A Course in Digital Signal Processing

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BOAZ PORAT

Technion, Israel Institute of Technology
Department of Electrical Engineering

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To Aliza
"The first time ever . . . "

To Ofer and Noga

and

In Memory of David, Tova, and Ruth Freud

The Author

Boaz Porat was born in Haifa, Israel, in 1945. He received the B.S. and M.S. degrees in electrical engineering from the Technion, in Haifa, Israel, in 1967 and 1975, respectively, and the M.S. degree in statistics and Ph.D. in electrical engineering from Stanford University in 1982. Since 1983, he has been with the Department of Electrical Engineering at the Technion, Haifa, where he is now a professor. He has held visiting positions at University of California at Davis, California; Yale University, New Haven, Connecticut; and Ben-Gurion University, Beer-Sheba, Israel. He also spent various periods with Signal Processing Technology, California, and served as a consultant to electronics industries in Israel on numerous occasions. He is a Fellow of the Institute of Electrical and Electronics Engineers.

Dr. Porat received the European Association for Signal Processing Award for the Best Paper of the Year in 1985; the Ray and Miriam Klein Award for Excellence in Research in 1986; the Technion's Distinguished Lecturer Award in 1989 and 1990; and the Jacknow Award for Excellence in Teaching in 1994. He was an Associate Editor of the IEEE TRANSACTIONS ON INFORMATION THEORY from 1990 to 1992, in the area of estimation. He is author of the book *Digital Processing of Random Signals: Theory and Methods,* published by Prentice Hall, and of 120 scientific papers. His research interests are in statistical signal processing, estimation, detection, and applications of digital signal processing in communications, biomedicine, and music.

The Software

The MATLAB software and the data files for this book are available by anonymous file transfer protocol (ftp) from:
`ftp.wiley.com /public/college/math/matlab/bporat`
or from:
`ftp.technion.ac.il /pub/supported/ee/Signal_processing/B_Porat`
See the file `readme.txt` for instructions.

Additional information on the book can be found on the World-Wide Web at:
`http://www-ee.technion.ac.il/~boaz`

The author welcomes comments, corrections, suggestions, questions, and any other feedback on the book; send e-mail to `boaz@ee.technion.ac.il`.

Preface

The last thing one discovers in composing a work is what to put first.

Blaise Pascal (1623–62)

This book is a text on digital signal processing, at a senior or first-year-graduate level. My purpose in writing it was to provide the reader with a precise, broad, practical, up-to-date exposition of digital signal processing. Accordingly, this book presents DSP theory in a rigorous fashion, contains a wealth of material—some not commonly found in general DSP texts, makes extensive use of MATLAB† software, and describes numerous state-of-the-art applications of DSP.

My students often ask me, at the first session of an undergraduate DSP course that I teach: "Is the course mathematical, or is it useful?" to which I answer: "It is both." To convince yourself that DSP is mathematical, take a moment to flip through the pages of this book. See? To convince yourself that DSP is useful, consider your favorite CD music recordings; your cellular phone; the pictures and sound you get on your computer when you connect to the Internet or use your multimedia CD-ROM software; the electronic medical instruments you might see in hospitals; radar systems used for air traffic control and for meteorology; the digital television you may have in the near future. All these rely to some extent on digital signal processing.

What does this book have to offer that other DSP texts don't? There is only one honest answer: my personal perspective on the subject and on the way it should be taught. So, here is my personal perspective, as it is reflected in the book.

1. Theory and practice should be balanced, with a slight tilt toward theory. Without theory, there is no practice. Accordingly, I always explain *why* things work before explaining *how* they work.
2. In explaining theories, accuracy is crucial. I therefore avoid cutting corners but spend the necessary time and effort to supply accurate and detailed derivations. Occasionally, there are results whose derivation is too advanced for the level of this book. In such cases, I only state the result, and alert the reader to the missing derivation.
3. Consistent notation is an indispensable part of accuracy; ambiguous notation leads to confusion. The theory of signals and systems is replete with mathematical objects that are similar, but not identical: signals in continuous and discrete time, convolutions of various kinds, Fourier transforms, Laplace transforms, z-transforms, and a host of discrete transforms. I have invested effort in developing a consistent notation for this book. Chapter 1 explains this notation in detail.
4. Examples should reflect real-life applications. Drill-type examples should not be ignored, but space should also be allocated to engineering examples. This is

†MATLAB is a registered trademark of The MathWorks, Inc., Natick, MA, U.S.A.

not easy, since the beginning student often has not been exposed to engineering reality. In constructing such examples, I have tried to be faithful to this reality, while keeping the discussion as elementary as possible.

