MF 796: Computational Methods of Mathematical Finance

Professors Christopher Kelliher and Eugene Sorets — Spring 2022

Problem set # 1

Due: Monday, February 7, by 2 pm

1. **What areas of finance**
   are of most interest to you (e.g. big data, trading, portfolio management, risk management, derivative pricing, analyzing complex derivatives)?

2. **What is the primary reason for your interest in this course?**

3. **List all programming languages you have used**
   and your level of familiarity with each.

4. **Option traders often say that when buying options we get gamma at the expense of theta.**
   What do you think they mean?

5. **Evaluation of a known integral using various quadratures:**
   In this problem we are going to compute the price of a European call option with 3 month expiry, strike 12, and implied vol 20%, Assume the underlying is 10 now and the interest rate is 1%.

   (a) Use Black-Scholes formula to compute the price of the call analytically.

   (b) Calculate the price of the call numerically using the following 3 quadrature methods applied to normal cdf inside the Black-Sholes formula:

      i. Left Riemann rule

      ii. Midpoint rule

      iii. Gauss nodes of your choice (say explicitly why you made that choice)

      with the number of nodes $N = 5, 10, 50$ and compute the calculation error as a function of N for each of the methods.

   (c) Estimate the experimental rate of convergence (i.e., as a function of N) of each method and compare it with the known theoretical estimate.

   (d) Which method is your favorite and why?

6. **Calculation of Contingent Options:**

Let S_1 be a random variable that takes on the value of the stock ABC one year from now and let S_2 take on the values of ABC 6 months from now. Assume that they are jointly normally distributed with

$$\begin{aligned} \sigma_1 &= 20 \\ \sigma_2 &= 15 \\ \rho &= 0.95 \end{aligned}$$

By ρ here we mean correlation between S_1 and S_2. Also, assume that interest rate is zero and the current value of the underlying, i.e., of ABC is 380.

(a) Evaluate the price of the one year call on ABC with the strike $K_1 = 380$. This is an example of a vanilla option.

(b) Evaluate the price of the one year call on ABC with the strike $K_1 = 380$, contingent on ABC at 6 months being below 375. This is a contingent option.

(c) Calculate the contingent option again, but with $\rho = 0.8$, $\rho = 0.5$, and $\rho = 0.2$.

(d) Does dependence on ρ make sense?

(e) Calculate the contingent option again, but with ABC at 6 months below 370 and 360.

(f) Does the dependence on the 6 month value make sense?

(g) Under what conditions do you think the price of the contingent option will equal the price of the vanilla one?