Illiquidity and Overindebtedness
–
Optimal Capital Structure under Realistic Default Triggers in a Double Barrier Option Framework

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Brownian Motion (BM) determines the nature of uncertainty

There is a filtered probability space $(\Omega, \mathcal{F}, \mathbb{P}, (\mathcal{F}_t)_{t\geq 0})$. We denote the available information at time t with $t \in [0, \infty)$ by the filtration $\mathcal{F}_t \subset \mathcal{F}_s$ with $0 \leq t < s$ where $\mathcal{F}_t$ describes the augmented σ-algebra generated by the Brownian Motion W_t, $t \geq 0$.

Stochastic process

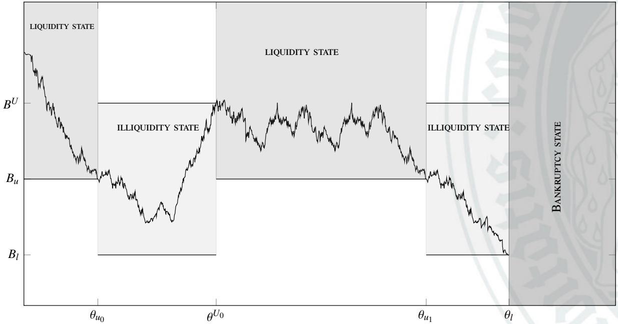
We consider a stochastic process $(R_t)_{t\in[0,\infty)}$, e.g. a revenue process characterized by the following stochastic differential equation (SDE)

$$dR_t = \mu R_t dt + \sigma R_t dW_t \tag{1}$$

where $\mu \in \mathbb{R}$ is the (constant) growth rate, $\sigma \in \mathbb{R}_0^+$ is the corresponding (constant) volatility. The initial value R_0 needs to be positive, i.e. $R_0 \in \mathbb{R}^+$.

The access to our general barrier model is intuitive

Figure: *Introduction to the General Model*



This figure depicts a stochastic process that starts in the liquidity state (LS). The process runs into illiquidity state (IS) at the moment θ_{u_0} when the lower-upper barrier B_u is hit. Continuing in IS, the process reenters LS in θ^{U_0} by hitting the upper-upper boundary B^U. In θ_{u_1} the process touches the lower-upper boundary B_u again and falls back into IS. Finally, the process is killed in θ_l, i.e. the process runs into bankruptcy state (BS) and hits thus the lower barrier B_l.

The hitting times are formally known as stopping times

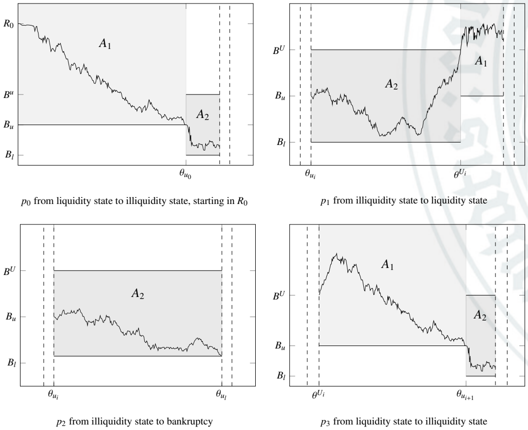
Definition (1 - Hitting Times)

Given three boundary constraints B_l, B_u, B^U with $B_l \leq B_u < B^U$, the corresponding **hitting times** are defined as follows for $i \in \mathbb{N}_0$:

$$
\begin{aligned}
\theta_l &:= \inf\{t \geq 0 \,|\, R_t = B_l\} \\
\theta_{u_0} &:= \inf\{t \geq 0 \,|\, R_t = B_u\} \\
\theta^{U_0} &:= \inf\{t \geq \theta_{u_0} \,|\, R_t = B^U \,\wedge\, R_s > B_l \text{ for all } s \in [\theta_{u_0}, t]\} \\
\theta_{u_1} &:= \inf\{t \geq \theta^{U_0} \,|\, R_t = B_u \,\wedge\, R_s > B_l \text{ for all } s \in [\theta^{U_0}, t]\} \\
&\dots \\
\theta_{u_i} &:= \inf\{t \geq \theta^{U_{i-1}} \,|\, R_t = B_u \,\wedge\, R_s > B_l \text{ for all } s \in [\theta^{U_{i-1}}, t]\} \\
\theta^{U_i} &:= \inf\{t \geq \theta_{u_i} \,|\, R_t = B^U \,\wedge\, R_s > B_l \text{ for all } s \in [\theta_{u_i}, t]\}.
\end{aligned}
$$

For all states of the model we can derive hitting times and state prices

Figure: *State Prices p_0, p_1, p_2, and p_3 in the General Model*



p_0 from liquidity state to illiquidity state, starting in R_0

p_1 from illiquidity state to liquidity state

p_2 from illiquidity state to bankruptcy

p_3 from liquidity state to illiquidity state

The formal definition of the state prices is now possible

Definition (2 - State Prices $p_0, ..., p_3$)

$\boldsymbol{p_0}$ is the price of a knock out barrier option that pays 1 \$ in θ_{u_0} starting in $t = 0$ (with the corresponding ordinate value R_0) when the stochastic process $(R_t)_{t\in[0,\infty)}$ **hits the lower-upper barrier** B_u, i.e. p_0 represents the **discounted probability of hitting** B_u **in** θ_{u_0}.