5. The understanding of DSP algorithms can be greatly enhanced by reading a piece of software code that implements the algorithm. A software code must be accurate, otherwise it will not work. Illustrating algorithms through software codes used to be a nightmare in the old days of FORTRAN and even during the present days of the C language. Not any more! Now we have MATLAB, which is as easy to read as plain English. I therefore have made the effort to illustrate every computational procedure described in the book by a MATLAB code. The MATLAB programs are also available via the Internet from the publisher or the author, see instructions preceding this preface. Needless to say, I expect every student to be MATLAB literate.

A problem in writing a textbook for a course on DSP is that the placement of such courses in the curriculum may vary, as also the level and background assumed of the students. In certain institutes (such as the one I am in), the first DSP course is taken at a junior or senior undergraduate level, right after a signals and systems course; therefore, mostly basic material should be taught. In other institutes, DSP courses are given at a first-year graduate level. Graduate students typically have better backgrounds and wider experiences, so they can be exposed to more advanced material. In trying to satisfy both needs, I have included much more material than can be covered in a single course. A typical course should cover about two thirds of the material, but undergraduate and graduate courses should not cover the same two thirds.

I tried to make the book suitable for the practicing engineer as well. A common misconception is that "the engineer needs practice, not theory." An engineer, after a few years out of college, needs updating of the theory, whether it be basic concepts or advanced material. The choice of topics, the detail of presentation, the abundance of examples and problems, and the MATLAB programs make this book well suited to self study by engineers.

The main prerequisite for this book is a solid course on signals and systems at an undergraduate level. Modern signals and systems curricula put equal (or nearly so) emphases on continuous-time and discrete-time signals. The reader of this book is expected to know the basic mathematical theory of signals and their relationships to linear time-invariant systems: convolutions, transforms, frequency responses, transfer functions, concepts of stability, simple block-diagram manipulations, and some applications of signals and systems theory.

I use the following conventions in the book:

1. Sections not marked include basic-level material. I regard them as a must for all students taking a first DSP course. I am aware, however, that many instructors disagree with me on at least two subjects in this class: IIR filters and the FFT. Instructors who do not teach one of these two subjects (or both) can skip the corresponding chapters (10 and 5, respectively).

2. Sections marked by an asterisk include material that is either optional (being of secondary importance) or more advanced (and therefore, perhaps, more suitable for a graduate course). Advanced problems are also marked by asterisks.

3. Superscript numerals denote end notes. End notes appear at the end of the chapter, in a section named "Complements." Each end note contains, in square brackets, backward reference to the page referring to it. Most end notes are of a more advanced nature.

4. Occasionally I put short paragraphs in boxes, to emphasize their importance. For example:

Read the Preface before you start Chapter 1!

5. Practical design procedures are highlighted; see page 284 for an example.
6. The symbol □ denotes the end of a proof (*QED*), as well as the end of an example.
7. The MATLAB programs are mentioned and explained at the points where they are needed to illustrate the material. However, the program listings are collected together in a separate section at the end of each chapter. Each program starts with a description of its function and its input-output parameters.

Here is how the material in the book is organized, and my recommendations for its usage.

1. Chapter 1, beside serving as a general introduction to the book, has two goals:
   (a) To introduce the system of notations used in the book.
   (b) To provide helpful hints concerning the use of summations.

