OSY.SSI[2018][8]

In the news...

- TODO

Please prepare for the lab

Bring your laptop.

We will try something new that may or may not work.

Also install WireShark + PuTTY (or `ssh`) on your *host* device.

In the previous episode...

$\emptyset$ · DiD · OTP/IND-1T

$\bullet_0$ $\bullet_1$ $\bullet_2$ $\bullet_3$ $\bullet_4$ $\bullet_5$ $\bullet_6$ $\bullet_7$ $\bullet_8$ $\bullet_9$ $\bullet_{10}$ $\bullet_{11}$ $\bullet_{12}$ $\bullet_{13}$ $\bullet_{14}$ $\bullet_{15}$ EXAM

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- On the other hand it is very impractical
- And in any case it only works for confidentiality

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- Trade security for practicability: replace "impossible" by "hard"
- Address data integrity (By defining a game)
- See what we need to establish a secure channel

Table of Contents

Statistics vs. complexity

Maybe instead of an “impossible to break” system, we just need “hard to break” systems.

What do we mean, “hard”?

Complexity

Algorithmic time complexity

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Examples:

- Finding the max element in a list of size n can be done in $O(n)$.
- Sorting a list of size n can be done in $O(n \log n)$.

P vs. NP

$\mathbf{P}$ and $\mathbf{NP}$

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- $\mathbf{NPTIME}$, or simply **NP** : Problems for which we can check a solution in time $O(\text{poly})$
- In particular, $\mathbf{P} \subset \mathbf{NP}$.

Three remarks:

- Problems in P or NP are always computable
- P and NP are well-defined thanks to the ECTT
- **P, NP $\subseteq$ PSPACE**

P and NP problems

Problems in $\mathbf{P}$:

- linear programming, greatest common divisor
- Type inference
- Determining if a number is prime

Problems in $\mathbf{NP}$:

- Hamiltonian path
- Traveling salesman problem
- Knapsack / Subset-sum

General intuition: $\mathbf{P}$ is "easy", $\mathbf{NP}$ is "interesting"

There are many, easier, harder, or unrelated classes: **L**, **PP**, **NP-hard**, **BQP**, **AM**, **#P**, **EXPTIME**, etc.

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Exercice: Prove that $\mathbf{P} = \mathbf{NP}$ or find a counter-example.

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But we can try.

Trying to get a one-way function

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$$\exp\left(\left(\sqrt[3]{\frac{64}{9}} + o(1)\right)(\ln n)^{\frac{1}{3}}(\ln\ln n)^{\frac{2}{3}}\right)$$

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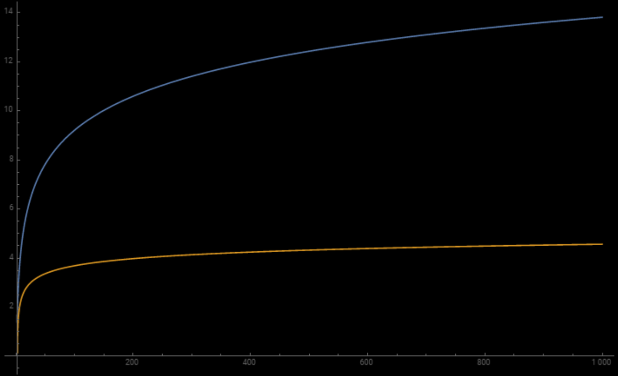
Difficulty (log scale) as a function of n:



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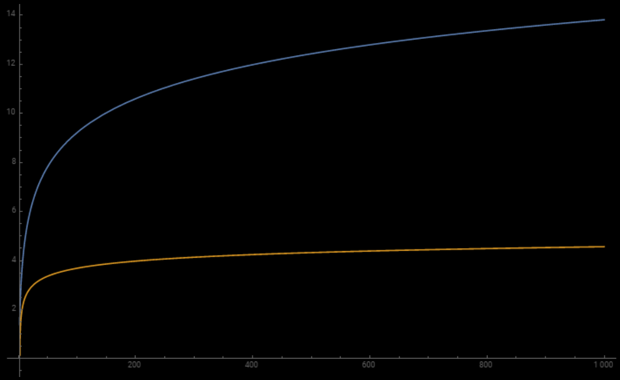

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So for instance, if we have $p, q > 2^{1600}$, the cost of factorisation is around... 2^{128}.

