

Detecting Path Anomalies in Sequential Data on Networks

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In collaboration with



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This Talk

Motivation: Understanding mechanisms behind sequential data on networks

Today:

Motivate the study of path anomalies

Introduce de Bruijn graph representation of sequential data

Develop tractable null model to measure deviation of path data from expectation

Validate null model in synthetic data + compare with naïve baseline method

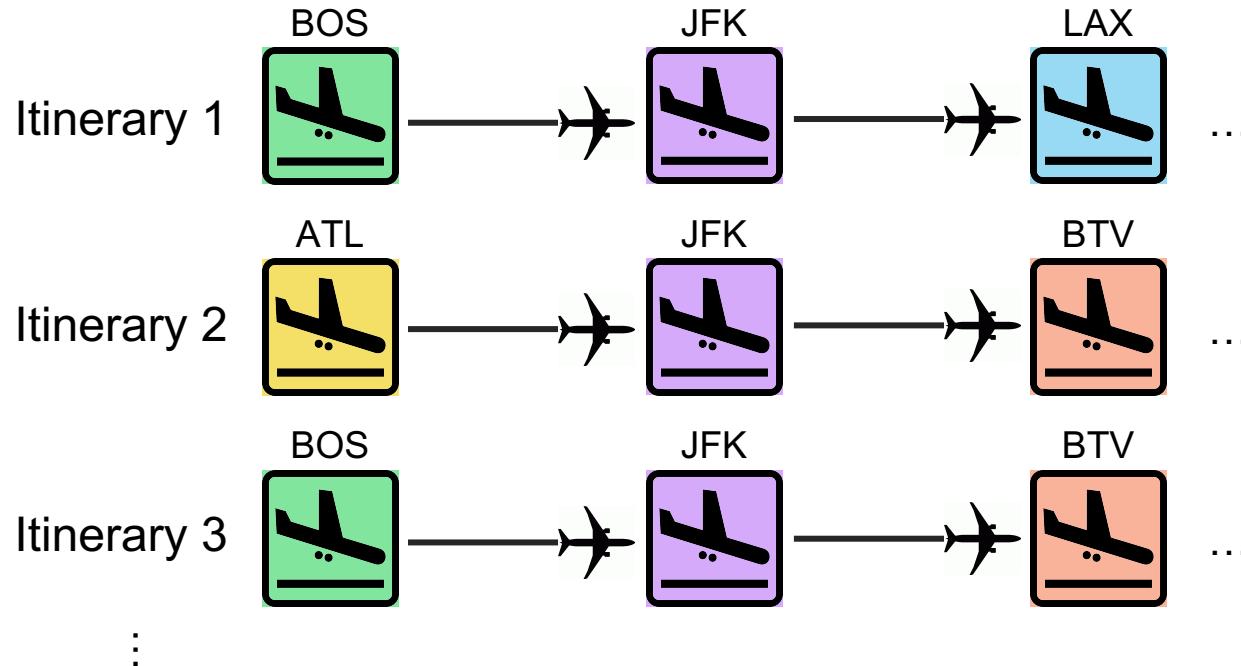
Application of methodology to a real system



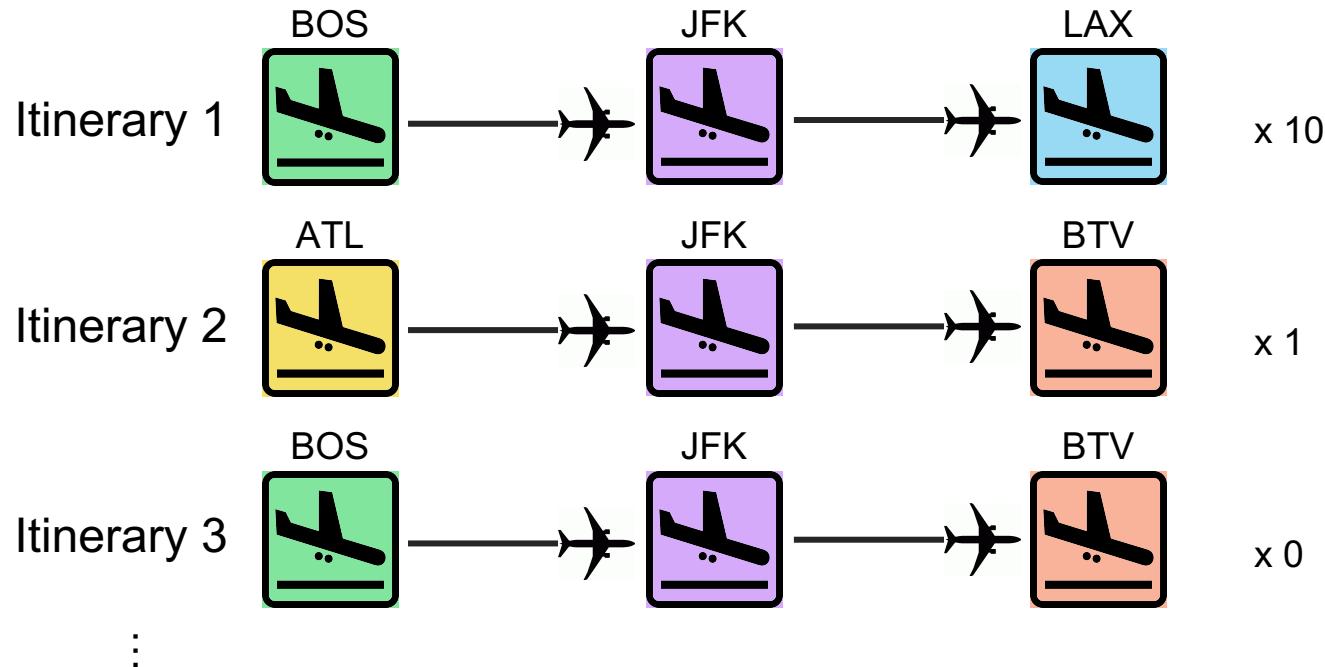
Intuitive Example: Passenger Flight Data



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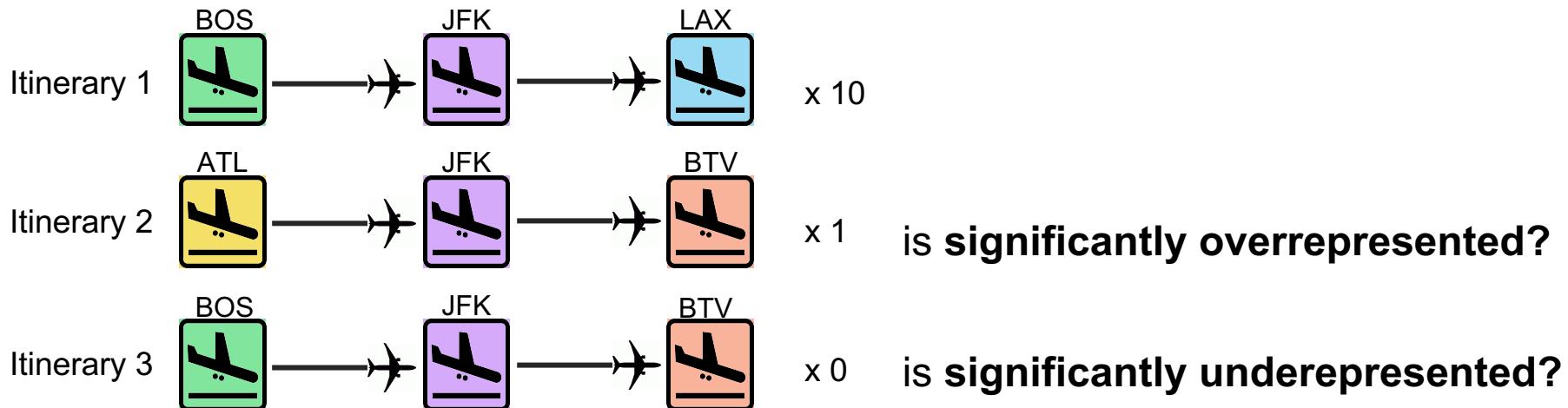


Intuitive Example: Passenger Flight Data



Research Question

Given this pathway dataset, can we determine whether...



