
THE WEAK SCALE FROM BBN

arXiv:1409.0551 [hep-ph] L. J. Hall, D. Pinner, J. T. Ruderman

E-lab. Lunch Journal (10th Oct.)

D1 Takumi KUWAHARA

LHC run 1

- The discovery of the Higgs boson
- No evidence for New Physics

The fine-tuning problem becomes much severer than before.

In the absence of any discovery of BSM and of any explanation for the dark energy..

a theoretical framework

to understand the fine-tuning of VEV of Higgs
and cosmological constant



The multiverse may provide such a framework

Multiverse

If observers are rare in the multiverse,
then those universes that have observers can contain parameters
that appear to be **finely tuned**.

cosmological constant: Λ_{CC}

relates to whether universes contain large scale structure

S.Weinberg (1987)

weak scale (Higgs VEV): v

relates to whether universes contain complex nuclei

V. Agrawal, S.M. Barr, J. F. Donoghue and D. Seckel (1998)

fine-tuning problems of Λ_{CC} and v : resisted solutions by symmetries

➡ This fact provides evidence for the multiverse

cosmological constant: Λ_{CC} S.Weinberg (1987)

weak scale (Higgs VEV): v V. Agrawal, S.M. Barr, J. F. Donoghue and D. Seckel (1998)



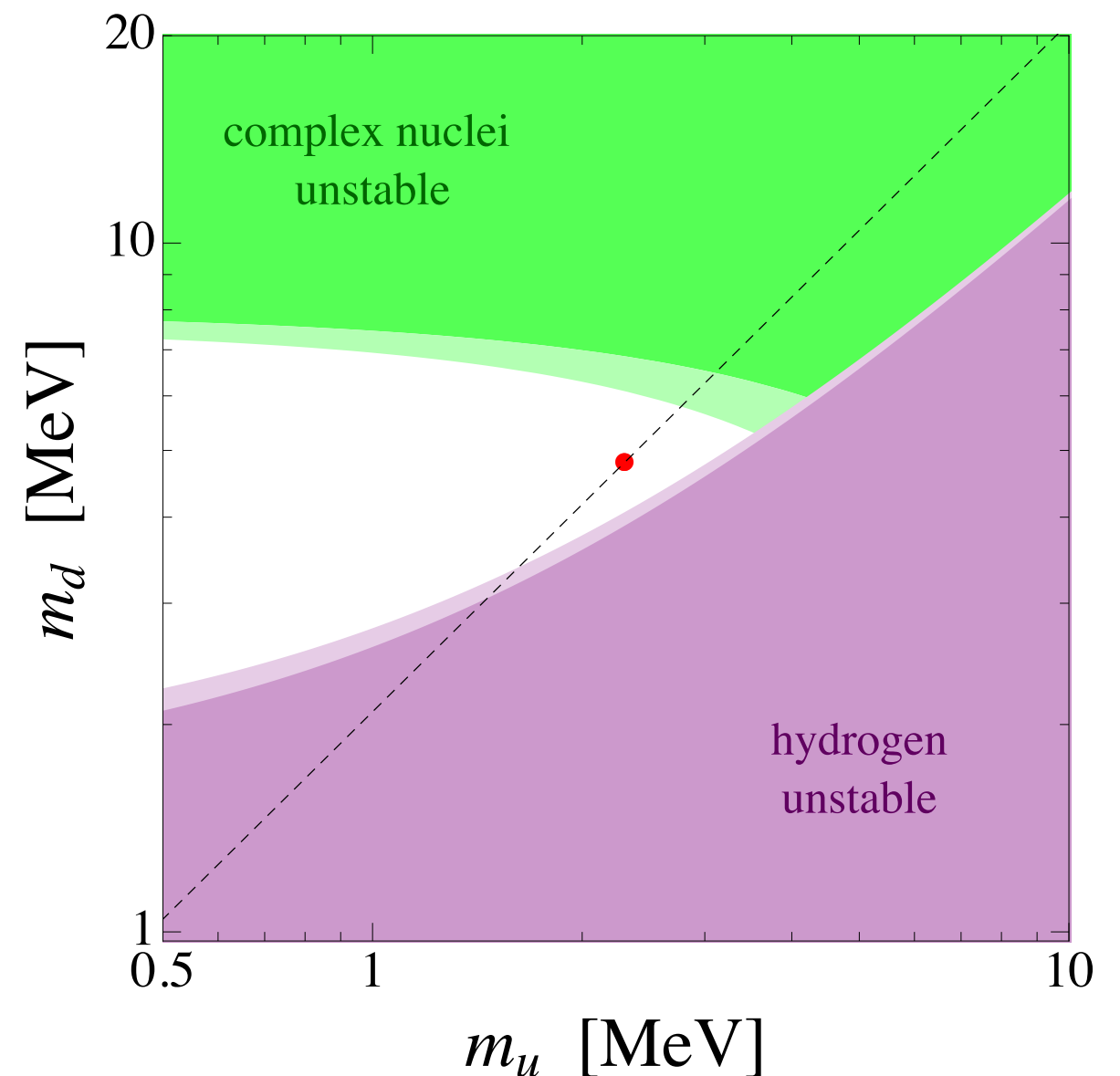
These results are obtained by using a **single**-parameter scan.
(SM+GR) have three dimensionful parameters: Λ_{CC} , v , M_{Pl}

In more general, dimensionless parameters scan.

these are treated as ratios of dimensionful couplings

We are living on the edge!

If the Yukawa couplings are only determined by symmetries, our place may be accidental.



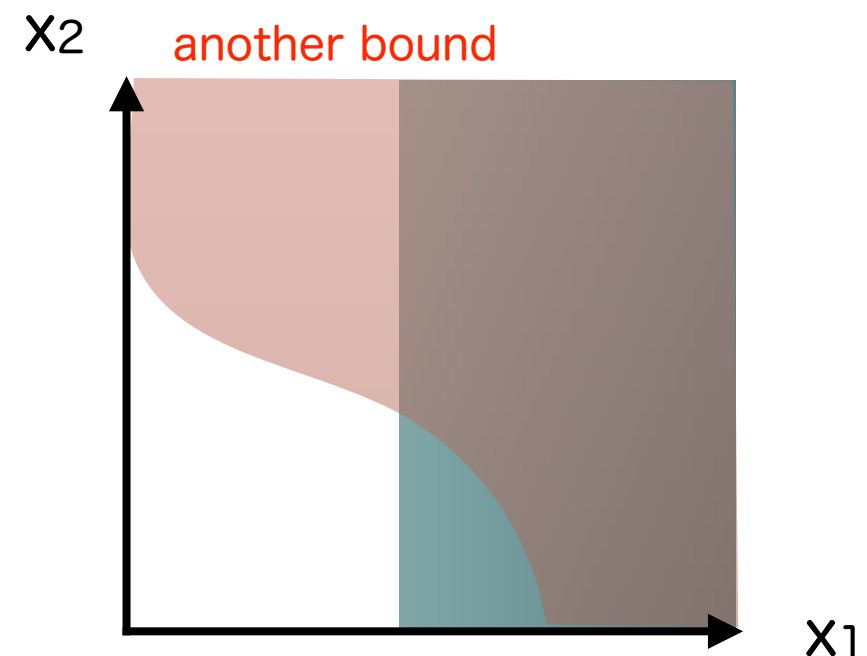
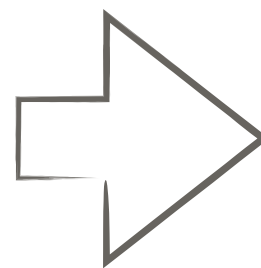
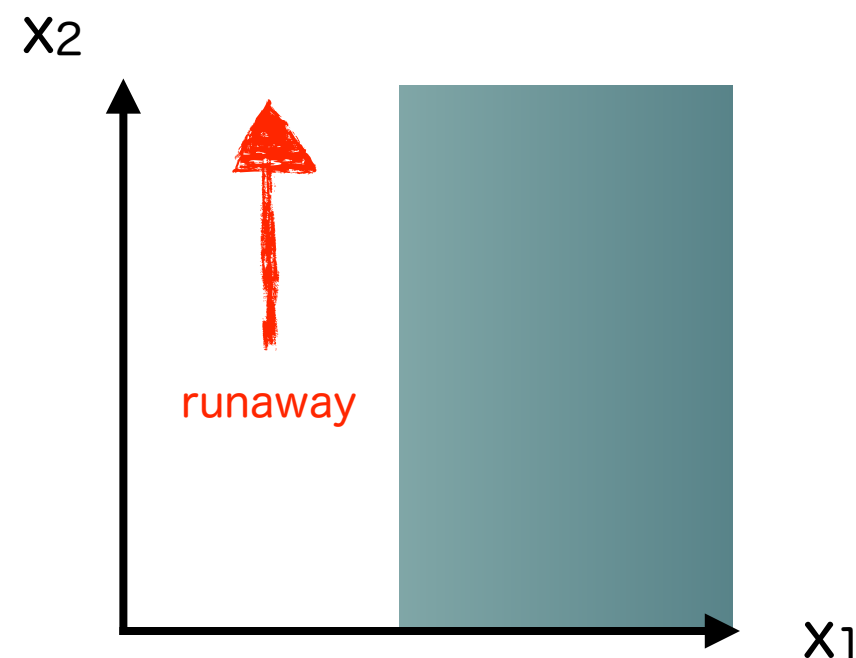
“Multiverse” may make us understand
the reason why we are living on the edge.

KEY QUESTION:

Whether the multiverse understanding of the finely-tuned values Λ_{cc} and v is robust in many parameters scanning.

In general,
the dimension of parameter space is enlarged in many parameters scanning.

