

# SU(5) GUT and Proton decay with High-scale SUSY

based on

J.Hisano, TK, N.Nagata : Phys. Lett. B **723** (2013) 324

J.Hisano, D.Kobayashi, TK, N.Nagata : JHEP **1307** (2013) 038

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# Motivation

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## High-Scale SUSY

- ⦿ The SM Higgs whose mass is about 126GeV was found at LHC in 2012.
- ⦿ In minimal SUSY models, Higgs mass tend to be lighter than Z boson at tree level. Therefore, we need large quantum corrections to Higgs mass.
- ⦿ One possible scenario : Heavy sparticle mass! (high-scale SUSY)

## Proton decay

- ⦿ Dim.5 proton decay rate can be small in this scenario.  
which is forbidden in low-energy SUSY scenario due to too short lifetime

# Outline

- ⦿ Motivation
- ⦿ Introduction
- ⦿ SUSY SU(5) GUT & GUT particle mass spectrum
- ⦿ Proton decay from dimension 5 operator
- ⦿ Summary & Future work

# High-scale SUSY

-without singlet-

If the ~~SUSY~~ messengers Z are not gauge singlet...

- the sfermion mass terms can be produced

$$\int d^4\theta \frac{1}{M_*^2} ZZ^\dagger \Phi \Phi^\dagger \rightarrow \frac{|\langle F_Z \rangle|^2}{M_*^2} \phi \phi^*$$

- the gaugino mass terms and the A terms can not be produced

$$\int d^2\theta \frac{1}{M_*} Z \text{tr} W W \quad \int d^2\theta \frac{1}{M_*} Z \Phi_i \Phi_j \Phi_k$$

Leading terms of the gaugino mass and the scalar trilinear  
can be induced w/ 1-loop suppression.

# High-scale SUSY

-gaugino mass-

Leading terms of the gaugino mass and the scalar trilinear  
can be induced w/ 1-loop suppression.

G.F.Giudice, M.A.Luty, H.Murayama and R.Rattazzi, JHEP **9812** (1998) 027

the gaugino mass

$$M_\lambda = -\frac{\beta_g(\mu)}{g(\mu)} m_{3/2} \quad \left( \beta_g(\mu) \simeq -\frac{bg^3(\mu)}{16\pi^2} \right)$$

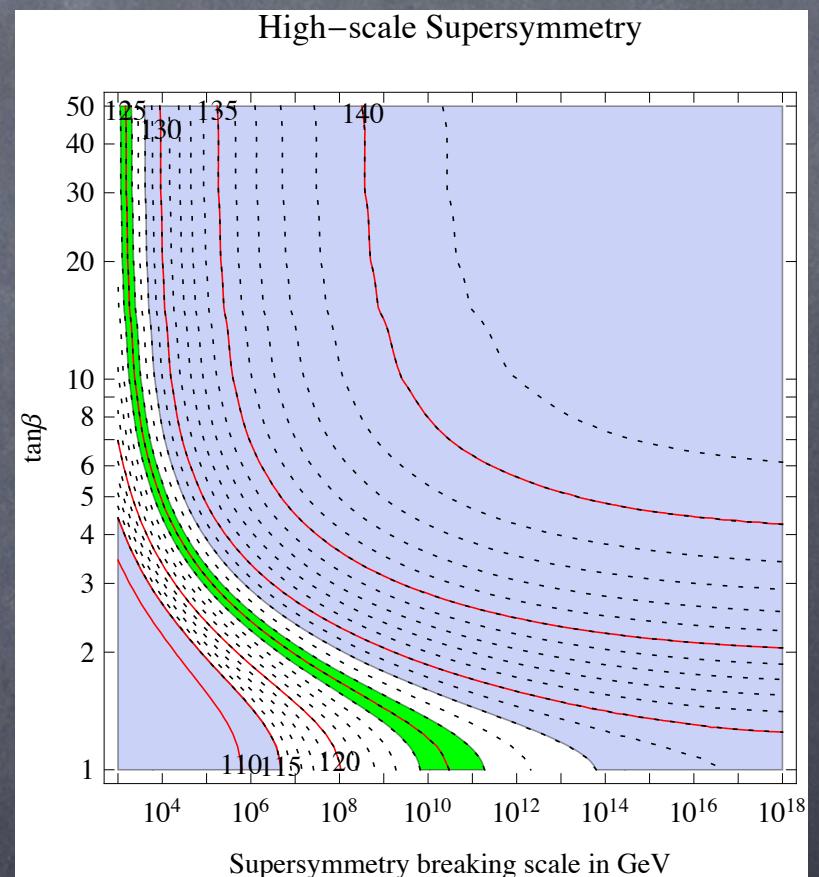
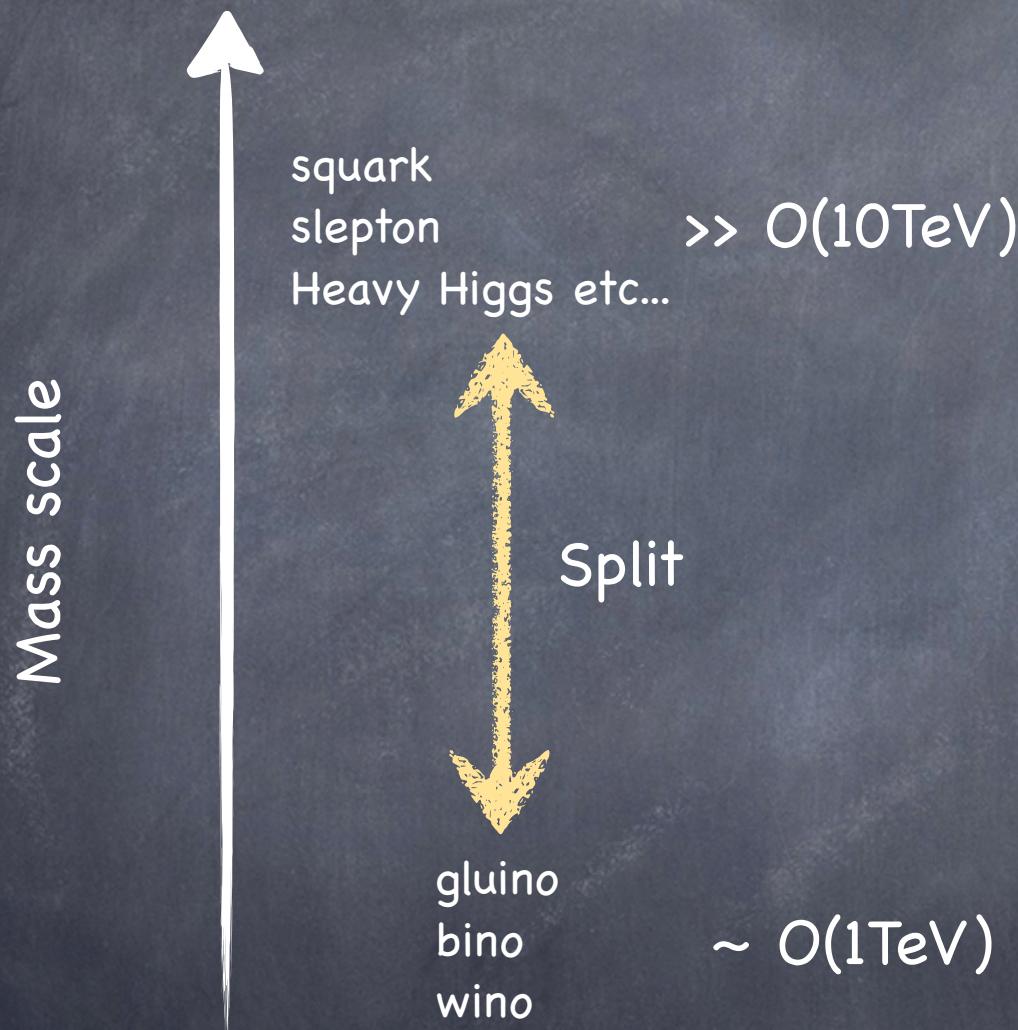
the Scalar trilinear coupling (A-term)

