# Synthesis of linear actuators

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***Abstract*** *The synthesis of electro-mechanic actuators is formulated as a constrained optimization problem where some performance function of the device is to be met, subject to the satisfaction of some constraints about its dimensions and supply conditions. The optimization problem is tackled by means of a genetic algorithm coupled to a multi-objective definition of the objective function that merge together objectives and constraints in one single scalar objective function. A fast magnetic analysis tool has been developed so that the computational cost of the genetic optimization run is acceptable. Some results about the synthesis of a tubular linear motor in two sizes are presented and discussed.*

## Introduction

Electro-mechanic linear actuators are used in several applications because of the advantage of their direct transmission of thrust and reduced strokes. Often these devices are required to increase force values while keeping a reduced size and weight. Design rules for these kinds of devices are not at the moment yet assessed because, even if their design can be similar to the one of rotating machines, some structures are quite innovative. In addition, design requirements can be very different from one application to the other, for instance forces, speed, positioning etc. Thus the problem of picking up the right dimensions for a particular application can become cumbersome and a trial and error procedure can be too long to be effective. Automated optimization procedures can aid to solve easily these kinds of problems, finding in the possible design configuration space the one that fulfills the best the design specifications.

Following this idea, this paper presents the application of a multiobjective optimization procedure based on genetic algorithm (GA) which faces the problem of the synthesis of one configuration of electromechanical linear actuator that meets a certain set of requirements. This process must take into account the whole engineering process with electric, magnetic and thermal constraints, thus it cannot be approached as an actual inverse problem but is tackled by means of optimization. The optimization procedure is coupled separately to magnetic solution, performed by means of lumped parameters network solved using numerical methods. The finite element method is then employed in order to estimate the objective final values and the other electrical and magnetic quantities of the device for the optimized configuration.

In the following, a brief description of the optimization algorithm adopted will be given, afterwards a presentation of the engineering problem will be given and some results on a specific problem will be discussed.

## Optimization algorithm

### *Genetic algorithm*

The synthesis problem under consideration cannot be approached with the classical inverse problem methodologies because it is not possible to express its

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performance function by means of analytical functions. In this case optimization algorithms can be a powerful tool in the determination of a configuration satisfying the requests of the problem. The synthesis problem can be stated as: given a device described by a set of $n$ design variables and whose performances are defined by a vector of $m$ objectives/constraints find, in the design parameters space $X$, the configuration that maximizes the degree of satisfaction of a given performance target. The performance target can regard both performances of the device computed by some analysis method (force, torque, losses, etc.) and dimensions of the device (length, width, weight, etc.). As a matter of fact, the problem is ill-posed in the sense that more than one configuration can have the same distance from the target and that the global objective can be a discontinuous function of the design parameters. This aspect hinders the use of deterministic optimization algorithms which can locate local minima only and call for the use of a global optimizer. In addition, the definition of the problem requires meeting different requirements at the same time and calls for the use of some efficient multi-objective algorithm.

Among the several stochastic optimization algorithms that can be chosen for this purpose, genetic algorithm (GA) was selected. This choice is mainly motivated by the fact that GA works in parallel on a population of individuals and thus can ensure a better exploration of the design variable space. The particular implementation of the GA used is based on the one presented in Alotto *et al.* (1998) with some slight changes and upgrades. The proposed GA works on continuous variables and introduces some operators that mimic the ones defined in the classical genetic method. In particular, the cross-over operator is substituted by a vector sum of two different configurations and mutation is obtained by adding a random vector to an existing one. A new operator similar to mutation is introduced which takes into account promising directions in the design variables space; a vector of random length is added to an existing configuration in the direction defined by the difference of the two best performing individuals. This fact adds to the method a "valley following" property that can be important when this kind of topological pattern is found in objective function. A schematic representation of the genetic operators is shown in Figure 1.

