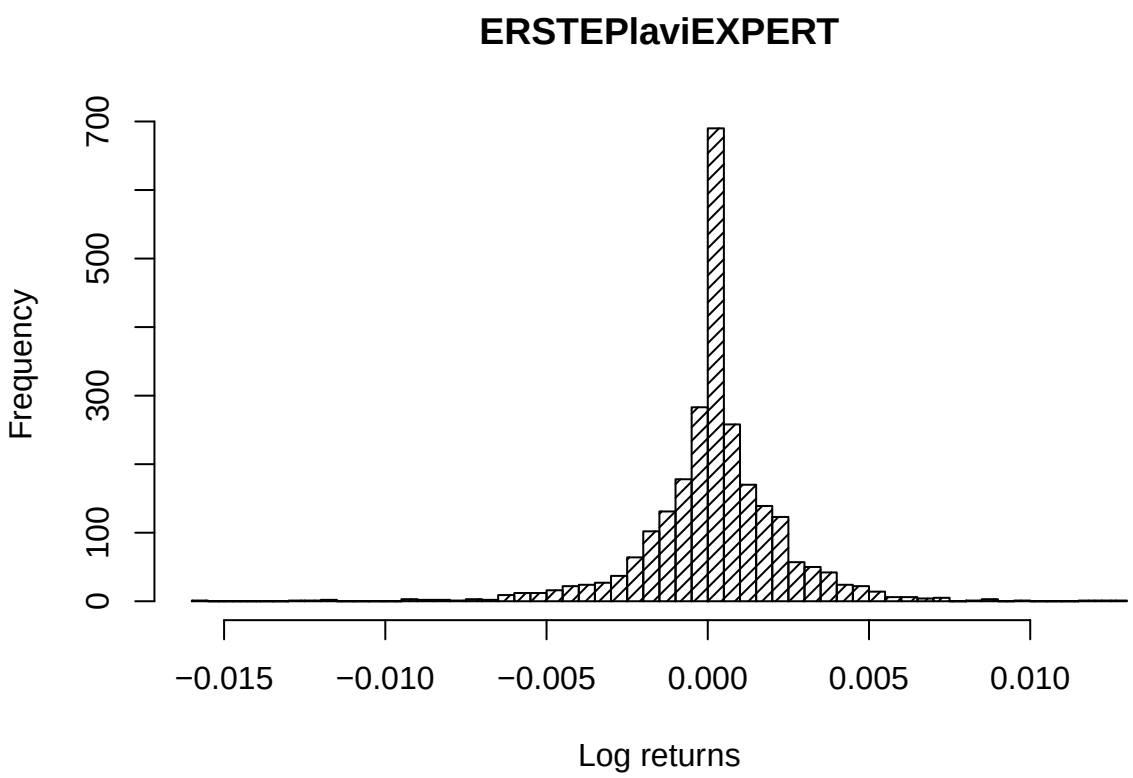
Provjera normalnosti dnevnih povrata fondova

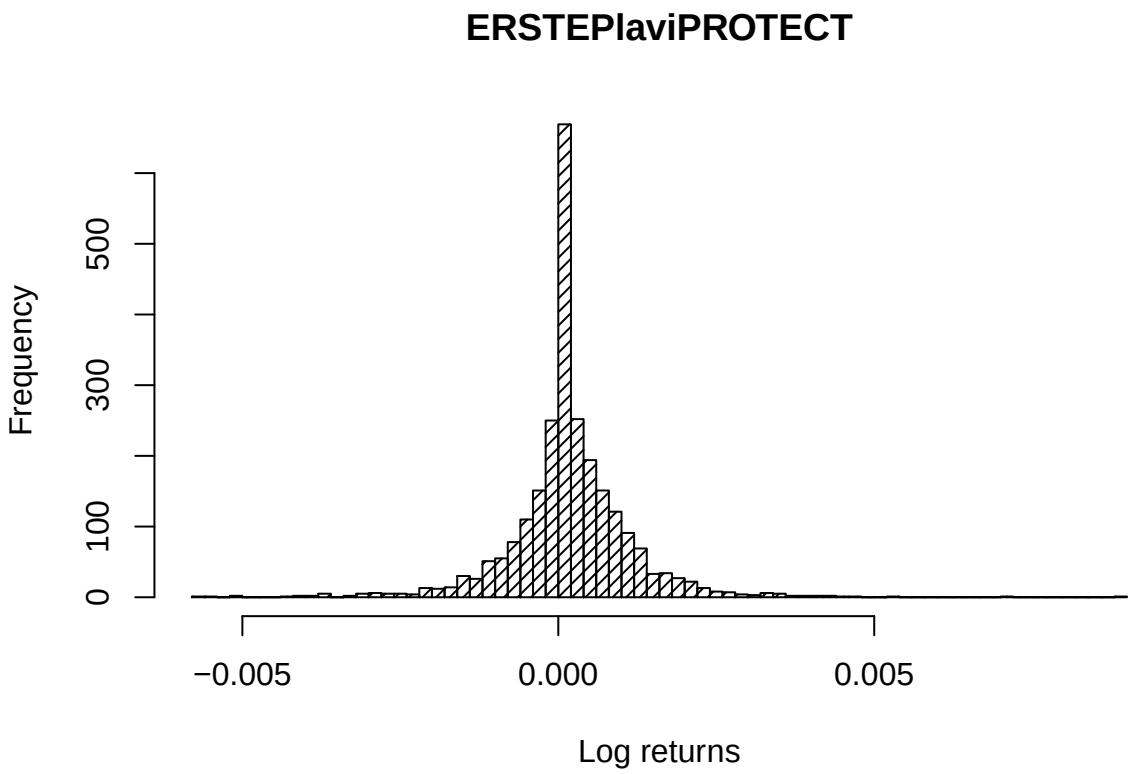
Histogramima

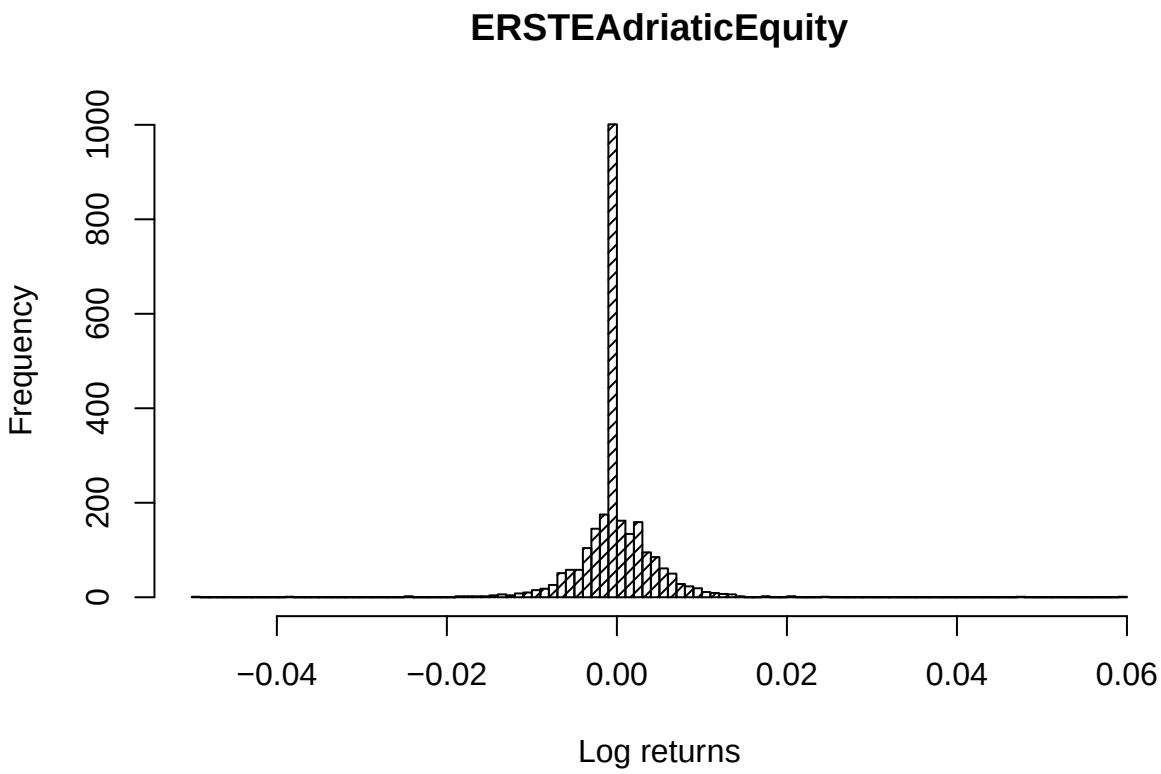
Iscrtavamo histograme povrata za svaki fond. Vidimo da imaju prilično teške repove, što ukazuje kako nisu baš normalno distribuirani.

