

# Mjerenje uspješnosti investicijskih fondova

## Učitavanje podataka i pomoćnih biblioteka

Prilikom proučavanja podataka primjetili smo da vrijednost fonda ErsteAdriaticEquity za 24.1.2016. poprilično odskaka od okolnih datuma. Pretragom na stranici Erste grupe ustvrdili smo pogrešku u unosu podataka te smo ručno ispravili vrijednost.

```
library(reshape2)
library(dplyr)
library(magrittr)
library(ggplot2)
library(stringr)
library(xts)
require(quantmod)
require(PerformanceAnalytics)
source('data_extraction.r')
xs <- read_normalize('./investicijski_fondovi_data.csv')
```

## Priprema i analiza podataka

### Podjela prema tipovima fondova

```
investment_funds <- c("ERSTAdriaticEquity", "OTPMeridian20", "ZBAktiv")
pension_funds <- c("RaiffeisenDMF", "ERSTEPlaviEXPERT", "ERSTEPlaviPROTECT")
market_portfolio <- c("CROBEX")

all_funds <- c(investment_funds, pension_funds)
data_columns <- c(pension_funds, investment_funds, market_portfolio)
```

## Povrati

Računanje dnevnih povrata prema formuli:  $R(t) = \log(S(t)/S(t-1))$

```
diff_function_log <- function(St, St_minus_one) log(St) - log(St_minus_one)
xs.returns <- to_time_series_diff_df(xs, data_columns, diff_function_log)
```

## Sažeci

Prikaz mjera centralne tendencije

```
xs.returns.summary <- summary(xs.returns[data_columns] * 365)
data.frame(unclass(xs.returns.summary), check.names = FALSE, stringsAsFactors = FALSE)
```

```
##           RaiffeisenDMF  ERSTEPlaviEXPERT  ERSTEPlaviPROTECT
## 1 Min.      :-5.79209    Min.      :-5.73634    Min.      :-2.06945
## 2 1st Qu.  :-0.18757    1st Qu.  :-0.19025    1st Qu.  :-0.05239
## 3 Median   : 0.02441    Median   : 0.02279    Median   : 0.04456
## 4 Mean     : 0.06451    Mean     : 0.07278    Mean     : 0.06709
```

```
## 5 3rd Qu.: 0.31443 3rd Qu.: 0.39346 3rd Qu.: 0.20759
## 6 Max. : 8.91872 Max. : 4.58776 Max. : 3.22798
## ERSTAdriaticEquity OTPMeridian20 ZBAktiv
## 1 Min. : -18.08756 Min. : -23.51025 Min. : -13.47776
## 2 1st Qu.: -0.48492 1st Qu.: -0.35673 1st Qu.: -0.41271
## 3 Median : 0.00000 Median : 0.00000 Median : 0.00000
## 4 Mean : 0.01423 Mean : 0.01395 Mean : 0.03645
## 5 3rd Qu.: 0.50246 3rd Qu.: 0.63048 3rd Qu.: 0.61738
## 6 Max. : 21.67018 Max. : 13.60614 Max. : 34.35281
## CROBEX
## 1 Min. : -17.43339
## 2 1st Qu.: -0.58382
## 3 Median : 0.00000
## 4 Mean : -0.00203
## 5 3rd Qu.: 0.67653
## 6 Max. : 31.25453
```

## Mjere raspršenosti

Prikaz vrijednosti standardne devijacije i varijance za svaki fond

```
variances <- apply(xs.returns[all_funds] * 365, 2, var, na.rm = T)
std.devs <- apply(xs.returns[all_funds] * sqrt(365), 2, sd, na.rm = T)

data.frame(std.devs, variances)
```

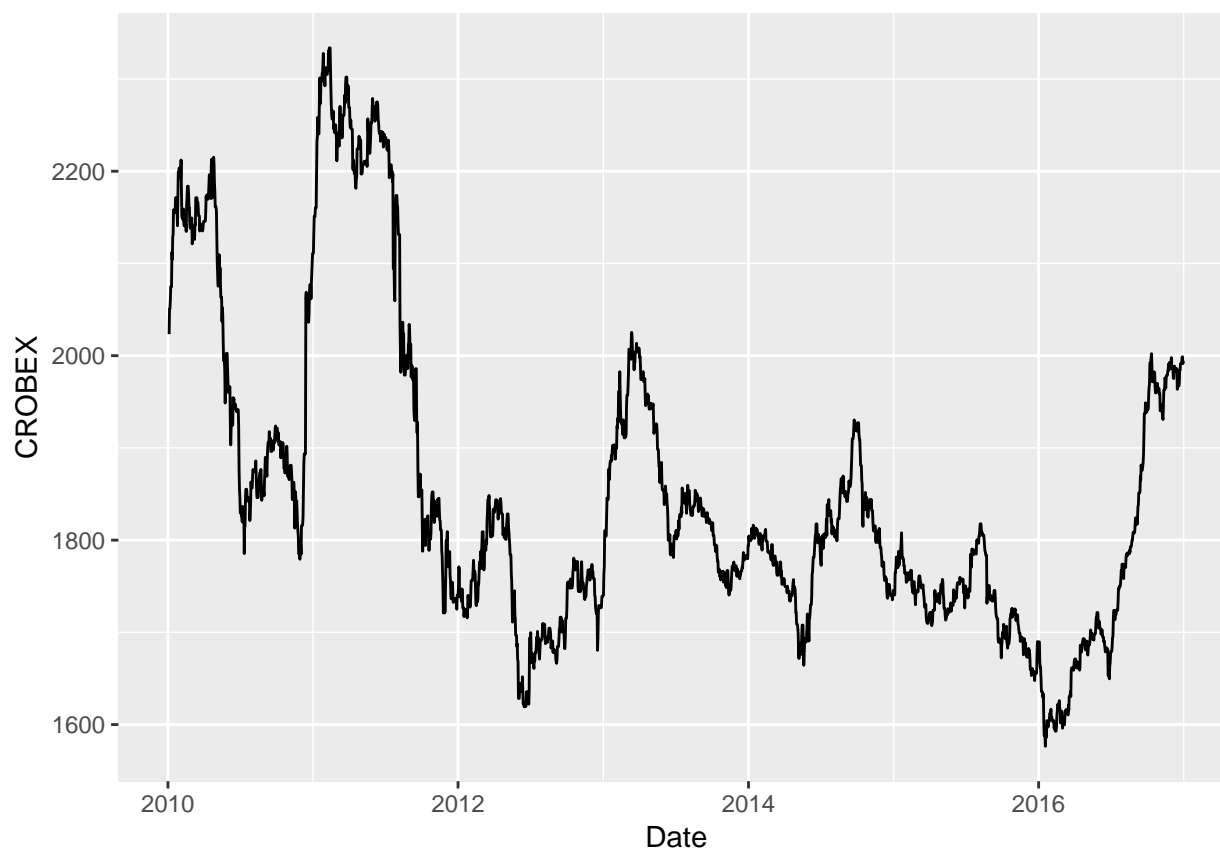
```
##          std.devs variances
## ERSTAdriaticEquity 0.08446841 2.6042430
## OTPMeridian20      0.09034363 2.9791195
## ZBAktiv            0.08987277 2.9481469
## RaiffeisenDMF      0.03555447 0.4614040
## ERSTEPlaviEXPERT    0.04020710 0.5900631
## ERSTEPlaviPROTECT   0.01835943 0.1230300
```

## Grafički prikaz podataka

### Prikaz vrijednosti CROBEX-a po danima

Kretanje vrijednosti burzovnog indeksa od početka 2010. godine do kraja 2016. godine.

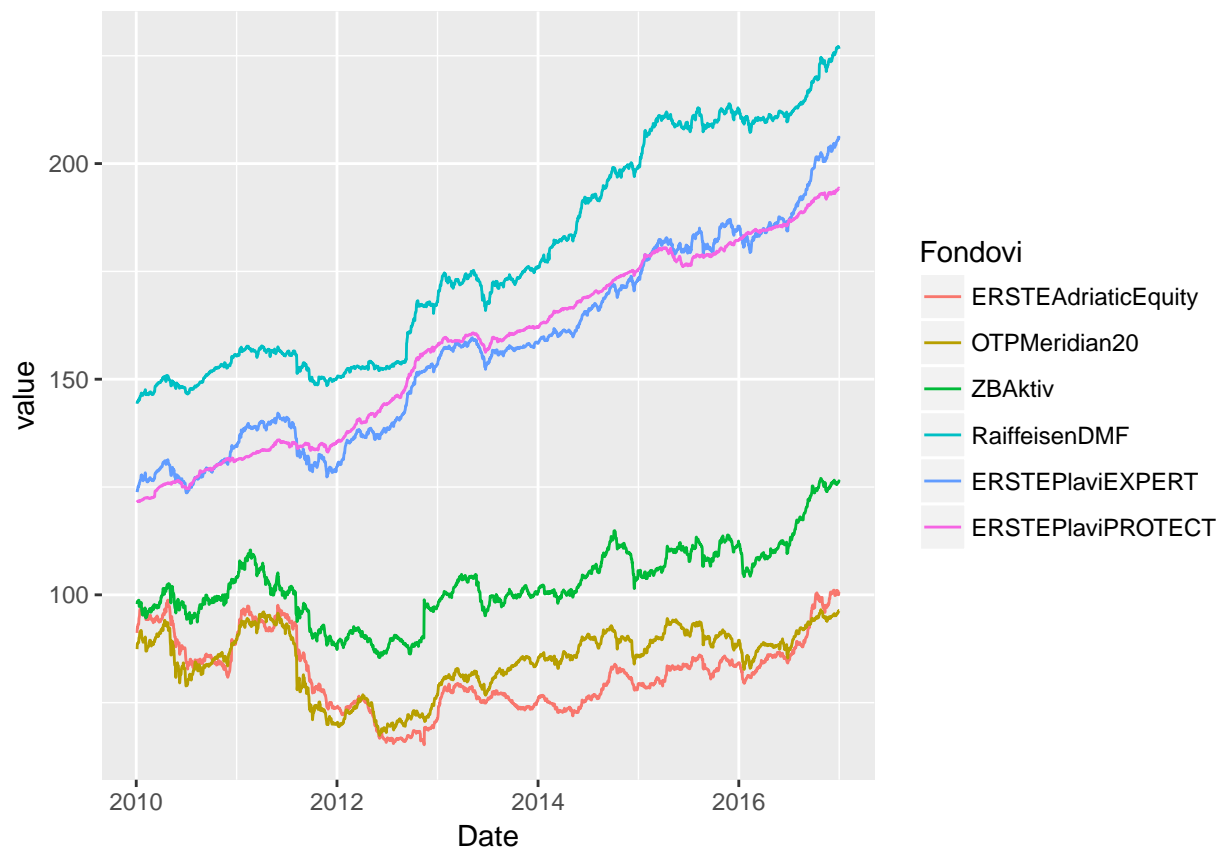
```
ggplot(xs, aes(Date, CROBEX)) + geom_line()
```



## Prikaz vrijednosti investicijskih i mirovinskih fondova po danima

Iz grafa se čini kako investicijski fondovi imaju veći apsolutan rast, uz veću prosječnu vrijednost.

```
df <- melt(xs[c("Date", investment_funds, pension_funds)],  
           id.vars = 'Date',  
           variable.name = 'Fondovi')  
ggplot(df, aes(Date, value)) + geom_line(aes(colour = Fondovi))
```



## Prikaz boxplotova za sve fondove

Iz ovog se grafa ne može zaključiti mnogo, ali vidi kako su investicijski fondovi (prva tri stupca) na dnevnoj bazi podložniji većim promjenama vrijednosti od mirovinskih, jer ima više stršećih vrijednosti.