➡ Need another catastrophic bound



Contents

- Introduction
- Scanning parameters and multiverse distributions
- The nuclear boundaries and $m_{u,d,e}$
- BBN with Freeze out below the QCD scale
- Scanning η and M_{Pl}
- BBN with Freeze out above the QCD scale
- Conclusions

Scanning parameters and multiverse distributions

Probability distribution “P” in the multiverse

$$dP = f(x_i)n(x_i)d \ln x_i$$

x_i : scanning parameter

n_i : weighting function (\propto # of observers)

f : a priori distribution of universes in the multiverse

For a multiverse explanation of weak scale via BBN, we should include

$$x_i = (m_a, v) \quad \text{with } a = (u, d, e)$$

where m_a is the masses of the light fermions,

since the observer region is determined by the nuclear stability.

The probability distribution for their analysis,

Helium production @BBN

$$dP = f(m_a/v, v) n_{\text{nuc}}(m_a) n_{\text{BBN}}(Y_4) n_{\text{other}}(m_a, v) d \ln m_a d \ln v$$

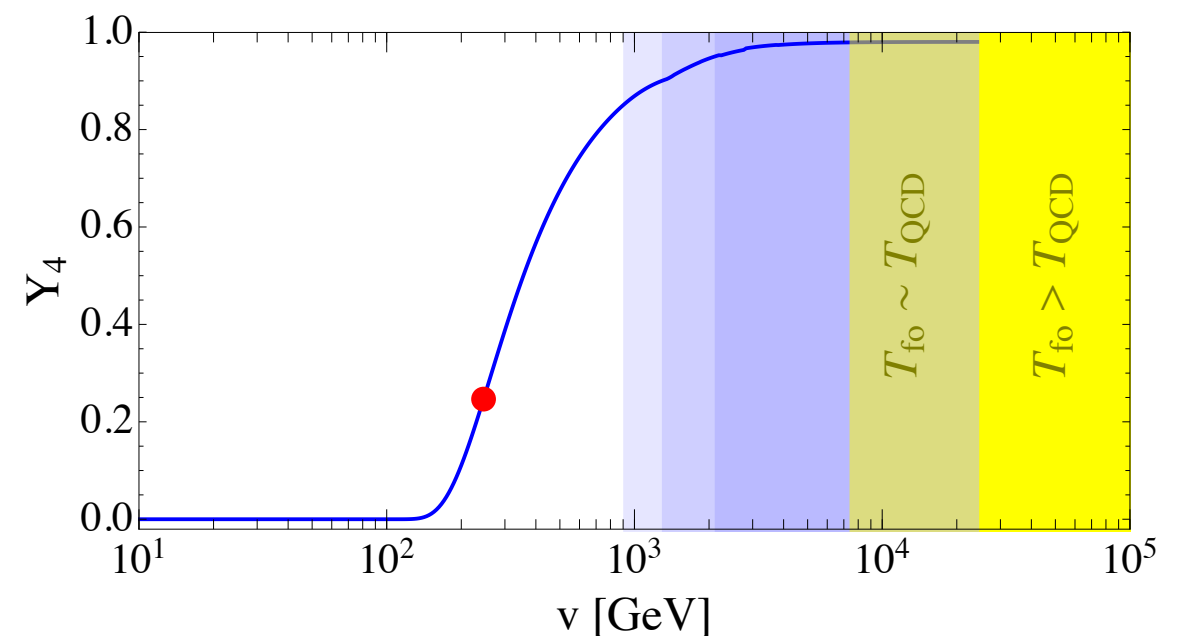
Nuclear stability

other effects

(SN explosion, stellar burning, etc..)

Y_4 is the mass fraction of helium-4 $Y_4 \equiv \frac{4n_{4\text{He}}}{n_N}$

- Nuclear stability is determined by fermion masses
- Assumed that n_{BBN} depends only on Y_4
- n_{other} is set to be unity

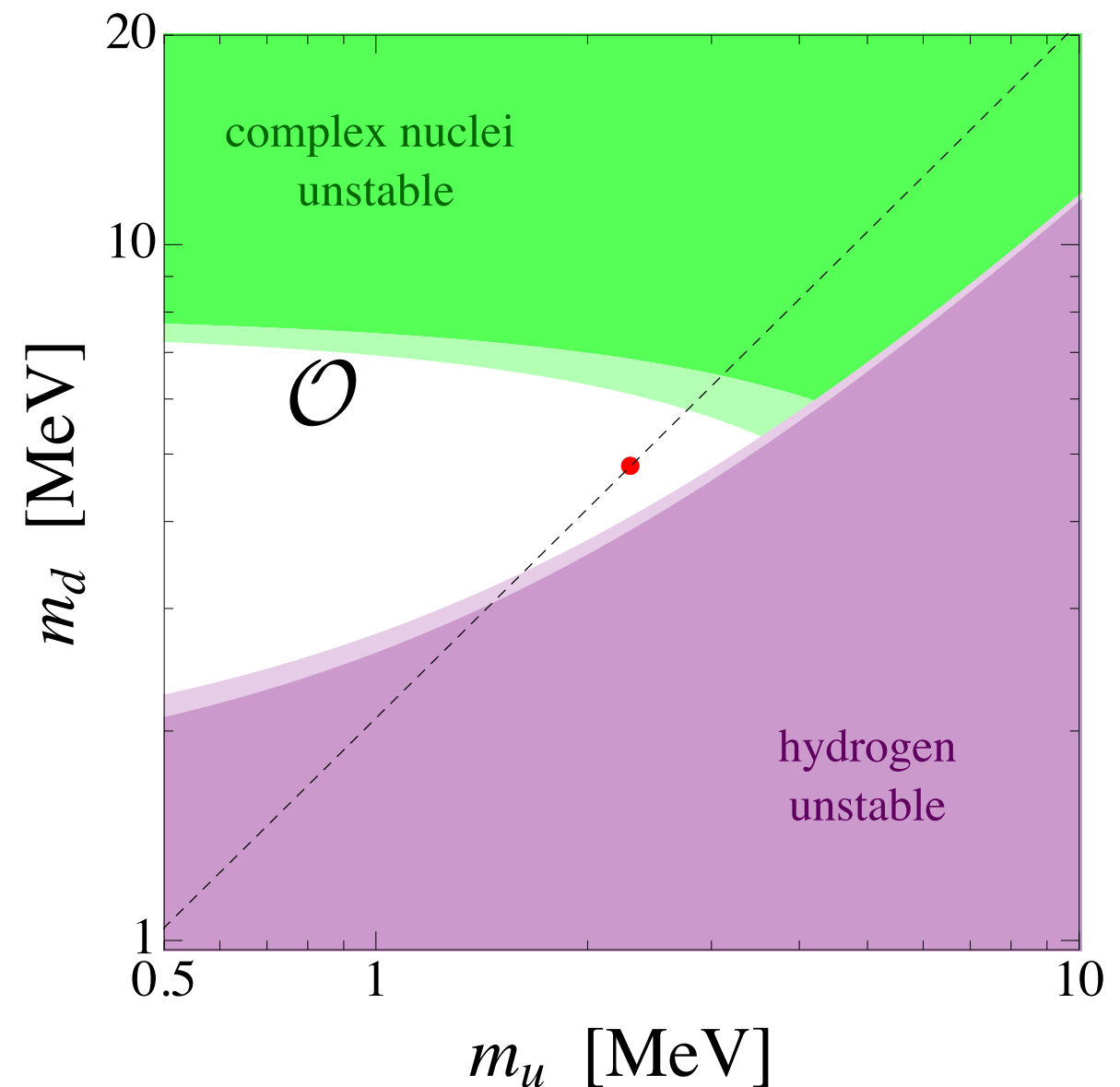


$$dP = f(m_a/v, v) n_{\text{nuc}}(m_a) n_{\text{BBN}}(Y_4) n_{\text{other}}(m_a, v) d \ln m_a d \ln v$$

The light shaded region ->
1 σ theoretical uncertainties

Nuclear stability boundaries: shape form

$$\Rightarrow n_{\text{nuc}} = \begin{cases} 1 & m_a \in \mathcal{O} \\ 0 & m_a \notin \mathcal{O} \end{cases}$$