Even if this implementation of GA has shown its capabilities in many applications, a major upgrade was added in the present implementation. In fact,

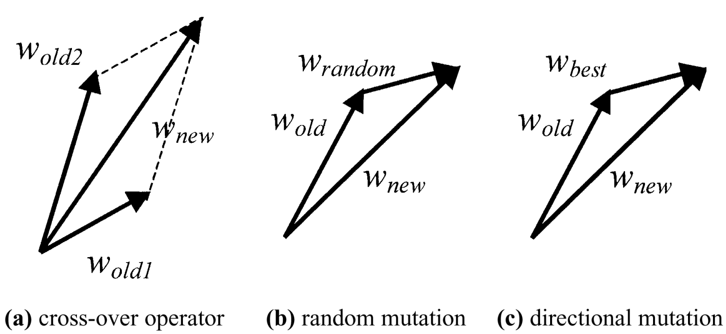



**Figure 1.**
Schematic representation of genetic operators used in the continuous variable implementation of GA: (a) cross-over operator; (b) random mutation; (c) directional mutation

one of the drawbacks of GA is that under selection pressure the algorithm can converge prematurely to a single good solution, losing other promising configurations. Against this behavior, corrections trying to preserve diversity in design variable space have been introduced with different schemes, as for instance in Zitzler and Thiele (1999); in our case a clustering technique followed by tournament selection was implemented. The population created by the genetic operators is naturally grouped in clusters: many new individuals are generated in the vicinity of most promising configurations. Uncontrolled generation of new individuals by parents very close to each other can lead to "inbreeding", spoiling the quality of the solution. To avoid this fact the population is grouped in clusters according to the distance among individuals in design variable space and interactions between individuals belonging to each cluster are inhibited. In fact, once the clusters are defined, only the individual with the best performance function from each cluster is entitled to enter the mating pool for the next generation. In this way genotypic diversity becomes a criterion to enter the mating pool, and afterwards the usual roulette wheel selection based on phenotypic values is applied. In Figure 2 the placement of 21 individuals in the degrees of freedom space is shown for the objective function

$$F(x,y) = -10 + \frac{1}{10}x^2 + \frac{1}{10}y^2 - 2\cos(2\pi x) - 2\cos(2\pi y) \tag{1}$$

*Tests on TEAM problem 22*

The TEAM problem 22 is based on the determination of the optimal configurations of two solenoids which have to fulfil two objectives: stored energy value of 180MJ and minimum value of stray magnetic field. The objectives are combined in one single objective function by means of weighted sum. The

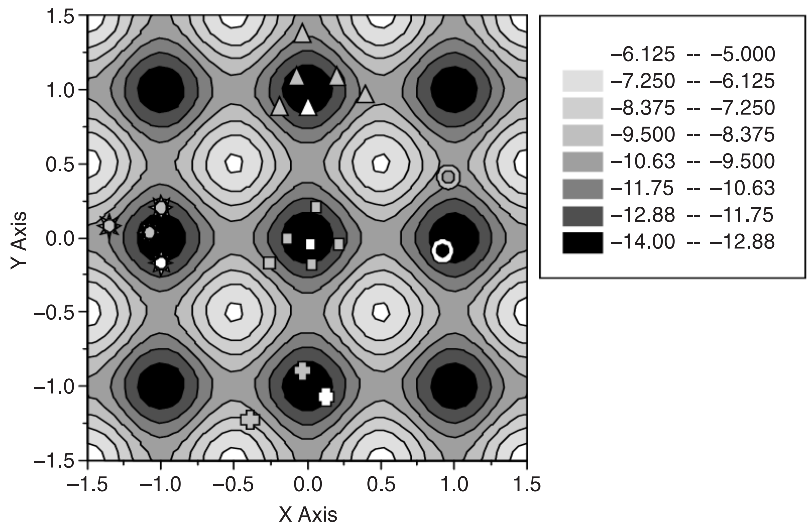



**Figure 2.**
Placement of 21 individuals for the function described in equation (1), different symbols identify different clusters and white symbols are the individuals selected for the mating pool

geometry and supply conditions of the two solenoids are described by 8 degrees of freedom. Under the hypothesis that the coils are superconductive, a constraint tying the current density value and the maximum value of magnetic flux density is set and is dealt with by means of a penalty function. Thanks to the absence of ferromagnetic materials, the evaluation of the objective function can be made by means of integration of the Biot-Savart law allowing a quite fast solution of the direct problem; a description of the problem can be found in Alotto *et al.* (1996) and on http://www-igte.tu-graz.ac.at/.