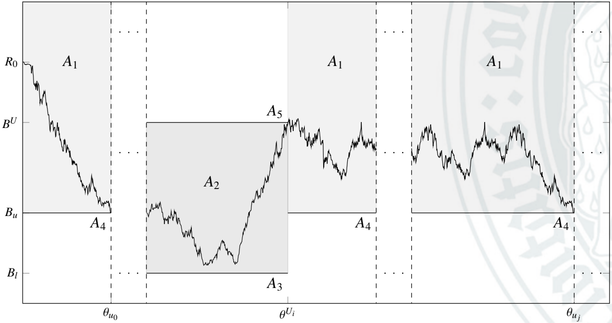
Analogously, $\boldsymbol{p_1}$ is the price of 1 \$ in θ^{U_i} starting in θ_{u_i} for all $i \in \mathbb{N}_0$ (with the corresponding ordinate value B_u) when the stochastic process $(R_t)_{t\in[0,\infty)}$ **hits the upper-upper barrier** B^U **without hitting the lower barrier** B_l.

$\boldsymbol{p_2}$ is the price of 1 \$ in θ_l starting in θ_{u_i} for all $i \in \mathbb{N}_0$ (with the corresponding ordinate value B_u) when the stochastic process $(R_t)_{t\in[0,\infty)}$ **hits the lower barrier** B_l **without hitting the upper-upper barrier** B^U.

Finally, $\boldsymbol{p_3}$ is the price of a knock out barrier option that pays 1 \$ in $\theta_{u_{i+1}}$ starting in θ^{U_i} for all $i \in \mathbb{N}_0$ (with the corresponding ordinate value B^U) when the stochastic process $(R_t)_{t\in[0,\infty)}$ **hits the lower-upper barrier** B_u.

The model allows for payoffs in any state or at any hitting time

Figure: *General Payoff Structure of a Stochastic Process*



The figure depicts a general payoff structure that can be generated in a double barrier framework with liquidity state (LS) , illiquidity state (IS) and bankruptcy state (BS). If the underlying process is in LS the payoff equals A_1. In case of IS the generated payoff is A_2.Hitting the lower boundary B_l the payoff accords with A_3. The same holds for the lower-upper barrier B_u and the payoff A_4 and the upper-upper barrier B^U with the payoff A_5, respectively.

Next we derive the expected values of the payoffs we introduced, and we start with A_1

$$
\begin{aligned}
\mathbb{E}[A_1] = A_1[(1-p_0)+ & \quad \text{, value until the first liquidity crisis } \theta_{u_0} \\
p_0 p_1 (1-p_3)+ & \quad \text{, value after leaving first IS } \theta^{U_0} \text{ and until } \theta_{u_1} \\
p_0 p_1 p_3 p_1 (1-p_3)+ & \quad \text{, value after } \theta^{U_1} \text{ and until } \theta_{u_2} \\
...]
\end{aligned}
$$

$$
\begin{aligned}
&= A_1[(1-p_0) + p_0 p_1 (1-p_3) \sum_{i=0}^{\infty} p_1^i p_3^i] \\
&= A_1[(1-p_0) + \frac{p_0 p_1 (1-p_3)}{1-p_1 p_3}] \\
&= pr_{A_1}^0 A_1,
\end{aligned}
$$

where $pr_{A_1}^0 := (1-p_0) + \frac{p_0 p_1 (1-p_3)}{1-p_3 p_1}$ denotes the state price of the payoff A_1 starting in $t = 0$.

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Our IO-model starts from the revenue process as we expect better empirical evidence

- We consider in an arbitrage-free market a firm whose instantaneous revenues $(R_t)_{t\in[0,\infty)}$ follow a gBm under the risk-neutral probability measure $\mathbb{Q}$:

$$dR_t = \mu R_t dt + \sigma R_t dW_t^{\mathbb{Q}}, \tag{2}$$

  where μ is the revenue's growth rate, σ is the corresponding volatility, W_t is a standard Bm under $\mathbb{Q}$, and the initial value of revenue is $R_0 > 0$
- Firm faces variable costs captured by a deterministic ratio of revenues γ and deterministic fixed costs F independent of revenues, thus:

$$EBIT_t := R_t(1-\gamma) - F \quad \forall t \in [0,\infty). \tag{3}$$

- Constant risk free rate is captured by r and the corporate tax rate τ is assumed to be flat (no personal taxes considered so far)

Our main technical contribution: Combining illiquidity and overindebtedness

Assumption (3)

The revenue process $(R_t)_{t\in[0,\infty)}$ starts in liquidity state (LS) at R_0 s.t. $R_0 > B_u$. When R_t hits B_u for some $t \in [0,\infty)$ the firm switches into illiquidity state (IS), and R_s continues facing the two boundaries B^U and B_l for some $t < s$. The firm reenters LS iff R_s hits B^U before it hits B_l for $t < s$. The number of switching events between LS and IS is not restricted. Given the firm stays in IS, the bankruptcy state BS is triggered iff R_s hits B_l before it hits B^U for $t < s$. The firm runs into bankruptcy iff $R_s = B_l$ for $t < s$ and the stochastic process stops.

