インフレーション理論の基礎と近年の観測に関して

E-ken Summer-school @ 勝山ニューホテル 2014.9.2

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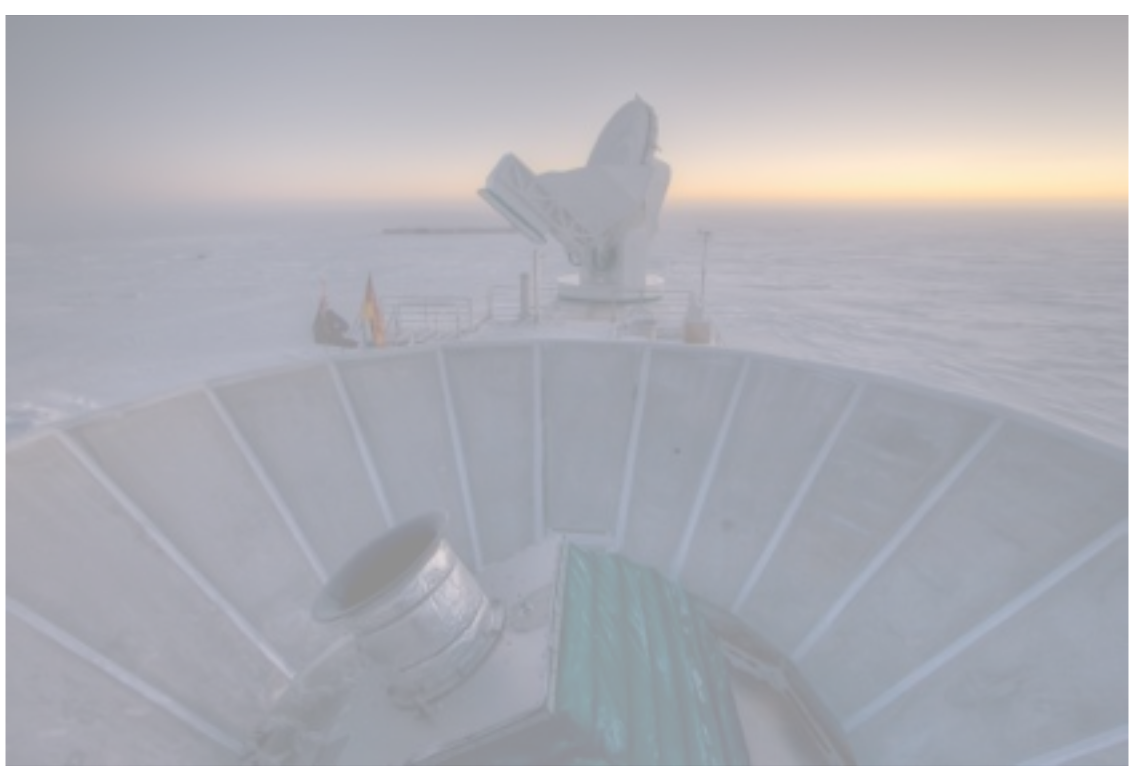
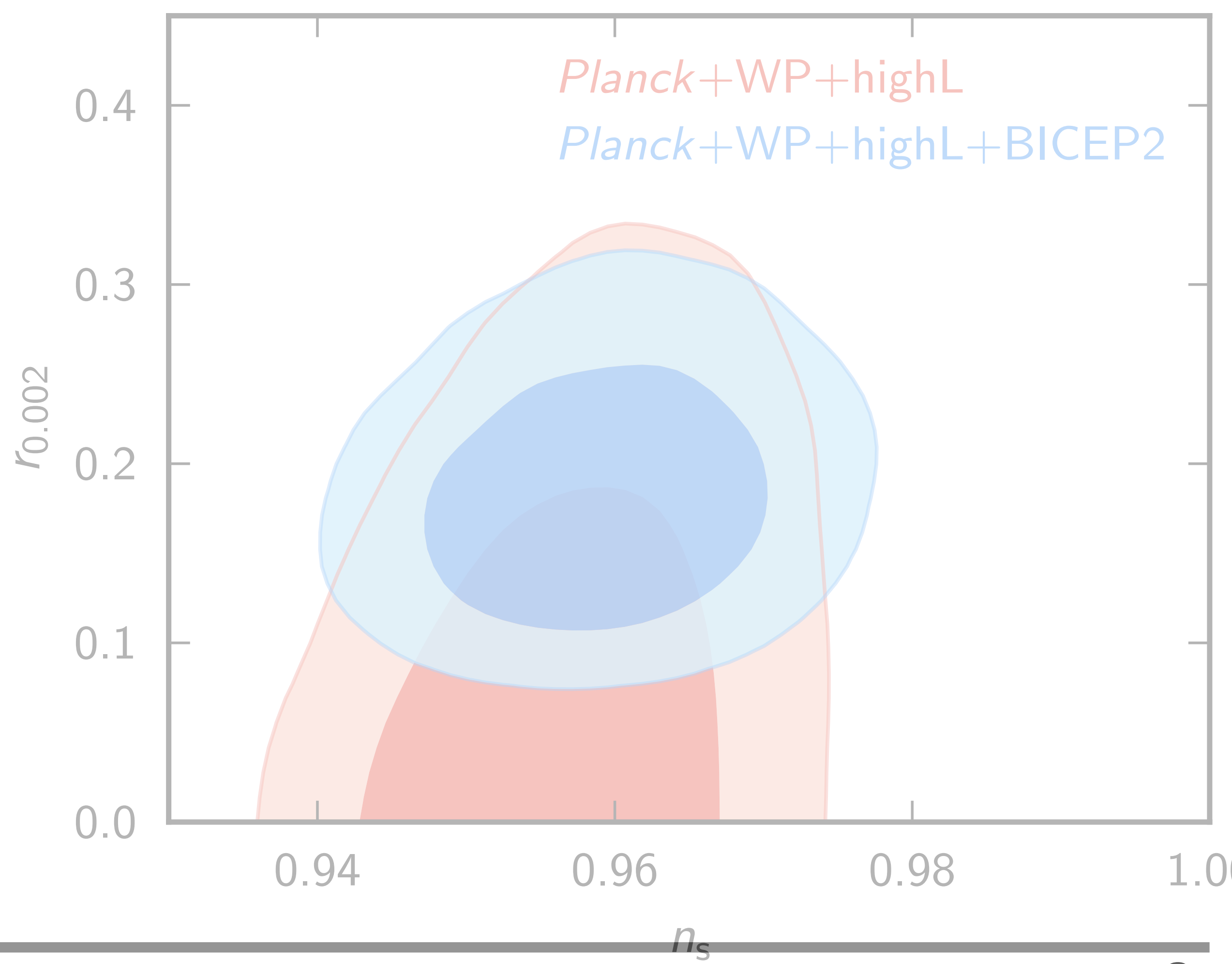
BICEP2 observation..

The indirect evidence of the gravitational wave of whose origin is INFLATION!!

In this opportunity, I want to know inflation mechanisms.





Contents

- Basic Picture of Inflation
- Inflation Models
- Cosmological Perturbations
- Observables for Some Models
- CMB Physics
- Constraints on Inflation models from Observations

Basic Picture of Inflation

MOTIVATION

The standard cosmology explains our universe well.

- The Big-Bang Cosmology

The age of the nucleosynthesis : t =10^{-2}~10^{2} sec. (T = 10~0.1 MeV)

↓

Now!! : t = 10^{10} years (T = 2.7K)

The standard cosmology is based on

the Freedman-Robertson-Walker (FRW) metric

$$ds^2 = dt^2 - a(t)^2 \left\{ \frac{dr^2}{1 - Kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \right\}$$

a: scale factor

K: curvature of our universe

This metric describes the spatially homogeneous and isotropic universe.

MOTIVATION

However, there are several problems in the standard cosmology

- Large-scale smoothness / Horizon problem
- Small-scale inhomogeneity
- Spatial flatness
- Unwanted (topological) relics
- Cosmological constant
- etc..

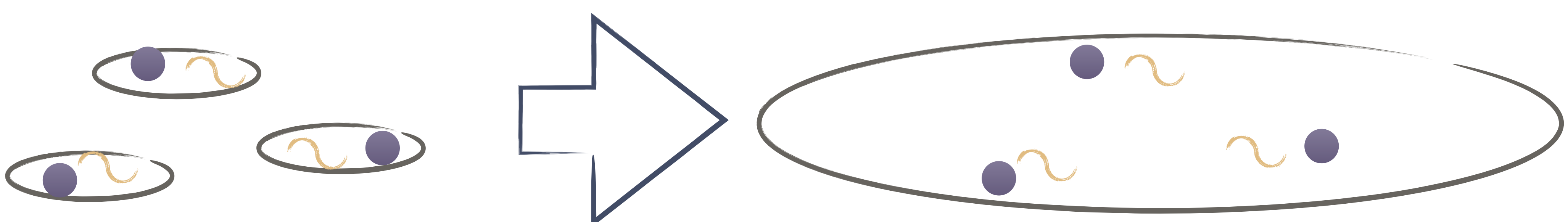
MOTIVATION

However, there are several problems in the standard cosmology

- Large-scale smoothness / Horizon problem

Why our universe is described by the FRW metric (homogeneous and isotropy)?

Entropy in particle horizon (causal horizon) today is much larger than that early universe..



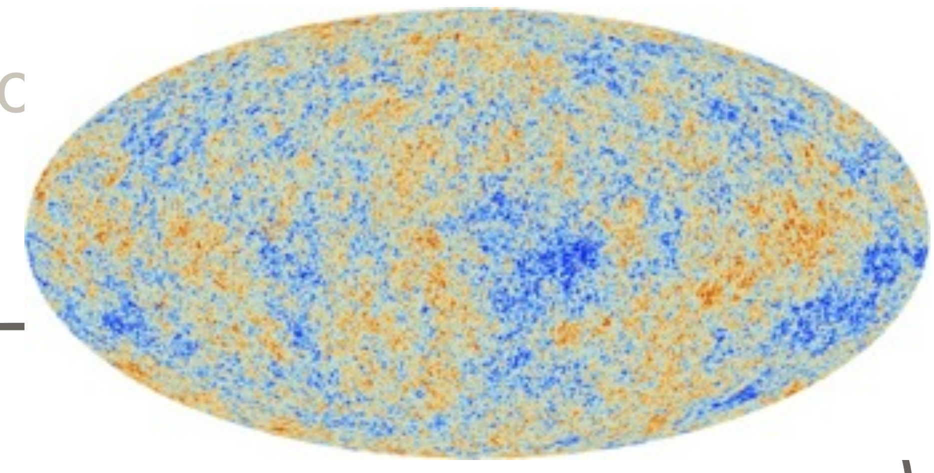
Most of matter and radiation are causally independent in early universe.

By growing the comoving horizon, these are in horizon today.
=> these are causally independent ???

MOTIVATION

However, there are several problems in the standard cosmology

- Large-scale smoothness / Horizon prc
- Small-scale inhomogeneity



Cosmic Microwave Background (CMB) tells us that small temperature (density) fluctuations existed in early universe.

These glow to the scale which explains structure of the universe.

What is the origin of these fluctuations?

MOTIVATION

However, there are several problems in the standard cosmology

- Large-scale smoothne
- Small-scale inhomoge
- Spatial flatness

Einstein eqs. w/ FRW metric

$$H^2 + \frac{K}{a^2} = \frac{8\pi G}{3}\rho$$

Hubble parameter

$$H \equiv \frac{1}{a}\frac{da}{dt}$$

Why is the universe so closely approximated by flat Euclidean space?

$$\Omega_K = \Omega - 1 = \frac{\rho - \rho_{\rm crit}}{\rho_{\rm crit}} = \frac{K}{(aH)^2} \qquad \rho_{\rm crit} = \frac{3H^2}{8\pi G}$$

Observed value (today)

$$100\Omega_K \sim -\mathcal{O}(1)$$

CMB only @ Planck&WMAP

This implies that a fine-tuning of Ω close to 1 in the early universe is required.

MOTIVATION

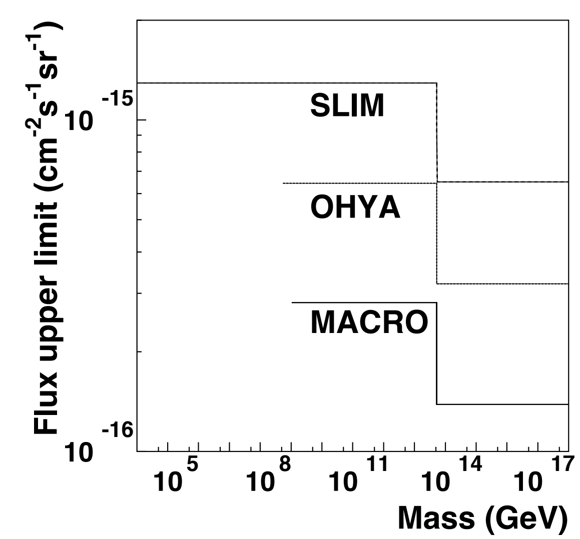
However, there are several problems in the standard cosmology

- Large-scale smoothness / Horizon problem
- Small-scale inhomogeneity
- Spatial flatness
- Unwanted (topological) relics

Within the context of GUTs, the superheavy monopoles are generically produced. => over close universe?

Cosmic ray monopole searches

=> No monopole is detected!



Why?

MOTIVATION

However, there are several problems in the standard cosmology

- Large-scale smoothness / Horizon problem
- Small-scale inhomogeneity
- Spatial flatness
- Unwanted (topological) relics
- Cosmological constant

What is the origin of the cosmological constant..

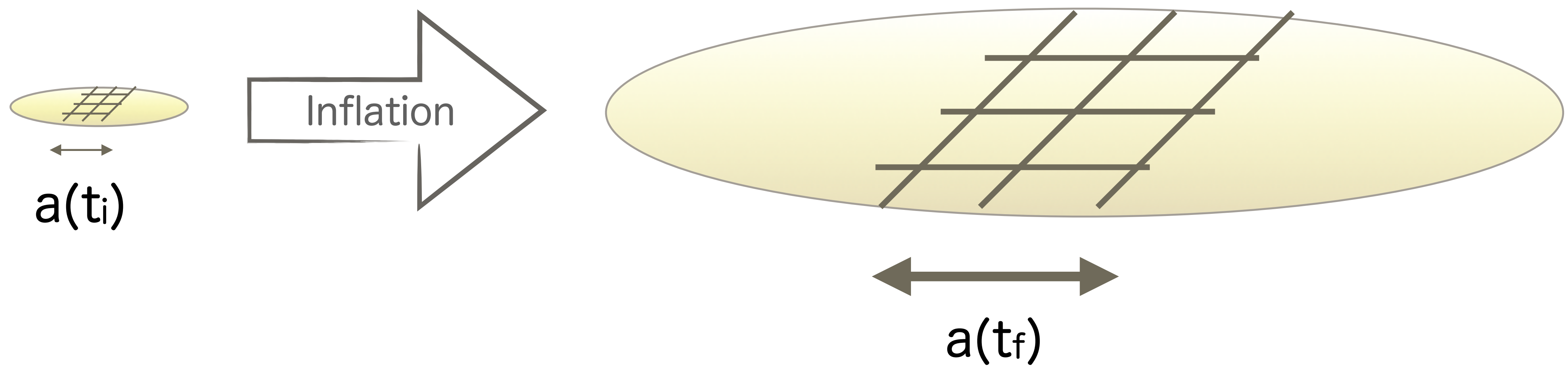
BASIC PICTURE OF INFLATION

Basic idea

There was the age in which the vacuum energy dominated the universe.

In this age, the scale factor increases exponentially.

the region which is small ($\sim H^{-1}$), smooth, and causally coherent glows to the size explaining the size of the current universe.



e-foldings number: describes the accelerating expansion

$$N(t) \equiv \ln \frac{a(t_{\rm end})}{a(t)} \simeq 50 \sim 60$$

t_{end} : finish time of inflation

t : time during the inflation

Who does play a role in inflation?

