Thomas Kuntz: 796 Homework 2

February 23, 2022

1 Exploring FFT Technique Parameters Consider a European Call Option with strike 250 expiring in six months.

1.0.1 i)

The equation that we need to worry about in terms of alpha (α) or the damping factor, is:

$$C_T(k_j) = \frac{1}{\pi}\exp\Big(-\alpha\Big[\ln S_0 - \Delta k\Big(\frac{N}{2} - j + 1\Big)\Big]\Big)R(y_i) \tag{1}$$

For the call price. Then we can see in graph 1 to that for any values between 0.25 & and 30 the price is largely stable. For very small values of 0.01, and 0.02 the model prices are abnormally large, and for values above 30, the values begin to go to extreme negative values. The full range of values is [0.01, 0.02, 0.25, 0.5, 0.75, 0.9, 1 ,1.25, 1.5, 1.75, 2, 5, 7, 10, 15, 20, 30, 38]. The stable price given the Fast Fourier Transform is 21.26886, and the run-time is 0.01817.

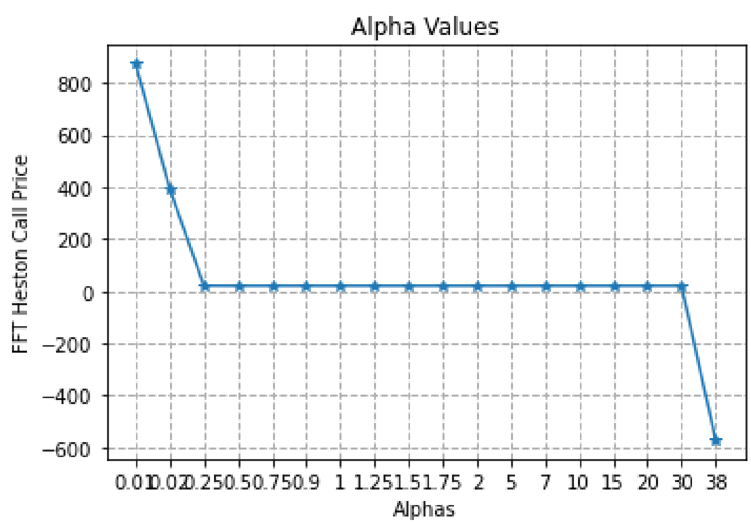
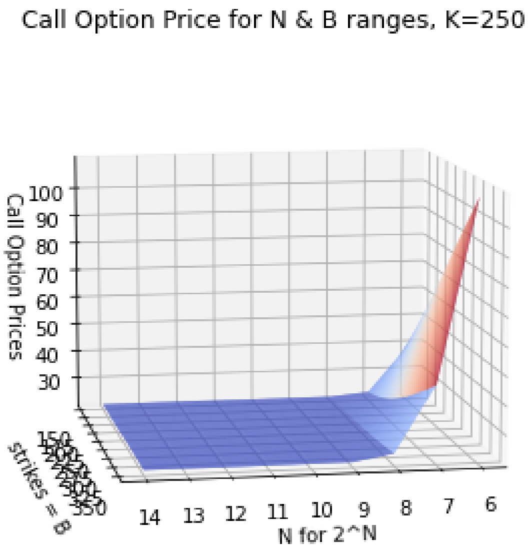


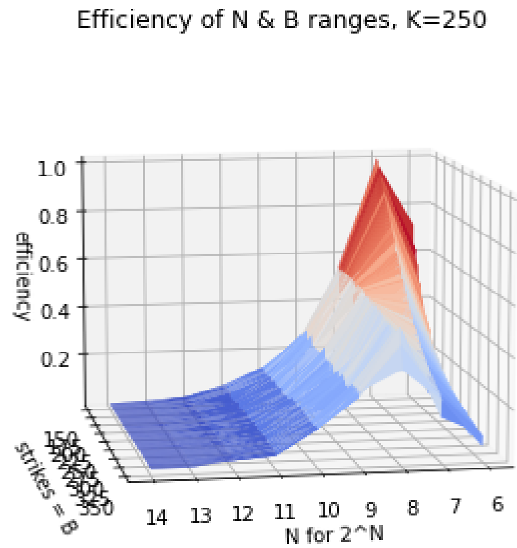
Figure 1: Model Alphas

1.0.2 ii)