Problem: Path anomaly detection

For a given graph G and integer k , identify all paths of length k through G whose observed frequencies deviate significantly from random expectation in a $(k - 1)$ -order model of paths through G .



Problem: Path anomaly detection

For a given graph G and integer k , identify all paths of length k through G whose observed frequencies deviate significantly from random expectation in a $(k - 1)$ -order model of paths through G .

When $k = 2$, this corresponds to comparing a random walk with a single step of memory to a memoryless (Markovian) random walk on the weighted network topology of G .

Toy Example

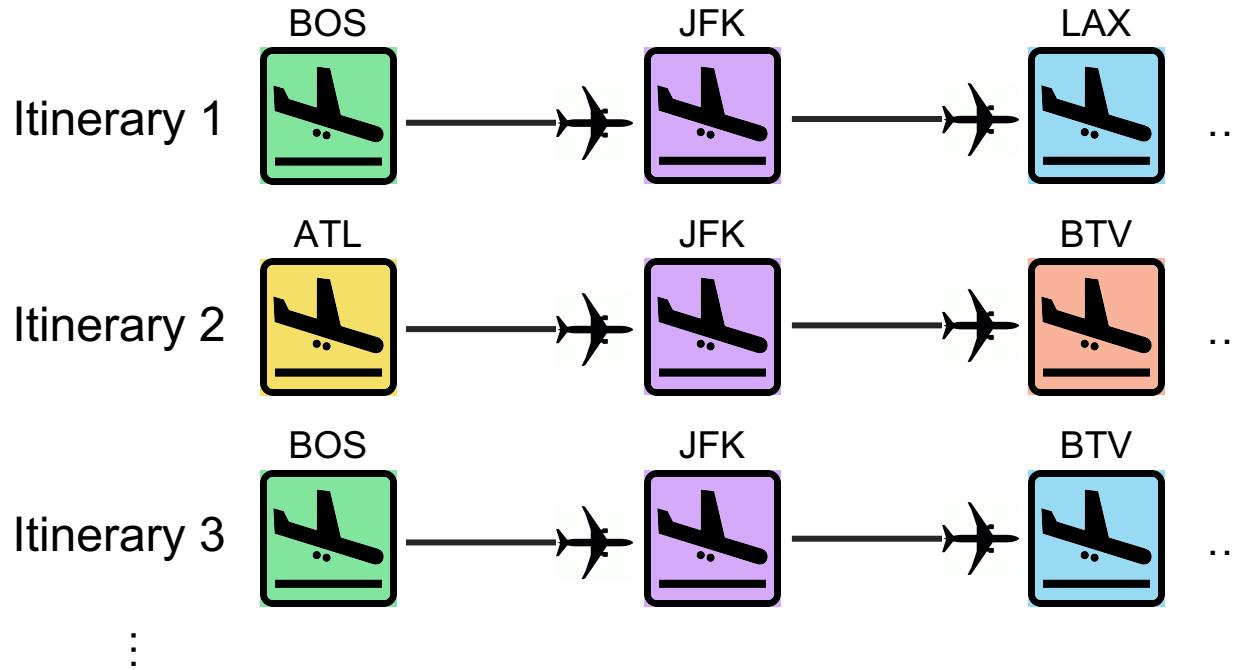
Dual Purpose:

1. Introduce de Bruijn graphs as representations of sequential data
2. Show how path anomalies can be detected through a random walk simulation approach

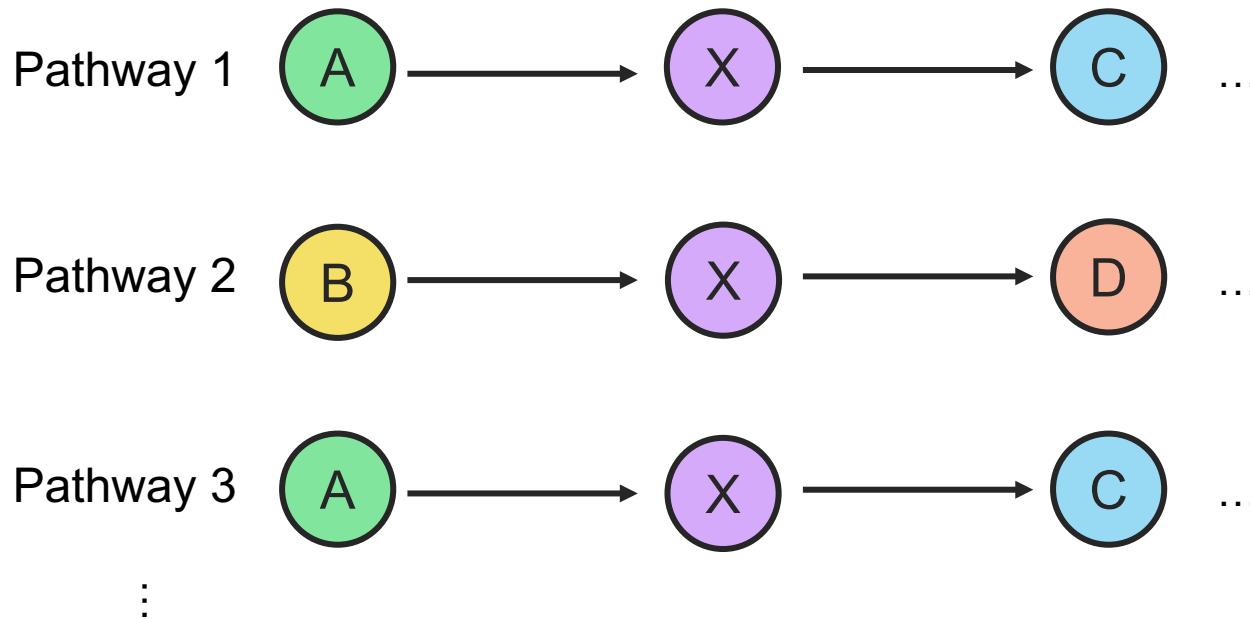
(Spoiler: Simulation approach is infeasible for real world datasets!)