Old-type inflation

Starobinsky-model/R^2 inflation (A.A.Starobinsky: 1980)

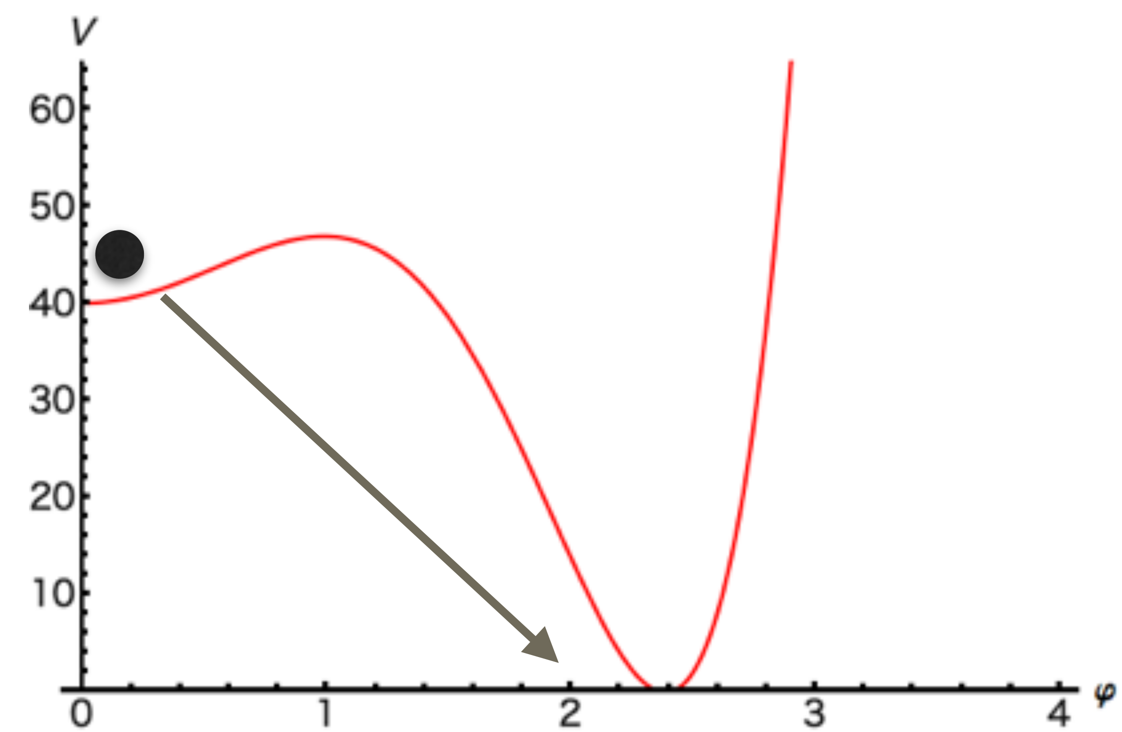
$$S = \int d^4x \sqrt{-g} \frac{M_{Pl}^2}{2} \left(R + \frac{R^2}{6M^2} \right)$$

GUT inflation (Kazanas:1980 / Sato:1981 / Guth:1981)

GUT Higgs fields lead to inflation.

There is a serious problem:

- Since we use the 1st phase transition of the “Higgs”, vacuum bubbles break homogeneity and isotropy of the universe..



New-type models!!

Inflaton fields lead to inflation

Slow-roll approx.

We define the slow-roll parameter

$$\epsilon \equiv -\frac{1}{H^2}\frac{dH}{dt} = -\frac{d\ln H}{dN}$$

describing the evolution of Hubble parameter.

H changes slowly $\Longleftrightarrow$ $\epsilon \ll 1$

The condition for the accelerating expansion universe is $\varepsilon < 1$ since;

$$\frac{\ddot{a}}{a} = H^2(1-\epsilon)$$

In an epoch of inflation, $\varepsilon \ll 1$ is satisfied since Hubble parameter change slowly.

Slow-roll approx.

If the Lagrangian for the inflaton field

$$P = P(X, \phi) \qquad X \equiv \frac{1}{2} g_{\mu\nu} \partial^\mu \phi \partial^\nu \phi$$

In particular, for the canonical inflaton field $P = X - V(\phi)$

We obtain

$$3M_{Pl}^2 H^2 = X + V(\phi),$$
$$M_{Pl}^2 \dot{H} = -X,$$
$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = 0$$

In general models!

The Friedmann eq. is given by

$$3M_{Pl}^2 H^2 = \rho = 2XP_{,X} - P,$$
$$M_{Pl}^2 \dot{H} = -XP_{,X},$$

and the continuous equation

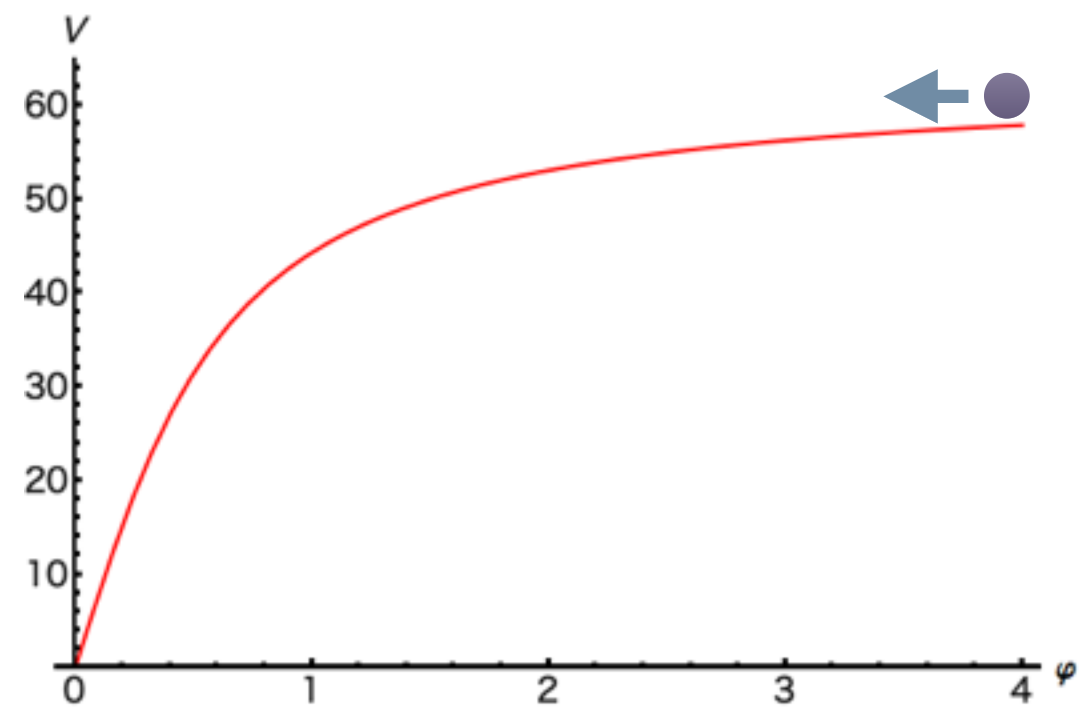
$$(P_{,X} + 2XP_{,XX})\ddot{\phi} + 3HP_{,X}\dot{\phi} + 2XP_{,X\phi} - P_{,\phi} = 0$$

In this model, the slow-roll parameter is obtained as follows:

$$\epsilon = -\frac{\dot{H}}{H^2} = \frac{3X}{X + V(\phi)}$$

In the slow-roll approx. this implies that:

$$\epsilon \ll 1 \iff X \ll V(\phi)$$



The parameters explain flatness of the inflaton potential

$$\epsilon_V \equiv \frac{M_{Pl}^2}{2}\left(\frac{V_{,\phi}}{V}\right)^2, \quad \eta_V \equiv \frac{M_{Pl}^2 V_{,\phi\phi}}{V}, \quad \xi_V^2 \equiv \frac{M_{Pl}^4 V_{,\phi} V_{,\phi\phi\phi}}{V^2}$$

Potential: Flat $\Longleftrightarrow$ $\{\epsilon_V, \eta_V, \xi_V\} \ll 1$

Under the slow-roll approx., the slow-roll parameters for potential relate to ε as;

$$\epsilon_V \simeq \epsilon, \quad \eta_V \simeq 2\epsilon - \frac{\dot{\epsilon}}{2H\epsilon}, \quad \xi_V^2 \simeq \left(2\epsilon_V - \frac{\dot{\eta}_V}{2H\eta_V}\right)\eta_V$$

Some observable is described by these parameters.

SOLVING PROBLEMS IN INFLATION SCENARIOS

By the exponentially accelerating universe

- Large-scale smoothness / Horizon problem
- Spatial flatness
- Unwanted (topological) relics

By the quantum fluctuation of inflaton

- Small-scale inhomogeneity

I will show later

Nobody knows (maybe)

- Cosmological constant

Inflation Models

INFLATION MODELS

Contents

– i –

– ii –

Too many models

(Encyclopaedia Inflationaris) J. Martin, C. Ringeval, V. Vennin [arXiv:1303.3787]

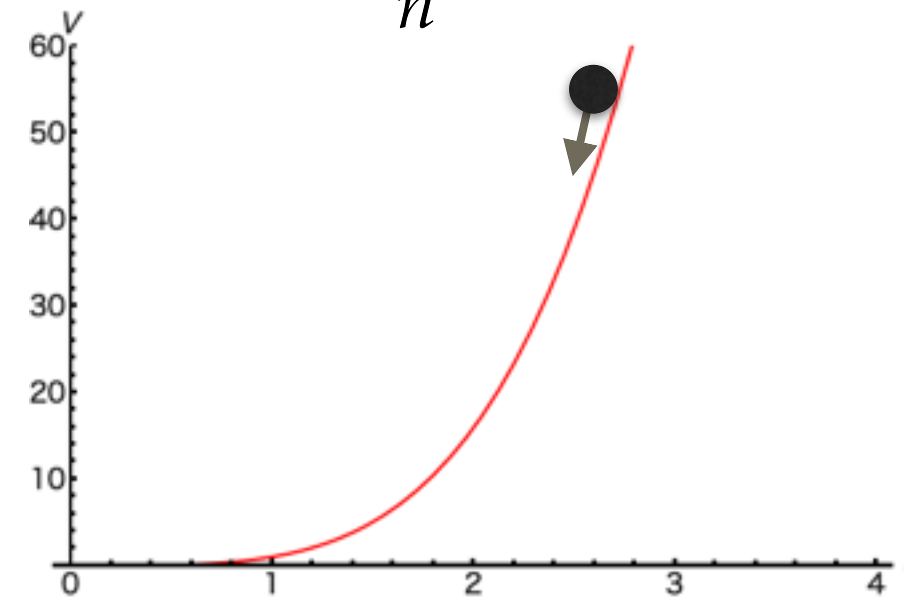
Now we consider the inflation models

Inflation models are roughly classified as..

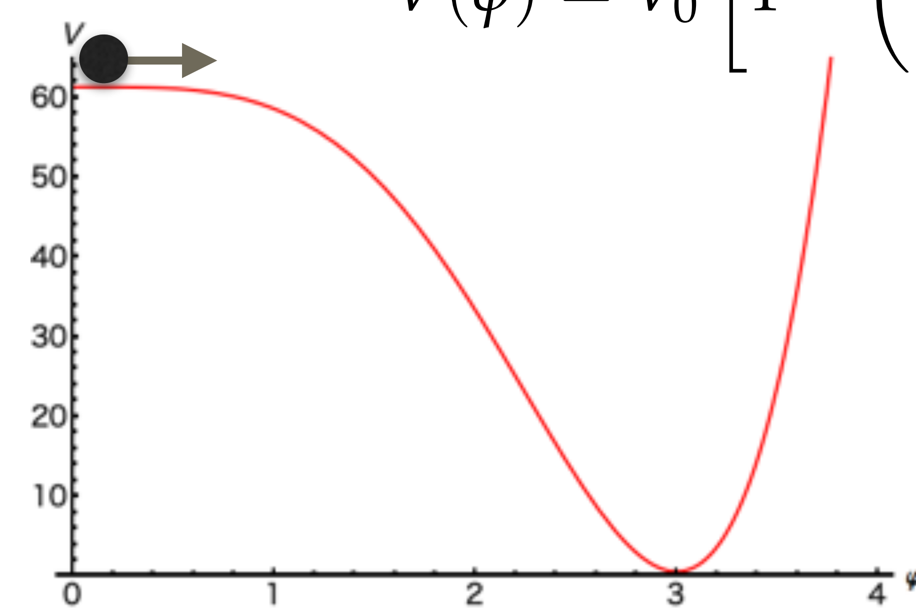
E.W.Kolb 1999

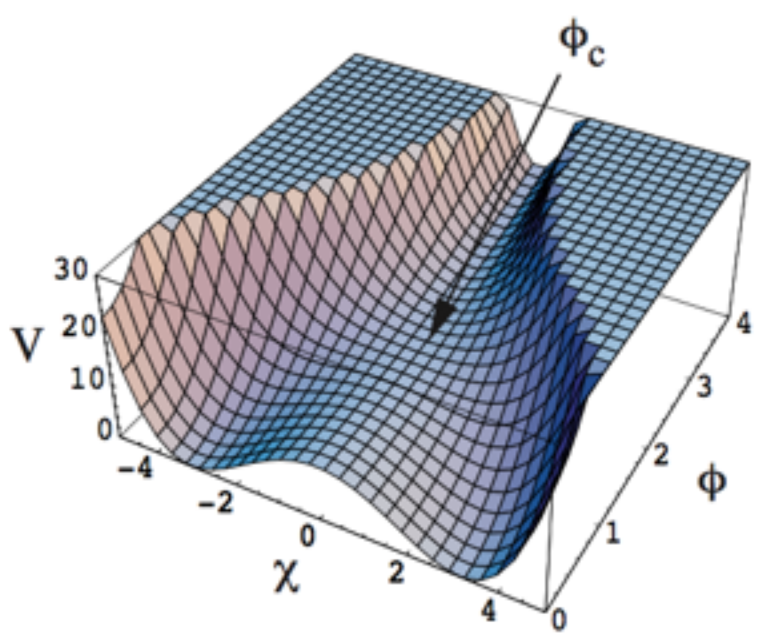
- Large-field models
- Small-field models
- Hybrid inflation models
- Multi-inflaton models

$$V(\phi) = \frac{\lambda}{n}\phi^n$$



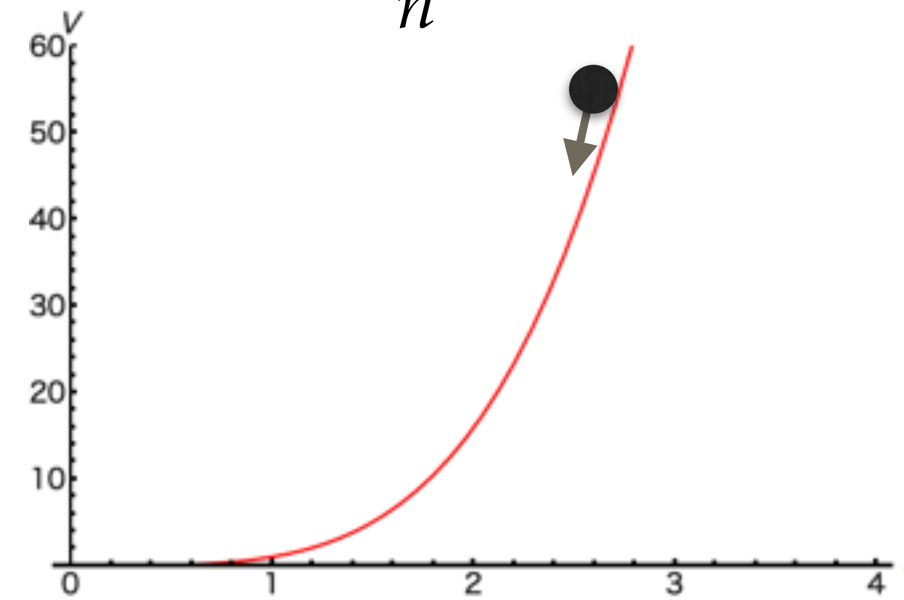
$$V(\phi) = V_0\left[1 - \left(\frac{\phi}{\mu}\right)^n\right]$$





$$V(\phi,\chi) = \frac{\lambda}{4}\left(\chi^2 - \frac{M^2}{\lambda}\right)^2 + \frac{1}{2}g^2\phi^2\chi^2 + \frac{1}{2}m^2\phi^2$$

$$V(\phi) = \frac{\lambda}{n}\phi^n$$



- Large-field models
- Small-field models
- Hybrid inflation models
- Multi-inflaton models

Initial value of the Inflaton field: $\phi > M_{Pl}$

$\rightarrow$ ϕ rolls down to the potential minimum

To realize the number of e-foldings: N>60

$\rightarrow$ $\phi_i > 3M_{Pl}$ is required

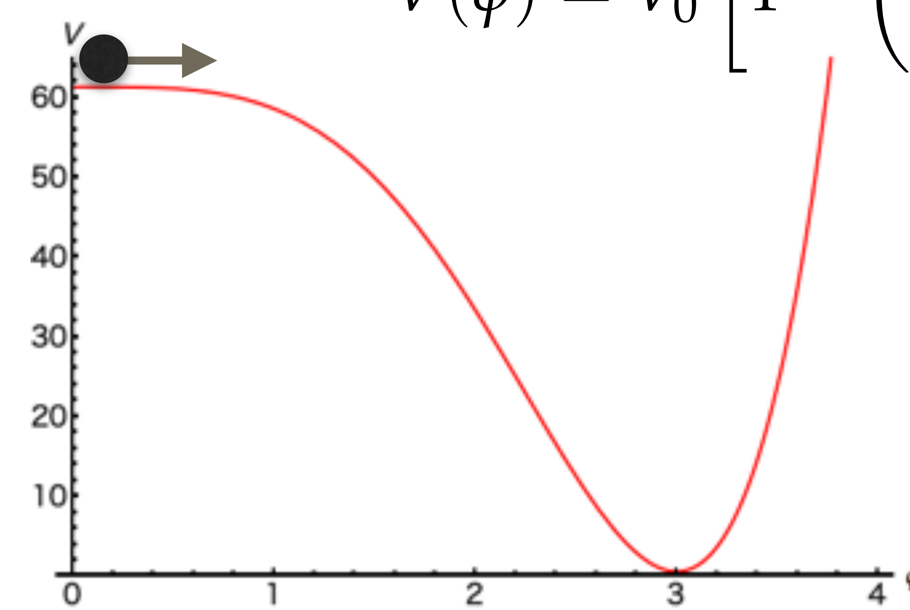
These models are characterized
by the potential expanded around ϕ=0 as below

- Large-field models
- Small-field models
- Hybrid inflation models
- Multi-inflaton models

Properties

- There are more parameters than large-field models
- initial condition for the inflaton: $\phi_i < M_{Pl}$

$$V(\phi) = V_0 \left[1 - \left(\frac{\phi}{\mu} \right)^n \right]$$



In this model,
no reason why the inflaton near the origin of the potential.