dotted line: variation of VEV for fixed Yukawas

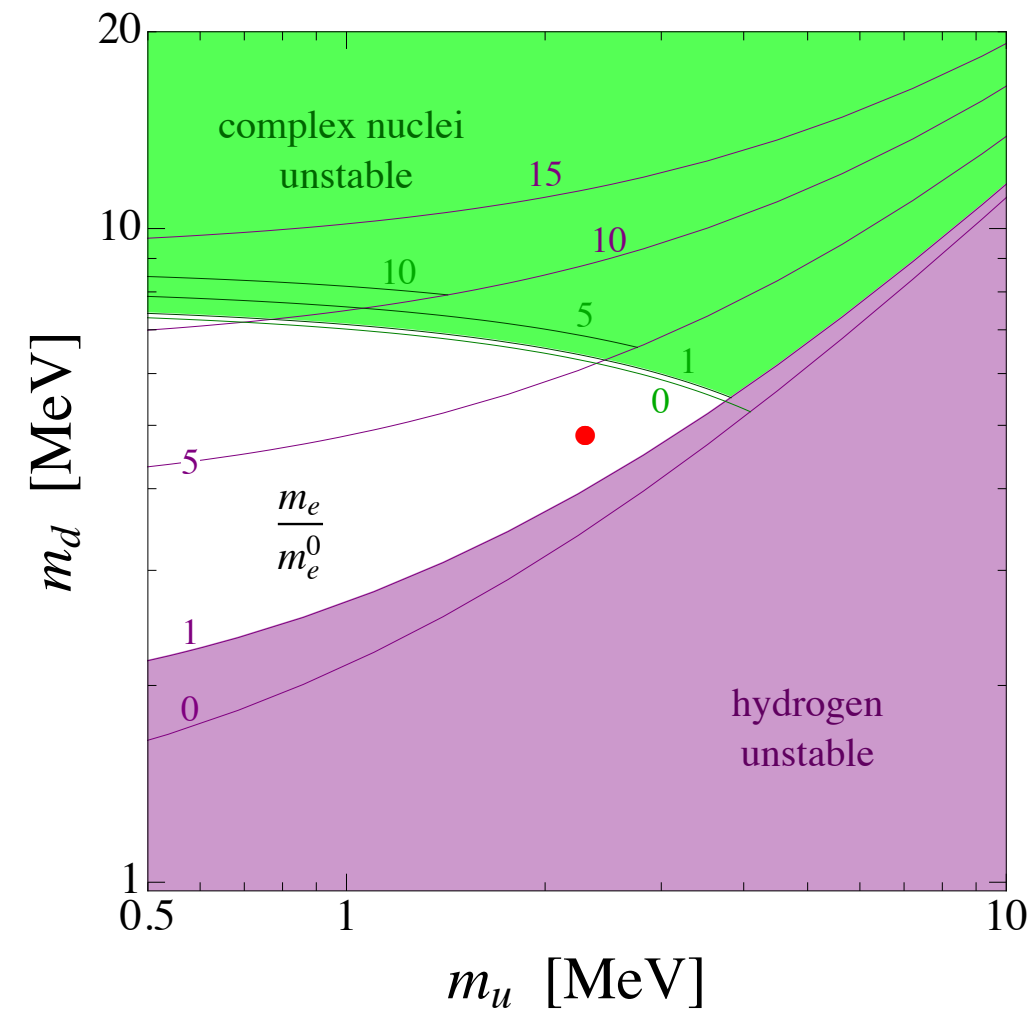
The nuclear boundaries and $m_{u,d,e}$

If flavor symmetries completely determine the hierarchy of Yukawas, it is surprising that

We are living on the edge of the nuclear stable region

It is convenient to rotate the (m_u, m_d) plane,

$$\begin{cases} m_d - m_u & \text{Hydrogen instability} \\ m_u + m_d & \text{Complex nuclei instability} \end{cases}$$



(m_u, m_d) plane with contours for varying m_e

The n-p mass splitting relates to u-d mass splitting

$$m_n - m_p \approx \delta_{\text{iso}} \frac{(m_d - m_u)}{(m_d - m_u)_0} + \delta_{\text{EM}}$$

δ_{iso} : isospin-violating contribution

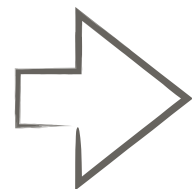
δ_{EM} : EM contribution

In the numerical analysis, they use lattice simulation result

$$\delta_{\text{iso}} = 2.39 \pm 0.21 \text{ MeV} \quad \text{A.Walker-Loud [arXiv:1401.8259]}$$

If $m_n - m_p < m_e$,

e^- capture of p was tend not to given rise to due to raising the rate of $p + e^- \rightarrow n + \nu$.



Hydrogen instability

Binding energy of nucleons depends on the mass of pion,

$$m_{\pi}^2 \propto m_u + m_d$$

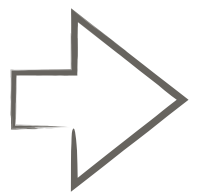


Requiring stable nuclei, the sum, $m_u + m_d$, is constrained.

If the binding energy per nucleon is sufficiently small,

$$\left| \frac{B}{A} \right| < m_n - m_p - m_e$$

bounded neutrons will decay.



Complex nuclei instability

m_{π} dependence of B

PRC 74 024002 J. F. Donoghue (2006)

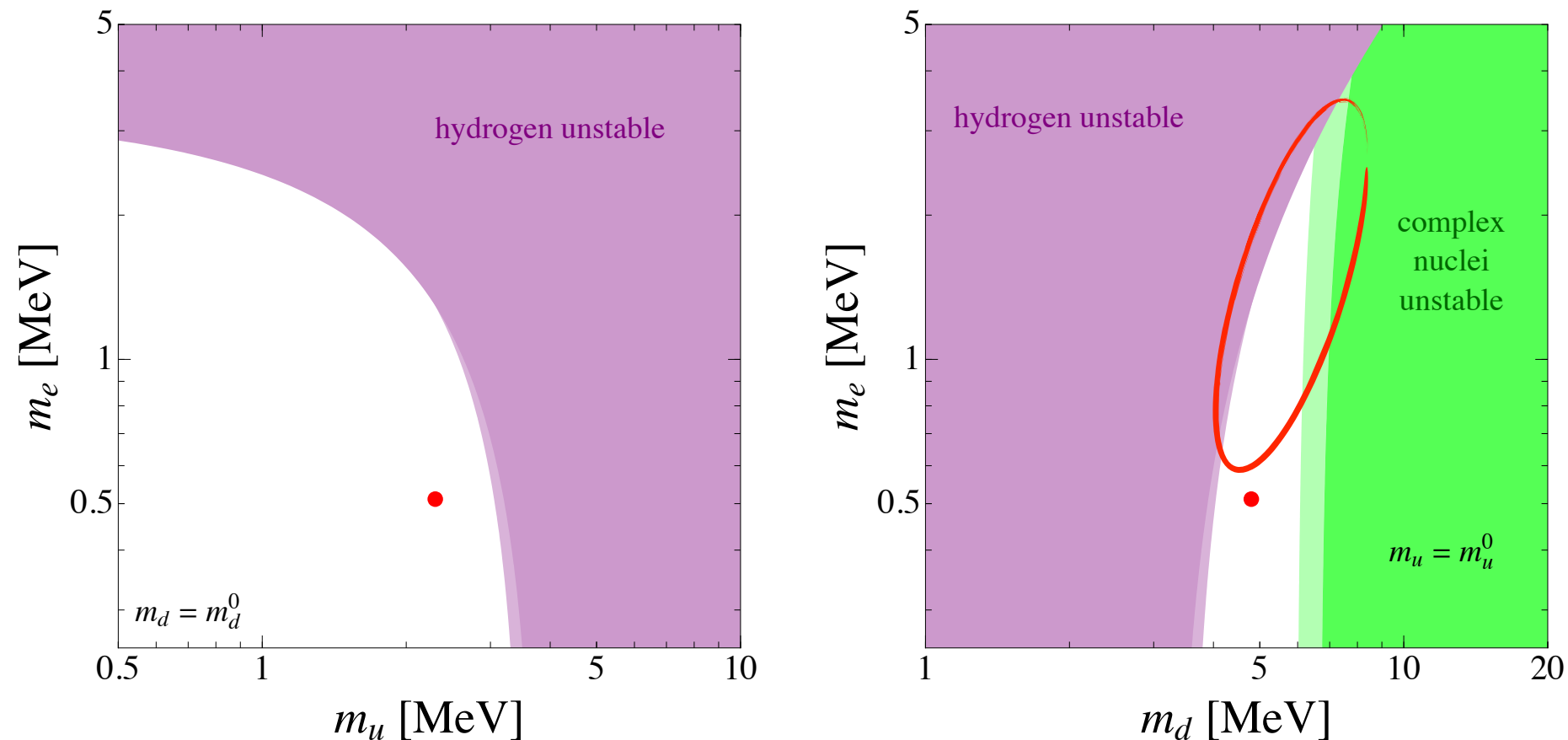
PRD 78 014014 T. Damour, J. F. Donoghue (2008)

IN THIS PAPER:

Complex nuclei: ^{16}O

Stability boundary does not almost depends on atomic number.

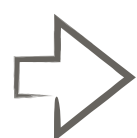
For fixed Yukawas,
3d (m_u, m_d, m_e) plot can not explain why we are living on the edge..



Unbounded region
orders of magnitude away from this tip in (m_u, m_e)
=>We are still close to the boundary..

We are on the edge up to $m_e \sim 2.5 m_{e,0}$

Motivation for the anthropic solution;



In order to bound allowed region, we use the abundance of ^4He .
BBN constraints on parameters of SM

BBN with Freeze out below the QCD scale

Helium-rich universe:

A large helium mass fraction has consequences that tend to suppress observers.

- Halo cooling becomes slower
- long-lived hydrogen burning stars becomes rarer
- hydrogen as a building block of life becomes rarer

Assumption:

10% of mass fraction of the universe is dominated by H at least.

Primordial abundance of ^4He in the full space $(m_{u,d,e}, v)$ with $v/v_0 < 10^2$.

p-n interconvert and freeze-out temp.

$$\Gamma_{p+e^- \rightarrow \nu+n}/H = \left(\frac{T}{0.9\text{MeV}}\right)^3 \left(\frac{v_0}{v}\right)^4 \left(\frac{M_{Pl}}{M_{Pl,0}}\right) \Rightarrow T_{\text{freeze-out}} \simeq (0.9\text{MeV}) \left(\frac{v}{v_0}\right)^{4/3} \left(\frac{M_{Pl,0}}{M_{Pl}}\right)^{1/3}$$

Below this temp., n-p ratio estimated as;