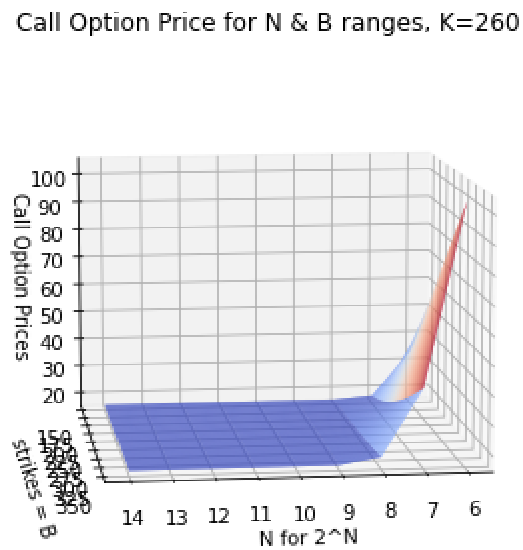
We can set the alpha that we use for the rest of the calculation as 1, and we will let the strike B and N be values that are arbitrarily large enough to mimic converging to ∞, but do not cause run-time errors. That means values between 2^6-2^{14}. For values of N greater then 9, we see a consistent option price. Efficiency can be read two ways, and we can see that for N close to 8, and strikes closer to the low and of the range where the $\min B$ is 150.



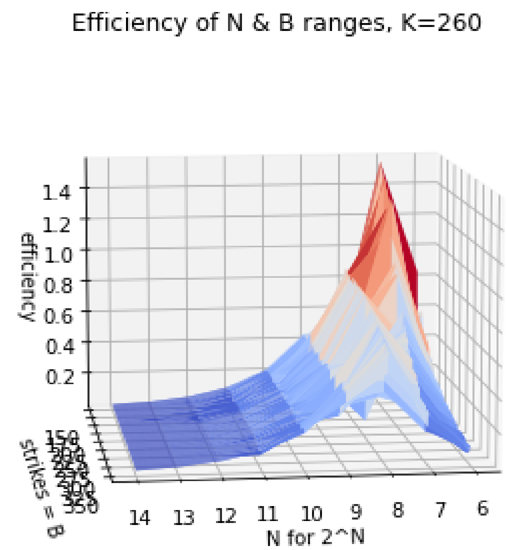
(a) Call Price Surface



(b) efficiency Surface



(c) Call Price Surface



(d) efficiency Surface

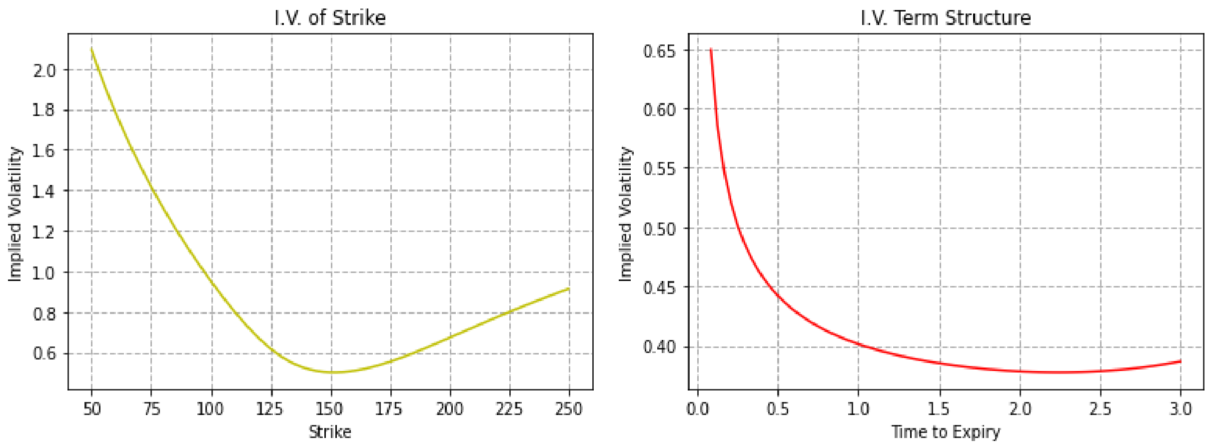
Figure 2: Surfaces

For the strike of 260, the call values are slightly lower then the strike of 250. The option values drops to 17.13632, and the run-time goes to 0.01894. From an optimization standpoint, the results do not change that much which is not entirely surprising since we are not changing the call value by that much in percentage terms. What this means is that we use a B of 1000, and a N of 9 for all the implied volatility graphs below.

2 (b) Exploring Heston Parameters Assume the risk-free rate is 2.5%, the initial asset price is 150 and that the asset pays no dividends.

2.0.1 i) & ii)

The pictures explain most of the interesting information, but there are some interesting differences.