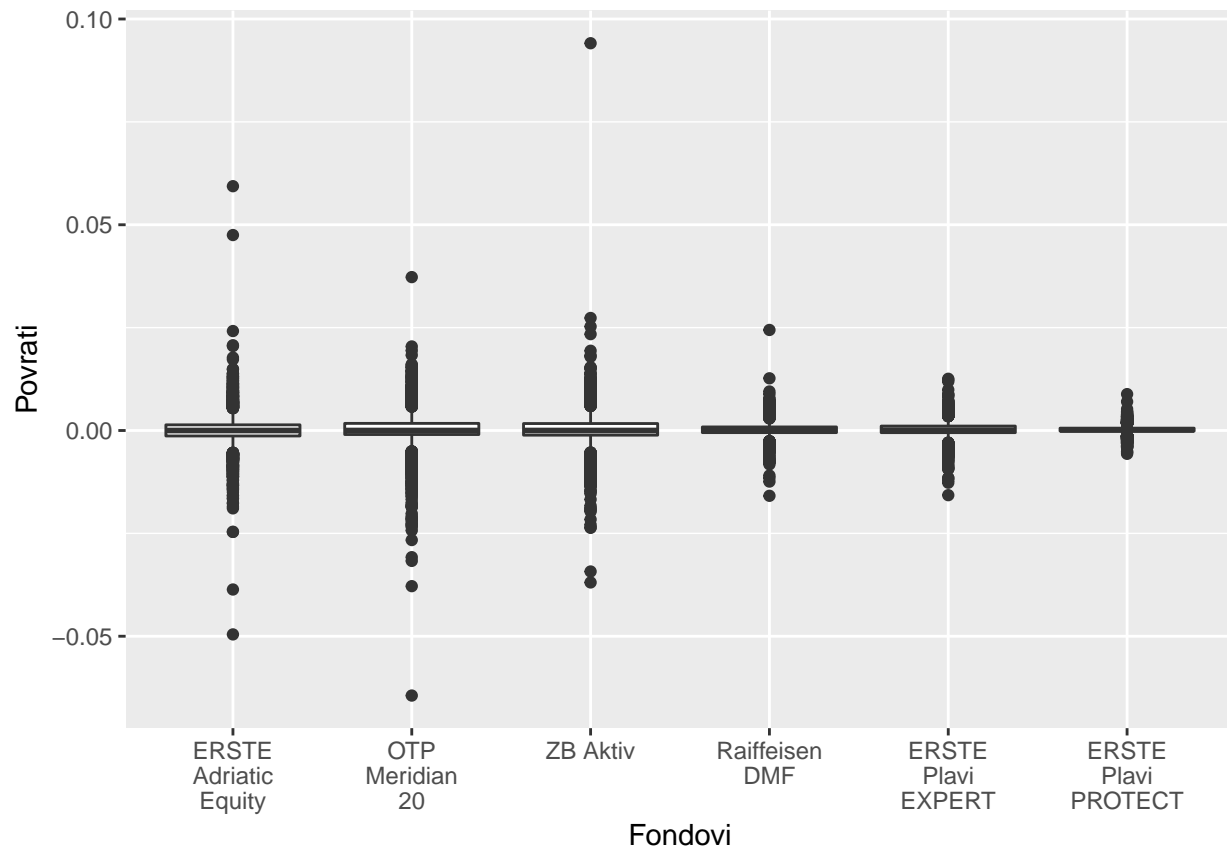
```
df.returns <- melt(xs.returns[c("Date", investment_funds, pension_funds)],
                  id.vars = 'Date',
                  variable.name = 'Fondovi')

label_prettify <- function(label) {
  first_matches <- str_match(label, "(^[A-Z]+)([A-Z][a-z]+)(.*)")
  second_matches <- str_match(label, "(^[A-Z][a-z]+)([A-Z]+)")

  first_word <- ifelse(!is.na(first_matches[1, 1]), first_matches[1, 2],
                     second_matches[1, 2])
  second_word <- ifelse(!is.na(first_matches[1, 1]), first_matches[1, 3],
                      second_matches[1, 3])
  second_word <- ifelse(!is.na(first_matches[1, 4]),
                      str_c(second_word, first_matches[1, 4], sep = " "),
                      second_word)

  return(str_c(first_word, second_word, sep = " ") %>% str_wrap(width = 10))
}

ggplot(df.returns, aes(Date, value)) +
  geom_boxplot(aes(Fondovi)) +
  xlab("Fondovi") +
  ylab("Povrati") +
  scale_x_discrete(labels = function(labels) lapply(labels, label_prettify))
```



## Provjera normalnosti dnevnih povrata fondova

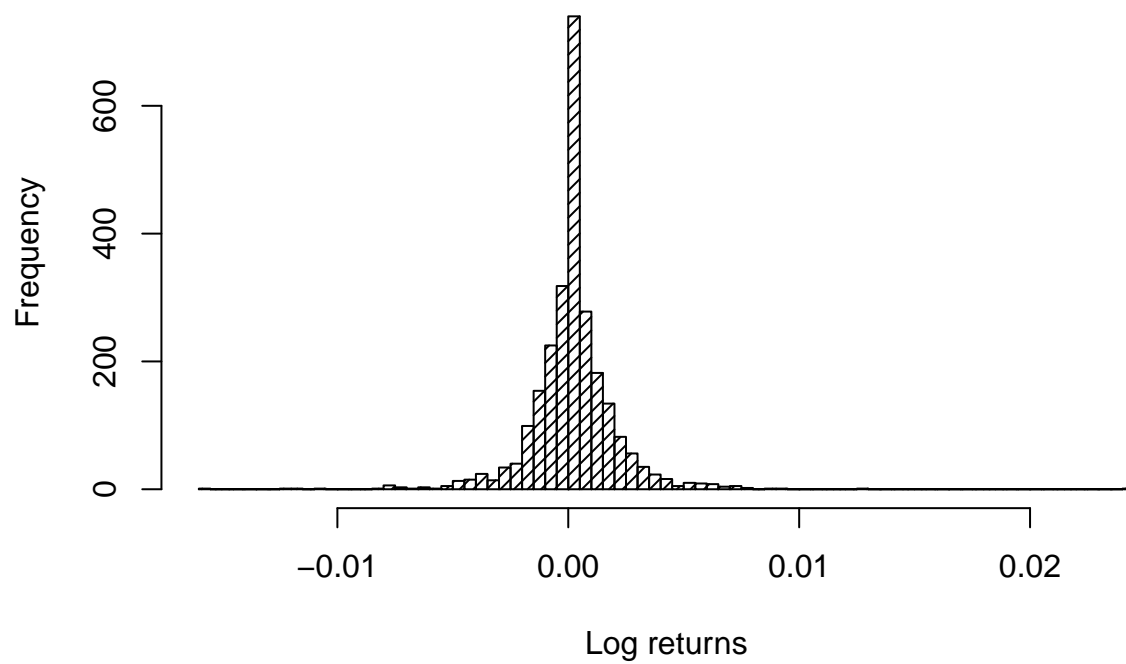
### Histogramima

Iscrtavamo histograme povrata za svaki fond. Vidimo da imaju prilično teške repove, što ukazuje kako nisu baš normalno distribuirani.

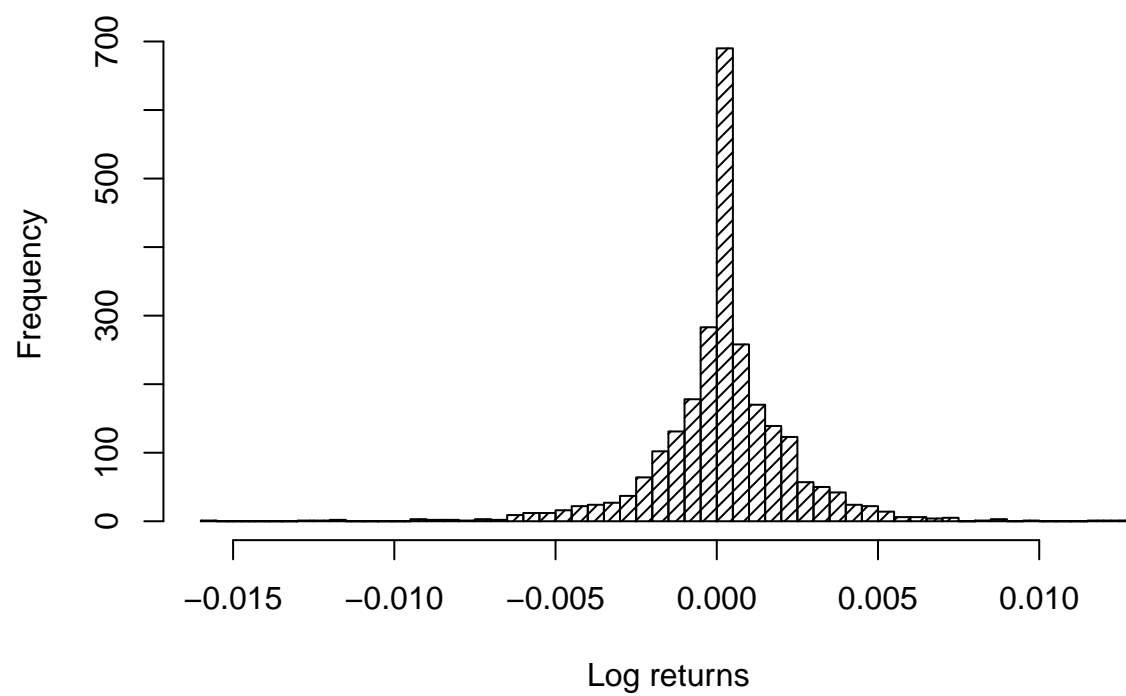
```
plot_returns <- function(fund.returns, fund.name) hist(fund.returns,
  main = fund.name,
  density=20,
  xlab='Log returns',
  labels=FALSE,
  breaks=100)

mapapply(plot_returns,
  c(xs.returns[c(pension_funds, investment_funds)]),
  c(pension_funds, investment_funds)) %>%
  invisible
```