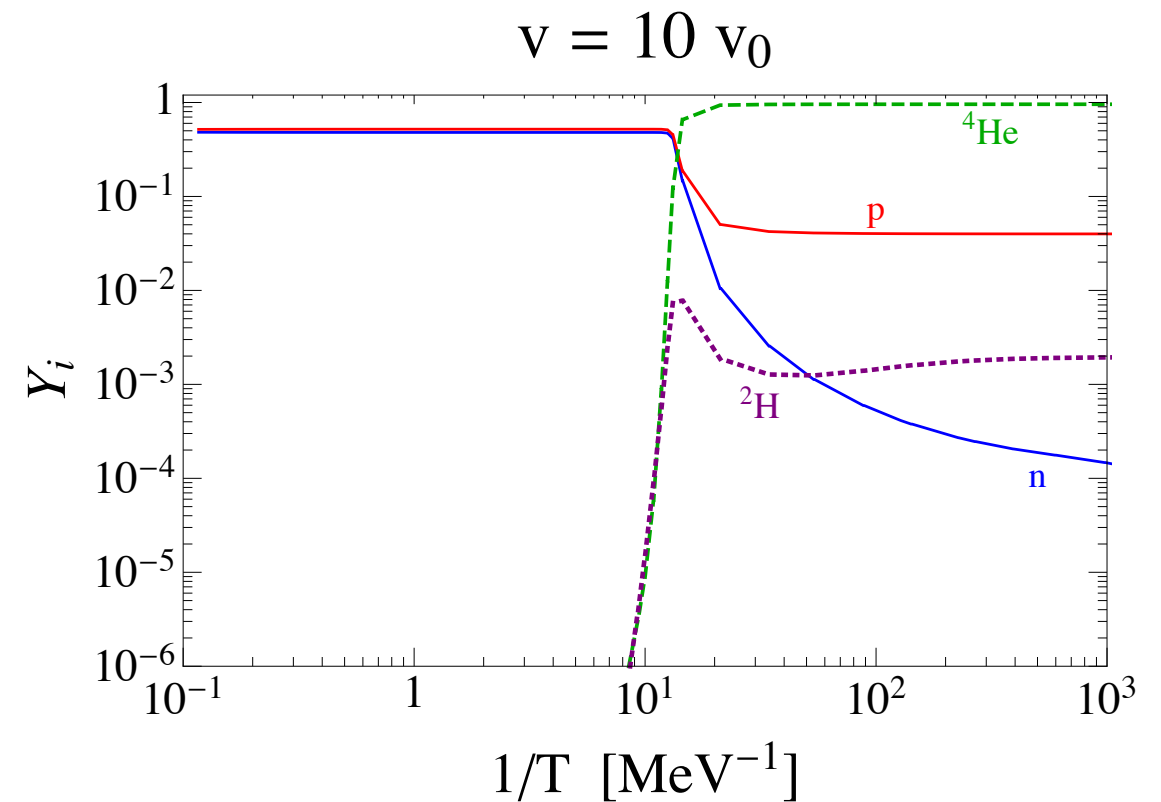
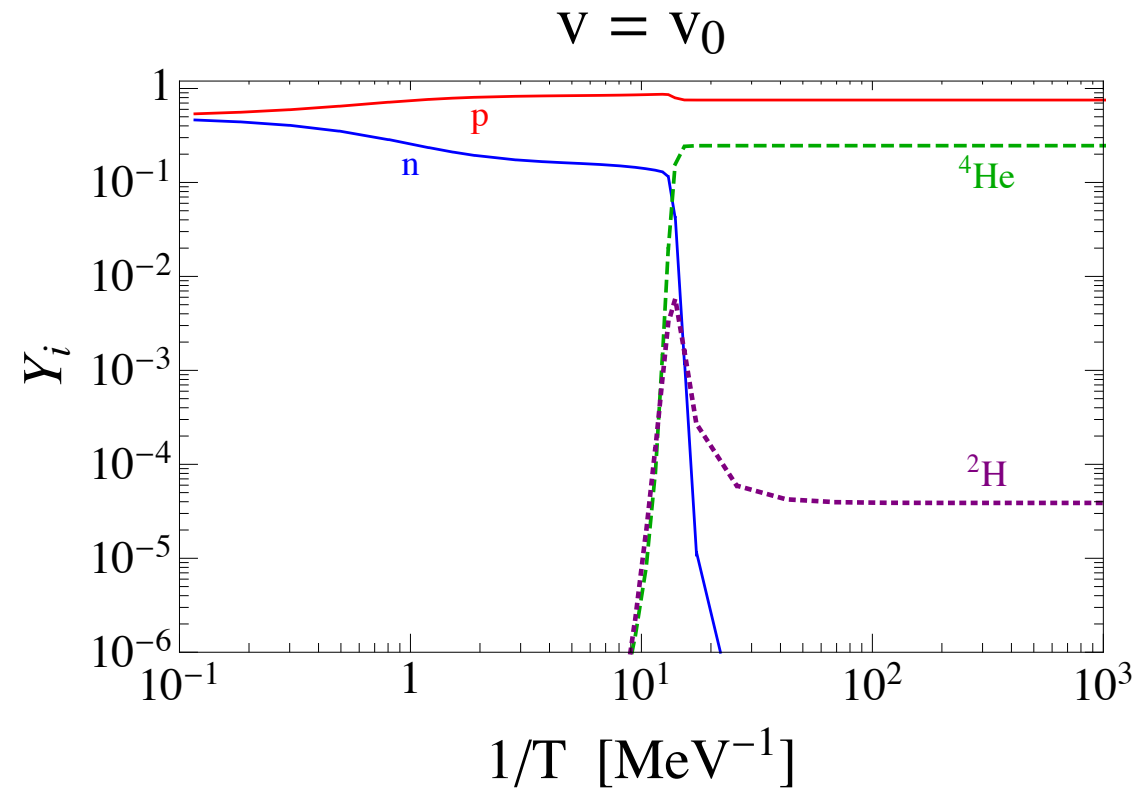
$$\frac{n}{p} \approx e^{-(m_n - m_p)/T_{\text{freeze-out}}} e^{-\Gamma_n t}$$

At around $T=0.1\text{MeV}$, a helium mass fraction is given by $Y_4 \approx \frac{2(n/p)}{1 + (n/p)}$

Large v implies;

- large $T_{\text{freeze-out}}$
- large (n/p) and $Y_4=1$

History of mass fraction; comparison with large v



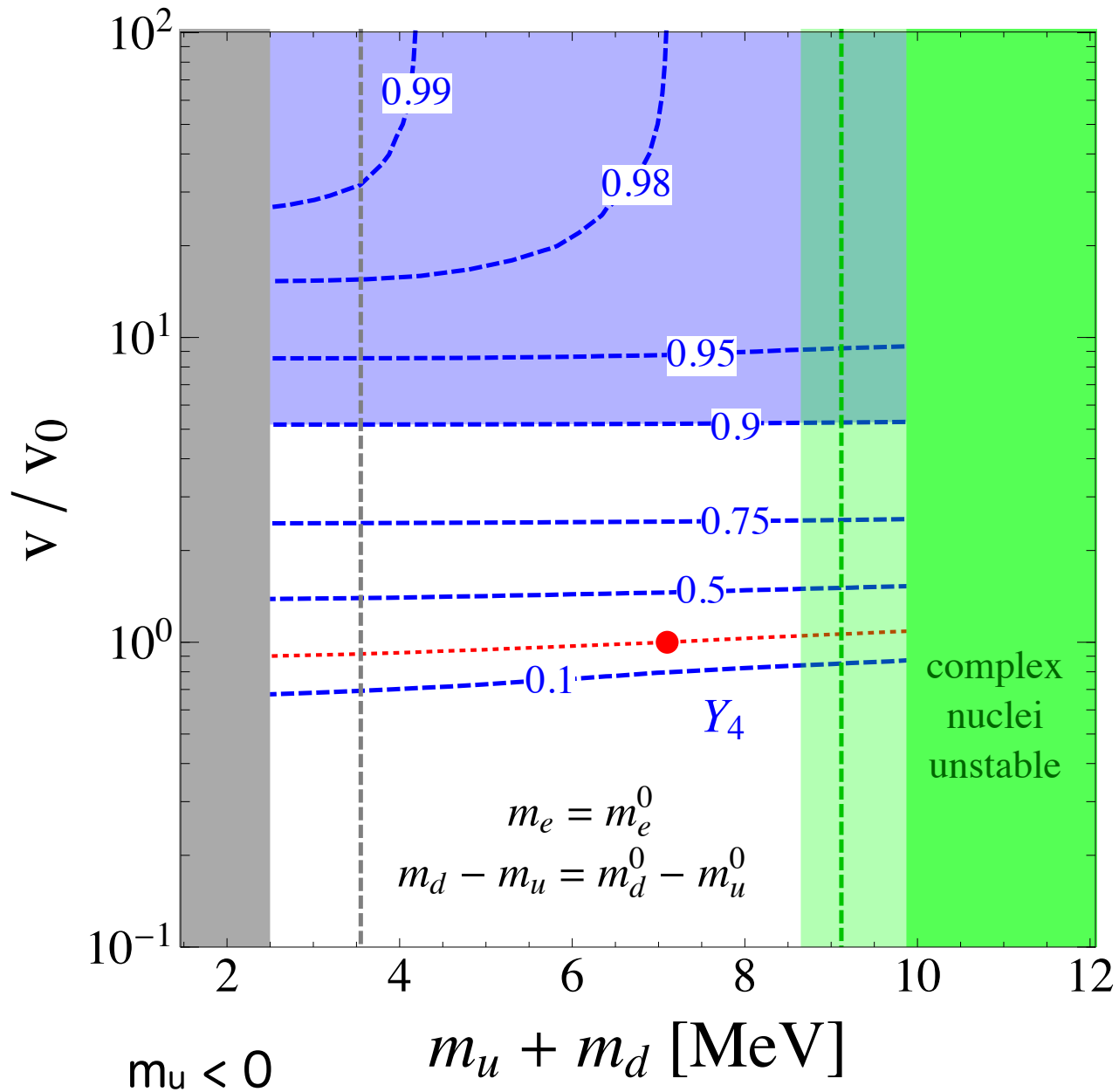
$T_{\text{freeze-out}} ; \text{ above } 10\text{MeV}$
 $m_n - m_p ; \text{ fixed}$

Features

- A finite abundance of neutrons @ very low temp. due to long lifetime of n
- The abundance of ^4He flattens off due to the freeze out of D production
- Large v implies over abundance of ^4He

$$\Gamma_n \approx \frac{1}{880s} \left(\frac{v}{v_0} \right)^4 \left(\frac{(m_n - m_p)_0}{(m_n - m_p)} \right)^5$$

$(m_u+m_d) - v$ plot



Assumption;

