# Dynamic Performance of Tubular Linear Actuator With Halbach Array and Mechanical Spring Driven by PWM Inverter

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This paper describes dynamic performance of a tubular linear actuator with Halbach array and mechanical spring driven by a pulse wide modulation inverter. By expressing mechanical components as motional impedance from a motion equation, this paper investigates the variation of not only active and reactive power dissipated in the actuator but also current and power factor for the actuator for various values of frequency and predicts theoretical resonant frequency. Finally, experimental results for dynamic characteristics such as current and stroke are presented for various values of frequency including resonant frequency.

Index Terms—Dynamic performance, resonant frequency, tubular linear actuator.

# I. INTRODUCTION

ITH its simple structure, a linear oscillatory actuator (LOA) as an actuator for small displacement round-trip motion, is used in various field such as vibrators, air pumps, compressors, and more recently, artificial hearts [1].

The normal mode of operation is to drive the actuator at a mechanical resonant frequency so that no work is done in accelerating/decelerating a moving mass, because a mechanical resonance makes the linear actuator not only realize small size and high speed, but also achieve optimum efficiency and displacement [1], [2]. The mechanical resonance originates from interaction between a spring and the moving mass and can be utilized in only bidirectional drive systems.

In previous work [3], we performed simulations and experiments for dynamic characteristics such as current and displacement of the tubular linear actuator with mechanical spring driven by unidirectional square-wave switching signals. However, as stated above, although the tubular linear actuator presented in [3] was equipped with spring, since that was driven in one-directional drive systems, we could not use the mechanical resonance. Therefore, this paper deals with the dynamic performance of the tubular linear actuator with spring driven by a pulsewidth modulation (PWM) inverter for bidirectional drive. First, we obtain an equivalent electrical circuit of the tubular linear actuator considering mechanical components by representing a mass/spring system as motional impedance. Second, by analyzing this circuit using control parameters presented in [3], this paper predicts theoretical resonant frequency and investigates the variation of power factor, current, active and reactive power versus frequency. Finally, experimental results for dynamic characteristics such as current and stoke are presented for various values of frequency. In particular, the methods to predict an accurate resonant frequency are discussed fully in terms of moving mass and spring.

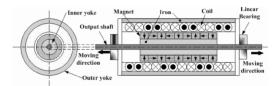


Fig. 1. Tubular linear actuator with Halbach array.

### II. STRUCTURES OF TUBULAR LINEAR ACTUATOR

As shown in Fig. 1, we chose the Halbach array as a mover and a slotless type as a stator of the tubular linear actuator for the reasons as follows. First, the fundamental field of the Halbach array is 1.4 times stronger than that of a conventional array, and thus the power efficiency of the actuator with Halbach array is doubled. Moreover, the magnetic field of the Halbach array is more purely sinusoidal than that of the conventional array, resulting in a simple control structure [4]. Second, the slotless stator eliminates the tooth ripple cogging effect, and thereby improves the dynamic performance and servo characteristics at the expense of a reduction in specific force capability [5]. Although the slotless stator sacrifices the higher force capabilities, with the advent of the modern high-energy rare-earth permanent magnets (PMs) and employment of the Halbach array, the problem stated above can be solved.

#### III. IMPEDANCE MODELING OF MASS/SPRING SYSTEMS

The motion equation for the actuator is given by [3]

$$M\frac{d^2x}{dt^2} = K_T i - kx - C_d \frac{dx}{dt} \tag{1}$$

where M and  $K_T$  are the mass of mover and thrust constant, respectively, and dx/dt is the velocity of mover. k and  $C_d$  are the coefficient of elasticity for spring and friction, respectively. The values of  $M, K_T, k$ , and  $C_d$  used in this paper are 3.8 [kg], 48 [N/A], 2400 [N/m], and 0.001 [N·s/m], respectively. Since the reciprocation of mover is similar to that of a pendulum, its displacement x and electromagnetic force  $K_T i$  can be assumed to  $x_m e^{j\omega t}$  and  $F = F_m e^{j\omega t}$ , respectively. Therefore, (1) can be rewritten by

$$F_m = (k - M\omega^2 + jC_d\omega)x_m \tag{2}$$

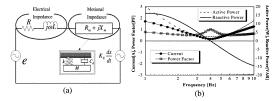


Fig. 2. (a) Equivalent electrical circuit including mechanical components and (b) the variation of current, power factor, active and reactive power versus frequency for the tubular linear actuator with spring.

