# Analysis of an E-Core Interior Permanent Magnet Linear Oscillating Actuator

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This paper presents the design, analysis and experimental validation of a single-phase tubular E-core interior permanent magnet (IPM) linear oscillating actuator (LOA). Firstly, the topology is described and its electromagnetic performance, in terms of the air-gap flux density, flux-linkage, back-emf coefficient and thrust force-displacement characteristic, is analyzed by finite element analyses. It shows that it is simple to manufacture compared to E-core SPM actuators and solves the problem of significant leakage flux in C-core actuators. In addition, the E-core IPM actuator is capable of producing reluctance force due to the salient mover structure, which enhances the spring stiffness and is conducive to the oscillation, although has no contribution to the output power. The influence of the magnet pole ratio and split ratio on the performance of the IPM LOA is investigated, and the optimal parameters have been identified with reference to the peak value and flatness of the excitation force-displacement characteristic. Finally, the predicted thrust force characteristics are validated by measurements on a prototype actuator.

Index Terms—Actuator, interior magnet, linear motor, oscillating, permanent magnet, tubular motor.

## I. INTRODUCTION

ECENTLY, single-phase permanent magnet (PM) linear oscillating actuators (LOAs) become increasingly popular in many industrial applications, such as compressors, pumps, vibrators, etc. [1]-[6], mainly because of their inherent high power density and high efficiency. In [6] a planar flux concentration type IPM LOA was proposed, and its performance further was further analyzed in [7]. While due to the advance of soft magnetic composite material [8] proposed a tubular quasi-Halbach magnetized PM LOA, the performance of machines equipped with non-magnetic and ferromagnetic mover back-iron was compared, the efficiency of >93\% being recorded for both cases. In [9] alternative actuator topologies were described, and their finite element (FE) predicted electromagnetic performance was compared. Consequently, a 3-pole interior PM (IPM) topology was identified and further analyzed in [10]. However, the investigation in [8]–[10] only focused on the C-core PM actuators.

In contrast, [11] presented alternative design concepts of single-phase tubular moving-magnet LOAs, both C-core and E-core, and concluded that the preferred stator tooth number  $N_t$  and the mover pole number  $N_p$  were defined by  $|N_t-N_p|=1$ . In addition, a 2-pole, E-core tubular LOA equipped with Halbach surface-mounted permanent magnets (HSPM) was also studied and significant performance improvement was achieved, compared to the actuator with radially magnetized surface PM (SPM) mover.

In this paper, an E-core IPM actuator based on [11] is proposed, which, unlike the SPM and HSPM LOAs having complex mover structures, the mover in the IPM actuator is very simple and easy to manufacture. It also solves the problem of significant leakage flux in C-core actuators. The electromagnetic performance of the LOA is analyzed, and its key design parameters, i.e., the magnet pole ratio and the split ratio, are optimized. Finally, the predicted thrust force characteristics of the E-core

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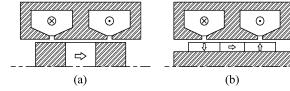


Fig. 1. Schematics of E-core moving-magnet LOAs. (a) IPM. (b) HSPM.

TABLE I
MAIN SPECIFICATIONS OF E-CORE IPM LOA

Items	Analyzed	Prototype
Axial length, $L_a$	40 mm	44 mm
Stator outer radius, $R_o$	32 mm	
Stator bore radius, $R_s$	12 mm	17 mm
Mover shaft radius, $R_r$	-	3 mm
Air-gap length, g	1 mm	
Stator slot pitch, $\tau_s$	20 mm	
Mover pole pitch, $\tau_p$	20 mm	
Axial length of iron pole, $\tau_{Fe}$	10 mm	12 mm
Axial length of PM ring, $\tau_{pm}$	10 mm	8 mm
NdFeB magnet remanence, $B_r$	1.2 T	
Rated stroke, $A_R$	±5 mm	

IPM LOA are compared with measurements on a prototype actuator.