   The first of these is a must for all readers. The second is mainly for the relatively inexperienced student.
2. Chapter 2 summarizes the prerequisites for the remainder of the book. It can be used selectively, depending on the background and level of preparation of the students. When I teach an introductory DSP course, I normally go over the material in one session, and assign part of it for self reading. The sections on random signals may be skipped if the instructor does not intend to teach anything related to random signals in the course. The section on real Fourier series can be skipped if the instructor does not intend to teach the discrete cosine transform.
3. Chapter 3 concerns sampling and reconstruction. These are the most fundamental operations of digital signal processing, and I always teach them as the first subject. Beside the basic-level material, the chapter contains a rather detailed discussion of physical sampling and reconstruction, which the instructor may skip or defer until later.
4. Chapter 4 is the first of three chapters devoted to frequency-domain analysis of discrete-time signals. It introduces the discrete Fourier transform (DFT), as well as certain related concepts (circular convolution, zero padding). It also introduces the geometric viewpoint on the DFT (orthonormal basis decomposition). Also introduced in this chapter is the discrete cosine transform (DCT). Because of its importance in DSP today, I have decided to include this material, although it is not traditionally taught in introductory courses. I included all four DCT types for completeness, but the instructor may choose to teach only type II, which is the most commonly used, and type III, its inverse.
5. Chapter 5 is devoted to the fast Fourier transform (FFT). Different instructors feel differently about this material. Some pay tribute to its practical importance by teaching it in considerable detail, whereas some treat it as a black box, whose details should be of interest only to specialists. I decided to present the Cooley-Tukey algorithms in detail, but omit other approaches to the FFT. The way I teach FFT is unconventional: Instead of starting with the binary case, I start with the general Cooley-Tukey decomposition, and later specialize to the binary case. I regard this as a fine example of a general concept being simpler than its special cases, and I submit to the instructor who challenges this approach to try it once.

This chapter also includes a few specialized topics: the overlap-add method of linear convolution, the chirp Fourier transform, and the zoom FFT. These are, perhaps, more suitable for a graduate course.

6. Chapter 6 is concerned with practical aspects of spectral analysis, in particular with short-time spectral analysis. It starts by introducing windows, the working tool of spectral analysis. It then discusses in detail the special, but highly important, problem of the measurement of sinusoidal signals. I regard these two topics, windows and sinusoid measurements, as a must for every DSP student. The last topic in this chapter is estimation of sinusoids in white noise. I have included here some material rarely found in DSP textbooks, such as detection threshold and the variance of frequency estimates based on a windowed DFT.
7. Chapter 7 provides the preliminary background material for the second part of the book, the part dealing with digital filters. It introduces the z-transform and its relationship to discrete-time, linear time-invariant (LTI) systems. The z-transform is usually taught in signals and systems courses. However, my experience has shown that students often lack this background. The placement of this material in this book is unconventional: in most books it appears in one of the first chapters. I have found that, on the one hand, the material on z-transforms is not needed until one begins to study digital filters; on the other hand, this material is not elementary, due to its heavy dependence on complex function theory. Teaching it within the middle of an introductory course, exactly at the point where it is needed, and after the student has developed confidence and maturity in frequency-domain analysis, has many pedagogical advantages. As in other books, the emphasis is on the two-sided transform, whereas the one-sided z-transform is mentioned only briefly.
8. Chapter 8 serves as an introduction to the subject of digital filters. It contains a mixture of topics, not tightly interrelated. First, it discusses the topic of filter types (low pass, high pass, etc.) and specifications. Next, it discusses in considerable detail, the phase response of digital filters. I decided to include this discussion, since it is missing (at least at this level of detail) from many textbooks. It represents, perhaps, more than the beginning student needs to know, but is suitable for the advanced student. However, the concept of linear phase and the distinction between constant phase delay and constant group delay should be taught to all students. The final topic in this chapter is an introductory discussion of digital filter design, concentrating on the differences between IIR and FIR filters.
9. Chapters 9 and 10 are devoted to FIR and IIR filters, respectively. I spent time trying to decide whether to put FIR before IIR or vice versa. Each of the two choices has its advantages and drawbacks. I finally opted for FIR first, for the following reasons: (1) this way there is better continuity between the discussion on linear phase in Chapter 8 and the extended discussion on linear phase in FIR filters at the beginning of Chapter 9; (2) there is also better continuity between Chapters 10 and 11; (3) since FIR filters appear more commonly than IIR filters in DSP applications, some instructors may choose to teach only FIR filters, or mention IIR filters only briefly. An introductory course that omits IIR is most likely to omit Chapters 11 and 12 as well. This enables the instructor to conveniently end the course syllabus with Chapter 9.