where  $F_m$  and  $x_m$  are the peak value of thrust and stroke, respectively, and  $\omega$  denotes the angular frequency. When the thrust has the peak value, the peak value of current, namely,  $I_p$  is obtained from  $F = K_T i$  and (2)

$$I_P = F_m/K_T = x_m(k - M\omega^2 + jC_d\omega)/K_T.$$
 (3)

The peak value  $(E_m)$  of the back-emf induced in stator coils due to motion of the PM mover can be expressed by

$$E = K_E(dx/dt) = j\omega x_m K_E e^{j\omega t} \to E_m = j\omega x_m K_E \quad (4)$$

where  $K_E$  represents the back-emf constant. From (3) and (4), the motional impedance  $Z_m$  is given by

$$Z_m = E_m/I_p = R_m + jX_m = (\beta\gamma + j\alpha\beta)/(\alpha^2 + \gamma^2) \quad (5)$$

where  $\alpha = k - M\omega^2$ ,  $\beta = \omega K_T K_E = \omega K_T^2$ , and  $\gamma = \omega C_d$ . Neglecting the coefficient of friction, (5) is identical with the equation for motional impedance presented in [6]. Since imaginary part is eliminated for the case when  $\alpha = 0$  in (5), the frequency in such a case is called resonant frequency and is given by

$$f_n = (1/2\pi)\sqrt{k/M}. (6)$$

Finally, the equivalent electrical circuit of the tubular linear actuator including mechanical components shown in Fig. 2(a) is obtained. From Fig. 2(a), total impedance of tubular linear actuator with mass/spring system is expressed as

$$Z_t = (R + R_m) + j(\omega L + X_m). \tag{7}$$

It is noted that (7) helps us to predict not only the active and reactive power dissipated in the tubular linear actuator, but also power factor and current, as shown in Fig. 2(b). Although the results shown in Fig. 2(b) are obtained under assumptions that the mover has the fixed stroke, it is noted that these results show the general characteristics that current and active power consumed by the actuator has the minimum at the theoretical resonant frequency calculated from (6).

## IV. DYNAMIC PERFORMANCE

### A. Testing Apparatus

Fig. 3 shows a testing apparatus which consists of direct current (dc) power supply, tubular linear actuator with spring, sensing part for measurement of current waveform, a position sensor for measurement of stroke, a gate driver which provides the functions for amplification of PWM signal, and a single-phase inverter for generating a single-phase alternating voltage from dc power supply.

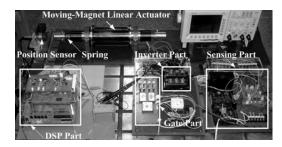


Fig. 3. Testing apparatus for measurements of dynamic characteristics.

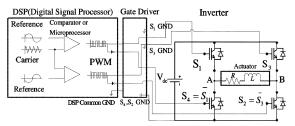


Fig. 4. Schematic of the drive systems shown in Fig. 3.

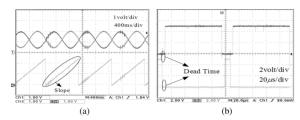


Fig. 5. Measured (a) reference waveforms and (b) PWM waveforms.

### B. Driving Circuit for Bidirectional Motion

Fig. 4 shows a driving circuit for bidirectional motion of the actuator. As shown in Fig. 4, since a triangular wave is applied to one of the two terminals of each comparator in order to modulate two different analogue signals (reference), two different PWM waveforms are obtained from the digital signal processor (DSP). These PWM waveforms are applied to a gate of each switch and make switches have the switching state as

$$S_1 + S_4 = 1$$
 and  $S_2 + S_3 = 1$ . (8)

It can be observed from (8) that two paired switches ( $S_1$  and  $S_4$  or  $S_2$ , and  $S_3$ ) are complementary to each other. However, when the switching state changes from one state to the other state between the two paired switches (insulated gate bipolar transistor module), both switches must be on the OFF states for a short time. This is to avoid the possibility of short-circuiting in the transient state in which the two switches can be simultaneously closing [7].