$$W = \lambda \Phi_1 \Phi_2 \Phi_3 \rightarrow A \phi_1 \phi_2 \phi_3 \in \mathcal{L}_{\text{soft}}$$

$$A(\mu) = -m_{3/2} \beta_\lambda(\mu) = -m_{3/2} \mu \frac{d\lambda}{d\mu}$$

# High-scale SUSY

-split mass spectrum and Higgs mass-



G.F.Giudice and A.Strumia,  
Nucl. Phys. B 858 (2012) 63

# SUSY SU(5) GUT

-particle contents-

The MSSM superfields are embedded in SU(5) multiplets.

matter

$$\psi^{\alpha\beta}(\mathbf{10}) = \begin{pmatrix} \epsilon^{abc} U_c^C & Q^{ar} \\ -Q^{sa} & \epsilon^{sr} E^C \end{pmatrix}, \quad \phi_\alpha(\mathbf{5}^*) = \begin{pmatrix} D_a^C \\ \epsilon_{rs} L^s \end{pmatrix}$$

Higgs

$$H(\mathbf{5}) = \begin{pmatrix} H_C \\ H_u \end{pmatrix}, \quad \bar{H}(\mathbf{5}^*) = \begin{pmatrix} \bar{H}_C \\ \bar{H}_d \end{pmatrix}$$

Gauge

$$\mathcal{V}_5 = \sum_{a=1}^{24} \mathcal{V}_5^a T^a = \frac{1}{\sqrt{2}} \begin{pmatrix} G^\alpha{}_\beta - \frac{2}{\sqrt{30}} B \delta^\alpha{}_\beta & X^{\dagger\alpha}{}_r \\ X^s{}_\beta & W^s{}_r + \frac{3}{\sqrt{30}} B \delta^s{}_r \end{pmatrix}$$

Gauge group SU(5) is broken by VEV of the adjoint Higgs  $\Sigma$ .

→  $X$ -boson,  $H_C$  and  $\Sigma$  have heavy masses.