# II. MACHINE TOPOLOGY

Fig. 1(a) shows the cross-section of the proposed E-core IPM actuator. The stator is formed by an E-shaped ferromagnetic stator core and two annular coils connected in series and carrying currents which flow in opposite directions, whilst the mover is composed of an axially magnetized PM ring sandwiched between two iron pole-pieces. Compared to the HSPM LOA described in [11] and shown in Fig. 1(b), this actuator has an essentially identical stator structure, but a much simpler mover. Mild steel is employed as the ferromagnetic core, together with NdFeB magnet. The main specifications of the IPM LOA are given in Table I.

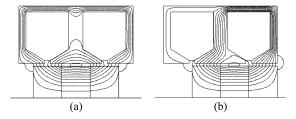


Fig. 2. Open-circuit field distributions of E-core IPM LOA. (a)  $Z_d=0\,$  mm. (b)  $Z_d=+5\,$  mm.

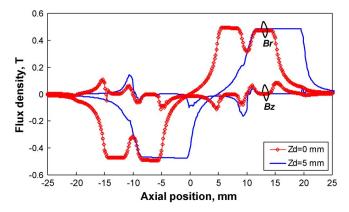


Fig. 3. Open-circuit air-gap field distributions (r = 11.5 mm).

#### III. ANALYSIS OF E-CORE IPM LOA

Fig. 2 shows the finite element predicted open-circuit field distributions when the mover is at the central rest and displacement  $(Z_d)=+5$  mm positions, respectively, and their corresponding air-gap flux density distributions are shown in Fig. 3, in which the waveforms of  $Z_d=+5$  mm are essentially shifted from that of  $Z_d=0$  mm, the discrepancies are mainly due to the slotting effect.

The predicted force-mover displacement characteristics are shown in Fig. 4. As can be seen, although the magnitude of cogging force is relatively small, the central rest point is a stable position due to the negative stiffness of cogging force being -356.4 N/m. The total thrust force and reluctance force are respectively calculated with and without permanent magnets, when the actuator is excited with DC current  $(J = 3 \text{ A/mm}^2)$ . The significant reluctance force is mainly caused by the variation of the winding inductance over the stroke range, Fig. 5. However, it is also evident that the reluctance force, similar to cogging force, will not contribute the output power during operation but will enhance the spring stiffness with its equivalent value being -5149 N/m under DC excitation and -1287 N/m under AC operation with the peak value of sinusoidal current equal to the DC current. The excitation force, produced by the interaction of permanent magnets and the current and the only force component contributing to the power output, is obtained by deducting the cogging and reluctance components from the total thrust force, which maintains a rather flat profile over the range of  $\pm 3$  mm at 45 N, but rolls off to 30 N at  $\pm 5$  mm, mainly due to the fringing flux at the axial ends of the iron polepieces, Fig. 3. The average of thrust force over the rated stroke,  $\pm 5$  mm, is  $\sim 42$  N. The flux-linkage characteristic varies almost linearly as the mover is displaced from the centre but becomes slightly saturated as it approaches the ends of stroke, Fig. 6. Thus, the back-emf coefficient,  $K_E$ , defined as the derivative of

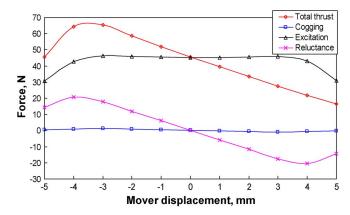


Fig. 4. FE predicted thrust force-mover displacement characteristics.

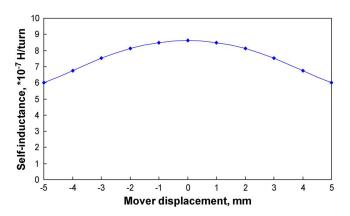


Fig. 5. Variation of winding inductance with mover displacement.

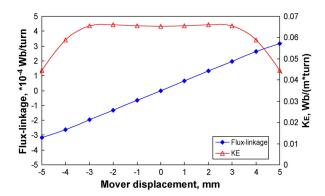


Fig. 6. Variations of flux-linkage and back-emf coefficient with mover displacement.

flux-linkage over mover displacement, is also shown in Fig. 6, and, as expected, exhibits essentially identical profile as the excitation force, Fig. 4, having an average of  $\sim 0.061$  Wb/m.