Toy Example

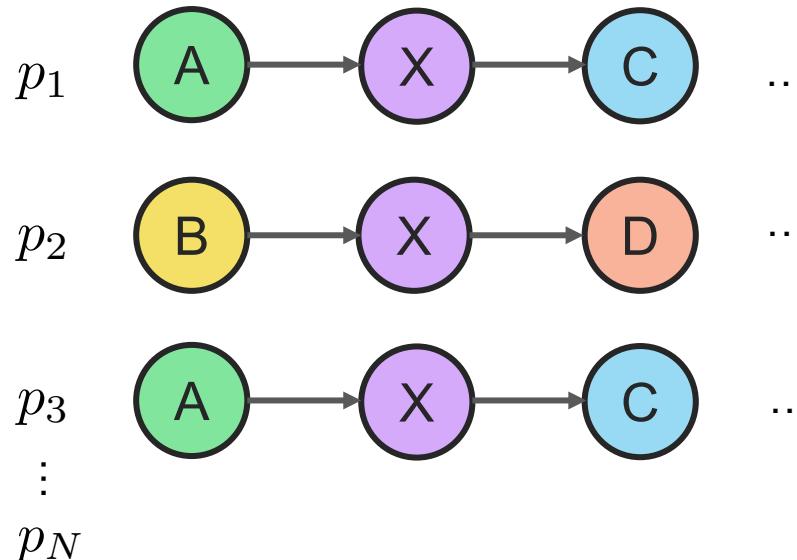


Toy Example

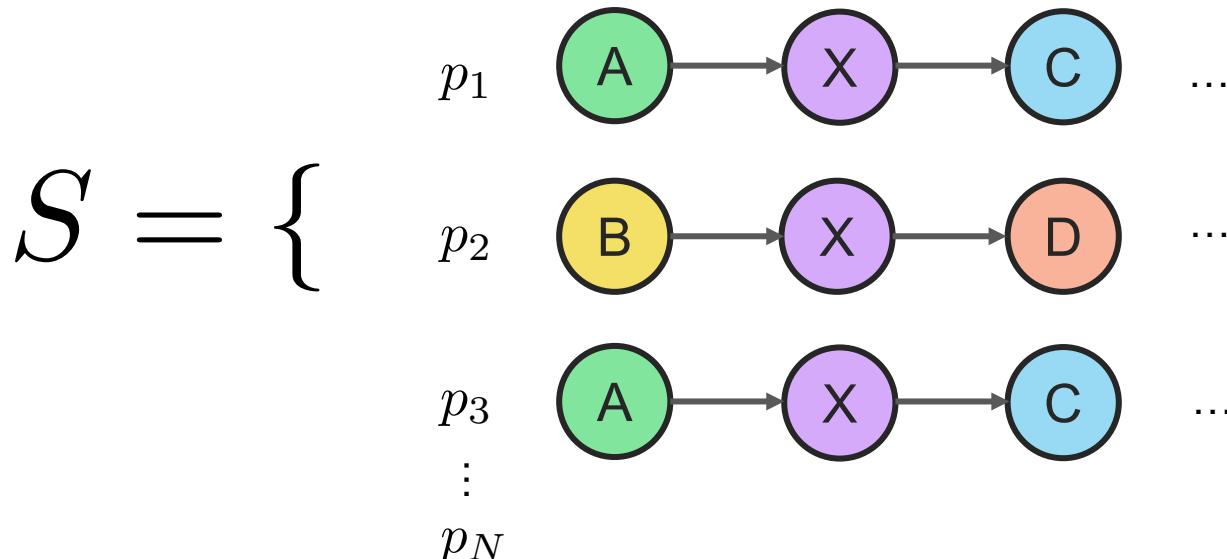


Toy Example: Data

$$S = \{$$

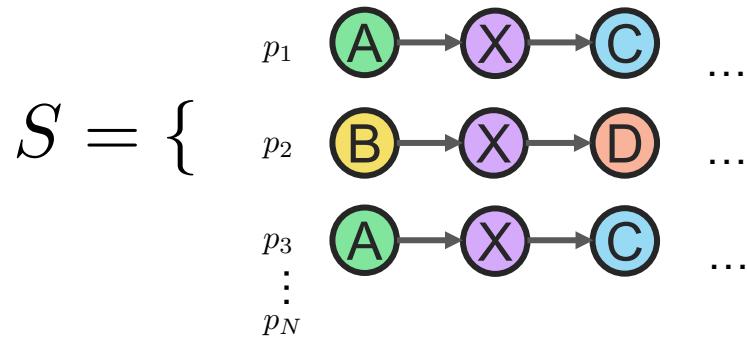


Toy Example: Data

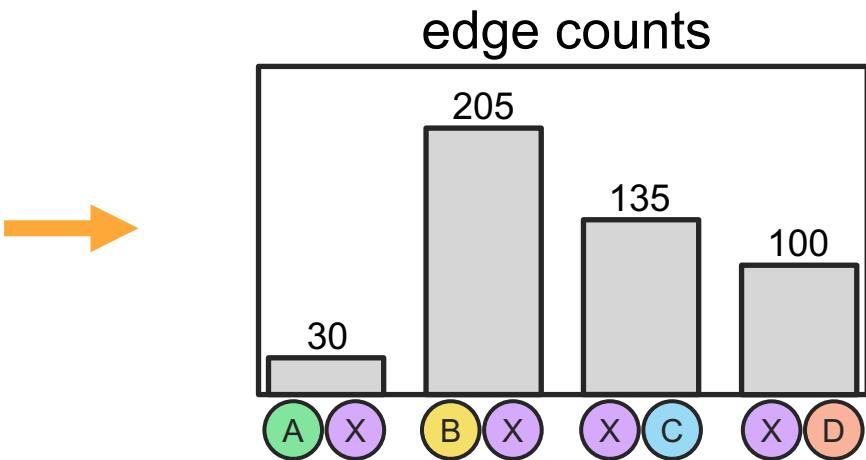


Total number of paths $N = |S| = 235$

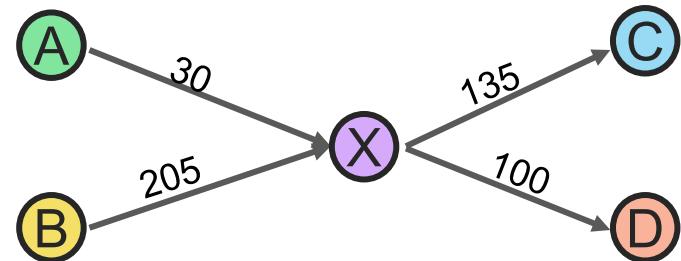
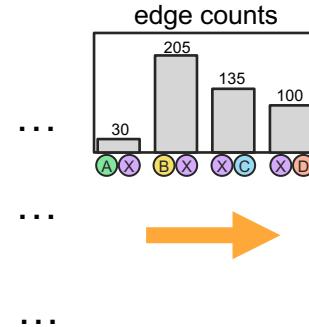
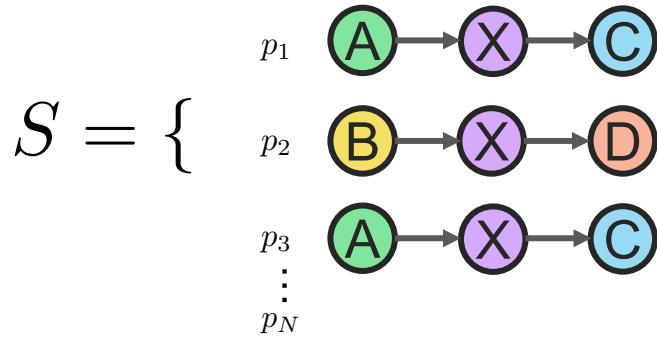
Toy Example: Data to (first-order) graph