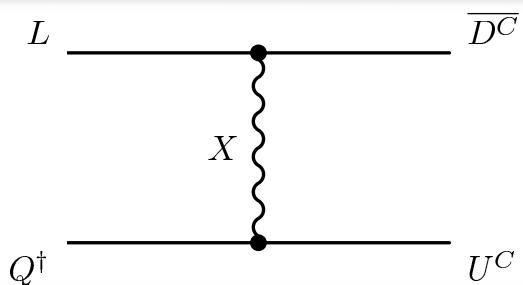
# SUSY SU(5) GUT

-dim.5 vs dim.6-

## Proton Decay in SUSY SU(5) GUT

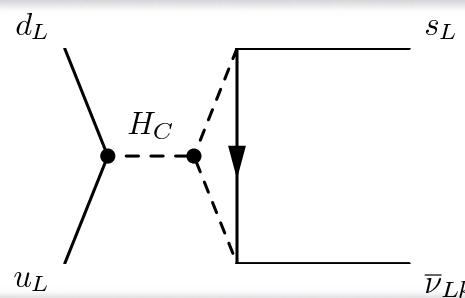
is caused by exchanging

- \* X-boson
- \* Colored Higgs



\* X-boson exchange

$$\tau_p(p \rightarrow \pi e^+) \propto (M_X)^4$$



\* Colored Higgs exchange

$$\tau_p(p \rightarrow K^+ \bar{\nu}) \propto (M_{H_C} M_S)^2$$

This can cause short lifetime for proton!!

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# RGE analysis

-threshold correction-

After integrating out the heavy particles

→ the gauge coupling relations between full and eff. theory

$$\mathcal{L}_{\text{full}} = -\frac{1}{4} F_{\mu\nu}^\alpha F^{\alpha\mu\nu} + \dots$$

where  $l_i = g_i^2(\lambda_i(\mu) + \lambda'_i(\mu)/\epsilon')$

$$\mathcal{L}_{\text{eff}} = \sum_i -\frac{1}{4} (1 - l_i) F_{\mu\nu}^{a_i} F^{a_i\mu\nu} + \dots$$

② “i” means residual symmetries.

③ The effects of heavy particles are included as “ $l_i$ ” in  $\mathcal{L}_{\text{eff}}$ .

By redefining fields and couplings to be canonical kinetic terms,  
we obtain the couplings in the effective theory.

$$-\frac{1}{4} (1 - l_i) F_{\mu\nu}^{a_i} F^{a_i\mu\nu} \rightarrow -\frac{1}{4} F'_{\mu\nu}^{a_i} F'^{a_i\mu\nu} \quad \text{by using}$$

$$A'_{a_i\mu} \equiv \sqrt{1 - l_i} A_{a_i\mu}$$

$$g_i \equiv g / \sqrt{1 - l_i}$$

# RGE analysis

-threshold correction-

We have already determined the relation between bare couplings of full theory and those of eff. theory.

Next, we consider renormalized couplings

$$g\mu^{\epsilon/2} = g(\mu) - b_G g^3(\mu)/\epsilon' + \dots$$

$$g_i\mu^{\epsilon/2} = g_i(\mu) - b_i g_i^3(\mu)/\epsilon' + \dots$$

$$g_i \equiv g/\sqrt{1-l_i}$$

$$\text{where } l_i = g_i^2(\lambda_i(\mu) + \lambda'_i(\mu)/\epsilon')$$

then we can obtain the relation @ 1-loop

$$\alpha_i^{-1}(\mu) = \alpha^{-1}(\mu) - 4\pi \lambda_i(\mu)$$

including the contributions  
from heavy particles

# GUT mass spectrum

-RGE analysis-

We assume that there are three thresholds.

- ▶ GUT scale
- ▶ SUSY breaking scale (squarks,sleptons,higgsinos and heavy Higgs)
- ▶ Gaugino threshold

By using these threshold corrections w/ the solution of 1-loop RGEs, we have

$$\frac{3}{\alpha_2(m_Z)} - \frac{2}{\alpha_3(m_Z)} - \frac{1}{\alpha_1(m_Z)} = \frac{1}{2\pi} \left[ \frac{12}{5} \ln \frac{M_{H_C}}{m_Z} - 2 \ln \frac{M_S}{m_Z} + 4 \ln \frac{M_3}{M_2} \right]$$

$$\frac{5}{\alpha_1(m_Z)} - \frac{3}{\alpha_2(m_Z)} - \frac{2}{\alpha_3(m_Z)} = \frac{1}{2\pi} \left[ 12 \ln \frac{M_V^2 M_\Sigma}{m_Z^3} + 4 \ln \frac{M_3 M_2}{m_Z^2} \right]$$