=> fine-tuning of the initial condition?

These models involve more than one scalar field.

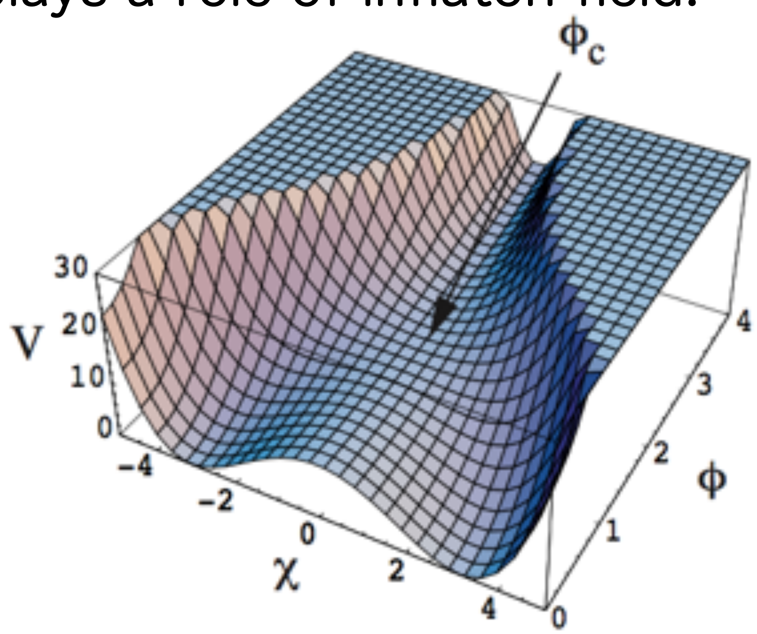
Not required:
ALL OF THEM ARE INFLATON!

- Large-field models
- Small-field models
- Hybrid inflation models
- Multi-inflaton models

One of the scalar field leads to phase transition (inducing fast-roll of the inflaton)

end of inflation $\epsilon \simeq 1$

ϕ plays a role of inflaton field.

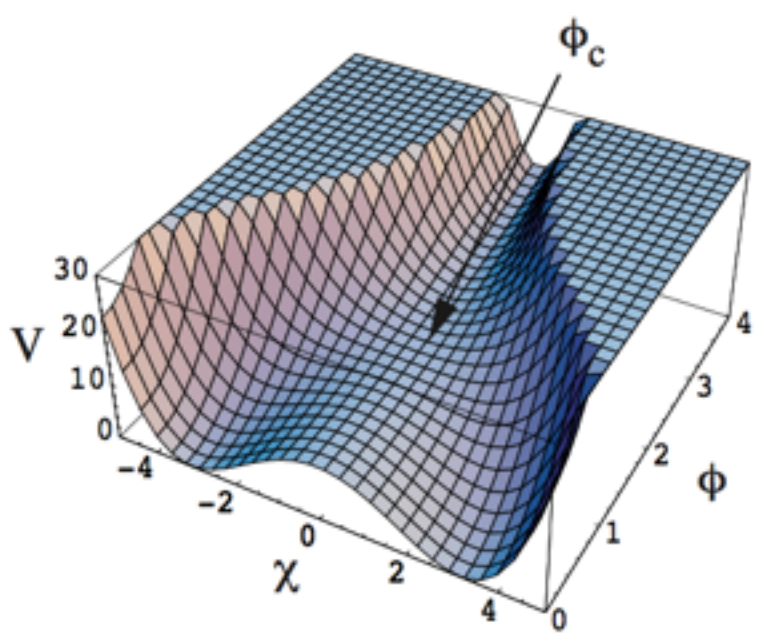


$$V(\phi,\chi) = \frac{\lambda}{4}\left(\chi^2 - \frac{M^2}{\lambda}\right)^2 + \frac{1}{2}g^2\phi^2\chi^2 + \frac{1}{2}m^2\phi^2$$

If we choose the parameter region in Hybrid inflation model appropriately, Inflation can be given rise to twice!

- Large-field models
- Small-field models
- Hybrid inflation models
- Multi-inflaton models

In such a model, they include more than two inflaton fields.



$$V(\phi,\chi) = \frac{\lambda}{4}\left(\chi^2 - \frac{M^2}{\lambda}\right)^2 + \frac{1}{2}g^2\phi^2\chi^2 + \frac{1}{2}m^2\phi^2$$

BUT!

Now we consider the models which involve only one-inflaton field.

Other models

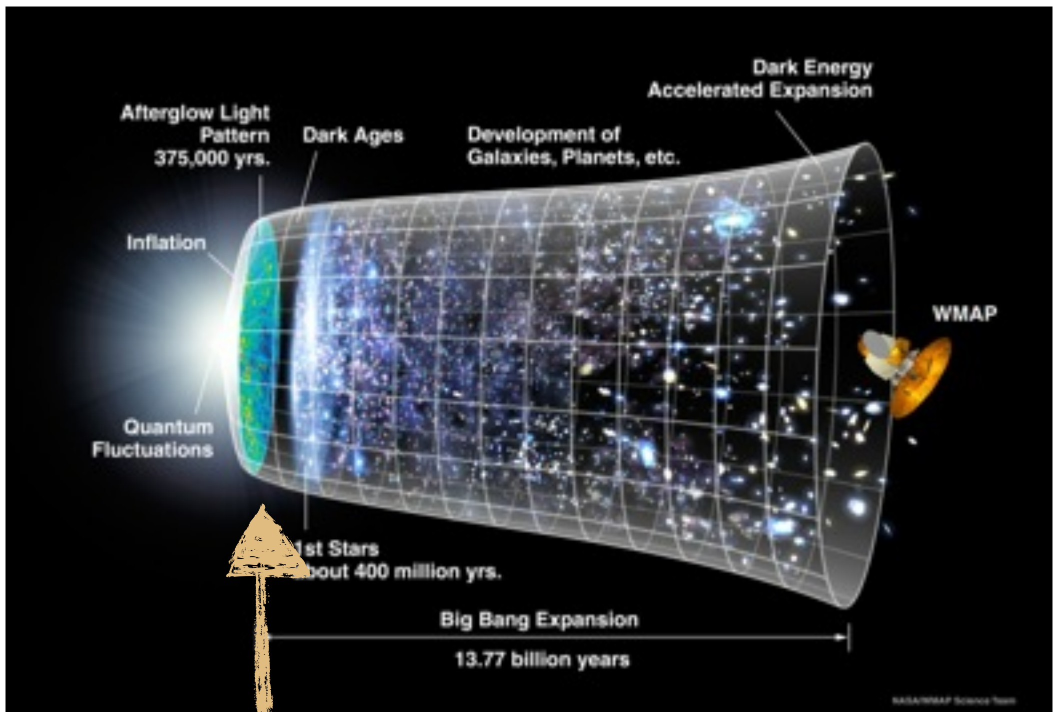
There also exist inflation models which are belong to four classification

- R^2 model / Higgs inflation: non-minimal coupling of matter and gravity
- Models which have no potential minimum
- Inflation is driven by the higher-order kinetic terms
- modified gravity models
- etc..

Cosmological Perturbations

REMNANTS OF THE INFLATION

How do we see (confirm) the inflation models?



Quantum fluctuations of the inflaton

affect the Cosmic Microwave Background (CMB).

Density and Temperature fluctuations in CMB

Polarization of CMB

In order to treat these fluctuations,

we use the perturbative method for cosmology

COSMOLOGICAL PERTURBATION

Now we add 1st perturbation to FRW metric.

$$g_{\mu\nu} = g^{(b)}_{\mu\nu} + \delta g_{\mu\nu}$$

The metric including perturbation is given as, $\delta g_{\mu\nu}$

$$ds^2 = a^2(\eta)\left\{-(1+2A)d\eta^2 + 2(\nabla_i B - S_i)d\eta dx^i + \left[(1+2\psi)\gamma_{ij} + 2\nabla_i\nabla_j E + 2\nabla_j F_i + h_{ij}\right]dx^i dx^j\right\}$$

where η is conformal time defined as $a(\eta)d\eta = dt$

Also, the energy-momentum tensor is expanded around Bkg scalar field

$$\phi(\eta, x^i) = \phi(\eta) + \delta\phi(\eta, x^i)$$

The relations btw $\delta\varphi$ and $\delta g_{\mu\nu}$ are obtained through the perturbative Einstein eqs.

$$\delta G^{\mu}{}_{\nu} = 8\pi G\, \delta T^{\mu}{}_{\nu}$$

Now, to avoid gauge (general coordinate) dependence on parameters, we introduce the gauge invariant valuables.

Beginning of the Gauge invariant cosmological perturbation: J.M.Bardeen (1980)

$$\Phi \equiv \psi - \mathcal{H}(E' - B), \quad \Psi \equiv A - \frac{1}{a}\left[a(E' - B)\right]'$$

$$\mathcal{R} \equiv \psi + \frac{\mathcal{H}}{\phi'}\delta\phi, \quad \zeta \equiv \psi - \frac{\mathcal{H}}{\rho'}\delta\rho$$

$$\delta\phi_\psi \equiv \delta\phi - \frac{\phi'}{\mathcal{H}}\psi, \quad \delta\rho_m \equiv \delta\rho - 3H\delta q$$

where $\delta q = -\frac{1}{a}P_{,X}\phi'\delta\phi$

By using the perturbative Einstein eqs., ψ relates to ${}^{(3)}R$ (spatial curvature)

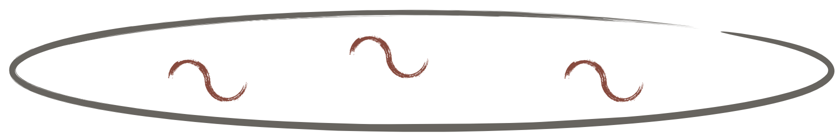
Gauge Fixing: $\delta\varphi = 0$ (Comoving gauge)

$\mathcal{R}$ is called the comoving curvature fluctuation

Outline for fluctuations:

The (quantum) fluctuation can be induced in the early time of inflation

k >> aH



Hubble radius: $(aH)^{-1}$

Quantum fluctuation with wavelength shorter than Hubble radius seeds the small inhomogeneity.

Assumption: This fluctuation begin to behave as classical fluctuation if wave-length (fluctuation) has the same order of Hubble radius

k ~ aH

This fluctuation freeze-out after exceeding the Hubble radius

k << aH

Now we consider the model that Lagrangian of inflaton field is written as..

$$P = P(X, \phi) \qquad X \equiv \frac{1}{2} g_{\mu\nu} \partial^\mu \phi \partial^\nu \phi$$

In general, the inflaton Lagrangian can include the higher power of X.
cf.) K-inflation model

For the scalar perturbations,

Now we consider

- Gauge Fixing: $\delta\varphi = 0$ (Comoving gauge)
- Flat space: K=0

we obtain

$$\left[Q_s a^2 \mathcal{R}' \right]' - Q_s a^2 c_s^2 \nabla^2 \mathcal{R} = 0$$

$$Q_s \equiv \frac{\phi'^2}{2\mathcal{H}} (P_{,X} + 2XP_{,XX})$$

$$c_s^2 \equiv \frac{P_{,X}}{P_{,X} + 2XP_{,XX}}$$

from perturbative Einstein equations.

$$\left[Q_s a^2 \mathcal{R}'\right]' - Q_s a^2 c_s^2 \nabla^2 \mathcal{R} = 0$$

$$Q_s \equiv \frac{\phi'^2}{2\mathcal{H}}(P_{,X} + 2XP_{,XX})$$

$$c_s^2 \equiv \frac{P_{,X}}{P_{,X} + 2XP_{,XX}}$$

Now we expand R in the Fourier space,

$$\mathcal{R}(\eta, \mathbf{x}) = \frac{1}{(2\pi)^3} \int d^3\mathbf{k} \left(u(\eta, \mathbf{k}) a(\mathbf{k}) + u^*(\eta, -\mathbf{k}) a^\dagger(\mathbf{k}) \right) e^{i\mathbf{k}\cdot\mathbf{x}}$$

we quantize this fluctuation and canonical field (in Momentum sp.) is defined as;

$$v = zu, \quad (z \equiv a\sqrt{2Q_s})$$

v satisfies Mukhanov-Sasaki equation (obtained from above equation)

$$v'' + \left(c_s^2 k^2 + \frac{z''}{z} \right) v = 0$$

we can solve this eq. by using Hankel function.

$$v'' + \left(c_s^2 k^2 + \frac{z''}{z} \right) v = 0$$

Solution for this equation is written as;

$$u(\eta, \mathbf{k}) = \frac{iH}{2(c_s k)^{3/2}} (1 + i c_s k \eta) e^{-i c_s k \eta}$$

Assumptions

- de Sitter limit

Corrections can be the order of $O(\varepsilon)$ if we consider the deviation from the de Sitter spacetime.

- Bunch-Davies vacuum

the solution (mode function) behaves as a plain-wave solution in asymptotically past ($\eta \to -\infty$).

This mode function "u" is important when we consider (calculate) the correlation function of the curvature fluctuation:

$$\langle 0 | \mathcal{R}(0, \mathbf{k}_1) \mathcal{R}(0, \mathbf{k}_2) | 0 \rangle , \qquad \langle 0 | \mathcal{R}(0, \mathbf{k}_1) \mathcal{R}(0, \mathbf{k}_2) \mathcal{R}(0, \mathbf{k}_3) | 0 \rangle$$

- power spectrum
- spectrum index
- running of index

- Non-Gaussianity

Observables:

$$\langle 0 | \mathcal{R}(0,\mathbf{k}_1)\mathcal{R}(0,\mathbf{k}_2) | 0 \rangle = \frac{1}{(2\pi)^3} P_{\mathcal{R}} \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2)$$

So-called "Power Spectrum" for the scalar fluctuations

$$\Delta^2_{\mathcal{R}}(\mathbf{k}) \equiv \frac{k^3}{(2\pi)^3} P_{\mathcal{R}}(\mathbf{k}) = \frac{H^2}{8\pi^2 M^2_{Pl} \epsilon c_s}$$

by using the mode function $u(\eta,k)$!!

Spectral index is defined as the k-dependence of the Δ_s^2.

$$n_s - 1 \equiv \left. \frac{d \ln \Delta^2_{\mathcal{R}}}{d \ln k} \right|_{c_s k = aH}$$

Spectral index also depends on scale k:
def. of running

$$\alpha_s \equiv \left. \frac{d \ln n_s}{d \ln k} \right|_{c_s k = aH}$$

and so on..

so,

$$\Delta^2_{\mathcal{R}}(\mathbf{k}) = \Delta^2_{\mathcal{R}}(\mathbf{k}_0) \left(\frac{k}{k_0} \right)^{n_s - 1 + \frac{1}{2}\alpha_s \ln k/k_0 + \ldots}$$

In order to compare with the observation,
we estimate these Observables at $c_s k = aH$.