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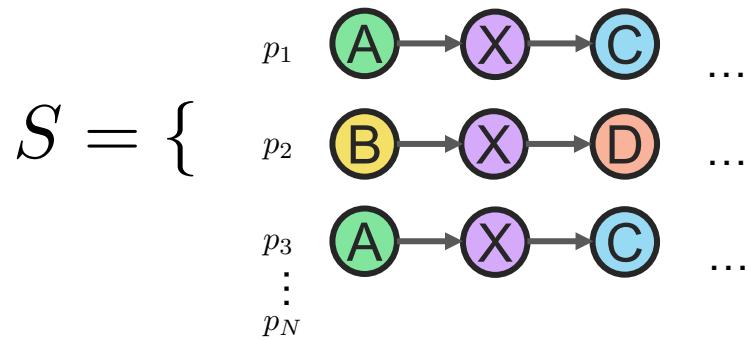


Toy Example: Data to (first-order) graph



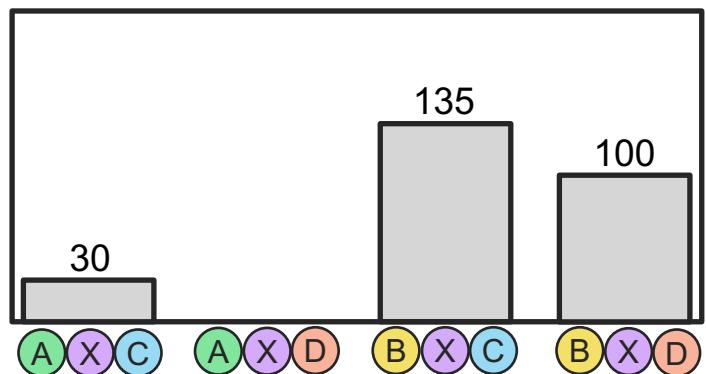
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Toy Example: Data to 2nd order de Bruijn graph



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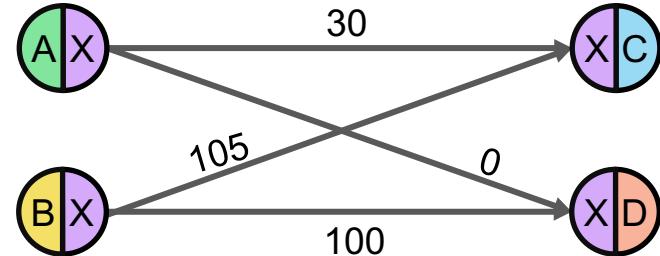
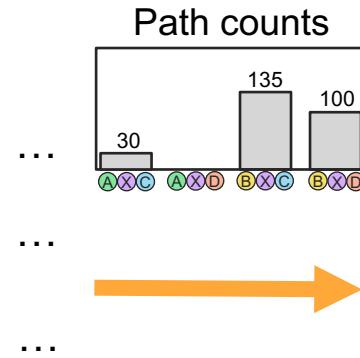
Path counts



Toy Example: Data to 2nd order de Bruijn graph

$S = \{$

p_1 
 p_2 
 p_3 
...
 p_N



Total number of paths $N = |S| = 235$

Toy Example: Path Anomalies via Simulations

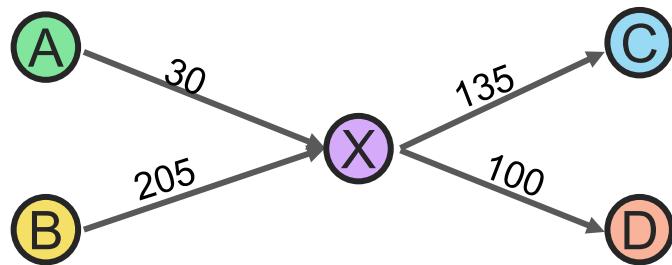
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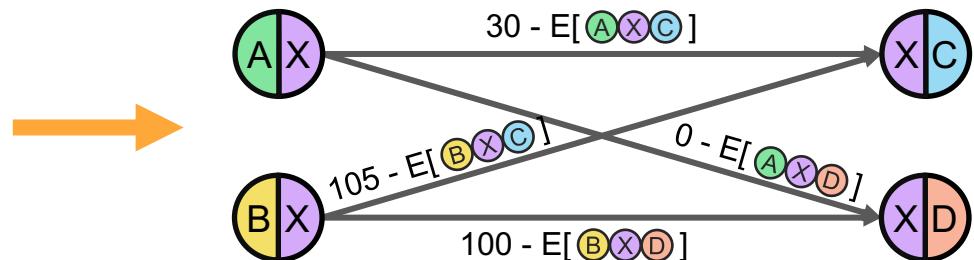
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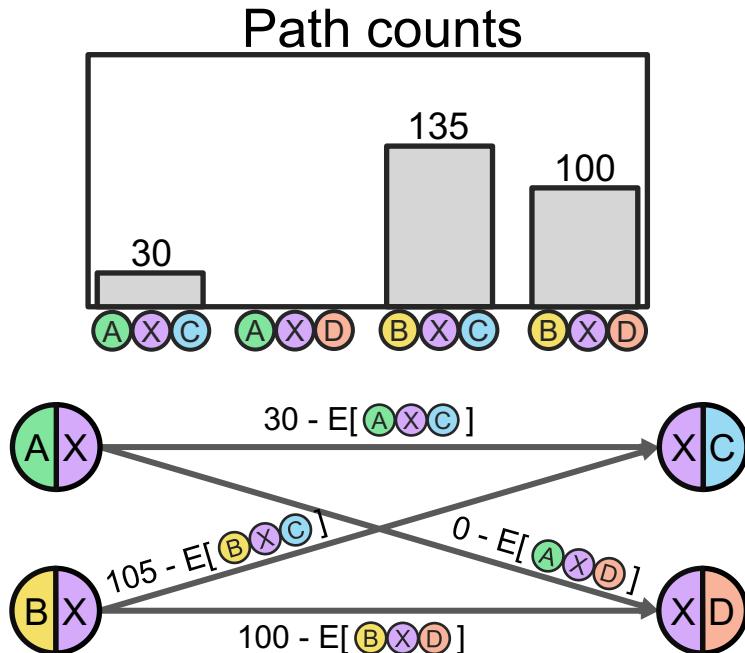
Simulate many random walk datasets



Compute expected frequency of each pathway and subtract from observed value



Toy Example: Path Anomalies via Simulations



Challenges



Path Anomaly Detection: Challenges

Detecting path anomalies via simulations → computationally intensive

Result is expected value, no concrete notion of significance

Alternative: detect path anomalies analytically by developing a tractable null model

Null Model: Challenges

Traditional null models (e.g. configuration model) cannot be applied directly

We want to randomize edge weight distribution in de Bruijn graph models, not the connectivity structure (fixed by the network topology)

Higher-order nodes can not be randomized by stub-matching across whole network, because transitions must match suffix-prefix: A-B → B-C is possible, but not A-B→C-D

HYPA: Efficient Detection of Path Anomalies



Northeastern University
Network Science Institute

Generalized Hypergeometric Ensemble

Generalization of the configuration model to weighted, directed networks.