   The chapter on FIR filters contains most of what is normally taught on this subject, except perhaps design by frequency sampling. Design by windows is explained in detail, as well as least-squares design. Equiripple design is covered, but in less detail than in some books, since most engineers in need of equiripple filters would have to rely on canned software anyway.

The chapter on IIR filters starts with low-pass analog filter design. Butterworth and Chebyshev filters are suitable for a basic course, whereas elliptic filters should be left to an advanced course. Analog filters, other than low pass, are constructed through frequency transformations. The second half of the chapter discusses methods for transforming an analog filter to the digital domain. Impulse invariant and backward difference methods are included for completeness. The bilinear transform, on the other hand, is a must.

10. Chapter 11 represents the next logical step in digital filter design: constructing a realization from the designed transfer function and understanding the properties of different realizations. Certain books treat digital system realizations before they teach digital filters. It is true that realizations have uses other than for digital filters, but for the DSP student it is the main motivation for studying them.

    I decided to include a brief discussion of state space, following the material on realizations. Beside being a natural continuation of the realization subject, state space has important uses for the DSP engineer: impulse response and transfer function computations, block interconnections, simulation, and the like. I realize, however, that many instructors will decide not to teach this material in a DSP course.

    The bulk of Chapter 11 is devoted to finite word length effects: coefficient quantization, scaling, computation noise, and limit cycles. Much of the material here is more for reference than for teaching. In a basic course, this material may be skipped. In an advanced course, selected parts can be taught according to the instructor's preferences.

11. Chapter 12 concerns multirate signal processing. This topic is usually regarded as specialized and is seldom given a chapter by itself in general DSP textbooks (although there are several books completely devoted to it). I believe that it should be included in general DSP courses. The chapter starts with elementary material, in particular: decimation, interpolation, and sampling-rate conversion. It then moves on to polyphase filters and filter banks, subjects better suited to a graduate course.

12. Chapter 13 is devoted to the analysis and modeling of random signals. It first discusses nonparametric spectrum estimation techniques: the periodogram, the averaged (Welch) periodogram, and the smoothed (Blackman-Tukey) periodogram. It then introduces parametric models for random signals and treats the autoregressive model in detail. Finally, it provides a brief introduction to Wiener filtering by formulating and solving the simple FIR case. The extent of the material here should be sufficient for a general graduate DSP course, but not for a specialized course on statistical signal processing.

13. Chapter 14 represents an attempt to share my excitement about the field with my readers. It includes real-life applications of DSP in different areas. Each application contains a brief introduction to the subject, presentation of a problem to be solved, and its solution. The chapter is far from being elementary; most beginning students and a few advanced ones may find it challenging on first reading. However, those who persist will gain (I hope) better understanding of what DSP is all about.