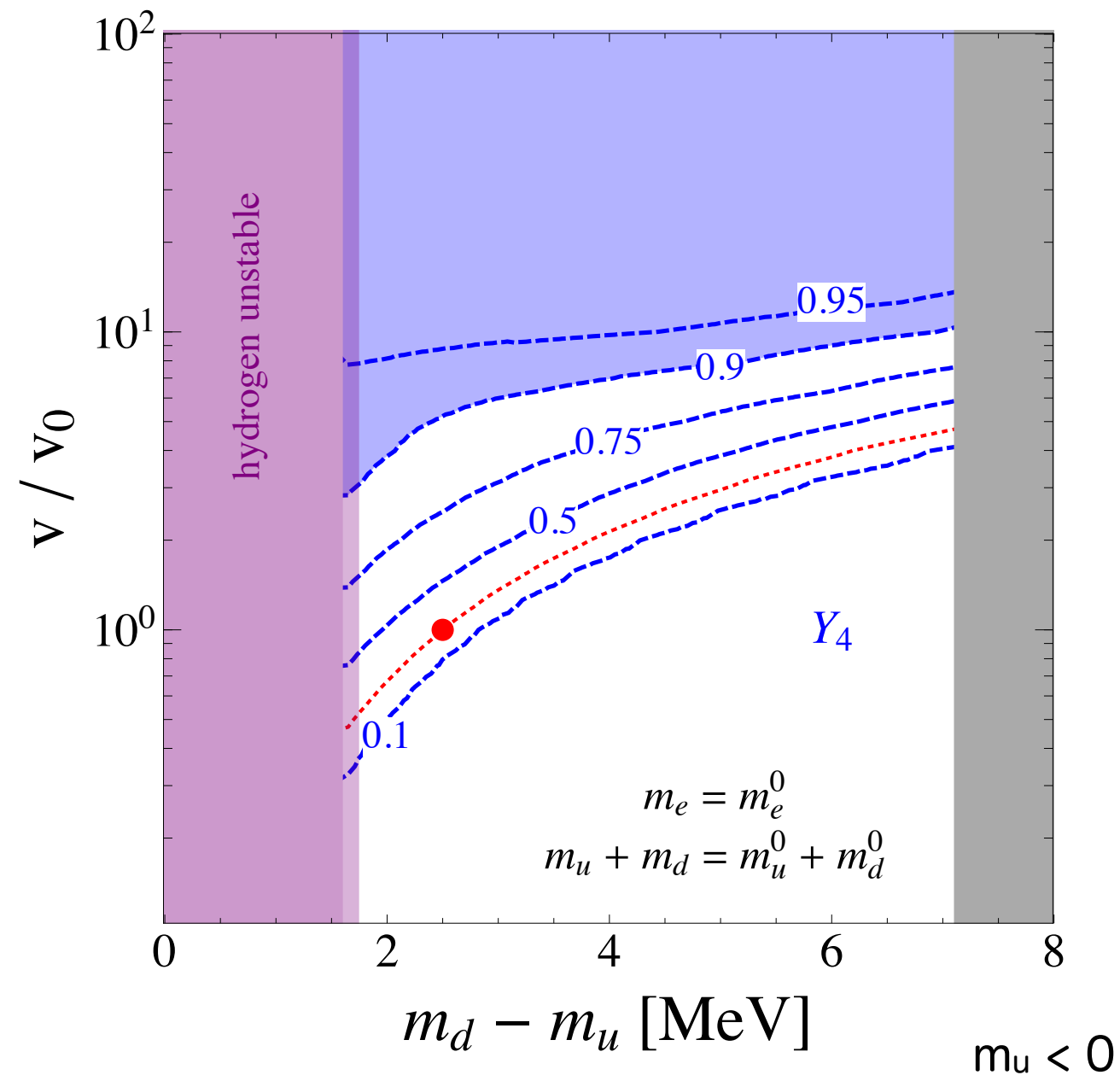
10% of mass fraction of the universe is dominated by H at least.

$$m_\pi^2 \propto m_u + m_d$$

Heavy pion mass

-> unstable complex nuclei

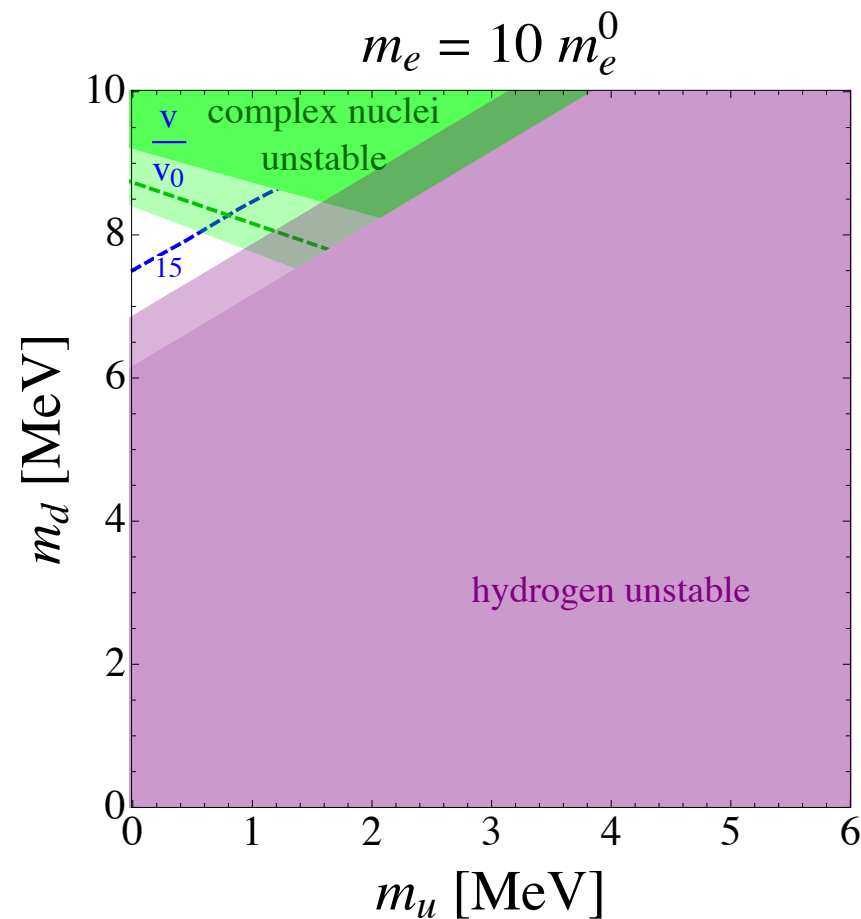
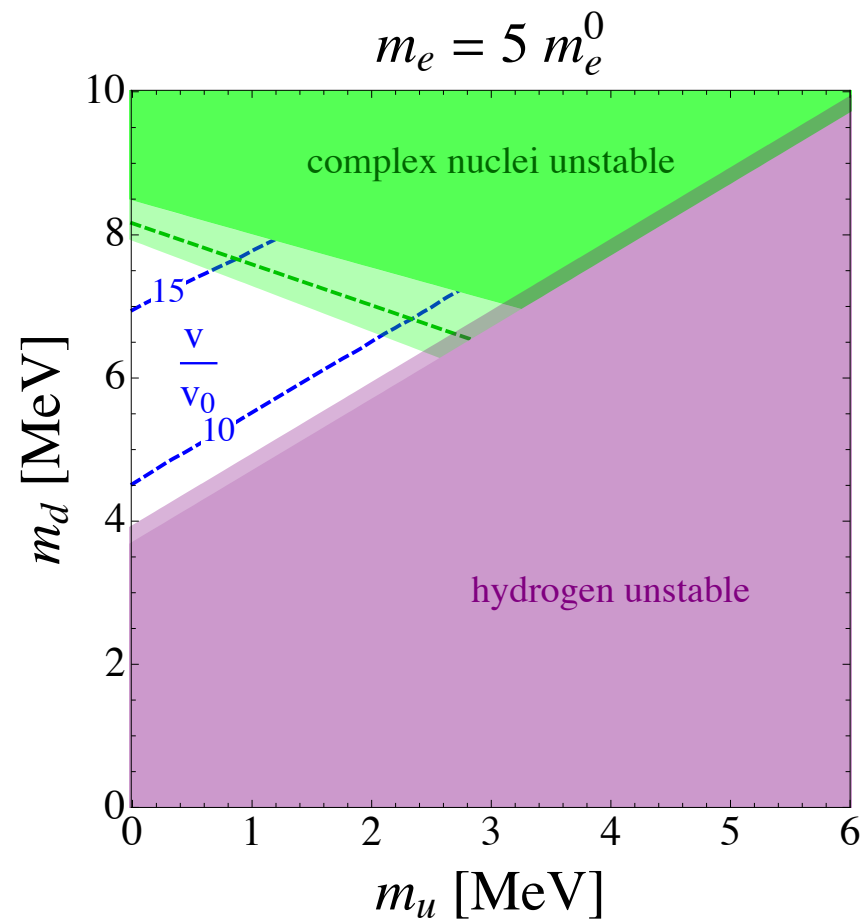
$(m_d - m_u) - v$ plot



Assumption;
 10% of mass fraction of the
 universe is dominated by H at least.

large $m_u - m_d$
 -> unstable proton (hydrogen)

m_u - m_d plot for fixed Y_4 ($Y_4 = 0.9$)



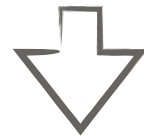
Important;

v remains bounded even if the large value of the mass of electron
by the requirement that Y_4 not exceed 90%

Scanning η and M_{Pl}

Previous analysis:

^4He and nuclear physics boundaries



Our universe lives in a finite volume in 4-dimensional parameter space (m_a, v)

