Mjerenje uspješnosti investicijskih fondova

Učitavanje podataka i pomoćnih biblioteka

Prilikom proučavanja podataka primjetili smo da vrijednost fonda ErsteAdriaticEquity za 24.1.2016. poprilično odskače od okolnih datuma. Pretragom na stranici Erste grupe ustvrdili smo pogrešku u unosu podataka te smo ručno ispravili vrijednost.

```
library(reshape2)
library(dplyr)
library(magrittr)
library(ggplot2)
library(stringr)
library(xts)
require(quantmod)
require(PerformanceAnalytics)
source('data_extraction.r')
xs <- read_normalize('./investicijski_fondovi_data.csv')</pre>
```

Priprema i analiza podataka

Podjela prema tipovima fondova

```
investment_funds <- c("ERSTEAdriaticEquity", "OTPMeridian20", "ZBAktiv")
pension_funds <- c("RaiffeisenDMF", "ERSTEPlaviEXPERT", "ERSTEPlaviPROTECT")
market_portfolio <- c("CROBEX")

all_funds <- c(investment_funds, pension_funds)
data_columns <- c(pension_funds, investment_funds, market_portfolio)</pre>
```

Povrati

```
Računanje dnevnih povrata prema formuli: R(t) = \log(S(t)/S(t-1))

diff_function_log <- function(St, St_minus_one) log(St) - log(St_minus_one)

xs.returns <- to_time_series_diff_df(xs, data_columns, diff_function_log)
```

Sažeci

Prikaz mjera centralne tendencije

```
xs.returns.summary <- summary(xs.returns[data_columns] * 365)
data.frame(unclass(xs.returns.summary), check.names = FALSE, stringsAsFactors = FALSE)</pre>
```

```
## RaiffeisenDMF ERSTEPlaviEXPERT ERSTEPlaviPROTECT
## 1 Min. :-5.79209 Min. :-5.73634 Min. :-2.06945
## 2 1st Qu.:-0.18757 1st Qu.:-0.19025 1st Qu.:-0.05239
## 3 Median : 0.02441 Median : 0.02279 Median : 0.04456
## 4 Mean : 0.06451 Mean : 0.07278 Mean : 0.06709
```

```
## 5 3rd Qu.: 0.31443
                      3rd Qu.: 0.39346
                                        3rd Qu.: 0.20759
## 6 Max. : 8.91872
                      Max. : 4.58776
                                        Max. : 3.22798
                            OTPMeridian20
                                                     ZBAktiv
    ERSTEAdriaticEquity
         :-18.08756 Min.
                                                :-13.47776
## 1 Min.
                            :-23.51025
                                         Min.
## 2 1st Qu.: -0.48492
                       1st Qu.: -0.35673
                                         1st Qu.: -0.41271
## 3 Median : 0.00000
                      Median: 0.00000
                                        Median: 0.00000
## 4 Mean : 0.01423
                       Mean : 0.01395
                                         Mean : 0.03645
## 5 3rd Qu.: 0.50246
                       3rd Qu.: 0.63048
                                         3rd Qu.: 0.61738
## 6 Max.
         : 21.67018
                       Max. : 13.60614
                                         Max. : 34.35281
##
                CROBEX
## 1 Min.
          :-17.43339
## 2 1st Qu.: -0.58382
## 3 Median : 0.00000
## 4 Mean
         : -0.00203
## 5 3rd Qu.: 0.67653
## 6 Max.
         : 31.25453
```

Mjere raspršenosti

Prikaz vrijednosti standardne devijacije i varijance za svaki fond

```
variances <- apply(xs.returns[all_funds] * 365, 2, var, na.rm = T)
std.devs <- apply(xs.returns[all_funds] * sqrt(365), 2, sd, na.rm = T)
data.frame(std.devs, variances)</pre>
```

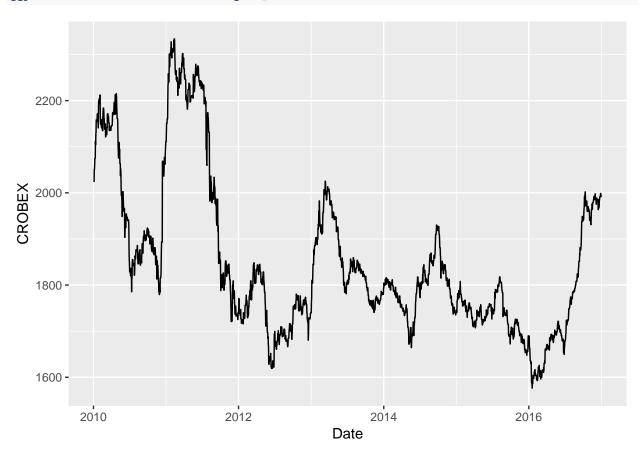
```
## ERSTEAdriaticEquity 0.08446841 2.6042430  
## OTPMeridian20 0.09034363 2.9791195  
## ZBAktiv 0.08987277 2.9481469  
## RaiffeisenDMF 0.03555447 0.4614040  
## ERSTEPlaviEXPERT 0.04020710 0.5900631  
## ERSTEPlaviPROTECT 0.01835943 0.1230300
```

