

Problem set # 6

Due: Monday, April 11, by 2pm

Problem 1: Simulation in the Heston Model:

Suppose that the underlying security SPY evolves according to the Heston model. That is, we know its dynamics are defined by the following system of SDEs:

$$\begin{aligned} dS_t &= (r - q)S_t dt + \sqrt{\nu_t}S_t dW_t^1 \\ d\nu_t &= \kappa(\theta - \nu_t) dt + \sigma\sqrt{\nu_t} dW_t^2 \\ \text{Cov}(dW_t^1, dW_t^2) &= \rho dt \end{aligned} \tag{1}$$

You know that the last closing price for SPY was 440. You also know that the dividend yield for SPY is 1.77% and the corresponding risk-free rate is 1.5%.

Using this information, you want to build a simulation algorithm to price a knock-out option on SPY, where the payoff is a European call option contingent on the option not being knocked out, and the knock-out is an upside barrier that is continuously monitored. We will refer to this as an **up-and-out call**.

This payoff can be written as:

$$c_0 = \mathbb{E} \left[(S_T - K_1)^+ 1_{\{M_T < K_2\}} \right] \tag{2}$$

where M_T is the maximum value of S over the observation period, and $K_1 < K_2$ are the strikes of the European call and the knock-out trigger respectively.

- (a) Find a set of Heston parameters that you believe govern the dynamics of SPY. You may use results from a previous Homework, do this via a new calibration, or some other empirical process. Explain how you got these and why you think they are reasonable.
- (b) Choose a discretization for the Heston SDE. In particular, choose the time spacing, ΔT as well as the number of simulated paths, N . Explain why you think these choices will lead to an accurate result.
- (c) Write a simulation algorithm to price a European call with strike $K = 282$ and time to expiry $T = 1$. Calculate the price of this European call using FFT and comment on the difference in price.
- (d) Update your simulation algorithm to price an up-and-out call with $T = 1$, $K_1 = 285$, and $K_2 = 315$. Try this for several values of N . How many do you need to get an accurate price?

- (e) Re-price the up-and-out call using the European call as a control variate. Try this for several values of N . Does this converge faster than before?