**MF 796: Computational Methods of Mathematical Finance**

Professors Christopher Kelliher and Eugene Sorets                                                                                Spring 2022

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**Problem set # 4**

Due: Tuesday, March 21, by 2pm

**Problem 1: Covariance Matrix Decomposition:** Download historical data from your favorite source for 5 years and at least 100 companies or ETFs. In this problem we will look at the covariance matrix for these assets and its properties.

(a) Clean the data so that your input pricing matrix is as full as possible. Fill in any gaps using a reasonable method of your choice. Explain why you chose that particular method.

(b) Generate a sequence of daily returns for each asset.

(c) Calculate the covariance matrix of daily returns and perform an eigenvalue decomposition on the matrix. How many positive eigenvalues are there? How many were negative? If any are negative, what happened?

(d) How many eigenvalues are required to account for 50% of the variance of the equally-weighted portfolio of the securities you picked? How about 90%? Does this make sense to you?

(e) Create the daily residual returns after the principal components that correspond to the top 90% eigenvalues have been removed from the equally-weighted portfolio. Plot this return stream and comment on its properties.

**Problem 2: Portfolio Construction:** In Lecture 5a, we defined a Lagrangian for portfolio with constraints in matrix form by

$$L(w, \lambda) = \langle R, w \rangle - a \langle w, Cw \rangle - \langle \lambda, Gw - c \rangle \tag{1}$$

(a) Form the matrix $G$ by imposing the budget constraint, which is $\langle 1, w \rangle = 1$, and another constraint that allocates 10% of the portfolio to the first 17 securities (to simulate sector allocation). Using $C$ from Problem 1, use your favorite method and the software package of your choice to invert $GC^{-1}G^T$ in a nice, stable way. (Hint: consider my favorite method).

(b) What does the resulting portfolio look like? Would it be acceptable to most mutual funds? If not, what would you to do to fix that?

**Problem 3: Portfolio Stability** For this problem please refer to the file `DataForProblem3.csv`.

Each column refers to a security and each row represent a day in history.

Each cell contains that day's return of that security in decimals.

We also view each day as a possible state of the world and assume that all these states are equally likely.

Also assume that your total wealth is $1,000,000 and no shorting is allowed.

Come up with your own reasonable guess at the expected return of each security and explain why you made this particular choice.

Justify each of your answers below by setting up the optimization problem you are planning to solve, explaining why you think this choice is the right one, and then solve it.

(a) Using the history of the returns of the instruments Sec1 through Sec10 in `DataForProblem3.csv`, construct the fully-invested minimum variance portfolio.

(b) Find the fully-invested optimal portfolio, assuming your risk aversion coefficient is 0.5.

(c) Find the portfolio that would maximize the expected return.

(d) Do you think the portfolio in (b) will be stable under small changes to expected returns? Please explain why or why not.