Grafički prikaz podataka

Prikaz vrijednosti CROBEX-a po danima

Kretanje vrijednosti burzovnog indeksa od početka 2010. godine do kraja 2016. godine.

ggplot(xs, aes(Date, CROBEX)) + geom_line()



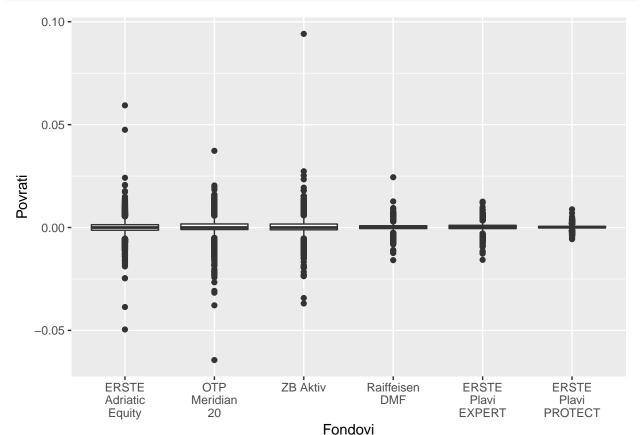
Prikaz vrijednosti investicijskih i mirovinskih fondova po danima

Iz grafa se vidi kako investicijski fondovi imaju veću tendenciju rasta.



Prikaz boxplotova za sve fondove

Iz ovog se grafa ne može zaključiti mnogo, ali vidi kako su investicijski fondovi (prva tri stupca) na dnevnoj bazi podložniji većim promjenama vrijednosti od mirovinskih, jer ima više stršećih vrijednosti.

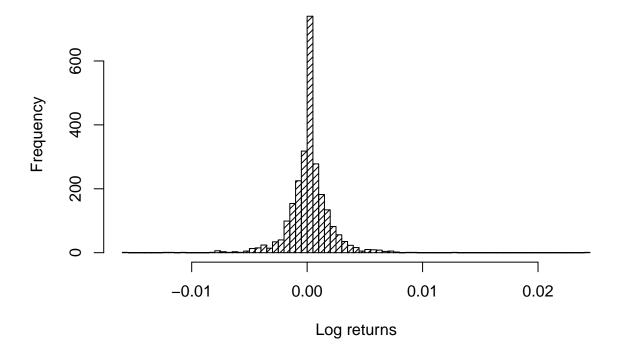


Provjera normalnosti dnevnih povrata fondova

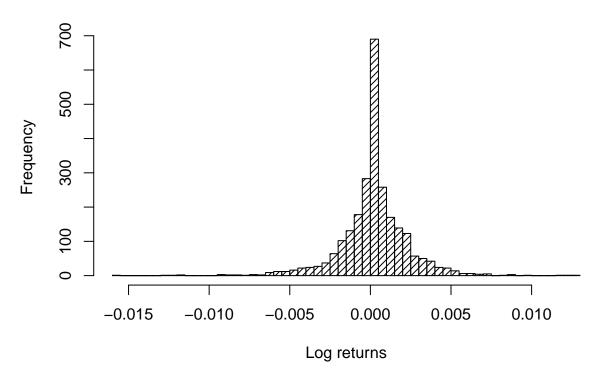
Histogramima

Iscrtavamo histograme povrata za svaki fond. Vidimo da imaju prilično teške repove, što ukazuje kako nisu baš normalno distribuirani.

