Effects of dim. 5 operator on proton decay

Phys.Rev. D59 (1999) 115009 [hep-ph/9808255v2] T.Goto, T.Nihei

Phys.Rev. D65 (2002) 055009 [hep-ph/0108104v1] H.Murayama, A.Pierce

Takumi Kuwahara

Content

- Introduction
  - Superpotential in SUSY SU(5) GUT, MSSM
  - dim.5 operator (LLLL, RRRR operator)
  - nucleon matrix element @ hadron level
- RGE analysis
  - Threshold correction
  - constraint on colored Higgs mass
- the effect of RRRR operator (Goto-Nihei)
- Decoupling scenario (Murayama-Pierce)
- Summary

Introduction

Superpotential in SUSY SU(5)

$$W = \frac{f}{3}\mathrm{Tr}\Sigma^3 + \frac{fV}{2}\mathrm{Tr}\Sigma^2 + \lambda \overline{H}_\alpha(\Sigma^\alpha_\beta + 3V\delta^\alpha_\beta)H^\beta + \frac{h^{ij}}{4}\epsilon_{\alpha\beta\gamma\delta\epsilon}\psi_i^{\alpha\beta}\psi_j^{\gamma\delta}H^\epsilon + \sqrt{2}f^{ij}\psi_i^{\alpha\beta}\phi_{j\alpha}\overline{H}_\beta$$

α,β,γ,δ,ε,... = 1,2,...,5 (SU(5) indices)

i,j = 1,2,3 (gen. indices)

we assume that
- R-parity
- the third term

@ ~~GUT~~ scale SU(5) → SU(3)×SU(2)×U(1) by VEV of adjoint Higgs Σ

by this assumption, only colored Higgs in H, $\overline{\text{H}}$ has heavy mass

Introduction

we can reparameterize h^{ij} and f^{ij} by using global sym. $U(3)\times U(3)$

d.o.f. : $(6+9)\times 2$

d.o.f. : 9×2

$\longrightarrow$ net d.o.f. : 12

Yukawa term of superpotential in MSSM

$$W = h^i V_{ij} u_i^C e_j^C H_C + \frac{1}{2} h^i e^{i\varphi_i} (Q_i Q_i) H_C$$
$$+ h^i u_i^C (Q_i H_f) + V_{ij}^* f^j e^{-i\varphi_i} u_i^C d_j^C \overline{H}_C$$
$$+ V_{ij}^* f^j Q_i L_j \overline{H}_C + V_{ij}^* f^j Q_i \overline{H}_f d_j^C + f^i e_i^C L_i \overline{H}_f$$

V : CKM matrix

h,f : Yukawa couplings @ GUT scale

φ : additional phases w/ $\varphi_1+\varphi_2+\varphi_3=0$

dim.5 operator

if colored Higgs has mass term as $M_{H_C}\overline{H}_C H_C$

By integrating out the heavy colored Higgs, we can obtain the superpotential for the dim.5 operators

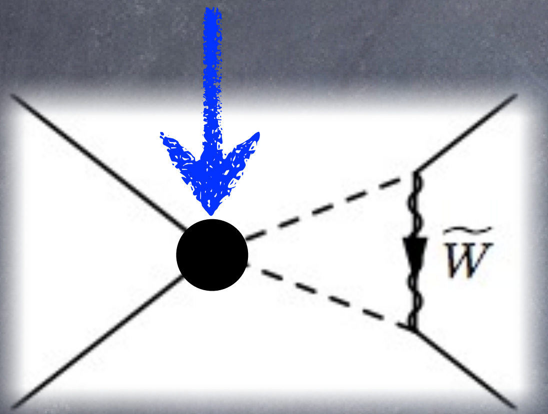
$$W_5 = -\frac{1}{M_{H_C}}\left\{\frac{1}{2}C_L^{ijkl}Q_kQ_lQ_iL_j + C_R^{ijkl}e_k^C u_l^C u_i^C d_j^C\right\}$$

colored Higgs mass LLLL operator RRRR operator

where these coefficients are
$$C_L^{ijkl} = h^i V_{kl}^* f_l e^{i\varphi_i}$$
$$C_R^{ijkl} = V_{kl}^* f_l h^i V_{ij} e^{-i\varphi_k}$$

Introduction

LLLL operator



$$W_{LLLL} = -\frac{h^i f^l}{2M_{H_C}} V^*_{kl} e^{i\varphi_i} Q_i Q_i Q_k L_l$$

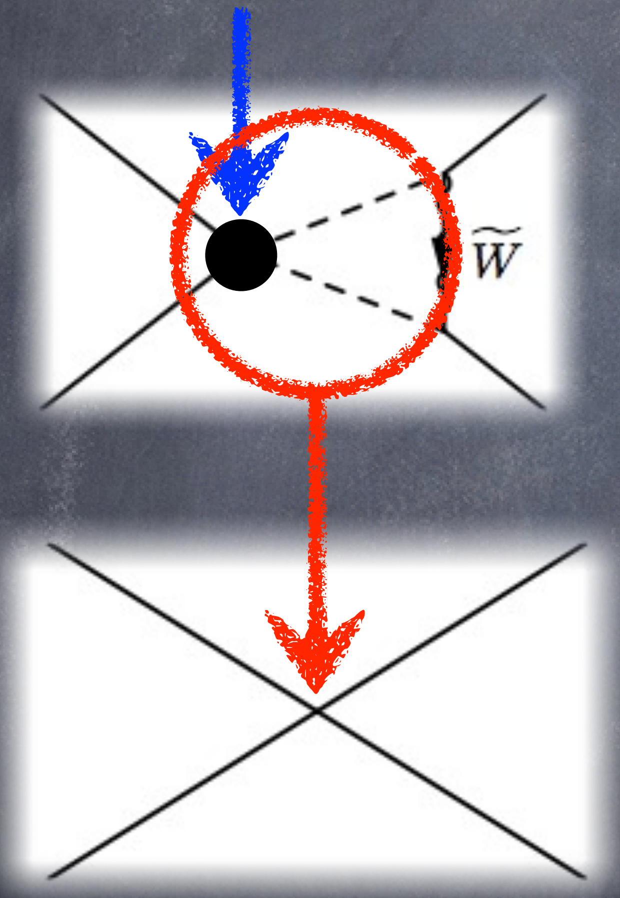
The charged winos contribute to p decay dominantly.
If masses of squarks and sleptons are degenerated,

f is defined as this triangle diagram contribution

$$\frac{\alpha_2}{2\pi} f(u,d) \approx \frac{\alpha_2}{2\pi} \frac{m_{\widetilde{W}}}{m^2_{\widetilde{Q}}} \qquad (m^2_{\widetilde{Q}} \gg m^2_{\widetilde{W}})$$