However, the abundance of ^4He depends on

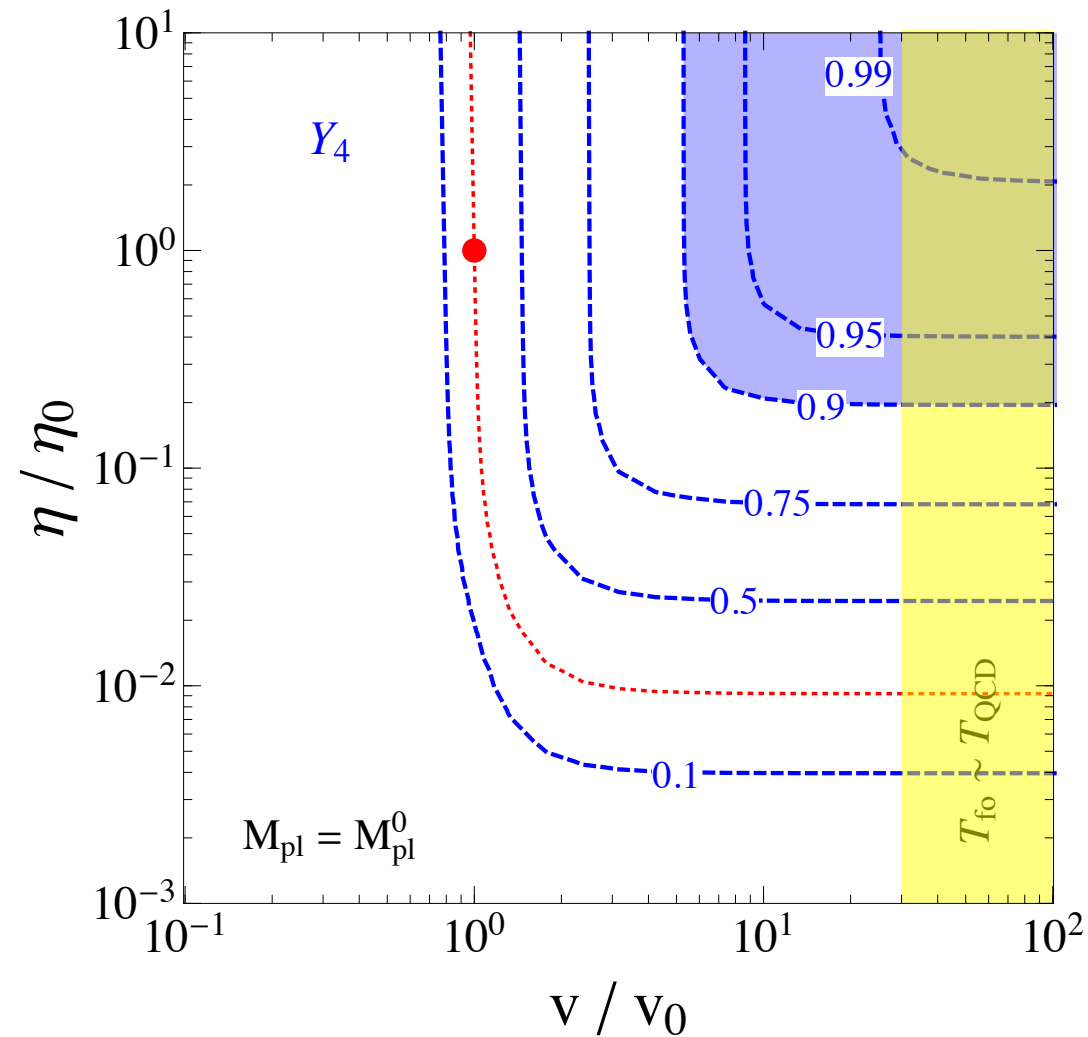
- baryon to photon ratio η
- Planck mass M_{Pl}

Now, we show whether varying them allows VEV to runaway to large values.

From previous analysis,

we assume that $m_{u,d,e}$ are selected by nuclear boundaries close to obs. values.

Varying η plot



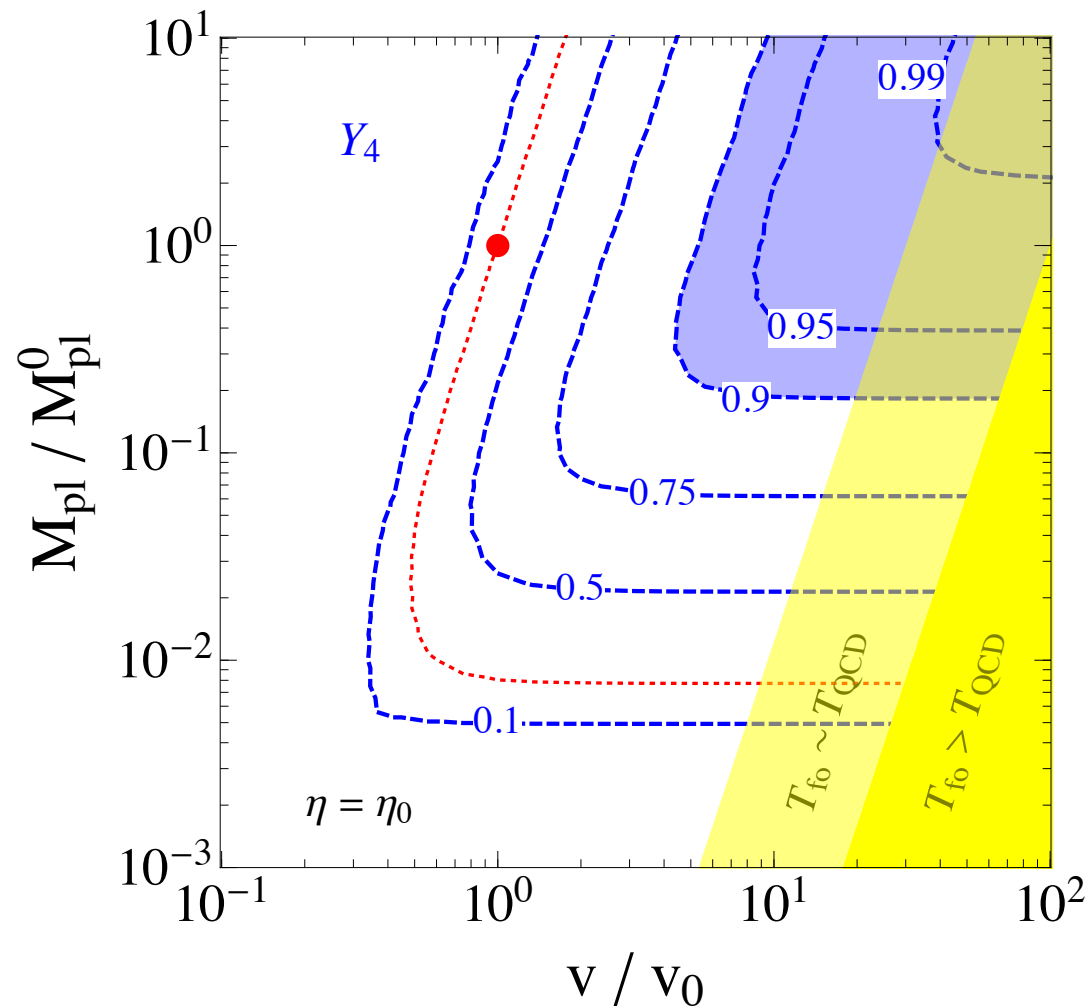
observed value: $M_{Pl} = 2 \times 10^{18} \text{ GeV}$

There is one-direction for a runaway VEV.

Comment:

- For lower η , the production of deuterium D becomes slowly due to small baryon number (which leads Y_4 being independent of VEV).

Varying M_{Pl} plot



There are two directions for a runaway VEV.

- Large v @ lower (fixed) M_{Pl}
- Large v by raising M_{Pl}

Comment:

- For large M_{Pl} , the Hubble expansion speeds up since $H \sim T^2/M_{\text{Pl}}$.

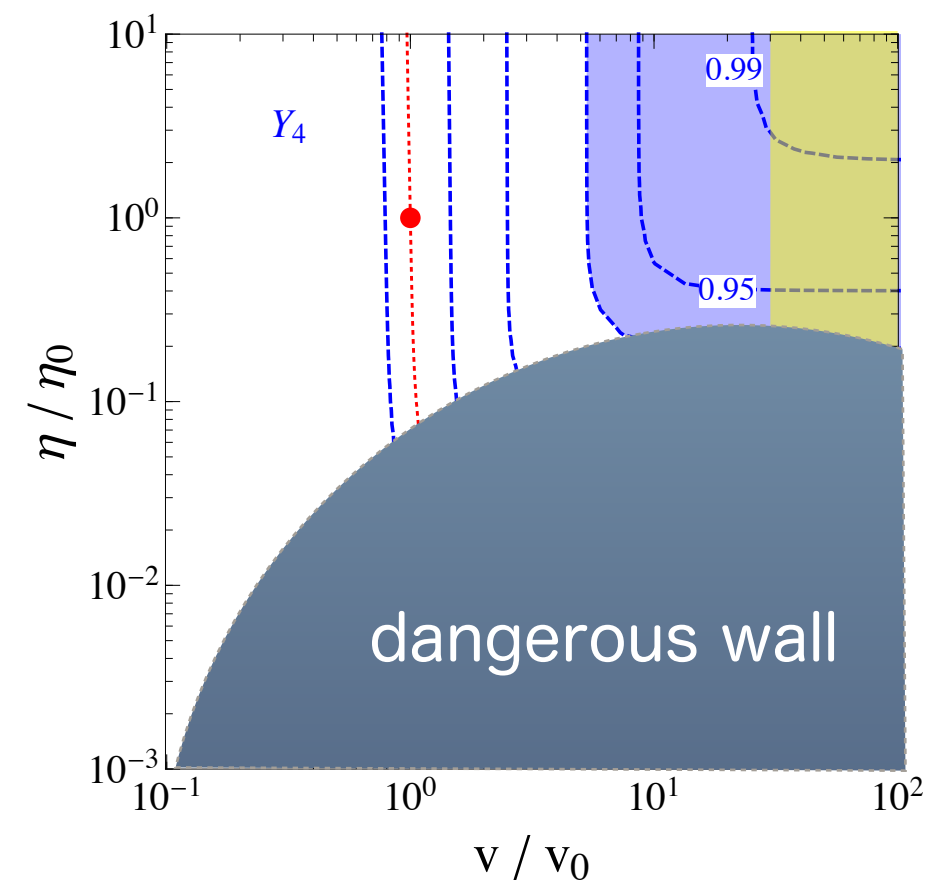
Once η and/or M_{PI} are varied,
BBN does not restrict the weak scale near the obs. value.