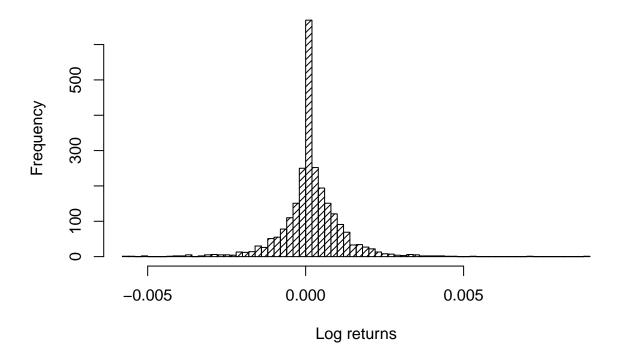
RaiffeisenDMF



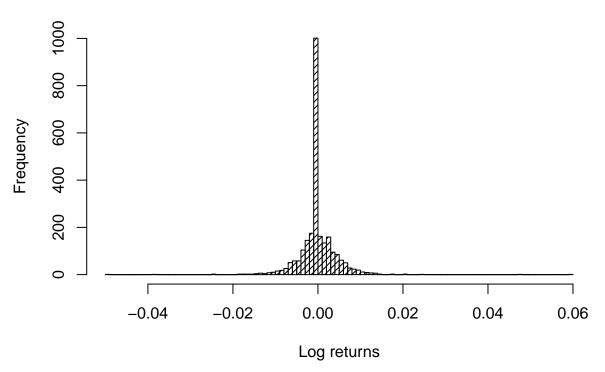
ERSTEPlaviEXPERT

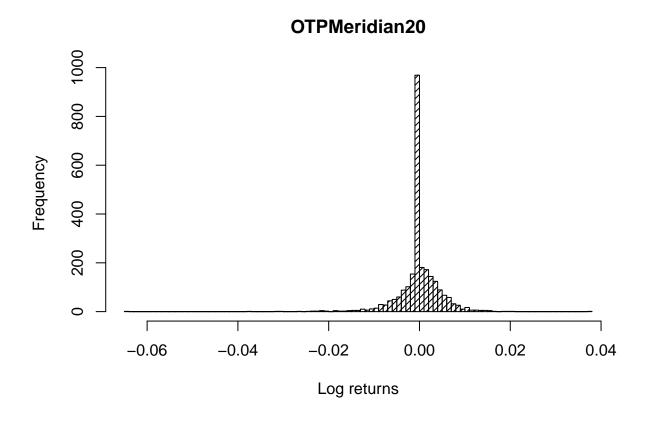


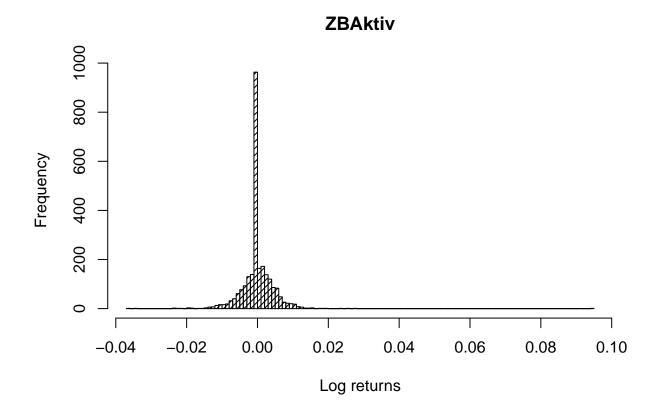
ERSTEPlaviPROTECT



ERSTEAdriaticEquity



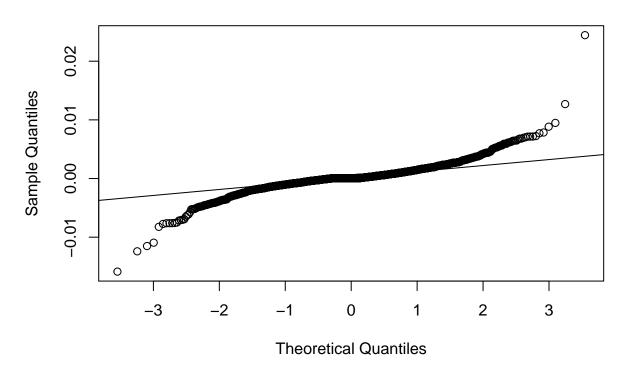




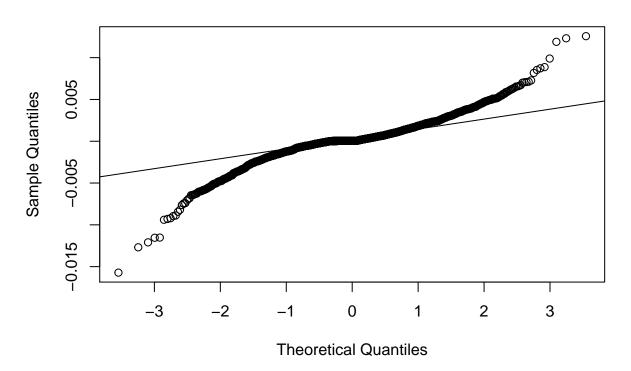
${f Q}{f Q}$ grafovima

Sljedećim QQ grafovima želimo ispitati normalnost distribucije povrata svih fondova. Teške repove primjećujemo radi sitne granulacije, tj. dnevnog računanja prinosa; u tako kratkom roku zna se dogoditi da pojedina dionica ili naglo naraste ili naglo padne u vrijednosti.