# GUT mass spectrum

-w/ 1-loop RGEs-

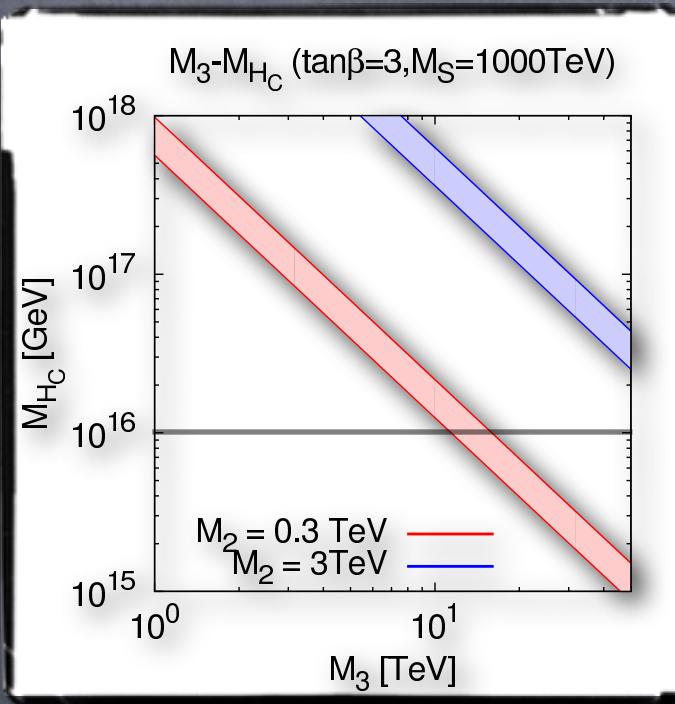
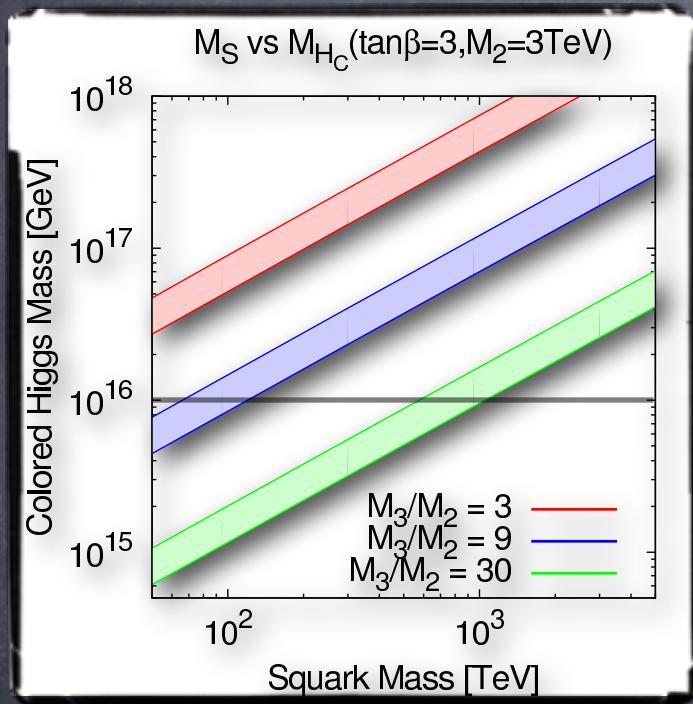
$$\frac{3}{\alpha_2(m_Z)} - \frac{2}{\alpha_3(m_Z)} - \frac{1}{\alpha_1(m_Z)} = \frac{1}{2\pi} \left[ \frac{12}{5} \ln \frac{M_{H_C}}{m_Z} - 2 \ln \frac{M_S}{m_Z} + 4 \ln \frac{M_3}{M_2} \right]$$
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- ⌚ The higher the SUSY breaking scale becomes, the heavier color-triplet Higgs becomes.
- ⌚ This triplet mass depends on the ratio of the gaugino mass.
- ⌚  $M_{\text{GUT}}^3 \equiv M_V^2 M_\Sigma$  becomes lighter if the gauginos are heavy.

# Numerical results

-Colored Higgs mass-

## Colored Higgs mass from 2-loop RGEs and Threshold corrections



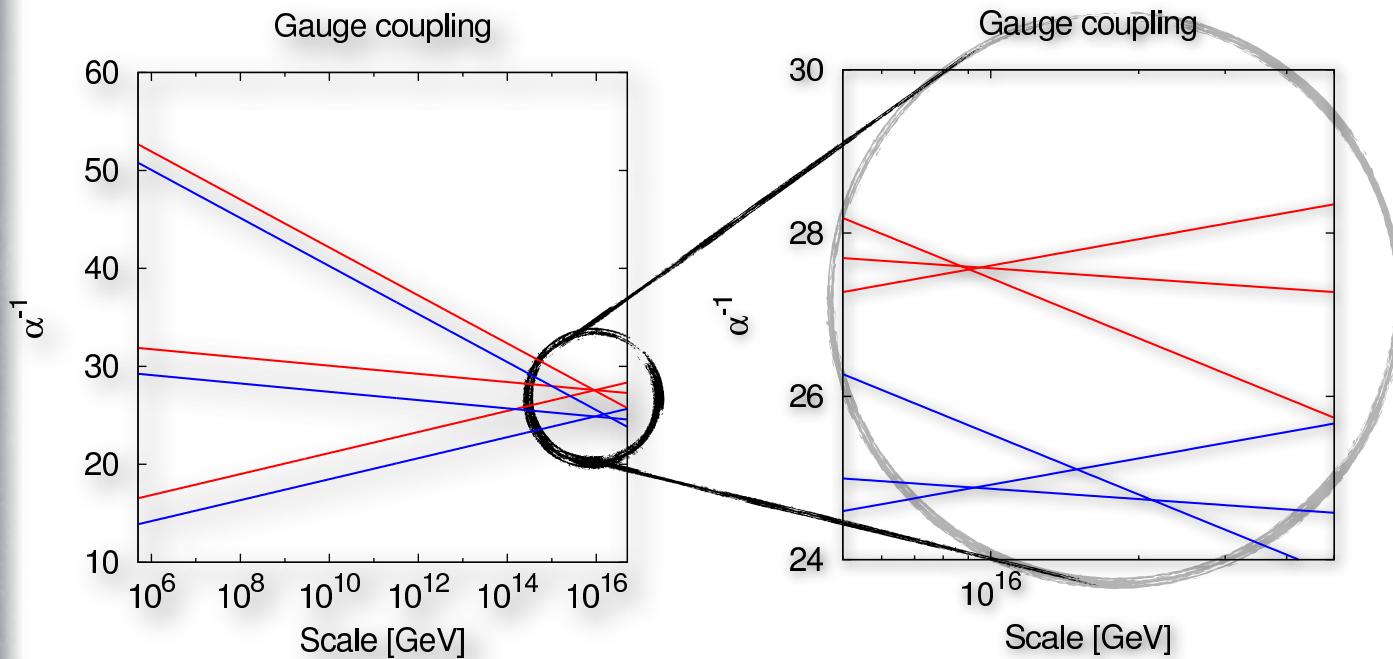
Colored Higgs can have  $10^{16}$  GeV mass !!!