Introduction

LLLL operator



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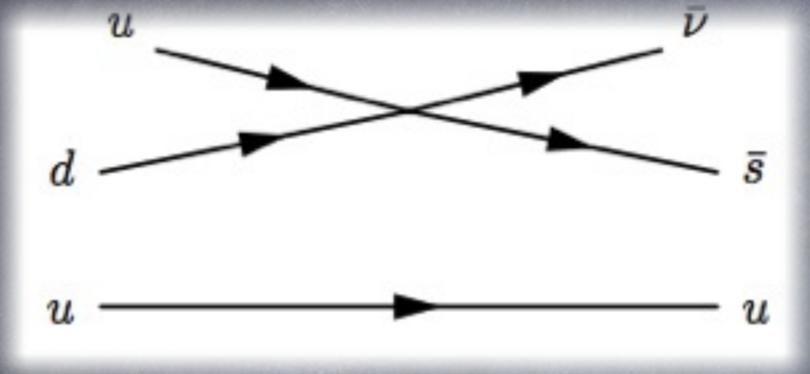
Introduction



Since color indices are totally antisym. in QQQL

$\rightarrow$ dominant decay $p \rightarrow K^{+} \bar{\nu}$

in SUSY GUT

the effective Lagrangian from LLLL operator

$$\mathcal{L} = \left[e^{i\varphi_2} A_e(\tilde{c}_L) + e^{i\varphi_3} A_e(\tilde{t}_L) \right] (u_L d_L)(s_L \nu_e)$$
$$+ (e \longleftrightarrow \mu)$$
$$+ (e \longleftrightarrow \tau)$$

where $A_i(\tilde{q}_{jL}) = \dfrac{1}{M_{H_C}} \dfrac{\alpha_2}{2\pi} h^j e^{i\varphi_j} V^*_{ui} f^j V_{js} V_{jd} \left(f(e_i, q_j) + f(d, q_j) \right)$

Introduction

-p-decay from LLLL-

$$A_i(\tilde{q}_{jL}) = \frac{1}{M_{H_C}}\frac{\alpha_2}{2\pi}h^j e^{i\varphi_j}V^*_{ui}f^j V_{js}V_{jd}\left(f(e_i,q_j)+f(d,q_j)\right)$$

we know the Yukawa couplings as masses only @ low energy scale

By using RGE for Yukawa couplings, we can write this coefficient in terms of quark mass @ 1GeV

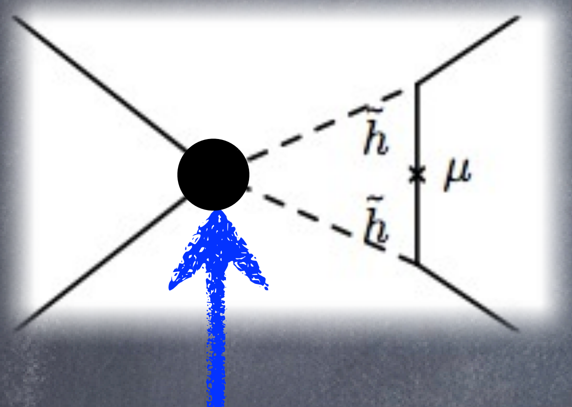
quark mass @ 1GeV

$$A_i(\tilde{q}_{jL}) = \frac{2\alpha_2^2}{M_{H_C}}\frac{m_{u_j}m_{d_j}}{m_W^2\sin 2\beta}e^{i\varphi_j}V^*_{ui}V_{js}V_{jd}$$

$$\times \underline{A_L A_S(\tilde{q}_j)}\left(f(e_i,q_j)+f(d,q_j)\right)$$

renormalization effects

RRRR operator



RRRR operator

$$W_{RRRR} = -\frac{f^l h^i}{M_{H_C}} V^*_{kl} V_{ij} e^{-i\varphi_k} u^C_i e^C_j u^C_k d^C_l$$

RRRR operator has 2 Yukawa couplings.

$\rightarrow$ the contribution of third gen. is only large.

g is defined as this triangle diagram contribution

$$g(u,d) \approx \frac{1}{(4\pi)^2} \frac{\mu}{m^2_{\tilde{Q}}} \qquad (m^2_{\tilde{Q}} \gg \mu^2)$$

the effective Lagrangian for $p \rightarrow K^+ \bar{\nu}$ from RRRR operator

$$\mathcal{L} = \frac{1}{M_{H_C}} f^1 f^3 (h^3)^2 V^*_{ud} V_{tb} V_{cs} e^{-i\varphi_1} g(c,\tau)(s_L \nu_{\tau L})(u^C d^C)$$

Introduction

We have already obtained quark level Lagrangian.

By using chiral Lagrangian technique, hadronic matrix element is determined as

$$\alpha u_L(\mathbf{k}) \equiv \varepsilon_{\alpha\beta\gamma} \langle 0| \, (d_R^\alpha u_R^\beta) u_L^\gamma \ \ |p, \mathbf{k}\rangle$$

$$\beta u_L(\mathbf{k}) \equiv \varepsilon_{\alpha\beta\gamma} \langle 0| \, (d_L^\alpha u_L^\beta) u_L^\gamma \ \ |p, \mathbf{k}\rangle$$

α,β are free parameter of which absolute value ranges

$$\alpha, \beta = (0.003 - 0.03)\mathrm{GeV}^3$$

Nucl.Phys B238(1984)561-581
Stanley J. BRODSKY, John ELLIS, John S. HAGELIN and C.T. SACHRAJDA

to obtain the maximum proton lifetime
-> we use the smallest value of the hadronic matrix element

RGE analysis

-threshold correction-

After integrating out the heavy particles,
we must consider gauge coupling relations between full and effective theory

$$\mathcal{L}_{\rm full} = -\frac{1}{4} F^{\alpha}_{\mu\nu} F^{\alpha\mu\nu} + \cdots$$

$$\mathcal{L}_{\rm eff} = \sum_i -\frac{1}{4}(1-l_i) F^{a_i}_{\mu\nu} F^{a_i\mu\nu} + \cdots$$

where $l_i = g_i^2(\lambda_i(\mu) + \lambda'_i(\mu)/\epsilon')$

- "i" means residual symmetries (now SU(3)×SU(2)×U(1)).
- The effects of heavy particles are included as "l_i" in $L_{\rm eff}$.
- To determine the "l_i", after we calculate 1 loop diagram from each Lagrangian, we match the two results.

redefine the fields and couplings to make kin. term canonical,

$$-\frac{1}{4}(1-l_i) F^{a_i}_{\mu\nu} F^{a_i\mu\nu} \to -\frac{1}{4} F'^{a_i}_{\mu\nu} F'^{a_i\mu\nu}$$

by using

$$A'_{a_i\mu} \equiv \sqrt{1-l_i} A_{a_i\mu}$$
$$g_i \equiv g/\sqrt{1-l_i}$$

RGE analysis

-threshold correction-

We have already determined the relation between bare g in full and eff. theory.

