FX Markets and Their Option Implied Risk-Neutral Densities

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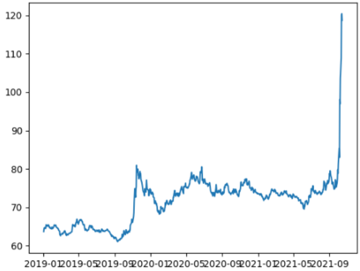
May 2021

Exchange Rates & Options

1. G10, EMEA & LATAM
   - USDNOK, USDEUR & USDAUD
   - USDHUF & USDILS
   - USDBRL & USDARS
2. Some currencies have a globally demanded commodity that helps demand for that currency, the Norwegian Krone and the Russian Ruble are two good examples.

1. Asian Options
   - Fixed Strike, Floating strike & Conditional
2. Digital options
3. European options



The Models + Black Scholes

Variance-Gamma

Can be equated to a pure jump process which fits short dated options. There is a characteristic function that we can use to pull a density with.

Difference of gammas

$X(t_i) = X(t_{i-1}) + \Gamma_i^+(t) - \Gamma_i^-(t)$.

works well with simulation, can't be inverted.

Time changed gamma

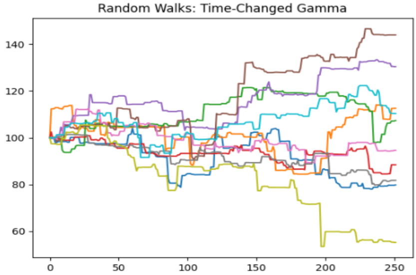
$X^{VG}(t;,\sigma,\nu,\theta) = \theta\Gamma(t;1,\nu) + \sigma W(\Gamma(t;1,\nu))$

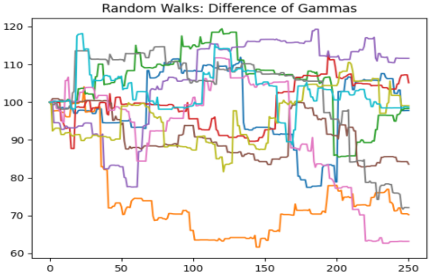
works well with simulation, can't be inverted.

Black-Scholes

$dS_t = rS_t dt + \sigma S_t dW_t$ does not have jumps, easy to invert

Simulations of Variance-Gamma





Asian Options

Fixed Strike:

$$Call_T = \max(A(S_{0,T}) - K, 0)$$

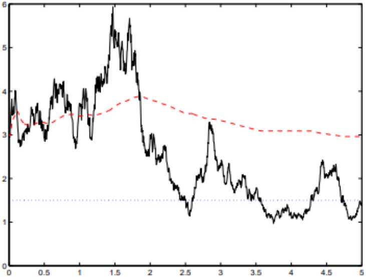
Floating Strike:

$$Call_T = \max(S_T - \kappa A(S_{0,T}), 0)$$

Conditional:

$$Put_T = \max\left(K - \frac{\int_0^T S_t I_{\{S_t > b\}} dt}{\int_0^T I_{\{S_t > b\}} dt}, 0\right)$$

The point of a conditional is that it is supposed to make the premium smaller by up to 40% so that more hedge funds will go buy them from BNP Paribas.



Conditional Asian Put Example

Pricing Differences

Analytic prices(European):

1. euro_put = 7.9655 and euro_call = 7.96556
2. euro_put(VG) = 9.9838 and euro_call(VG) = 9.0398

Simulations(Asian Options)

1. Asian call(geo.) = 4.4318 and Asian put(geo.) = 4.7646
2. Asian call(GBM) = 4.5951 and Asian put(GBM) = 4.6917
3. Asian call(TC) = 4.7451 and Asian put(TC) = 4.6077
4. Asian call(DG) = 4.5531 and Asian put(DG) = 4.4545

Payoff Differences:

1. calls → floating(4.6015) vs. fixed(4.5531) vs.conditional(3.9551)
2. puts → floating(4.5213) vs. fixed(4.5629) vs.conditional(3.8691)

Breeden-Litzenberger & Risk-Neutral Densities

Refresher:

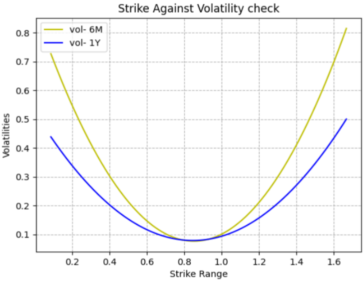
1. Same basic methods that we have seen in class
2. We can't invert the Monte Carlo simulations, so we need to look at the analytic solutions
3. This means we are reduced to the geometric mean for the Asian options, and the Black-Scholes formula

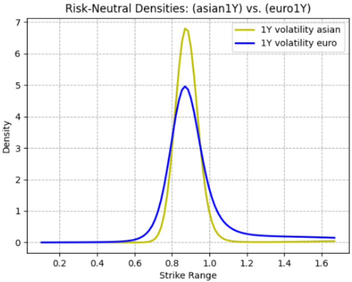
Basic Method:

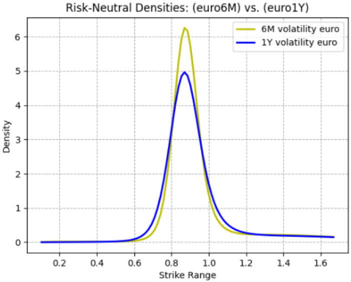
1. Look at the market implied densities at different points in time (01/01/2008, 01/02/2019)
2. Hope that we find a mis-match that we can take advantage of or a fat tail vs. normal tail

Breeden-Litzenberger & Risk-Neutral Densities

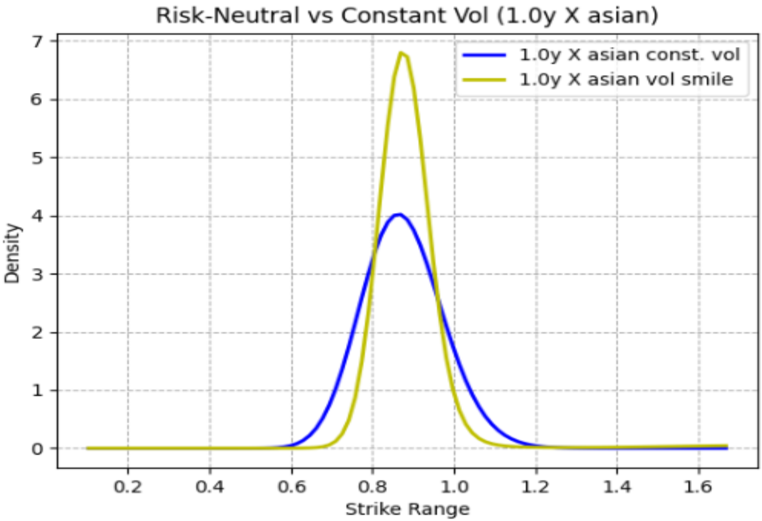
The densities of the USDEUR on (01/02/2019) are all largely centered with low skew. The volatility smile is also not very skewed.







Breeden-Litzenberger & Risk-Neutral Densities

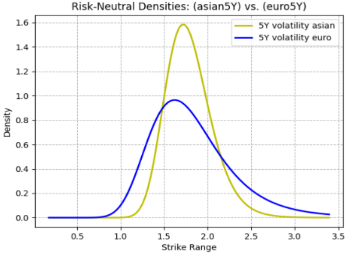


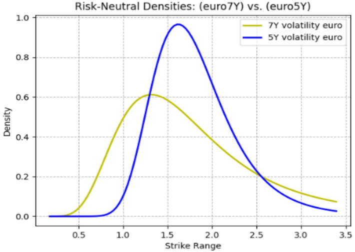
Triangle Trade: Isolate a Carry From Oil Production

NOK vs. BRL vs. USD: (from 01/01/2008)

USDBRL trade results

results (5Y expiry)		
option	premium	P/L
Asian float	0.0846	-0.01804
Asian fixed	0.0938	-0.0272
digital call	0.4555	0.5445
euro call	0.1589	0.0272

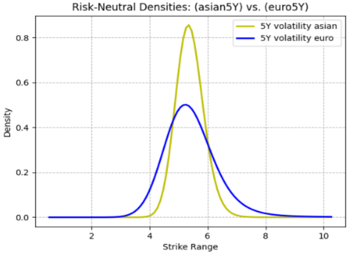


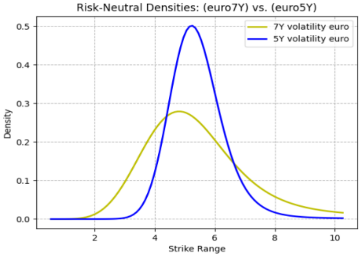


Triangle Trade: Isolate a Carry From Oil Production

USDNOK trade results

results (5Y expiry)		
option	premium	P/L
Asian float	0.2577	0.2230
Asian fixed	0.2901	-0.6251
digital put	0.5461	-0.5461
euro put	0.4539	-0.4539

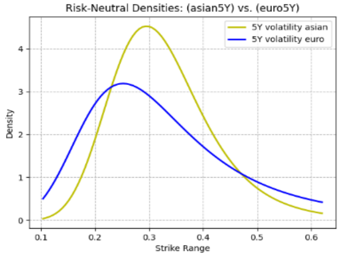


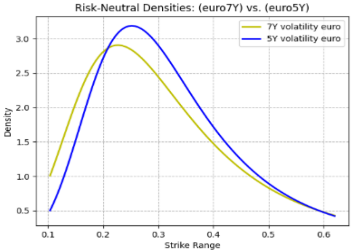


Triangle Trade: Isolate a Carry From Oil Production

NOKBRL trade results

results (5Y expiry)		
option	premium	P/L
Asian float	0.0342	-0.0341
Asian fixed	0.0302	0.0247
digital call	0.4154	0.5845
euro call	0.0552	-0.0130





Triangle Trade: Isolate a Carry From Oil Production