# RaiffeisenDMF

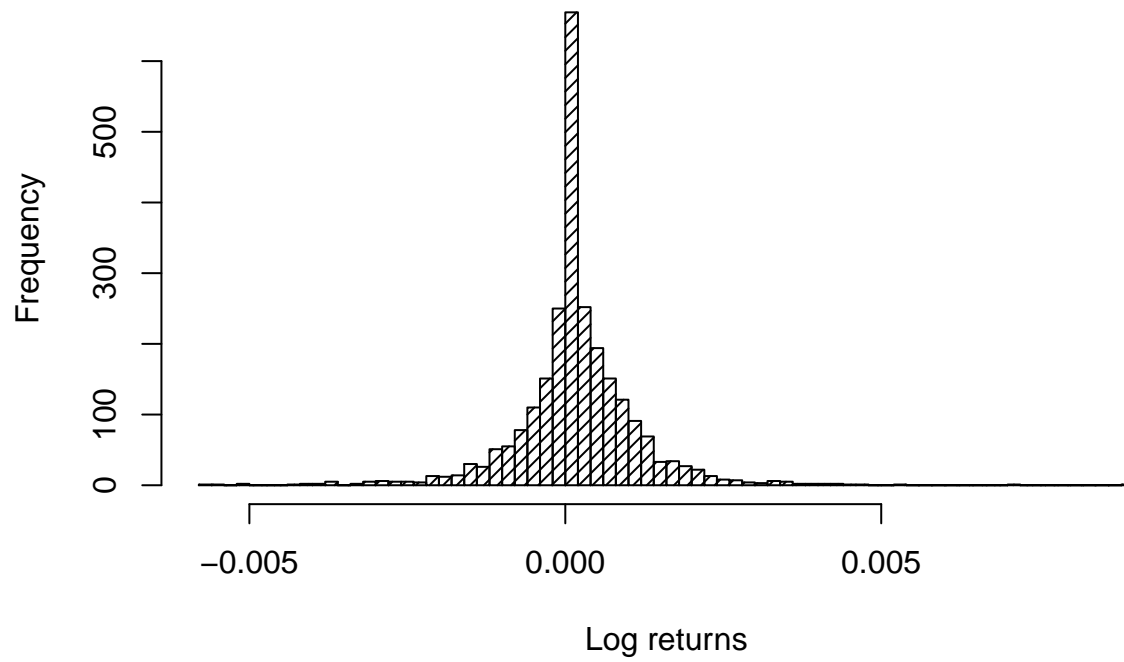


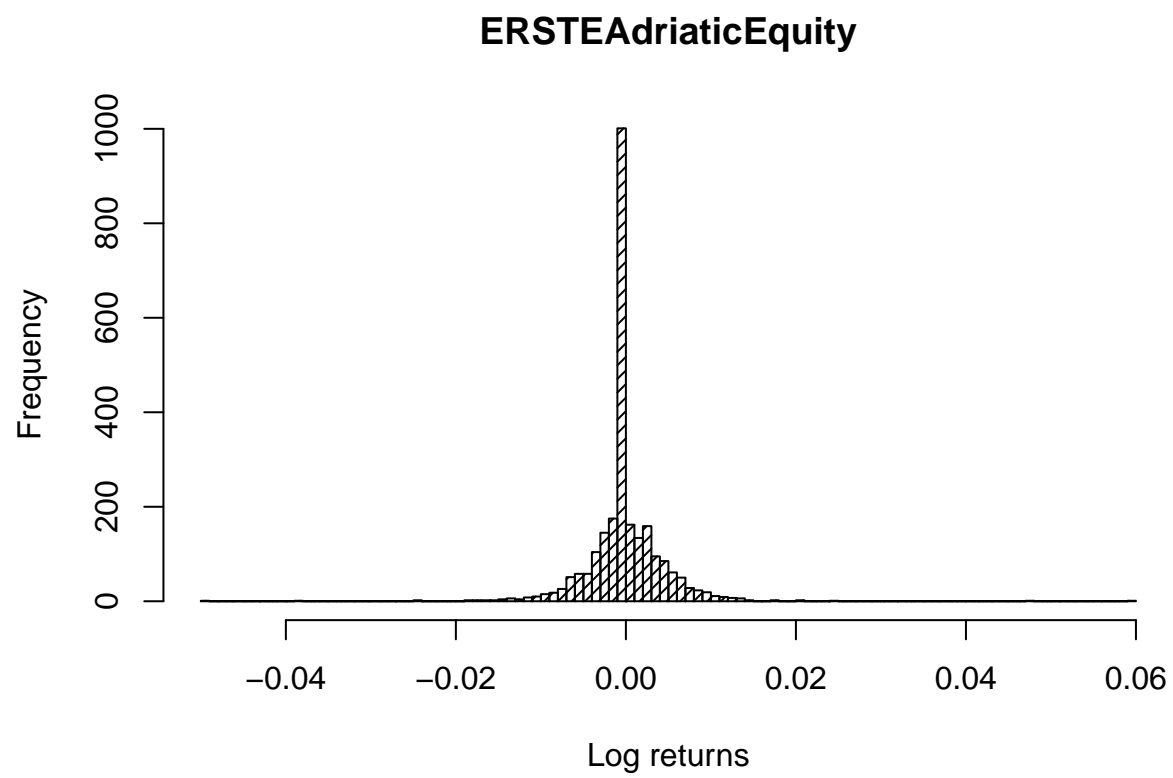
## ERSTEPlaviEXPERT

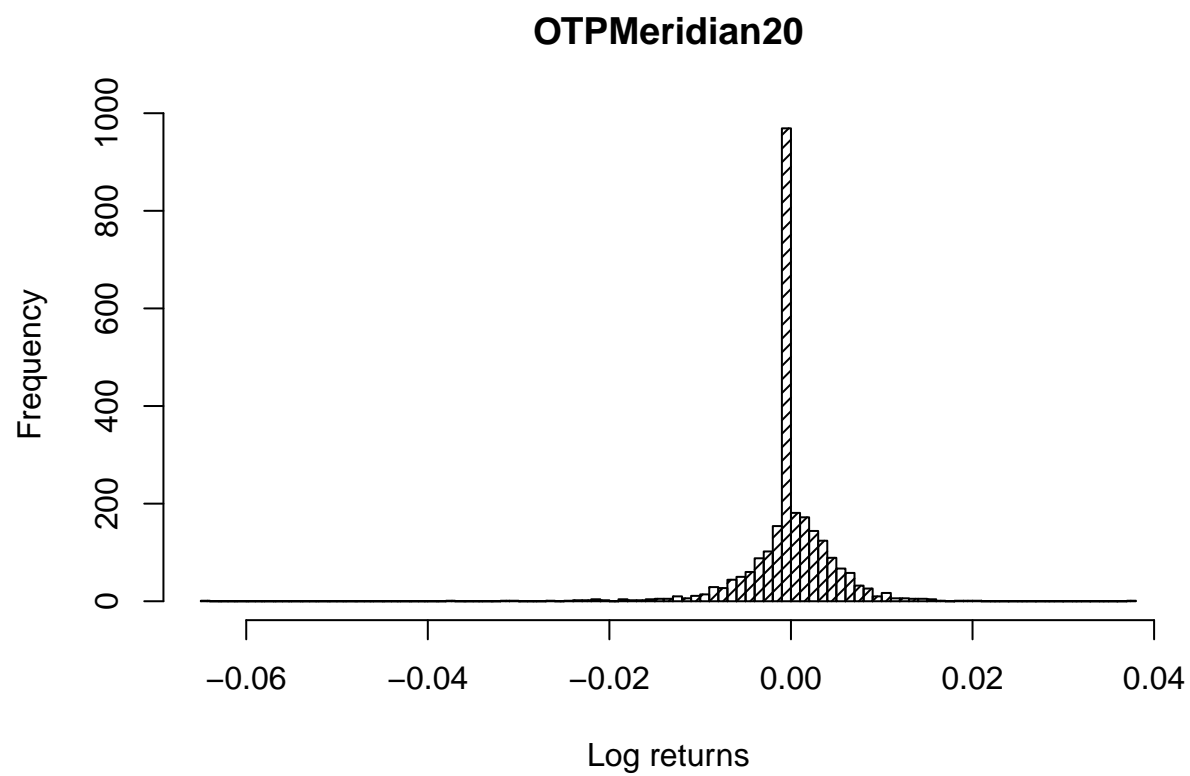


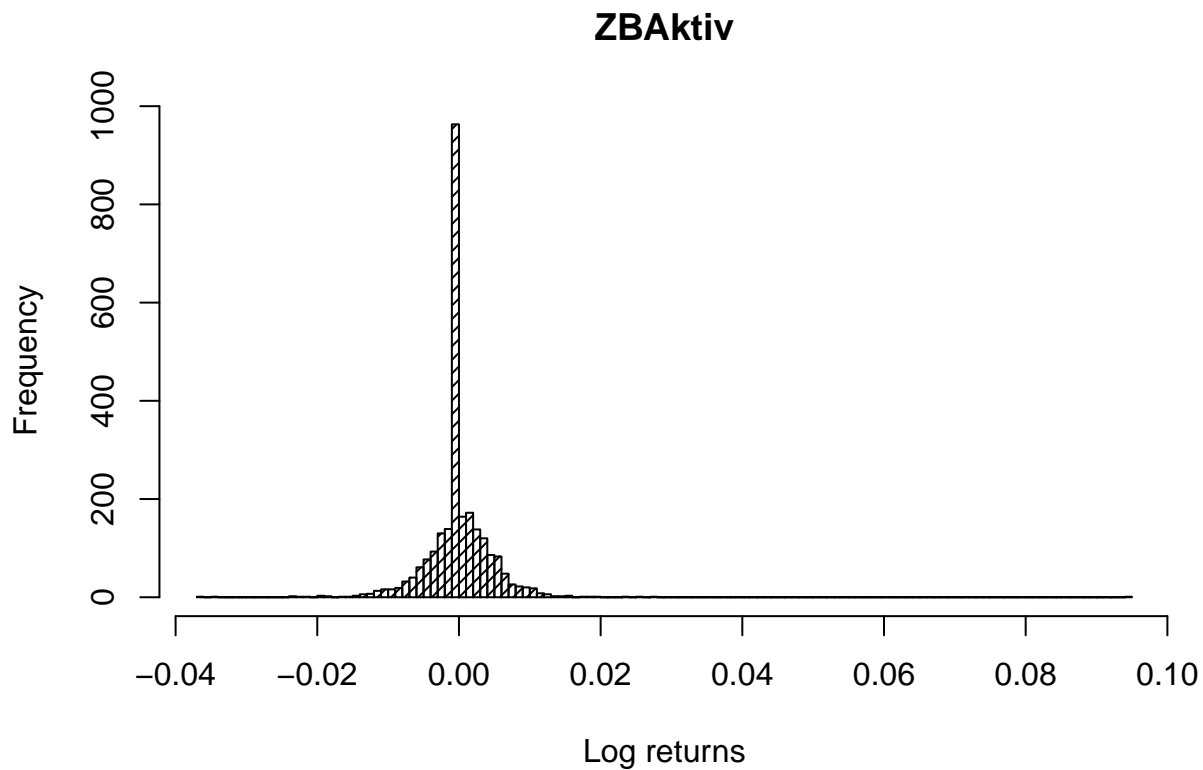


# ERSTEPlaviPROTECT







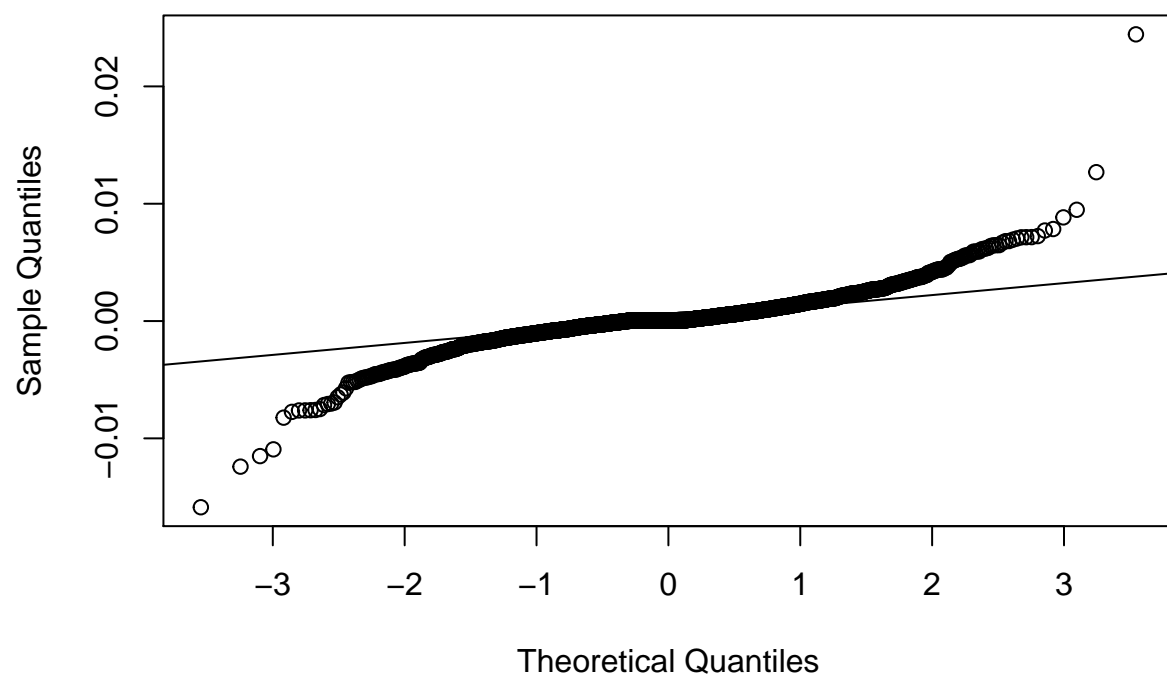


### QQ grafovima

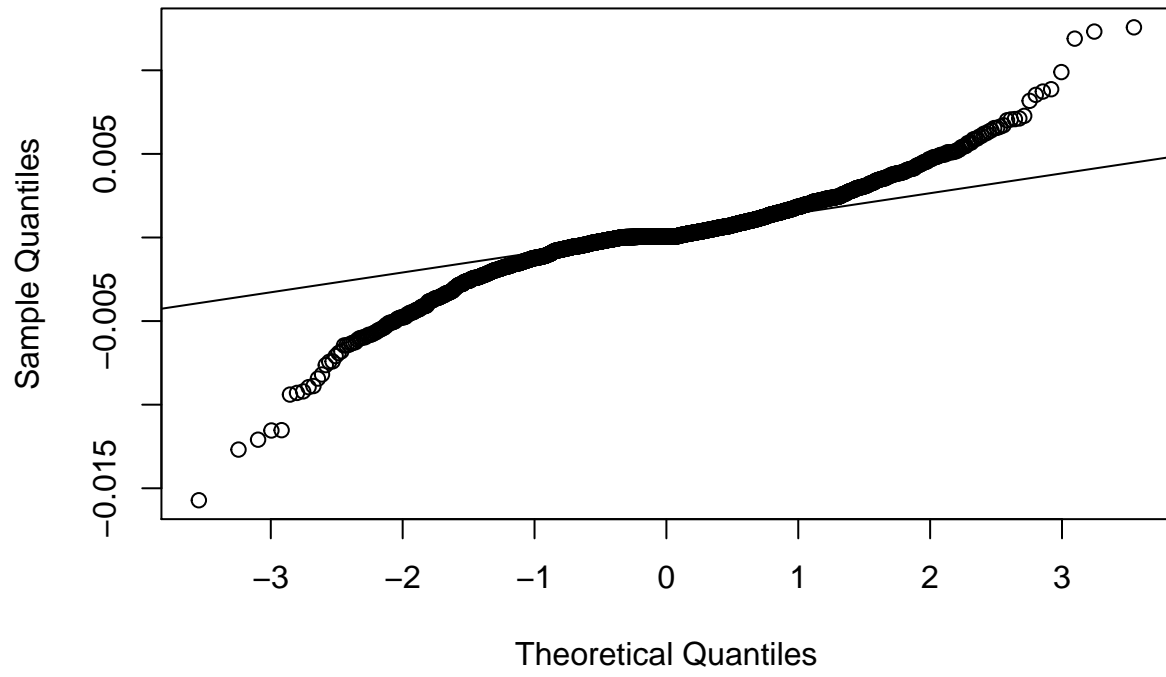
Sljedećim QQ grafovima želimo ispitati normalnost distribucije povrata svih fondova. Teške repove primjećujemo radi sitne granulacije, tj. dnevnog računanja prinosa; u tako kratkom roku zna se dogoditi da pojedina dionica ili naglo naraste ili naglo padne u vrijednosti.

```
qqplots <- function(fund.returns, fund.name) {  
  qqnorm(fund.returns, main = fund.name)  
  qqline(fund.returns)  
}  
  
mapply(qqplots,  
  c(xs.returns[c(pension_funds, investment_funds)]),  
  c(pension_funds, investment_funds)) %>%  
  invisible
```