Many people helped me to make this book a better one—Guy Cohen, Orli Gan, Isak Gath, David Malah, Nimrod Peleg, Leonid Sandomirsky, Adam Shwartz, David Stanhill, Virgil Stokes, Meir Zibulsky—read, found errors, offered corrections, criticized, enlightened. Benjamin Friedlander took upon himself the tedious and unrewarding task of teaching from a draft version of the book, struggling with the rough edges and helping smooth them, offering numerous suggestions and advice. Shimon Peleg read the book with the greatest attention imaginable; his detailed feedback on almost every page greatly improved the book. Simon Haykin was instrumental in having this book accepted for publication, and gave detailed feedback both on the early draft and later. William J. Williams and John F. Doherty reviewed the book and made many helpful suggestions. Irwin Keyson, Marge Herman, and Lyn Dupré, through her excellent book [Dupré, 1995], helped me improve my English writing. Brenda Griffing meticulously copyedited the book. Aliza Porat checked the final manuscript. Ezra Zeheb provided me with Eliahu Jury's survey on the development of the z-transform. James Kaiser helped me trace the original reference to the Dolph window. Thomas Barnwell kindly permitted me to quote his definition of digital signal processing; see page 1. Steven Elliot, the former acquisition editor at Wiley, and Charity Robey, who took over later, gave me a lot of useful advice. Jennifer Yee, Susanne Dwyer, and Paul Constantine at Wiley provided invaluable technical assistance. Yehoshua Zeevi, chairman of the Department of Electrical Engineering at the Technion, allowed me to devote a large part of my time to writing during 1996. Yoram Or-Chen provided moral support. Toshiba manufactured the T4800CT notebook computer, Y&Y, Inc. provided the LaTeX software, and Adobe Systems, Inc. created PostScript. Ewan MacColl wrote the song and Gordon Lightfoot and the Kingston Trio (among many others) sang it. I thank you all.

I try never to miss an opportunity to thank my mentors, and this is such an opportunity: Thank you, Tom Kailath and Martin Morf, for changing my course from control systems to signal processing and, indirectly, from industry to academia. If not for you, I might still be closing loops today! And thank you, Ben, for expanding my horizons in so many ways and for so many years.

And finally, to Aliza: The only regret I may have for writing this book is that the hours I spent on it, I could have spent with you!

Boaz Porat

Haifa, August 1996

Contents

Symbols and Abbreviations

Symbols

1. The symbols are given in an alphabetical order. Roman symbols are given first, followed by Greek symbols, and finally special symbols.
2. Page numbers are the ones in which the symbol is either defined or first mentioned. Symbols for which there are no page numbers are used throughout the book.
3. Section 1.2 explains the system of notation in detail.

Symbol	Meaning	Page
$a_1, \ldots, a_p$	denominator coefficients of a difference equation	214
$\boldsymbol{a}_i$	solution of ith-order Yule-Walker equation	527
$\boldsymbol{b}_i$	solution of the ith-order Wiener equation	538
$\tilde{\boldsymbol{a}}_i$	the vector $\boldsymbol{a}_i$ in reversed order	527
$A(\theta)$	amplitude function of a digital filter	256
A_p	pass-band ripple	246
A_s	stop-band attenuation	247
$\mathcal{A}_\mathrm{c}(N)$	number of complex additions in FFT	138
$\mathcal{A}_\mathrm{r}(N)$	number of real additions in FFT	141
$\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C}, \boldsymbol{D}$	state-space matrices	403
$a(z), b(z)$	denominator and numerator polynomials of a rational transfer function	215
adj	adjugate of a matrix	407
$b_0, \ldots, b_q$	numerator coefficients of a difference equation	214
B	number of bits (Chapter 11)	
$\mathbb{C}$	the complex plane	
$\boldsymbol{C}_N$	the discrete cosine transform matrix	114
CG	coherent gain of a window	187
d	discrimination factor	329
D	duration of a signal	189
$\mathcal{D}$	discrete Fourier transform (DFT) operator	94
$D(\theta, N)$	Dirichlet kernel	167
det	determinant of a matrix	407
e	$\sum_{n=0}^{\infty} \frac{1}{n!}$	
$e[n]$	quantization noise in a digital filter	429
e_i, f_i	coefficients in parallel realization	397
$E(\cdot)$	expectation	
$\exp\{a\}$	e^a	
f	continuous-time frequency	