Next, we consider $\overline{\text{MS}}$ (similarly $\overline{\text{DR}}$) renormalized couplings

$$g\mu^{\epsilon/2} = g(\mu) - b_G g^3(\mu)/\epsilon' + \ldots$$
$$g_i\mu^{\epsilon/2} = g_i(\mu) - b_i g_i^3(\mu)/\epsilon' + \ldots$$

$$g_i \equiv g/\sqrt{1-l_i}$$

so we can obtain the relation @ 1 loop

$$g_i(\mu) = g(\mu) + \frac{1}{2}\lambda_i(\mu)g^3(\mu)$$

where $l_i = g_i^2(\lambda_i(\mu) + \lambda_i'(\mu)/\epsilon')$

RGE analysis

$$\frac{dg_i}{d\ln\mu} = \frac{1}{16\pi^2} b_i g_i^3$$

RGEs for gauge coupling @ 1 loop

the solution of RGEs for gauge coupling

$$\alpha_i^{-1}(m_Z) = \alpha_5^{-1}(\Lambda) - \frac{1}{2\pi} b_i \ln\frac{\Lambda}{m_Z} - \underline{4\pi\lambda_i'(m_{SUSY}) - 4\pi\lambda_i(\Lambda)}$$

Threshold Corrections @ ~~SUSY~~ ~~GUT~~

Λ : the scale larger than any of the masses @GUT

we can obtain M_{H_C} independently of the other masses @ GUT scale

$$3\alpha_2^{-1}(m_Z) - 2\alpha_3^{-1}(m_Z) - \alpha_1^{-1}(m_Z) = \frac{1}{2\pi}\left(\frac{12}{5}\ln\frac{M_{H_C}}{m_Z} - 2\ln\frac{m_{SUSY}}{m_Z}\right)$$

RGE analysis

$$3\alpha_2^{-1}(m_Z) - 2\alpha_3^{-1}(m_Z) - \alpha_1^{-1}(m_Z) = \frac{1}{2\pi}\left(\frac{12}{5}\ln\frac{M_{H_C}}{m_Z} - 2\ln\frac{m_{SUSY}}{m_Z}\right)$$

By taking into account the mass splitting of the SUSY particles
(w/ the approx. that squark,slepton masses are degenerated)

$$-2\ln\frac{m_{SUSY}}{m_Z} \to 4\ln\frac{m_{\tilde{g}}}{m_{\widetilde{W}}} - \frac{8}{5}\ln\frac{\mu}{m_Z} - \frac{2}{5}\ln\frac{m_H}{m_Z}$$

M_{H_C} is varied by varying higgsino mass μ

$$3.5 \times 10^{14}\mathrm{GeV} \leq M_{H_C} \leq 3.6 \times 10^{15}\mathrm{GeV}$$

range of μ : 100GeV - 400GeV

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Goto-Nihei argument

-setups-

"Effect of an RRRR dim. 5 operator on the p decay
in the minimal SU(5) SUGRA GUT model"

Now we consider the contributions of RRRR operator.

In this paper,

- mSUGRA model for soft ~~SUSY~~
  (universal scalar mass, gaugino mass, trilinear coupling)
- RRRR contributes to only $p \to K^+ \bar{\nu}_\tau$
  because of Yukawa coupling
  (1^{st},2^{nd} contributions are small)

Goto-Nihei argument

-RRRR contribution-

each contribution to p-decay from LLLL & RRRR

$$A(p \to K^+ \overline{\nu}_e) \approx \left[e^{i\varphi_2} A_e(\tilde{c}_L) + e^{i\varphi_3} A_e(\tilde{t}_L)\right]_{LLLL}$$
$$A(p \to K^+ \overline{\nu}_\mu) \approx \left[e^{i\varphi_2} A_\mu(\tilde{c}_L) + e^{i\varphi_3} A_\mu(\tilde{t}_L)\right]_{LLLL}$$
$$A(p \to K^+ \overline{\nu}_\tau) \approx \left[e^{i\varphi_2} A_\tau(\tilde{c}_L) + e^{i\varphi_3} A_\tau(\tilde{t}_L)\right]_{LLLL} + e^{-i\varphi_1} A_\tau(\tilde{t}_R)_{RRRR}$$

Notes

- the magnitude of $A(c_L)$ is comparable with that of $A(t_L)$
- the magnitude of $A(t_R)$ is larger than that of $A(c_L)$, $A(t_L)$

⇒ if $\varphi_2 - \varphi_3 \approx \pi$ ⇒ LLLL contributions are "small" !

Even if LLLL contributions cancell out,

3rd gen. contribution remains !

Goto-Nihei argument

-numerical results-

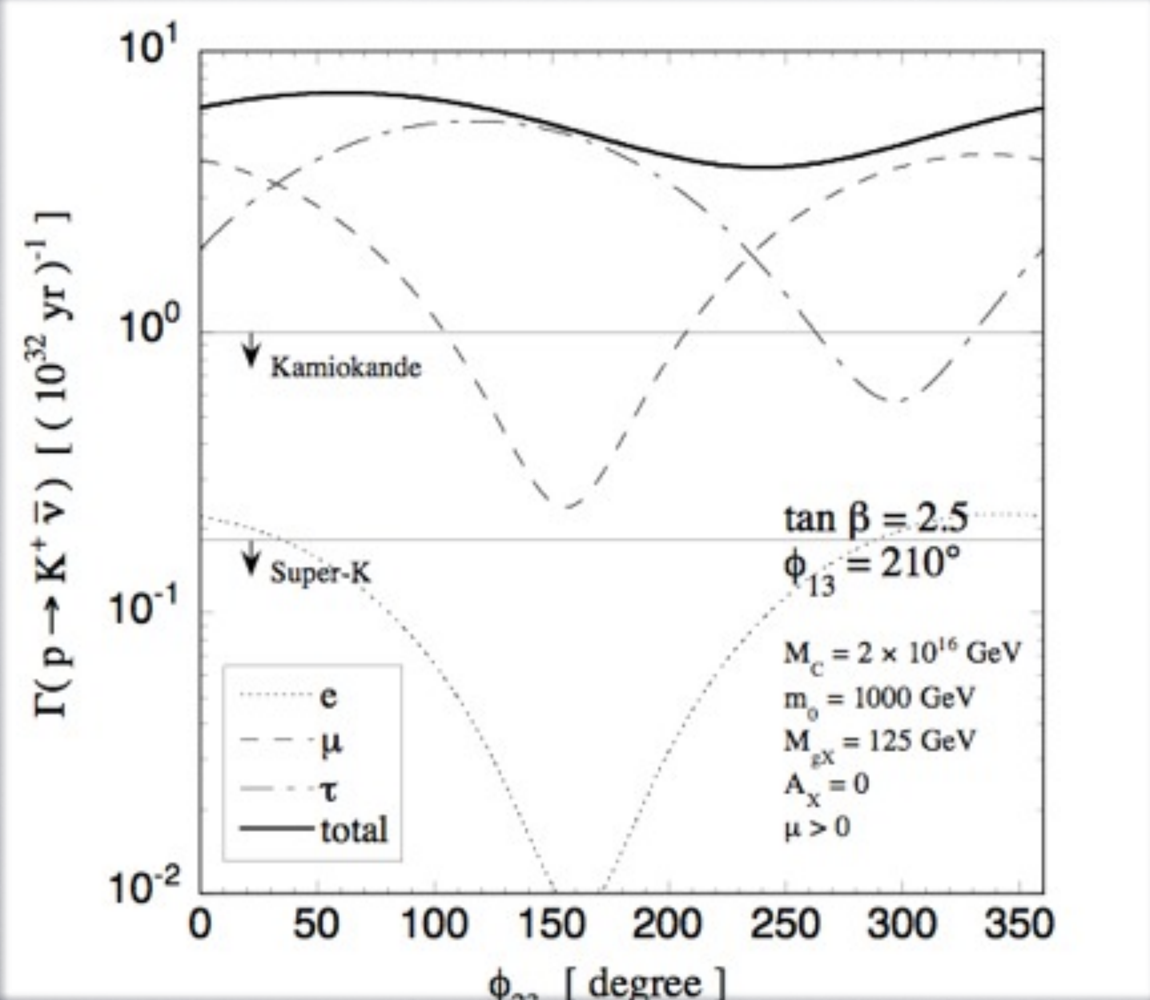


Fig: the total decay rate for p → Kν described in Goto-Nihei's paper

fixed φ_1–φ_3 case (decay rate)