Error bar →  
the uncertainty of the  
strong coupling.  
 $\alpha_s(M_Z)=0.1184(7)$

$M_S=1\text{TeV}, M_3=1\text{TeV}, M_2=0.3\text{TeV}$

$$M_{H_C} = (8.774 \pm 2.251) \times 10^{14}\text{GeV}$$

# Improvement in gauge coupling unification



Red: High-scale SUSY

$$M_S = 10^2 \text{ TeV}$$

$$M_2 = 3 \text{ TeV}$$

$$M_3/M_2 = 9$$

Blue: Low-scale SUSY

$$M_S = 1 \text{ TeV}$$

$$M_2 = 300 \text{ GeV}$$

$$M_3/M_2 = 3$$

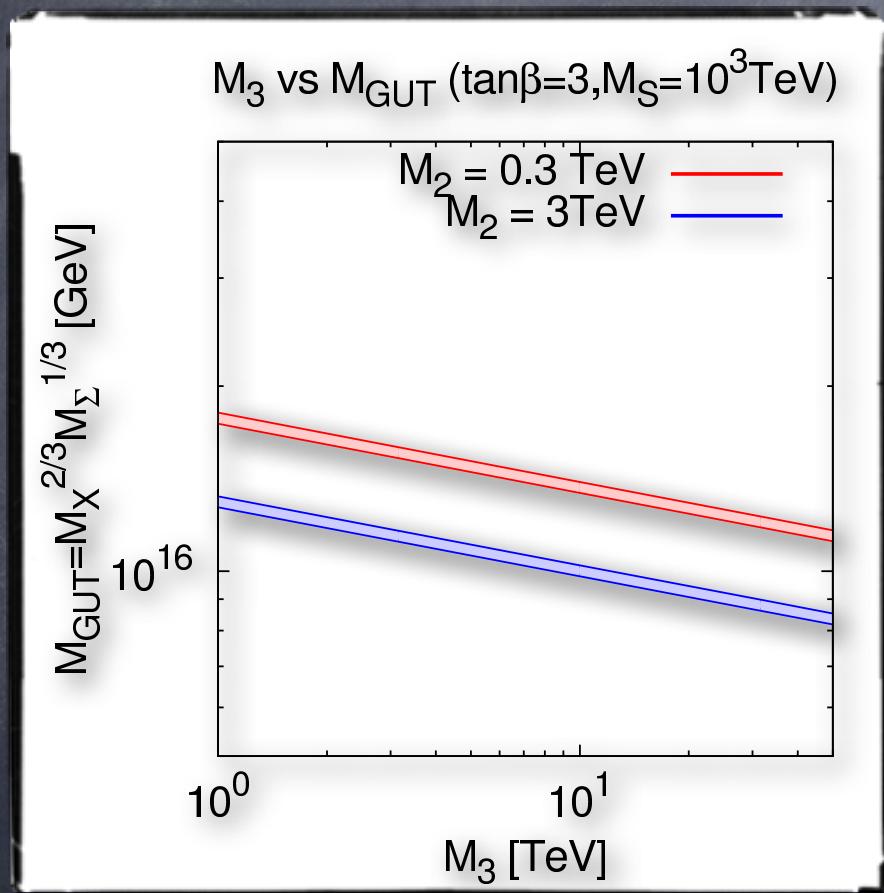
If Colored-Higgs can have  $10^{16}$  GeV mass  
 => threshold correction becomes small @ GUT scale.  
 => gauge couplings unify accurately

# Numerical results

-GUT scale-

GUT scale ( $M_{\text{GUT}}^3 \equiv M_X^2 M_\Sigma$ ) from 2-loop RGEs

and Threshold corrections



Error:  $\alpha_s(M_Z) = 0.1184(7)$

X-boson can be lighter than  $10^{16} \text{ GeV}$ .

This result implies that  
Dim.6 Proton decay rate may be  
enhanced slightly...

If...  $M_X = 0.8 \times 10^{16} \text{ GeV}$

$\tau(p \rightarrow \pi^0 e^+) \sim 5 \times 10^{34} \text{ yrs}$

Current experimental limit

$\tau_{\text{exp}}(p \rightarrow \pi^0 e^+) \sim 1.3 \times 10^{34} \text{ yrs}$

# Brief summary for threshold correction

The Colored-Higgs can have mass around GUT scale( $\sim 10^{16}$ GeV).

2 possibilities.

1. Gauge coupling unification can be improved in High-scale SUSY.
2. The proton decay by intermediating  $H_C$  can be evaded from exp. limits.

X-boson can be lighter than  $10^{16}$ GeV slightly.

The proton decay (due to X) can be caught by next generation detector??