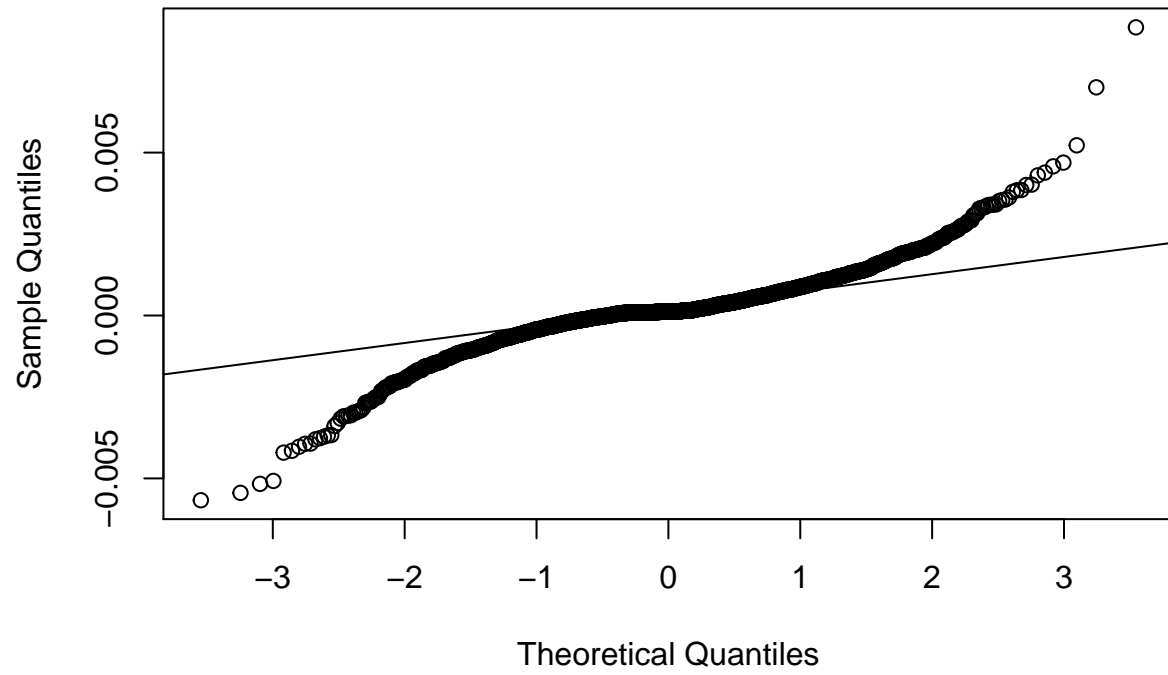
# RaiffeisenDMF



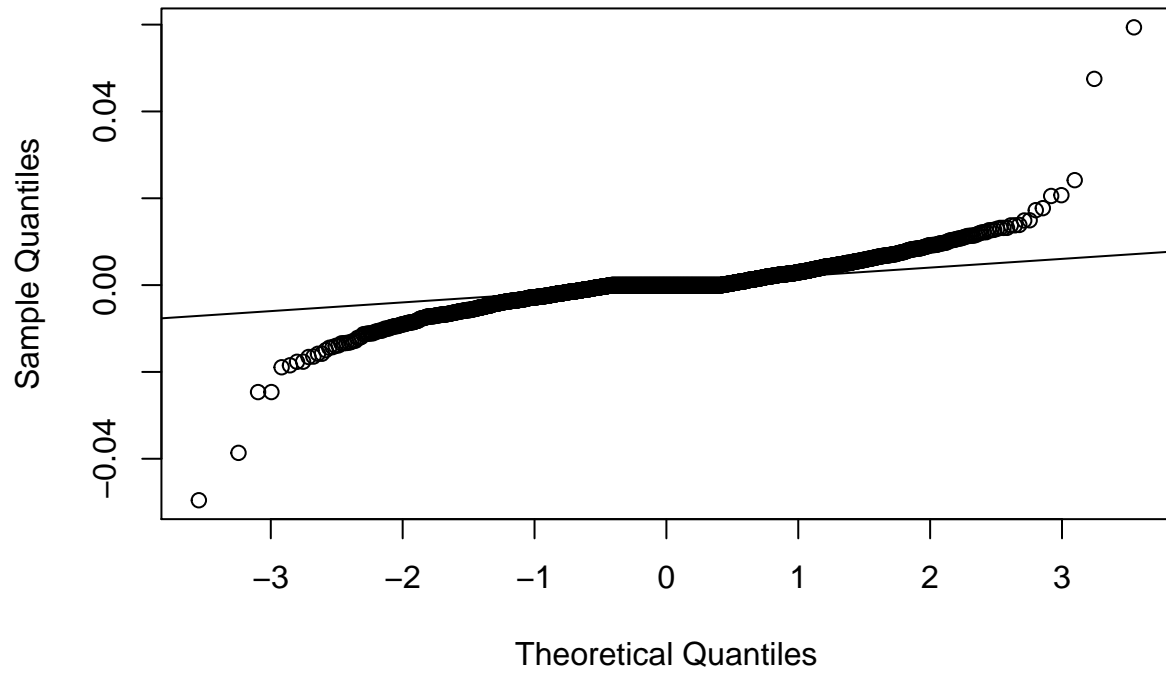
# ERSTEPlaviEXPERT



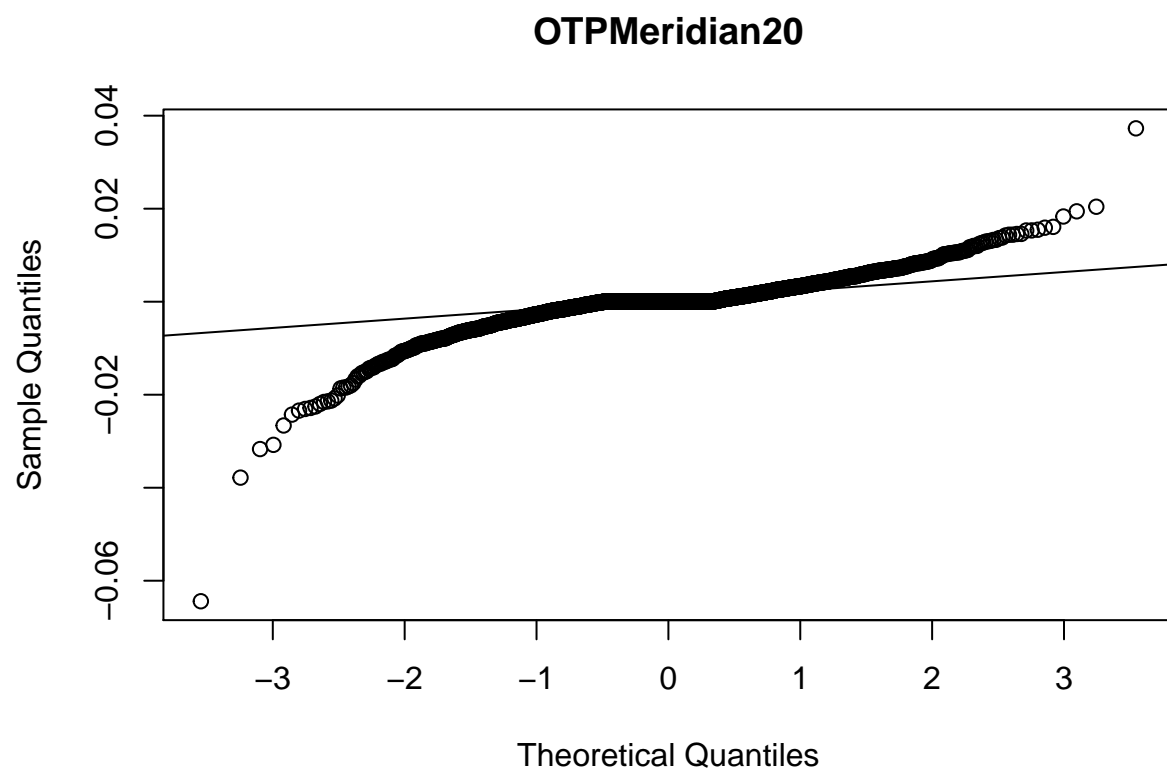
# ERSTEPlaviPROTECT

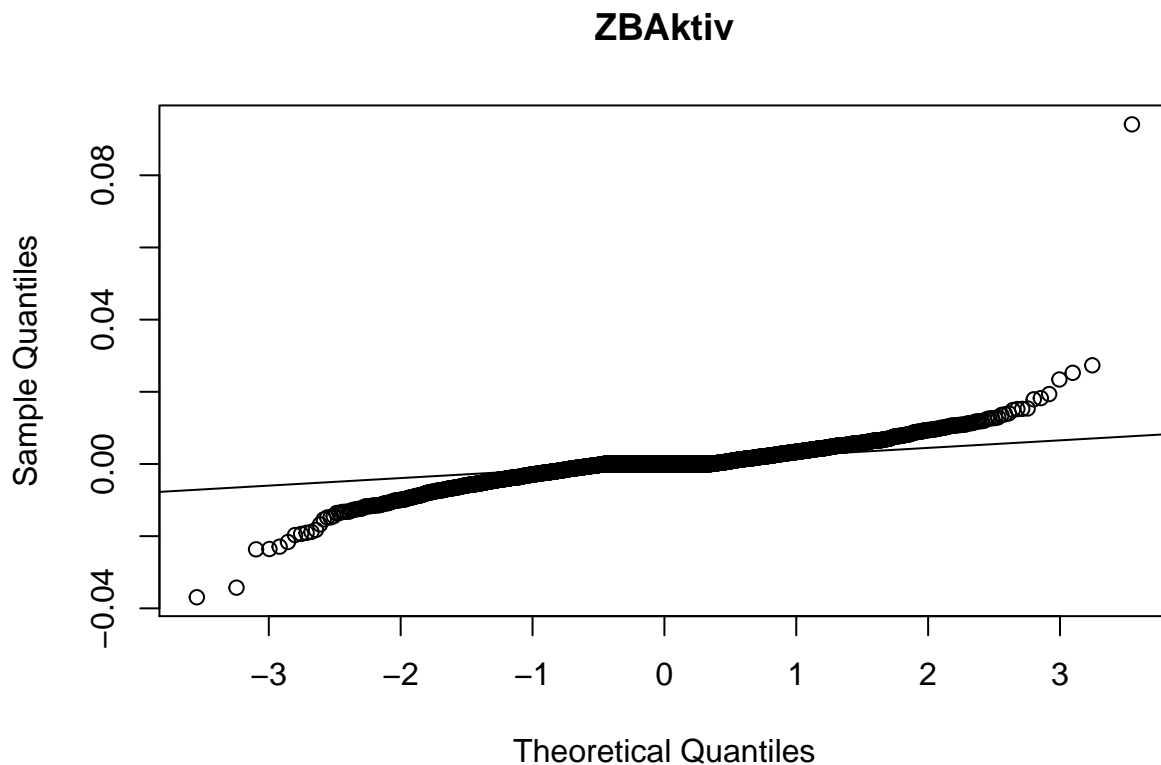


# ERSTAdriaticEquity









## Testovi fondova

Iako QQ grafovi pokazuju da povrati nisu normalno raspodijeljeni, radimo tu pretpostavku s obzirom na robusnost T-testa. Jasno je da globalni događaji (kriza, teroristički napadi, ...) često utječu na cijelo tržište odjednom, pa koristimo T-testove za uparene podatke.