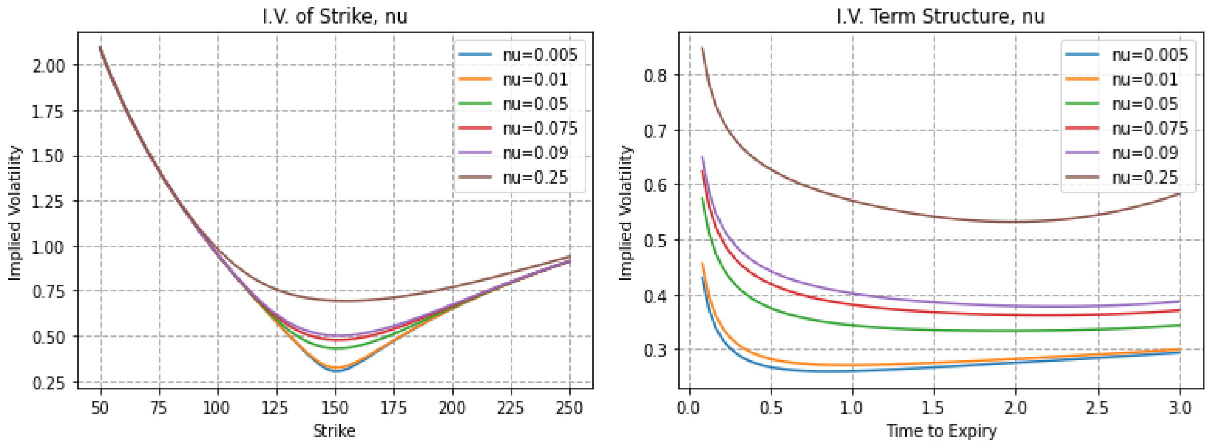
(a) Volatility Smile of Strikes

(b) Term Structure of Volatility

Figure 3: Smile(K), Term-Structure(T)

The strike smile is much more sharp and angular around the at-the-money strike, and the term-structure is a lot smoother and does not increase as much at three years as the strike smile does at K = 250. We expect to see the downward skew on the term-structure, so for a sanity check, this makes sense, one point that is worth making is that the implied Volatility of the strike smile is significantly higher as a range which I don't have a good explanation for.

2.0.2 iii)

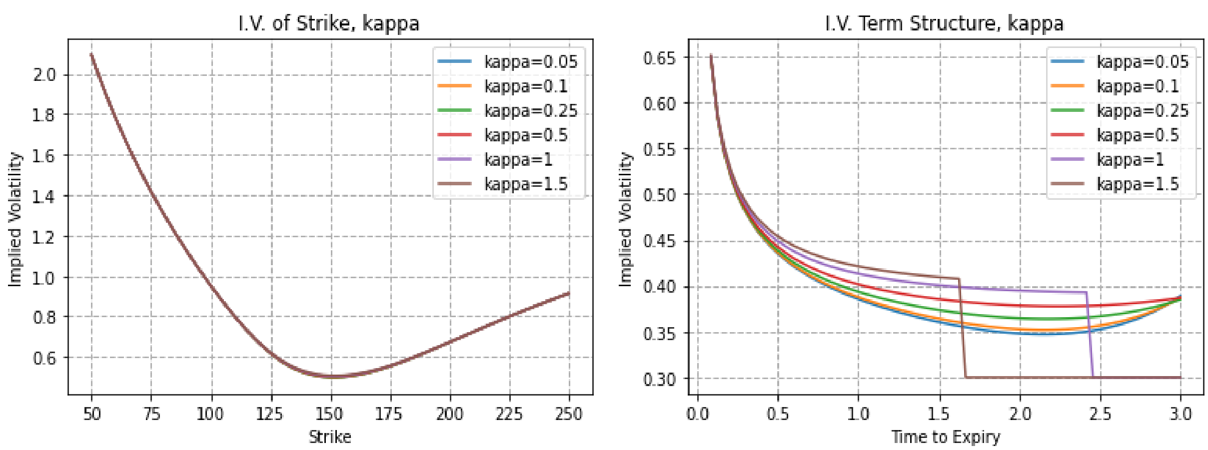


(a) Volatility Smile of Strikes

(b) Term Structure of Volatility

Figure 4: nu changes

These two plots both have some interesting characteristics, the smiles all converge at the tails, but they get more V-shaped as nu goes towards zero. The term-structures are a little less obvious, but as nu goes to zero again, the plots seem to also flatten and go to smaller values of implied volatility. A takeaway could be that as nu goes to zero, the term-structure is affected more then the smile, hence nu is correlated to time more then strike, but has a noticeable effect on both.