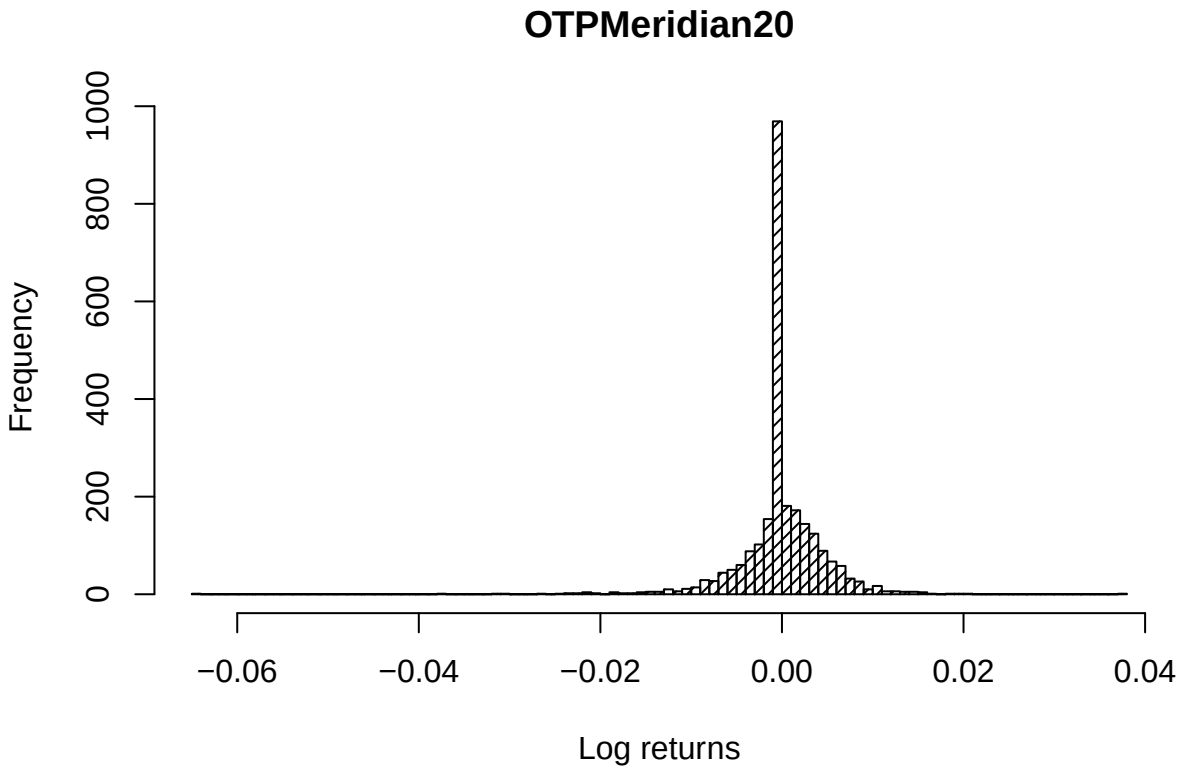
```
plot_returns <- function(fund.returns, fund.name) hist(fund.returns,
                                                       main = fund.name,
                                                       density=20,
                                                       xlab='Log returns',
                                                       labels=FALSE,
                                                       breaks=100)


mapply(plot_returns,
       c(xs.returns[c(pension_funds, investment_funds)]),
       c(pension_funds, investment_funds)) %>%
  invisible
```

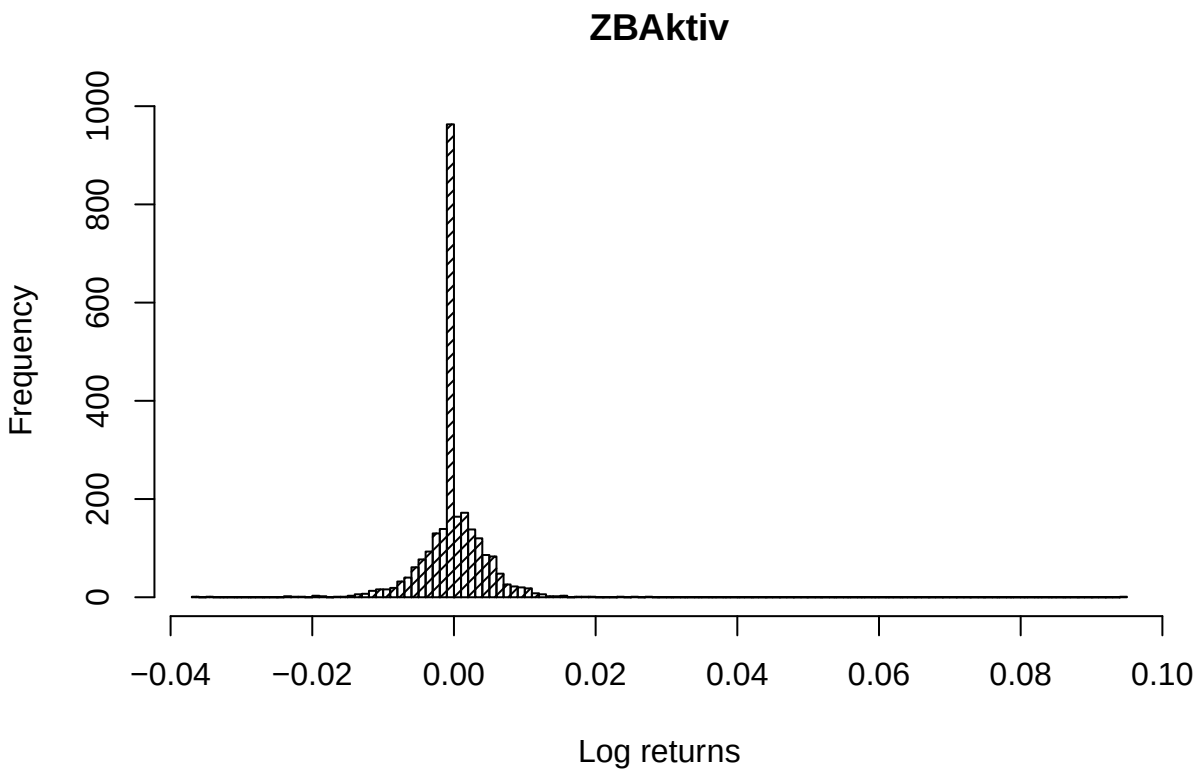

RaiffeisenDMF
Frequency
0
200
400
600
−0.01
0.00
0.01
0.02
Log returns


ERSTEPlaviEXPERT
Frequency
0
100
300
500
700
−0.015
−0.010
−0.005
0.000
0.005
0.010
Log returns


ERSTEPlaviPROTECT
Frequency
0
100
300
500
−0.005
0.000
0.005
Log returns


ERSTEAdriaticEquity
Frequency
0
200
400
600
800
1000
−0.04
−0.02
0.00
0.02
0.04
0.06
Log returns