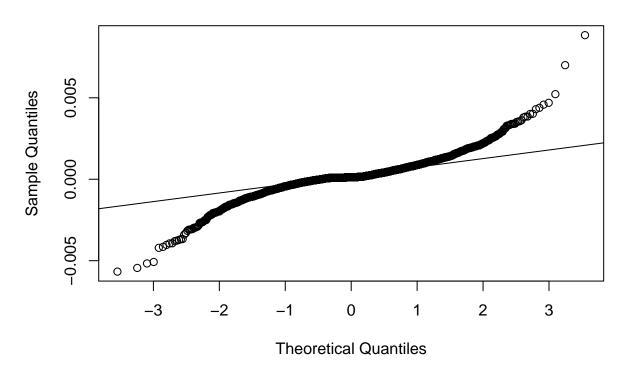
RaiffeisenDMF



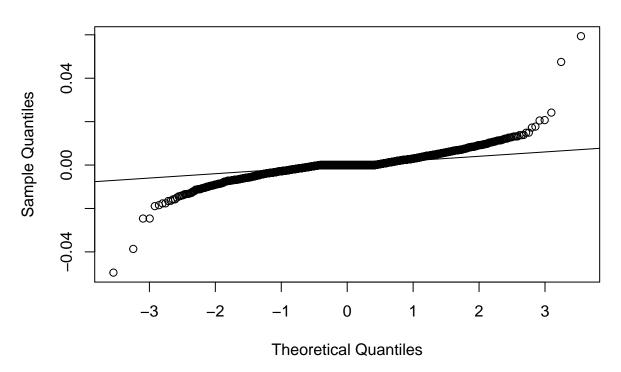
ERSTEPlaviEXPERT



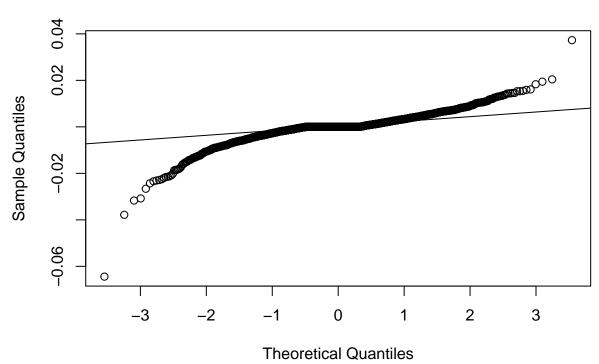
ERSTEPlaviPROTECT



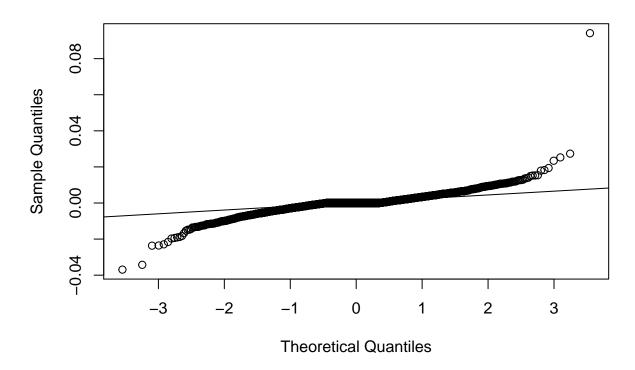
ERSTEAdriaticEquity



OTPMeridian20



ZBAktiv



Testovi fondova

Iako QQ grafovi pokazuju da povrati nisu normalno raspodijeljeni, radimo tu pretpostavku s obzirom na robusnost T-testa. Jasno je da globalni događaji (kriza, teroristički napadi,...) često utječu na cijelo tržište odjednom, pa koristimo T-testove za uparene podatke.

Testovi povrata investicijskih fondova u odnosu na CROBEX

Za H0 hipotezu uzimamo kako su sredine investicijskih fondova i CROBEX indeksa jednake. Iz sljedećih testova vidimo da ne možemo opovrgnuti tu hipozetu.

```
ERSTEAdriaticEquity
                                          OTPMeridian20
## statistic
               -0.4774079
                                          -0.4286687
               2552
                                          2552
## parameter
## p.value
               0.6331126
                                          0.6682005
                                         Numeric,2
## conf.int
               Numeric, 2
                                          -4.377476e-05
## estimate
               -4.454166e-05
## null.value
               0
                                          "two.sided"
## alternative "two.sided"
```

```
## method
               "Paired t-test"
                                         "Paired t-test"
## data.name
               "index and fund.returns" "index and fund.returns"
##
               ZBAktiv
               -0.8207028
## statistic
## parameter
               2552
               0.4118922
## p.value
## conf.int
               Numeric.2
## estimate
               -0.0001054245
## null.value
## alternative "two.sided"
## method
               "Paired t-test"
## data.name
               "index and fund.returns"
```

Testovi povrata mirovinskih fondova u odnosu na CROBEX

Za H0 hipotezu uzimamo kako su sredine mirovinskih fondova i CROBEX indeksa jednake. Iz sljedećih testova zaključujemo da ne možemo odbaciti H0 hipotezu uz nivo značajnosti 5% za fondove RaiffeisenDMF i ERSTEPlaviPROTECT, dok za ERSTEPlaviEXPERT možemo.

```
mapply(compare.to.index(xs.returns$CROBEX), xs.returns[pension_funds])
```

```
##
                                         ERSTEPlaviEXPERT
               RaiffeisenDMF
               -1.789106
                                         -2.125481
## statistic
## parameter
               2552
                                         2552
## p.value
               0.07371632
                                         0.03364232
## conf.int
                                         Numeric, 2
               Numeric, 2
                                         -0.0002049503
## estimate
               -0.0001823123
## null.value 0
## alternative "two.sided"
                                         "two.sided"
## method
               "Paired t-test"
                                         "Paired t-test"
## data.name
               "index and fund.returns" "index and fund.returns"
##
               ERSTEPlaviPROTECT
               -1.711485
## statistic
## parameter
               2552
## p.value
               0.08711319
## conf.int
               Numeric,2
               -0.0001893649
## estimate
## null.value
## alternative "two.sided"
## method
               "Paired t-test"
               "index and fund.returns"
## data.name
```