From the point of view of the behavior of the objective function, it can be said that, as in almost all the problems involving solenoids, where narrow valleys along which several minimal points can be found characterize the objective function. In this case the quenching condition can make the problem tougher by interrupting the valley when it intersects some unfeasible regions.

The proposed genetic algorithm procedure has been tested on this; the main parameters used are:

- number of population: 51;
- percentage of x-over: 20 percent;
- percentage of random mutation: 25 percent;
- percentage of directional mutation: 25 percent;
- percentage of reproduction: 30 percent;
- maximum number of generations: 300;
- number of generations without update of the best configuration: 30.

In average the runs performed led to the formation of 12-16 clusters in the degrees of freedom space, of which three to four, with the subsequent use of a pattern search routine, gave rise to very good minima. For example, the results obtained in one of the best runs are reported in Table I.

The same data are shown in the graph of Figure 3, where for each degree of freedom the normalized value of each variable is shown. As it can be seen from

**Table I.** Values of degrees of freedom and of objectives of TEAM problem 22 in four configurations got from the optimization

| | Conf0. A | Conf0. B | Conf0. C | Conf0. D |
|---|---|---|---|---|
| R1 [m] | 0.12936E+01 | 0.15447E+01 | 0.19252E+01 | 0.10442E+01 |
| R2 [m] | 0.19293E+01 | 0.19554E+01 | 0.23762E+01 | 0.18035E+01 |
| H1/2 [m] | 0.56975E+00 | 0.81337E+00 | 0.66806E+00 | 0.11816E+01 |
| H2/2 [m] | 0.13020E+01 | 0.12188E+01 | 0.12532E+01 | 0.16338E+01 |
| D1 [m] | 0.73108E+00 | 0.63449E+00 | 0.66942E+00 | 0.64768E+00 |
| D2 [m] | 0.10015E+00 | 0.18681E+00 | 0.23264E+00 | 0.12056E+00 |
| J1 [A/m$^2$] | 0.19695E+08 | 0.18325E+08 | 0.16641E+08 | 0.14959E+08 |
| J2 [A/m$^2$] | –0.29055E+08 | –0.26307E+08 | –0.16948E+08 | –0.20096E+08 |
| Energy [J] | 180.00E+06 | 180.00E+06 | 180.02E+06 | 180.57E+06 |
| Bstray [T] | 8.1923E–06 | 35.733E–06 | 65.345E–06 | 43.288E–06 |

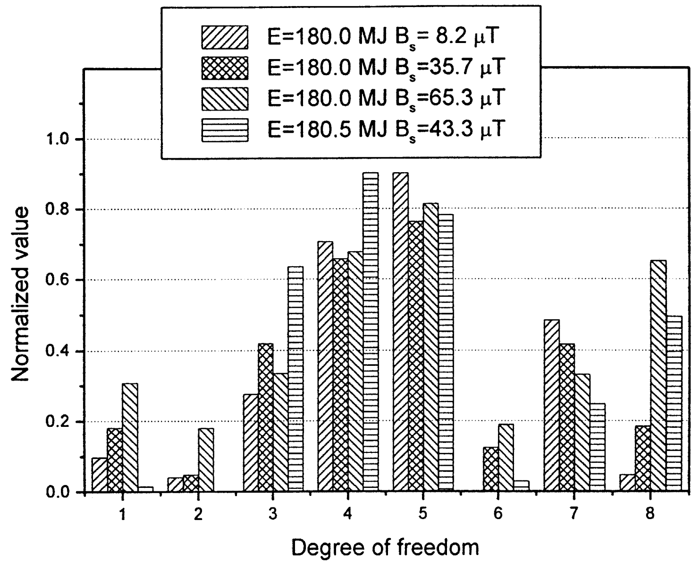



**Figure 3.** Comparison in the design variable space of the four configurations reported in Table I

the graph, the four configurations are quite different in the variable space even if they have objective values which are quite comparable.