Lemma (4)

The firm will enter illiquidity state (IS) if $EBIT(1-\tau) \leq \delta C$, which corresponds to $R_t \leq B_u$ where $B_u = (\delta C + F(1-\tau))\,/\,((1-\gamma)(1-\tau))$.

Default expenses occur once the IS is reached

Assumption (5)

When the firm enters IS, certain default expenses occur, e.g. due to customers that stop buying the firms' products, which we assume to be a proportion ϵ *of* $\mathbb{E}[V_{\theta_{u_i}} | \mathcal{F}_{\theta_{u_i}}]$*. Moreover, as long as the firm remains in IS (*$B_l < R_t < B^U$ *with* $t \geq \theta_{u_i}$*) the debtholders demand penalty interest payments* C_{il} *with* $C_{il} > C$*. Consequently,* $B_u < B^U$*. If the firm reenters LS, the penalty interest payments will stop and the regular coupon payment* C *will be enforced.*

Lemma (6)

The firm will reenter liquidity state LS if $EBIT(1-\tau) = \delta C_{il}$ *with* $t \geq \theta_{u_i}$*, which corresponds to* $R_t = B^U$ *where* $B^U = (\delta C_{il} + F(1-\tau)) / ((1-\gamma)(1-\tau))$ *with* $t \geq \theta_{u_i}$*.*

Lemma (7)

The firm will file for bankruptcy if $\mathbb{E}[V_t] = V_B$ *with* $t \geq \theta_{u_i}$*, which corresponds to* $R_t = B_l$ *where* $B_l = \left(\left(V_B + \frac{F(1-\tau)}{r}\right)(r-\mu)\right) / ((1-\gamma)(1-\tau))$ *with* $t \geq \theta_{u_i}$*.*

The single-barrier state prices in the IO-model are straightforward

- Recap: p_0 and p_3 can be seen as assets, or more specifically as perpetual, down-and-in, cash-at-hit-or-nothing, single-barrier options which pay \$1 when the stochastic process R_t hits the barrier B_u which is below the initial value of the stochastic process
- Both options only differ with respect to its initial values which are R_0 and $R_{\theta^U} = B^U$, respectively
- Pricing formulae for such options is well known (Rubinstein and Reiner, 1991):[1]

$$p_0 = \left(\frac{B_u}{R_0}\right)^y \tag{4}$$

and analogously to

$$p_3 = \left(\frac{B_u}{B^U}\right)^y, \tag{5}$$

where

$$a := \mu - \frac{1}{2}\sigma^2, \qquad b := \sqrt{a^2 + 2\sigma^2 \cdot r}, \qquad y := \frac{a+b}{\sigma^2}. \tag{6}$$

The double-barrier state prices are less trivial

- Pelsser (2000) provides a pricing formula for both structures in finite time which can be easily extended to a perpetual setting and applied to our specific problem:

$$p_1 = \exp\left\{\frac{a(l-x)}{\sigma^2}\right\}\frac{\sinh(\frac{b}{\sigma^2}x)}{\sinh(\frac{b}{\sigma^2}l)} \tag{7}$$

and analogously

$$p_2 = \exp\left\{\frac{-ax}{\sigma^2}\right\}\frac{\sinh(\frac{b}{\sigma^2}(l-x))}{\sinh(\frac{b}{\sigma^2}l)}, \tag{8}$$

where

$$x := \log\left(\frac{B_u}{B_l}\right) := \log\left(\frac{\delta C + F(1-\tau)}{V_B + \frac{F(1-\tau)}{r}(r-\mu)}\right), \tag{9}$$

$$l := \log\left(\frac{B^U}{B_l}\right) := \log\left(\frac{\delta C_{il} + F(1-\tau)}{V_B + \frac{F(1-\tau)}{r}(r-\mu)}\right), \tag{10}$$

and a as well as b are as defined in (6). Please note that x and l are functions of V_B.

