Analysis of the Tubular Motor with Halbach and Radial Magnet Array

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Abstract— **In the machine tool industry, direct drive linear motor technology is of increasing interest as a means to achieve high acceleration, and to increase reliability [1]. This paper analyzes and compares the characteristics of tubular motor with halbach and radial magnet array respectively. The governing equations established analytically in terms of vector potential, two dimensional cylindrical coordinate system and Maxwell's equations. We derive magnetic field solutions due to the PMs and to the currents. Motor thrust, flux linkage, back emf are then derived. The results are shown in good conformity with those obtained from the commonly used finite element method.**

I. INTRODUCTION

Our work is motivated by the desire to develop a direct drive linear actuator for machine tool applications. The applications for such a motor range from material handling devices to semi-conductor wafer stepping applications, diamond turning machines and other precision applications [1]. *Tubular structures are very attractive compared with flat* linear motors, owing to no existence of end-turn effects. In addition to this advantage, tubular motor has many advantages: 1) higher maximum speeds and acceleration limits, 2) higher position accuracy without anti-backlash devices, 3) no direct physical constraint in the axial direction of propulsion, 4) no power loss in rotary-linear power conversion, 5) no friction except in the ball bearings that support the platen weight [1].

In this paper, two structures of PM tubular motors are analyzed. One is the tubular motor with the Halbach magnet array and the other is the tubular motor with radial magnet array. Two types of PM tubular motors are analyzed, with reference to the following parameters as variables: magnetic field, flux linkage, motor thrust and back emf. These variables are derived by the use of analytical method in terms of two-dimensional cylindrical coordinate system. The results are validated extensively by comparison with finite element method.

II. TUBULAR MOTOR STRUCTURES AND ANALYTICAL MODEL

Fig.1 shows that tubular linear motor which we analyze. Fig.2 shows that the cross section, excluding stator windings, of tubular motor sketched in Fig.1. Actually, the permeability of the winding region is the same as it of the air region. So, analyzing the magnetic fields due to PMs, we regard the winding region as the air region. Consequently, the magnetic field analysis is confined to two regions. Roma letters *I* and *II* in Fig.2 represent the air region and the magent region respectively. Therefore,

$$B = \begin{cases} \mu_0 H & \text{in the air/stator windings} \\ \mu_0\mu_r H + \mu_0 M & \text{in the magnets} \end{cases} \qquad (1)$$

where μ_r is the relative recoil permeability of the magnets and is supposed unity, that is, $\mu_r = 1$. M is the remanent magnetization. Fig.3 (a) and (b) shows that single and three phase current density distribution respectively. We assume that the stator current flows through an infinitesimally thin sheet on the interior surface of the stator, at $r=r_s$. We also assume that both iron permeability and motor length are infinite.

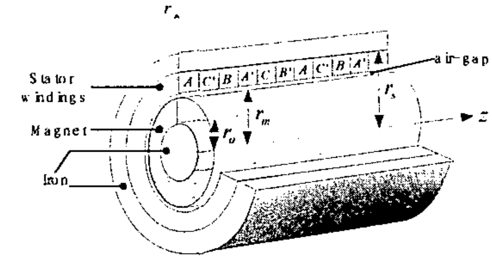

Fig.1. Tubular Motor Model

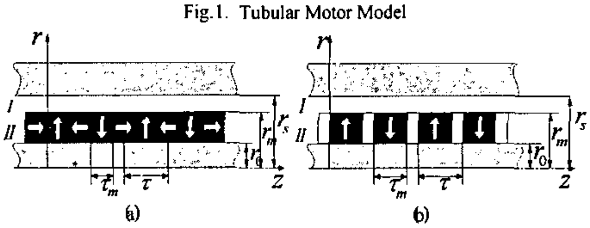

Fig.2. The cross section of tubular motor with Halbach array of PMs (a) Radial array of PMs (b)

