MF728-Project-Outline

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First, I think it is important to keep the context of our project in mind while we work our way through this, this is essentially a sell-side hedging problem. We are taking what could presumably be considered our book (whatever instruments we need to price and deal) and then we need to hedge it however we see is most optimal on the yield curve. This could be going long(short) Gamma, or long(short) Vega...etc. Ideally we can build a class or a function that lets us select how we want to approach the hedging. I think for our own sanity we should not go past **European and American swaptions**, Bermudan swaptions are something that even Eugene and Chris don't entirely know how to price. Breakdown of what I think we need to do is below.

Short outline Classes:

- Base: *(helper functions that should be global to the project) We may find that we don't need this but it can be helpful, and these are thing like a daycount function, or print and graphing functions so that outputs are uniform.*
- Market Data: *this will be all of the different bits of data that we need.*
- Curves: *constructing the various curves we will need and use, this was essentially the project for 703, need to add some things like daycount but otherwise its already done.*
- Stochastic Processes: *we may not need all of the ones I'll list but hard to know till we get there.*
- Swaptions/Swaps: *This will probably be the second hardest part.*
- Volatility Calibration: *heston-SABR...etc. We just need to figure out what the best way to calibrate to market data is.*
- Greeks/Sensitivities: *This will be essential to getting the hedging correct.*
- Hedging Strategy: *This is the part that I don't exactly know how to or what to do for this yet....should be fun!*
- Validation: *WE can use QuantLib to validate a lot of our outputs.*

There are some parts that will be fairly quick to do, the Market Data, Curves and Stochastic processes shouldn't be two hard. The Hedging and the volatility Calibration I'm guessing is where we'll run into problems.

Base

- DayCount(): This will get called a lot because we will need to take into account the exact number of days to expiry for the Yield Curves and the Swaps/Swaptions.
- Print():
- Graph():
- 3D Graph():

Market Data

I am planning on pulling the data from Bloomberg simply because it will give us a consistent data source. I am almost positive we can't get the data from anywhere else.

- Yield_Curve_Data():
- Underlying_Asset_Data():
- Inflation_Data():
- Swap_Swaptions_Data(): This will be for model validation.
- Backtesting_Data(): potentially(still not sure well use this.

Curves

This will call **Base & Market Data** mostly for the daycount function.

- Discount_Curve():
- OIS_Curve():
- FRA_Futures_Conversion():
- Treasury_Curve():
- Any_Other_curve():
- PCA_Build(): we may not want to deal with this at all, but it may also make our lives easier. Not sure yet.

Stochastic Processes

This will be kind of a helper function, but we will see how it goes.

- Gamma_Variance():
  The basic gamma process for Monte Carlo simulation is:

$$X(t_i) = X(t_{i-1}) + \Gamma_i^+(t) - \Gamma_i^-(t).$$

  This might work, it might not, we will see. SABR is the most common.

$$X^{VG}(t; , \sigma, \nu, \theta) := \theta\Gamma(t; 1, \nu) + \sigma W(\Gamma(t; 1, \nu)), \tag{1}$$

  and we can put this into a difference of two independent gamma processes:

$$X^{VG}(t; , \sigma, \nu, \theta) := \Gamma(t; \mu_p, \mu_p^2\nu) - \Gamma(t; \mu_p, \mu_p^2\nu) \tag{2}$$

  where

$$\mu_p := \frac{1}{2}\sqrt{\theta^2 + \frac{2\sigma^2}{\nu}} + \frac{1}{2} \quad \text{and} \quad \mu_p := \frac{1}{2}\sqrt{\theta^2 + \frac{2\sigma^2}{\nu}} - \frac{1}{2}. \tag{3}$$

  We will use this for American style options if we get to those.

  references:
  Hirsa, Ali; Madan, Dilip B. (2003). "Pricing American Options Under Variance Gamma". Journal of Computational Finance. 7 (2): 63–80. doi:10.21314/JCF.2003.112

  Ralf Korn; Elke Korn & Gerald Kroisandt (2010). Monte Carlo Methods and Models in Finance and Insurance. Boca Raton, Fla.: Chapman and Hall/CRC. ISBN 978-1-4200-7618-9. (Section 7.3.3)

- SABR():
  The SABR model is essentially as follows:

$$\begin{aligned}
DS_t &= rS_t dt + \sigma S_t^\beta dW_t^1 && (4)\\
d\sigma_t &= \alpha dW_t^2 && (5)\\
(dW_t^1, dW_t^2) &= \rho dt && (6)\\
r &= riskfree && (7)\\
\alpha &= && (8)\\
\beta &= 0.5 \text{ (if we go off of Chris's preference)} && (9)\\
\rho &= && (10)\\
\sigma &= && (11)
\end{aligned}$$

- Local_Volatility():

$$\begin{aligned}
ds_t &= r(t)S_t dt + \sigma(S_t, t)S_t dW_t && (12)\\
r(t) &= riskfree && (13)\\
\sigma(S_t, t) &= && (14)
\end{aligned}$$

All that follows is my best guess, could be wildly wrong, if anyone has a better idea please share.