Now we have all prerequisites to derive the firm's contingent claims (1/3)

1. Debt Value $D(V, C, C_{il}) = D(V)$

- In our setting: Debt promises a perpetual coupon payment C whose level remains constant unless the firm enters IS
- In LS: Debt value equals $\frac{C}{r}$ (c.f. A_1) as long as B_u is not hit
- In IS: Debt value equals $\frac{C_{il}}{r}$ (c.f. A_2) as long as neither B^U nor B_l is hit
- If bankruptcy occurs (B_l is hit), a fraction $0 \leq \alpha \leq 1$ of value will be lost to bankruptcy costs, thus, debt value equals $(1-\alpha)V_B$ (c.f. A_3)

$$\vec{D}^{\intercal} = \begin{pmatrix} \frac{C}{r} & \frac{C_{il}}{r} & (1-\alpha)V_B & 0 & 0 \end{pmatrix}. \tag{11}$$

- To obtain the expected debt value $\mathbb{E}[DV(V)]$ we need to multiply the payoff vector $\vec{D}$ with the state price vector $\vec{pr}_0$:

$$D(V) := \mathbb{E}[DV(V)] = \vec{D}^{\intercal}\vec{pr}_0. \tag{12}$$

Now we have all prerequisites to derive the firm's contingent claims (2/3)

2. Tax Benefits $TB(V)$

$$\vec{TB}^{\top} = \begin{pmatrix} \frac{\tau C}{r} & \frac{\tau C_{il}}{r} & 0 & 0 & 0 \end{pmatrix}. \tag{13}$$

Thus, we have:

$$TB(V) = \vec{TB}^{\top} \vec{pr}_0. \tag{14}$$

3. Bankruptcy Costs $BC(V)$

- Unlevered firm value at θ_l is represented by $V_B = \frac{B_l(1-\gamma)(1-\tau)}{r-\mu} - \frac{F(1-\tau)}{r}$ and αV_B reflects the bankruptcy costs if bankruptcy is triggered (A_3)
- In no other states bankruptcy costs occur leaving us with

$$\vec{BC}^{\top} = \begin{pmatrix} 0 & 0 & \alpha V_B & 0 & 0 \end{pmatrix}. \tag{15}$$

- In vectorial writing

$$BC(V) = \vec{BC}^{\top} \vec{pr}_0 \tag{16}$$

Now we have all prerequisites to derive the firm's contingent claims (3/3)

4. Illiquidity Expenses $IE(V)$

- Illiquidity expenses IE may occur whenever the firm enters IS (A_4)
- Two key reasons: Direct costs of lawyers, banking fees and so on and on the other hand indirect costs, such as loss of investors' or customers' confidence
- This will be priced with a fee in portion ϵ to the then prevailing unlevered firm value $\mathbb{E}[V_{\theta_{u_i}}]$
- Thus, we have the following payoff structure for IE:

$$\vec{IE}^{\top} = \begin{pmatrix} 0 & 0 & 0 & \varepsilon \cdot \mathbb{E}[V_{\theta_{u_i}}] & 0 \end{pmatrix}. \tag{17}$$

- Again, multiplication with the state price vector yields the value of the illiquidity expenses $IE(V)$

$$IE(V) = \vec{IE}^{\top} \vec{pr}_0. \tag{18}$$

The net benefit $NB(V)$ and the levered firm value $V^L(V)$ are a direct result of the previous components

- Total firm value $V^L(V)$ is the sum of the previous terms: the firms' asset value (V), less the bankruptcy costs $(BC(V))$ and illiquidity expenses $(IE(V))$, plus value of tax benefits $(TB(V))$

$$\vec{NB} := \vec{TB} - \vec{IE} - \vec{BC} = \begin{pmatrix} \frac{\tau C}{r} \\ \frac{\tau C_{il}}{r} \\ 0 \\ 0 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 0 \\ \varepsilon \cdot \mathbb{E}[V_{\theta_{u_i}}] \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ \alpha V_B \\ 0 \\ 0 \end{pmatrix} \tag{19}$$

$$= \begin{pmatrix} \frac{\tau C}{r} & \frac{\tau C_{il}}{r} & -\alpha V_B & -\varepsilon \mathbb{E}[V_{\theta_{u_i}}] & 0 \end{pmatrix}^{\intercal}. \tag{20}$$

- Taking the conditional expected value V into consideration we have the following total firm value:

$$V^L(V) = V + \vec{NB}^{\intercal} \vec{pr}_0. \tag{21}$$

- For the value of equity we have

$$EV(V) = V + \vec{NB}^{\intercal} \vec{pr}_0 - \vec{D}^{\intercal} \vec{pr}_0. \tag{22}$$

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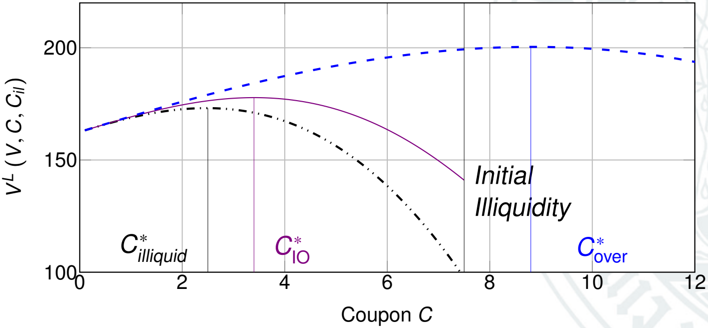
The optimal capital structure depends on the endogenous coupon payments C and the optimal bankruptcy level V_B