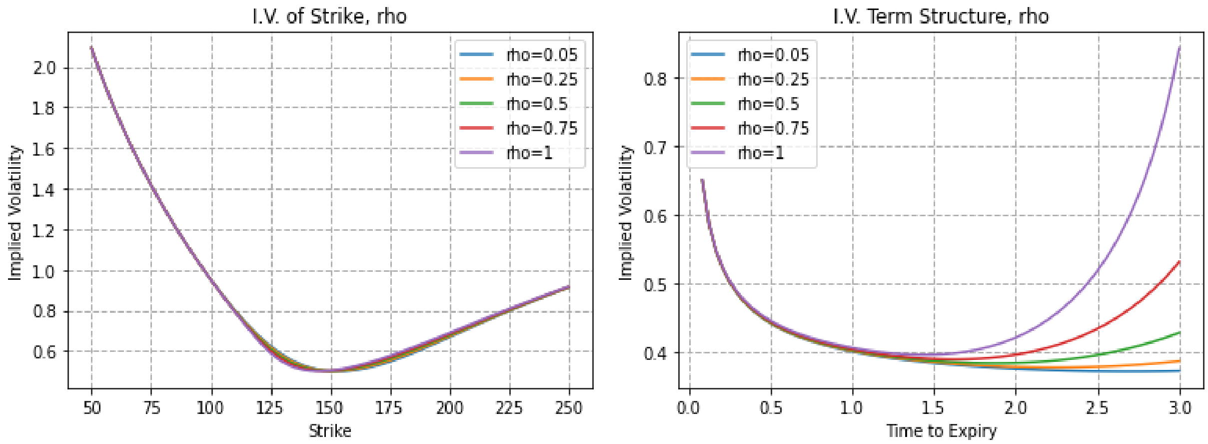


(a) Volatility Smile of Strikes

(b) Term Structure of Volatility

Figure 5: kappa changes

Here, the smile is visually unaffected, but the term-structure has some weird changes. For kappa ≥ 1, the heston breaks for time greater then 1.5 years, which is not necessarily a bad thing, but just worth making a mental note of.

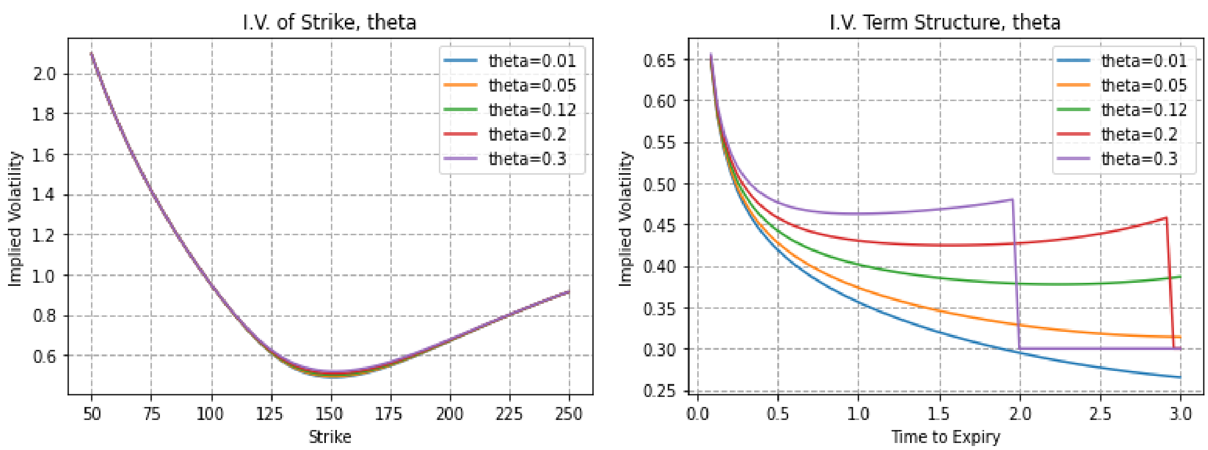


(a) Volatility Smile of Strikes

(b) Term Structure of Volatility

Figure 6: rho changes

For rho greater then 0.25, the term-structure blows up after 2 years, but nothing else is really noteworthy.



(a) Volatility Smile of Strikes

(b) Term Structure of Volatility

Figure 7: theta changes

Theta really only effects the term-structure again, and for theta ≥ 0.2 the graphs become weird after two years. For the rest of the values, the skew gets progressively steaper as theta goes to 0.01.