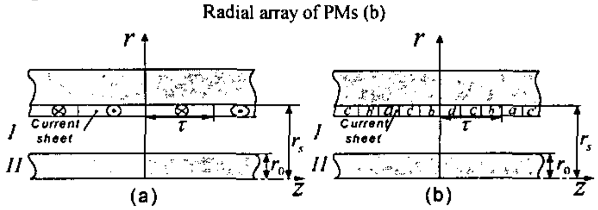

Fig.3. Single-phase (a) and three-phase (b) current density distribution

III. MAGNETIC FIELDS DUE TO THE PMs

A. Governing Equations

Since there is no free current in the magnet region, curlH=0. So, $\nabla\times B=\mu_0\nabla\times M$. The magnetic vector potential A is defined as ∇×A=B. By the geometry of the tubular motor, the vector potential has only θ-components. Therefore, The Poisson's equation for radial magnet array, in terms of the Coulomb gauge, ∇•A=0, is given by

$$\frac{\partial^2}{\partial r^2}A_\theta + \frac{1}{r}\frac{\partial}{\partial r}A_\theta - \left(k_n^2 + \frac{1}{r^2}\right)A_\theta = -\mu_0 k_n\left(\frac{c_1}{r} + c_2 r\right)M_{rn} \quad (2)$$

where the pole pitch of the motor is τ, and the spatial wavenumber of the nth harmonic is $k_n = n\pi/\tau$. Here M_{rn} is the Fourier coefficient of nth-order radial magnetization components in radial and Halbach magnet array. The coefficients c_1 and c_2 are appropriately selected to reduce the modification of radial component M_r in magnetization M [2]. On the other hand, both governing equation of radial and Halbach magnetized tubular motor model are given by (2) and in the case of M_{rn}=0, (2) reduces to a Laplace's equation in air region.

B. Boundary Conditions.

Boundary conditions of the Halbach and conventional magnetized tubular motor model shown in Fig.2 are given by

Halbach magnetized model	radial magnetized model
$B_z^{II}(r_o,z) = -\mu_0 M_{zn}$	$B_z^{II}(r_o,z) = 0$
$B_r^{II}(r_m,z) = B_r^{I}(r_m,z)$	$B_r^{II}(r_m,z) = B_r^{I}(r_m,z)$
$B_z^{II}(r_m,z) - B_z^{I}(r_m,z) = -\mu_0 M_{zn}$	$B_z^{II}(r_m,z) = B_z^{I}(r_m,z)$
$B_z^{I}(r_s,z) = 0$	$B_z^{I}(r_s,z) = 0$

(3)

where M_{zn} is the Fourier coefficient of nth-order axial magnetization components in halbach magnet array.

C. Characteristic equations of flux density.

The resulting axial and radial components of flux density are given by

$$B_{zn}^{I} = k_n[A_n^{I} I_0(k_n r) - B_n^{I} K_0(k_n r)]\cos(k_n z)$$
$$B_{zn}^{II} = k_n[A_n^{II} I_0(k_n r) - B_n^{II} K_0(k_n r) + \frac{2c_2\mu_0 M_{rn}}{k_n^2}]\cos(k_n z)$$
$$B_{rn}^{I} = k_n[A_n^{I} I_1(k_n r) + B_n^{I} K_1(k_n r)]\sin(k_n z)$$
$$B_{rn}^{II} = k_n[A_n^{II} I_1(k_n r) + B_n^{II} K_1(k_n r) + \frac{\mu_0 M_{rn}}{k_n}\left(\frac{c_1}{r} + c_2 r\right)]\sin(k_n z) \quad (4)$$

where I_1 and K_1 are modified Bessel functions of the first and second kind, of order one and I_0 and K_0 are also modified Bessel functions of the first and second kind, of order zero [3]. Equation (4) can be applied to both Halbach and conventional magnetized tubular motor model, the coefficients A_n^{I}, B_n^{I}, A_n^{II} and B_n^{II} are only different in each model. These coefficients are determined by substituting (3) for (4).