f_{sam}	sampling frequency	
$\mathcal{F}$	Fourier transform operator (in either continuous or discrete time)	11
$\boldsymbol{F}, \boldsymbol{G}, \boldsymbol{H}, \boldsymbol{K}$	interconnection matrices in a digital network	408
$\boldsymbol{F}_N$	the discrete Fourier transform matrix	99
g_i, h_i	coefficients in cascade realization	399
$h[n]$	impulse response of a discrete-time system	29
$h(t)$	impulse response of a continuous-time system	13
$H^{\mathrm{f}}(\theta)$	frequency response of a discrete-time system	29
$H^{\mathrm{F}}(\omega)$	frequency response of a continuous-time system	14
$\boldsymbol{I}_N$	$N \times N$ identity matrix	
$I_0(x)$	modified Bessel function of order zero	175
j	$\sqrt{-1}$	
J_w	window parameter	190
k	DFT sequence index, general discrete index	
k	selectivity factor (Chapter 10)	329
$\boldsymbol{\kappa}_i$	vector of covariances	527
$\tilde{\boldsymbol{\kappa}}_i$	the vector $\boldsymbol{\kappa}_i$ in reversed order	527
$\boldsymbol{K}_i$	symmetric Toeplitz matrix of covariances	527
$K(m)$	complete elliptic integral of the first kind	343
$K_x^{\mathrm{f}}(\theta)$	power spectral density of a discrete-time random signal $x[n]$	30
$\hat{K}_x^{\mathrm{f}}(\theta)$	estimated power spectral density	515
$K_x^{\mathrm{F}}(\omega)$	power spectral density of a continuous-time random signal $x(t)$	24
L	expansion ratio (Chapter 12)	462
$\mathcal{L}$	Laplace transform operator	7
M	decimation ratio (Chapter 12)	462
$\mathcal{M}_c(N)$	number of complex multiplications in FFT	138
$\mathcal{M}_r(N)$	number of real multiplications in FFT	141
n	discrete time index	
N	length of a discrete-time signal	
N	order of a filter (Chapters 9 and 10)	
N_0	power spectral density of white noise	24
NG	noise gain	32
$p_T(t)$	impulse train	18
PG	processing gain of a window	188
$\mathbb{R}$	the real line	
$R_N(x)$	Chebyshev rational function	343
$\boldsymbol{R}_{i+1}(z)$	transfer matrix of an FIR lattice section	529
$\mathrm{rect}(t)$	the rectangular function	16
res	residue of a complex function	221
s	Laplace transform variable	
S	Fourier series operator	17
s_i	variance of ith-order prediction error	527
$s_1[n], s_2[n], \ldots$	state vector components	403
$\boldsymbol{s}[n]$	state vector	403
$\boldsymbol{S}_N$	the discrete sine transform matrix	120
$S(\theta)$	sensitivity bound	416
$\boldsymbol{S}_{i+1}(z)$	transfer matrix of an IIR lattice section	531