Possibility that prevents this runaway behavior:

- η and/or M_{PI} either do not scan or have prior distributions $f(\eta, M_{\text{PI}})$ sufficiently to prevent runaways.
- there are other dangerous wall.

For example

DM energy and star formation



DM energy and star formation

For example,

Suppose that DM is a WIMP (annihilation rate: related weak scale)

$$\sigma_{\text{DM}} = \frac{\alpha}{4\pi v^2} \quad \rho_{\text{DM}} = \left(\frac{v}{v_0}\right)^2 \left(\frac{M_{Pl}}{M_{Pl,0}}\right)^{-1} \rho_{\text{DM}}^0$$

Constraint:

too large DM energy density leads to fragmentation of baryonic discs with in galaxies.

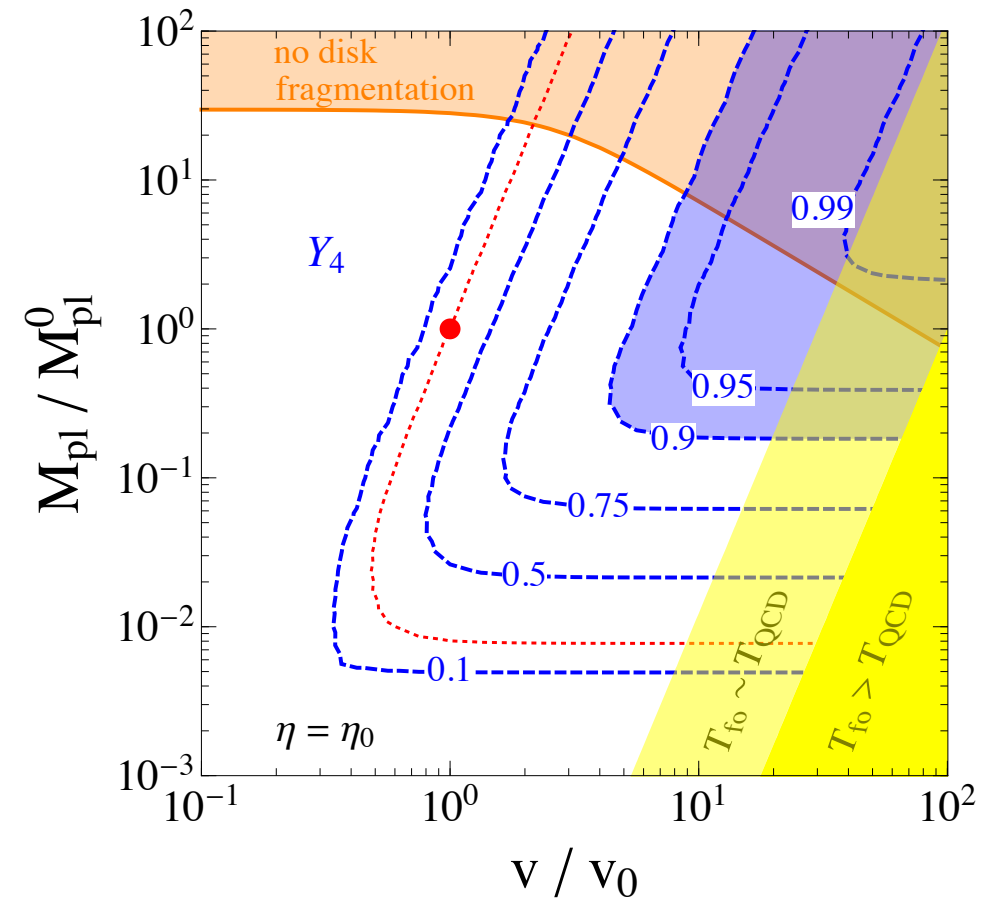
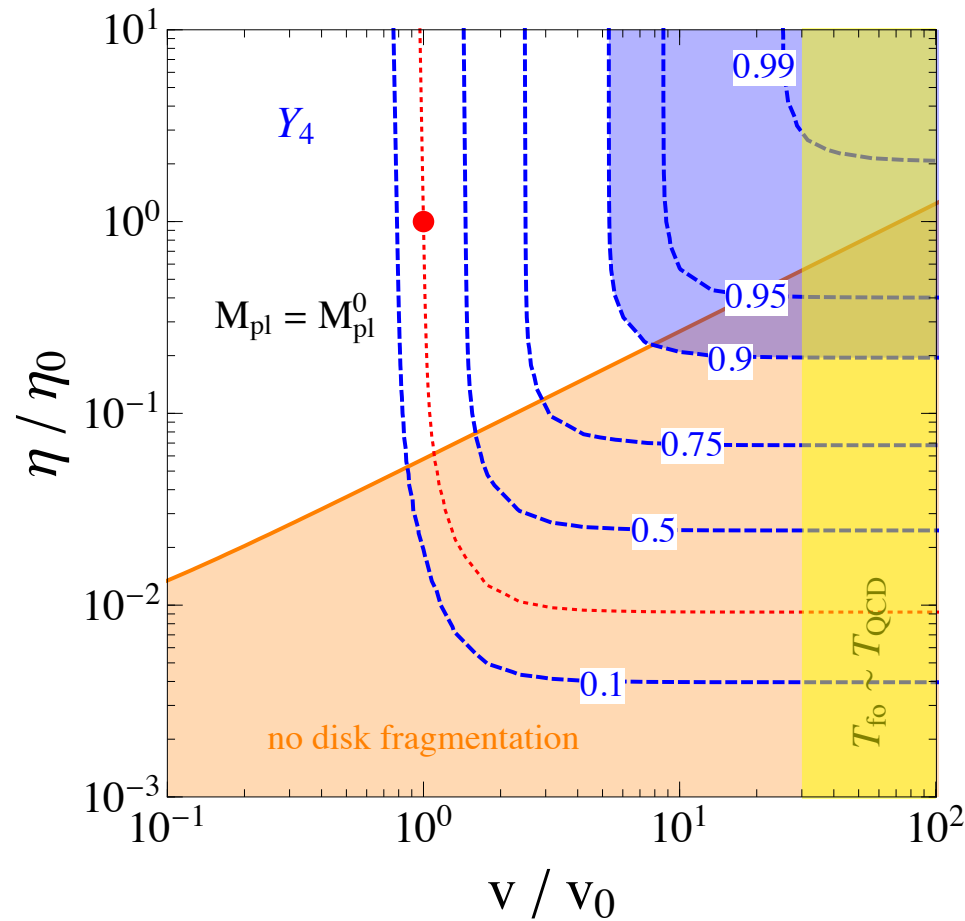
This constraint is described as:

$$\frac{\rho_B}{\rho_{B,0}} > 0.014 \frac{M_{Pl}}{M_{Pl,0}} \left(\frac{\rho_B + \rho_{\text{DM}}}{\rho_{B,0} + \rho_{\text{DM},0}} \right)^{1/3}$$

A. Aguirre, M. Rees, M. Tegmark, F. Wilczek (2006)

$\rho_B = \rho_{B,0} \frac{\eta}{\eta_0}$ is the energy density of baryonic matter.

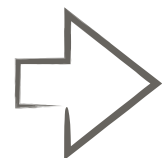
Varying η and M_{Pl} plot with DM constraint



no disk fragmentation $\frac{\rho_B}{\rho_{B,0}} > 0.014 \frac{M_{Pl}}{M_{Pl,0}} \left(\frac{\rho_B + \rho_{DM}}{\rho_{B,0} + \rho_{DM,0}} \right)^{1/3}$

Comment:

- There remains a runaway direction: large values of VEV @ small M_{Pl}



New regime of high $T_{\text{freeze-out}}$ -> Next

BBN with Freeze out above the QCD scale

Large value of v (more than $100v_0$)

freeze-out of weak interaction occurs in the QGP.

$$n/p \text{ only depends on } x \equiv u/d$$

Baryon contents:

$$\frac{n}{p} = \begin{cases} 0, & x > 2, x < -1 \\ \frac{2-x}{2x-1}, & 1/2 < x < 2 \\ \infty, & -1 < x < 1/2 \end{cases}$$

hydrogen universe: no BBN
BUT!: expected to have similar # of observers

Like our universe:
for $1/2 < x < 1$ ($n/p > 1$): neutron/He universe
for $1 < x < 2$ ($n/p < 1$): H/He universe
around $x=1$: helium dominated universe
(previous analysis: suppressed observers)

neutron universe: no BBN

Neutron universes and neutron/He universes

In this scenario ($v > 10^2 v_0$), the lifetime of neutron

$$\tau_n \sim 10^{11} \left(\frac{v/v_0}{10^2} \right)^4 \text{ sec.} \quad \rightarrow \quad v/v_0 > \begin{cases} 10^4 & : (\tau_n > \tau_{\text{our universe}}) \\ 10^3 & : (\tau_n \sim \tau_{\text{star formation}}) \end{cases}$$

From this fact, we find:

For $-1 < x = \frac{u}{d} < 1$

$$v/v_0 > 10^4$$

Neutrons do NOT decay

=> **p, H are not produced**

$$10^3 < v/v_0 < 10^4$$

less heavy elements
compared to our universe

$$10^2 < v/v_0 < 10^3$$

- $T_{\text{freeze-out}}$ is not far above Λ_{QCD}
- expected to contain observers

Determination of u/d ratio

A particular simple case:

the masses of u,d,e, ν_i are only smaller than $T_{\text{freeze-out}}$.