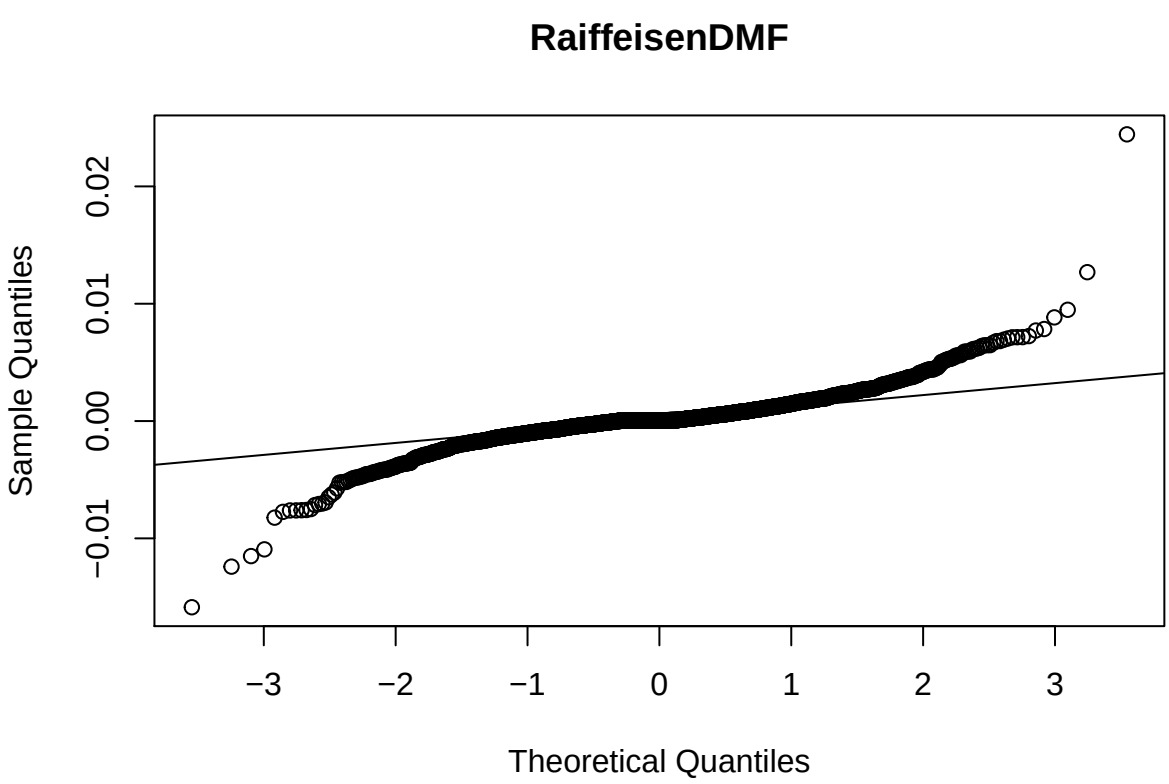

OTPMeridian20
Frequency
0
200
400
600
800
1000
−0.06
−0.04
−0.02
0.00
0.02
0.04
Log returns