IV. MAGNETIC FIELDS DUE TO THE STATOR CURRENTS

The linear current density J may be expanded into a Fourier series, viz

$$J_\theta(z) = \sum_{n=1,odd}^{\infty} J_n \sin(k_n z) \quad (5)$$

where J_n are function of the current value and the winding distribution. Since this paper assumes that the current is distributed in an infinitesimal thin sheet, both air/stator windings and iron regions remain characterized by curlH=0 and then magnetic fields are computed by means of the vector potential. Governing equation is represented by Laplace's equation (2), with M_{rn}=0. The boundary conditions are given by

$$B_z^{I}(r_s,z) = \mu_0 J_\theta(z)$$
$$\mu_r B_z^{I}(r_o,z) = B_z^{II}(r_o,z)$$
$$B_r^{II}(r_o,z) = B_r^{I}(r_o,z) \quad (6)$$

where μ_r is the relative recoil permeability of the iron and is supposed infinity, that is, $\mu_r=\infty$

The resulting characteristic equations of flux density is given by

$$B_{rn}^{I} = \mu_0 k_n[-I_1(k_n r) + \chi_n k_1(k_n r)]\upsilon_n J_n \cos(k_n z)$$
$$B_{zn}^{I} = \mu_0 k_n[I_0(k_n r) + \chi_n k_0(k_n r)]\upsilon_n J_n \sin(k_n z) \quad (7)$$

where the coefficients χ_n, υ_n are given by

$$\upsilon_n = \frac{1}{k_n[I_0(k_n r_s) + \chi_n K_0(k_n r_s)]} \quad \chi_n = -\frac{I_0(k_n r_o)}{K_0(k_n r_o)} \quad (8)$$

V. THE FLUX LINKAGES, THE BACK EMF AND THRUST FORCE

The flux linkage of each phase is given by [2]

$$\lambda = \int_i^f \frac{N}{\tau}\int_z^{z+\tau} 2\pi r_s B_r(z)dzdz \quad (9)$$

where N is the number of conductors per pole, i and f are the initial and final position of the considered phase respectively.

The back electromotive force of each phase is given by [4]

$$e_b = \frac{d\lambda}{dt} = \frac{dz}{dt}\frac{d\lambda}{dz} = v\frac{d\lambda}{dz} \quad (10)$$

Thus back emf is given by the product of velocity v and the rate of change in flux linkage with respect to position.

The axial thrust exerted on the stator winding, resulting from the interaction between the current density and the permanent magnet field. In the general position z, the thrust in an infinitesimal tubular motor length dz is given by [2]

$$dF_z(z) = -2\pi r_s J_\theta(z) B_r^{I}(r_s,z)dz \quad (11)$$

It is advantageous that (11) is free from integrals of Bessel functions related to a formula $F=\int_v (J\times B)dv$ that cause a significant analytical burden

VI. Comparison Of Analytical And FE Results

The main design parameters of two different analytical models of tubular motor shown in Fig.2, for which characteristic equations have been obtained, are given Table I. Then, this paper deals with comparison of analytical and FE results. Fig.4 and 5 shows analytical and FE results for flux density of Halbach and radial magnetized topologies respectively. Fig.6 shows analytical and FE results for flux density due to excitation of single and three-phase windings. Fig 7, 8 and 9 show that force, flux linkages and back emf for Halbach and radial magnetized topologies respectively.

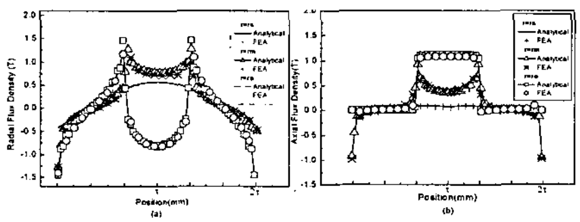

Fig.4. Comparison of analytical and FE results: radial (a) and axial (b) flux density due to Halbach magnetized PMs