Test povrata investicijskih fondova u odnosu na mirovinske fondove

Izračunate su sredine mirovinskih i investicijskih fondova pa je sproveden test njihovih vrijednosti. Dobivamo izrazito malu p-vrijednost, stoga uz relativno veliku sigurnost zaključujemo da možemo odbaciti nul-hipotezu koja tvrdi da su sredine jednake.

pension vs. Investment fund means 0.02846181 two.sided 0.0001276289

CAPM model

CAPM (Capital Asset pricing model) je model koji opisuje odnos između kamatne stope i očekivanog povrata sredstava. Svodi se na linearnu regresiju:

$$R_p - R_f = \alpha + \beta (R_m - R_f) + \epsilon$$

gdje je R_p prinos promatranog fonda (portfelja), R_m prinos tržišnog (referentnog) portfelja, a R_f je bezrizična kamatna stopa. Koeficijent α mjeri koliko je prinos promatranog fonda veći od prinosa tržišnog portfelja, a β mjeri osjetljivost fonda na tržišne prinose i predstavlja rizičnost.

Promatrajući koeficijente α i β svakog fonda zaključujemo da bi najbolje bilo uložiti u fond ERSTEPlaviPROTECT jer ima najmanji koeficijent β od svih fondova i poprilično visok koeficijent α .

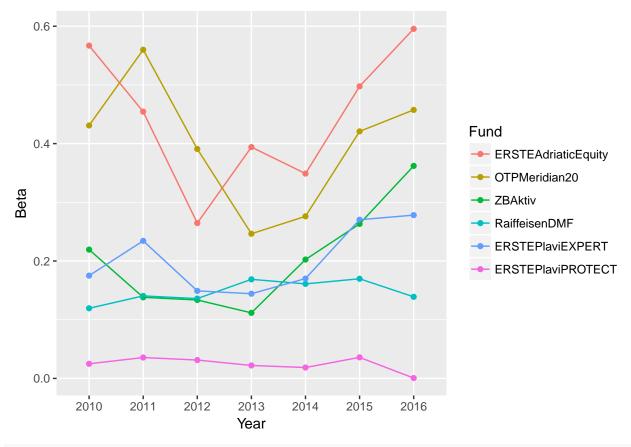
```
year <- function(date) format(date, "%Y")</pre>
get_for_year <- function(df, dates, desired_year) df[year(dates) == desired_year, ]</pre>
get_capm_for_year <- function(df, fund, desired_year){</pre>
  xs.year = get_for_year(df, df$Date, desired_year)
  fund.year <- xs.year[c('Date', fund)]</pre>
  fund.ts <- xts(fund.year[, -1], order.by=fund.year$Date)</pre>
  capm.index.year <- xs.year[c('Date', 'CROBEX')]</pre>
  capm.index.ts <- xts(capm.index.year[, -1], order.by=capm.index.year$Date)</pre>
  capm.risk_free.year <- xs.year[c('Date', 'InterestRate.daily')]</pre>
  capm.risk free.year <- capm.risk free.year[1, -1]</pre>
  data.frame(fund, as.factor(desired year),
              CAPM.alpha(fund.ts, capm.index.ts, capm.risk_free.year),
              CAPM.beta(fund.ts, capm.index.ts, capm.risk_free.year))
}
get_capm_for_fund <- function(df, selected_fund){</pre>
  fund <- df[c('Date', selected_fund)]</pre>
  fund.ts <- xts(fund[, -1], order.by=fund$Date)</pre>
  capm.index <- df[c('Date', 'CROBEX')]</pre>
  capm.index.ts <- xts(capm.index[, -1], order.by=capm.index$Date)</pre>
  capm.risk free <- df[c('Date', 'InterestRate.daily')]</pre>
  capm.risk_free.ts <- capm.risk_free[1, -1]</pre>
  data.frame(selected_fund,
              CAPM.alpha(fund.ts, capm.index.ts, capm.risk_free.ts),
              CAPM.beta(fund.ts, capm.index.ts, capm.risk free.ts))
}
xs.years = seq(from = 2010, by = 1, length = 7)
xs.fund.names = c(investment_funds, pension_funds)
xs.capm <- data.frame(matrix(ncol = 3, nrow = 0))</pre>
xs.capm.all <- data.frame(matrix(ncol = 3, nrow = 0))</pre>
for (i in 1:length(xs.fund.names)){
  for (j in 1:length(xs.years)){
    xs.capm <- rbind(xs.capm, get_capm_for_year(xs.returns, xs.fund.names[i], xs.years[j]))</pre>
  }
```

```
for (i in 1:length(xs.fund.names)){
    xs.capm.all <- rbind(xs.capm.all, get_capm_for_fund(xs.returns, xs.fund.names[i]))
}

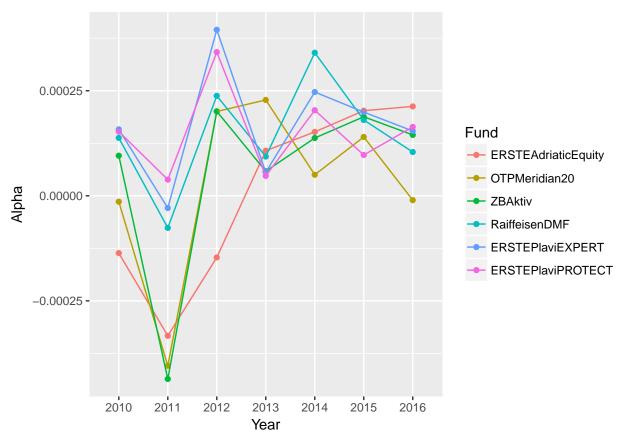
colnames(xs.capm) <- c("Fund", "Year", "Alpha", "Beta")

colnames(xs.capm.all) <- c("Fund", "Alpha", "Beta")

ggplot(xs.capm, aes(Year, Beta, color= Fund, group = Fund)) +
    geom_point() + geom_line()</pre>
```