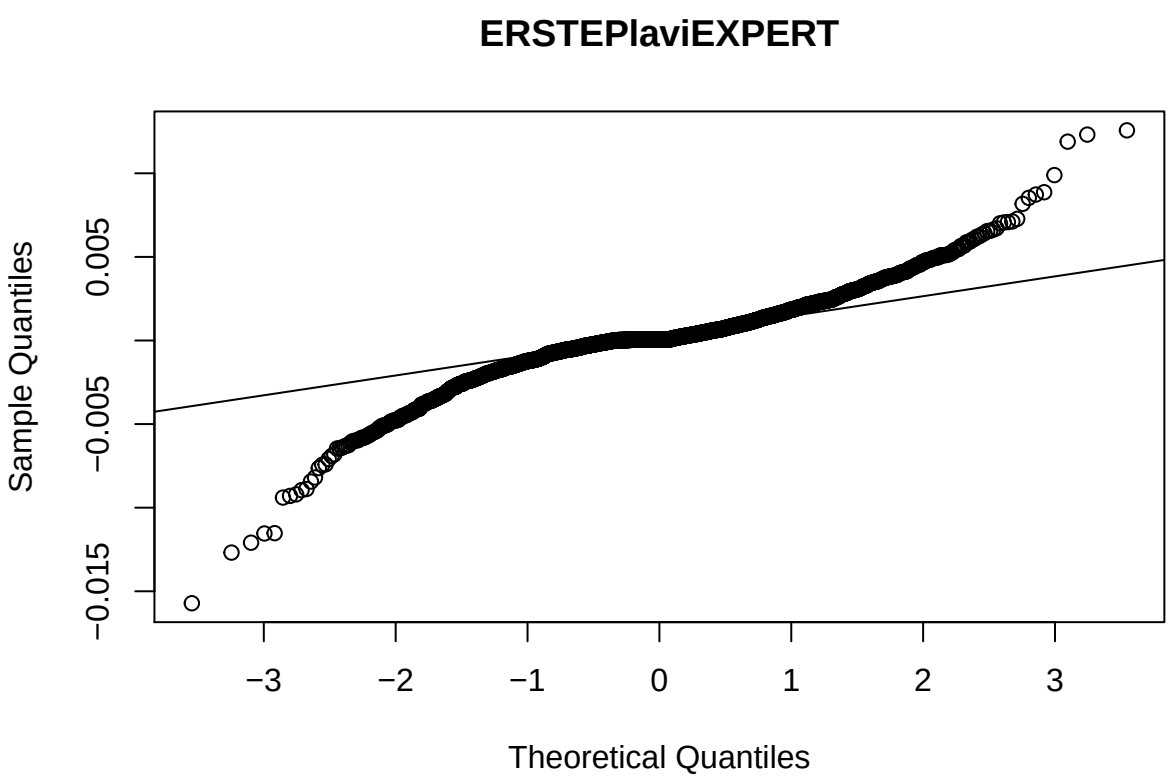


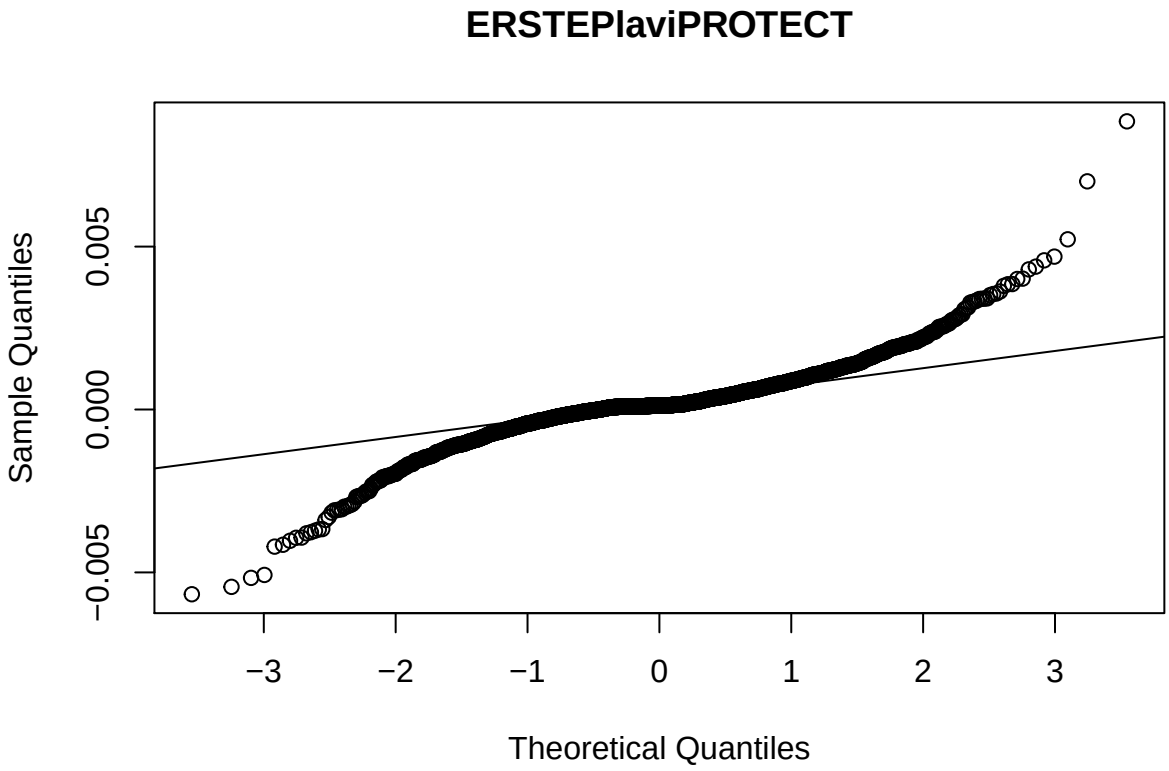
QQ grafovima

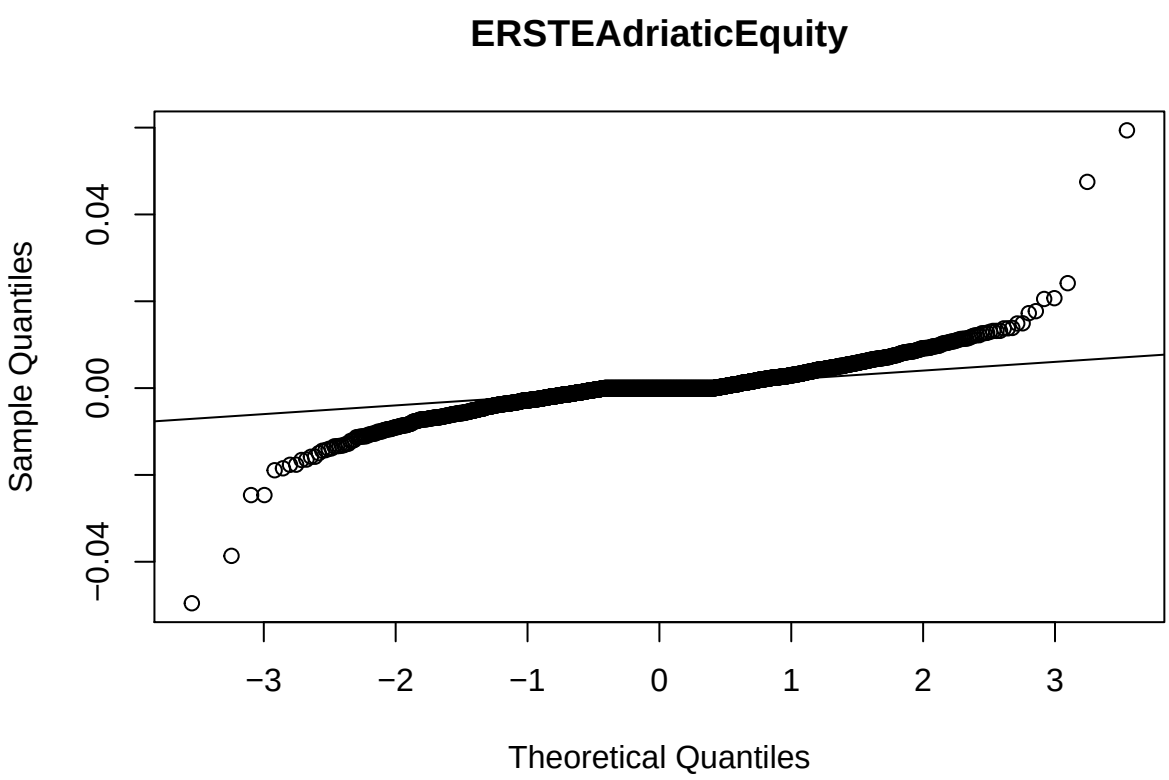
Sljedećim QQ grafovima želimo ispitati normalnost distribucije povrata svih fondova. Teške repove primjećujemo radi sitne granulacije, tj. dnevnog računanja prinosa; u tako kratkom roku zna se dogoditi da pojedina dionica ili naglo naraste ili naglo padne u vrijednosti.

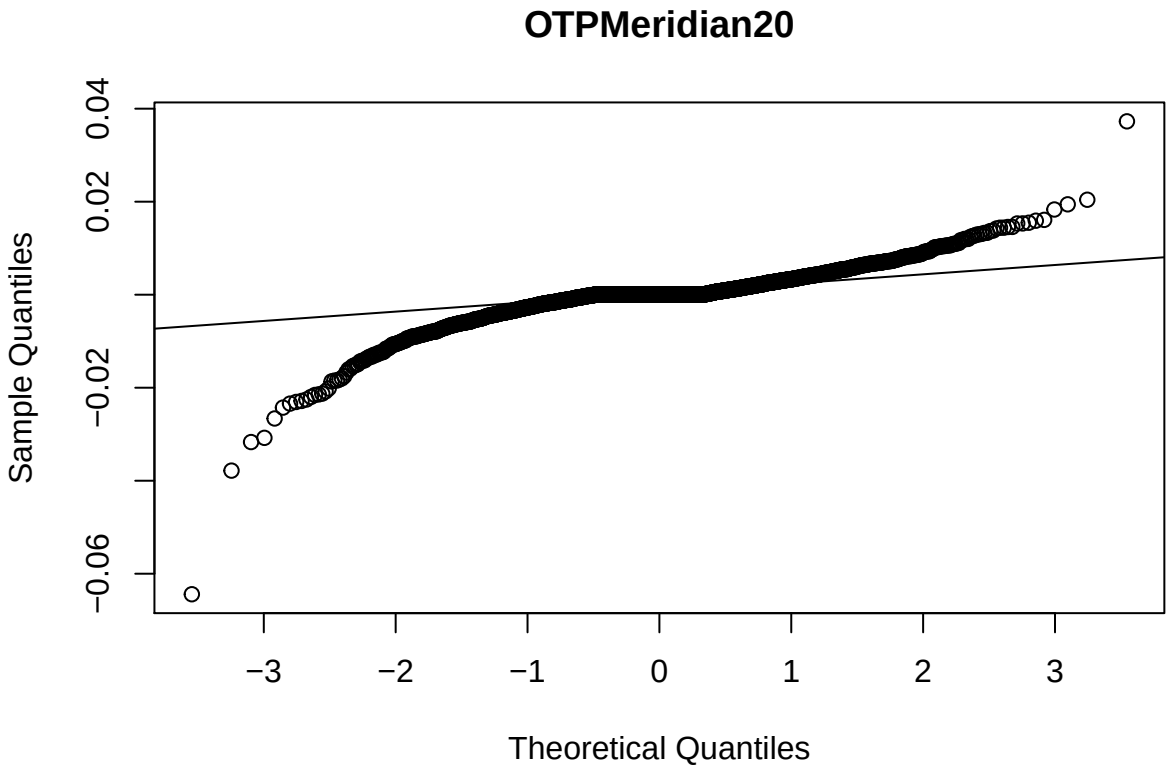
```
qqplots <- function(fund.returns, fund.name) {
  qqnorm(fund.returns, main = fund.name)
  qqline(fund.returns)
}

mapply(qqplots,
       c(xs.returns[c(pension_funds, investment_funds)]),
       c(pension_funds, investment_funds)) %>%
  invisible
```

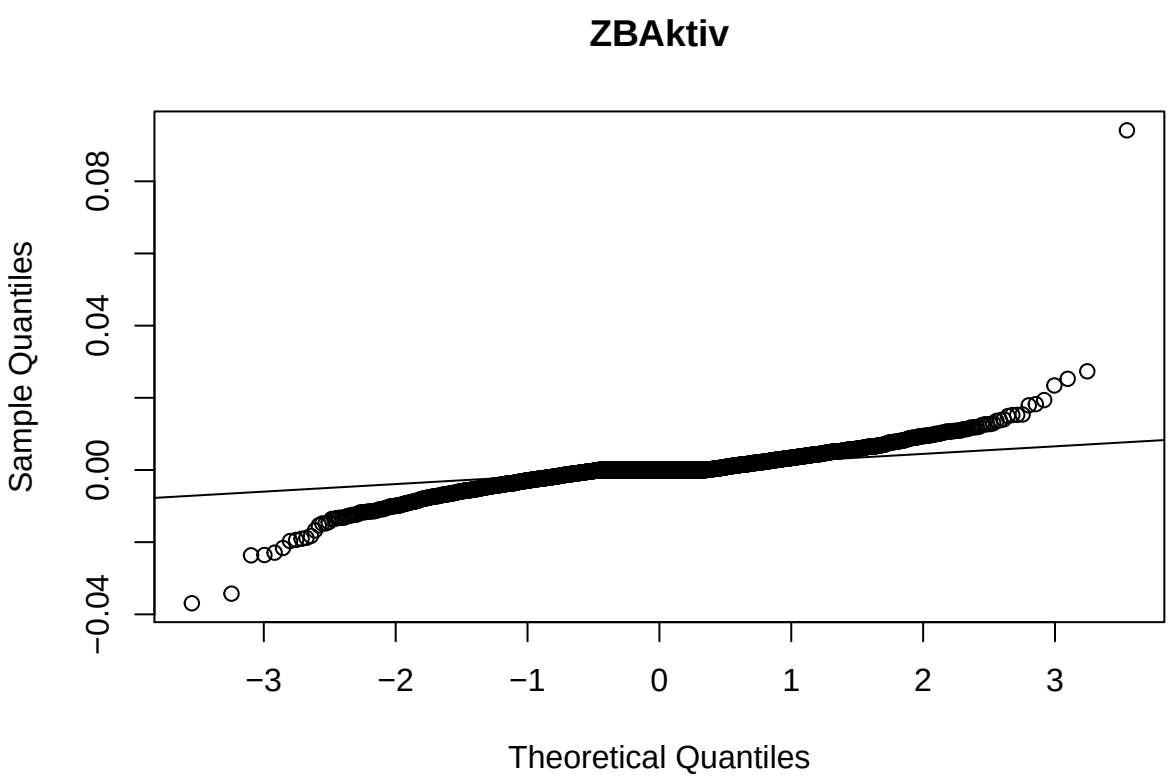

RaiffeisenDMF
Sample Quantiles
0.02
0.01
0.00
−0.01
−3
−2
−1
0
1
2
3
Theoretical Quantiles