Applying the conditions of chemical equilibria and charge neutrality:

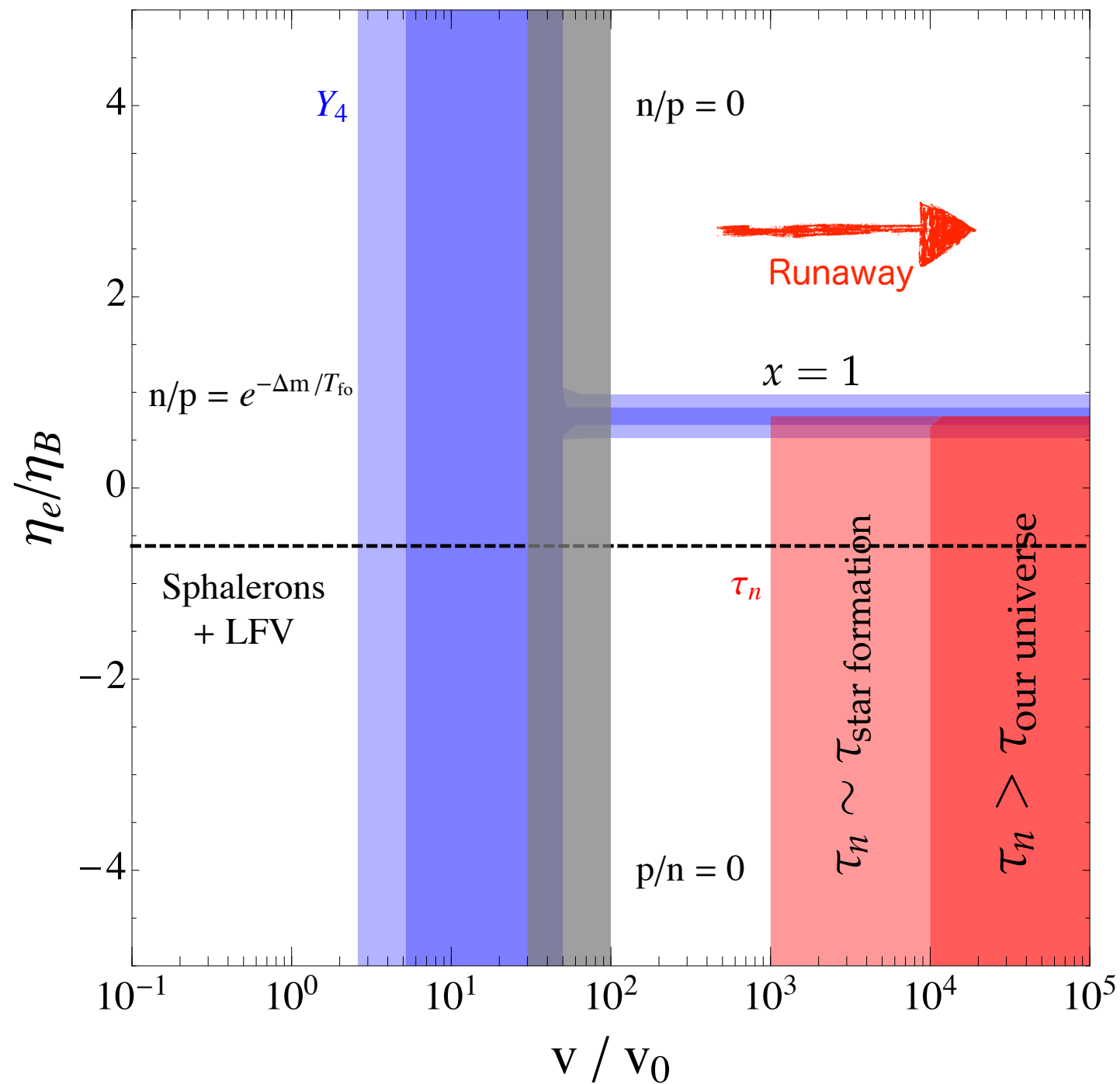
$$x = \frac{u}{d} = \frac{4\eta_B + 2\eta_e}{7\eta_B - 2\eta_e}$$

For each baryon content, we obtain regions of parameter space in terms of η_e/η_B .

For example, neutron universes and neutron/He universes

$$-1 < x \lesssim 1 \qquad \frac{\eta_e}{\eta_B} \lesssim \frac{3}{4}$$

^4He abundance and n lifetime



Blue: Helium dominated universe

Gray: $T_{\text{freeze-out}} \sim T_{\text{QCD}}$

neutron universes and neutron/He universes

$$\frac{\eta_e}{\eta_B} \lesssim \frac{3}{4}$$

Unshaded: expected observers

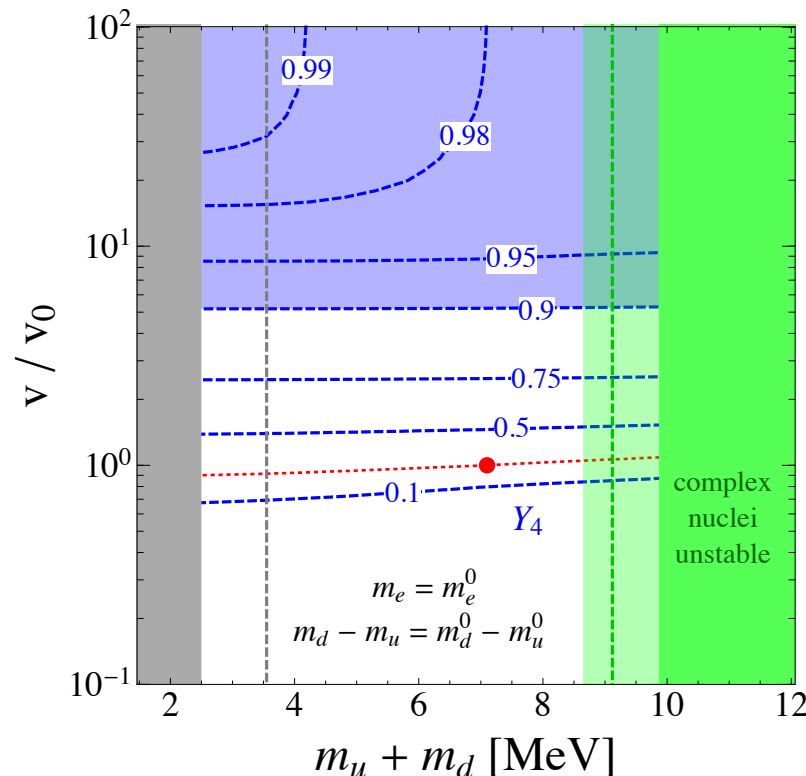
@ large v

there is no anthropic constraint on v

Possibility:

- To explain our proximity to the nearby helium boundary,

Recall



$$dP = f(m_a/v, v) n_{\text{nuc}}(m_a) n_{\text{BBN}}(Y_4) n_{\text{other}}(m_a, v) d \ln m_a d \ln v$$

m_a dependence of Y_4 can be ignored:

$$dP = f_v(v) n_{\text{BBN}}(v) d \ln v, \quad f_v(v) = \int_{\mathcal{O}} f(m_a/v, v) d \ln m_a$$

If the probability force

$$\frac{\partial f_v}{\partial v} \begin{cases} > 0 & \text{for } v < 10^2 v_0 \\ < 0 & \text{for } v > 10^2 v_0 \end{cases}$$

we can understand that observers such as ourselves are more probable than large v .

Change of sign of the probability force (POSSIBILITY):

If new physics cuts off the fine-tuning at scale m btw v_0 and $10^2 v_0$,

then probability force may decrease at m .

➡ Possibility of the new physics

Another possibility:

- the assumption that $n_{\text{other}}=1$ is invalid.

The weak scale may affect the environment for observers by changing

- the physics of stars
- the strength of shock waves in SN explosion (ejection of heavy elements)

Conclusions

- The observed value of the weak scale lies within the critical regime where BBN transitions from producing all hydrogen to almost all helium.
- In 3-dim. parameter space (m_u, m_d, m_e), we are in the finite volume which determined anthropically from the existence of complex nuclei and hydrogen.
- There is a finite volume in (v, m_a) space where..
 - ◆ BBN does not produce too much helium
 - ◆ hydrogen and complex nuclei are stable
- η and M_{Pl} scan: runaway v (other catastrophe boundary)
- $v/v_0 > 10^2$ case:

there must be some other reason why universes with these large v are disfavored (possibility a little hierarchy, such as multi-TeV SUSY/ another cosmological explanation)

Backups

Scanning heavy flavor

In the case of $v/v_0 > 10^2$, we may consider the heavy flavors.

- Decay after 100s could lead to decay products which dissociate He (He \rightarrow p).

muon

If $v/v_0 \gg 10^2$ ($\eta_\mu \gg \eta_B$), EM shower leads to significant dissociation.

- Heavy flavors decay charged pion btw 1s and 100s, then

$$\pi^+ n \rightarrow \pi^0 p, \quad \pi^- p \rightarrow \pi^0 n$$

universes (with $n/p \sim 1$) end up not being He dominated.

muon case: $10 < v/v_0 < 10^2, \quad y_\mu/y_{\mu,0} \sim 10^{-2}$

- Heavy flavor decays directly produce protons (avoiding helium domination)

strange case: $\Lambda(uds) \rightarrow p\pi^- \quad 10 < v/v_0 < 10^2, \quad y_s/y_{s,0} \sim 10^{-2}$

Deuterium bottleneck with varying η

Deuterium bottleneck:

Due to the large entropy-per-baryon in the early universe, deuterium is efficiently photo-dissociated until the temperature drops such that the fraction of photons with sufficient energy to destroy deuterium is of order the baryon-to-photon ratio.

$$\eta \exp(-B_D / T_D) \sim 1 \quad \text{so,} \quad T_D \sim \ln \eta$$

Binding energy of complex nuclei