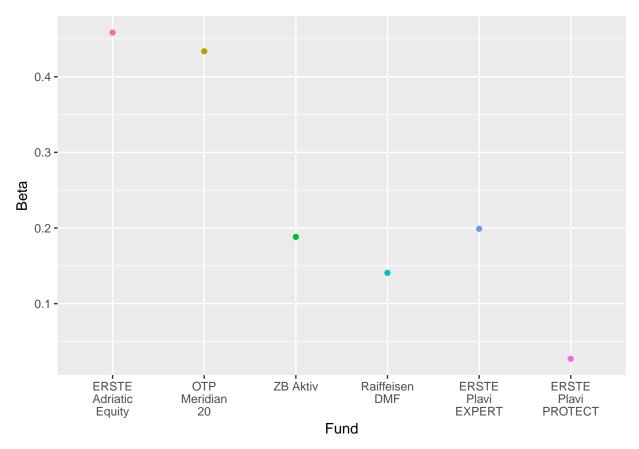


ggplot(xs.capm, aes(Year, Alpha, color= Fund, group = Fund)) +
geom_point() + geom_line()



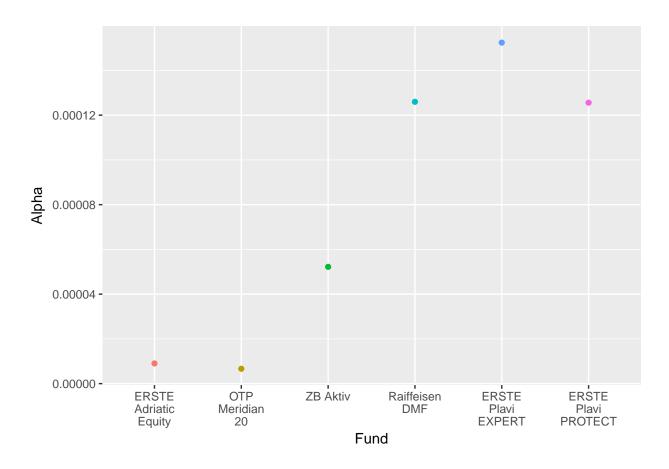
```
ggplot(xs.capm.all, aes(Fund, Beta, color= Fund, group = Fund)) +
  geom_point() +
  geom_line() +
  scale_x_discrete(labels = function(labels) lapply(labels, label_prettify)) +
  theme(legend.position="none")
```

geom_path: Each group consists of only one observation. Do you need to
adjust the group aesthetic?



```
ggplot(xs.capm.all, aes(Fund, Alpha, color= Fund, group = Fund)) +
  geom_point() +
  geom_line() +
  scale_x_discrete(labels = function(labels) lapply(labels, label_prettify)) +
  theme(legend.position="none")
```

geom_path: Each group consists of only one observation. Do you need to
adjust the group aesthetic?