ERSTEPlaviEXPERT
Sample Quantiles
0.005
−0.005
−0.015
−3
−2
−1
0
1
2
3
Theoretical Quantiles


ERSTEPlaviPROTECT
Sample Quantiles
0.005
0.000
−0.005
−3
−2
−1
0
1
2
3
Theoretical Quantiles


ERSTEAdriaticEquity
Sample Quantiles
0.04
0.00
−0.04
−3
−2
−1
0
1
2
3
Theoretical Quantiles


OTPMeridian20
Sample Quantiles
0.04
0.02
−0.02
−0.06
−3
−2
−1
0
1
2
3
Theoretical Quantiles



Testovi fondova

Iako QQ grafovi pokazuju da povrati nisu normalno raspodijeljeni, radimo tu pretpostavku s obzirom na robusnost T-testa. Jasno je da globalni događaji (kriza, teroristički napadi,...) često utječu na cijelo tržište odjednom, pa koristimo T-testove za uparene podatke.

Testovi povrata investicijskih fondova u odnosu na CROBEX

Za H0 hipotezu uzimamo kako su sredine investicijskih fondova i CROBEX indeksa jednake. Iz sljedećih testova vidimo da ne možemo opovrgnuti tu hipozetu.

```
compare.to.index <- function(index) function(fund.returns) t.test(index, fund.returns,
                                                                   paired = TRUE)

mapply(compare.to.index(xs.returns$CROBEX), xs.returns[investment_funds])
```

```
##             ERSTEAdriaticEquity      OTPMeridian20
## statistic   -0.4774079              -0.4286687
## parameter   2552                     2552
## p.value     0.6331126                0.6682005
## conf.int    Numeric,2                Numeric,2
## estimate    -4.454166e-05            -4.377476e-05
## null.value  0                        0
## alternative "two.sided"              "two.sided"
```

```
## method      "Paired t-test"          "Paired t-test"
## data.name   "index and fund.returns" "index and fund.returns"
##             ZBAktiv
## statistic   -0.8207028
## parameter   2552
## p.value     0.4118922
## conf.int    Numeric,2
## estimate    -0.0001054245
## null.value  0
## alternative "two.sided"
## method      "Paired t-test"
## data.name   "index and fund.returns"
```

Testovi povrata mirovinskih fondova u odnosu na CROBEX

Za H0 hipotezu uzimamo kako su sredine mirovinskih fondova i CROBEX indeksa jednake. Iz sljedećih testova zaključujemo da ne možemo odbaciti H0 hipotezu uz nivo značajnosti 5% za fondove RaiffeisenDMF i ERSTEPlaviPROTECT, dok za ERSTEPlaviEXPERT možemo.

```
mapply(compare.to.index(xs.returns$CROBEX), xs.returns[pension_funds])
```

```
##             RaiffeisenDMF            ERSTEPlaviEXPERT
## statistic   -1.789106                -2.125481
## parameter   2552                     2552
## p.value     0.07371632               0.03364232
## conf.int    Numeric,2                Numeric,2
## estimate    -0.0001823123            -0.0002049503
## null.value  0                        0
## alternative "two.sided"              "two.sided"
## method      "Paired t-test"          "Paired t-test"
## data.name   "index and fund.returns" "index and fund.returns"
##             ERSTEPlaviPROTECT
## statistic   -1.711485
## parameter   2552
## p.value     0.08711319
## conf.int    Numeric,2
## estimate    -0.0001893649
## null.value  0
## alternative "two.sided"
## method      "Paired t-test"
## data.name   "index and fund.returns"
```

Test povrata investicijskih fondova u odnosu na mirovinske fondove

Izračunate su sredine mirovinskih i investicijskih fondova pa je sproveden test njihovih vrijednosti. Dobivamo izrazito malu p-vrijednost, stoga uz relativno veliku sigurnost zaključujemo da možemo odbaciti nul-hipotezu koja tvrdi da su sredine jednake.

```
grouped.return.means = data.frame(Date = xs.returns$Date,
                                  MeansPension = rowMeans(xs.returns[pension_funds]),
                                  MeansInvestment = rowMeans(xs.returns[investment_funds]))

t <- t.test(grouped.return.means$MeansPension,
```

```
            grouped.return.means$MeansInvestment, paired = TRUE)
data.frame(p=t$p.value, type=t$alternative, null=t$estimate,
           row.names = "Pension vs. Investment fund means")
```

```
##                                           p      type         null
## Pension vs. Investment fund means 0.02846181 two.sided 0.0001276289
```

CAPM model

CAPM (Capital Asset pricing model) je model koji opisuje odnos između kamatne stope i očekivanog povrata sredstava. Svodi se na linearnu regresiju:

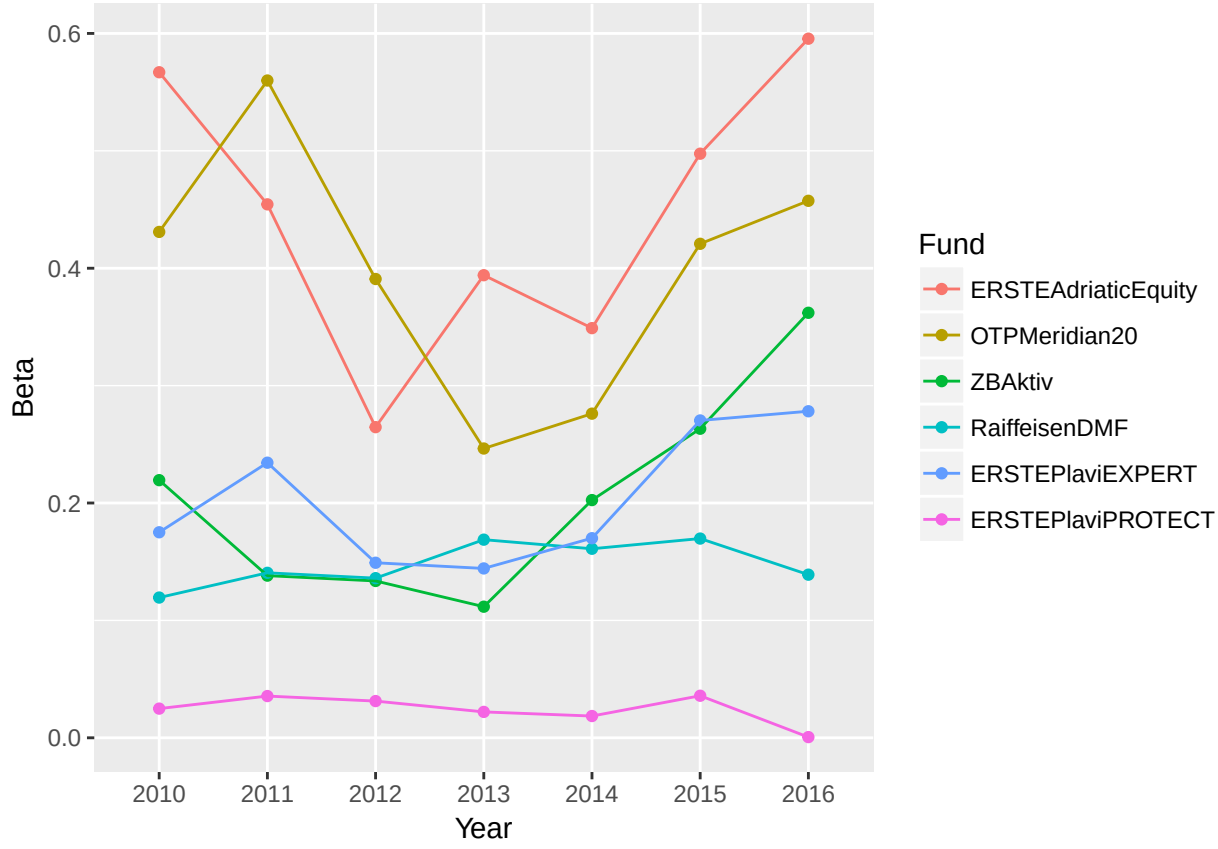
$$R_p - R_f = \alpha + \beta(R_m - R_f) + \epsilon$$

gdje je R_p prinos promatranog fonda (portfelja), R_m prinos tržišnog (referentnog) portfelja, a R_f je bezrizična kamatna stopa. Koeficijent α mjeri koliko je prinos promatranog fonda veći od prinosa tržišnog portfelja, a β mjeri osjetljivost fonda na tržišne prinose i predstavlja rizičnost.

