

**Problem set # 5**

Due: Thursday, October 28th, by 4pm.

**1. Bond Pricing in C++:**

Consider the following table of bond yields by maturity:

Maturity(years)	Yield
1	0.025
2	0.026
3	0.027
5	0.03
10	0.035
30	0.04

**Note:** these are yield to maturities for each bond can be used for all its underlying cashflows. For example, for a 5-year coupon (or zero-coupon) bond, use a constant yield of 3% for all the bonds cashflows.

- (a) Calculate prices of a zero coupon bond that pays \$100 at maturity for each maturity & yield combination. Which price is the highest? Is this reasonable?
- (b) Calculate the duration of each zero coupon bond, or sensitivity of the bond price to a change in bond yield, using using finite differences. What is the relationship between bond prices and bond yields?
- (c) Calculate prices of coupon bonds that pay \$100 at maturity at 3% annually until maturity. Which prices are below \$100? Which prices are above? Why?
- (d) Calculate the duration of each coupon bond using finite differences. Do zero-coupon bonds or coupon bonds have higher duration? Why?
- (e) Calculate the second derivative of each bond price with respect to yield (commonly known as convexity). Are the second derivatives positive or negative?
- (f) Consider a portfolio that is long one unit of the 1 year zero-coupon bond, long one unit of the 3 year zero-coupon bond and short two units of the 2 year zero-coupon bond. Calculate the initial value of the portfolio.
- (g) Calculate the duration of this portfolio. Calculate the convexity of the portfolio as well. Which quantity is bigger?

- (h) Adjust the number of units of the short position in the two year zero-coupon bond so that the portfolio is duration neutral (leaving the units of the one and three year zero-coupon bonds unchanged). How many units of the two year zero-coupon bond are required to do this?
- (i) Suppose you own this adjusted portfolio and rates sell off by 100 basis points (each yield rises by 1%). What happens to the value of your portfolio?
- (j) Now suppose you own this adjusted portfolio and rates rally by 100 basis points (each yield decreases by 1%). What happens to the value of your portfolio? Is this a portfolio you would want to own? What are the risks of owning this portfolio?
- (k) Print the cashflows of a 5-year amortizing bond that repays 20% of its principal annually and pays a 3% coupon (annually).
- (l) Calculate the price and duration of the amortizing bond using finite differences. Comment on the difference between this bond and its zero coupon and coupon equivalents.

**NOTE: All code for completing these exercises should be completed in C++ and should be written generically.**