*Multi-objective optimization problem*

The proposed optimization algorithm, as it has been presented, can deal with scalar optimization problems only. As it is evident from its definition, the synthesis problem under analysis is essentially a vector one since it must take into account several objectives/constraints at the same time. Multi-objective optimization problems can be approached by means of a generation of a scalar function taking into account all the single objectives. This process, named scalarization or aggregation, is not univocal and several approaches can be devised like weighted sum, goal attainment (Sakawa and Yano, 1984), fuzzy treatment of objectives (Bellman and Zadeh, 1970) etc. In addition, GA allows the treatment of vector optimization problems by means of Pareto based population approaches (Zitzler and Thiele, 1999), but this method has not been taken into account in this work.

In the authors' experience, the fuzzy treatment of multi-objective problems is very effective because it allows to combine together objectives and constraints in a way which is easy to use in a design environment (Chiampi *et al.*, 1996). In summary, each objective or constraint of the problem is translated in a normalized logical value by means of a fuzzy membership function. The degrees of satisfaction are then mingled together by the use of the logical AND operator, which in fuzzy logic can be defined as the min operator. Thus the optimization problem is defined by the formula:

$$\max_{x \in X}\left\{\min_{i=1,..,N_{obj}}[\mu_i(O_i(x))]\right\} \qquad (2)$$

where $\mu_i$ is the membership function of $O_i$ the i-th objective and $N_{obj}$ is the number of objectives/constraints of the problem.

**Linear actuators**

Linear electromagnetic machines are increasingly employed in industrial application in a great range of applications because they offer numerous advantages with respect to the use of rotating machines coupled to mechanical gears. Many configurations of these devices are developed to respond to a variety of specifications such as size, weight, thrust and stroke. Electromagnets and tubular linear motors are two examples of actuators: the first ones are used in applications in an exact control of position which is not requested, while the second ones are chosen for their controllability. The characteristics of the two types of device are different as the magnetic circuits are different, but the proposed fast analysis tool allows to study both devices. In this study the attention is devoted to the design of tubular linear motors.

This device can be realized with different kinds of magnetic configurations, as traditional radial flux machine (permanent magnets, induction and reluctance machine).

The optimized design of these kinds of devices can take into account different aspects, for instance:

- average value of force;
- cogging force due to the permanent magnets and anisotropic magnetic core;
- weight and the global size;
- in addition, several engineering constraints must be taken into account, like for instance saturation of ferromagnetic core, demagnetization of the permanent magnet, thermal dissipation of Joule losses, etc.

The evaluation of the performances of a given machine structure is commonly performed by means of finite element codes. Even if this method can provide very high accuracy performance indicators it is onerous in term of calculus operation, so that coupling it with optimisation techniques as genetic algorithm can result in it being unbearably expensive in terms of computational cost.

To overcome this problem, a fast magnetic analysis tool has been thus developed.

*Fast magnetic analysis tool*

An approach based on the construction and solution of lumped parameters networks applied to magnetic model is devised. In the magnetic equivalent circuit (MEC) a lumped parameter electrical network represents the magnetic system. The model of the device is carried out by choosing appropriate flux tubes in the magnetic circuit, which are described by reluctances and by a mmf value. The evaluation of flux tubes repartition is a crucial node of the model definition so the permeance network must be accurate in taking into account each one of the flux tubes. Nonlinearities are taken into account by means of the fixed point method. In Figure 4 an example of the magnetic circuit associated to a slot-teeth configuration is shown.