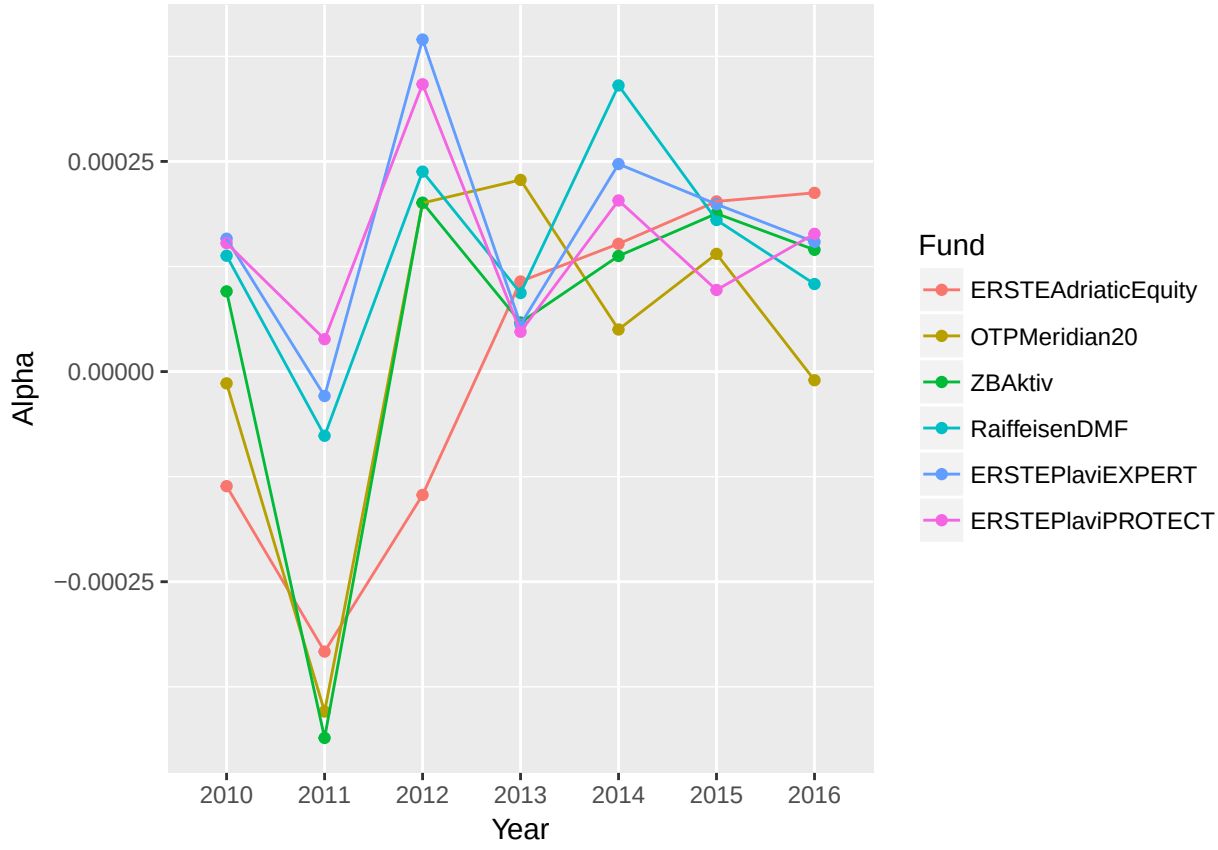
Promatrajući koeficijente α i β svakog fonda zaključujemo da bi najbolje bilo uložiti u fond ERSTEPlaviPROTECT jer ima najmanji koeficijent β od svih fondova i poprilično visok koeficijent α.

```
year <- function(date) format(date, "%Y")
get_for_year <- function(df, dates, desired_year) df[year(dates) == desired_year, ]

get_capm_for_year <- function(df, fund, desired_year){
  xs.year = get_for_year(df, df$Date, desired_year)

  fund.year <- xs.year[c('Date', fund)]
  fund.ts <- xts(fund.year[, -1], order.by=fund.year$Date)

  capm.index.year <- xs.year[c('Date', 'CROBEX')]
  capm.index.ts <-  xts(capm.index.year[, -1], order.by=capm.index.year$Date)

  capm.risk_free.year <- xs.year[c('Date', 'InterestRate.daily')]
  capm.risk_free.year <- capm.risk_free.year[1, -1]

  data.frame(fund, as.factor(desired_year),
             CAPM.alpha(fund.ts, capm.index.ts, capm.risk_free.year),
             CAPM.beta(fund.ts, capm.index.ts, capm.risk_free.year))
}

get_capm_for_fund <- function(df, selected_fund){
  fund <- df[c('Date', selected_fund)]
  fund.ts <- xts(fund[, -1], order.by=fund$Date)
  capm.index <- df[c('Date', 'CROBEX')]
  capm.index.ts <- xts(capm.index[, -1], order.by=capm.index$Date)
  capm.risk_free <- df[c('Date', 'InterestRate.daily')]
  capm.risk_free.ts <- capm.risk_free[1, -1]
  data.frame(selected_fund,
             CAPM.alpha(fund.ts, capm.index.ts, capm.risk_free.ts),
             CAPM.beta(fund.ts, capm.index.ts, capm.risk_free.ts))
}

xs.years = seq(from = 2010, by = 1, length = 7)
xs.fund.names = c(investment_funds, pension_funds)
xs.capm <- data.frame(matrix(ncol = 3, nrow = 0))
xs.capm.all <- data.frame(matrix(ncol = 3, nrow = 0))

for (i in 1:length(xs.fund.names)){
  for (j in 1:length(xs.years)){
    xs.capm <- rbind(xs.capm, get_capm_for_year(xs.returns, xs.fund.names[i], xs.years[j]))
  }
```

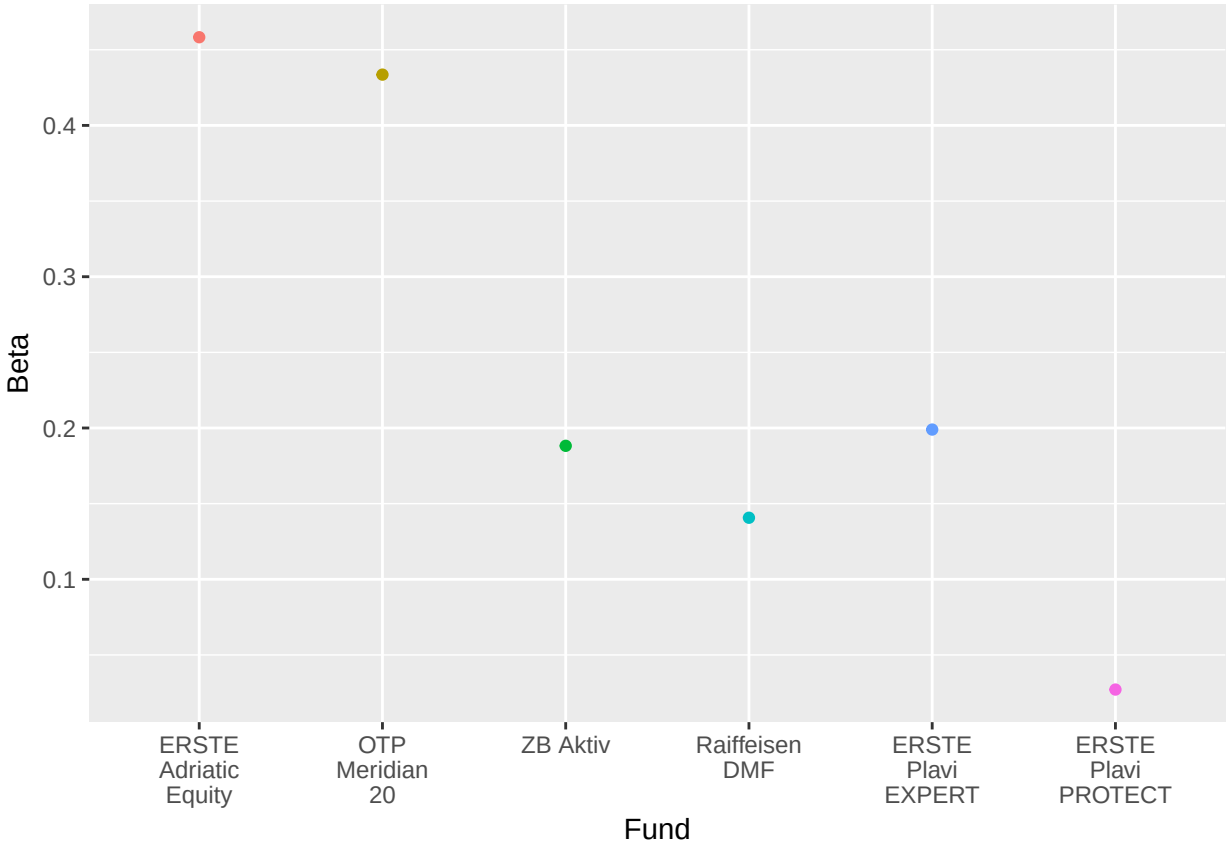
```
}

for (i in 1:length(xs.fund.names)){
  xs.capm.all <- rbind(xs.capm.all, get_capm_for_fund(xs.returns, xs.fund.names[i]))
}

colnames(xs.capm) <- c("Fund", "Year", "Alpha", "Beta")
colnames(xs.capm.all) <- c("Fund", "Alpha", "Beta")

ggplot(xs.capm, aes(Year, Beta, color= Fund, group = Fund)) +
  geom_point() + geom_line()
```