fixed soft ~~SUSY~~ parameter

squark universal mass : 1 TeV

gaugino universal mass : 125 GeV

A-term coefficient : 0

sign(μ) > 0

masses of GUT particles

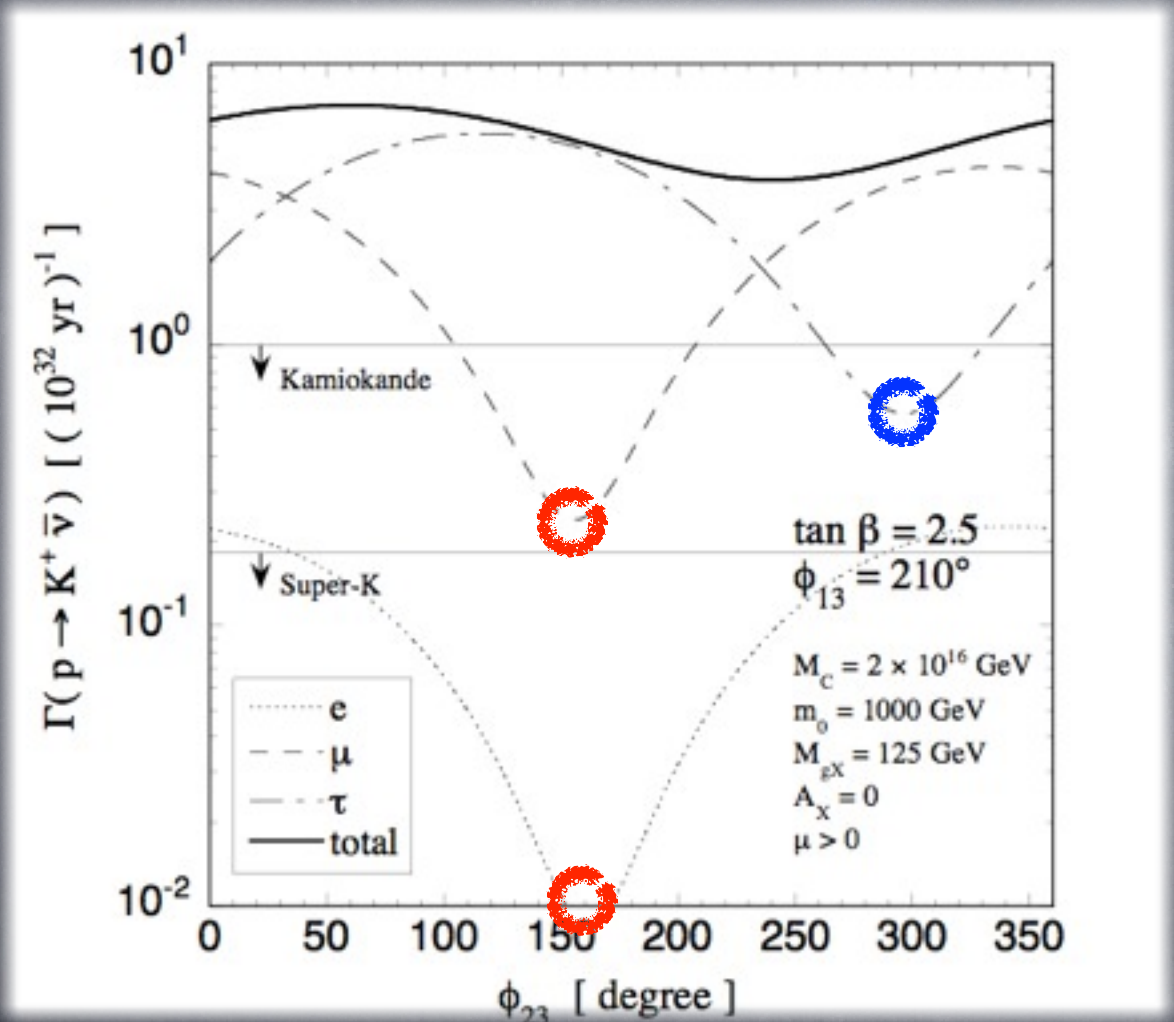
-> 2.0 $\times 10^{16}$GeV

minimum value for each decay mode

- $\varphi_{23} \sim 160°$ for ν_e or ν_μ decay mode
- $\varphi_{23} \sim 300°$ for ν_τ decay mode