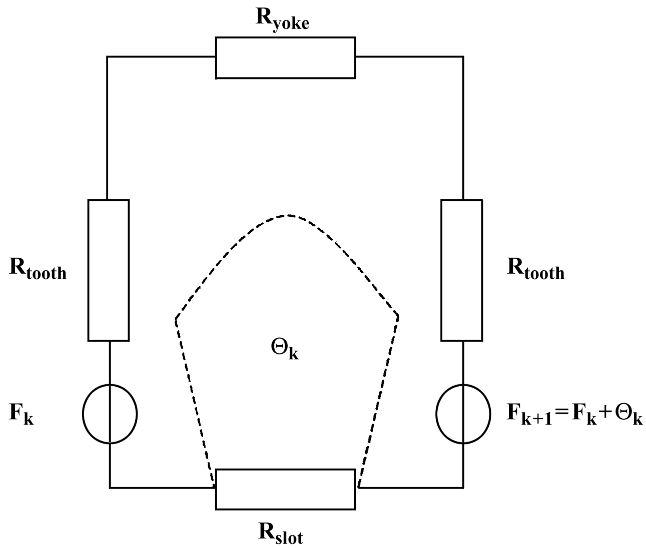



**Figure 4.**
Electrical machine tooth equivalent : $\Theta_k$ is the magnetomotive force of k-th slot

The analysis tool is based on a library of predefined magnetic blocks that allow to calculate the reluctance taking into account the geometry of the branch and of the non linear material behaviour. The library permits a full parametric input of blocks, geometry and of supply conditions in order to couple the analysis tools with optimisation one. The solution of the equivalent magnetic network is obtained by means of nodal analysis. The outputs of each block are the flux and the average flux density, from which is possible to calculate force and coils linked fluxes.

*Tubular linear motor*

The proposed tool is applied to the analysis of permanent magnet linear tubular motors. As traditional machines, tubular motors are made by a stator consisting of coils cylindrically wound and by a cursor consisting of cylindrical assembly of rare hearths permanent magnets. The magnetic core is slotted in the stator part. A three-dimensional view of the structure is reported in Figure 5 while a cross-section of the machine is shown in Figure 6 together with the design variables.

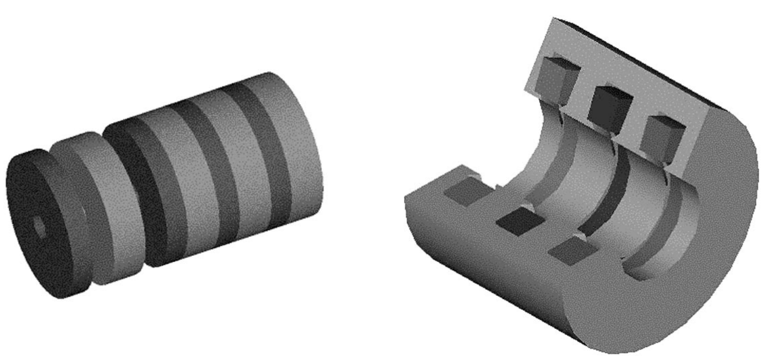


**Figure 5.**
Buried magnet cursor and slotted stator of a tubular linear motor

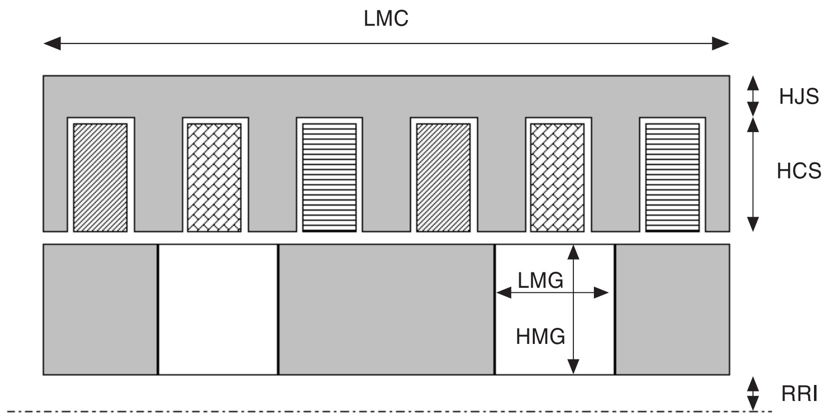



Figure 6.
2D axial-symmetric model of tubular machine and design parameters

**Key**
LMC – pole pair length
HJS – height of stator yoke
HCS – height of stator slot
RRI – inner radius
LMG – axial permanent magnet length
HMG – permanent magnet height

By a preliminary analysis of the machine, the most influencing parameters of the machine are:

- the inner radius of the rotor;
- the height of the stator slot;
- the magnet axial dimension;
- the magnet radial dimension;
- the length of the machine.