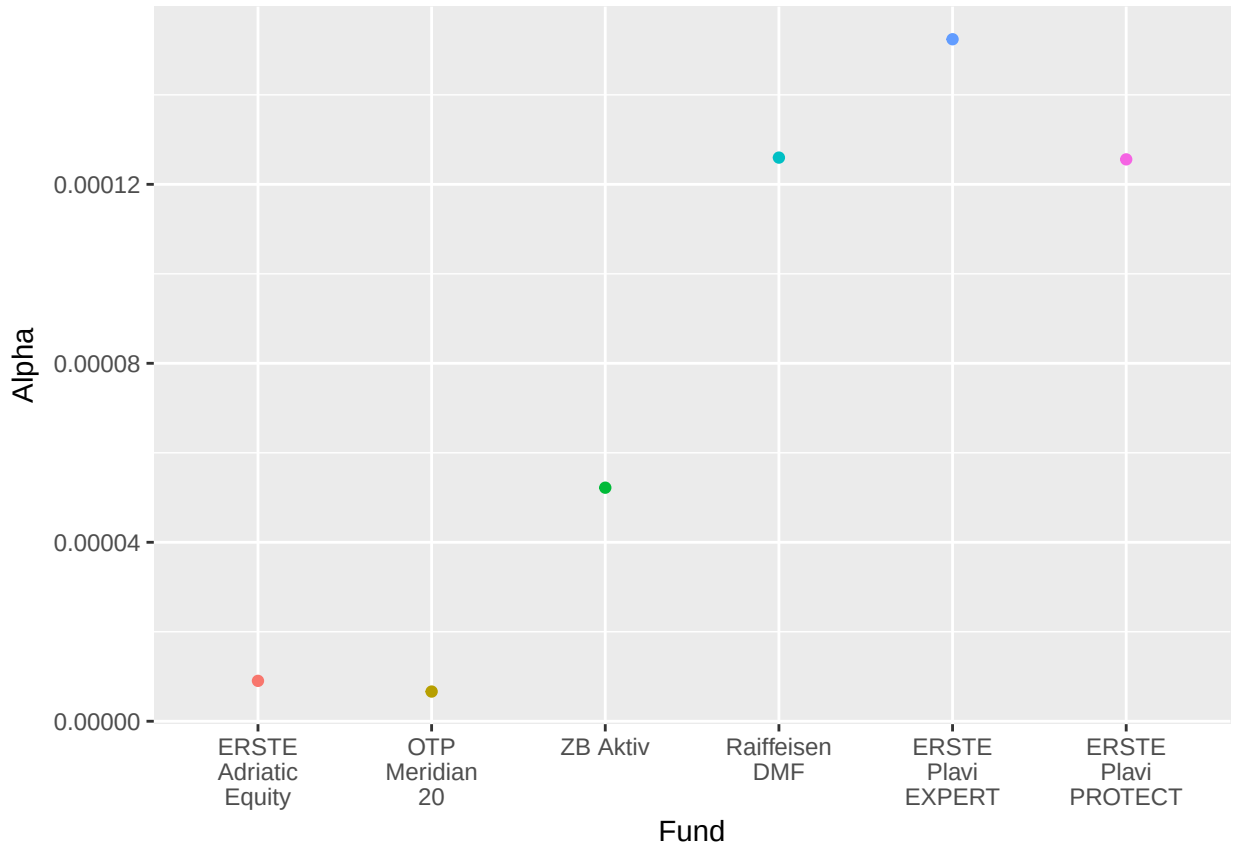

```
ggplot(xs.capm, aes(Year, Alpha, color= Fund, group = Fund)) +
  geom_point() + geom_line()
```




```
ggplot(xs.capm.all, aes(Fund, Beta, color= Fund, group = Fund)) +
  geom_point() +
  geom_line() +
  scale_x_discrete(labels = function(labels) lapply(labels, label_prettify)) +
  theme(legend.position="none")
```




```
ggplot(xs.capm.all, aes(Fund, Alpha, color= Fund, group = Fund)) +
  geom_point() +
  geom_line() +
  scale_x_discrete(labels = function(labels) lapply(labels, label_prettify)) +
  theme(legend.position="none")
```



Provjera reziduala

Jedna od pretpostavki pri regresijskoj analizi jest normalnost razdiobe reziduala. Sljedećom tablicom dane su p-vrijednosti, kad Kolmogorov-Smirnovljevim testom usporedimo distribucije reziduala za svaki fond po godinama i normalnu distribuciju. Kako su te vrijednosti male, odbacujemo hipotezu kako su reziduali normalno distribuirani.

```
nrows = nrow(xs.capm)
get_residuals_norm <- function(df, row) {
  xs.tmpYear <- get_for_year(xs.returns, xs.returns$Date, df[row,'Year'])
  fund <- as.character(df[row, 'Fund'])
  xs.tmpYearFund <- xs.tmpYear[,fund]

  capm.risk_free.year <- xs.tmpYear$InterestRate.daily
  capm.market.year <- xs.tmpYear$CROBEX

  residuals.tmp <- vector(mode="numeric", length=length(xs.tmpYearFund))

  for (i in 1:length(xs.tmpYearFund)) {
    residuals.tmp[i] <- ((xs.tmpYearFund[i] - capm.risk_free.year[i]) -
      (df[row,3] + df[row, 4] * (capm.market.year[i] - capm.risk_free.year[i])))
  }

  scaled.residuals.tmp <- scale(residuals.tmp)
  x <- ks.test(scaled.residuals.tmp, 'pnorm')
```

```
  df[row, 'KS.p'] <- x$p.value

  return(df)
}

for(i in 1:nrows) {
  xs.capm <- suppressWarnings( get_residuals_norm(xs.capm, i) )
}
```

