2024

Certificate Authority Cup International Mathematical Contest Modeling

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Problem A (MCM)
Jupiter: Protector or Threatener?

An impact on Earth by an asteroid or comet from the Oort Cloud would have a significant adverse effect on the planet. It has long been assumed that the planet Jupiter acts as a giant shield, significantly lowering the impact rate of minor bodies on the Earth[1]. However, some researchers have put forth the hypothesis that Jupiter's perturbing effect on asteroid and comet orbits increases the instability of these orbits, thereby potentially elevating the probability that they will enter the inner solar system under minor perturbations[2]. Consequently, the threat to Earth of being struck by such objects may be heightened. We would be grateful if you could develop a reasonable mathematical model to study this problem and answer the following three questions.

Tasks:

1. It is assumed that the threat originates solely from the asteroid belt. The objective is to assess which has the greater protective or threatening effect of Jupiter on Earth, under distribution conditions in which the asteroid belt is located on the inner side of Jupiter's orbit.
2. Evaluate which is the greater protective or threatening effect of Jupiter on the Earth, assuming that the threat originates only from the Oort Cloud.
3. If Jupiter's mass and orbit could be altered slightly, would this result in a significant influx of asteroids or comets into the inner solar system, thereby causing the late heavy bombardment effect? If so, at what threshold could such an effect occur?

References

[1] https://xkcd.com/681_large

[2] J Horner, B W Jones, Jupiter: friend or foe? An answer, Astronomy & Geophysics, Volume 51, Issue 6, December 2010, Pages 6.166.22