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Generalization of the configuration model to weighted, directed networks.

Fixes the *expected* weight of every node, rather than the *exact* degree sequence.



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Intuition: Urn problem where each pair of nodes i, j that can possibly connect is assigned a color, and K_{ij} balls are added, where K_{ij} is the total possible multi-edges between the nodes (e.g. $K_{ij} = k_i^{\text{out}} k_j^{\text{in}}$, as in the config model).

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To sample from the ensemble, draw $m = \sum_{ij} W_{ij}$ (total observed edges) multi-edges randomly from the urn.

$$m = \sum_{ij} W_{ij}$$
$$K_{ij} = k_i^{\text{out}} k_j^{\text{in}}$$

Hypergeometric Ensemble

$$\Pr(X_{vw} = f(v, w)) \propto \binom{K_{vw}}{f(v, w)} \binom{\sum_{ij} K_{ij} - K_{vw}}{m - f(v, w)}$$

$$m = \sum_{ij} W_{ij}$$

Hypergeometric Ensemble

$$K_{ij} = k_i^{\text{out}} k_j^{\text{in}}$$

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Probability of observing frequency $f(v,w)$ given the entire weighted network structure.

$$m = \sum_{ij} W_{ij}$$

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Number of ways to pick $f(v,w)$ multiedges from K_{vw} possible.

$$m = \sum_{ij} W_{ij}$$

Hypergeometric Ensemble

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$$Pr(X_{vw} = f(v, w)) \propto \binom{K_{vw}}{f(v, w)} \binom{\sum_{ij} K_{ij} - K_{vw}}{m - f(v, w)}$$

Number of ways to pick everything else.

Putting it all together: HYPA scores

$$\text{HYPA}^{(k)}(\vec{v}, \vec{w}) := \Pr(X_{\vec{v}\vec{w}} \leq f(\vec{v}, \vec{w}))$$

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If HYPA is “close” to **0**, then the pathway is **underrepresented**.

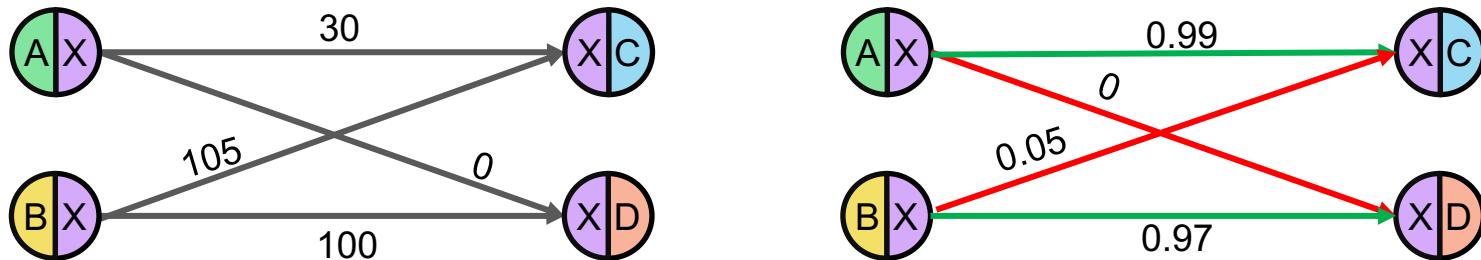
If HYPA is “close” to **1**, then pathway is **overrepresented**.

Putting it all together: HYPA scores

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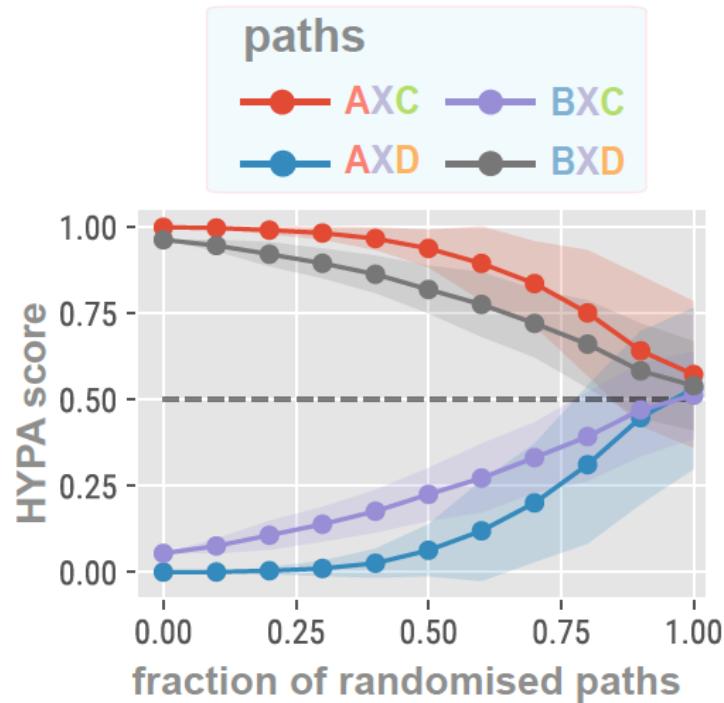
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Validation



Robustness to noise



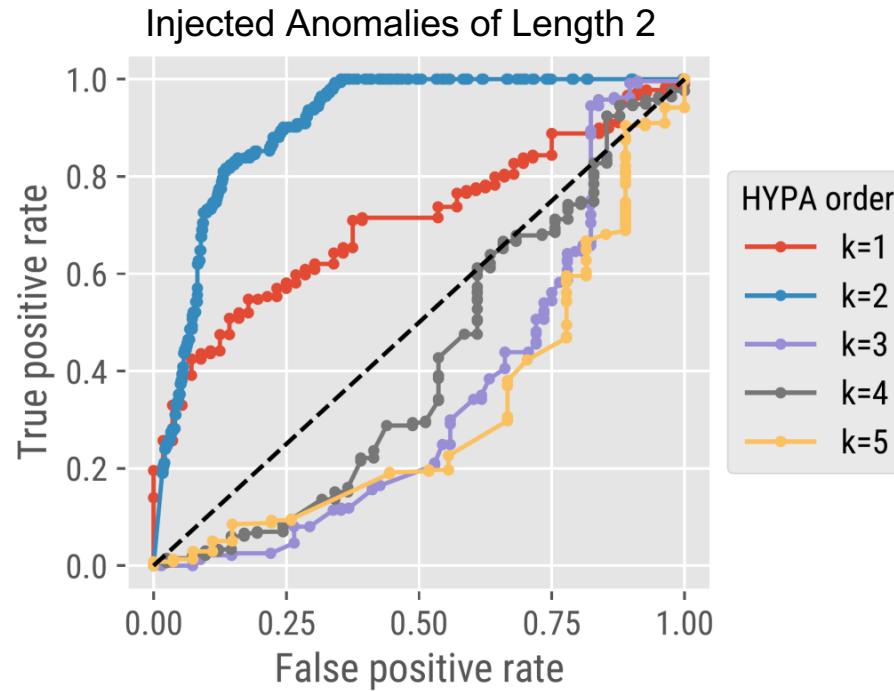
Synthetic Anomalies: Setup

Given an order k and a random Erdos-Renyi topology, construct a k th-order de Bruijn graph model and randomly increase the frequency of some transitions.