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# Proton decay

-problems for dim.5-

The minimal SUSY SU(5) GUT(+ low-scale SUSY)  
has predicted that proton lifetime is too short.

$$\tau_p(p \rightarrow K^+ \bar{\nu}) \sim 10^{30} \text{ yrs}$$

T.Goto and T.Nihei, Phys. Rev. D **59** (1999) 115009

Current experimental limit

$$\tau_{\text{exp}}(p \rightarrow K^+ \bar{\nu}) > 4 \times 10^{33} \text{ yrs}$$

To solve this problem...

- Forbid or Suppress the dim.5 operator by symmetry.
- “high-scale SUSY” can revive the minimal SUSY SU(5).

## Superpotential in SUSY SU(5)

$$W = \frac{f}{3} \text{Tr} \Sigma^3 + \frac{fV}{2} \text{Tr} \Sigma^2 + \lambda \overline{H}_\alpha (\Sigma_\beta^\alpha + 3V \delta_\beta^\alpha) H^\beta \\ + \frac{h^{ij}}{4} \epsilon_{\alpha\beta\gamma\delta\epsilon} \psi_i^{\alpha\beta} \psi_j^{\gamma\delta} H^\epsilon + \sqrt{2} f^{ij} \psi_i^{\alpha\beta} \phi_{j\alpha} \overline{H}_\beta$$

$\alpha, \beta, \gamma, \delta, \epsilon, \dots = 1, 2, \dots, 5$  (SU(5) indices)

$i, j = 1, 2, 3$  (gen. indices)

SU(5)  $\rightarrow$  SU(3)  $\times$  SU(2)  $\times$  U(1) by VEV of adjoint Higgs  $\Sigma$

## Assumptions

- ⦿ R-parity
- ⦿ the third term (doublet-triplet splitting)

# Proton decay

## - $p$ decay in mSUSY GUT-

# Yukawa terms in superpotential in terms of MSSM superfields

$$\begin{aligned} W = & h^i V_{ij} U_i^C E_j^C H_C + \frac{1}{2} h^i e^{i\varphi_i} (Q_i Q_i) H_C \\ & + h^i U_i^C (Q_i H_f) + V_{ij}^* f^j e^{-i\varphi_i} U_i^C D_j^C \overline{H}_C \\ & + V_{ij}^* f^j Q_i L_j \overline{H}_C + V_{ij}^* f^j Q_i \overline{H}_f D_j^C + f^i E_i^C L_i \overline{H}_f \end{aligned}$$

# $V$ : CKM matrix

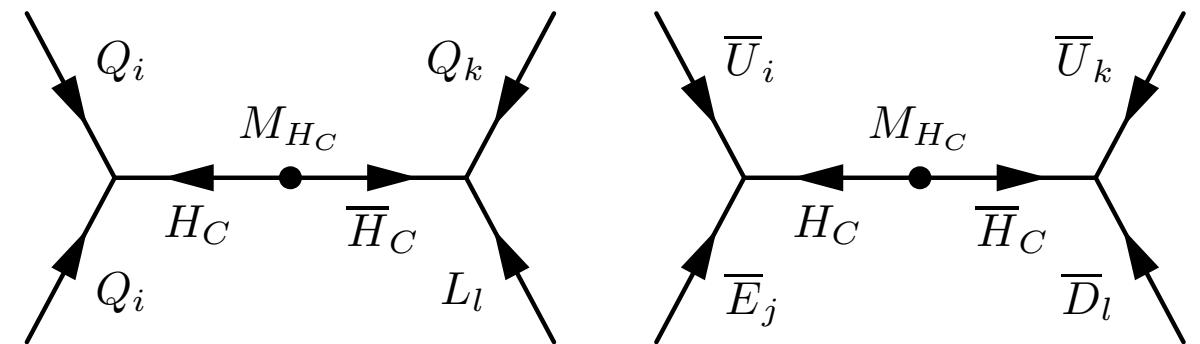
# $h, f$ : Yukawa couplings @ GUT scale

$\varphi$  : additional phases w/  $\varphi_1 + \varphi_2 + \varphi_3 = 0$

# Proton decay

-p decay in mSUSY GUT-

dim.5 operator



By integrating out the heavy colored Higgs, we can obtain the superpotential for the dim.5 operators

$$W_5 = -\frac{1}{M_{H_C}} \left\{ \frac{1}{2} C_L^{ijkl} Q_k Q_l Q_i L_j + C_R^{ijkl} E_k^C U_l^C U_i^C D_j^C \right\}$$

colored Higgs mass

LLLL operator

RRRR operator

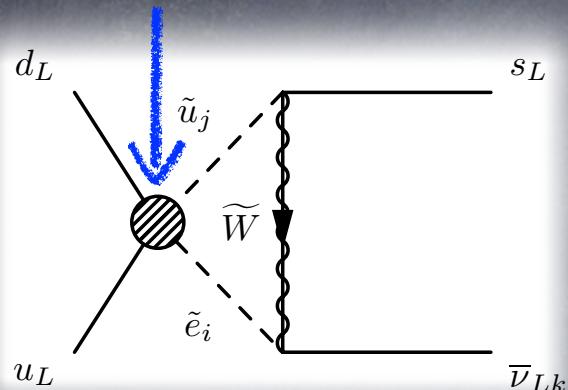
$$C_L^{ijkl} = h^i V_{kl}^* f_l e^{i\varphi_i}$$