- In general, we are concerned with maximizing the levered firm value with respect to C subject to certain constraints
- The classic constraint introduced by Leland (1994) is that equityholders choose V_B, the asset value where the firm files for bankruptcy, in order to maximize the equity value
- We denote this optimal level of bankruptcy asset value with V_B^*
- An additional constraint in our setting is that C_{il} needs to reflect a certain risk spread $\varphi \geq r$. Thus, our optimization problem can be formally stated as follows:

$$
\begin{aligned}
V^L(V, C, C_{il}) &\rightarrow \max \\
\text{s.t. } \frac{\partial EV(V, C, C_{il})}{\partial V_B} &= 0 \\
C_{il} - \varphi DV(V, C, C_{il}) &= 0.
\end{aligned}
\tag{23}
$$

We compare our model to the classic Leland and Couch model

Figure: *Optimal Capital Structure under the IO-Model, pure Illiquidity Model and pure Overindebtedness Model*



This figure analyzes the firm value ($V^L(V)$)-maximizing choice of coupon payments C for the IO-model in comparison to the classic models of illiquidity and overindebtedness. The blue, dashed line represents $V^L(V)$ for different C with overindebtedness as a bankruptcy trigger. The violet, solid line represents the IO-model and the black, dashed-dotted line depicts the case of illiquidity. The chosen model parameters are as follows: $r = 0.05$, $\tau = 0.35$, $R_0 = 25$, $\mu = 0.02$, $\sigma = 0.20$, $\gamma = 0.70$, $F = 0$, $\delta = 1 - \tau$, $\epsilon = 0.00$, and $\varphi = 2.5$.

The IO-model outperforms the other two models significantly

Table: *Optimal Capital Structure Estimates versus Observed Leverage for NAICS Sectors*

	Observed		Illiquidity		IO-Model		Overindebtedness	
NAICS Sector	***L***	**1 Std. Err.**	***L****	**Abs. Dev.**	***L****	**Abs. Dev.**	***L****	**Abs. Dev.**
Accommodation and Food Services	0.4594	0.0752	0.1861	0.2734	0.3096**	0.1498	0.7942	0.3348
Administrative, Support, Waste, Remediation	0.1994	0.0380	0.1599	0.0394	0.2985*	0.0991	0.8064	0.6071
Construction	0.4405	0.0376	0.1442	0.2962	0.3658**	0.0747	0.6620	0.2215
Health Care and Social Assistance	0.5497	0.0440	0.0662	0.4835	0.4627**	0.0871	0.6554	0.1057
Information	0.2927	0.0160	0.0316	0.2611	0.2236	0.0691	0.8629	0.5703
Manufacturing	0.3071	0.0096	0.0472	0.2599	0.3252**	0.0181	0.6746	0.3675
Mining, Quarrying, and Oil and Gas Extraction	0.2535	0.0142	0.0125	0.2410	0.2450***	0.0085	0.6359	0.3824
Professional, Scientific, and Technical Services	0.2225	0.0127	0.0974	0.1251	0.2227***	0.0003	0.6897	0.4672
Real Estate and Rental and Leasing	0.5412	0.0268	0.0149	0.5263	0.1584	0.3828	0.7198	0.1786
Retail Trade	0.2714	0.0170	0.0412	0.2302	0.3017**	0.0303	0.6859	0.4145
Transportation and Warehousing	0.4286	0.0252	0.0452	0.3834	0.2446	0.1840	0.7138	0.2852
Utilities	0.4531	0.0259	0.0104	0.4427	0.4583***	0.0051	0.6323	0.1791
Wholesale Trade	0.2923	0.0337	0.0459	0.2464	0.1329	0.1594	0.6626	0.3703
Others	0.2683	0.0242	0.0936	0.1747	0.1689	0.0994	0.6880	0.4197

This table summarizes the optimal leverage ratios $L^* = D(V)/V^{L,*}(V)$ generated by the IO-model, and for a pure illiquidity or overindebtedness trigger. The results are compared to the observed average leverage L for all NAICS sectors. The absolute deviation towards the observed leverage is depicted for each of the three models (Abs. Dev.).
$***$ = *Within 1 standard error;* $**$ = *Within 2 standard errors;* $*$ = *Within 3 standard errors*

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To sum up the presented paper...

- First dynamic corporate valuation model incorporating an illiquidity and a bankruptcy trigger in a double barrier framework
- The introduction of the general model is carefully developed towards definitions of state prices and payoff structures
- Application of the general model to corporate valuation and the problem of optimal capital structure
- Comparison of our solution to the two classic cases of only considering one of the two boundaries
- Our results lie in-between and explain observed capital structure choices much better than the existing models as we demonstrate by an empirical study of the US market
- General model applicable in other research areas (e.g. macroeconomics, biology)
  - **Thank you very much for your attention! If you have feedback or further questions, please let us know.**

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Research in dynamic corporate finance attempts to explain the optimal choice of leverage