$\mathrm{Si}(x)$	sine integral	292
$\mathrm{sign}(t)$	the sign function	37
$\mathrm{sinc}(t)$	the sinc function	16
$\mathrm{sn}(u, m)$	Jacobi elliptic sine function	343
$\mathrm{SNR_i}$	input signal-to-noise ratio	188
$\mathrm{SNR_o}$	output signal-to-noise ratio	188
t	time	
T	sampling interval	
$\boldsymbol{T}$	state-space transformation	405
$T_N(x)$	Chebyshev polynomial	333
$u(\phi, m)$	elliptic integral of the first kind	343
$w[n]$	a window	168
$w_\mathrm{b}[n]$	Blackman window	174
$w_\mathrm{d}[n]$	Dolph window	175
$w_\mathrm{hm}[n]$	Hamming window	172
$w_\mathrm{hn}[n]$	Hann window	171
$w_\mathrm{k}[n]$	Kaiser window	175
$w_\mathrm{r}[n]$	rectangular window	165
$w_\mathrm{t}[n]$	triangular (Bartlett) window	169
W_N	$\exp\{j2\pi/N\}$	94
$x[n], y[n]$, etc.	discrete-time signals	
$x_{(\downarrow M)}[n]$	M-fold decimation	464
$x_{(\uparrow L)}[n]$	L-fold expansion	464
$x(t), y(t)$, etc.	continuous-time signals	
$\tilde{x}(t), \tilde{x}[n]$	periodic extension of a signal	6
$x(t_0^+)$	limit from the right of $x(t)$ at t_0	
$x(t_0^-)$	limit from the left of $x(t)$ at t_0	
$x_\mathrm{p}(t)$	impulse sampling of $x(t)$	46
$X^{\mathrm{c1}}[k]$	type-I discrete cosine transform (DCT-I)	116
$X^{\mathrm{c2}}[k]$	type-II discrete cosine transform (DCT-II)	118
$X^{\mathrm{c3}}[k]$	type-III discrete cosine transform (DCT-III)	118
$X^{\mathrm{c4}}[k]$	type-IV discrete cosine transform (DCT-IV)	119
$X^{\mathrm{d}}[k]$	discrete Fourier transform (DFT)	94
$X^{\mathrm{f}}(\theta)$	discrete-time Fourier transform	27
$X^{\mathrm{h}}[k]$	discrete Hartley transform	131
$X^{\mathrm{s1}}[k]$	type-I discrete sine transform (DST-I)	120
$X^{\mathrm{s2}}[k]$	type-II discrete sine transform (DST-II)	120
$X^{\mathrm{s3}}[k]$	type-III discrete sine transform (DST-III)	120
$X^{\mathrm{s4}}[k]$	type-IV discrete sine transform (DST-IV)	120
$X^{\mathrm{z}}(z)$	z-transform	206
$X^{\mathrm{z}}_{+}(z)$	unilateral z-transform	226
$X^{\mathrm{F}}(\omega)$	continuous-time Fourier transform	11
$X^{\mathrm{L}}(s)$	Laplace transform	7
$X^{\mathrm{S}}[k]$	Fourier series coefficients	17
$y_\mathrm{zir}[n]$	zero-input response of a difference equation	228
$y_\mathrm{zsr}[n]$	zero-state response of a difference equation	228
z	z-transform variable	
$\mathbb{Z}$	the set of integers	
$\mathcal{Z}$	z-transform operator	206

$\alpha_1, \ldots, \alpha_p$	poles of a rational transfer function	216
$\alpha_{\mathrm{r}}, \alpha_{\mathrm{i}}$	real and imaginary parts of the pole α	401
$\beta_1, \ldots, \beta_r$	zeros of a rational transfer function	216
γ_x	variance of the random variable x	21
$\gamma_{x,y}$	covariance of the random variables x, y	22
$\Gamma_w(x)$	integral of an amplitude function of a window	294
$\delta^+, \delta^-, \delta_{\mathrm{p}}$	pass-band tolerances	246
δ_{s}	stop-band tolerance	246
$\delta(t)$	the Dirac delta (impulse) function	12
$\delta[n]$	the unit sample (discrete-time impulse)	29
ε	parameter for Chebyshev and elliptic filters	335
ζ	angle of a complex zero	282
θ	discrete-time angular frequency	
θ_{p}	pass-band edge frequency	246
θ_{s}	stop-band edge frequency	246
$\kappa_x(\tau)$	covariance function of a continuous-time random signal $x(t)$	22
$\kappa_x[m]$	covariance sequence of a discrete-time random signal $x[n]$	30
$\hat{\kappa}_x[m]$	estimated covariance	521
$\kappa_{yx}[m]$	cross-covariance sequence of two discrete-time random signals $x[n]$, $y[n]$	537
λ_k	scaling parameters in filter realizations	424
$\boldsymbol{\lambda}_i$	right side of the ith-order Wiener equation	538
$\Lambda(\cdot)$	attenuation function	329
μ_x	mean of the random variable x	21
$\hat{\mu}_x$	estimated mean	521
$\boldsymbol{\xi}_p$	vector used in least-squares AR modeling	536
Ξ_p	matrix used in least-squares AR modeling	536
π	the ratio between the circumference and the diameter of a circle	
ρ_i	partial correlations, or reflection coefficients	528
σ_x	standard deviation of the random variable x	21
τ_{g}	group delay of a digital filter	259
τ_{p}	phase delay of a digital filter	258
$\phi(\theta)$	continuous phase of a digital filter	256
$\psi(\theta)$	phase response of a digital filter	253
$\upsilon(t)$	the continuous-time unit-step function	48
$\upsilon[n]$	the discrete-time unit-step function	48
ω	continuous-time angular frequency	
$\omega_{3\mathrm{db}}$	-3 dB angular frequency	329
$\omega_{\mathrm{c}}, \omega_{\mathrm{l}}, \omega_{\mathrm{h}}$	parameters in frequency transformations	347
(a,b)	the set $\{x:\ a < x < b\}$ (open interval)	
$(a,b]$	the set $\{x:\ a < x \le b\}$ (left semiopen interval)	
$[a,b)$	the set $\{x:\ a \le x < b\}$ (right semiopen interval)	
$[a,b]$	the set $\{x:\ a \le x \le b\}$ (closed interval)	
$\bar{x}$	complex conjugate of x	
$\Im\{x\}$	imaginary part of the complex number x	
$\Re\{x\}$	real part of the complex number x	