$$C_R^{ijkl} = V_{kl}^* f_l h^i V_{ij} e^{-i\varphi_k}$$

# Proton decay

-p-decay from dim.5 operator-

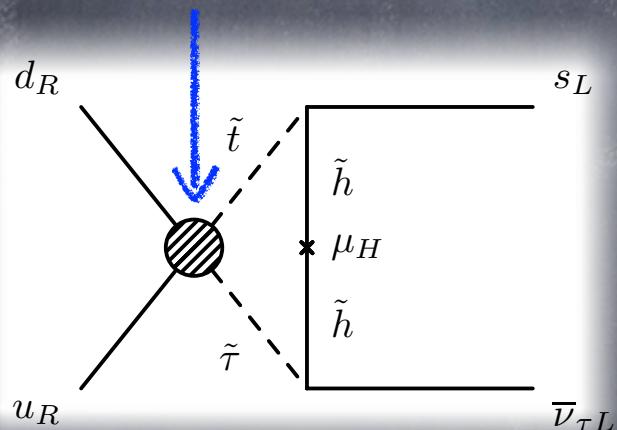
LLLL operator



$$W_{LLLL} = -\frac{h^i f^l}{2M_{H_C}} V_{kl}^* e^{i\varphi_i} Q_i Q_i Q_k L_l$$

The **charged winos** contribute to proton decay dominantly.

RRRR operator



$$W_{RRRR} = -\frac{f^l h^i}{M_{H_C}} V_{kl}^* V_{ij} e^{-i\varphi_k} U_i^C E_j^C U_k^C D_l^C$$

higgsino-dressed diagrams need the Yukawa coupling.  
 → the contribution of third gen. is only large.

# Full Lagrangian for proton decay

4-Fermi operator from wino-dressed diagram

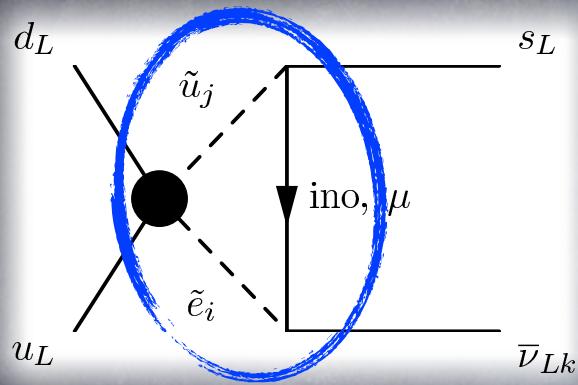
$$\frac{2}{M_{H_C}} \frac{\alpha_2(M_S)\alpha_2(m_Z)}{m_W^2 \sin 2\beta} \sum_{i,l} m_{u_i} m_{d_l} A_{S_W}^{(i,l)} e^{i\varphi_i} V_{ul}^* V_{is} V_{id} \\ \times F(M_2, M_S) \epsilon_{\alpha\beta\gamma} \left[ (d_L^\beta u_L^\alpha)(s_L^\gamma \nu_{lL}) + (s_L^\beta u_L^\alpha)(d_L^\gamma \nu_{lL}) \right]$$

from higgsino-dressed diagram

$$-\frac{1}{M_{H_C}} \frac{\alpha_2^2 e^{i\varphi_1}}{m_W^4 \sin^2 2\beta} F(\mu_H, M_S) (m_t)^2 m_\tau V_{tb}^* \epsilon^{\alpha\beta\gamma} \\ [m_d V_{ud} V_{ts} A_{S_h}^1 (d_{R\gamma} u_{R\alpha})(s_{L\beta} \nu_{L\tau}) + m_s V_{us} V_{td} A_{S_h}^2 (s_{R\gamma} u_{R\alpha})(d_{L\beta} \nu_{L\tau})]$$

It is important to understand  
the behavior of the loop-function F

# Sparticle Mass dependence



$$F(\mu, M_S) = \frac{\mu}{M_S^2 - \mu^2} \left[ 1 - \frac{\mu^2}{M_S^2 - \mu^2} \ln \frac{M_S^2}{\mu^2} \right]$$

$$\sim \begin{cases} \frac{\mu}{M_S^2} & (M_S \gg \mu) \\ \frac{1}{2M_S} & (M_S \sim \mu) \end{cases}$$

In a splitting case ( $\mu \ll M_S$ )

$F \ll 1/M_S$  i.e. dim.5 proton lifetime can be longer.

Approximately, proton lifetime given as,

$$\tau_p \sim (4 \times 10^{35}) \times \sin^4 2\beta \left( \frac{0.1}{A_R} \right)^2 \left( \frac{M_S}{10^2 \text{ TeV}} \right)^2 \left( \frac{M_{H_C}}{10^{16} \text{ GeV}} \right)^2 \text{ [yrs]}$$