## Testovi povrata investicijskih fondova u odnosu na CROBEX

Za  $H_0$  hipotezu uzimamo kako su sredine investicijskih fondova i CROBEX indeksa jednake. Iz sljedećih testova vidimo da ne možemo opovrgnuti tu hipotezu.

```
compare.to.index <- function(index) function(fund.returns) t.test(index, fund.returns,
                                                                    paired = TRUE)

mapply(compare.to.index(xs.returns$CROBEX), xs.returns[investment_funds])
```

##	ERSTAdriaticEquity	OTPMeridian20
## statistic	-0.4774079	-0.4286687
## parameter	2552	2552
## p.value	0.6331126	0.6682005
## conf.int	Numeric,2	Numeric,2
## estimate	-4.454166e-05	-4.377476e-05
## null.value	0	0
## alternative	"two.sided"	"two.sided"

```
## method      "Paired t-test"      "Paired t-test"
## data.name    "index and fund.returns" "index and fund.returns"
##             ZBAktiv
## statistic    -0.8207028
## parameter    2552
## p.value      0.4118922
## conf.int     Numeric,2
## estimate     -0.0001054245
## null.value   0
## alternative   "two.sided"
## method      "Paired t-test"
## data.name    "index and fund.returns"
```

## Testovi povrata mirovinskih fondova u odnosu na CROBEX

Za  $H_0$  hipotezu uzimamo kako su sredine mirovinskih fondova i CROBEX indeksa jednake. Iz sljedećih testova zaključujemo da ne možemo odbaciti  $H_0$  hipotezu uz nivo značajnosti 5% za fondove RaiffeisenDMF i ERSTEPlaviPROTECT, dok za ERSTEPlaviEXPERT možemo.

```
mapply(compare.to.index(xs.returns$CROBEX), xs.returns[pension_funds])
```

```
##           RaiffeisenDMF           ERSTEPlaviEXPERT
## statistic    -1.789106           -2.125481
## parameter    2552                2552
## p.value      0.07371632           0.03364232
## conf.int     Numeric,2           Numeric,2
## estimate     -0.0001823123        -0.0002049503
## null.value   0                   0
## alternative   "two.sided"         "two.sided"
## method      "Paired t-test"      "Paired t-test"
## data.name    "index and fund.returns" "index and fund.returns"
##           ERSTEPlaviPROTECT
## statistic    -1.711485
## parameter    2552
## p.value      0.08711319
## conf.int     Numeric,2
## estimate     -0.0001893649
## null.value   0
## alternative   "two.sided"
## method      "Paired t-test"
## data.name    "index and fund.returns"
```

## Test povrata investicijskih fondova u odnosu na mirovinske fondove

Izračunate su sredine mirovinskih i investicijskih fondova pa je sproveden test njihovih vrijednosti. Dobivamo izrazito malu p-vrijednost, stoga uz relativno veliku sigurnost zaključujemo da možemo odbaciti nul-hipotezu koja tvrdi da su sredine jednake.

```
grouped.return.means = data.frame(Date = xs.returns$Date,
                                   MeansPension = rowMeans(xs.returns[pension_funds]),
                                   MeansInvestment = rowMeans(xs.returns[investment_funds]))

t <- t.test(grouped.return.means$MeansPension,
```

```
        grouped.return.means$MeansInvestment, paired = TRUE)
data.frame(p=t$p.value, type=t$alternative, null=t$estimate,
           row.names = "Pension vs. Investment fund means")
```

```
##                                p          type          null
## Pension vs. Investment fund means 0.02846181 two.sided 0.0001276289
```

## CAPM model

CAPM (Capital Asset pricing model) je model koji opisuje odnos između kamatne stope i očekivanog povrata sredstava. Svodi se na linearnu regresiju:

$$R_p - R_f = \alpha + \beta(R_m - R_f) + \epsilon$$

gdje je  $R_p$  prinos promatranog fonda (portfelja),  $R_m$  prinos tržišnog (referentnog) portfelja, a  $R_f$  je bezrizična kamatna stopa. Koeficijent  $\alpha$  mjeri koliko je prinos promatranog fonda veći od prinosa tržišnog portfelja, a  $\beta$  mjeri osjetljivost fonda na tržišne prinose i predstavlja rizičnost.

Promatrajući koeficijente  $\alpha$  i  $\beta$  svakog fonda zaključujemo da bi najbolje bilo uložiti u fond ERSTEPlaviPROTECT jer ima najmanji koeficijent  $\beta$  od svih fondova i poprilično visok koeficijent  $\alpha$ .

```
year <- function(date) format(date, "%Y")
get_for_year <- function(df, dates, desired_year) df[year(dates) == desired_year, ]

get_capm_for_year <- function(df, fund, desired_year){
  xs.year = get_for_year(df, df$Date, desired_year)

  fund.year <- xs.year[c('Date', fund)]
  fund.ts <- xts(fund.year[, -1], order.by=fund.year$Date)

  capm.index.year <- xs.year[c('Date', 'CROBEX')]
  capm.index.ts <- xts(capm.index.year[, -1], order.by=capm.index.year$Date)

  capm.risk_free.year <- xs.year[c('Date', 'InterestRate.daily')]
  capm.risk_free.year <- capm.risk_free.year[1, -1]

  data.frame(fund, as.factor(desired_year),
             CAPM.alpha(fund.ts, capm.index.ts, capm.risk_free.year),
             CAPM.beta(fund.ts, capm.index.ts, capm.risk_free.year))
}

get_capm_for_fund <- function(df, selected_fund){
  fund <- df[c('Date', selected_fund)]
  fund.ts <- xts(fund[, -1], order.by=fund$Date)
  capm.index <- df[c('Date', 'CROBEX')]
  capm.index.ts <- xts(capm.index[, -1], order.by=capm.index$Date)
  capm.risk_free <- df[c('Date', 'InterestRate.daily')]
  capm.risk_free.ts <- capm.risk_free[1, -1]
  data.frame(selected_fund,
             CAPM.alpha(fund.ts, capm.index.ts, capm.risk_free.ts),
             CAPM.beta(fund.ts, capm.index.ts, capm.risk_free.ts))
}

xs.years = seq(from = 2010, by = 1, length = 7)
xs.fund.names = c(investment_funds, pension_funds)
xs.capm <- data.frame(matrix(ncol = 3, nrow = 0))
xs.capm.all <- data.frame(matrix(ncol = 3, nrow = 0))

for (i in 1:length(xs.fund.names)){
  for (j in 1:length(xs.years)){
    xs.capm <- rbind(xs.capm, get_capm_for_year(xs.returns, xs.fund.names[i], xs.years[j]))
  }
}
```

```

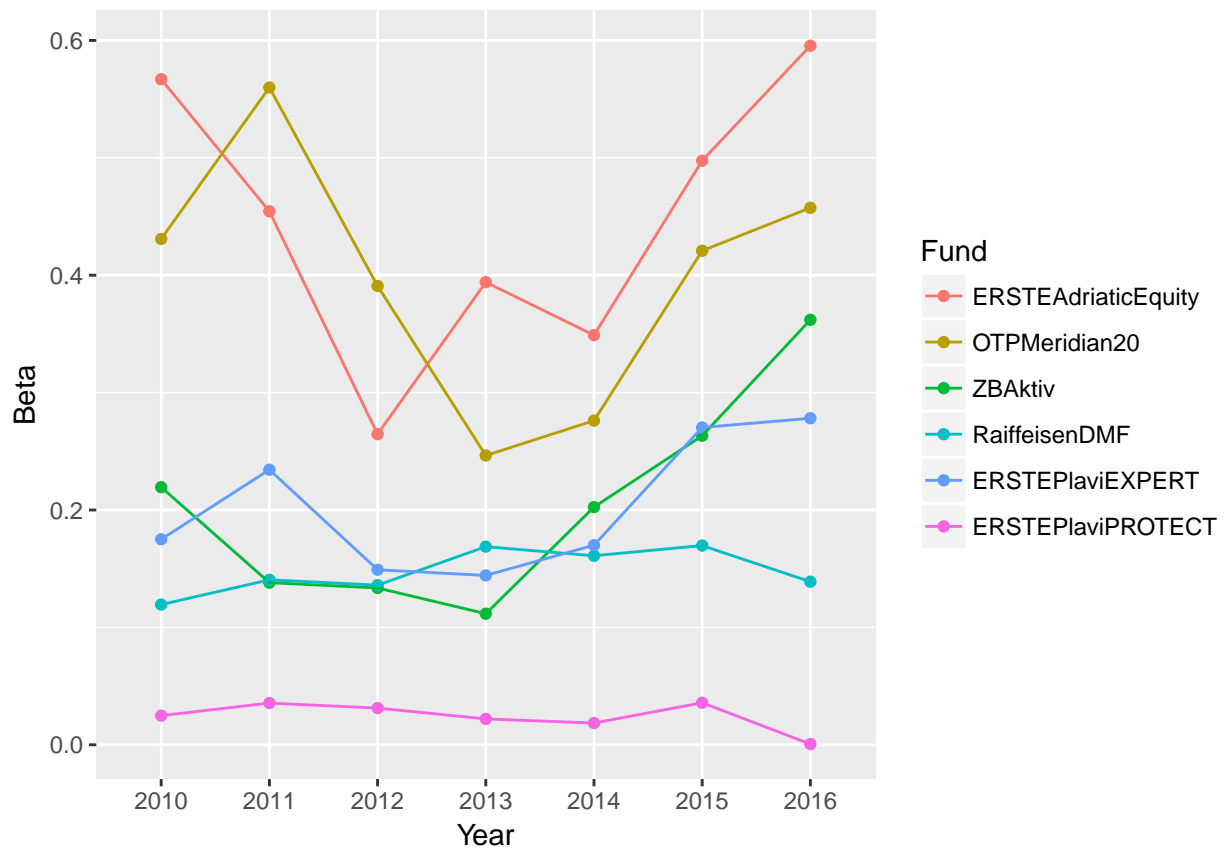
}

for (i in 1:length(xs.fund.names)){
  xs.capm.all <- rbind(xs.capm.all, get_capm_for_fund(xs.returns, xs.fund.names[i]))
}

colnames(xs.capm) <- c("Fund", "Year", "Alpha", "Beta")
colnames(xs.capm.all) <- c("Fund", "Alpha", "Beta")

ggplot(xs.capm, aes(Year, Beta, color= Fund, group = Fund)) +
  geom_point() + geom_line()

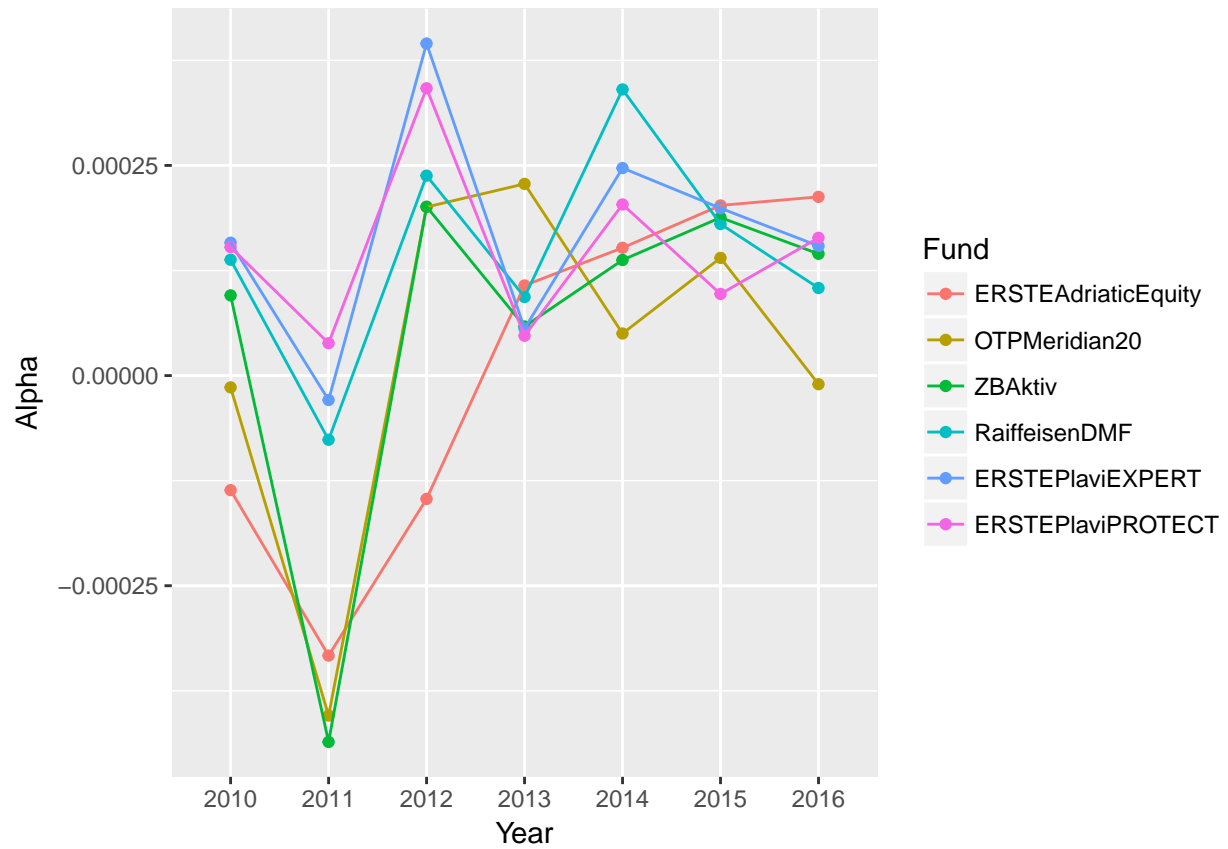
```