- Interest_Rate_Swap():

$$PV_{fixed} = \sum_{n=1}^{N} S\Delta(t_{n-1}, t_n) DF(t_n) \tag{15}$$

$$PV_{float} = \sum_{n=1}^{N} R(t_n)\Delta(T_{n-1}, t_n) DF(t_n) \tag{16}$$

$$PV_{irs} = PV_{fixed} - PV_{float} \tag{17}$$

  where

$$DF(t_n) = \prod_{k=1}^{n}(1 + R(t_k)\Delta(t_{k-1}, t_k)) \quad n = 1, ..., N \tag{18}$$

  For $DF(t_0) = 1$. Daycount will need to fit in here because we need to be accurate to the exact day.

- Interest_Rate_Swaption(): from Paul Wilmott
  The Market practice is $\max(r_f - r_E, 0) * PV$ of all future cash flows.

  Payer Swaption:

$$\frac{1}{F} e^{-r(T-t)} \big(1 - \big(1 + \frac{1}{2}F\big)^{-2(T_s - T)}\big)(FN(d_1^{'}) - EN(d_2^{'}) \tag{19}$$

  Reciever Swaption:

$$\frac{1}{F} e^{-r(T-t)} \big(1 - \big(1 + \frac{1}{2}F\big)^{-2(T_s - T)}\big)(EN(d_2^{'}) - FN(d_1^{'})) \tag{20}$$

  where F is the forward rate of the swap, T_S is the maturity of the swap and

$$d_1^{'} = \frac{\log(F/E) + \frac{1}{2}\sigma^2(T-t)}{\sigma\sqrt{T-t}} \quad \text{and} \quad d_2^{'} = d_1^{'} - \sigma\sqrt{T-t} \tag{21}$$

  assuming semi-annual coupon payments.

- Inflation_SwapYoY():

$$X_{fixed} = N\phi_i K \tag{22}$$

$$X_{floating} = N\psi_i \big[\frac{I(T_i)}{I(T_{i-1})} - 1\big] \tag{23}$$

  still working on this.....

- Inflation_Swap_Zero():

$$X_{fixed} = N[(1+K)^M - 1] \tag{24}$$

$$X_{floating} = N\Big[\frac{I(T_M)}{I(T_M 0)} - 1\Big] \tag{25}$$

still working on this.....

- Inflation_Swaption():
still working on this.....

Volatility Calibration

- Heston(): We all know what to do for this one.
- Lattice(): This is what Eugene was going over in the last class, we can probably GitHub this or get close to what we need from something on Github.

Greeks/Sensitivities:

————————————————-calls————————puts———————————————-

- Delta():

$$e^{-q\tau}\Phi(d_1) \quad \text{and} \quad -e^{-q\tau}\Phi(-d_1) \tag{26}$$

- Gamma():

$$e^{-q\tau}\frac{\Phi(d_1)}{S\sigma\sqrt{\tau}} = Ke^{-r\tau}\frac{\Phi(d_2)}{S^2\sigma\sqrt{\tau}} \tag{27}$$

- Theta():

$$-e^{-q\tau}\frac{S\Phi(d_1)\sigma}{s\sqrt{2}} - rKe^{-r\tau}\Phi(d_2) + qSe^{-q\tau}\Phi(d_1) \text{ and } -e^{-q\tau}\frac{S\Phi(d_1)\sigma}{s\sqrt{2}} + rKe^{-r\tau}\Phi(d_2) - qSe^{-q\tau}\Phi(-d_1) \tag{28}$$

- Vega():

$$Se^{-q\tau}\Phi(d_1)\sqrt(\tau) = Ke^{-r\tau}\Phi(d_2)\sqrt(\tau) \tag{29}$$

- Vanna():

$$e^{-q\tau}\Phi(d_1)\frac{d_2}{\sigma} = \frac{\nu}{S}\Big[1 - \frac{d_1}{\sigma\sqrt{\tau}}\Big] \tag{30}$$

- Vomma():

$$Se^{-q\tau}\Phi(d_1)\sqrt{\tau}\frac{d_1 d_2}{\sigma} = \nu\frac{d_1 d_2}{\sigma} \tag{31}$$

- Veta():

$$-Se^{q\tau}\phi(d_1)\sqrt{\tau}\Big[q + \frac{(r-q)d_1}{\sigma\sqrt{\tau}} - \frac{1+d_1 d_2}{2\tau}\Big] \tag{32}$$

- Duration():
$$wedidthis \tag{33}$$
- Convexity():
$$wedidthis \tag{34}$$
- DV01():
$$wedidthis \tag{35}$$

Hedging Strategy

- Delta_Neutral():
- Gamma_Neutral():
- Curve_Tenor_Hedge():
- Multi_Tenor_Hedge():

Validation

- QuantLib():
- Bloomberg_Values():