for  $M_S \sim \mu_H$

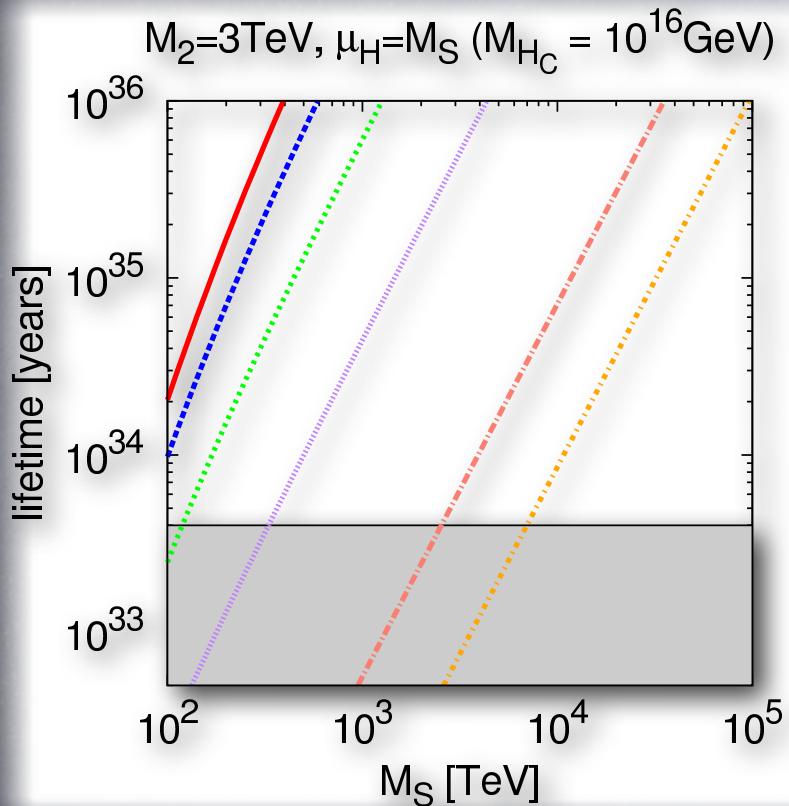
# Setups for calc.

-p-decay from dim.5 operator-

- ⦿ Squarks and Sleptons are degenerated( $M_S$ )
- ⦿ Gaugino masses are lighter than  $M_S$ .
- ⦿ Higgsino mass is model-dependent
  - (→degenerated to  $M_S$  or lighter than  $M_S$ )
- ⦿ A-terms are negligible due to the 1-loop suppressed coupling.
- ⦿ the Colored Higgs mass is set to be  $10^{16}\text{GeV}$
- ⦿ We choose the additional phase as the proton lifetime is short.

# Results

- $\mu_H = M_S$ -



case :  $\mu_H = M_S$

From left top to right bottom,

$\tan\beta$  is 2,3,5,10,30 and 50, respectively.

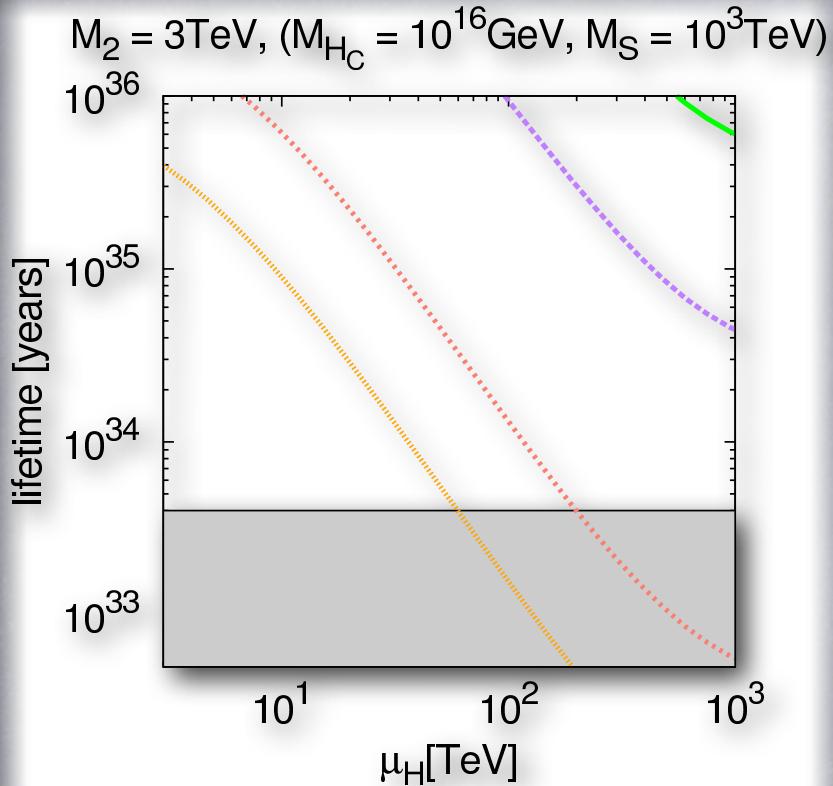
$$\tau_p \propto \frac{1}{\tan^4 \beta}$$

The proton lifetime can be consistent with experimental result even the sfermion mass is about 100TeV.

the shaded region is excluded by the Super-Kamiokande experiment

# Results

$-\mu_H < M_S -$



case :  $\mu_H < M_S$

From right top to left bottom,  
 $\tan\beta$  is 5,10,30 and 50, respectively.

$$\tau_p \propto \frac{1}{\tan^4 \beta}$$

The lighter Higgsino mass becomes,  
the longer lifetime proton has.

Corresponding to Splitting case

the shaded region is excluded by  
the Super-Kamiokande experiment

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# Summary

- high-scale SUSY may be fascinating.
- The gauge coupling unification will be improved in high-scale SUSY.
- The dim.5 proton decay w/ high-scale SUSY can be evaded from experimental limits(Super-K).
- This decay modes ( $p \rightarrow K^+ + \nu$ ) may be discovered by next generation detector(Hyper-K :  $\tau > 2.5 \times 10^{34}$  yrs).