# C. PWM Waveforms

Fig. 5(a) shows the measurements for two reference waveforms used in generation of PWM waveforms and sawtooth waveforms which changes frequency of reference wave by adjusting its slope. As shown in Fig. 5(a), each reference wave has the same magnitude, and its phase is displaced 180° each other. Fig. 5(b) shows the measurements for PWM waveforms generated from the DSP. As stated above, it can be seen that a dead time exists in order to prevent the possibility of short-circuiting in two paired switches.

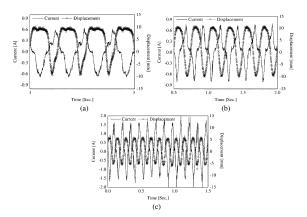


Fig. 6. Measured dynamic performance for various values of frequency: (a) 2 Hz, (b) 4 Hz, and (c) 8 Hz.

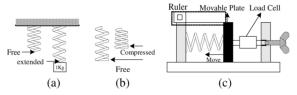


Fig. 7. (a) The measurement method of k in this paper, (b) the characteristic of spring used in this paper, and (c) the proposed measurement method of k.

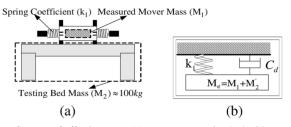


Fig. 8. Concept of effective mass: (a) actuator on testing bed with mass  $M_2$  and (b) equivalent mass/spring system for the schematic of Fig. 8(a).

#### D. Experimental Results and Discussion

Fig. 6 shows the experimental results for dynamic characteristics such as current and stroke for various values of frequency. It can be observed that stroke of the linear actuator decreases, as the frequency is increased. It can also be observed that the peak value of current measured at 4 Hz is higher than that at 2 Hz. This result implies that 4 Hz (theoretical resonant frequency) is not actual resonant frequency, judging from the general fact that the current consumed in linear actuator has the minimum at the resonant frequency. So, although we do not find the resonant frequency experimentally, it can be predicted that experimental resonant frequency is not identical with theoretical resonant frequency.

Therefore, we discuss the major causes of this disagreement in terms of moving mass and spring. Fig. 7(a) shows the measurement method of the coefficient of elasticity for spring (k) used in this paper. However, as shown in Fig. 7(b), since a spring used in this paper is compressed spring, the measurement method presented in Fig. 7(a) can cause an error in estimation of k. However, the problem stated above can be solved by applying the method shown in Fig. 7(c) to measurement of k.

In order to calculate the resonant frequency, we substitute only the mover mass  $(M_1)$  of the actuator for M in (6). However, for the case when the actuator is on a testing bed with mass  $M_2$  shown in Fig. 8(a), since the testing bed is vibrated by recip-

rocation of the mover, not only the mover mass  $(M_1)$  but also testing bed mass  $(M_2)$  must be considered in order to calculate the resonant frequency accurately. Fig. 8(b) shows an equivalent mass/spring system of the schematic shown in Fig. 8(a). Compared with the stoke of the actuator, since that of the testing bed is at the level of vibration, the mass of testing bed considered in calculation of the resonant frequency should be not  $M_2$ but  $M_2$ ', as shown in Fig. 8(b). Therefore, the effective mass of the actuator system presented in this paper can be calculated by the sum of mover mass  $M_1$  and  $M_2$ ' obtained by solving a differential equation governed by mechanical system. Actually, the effective mass is very important issue in industrial applications. For instance, for the case when linear actuators are used in compressors for air conditioners, since its entity is vibrated in detail by reciprocation of the mover, the effective mass must be considered in order to determine the range of drive frequency including the resonant frequency.

#### V. CONCLUSION

Dynamic performance of the tubular linear actuator with mechanical spring and Halbach array driven by PWM inverter has been described. By representing the mass/spring system as motional impedance equivalently from the motion equation, this paper has predicted the theoretical resonant frequency and has investigated the variation of power factor, current, active and reactive power versus frequency for tubular linear actuator with spring. And then, experimental results for the dynamic characteristics such as current and displacement has been presented for various values of frequency. Finally, the major causes of the disagreement between the actual and the theoretical resonant frequency has been discussed in terms of moving mass and spring. In our future work, we will calculate the accurate theoretical resonant frequency by using methods presented in this paper, and then will find the resonant frequency experimentally on the basis of the calculated result.

#### ACKNOWLEDGMENT

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