1. Model derivation - The levered firm value

1.1. The case of the simple EBIT-model

Before we derive the closed form valuation equations for the firm subject to an ESR, we provide the formulas of the standard EBIT model for further usage and as yardstick.

Consider an otherwise identical but levered firm whose management has decided to finance the operations by equity and debt in order to maximize the wealth of its equityholders. The debt issue, a single console bond, promises a fixed coupon payment C to the debtholders, as long as the firm remains solvent. The value $F(V,t)$ of any claim underlying the EBIT with a constant payment flow C, i.e., any claim on the firm that continuously pays a nonnegative coupon C per instant of time when the firm is solvent, must satisfy

$$\mu V F_V(V) + \frac{\sigma^2}{2} V^2 F_{VV}(V) + F_t + P(V) = rF(V), \tag{1}$$

where $P(V)$ is the income or payout flow. Notice that we have suppressed the time and value dependence (V,t). Since the firm issues perpetual debt the corporate claims have no explicit time dependence, the term $F_t(V,t) = 0$ and (1) reduces to an ordinary differential equation (ODE) of the form

$$\mu V F_V(V) + \frac{\sigma^2}{2} V^2 F_{VV}(V) + P(V) = rF(V). \tag{2}$$

The general solution to the aforementioned nonhomogenous ODE is given by

$$F(V) = A_0 + A_1 V^{-y} + A_2 V^{-x}, \tag{3}$$

with

$$x = \frac{1}{\sigma^2}\left[(\mu - \frac{\sigma^2}{2}) + \sqrt{(\mu - \frac{\sigma^2}{2})^2 + 2r\sigma^2}\right], \tag{4}$$

$$y = \frac{1}{\sigma^2}\left[(\mu - \frac{\sigma^2}{2}) - \sqrt{(\mu - \frac{\sigma^2}{2})^2 + 2r\sigma^2}\right], \tag{5}$$

where the constants A_i, $i = 1,2$ are the solution of the homogenous ODE (1) with $P(V) = 0$ and determined by the subsequently discussed boundary conditions. A_0 is the equivalent particular solution. For $P(V) = C$ we derive $A_0 = \frac{C}{r}$.

Let $D(V)$ denote the value of debt and V_B the endogenously determined level of the asset value that triggers the bankruptcy. In case of bankruptcy a fraction $0 \leq \alpha \leq 1$ of the asset value is lost to bankruptcy costs, leaving the debtholders with $(1-\alpha)V_B$ and the equityholders with nothing. Since the optimal bankruptcy level depends on the debt value maximizing the value of equity, we first derive expressions for the debt value, the bankruptcy costs and the tax shield value.

The boundary conditions for determining the market value of debt $D(V)$ are given by:

$$\text{at } V = V_B: \qquad D(V) = (1-\alpha)V_B, \tag{6}$$

$$\text{as } V \to \infty: \qquad D(V) = \frac{C}{r}. \tag{7}$$

Using (6) we are able to determine the constants A_0, A_1 and A_2 for equation (3). Since $y < 0$ it follows that $-y > 0$, so A_1 is equal to zero for all claims of interest in this section. From the infinity boundary condition we derive $A_0 = \frac{C}{r}$ and A_2 is arbitrary. Putting this into the first boundary condition yields

$$(1-\alpha)V_B = \frac{C}{r} + 0 + A_2(V_B)^{-x} \tag{8}$$

$$A_2 = [(1-\alpha)V_B - \frac{C}{r}](V_B)^{x}. \tag{9}$$

Thus, the value of debt is given by

$$D(V) = \frac{C}{r} + \left[(1-\alpha)V_B - \frac{C}{r}\right]\left(\frac{V}{V_B}\right)^{-x}. \tag{10}$$

The boundary conditions for the value of the bankruptcy costs are given by

$$\text{at } V = V_B: \qquad BC(V) = \alpha V_B, \tag{11}$$

$$\text{as } V \to \infty: \qquad BC(V) \to 0. \tag{12}$$

The solution of equation (3) using the aforementioned boundary conditions is then given by

$$BC(V) = \alpha V_B \left(\frac{V}{V_B}\right)^{-x}. \tag{13}$$

Since the bankruptcy costs vanish in case of $V \to \infty$, A_0 is equal to 0. It remains $\alpha V_B = A_2(V_B)^{-x} \Leftrightarrow A_2 = \alpha V_B(\frac{V}{V_B})^{x}$.