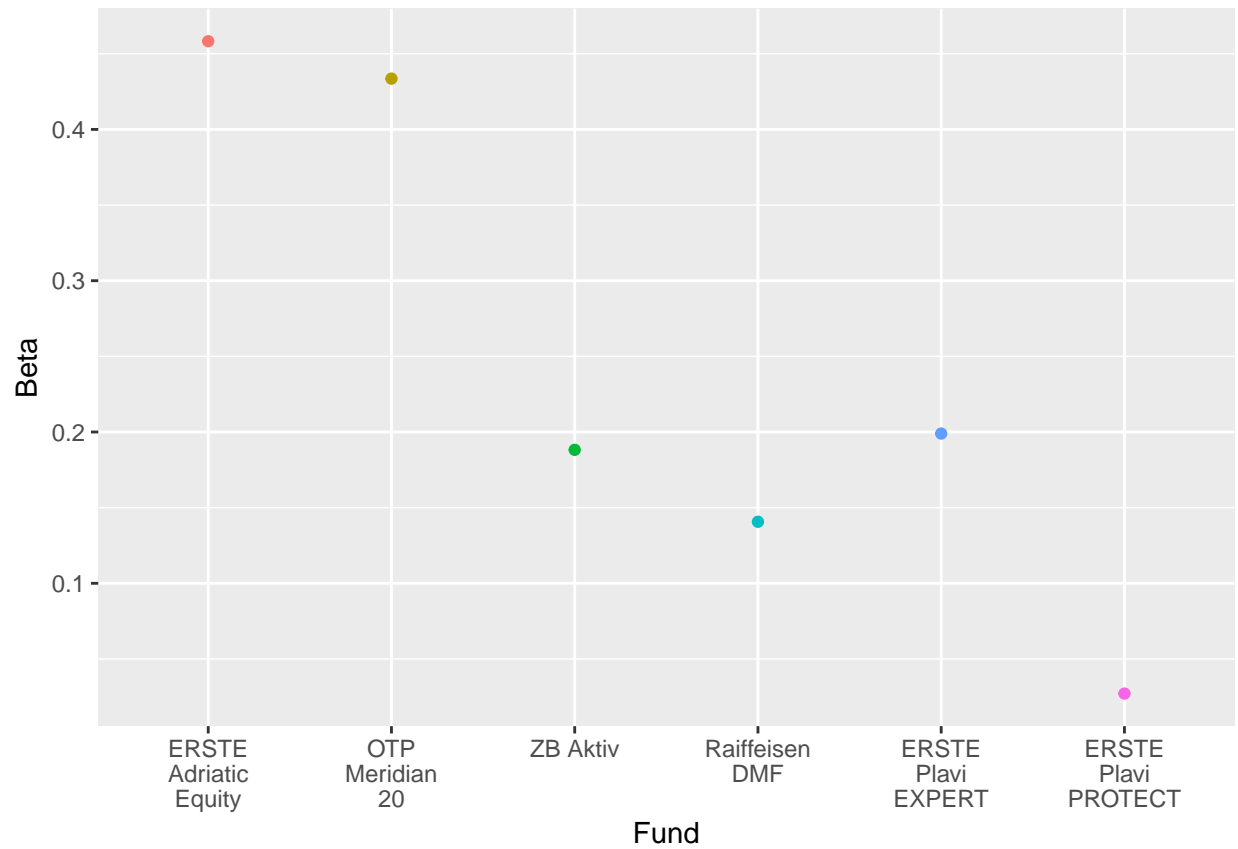
```

ggplot(xs.capm, aes(Year, Alpha, color= Fund, group = Fund)) +
  geom_point() + geom_line()

```

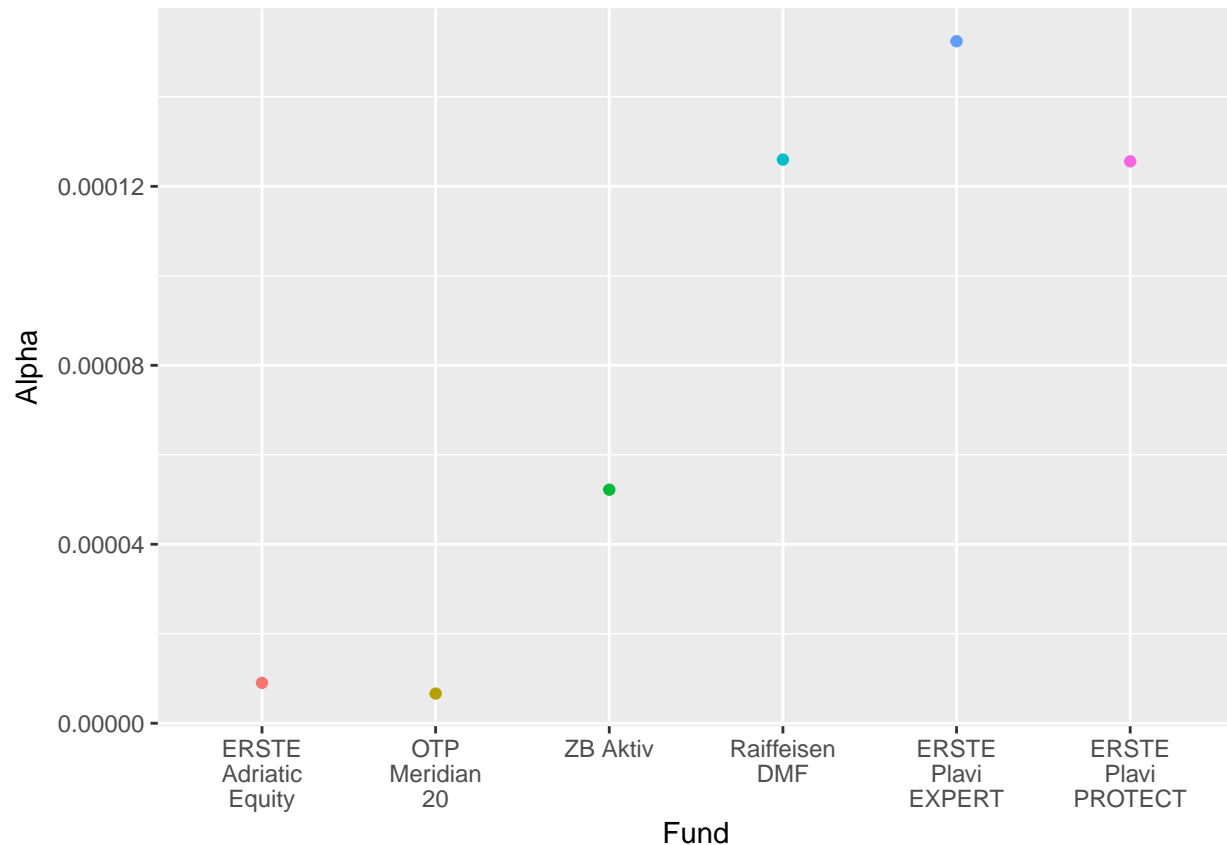


```
ggplot(xs.capm.all, aes(Fund, Beta, color= Fund, group = Fund)) +
  geom_point() +
  geom_line() +
  scale_x_discrete(labels = function(labels) lapply(labels, label_prettify)) +
  theme(legend.position="none")
```



```
ggplot(xs.capm.all, aes(Fund, Alpha, color= Fund, group = Fund)) +
  geom_point() +
  geom_line() +
  scale_x_discrete(labels = function(labels) lapply(labels, label_prettify)) +
  theme(legend.position="none")
```





