Logistic Regression

$K=2$, $Y=1$ or 0

linear regression for $y \in (-\infty, \infty)$

classification $\longrightarrow$ want $P(\hat{Y}=1|X)$, probability $[0,1]$

$P(X) = P(\hat{Y}=1|X)$

$$\underbrace{\log\left(\frac{p(x)}{1-p(x)}\right) \text{ in } (-\infty, \infty)}_{\text{linear regression okay}}$$

$$\log\left(\frac{p(x)}{1-p(x)}\right) = \hat{\beta}_0 + \hat{\beta}_1 X$$

$$\hat{p}(x) = \frac{\exp(\hat{\beta}_0 + \hat{\beta}_1 x)}{1+\exp(\hat{\beta}_0 + \hat{\beta}_1 x)}$$

$K > 2$:

$k = 1, \ldots, K-1$

$$\log\left(\frac{P_k(x)}{1-P_k(x)}\right) = \beta_{k_0} + \beta_{k_1} X$$

$P_k(x) = P(\hat{Y}=k|X)$ $\quad$ K a baseline category

$P_K(x) = P(\hat{Y}=K|X)$

Bayes' Rule

$$P(Y|X) = \frac{P(X \text{ and } Y)}{P(X)}$$

regression models estimate this directly

$$= \frac{P(X|Y)P(Y)}{P(X)}$$

LDA estimates all these

$P(Y|X)$