Introduction to Numerical Analysis and Applications

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Homework #4 (**DUE**: June 9th)

This homework includes the materials from the past weeks, including *Curve fitting, Interpolation, Numerical Integration,* and *Numerical Differentiation*.

1. (10%) Derive the centered finite-difference formula for 1st derivative with high-accuracy:

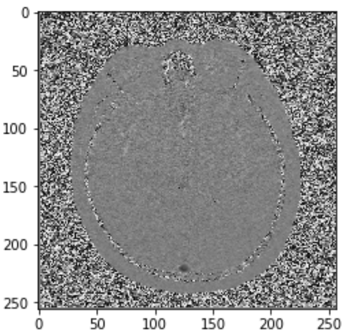
$$f'(x_i) = \frac{-f(x_{i+2}) + 8f(x_{i+1}) - 8f(x_{i-1}) + f(x_{i-2})}{12h}$$

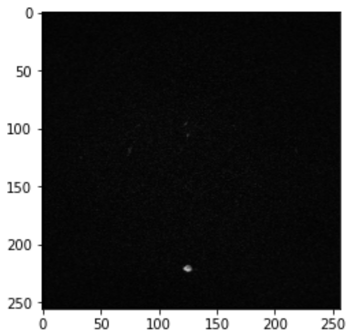
2. (15%) The two-point Gauss-Legendre formula has a truncation error $\sim f^{(4)}(\xi)$. Validate this statement by integrating the following polynomials over [0 0.8]: $f_1(x) = 0.2 + 25x - 200x^2 + 675x^3$ and $f_2(x) = 0.2 + 25x - 200x^2 + 675x^3 - 900x^4$. What are the true percent relative error ($\varepsilon_t = \frac{true\ value - approx.}{true\ value}$) in each case?

3. Up to this date the number of COVID-19 infection is over 5 million worldwide, with over 345,000 deaths already. Scientists utilize daily reported numbers to build infection model to help make public health policies. Although we are no epidemiologists, we should be able to tell which methods to be used and which others could lead to incorrect results. In this exercise, you are asked to process the open source data, and make educated observations.
   a. (15%) Daily COVID-19 numbers of US can be found here: https://covidtracking.com, and let's have a look at "Historic US data". Based on the downloaded datasheet, plot daily number of death (💔), along with the 7-day average curve.
   b. (15%) Curve fitting may be utilized to smooth the data, and might help reveal the trend. Perform curve fitting with polynomials of 2~20 degrees on the same data, and how do they predict the number of deaths on June 1st. Do you think it makes sense to make prediction in this fashion? Discuss what you observe from your results.

4. Many of the integration techniques are already implemented, so let's give them a shot. The scipy.integrate sub-package could be found in the following link and used in this exercise. https://docs.scipy.org/doc/scipy/reference/tutorial/integrate.html

   A series of MRI phase-contrast images is packed into "pcMRI.zip", with dimension (x, y, t) of 256×256×51. The field-of-view of the images is 220mm×220mm. The pixel value represents the through-plane velocity in cm/s (value within ±40), while the frame rate of dataset is 50 frame/s. Another set of data is helpful to identify the region of the superior sagittal sinus (SSS, the vein in interest). The "magMRI.zip" gives the magnitude information of the same data, and the pixel value represents how likely this pixel contains flowing blood. The following two figures show the velocity map and the magnitude image, respectively.





   a. (15%) Perform spatial integration over the SSS region, and plot the average blood flow velocity with respect of time.
   b. (15%) Perform the integration over the whole time interval (i.e. 1 s), and find out the average blood flow rate in ml/s. Use composite trapezoidal rule and composite Simpson's rule and how different are your answers?
   c. (15%) From the plot in (b), what is the time when the averaged blood flow velocity reaches to its maximum? Show the corresponding flow profile at that specific time. *Hint: a 3^{rd}-order polynomial that fits the first half of the time series could indicate the time when it reaches to the maximal flow velocity.*

More on this homework:

We all know that extrapolation is much less reliable that interpolation, and making predictions based on simple polynomials could lead to very confusing results. It is reported that polynomial fitting with was adopted for "better visualization of the data", as well as implying prediction that would affect the public health policy. While the politics behind is never the topic of this course, the science is. Q3 is meant to give an example how the data should or should not be manipulated.