Tensor perturbations satisfies the similar equation

$$v''_\lambda + \left(k^2 + \frac{z''_t}{z_t}\right) v_\lambda = 0$$

in order to normalize the perturbation tensor field canonically,

$$z_t \equiv a\sqrt{2Q_t} = \frac{a}{\sqrt{2}} M_{Pl}$$

where

$$v_\lambda = z_t h_\lambda, \qquad h_{ij} = e^{+}_{ij} h_+ + e^{\times}_{ij} h_\times$$

$\lambda = +, \times$

Polarization vector

We also obtain the correlation function for the tensor perturbation;

$$\langle 0 | \, h_{ij}(0, \mathbf{k}_1) h^{ij}(0, \mathbf{k}_2) \, | 0 \rangle = \frac{1}{k_1^3} \Delta_h^2(\mathbf{k}_1) \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2)$$

Power Spectrum, spectral index, and its running for tensor pert.;

$$\Delta_h^2(\mathbf{k}) = \frac{2H^2}{\pi^2 M_{Pl}^2}, \qquad n_t \equiv \left. \frac{d \ln \Delta_h^2}{d \ln k} \right|_{k=aH}, \qquad \alpha_t \equiv \left. \frac{d n_t}{d \ln k} \right|_{k=aH}$$

In particular in the case Lagrangian written as

$$P = X - V(\phi)$$

In this type model

$$c_s = 1$$

by using the slow-roll parameters,

$$n_s = 1 - 6\epsilon_V + 2\eta_V$$
$$n_t = -2\epsilon_V$$

and, we define the tensor-to-scalar ratio

$$r \equiv \frac{\Delta_h^2}{\Delta_{\mathcal{R}}^2} = 16\epsilon$$

These parameters are calculable if you define the shape of the inflaton potential.

Note:

The energy scale of inflation can be estimated as a function of the tensor-to-scalar ratio

$$\Delta_h^2 = \frac{2H^2}{\pi^2 M_{Pl}^2} \sim \frac{2}{3\pi^2}\frac{V}{M_{Pl}^4}$$

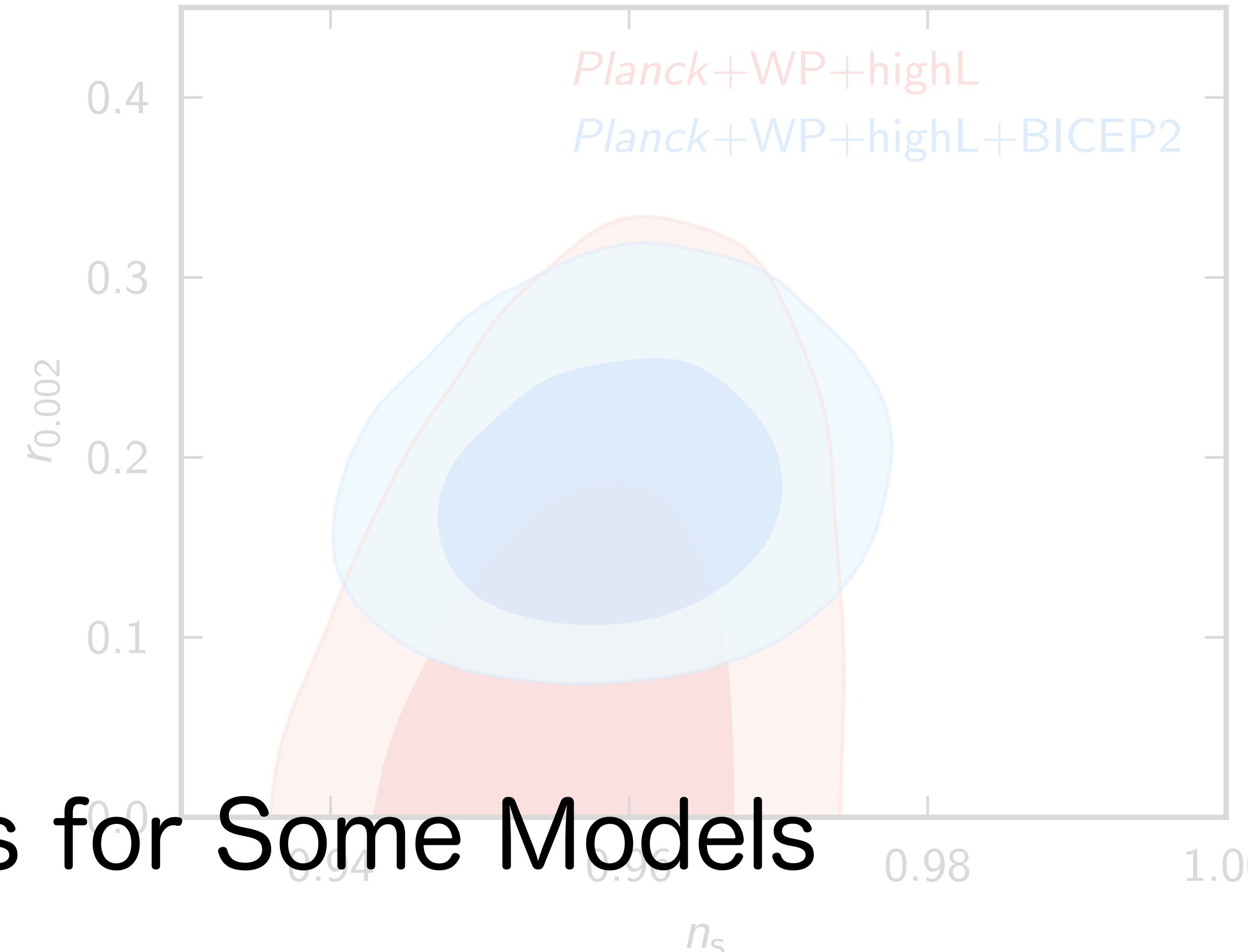
$$\Delta_{\mathcal{R}}^2 \sim 10^{-9} \qquad \text{(measured)}$$

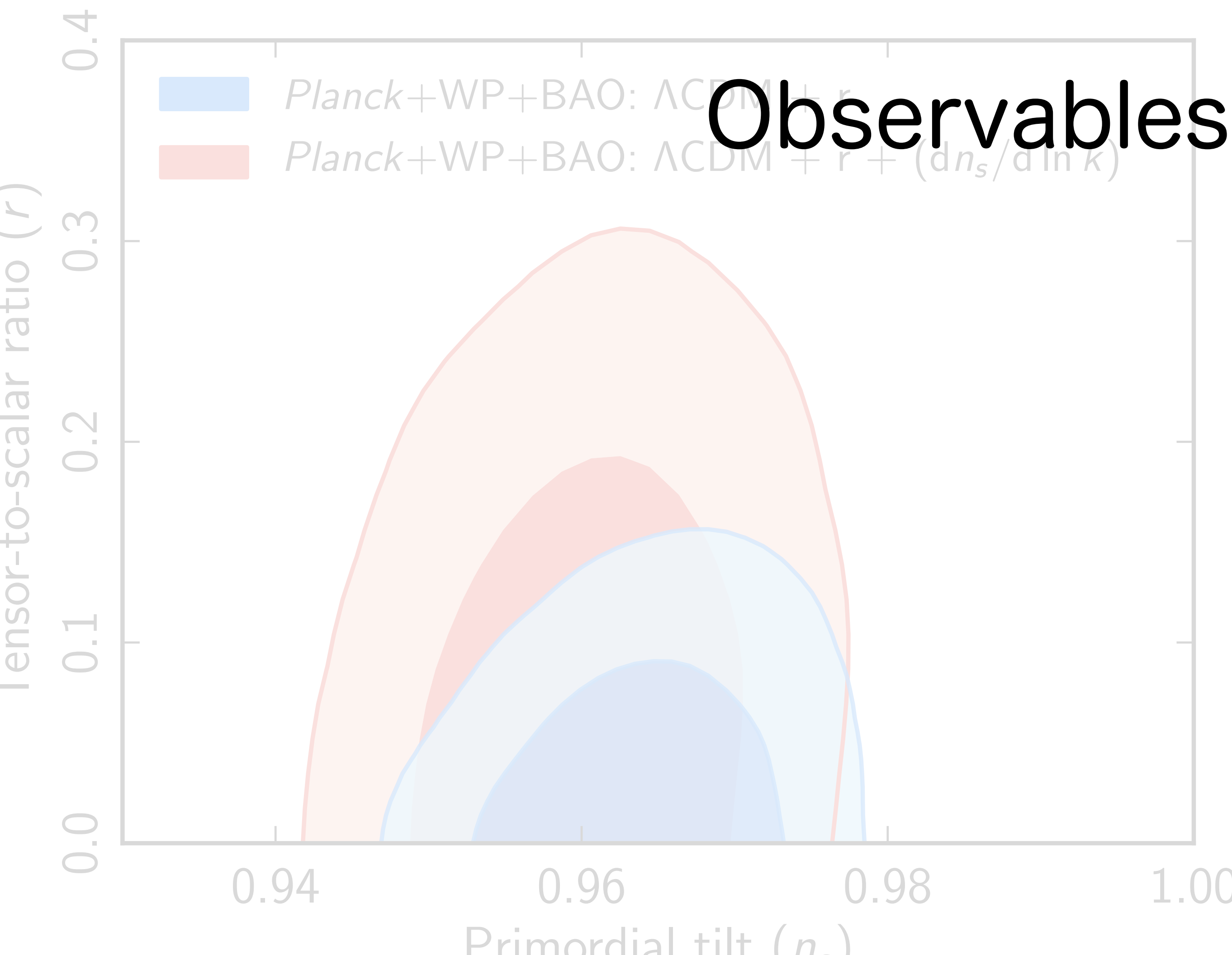
$$\Rightarrow \quad V \sim \left(\frac{r}{0.01}\right)(8.5 \times 10^{15}\mathrm{GeV})^4$$

SHORT SUMMARY

- Motivation (difficulty of the standard cosmology).
- Basic picture of the inflation mechanism.
- Calculating the small inhomogeneity induced by the quantum fluctuation of the inflaton.

Observables for Some Models





Lyth bound

Lyth (1997)

Recall;
$$N(t) = -\int_t^{t_{\rm end}} H d\tilde{t} \simeq \frac{1}{M_{Pl}^2} \int_{\phi_{\rm end}}^{\phi} \frac{V}{V_{,\tilde{\phi}}} d\tilde{\phi}$$

then, by using the relation btw the tensor/scalar ratio and the slow-roll parameter, we obtain,

$$r \simeq 16\epsilon_V = 8M_{Pl}^2 \left(\frac{V_{,\phi}}{V}\right)^2$$

In the inflation age, since the tensor/scalar ratio does not change dramatically, we obtain

$$\frac{\Delta\phi}{M_{Pl}} \simeq \mathcal{O}(1) \times \left(\frac{r}{0.01}\right)^{1/2}$$

Lyth bound

This implies that..

large r <=> large-field model
small r <=> small-field model

in one-inflaton models

ϕ^n potential / Chaotic inflation

If we consider the inflaton potential;

$$V(\phi) = \frac{\lambda}{n}\phi^n$$

slow-roll parameters:

$$\epsilon_V = \frac{n^2 M_{Pl}^2}{2\phi^2}, \quad \eta_V = \frac{n(n-1)M_{Pl}^2}{\phi^2}$$

In order to estimate the value of the inflaton field with e-folding N and the value of the inflaton field in the end of inflation.

The finish time of the inflation: we obtain the field value by setting $\varepsilon = 1$

$$\phi_{\rm end} = \frac{nM_{Pl}}{\sqrt{2}}$$

The value of the inflaton field with e-folding N

$$N \simeq \frac{1}{M_{Pl}^2}\int_{\phi_{\rm end}}^{\phi} \frac{V}{V_{,\tilde{\phi}}} d\tilde{\phi}$$

$$\phi(N) = \sqrt{2n\left(N + \frac{n}{4}\right)} M_{Pl}$$

Then,

$$n_s = 1 - \frac{2(n+2)}{4N+n}, \quad r = \frac{16n}{4N+n}$$

ϕ^n potential / Chaotic inflation

$$\phi(N) = \sqrt{2n\left(N + \frac{n}{4}\right)} M_{Pl} \qquad n_s = 1 - \frac{2(n+2)}{4N+n}, \quad r = \frac{16n}{4N+n}$$

For N=60,

$$\begin{cases} \phi(N=60) = 15.6 M_{Pl} & (n=2) \\ \phi(N=60) = 22.1 M_{Pl} & (n=4) \end{cases}$$

Spectral index and tensor-to-scalar ratio,

for n=2

$$n_s = 0.967$$
$$r = 0.132$$

for n=4

$$n_s = 0.951$$
$$r = 0.262$$

Natural inflation

If we consider the inflaton potential;

$$V(\phi) = V_0 \left[1 + \cos\frac{\phi}{f}\right]$$

which is generated as the potential of pseudo-Nambu-Goldstone Boson (PNGB) like axion effective potential.

$$\epsilon_V = \frac{M_{Pl}^2}{2f^2}\frac{\sin^2(\frac{\phi}{f})}{\left[1+\cos(\frac{\phi}{f})\right]^2}, \quad \eta_V = -\frac{M_{Pl}^2}{f^2}\frac{\cos(\frac{\phi}{f})}{1+\cos(\frac{\phi}{f})}$$

also, we can calculate the e-folding # for this potential.

$$N \simeq \frac{f^2}{M_{Pl}^2}\ln\left[\frac{\sin(\frac{\phi_{\rm end}}{2f})}{\sin(\frac{\phi(N)}{2f})}\right]$$

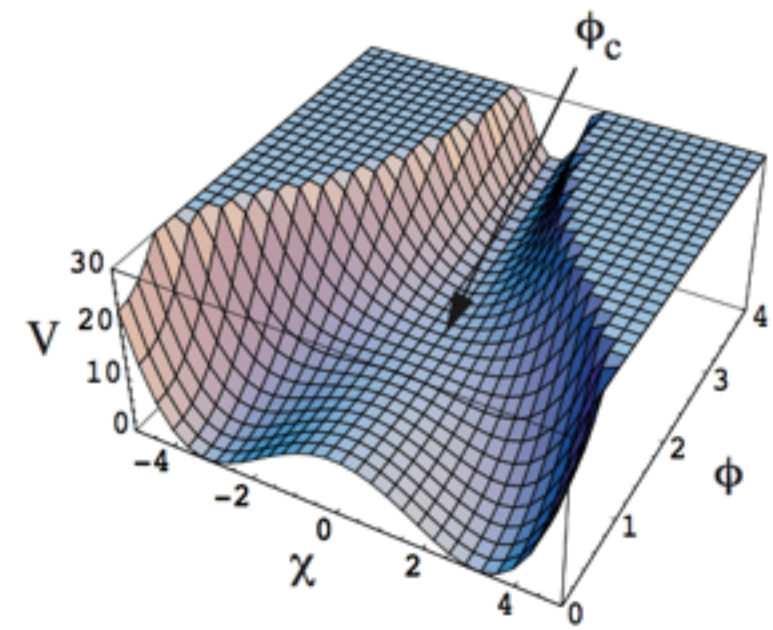
IN THIS MODEL!

Since there exists additional parameter f, both of large-inflation and small-inflation can be realized.

$$\begin{cases} f \gtrsim 1.5 M_{Pl} & \text{(large-field model)} \\ f \lesssim 1.5 M_{Pl} & \text{(small-field model)} \end{cases}$$

Hybrid inflation

During inflation, another scalar field is around the origin $\chi=0$



If we consider the potential as:

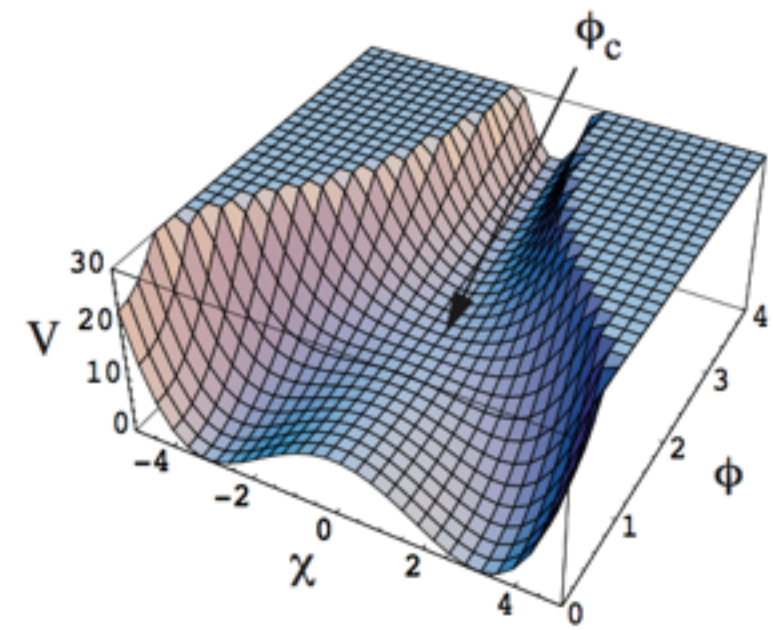
$$V(\phi,\chi)=\Lambda^4\left(1-\frac{\chi^2}{\mu^2}\right)^2+U(\phi)+\frac{1}{2}g\phi^2\chi^2$$

then,

$$\epsilon_V=\frac{M_{Pl}^2}{2}\frac{U_{,\phi}^2}{(\Lambda^4+U)^2}\,,\qquad \eta_V=\frac{M_{Pl}^2U_{,\phi\phi}}{(\Lambda^4+U)}$$

so, we can calculate slow-roll parameters if we determine the inflaton potential.