Goto-Nihei argument

-numerical results-



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Goto-Nihei argument

-numerical results-

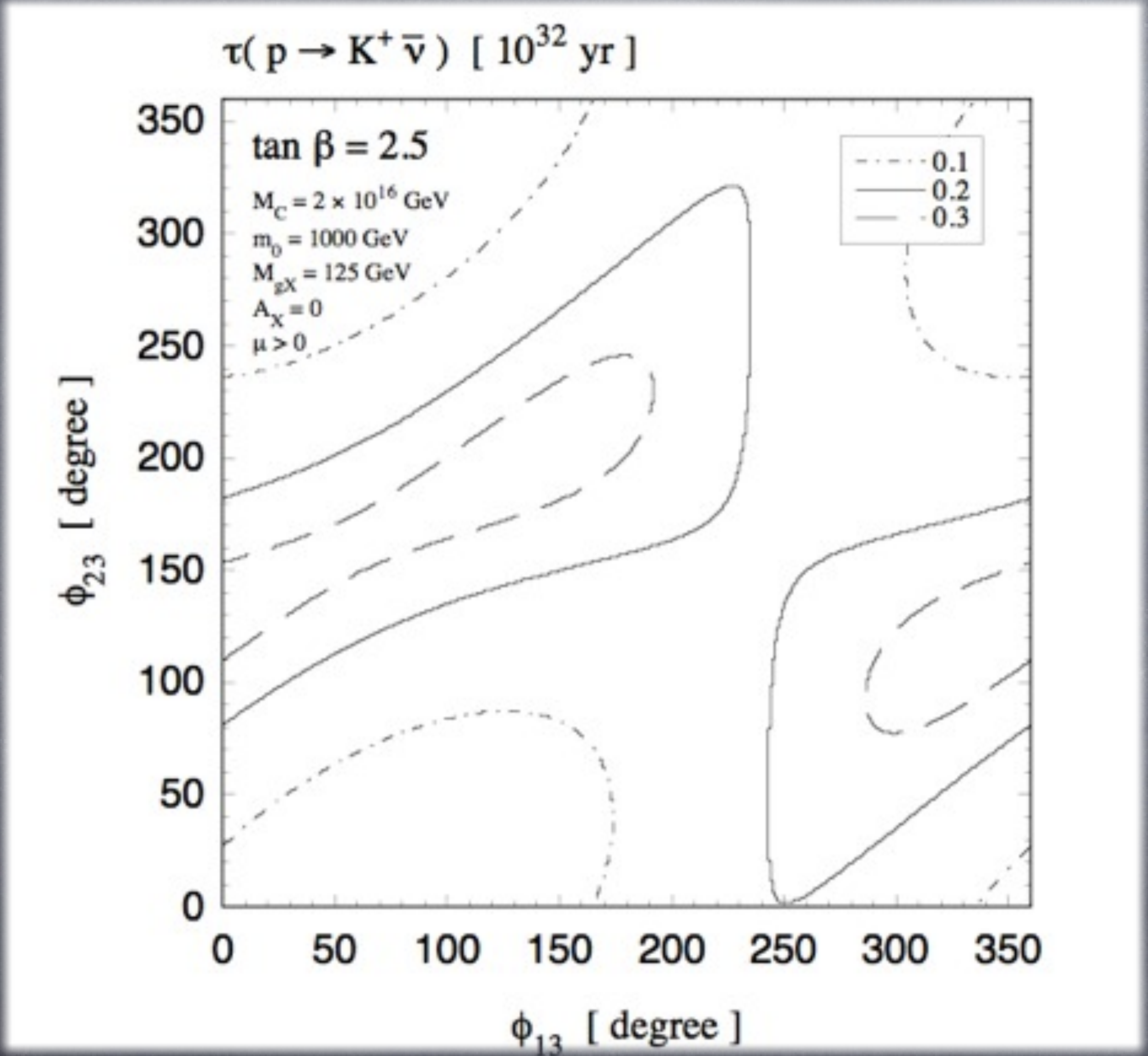

Fig: the partial lifetime for p described in Goto-Nihei's paper

proton lifetime

contour plot

there is no region making p lifetime longer than 0.5×10^{32} yrs

On the other hand, Kamiokande limit

$\tau(p \to K\nu) > 1.0\times10^{32}$ yrs

in 1999

Goto-Nihei argument

-numerical results-

Note: the recent data for p-decay $\tau(p \to K\nu) > 2.3{\times}10^{33}$ yrs

Phys. Rev. D **72**, 052007 (2005), hep-ex/0502026

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Murayama-Pierce argument

-decoupling scenario intro.-

"Not Even Decoupling Can Save minimal SUSY SU(5)"

Decoupling scenario

In the MSSM avoiding FCNC requires very precise squark universality or alignment. We have argued that such a scenario is neither minimal nor necessarily superior. Instead we have advocated that physics beyond the Z be constructed from a low energy, effective field theory perspective.

in summary

Effective Supersymmetry can be implemented with automatic suppression of baryon and lepton number violation and a dynamically ...

in abst.

A. G. Cohen, D. B. Kaplan and A. E. Nelson,
Phys. Lett. B 388, 588 (1996) [hep-ph/9607394]

Murayama-Pierce argument -decoupling scenario setups-

"Not Even Decoupling Can Save minimal SUSY SU(5)"

Decoupling scenario

A. G. Cohen, D. B. Kaplan and A. E. Nelson,
Phys. Lett. B 388, 588 (1996) [hep-ph/9607394]

@ $\Lambda \sim$5-20 TeV

squarks and sleptons in 1st, 2nd generation decouple

→ 1st, 2nd gen. sparticle masses → ~ 5-20TeV

other superparticle masses → ~ 1TeV

especially 3rd gen. sparticles

Since Amp. for p decay depends on M_q like M_χ/M_q^2,

Can Decoupling scenario save mSUSY SU(5) from p decay???

Murayama-Pierce argument

-decoupling scenario-

Since scharm is heavier than wino in this scenario,
contribution of triangle diagram is suppressed.

$$A_i(\tilde{c}_L)_{LLLL} \sim 0$$

But stop mass is around 400~800 GeV

In this limit, Amp. for p -> $K^+ \bar{\nu}$ are rewritten as

$$A(p \to K^+ \overline{\nu}_e) \approx e^{i\varphi_3} A_e(\tilde{t}_L)_{LLLL}$$

$$A(p \to K^+ \overline{\nu}_\mu) \approx e^{i\varphi_3} A_\mu(\tilde{t}_L)_{LLLL}$$

$$A(p \to K^+ \overline{\nu}_\tau) \approx e^{i\varphi_3} A_\tau(\tilde{t}_L)_{LLLL} + e^{-i\varphi_1} A_\tau(\tilde{t}_R)_{RRRR}$$

Murayama-Pierce argument -constraint on p decay in d.s.-

By using the colored Higgs mass, 3.6×10^{15}GeV (from RGE analysis).

upper limit for proton partial lifetime

$$\tau(p \to K^+\bar{\nu}) \leq 2.9 \times 10^{30}\mathrm{yrs}$$

Therefore, even decoupling scenario is excluded.

Note: the recent data for p-decay $\tau(p \to K\nu) > 2.3\times10^{33}$ yrs

Phys. Rev. D **72**, 052007 (2005), hep-ex/0502026

Murayama-Pierce argument -avoiding the constraint-

There are two ways to avoid the constraint

- To push the colored Higgs mass very heavy.
  → by adding the contribution to the GUT threshold correction
  ex) including a pair of Higgs multiplet in the $5+\bar{5}$ representation

- To suppress the dim.5 operator.
  → by additinnal sym. which forbid the $M_{H_C}\bar{H}_C H_C$ term.

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Summary

- minimal SU(5) SUSY GUT is excluded by the inconsistency between p decay and RGE analysis.

- RRRR operator largely contributes to p-decay with different phase φ from LLLL.

  -> $\tau(p \to K\nu_\mu)$ and $\tau(p \to K\nu_\tau)$ are not reduced simultaneously.

- Since 3rd gen. contributes to p decay dominantly, the decoupling scenario can't save mSUSY SU(5) GUT.

- These arguments don't claim that SU(5) SUSY GUT is excluded. (ex. additional sym.,additional fields etc...)

Thank you for your attentions.

Backup slides

What mediate to make 4-fermi from LLLL

- gluino, neutral gaugino, neutral higgsino

flavor-diagonal int. because of the Yukawa couplings of 1st, 2nd gen., ⇒ small.

But strong coupling α_3 for gluino

if squark masses are degenerate. ⇒ completely vanishes.

- charged higgsino

Since we need Yukawa couplings ⇒ small.

- charged gaugino

dominant contribution.

What mediate to make 4-fermi from RRRR

Since there is no Left-handed particles,

SU(2) gaugino can't mediate.

- gluino, neutral higgsino

flavor-diagonal int. because of the Yukawa couplings of 1st, 2nd gen., $\Rightarrow$ small.

But strong coupling α_3 for gluino

if squark masses are degenerate. $\Rightarrow$ completely vanishes.

- charged higgsino

Since we need Yukawa couplings $\Rightarrow$ small.

3rd gen. contribution only contributes dominantly.

