

The equations of a Convolutional Neural Network of 4 layers:

$$\begin{aligned}
L_0 &= \chi \\
T_0[x][y] &= \sum_{i=0}^{\lambda_0-n_0+1} \sum_{j=0}^{\lambda_0-n_0+1} L_0[x+i][y+i] \cdot w_0[i][j] \\
L_1[x][y] &= \Delta(T_0[x][y]) \\
T_1[x][y] &= \sum_{i=0}^{\lambda_1-n_1+1} \sum_{j=0}^{\lambda_1-n_1+1} L_1[x+i][y+i] \cdot w_1[i][j] \\
L_2[x][y] &= \Delta(T_1[x][y]) \\
L_2[x][y] &\rightarrow L_2[i] \\
T_2[i] &= \sum_{j=0}^{\lambda_2} L_2[j] \cdot w_2[j][i] \\
L_3[i] &= \Delta(T_2[i]) \\
T_3[i] &= \sum_{j=0}^{h_3} L_3[j] \cdot w_3[j][i] \\
L_4[i] &= \Delta(T_3[i]) \\
\hat{y} &= L_4
\end{aligned}$$

The derivatives would be:

$$\begin{aligned}
\frac{\partial T_0[x][y]}{\partial w_0[i][j]} &= L_0[x+i][y+j], \\
\frac{\partial L_1[x][y]}{\partial T_0[x][y]} &= \Delta'(T_0[x][y]), \\
\frac{\partial T_1[x][y]}{\partial L_1[x+i][y+j]} &= w_1[i][j], \\
\frac{\partial L_2[x][y]}{\partial T_1[x][y]} &= \Delta'(T_1[x][y]), \\
\frac{\partial T_2[i]}{\partial L_2[x][y]} &= w_2[\mathbf{m}(x, y)][i], \\
\frac{\partial L_3[i]}{\partial T_2[i]} &= \Delta'(T_2[i]), \\
\frac{\partial T_3[i]}{\partial L_3[j]} &= w_3[j][i], \\
\frac{\partial L_4[i]}{\partial T_3[i]} &= \Delta'(T_3[i]), \\
\frac{\partial C}{\partial L_4[i]} &= L_4[i] - y[i].
\end{aligned}$$

$$\begin{aligned}
\frac{\partial C}{\partial w_0[w_x][w_y]} &= \frac{\partial C}{\partial L_4[i]} \frac{\partial L_4[i]}{\partial T_3[i]} \frac{\partial T_3[i]}{\partial w_0[w_x][w_y]}, \\
\frac{\partial T_3[i]}{\partial w_0[w_x][w_y]} &= \sum_j \frac{\partial T_3[i]}{\partial L_3[j]} \frac{\partial L_3[j]}{\partial T_2[j]} \frac{\partial T_2[j]}{\partial w_0[w_x][w_y]}, \\
\frac{\partial T_2[i]}{\partial w_0[w_x][w_y]} &= \sum_{x,y} \frac{\partial T_2[i]}{\partial L_2[x][y]} \frac{\partial L_2[x][y]}{\partial T_1[x][y]} \frac{\partial T_1[x][y]}{\partial w_0[w_x][w_y]}, \\
\frac{\partial T_1[x][y]}{\partial w_0[w_x][w_y]} &= \sum_{q,z} \frac{\partial T_1[x][y]}{\partial L_1[x+q][y+z]} \frac{\partial L_1[x+q][y+z]}{\partial T_0[x+q][y+z]} \frac{\partial T_0[x+q][y+z]}{\partial w_0[w_x][w_y]}.
\end{aligned}$$

$$\begin{aligned}
\frac{\partial C}{\partial w_0[w_x][w_y]} &= \sum_i (L_4[i] - y[i]) \cdot \Delta'(T_3[i]) \cdot \frac{\partial T_3[i]}{\partial w_0[w_x][w_y]}, \\
\frac{\partial T_3[i]}{\partial w_0[w_x][w_y]} &= \sum_j w_3[j][i] \cdot \Delta'(T_2[j]) \cdot \frac{\partial T_2[j]}{\partial w_0[w_x][w_y]}, \\
\frac{\partial T_2[i]}{\partial w_0[w_x][w_y]} &= \sum_{x,y} w_2[\mathfrak{m}(x,y)][i] \cdot \Delta'(T_1[x][y]) \cdot \frac{\partial T_1[x][y]}{\partial w_0[w_x][w_y]}, \\
\frac{\partial T_1[x][y]}{\partial w_0[w_x][w_y]} &= \sum_{q,z} w_1[q][z] \cdot \Delta'(T_0[x+q][y+z]) \cdot L_0[x+q+w_x][y+z+w_y].
\end{aligned}$$