All the other defining parameters are calculated introducing some constraints due to the desired flux density in teeth and yoke and to the maximum current density allowed in the coils according with thermal constraints.

The magnetic circuit model is obtained with a 34 blocks magnetic network, as shown in Figure 7. The topology of the network allows to estimate the mean flux tubes in different supply conditions.

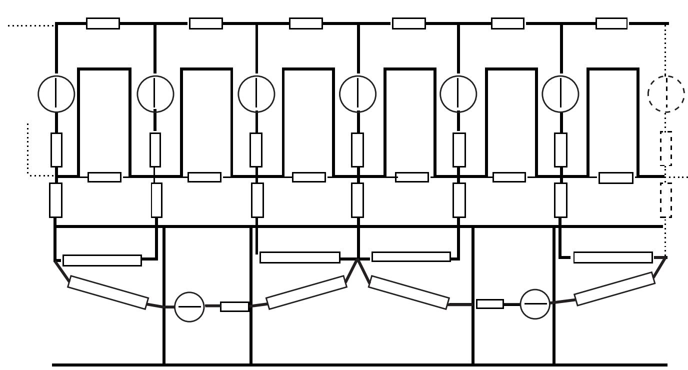


Figure 7.
Equivalent magnetic network

The MEC approach has been validated versus FEM analysis, performing several comparisons in different configurations, changing the design parameters within the feasibility area. In all these cases the maximum error on force computation was lower then 20 percent.

*Synthesis of two sizes of linear motors*

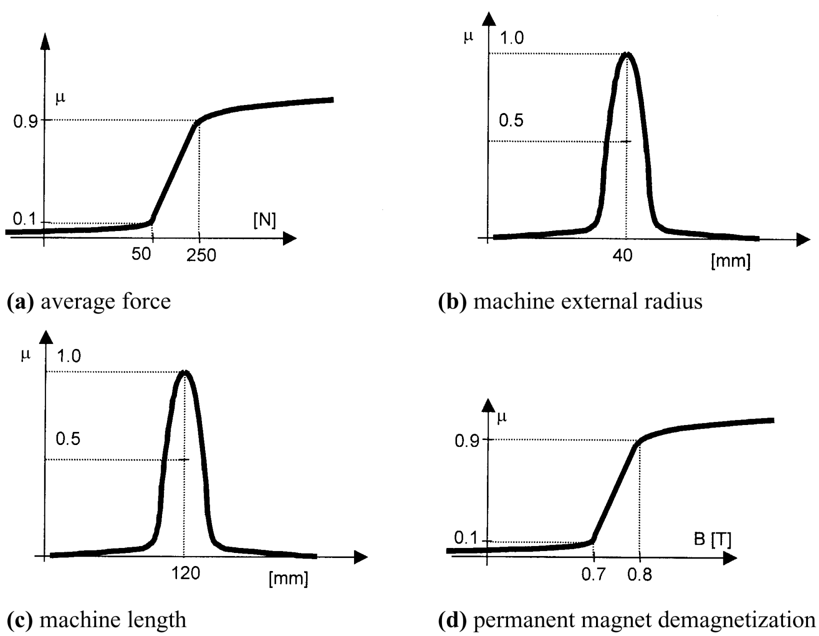
The synthesis problem is applied to the design of two sizes of linear motors. Two different sets of requirements are defined for a "big" size and one "small" size motor. Design specifications are set on:

- average force;
- machine external radius;
- machine length.