How to make the dim. 5 operator

$$K_{H_C} + W_{H_C} = M_{H_C} H_C \overline{H}_C + h^i e^{i\varphi_i} u_i^C e_i^C H_C + \frac{h^i}{2} e^{i\varphi_i} Q_i Q_i H_C + V_{ij}^* f_j Q_i L_j \overline{H}_C + V_{ij}^* f_j u_i^C d_j^C \overline{H}_C$$

$$= M_{H_C} \left\{ H_C^\alpha + \frac{1}{M_{H_C}} \left(V^* f Q_i^\alpha L_j + \epsilon^{\alpha\beta\gamma} V^* f u_{i\beta} d_{j\gamma} \right) \right\}$$

$$\times \left\{ \overline{H}_{C\alpha} + \frac{1}{M_{H_C}} \left(h e^{i\varphi_i} u_{i\alpha} e_i + \frac{h}{2} \epsilon_{\alpha\beta\gamma} e^{i\varphi_i} Q_i^\beta Q_i^\gamma \right) \right\}$$

$$- \frac{1}{M_{H_C}} \left(V^* f Q_i^\alpha L_j + \epsilon^{\alpha\beta\gamma} V^* f u_{i\beta} d_{j\gamma} \right) \left(h e^{i\varphi_i} u_{i\alpha} e_i + \frac{h}{2} \epsilon_{\alpha\beta\gamma} e^{i\varphi_i} Q_i^\beta Q_i^\gamma \right)$$

where we ignore the indices of Yukawa coupling and CKM matrix for simplicity

we can obtain dim. 5 operators

$$W_5 = -\frac{1}{M_{H_C}} \left(\underline{V_{kl}^* f_l h^i e^{i\varphi_i} u_i^C e_i^C Q_k L_l} + V_{kl}^* f_l h^i e^{i\varphi_i} u_i^C e_i^C u_k^C d_l^C + \frac{h^i}{2} V_{kl}^* f_l e^{i\varphi_i} Q_i Q_i Q_k L_l \right)$$

doesn't contribute to p-decay

where we use the relation as

$$\epsilon^{\alpha\beta\gamma} \epsilon_{\alpha\delta\epsilon} u_\beta d_\gamma Q^\delta Q^\epsilon = (\delta_\delta^\beta \delta_\epsilon^\gamma - \delta_\epsilon^\beta \delta_\delta^\gamma) u_\beta d_\gamma Q^\delta Q^\epsilon = 0$$

full 4-fermi operator for p → Kν from LLLL

$$\frac{1}{M_{H_C}} h^i e^{i\varphi_i} V_{kl}^* f^l \left[V_{ij} V_{im} (f(e_l, u_i) + f(u_i, d_k))(d_j \nu_l)(d_m u_k)\right.$$
$$\left. + V_{ij} V_{km} (f(e_l, u_k) + f(u_i, d_i))(u_i d_j)(d_m \nu_l)\right]$$

we set m=k=1 and j=2 in 1st line, i=j=1 and m=2 in 2nd line

$$\frac{1}{M_{H_C}} \left[h^i e^{i\varphi_i} V_{ul}^* f^l V_{is} V_{id} (f(e_l, u_i) + f(u_i, d))\right.$$
$$\left. + h^1 e^{i\varphi_1} V_{kl}^* f^l V_{ud} V_{kd} (f(e_l, u_k) + f(u, d))\right] (u_L d_L)(s_L \nu_{lL})$$

2nd line has h^1,small Yukawa coupling, therefore main contribution is from 1st line.

full 4-fermi operator for p → Kν from RRRR

$$\frac{1}{M_{H_C}} h^i e^{-i\varphi_k} V^*_{kl} f^l V_{ij} \left[h^k f^j V_{kn} g(u_k, e_j) d_n \nu_j d^C_l u^C_i \right.$$
$$\left. + h^i f^j V_{in} g(u_i, e_j) u^C_k d^C_l d_{nL} \nu_{Lj} \right]$$

we set l=i=1 and n=2 in 1st line, l=k=1 and n=2 in 2nd line

$$\frac{1}{M_{H_C}} \left[f^1 h^1 h^k f^j V^*_{kd} V_{uj} V_{kd} e^{-i\varphi_k} g(u_k, e_j) \right.$$
$$\left. + f^1 h^i h^i f^j V^*_{ud} V_{ij} V_{id} e^{-i\varphi_1} g(u_i, e_j) \right] (s_L \nu_{Lj})(u^C d^C)$$

1st line has h^1,small Yukawa coupling, therefore main contribution is from 2nd line.

Threshold correction

After integrating out the heavy particles,
we must consider gauge coupling relations between full and effective theory

$$\mathcal{L}_{\text{full}} = -\frac{1}{4} F^{\alpha}_{\mu\nu} F^{\alpha\mu\nu} + \cdots$$

$$\mathcal{L}_{\text{eff}} = \sum_i -\frac{1}{4}(1 - l_i) F^{a_i}_{\mu\nu} F^{a_i\mu\nu} + \cdots$$

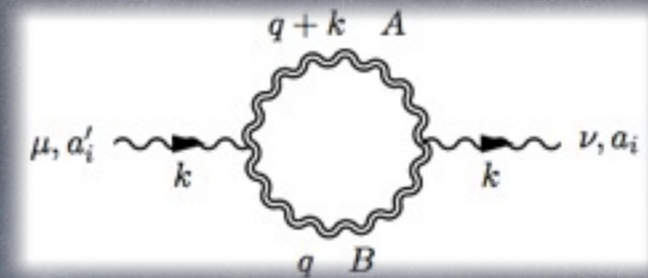
- "i" means residual symmetries (now SU(3)×SU(2)×U(1)).
- Heavy particles don't appear in L_{eff}.
- The effects of heavy particles are included as "l_i" in L_{eff}.

To determine the "l_i", after we calculate 1 loop diagram...

Threshold correction

-intro. for t.c.-

To determine the "l_i", after we calculate 1 loop diagram...



etc...

matching the coefficients,

$$\underbrace{-l_i(k^2 g^{\mu\nu} - k^\mu k^\nu)}_{\text{from } L_{eff}} = \underbrace{(\text{heavy particles contributions})(k^2 g^{\mu\nu} - k^\mu k^\nu)}_{\text{from } L_{full}}$$

redefine the fields and couplings to make kin.term canonical,

$$-\frac{1}{4}(1-l_i)F^{a_i}_{\mu\nu}F^{a_i\mu\nu} \to -\frac{1}{4}F'^{a_i}_{\mu\nu}F'^{a_i\mu\nu} \quad \text{by using} \quad \begin{array}{c} A'_{a_i\mu} \equiv \sqrt{1-l_i}A_{a_i\mu} \\ g_i \equiv g/\sqrt{1-l_i} \end{array}$$

Threshold correction

We have already determined the relation between bare g in full and eff. theory.