$$\begin{aligned}
\frac{\partial C}{\partial w_1[w_x][w_y]} &= \frac{\partial C}{\partial L_4[i]} \frac{\partial L_4[i]}{\partial T_3[i]} \frac{\partial T_3[i]}{\partial w_1[w_x][w_y]}, \\
\frac{\partial T_3[i]}{\partial w_1[w_x][w_y]} &= \sum_j \frac{\partial T_3[i]}{\partial L_3[j]} \frac{\partial L_3[j]}{\partial T_2[j]} \frac{\partial T_2[j]}{\partial w_1[w_x][w_y]}, \\
\frac{\partial T_2[i]}{\partial w_1[w_x][w_y]} &= \sum_{x,y} \frac{\partial T_2[i]}{\partial L_2[x][y]} \frac{\partial L_2[x][y]}{\partial T_1[x][y]} \frac{\partial T_1[x][y]}{\partial w_1[w_x][w_y]}, \\
\frac{\partial T_1[x][y]}{\partial w_1[w_x][w_y]} &= \sum_{q,z}
\end{aligned}$$

$$\begin{aligned}
\frac{\partial T_1[x][y]}{\partial w_1[i][j]} &= L_1[x+i][y+1], \\
\frac{\partial L_2[x][y]}{\partial T_1[x][y]} &= \Delta'(T_1[x][y]), \\
\frac{\partial T_2[i]}{\partial L_2[x][y]} &= w_2[\mathfrak{m}(x,y)][i], \\
\frac{\partial L_3[i]}{\partial T_2[i]} &= \Delta'(T_2[i]), \\
\frac{\partial T_3[i]}{\partial L_3[j]} &= w_3[j][i], \\
\frac{\partial L_4[i]}{\partial T_3[i]} &= \Delta'(T_3[i]), \\
\frac{\partial C}{\partial L_4[i]} &= L_4[i] - y[i].
\end{aligned}$$

Or, in general:

$$\begin{aligned}
L_0 &= \chi \\
T_k[x][y] &= \sum_{i=k}^{\lambda_k - n_k + 1} \sum_{j=0}^{\lambda_k - n_k + 1} L_k[x+i][y+j] \cdot w_k[i][j] \\
L_k[x][y] &= \Delta(T_{k-1}[x][y]) \\
L_{c-1}[x][y] &\rightarrow L_{c-1}[i] \\
T_k[i] &= \sum_{j=0}^{h_k} L_k[j] \cdot w_k[j][i] \\
L_k[i] &= \Delta(T_{k-1}[i]) \\
\hat{y} &= L_{s-1} \\
\\
\frac{\partial C}{\partial w_\psi[w_x][w_y]}[i] &= \frac{\partial C}{\partial L_s[i]} \frac{\partial L_s[i]}{\partial T_{s-1}[i]} \sum_j \frac{\partial T_{s-1}[j]}{\partial w_\psi[w_x][w_y]}[j], \\
\frac{\partial T_k[i]}{\partial w_\psi[w_x][w_y]}[j] &= \frac{\partial T_k[i]}{\partial L_k[j]} \frac{\partial L_k[j]}{\partial T_{k-1}[j]} \cdot \sum_r \frac{\partial T_{k-1}[j]}{\partial w_\psi[w_x][w_y]}[r], \\
\frac{\partial T_{c-1}[i]}{\partial w_\psi[w_x][w_y]}[m(x, y)] &= \frac{\partial T_{c-1}[i]}{\partial L_{c-1}[x][y]} \frac{\partial L_{c-1}[x][y]}{\partial T_{c-2}[x][y]} \cdot \sum_{n, m} \frac{\partial T_{c-2}[x][y]}{\partial w_\psi[w_x][w_y]}[n][m], \\
\frac{\partial T_k[x][y]}{\partial w_\psi[w_x][w_y]}[i][j] &= \frac{\partial T_k[x][y]}{\partial L_k[x+i][y+j]} \frac{\partial L_k[x+i][y+j]}{\partial T_{k-1}[x+i][y+j]} \cdot \sum_{n, m} \frac{\partial T_{k-1}[x+i][y+j]}{\partial w_\psi[w_x][w_y]}[n][m], \\
\frac{\partial T_{\psi+1}[x][y]}{\partial w_\psi[w_x][w_y]}[i][j] &= \frac{\partial T_{\psi+1}[x][y]}{\partial L_{\psi+1}[x+i][y+j]} \frac{\partial L_{\psi+1}[x+i][y+j]}{\partial T_\psi[x+i][y+j]} \frac{\partial T_\psi[x+i][y+j]}{\partial w_\psi[w_x][w_y]}, \\
\\
\delta_s[i] &= (L_s[i] - y[i]) \cdot \Delta'(T_{s-1}[i]), \\
\delta_k[i][j] &= w_k[j][i] \cdot \Delta'(T_{k-1}[j]), \\
\xi_k[i] &= \sum_j \delta_{k-1}[i][j], \\
\delta_k[x][y][i][j] &= w_k[i][j] \cdot \Delta'(T_{k-1}[x+i][y+j]), \\
\xi_k[x][y] &= \sum_j \delta_{k-1}[x][y][i][j], \\
\delta_{\psi+1}[x][y][i][j] &= w_{\psi+1}[i][j] \cdot \Delta'(T_\psi[x+i][y+j]) \cdot L_\psi[x+i][y+j],
\end{aligned}$$