## Provjera reziduala

Jedna od pretpostavki pri regresijskoj analizi jest normalnost razdiobe reziduala. Sljedećom tablicom dane su p-vrijednosti, kad Kolmogorov-Smirnovljevim testom usporedimo distribucije reziduala za svaki fond po godinama i normalnu distribuciju. Kako su te vrijednosti male, odbacujemo hipotezu kako su reziduali normalno distribuirani.

```
nrows = nrow(xs.capm)
get_residuals_norm <- function(df, row) {
  xs.tmpYear <- get_for_year(xs.returns, xs.returns$Date, df[row, 'Year'])
  fund <- as.character(df[row, 'Fund'])
  xs.tmpYearFund <- xs.tmpYear[, fund]

  capm.risk_free.year <- xs.tmpYear$InterestRate.daily
  capm.market.year <- xs.tmpYear$CROBEX

  residuals.tmp <- vector(mode="numeric", length=length(xs.tmpYearFund))

  for (i in 1:length(xs.tmpYearFund)) {
    residuals.tmp[i] <- ((xs.tmpYearFund[i] - capm.risk_free.year[i]) -
      (df[row, 3] + df[row, 4] * (capm.market.year[i] - capm.risk_free.year[i])))
  }

  scaled.residuals.tmp <- scale(residuals.tmp)
  x <- ks.test(scaled.residuals.tmp, 'pnorm')
```

```

df[row, 'KS.p'] <- x$p.value

return(df)
}

for(i in 1:nrows) {
  xs.capm <- suppressWarnings( get_residuals_norm(xs.capm, i) )
}

```

## Test prilagodbe modela

Izračunali smo i ANOVA test na prilagodbu modela, no ne obraćamo preveliku pažnju na njega radi loših rezultata KS testa na normalnost reziduala.

```

alpha_beta_r <- function(vals, alpha, beta, index, risk.free) {

  model <- alpha + beta * (index - risk.free) + risk.free

  SSE = (vals - model)^2 %>% sum
  SST = (vals - mean(vals))^2 %>% sum

  return(1 - SSE / SST)
}

rsquared.from_returns <- function(desired_year, fund, alpha, beta) {
  xs.curr <- xs.returns[year(xs.returns$Date) == desired_year, ]
  fund.actual.vals <- xs.curr[, as.character(fund)]

  return(alpha_beta_r(fund.actual.vals,
                      alpha, beta,
                      xs.curr$CROBEX,
                      xs.curr$InterestRate.daily))
}

xs.capm$R.squared <- mapply(rsquared.from_returns,
                           xs.capm$Year,
                           xs.capm$Fund,
                           xs.capm$Alpha,
                           xs.capm$Beta)

xs.capm

```

##		Fund	Year	Alpha	Beta	KS.p
## 1	ERST	AdriaticEquity	2010	-1.363158e-04	0.5669352275	7.606159e-08
## 2	ERST	AdriaticEquity	2011	-3.331016e-04	0.4543998614	8.972995e-10
## 3	ERST	AdriaticEquity	2012	-1.467188e-04	0.2646270122	3.976819e-13
## 4	ERST	AdriaticEquity	2013	1.072103e-04	0.3940627807	3.095202e-11
## 5	ERST	AdriaticEquity	2014	1.520230e-04	0.3489534496	5.271628e-11
## 6	ERST	AdriaticEquity	2015	2.024548e-04	0.4975787045	2.888248e-09
## 7	ERST	AdriaticEquity	2016	2.126424e-04	0.5955184335	1.182900e-08
## 8		OTPMeridian20	2010	-1.401558e-05	0.4309319208	2.925515e-10
## 9		OTPMeridian20	2011	-4.044461e-04	0.5599034422	3.870977e-08
## 10		OTPMeridian20	2012	2.005561e-04	0.3908358348	2.004861e-09

## 11	OTPMeridian20	2013	2.280547e-04	0.2464292424	2.003044e-10
## 12	OTPMeridian20	2014	5.006249e-05	0.2761167134	7.392093e-09
## 13	OTPMeridian20	2015	1.400398e-04	0.4208172130	2.578632e-08
## 14	OTPMeridian20	2016	-1.026420e-05	0.4574313828	5.335510e-12
## 15	ZBAktiv	2010	9.544554e-05	0.2194612955	2.049980e-04
## 16	ZBAktiv	2011	-4.358250e-04	0.1381375343	2.527614e-05
## 17	ZBAktiv	2012	2.009252e-04	0.1336074631	1.354321e-10
## 18	ZBAktiv	2013	5.840394e-05	0.1116804387	2.323343e-06
## 19	ZBAktiv	2014	1.375125e-04	0.2024930920	6.164473e-06
## 20	ZBAktiv	2015	1.880294e-04	0.2632705419	2.186302e-05
## 21	ZBAktiv	2016	1.448745e-04	0.3620458129	1.539224e-04
## 22	RaiffeisenDMF	2010	1.378239e-04	0.1194627941	2.717111e-03
## 23	RaiffeisenDMF	2011	-7.635330e-05	0.1405371249	3.224060e-06
## 24	RaiffeisenDMF	2012	2.378826e-04	0.1359857205	1.741118e-11
## 25	RaiffeisenDMF	2013	9.345165e-05	0.1687797266	2.118944e-04
## 26	RaiffeisenDMF	2014	3.404221e-04	0.1610284601	2.405696e-04
## 27	RaiffeisenDMF	2015	1.803462e-04	0.1697165854	1.146525e-03
## 28	RaiffeisenDMF	2016	1.041926e-04	0.1389678904	2.727515e-06
## 29	ERSTEPlaviEXPERT	2010	1.580814e-04	0.1750763495	2.160262e-04
## 30	ERSTEPlaviEXPERT	2011	-2.905792e-05	0.2343404356	1.988193e-04
## 31	ERSTEPlaviEXPERT	2012	3.950473e-04	0.1491048287	7.504533e-05
## 32	ERSTEPlaviEXPERT	2013	5.594323e-05	0.1442715157	3.300715e-04
## 33	ERSTEPlaviEXPERT	2014	2.469191e-04	0.1700392710	2.902686e-03
## 34	ERSTEPlaviEXPERT	2015	1.991195e-04	0.2703175563	3.123242e-03
## 35	ERSTEPlaviEXPERT	2016	1.542028e-04	0.2781643675	7.591789e-05
## 36	ERSTEPlaviPROTECT	2010	1.528344e-04	0.0248670367	1.210117e-05
## 37	ERSTEPlaviPROTECT	2011	3.855290e-05	0.0355562101	4.594969e-05
## 38	ERSTEPlaviPROTECT	2012	3.418137e-04	0.0312794690	1.652921e-05
## 39	ERSTEPlaviPROTECT	2013	4.735975e-05	0.0220619941	8.751553e-07
## 40	ERSTEPlaviPROTECT	2014	2.035929e-04	0.0185440353	3.416949e-03
## 41	ERSTEPlaviPROTECT	2015	9.710402e-05	0.0358210528	4.620251e-04
## 42	ERSTEPlaviPROTECT	2016	1.639057e-04	0.0005997816	2.693304e-05
##	R.squared				
## 1			6.639383e-01		
## 2			3.595653e-01		
## 3			9.948311e-02		
## 4			2.788338e-01		
## 5			2.204703e-01		
## 6			3.111983e-01		
## 7			3.610346e-01		
## 8			3.027301e-01		
## 9			4.105272e-01		
## 10			2.900392e-01		
## 11			1.049314e-01		
## 12			1.008854e-01		
## 13			1.860068e-01		
## 14			2.315158e-01		
## 15			9.085646e-02		
## 16			3.700341e-02		
## 17			1.484719e-02		
## 18			2.126078e-02		
## 19			5.614227e-02		
## 20			6.677313e-02		
## 21			1.805736e-01		

```
## 22 3.803706e-01
## 23 3.017203e-01
## 24 1.256158e-01
## 25 1.440430e-01
## 26 1.489808e-01
## 27 1.418445e-01
## 28 1.094703e-01
## 29 3.765089e-01
## 30 3.700067e-01
## 31 1.752146e-01
## 32 1.982458e-01
## 33 2.481500e-01
## 34 2.457412e-01
## 35 3.209855e-01
## 36 4.085443e-02
## 37 6.580049e-02
## 38 1.991347e-02
## 39 1.191934e-02
## 40 1.559203e-02
## 41 2.075516e-02
## 42 1.035892e-05
```

## ANOVA

ANOVA (ANalysis Of VAriance) je metoda koja nam pomaže da donesemo neke zaključke o razlikama između sredina više od dvije populacije. Ovdje analiziramo srednje vrijednosti prinosa svakog fonda, uz hipotezu da su im srednji prinosi svima jednaki (drugim riječima, želimo pokazati da u konačnici nije bitno u koji se fond ulaže). Rezultat testa potvrđuje našu hipotezu, no ipak ga uzimamo s oprezom radi nesavršenih uvjeta za obavljanje ovakvog testa.

```
num.items <- xs.returns %>% dim %>% first
anova.subset <- all_funds
anova.returns <- xs.returns %>%
  subset.data.frame(select=anova.subset) %>%
  unlist
anova.factors.funds <- anova.subset %>%
  rep(rep(num.items, length(.))) %>%
  as.factor

anova(lm(anova.returns ~ anova.factors.funds))

## Analysis of Variance Table
##
## Response: anova.returns
##              Df    Sum Sq   Mean Sq F value Pr(>F)
## anova.factors.funds    5 0.000069 1.3891e-05    1.144 0.3345
## Residuals          15312 0.185924 1.2142e-05
```

## Dvofaktorska ANOVA

Usprkos narušenim uobičajenim pretpostavkama za dvofaktorsku ANOVu (ponajviše nezavisnosti, a zatim i normalnosti srednjih vrijednosti prinosa, što je već pokazano da ne vrijedi u ranijim odjeljcima), zanimljivo

je primjetiti da dvofaktorska ANOVA ovdje pokazuje kako ukupno stanje tržišta tijekom neke godine ipak snažno utječe na prinose fondova.

```
anova.factors.years <- xs.returns$Date %>%
  year %>%
  rep(length(anova.subset)) %>%
  as.factor

anova(lm(anova.returns ~ anova.factors.funds * anova.factors.years))
```

```
## Analysis of Variance Table
##
## Response: anova.returns
##
##              Df    Sum Sq   Mean Sq F value
## anova.factors.funds      5 0.000069 1.3891e-05  1.1458
## anova.factors.years      6 0.000483 8.0530e-05  6.6425
## anova.factors.funds:anova.factors.years    30 0.000242 8.0640e-06  0.6651
## Residuals             15276 0.185199 1.2124e-05
##              Pr(>F)
## anova.factors.funds      0.3336
## anova.factors.years      4.968e-07 ***
## anova.factors.funds:anova.factors.years    0.9176
## Residuals
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

## Zaključak

Iz svega ovoga odlučili smo da bi kao grupa investirali u ERSTEPlaviPROTECT, jer ima najbolji omjer alfa i beta parametra. Ima visoku alfu, uz nisku betu.