#### IV. DESIGN OPTIMIZATION

The magnet pole ratio,  $\alpha_p = \tau_{Fe}/(\tau_{Fe} + \tau_{pm})$ , and the actuator split ratio,  $\alpha_s = R_s/R_o$ , greatly influence the specific thrust force capability of the LOA. Thus,  $\alpha_p$  and  $\alpha_s$  have been optimized with reference to the LOA studied earlier, by fixing the actuator outer radius, the copper loss, the stator tooth width, the stator back-iron thickness, the stator slot opening, the actuator axial length, the slot- and pole-pitches. Based on the sinusoidal current control, it is assumed the mover velocity is also sinusoidal and in-phase with the current due to the mechanical low-pass filter effect. The maximum excitation force will occur

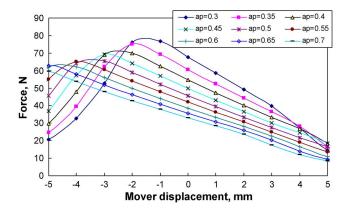


Fig. 7. Variations of force with mover displacement at different  $\alpha_p$  when  $\alpha_s =$ 0.375 and  $p_{Cu} = 5$  W.

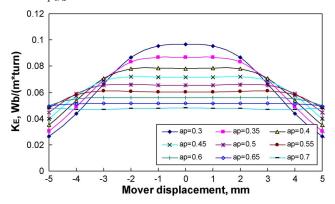


Fig. 8.  $K_E$ -mover displacement characteristics ( $\alpha_s = 0.375$ ).

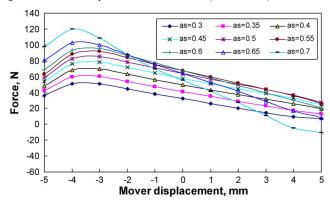


Fig. 9. Variations of force with mover displacement at different  $\alpha_s$  when  $\alpha_p$ 0.5 and  $p_{Cu} = 5$  W.

at the central position. Therefore, the aim of the optimization is to find a good balance between the maximum value at 0 position and the flatness of the excitation force.

Fig. 7 shows the various force-mover displacement characteristics predicted by FE analyses, when  $\alpha_p$  varies between 0.3 and 0.7 but  $\alpha_s = 0.375$  and fixed copper loss  $(p_{Cu} = 5 \text{ W})$ . It is evident that the slope and peak force increase as  $\alpha_p$  decreases. Since the excitation force is critical to the production of output power during operation, it is worth to be studied further. However, because of the difficulty to separate the excitation and reluctance forces especially in saturated scenarios, the  $K_E$ -mover displacement characteristic, Fig. 8, is used to indicate the influence of  $\alpha_p$  on the excitation force. As can be seen, both the peak value and the non-linearity of  $K_E$  become more significant with

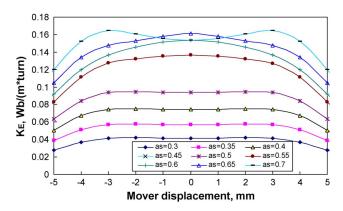


Fig. 10.  $K_E$ -mover displacement characteristics ( $\alpha_p = 0.5$ ).

smaller  $\alpha_p$ , mainly due to the enhanced air-gap flux density but shortened axial pole length.

Fig. 9 shows the variation of the thrust force with  $\alpha_s$  when  $\alpha_p = 0.5$  and  $p_{Cu} = 5$  W, whilst Fig. 10 shows the  $K_E$ -mover displacement characteristics with different  $\alpha_s$ . It can be seen that both the slope and peak force increase as  $\alpha_s$  increases due to the increase of air-gap flux density and the increase of cogging force, respectively. Similarly, the peak value and the non-linearity of  $K_E$  increase as  $\alpha_s$  increases. Compared to the rest, the different curve profile with  $\alpha_s=0.7$  is mainly caused by the magnetic saturation in the stator back-iron.