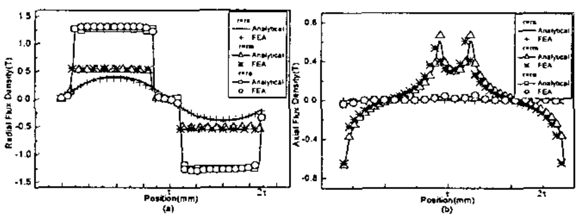

Fig.5. Comparison of analytical and FE results: radial (a) and axial (b) flux density due to radial magnetized PMs

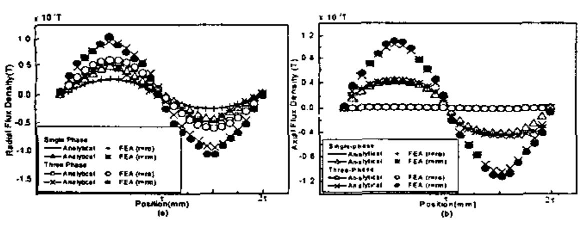

Fig.6. Comparison of analytical and FE results: radial (a) and axial (b) flux density due to excitation of single and three-phase windings

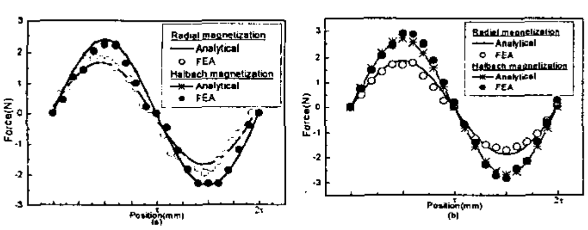

Fig. 7. Thrust on the mover with Halbach and radial magnetized PMs : single-phase windings with i=1(A) (a) and three-phase windings with i_a=1(A), i_b=-0.5(A), i_c=-0.5(A) (b)

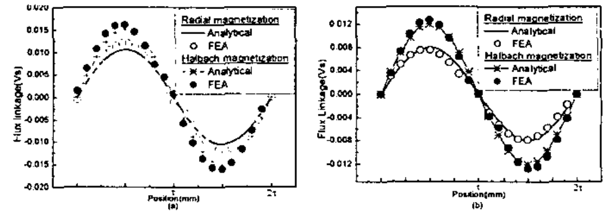

Fig. 8. The flux linkage due to Halbach and radial magnetized PMs; single-phase windings with i=1(A) (a) and three-phase windings with i_a=1(A), i_b= -0.5(A), i_c= -0.5(A) (b)

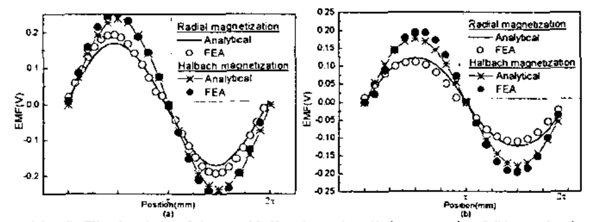

Fig. 9. The back emf due to Halbach and radial magnetized PMs; single-phase windings with i=1(A) (a) and three-phase windings with i_a=1(A), i_b= -0.5(A), i_c= -0.5(A) (b)

TABLE I
DESIGN PARAMETERS OF THE TUBULAR LINEAR MOTOR

Parameters	Radial magnetized topologies	Halbach magnetized topologies
τ (pole pitch)	20(mm)	20(mm)
τ_m (PM pole pitch)	15(mm)	10(mm)
r_o (inner PM radius)	10(mm)	10(mm)
r_m (outer PM radius)	20(mm)	20(mm)
r_r (outer airgap radius)	25(mm)	25(mm)
B_r (residual PM flux density)	1.1(T)	1.1(T)

VII. Conclusions

In this paper, magnetic fields, forces, flux linkages and back emf of tubular motor model with Halbach and radial magnetized PMs are given. The analytical results have been verified by finite element analyses, confirming the goodness of the proposed analysis. The analytical model can be applicable to both slot and slotless topologies and is also very attractive in terms of rapid analysis of the tubular linear motor.

References

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