Hybrid inflation



$$V(\phi,\chi) = \Lambda^4\left(1-\frac{\chi^2}{\mu^2}\right)^2 + U(\phi) + \frac{1}{2}g\phi^2\chi^2$$

$$\epsilon_V = \frac{M_{Pl}^2}{2}\frac{U_{,\phi}^2}{(\Lambda^4+U)^2}, \quad \eta_V = \frac{M_{Pl}^2 U_{,\phi\phi}}{(\Lambda^4+U)}$$

if we assume

$$U(\phi) = \frac{1}{2}m^2\phi^2$$

$$r = 4M_{Pl}^2\frac{m^4\phi^2}{(\Lambda^4+\frac{1}{2}m^2\phi^2)^2}, \quad n_s = 1 + \frac{2m^2M_{Pl}^2}{(\Lambda^4+\frac{1}{2}m^2\phi^2)^2}(\Lambda^4 - m^2\phi^2)$$

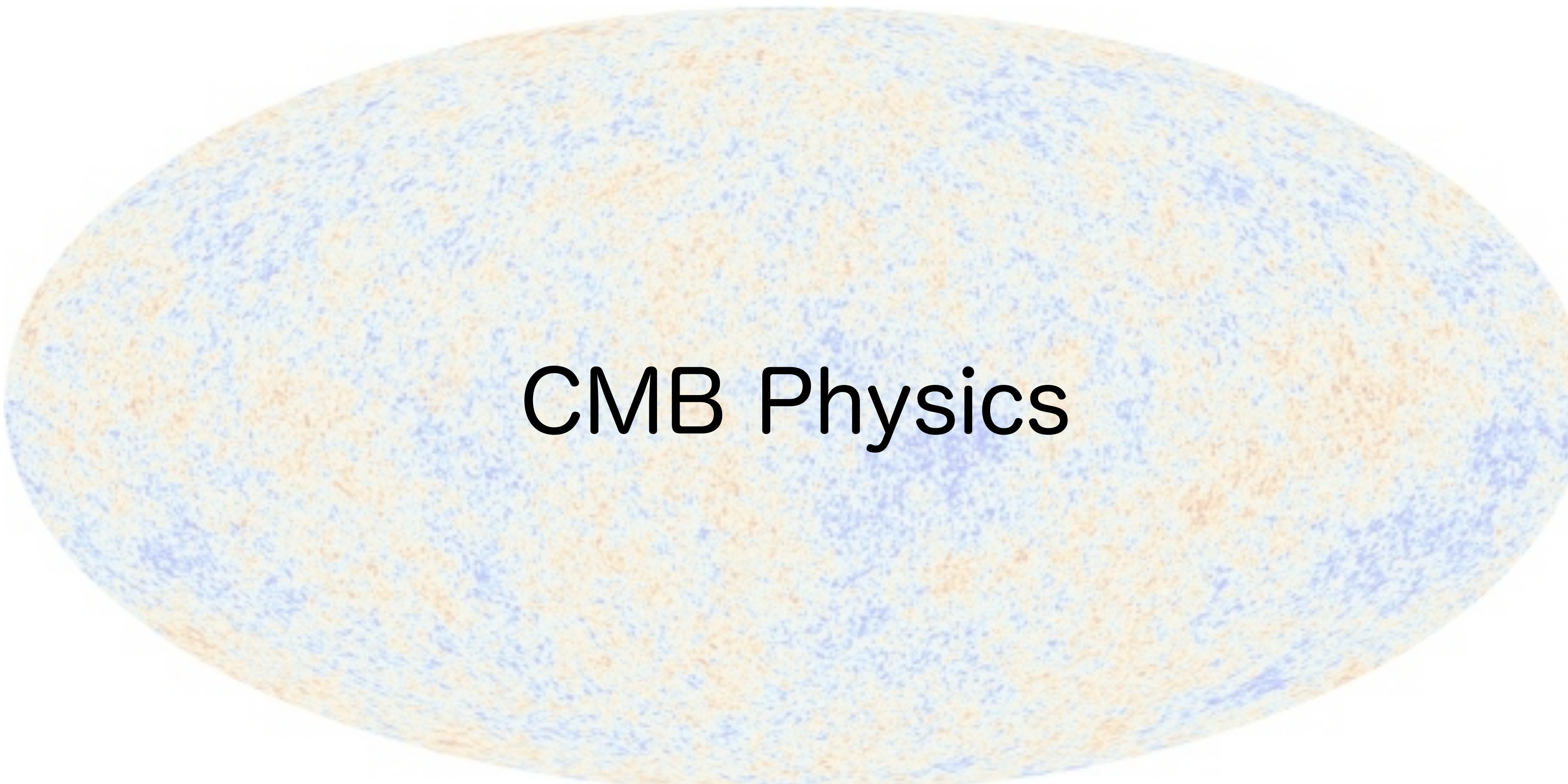
IF..

$$M_{Pl}^4 \sim \Lambda^4 \gg m^2\phi^2$$

$$r \sim 0, \quad n_s > 1$$

$$M_{Pl}^4 \sim \Lambda^4 \sim \frac{1}{2}m^2\phi^2$$

$$r > 1, \quad n_s \sim 1$$



CMB Physics

In BICEP2 experiment,

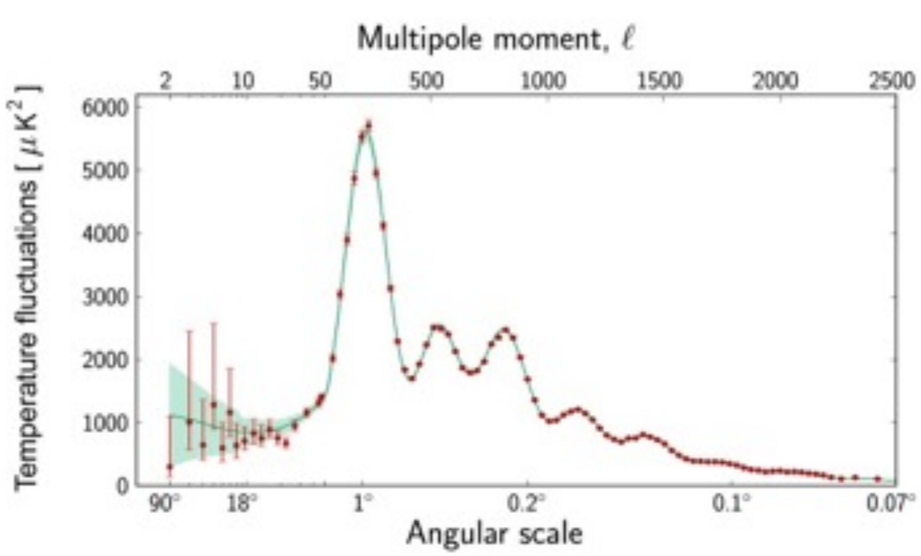
BICEP2 I: DETECTION OF *B*-mode POLARIZATION AT DEGREE ANGULAR SCALES

BICEP2 COLLABORATION - P. A. R. ADE[1], R. W. AIKIN[2], D. BARKATS[3], S. J. BENTON[4], C. A. BISCHOFF[5], J. J. BOCK[2,6], J. A. BREVIK[2], I. BUDER[5], E. BULLOCK[7], C. D. DOWELL[6], L. DUBAND[8], J. P. FILIPPINI[2], S. FLIESCHER[9], S. R. GOLWALA[2], M. HALPERN[10], M. HASSELFIELD[10], S. R. HILDEBRANDT[2,6], G. C. HILTON[11], V. V. HRISTOV[2], K. D. IRWIN[12,13,11], K. S. KARKARE[5], J. P. KAUFMAN[14], B. G. KEATING[14], S. A. KERNASOVSKIY[12], J. M. KOVAC[5,17], C. L. KUO[12,13], E. M. LEITCH[15], M. LUEKER[2], P. MASON[2], C. B. NETTERFIELD[4,16], H. T. NGUYEN[6], R. O'BRIENT[6], R. W. OGBURN IV[12,13], A. ORLANDO[14], C. PRYKE[9,7,17], C. D. REINTSEMA[11], S. RICHTER[5], R. SCHWARZ[9], C. D. SHEEHY[9,15], Z. K. STANISZEWSKI[2,6], R. V. SUDIWALA[1], G. P. TEPLY[2], J. E. TOLAN[12], A. D. TURNER[6], A. G. VIEREGG[5,15], C. L. WONG[5], AND K. W. YOON[12,13]

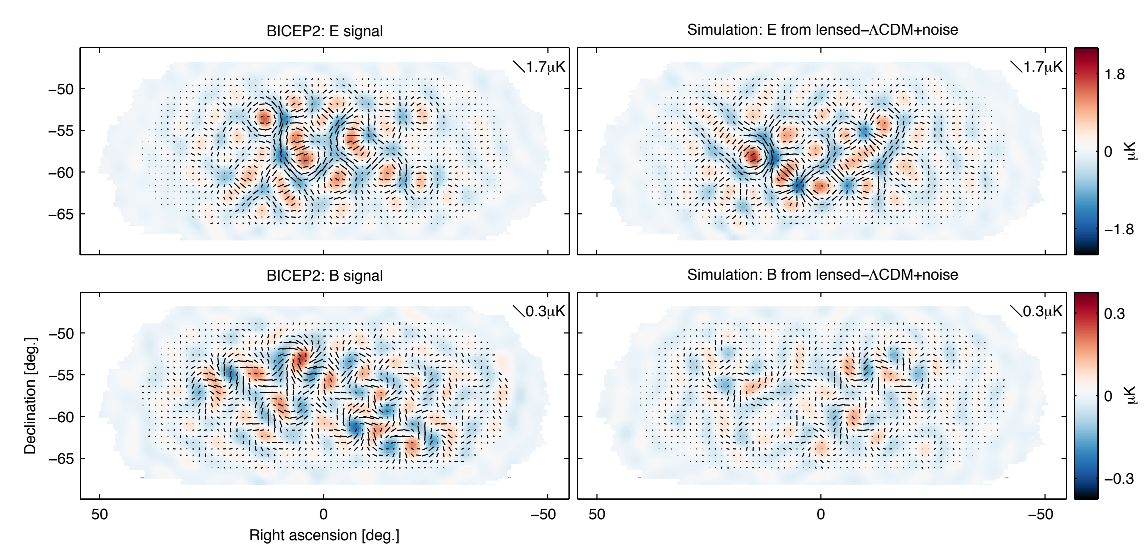
to be submitted to a journal TBD

Important observables of CMB

Temperature fluctuation

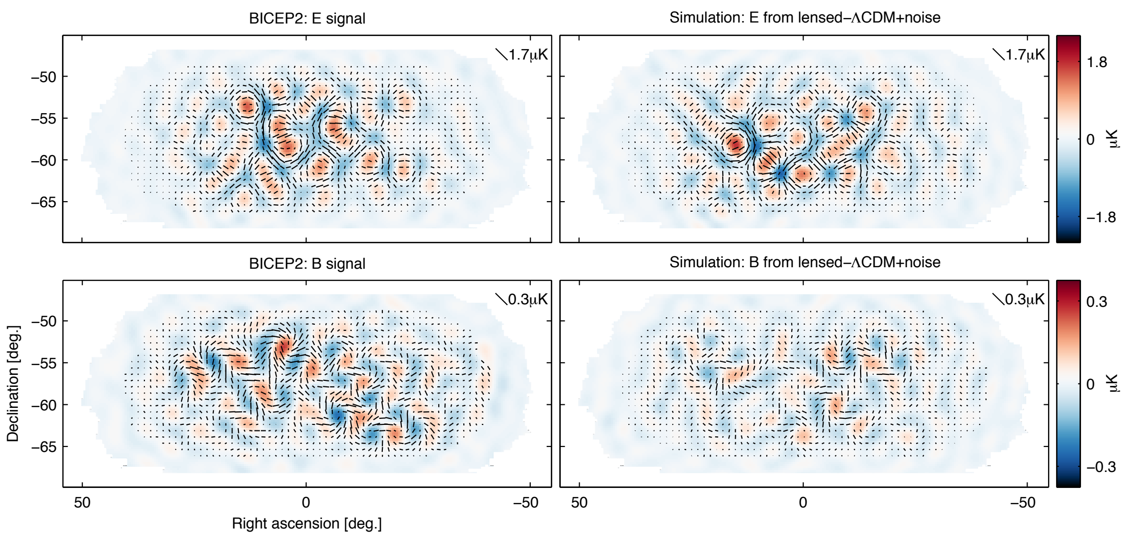


Polarization



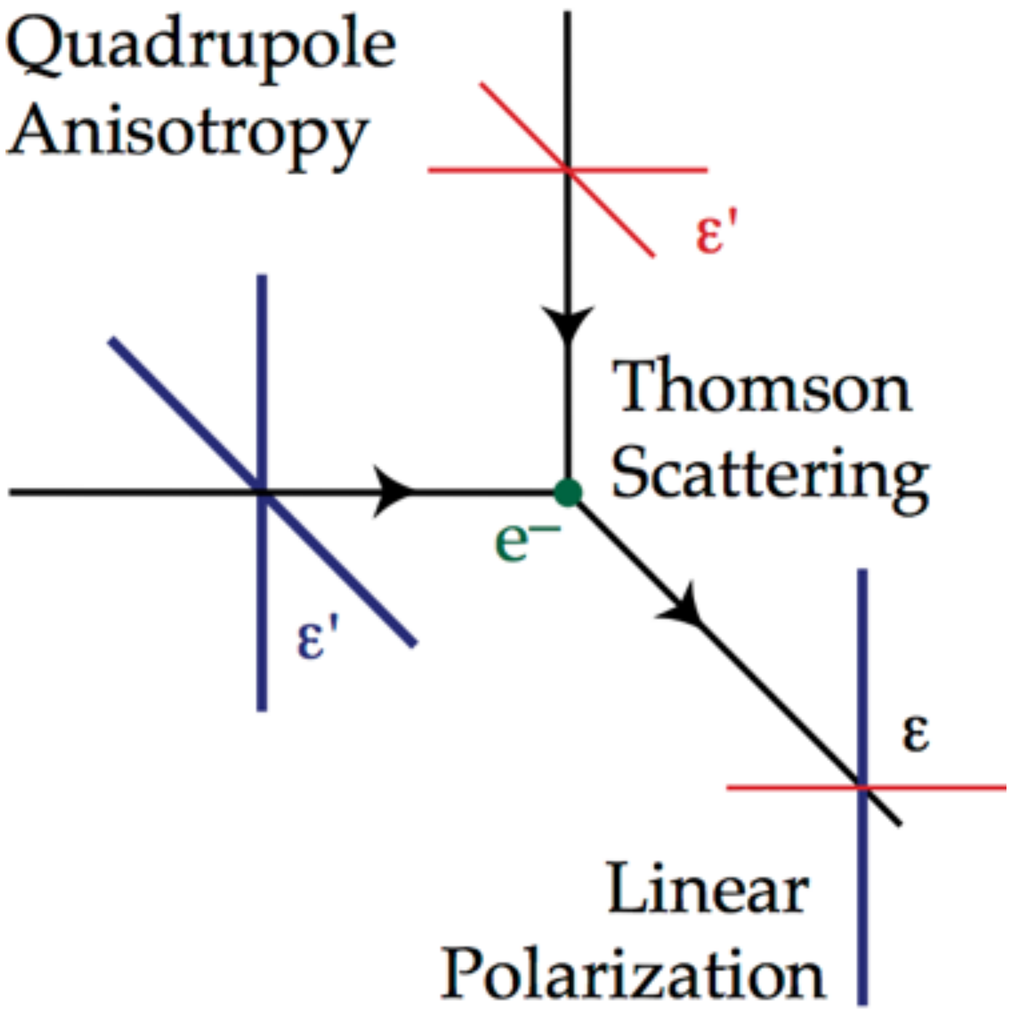
Polarization of CMB

The age of decoupling of photon with electron



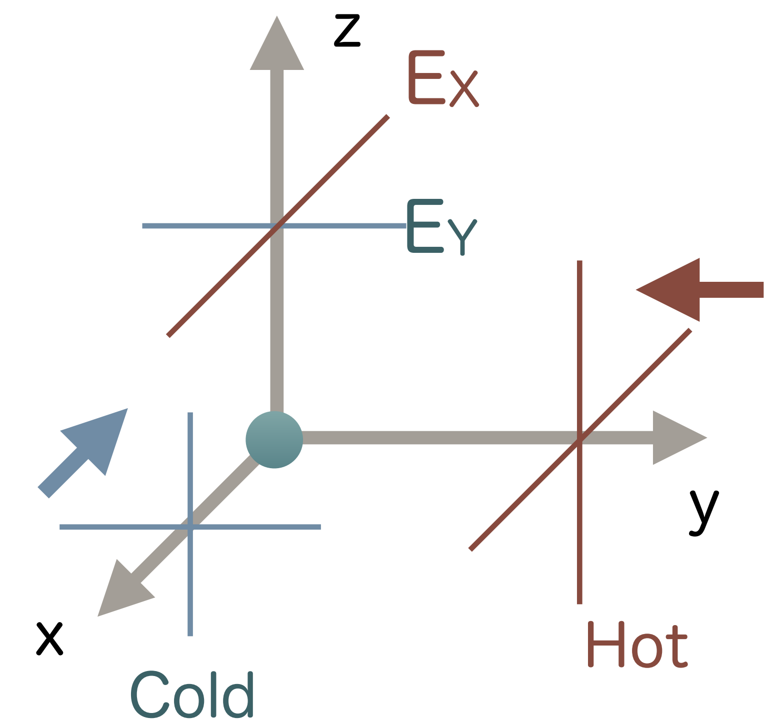
By Thomson scattering (long wave-length scattering), vertical component to the following direction remains.