Let us consider the tax shield value. We regard the typical structural assumptions: As long as the firm is solvent, period specific tax savings amounting to τC are generated. In case the firm is bankrupt no tax savings are generated. Therefore, taking the adapted solution of the ODE (1) into account the value of the tax benefits of debt $TB(V)$ must satisfy equation (3) with the boundary conditions

$$\text{at } V = V_B: \qquad TB(V) = 0, \tag{14}$$

$$\text{as } V \to \infty: \qquad TB(V) = \frac{\tau C}{r}. \tag{15}$$

We solve the following equation system:

$$0 = A_0 + A_1(V_B)^{-y} + A_2(V_B)^{-x} \tag{16}$$

$$\frac{\tau C}{r} = A_0 + A_1 \lim_{V\to\infty} V^{-y} + A_2 \lim_{V\to\infty} V^{-x}. \tag{17}$$

Since $\lim_{V\to\infty} V^{-y} = \infty$, and $\lim_{V\to\infty} V^{-x} = 0$ we derive $A_1 = 0$ and A_2 is arbitrary. It remains $A_0 = \frac{\tau C}{r}$. Using (16) we finally state

$$0 = \frac{\tau C}{r} + A_2(V_B)^{-x}$$

$$A_2 = -\frac{\tau C}{r} \cdot (V_B)^{x}$$

The general solution is given by

$$
\begin{aligned}
TB(V) &= \frac{\tau C}{r} - \frac{\tau C}{r}\left(\frac{V}{V_B}\right)^{-x} \\
&= \frac{\tau C}{r}[1 - \left(\frac{V}{V_B}\right)^{-x}].
\end{aligned} \tag{18}
$$

The equations (10), (13) and (18) are equivalent to the respective equations of **?** while in our analysis V is driven by ζ implying a different x. The total value of the levered firm consisting of the equity, the debt value and the tax shield is given by

$$
v(V) = V + TB(V) - BC(V) \tag{19}
$$

$$
= V + \frac{\tau C}{r} - [\frac{\tau C}{r} + \alpha V_B]\left(\frac{V}{V_B}\right)^{-x}. \tag{20}
$$

For determining the optimal capital structure we have to find the optimal bankruptcy trigger V_B by maximizing the value of equity:

$$
\begin{aligned}
E(V) &= v(V) - D(V) \\
&= V + \frac{C}{r}(1-\tau) - \left[\frac{\tau C}{r} + V_B - \frac{C}{r}\right]\left(\frac{V}{V_B}\right)^{-x}
\end{aligned} \tag{21}
$$

using the smooth-pasting condition

$$
\frac{dE(V)}{d(V)}|_{V=V_B} = 0. \tag{22}
$$

In turn, the optimal bankruptcy trigger is given by

$$
V_B^{LEBIT} = \frac{Cx(1-\tau)}{r(1+x)}. \tag{23}
$$

1.2. The case of a limitation of interest deductibility

The standard EBIT-driven model on optimal capital structure assumes that the interest payments are tax deductible and generate tax savings as long as the firm is solvent.[1] Several countries adopted an ESR limiting the tax deductibility of interest payments to a certain proportion γ of the EBITDA. Since our basic setting considers an EBIT-generating machine, i.e., without any depreciation and (re-)investments, the EBITDA is equivalent to our underlying stochastic EBIT variable, ζ_t.[2]

We have the following dynamics for the tax savings at a certain point in time: Whenever $C > \gamma\zeta$, i.e., the earnings stripping rule applies, the tax savings amount to $\tau\gamma\zeta$. In most tax jurisdiction, e.g., Germany or France, the EBIT exceeding part of the interest payments might be carried forward to future points in time (interest carryforward). In order to keep the typical time independent setting, we assume that there

[1] Under several tax codes, e.g., the US, full tax savings can only be achieved as long as the interest payments C are less than the EBIT. **?** provides a solution for this scenario, under the assumption of deterministic relation between the EBIT and V.