The choice of capital structure is an important driver of firm values

- Starting with Brennan and Schwartz (1984) and brought to wider attention by Leland (1994) a whole strand of literature emerged dealing with optimal capital structure models in a stochastic framework
- Before Leland (1994): Exogenous default trigger and static debt
- Leland (1994) and thereafter: Endogenous default trigger and static debt
- Goldstein, Ju, and Leland (2001): Endogenous default trigger and dynamic adjustments of debt (but for the special case of constant leverage)
- Many strong contributions followed adding to capital structure theory, e.g.: Hennessy and Whited (2005) (Debt dynamics); Strebulaev (2007) (Tests of capital structure models); Titman and Tsyplakov (2007) (Optimal capital structure with dynamic debt and investment choices); Danis, Rettl, and Whited (2014) (Refinancing and capital structure)
- **However, empirical evidence with respect to explaining observed capital structures remained weak**[2]

DISCUSSION

Some open issues are already known to us

Open issues to be addressed:

- Validate basic setup of process, debt policy, barriers and payoffs in the IO-model:
    - Stochastic/deterministic jump at illiquidity?
    - Incorporation of dynamic debt policies?
    - Is there real evidence for illiquidity expenses etc.?
- Extend empirical analysis:
    - Switch to SIC codes
    - Understand and implement refinancing logic
    - Compare results also to Titman, Tsyplakov (2007) and Strebulaev (2007)
- Rework paper, build appendices/proofs

APPENDIX

H_0: Including both default constraints explains observed capital structures significantly better

Trigger discussion

- Trigger 1 - Indebtedness:
  - Certain market value of assets where equityholders choose to file for bankruptcy
  - Barrier level is determined endogenously by maximizing equity value
  - Literature: Leland (1994), Goldstein et al. (2001)
  - Implicit assumptions: equityholders with "deep pockets"; no default mechanism for debtholders
- Trigger 2 - Illiquidity/ breach of covenant:
  - Firm defaults if cash flow is not sufficient to cover cash obligation or to fulfil covenant
  - Barrier level is determined exogenously either by covenant or cash obligation
  - Literature: Kim et al. (1993), Couch et al. (2012)
  - Implicit assumptions: no "deep pockets"; no debt restructuring or covenant tolerance

State prices $pr^0_{A_2}, ..., pr^0_{A_5}$

State price of the payoff A_2 in $t = 0$: $pr^0_{A_2} := \frac{p_0(1-p_1-p_2)}{1-p_1 p_3}$

State price of the payoff A_3 in $t = 0$: $pr^0_{A_3} := \frac{p_0 p_2}{1-p_1 p_3}$

State price of the payoff A_4 in $t = 0$: $pr^0_{A_4} := \frac{p_0}{1-p_1 p_3}$

State price of the payoff A_5 in $t = 0$: $pr^0_{A_5} := \frac{p_0 p_1}{1-p_1 p_3}$

The vectorial calculus

Next to a better readability this brings the advantage that we can compress our notation to a minimum. Therefore, let $\vec{PO}$ denote the general payoff structure and $\vec{pr}_0$ the according state prices starting in $t = 0$. The first row represents the payoff A_1 and the state price $pr^0_{A_1}$, respectively. In conclusion, we have

$$\vec{PO} := \begin{pmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{pmatrix} \qquad \vec{pr}_0 := \begin{pmatrix} pr^0_{A_1} \\ pr^0_{A_2} \\ pr^0_{A_3} \\ pr^0_{A_4} \\ pr^0_{A_5} \end{pmatrix} = \begin{pmatrix} (1 - p_0) + \frac{p_0 p_1 (1-p_3)}{1-p_1 p_3} \\ \frac{p_0(1-p_1-p_2)}{1-p_1 p_3} \\ \frac{p_0 p_2}{1-p_1 p_3} \\ \frac{p_0}{1-p_1 p_3} \\ \frac{p_0 p_1}{1-p_1 p_3} \end{pmatrix}. \tag{24}$$

APPENDIX

We base the covenant/illiquidity boundary on the interest coverage ratio

Lemma (1)

The firm will enter illiquidity state (IS) if $EBIT(1-\tau) \leq \delta C$*, which corresponds to* $R_t \leq B_u$ *where* $B_u = (\delta C + F(1-\tau)) \,/\, ((1-\gamma)(1-\tau))$*.*

Proof.

We substitute Equation (3) *into the covenant definition from above and rearrange for* R_t*:*

$$\begin{aligned} EBIT(1-\tau) &= \delta C \\ \left(R_t(1-\gamma) - F\right)(1-\tau) &= \delta C \\ R_t &= \frac{\delta C + F(1-\tau)}{(1-\gamma)(1-\tau)}. \end{aligned}$$

Since the covenant definition $(1-\tau)EBIT_t = \delta C$ *corresponds to* $R_t = B_u$*, we have:*

$$B_u := \frac{\delta C + F(1-\tau)}{(1-\gamma)(1-\tau)} \qquad \square$$

Assumption 2.2 allows us to derive B^U explicitly

Lemma (2)

The firm will reenter liquidity state LS if $EBIT(1-\tau) = \delta C_{il}$ *with* $t \geq \theta_{u_i}$*, which corresponds to* $R_t = B^U$ *where* $B^U = (\delta C_{il} + F(1-\tau)) \,/\, ((1-\gamma)(1-\tau))$ *with* $t \geq \theta_{u_i}$*.*

Proof.