If there exist temp. fluctuations of CMB, the polarization of CMB is induced through Thomson scatt.



http://background.uchicago.edu/~whu/

Quantitative analysis for polarization



Scattered photon (z-direction)

$$E_x = A_x e^{i(\omega t - kz + \delta_x)}, \qquad E_y = A_y e^{i(\omega t - kz + \delta_y)}$$

We define the strength tensor

$$S_{ij} \equiv \left\langle E_i E_j^* \right\rangle$$

and Stokes parameters

$$I \equiv \left\langle A_x^2 \right\rangle + \left\langle A_y^2 \right\rangle, \quad Q \equiv \left\langle A_x^2 \right\rangle - \left\langle A_y^2 \right\rangle,$$
$$U \equiv 2 \left\langle A_x A_y \cos\delta \right\rangle, \quad V \equiv 2 \left\langle A_x A_y \sin\delta \right\rangle$$

$$\delta \equiv \delta_x - \delta_y$$

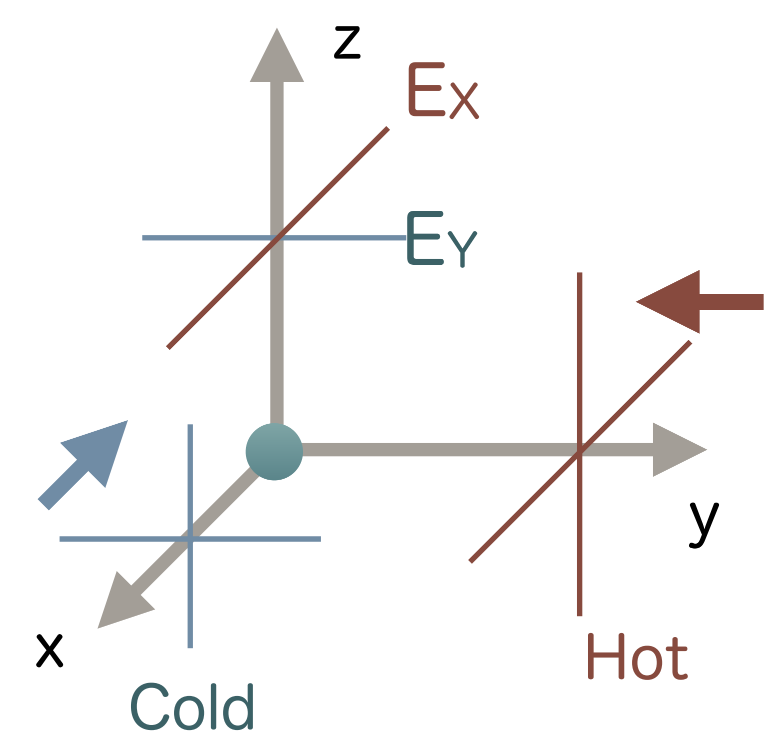
Thus, the strength tensor (matrix rep.)

$$S = \frac{1}{2} \begin{pmatrix} I + Q & U - iV \\ U + iV & I - Q \end{pmatrix}$$

Under rotation on xy-plane, Q mixes with U.

I: isotropic strength
Q: anisotropic strength
U: strength along x=±y
V: circular strength

Quantitative analysis for polarization



Under rotation on xy-plane (phase: φ), Q^2+U^2 is invariant,

and $Q\pm iU$ behave as spin-2 field under xy-rotation.

$$Q \pm iU \rightarrow \hat{Q} \pm i\hat{U} = e^{\mp 2i\varphi}(Q \pm iU)$$

thus, these values are expanded by using the spin±2 spherical harmonic function ${}_{\pm 2}Y_l^m$

$$Q \pm iU = \sum_{l,m} a_{lm}^{(\pm 2)} \left[{}_{\pm 2}Y_l{}^m(\mathbf{n})\right]$$

By using the ladder operator $\partial^{\pm}$, we obtain the scalar Y_l^m

$$(\eth^{\pm})^2[Q \pm iU] = \sum_{l,m} \sqrt{\frac{(l+2)!}{(l-2)!}} a_{lm}^{(\pm 2)} \left[Y_l{}^m(\mathbf{n})\right]$$

Then, we define E-mode and B-mode as follows

$$E(\mathbf{n}) \equiv -\frac{1}{2}\left[(\eth^-)^2(Q+iU) + (\eth^+)^2(Q-iU)\right] = \sum_{l,m} \sqrt{\frac{(l+2)!}{(l-2)!}} a_{lm}^{E} \left[Y_l{}^m(\mathbf{n})\right]$$

$$B(\mathbf{n}) \equiv -\frac{1}{2}\left[(\eth^-)^2(Q+iU) - (\eth^+)^2(Q-iU)\right] = \sum_{l,m} \sqrt{\frac{(l+2)!}{(l-2)!}} a_{lm}^{B} \left[Y_l{}^m(\mathbf{n})\right]$$

Similarly, we expand the temperature fluctuation

by using the spherical harmonic function.

$$\Theta \equiv \frac{\delta T}{T} = \sum_{l,m} a^T_{lm} Y_l{}^m(\theta, \phi)$$

The angular power spectra are defined as;

$$C_l^{XY} \equiv \frac{1}{2l+1} \sum_m \left\langle a^{X*}_{lm} a^Y_{lm} \right\rangle, \quad X, Y = T, E, B.$$

Then, these spectra can be written as;

$$C_l^{TT} \simeq \frac{2}{\pi} \int k^2 dk P_{\mathcal{R}} \Delta^2_{Tl}(k)$$

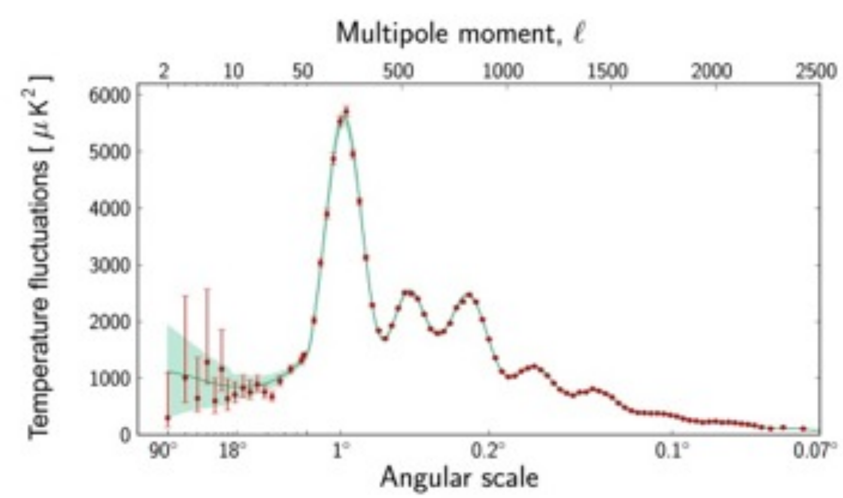
$$C_l^{EE} \simeq (4\pi)^2 \int k^2 dk P_{\mathcal{R}} \Delta^2_{El}(k)$$

$$C_l^{TE} \simeq (4\pi)^2 \int k^2 dk P_{\mathcal{R}} \Delta_{Tl}(k) \Delta_{El}(k)$$

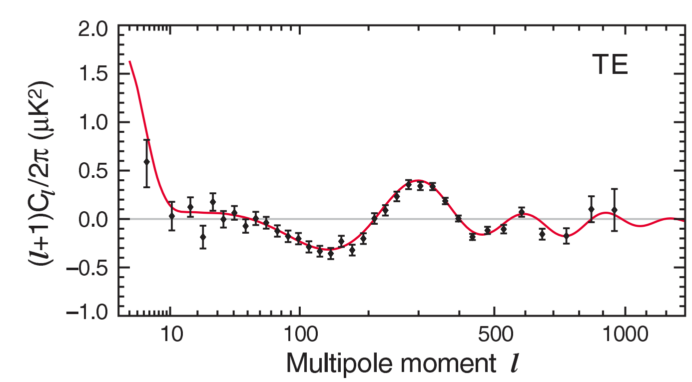
$$C_l^{BB} = (4\pi)^2 \int k^2 dk P_h \Delta^2_{Bl}(k)$$

Seljak et al. (1996)

C^{TT} is observed by Planck satellite.



C^{TE}, C^{EE} are also observed by WMAP.



So, these spectra are determined by

- power spectra (P_R, P_h)
- anisotropy (Δ_{Tl}, etc.. composed by perturbations)

Properties

E-mode

- This mode can be induced by the scalar perturbation and the tensor perturbation (BUT! the tensor contribution is small if the tensor-to-scalar ratio is small)

B-mode

- This mode can be induced by the vector perturbation and the tensor perturbation => Exponentially expansion leads attenuation of the amplitude of primordial vector pert..
- C^{BB} is small if the tensor-to-scalar ratio r<<1.
- The detection of the primordial tensor perturbation
  => the evidence of the gravitational wave whose origin is INFLATION.

Kamionkowski et al. (1996) / Seljak et al. (1996)

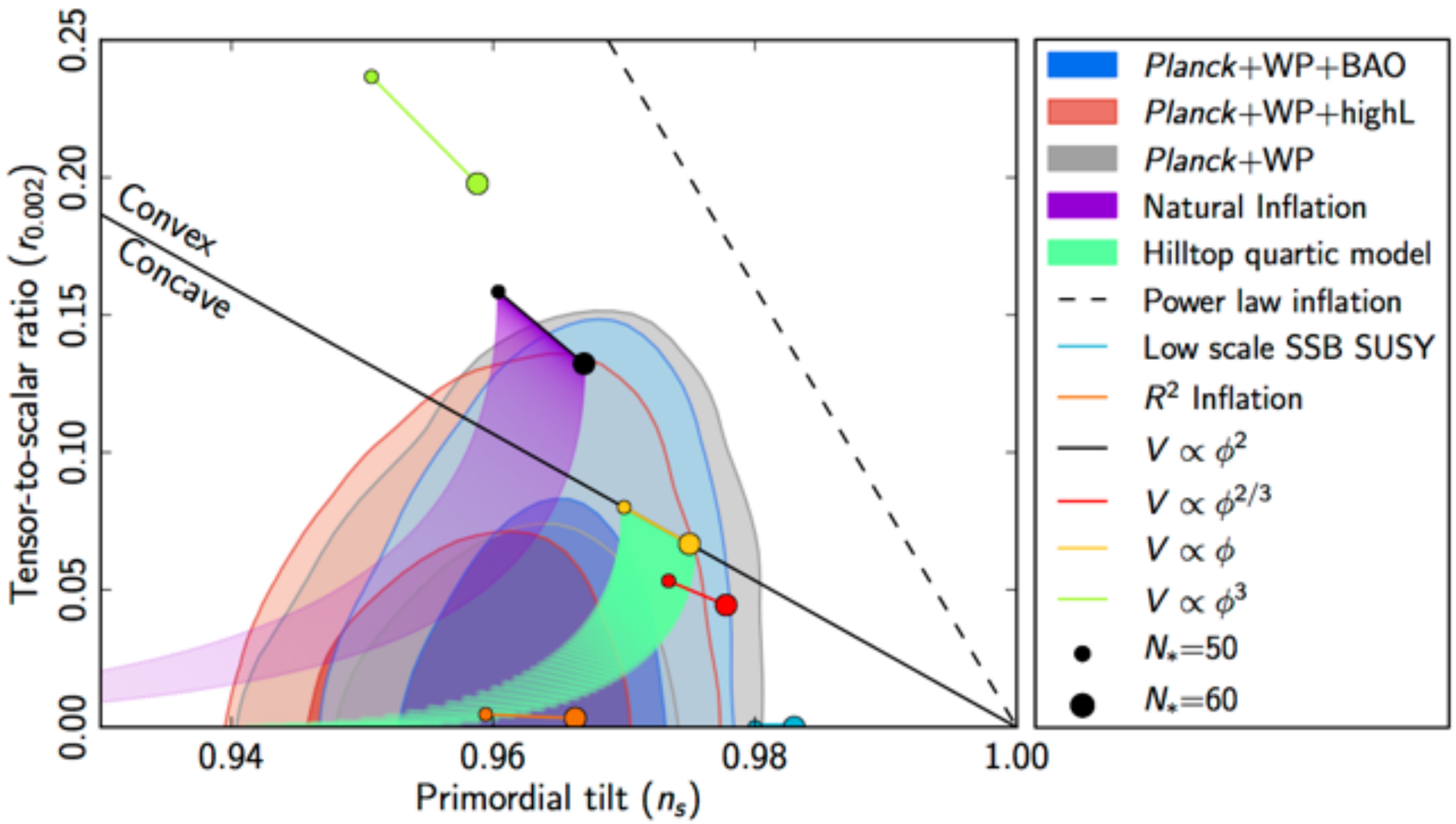
Constraints on Inflation Models from Observations

Planck 2013 results

much data for cosmology

Planck 2013 results. I. Overview of products and results
Planck 2013 results. II. Low Frequency Instrument data processing
Planck 2013 results. III. LFI systematic uncertainties
Planck 2013 results. IV. LFI beams
Planck 2013 results. V. LFI calibration
Planck 2013 results. VI. High Frequency Instrument data processing
Planck 2013 results. VII. HFI time response and beams
Planck 2013 results. VIII. HFI calibration and mapmaking
Planck 2013 results. IX. HFI spectral response
Planck 2013 results. X. HFI energetic particle effects
Planck 2013 results. XI. All-sky model of thermal dust emission
Planck 2013 results. XII. Component separation
Planck 2013 results. XIII. Galactic CO emission
Planck 2013 results. XIV. Zodiacal emission
Planck 2013 results. XV. CMB power spectra and likelihood
Planck 2013 results. XVI. Cosmological parameters
Planck 2013 results. XVII. Gravitational lensing by large-scale structure
Planck 2013 results. XVIII. The gravitational lensing-infrared background correlation
Planck 2013 results. XIX. The integrated Sachs-Wolfe effect
Planck 2013 results. XX. Cosmology from Sunyaev-Zeldovich cluster counts
Planck 2013 results. XXI. All-sky Compton-parameter map and characterization
Planck 2013 results. XXII. Constraints on inflation
Planck 2013 results. XXIII. Isotropy and statistics of the CMB
Planck 2013 results. XXIV. Constraints on primordial non-Gaussianity
Planck 2013 results. XXV. Searches for cosmic strings and other topological defects
Planck 2013 results. XXVI. Background geometry and topology of the Universe
Planck 2013 results. XXVII. Special relativistic effects on the CMB dipole
Planck 2013 results. XXVIII. The Planck Catalogue of Compact Sources
Planck 2013 results. XXIX. The Planck catalogue of Sunyaev-Zeldovich sources
Planck 2013 results. XXX. Cosmic infrared background measurements and implications for star formation
Planck 2013 results. XXXI. Consistency of the data
Planck 2013 results. Explanatory supplement
Planck 2013 results. Web-based explanatory supplement