[2] Usually, rules limiting the tax deductibility of interests use the EBITDA as basis. For example, the German ESR limits the interest deductibility to 30% of the EBITDA, i.e., the maximum deductible interest amount is determined by max($0.3EBITDA$, Interest). Since we regard an EBIT-generating machine which is not depreciated (e.g. **?**) $EBIT = EBITDA$. Nevertheless, a difference mapping the depreciation could be easily modelled by a constant depreciation factor, i.e., $EBIT = EBITDA(1-\delta)$, which would only scale the initial value of the EBITDA up.

are no interest carryforwards. In case of solvency and $C < \gamma\zeta$ full tax savings are generated, i.e., τC and no tax benefits are generated in the case of bankruptcy. The tax shield value with a possible application of an ESR, $TB_{ESR}(V)$, is time independent. Since $\zeta = \frac{r-\mu}{1-\tau}V$ we derive the following adapted boundary conditions with respect to (1):

$$\text{at } V \leq \overline{V}_B: \qquad TB_{ESR}(V) = 0, \tag{24}$$

$$\text{as } \overline{V}_B < V \leq V_{ESR} \qquad TB_{ESR}(V) = \frac{\tau\gamma}{1-\tau}V, \tag{25}$$

$$\text{as } V_{ESR} \leq V \qquad TB_{ESR}(V) = \frac{\tau C}{r}, \tag{26}$$

where $\overline{V}_B$ denotes the endogenously bankruptcy trigger for the ESR case and V_{ESR} the threshold value for an application of the earnings stripping rule. Notice that the boundary conditions are adapted to the appropriate particular solution of the ODE (1). Since V_{ESR} is the point at which the tax savings are invariant, i.e. $\tau C = \tau\gamma\zeta$, we obtain

$$V_{ESR} = \frac{1-\tau}{\gamma(r-\mu)}C. \tag{27}$$

Thus, summarized we have the following system of differential equations

$$TB_{ESR}(V) = \begin{cases} A_2 V^{-y}, & \text{if } V \leq \overline{V}_B, \\ B_1 V^{-x} + B_2 V^{-y} + \frac{\tau\gamma}{1-\tau}V, & \text{if } \overline{V}_B < V \leq V_{ESR}, \\ C_1 V^{-x} + \frac{\tau C}{r}, & \text{if } V_{ESR} \leq V. \end{cases} \tag{28}$$

Note that the terms A_0, A_1, C_2 were suppressed, since $x > 0$, $y < 0$ and thus $\lim_{V\to 0} V^{-x} = \lim_{V\to\infty} V^{-y} = \infty$ and $A_0 = 0$. This still leaves four constants, for which we consider the points $\overline{V_B} = V$ and $V = V_{ESR}$ where the appropriate regions meet. Since the Brownian motion of V can diffuse freely across these boundaries, the value function cannot change abruptly across them. In fact the solution $TB(V)$ must be continuously differentiable across V_B and V_{ESR}.[3] Thus, we get

$$A_2 V_B^{-y} = B_1 V_B^{-x} + B_2 V_B^{-y} + HV_B \tag{29}$$

$$-yA_2 V_B^{-y-1} = -xB_1 V_B^{-x-1} - yB_2 V_B^{-y-1} + H \tag{30}$$

$$B_1 V_{ESR}^{-x} + B_2 V_{ESR}^{-y} + HV_{ESR} = C_1 V_{ESR}^{-x} + \frac{\tau C}{r} \tag{31}$$

$$-xB_1 V_{ESR}^{-x-1} - yB_2 V_{ESR}^{-y-1} + H = -xC_1 V_{ESR}^{-x-1}. \tag{32}$$

[3] For a heuristic argument see **?**[Section 3.8] or for a rigorous proof **?**[Theorem 4.4.9].

Solving this system yields

$$A_2 = B_1 V_B^{y-x} + HV_B^{y+1} + B_2 \tag{33}$$

$$B_1 = HV_B^{x+1}[\frac{y+1}{x-y}] \tag{34}$$

$$B_2 = HV_{ESR}^{y+1}[\frac{x+1}{y-x}] - \frac{\tau C x}{r(y-x}V_{ESR} \tag{35}$$

$$C_1 = B_1 + \frac{y}{x}B_2 V_{ESR}^{x-y} - \frac{\tau\gamma}{1-\tau}V_{ESR}^{x+1}. \tag{36}$$

Due to readability the exact calculation is removed to the appendix. To find the optimal capital structure based upon the trade-off between the tax shield value considering a possible application of the ESR and the value of the bankruptcy costs we have to determine the endogenous bankruptcy trigger by maximizing the equity value. Since the $D(V)$ and $BC(V)$ are only indirectly affected via $\overline{V}_B$ by an ESR the equations (10) and (13) still hold.