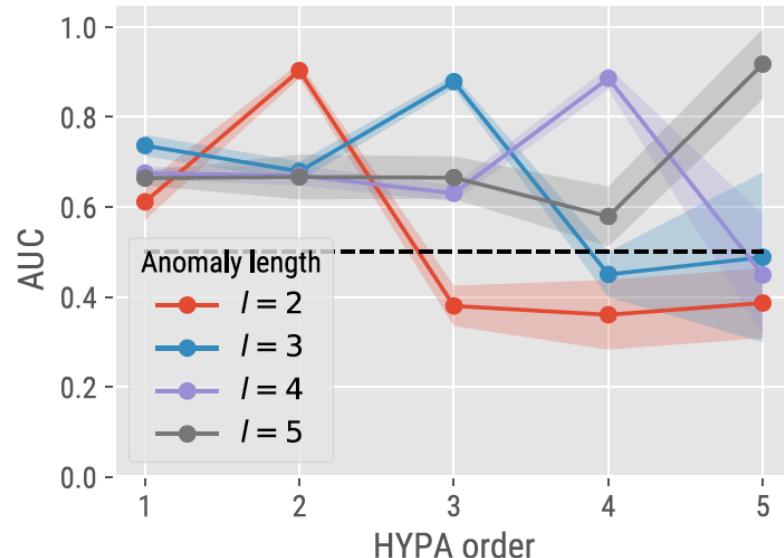
Generate many random walks on this k th-order model, resulting in data that includes injected path anomalies.

Apply Hypa at various orders and measure detection accuracy.

Synthetic Anomalies: ROC Example



Synthetic Anomalies: AUC Results



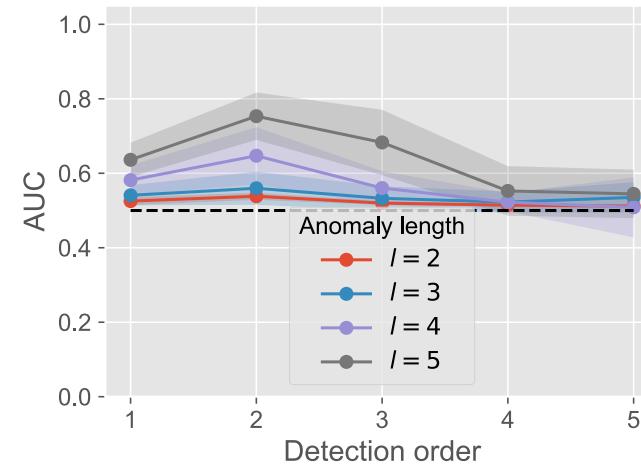
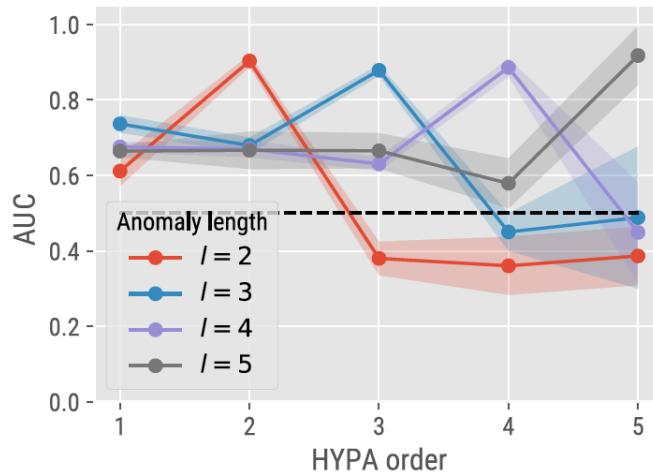
Naïve Baseline Comparison

Frequency-Based Anomaly Detection (FBAD)

Compute mean, μ , and standard deviation, σ , of kth order edge weights

Given a scaling factor α , label edges with frequencies larger than $\mu + \sigma^* \alpha$ over-represented, and smaller than $\mu - \sigma^* \alpha$ under-represented

Synthetic Anomalies



Application to Flight Data



Airlines



Northeastern University
Network Science Institute

Airlines

Hypotheses:

1. Return flights should be over-represented, since people most often travel round trip.



Airlines: Return trips are over-represented

α	Return	Non-return
0.05	0.915	0.340
0.01	0.851	0.130
0.001	0.760	0.023
0.0001	0.688	0.004
0.00001	0.628	0.001

Fraction of over-represented return/non-return flights for various discrimination thresholds.

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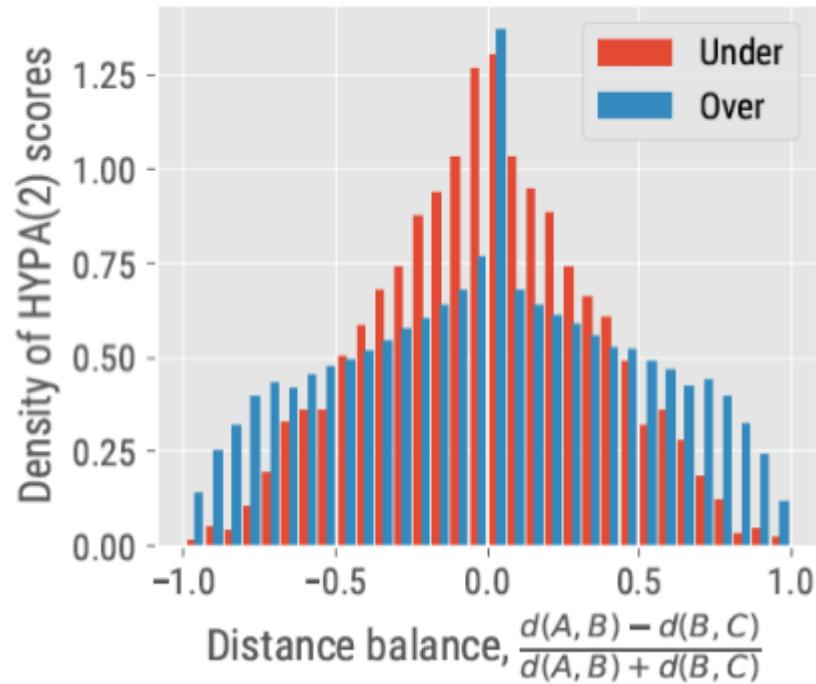
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Airlines

Hypotheses:

1. Return flights should be over-represented, since people most often travel round trip.
 - a. True, but many non-return are still over-represented.
2. Over-represented non-return flights are due to regional/national hubs, since people need to fly from small airports → regional hub → large airport.