We substitute Equation (3) *into the adjusted covenant definition from above and rearrange for* R_t*:*

$$
\begin{aligned}
EBIT(1-\tau) &= \delta C_{il} \\
(R_t(1-\gamma) - F)\,(1-\tau) &= \delta C_{il} \\
R_t &= \frac{\delta C_{il} + F(1-\tau)}{(1-\gamma)(1-\tau)}.
\end{aligned}
$$

Since the covenant definition $EBIT(1-\tau) = \delta C_{il}$ *corresponds to* $R_t = B^U$*, we have:*

$$B^U := \frac{\delta C_{il} + F(1-\tau)}{(1-\gamma)(1-\tau)}. \qquad \square$$

APPENDIX

The bankruptcy trigger of our model is standard but needs adjustment for the revenue process

Lemma (3)

The firm will file for bankruptcy if $\mathbb{E}[V_t] = V_B$ *with* $t \geq \theta_{u_i}$*, which corresponds to* $R_t = B_l$ *where* $B_l = \left(\left(V_B + \frac{F(1-\tau)}{r}\right)(r-\mu)\right) / \left((1-\gamma)(1-\tau)\right)$ *with* $t \geq \theta_{u_i}$*.*

Proof.

*We substitute equation (**??**) into the bankruptcy trigger definition from above and rearrange for* R_t*:*

$$\mathbb{E}[V_t \,|\, \mathcal{F}_t] = V_B$$

$$\frac{R_t(1-\gamma)(1-\tau)}{r-\mu} - \frac{F(1-\tau)}{r} = V_B.$$

Since the bankruptcy definition $\mathbb{E}[V_t] = V_B$ *corresponds to* $R_t = B_l$*, we have:*

$$B_l := \frac{\left(V_B + \frac{F(1-\tau)}{r}\right)(r-\mu)}{(1-\gamma)(1-\tau)}$$

□

All other parameters are exogenous and need to be estimated (1/2)

Table: *Exogenous Parameters of the IO-Model*

Para-meter	**Description**	**Rationale**	**Exemplary reasonable values**
r	risk free rate	Average of 10-year Treasury rate (1/1989-7/2016) Approach similar to Leland (2004), Huang and Huang (2012)	0.05
τ	corporate tax rate	Federal corporate income tax rate in the US for bigger companies Approach similar to similar to Leland and Toft (1996), Strebulaev (2007)	0.35
R_0	initial value of the revenue process	Firm individual observable parameter	\$25 bn
μ	risk-neutral drift of the revenue process	Firm individual empirical estimation of the real drift μ_P and risk-neutral adjustment by $\mu = \mu_P - (r_A - r)$ Adjustment similar to Goldstein et al. (2001), Couch et al. (2012)	0.02
σ	volatility of the revenue process	Firm individual empirical estimation of the revenue's volatility	0.25
γ	variable cost ratio	Firm individual empirical estimation of the costs of goods sold ratio	0.70

This table contains all exogenously set parameters of the IO-model. It also provides suggestions how to observe or estimate the parameters and gives indications with respect to reasonable values.

All other parameters are exogenous and need to be estimated (2/2)

Table: *Exogenous Parameters of the IO-Model cont'd*

Parameter	**Description**	**Rationale**	**Exemplary reasonable values**
F	fixed costs	Firm individual empirical estimation of selling, general and administrative expenses	0.00
δ	interest coverage ratio	Firm or debt tranche individual covenant defined in the debt contract. Natural lower boundary: $1 - \tau$ as this reflects illiquidity.	$1 - \tau$
φ	spread factor for illiquid firms vs. r	Estimation based on average spread between the promised yield of Caa-rated firms (highly vulnerable to nonpayment) and the risk free rate with 10 years maturity (source: Moody's)	2.50
α	bankruptcy cost ratio	Firm or industry-specific estimation based on empirical models We use findings of Glover (2016)	e.g. 0.39 (Food) 0.49 (machinery)
ϵ	illiquidity cost ratio	Firm or industry-specific estimation based on emprical models with respect to technical defaults We use findings of Ertan and Karolyi (2016)	0.04

This table contains all exogenously set parameters of the IO-model. It also provides suggestions how to observe or estimate the parameters and gives indications with respect to reasonable values.