Planck 2013 results



Spectral index (Planck+WMAP)

n_s=0.9603±0.0073

Large-field models like

$V \propto \phi^n \quad (n \in Z)$

=> severe constrained

Small-field models like

Natural inflation, R^2 inflation

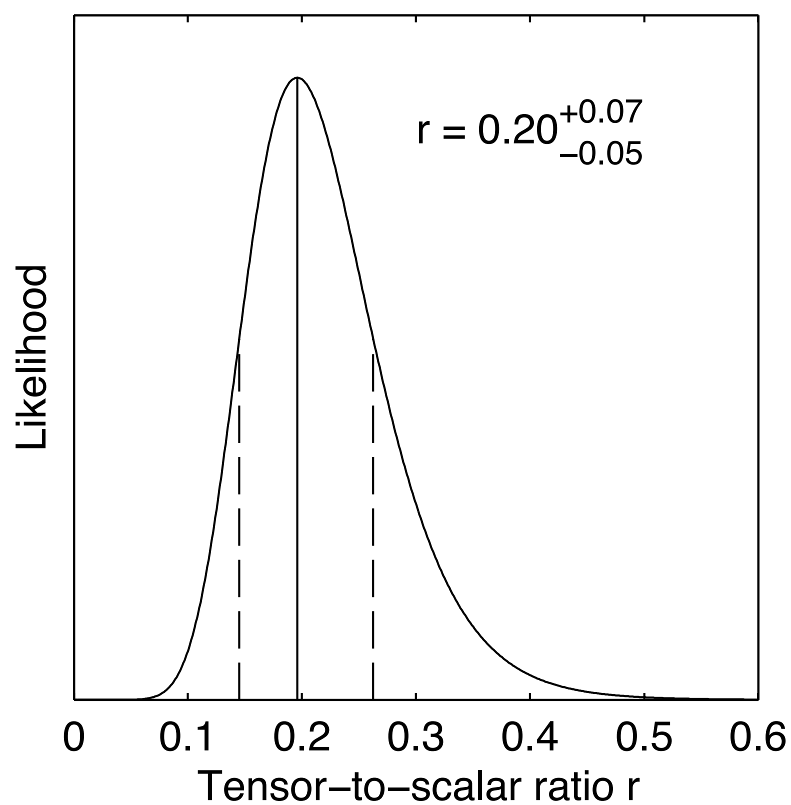
=> favored

Hybrid inflation models

Most of models are disfavored

due to predicting a high tensor-to-scalar ratio and n_s>1

BICEP2 results

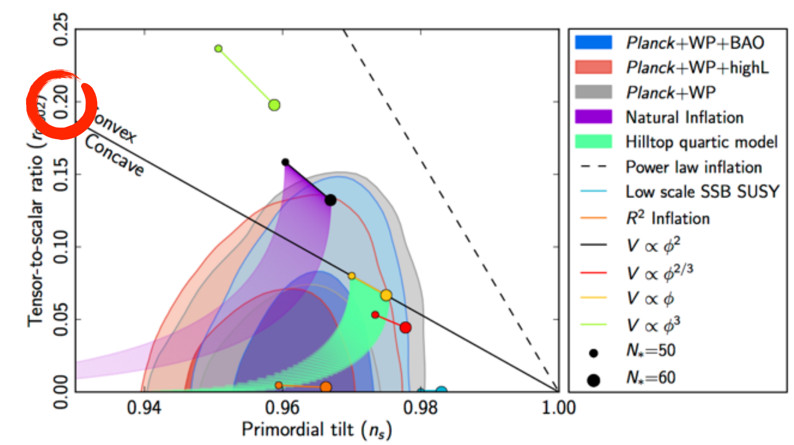


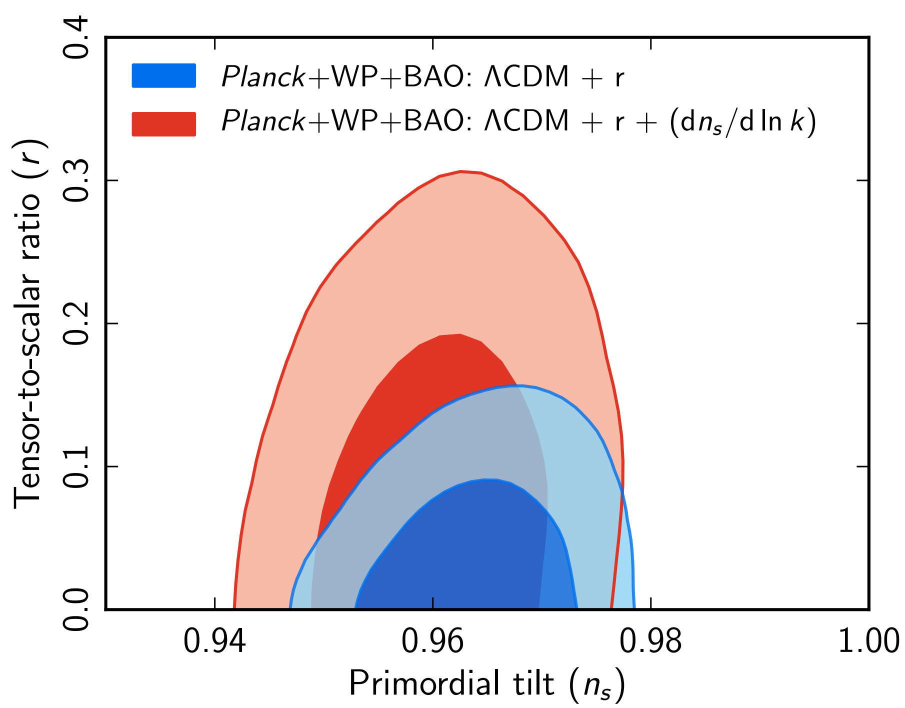
tensor-to-scalar ratio

r=0: ruled out at significance of 7.0 σ

This does not mean that

BICEP2 result is inconsistent with Planck result





If we include the running of n_s,

allowed region for r can be expanded.

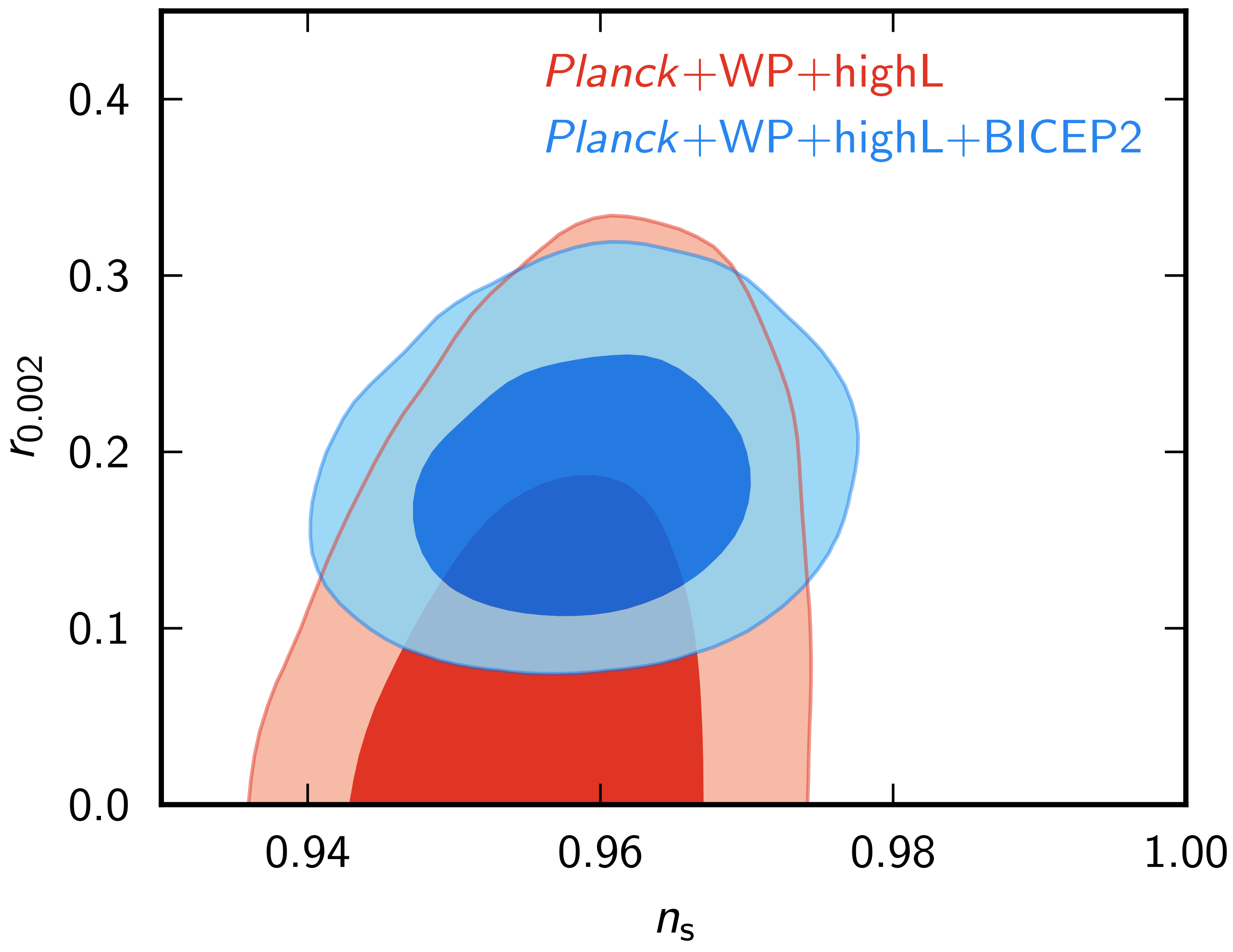
(Planck 2013 results)

BICEP2 data implies that

Large r and non-zero α_s

BICEP2 results

Combined result (Planck and BICEP2)



BICEP2 data implies that

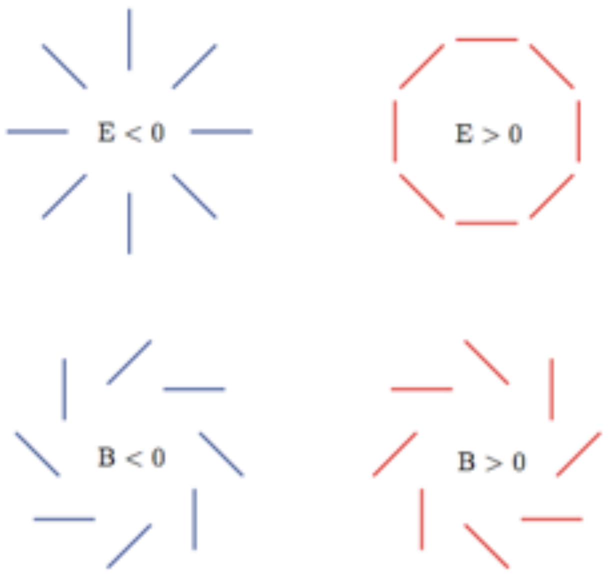
Large r and non-zero α_s

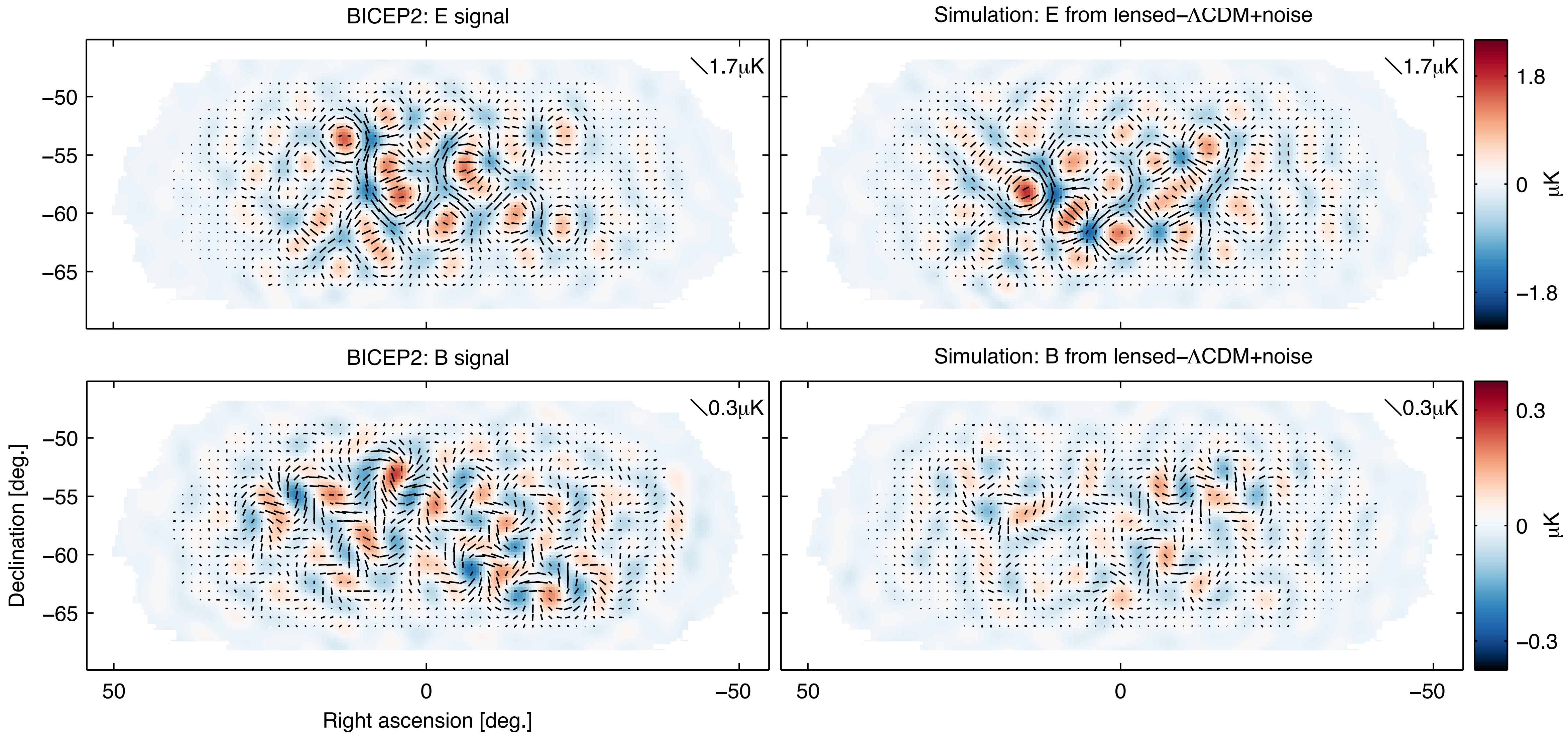
Important feature;

- Chaotic inflation like models are favored
- Small-field models are excluded

BICEP2 results

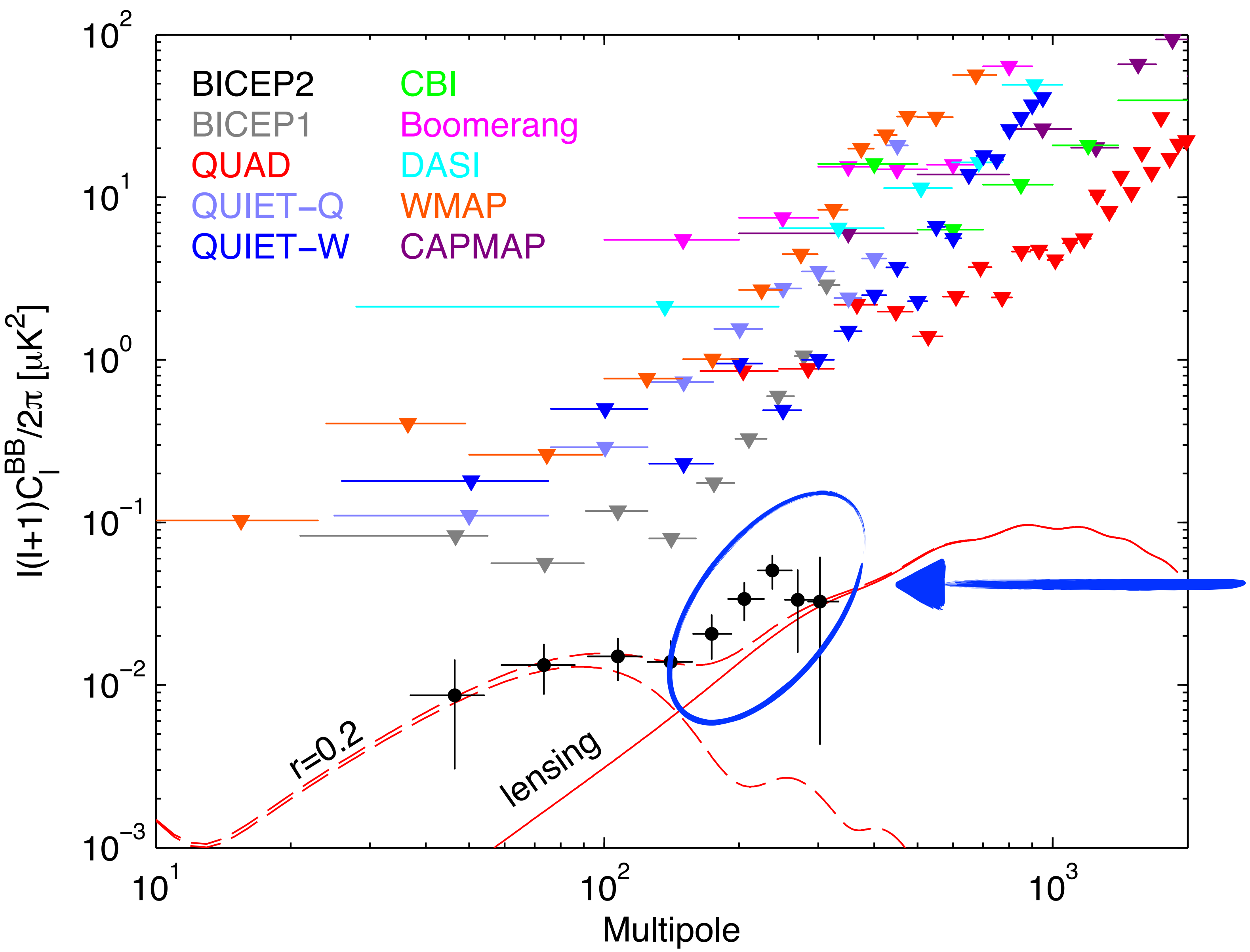
observed E-mode and B-mode





BICEP2 results

Observation and Fitting of the angular power spectrum (B-B mode)



r=0.2 + lensing ΛCDM

=> Fitting OK !?

excess !?