Noting that $TB_{ESR}(V)$ and $E(V)$ are monotonically increasing functions, there exists only one optimal bankruptcy trigger V_B^*. Since $\overline{V}_B \leq V_{ESR}$ always holds, $\overline{V}_B^*$ has to be in the interval $[0, V_{ESR}]$. The maximum of $E(V)$ cannot lie within the interval $[0, \overline{V}_B]$ since this case would already imply bankruptcy. Consequently, it suffices that we regard the case $\overline{V}_B \leq V \leq V_{ESR}$. Following this line of argumentation we maximize the equity value given by

$$E(V) = V + B_1 V^{-x} + B_2 V^{-y} + \frac{\tau\gamma}{1-\tau}V - BC(V) - D(V). \tag{37}$$

Using the smooth-pasting condition (22) we obtain

$$\frac{dE(V)}{d(V)}|_{V=\overline{V}_B} = 0 = 1 - xB_1\overline{V}_B^{-x-1} - yB_2\overline{V}_B^{-y-1} + \frac{\tau\gamma}{1-\tau} + x - \frac{Cx}{r\overline{V}_B} \tag{38}$$

$$1 + \frac{\tau\gamma}{1-\tau} + x = xB_1\overline{V}_B^{-x-1} + yB_2\overline{V}_B^{-y-1} + \frac{Cx}{r\overline{V}_B}. \tag{39}$$

Equation (38) does not allow for an analytical closed form solution for the optimal endogenous bankruptcy trigger, V_B^*. Therefore, we numerically solve for V_B^*.

1.2.1. Heuristic Solution of the simple EBIT-model

Adjusting the boundary conditions and abandoning the continuous differentiability at V_{ESR} permits a closed form solution of the tax benefit function TB, i.e. using (29) – (31) as boundary conditions only. Since $A_1 = A_2 = 0$ is a general solution of the homogeneous ODE (1) we receive by simple rearrangements

$$B_1 = HV_B^{x+1}\frac{1+y}{x-y} \tag{40}$$

$$B_2 = -HV_B^{y+1}\frac{1+x}{x-y}, \tag{41}$$

where $H := \frac{\tau\gamma}{1-\tau}$ [4]. Computing the optimal endogenous bankruptcy trigger by maximizing the equity value yields

$$\frac{dE(V)}{d(V)}|_{V=V_B} = 0 = 1 + TB'(V) + x - \frac{Cx}{rV_B}. \tag{42}$$

Simple rearrangements provide[5]

$$V_B = \frac{Cx}{r(1+x)}. \tag{43}$$

As the following section shows the closed form solution of the endogenous bankruptcy trigger provides good approximation compared to the numerical result (38) in a real world setting[6]. Degenerated tax deductibility of interest payments to a certain portion γ or extremely high taxes ($\tau > 75\%$) distort the approximation due to the lag of τ and γ in the closed form solution (43). Moreover, we observe that the closed form solution differs only in $\tau\frac{Cx}{r(1+x)}$ compared to Leland's solution.

Appendix A

Considering the following system of equations

$$A_2V_B^{-y} = B_1V_B^{-x} + B_2V_B^{-y} + HV_B \tag{44}$$

$$-yA_2V_B^{-y-1} = -xB_1V_B^{-x-1} - yB_2V_B^{-y-1} + H \tag{45}$$

$$B_1V_T^{-x} + B_2V_T^{-y} + H(V_T) = C_1V_T^{-x} + \frac{\tau C}{r} \tag{46}$$

$$-xB_1V_T^{-x-1} - yB_2V_T^{-y-1} + H = -xC_1V_T^{-x-1}, \tag{47}$$

with $H := \frac{\tau\gamma}{1-\tau}$. Rearranging (44) and (45) yield

$$A_2 - B_2 = B_1V_B^{y-x} + HV_B^{y+1} \tag{48}$$

$$A_2 - B_2 = \frac{x}{y}B_1V_B^{y-x} - \frac{1}{y}HV_B^{y+1} \tag{49}$$

4 Due to readability and simplicity the derivation of the parameters can be found in the appendix
5 A detailed derivation can be found in the appendix.
6 i.e. there are no degenerated taxes, interests and so on