Based on the foregoing analyses, the optimal actuator design should exhibit big peak  $K_E$ /excitation force and rather flat profiles over the rated stroke. Hence, the global optimization with regard to  $\alpha_p$  and  $\alpha_s$  is undertaken by calculating two parameters:

$$F(0) = F(Z_d = 0) \tag{1}$$

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$$\varsigma = \max\left(\frac{|K_E(Z_d) - K_E(0)|}{K_E(0)}\right)$$
(2)

where F(0), used as the approximation of peak excitation force, is the thrust force at the central position with current injections, as both the cogging and reluctance forces are  $\sim 0$ , whilst  $\varsigma$  is employed as an index for the non-linearity of  $K_E$ -displacement characteristics. Fig. 11 shows the variation of the magnet pole ratio and split ratio on the FE predicted F(0) and  $\varsigma$ , when  $\alpha_n$ and  $\alpha_s$  both being varied from 0.3 to 0.7. It can be seen that the optimal  $\alpha_s$  is essentially identical,  $\sim$ 0.58, irrespective of  $\alpha_p$ changes, and for specific  $\alpha_p$  the variation of  $\varsigma$  is insignificant especially when  $\alpha_s$  is small. When combining the two parameters, Fig. 12 indicates that the combination of  $\alpha_p = 0.6$  and  $\alpha_s = 0.5$  is close to the optimal.

## V. Comparison With Measurements

In order to validate the foregoing FE analyses, a prototype actuator, Fig. 13(a), has been designed, with the resonant frequency = 50 Hz, the output power = 42 W and the predicted efficiency = 89%. The total and moving masses are 866 g and 230 g, respectively. However, for the consideration of application dimensions of the fabricated prototype are slightly different from those used in the foregoing analyses, as highlighted in Table I. Further, in order to enhance the mover integrity the magnet and mild steel are made of rings, rather than solid disks, mounted on a shaft of 3 mm radius, as can

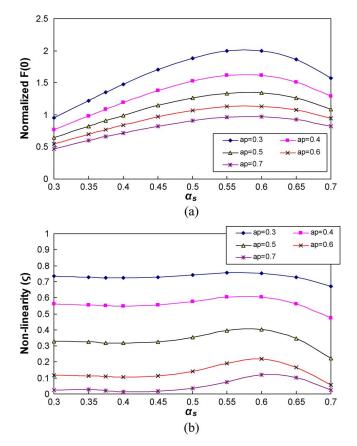


Fig. 11. Variations of the FE predicted F(0) and  $\varsigma$ . (a) F(0); (b)  $\varsigma$ .

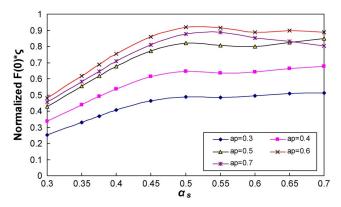


Fig. 12. Variation of  $F(0)^*\varsigma$ .

be seen in Fig. 13(a). Fig. 13(b) shows the static test bench used to measure the cogging force and static thrust force-mover displacement characteristics. As can be seen in Fig. 14, good agreement between the measured and corresponding FE predicted results is achieved.

#### VI. CONCLUSION

A tubular E-core IPM linear oscillating actuator has been designed and its electromagnetic performance, in terms of the air-gap flux density, flux-linkage, back-emf coefficient and thrust force-displacement characteristic is analyzed by finite element analyses. It shows that the E-core IPM actuator yields a reluctance force due to the salient mover structure, which enhances the spring stiffness and is conducive to the oscillation, although has no contribution to the output power. The influence

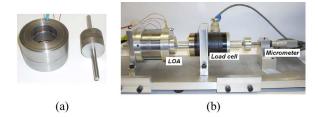


Fig. 13. Prototype actuator and static test bench. (a) Prototype stator and mover. (b) Test bench.

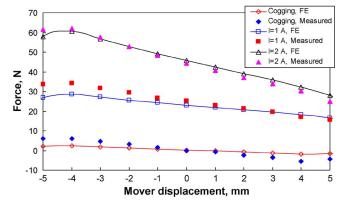


Fig. 14. Comparison of predicted and measured thrust force-mover displacement characteristics.

of the magnet pole ratio and split ratio on the performance of the IPM LOA is investigated, and the optimal parameters have been identified with reference to the peak value and flatness of the excitation force-displacement characteristic. Finally, the predicted thrust force characteristics are validated by measurements on a prototype actuator.

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