Test prilagodbe modela

Izračunali smo i ANOVA test na prilagodbu modela, no ne obraćamo preveliku pažnju na njega radi loših rezultata KS testa na normalnost reziduala.

```
alpha_beta_r <- function(vals, alpha, beta, index, risk.free) {

  model <- alpha + beta * (index - risk.free) + risk.free

  SSE = (vals - model)^2 %>% sum
  SST = (vals - mean(vals))^2 %>% sum

  return(1 - SSE / SST)
}

alpha_beta_anova <- function(vals, alpha, beta, index, risk.free) {

  model <- alpha + beta * (index - risk.free) + risk.free

  n_i <- length(vals)
  N <- 2 * n_i

  SSA <- (n_i * (vals - model)^2) %>% sum
  SSE <- ((n_i -1) * var(vals)) %>% sum

  f <- SSA / SSE / N

  return(1 - pf(f, 1, N))
}

iterate.returns <- function(apply_fn) function(desired_year, fund, alpha, beta) {
  xs.curr <- xs.returns[year(xs.returns$Date) == desired_year, ]
  fund.actual.vals <- xs.curr[, as.character(fund)]

  return(apply_fn(fund.actual.vals,
                  alpha, beta,
                  xs.curr$CROBEX,
                  xs.curr$InterestRate.daily))
}

xs.capm$anova <- mapply(iterate.returns(alpha_beta_anova),
                        xs.capm$Year,
                        xs.capm$Fund,
```

```
                      xs.capm$Alpha,
                      xs.capm$Beta)
xs.capm
```

```
##                   Fund Year         Alpha         Beta         KS.p
## 1  ERSTEAdriaticEquity 2010 -1.363158e-04 0.5669352275 7.606159e-08
## 2  ERSTEAdriaticEquity 2011 -3.331016e-04 0.4543998614 8.972995e-10
## 3  ERSTEAdriaticEquity 2012 -1.467188e-04 0.2646270122 3.976819e-13
## 4  ERSTEAdriaticEquity 2013  1.072103e-04 0.3940627807 3.095202e-11
## 5  ERSTEAdriaticEquity 2014  1.520230e-04 0.3489534496 5.271628e-11
## 6  ERSTEAdriaticEquity 2015  2.024548e-04 0.4975787045 2.888248e-09
## 7  ERSTEAdriaticEquity 2016  2.126424e-04 0.5955184335 1.182900e-08
## 8        OTPMeridian20 2010 -1.401558e-05 0.4309319208 2.925515e-10
## 9        OTPMeridian20 2011 -4.044461e-04 0.5599034422 3.870977e-08
## 10       OTPMeridian20 2012  2.005561e-04 0.3908358348 2.004861e-09
## 11       OTPMeridian20 2013  2.280547e-04 0.2464292424 2.003044e-10
## 12       OTPMeridian20 2014  5.006249e-05 0.2761167134 7.392093e-09
## 13       OTPMeridian20 2015  1.400398e-04 0.4208172130 2.578632e-08
## 14       OTPMeridian20 2016 -1.026420e-05 0.4574313828 5.335510e-12
## 15             ZBAktiv 2010  9.544554e-05 0.2194612955 2.049980e-04
## 16             ZBAktiv 2011 -4.358250e-04 0.1381375343 2.527614e-05
## 17             ZBAktiv 2012  2.009252e-04 0.1336074631 1.354321e-10
## 18             ZBAktiv 2013  5.840394e-05 0.1116804387 2.323343e-06
## 19             ZBAktiv 2014  1.375125e-04 0.2024930920 6.164473e-06
## 20             ZBAktiv 2015  1.880294e-04 0.2632705419 2.186302e-05
## 21             ZBAktiv 2016  1.448745e-04 0.3620458129 1.539224e-04
## 22       RaiffeisenDMF 2010  1.378239e-04 0.1194627941 2.717111e-03
## 23       RaiffeisenDMF 2011 -7.635330e-05 0.1405371249 3.224060e-06
## 24       RaiffeisenDMF 2012  2.378826e-04 0.1359857205 1.741118e-11
## 25       RaiffeisenDMF 2013  9.345165e-05 0.1687797266 2.118944e-04
## 26       RaiffeisenDMF 2014  3.404221e-04 0.1610284601 2.405696e-04
## 27       RaiffeisenDMF 2015  1.803462e-04 0.1697165854 1.146525e-03
## 28       RaiffeisenDMF 2016  1.041926e-04 0.1389678904 2.727515e-06
## 29    ERSTEPlaviEXPERT 2010  1.580814e-04 0.1750763495 2.160262e-04
## 30    ERSTEPlaviEXPERT 2011 -2.905792e-05 0.2343404356 1.988193e-04
## 31    ERSTEPlaviEXPERT 2012  3.950473e-04 0.1491048287 7.504533e-05
## 32    ERSTEPlaviEXPERT 2013  5.594323e-05 0.1442715157 3.300715e-04
## 33    ERSTEPlaviEXPERT 2014  2.469191e-04 0.1700392710 2.902686e-03
## 34    ERSTEPlaviEXPERT 2015  1.991195e-04 0.2703175563 3.123242e-03
## 35    ERSTEPlaviEXPERT 2016  1.542028e-04 0.2781643675 7.591789e-05
## 36   ERSTEPlaviPROTECT 2010  1.528344e-04 0.0248670367 1.210117e-05
## 37   ERSTEPlaviPROTECT 2011  3.855290e-05 0.0355562101 4.594969e-05
## 38   ERSTEPlaviPROTECT 2012  3.418137e-04 0.0312794690 1.652921e-05
## 39   ERSTEPlaviPROTECT 2013  4.735975e-05 0.0220619941 8.751553e-07
## 40   ERSTEPlaviPROTECT 2014  2.035929e-04 0.0185440353 3.416949e-03
## 41   ERSTEPlaviPROTECT 2015  9.710402e-05 0.0358210528 4.620251e-04
## 42   ERSTEPlaviPROTECT 2016  1.639057e-04 0.0005997816 2.693304e-05
##        anova
## 1  0.6819894
## 2  0.5716510
## 3  0.5024239
## 4  0.5483689
## 5  0.5326181
## 6  0.5574813
```

```
## 7  0.5720919
## 8  0.5550716
## 9  0.5873669
## 10 0.5514915
## 11 0.5037197
## 12 0.5027573
## 13 0.5236979
## 14 0.5355345
## 15 0.5003877
## 16 0.4879652
## 17 0.4830034
## 18 0.4844315
## 19 0.4923193
## 20 0.4947660
## 21 0.5223154
## 22 0.5779664
## 23 0.5547833
## 24 0.5086889
## 25 0.5131886
## 26 0.5144059
## 27 0.5126482
## 28 0.5048025
## 29 0.5767841
## 30 0.5748023
## 31 0.5209586
## 32 0.5268349
## 33 0.5399832
## 34 0.5393349
## 35 0.5602932
## 36 0.4888386
## 37 0.4945413
## 38 0.4841304
## 39 0.4823546
## 40 0.4831694
## 41 0.4843187
## 42 0.4797274
```

ANOVA

ANOVA (ANalysis Of VAriance) je metoda koja nam pomaže da donesemo neke zaključke o razlikama između sredina više od dvije populacije. Ovdje analiziramo srednje vrijednosti prinosa svakog fonda, uz hipotezu da su im srednji prinosi svima jednaki (drugim riječima, želimo pokazati da u konačnici nije bitno u koji se fond ulaže). Rezultat testa potvrđuje našu hipotezu, no ipak ga uzimamo s oprezom radi nesavršenih uvjeta za obavljanje ovakvog testa.

```
num.items <- xs.returns %>% dim %>% first
anova.subset <- all_funds
anova.returns <- xs.returns %>%
  subset.data.frame(select=anova.subset) %>%
  unlist
anova.factors.funds <- anova.subset %>%
  rep(rep(num.items, length(.))) %>%
  as.factor
```

```
(anova.returns ~ anova.factors.funds) %>%
  lm %>% anova
```

```
## Analysis of Variance Table
##
## Response: anova.returns
##                         Df   Sum Sq    Mean Sq F value Pr(>F)
## anova.factors.funds      5 0.000069 1.3891e-05   1.144 0.3345
## Residuals            15312 0.185924 1.2142e-05
```

Dvofaktorska ANOVA

Usprkos narušenim uobičajenim pretpostavkama za dvofaktorsku ANOVu (ponajviše nezavisnosti, a zatim i normalnosti srednjih vrijednosti prinosa, što je već pokazano da ne vrijedi u ranijim odjeljcima), zanimljivo je primjetiti da dvofaktorska ANOVa ovdje pokazuje kako ukupno stanje tržišta tijekom neke godine ipak snažno utječe na prinose fondova.

```
anova.factors.years <- xs.returns$Date %>%
  year %>%
  rep(length(anova.subset)) %>%
  as.factor

(anova.returns ~ anova.factors.funds * anova.factors.years) %>%
  lm %>% anova
```

```
## Analysis of Variance Table
##
## Response: anova.returns
##                                          Df   Sum Sq    Mean Sq F value
## anova.factors.funds                       5 0.000069 1.3891e-05  1.1458
## anova.factors.years                       6 0.000483 8.0530e-05  6.6425
## anova.factors.funds:anova.factors.years  30 0.000242 8.0640e-06  0.6651
## Residuals                             15276 0.185199 1.2124e-05
##                                             Pr(>F)
## anova.factors.funds                         0.3336
## anova.factors.years                      4.968e-07 ***
## anova.factors.funds:anova.factors.years     0.9176
## Residuals
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Zaključak

Iz svega ovoga odlučili smo da bi kao grupa investirali u ERSTEPlaviPROTECT, jer ima najbolji omjer alfa i beta parametra. Ima visoku alfu, uz nisku betu.