Finally, we apply our model to the US market sectors defined by NAICS - First step process testing

Table: *Normal-Distribution Test of the log-changes of R_t*

		Jarque-Bera Test					
		N, Norm.-Dist.		in %, Norm.-Dist.			
NAICS Sector	**No. Of Firms (N)**	$\alpha = 0.05$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.10$	μ	σ
Accommodation and Food Services	96	44	36	0.4583	0.3750	0.0264	0.1030
Administrative, Support, Waste, Remediation	105	42	31	0.4000	0.2952	0.0253	0.2175
Construction	77	33	29	0.4286	0.3766	0.0110	0.2190
Health Care and Social Assistance	103	31	27	0.3010	0.2621	0.0074	0.1824
Information	529	256	216	0.4839	0.4083	0.0199	0.1776
Manufacturing	1988	948	785	0.4769	0.3949	-0.0026	0.1526
Mining, Quarrying, and Oil and Gas Extraction	264	157	126	0.5947	0.4773	0.0117	0.2622
Professional, Scientific, and Technical Services	408	221	181	0.5417	0.4436	0.0014	0.1454
Real Estate and Rental and Leasing	204	77	66	0.3775	0.3235	0.0392	0.1912
Retail Trade	242	122	107	0.5041	0.4421	0.0438	0.1270
Transportation and Warehousing	143	56	46	0.3916	0.3217	0.0188	0.1607
Utilities	103	41	33	0.3981	0.3204	-0.0054	0.1780
Wholesale Trade	147	69	55	0.4694	0.3741	0.0269	0.2304
Others	110	48	41	0.4364	0.3727	0.0274	0.2014

The table depicts the results of the Jarque-Bera test for normal distribution which we apply to examine the log-changes of the stochastic process R_t. The null hypothesis of the test is that the underlying process is normally distributed. Thus, choosing a higher significance level α leads to a higher number of firms for which normal distribution is ruled out. The last two columns provide our estimations of the risk-neutral drift of the revenue process μ and its standard deviation σ.

For each sector we estimate the exogenous parameters and retrieve the corresponding leverages

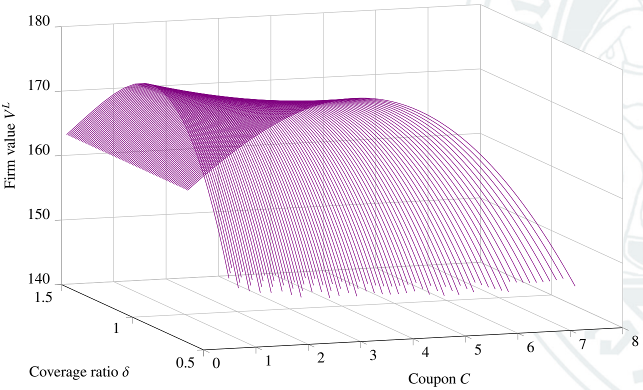
Table: *Input Parameters of the IO-Model and Observed Leverage*

NAICS Sector	α	ϵ	δ	γ	F	R_0	$L = D(V)/V^L(V)$
Accommodation and Food Services	0.3890	0.04	1.00	0.6195	14.00	100	0.4594
Administrative, Support, Waste, Remediation	0.4740	0.04	1.00	0.5110	23.04	100	0.1994
Construction	0.3740	0.04	1.00	0.7220	18.83	100	0.4405
Health Care and Social Assistance	0.4740	0.04	1.00	0.2483	51.35	100	0.5497
Information	0.4740	0.04	1.00	0.3941	25.37	100	0.2927
Manufacturing	0.3970	0.04	1.00	0.6915	15.85	100	0.3071
Mining, Quarrying, and Oil and Gas Extraction	0.4630	0.04	1.00	0.5165	11.42	100	0.2535
Professional, Scientific, and Technical Services	0.4740	0.04	1.00	0.5131	33.90	100	0.2225
Real Estate and Rental and Leasing	0.4740	0.04	1.00	0.4066	14.27	100	0.5412
Retail Trade	0.4420	0.04	1.00	0.7026	19.19	100	0.2714
Transportation and Warehousing	0.4130	0.04	1.00	0.4513	27.16	100	0.4286
Utilities	0.4740	0.04	1.00	0.3518	33.02	100	0.4531
Wholesale Trade	0.4420	0.04	1.00	0.7382	23.74	100	0.2923
Others	0.4598	0.04	1.00	0.5824	23.69	100	0.2683

The table provides an overview of the chosen input parameters for each NAICS sector. For the bankruptcy costs α we follow the estimates of Glover (2016). Regarding the illiquidity expenses ϵ and the average covenant ratio δ industry-specific estimates are not yet available. Thus, we apply the general estimates of Ertan and Karolyi (2016) to all industries. The starting point of the stochastic revenue process R_0 is indexed to 100. The estimates for the variable cost ratio γ and the fixed costs F are based on all normally distributed firms in our sample from NASDAQ, NYSE, NYSE ARCA, and NYSE MKT. F has been related to the index of R_0. The leverage ratio $L = D(V)/V^L(V)$ is based on our sample, too.

A possible extension is to endogenize the chosen covenant δ

Figure: *Levered Firm Value $V^L(V)$ in dependence of Covenant Ratio δ and Coupon Payment C*



The graph depicts how changing δ and C impacts $V^L(V)$. For lower delta values the maximum levered firm value $V^{L,*}(V)$ is achieved with higher choices of C^* and vice versa. The global optimum is at the minimum δ of $1-\tau$. The chosen model parameters are as follows: $r = 0.05$, $\tau = 0.35$, $R_0 = 25$, $\mu = 0.02$, $\sigma = 0.20$, $\gamma = 0.70$, $F = 0$, $\epsilon = 0.00$, and $\varphi = 2.5$.