Physics 112 – Fall 2017

Thermodynamics and Statistical Mechanics

Homework 2 due in class on October 17, 2017.

THIS ASSIGNMENT CONTAINS 6 PROBLEMS + 1 (OPTIONAL) BONUS PROBLEM
CHECK ALL THE PAGES.

Problem 1:

Use a Taylor series to derive the useful approximation $\ln(1+x) \approx x$, for $|x| \ll 1$. Test it numerically for $x = 0.1, -0.01, 0.001$.

Problem 2:

Suppose you flip 10000 identical fair coins (obtain a non-zero numerical value for parts). Use Stirling's approximation to calculate the probability of getting exactly…

(a) 5000 heads and 5000 tails.
(b) 5100 heads and 4900 tails.
(c) 5200 heads and 4800 tails.
(d) 5300 heads and 4700 tails.
(e) Would you be surprised to see 6000 heads and 4000 tails?

Problem 3 (Schroeder 2.5):

Do (a), (d), (e), (f) and (g).

Problem 2.5. For an Einstein solid with each of the following values of N and q, list all of the possible microstates, count them, and verify formula 2.9.

(a) $N = 3$, $q = 4$
(b) $N = 3$, $q = 5$
(c) $N = 3$, $q = 6$
(d) $N = 4$, $q = 2$
(e) $N = 4$, $q = 3$
(f) $N = 1$, $q =$ anything
(g) $N =$ anything, $q = 1$

Problem 4 (Schroeder 2.7):

Problem 2.7. For an Einstein solid with four oscillators and two units of energy, represent each possible microstate as a series of dots and vertical lines, as used in the text to prove equation 2.9.

Problem 5 (Schroeder 2.17):

Problem 2.17. Use the methods of this section to derive a formula, similar to equation 2.21, for the multiplicity of an Einstein solid in the "low-temperature" limit, $q \ll N$.

Problem 6 (Schroeder 2.23):

Do (a) and (b) only.

Problem 2.23. Consider a two-state paramagnet with 10^{23} elementary dipoles, with the total energy fixed at zero so that exactly half the dipoles point up and half point down.

(a) How many microstates are "accessible" to this system?

(b) Suppose that the microstate of this system changes a billion times per second. How many microstates will it explore in ten billion years (the age of the universe)?

Bonus Problem (Schroeder B.14):

This is optional and worth an extra 20% bonus of the assignment total. You will need to read sections B.2, B.3 and B.4 in the appendix of the book to solve this problem.

Problem B.14. The proof of formula B.28 is by induction.

(a) Check formula B.28 for $n = 0$ and for $n = 1$.

(b) Show that

$$\int_0^\pi (\sin\theta)^n \, d\theta = \left(\frac{n-1}{n}\right) \int_0^\pi (\sin\theta)^{n-2} \, d\theta.$$

(Hint: First write $(\sin\theta)^n$ as $(\sin\theta)^{n-2}(1 - \cos^2\theta)$. Integrate the second term by parts, differentiating one factor of $\cos\theta$ and integrating everything else.)

(c) Use the results of parts (a) and (b) to prove formula B.28 by induction.