Airlines: Trip Balance

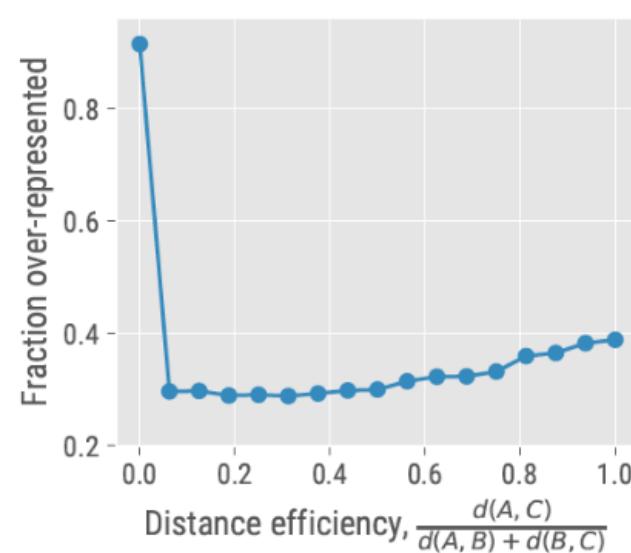
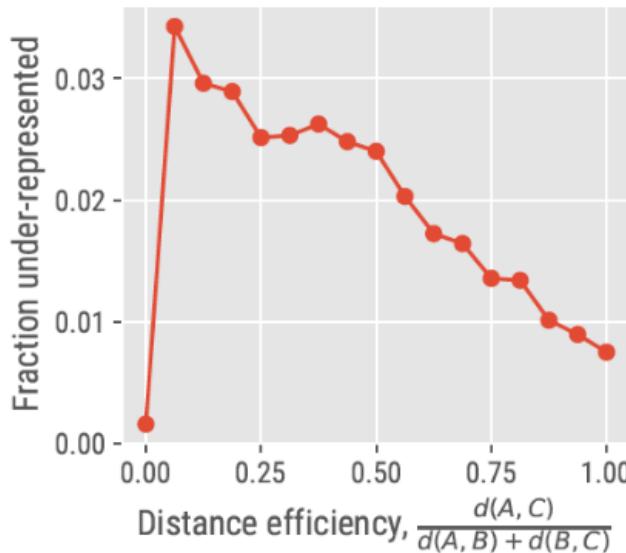


Airlines

Hypotheses:

1. Return flights should be over-represented, since people most often travel round trip.
 - a. True, but many non-return are still over-represented.
2. Over-represented non-return flights are due to regional/national hubs, since people need to fly from small airports → regional hub → large airport.
3. “Efficient” paths are more likely to be over-represented.

Airlines: Efficiency



Thanks!

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References

Scholtes, Ingo. "When is a network a network?: Multi-order graphical model selection in pathways and temporal networks." *Proceedings of the 23rd ACM SIGKDD International Conference on Knowledge Discovery and Data Mining*. ACM, 2017.

Casiraghi, Giona, et al. "Generalized hypergeometric ensembles: Statistical hypothesis testing in complex networks." *arXiv preprint arXiv:1607.02441* (2016).

Transport for London Open Data: <https://tfl.gov.uk/info-for/open-data-users/>

Definition: kth-order de Bruijn Graph

For a given graph $G = (V, E)$ and positive integer k we define a k -th order De Bruijn graph of paths in G as a graph $G^k = (V^k, E^k)$, where (i) each node $\vec{v} := v_0v_1 \dots v_{k-1} \in V^k$ is a path of length $k - 1$ in G , and (ii) $(\vec{v}, \vec{w}) \in E^k$ iff $v_{i+1} = w_i$ for $i = 0, \dots, k - 2$.

Pseudocode

Algorithm 1 ComputeHYPA(S, k): *Compute kth order HYPA scores for sequence dataset S.*

Input: S (sequences), k (desired order)

Output: HYPA $^{(k)}$ score for all k -th order paths

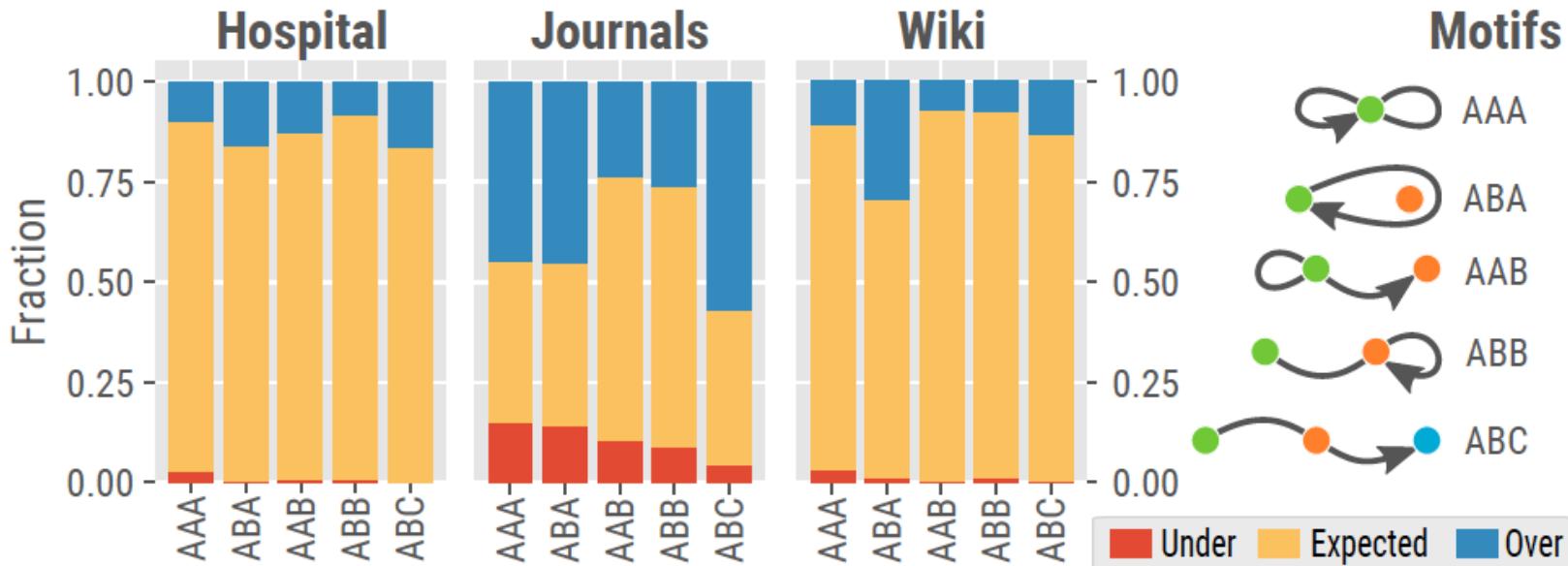
- 1: $G^k \leftarrow \text{DeBruijnGraph}(S, k)$ # Construct k th order graph
 - 2: $\Xi \leftarrow \text{fitXi}(G^k, \text{tolerance})$ # Optimization (Algorithm 2 in Appendix A.1)
 - 3: **for** $(\vec{v}, \vec{w}) \in G^k$ **do**
 - 4: $\text{HYPA}^{(k)}(\vec{v}, \vec{w}) \leftarrow \Pr(x_{vw} \leq (\vec{v}, \vec{w}) \mid m, \Xi)$
 # Compute CDF
 - 5: **return** HYPA $^{(k)}$
-



Real Data

Data	Topology		Sequences			l^{\max}	$\langle l \rangle$
	Nodes	Edges	Total	Unique			
Tube	268	646	4295731	67015	35	6.75	
Flights	382	6933	185871	88539	10	2.48	
Journals	283	1743	480496	309565	35	14.8	
Hospital	75	1138	28422	2561	5	1.19	
Wiki	100	1598	29682	7431	21	1.64	