BUT!

Note added: results of BICEP2 published paper

Note added

Since we submitted this paper new information on polarized dust emission has become available from the *Planck* experiment in a series of papers [96,108–110]. While these confirm that the modal polarization fraction of dust is ~4%, there is a long tail to fractions as high as 20% (see Fig. 7 of [96]). There is also a trend to higher polarization fractions in regions of lower total dust emission [see Fig. 18 of [96] noting that the BICEP2 field has a column density of $\sim(1\text{–}2) \times 10^{20}\ \mathrm{H\,cm^{-2}}$]. We note that these papers restrict their analysis to regions of the sky where "systematic uncertainties are small, and where the dust signal dominates total emission," and that this excludes 21% of the sky that includes the BICEP2 region. Thus while these papers do not offer definitive information on the level of dust contamination in our field, they do suggest that it may well be higher than any of the models considered in Sec. IX.

The possibility of the dust polarization

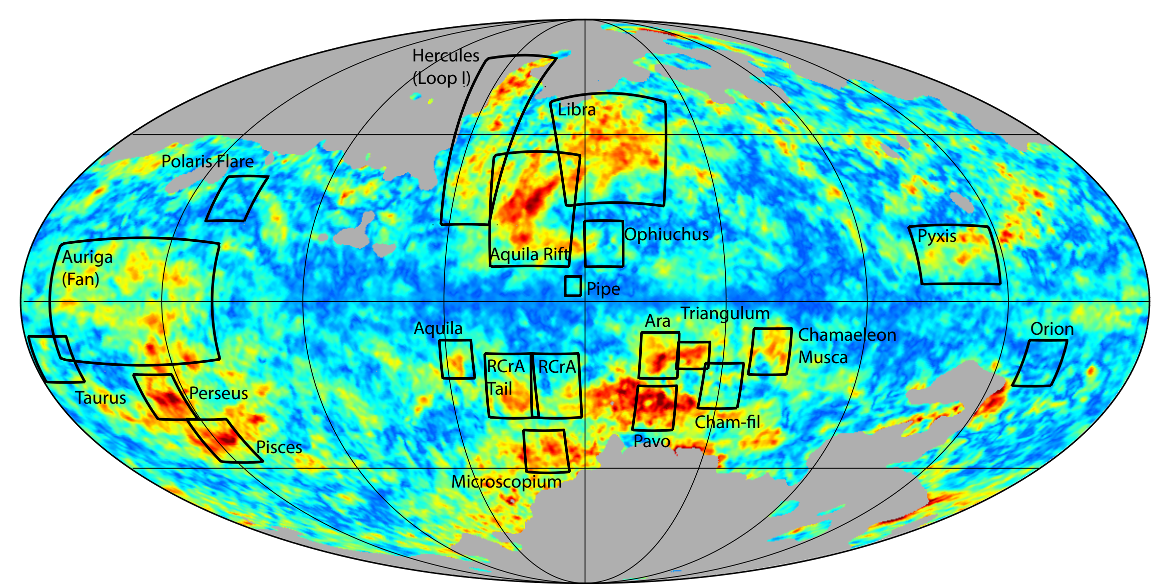
In BICEP2 experiment;
Dust polarization fraction is assumed

$$\delta\Delta^2_{BB,\mathrm{dust}} \equiv \frac{\Delta^2_{BB,\mathrm{dust}}}{\Delta^2_{BB,\mathrm{tot}}} \simeq 5-6\%$$

Planck intermediate results. XIX. [e-print:1405.0871]
XIX

Dust polarization fraction is observed to range from 0 to more than 15%

$$\delta\Delta^2_{BB,\mathrm{dust}} \gtrsim 15\%$$



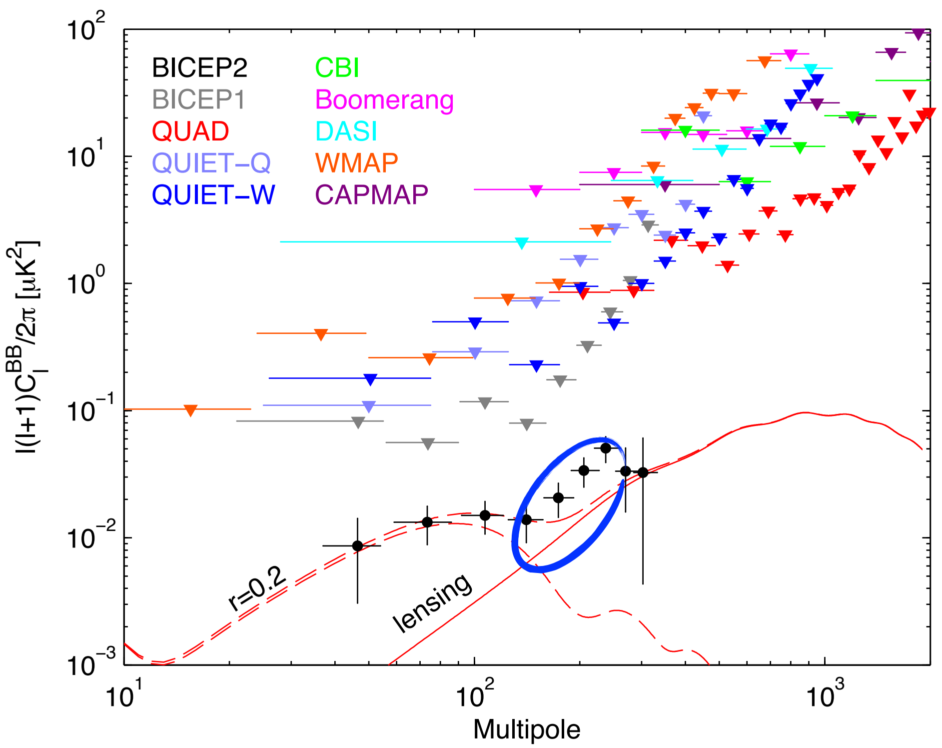
Recently work [arXiv:1405.5857]

A joint analysis of Planck and BICEP2 B modes including dust polarization uncertainty

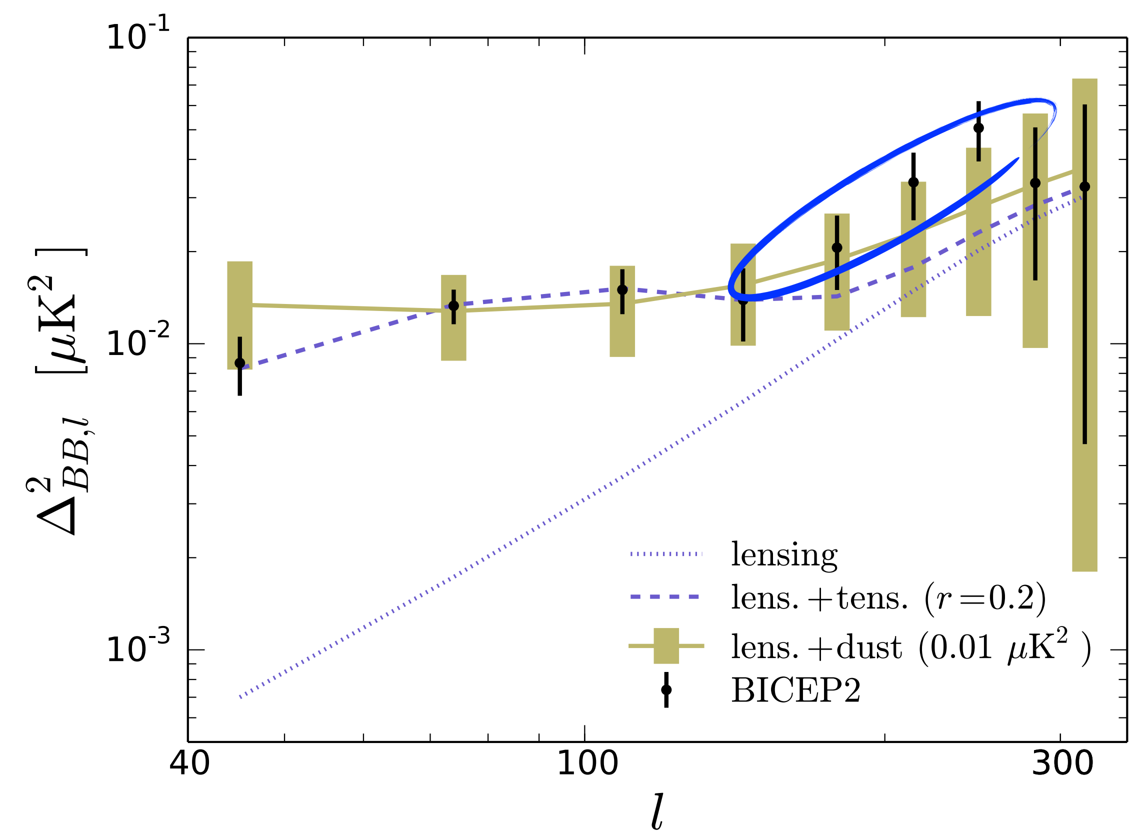
Michael J. Mortonson,[a] Uroš Seljak[b]

They assume that the polarized dust fraction has the form of..

$$\Delta^2_{BB,\mathrm{dust},l} = \frac{1}{2\pi} l^2 C_l^{BB,\mathrm{dust}} \propto l^{-0.3}$$



They explain the excess near l ~ 250 by using dust polarization.



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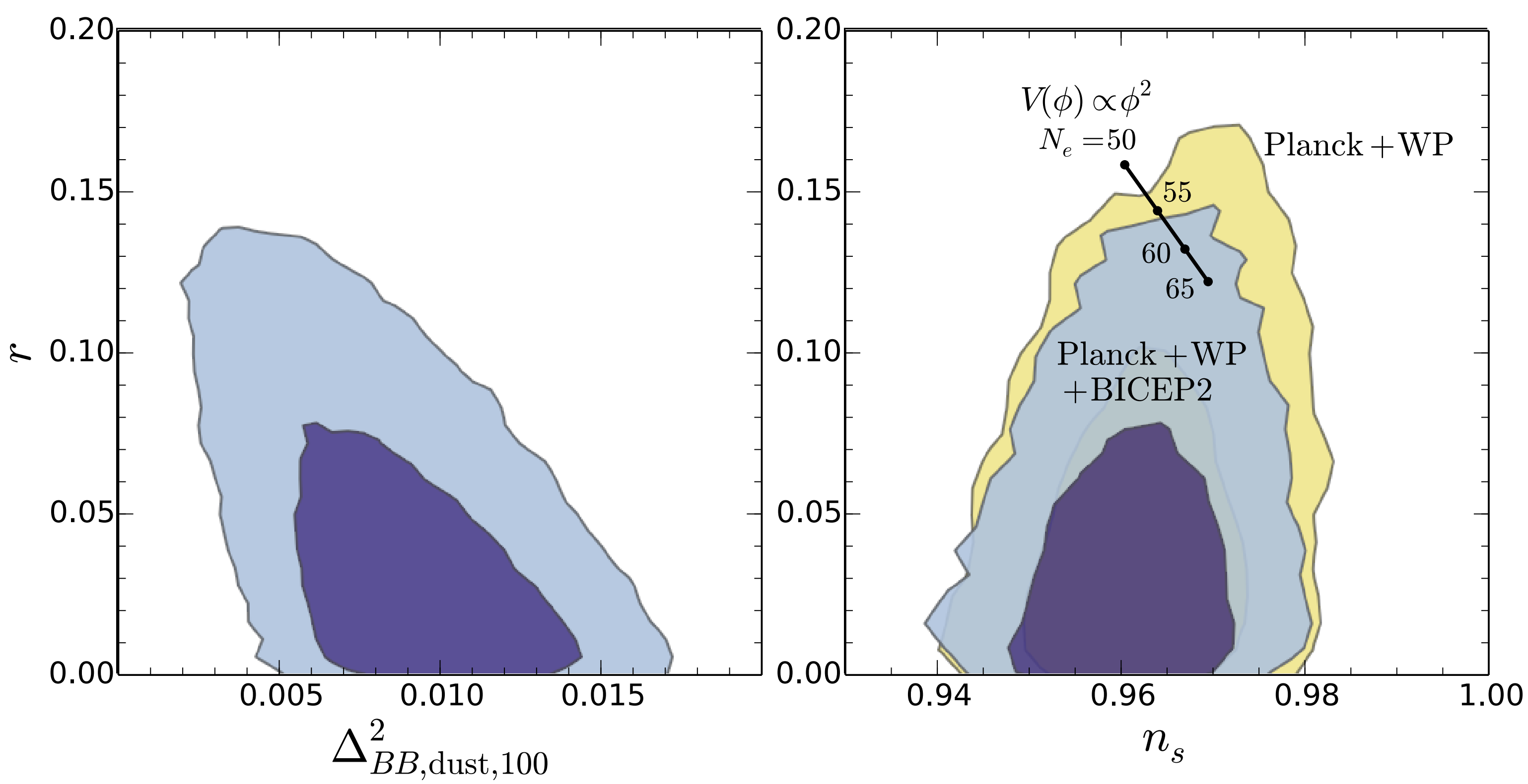


They also combine the result (the n_s-r plot) of the Planck (including dust polarization) and BICEP2 .

RESULT: r<0.11

Slightly improved from the result only Planck (r<0.13)

SUMMARY

- I reviewed foundations for inflation.

  slow-roll parameters, models, observables, and so on..

- The results of Planck

  tensor-to-scalar ratio: $r<0.13$

  spectral index: $n_s = 0.96$

  etc..

- The results of BICEP2

  tensor-to-scalar ratio: $r = 0.2$

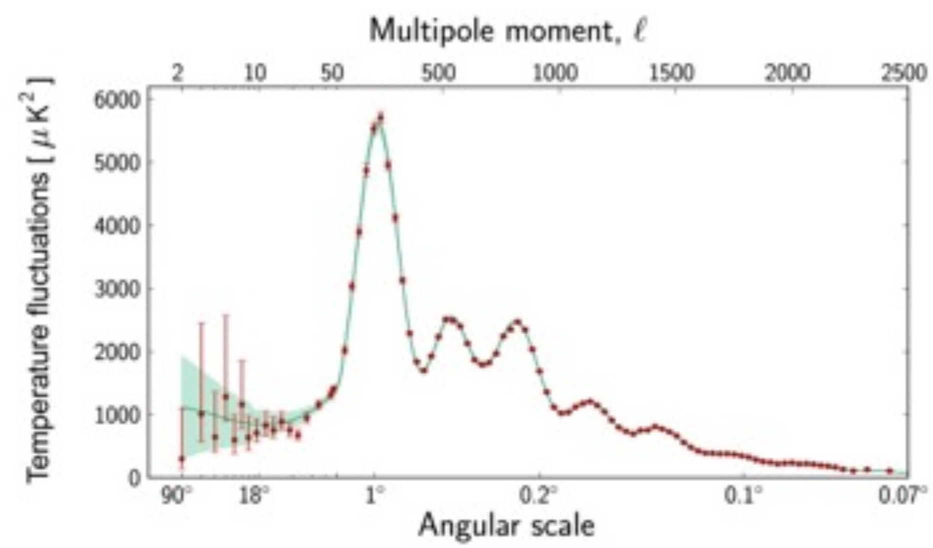
  non-zero running of the spectral index

  etc..

- The possibility of the large dust polarization fraction

  also, combined results of Planck (w/ dust) and BICEP2

THE ISSUES I WANTED TO TALK..



- Temperature fluctuations

  Foundation & Planck results on T fluctuations

- Non-gaussiannity

  Foundation & Planck constraints

  also, I found an interesting paper:

  Camenho, Edelstein, Maldacena, and Zhiboedov [arXiv:1407.5597]

  "*For inflation, or de Sitter-like solutions, it indicates the existence of massive higher spin particles if the gravity wave non-gaussianity deviates significantly from the one computed in the Einstein theory.*"

- How Primordial Quantum Fluctuations Go Classical?

  Recently the corresponding paper appears..

  Burgess, Holman, Tasinato, and Williams[arXiv:1408.5002]

- Other experiments [POLARBEAR, LiteBIRD, etc..(B-mode detection)]