Next, we consider $\overline{\text{MS}}$ (similarly $\overline{\text{DR}}$) renormalized couplings

$$g\mu^{\epsilon/2} = g(\mu) - b_G g^3(\mu)/\epsilon' + \dots$$
$$g_i\mu^{\epsilon/2} = g_i(\mu) - b_i g_i^3(\mu)/\epsilon' + \dots$$

$$g_i \equiv g/\sqrt{1 - l_i}$$

so we can obtain the relation @ 1 loop

$$g_i(\mu) = g(\mu) + \frac{1}{2}\lambda_i(\mu) g^3(\mu)$$

where $\quad l_i = g_i^2(\lambda_i(\mu) + \lambda_i'(\mu)/\epsilon')$

Threshold correction

$$\frac{dg_i}{d\ln\mu} = \frac{1}{16\pi^2} b_i g_i^3$$

RGEs for gauge coupling @ 1 loop

the solutions of RGEs

$$\alpha_i^{-1}(m_Z) - \alpha_i^{-1}(\Lambda) = \frac{1}{2\pi} b_i \ln\frac{\Lambda}{m_Z}$$

$$\alpha_i^{-1}(\Lambda) = \alpha_G^{-1}(\Lambda) - 4\pi\lambda_i(\Lambda)$$

the threshold correction @ ~~GUT~~

$$\alpha_i^{-1}(m_{SUSY}) = \alpha_i^{-1}(m_{SUSY}) - 4\pi\lambda_i'(m_{SUSY})$$

the threshold correction @ ~~SUSY~~

Threshold correction

$$\alpha_3^{-1}(m_Z) = \alpha_5^{-1}(\Lambda) + \frac{1}{2\pi}\left(\left(-2 - \frac{2}{3}N_g\right)\ln\frac{m_{SUSY}}{m_Z}\right.$$
$$\left. +(-9 + 2N_g)\ln\frac{\Lambda}{m_Z} - 4\ln\frac{\Lambda}{M_V} + 3\ln\frac{\Lambda}{M_\Sigma} + \ln\frac{\Lambda}{M_{H_C}}\right)$$

$$\alpha_2^{-1}(m_Z) = \alpha_5^{-1}(\Lambda) + \frac{1}{2\pi}\left(\left(-\frac{4}{3} - \frac{2}{3}N_g - \frac{5}{6}\right)\ln\frac{m_{SUSY}}{m_Z}\right.$$
$$\left. +(-6 + 2N_g + 1)\ln\frac{\Lambda}{m_Z} - 6\ln\frac{\Lambda}{M_V} + 2\ln\frac{\Lambda}{M_\Sigma}\right)$$

$$\alpha_1^{-1}(m_Z) = \alpha_5^{-1}(\Lambda) + \frac{1}{2\pi}\left(\left(-\frac{2}{3}N_g - \frac{1}{2}\right)\ln\frac{m_{SUSY}}{m_Z}\right.$$
$$\left. +(2N_g + \frac{3}{5})\ln\frac{\Lambda}{m_Z} - 10\ln\frac{\Lambda}{M_V} + \frac{2}{5}\ln\frac{\Lambda}{M_{H_C}}\right)$$

Gaugino mass relation

-in RGE analysis-

$$\frac{d}{d\ln\mu}M_a = \frac{1}{8\pi^2}b_i g_i^2 M_a$$ RGE for gaugino mass @ 1 loop

$$\frac{d}{d\ln\mu}g_i = \frac{1}{16\pi^2}b_i g_i^3$$ RGE for gauge coupling @ 1 loop

So, $$\frac{d}{d\ln\mu}\frac{M_a}{g_i^2} = 0$$

It means that $$\frac{M_3}{g_3^2} = \frac{M_2}{g_2^2} = \frac{M_1}{g_1^2} = \frac{m_{1/2}}{g_U^2}$$

where $m_{1/2}$ means universal mass

and g_U means unified coupling @ GUT scale .

ratio of the amplitudes

-tau loop case-

we assume that squark mass is much heavier than wino etc...

to simplify our notation.

$$A_\tau(\tilde{c}_L) \sim \frac{M_2 g_2^2}{M_{H_C} m_{\tilde{Q}}^2} Y_c Y_b V_{ub}^* V_{cd} V_{cs}$$

$$A_\tau(\tilde{t}_L) \sim \frac{M_2 g_2^2}{M_{H_C} m_{\tilde{Q}}^2} Y_t Y_b V_{ub}^* V_{td} V_{ts}$$

$$A_\tau(\tilde{t}_R) \sim \frac{\mu}{M_{H_C} m_{\tilde{Q}}^2} Y_d Y_t^2 Y_\tau V_{tb}^* V_{ud} V_{ts}$$

we can obtain the ratio between each amplitude

$$A_\tau(\tilde{c}_L)/A_\tau(\tilde{t}_L) \sim m_c V_{cd} V_{cs} / m_t V_{td} V_{ts} \sim 4.73$$

$$A_\tau(\tilde{t}_R)/A_\tau(\tilde{t}_L) \sim \frac{\mu}{M_2} \frac{m_d}{m_b} \frac{V_{tb}^* V_{ud}}{V_{ub}^* V_{td}} Y_t Y_\tau \sim \frac{\mu}{M_2} \times 661.6$$

note: now we consider the case which $\mu > M_2$

D.S. affect RGE?