$\measuredangle x$	angle of the complex number x	
$\lvert x \rvert$	absolute value of x	
$\lVert x \rVert$	norm of x (Chapter 11)	
$\boldsymbol{x}'$	transpose of the vector (or the matrix) $\boldsymbol{x}$	
$\lfloor x \rfloor$	the floor of x (rounding downward)	
$\lceil x \rceil$	the ceiling of x (rounding upward)	
$\langle x \rangle$	empirical mean of x	
$1(t)$	the DC function	14
$\triangleq$	equality by definition	
$*$	convolution (in continuous or discrete time)	6
$\circledast$	circular convolution (in discrete time)	108
$\star$	correlation (in continuous or discrete time)	41
$\cup$	set union	
$\cap$	set intersection	

Abbreviations

Abbreviation	**Meaning**	**Page**
A/D	analog-to-digital (converter)	65
AM	amplitude modulation	42
AR	autoregressive (model or signal)	523
ARMA	autoregressive moving-average (model or signal)	523
BIBO	bounded-input, bounded-output	213
BP	band-pass (filter)	249
BPSK	binary phase-shift keying	75
BS	band-stop (filter)	250
CQF	conjugate quadrature filter (bank)	490
D/A	digital-to-analog (converter)	63
DC	direct current (function)	14
DCT	discrete cosine transform	114
DFT	discrete Fourier transform	93
DHT	discrete Hartley transform	131
DSB	double-side-band (modulation)	38
DSP	digital signal processing	2
DST	discrete sine transform	120
ECG	electrocardiogram	580
EEG	electroencephalogram	21
FFT	fast Fourier transform	133
FIR	finite impulse response (filter)	244
GLP	generalized linear phase	259
GSM	Groupe Special Mobile	550
HP	high-pass (filter)	249
IDFT	inverse discrete Fourier transform	97
IF	intermediate frequency	73
IIR	infinite impulse response (filter)	244
IRT	impulse response truncation	284
ISI	intersymbol interference	539
ISO	International Standards Organization	503

LAR	log-area ratio	562
LP	low-pass (filter)	246
LPC	linear predictive coding	556
LSB	least significant bit	412
LTI	linear time-invariant (system)	13
MA	moving-average (model or signal)	523
MAC	multiplier-accumulator	583
MMSE	minimum mean-square error	525
MOS	mean opinion score	493
MPEG	Moving Picture Expert Group	495
MSB	most significant bit	67
NRZ	non-return to zero	75
OLA	overlap-add (convolution)	149
OQPSK	offset quadrature phase-shift keying	517
PAM	pulse amplitude modulation	539
PSD	power spectral density	24
QMF	quadrature mirror filter (bank)	489
RAM	random-access memory	585
RCSR	real, causal, stable, rational (filter)	253
RMS	root mean square	191
ROC	region of convergence	206
ROM	read-only memory	585
S/H	sample and hold	65
SISD	single-instruction, single-data	582
SISO	single-input, single-output (system)	13
SLL	side-lobe level	189
SNR	signal-to-noise ratio	188
SSB	single-side-band (modulation)	274
WSS	wide-sense stationary (signal)	13
ZOH	zero-order hold	59