Provjera reziduala

Jedna od pretpostavki pri regresijskoj analizi jest normalnost razdiobe reziduala. Sljedećom tablicom dane su p-vrijednosti, kad Kolmogorov-Smirnovljevim testom usporedimo distribucije reziduala za svaki fond po godinama i normalnu distribuciju. Kako su te vrijednosti male, odbacujemo hipotezu kako su reziduali normalno distribuirani.

```
df[row, 'KS.p'] <- x$p.value
  return(df)
}

for(i in 1:nrows) {
    xs.capm <- suppressWarnings( get_residuals_norm(xs.capm, i) )
}</pre>
```

Test prilagodbe modela

Izračunali smo i ANOVA test na prilagodbu modela, no ne obraćamo preveliku pažnju na njega radi loših rezultata KS testa na normalnost reziduala.

```
alpha_beta_r <- function(vals, alpha, beta, index, risk.free) {</pre>
  model <- alpha + beta * (index - risk.free) + risk.free</pre>
  SSE = (vals - model)^2 \% sum
  SST = (vals - mean(vals))^2 %>% sum
  return(1 - SSE / SST)
}
rsquared.from_returns <- function(desired_year, fund, alpha, beta) {</pre>
  xs.curr <- xs.returns[year(xs.returns$Date) == desired_year, ]</pre>
  fund.actual.vals <- xs.curr[, as.character(fund)]</pre>
  return(alpha_beta_r(fund.actual.vals,
                        alpha, beta,
                        xs.curr$CROBEX,
                       xs.curr$InterestRate.daily))
}
xs.capm$R.squared <- mapply(rsquared.from_returns,</pre>
                              xs.capm$Year,
                              xs.capm$Fund,
                              xs.capm$Alpha,
                              xs.capm$Beta)
xs.capm
```

```
##
                    Fund Year
                                       Alpha
                                                     Beta
                                                                  KS.p
## 1
     ERSTEAdriaticEquity 2010 -1.363158e-04 0.5669352275 7.606159e-08
     ERSTEAdriaticEquity 2011 -3.331016e-04 0.4543998614 8.972995e-10
     ERSTEAdriaticEquity 2012 -1.467188e-04 0.2646270122 3.976819e-13
## 3
## 4 ERSTEAdriaticEquity 2013 1.072103e-04 0.3940627807 3.095202e-11
## 5 ERSTEAdriaticEquity 2014 1.520230e-04 0.3489534496 5.271628e-11
     ERSTEAdriaticEquity 2015
## 6
                               2.024548e-04 0.4975787045 2.888248e-09
## 7
     ERSTEAdriaticEquity 2016 2.126424e-04 0.5955184335 1.182900e-08
## 8
            OTPMeridian20 2010 -1.401558e-05 0.4309319208 2.925515e-10
## 9
            OTPMeridian20 2011 -4.044461e-04 0.5599034422 3.870977e-08
            OTPMeridian20 2012 2.005561e-04 0.3908358348 2.004861e-09
## 10
```

```
## 11
            OTPMeridian20 2013 2.280547e-04 0.2464292424 2.003044e-10
## 12
            OTPMeridian20 2014 5.006249e-05 0.2761167134 7.392093e-09
## 13
            OTPMeridian20 2015 1.400398e-04 0.4208172130 2.578632e-08
            OTPMeridian20 2016 -1.026420e-05 0.4574313828 5.335510e-12
## 14
## 15
                  ZBAktiv 2010
                                9.544554e-05 0.2194612955 2.049980e-04
                  ZBAktiv 2011 -4.358250e-04 0.1381375343 2.527614e-05
## 16
                  ZBAktiv 2012 2.009252e-04 0.1336074631 1.354321e-10
## 17
                  ZBAktiv 2013 5.840394e-05 0.1116804387 2.323343e-06
## 18
## 19
                  ZBAktiv 2014
                                1.375125e-04 0.2024930920 6.164473e-06
                                1.880294e-04 0.2632705419 2.186302e-05
## 20
                  ZBAktiv 2015
## 21
                  ZBAktiv 2016
                               1.448745e-04 0.3620458129 1.539224e-04
            RaiffeisenDMF 2010
## 22
                               1.378239e-04 0.1194627941 2.717111e-03
## 23
            RaiffeisenDMF 2011 -7.635330e-05 0.1405371249 3.224060e-06
            RaiffeisenDMF 2012 2.378826e-04 0.1359857205 1.741118e-11
## 24
## 25
            RaiffeisenDMF 2013
                                9.345165e-05 0.1687797266 2.118944e-04
## 26
            RaiffeisenDMF 2014
                                3.404221e-04 0.1610284601 2.405696e-04
## 27
            RaiffeisenDMF 2015
                                1.803462e-04 0.1697165854 1.146525e-03
## 28
            RaiffeisenDMF 2016
                                1.041926e-04 0.1389678904 2.727515e-06
## 29
         ERSTEPlaviEXPERT 2010
                               1.580814e-04 0.1750763495 2.160262e-04
## 30
         ERSTEPlaviEXPERT 2011 -2.905792e-05 0.2343404356 1.988193e-04
         ERSTEPlaviEXPERT 2012
## 31
                                3.950473e-04 0.1491048287 7.504533e-05
## 32
         ERSTEPlaviEXPERT 2013
                                5.594323e-05 0.1442715157 3.300715e-04
                                2.469191e-04 0.1700392710 2.902686e-03
## 33
         ERSTEPlaviEXPERT 2014
         ERSTEPlaviEXPERT 2015
                                1.991195e-04 0.2703175563 3.123242e-03
## 34
         ERSTEPlaviEXPERT 2016
                                1.542028e-04 0.2781643675 7.591789e-05
## 35
  36
        ERSTEPlaviPROTECT 2010
                                1.528344e-04 0.0248670367 1.210117e-05
## 37
        ERSTEPlaviPROTECT 2011
                                3.855290e-05 0.0355562101 4.594969e-05
        ERSTEPlaviPROTECT 2012
                                3.418137e-04 0.0312794690 1.652921e-05
##
  38
##
  39
        ERSTEPlaviPROTECT 2013
                                4.735975e-05 0.0220619941 8.751553e-07
## 40
        ERSTEPlaviPROTECT 2014
                                2.035929e-04 0.0185440353 3.416949e-03
## 41
        ERSTEPlaviPROTECT 2015
                                9.710402e-05 0.0358210528 4.620251e-04
##
  42
        ERSTEPlaviPROTECT 2016
                               1.639057e-04 0.0005997816 2.693304e-05
##
         R.squared
## 1
      6.639383e-01
##
  2
      3.595653e-01
## 3
     9.948311e-02
## 4
     2.788338e-01
## 5
     2.204703e-01
## 6
      3.111983e-01
     3.610346e-01
## 7
     3.027301e-01
## 8
## 9 4.105272e-01
## 10 2.900392e-01
## 11 1.049314e-01
## 12 1.008854e-01
## 13 1.860068e-01
## 14 2.315158e-01
## 15 9.085646e-02
## 16 3.700341e-02
## 17 1.484719e-02
## 18 2.126078e-02
## 19 5.614227e-02
## 20 6.677313e-02
## 21 1.805736e-01
```

```
## 22 3.803706e-01
## 23 3.017203e-01
## 24 1.256158e-01
## 25 1.440430e-01
## 26 1.489808e-01
## 27 1.418445e-01
## 28 1.094703e-01
## 29 3.765089e-01
## 30 3.700067e-01
## 31 1.752146e-01
## 32 1.982458e-01
## 33 2.481500e-01
## 34 2.457412e-01
## 35 3.209855e-01
## 36 4.085443e-02
## 37 6.580049e-02
## 38 1.991347e-02
## 39 1.191934e-02
## 40 1.559203e-02
## 41 2.075516e-02
## 42 1.035892e-05
```