By equalizing (48) and (49) we receive

$$B_1 V_B^{y-x} + HV_B^{y+1} = \frac{x}{y} B_1 V_B^{y-x} - \frac{1}{y} HV_B^{y+1} \tag{50}$$

$$B_1 V_B^{y-x} - \frac{x}{y} B_1 V_B^{y-x} = -\frac{1}{y} HV_B^{y+1} - HV_B^{y+1} \tag{51}$$

$$B_1 V_B^{y-x} [1 - \frac{x}{y}] = -HV_B^{y+1} [1 + \frac{1}{y}] \tag{52}$$

$$B_1 [\frac{y-x}{y}] = -HV_B^{x+1} [\frac{y+1}{y}] \tag{53}$$

$$B_1 = HV_B^{x+1} [\frac{y+1}{x-y}] \tag{54}$$

Utilizing the same procedure for (46) and (47) yields

$$B_1 - C_1 = V_T^x \frac{\tau C}{r} - B_2 V_T^{x-y} - HV_T^{x+1} \tag{55}$$

$$B_1 - C_1 = \frac{-y}{x} B_2 V_T^{x-y} + \frac{H}{x} V_T^{x+1} \tag{56}$$

By equalizing (48) and (49) we receive

$$V_T^x [\frac{\tau C}{r} - B_2 V_T^{-y} - HV_T] = \frac{-y}{x} B_2 V_T^{x-y} + \frac{H}{x} V_T^{x+1} \tag{57}$$

$$\frac{\tau C}{r} - B_2 V_T^{-y} - HV_T = \frac{-y}{x} B_2 V_T^{-y} + \frac{H}{x} V_T \tag{58}$$

$$B_2 V_T^{-y} [\frac{y}{x} - 1] = HV_T [1 + \frac{1}{x}] - \frac{\tau C}{r} \tag{59}$$

$$B_2 = HV_T^{y+1} [\frac{x+1}{y-x}] - \frac{\tau C x}{r(y-x)} V_T^y. \tag{60}$$

Moreover we obtain from (44) and (47)

$$A_2 = B_1 V_B^{y-x} + HV_B^{y+1} + B_2 \tag{61}$$

$$C_1 = B_1 + \frac{y}{x} B_2 V_T^{x-y} - HV_T^{x+1}. \tag{62}$$

Appendix B

Considering the following system of equations with $H := \frac{\tau\gamma}{1-\tau}$

$$0 = B_1 V_B^{-x} + B_2 V_B^{-y} + HV_B \tag{63}$$

$$0 = -xB_1 V_B^{-x-1} - yB_2 V_B^{-y-1} + H \tag{64}$$

$$B_1 V_T^{-x} + B_2 V_T^{-y} + b_0(V_T) = C_1 V_T^{-x} + \frac{\tau C}{r} \tag{65}$$

Rearranging (63) yields

$$B_1 = -B_2 V_B^{x-y} - HV_B^{x+1} \tag{66}$$

By inserting (66) in (64) we receive

$$0 = -xV_B^{-x-1}[-B_2V_B^{x-y} - HV_B^{x+1}] - yB_2V_B^{-y-1} + H \tag{67}$$

$$B_2 = -HV_B^{y+1}\frac{1+x}{x-y} \tag{68}$$

Finally, we can determine B_1

$$B_1 = HV_B^{x+1}(\frac{1+y}{x-y}). \tag{69}$$

Appendix C

First, we recapitulate the required functions for determining the optimal endogenous bankruptcy trigger V_B

$$D(V) = \frac{C}{r} + \left((1-\beta)V_B - \frac{C}{r}\right)\left(\frac{V}{V_B}\right)^{-x} \tag{70}$$

$$BC(V) = \beta V_B\left(\frac{V}{V_B}\right)^{-x} \tag{71}$$

$$\tag{72}$$

Deviating the sum of the market value of debt and the bankruptcy costs at the point V_B yields

$$D'(V) + BC'(V)|_{V=V_B} = -x + \frac{Cx}{rV_B}. \tag{73}$$

We conclude by deviating the equity value at V_B and solving the equation for V_B:

$$\frac{dE(V)}{d(V)}|_{V=V_B} = 0 = 1 - xB_1V_B^{-x-1} + -yB_2V_B^{-y-1} + H + x - \frac{Cx}{rV_B} \tag{74}$$