Exploring Motifs



Case Study: London Tube

Data:

- Origin → destination statistics between London Tube stations
 - (origin, destination, #observations)
- Shortest paths between stations
 - Assume people follow shortest paths



London Tube

Hypothesis:

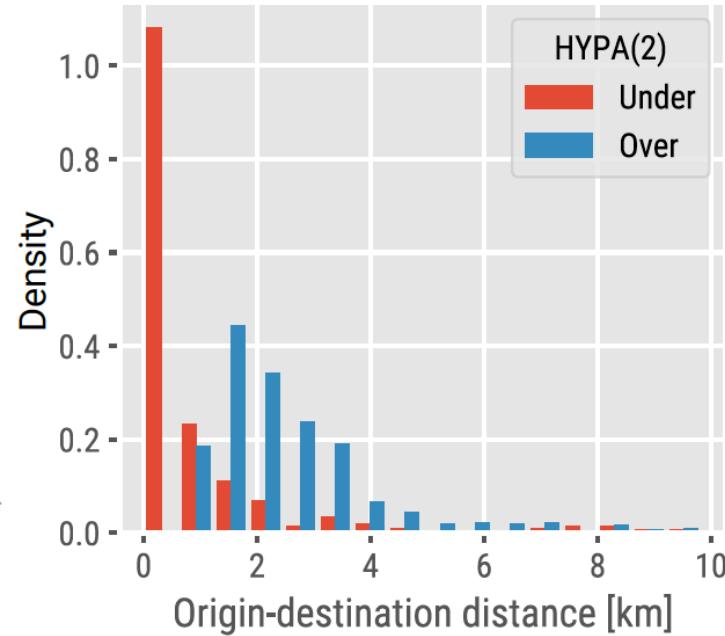
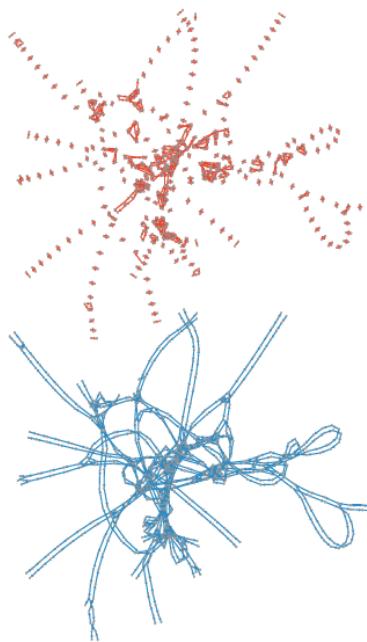
- People typically use public transportation to travel large geographic distances
- *Overrepresented pathways* should cover *larger* distances

Test:

- Measure distance between every station
- For 2nd order transitions A-B-C, compute distance between nodes A and C
- Analyze distributions of distance in over vs. under represented transitions
 - Expect to see distribution shifted towards higher values for over-represented transitions



London Tube



London Tube

HYP_A^(k)	$k = 2$	$k = 3$	$k = 4$	$k = 5$	$k = 6$
Under [km]	0.00	2.38	3.29	4.60	5.43
Over [km]	2.20	2.93	3.79	5.21	5.63
p-value	$< 10^{-170}$	$< 10^{-7}$	$< 10^{-4}$	0.006	0.08

Median distance between source and destination nodes in under/over represented transitions.

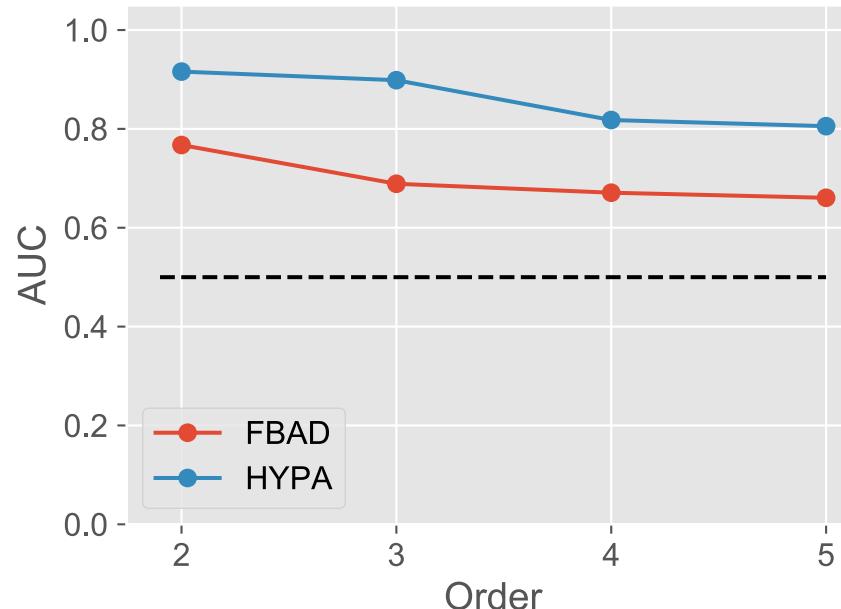
Constructing Ground Truth

Construct ground truth based on the method discussed earlier:

- Randomize path data using $k-1^{\text{st}}$ order random walks
- Compute k^{th} -order path statistics
- Repeat m times, noting the frequency of each path
- Estimate multinomial distribution and its CDF from these statistics
- If $\text{CDF}(\text{path}) > \text{threshold}$, label over-represented



Tube Data - Ground Truth



$$m = \sum_{ij} W_{ij}$$

$$\Xi_{ij} = k_i^{out} k_j^{in}$$

Hypergeometric Ensemble

$$\Pr(X_{\vec{v}\vec{w}} = f(\vec{v}, \vec{w})) = \binom{\sum_{ij} \Xi_{ij}}{m}^{-1} \binom{\Xi_{vw}}{f(\vec{v}, \vec{w})} \binom{\sum_{ij} \Xi_{ij} - \Xi_{vw}}{m - f(\vec{v}, \vec{w})}$$

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Normalization. Total number of ways to pick m multiedges from total possible.

$$m = \sum_{ij} W_{ij}$$

$$\Xi_{ij} = k_i^{out} k_j^{in}$$

Hypergeometric Ensemble

$$\Pr(X_{\vec{v}\vec{w}} = f(\vec{v}, \vec{w})) = \binom{\sum_{ij} \Xi_{ij}}{m}^{-1} \binom{\Xi_{vw}}{f(\vec{v}, \vec{w})} \binom{\sum_{ij} \Xi_{ij} - \Xi_{vw}}{m - f(\vec{v}, \vec{w})}$$

Number of ways to pick $f(\vec{v}, \vec{w})$ multiedges from $\Xi_{\vec{v}\vec{w}}$ possible.

$$m = \sum_{ij} W_{ij}$$

$$\Xi_{ij} = k_i^{out} k_j^{in}$$

Hypergeometric Ensemble

$$\Pr(X_{\vec{v}\vec{w}} = f(\vec{v}, \vec{w})) = \binom{\sum_{ij} \Xi_{ij}}{m}^{-1} \binom{\Xi_{vw}}{f(\vec{v}, \vec{w})} \binom{\sum_{ij} \Xi_{ij} - \Xi_{vw}}{m - f(\vec{v}, \vec{w})}$$



Number of ways to pick everything else.