MF 796: Computational Methods of Mathematical Finance

Professors Christopher Kelliher and Eugene Sorets Spring 2022

Problem set # 5

Due: Monday, March 28, by 2pm

Problem 1: Numerical PDEs: Suppose that the underlying security SPY evolves according to standard geometric brownian motion. Then its derivatives obey the Black-Scholes equation:

$$\frac{\partial c}{\partial t} + \frac{1}{2}\sigma^2 s^2 \frac{\partial^2 c}{\partial s^2} + rs\frac{\partial c}{\partial s} - rc = 0 \tag{1}$$

Use SPY's closing price of 440.

We are going to find the price of a call spread with the right of early exercise. The two strikes of the call spread are $K_1 = 445$ and $K_2 = 450$ and the expiry is September 16, 2022.

(a) Explain why this instrument is not the same as being long an American call with strike 445 and short an American call with strike 450, both with expiry September 16, 2022.

(b) For riskless rate r, use the 3-month US Treasury bill at the close of March 23, 2022. Say where you got the rate and why you consider it a reliable source.

(c) Let's assume that we are not able to find σ by calibrating to the European call spread price and must find it by other means. Find a way to pick the σ, explain why you chose this method, and then find the σ.

(d) Set up an explicit Euler discretization of (1). You will need to make decisions about the choice of $s_{\max}$, h_s, h_t, etc. Please explain how you arrived at each of these choices.

(e) Let A be the update matrix that you created in the previous step. Find out its eigenvalues and check their absolute values.

(f) Apply your discretization scheme to find today's price of the call spread *without* the right of early exercise. The scheme will produce a whole vector of prices at time 0. Explain how you chose the one for today's price.

(g) Modify your code in the previous step to calculate the price of the call spread *with* the right of early exercise. What is the price?

(h) Calculate the early exercise premium as the difference between the American and European call spreads. Is it reasonable?