$$-1 - H - x = -xV_B^{-x-1}[HV_B^{x+1}\frac{1+y}{x-y}] - yV_B^{-y-1}[-HV_B^{y+1}\frac{1+x}{x-y}] - \frac{Cx}{rV_B} \tag{75}$$

$$1 + H + x = H[\frac{x(1+y) - y(1+x)}{x-y}] + \frac{Cx}{rV_B} \tag{76}$$

$$1 + H + x = H + \frac{Cx}{rV_B} \tag{77}$$

$$V_B = \frac{Cx}{r(1+x)} \tag{78}$$

2. Appendix D

Let us consider the tax shield value for SEM in the TSWB case. We regard the typical structural assumptions:

$$\text{at } V \leq V_B: \qquad TB(V) = -\delta\tau V, \tag{79}$$

$$\text{as } V \to \infty: \qquad TB(V) = \frac{\tau C}{r}. \tag{80}$$

We have to solve the following equation system:

$$A_0 = -\frac{\delta\tau}{r-\mu}V \tag{81}$$

$$B_0 + B_1 V^{-x} = \frac{\tau C}{r}. \tag{82}$$

Since $\lim_{V\to\infty} V^{-x} = 0$ we derive $B_0 = \frac{\tau C}{r}$. Since the two ODE functions meet at the point V_B the constraint

$$-\frac{\delta\tau}{r-\mu}V_B = \frac{\tau C}{r} + B_1 V_B^{-x} \tag{83}$$

leads to the determination of B_1.

3. Appendix E

Regarding the system of equations

$$-GV_B = HV_B + B_1 V_B^{-x} + B_2 V_B^{-y} \tag{84}$$

$$HV_{ESR} + B_1 V_{ESR}^{-x} + B_2 V_{ESR}^{-y} = \frac{\tau C}{r} + C_1 V_{ESR}^{-x} \tag{85}$$

$$-xB_1 V_{ESR}^{-x-1} - yB_2 V_{ESR}^{-y-1} = -xC_1 V_{ESR}^{-x-1}. \tag{86}$$

we obtain by rearranging (84) and inserting (87) into (85)

$$(-G-H)V_B = B_1 V_B^{-x} + B_2 V_B^{-y} \tag{87}$$

$$C_1 = V_{ESR}^{-x}[HV_{ESR} - \frac{\tau C}{r} - V_B(G+H)]. \tag{88}$$

Rearranging (84) again yields

$$B_1 = (-G-H)V_B^{x+1} - B_2 V_B^{-y+x}. \tag{89}$$

Rearranging (86)

$$B_2 = \frac{y}{x}[(C_1 - B_1)V_{ESR}^{y-x}] \tag{90}$$

and inserting (89) and (88) into (90) yields

$$B_2 = \frac{y}{x}[(V_{ESR}^{-x}[HV_{ESR} - \frac{\tau C}{r} - V_B(G+H)] + (G+H)V_B^{x+1} + B_2 V_B^{-y+x})V_{ESR}^{y-x}] \tag{91}$$

$$B_2[1 - \frac{y}{x}V_B^{-y+x}V_{ESR}^{y-x}] = \frac{y}{x}V_{ESR}^{y-x}\left(V_{ESR}^{-x}[HV_{ESR} - \frac{\tau C}{r} - V_B(G+H)] + (G+H)V_B^{x+1}]\right) \tag{92}$$

$$B_2 = \frac{y}{x}V_{ESR}^{y-x}\left(V_{ESR}^{-x}[HV_{ESR} - \frac{\tau C}{r} - V_B(G+H)] + (G+H)V_B^{x+1}]\right)\cdot\left([1 - \frac{y}{x}V_B^{-y+x}V_{ESR}^{y-x}]\right)^{-1} \tag{93}$$

$$B_2 = yV_{ESR}^{y-x}\left(C_1 + (G+H)V_B^{x+1}]\right)\cdot\left([x - yV_B^{-y+x}V_{ESR}^{y-x}]\right)^{-1} \tag{94}$$

$$B_2 = \frac{yV_{ESR}^{y-x}\left(C_1 + (G+H)V_B^{x+1}]\right)}{[x - yV_B^{-y+x}V_{ESR}^{y-x}]} \tag{95}$$