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Example: $G = \mathbb{F}_{17}^{\times} = \{3^k \bmod 17\} = \{1, 3, 9, 10, 13, 5, 15, 11, 16, 14, 8, 7, 4, 12, 2, 6\}$

(Other examples: $\mathbb{F}_{p^k}^{\times}$, $E(\mathbb{F}_{p^k})$.)

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- If $G = E(\mathbb{F}_p)$ the best known classical algorithm is Pollard's ρ $\qquad O(\sqrt{p})$

Candidate hard problems in crypto

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So we want to make it as hard to invert f while keeping it easy to compute f.

Example: $p \approx 2^{256}$ gives a user cost of $\approx 2^8$ and an adversarial cost of $\approx 2^{128}$.

There are many other candidates out there, we focus on these two which are the most widely in use today.

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In other terms, only the signatory should be able to compute σ. Can we make this desire precise?

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Adversary		Alice
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Goal: design an algorithm σ such that the probability of the adversary winning is negligible!

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Best known generic algorithm requires factorisation of n.

Rivest–Shamir–Adleman Signatures

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$$\mathrm{OAEP}(m) = (m \oplus G(r)) \| (H(m \oplus G(r)) \oplus r), \quad H \text{ and } G \text{ are "random oracles"}$$

Random what?

The security of digital signatures is a famously difficult problem, that often makes use of one or several random oracles.

An oracle is a black-box entity that can be queried and provides a response according to some prescription.

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ROs are impossible to implement but they provide useful “ideal” models. In practice: hash functions.

Superb so now we have provably secure communications

- ECDHE: key exchange assuming DLOG in the GGM with uniform KDF
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Under these assumptions we can perform authenticated key exchange that allows us to negociate a session key.

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- How do I generate the randomness? TRNG
- How fast is it? Slow.

Hybrid PK-SK

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- Symmetric encryption (e.g. Chacha20, AES) instead of public-key encryption
- MACs (e.g. Poly1305) instead of public-key signatures
- A mode of operation must be chosen.

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Go to a HTTPS website and look at what they're using!

1 slide summary

- PKI certification $\rightarrow$ long term, personal signature key
- Authenticated key exchange $\rightarrow$ ephemeral, shared session key
- Symmetric encryption+MAC (or AEAD) $\rightarrow$ confid + integr. on channel
- Key sizes chosen to ensure heuristic effort of about 2^{128} $\rightarrow$ security level
- Key management is essential and critical Use HSMs
- Randomness is essential and critical Use TRNGs

E.g.: Use Curve25519 + EdDSA + SHA3 + AES-GCM-SIV

Take away

- Learn about cryptography (I can give references)
- It is very easy to design a system that the designers themselves can't break, especially if the designers have little experience in breaking things. (Schneier's Law)
- Proofs have preconditions and precise scope. GAME-MEANS notation.
- Using proper cryptography is necessary but not sufficient for security

Pause

After the pause: Internet Lab!

Bonus: flipping a coin over a telephone

Note f is a one-way function, $\|$ denotes concatenation.

1. Alice chooses random numbers a_0, a_1 and computes $b_i = f(i\|a_i)$.
2. Alice sends Bob b_0 and b_1, and indicates that she bets on b_A, $A \in \{0, 1\}$.
3. Bob flips a coin and announces the result B.
4. Alice reveals A and a_0, a_1.
5. Bob can check that $f(A\|a_A) = b_A$, and if $A = B$ then Alice wins otherwise she looses.