ANOVA

ANOVA (ANalysis Of VAriance) je metoda koja nam pomaže da donesemo neke zaključke o razlikama između sredina više od dvije populacije. Ovdje analiziramo srednje vrijednosti prinosa svakog fonda, uz hipotezu da su im srednji prinosi svima jednaki (drugim riječima, želimo pokazati da u konačnici nije bitno u koji se fond ulaže). Rezultat testa potvrđuje našu hipotezu, no ipak ga uzimamo s oprezom radi nesavršenih uvjeta za obavljanje ovakvog testa.

```
num.items <- xs.returns %>% dim %>% first
anova.subset <- all_funds
anova.returns <- xs.returns %>%
    subset.data.frame(select=anova.subset) %>%
    unlist
anova.factors.funds <- anova.subset %>%
    rep(rep(num.items, length(.))) %>%
    as.factor

anova(lm(anova.returns ~ anova.factors.funds))

## Analysis of Variance Table
##
## Response: anova.returns
##
Df Sum Sq Mean Sq F value Pr(>F)
```

Dvofaktorska ANOVA

anova.factors.funds

Residuals

Usprkos narušenim uobičajenim pretpostavkama za dvofaktorsku ANOVu (ponajviše nezavisnosti, a zatim i normalnosti srednjih vrijednosti prinosa, što je već pokazano da ne vrijedi u ranijim odjeljcima), zanimljivo

1.144 0.3345

5 0.000069 1.3891e-05

15312 0.185924 1.2142e-05

je primjetiti da dvofaktorska ANOVa ovdje pokazuje kako ukupno stanje tržišta tijekom neke godine ipak snažno utječe na prinose fondova.

```
anova.factors.years <- xs.returns$Date %>%
  year %>%
  rep(length(anova.subset)) %>%
  as.factor
anova(lm(anova.returns ~ anova.factors.funds * anova.factors.years))
## Analysis of Variance Table
##
## Response: anova.returns
##
                                                   Sum Sq
                                                            Mean Sq F value
                                              5 0.000069 1.3891e-05 1.1458
## anova.factors.funds
## anova.factors.years
                                              6 0.000483 8.0530e-05 6.6425
## anova.factors.funds:anova.factors.years
                                              30 0.000242 8.0640e-06 0.6651
## Residuals
                                           15276 0.185199 1.2124e-05
##
                                              Pr(>F)
## anova.factors.funds
                                              0.3336
## anova.factors.years
                                           4.968e-07 ***
## anova.factors.funds:anova.factors.years
                                              0.9176
## Residuals
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
```