One physical constraint is set on permanent magnet demagnetization. These requirements are implemented in membership functions. For the "big" size linear motor these functions are reported in Figure 8.

The results of five runs of GA show a very good performance of the algorithm that was always able to find solutions very close to the design requirements. On average, the algorithm found convergence in 380 generations, each generation had a population of 51 individuals and percentage of genetic operators was the same as the TEAM 22 problem. In Table II the results obtained in one of the best configurations are reported.

For the "small" size motor, the external dimensions were reduced together with the target on average force. The results obtained are reported in Table III




**(a)** average force

**(b)** machine external radius

**(c)** machine length

**(d)** permanent magnet demagnetization

**Figure 8.** Membership functions defined on the four objectives/constraints

whereas the performances of the algorithm were approximately the same as in the previous case.

In both cases GA was able to find very good solutions that were only slightly modified by the subsequent deterministic search phase.

Finite element analyses were run on the final configurations to validate the magnetic circuit analysis and results on force values showed a good agreement: in the "big" size motor the difference between FE and magnetic circuit analysis was of 2 percent while in the "small" size it was of the 13 percent, in both cases the error was lower than the maximum error obtained in the validation phase cited in the previous paragraph. In Figures 9 and 10 the gray-scale map of magnetic flux density absolute values is shown on the geometry of both motors.

**Table II.**
Results in one of the best optimized configurations for the "big" size motor

| | |
|---|---|
| Machine length = 124mm | (required 120mm) |
| External radius = 43mm | (required 40mm) |
| Force = 170N | (required 50÷ 250N) |
| B in permanent magnet = 0.9T | (required over 0.7T) |

**Table III.**
Results in one of the best optimized configurations for the "small" size motor

| | |
|---|---|
| Machine length = 61.6mm | (required 60mm) |
| External radius = 41mm | (required 40mm) |
| Force = 121N | (required 30÷ 100N) |
| B in permanent magnet = 0.83T | (required over 0.7T) |

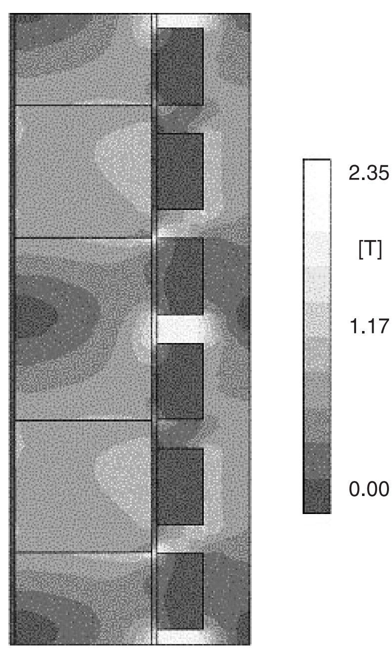



**Figure 9.**
Optimized "big" size tubular linear motor with absolute value of magnetic flux density

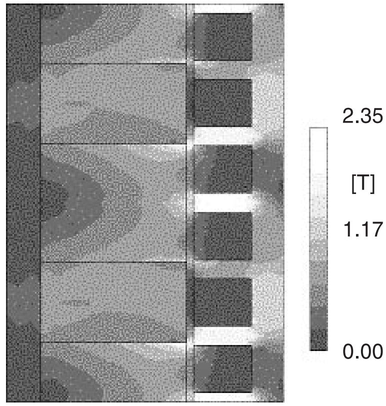



**Figure 10.**
Optimized "small" size tubular linear motor with absolute value of magnetic flux density

## Conclusions

The work performed has regarded the synthesis of linear actuators from a given set of design specifications. The approach of this inverse problem by means of GA was feasible and efficient. In addition, the use of fuzzy logic scalarization for treating multi-objective problems gave sensible and effective results. The set-up of a fast magnetic analysis tool for the computation of the performance function of the device was crucial in order to use GA in a reasonable CPU time.

Further work will be devoted to the integration of the magnetic analysis tool with other simulation tools such as mechanical dynamics, eddy currents and thermal analysis.

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