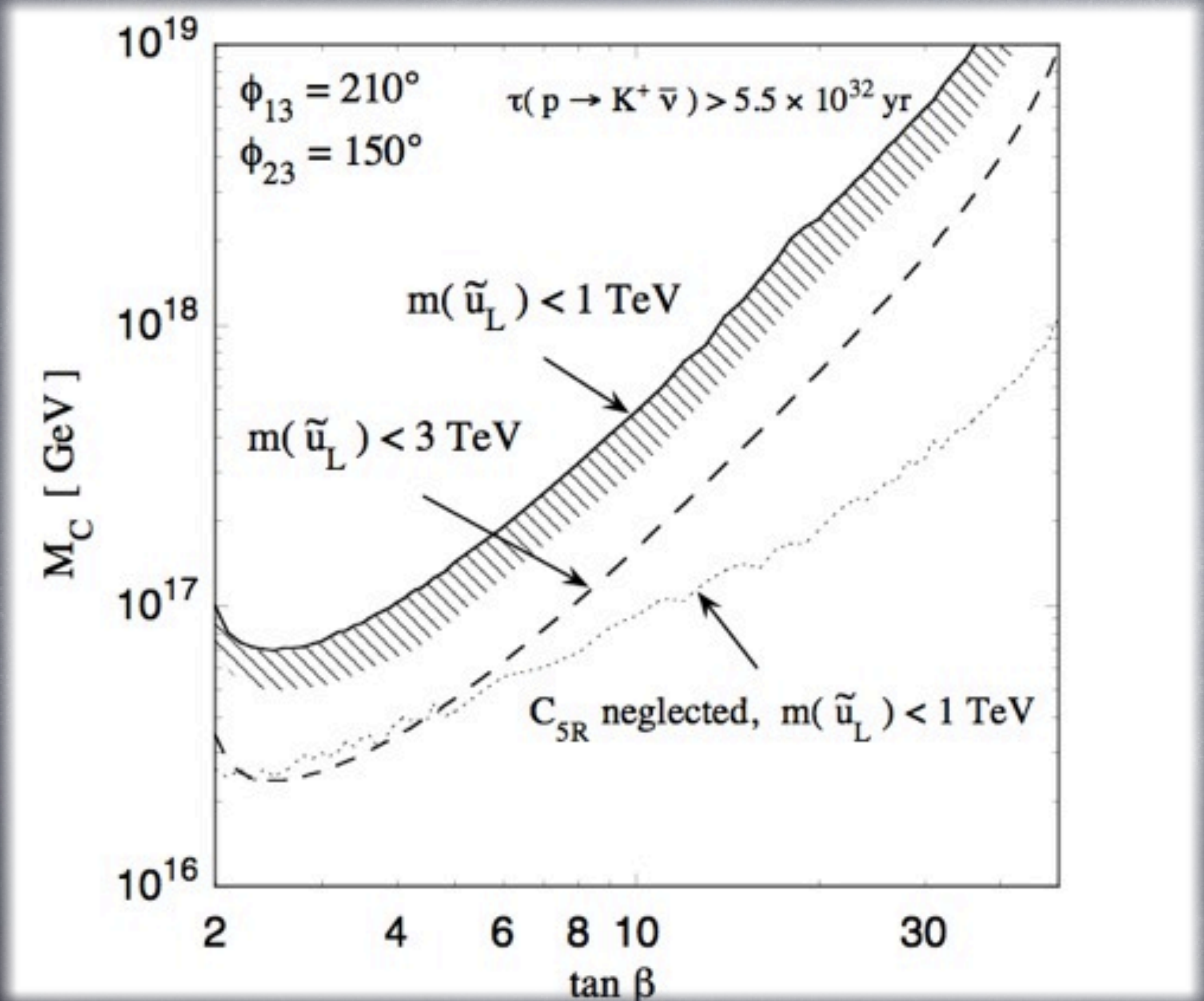
-decoupling scenario-

Does the Decoupling scenario affect RGE analysis??

- $\Lambda / \Lambda_{\text{SUSY}} \sim 10$

  Since energy scale dependence is logarithmic, splitting doesn't almost affect to RGE analysis.

tanβ sensitivity in G-N argument

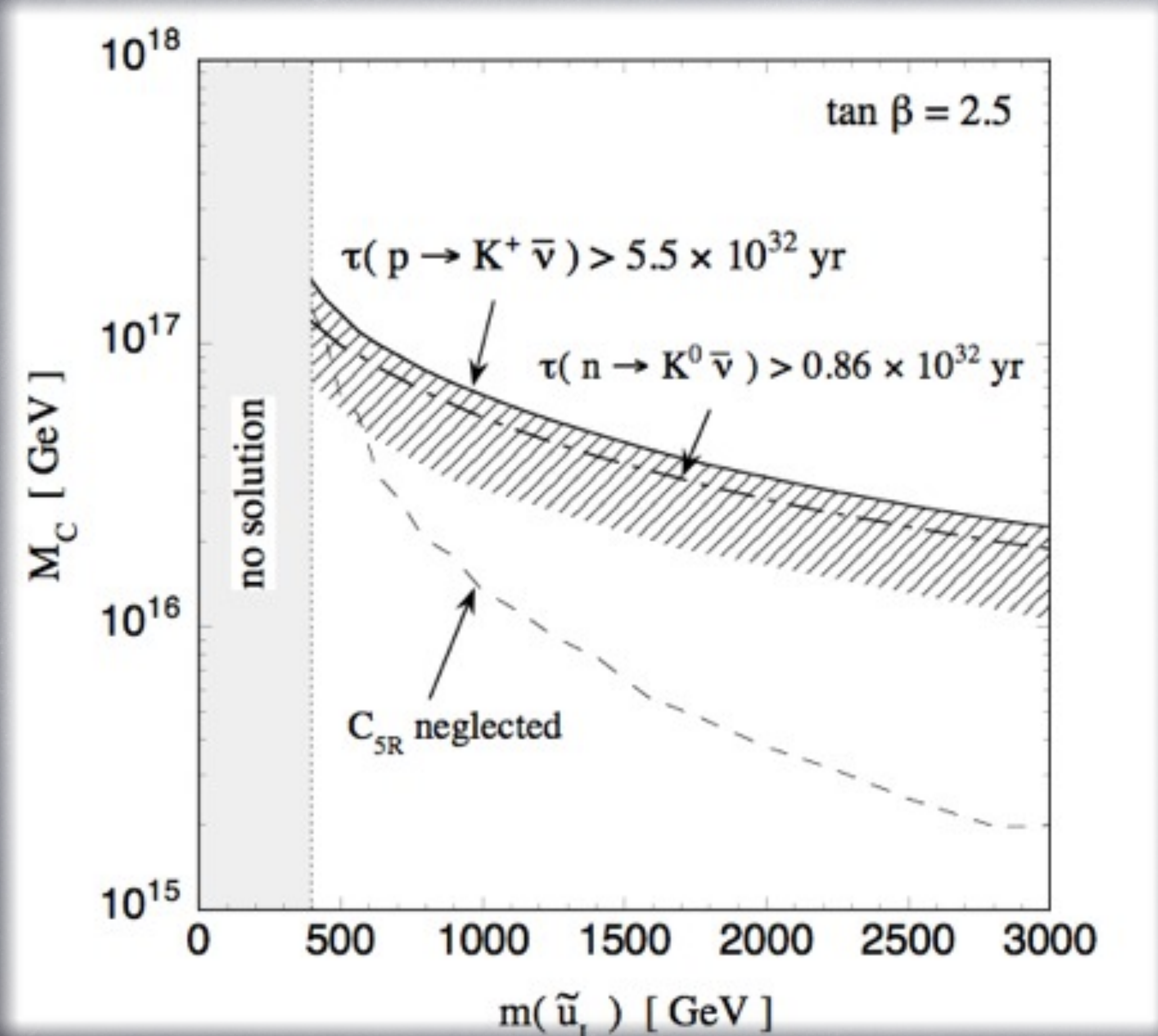


lower bound on
colored Higgs mass

large tanβ is not favored

squark mass scan in G-N argument

-squark m scan-



lower bound on
colored Higgs mass
as a function of the squark mass

large tanβ is not favored

RGE analysis

-my calc.-

$$3\alpha_2^{-1}(m_Z) - 2\alpha_3^{-1}(m_Z) - \alpha_1^{-1}(m_Z) = \frac{1}{2\pi}\left(\frac{12}{5}\ln\frac{M_{H_C}}{m_Z} - 2\ln\frac{m_{SUSY}}{m_Z}\right)$$

By taking into account the mass splitting of the SUSY particles (w/ the approx. that squark,slepton masses are degenerated)

$$-2\ln\frac{m_{SUSY}}{m_Z} \to 4\ln\frac{m_{\tilde{g}}}{m_{\widetilde{W}}} - \frac{8}{5}\ln\frac{\mu}{m_Z} - \frac{2}{5}\ln\frac{m_H}{m_Z}$$

input parameters

α_e^{-1} = 127.943±0.27

$\sin\theta_W$ = 0.23101

α_1^{-1} = 3 $\alpha^{-1}_e \sin\theta_W$ /5

α_2^{-1} = $\alpha^{-1}_e \sin\theta_W$

α_3^{-1} = 0.1183

m_Z = 91.1876GeV

m_H = 126.3GeV

$$3.5\times 10^{15}\mathrm{GeV} \le M_{H_C} \le 1.6\times 10^{16}\mathrm{GeV}$$